



# 4. Generalized Parton Distribution functions



<http://th.physik.uni-frankfurt.de/~jr/gif/phys/bohrpaul.jpg>

<http://f.hatena.ne.jp/ser/20070728164337>

Let's keep it inside, then.  
We may access the orbital motion.



## 4. Generalized Parton Distribution functions

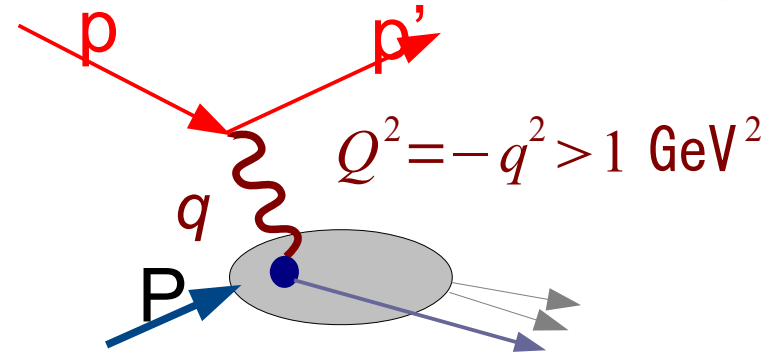
- Hard Exclusive Production and GPD
  - Marriage of PDF (DIS) and Form Factor (Elastic Scattering)
    - Deeply Virtual Compton Scattering
- Experimental highlights
- COMPASS II

- Scattered off “a free parton”
- Elastic scattering

だるま落とし  
Daruma  
Otoshi



## Deep Inelastic Scattering



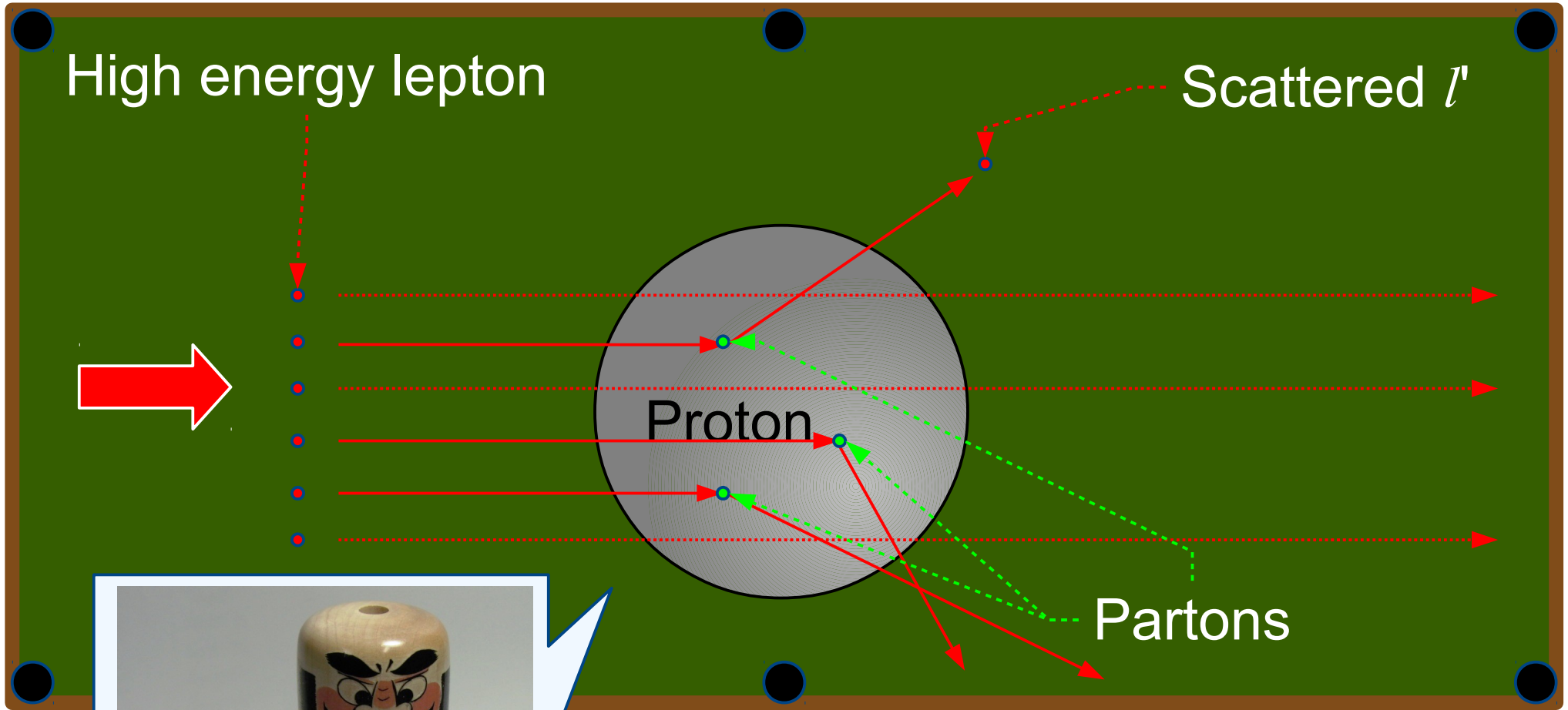
Parton distribution:  $q(x), \Delta q(x)$

$l$ - $N$  “Inelastic scattering”  
→ braking a target



# Deep Inelastic Scattering

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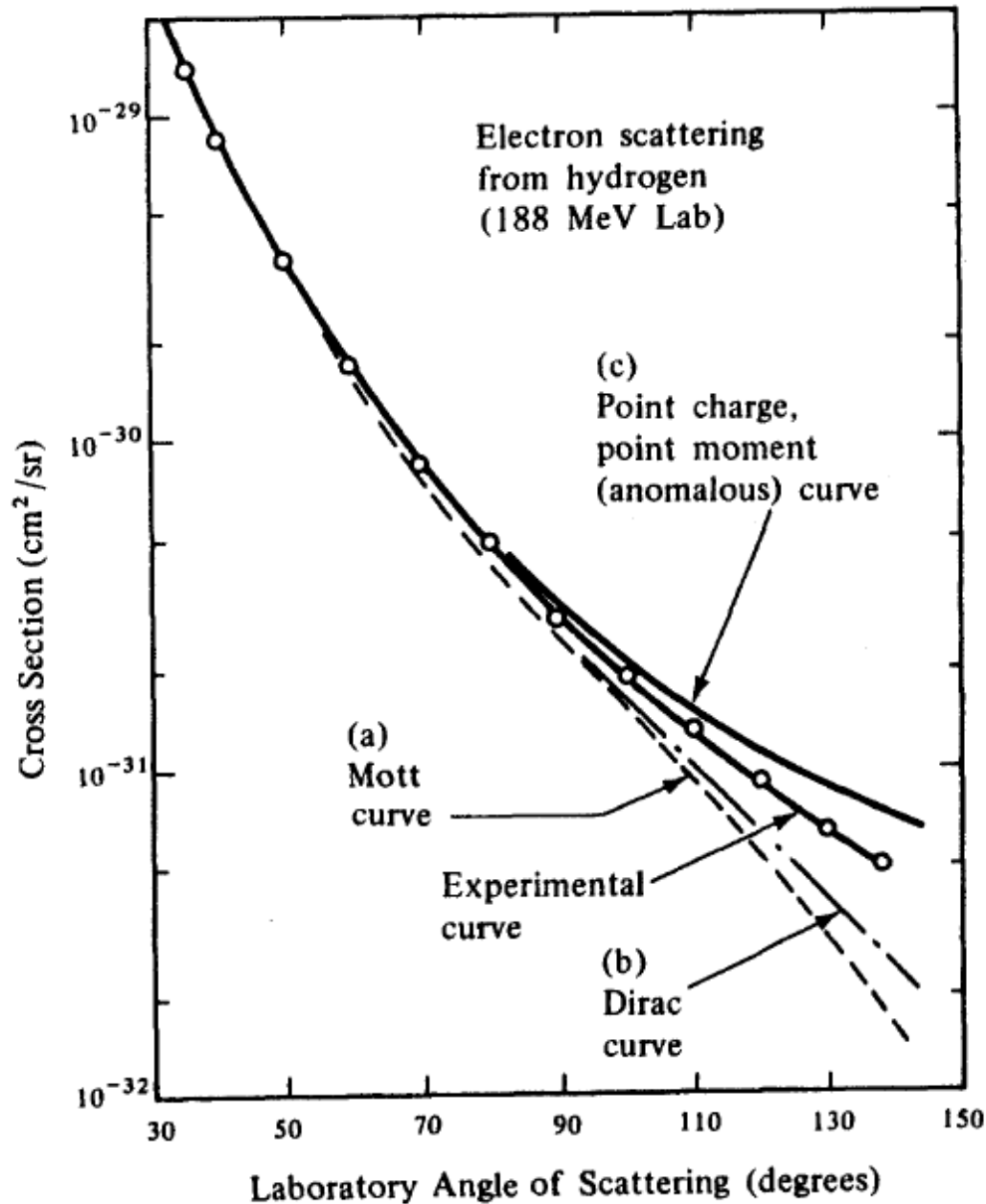


How many parton with  $x$  in the proton?

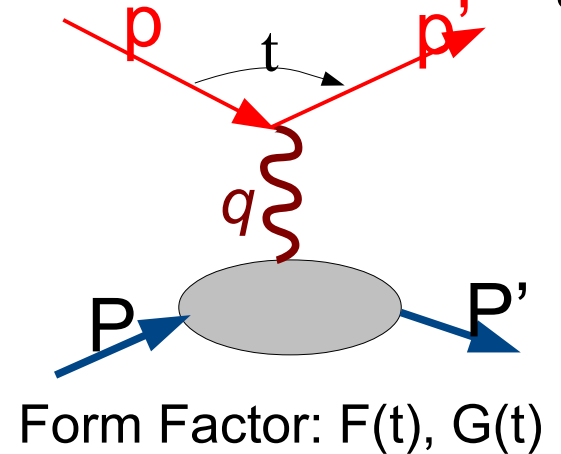


# Elastic scattering and Form Factor

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## Elastic scattering



$$\frac{d\sigma(\theta)}{d\Omega} = \frac{dY}{I \cdot \rho \cdot d\Omega}$$

$$= |F(q)|^2 \frac{d\sigma_0}{d\Omega}$$

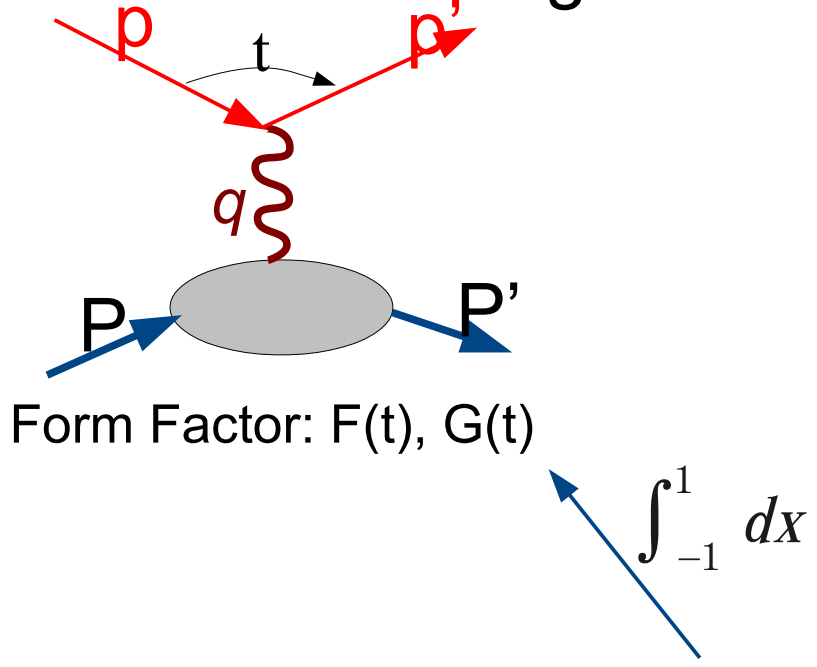
$$F(q) = \int dV \rho(r) e^{-i\vec{q} \cdot \vec{r}}$$



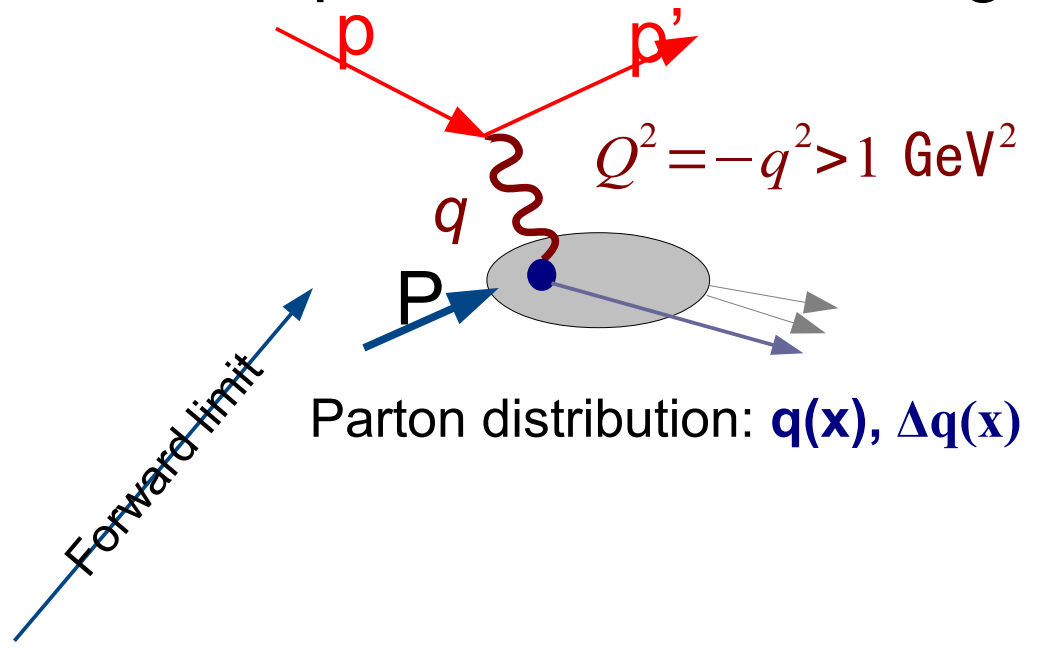
# Hard Exclusive Production and GPD

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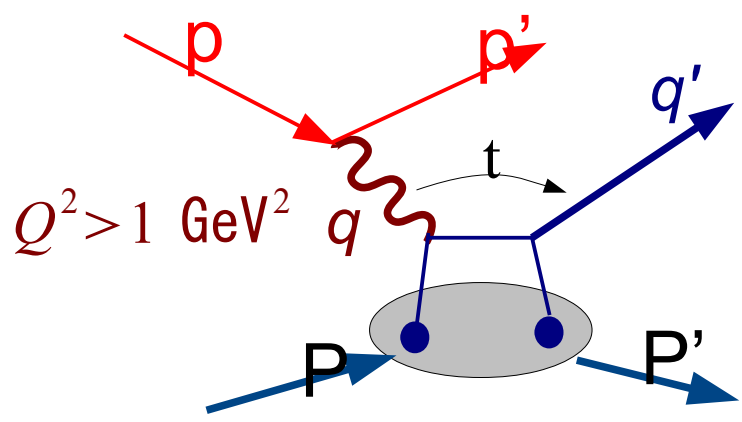
## Elastic scattering



## Deep Inelastic Scattering



## Hard Exclusive Production:

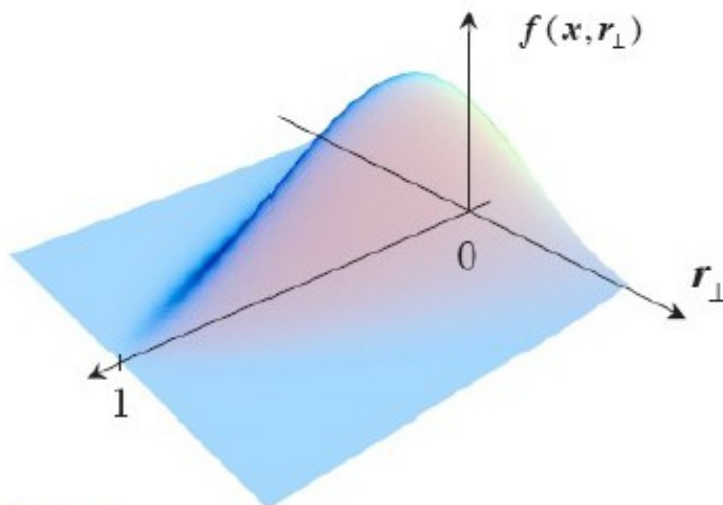
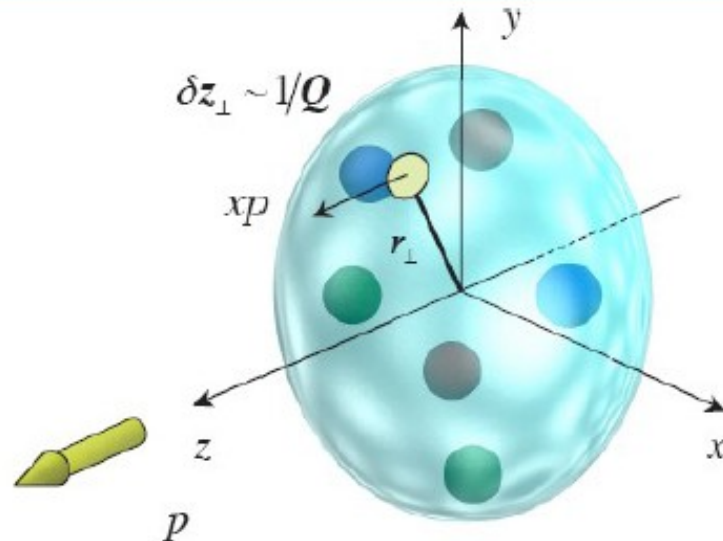


$$e + N \rightarrow e' + N' + \{\gamma, \rho, \pi, \dots\}$$

## Generalized Parton Distribution: $H, E, \tilde{H}, \tilde{E}$

$$J_q = \frac{1}{2} \int_{-1}^1 dx x [H^q(x, \xi, t \rightarrow 0) + E^q(x, \xi, t \rightarrow 0)]$$

## Generalized Parton Distributions (GPDs)



- Multidimensional description of nucleon structure (longitudinal momentum vs transverse position)
- Include parton distribution functions and form factors as forward limits and moments, respectively
- Can provide access to the total (and hence orbital) angular momentum of quarks in the nucleon via Ji relation:

$$J_q = \lim_{t \rightarrow 0} \int_{-1}^1 dx x [H_q(x, \xi, t) + E_q(x, \xi, t)]$$

- Four GPDs in case of proton target:

$$H, \tilde{H}, E, \tilde{E}$$



## big picture: the 3-D experience

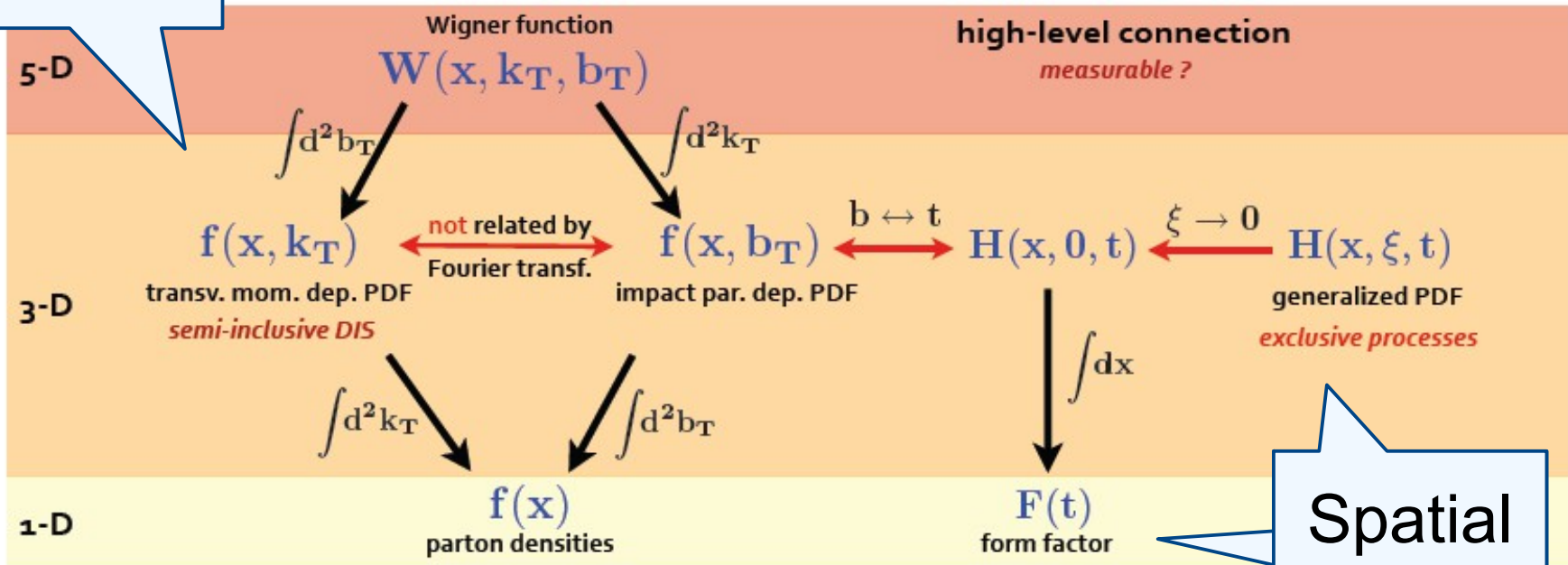
standard PDFs do not resolve transverse positions in the nucleon



### compelling questions

- how are quarks and gluons spatially distributed
- how do they move in the transverse plane
- do they orbit and do we have access to spin-orbit correlations

Momentum



Spatial

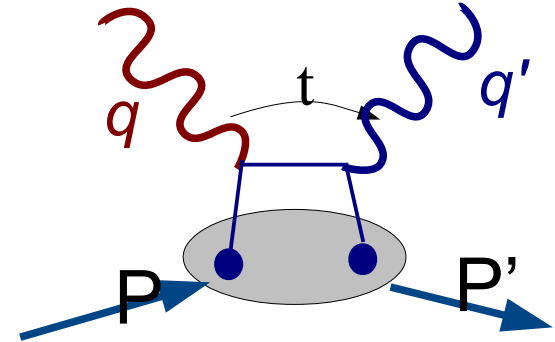




## Deeply Virtual Compton Scattering:

$$e + N \rightarrow e' + N' + \gamma$$

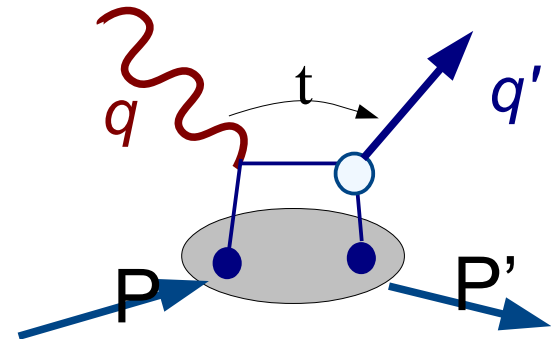
Involved GPDs:  $H, E, \tilde{H}, \tilde{E}$   
clean reaction

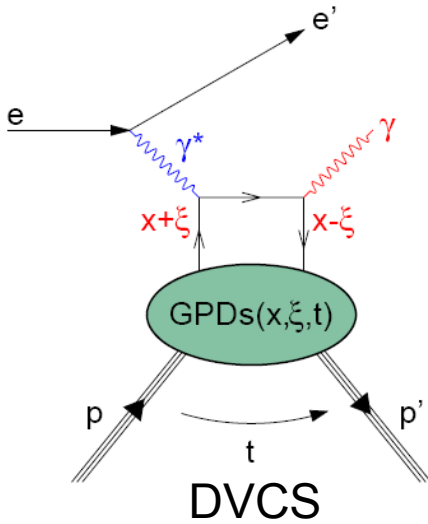


## Hard exclusive meson production:

$$e + N \rightarrow e' + N' + \{\rho, \pi, \dots\}$$

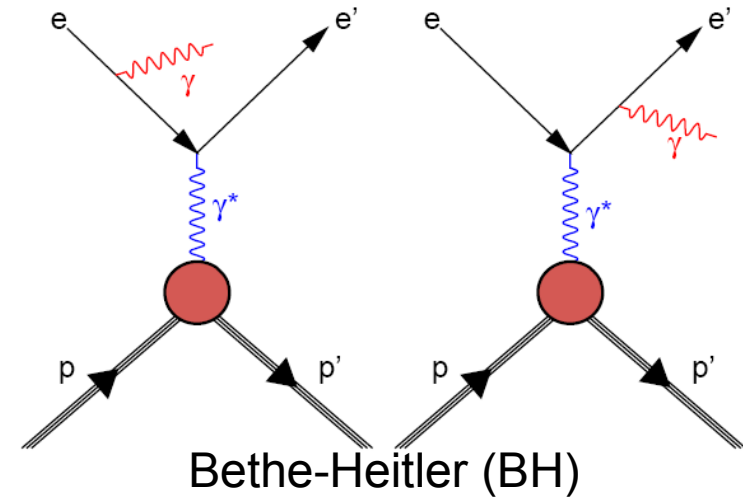
vector meson:  $H, E$   
pseudo-scalar meson:  $\tilde{H}, \tilde{E}$   
Meson amplitude involved





$$\sigma_{\text{DVCS}} \ll \sigma_{\text{BH}}$$

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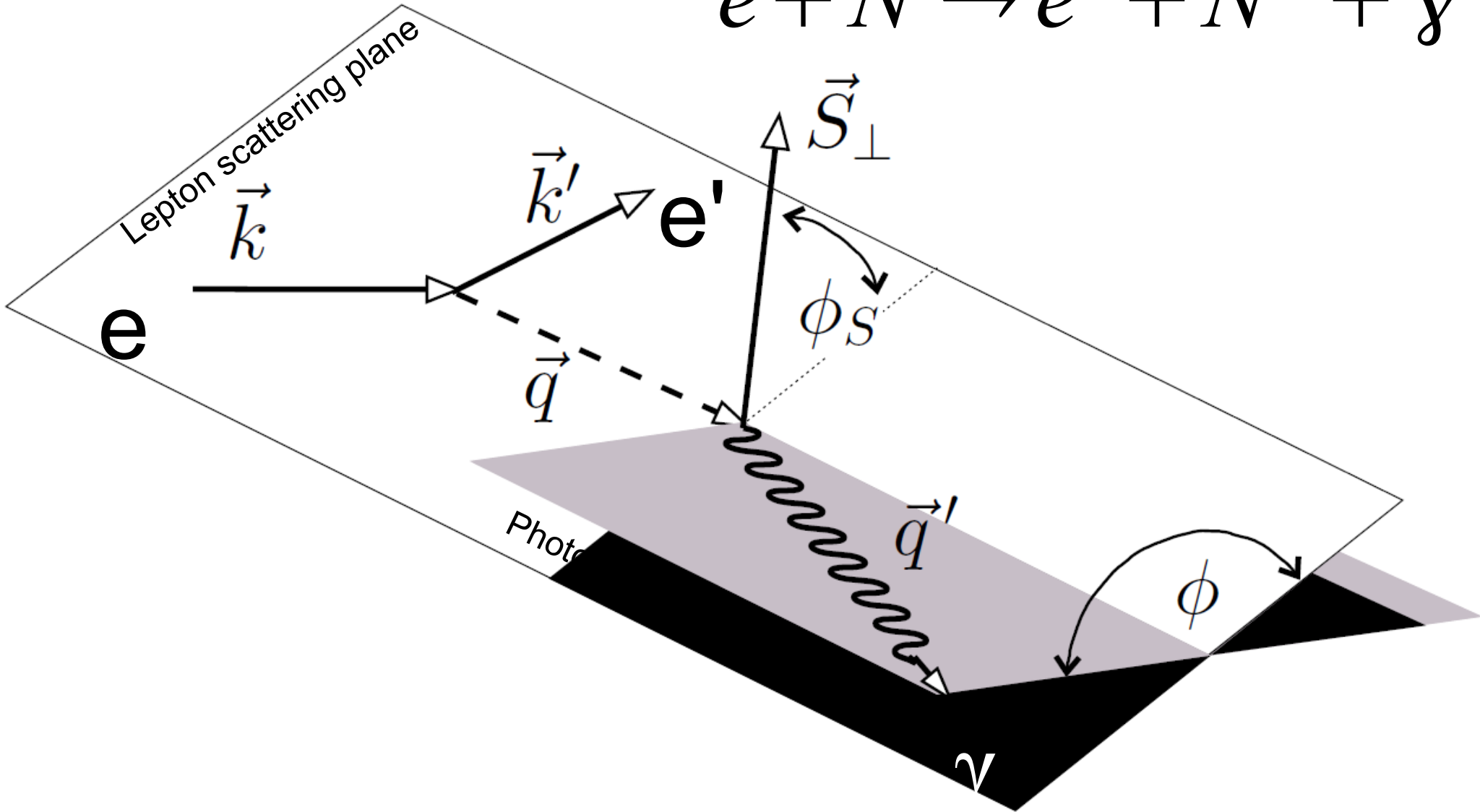
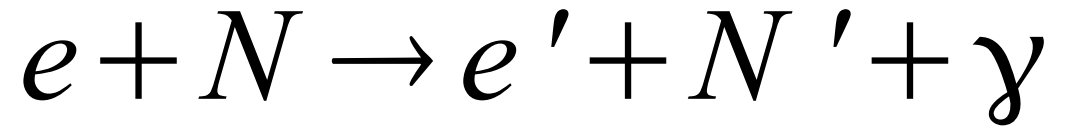
$$\frac{d^4\sigma}{dQ^2 dx_B dt d\phi} = \frac{x_B e^6}{32(2\pi)^4 Q^4 \sqrt{1 + \epsilon^2}} |\mathcal{T}_{ep \rightarrow ep\gamma}|^2,$$

$$|\mathcal{T}_{ep \rightarrow ep\gamma}|^2 = |\mathcal{T}_{\text{BH}}|^2 + |\mathcal{T}_{\text{DVCS}}|^2 + \mathcal{I},$$

$$\mathcal{I} = \mathcal{T}_{\text{BH}} \mathcal{T}_{\text{DVCS}}^* + \mathcal{T}_{\text{DVCS}} \mathcal{T}_{\text{BH}}^*.$$



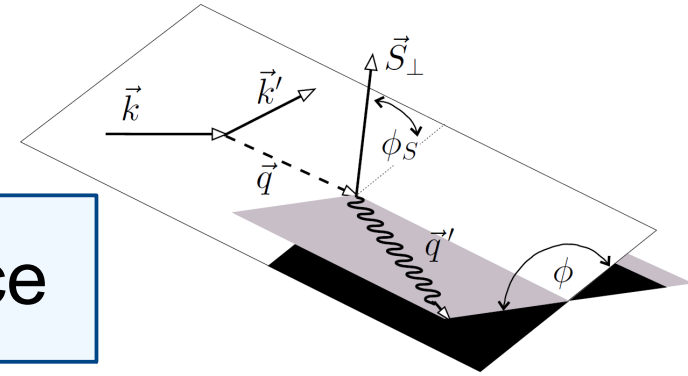
# Azimuthal angles in DVCS





$$d\sigma \propto |\mathcal{T}_{\text{BH}}|^2 + |\mathcal{T}_{\text{DVCS}}|^2 + \mathcal{I},$$

Azimuthal angle dependence



$$|\mathcal{T}_{\text{BH}}|^2 = \frac{K_{\text{BH}}}{\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left( c_0^{\text{BH}} + \sum_{n=1}^2 c_n^{\text{BH}} \cos(n\phi) \right),$$

$$|\mathcal{T}_{\text{DVCS}}|^2 = K_{\text{DVCS}} \left( c_0^{\text{DVCS}} + \sum_{n=1}^2 c_n^{\text{DVCS}} \cos(n\phi) + \lambda s_1^{\text{DVCS}} \sin \phi \right),$$

$$\mathcal{I} = \frac{-e_{\ell} K_{\mathcal{I}}}{\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left( c_0^{\mathcal{I}} + \sum_{n=1}^3 c_n^{\mathcal{I}} \cos(n\phi) + \lambda \sum_{n=1}^2 s_n^{\mathcal{I}} \sin(n\phi) \right).$$

Beam Charge

Beam helicity



# DVCS asymmetries

$A_C$

$A_{LU}$

$$\mathcal{I} = \frac{-e_\ell K_{\mathcal{I}}}{\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left( c_0^{\mathcal{I}} + \sum_{n=1}^3 c_n^{\mathcal{I}} \cos(n\phi) - \lambda \sum_{n=1}^2 s_n^{\mathcal{I}} \sin(n\phi) \right)$$

$$s_1^{\mathcal{I}} \approx 8 \frac{\sqrt{-t}}{Q} y(2-y) \Im m(C_{\text{unp}}^{\mathcal{I}}).$$

$$C_{\text{unp}}^{\mathcal{I}} = F_1 \mathcal{H} + \frac{x_B}{2-x_B} (F_1 + F_2) \tilde{\mathcal{H}} - \frac{t}{4M_p^2} F_2 \mathcal{E}.$$



$$C_{\text{unp}}^{\mathcal{I}} = F_1 \mathcal{H} + \frac{x_B}{2 - x_B} (F_1 + F_2) \tilde{\mathcal{H}} - \frac{t}{4M_p^2} F_2 \mathcal{E},$$

## Compton form factors

NPB629 (2002) 323-392

$$\{\mathcal{H}, \mathcal{E}, \mathcal{H}_+^3, \mathcal{E}_+^3, \tilde{\mathcal{H}}_-, \tilde{\mathcal{E}}_-\}(\xi) = \int_{-1}^1 dx C^{(-)}(\xi, x) \{H, E, H_+^3, E_+^3, \tilde{H}_-, \tilde{E}_-\}(x, \eta)|_{\eta=-\xi},$$

$$\{\tilde{\mathcal{H}}, \tilde{\mathcal{E}}, \tilde{\mathcal{H}}_+^3, \tilde{\mathcal{E}}_+^3, \mathcal{H}_-^3, \mathcal{E}_-^3\}(\xi) = \int_{-1}^1 dx C^{(+)}(\xi, x) \{\tilde{H}, \tilde{E}, \tilde{H}_+^3, \tilde{E}_+^3, H_-^3, E_-^3\}(x, \eta)|_{\eta=-\xi},$$

Sum of quark flavors

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$$C_{\text{unp}}^{\mathcal{I}} = F_1 \mathcal{H} + \frac{x_B}{2 - x_B} (F_1 + F_2) \tilde{\mathcal{H}} - \frac{t}{4M_p^2} F_2 \mathcal{E},$$

Negligible



ALU



$$\propto S_m H$$



GPD H

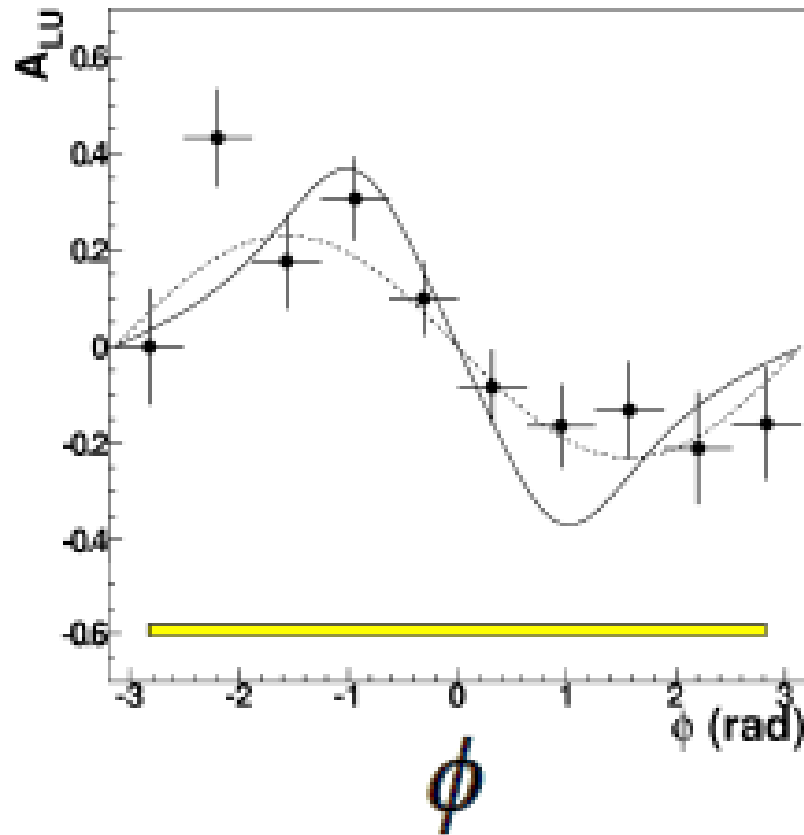


# Beam spin asymmetry in 2001

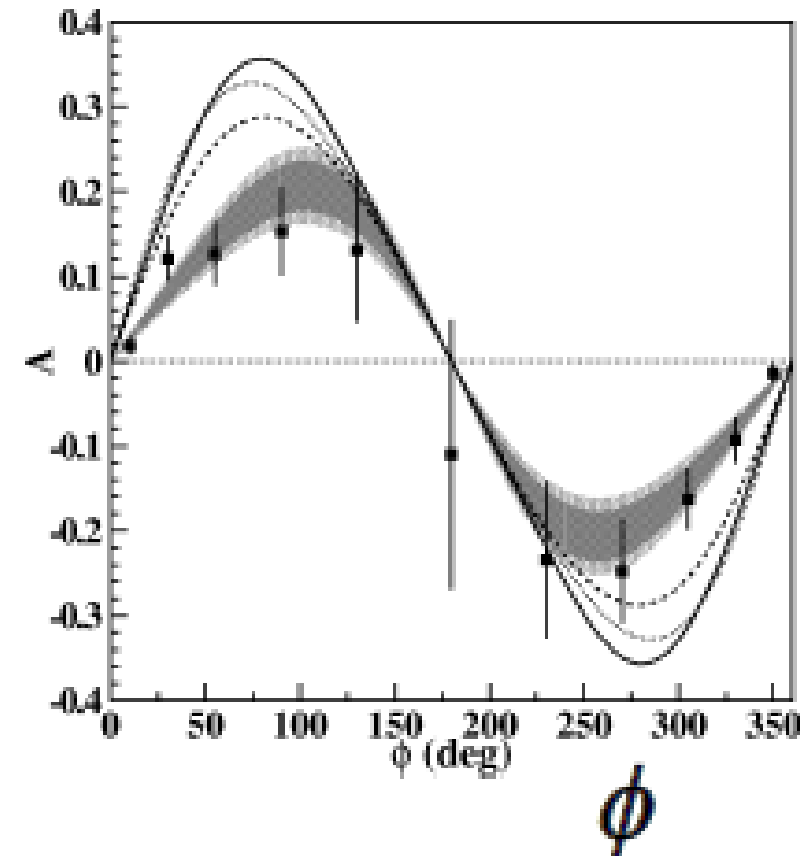
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## $A_{LU}$

### HERMES (DESY)



### CLAS (JLab)







# Beam spin, charge asymmetry

- Cross section  $\sigma_{LU}(\phi, P_B, C_B) = \sigma_{UU}[1 + P_B A_{LU}^{DVCS} + C_B P_B A_{LU}^I + C_B A_C]$

- Beam-charge asymmetry

$$A_C(\phi) = \frac{\sigma^+(\phi) - \sigma^-(\phi)}{\sigma^+(\phi) + \sigma^-(\phi)} \propto \Re \mathcal{H}$$

- Charge-difference beam-helicity asymmetry

$$A_{LU}^I(\phi) = \frac{(\sigma^{+\rightarrow}(\phi) - \sigma^{+\leftarrow}(\phi)) - (\sigma^{-\rightarrow}(\phi) - \sigma^{-\leftarrow}(\phi))}{(\sigma^{+\rightarrow}(\phi) - \sigma^{+\leftarrow}(\phi)) + (\sigma^{-\rightarrow}(\phi) - \sigma^{-\leftarrow}(\phi))} \propto \Im m \mathcal{H}$$

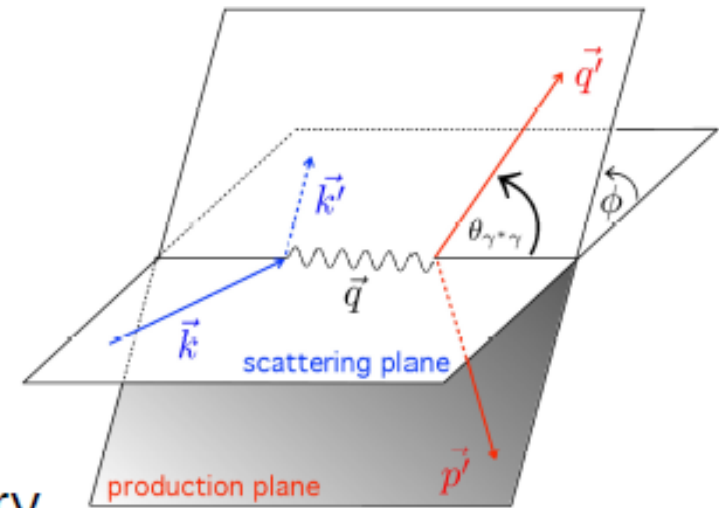
- Charge-averaged beam-helicity asymmetry

$$A_{LU}^{DVCS}(\phi) = \frac{(\sigma^{+\rightarrow}(\phi) + \sigma^{-\rightarrow}(\phi)) - (\sigma^{+\leftarrow}(\phi) + \sigma^{-\leftarrow}(\phi))}{(\sigma^{+\rightarrow}(\phi) + \sigma^{-\rightarrow}(\phi)) + (\sigma^{+\leftarrow}(\phi) + \sigma^{-\leftarrow}(\phi))} \propto \Im m [\mathcal{H}\mathcal{H}^* + \tilde{\mathcal{H}}\tilde{\mathcal{H}}^*]$$

- Separation of contribution from DVCS and interference term

- Impossible in case of single-charge asymmetry

$$A_{LU}(\phi) = \frac{\sigma^{\rightarrow} - \sigma^{\leftarrow}}{\sigma^{\rightarrow} + \sigma^{\leftarrow}}$$





# DVCS with a transversely polarized target

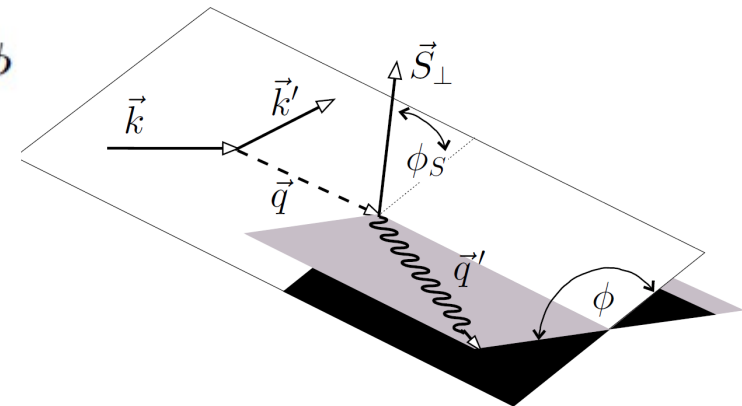
$$|T_{BH}|^2 = \frac{K_{BH}}{\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left( c_{0,UU}^{BH} + \left\{ c_{1,UU}^{BH} \cos \phi + \left\{ c_{2,UU}^{BH} \cos(2\phi) \right\} \right\} \right),$$

UU: Unpol. Beam & Target

$$|T_{DVCS}|^2 = K_{DVCS} \left( c_{0,UU}^{DVCS} + c_{2,UU}^{DVCS} \cos(2\phi) + \left\{ c_{1,UU}^{DVCS} \cos \phi \right. \right. \\ \left. \left. + S_{\perp} \left[ c_{0,UT}^{DVCS} \sin(\phi - \phi_S) + c_{2,UT}^{DVCS} \sin(\phi - \phi_S) \cos(2\phi) \right. \right. \right. \\ \left. \left. + s_{2,UT}^{DVCS} \cos(\phi - \phi_S) \sin(2\phi) \right. \right. \\ \left. \left. + \left\{ c_{1,UT}^{DVCS} \sin(\phi - \phi_S) \cos \phi + s_{1,UT}^{DVCS} \cos(\phi - \phi_S) \sin \phi \right\} \right] \right)$$

UT: Unpol. Beam T-pol. Target

$$I = \frac{-K_I e_l}{\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left( c_{1,UU}^I \cos \phi + c_{3,UU}^I \cos(3\phi) \right. \\ \left. + \left\{ c_{0,UU}^I + c_{2,UU}^I \cos(2\phi) \right\} \right. \\ \left. + S_{\perp} \left[ c_{1,UT}^I \sin(\phi - \phi_S) \cos \phi + s_{1,UT}^I \cos(\phi - \phi_S) \sin \phi \right. \right. \\ \left. \left. + c_{3,UT}^I \sin(\phi - \phi_S) \cos(3\phi) + s_{3,UT}^I \cos(\phi - \phi_S) \sin(3\phi) \right. \right. \\ \left. \left. + \left\{ c_{0,UT}^I \sin(\phi - \phi_S) + c_{2,UT}^I \sin(\phi - \phi_S) \cos(2\phi) \right. \right. \right. \\ \left. \left. \left. + s_{2,UT}^I \cos(\phi - \phi_S) \sin(2\phi) \right\} \right] \right).$$





# DVCS with a transversely polarized target

$$c_{0,UT}^{DVCS} \propto -\frac{\sqrt{-t}}{M} \text{Im} \left\{ \mathcal{H}\mathcal{E}^* - \mathcal{E}\mathcal{H}^* + \xi\tilde{\mathcal{E}}\tilde{\mathcal{H}}^* - \tilde{\mathcal{H}}\xi\tilde{\mathcal{E}}^* \right\},$$

$$c_{1,UU}^I \propto \frac{\sqrt{-t}}{Q} \text{Re} \left\{ F_1\mathcal{H} + \xi(F_1 + F_2)\tilde{\mathcal{H}} - \frac{t}{4M^2}F_2\mathcal{E} \right\},$$

$$c_{0,UU}^I \propto -\frac{\sqrt{-t}}{Q} c_{1,UU}^I,$$

Target spin asymmetry: UT

$$c_{1,UT}^I \propto -\frac{M}{Q} \text{Im} \left\{ \frac{t}{4M^2} \left[ (2 - x_B)F_1\mathcal{E} - 4\frac{1 - x_B}{2 - x_B}F_2\mathcal{H} \right] + x_B\xi \left[ F_1(\mathcal{H} + \mathcal{E}) - (F_1 + F_2) \left( \tilde{\mathcal{H}} + \frac{t}{4M^2}\tilde{\mathcal{E}} \right) \right] \right\}$$

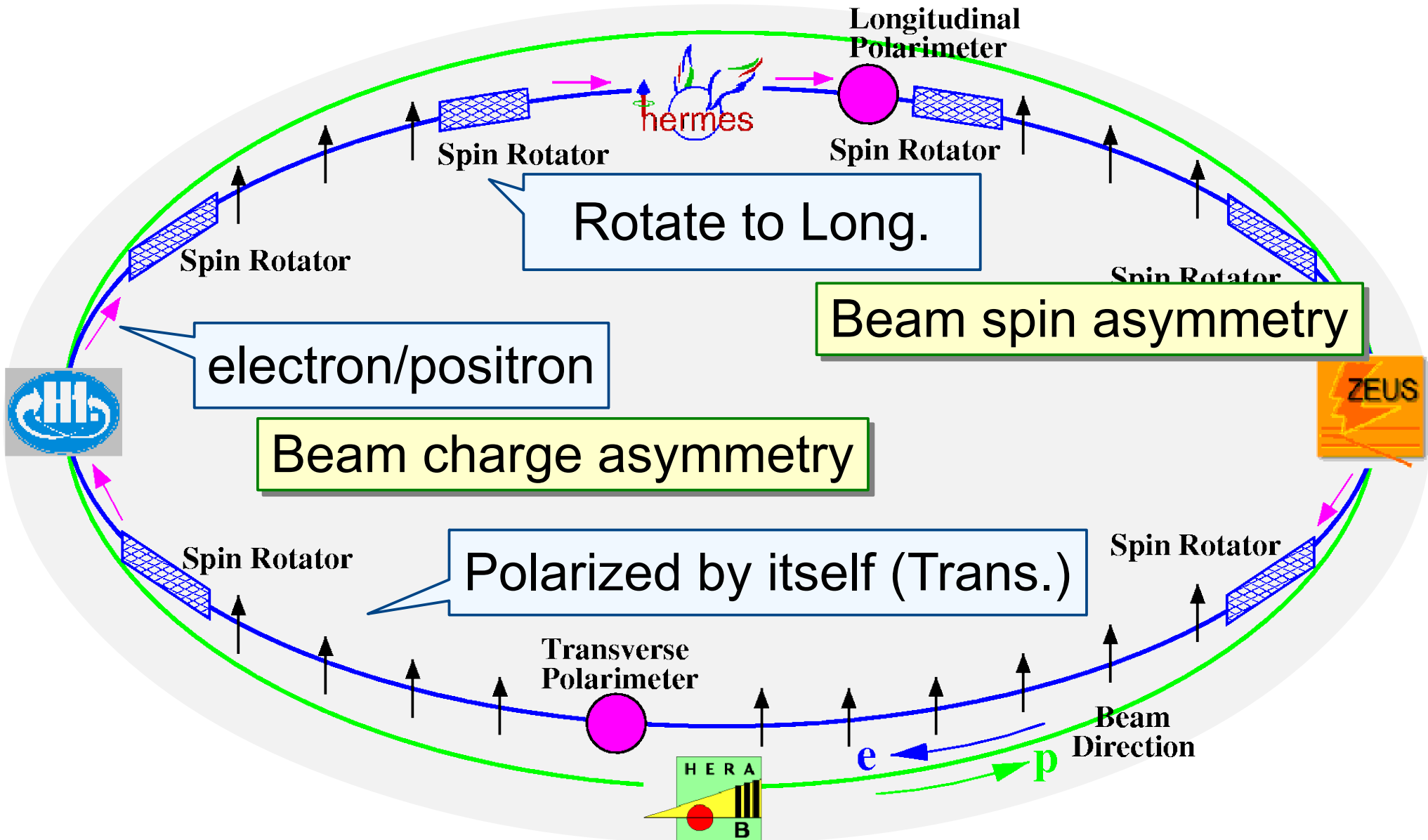
$$c_{0,UT}^I \propto -\frac{\sqrt{-t}}{Q} c_{1,UT}^I,$$

UT → GPD E

$$s_{1,UT}^I \propto -\frac{M}{Q} \text{Im} \left\{ \frac{t}{4M^2} \left[ 4\frac{1 - x_B}{2 - x_B}F_2\tilde{\mathcal{H}} - (F_1 + \xi F_2)x_B\tilde{\mathcal{E}} \right] + x_B \left[ (F_1 + F_2) \left( \xi\mathcal{H} + \frac{t}{4M^2}\mathcal{E} \right) - \xi F_1 \left( \tilde{\mathcal{H}} + \frac{x_B}{2}\tilde{\mathcal{E}} \right) \right] \right\}$$



# HERMES polarized electron/positron beam

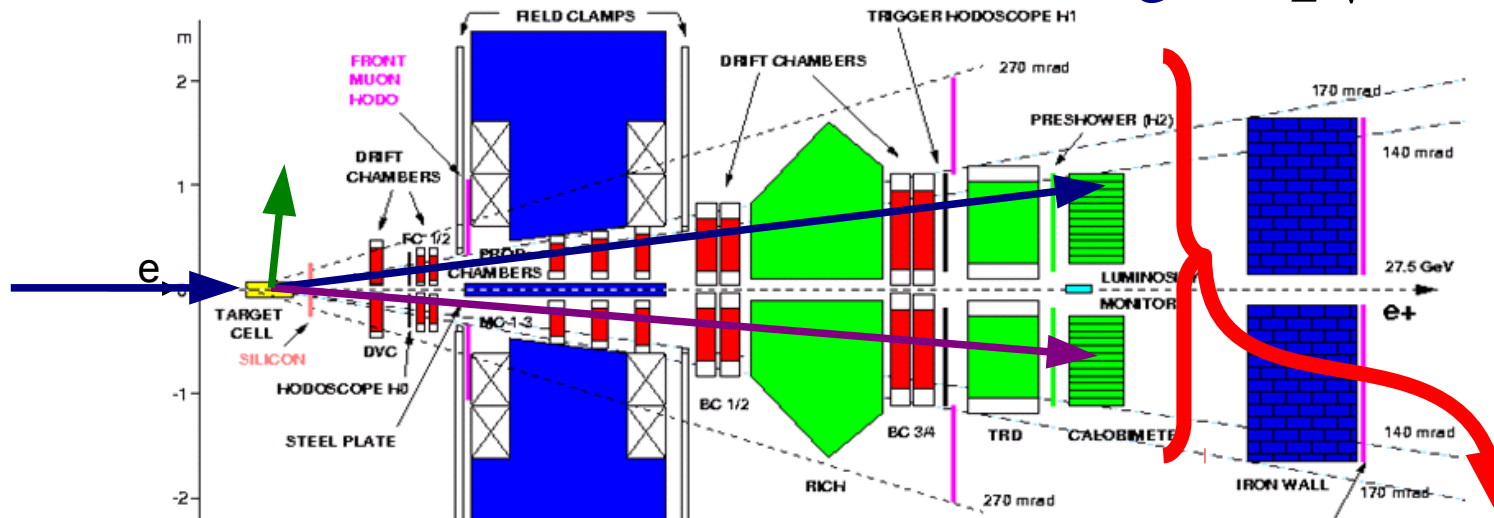




# Measurement of exclusive production at

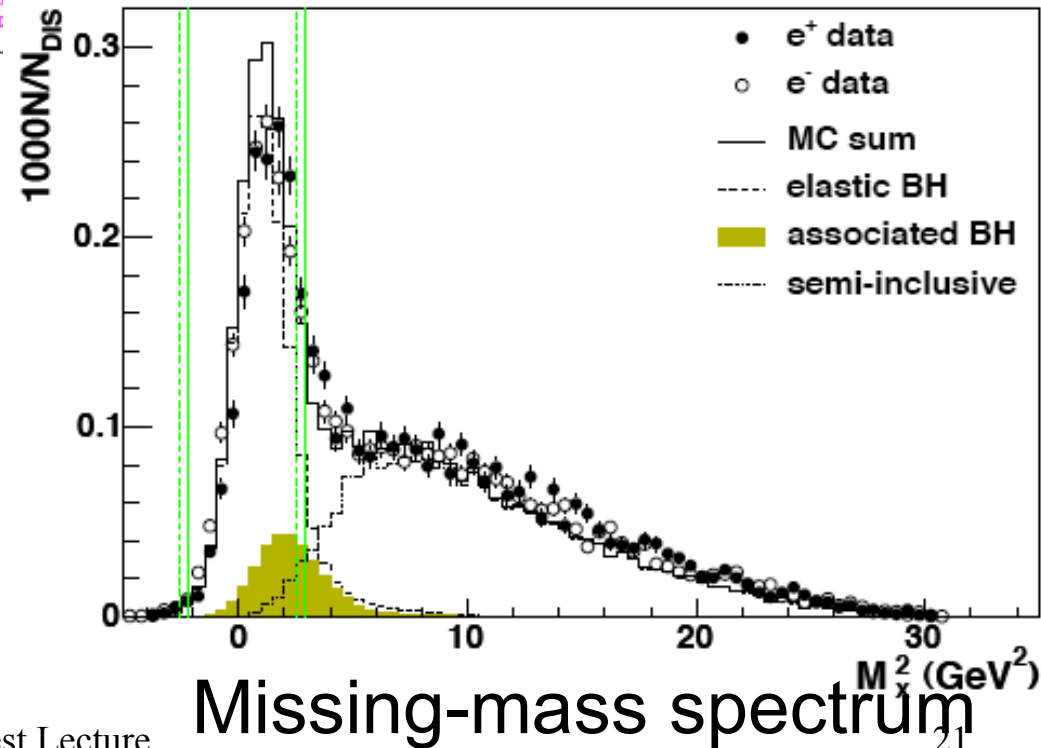


$$e + N \rightarrow e' + N + \gamma$$



1996 ~ 2005:

No detector for the recoil proton  
Cut on missing-mass spectrum





<u>Year</u>	<u>Beam</u>		<u>Targets</u>		<u>Detector</u>
	Charge	Helicity	Pol.	Unpol.	
1996	+	+	H(l)	H, D, $^3\text{He}$	
1997	+	+, -	H(l)	H, D, N	
1998	-	+, -	D(l)	H, D, Kr	
1999	+	+, -	D(l)	D, N, Kr	
2000	+	+, -	D(l)	H, D, $^4\text{He}$ , Ne, Kr	
2002	+	-	<b>H(t)</b>	H, D, Kr	
2003	+	+	<b>H(t)</b>	H, Kr	
2004	+	+, -	<b>H(t)</b>	H, D, Kr, Xe	
2005	-	+, -	<b>H(t)</b>	H, D, Xe	
2006	-, +	+, -		H, D	Recoil
2007	+	+, -		H, D	Recoil

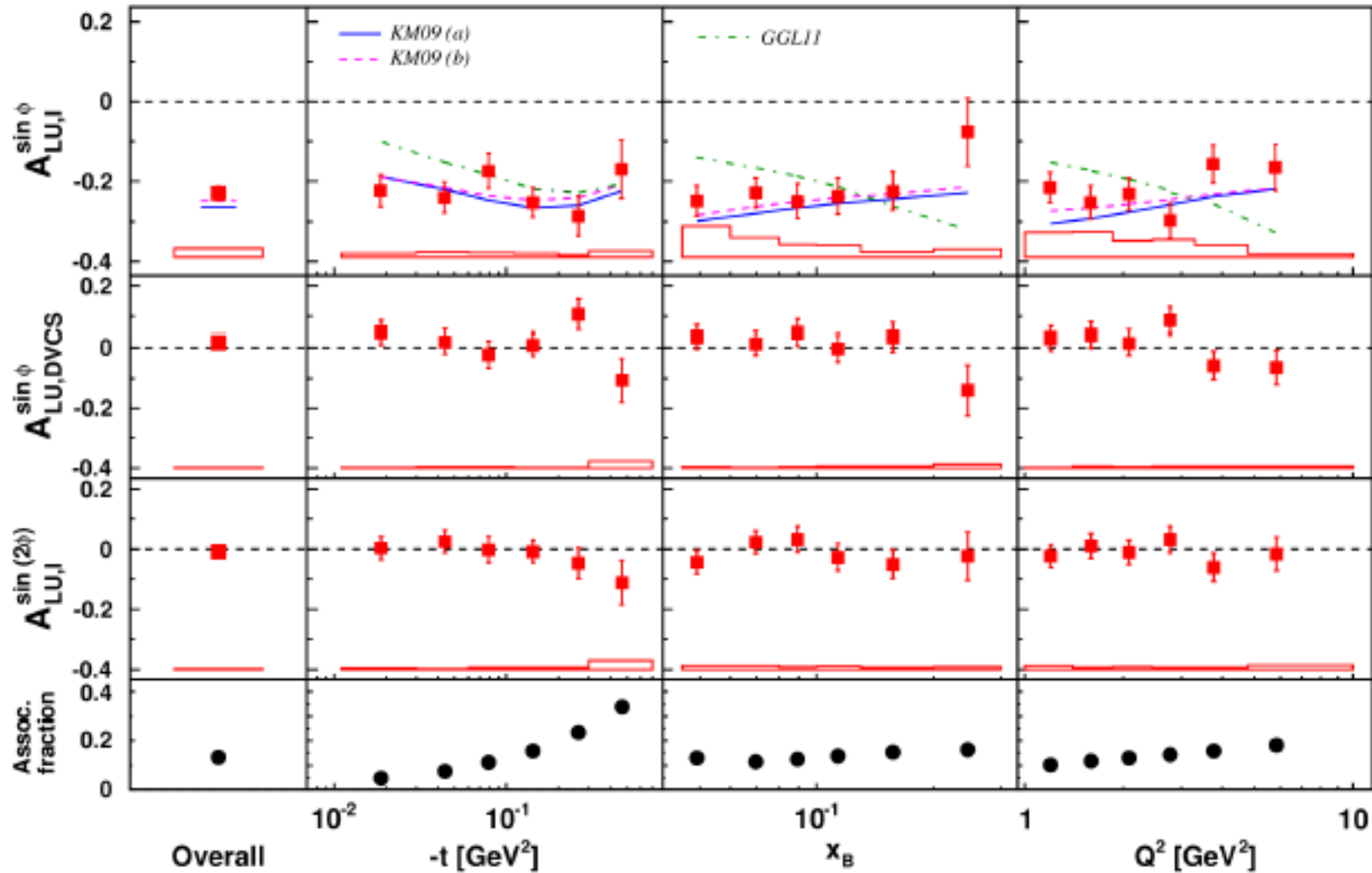
- Beam spin, charge asymmetry H PRL87 (2001) 182001  
H JHEP 11 (2009) 083  
H JHEP 07 (2012) 032  
D NPB829 (2010) 1-27
- Beam spin **with Recoil detector** H JHEP 10 (2012) 042
- Beam charge asymmetry H PRD75 (2007) 011103
- Longitudinal target spin asymmetry H JHEP 06 (2010) 019  
D NPB842 (2011) 265-298
- Transverse target spin asymmetry H JHEP 06 (2008) 066
- DVCS on nucleus targets PRC81 (2010) 035202



# Beam spin asymmetry

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Published: *A. Airapetian et al, JHEP 07 (2012) 032*



$\propto \Im m \mathcal{H}$

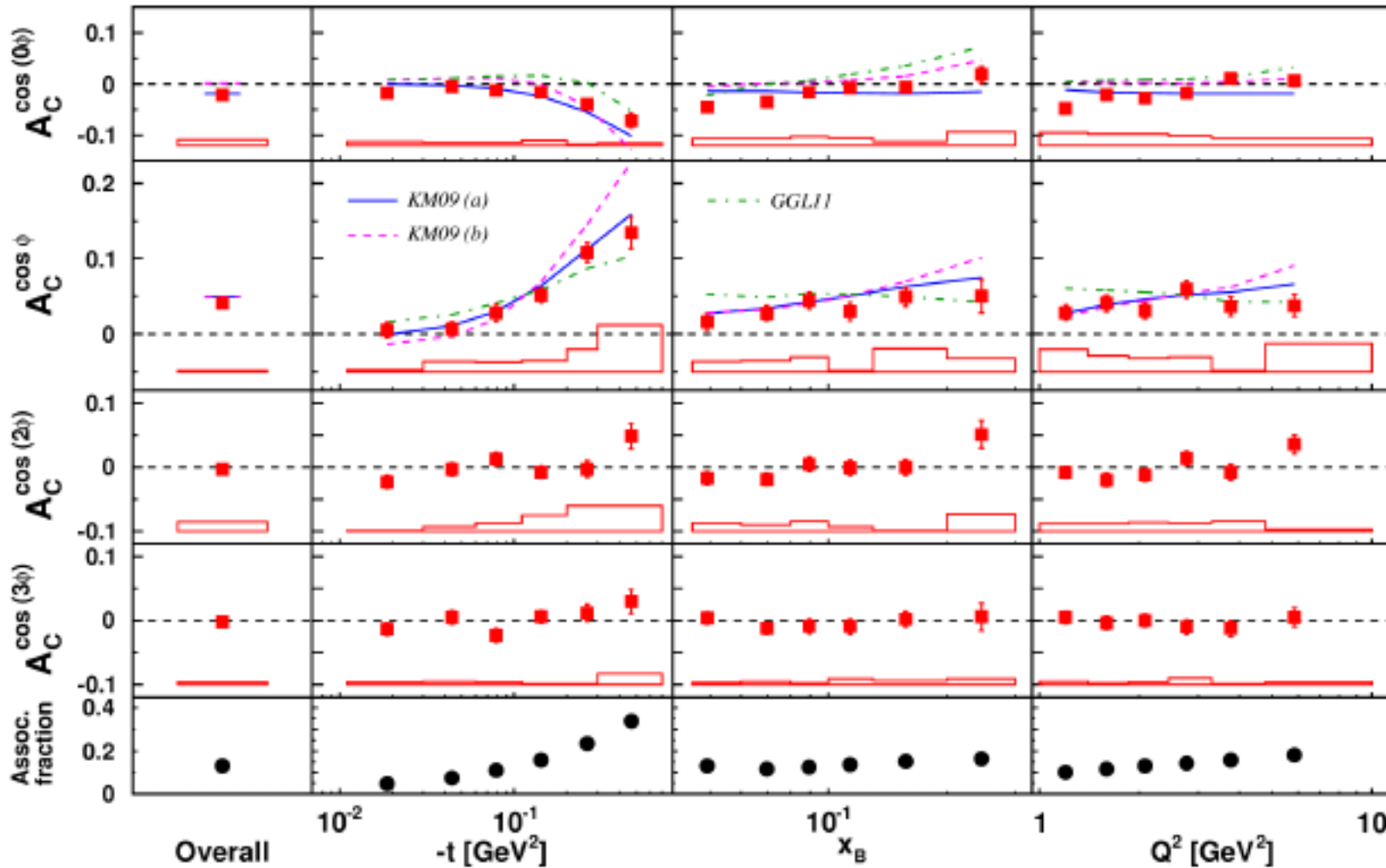




# Beam charge asymmetry

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Published: *A. Airapetian et al, JHEP 07 (2012) 032*

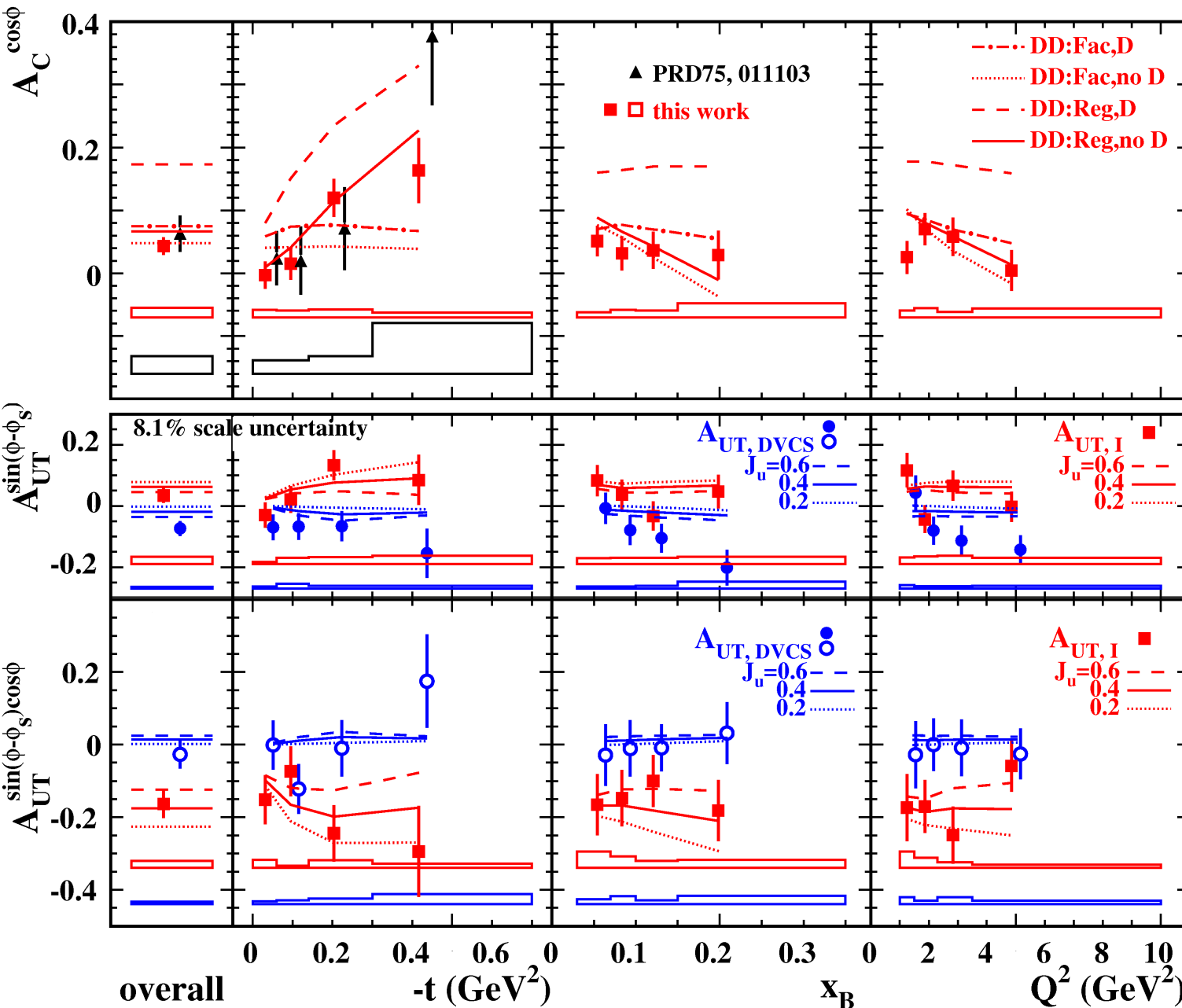


$\propto \text{Re}H$



# DVCS with T-pol. H target

HERMES Collaboration, JHEP06 (2008) 066



$$A_C^{\cos\phi} \propto \Re [ F \mathcal{H} ]$$

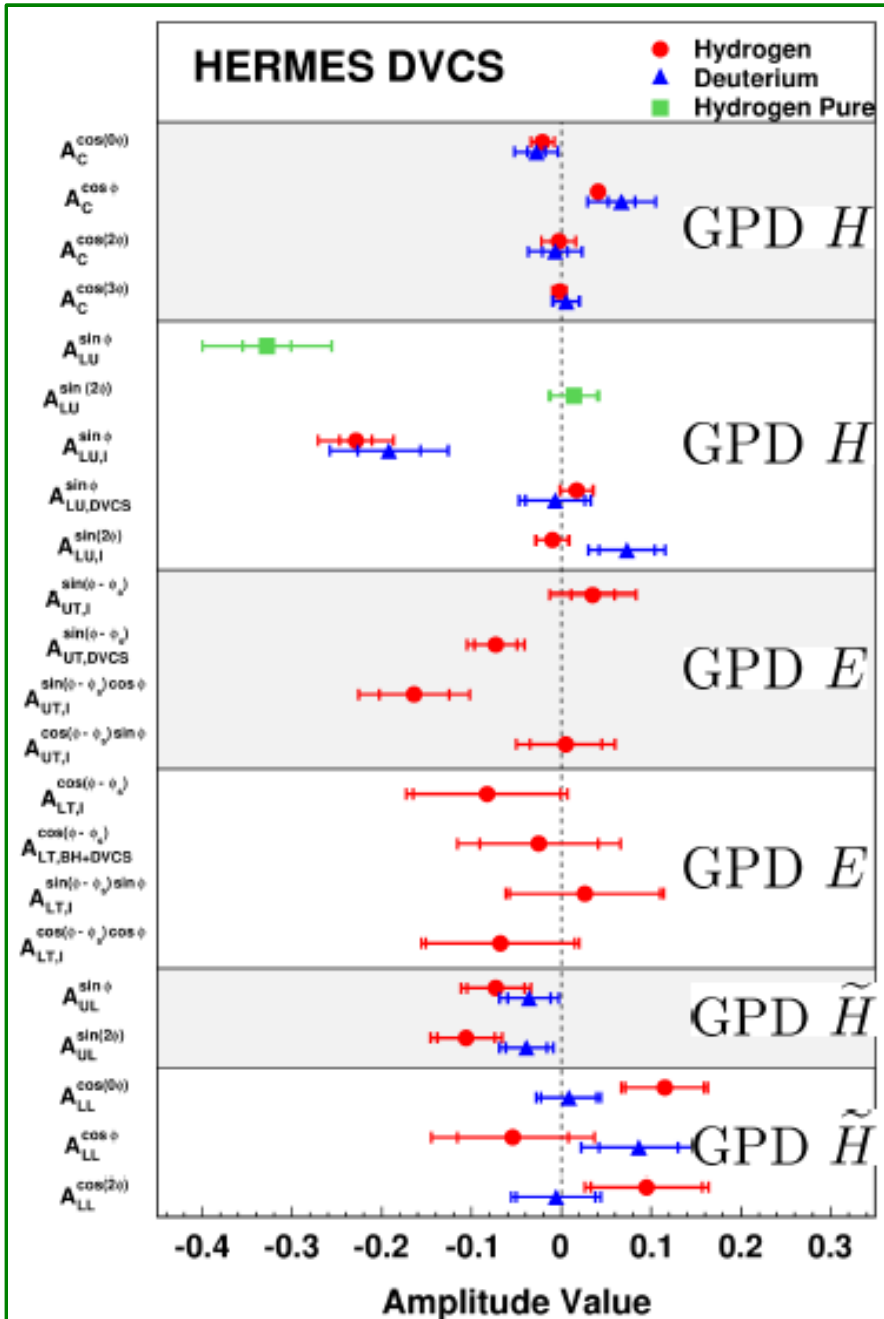
$$A_{UT, DVCS}^{\sin(\phi-\phi_s)} \propto \Im [ \mathcal{H} \mathcal{E}^* - \mathcal{E} \mathcal{H}^* + \xi \tilde{\mathcal{E}} \tilde{\mathcal{H}}^* - \tilde{\mathcal{H}} \xi \tilde{\mathcal{E}}^* ]$$

$$A_{UT, I}^{\sin(\phi-\phi_s)} \propto -A_{UT, I}^{\sin(\phi-\phi_s) \cos\phi} \propto \Im [ F_1 \mathcal{E} - F_2 \mathcal{H} ]$$



# DVCS amplitudes measured at HERMES

S. Yaschenko, DIS2013



● Beam-charge and beam-spin asymmetry

*PRL 87 (2001) 182001*

*PRD 75 (2007) 011103*

*JHEP 11 (2009) 083*

*JHEP 07 (2012) 032, JHEP 10 (2012) 042*

*Nucl. Phys. B 829 (2010) 1*

With Recoil

● Transverse target-spin asymmetry

*JHEP 06 (2008) 066*

● Transverse double-spin asymmetry

*Phys. Lett. B 704 (2011) 15*

● Longitudinal target spin asymmetry

*JHEP 06 (2010) 019*

● Longitudinal target & double spin asymmetry

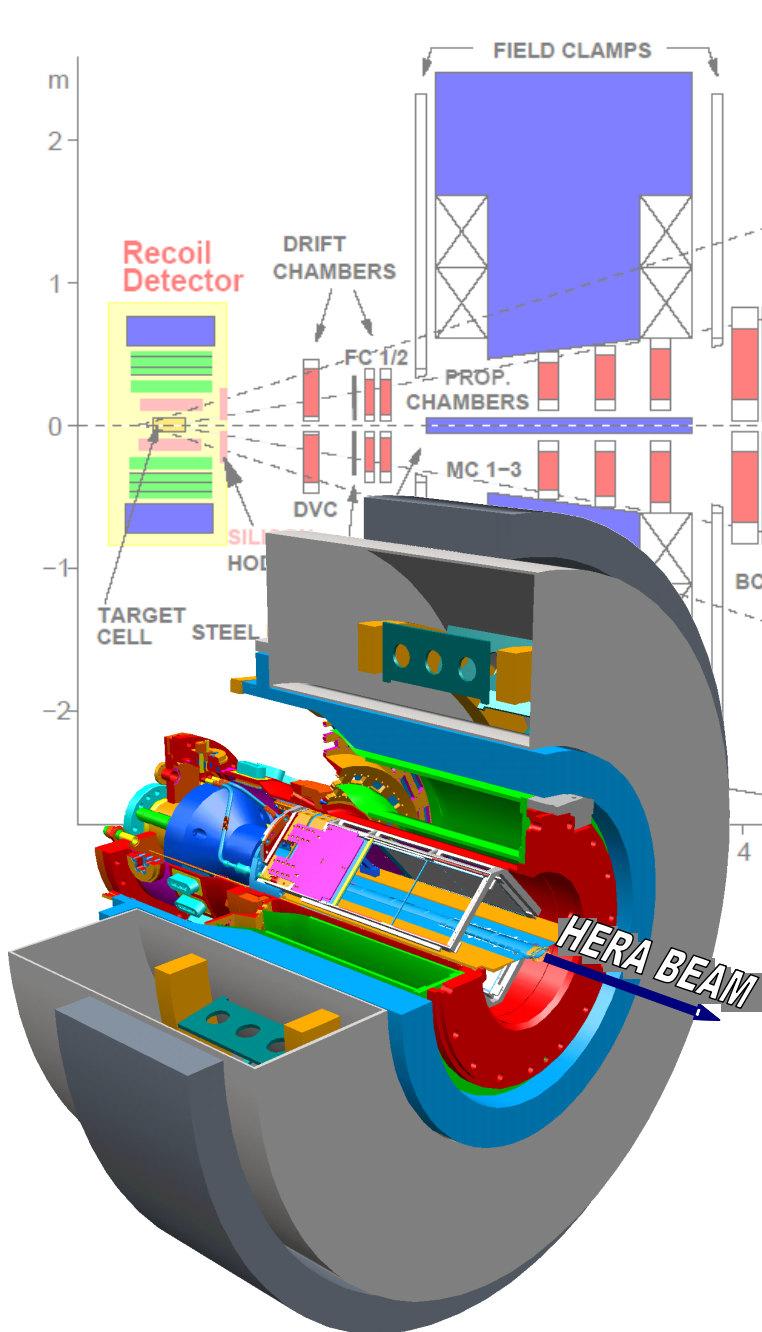
*Nucl. Phys. B 842 (2011) 265*





# HERMES Recoil Detector:

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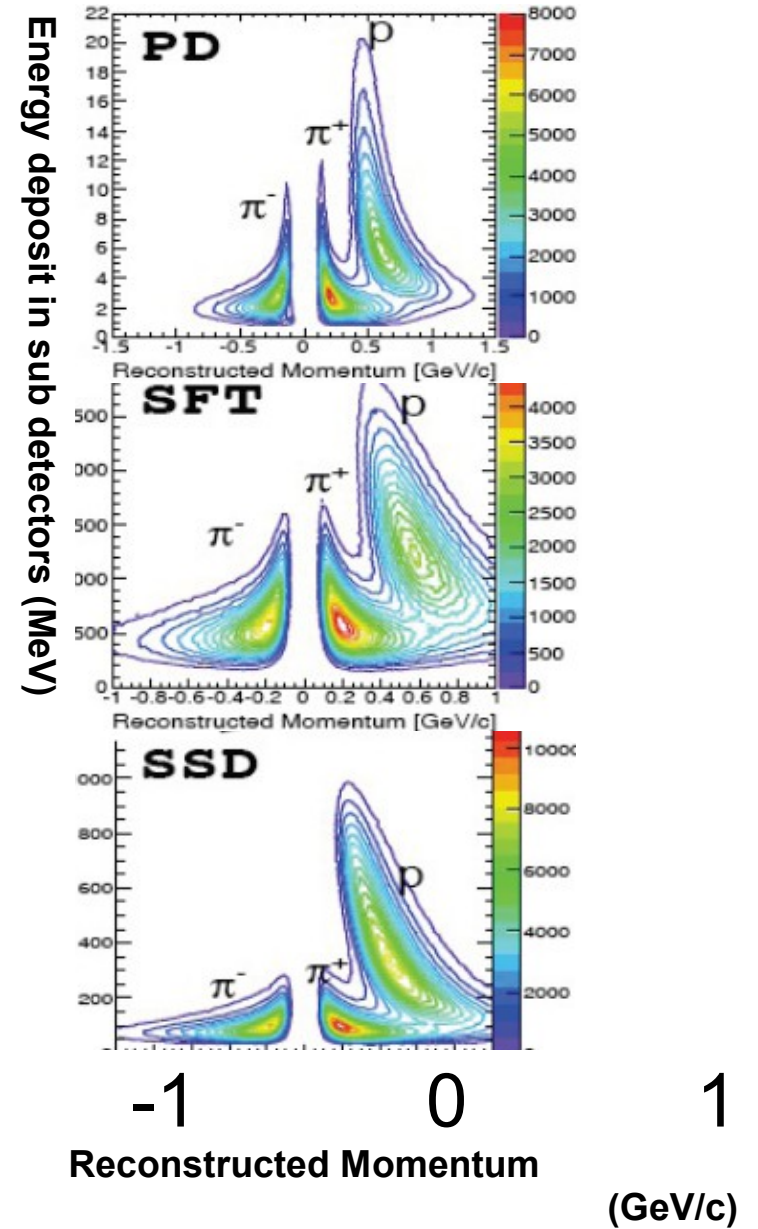
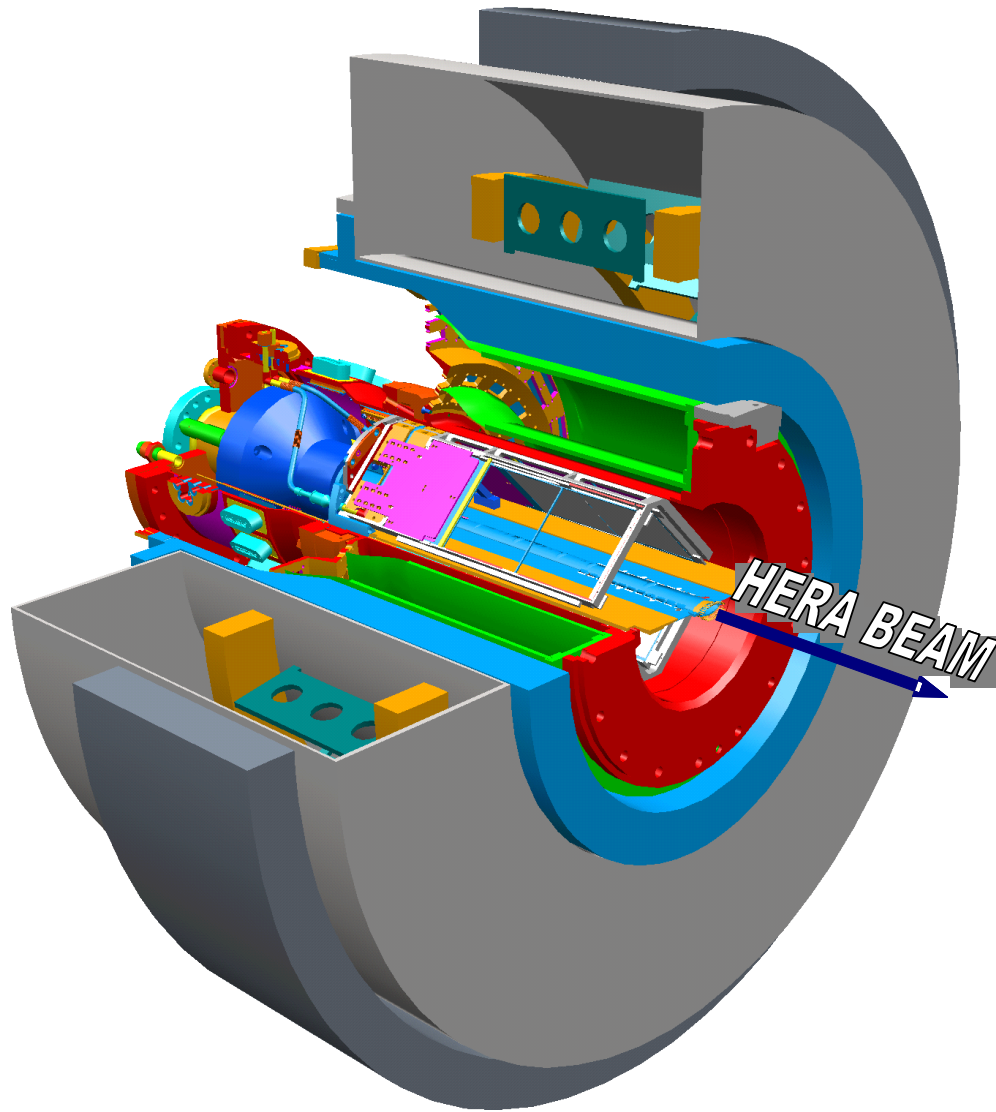


- Sub-detectors in 1 T Solenoid
  - Photon Detector (**PD**)
    - 3 Tungsten/Scintillator layers,  $\pi^0$ ,  $\pi/p$
  - Scintillation Fiber Tracker (**SFT**)
    - 2 x (2 Parallel and 2 Stereo layers), momentum reconstruction and  $\pi/p$
  - Silicon Strip Detector (**SSD**)
    - 2 layers of 16 double sided sensor, momentum reconstruction and  $\pi/p$
- Unpol. H&D targets: (2006, 2007)



# HERMES Recoil Detector

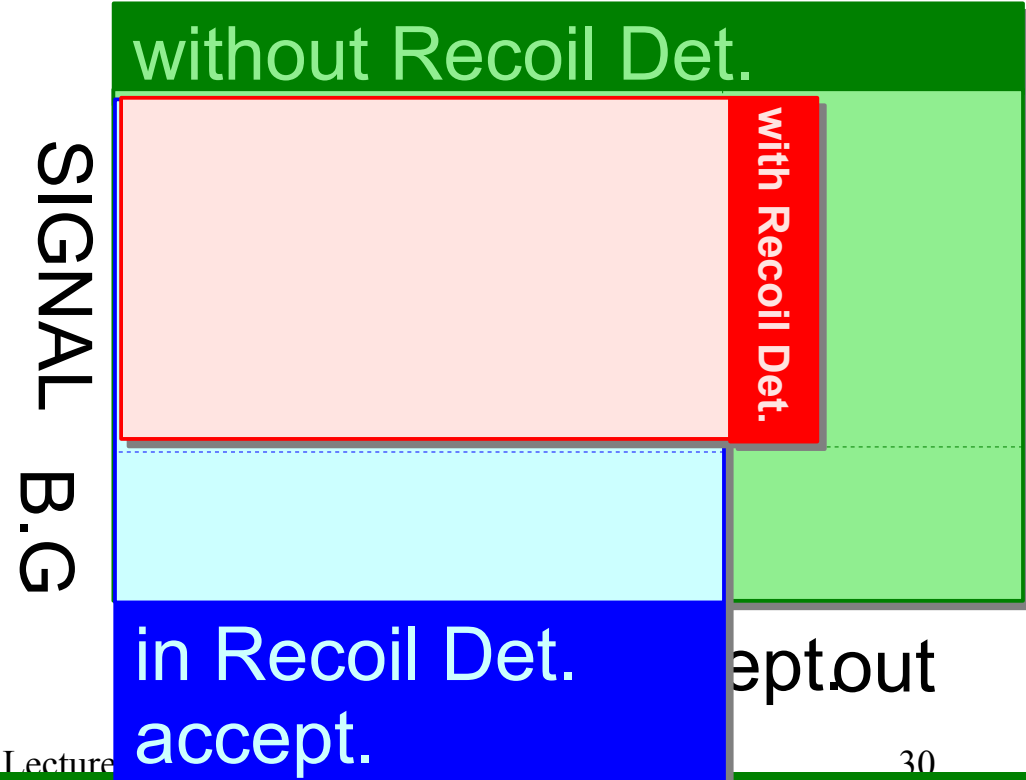
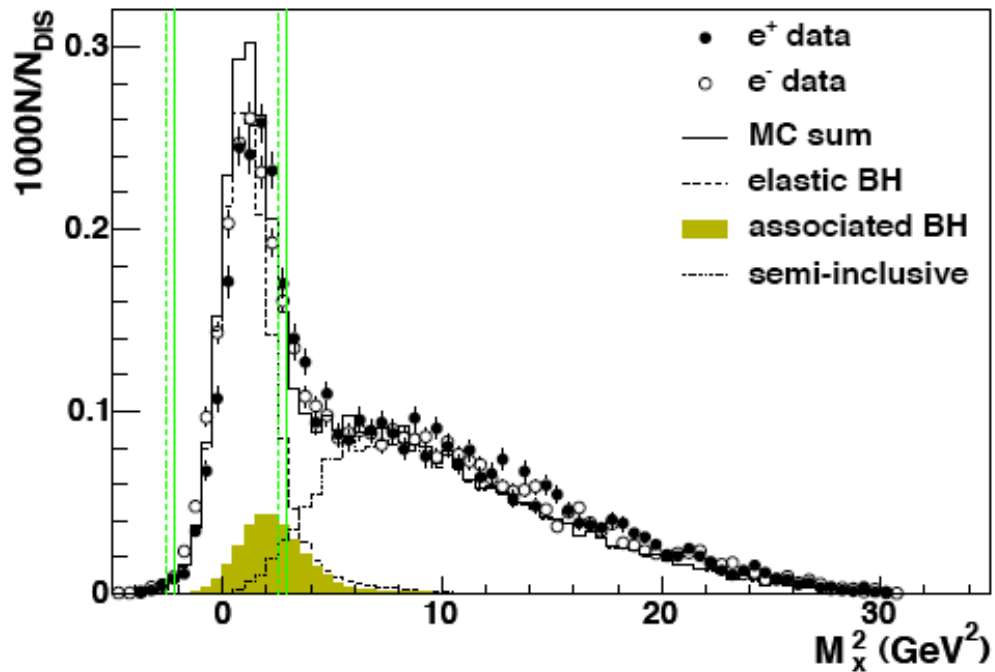
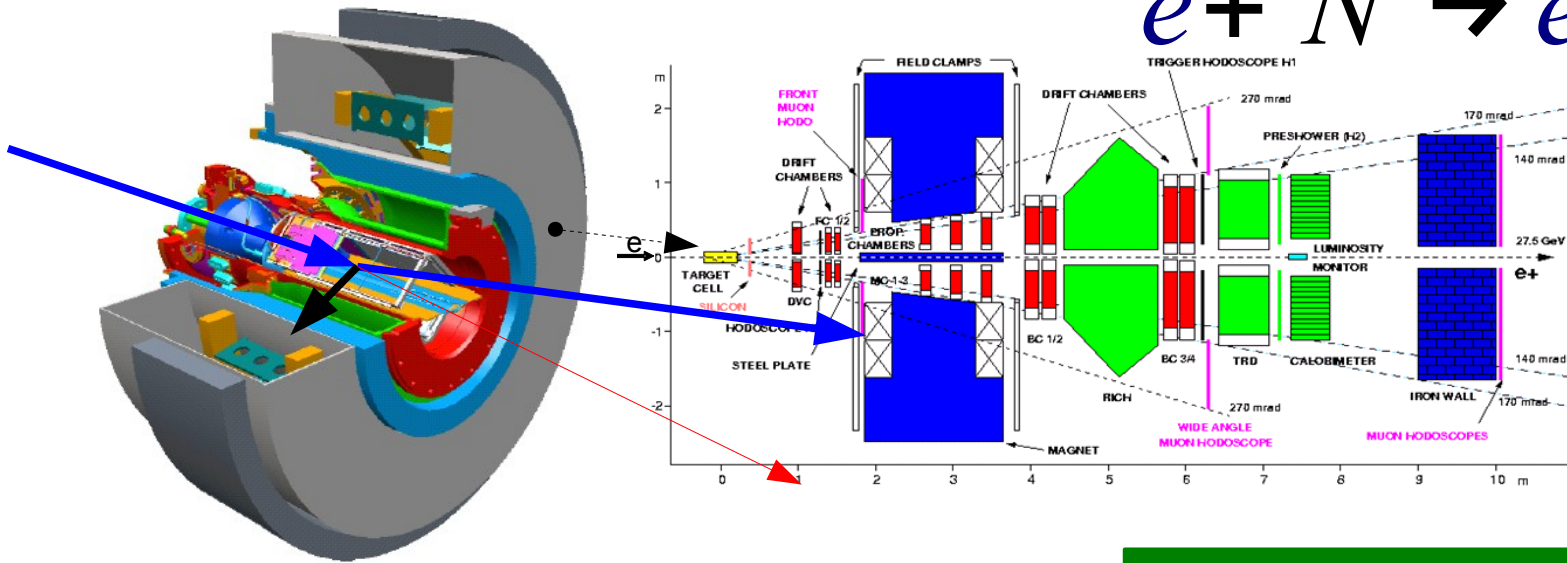
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# Exclusive production with Recoil Detector

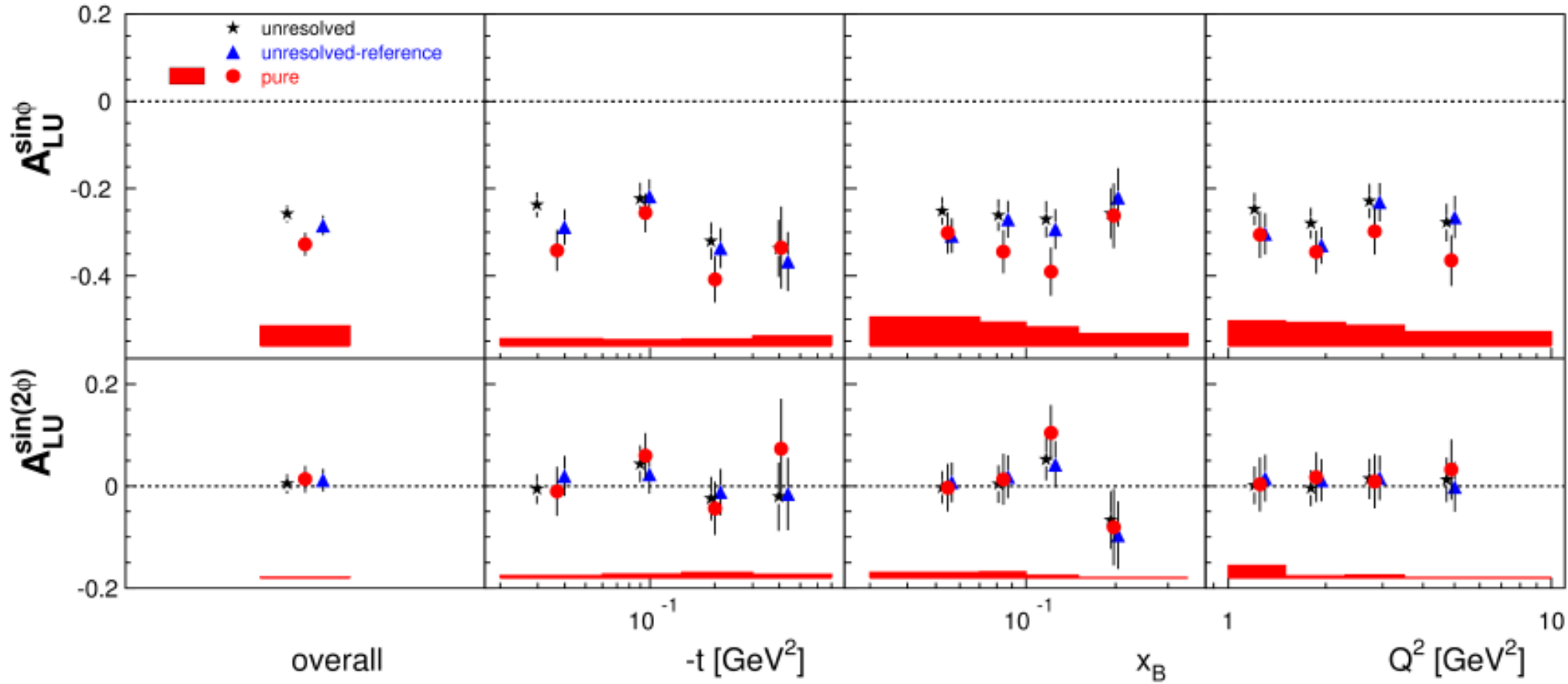
$$e + N \rightarrow e' + N + \gamma$$





# DVCS amplitudes with Recoil Detector at

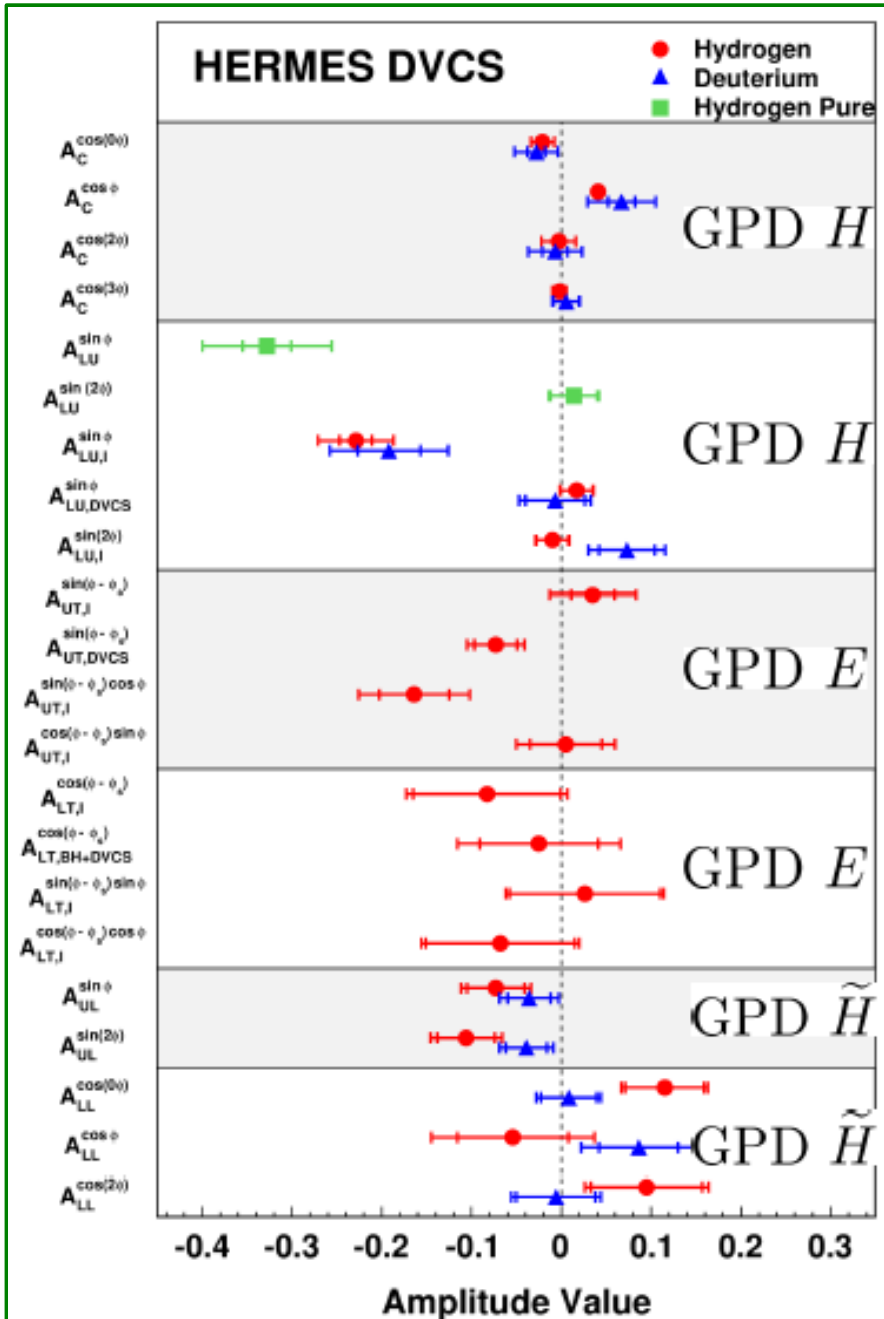
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# DVCS amplitudes measured at HERMES

S. Yaschenko, DIS2013



● Beam-charge and beam-spin asymmetry

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*JHEP 11 (2009) 083*

*JHEP 07 (2012) 032, JHEP 10 (2012) 042*

*Nucl. Phys. B 829 (2010) 1*

With Recoil

● Transverse target-spin asymmetry

*JHEP 06 (2008) 066*

● Transverse double-spin asymmetry

*Phys. Lett. B 704 (2011) 15*

● Longitudinal target spin asymmetry

*JHEP 06 (2010) 019*

● Longitudinal target & double spin asymmetry

*Nucl. Phys. B 842 (2011) 265*



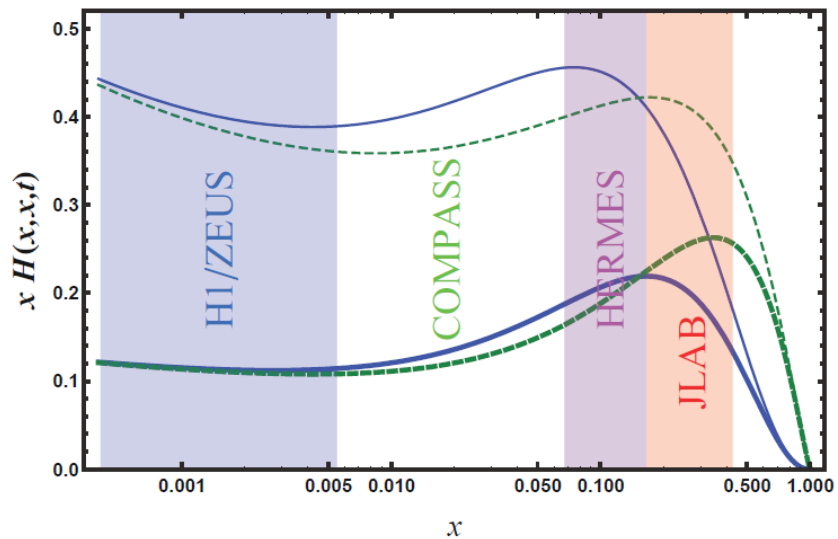
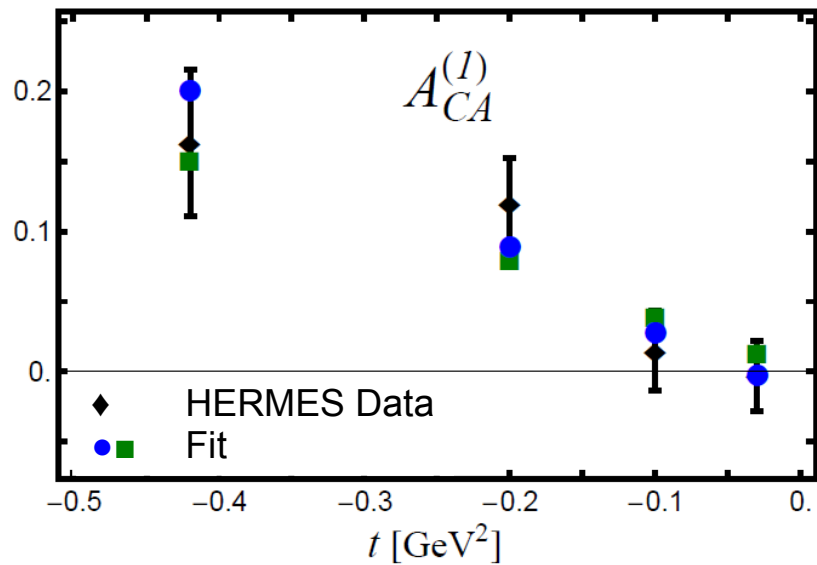




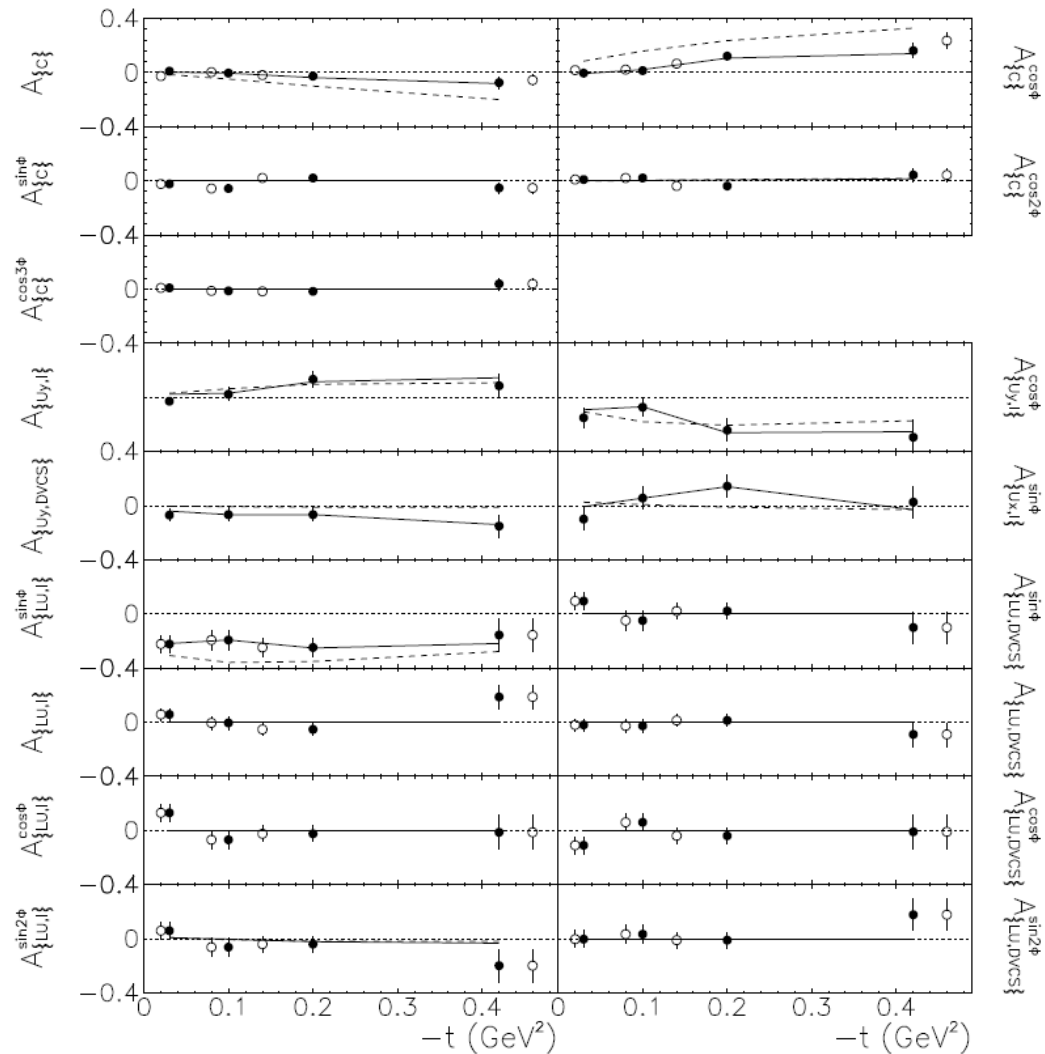
# Fit to the HERMES data

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K. Kumericki and D. Muller, arXiv:0904.0458



M. Guidal and H. Moutarde, arXiv:0905.1220



## New programs (COMPASS II)

approved by CERN Research Board in 2010

- Polarized Drell-Yan measurement

2014

TMD PDFs

$\pi^-$  beam with polarized proton target

- GPD measurement

Transverse imaging

$\mu^+ \mu^-$  beam with liquid hydrogen target

2015 - 2016

- Pion and Kaon polarizability

Chiral perturbation theory

$\pi^-, K^- (\mu^+)$  beam with nucleus target

With a upgraded COMPASS spectrometer



Proton beam from SPS

→ secondary hadrons

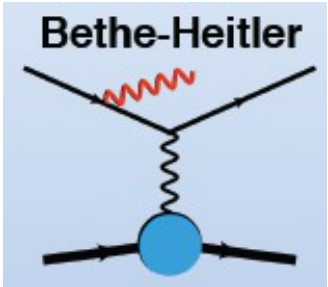
→ weak decay to muon

→ select muon charge & momentum by magnet

→ polarized muons

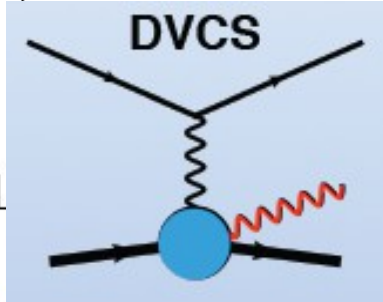
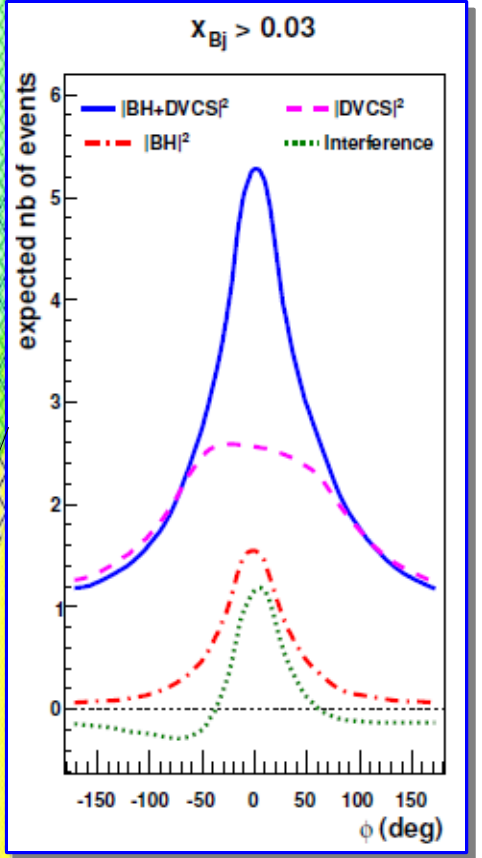
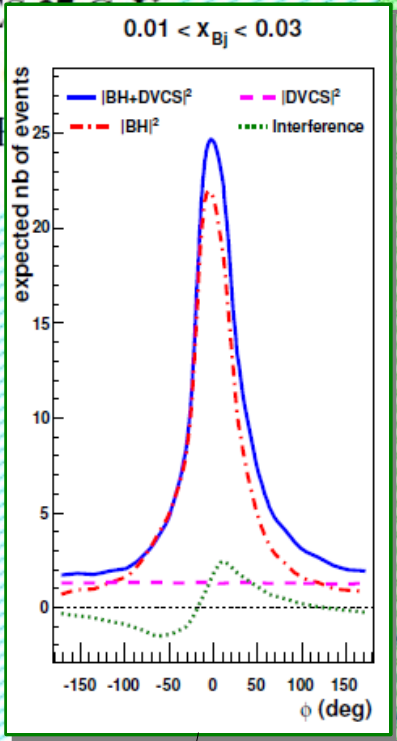
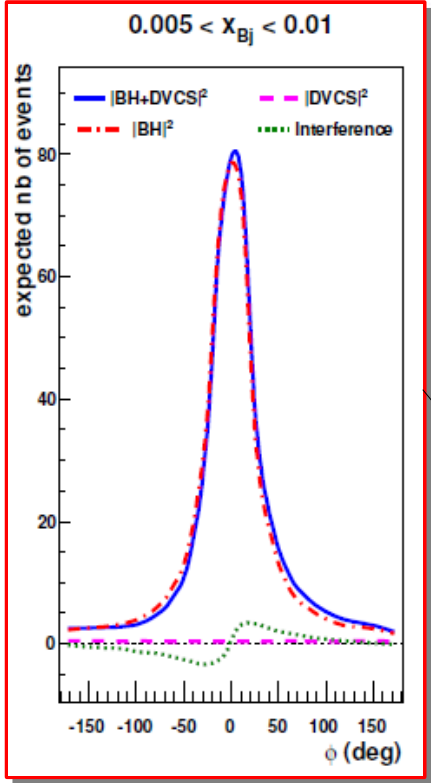
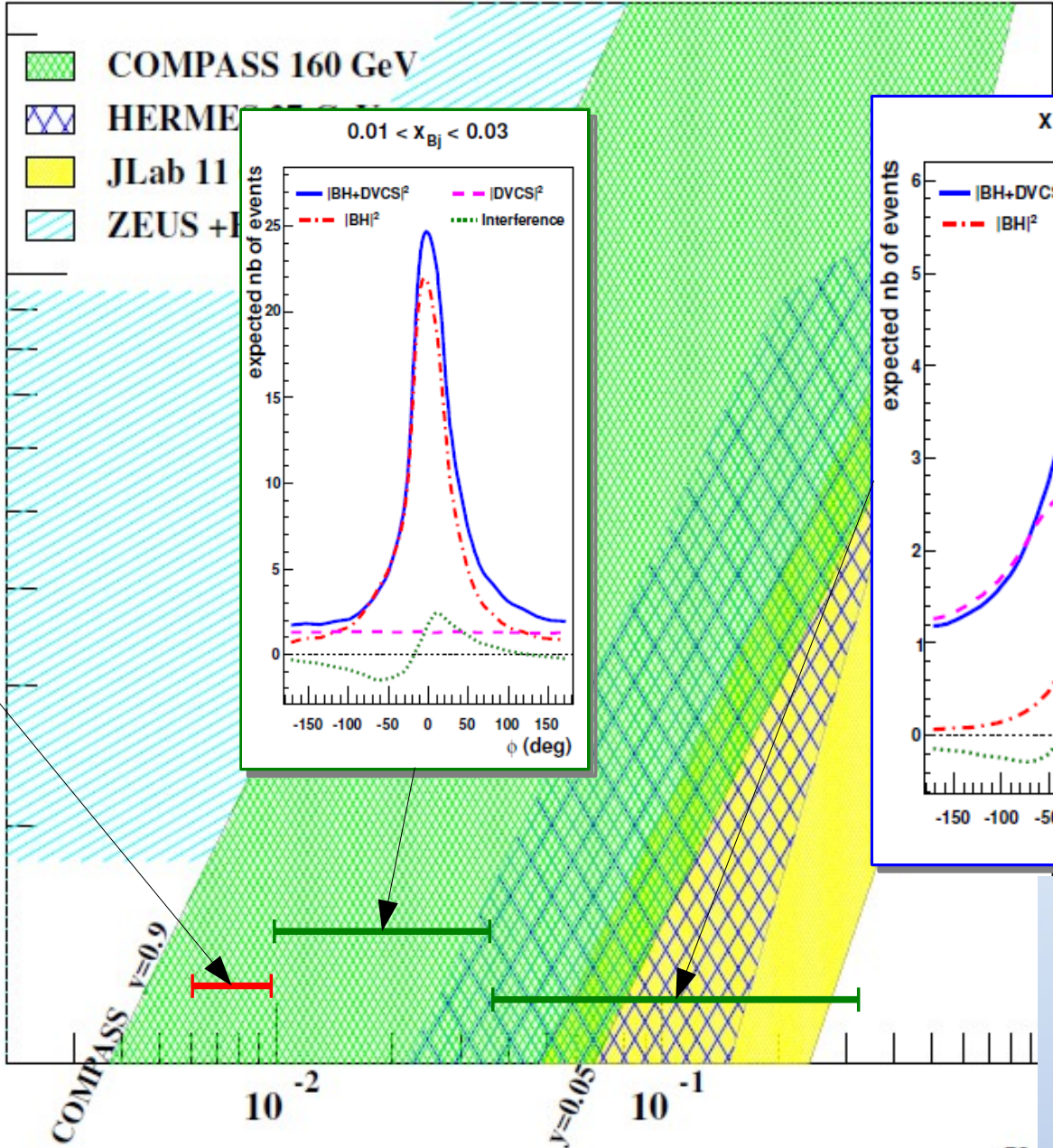


# COMPASS II : GPD



Bethe-Heitler

$Q^2$  (GeV<sup>2</sup>)



DVCS



(1) The beam charge & spin sum of cross sections

$$\mathcal{S}_{CS,U} \equiv d\sigma^{\leftrightarrow+} + d\sigma^{\leftrightarrow-} = 2(d\sigma^{BH} + d\sigma_{unpol}^{DVCS} + e_{\mu} P_{\mu} \text{Im } I),$$

(2) The beam charge & spin difference of cross sections

$$\mathcal{D}_{CS,U} \equiv d\sigma^{\leftrightarrow+} - d\sigma^{\leftrightarrow-} = 2(P_{\mu} d\sigma_{pol}^{DVCS} + e_{\mu} \text{Re } I),$$

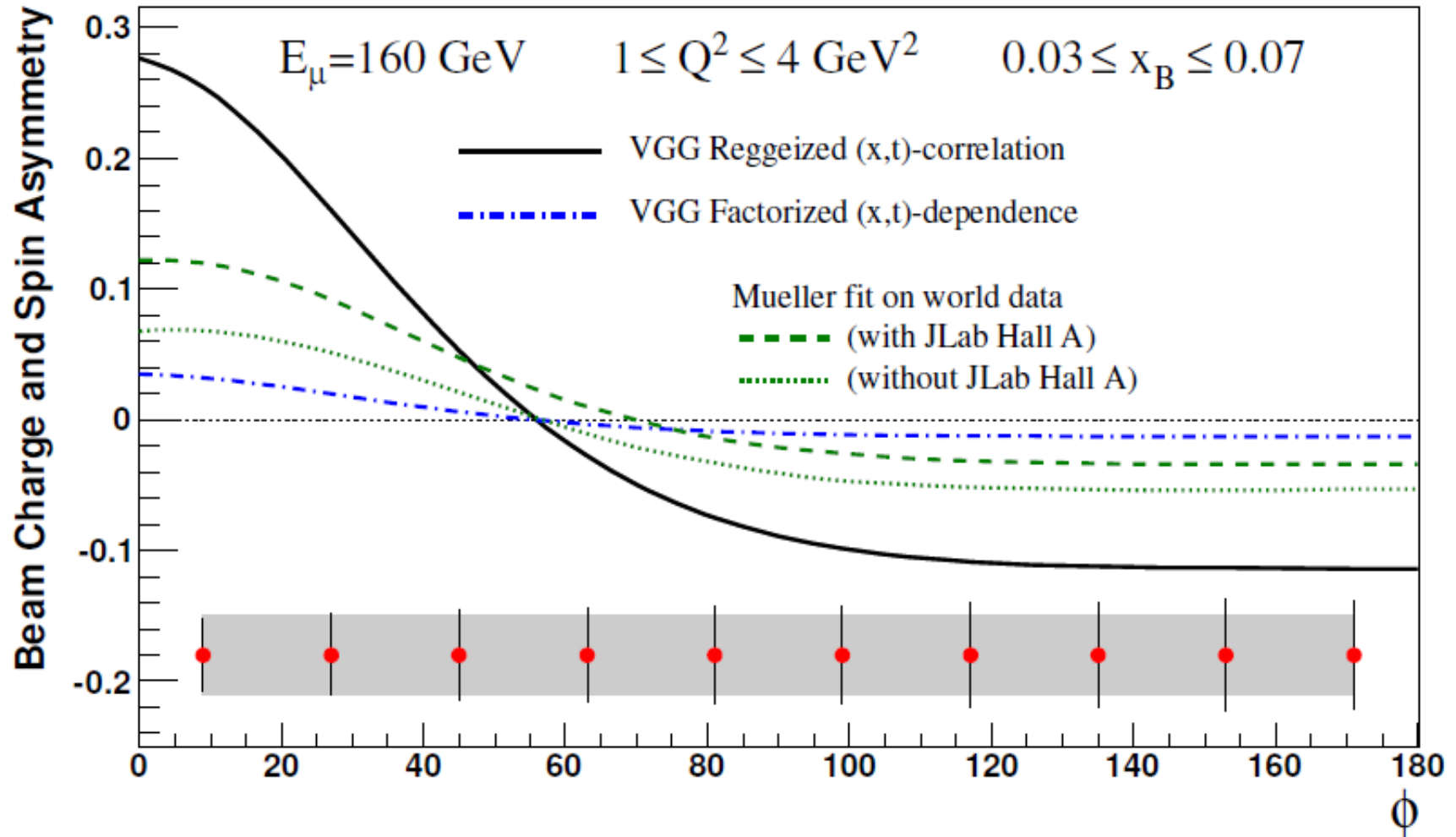
(3) The beam charge & spin asymmetry of cross sections

$$\mathcal{A}_{CS,U} \equiv \frac{d\sigma^{\leftrightarrow+} - d\sigma^{\leftrightarrow-}}{d\sigma^{\leftrightarrow+} + d\sigma^{\leftrightarrow-}} = \frac{\mathcal{D}_{CS,U}}{\mathcal{S}_{CS,U}},$$



# COMPASS II: GPD, expected asymmetry

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$$c_1^I \propto \text{Re} \left( F_1 \mathcal{H} + \xi (F_1 + F_2) \tilde{\mathcal{H}} - \frac{t}{4M^2} F_2 \mathcal{E} \right)$$

SF  
A. Ferrero

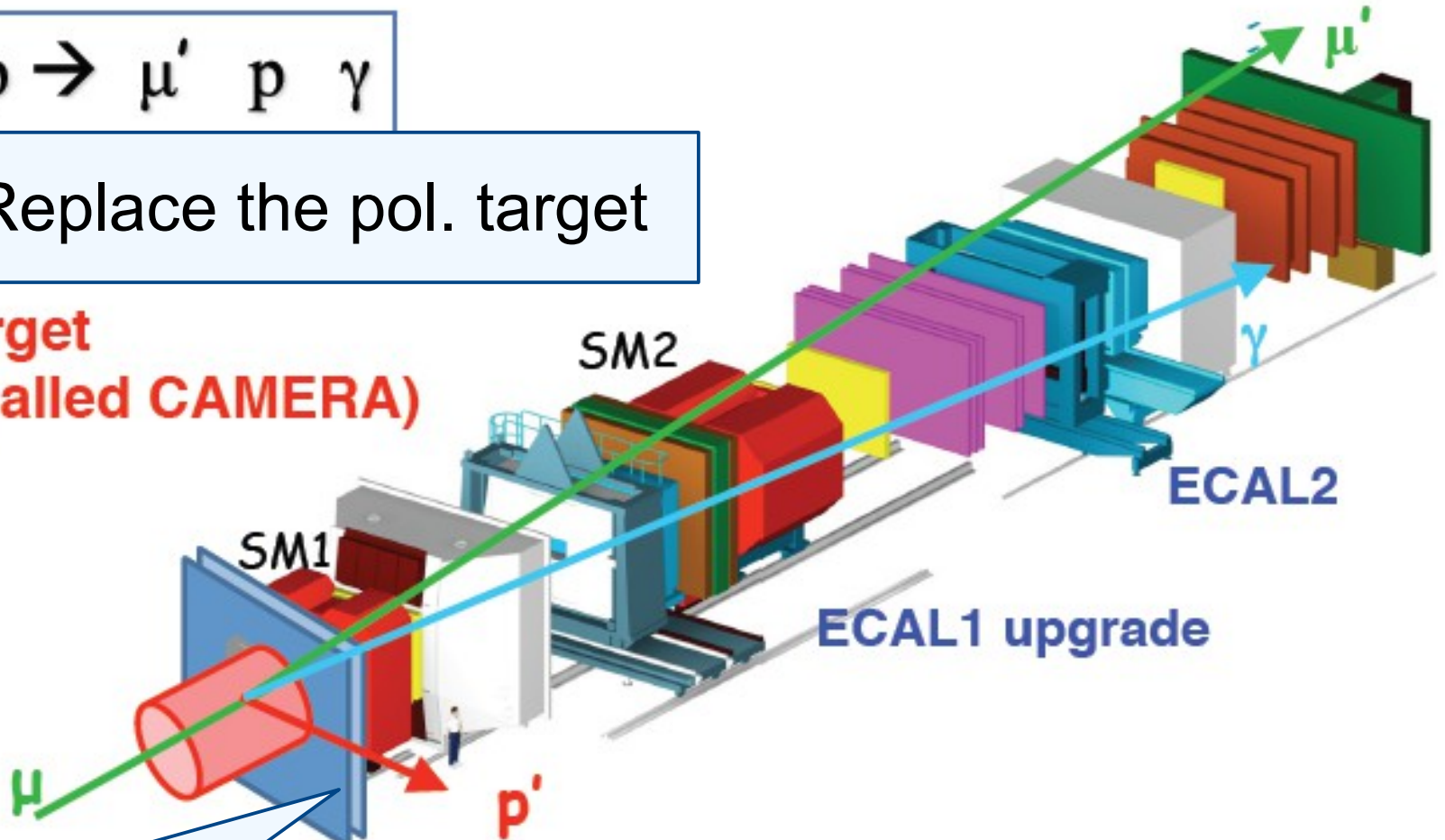


# COMPASS II: GPD setup

$$\mu p \rightarrow \mu' p \gamma$$

Replace the pol. target

2.5m LH2 target  
+ 4m RPD (called CAMERA)



Detect recoiled  $p$

+ ECAL0 before SM1  
(for higher acceptance in large  $X_B$ )



# COMPASS II: GPD 2012 “short” RUN

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Vacuum chamber for  
Liq. H target  
was tested at Yamagata

