# 中性子過剰核のE1応答と
# R過程元素合成の中性子捕獲断面積

M. Matsuo (Niigata U.)

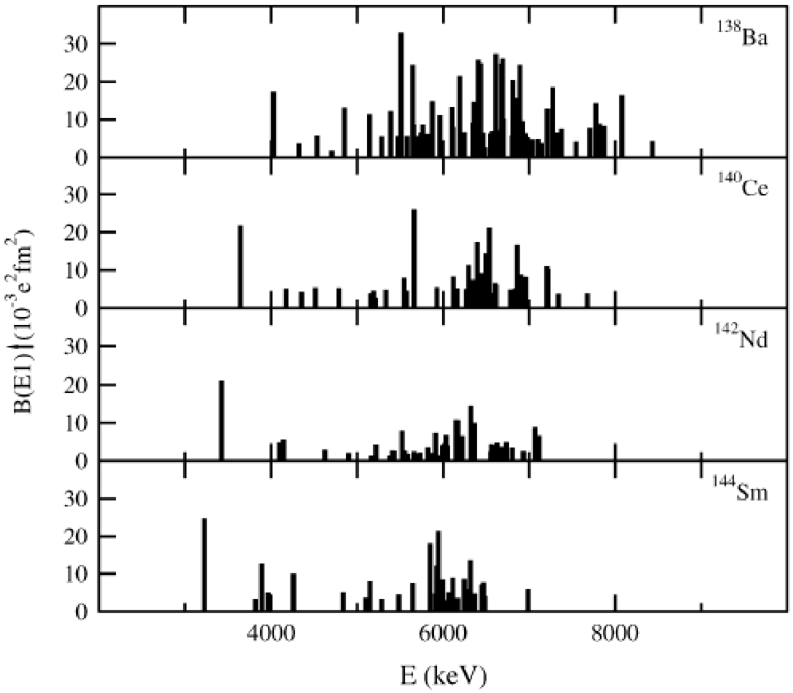
# 序論

# E1 応答実験の発展：安定核

Pygmy dipole resonance

Darmstadt (γ,γ') on stable N=82 isotones

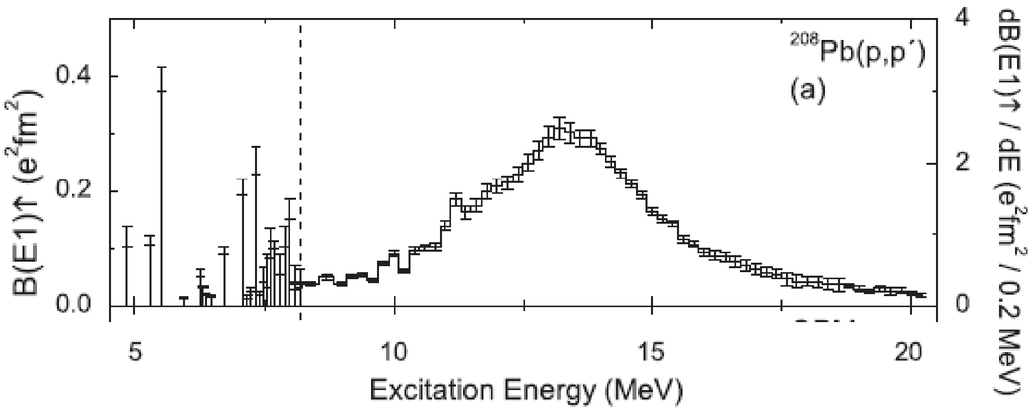
Volz et al. NPA779,1 (2006)




Whole E1 response : pygmy + GDR

RCNP (p,p') on 208Pb

Tamii et al. PRL107, 062502 (2011)

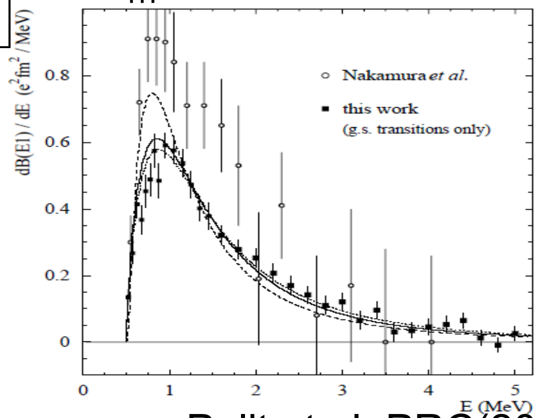


# E1 応答実験の発展: 不安定核、中性子過剰核

Threshold strength
very small $S_{1n}$
(light mass nuclei)

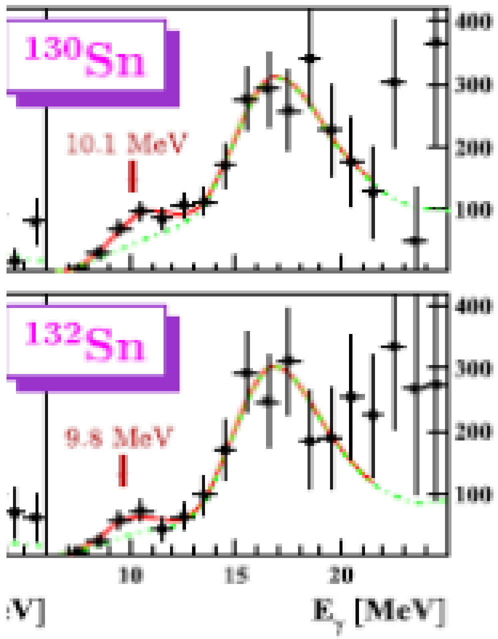
Pygmy dipole resonance
not necessarily small $S_{1n}$
(Heavy mass nuclei)

$^{11}$Be $S_{1n}$=0.5 MeV




Palit et al. PRC(2003)

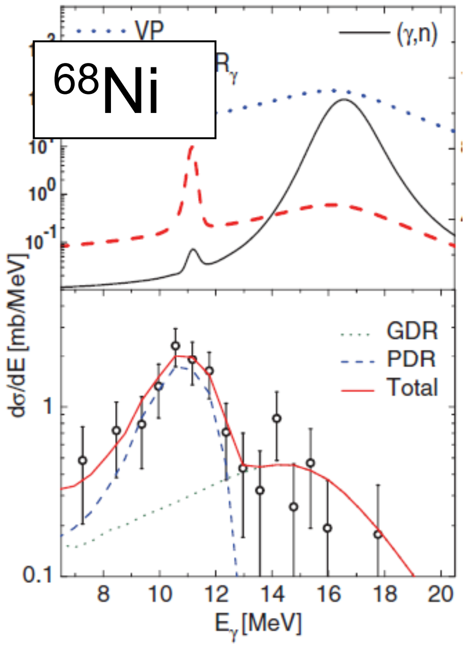
$S_{1n}$=7.3 MeV




Adrich et al. PRL (2006)

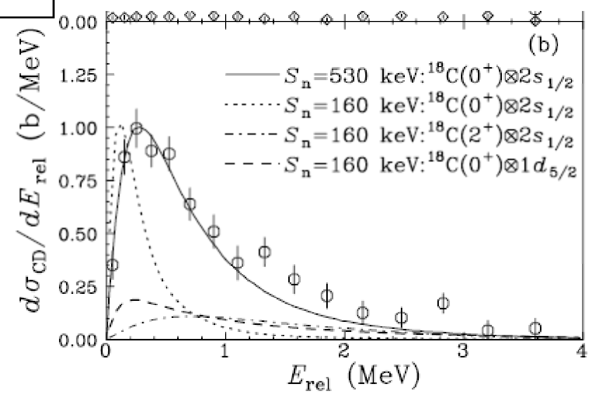
$S_{1n}$=7.8 MeV

$^{68}$Ni




Wieland et al. PRL (2009)

$^{19}$C $S_{1n}$~0.6 MeV



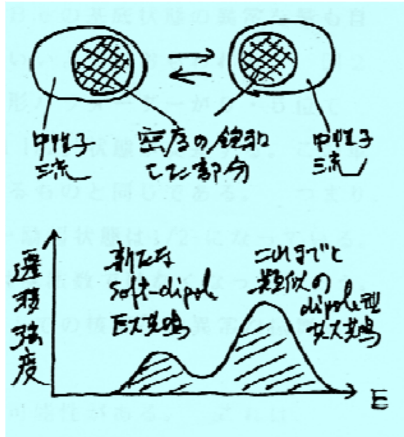

Nakamura et al. PRL (1999)

# ピグミー双極共鳴・ソフト双極励起の物理

1. “ドリップライン原子核から安定核まで”の多くの原子核で存在しているらしい

2. どんな励起モードなのか？

池田清美 in「大型ハドロン計画における核物理の魅力」1988




…の集団運動？
…性子の単一粒子解離？
…トロンの解離？集団運動？

スキンの集団運…
アイソスカラー／アイソ…

トロイダルモード？

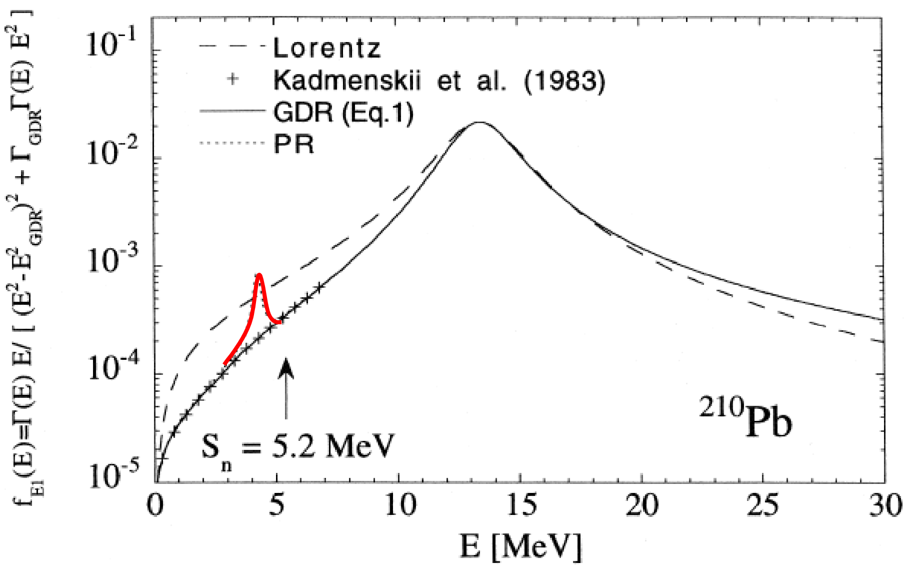
明確な集団モードではない（独立ph励起の…
アイソベクトル相互作用／表面対称エネルギー…ン厚を反映？

呼称も整理されていない
ソフト双極励起、　ソフト双極共鳴、　ピグミー双極共鳴 etc

4. R過程・S過程元素合成に重要な役割

# 例：Gorielyによる初期研究

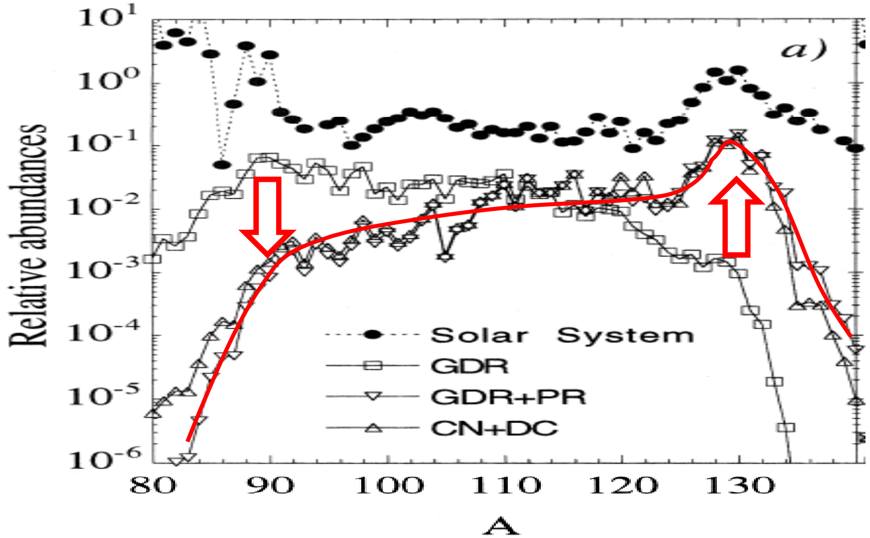
S. Goriely PLB436, 10 (1998)




Adopted E1 strength function (schematic)

Existence of pygmy resonance below $S_n$

in Hauser-Feshbach statistical (n,$\gamma$) model




Resultant r-process abundance

Accelerated neutron-capture

conditions

low $T$, low $\rho_n$

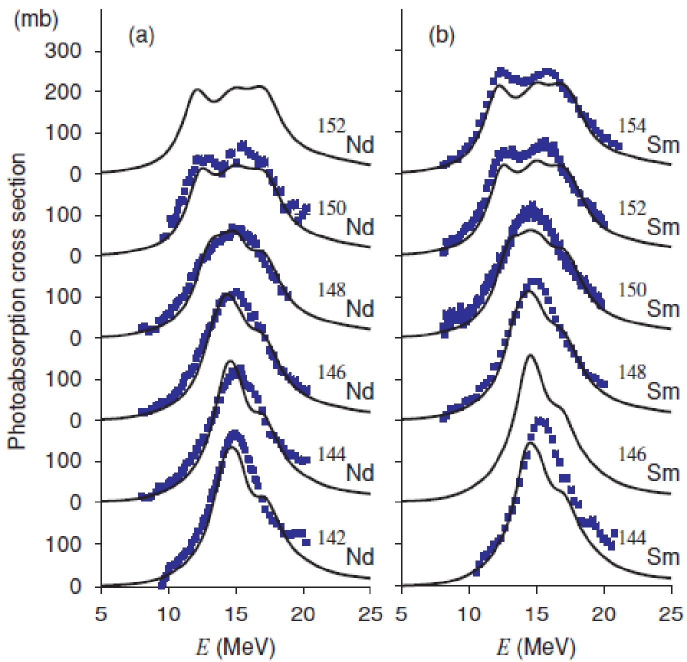
no $n$-$\gamma$ equilibrium

# E1応答の理論

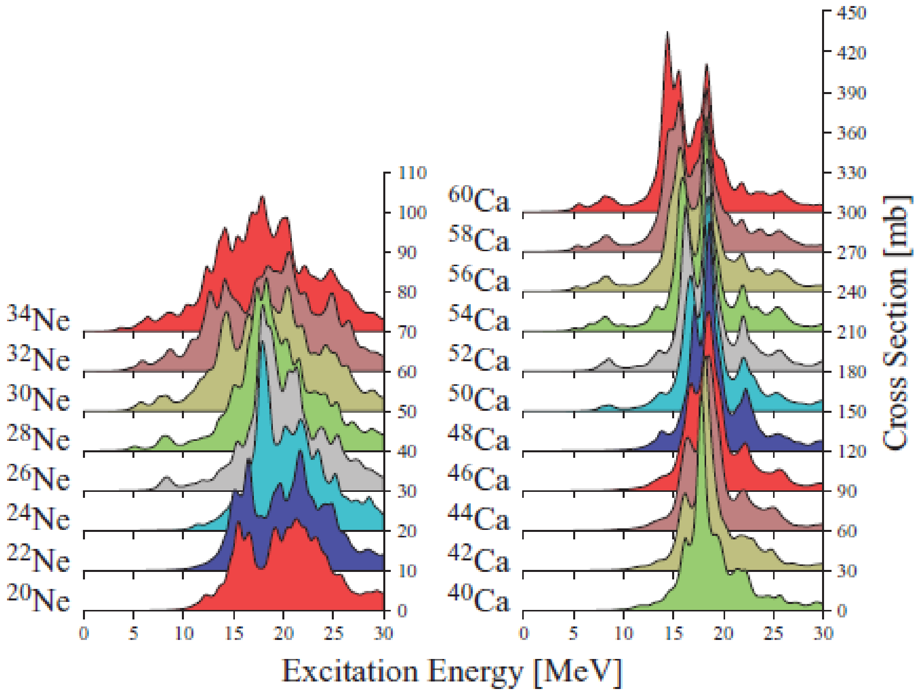
軽い核を除けば、"原子核密度汎関数模型＋乱雑位相近似(線形応答)"、
DFT+QRPA　(閉殻の場合は　DFT+RPA)
またはその拡張、で記述できるだろうと多くの人が考えている。

最近の発展を示す例2つを挙げる

Yoshida & Nakatsukasa, PRC83, 021304 (2011)




Inakura, Nakatsukasa & Yabana, PRC84, 021302 (2011)




実際には。。。。

- 非常に多数の計算・研究（文献省略）
- 模型によって異なる結果、解釈
- 標準模型・理論・解釈が確立しているわけではなく、研究途上

see eg. a review : Paar, Vretenar, Khan, Colo
Rep. Prog. Phys. 70, 691 (2007)

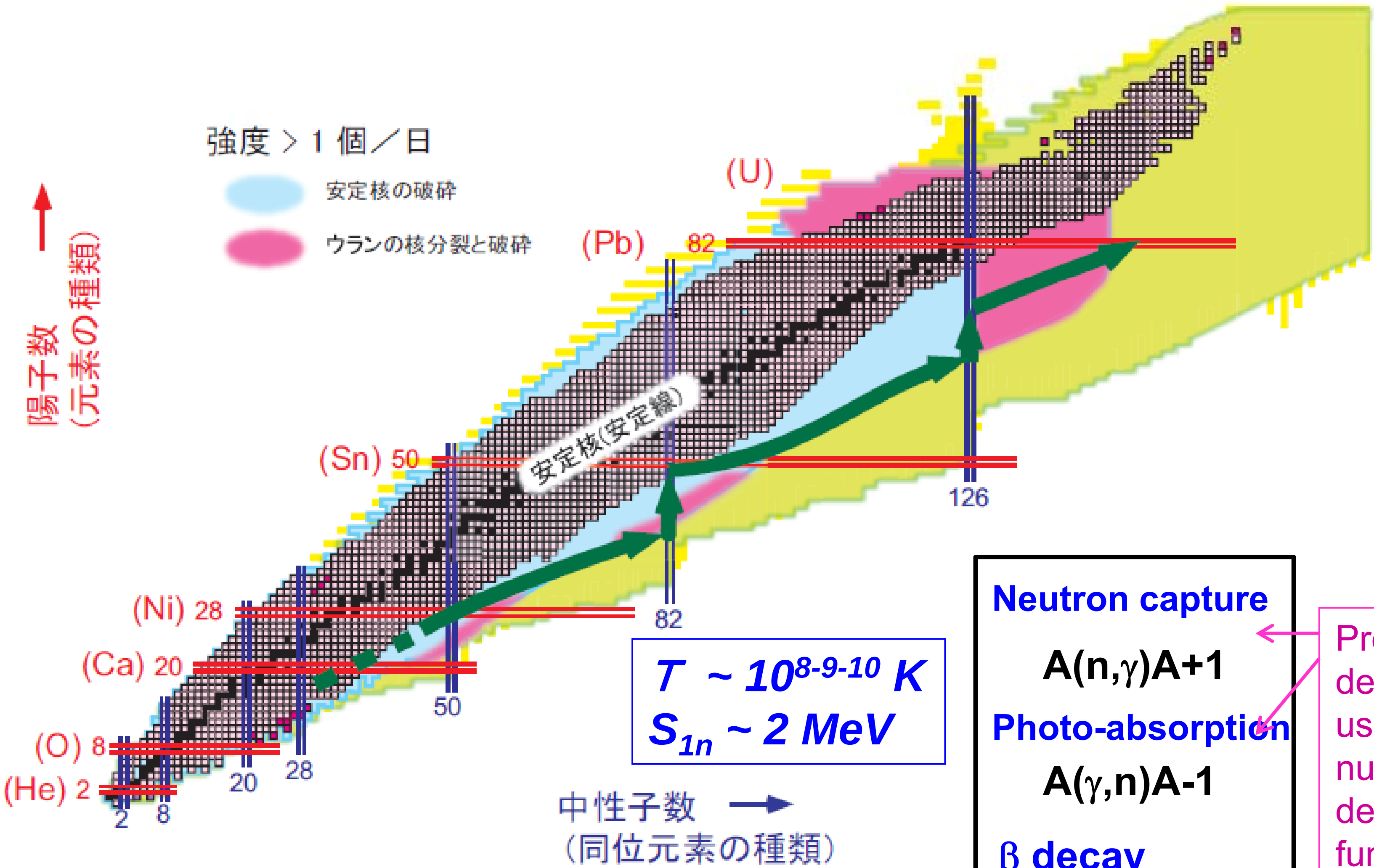
# E1応答とR過程元素合成

## 本講演のメッセージ

Nuclear DFT + QRPA理論 は、E1応答（光吸収断面積）だけでなく、R過程元素合成で大きな役割を果たす直接中性子捕獲反応の断面積を記述する能力を持つ。

巨大共鳴やピグミー共鳴・ソフト励起からの中性子崩壊を測定・記述することが重要である。

# R-process nucleo-synthesis




***$T \sim 10^{8\text{-}9\text{-}10}$ K***
***$S_{1n} \sim 2$ MeV***

**Neutron capture**

**A(n,γ)A+1**

**Photo-absorption**

**A(γ,n)A-1**

**β decay**

**$^{A}Z \rightarrow {}^{A}Z+1$**

Predictive description using the nuclear density functional theory

# Statistical vs. direct n-capture

### *1. Statistical model of neutron-capture*

- ●High $E_x$ (large $S_{1n}$) & high $\rho(E)$
- ●Formation of Bohr's compound states, and $\gamma$ cascade
  - Hauser-Feshbach statistical reaction theory
- ●E1-strength function below threshold energy
  - with Brink-Axel hypothesis

✓ phenomenology

⇨ microscopic QRPA / beyond-QRPA

Ex. S.Goriely, E.Khan, M.Shamyn, NPA739(2004)331
E. Litvinova, et al. NPA823 (2009) 26

### *2. Direct neutron-capture*

- ●Low $E_x$ (small $S_{1n}$) & low $\rho(E)$
- ●Direct neutron capture with emission of E1 $\gamma$

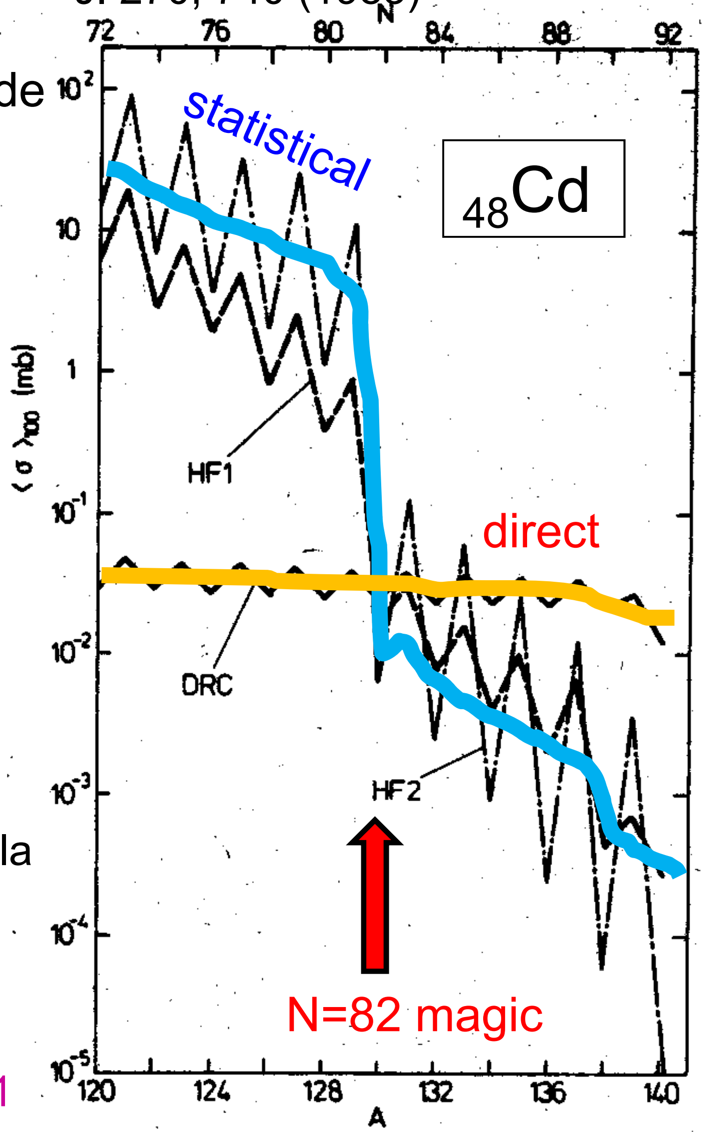
✓ Single-particle model (simplified analytic formula

S. Raman et al. PRC32 (1985) 18
S. Typel and G. Bauer NPA759 (2005) 247
M.A.Nagarajan, S.Lenzi, A.Vitturi EPJA24 (2005) 63

⇨ **microscopic QRPA**

Present work, to include collectivities in E1

G.J. Mathews et al. Astrophysical J. 270, 740 (1983)



# Statistical (n,γ) model

Formation of compound states

$n + (A\text{-}1) \rightarrow A_{CN}^*$

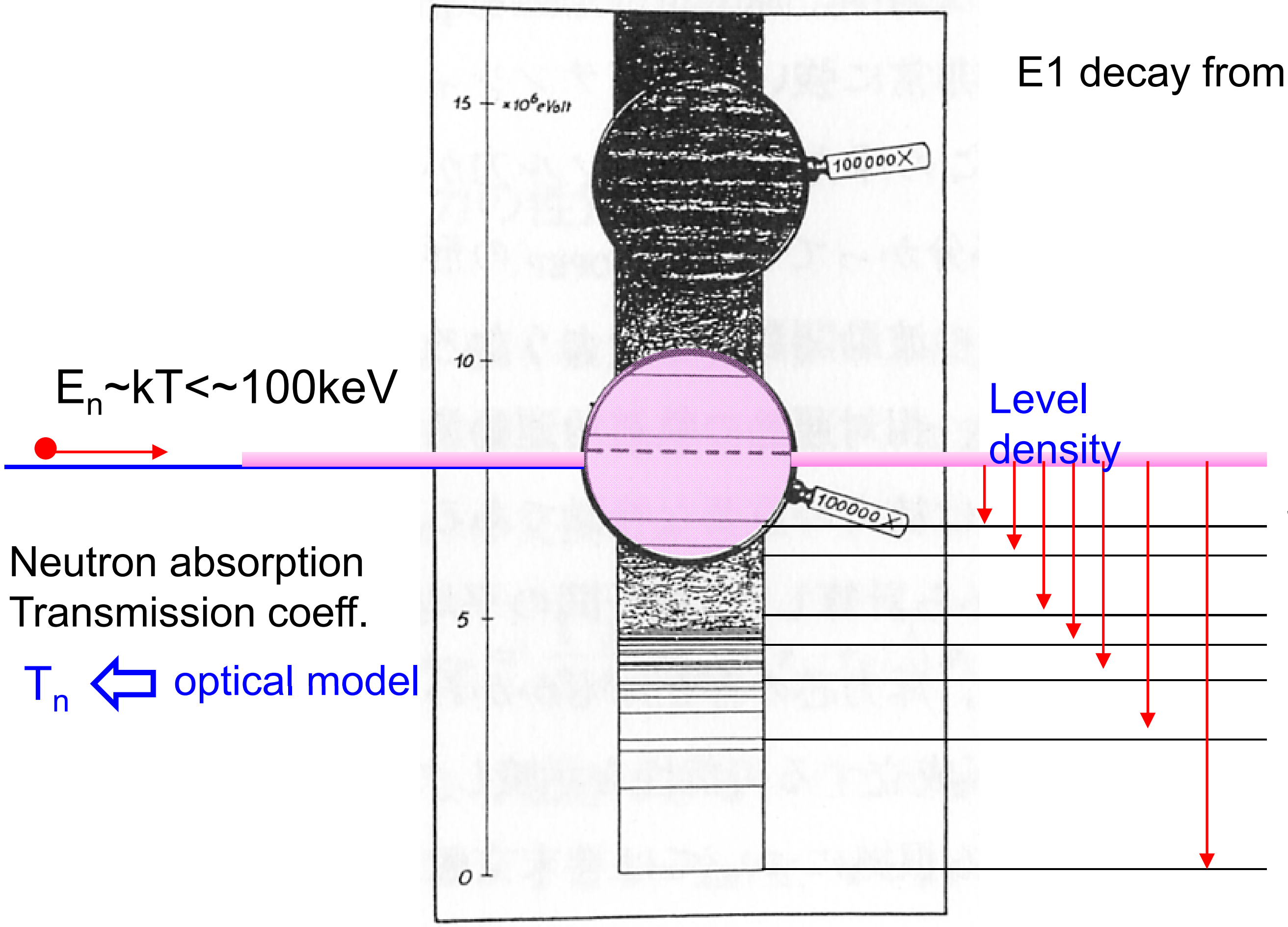
Statistical gamma-decay

$A_{CN}^* \rightarrow A_i + \gamma$

E1 decay from compound states

$E_n$~kT<~100keV

$S_n$




Neutron absorption
Transmission coeff.

$T_n$ ⇦ optical model

$S_{E1}(E\gamma)$ ⇦

E1 strength function

Below threshold

$E\gamma < S_n$

Brink-Axel hypothesis

excited states
~ ground state

# Single-particle model of direct n-capture

$$\frac{dB(E1)}{dE} \propto \left| \left\langle \phi_f \left| M(E1) \right| \phi_i(E) \right\rangle \right|^2$$

bound orbit

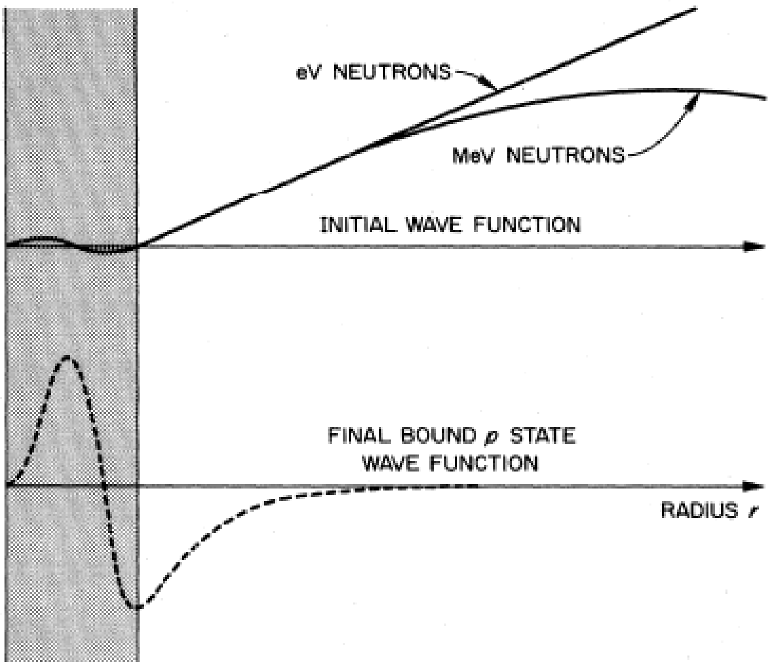
scattering state in continuum

Analytic expression using the asymptotic wave forms

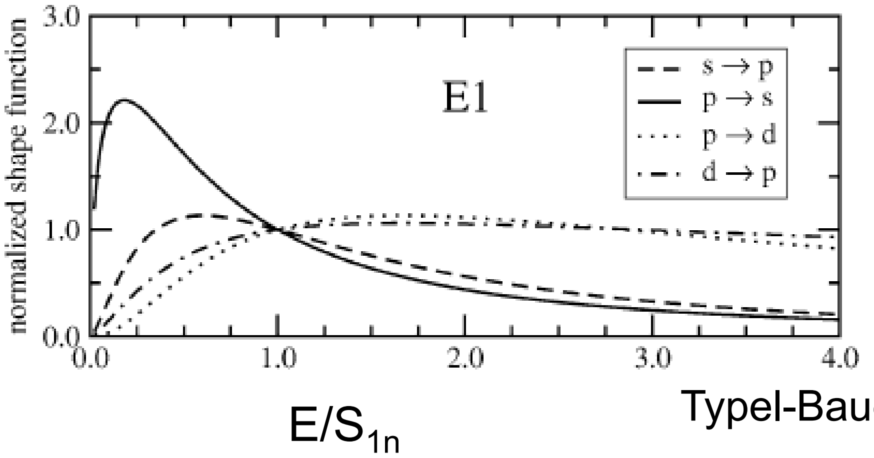
S. Raman et al. PRC32 (1985) 18

S. Typel and G. Bauer NPA759 (2005) 247

M.A.Nagarajan, S.Lenzi, A.Vitturi EPJA24 (2005) 63




S. Raman et al.



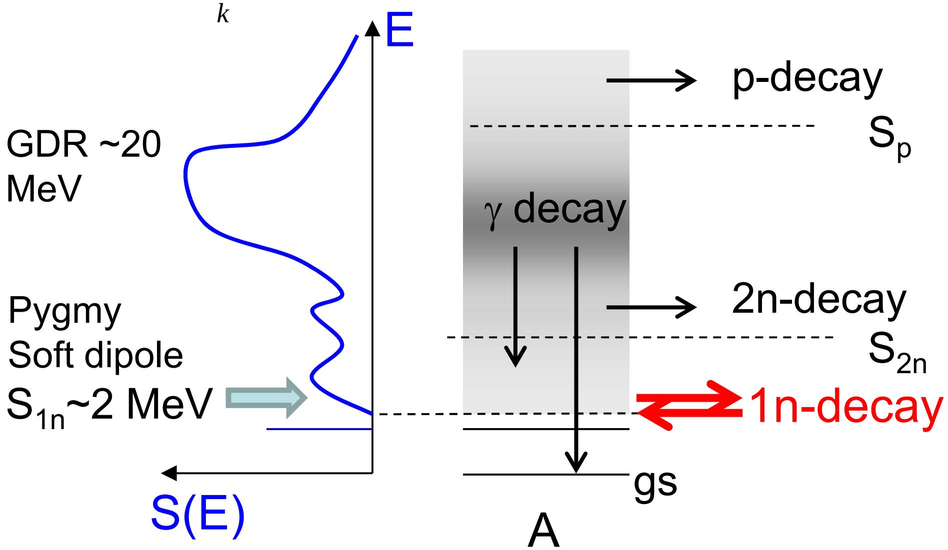

Typel-Bauer

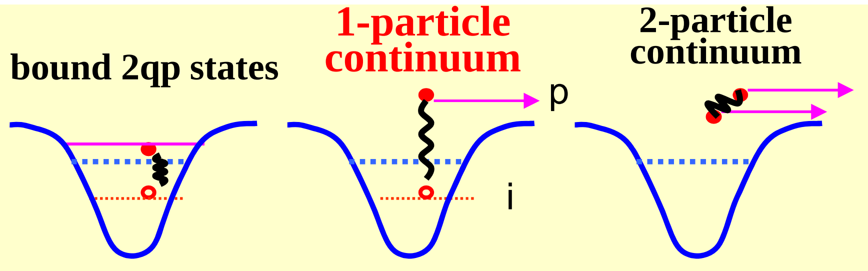
# 連続状態QRPA理論による
# 直接中性子捕獲断面積

# E1 response and (γ,n)/(n,γ) cross section

***E1 strength function (Photo-absorption cross section)***

$$S(E) = \sum_k \left|\langle \Omega_k | E1 | 0 \rangle\right|^2 \delta(E - \Omega_k)$$




- ***Partial*** **photo-absorption x sect. for** ***direct 1n decay*** γ + A → A* → (A-1) +n

$$\sigma(\gamma + A \to (A-1)_i + n_p) = \frac{16\pi^3 e^2}{9\hbar c} E_\gamma S_{1c(ip)}(E_\gamma)$$

- **Inverse** ***direct neutron capture*** **x sect.** n+(A-1) → A* → A + γ

$$\sigma((A-1)_i + n_p \to \gamma + A) = S_{J_A}^{J_{A-1}} \frac{k_\gamma^2}{k_n^2} \sigma(\gamma + A \to (A-1)_i + n_p)$$

NB. neutron energy ***$E_n$ ~ kT ~ 10-100-1000 keV***

# Continuum Linear Response (QRPA) in Density Functional Theory

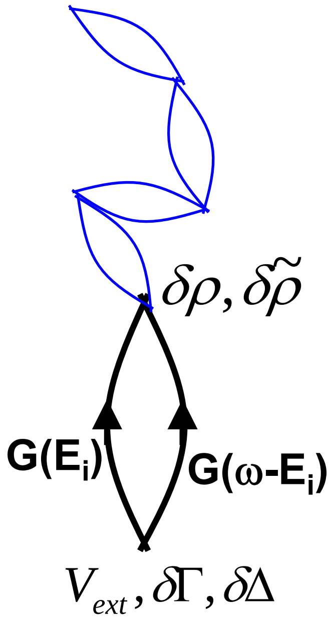
Matsuo, NPA696,2001
Mizuyama, et al PRC79,2009
Serizawa, et al PTP121,2009

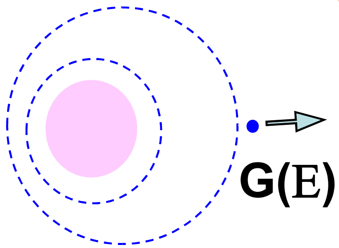
**Density & pair density oscillation**

$$\begin{pmatrix} \delta\rho(r,\omega) \\ \delta\tilde{\rho}(r,\omega) \\ \delta\tilde{\rho}^*(r,\omega) \end{pmatrix} = \int dr' \left(R^{\alpha\beta}{}_0(r,r',\omega)\right) \begin{pmatrix} \delta\Gamma(r',\omega)+V_{ext}(r') \\ \delta\Delta(r',\omega) \\ \delta\Delta^*(r',\omega) \end{pmatrix}$$

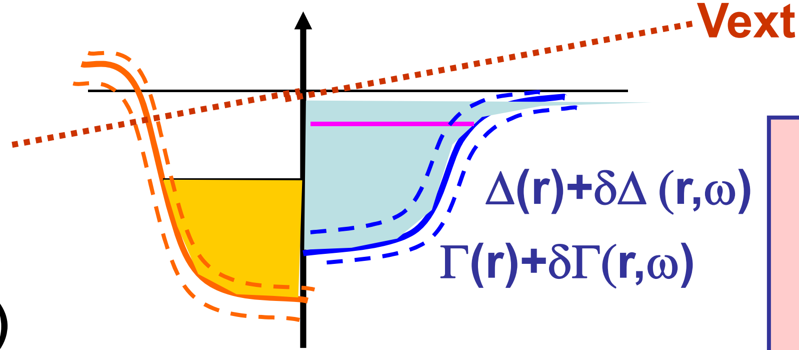
**Unperturbed response function**

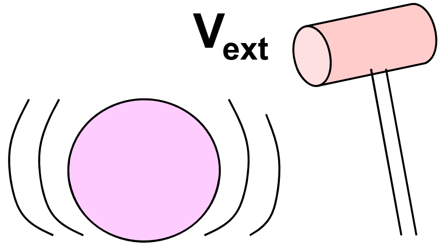
$$R_0(\vec{r},\vec{r}',\omega) = \int_C dE G(r,r',E) G(r',r,E+\omega) + backward$$




Propagation of quasi-particle wave in the continuum




**External field**

$$e^{-i\omega t} V_{ext}(r)$$

**Induced fields (polarization)**

$$\delta\Gamma(r,\omega) = \frac{\partial^2 E[\rho]}{\partial\rho^2} \delta\rho(r,\omega)$$

$$\delta\Delta(r,\omega) = \frac{\partial^2 E_{pair}[\tilde{\rho}]}{\partial\tilde{\rho}^2} \delta\tilde{\rho}(r,\omega)$$

**Selfconsistent field**

$$V_{scf}(\omega) = \begin{pmatrix} V_{ext} + \delta\Gamma(\omega) \\ \delta\Delta(\omega) \\ \delta\Delta^*(\omega) \end{pmatrix}$$

# Strength function and its decomposition

Cf . A.Zangwill, P.Soven, PRA 21(1980)1561
T.Nakatsukasa, K.Yabana JCP 114(2001)2550

## Zangwill-Soven decomposition

correlated response

External field

$$S(\omega) = -\tfrac{1}{\pi}\mathrm{Im}\iint drdr' V_{ext}^{+}(r) R_{\mathrm{RPA}}(r,r',\omega) V_{ext}(r') \quad = \sum_k \left|\langle \Omega_k | V_{ext} | 0 \rangle\right|^2 \delta(\omega - \Omega_k)$$

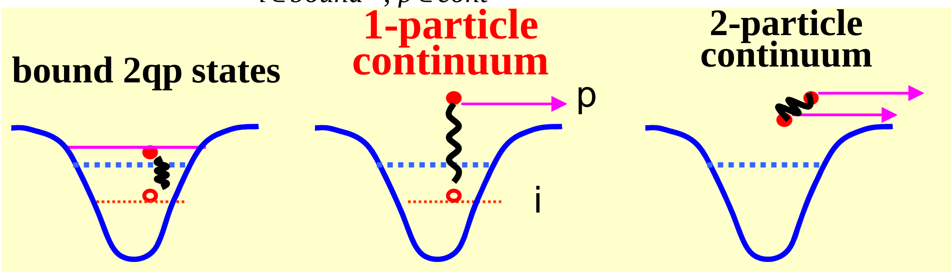
$$= -\tfrac{1}{\pi}\mathrm{Im}\iint drdr' \underline{V_{scf}^{+}(r,\omega)}\, \underline{R_0(r,r',\omega)}\, \underline{V_{scf}(r,\omega)} = \sum_{i>j} \underline{\left|\langle ij | V_{scf}(\omega) | 0 \rangle\right|^2 \delta(\omega - E_{ij})}$$

Selfconsistent field incl. correlation

uncorrelated response

- Decomposition w.r.t. uncorrl. 2qp states
- Matrix elements of selfconsistent field

$$= S_{bb}(E) + \sum_{i\in bound\ ,\ p\in cont} S_{1c(ip)}(E) + S_{2c}(E)$$




## Strength associated with one-particle continuum components

$$S_{1c,(ip)}(\omega) = \sum_{\substack{i\in bound \\ p\in continuum}} \left|\langle ip | V_{scf}(\omega) | 0 \rangle\right|^2 \delta(\omega - E_{ip})$$

$$= -\tfrac{1}{\pi}\mathrm{Im}\sum_{i\in bound} \langle \underline{\bar{\phi}_i} | \underline{V_{scf}^{+}(\omega)}\, \underline{G_{>}(\omega - i\varepsilon - E_i)}\, \underline{V_{scf}(\omega)} | \underline{\bar{\phi}_i} \rangle - backward$$

RPA correlation

Green's function for continuum qp state

proper asymptotic form

wave function of bound qp state

# Demonstration

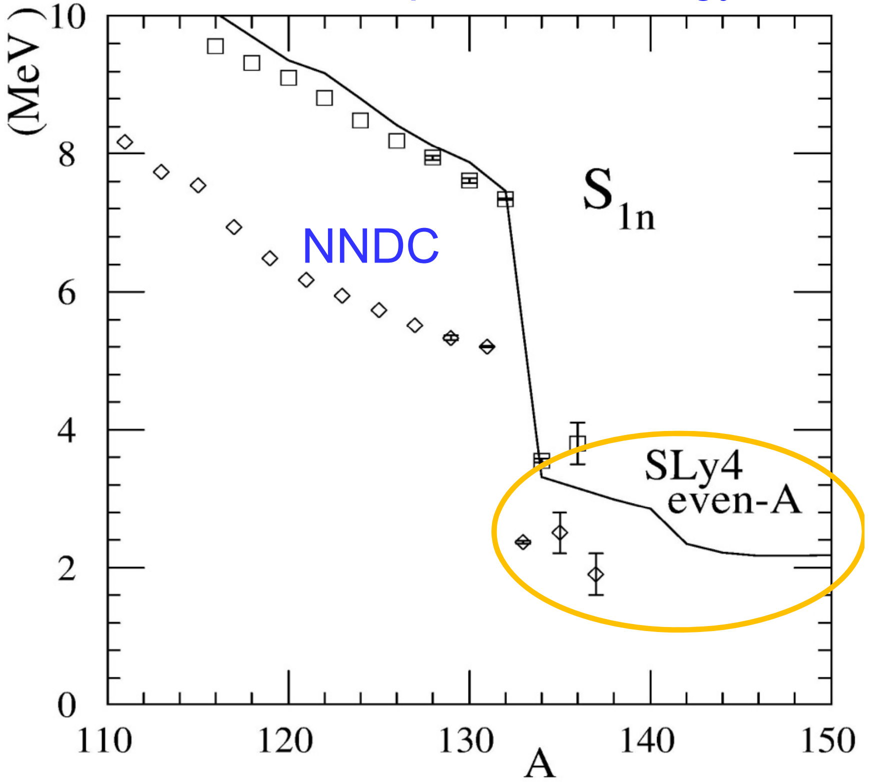
**1. Sn isotopes beyond A=132: expected r-process nuclei**

$^{134}$Sn,
$^{136}$Sn,
$^{138}$Sn,
$^{140}$Sn,
$^{142}$Sn,
$^{144}$Sn,
$^{146}$Sn,
$^{148}$Sn

$S_{1n}$ = 2-4 MeV

**expected r-process nuclei**

One-neutron separation energy




**2. HFB with Skyrme functional & DDDI**

SLy4, DDDI pairing (mix)

$E_{cut}$=60 MeV, $R_{max}$=20 fm

**3. Continuum QRPA in coordinate space**

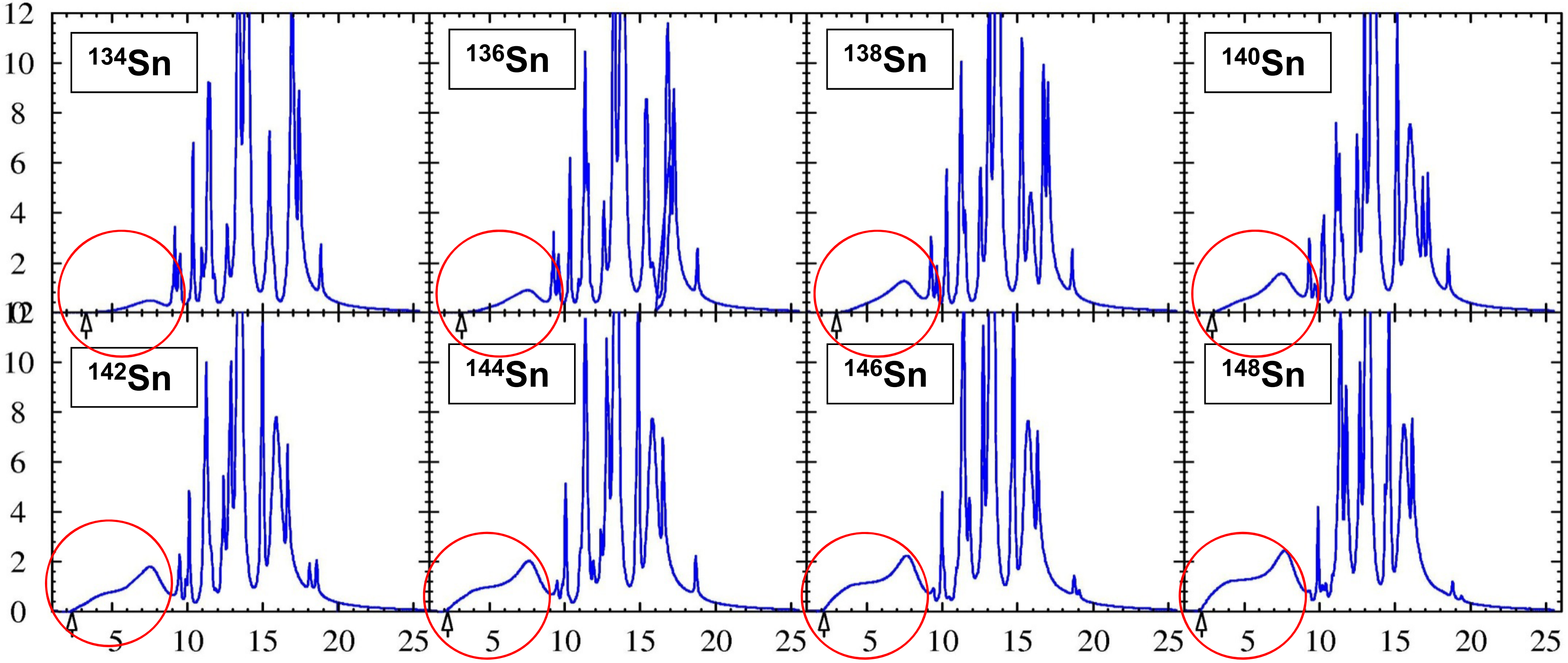
Landau-Migdal approx.

Spherical code:
Serizawa-Matsuo, Prog.Theor.Phys. 121 (2009) 97

# Skyrme HFB + Continuum QRPA

*Parameter set: SLy4 DDDI-mix*
*Landau-Migdal approx.*

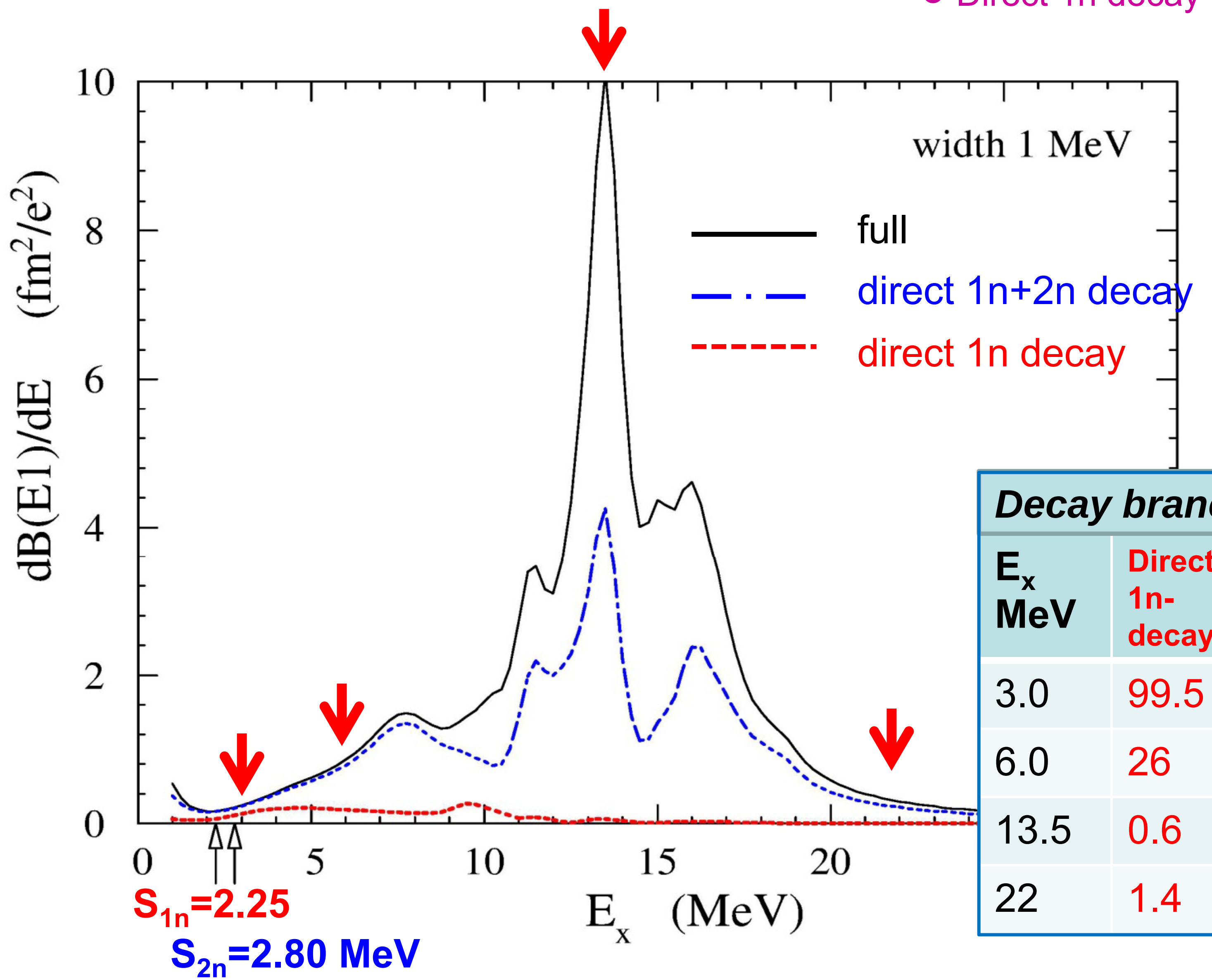
width=0.1MeV




Arrows: neutron separation energy $S_n$ ~ 2-3 MeV

# E1 strength in $^{142}Sn$

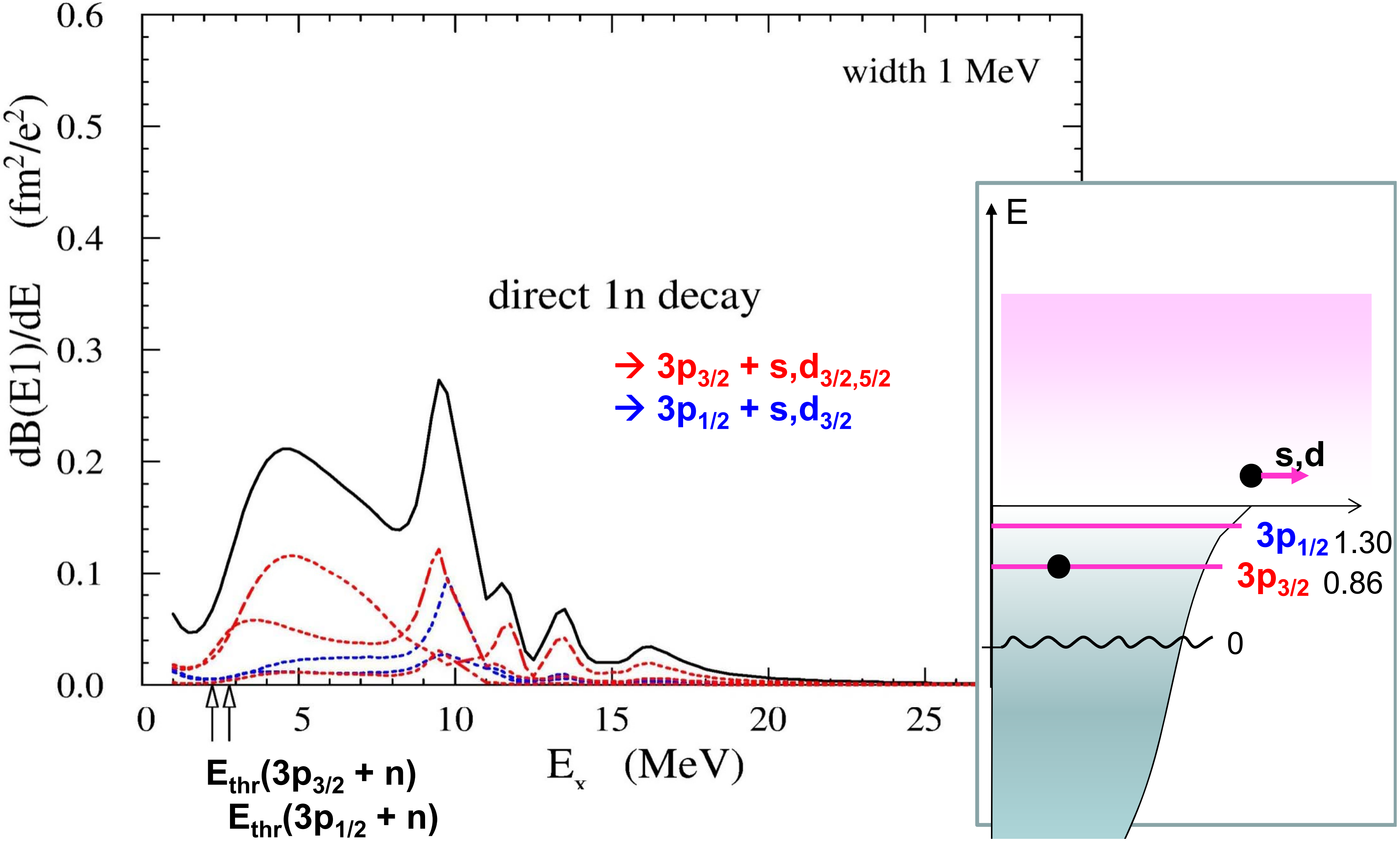
- Direct 2n decay dominates
- Direct 1n decay is a small fraction



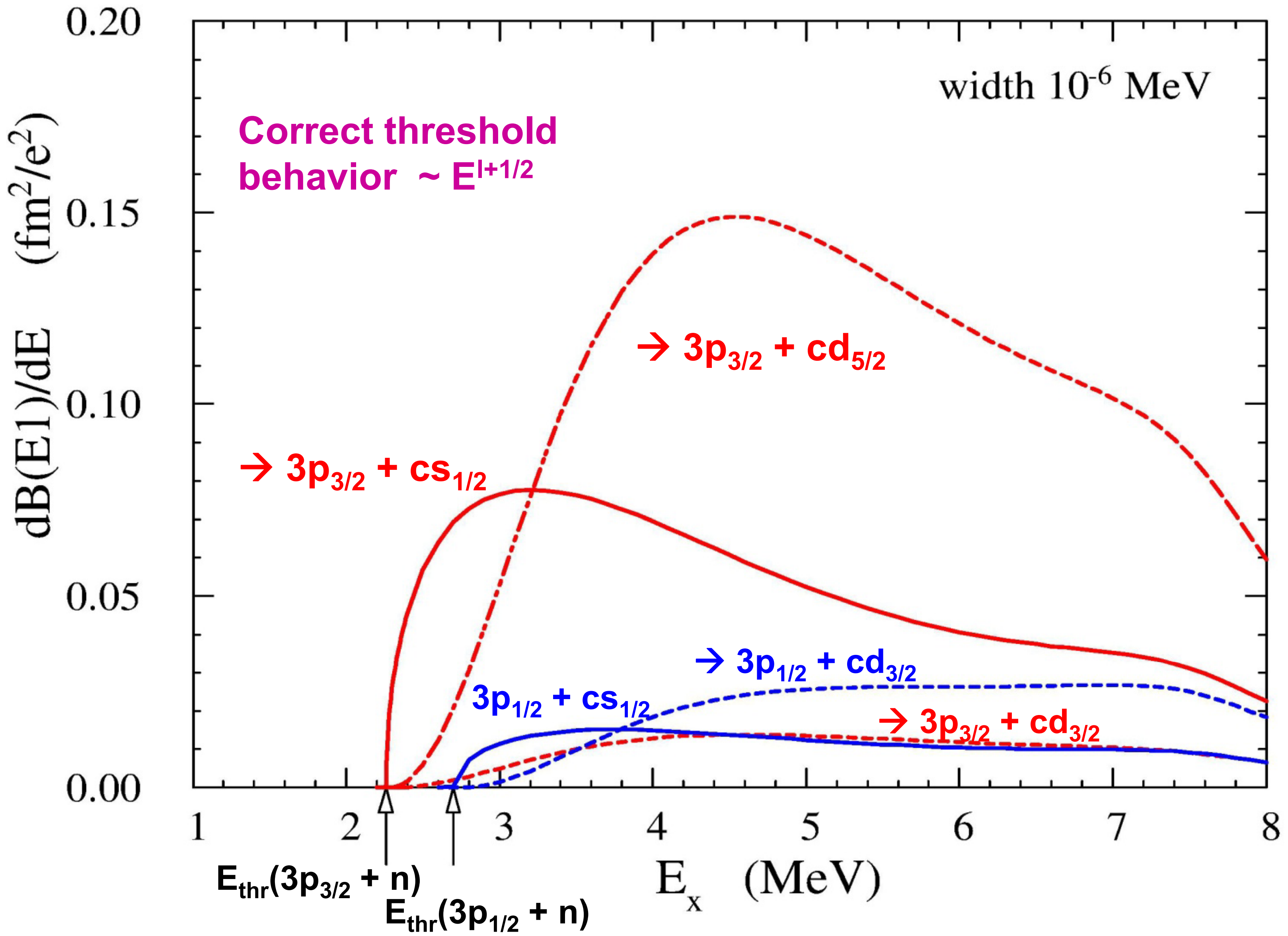

***Decay branching ratio***

| $E_x$ MeV | Direct 1n-decay | Direct 2n-decay | Other decays |
|---|---|---|---|
| 3.0 | 99.5 | 0.5 | 0 |
| 6.0 | 26 | 73 | 1.0 |
| 13.5 | 0.6 | 41 | 52 |
| 22 | 1.4 | 73 | 26 |

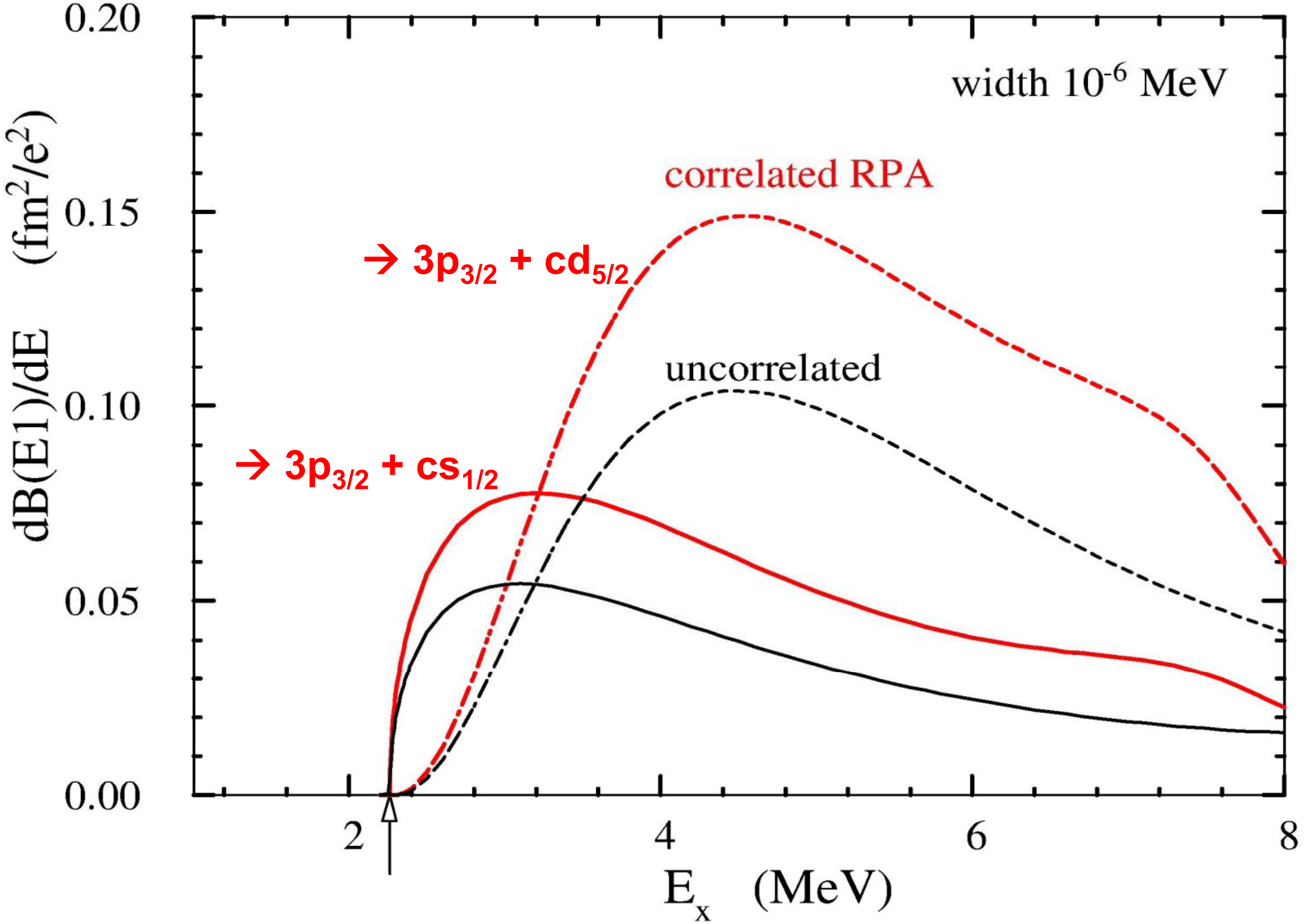
# E1 strength for direct 1n decay in $^{142}Sn$



# E1 strength for direct 1n decay in $^{142}Sn$



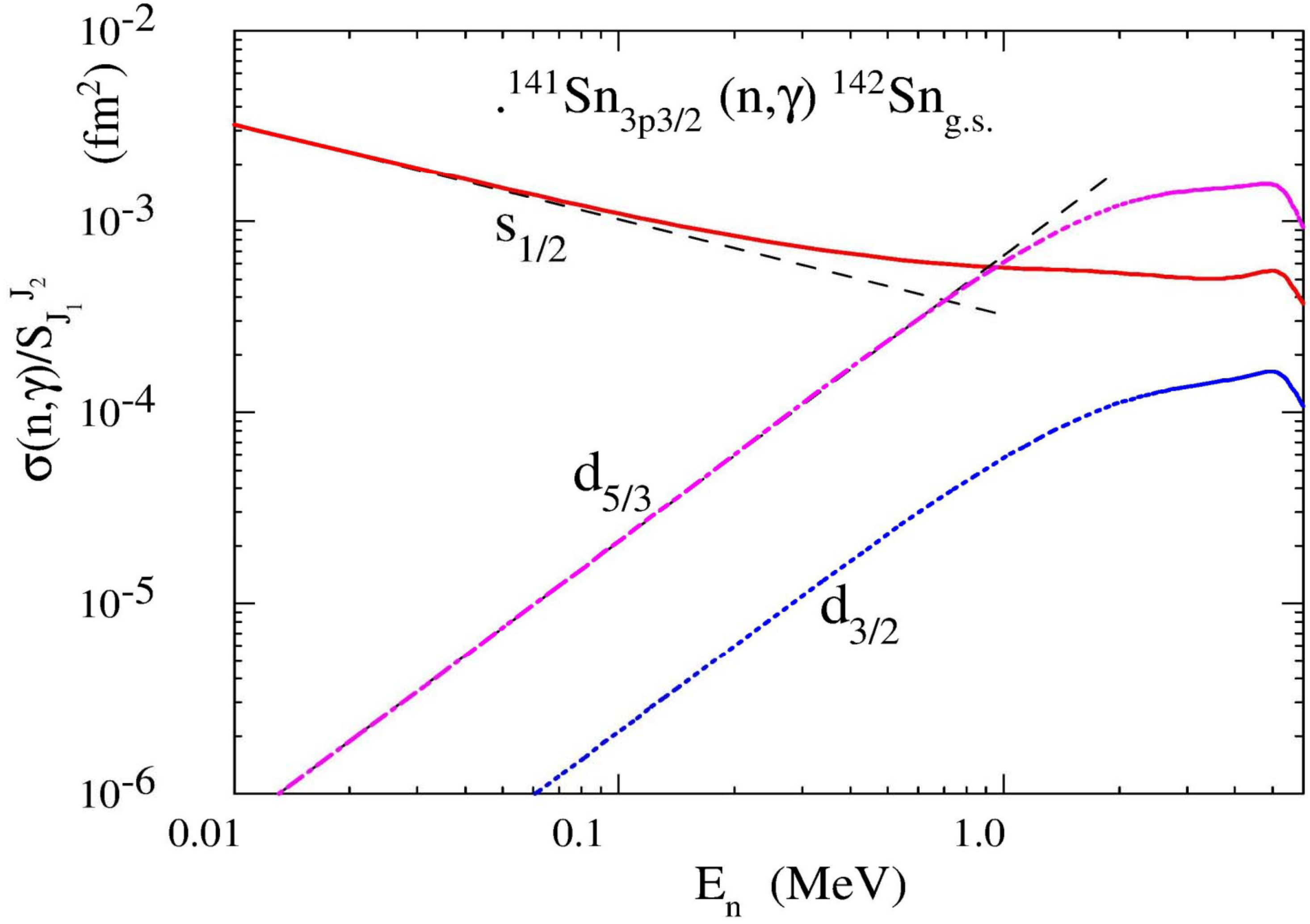
# Correlation effects



# Direct neutron capture cross section

$^{141}$Sn (3/2$^-$ gs) + n (s,d) → $^{142}$Sn (0$^+$ gs) + γ

Correct low-energy behavior $\sigma \sim E^{l-1/2}$



# Conclusions

- **The first attempt of describe the direct (radiative) neutron capture cross section $\sigma(n,\gamma)$ , relevant for the r-process by means of the density functional theory (the continuum QRPA) and the Zangwill-Soven decomposition.**

- **Decay branching ratio in photo-absorption process**

- **$\sigma(n,\gamma)$ at low neutron energies $E_n$~ 10keV--1MeV**

- **This will also provide us a useful method to discuss the effect of "the pygmy-above-the-threshold" on the neutron capture cross section $\sigma(n,\gamma)$**

- **Further developments to be done**

  Selfconsistency, decay to excited states, odd-A, other multipolarities