Properties of nuclear masses for heavy and superheavy nuclei

Hiroyuki KOURA
Advanced Science Research Center,
Japan Atomic Energy Agency (JAEA)

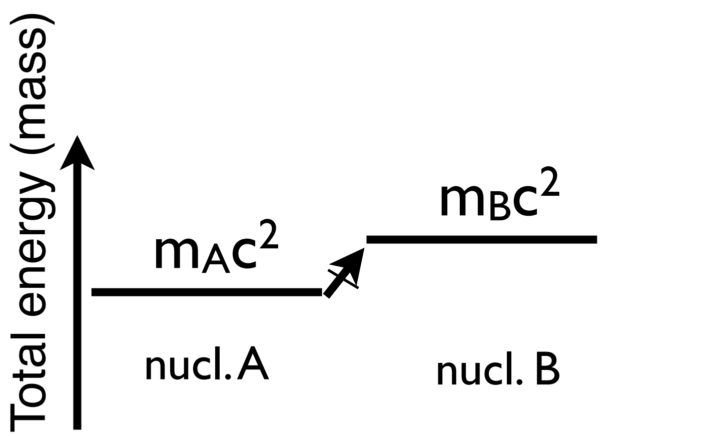
I. Introduction

Why nuclear mass?

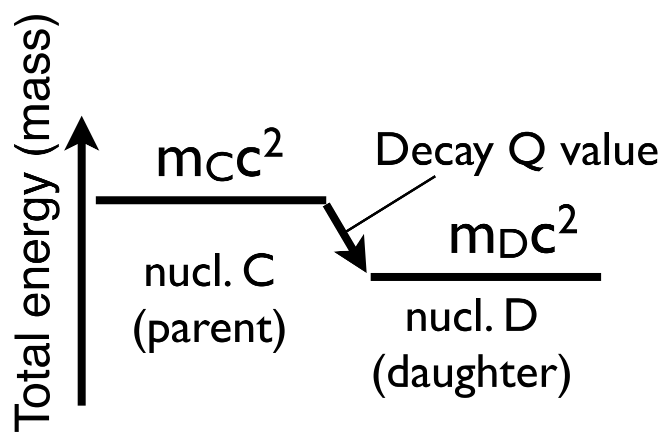
- Equivalence to **total energy** of nucleus: $E = mc^2$

  ➡ Governing nuclear reaction and decay modes

E=mc²

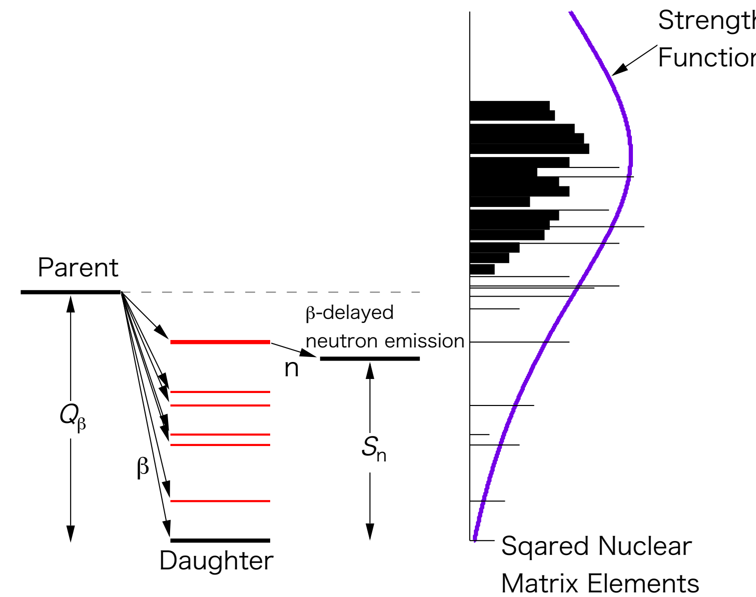


Nucleus A can not decay.



Nucleus C can decay.

Diff. of mass(total energy) determine the direction of nuclear decay.

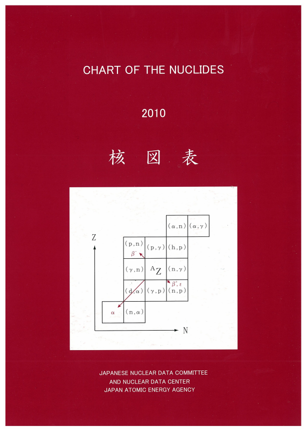


$$\lambda = \frac{1}{2\pi^3}\int_{-Q}^{0}\sum_{\Omega}|g_\Omega|^2 \cdot |M_\Omega(E_g)|^2 f(-E_g+1)dE_g$$

Decay rate of beta-decay

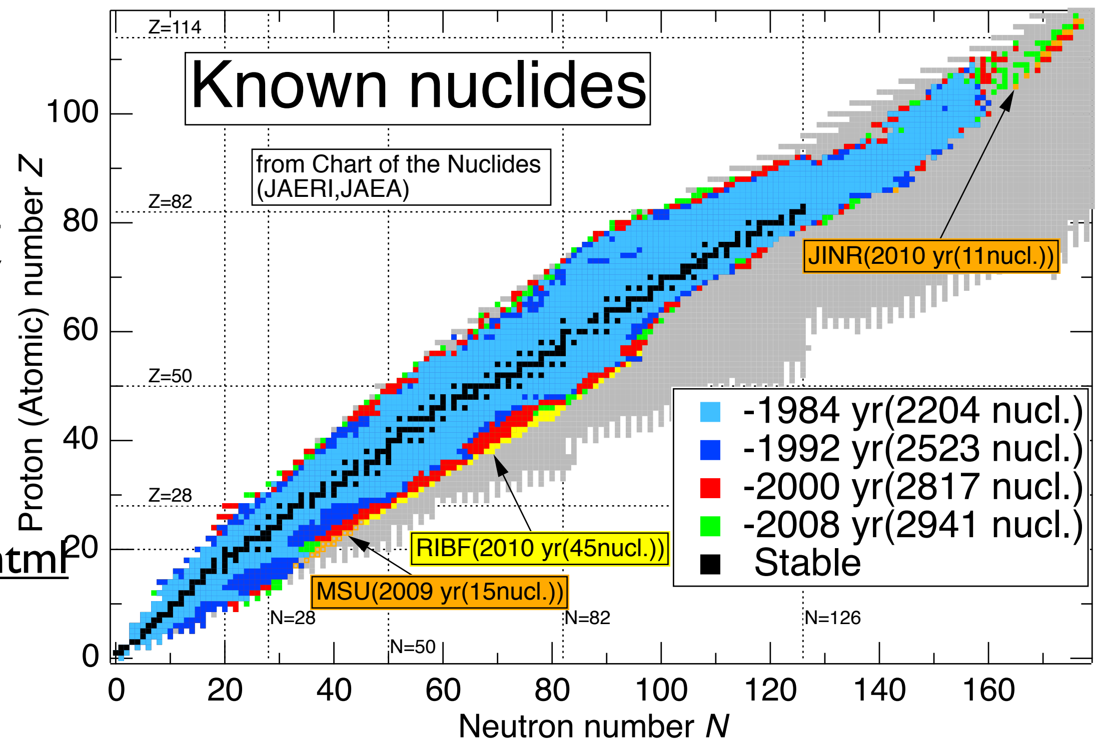
Mass-measured nuclei: current understandings

Identified



taken from Chart of the nuclides by JAERI and JAEA (Tachibana, HK, Katakura, 2010)

wwwndc.jaea.go.jp/CN10/index.html



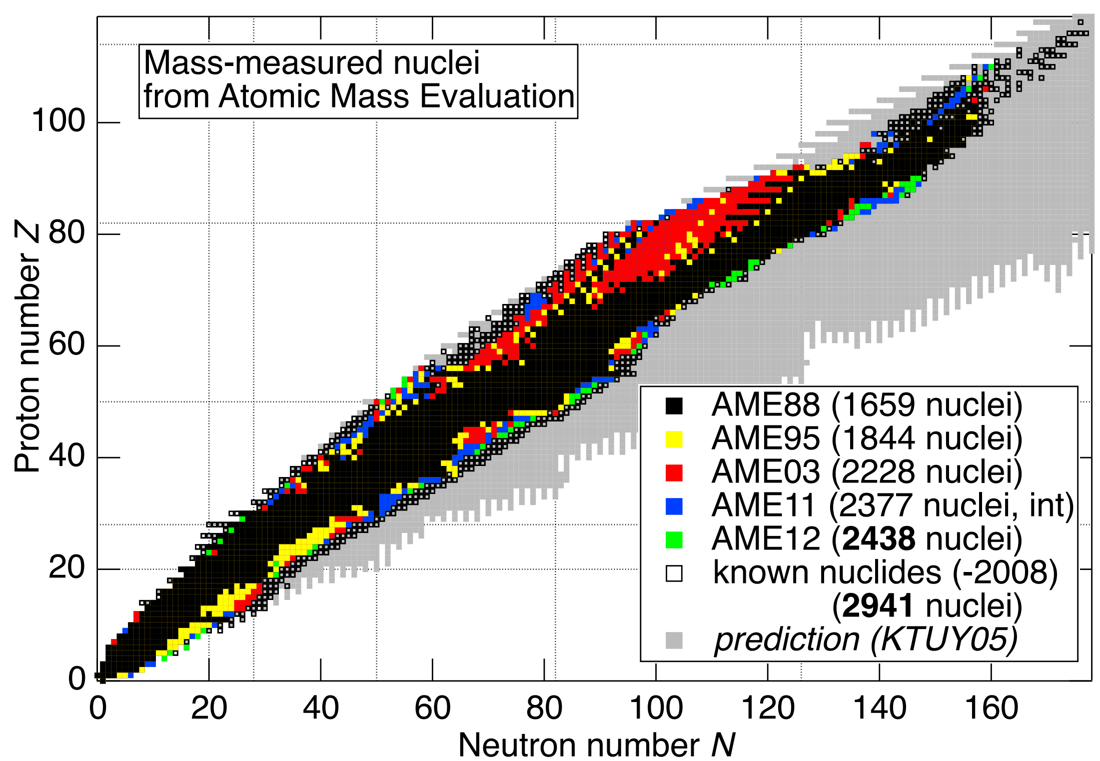
~3000 nuclei

Mass-measured

AMDC
Atomic Mass Data Center

AME2012 is updated last year!

amdc.in2p3.fr/mastables/filel.html



~2400 nuclei

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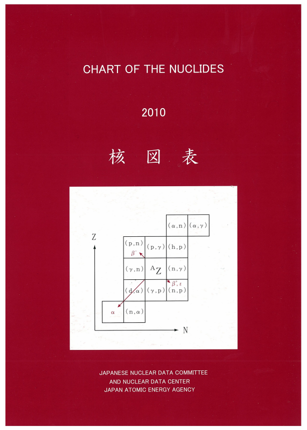

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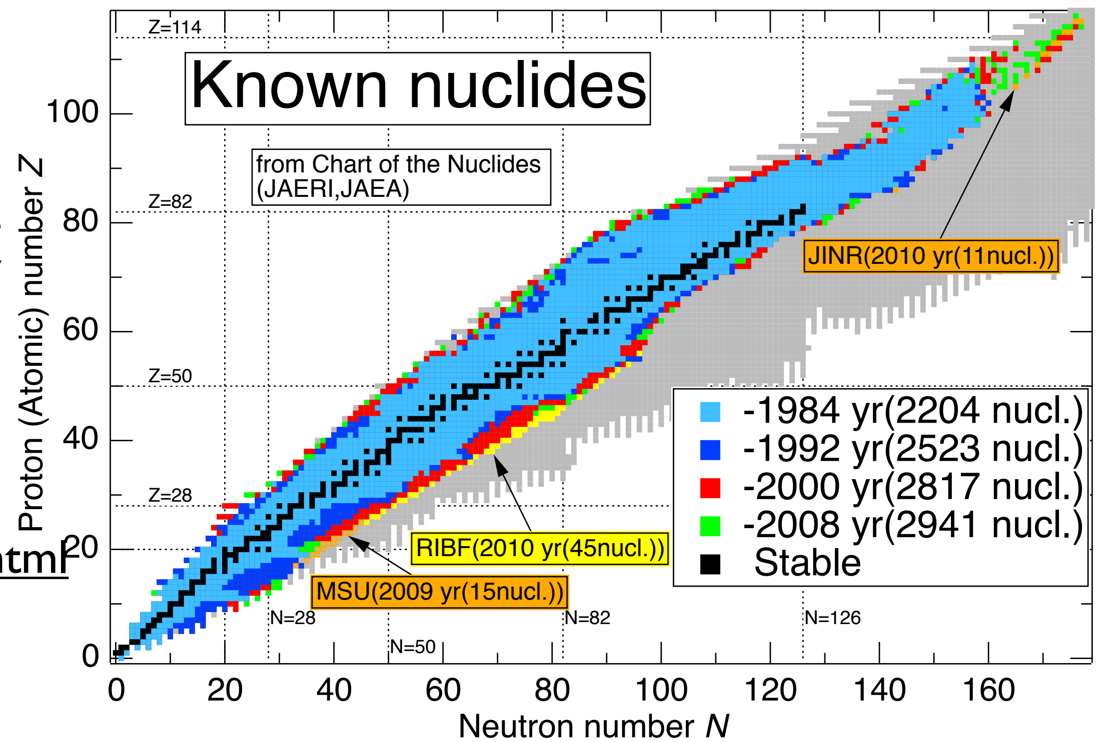

We know now

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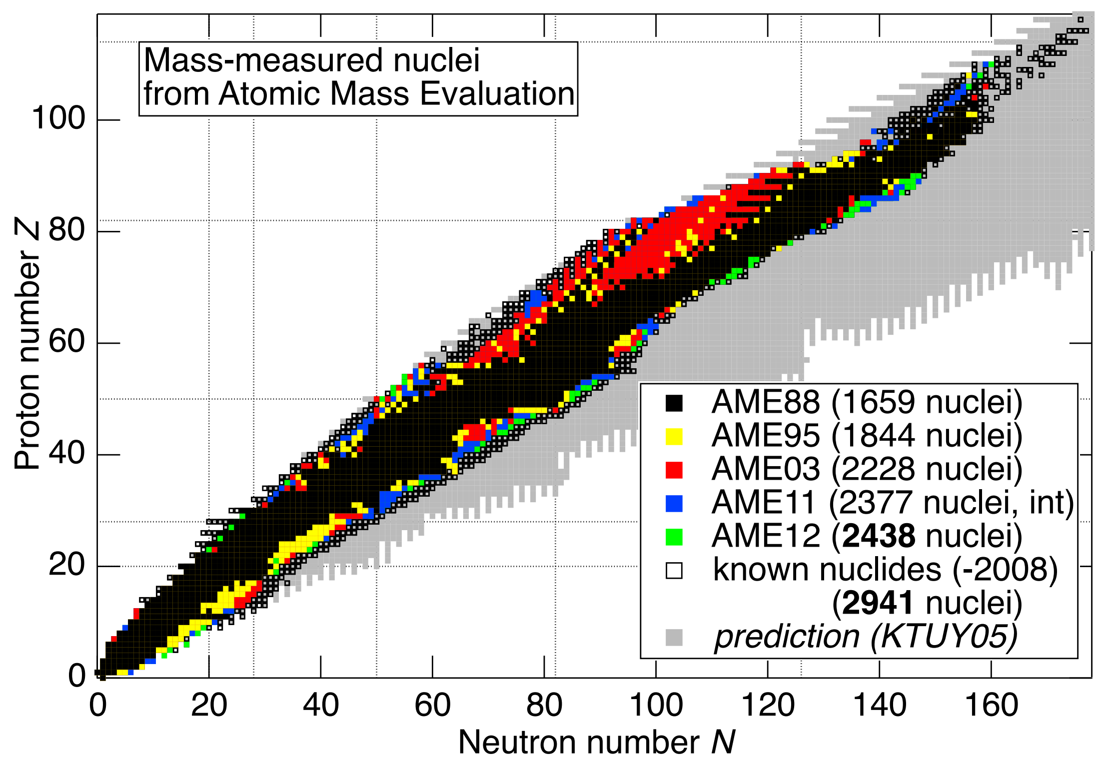

We know now masses of

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Mass-measured nuclei: current understandings

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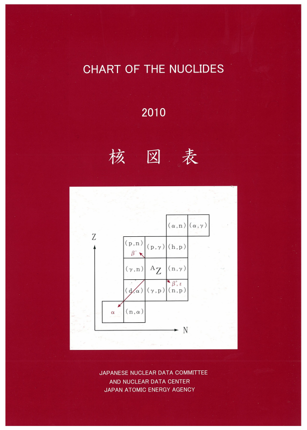

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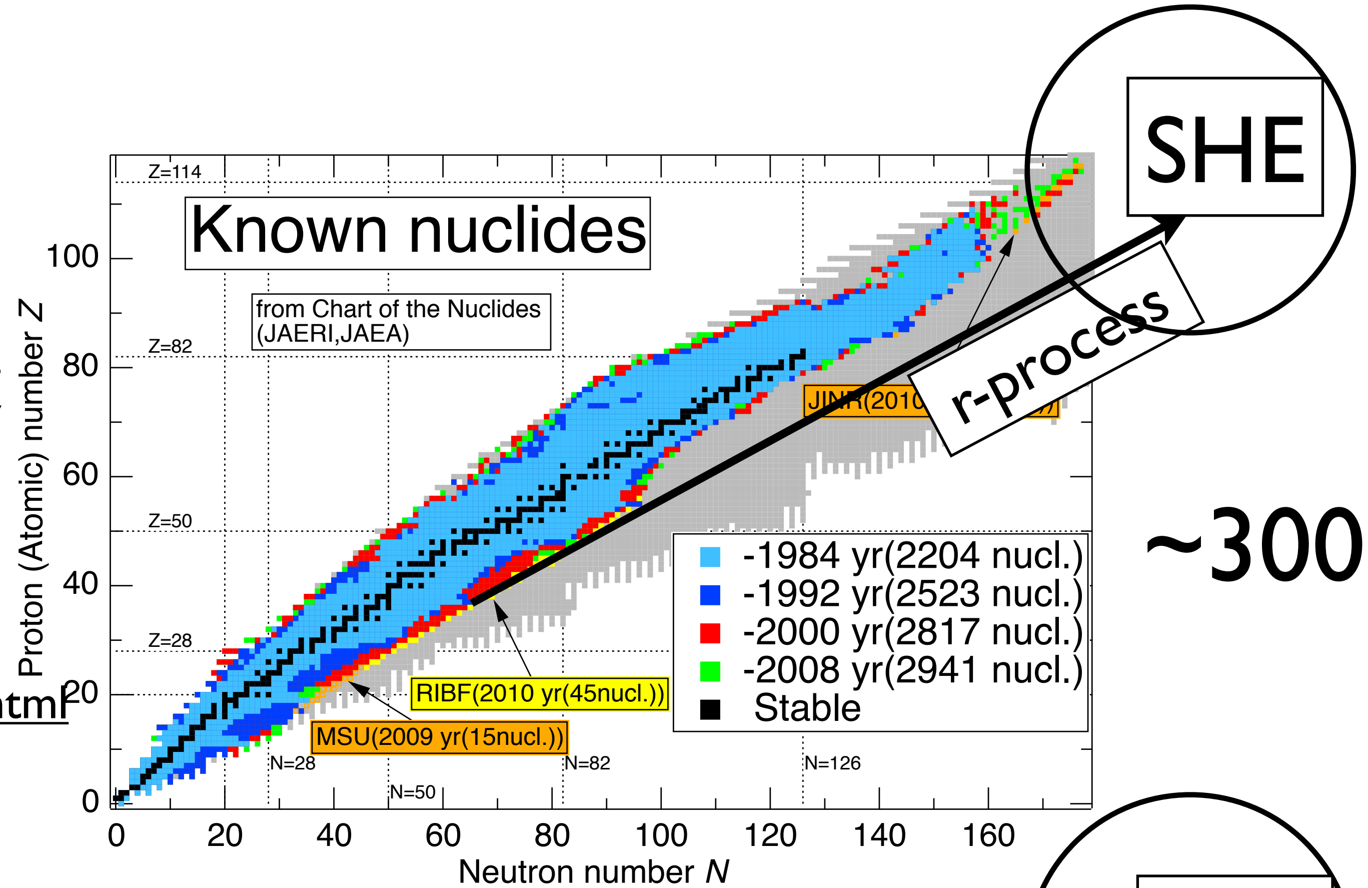

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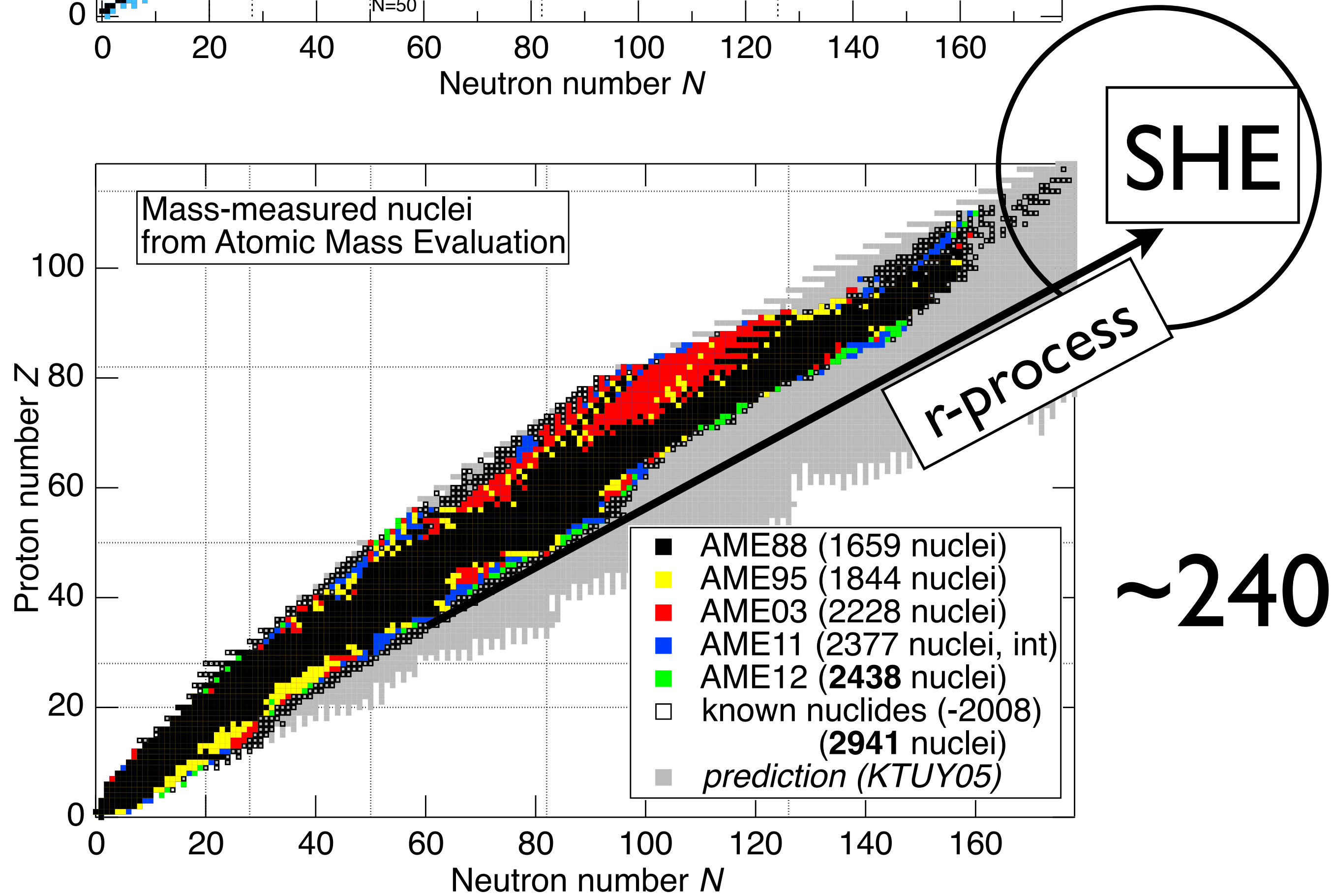


We know now masses of ~2400 nuclei

Importance of mass prediction : drip line

First measurement of ground-state di-proton decay (from ^{45}Fe)

VOLUME 89, NUMBER 10 PHYSICAL REV

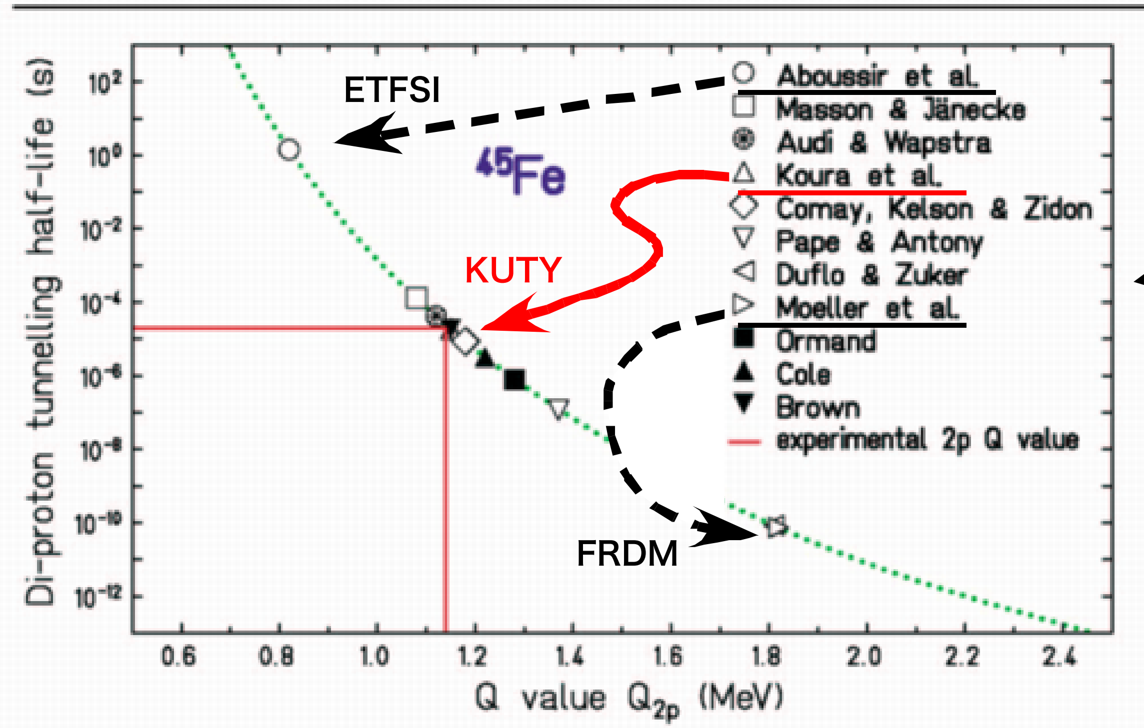
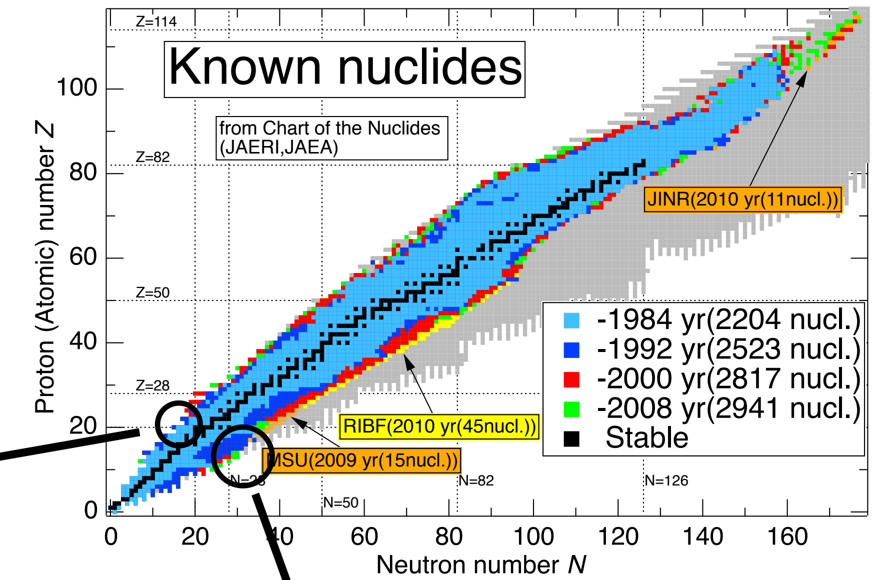


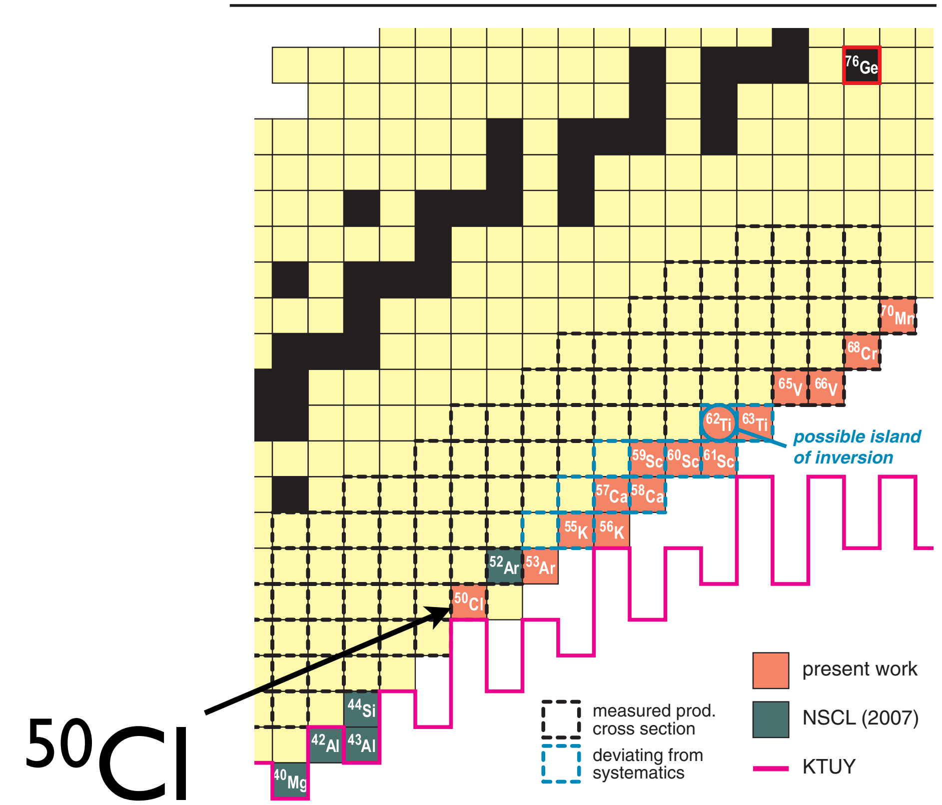
FIG. 5 (color online). Barrier-penetration half-life as a function of the two-proton Q value, Q_{2p}, for ^{45}Fe. The barrier-penetration was calculated by assuming a spectroscopic factor of unity. Different model predictions [5–7,16–21] were used for Q_{2p}. The experimentally observed Q value of ^{45}Fe implies a diproton barrier-penetration half-life of 0.024 ms.

Gioviazzo et al., PRC 89, 2002 (GANIL)
(and also done in GSI by Pfützner(2002))



New isotope ^{50}Cl etc. near the n-drip line

PRL 102, 142501 (2009) PHYSICAL R



O.V. Tarsov, et al., PRL 102, 2009 (MSU)

II. Bulk properties of nuclear mass

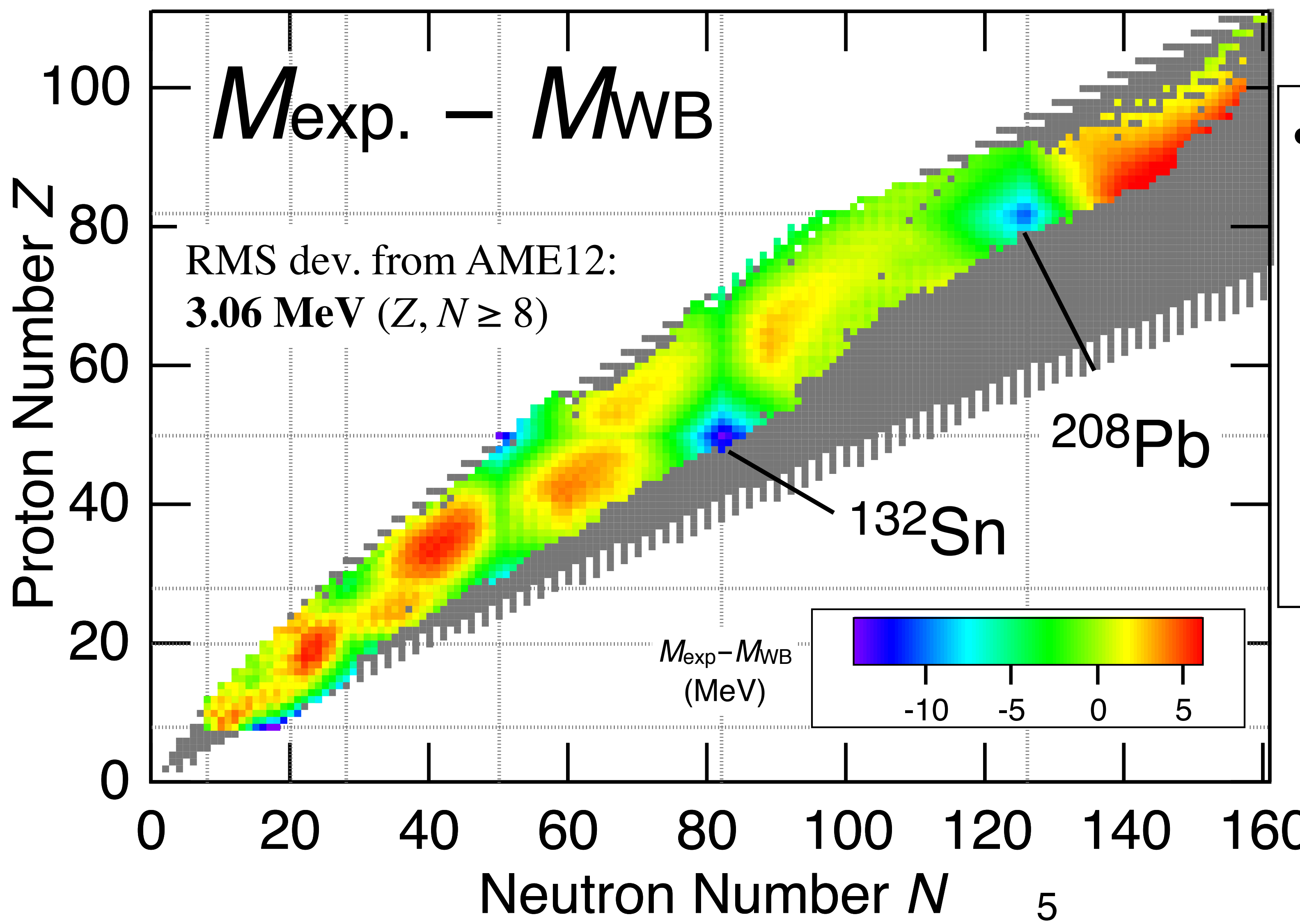
Weizsäcker-Bethe semi-empirical atomic mass formula

$$M_{\rm WB}(Z,N) = Z\,m_{\rm H} + N\,m_{\rm n} - B(Z,N)$$

$$= Z\,m_{\rm H} + N\,m_{\rm n} - a_{\rm V}A + a_{\rm s}A^{2/3} + a_{\rm I}(N-Z)^2/A + a_{\rm C}Z(Z-1)/A^{1/3} + \delta_{\rm eo}$$

$a_{\rm V}$	$a_{\rm s}$	$a_{\rm I}$	$a_{\rm C}$	$a_{\rm eo}$	
15.604	17.472	22.99	0.7073	12.338	(MeV)

$$\delta_{\rm eo} = \begin{cases} -a_{\rm eo}/A^{1/2} & \text{for even-}Z\text{ and even-}N \\ 0 & \text{for odd-}A \\ +a_{\rm eo}/A^{1/2} & \text{for odd-}Z\text{ and odd-}N \end{cases}$$



From the 'exp.' shell energy:

- Existence of magic number
  N=28,50,82,126
  Z=28,50,82

Mass data : 2012 Atomic mass evaluation (M. Wang, G. Audi, A.H. Wapstra *et al.*)

Trend in MeV-order

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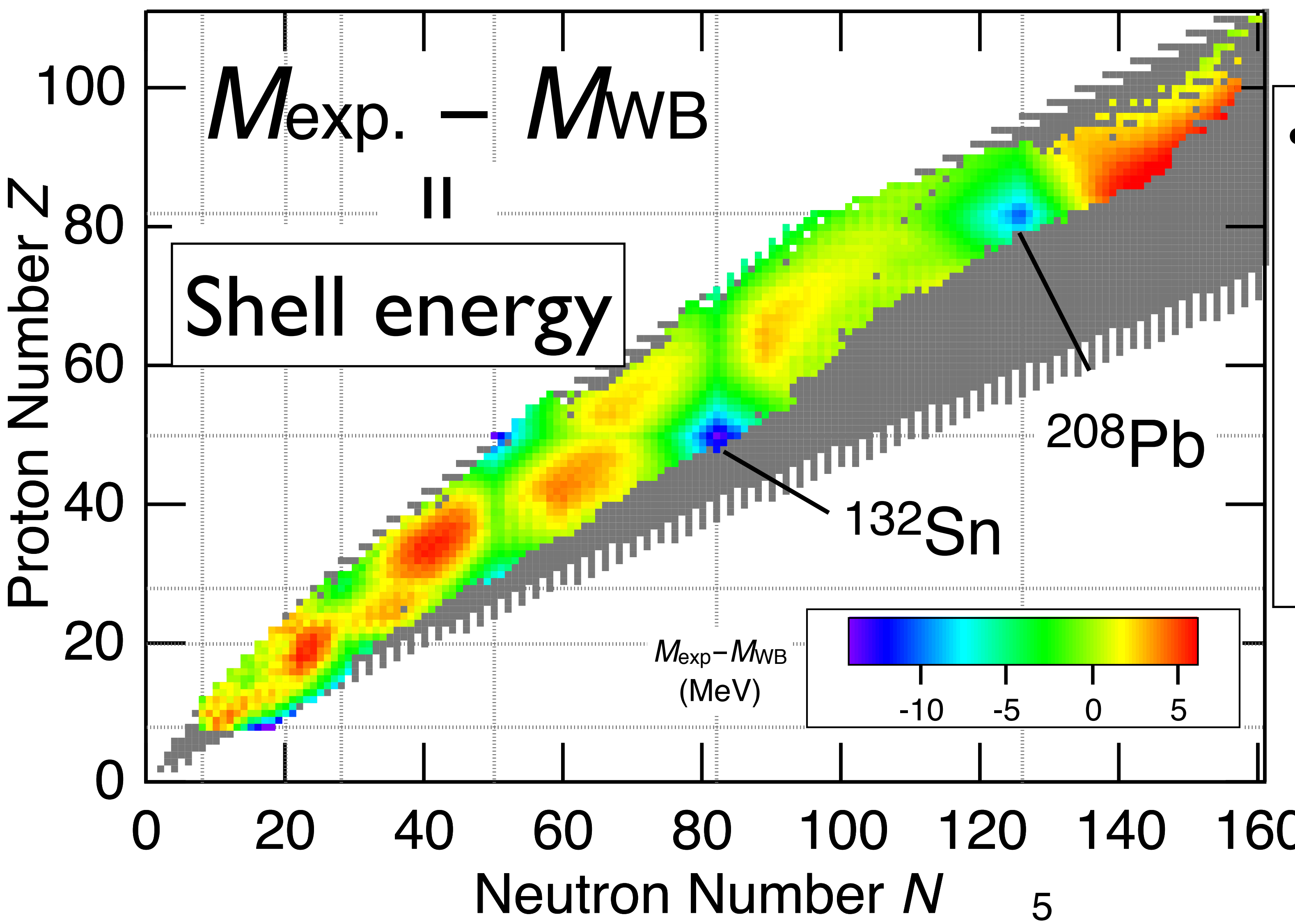

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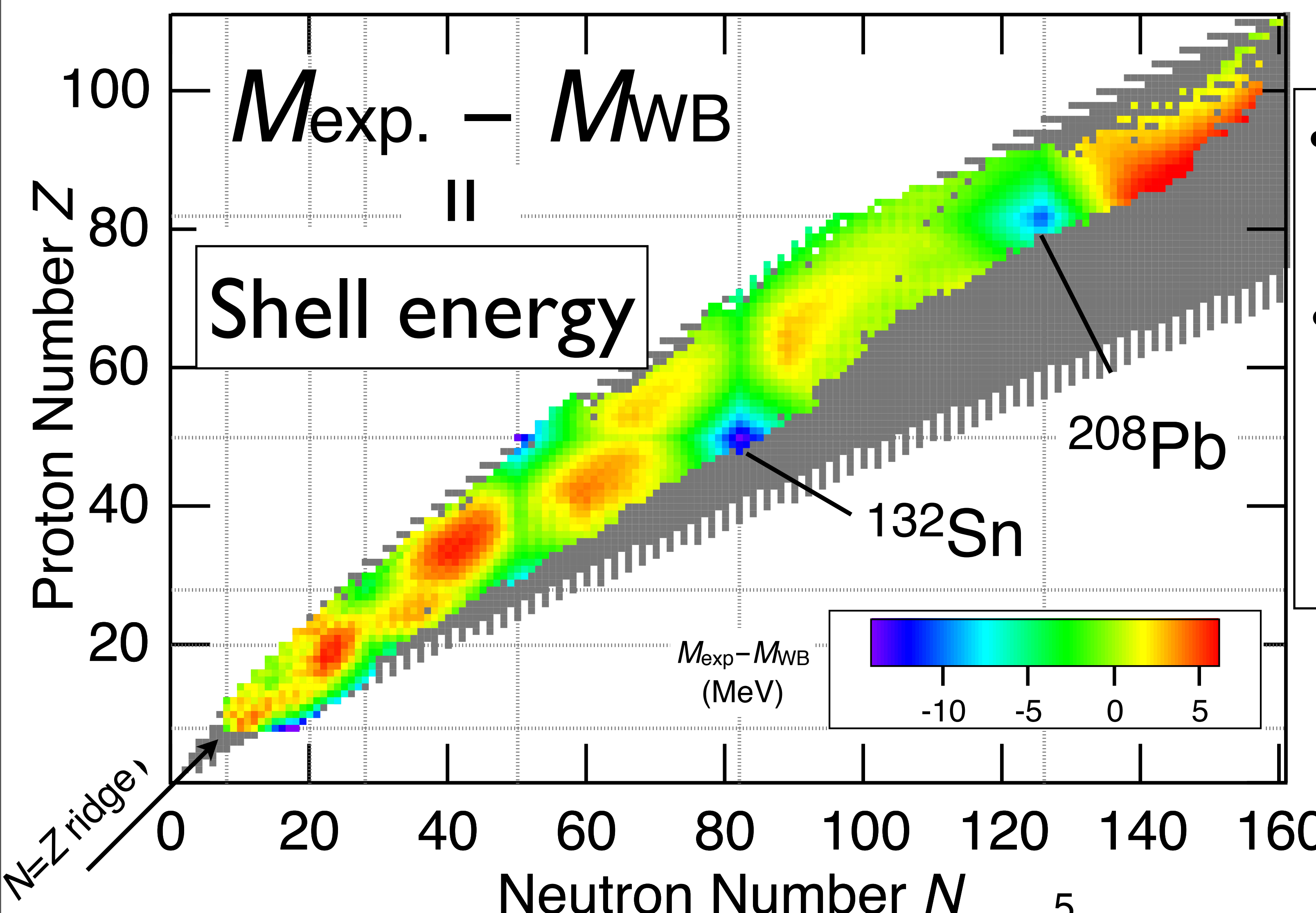

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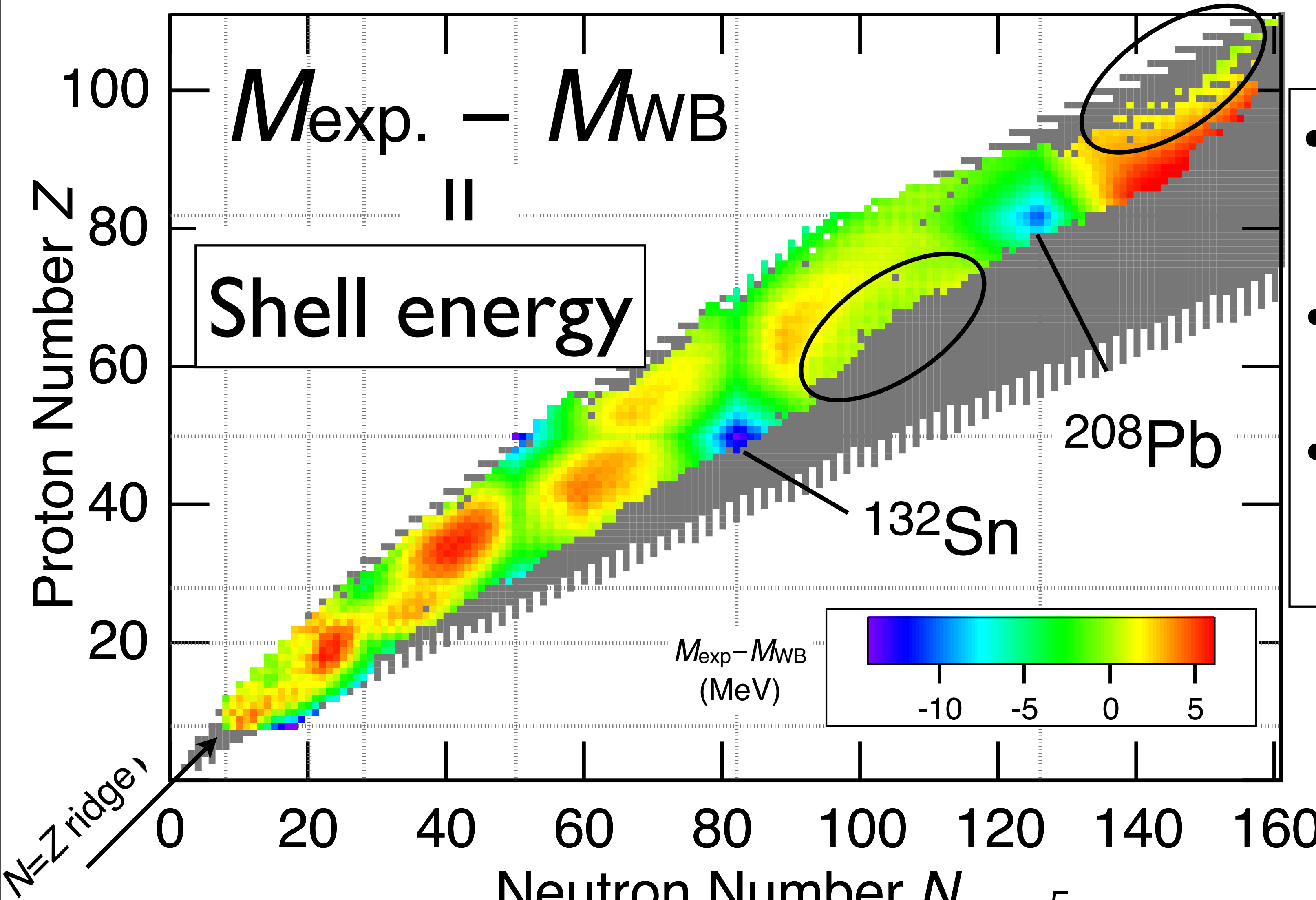

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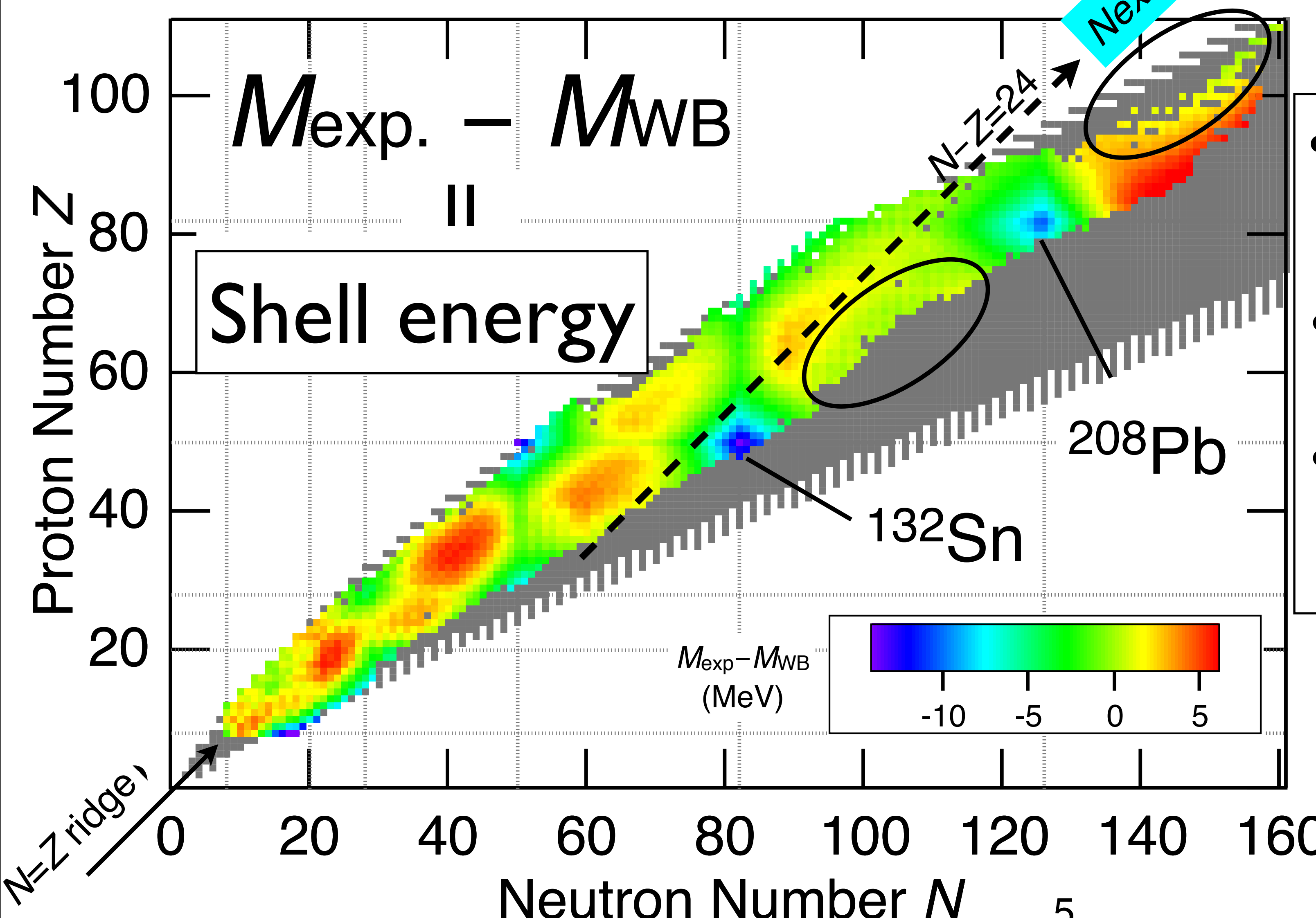

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Trend in MeV-order

Shape transition and shell energies

- Schematic -

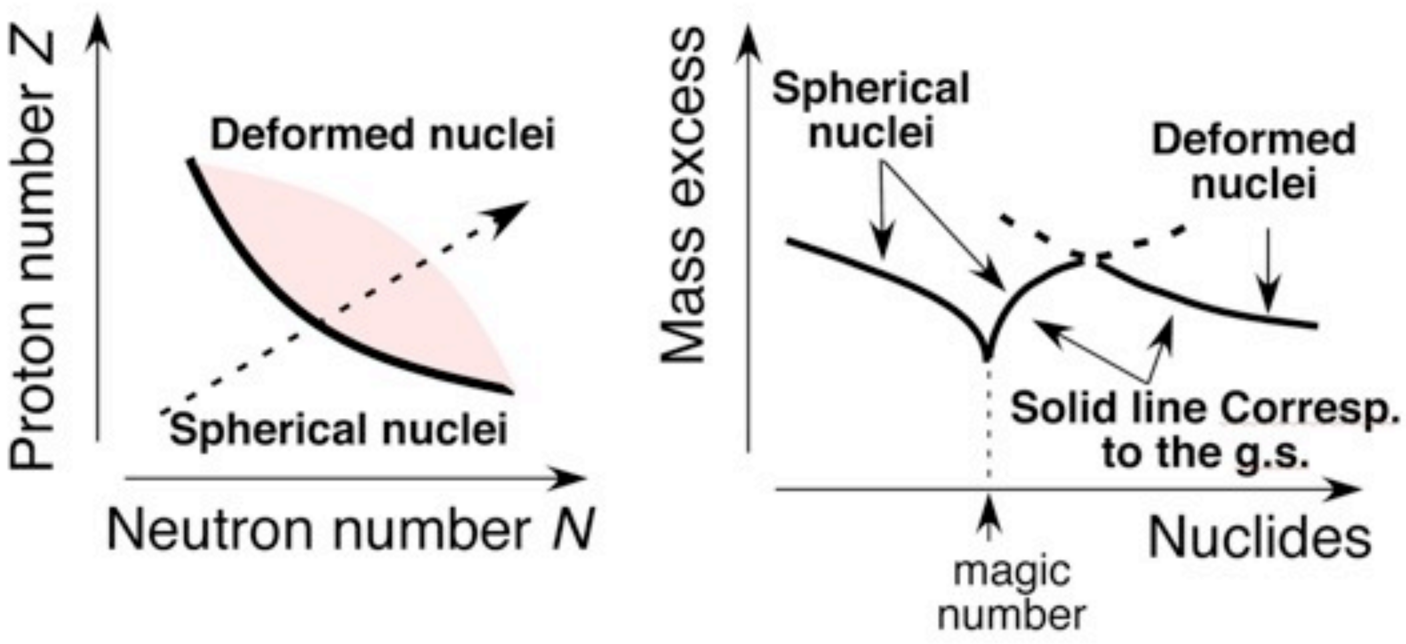
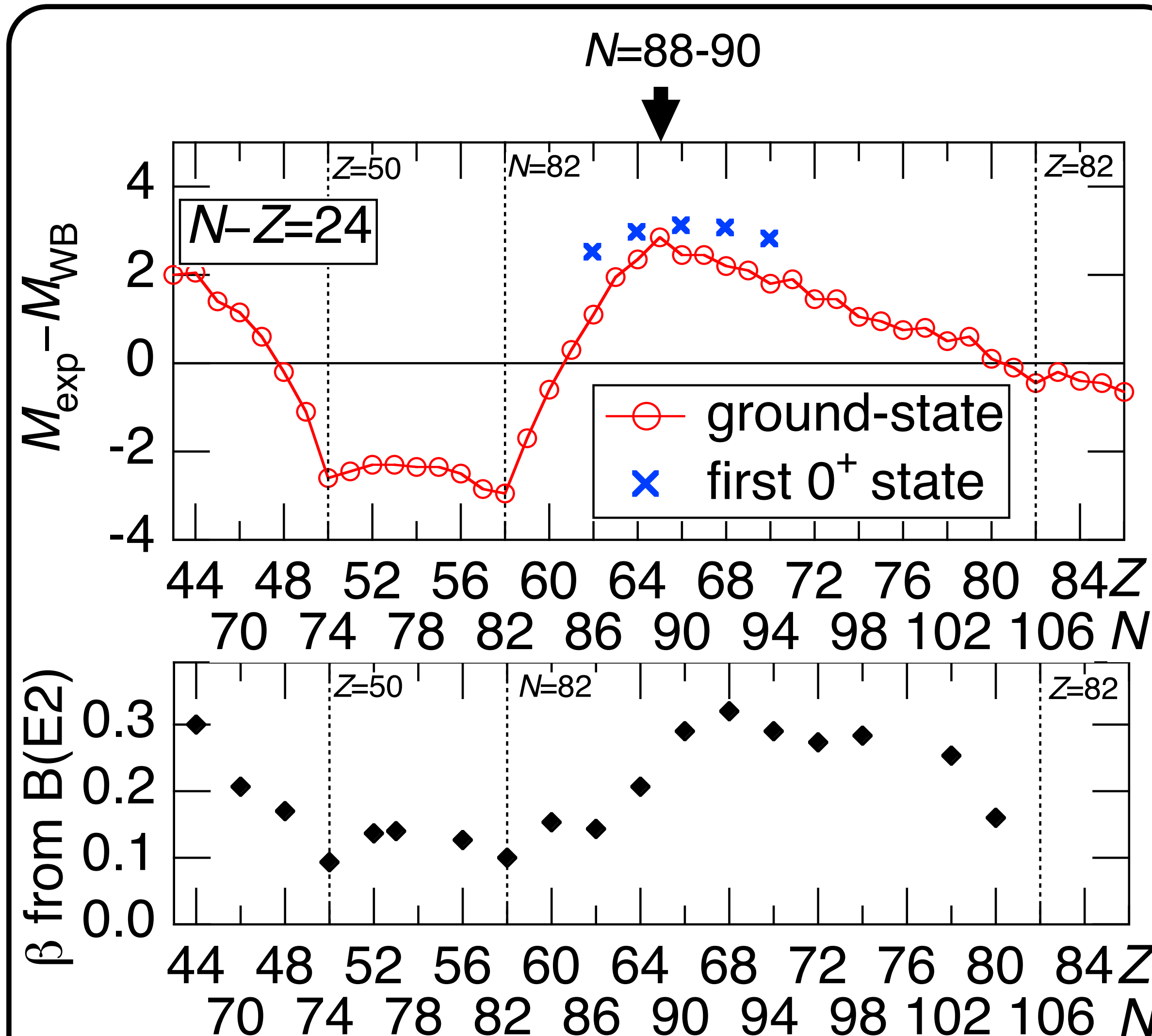


Fig. (a) N-Z plane

Fig. (b) cross section along dashed line in (a)

- Experiment -

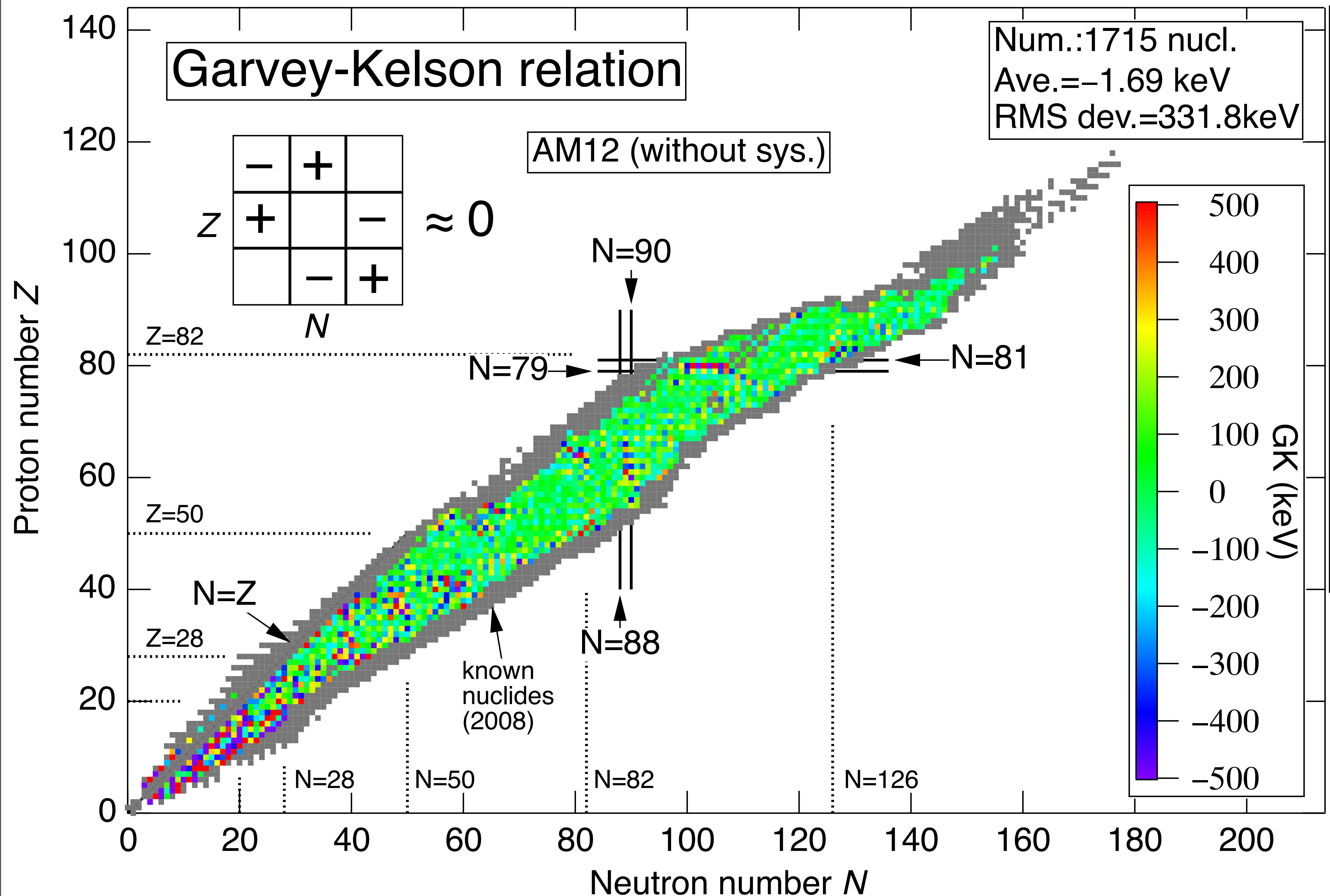


β from B(E2) : S. Raman et al., ADNDT78 (2001)

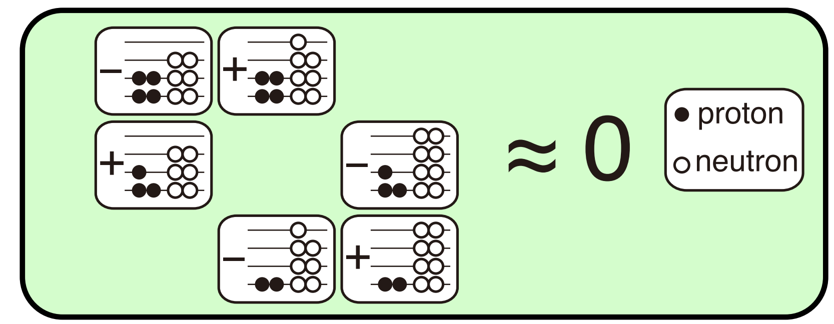
Notable feature on discontinuity of derivative of mass values

- Z=50, N=82 and Z=82 discontinuity of derivative: Spherical single-particle shell closure
- N=88-90 discontinuity: Shape transition

Mass relation: Garvey-Kelson systematics



A consideration of cancellation of core + valence nucleons (based on the shell model)



- Assumption: Cores among related six nuclei are the same.

Region	num.	Average	RMS dev.
All	**1715**	**-1.7 (keV)**	**331.8(keV)**
$A>100$	1202	-0.03	161.2
$A\leq100$	513	-29.3	554.1

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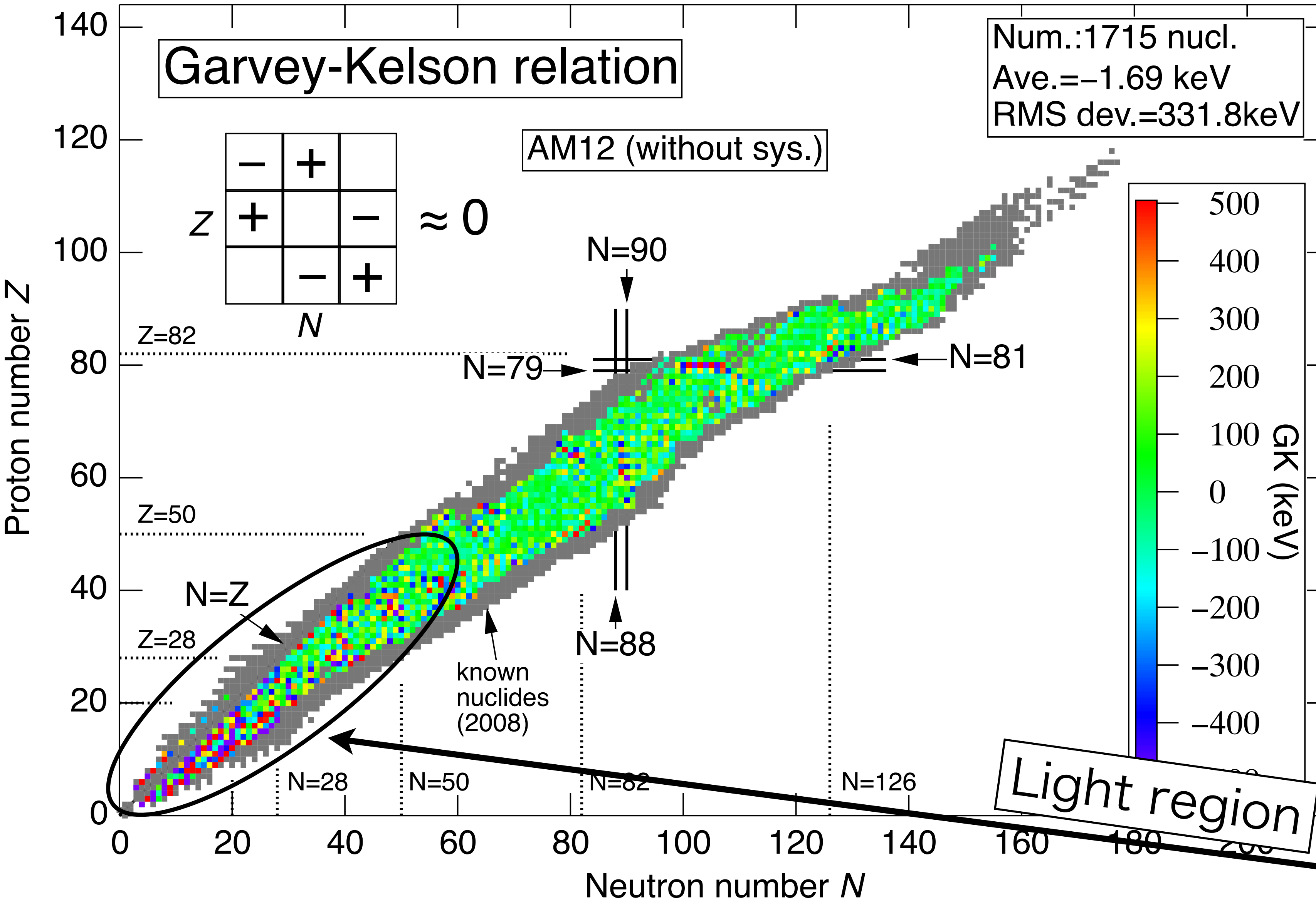

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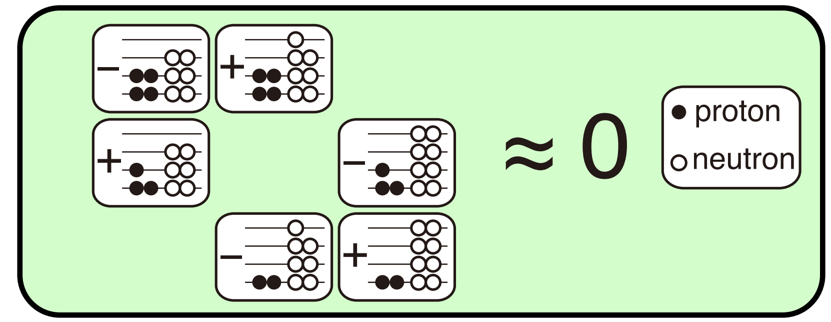

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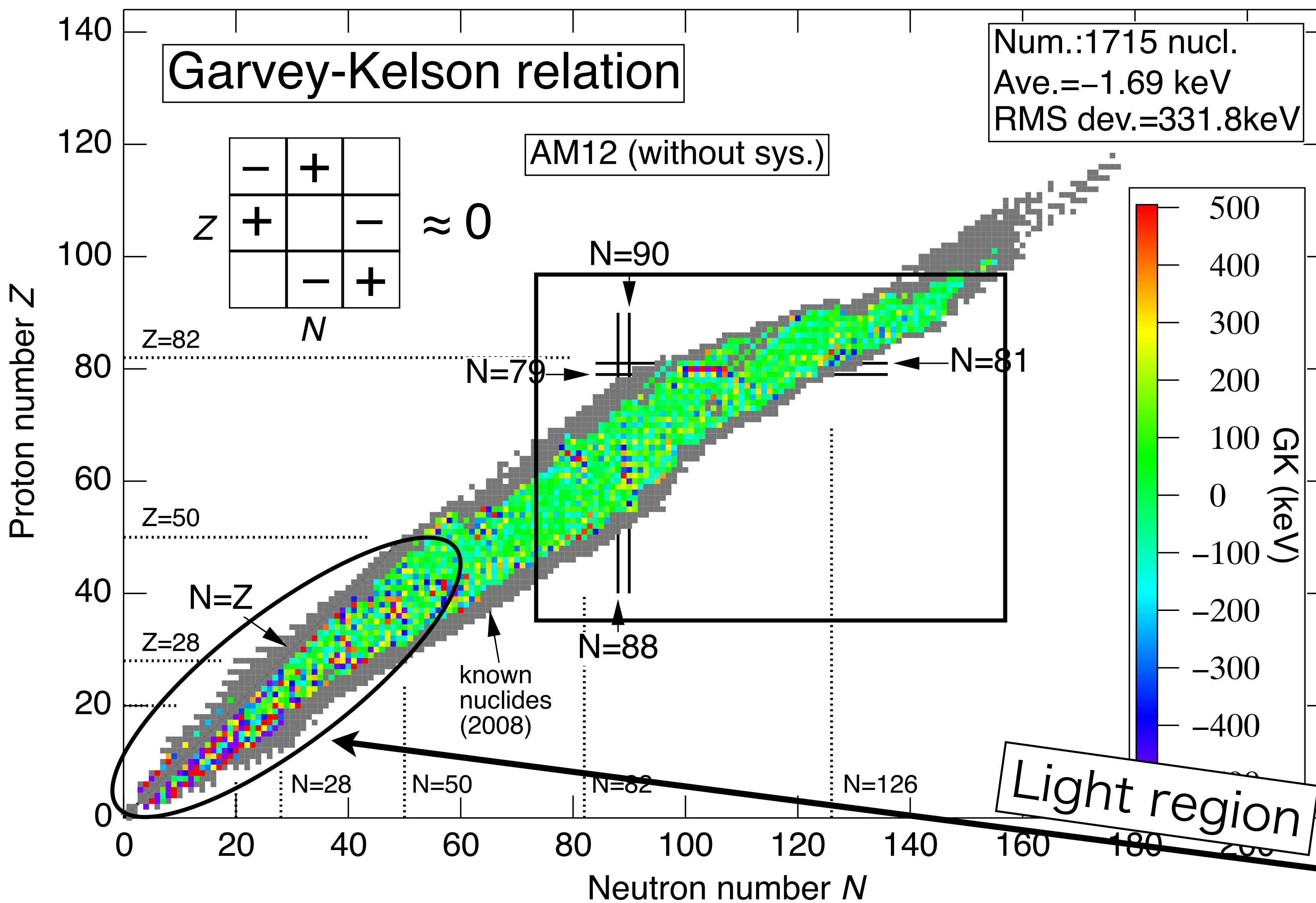

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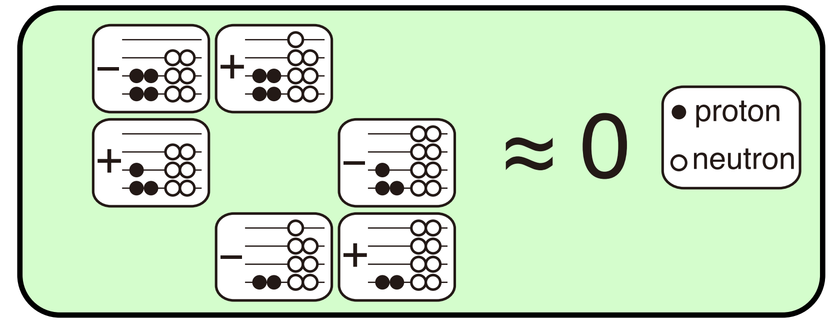

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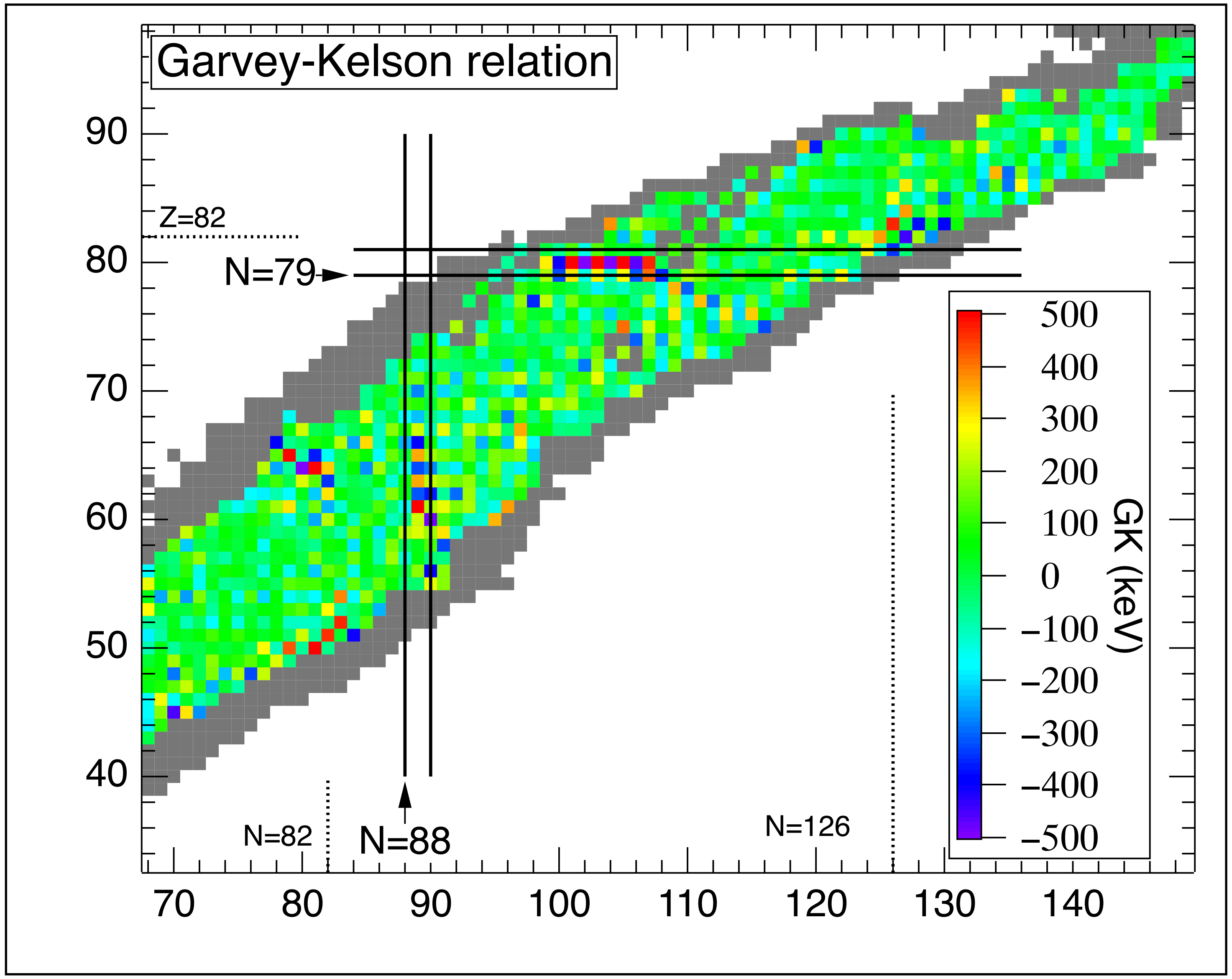

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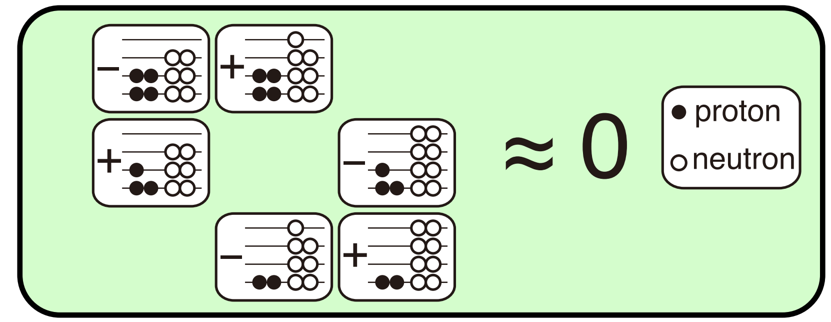

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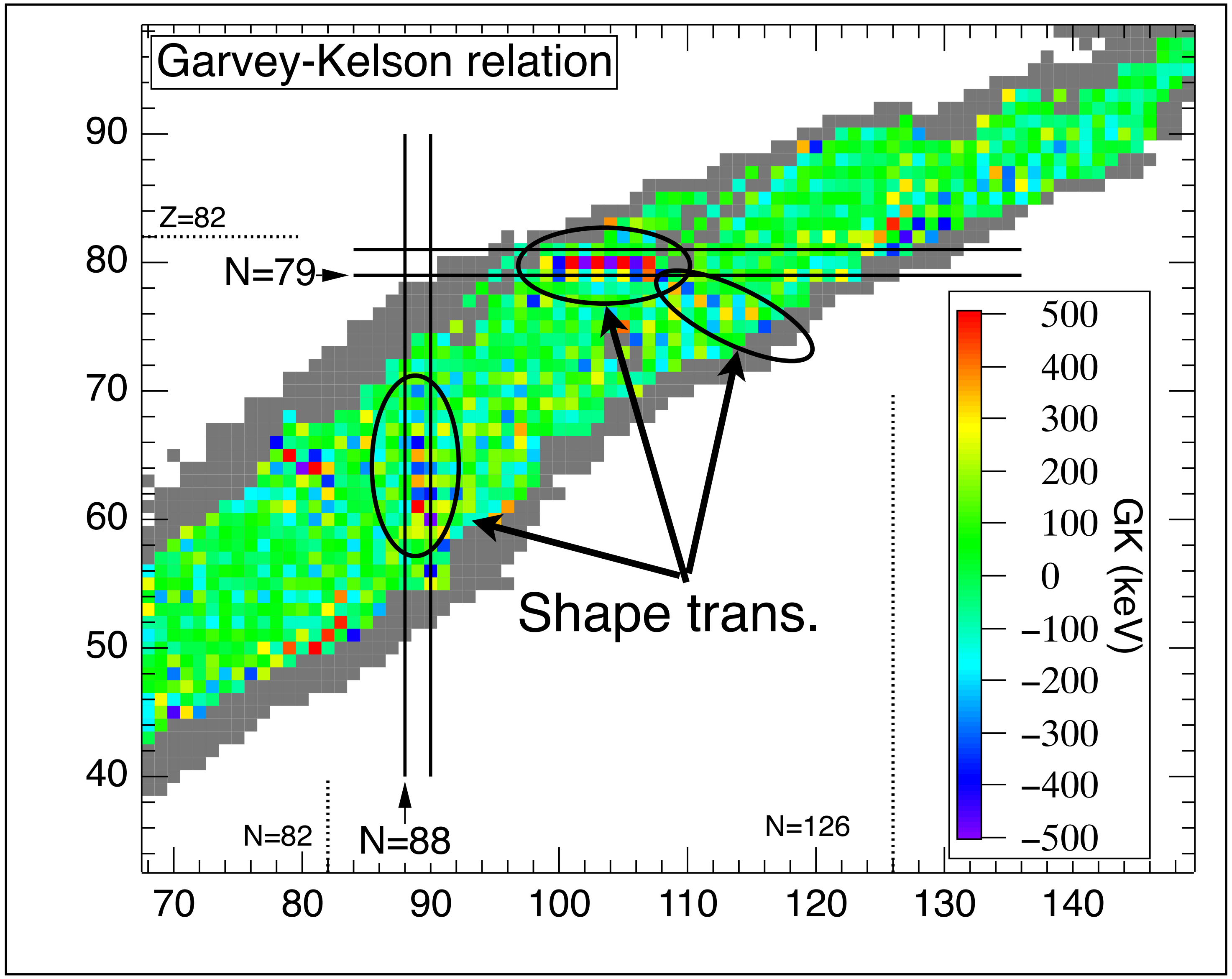

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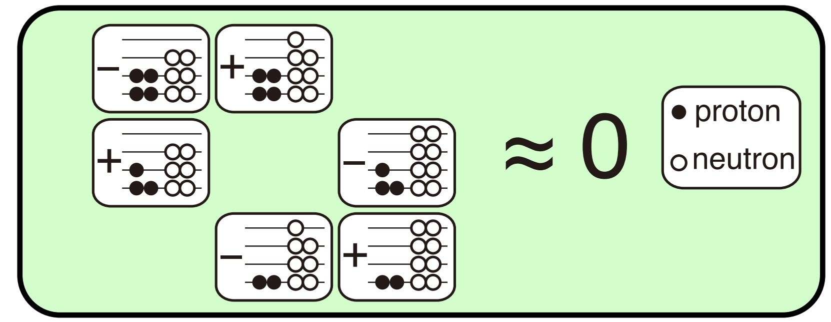

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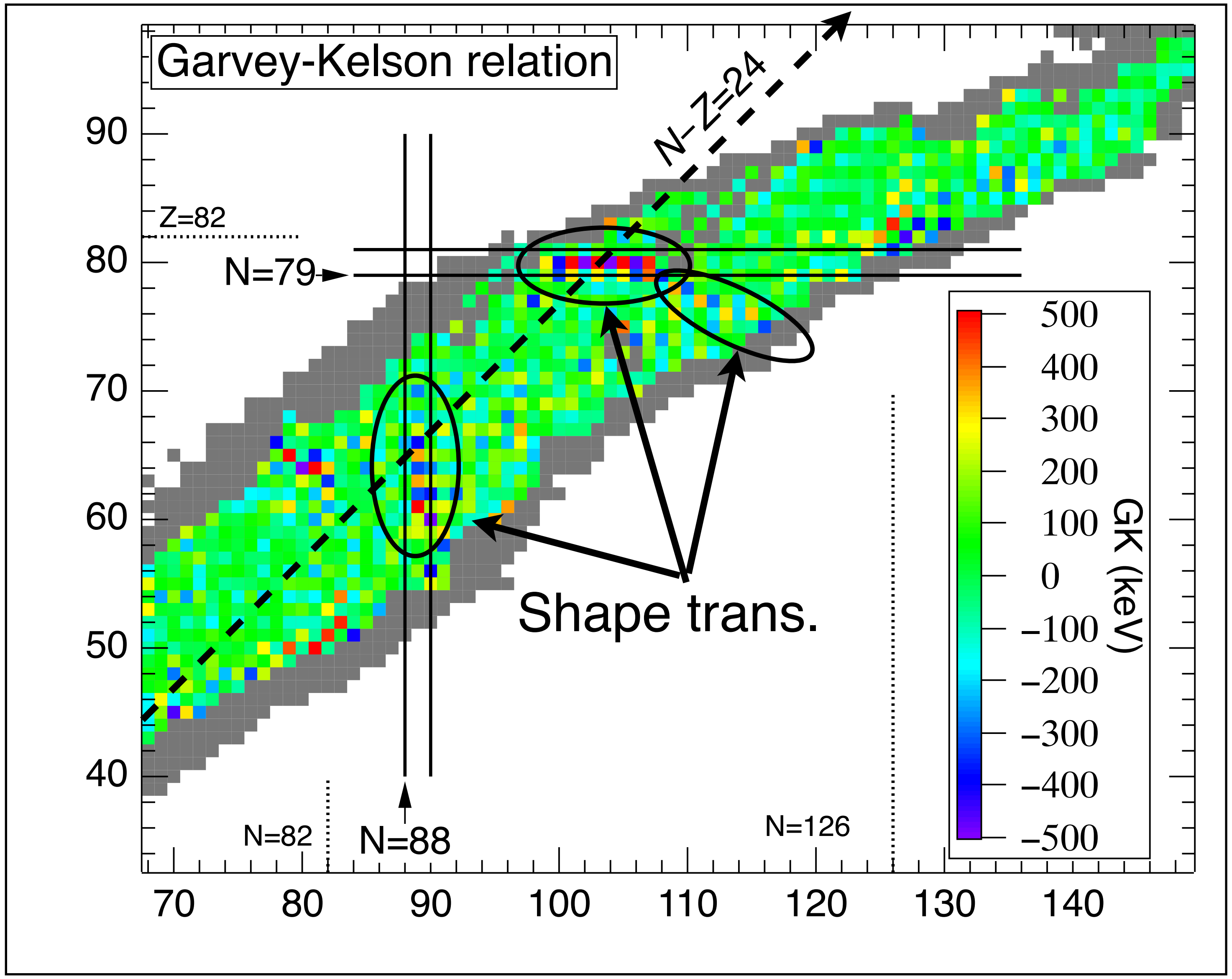

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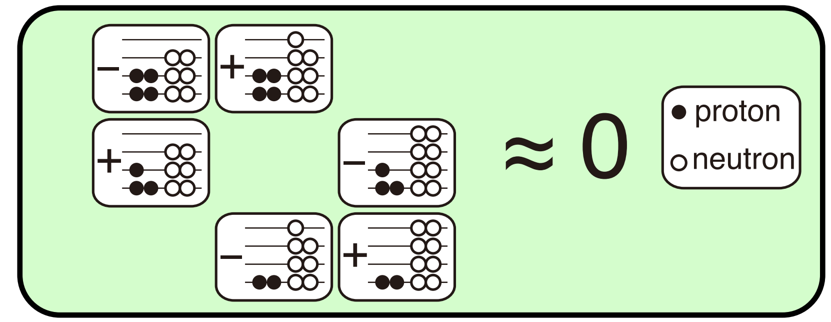

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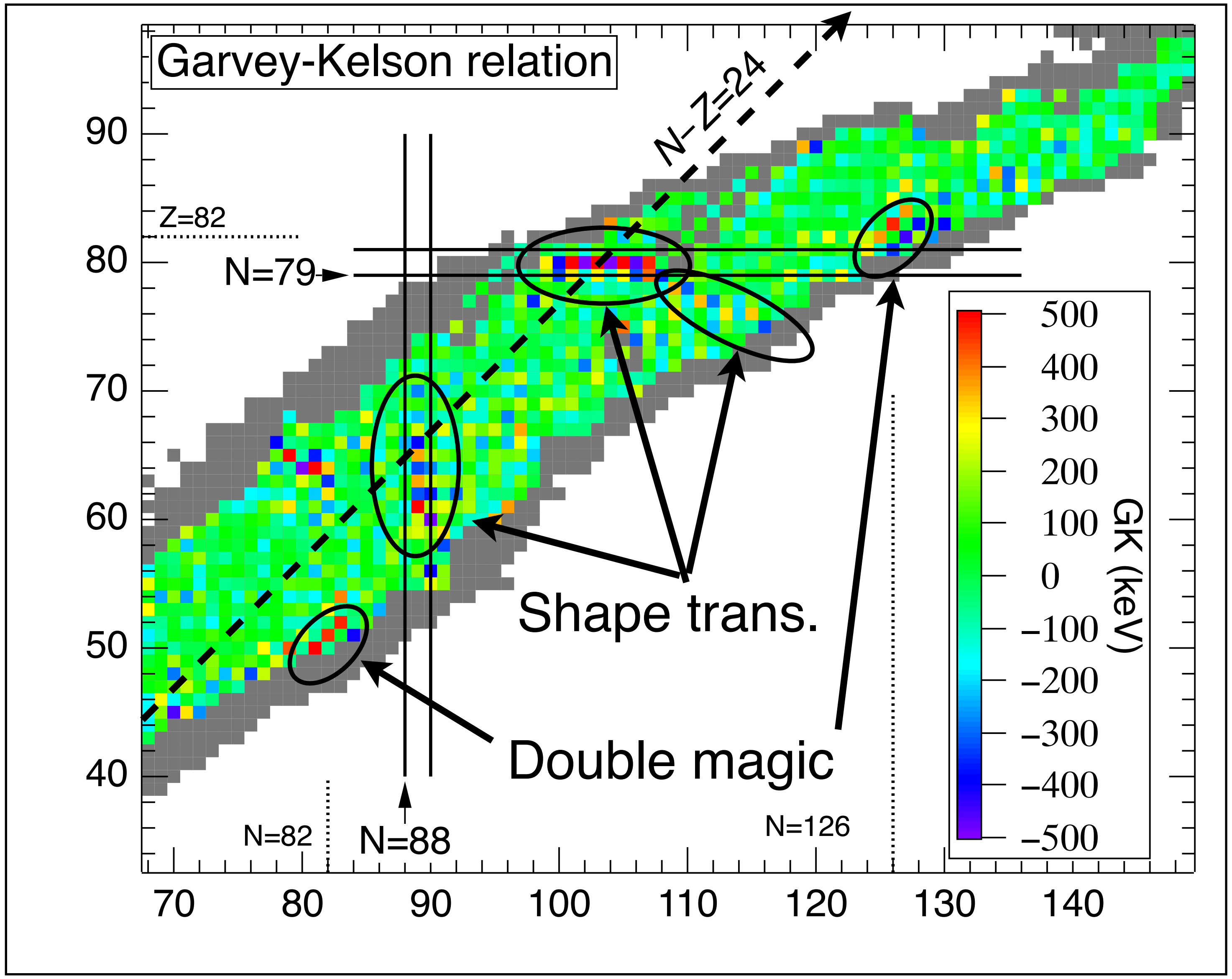

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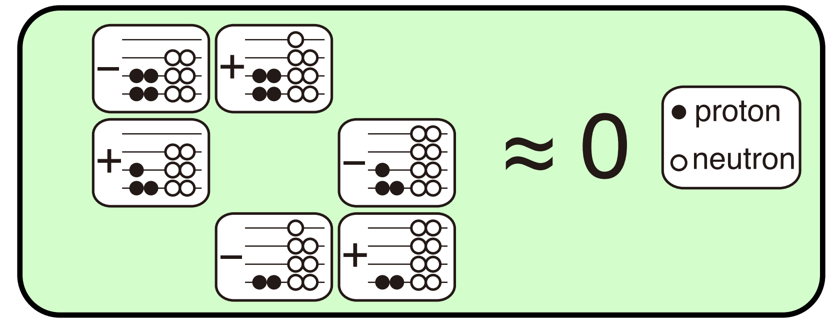

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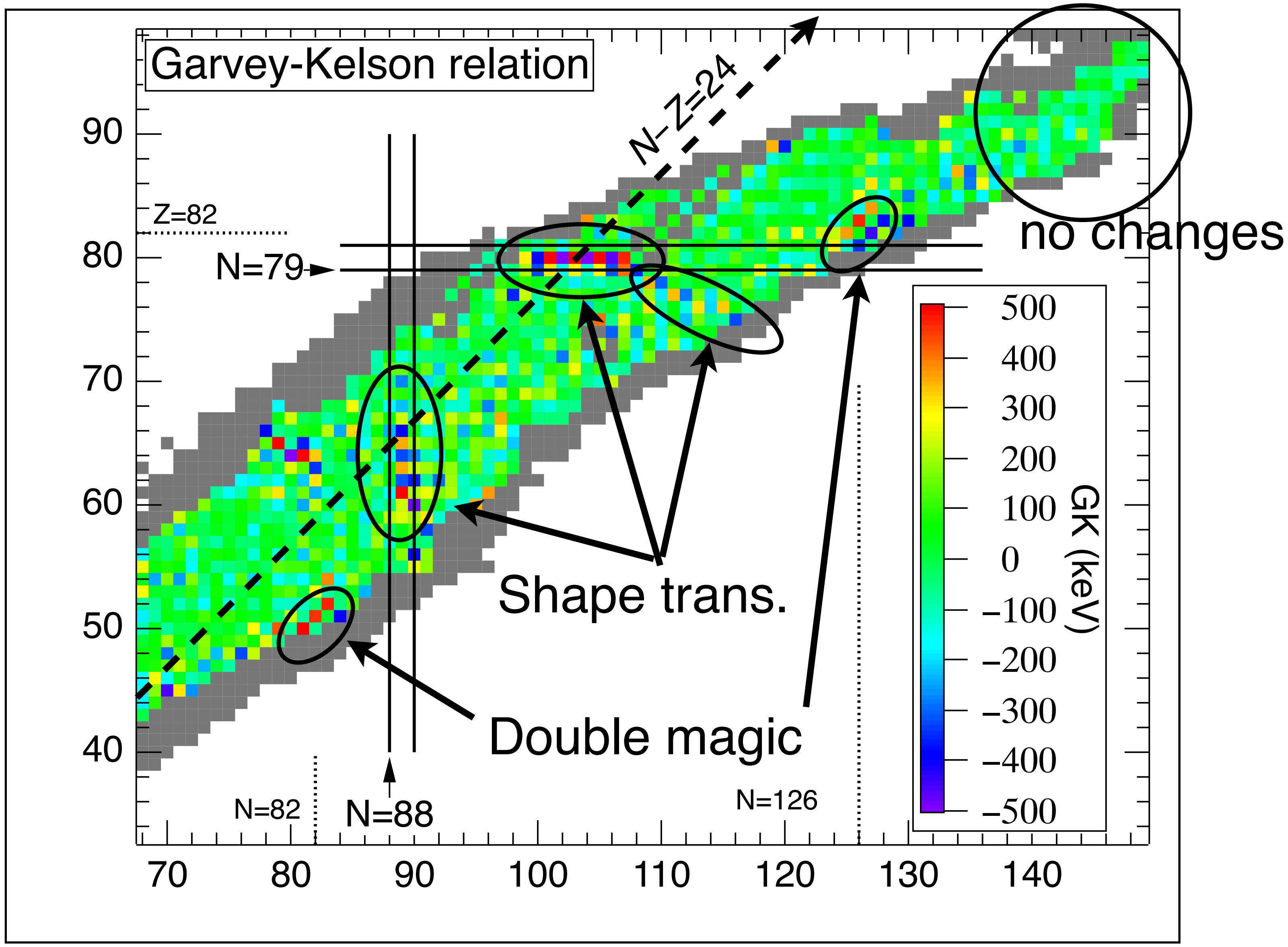

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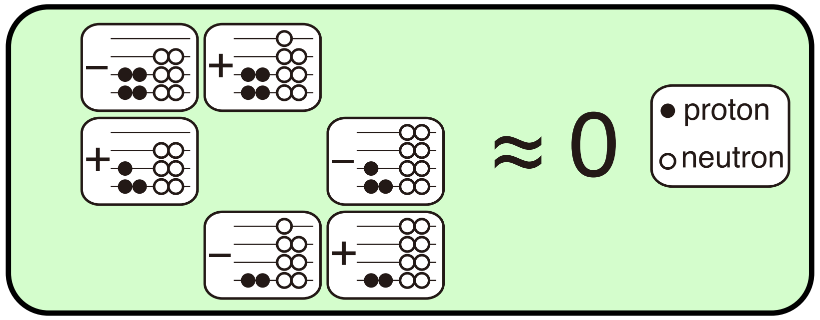

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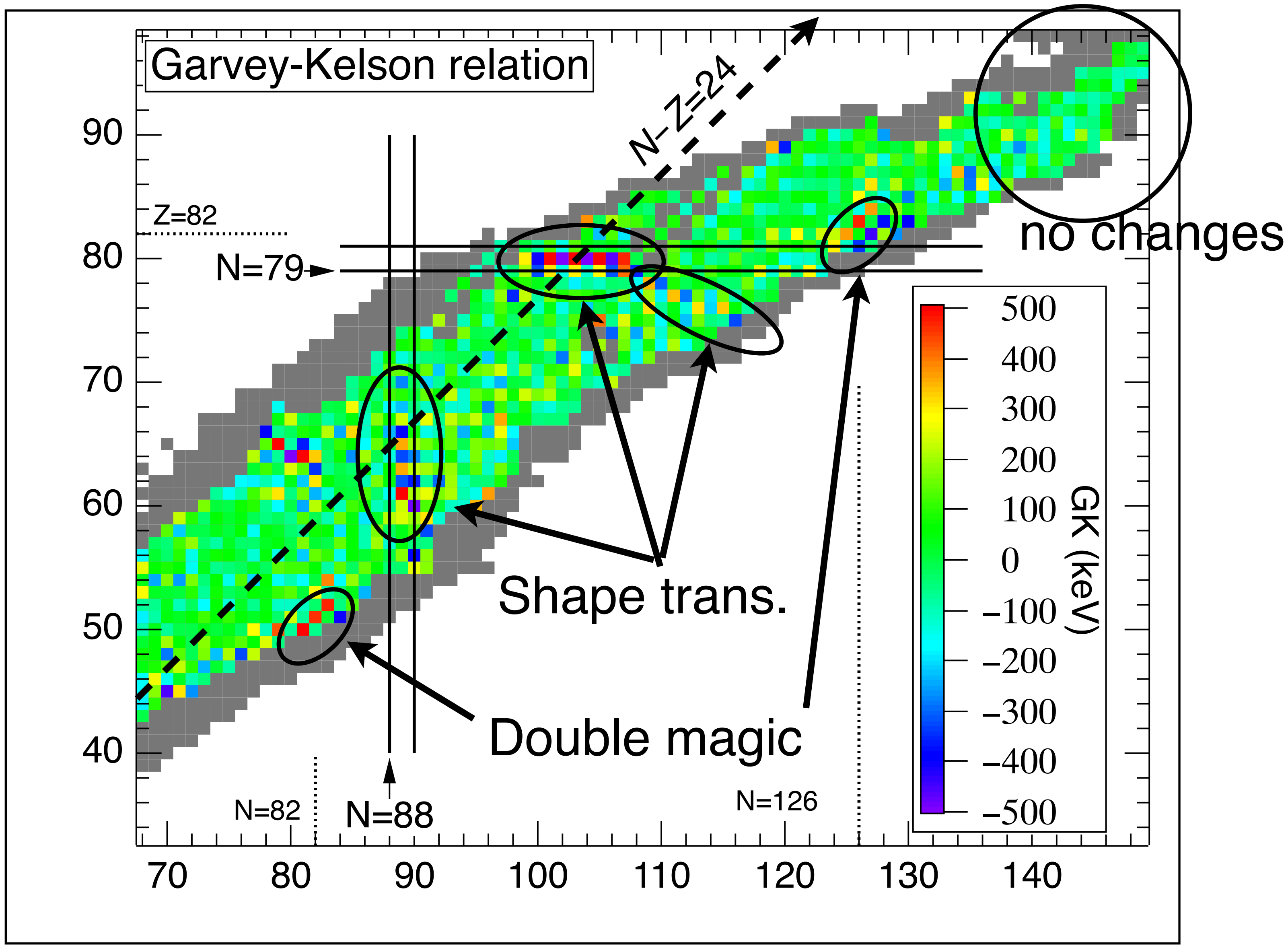
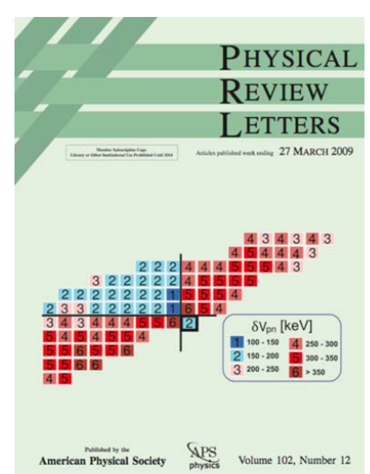


cf. Study for paring and proton-neutron interaction: neighboring doubly-magic nuclei.

Systematical trend of average p-n interaction crossing N=126.



PRL 102, 122503(2009)

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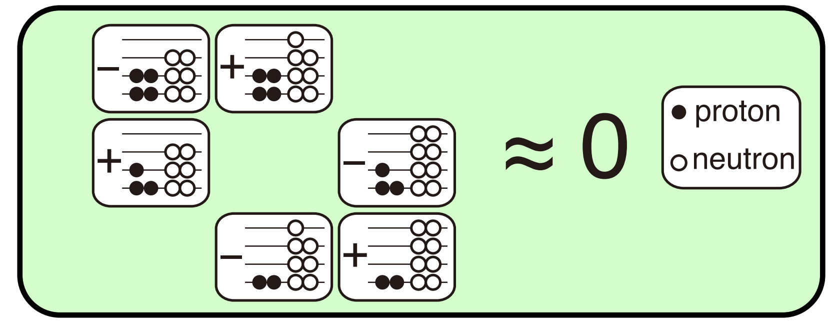

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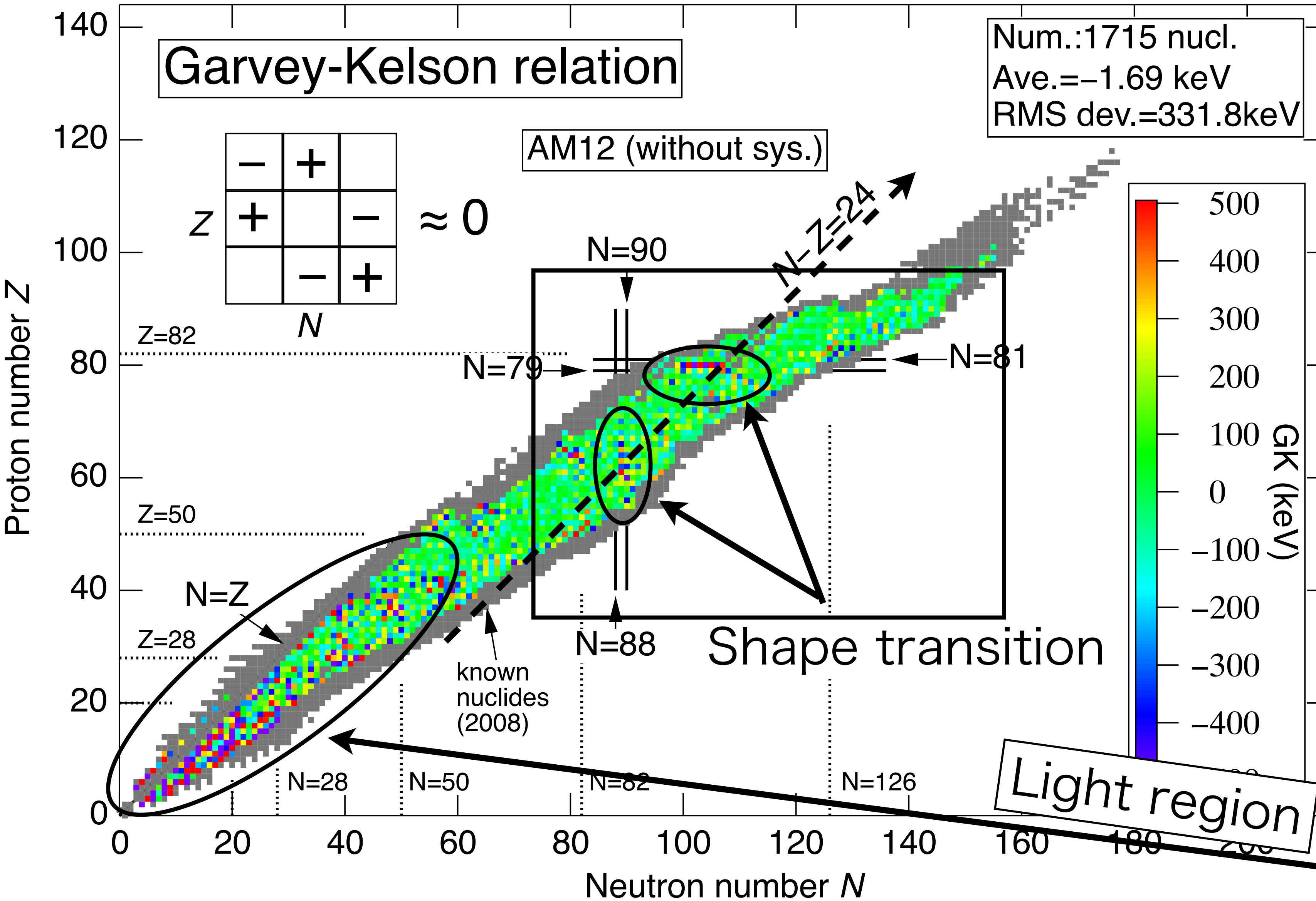
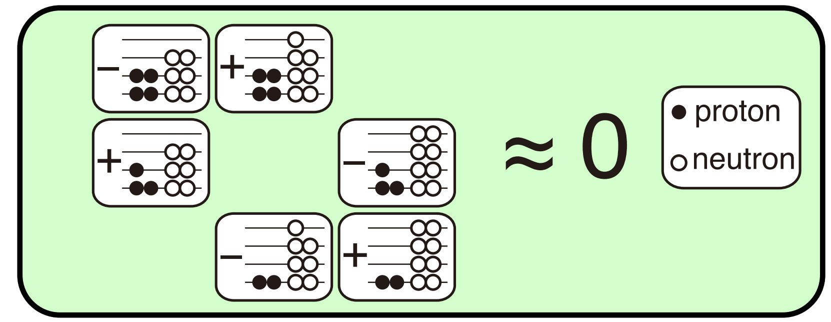
$A>100$	1202	-0.03	161.2
$A\leq100$	513	-29.3	554.1

larger

Mass relation: Garvey-Kelson systematics



A consideration of cancellation of core + valence nucleons (based on the shell model)



- Assumption: Cores among related six nuclei are the same.

Region	num.	Average	RMS dev.
All	**1715**	**-1.7 (keV)**	**331.8(keV)**
$A>100$	1202	-0.03	161.2
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larger

GK type mass formula:
Comay-Kelson-Zidon (CKZ1988)
Jänecke-Masson (JM1988)
Masson-Jänecke(MJ1988)
(ADNDT39, 1988)

In heavy region, the GK sys. gives some transition as changes of structures, especially nuclear shape.

Trend in 100 keV-order

Summary: Some properties of nuclear masses

- **Bulk properties of mass surface:**

- Volume energy, surface energy, symmetry energy, ...

- **Shell gaps:**

- N, Z=20, 28, 50, 82,126(only N) and a change of magicities (ex. N=14 to 16)

- **Transition of sphere to deformation:**

- Discontinuity of derivatives at N=88 to 90 near the β-stable region.

- **Wigner term:**

- Discontinuity at N=Z.

- **Averaged even-odd effect:**

- Staggering change of masses alternates even and odd-N/Z.

III. Nuclear mass formulae

There are many and various mass models.

- **Systematics:**

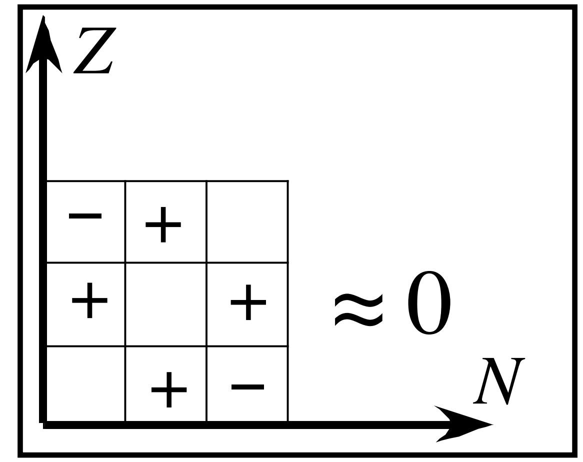
- Construction by focusing mass relation

- **Mass Model, Apploximation:**

- Macro-micro, Hybrid, or micro-like framework

Only mass data available to obtain are adopted.
(RMF, EDF mass formula are not included.)

Systematics・Phenomenology



- **Garvey-Kelson-type mass systematics**

  focusing on relation between mass values and Z, N

  Comay-Kelson-Zidon, Jänecke-Masson (1988)

- **Empirical shell term**

  focusing on Bulk part (**WB-like**)+deviation （**Shell term**）

  <u>Tachibana-Uno-Yamada-Yamada</u> (1988)

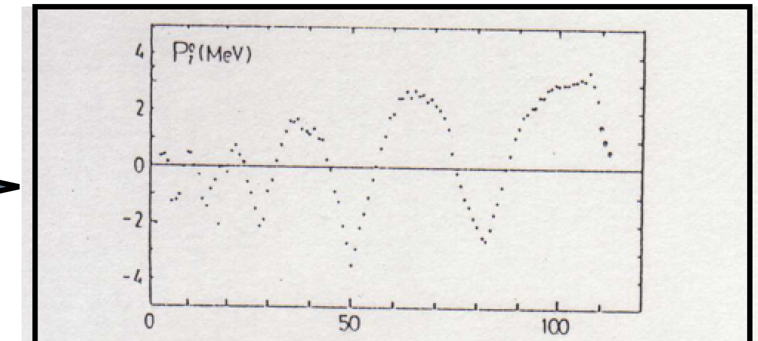
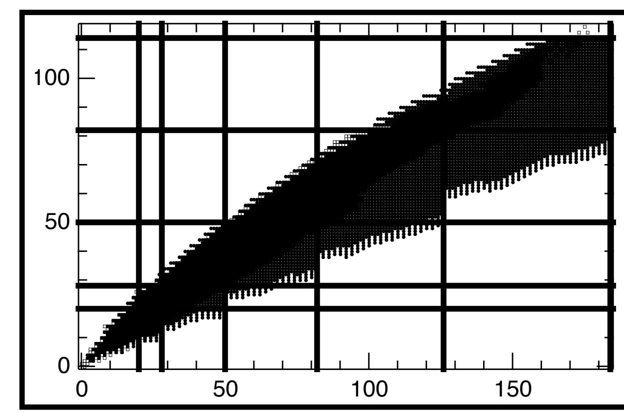


Figure 1. The proton shell parameters P_Z^0. The open circles are extrapolated values.

- **Phenomenological shell model calculation**

  Polynomials of particle and hole numbers,
  obliged to assume magic numbers in advance.

  Liran-Zeldes (1976), Duflo-Zuker (1995)



- ...

Properties

・Good reproduction of masses for known nuclei + good prediction for unknown nuclei (quite) near mass-measured nuclides. (300-600 keV)

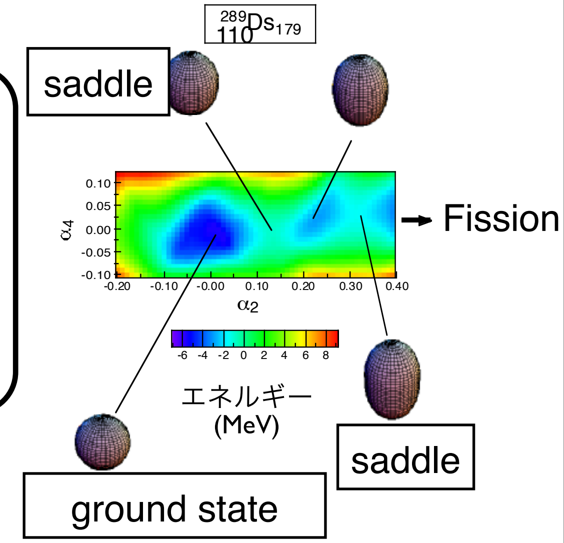
・<u>No predictable power for superheavy nuclei (next magic number, etc.)</u>

・<u>No deformation is obtained.</u>

Mass model, Approximation

Recent mass formulas:

- are designed for nuclei **with Z, N=8 to $^{310}[126]_{184}$ or more**
- have the RMS dev. from exp. masses. **of 600-800 keV**
- give **deformation parameters** β2, β4... and **fission barriers**



Potential energy surface

- **<u>Density functional theory</u>** <- recent project

by Dobaczewski et al.

- **<u>Hartree-Fock method with Skyrme force</u>**

Strong short-range force => δ-function => HF calc.
ETFSI (1995), HFBCS (2001), HFB (2002-)

by S.Goriely et al.

- **<u>Liquid-drop model</u>**

Deformed liquid-drop part+Micro. (folded Yukawa)
FRDM (1995), FRLDM (2002),

by P.Möller et al.

- **Mass formula with spherical-basis shell term**

Phenom. gross (WB-like)+spherical-basis shell part
KUTY (2000), KTUY (2005) Koura, Uno, Tachibana, Yamada

by H. Koura et al.

micro (-like) ↕ macro+micro ↕ phenom.

Skyrme-Hartree-Fock-Bogoliubov mass formula (2002-2010)

by S. Goriely et al.

$$E_{tot} = E_{HFB} + E_{wigner}$$

BSk21 force parameter set:

t_0=-3961.39 MeV fm^3, t_1=396.131 MeV fm^5
t_2=0 MeV fm^5, t_3=22588.2 MeV fm$^{3+3\alpha}$
t_4=-100.000 MeV fm$^{5+3\beta}$, t_5=-150.000 MeVfm$^{5+3\gamma}$
x_0=0.885231, x_1=-0.0648452, t_2x_2=1390.38 MeV fm^5
x_3=1.03928, x_4=2.00000, x_5=-11.0000
W_0=109.622 MeV fm^5, α=1/12, β=1/2, γ=1/12
f^+_n=1.00, f^+_p=1.07, f^-_n=1.05, f_p=1.13
V_W=-1.80 MeV, λ=280, V'_W=0.96, A_0=24

The long road in the HFB mass model development

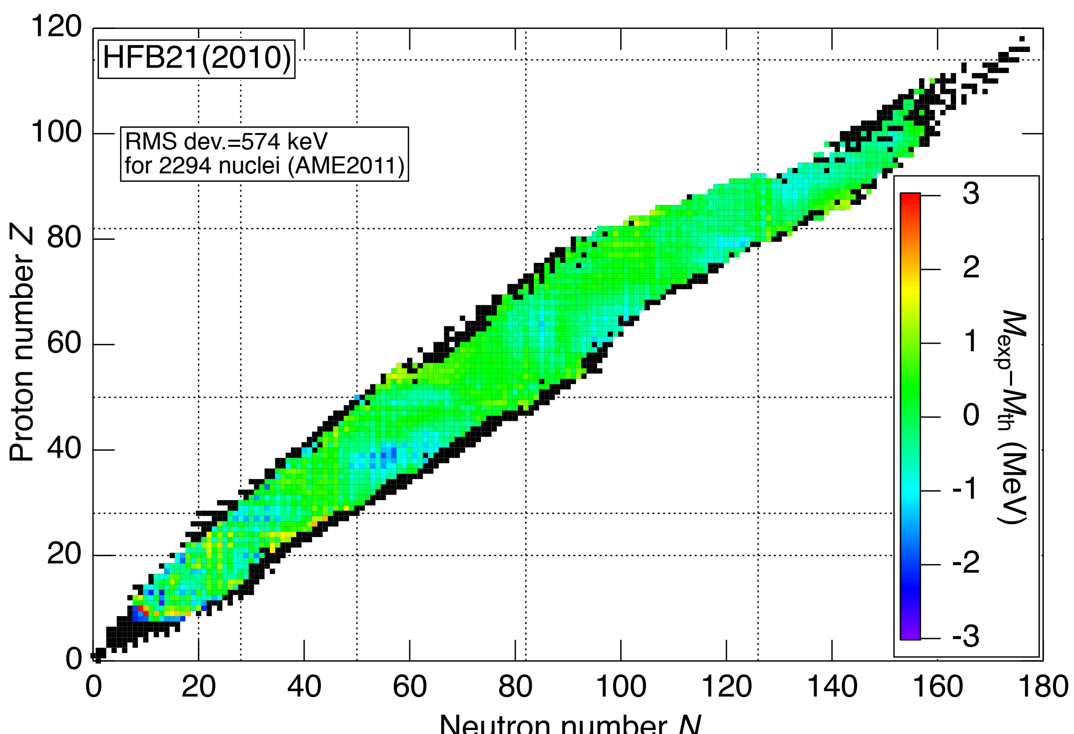
		Accuracy σ_{rms} (2149 nuc)
HFB-2 :	Possible to fit all 2149 exp masses Z≥8	659 keV
HFB-3:	Volume versus surface pairing	635 keV
HFB-4-5:	Nuclear matter EoS: M^*_s=0.92	660 keV
HFB-6-7:	Nuclear matter EoS: M^*_s=0.80	657 keV
HFB-8:	Particle-number projection	635 keV
HFB-9:	Neutron matter EoS	733 keV
HFB-10-13:	Low pairing & NLD	717 keV
HFB-14:	Collective correction and Fission B_f	729 keV
HFB-15:	Coulomb correlations / CSB	678 keV
HFB-16:	**Pairing constrained to NM**	**632 keV**

Current version: HFB-21 (2010)

HFB21 gives a less than 600 keV of the RMS dev.
In the light region there is some discrepancy in derivatives as S_n.

Referred mass data:AME03

Mass



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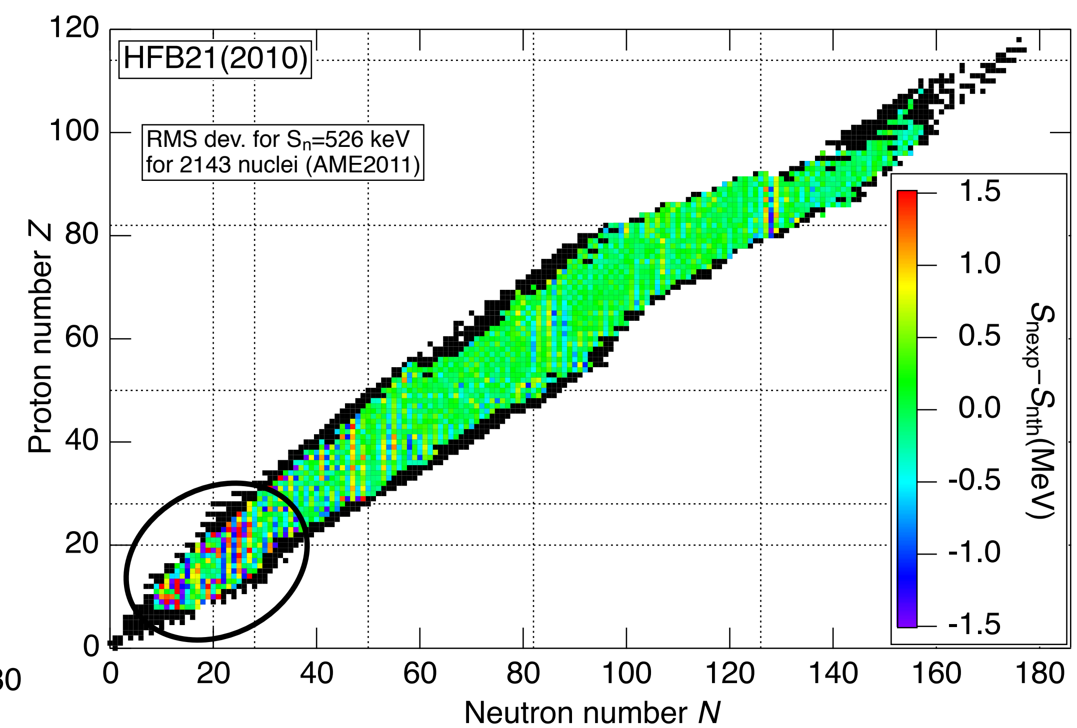
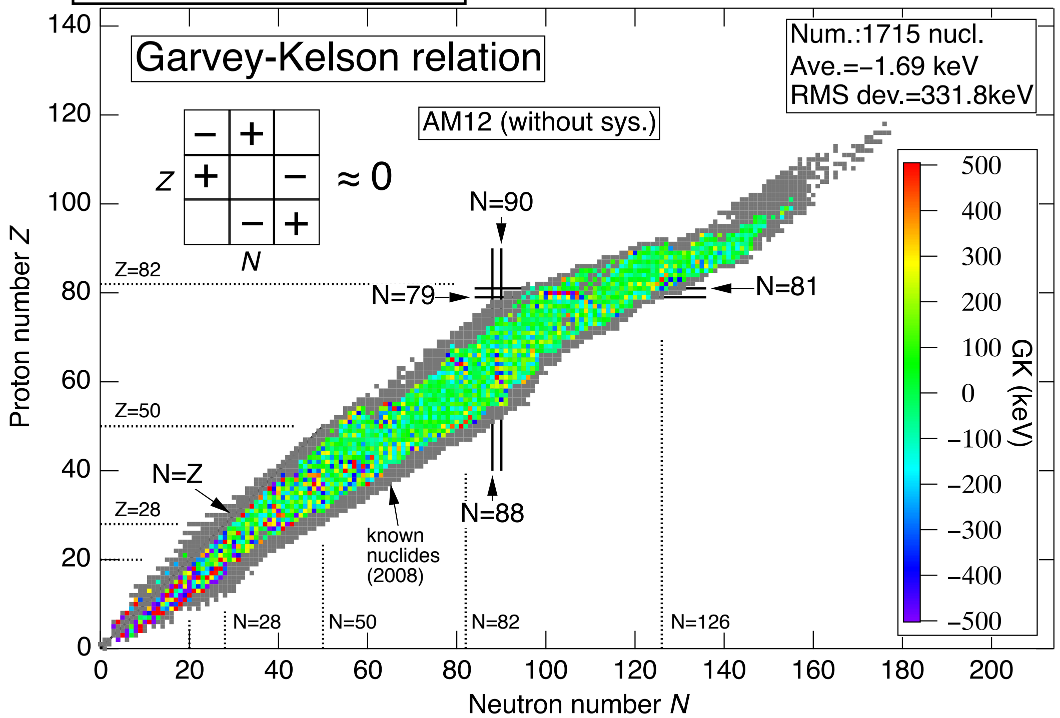
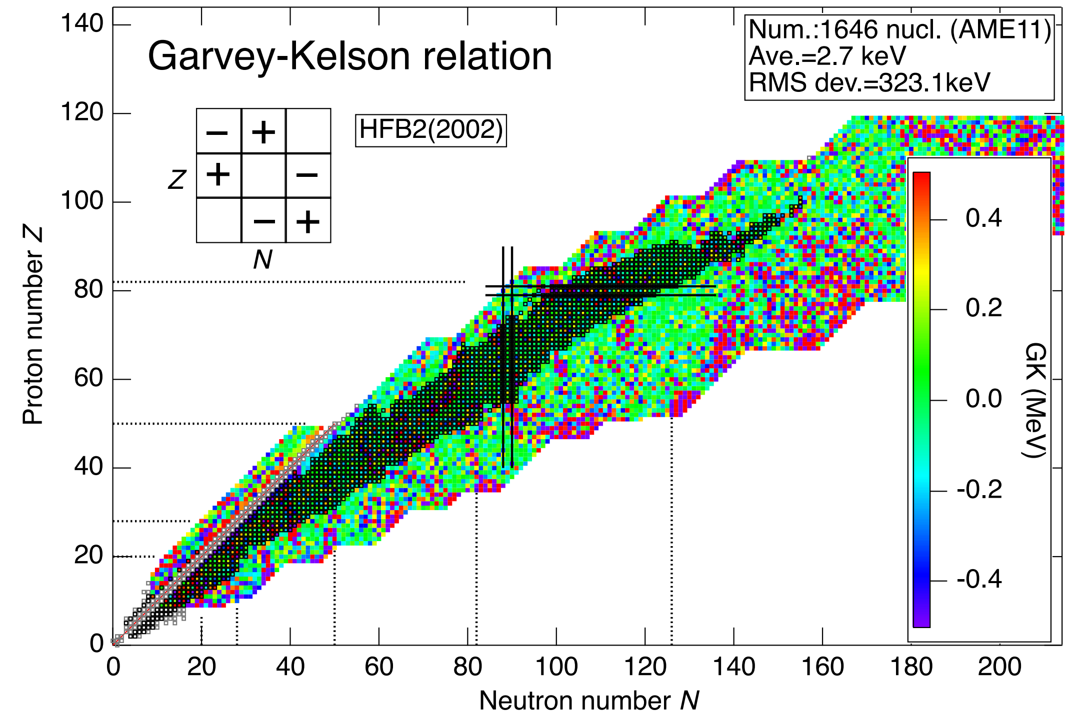


G-K sys. check for HFB formulae

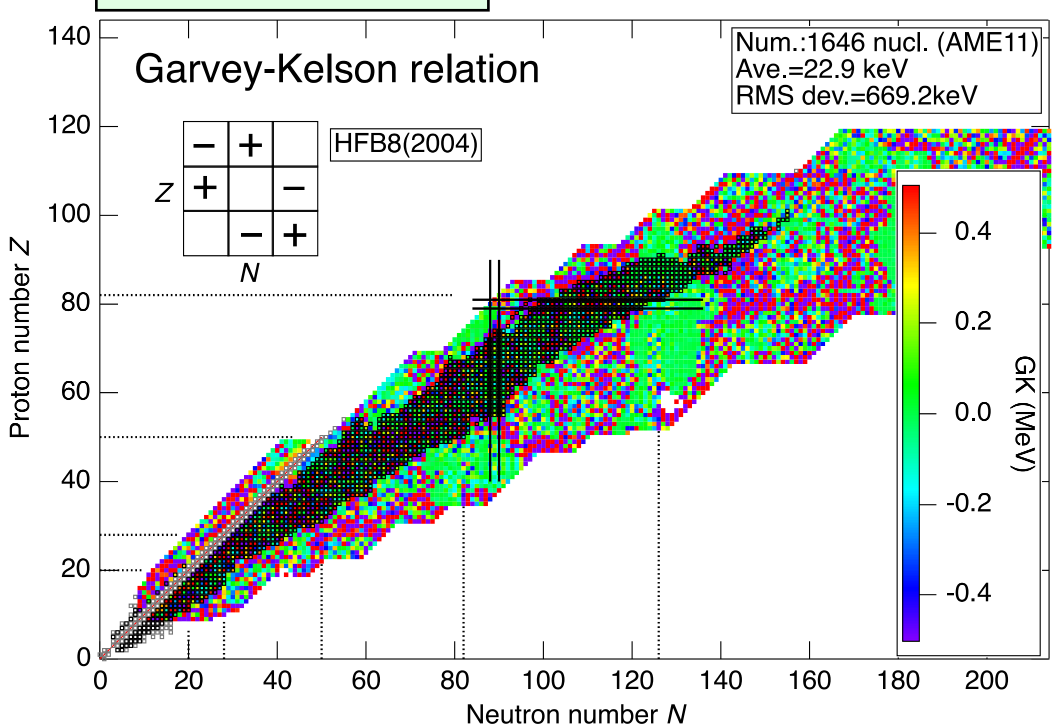
AME11(exp)



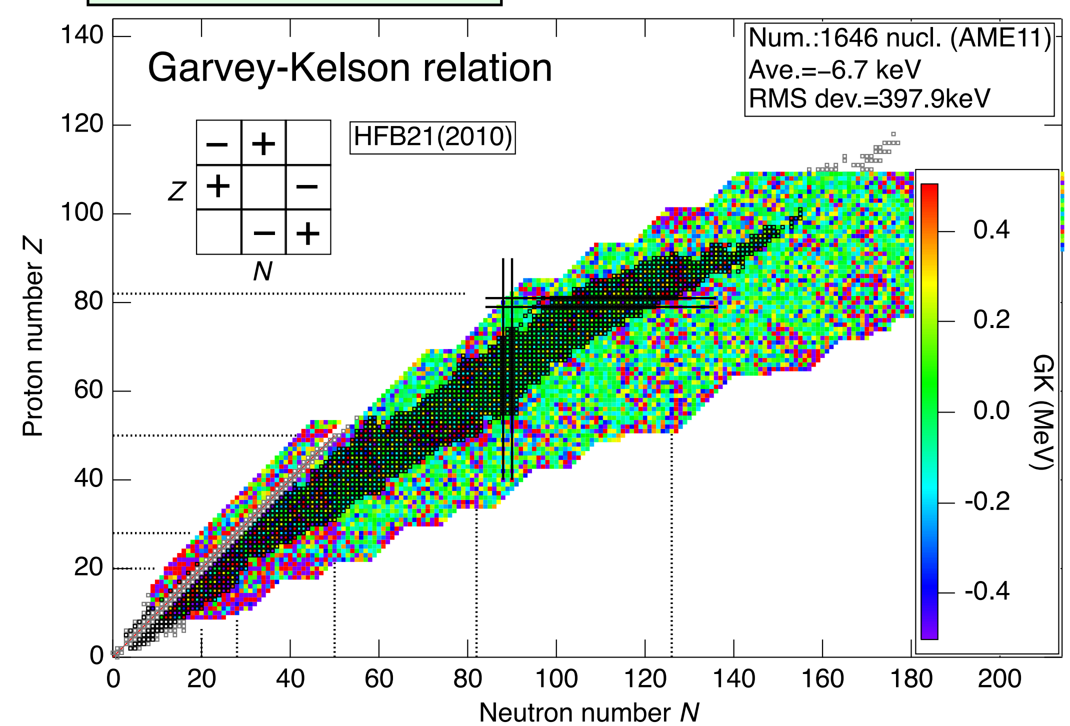
HFB2(2002)



HFB8(2004)



HFB21(2010)



- All the HFB calc. violate the GK summation everywhere!

Finite-Range-Droplet Model (FRDM) mass formula (1995)

by P. Möller et al.

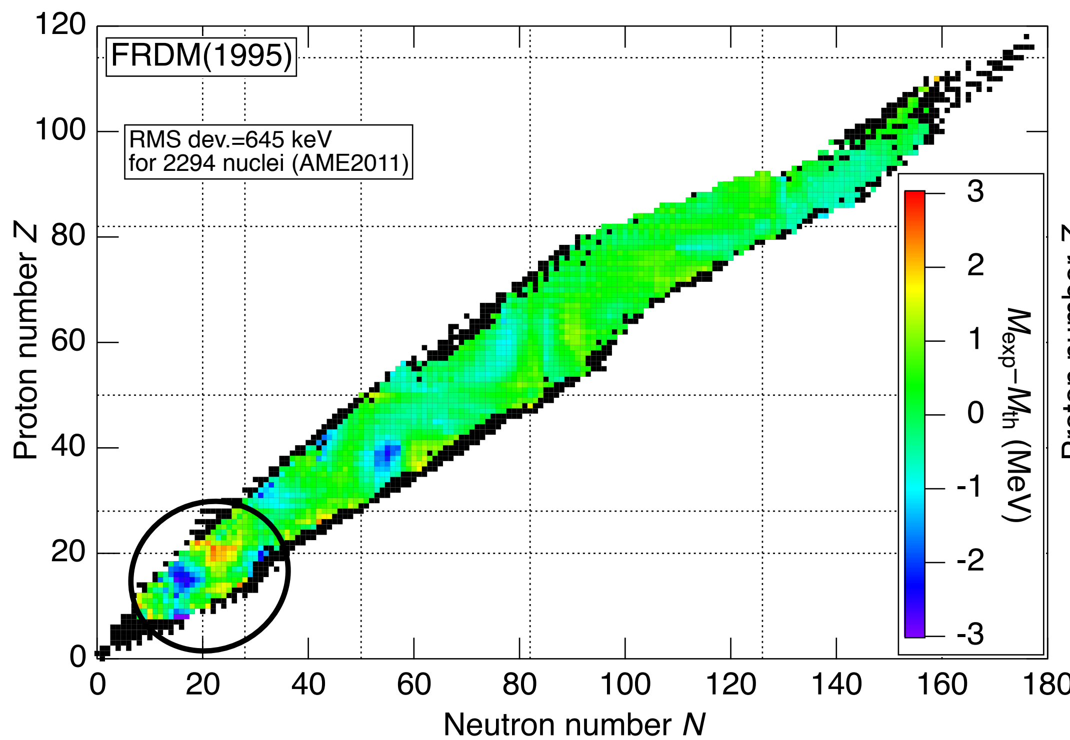
Current version is FRLDM (2003-)

$$E(Z, N, shape)=E_{macro}(Z, N, shape)+E_{micro}(Z, N, shape)$$

E_{macro}: Droplet part as a function of Z and N
E_{micro}: Folded Yukawa-type potential + Nilsson-Strutinsky method

- Deformation, fission barrier is obtained
- Good prediction on fission properties.

Mass



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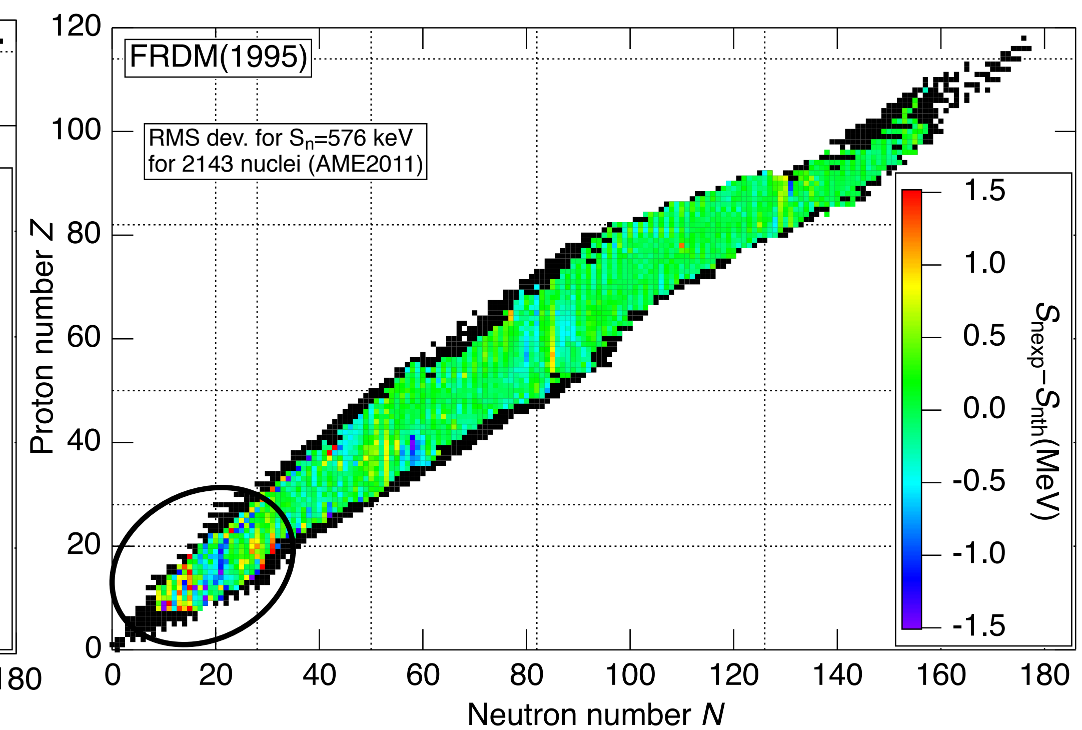


Good for the heavier mass region. Some large discrepancies appear in the light region.

Referred mass data:AME93

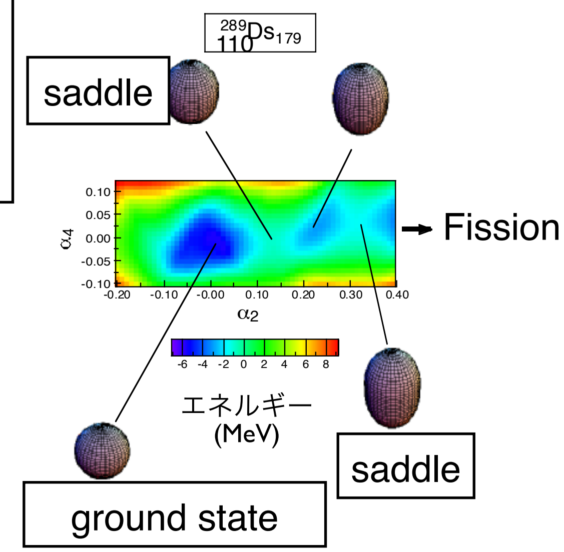
Spherical-Basis (KTUY) mass formula (2005)

$$M(Z, N) = M_{gross}(Z, N) + M_{eo}(Z, N) + M_{shell}(Z, N)$$

by H. Koura et al., PTP113 (2005)

M_{gross} smooth function of N and Z. (same as the TUYY formula)
M_{shell}: modified Woods-Saxon pot.+BCS+deform. config.

- Deformation, fission barrier is obtained
- Change of magicties in the n-rich nuclei is predicted. (N=20 -> 16, etc.)
- Topic: decay modes for superheavy nuclei are applied for.



Potential energy surface

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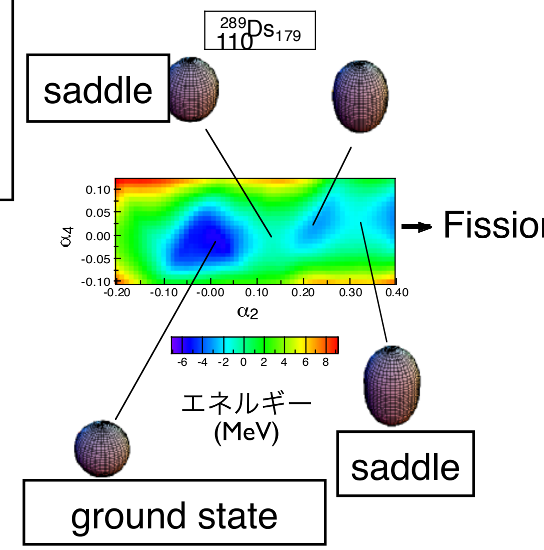
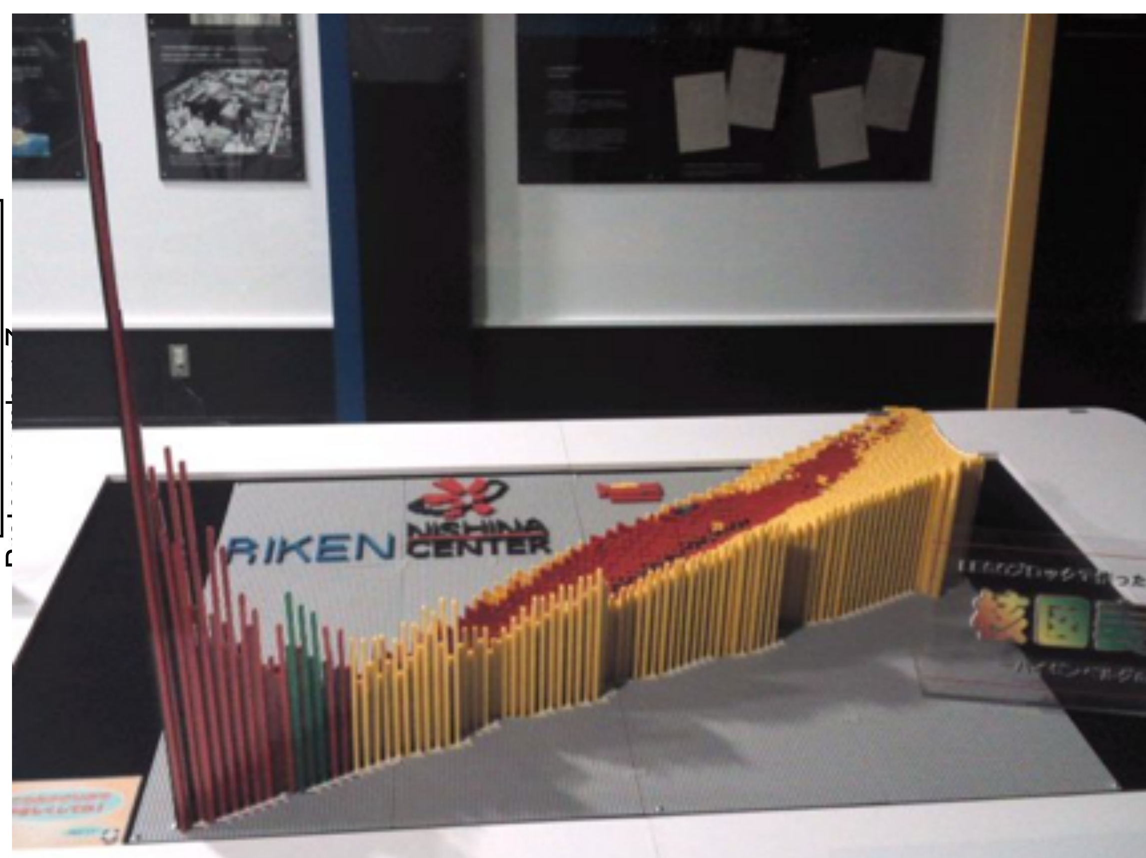


Potential energy surface

Derivatives of mass like S_n ,Q_α, Q_β, gives a good reproduction.



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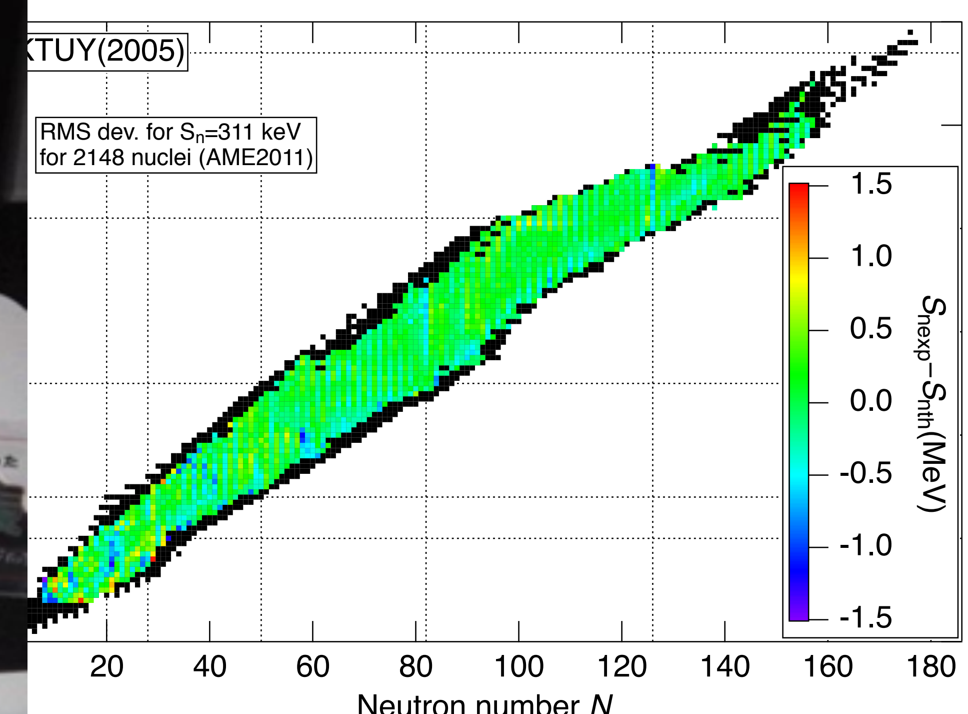

Referred Mass data:AME03

Spherical-Basis (KTUY) mass formula (2005)

$$M(Z, N) = M_{gross}(Z, N) + M_{eo}(Z, N) + M_{shell}(Z, N)$$

by H. Koura et al.,
PTP113 (2005)

M_{gross} smooth function of N and Z. (same as the TUYY formula)
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- Deformation, fission barrier is obtained
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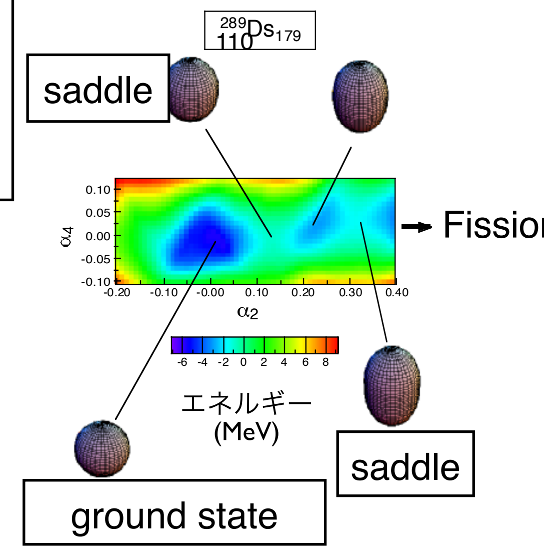
- Topic: decay modes for superheavy nuclei are applied for.

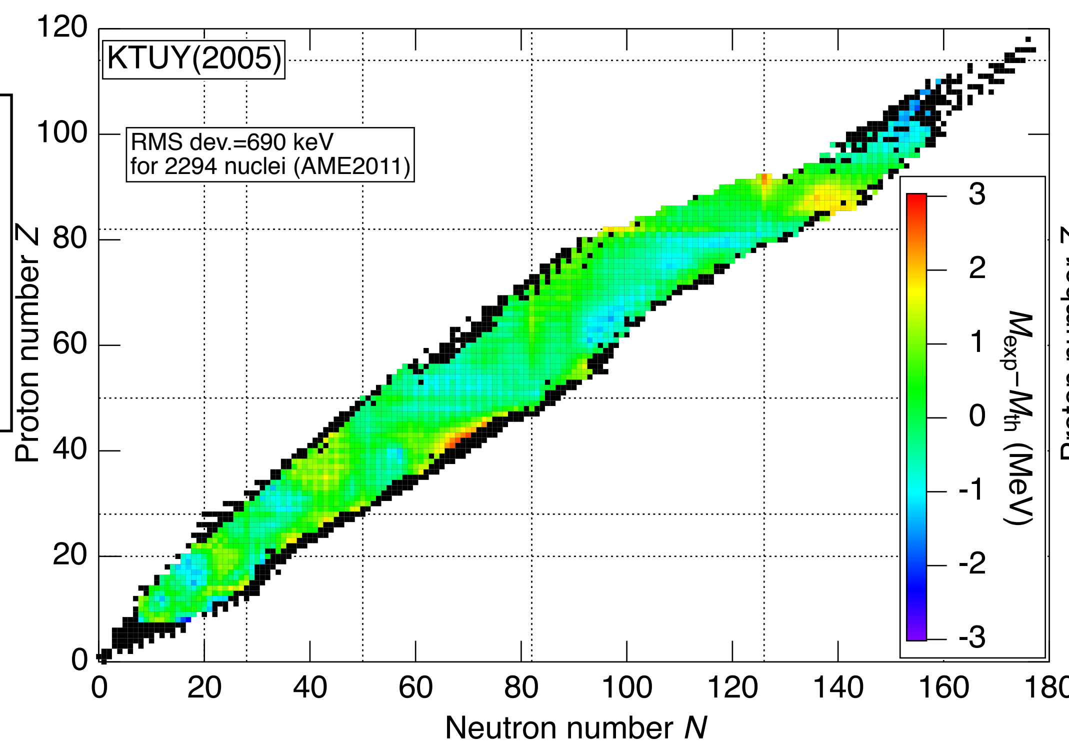


Potential energy surface

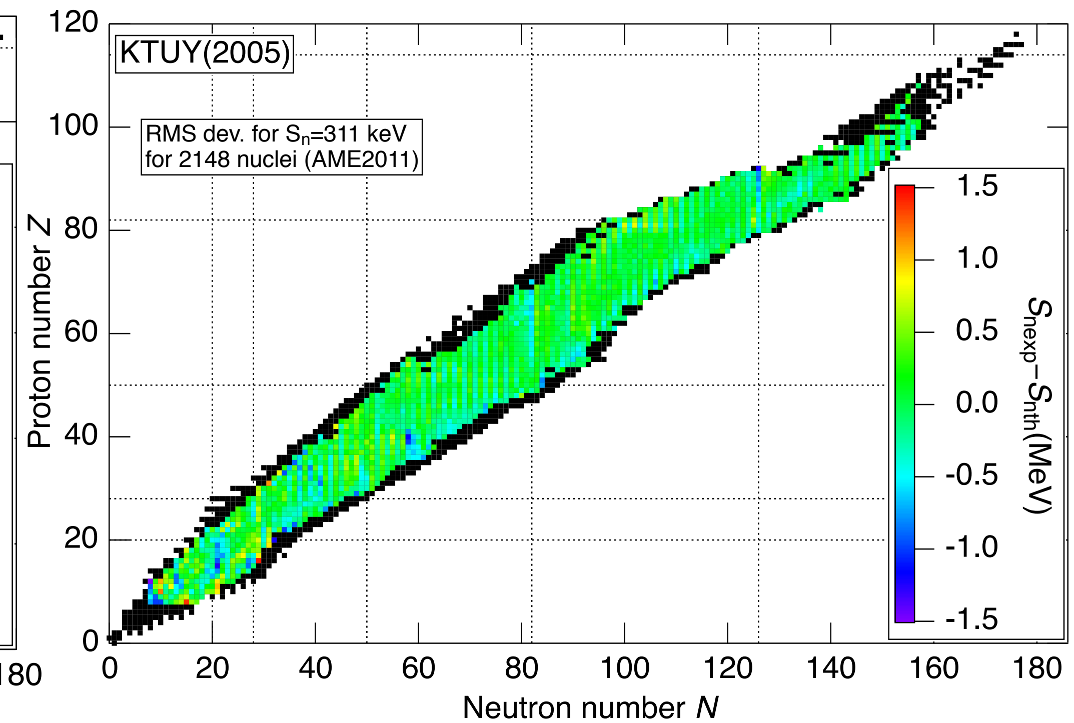
Derivatives of mass like S_n ,Q_α, Q_β, gives a good reproduction.

Referred Mass data:AME03

Mass

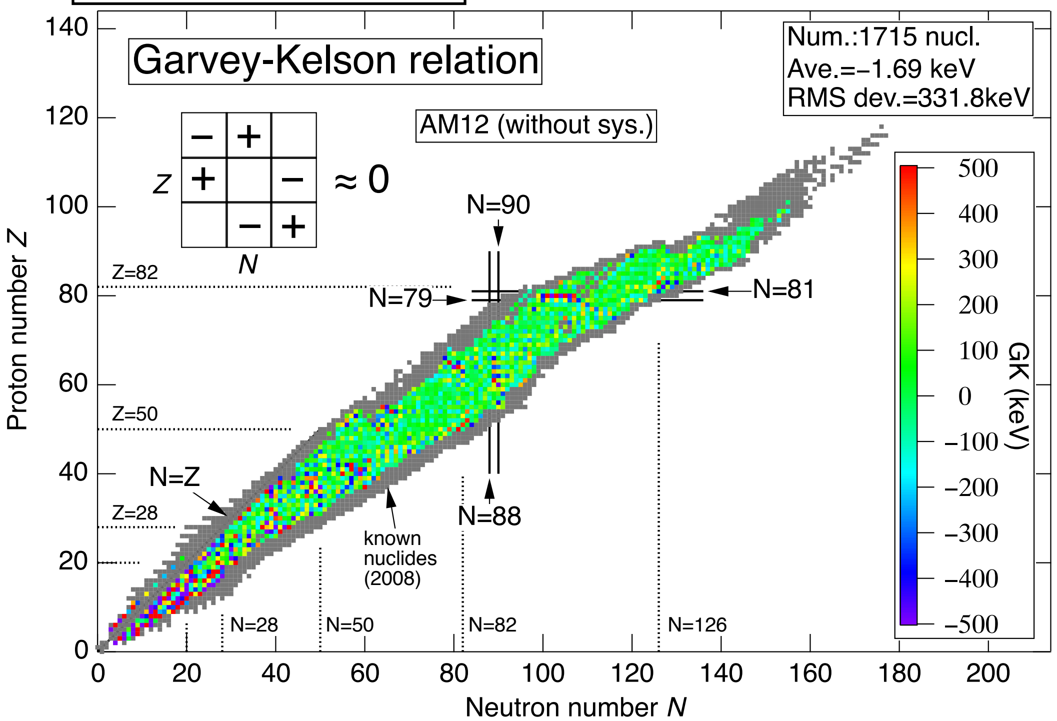


Sn

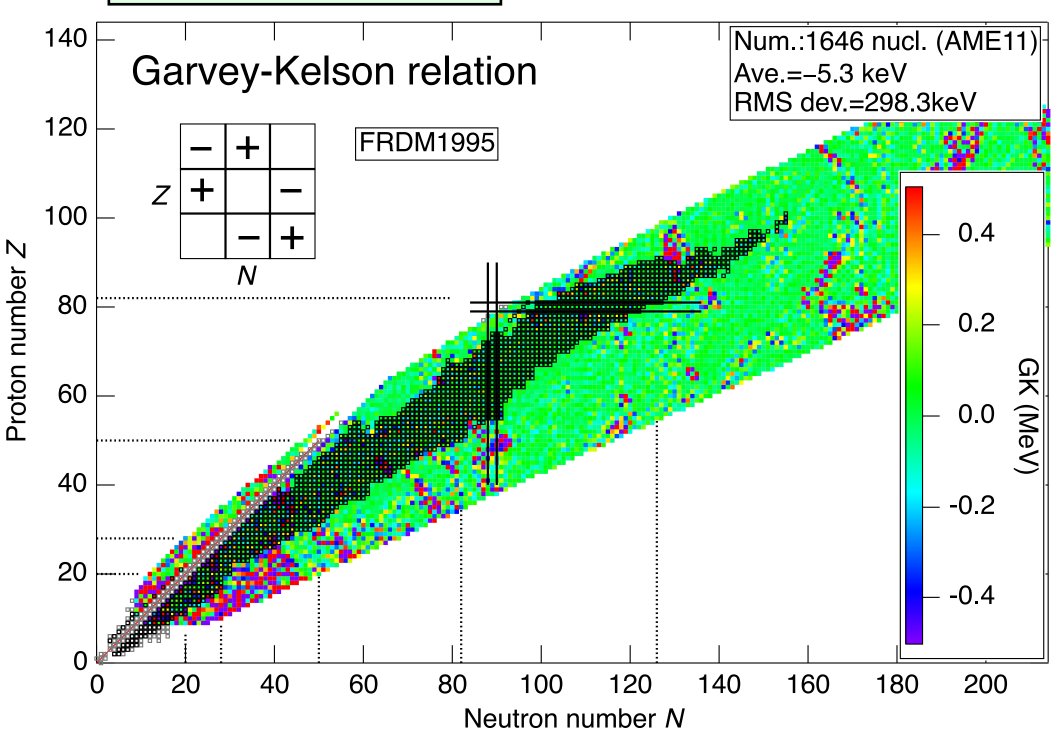


G-K sys. check for FRDM and KTUY

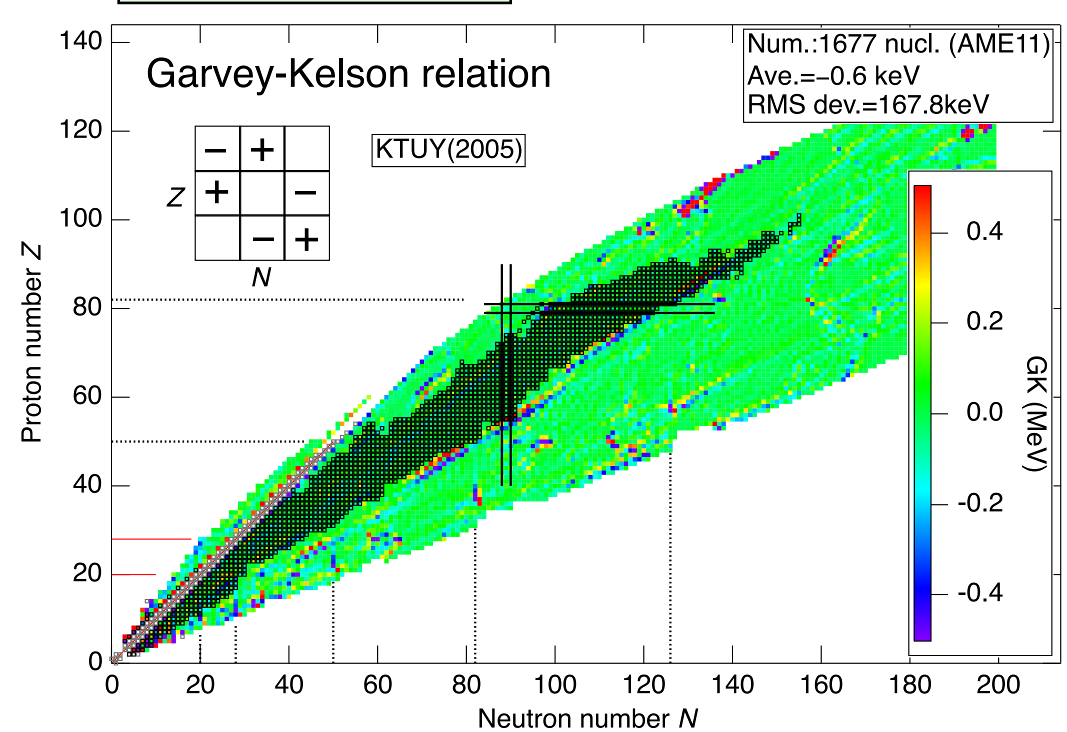
AME11(exp)



FRDM1995 : gives a smooth trends



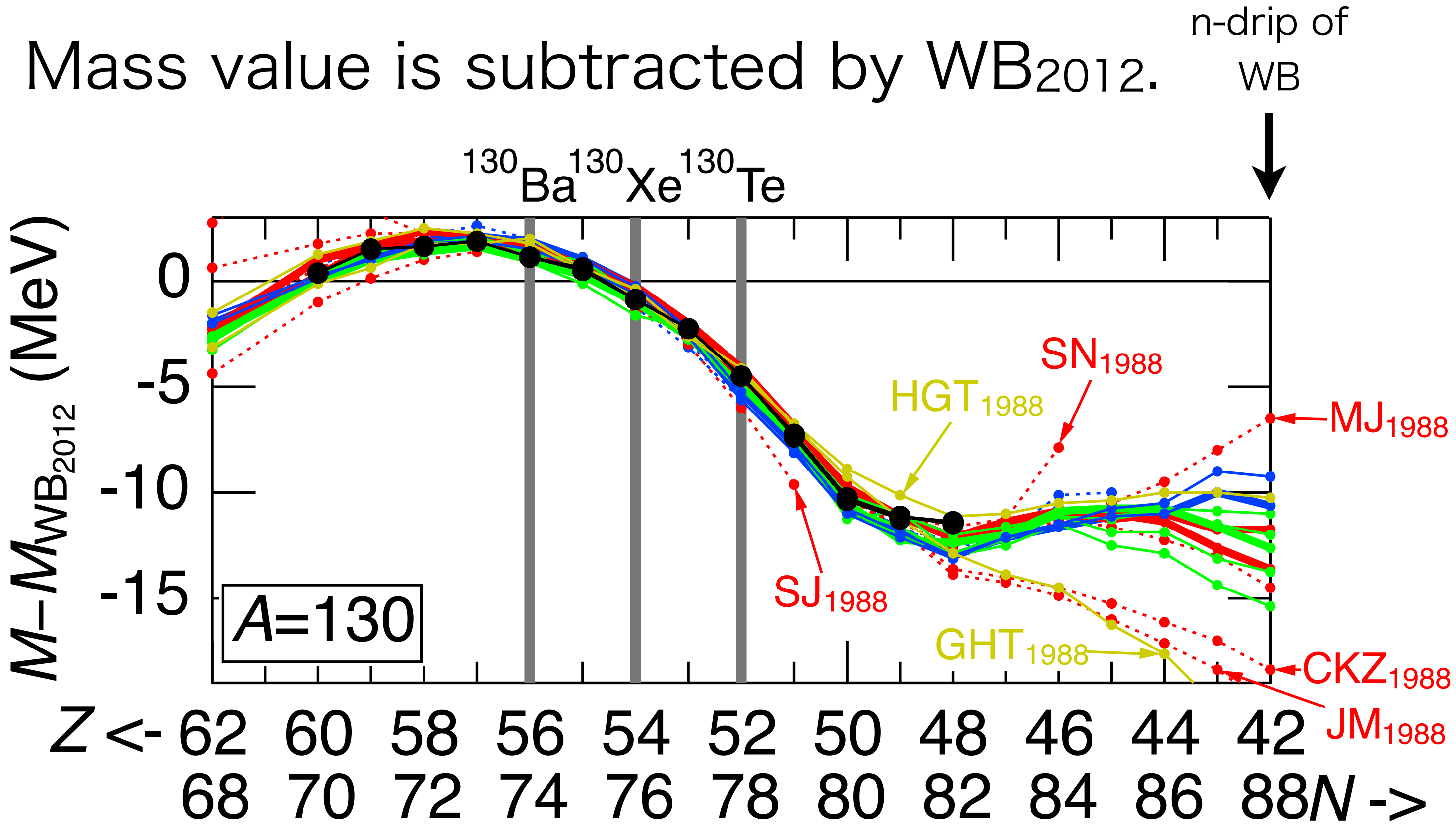
KTUY2005 : gives a smooth trends

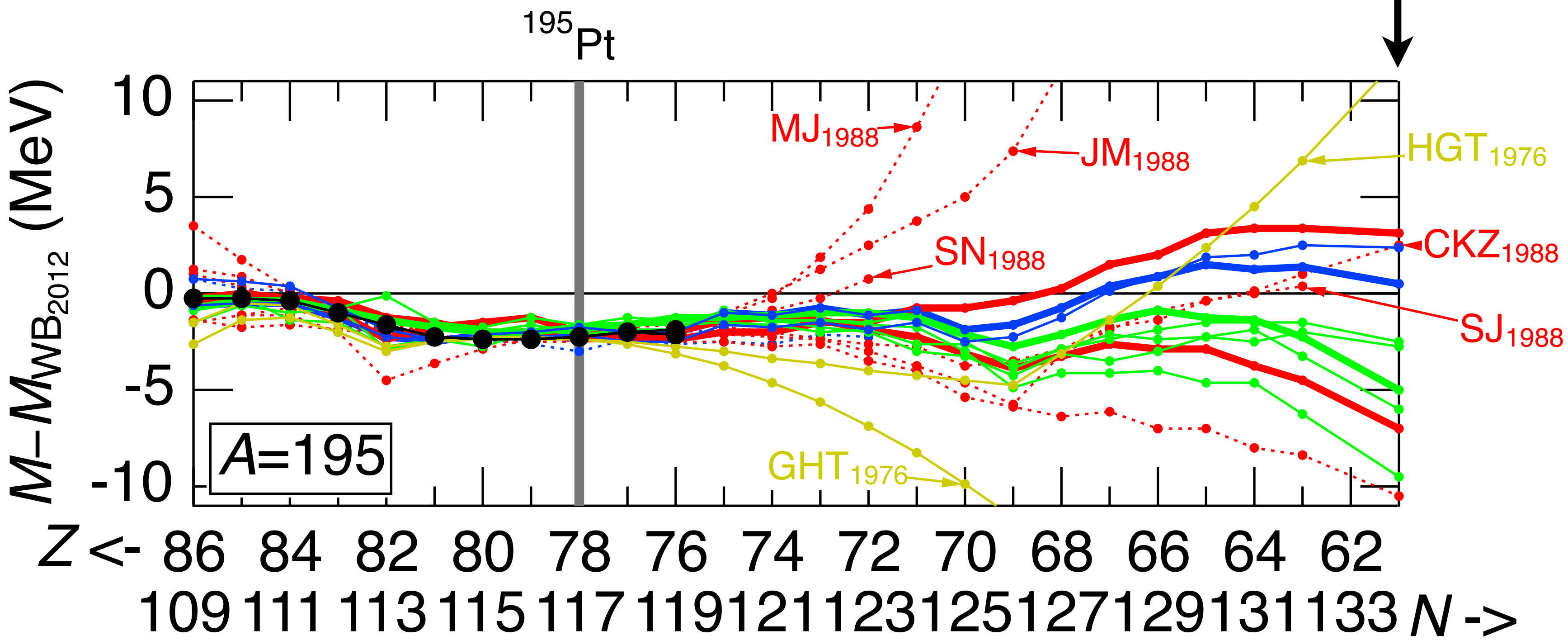


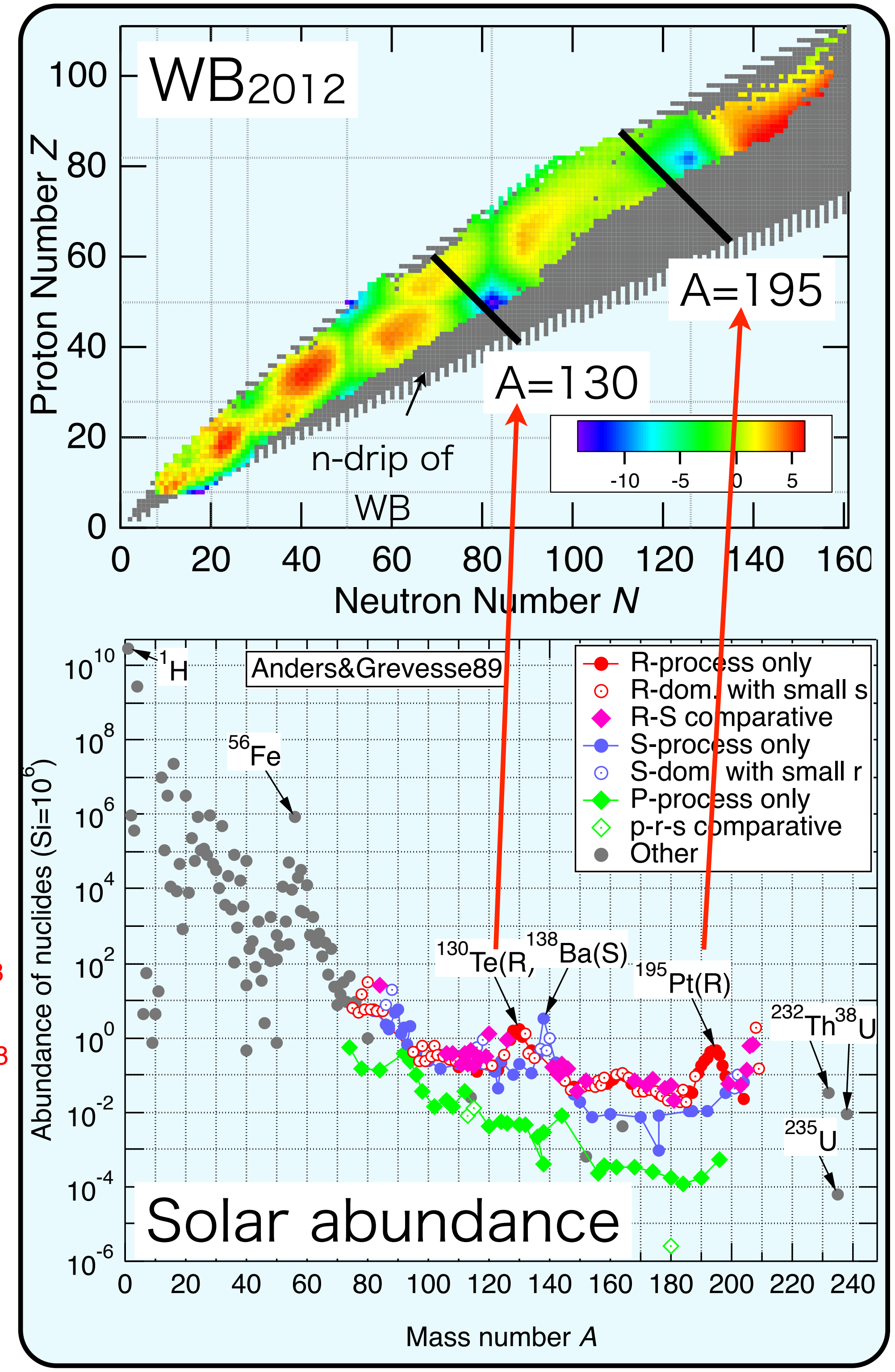
- FRDM and KTUY give a smooth trends.

Extrapolation to the n-rich nuclei

Mass value is subtracted by WB_{2012}.

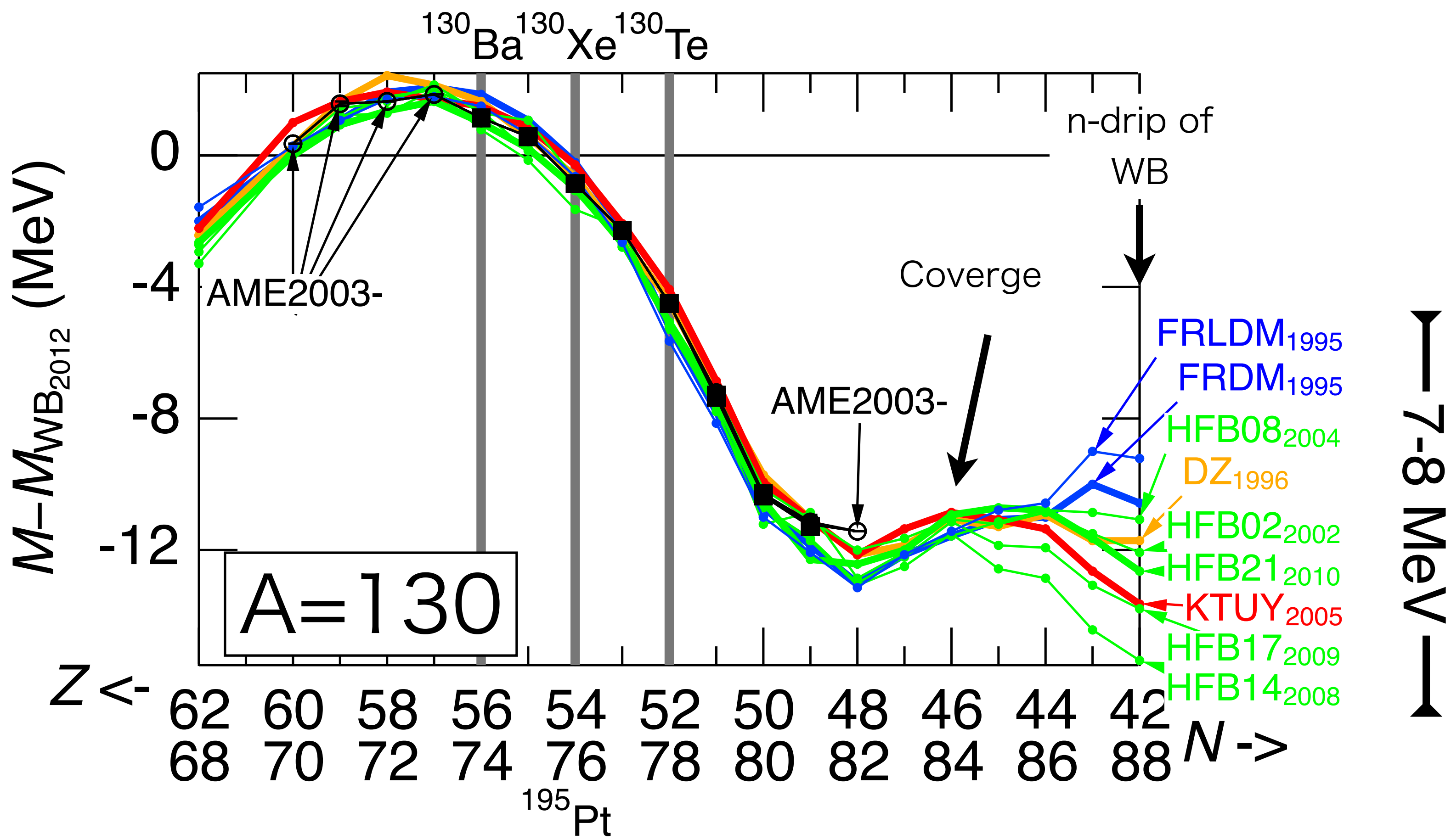






- In old-type mass formulae (-1988), mass values extremely diverge in the very neutron-rich region

Extrapolation to the n-rich nuclei

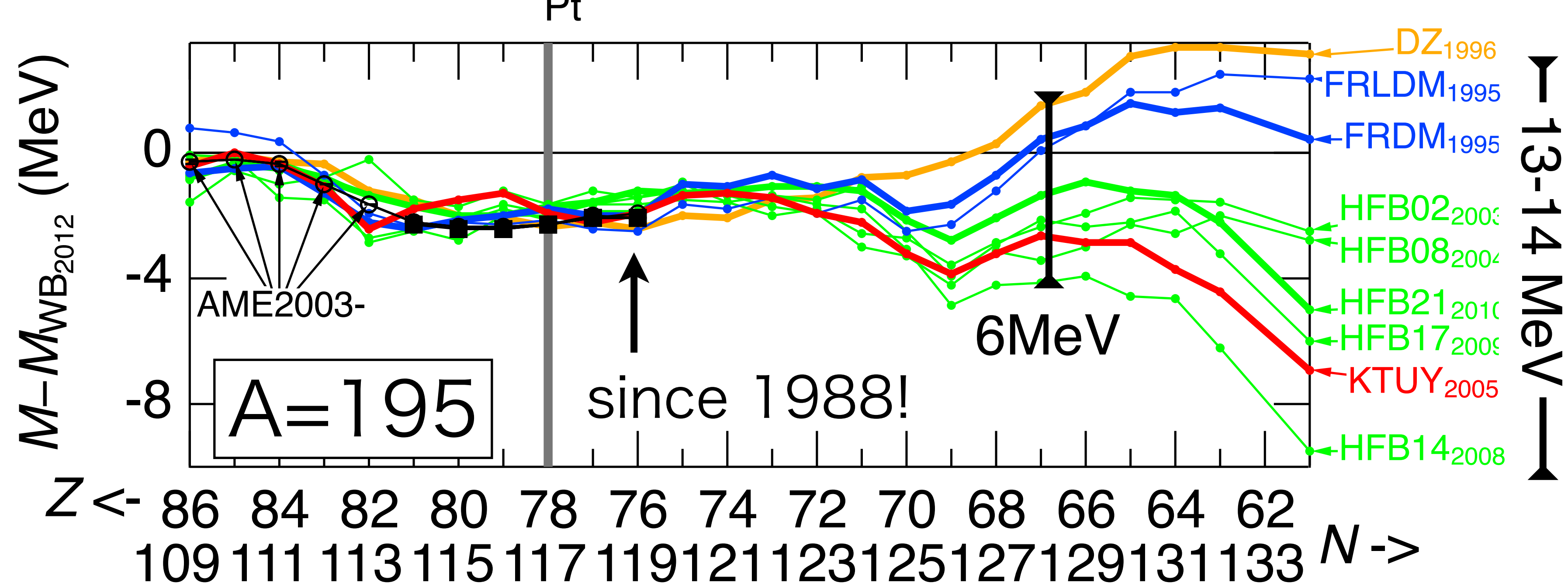


(Only since 1990-)

Predicted mass values still diverge.

Even among HFB's, mass values diverge in the n-rich region. (several MeV)

Poor experimental mass data.



R-process nucleosynthesis

-Check the mass formulae as astrophysical data-

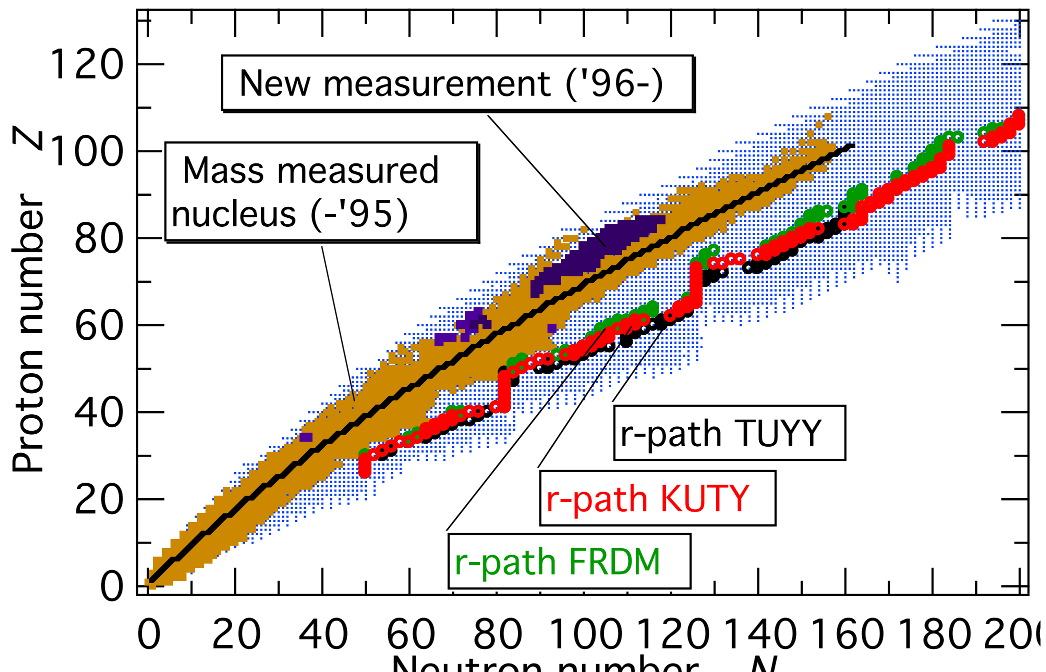
- **Canonical model** Steady flow +Waiting point Approximation

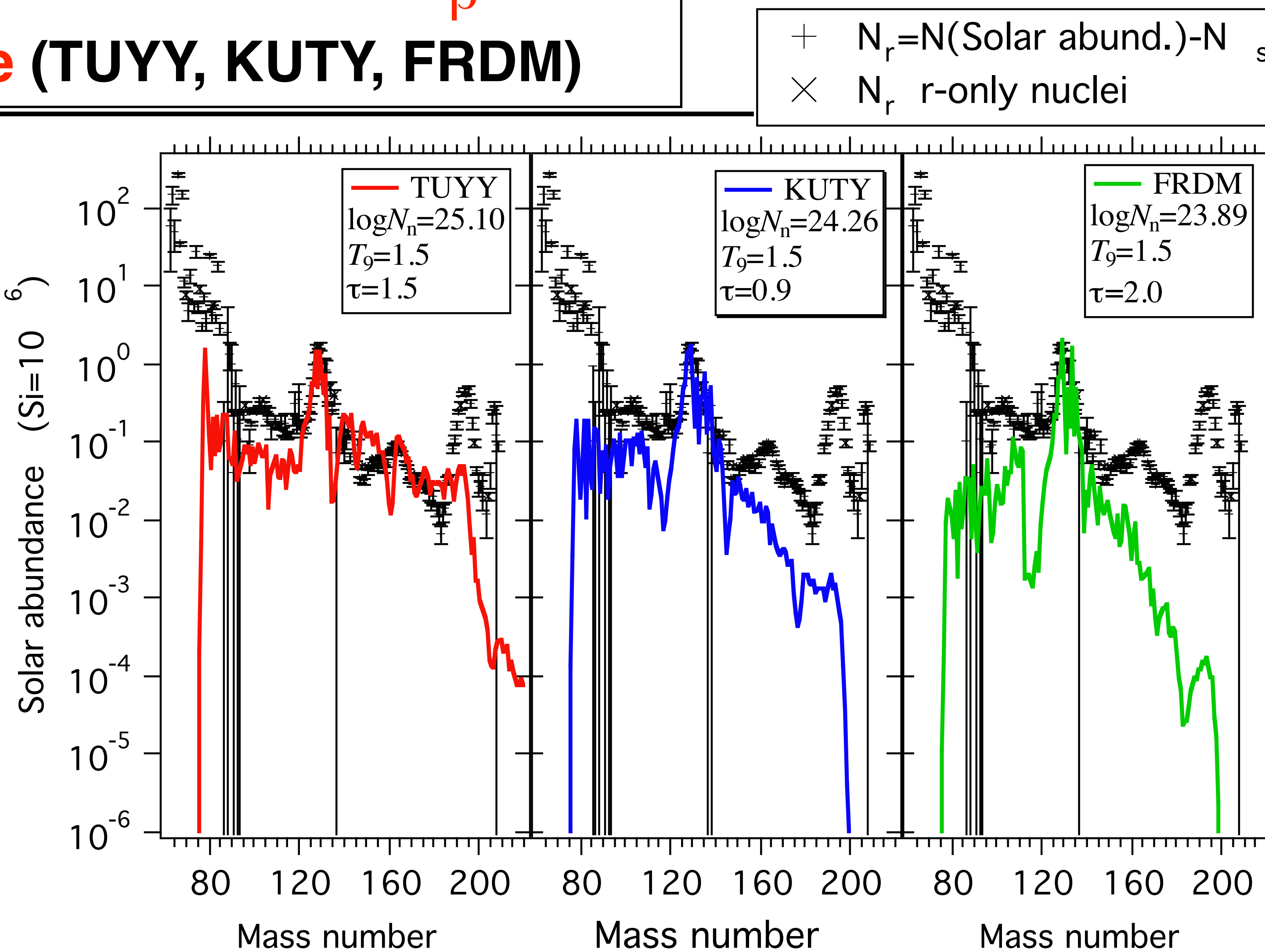
Neutron-number density (N_n) and temperature (T_9) are constants

(n,γ)-(γ,n) equilibrium is established over an irradiation time τ

N_n, T_9, τ: **chosen to reproduce the abundance peak at *A*=130 (obs.)**

S_{2n} for equilibrium eq. (determine the path) and Q_β for λ_β:
estimated from mass formulae (TUYY, KUTY, FRDM)





- **TUYY**: gross term (WB-like with higher expansion) + empirical shell term.
- **KTUY**: TUYY gross term + deformed shell with a modified Woods-Saxon pot.
- **FRDM**: Macroscopic Droplet + microscopic deformed shell with a folded Yukawa pot.

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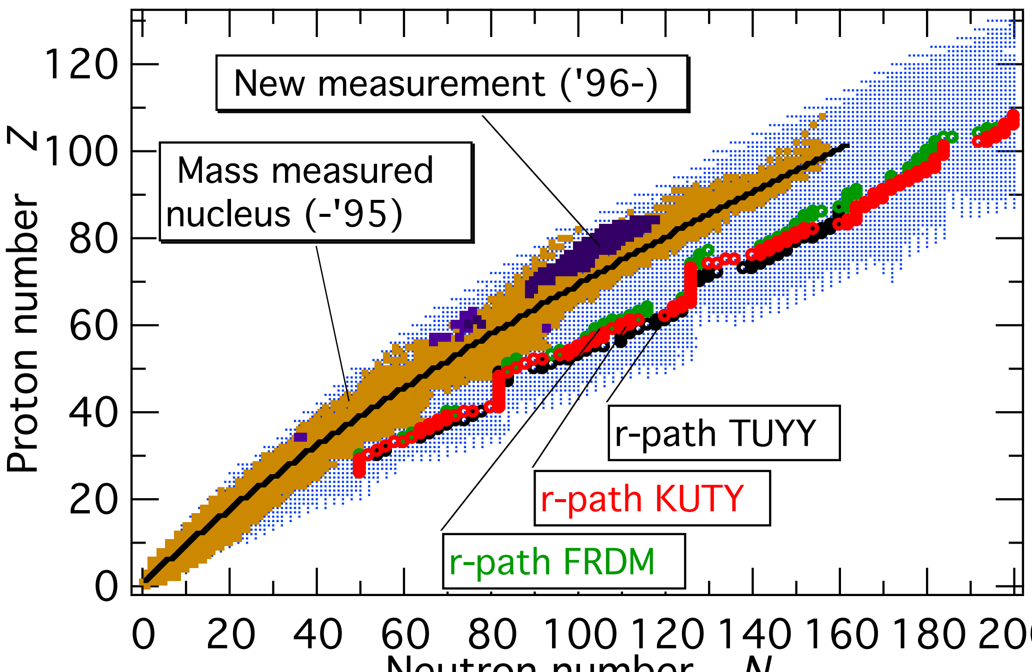

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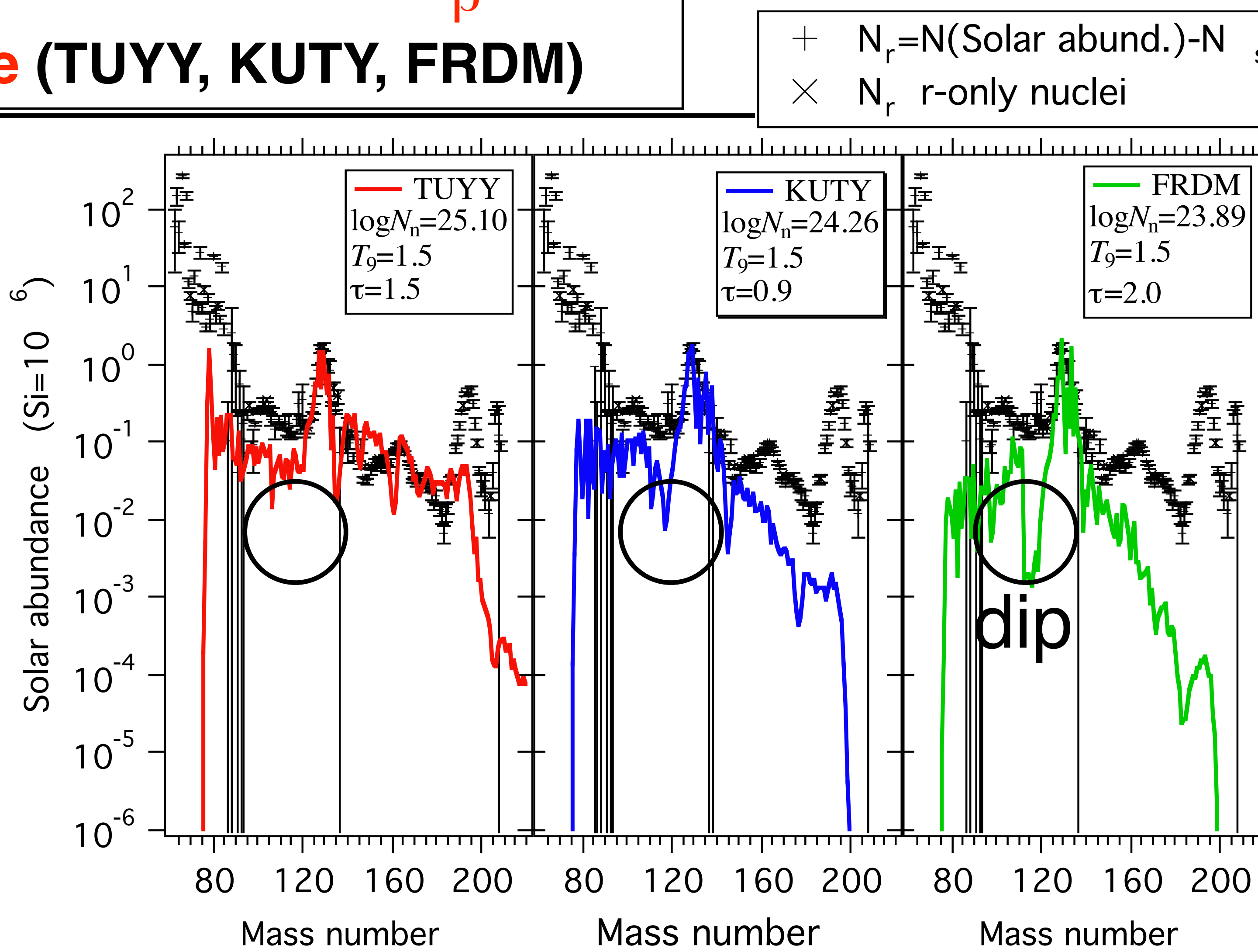
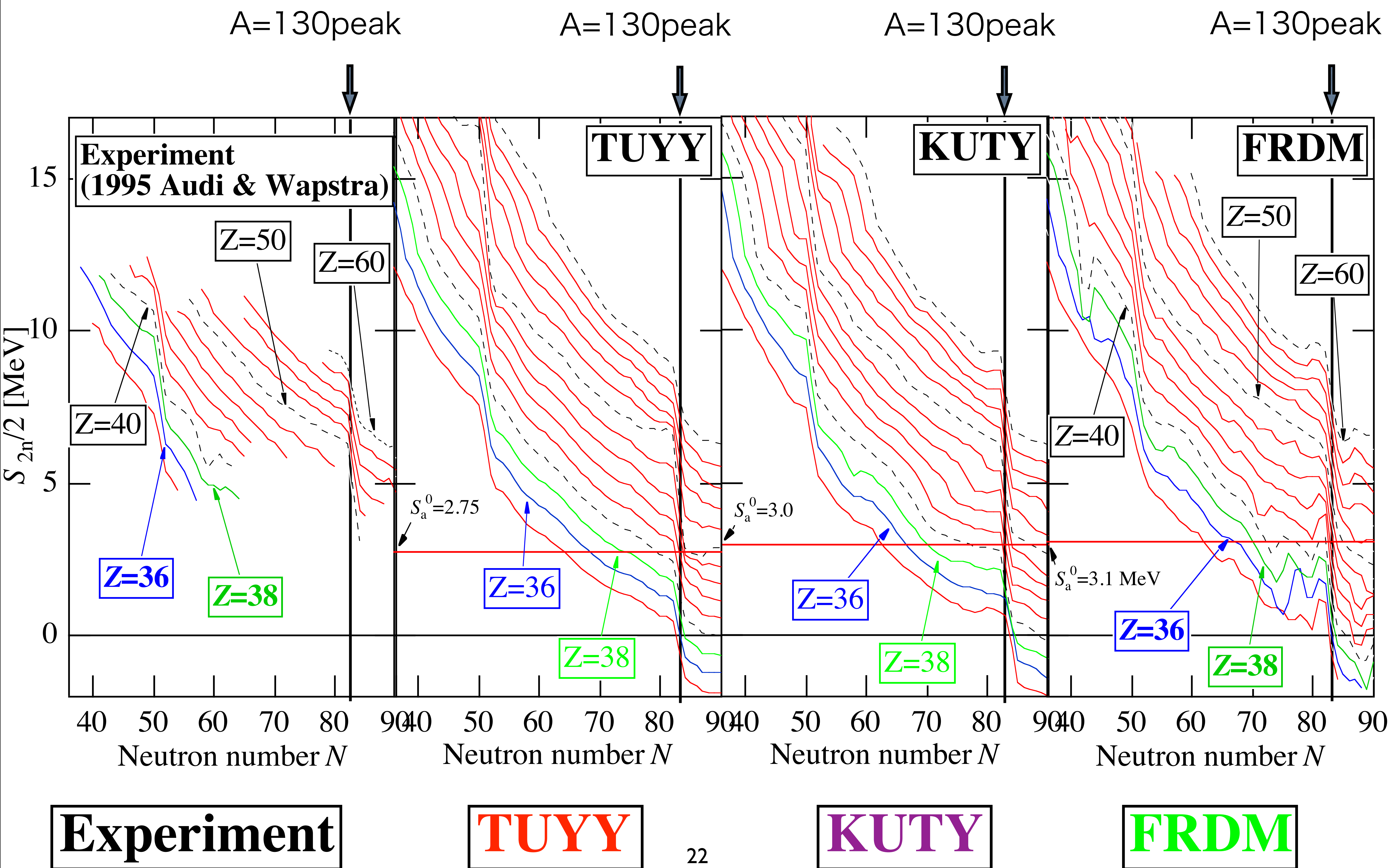
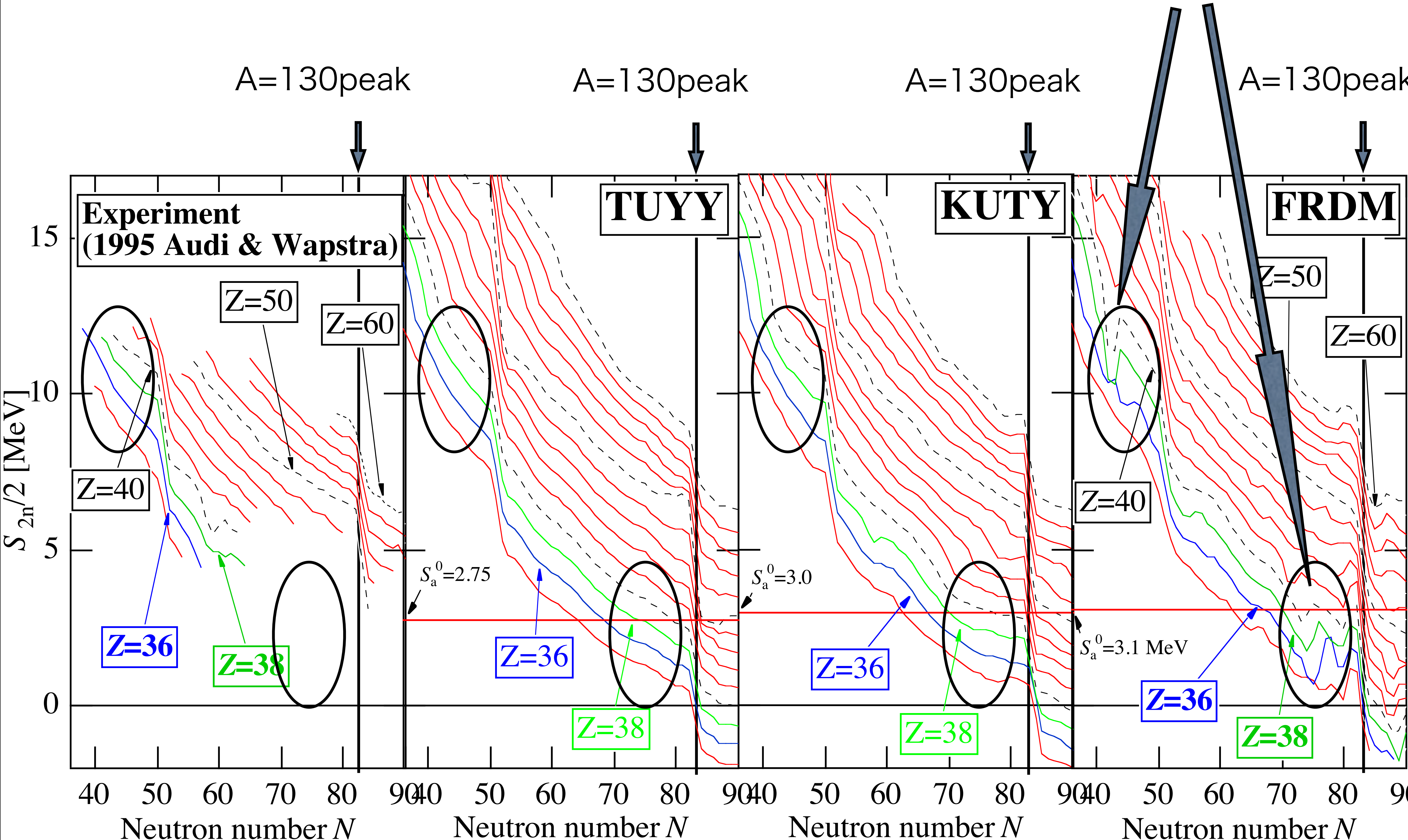


TUYY+GT2 KUTY+GT2 FRDM+GT2

S_{2n} systematics

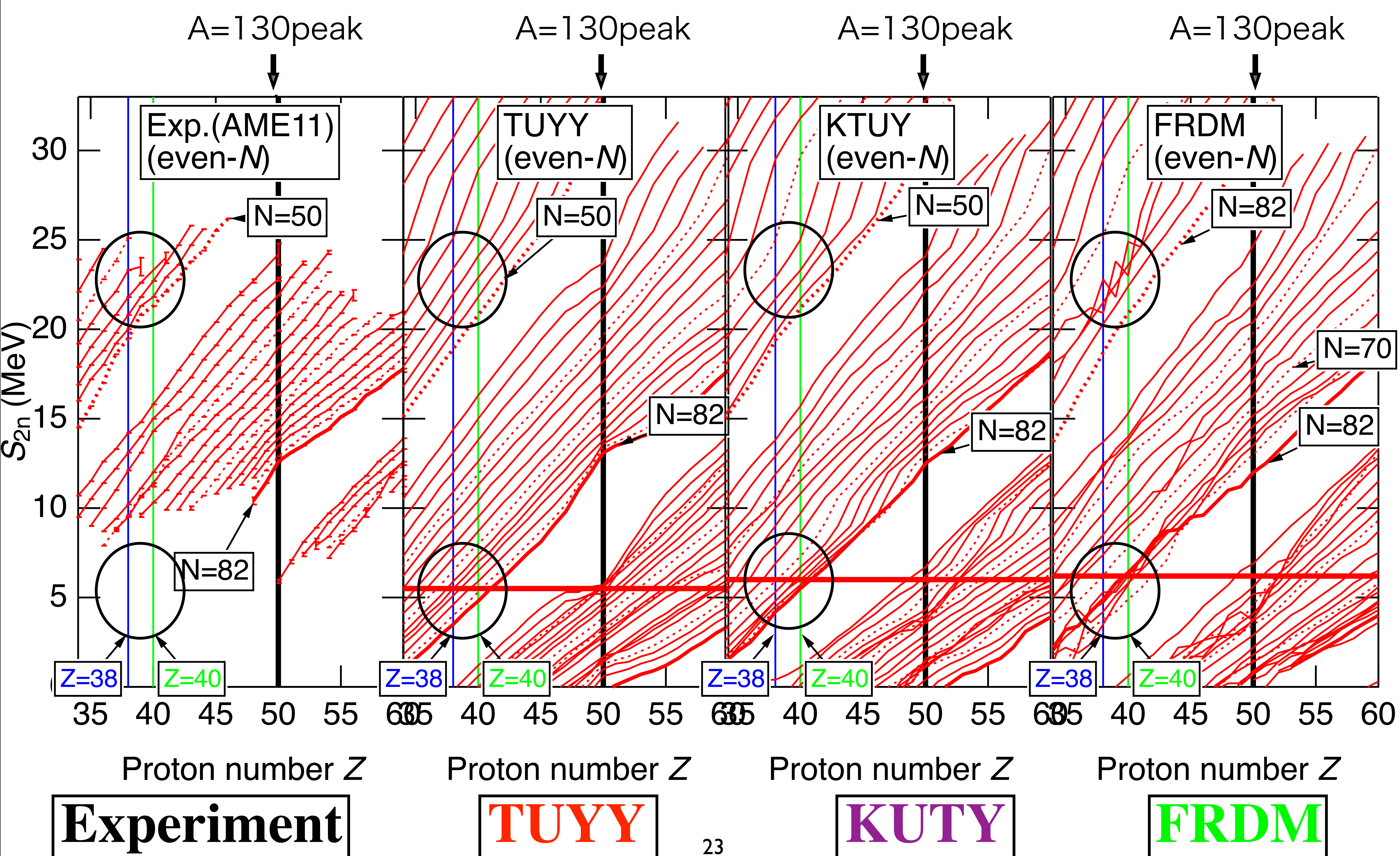


S_{2n} systematics



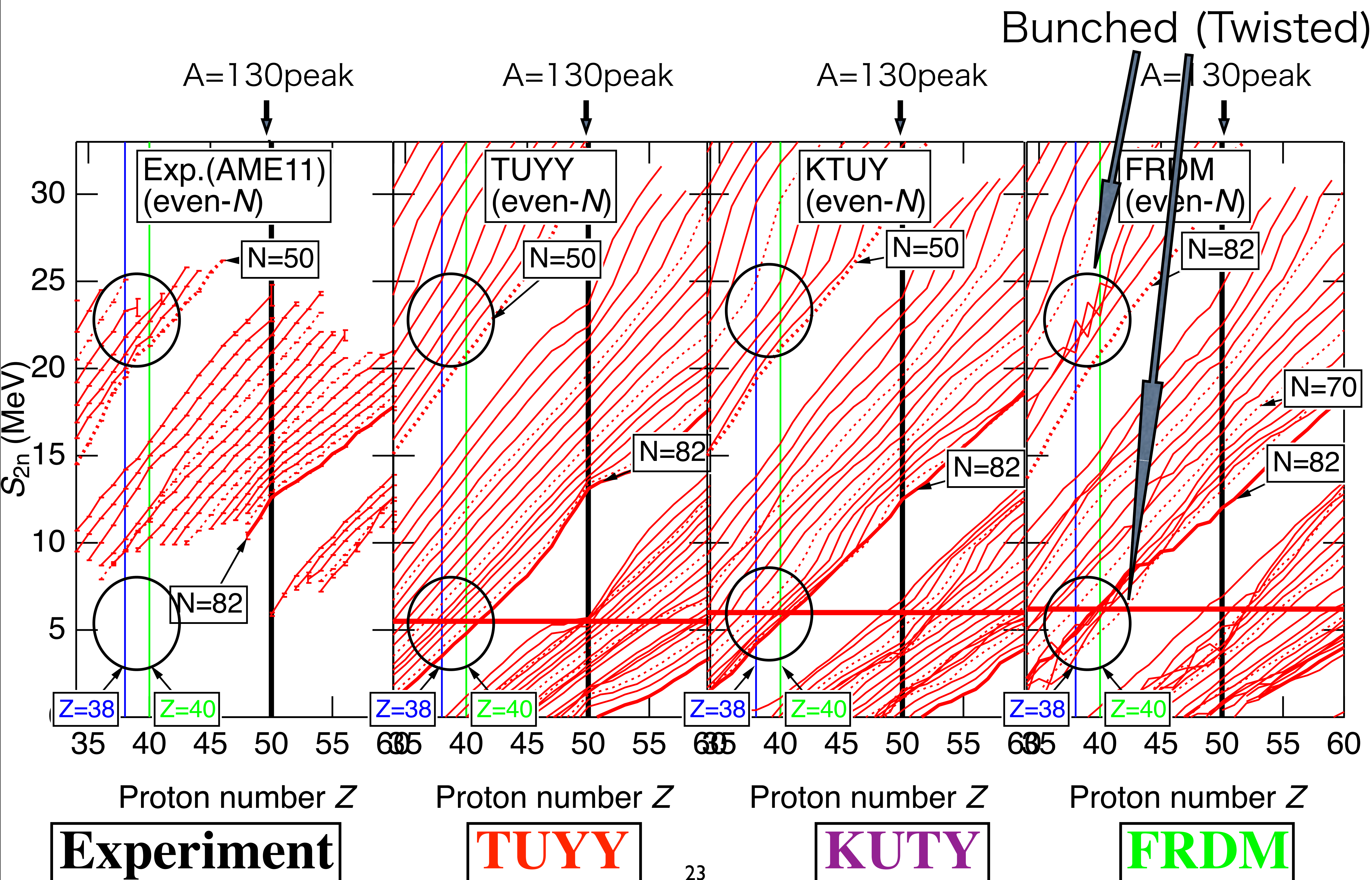
Experiment **TUYY** **KUTY** **FRDM**

S_{2n} systematics



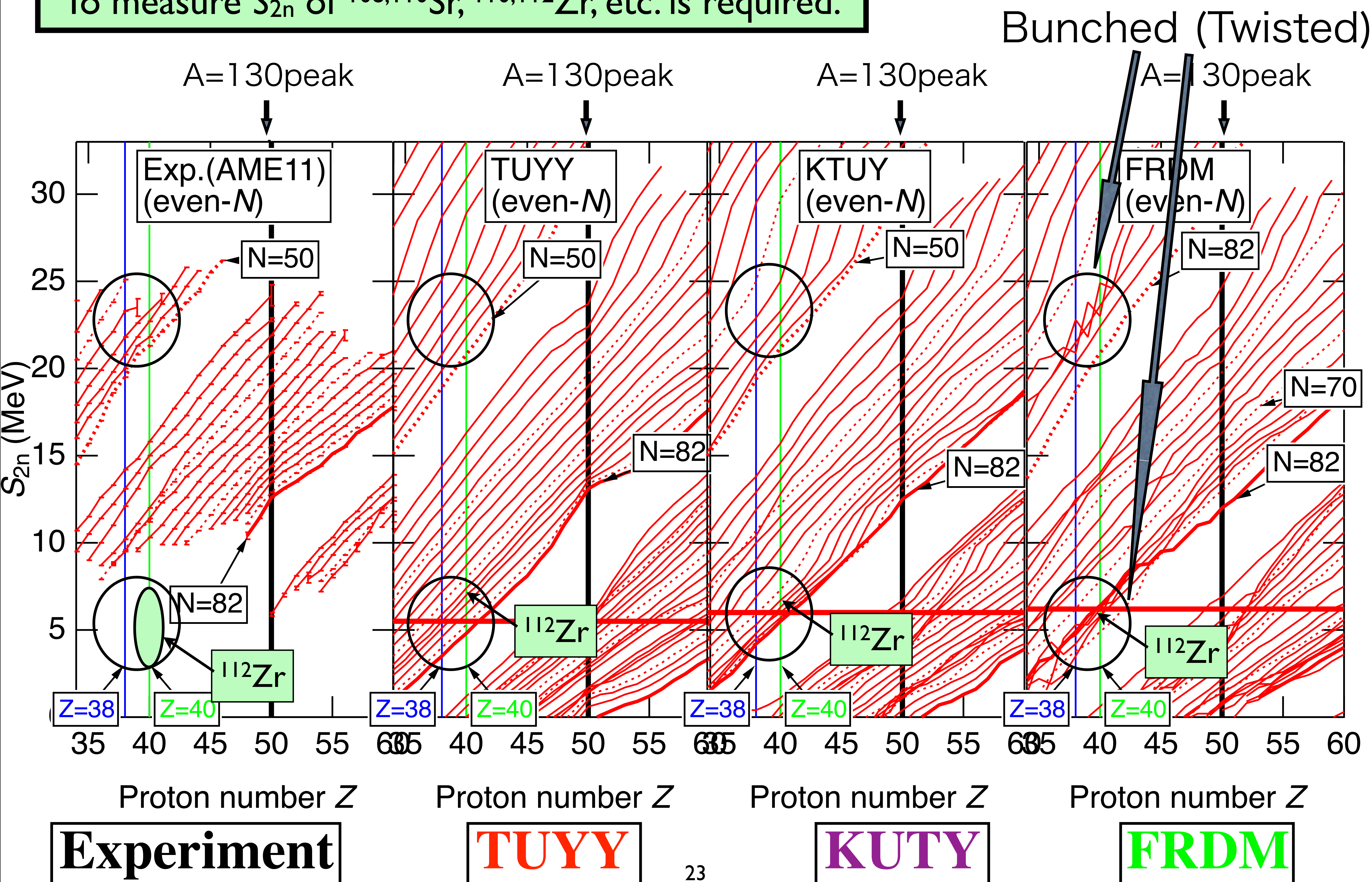
13年5月24日金曜日

S_{2n} systematics



S_{2n} systematics

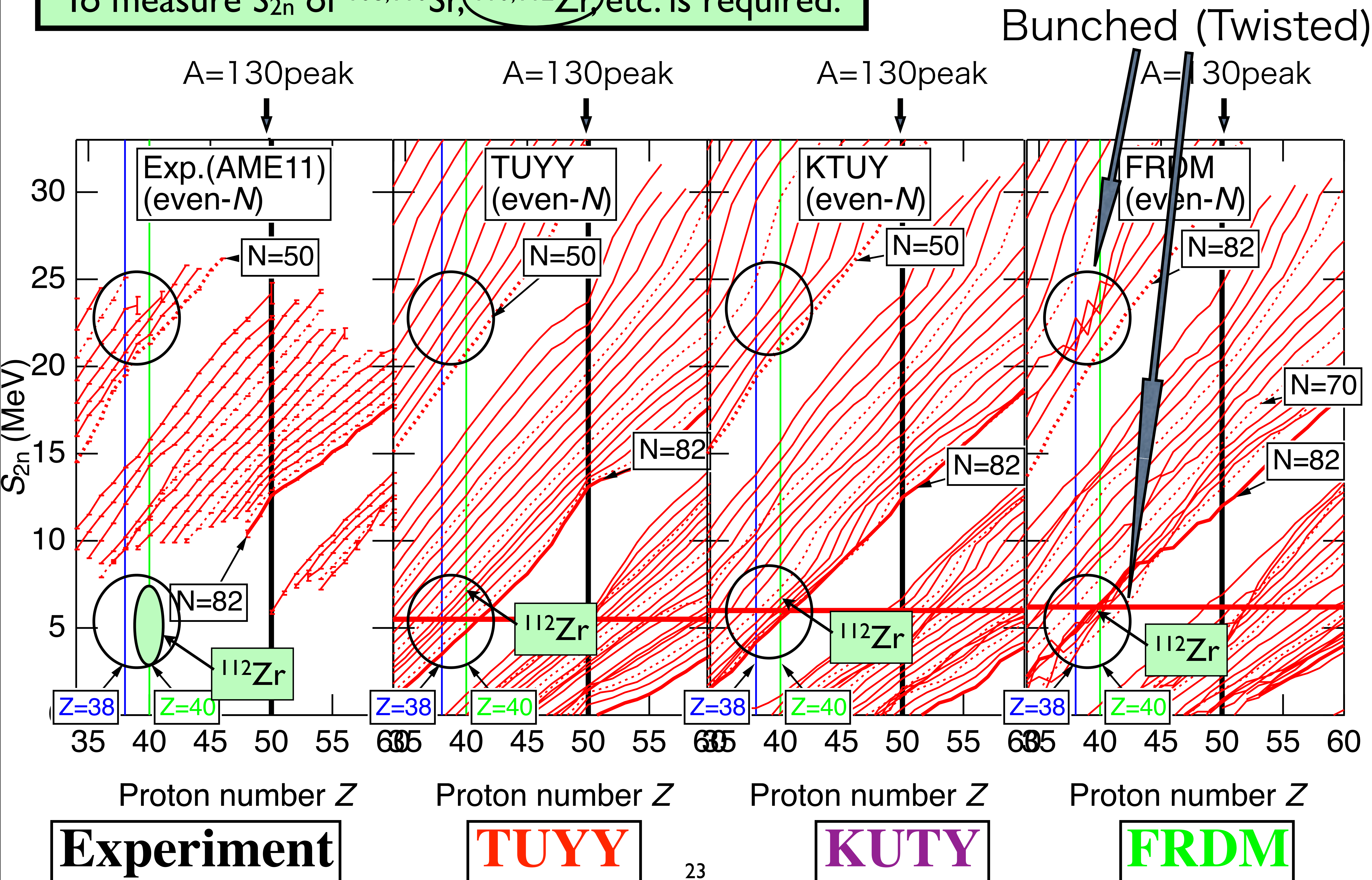
To measure S_{2n} of $^{108,110}Sr$, $^{110,112}Zr$, etc. is required.



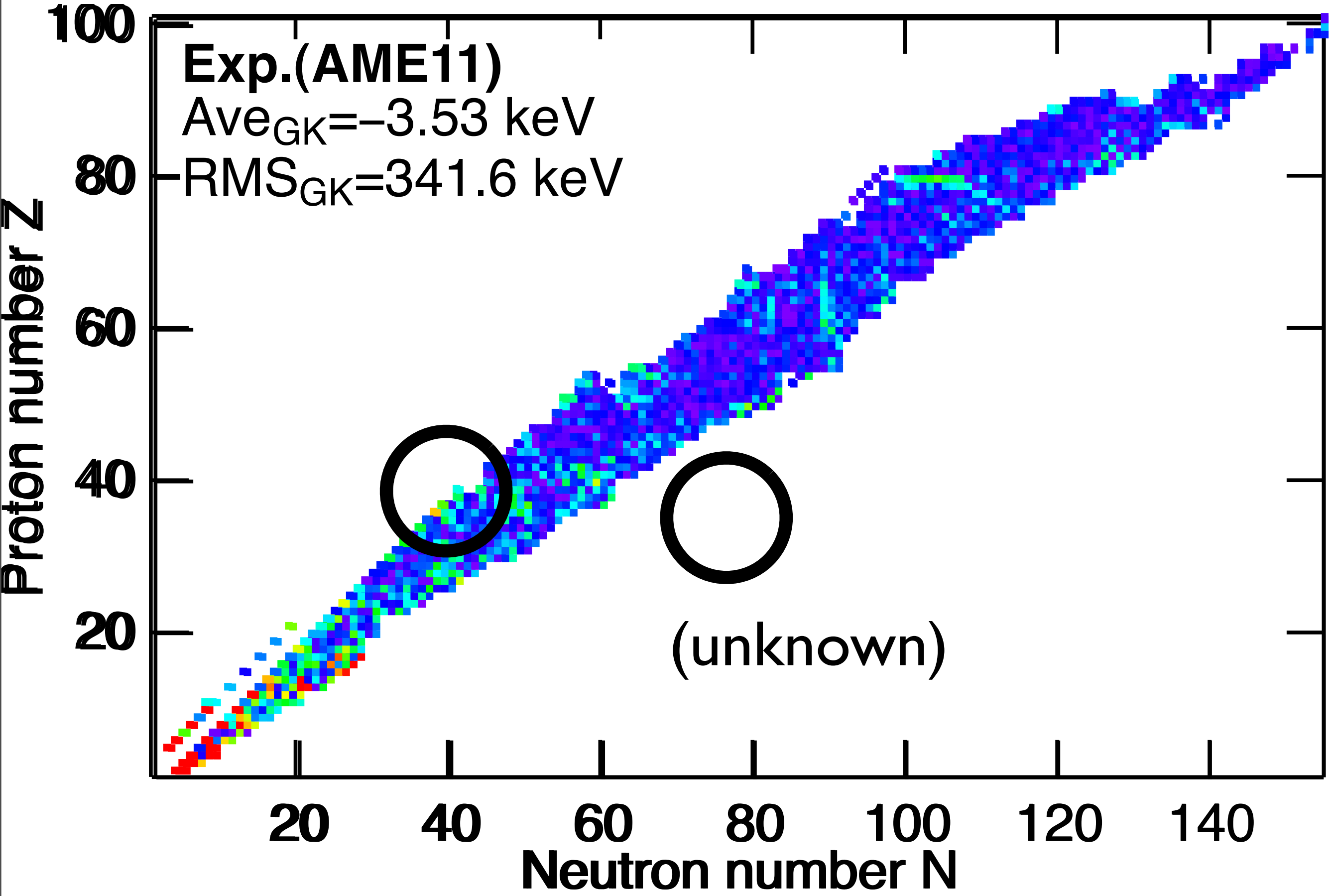
S_{2n} systematics

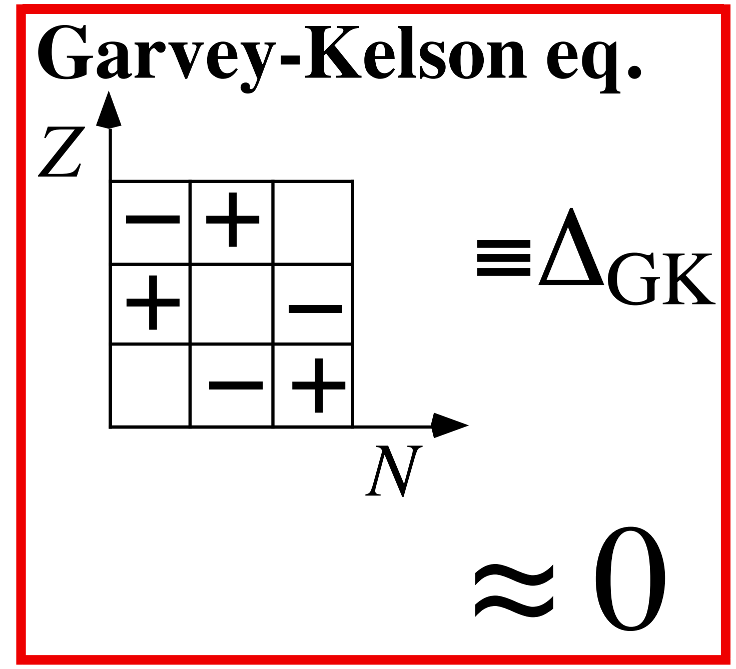
$T_{1/2}$ of ^{110}Zr was measured by Nishimura et al.

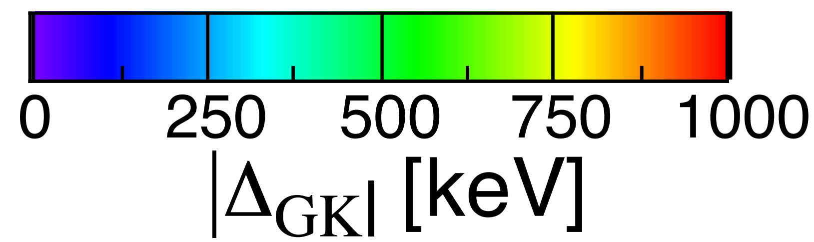
To measure S_{2n} of ^{108,110}Sr, ^{110,112}Zr, etc. is required.

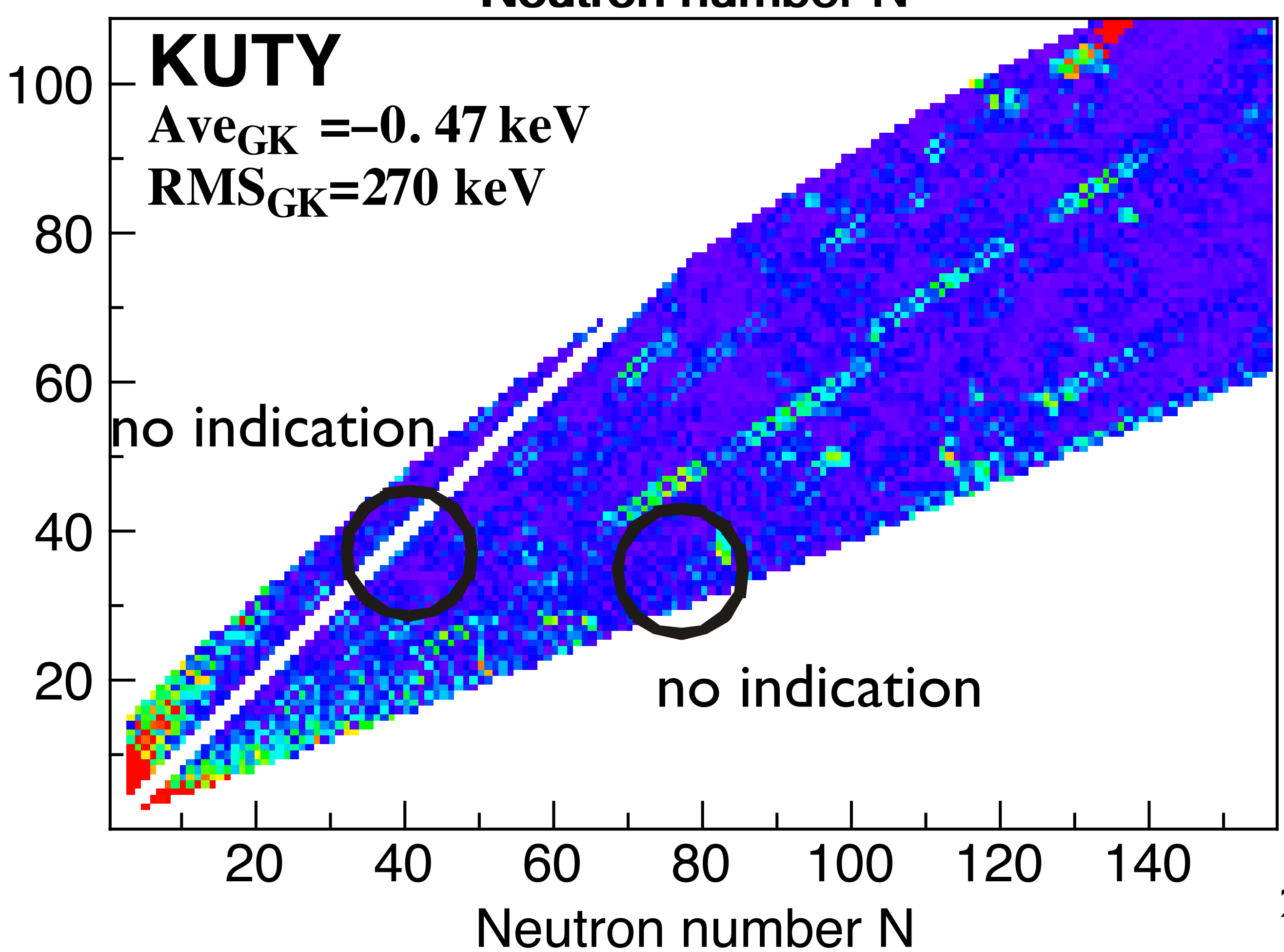


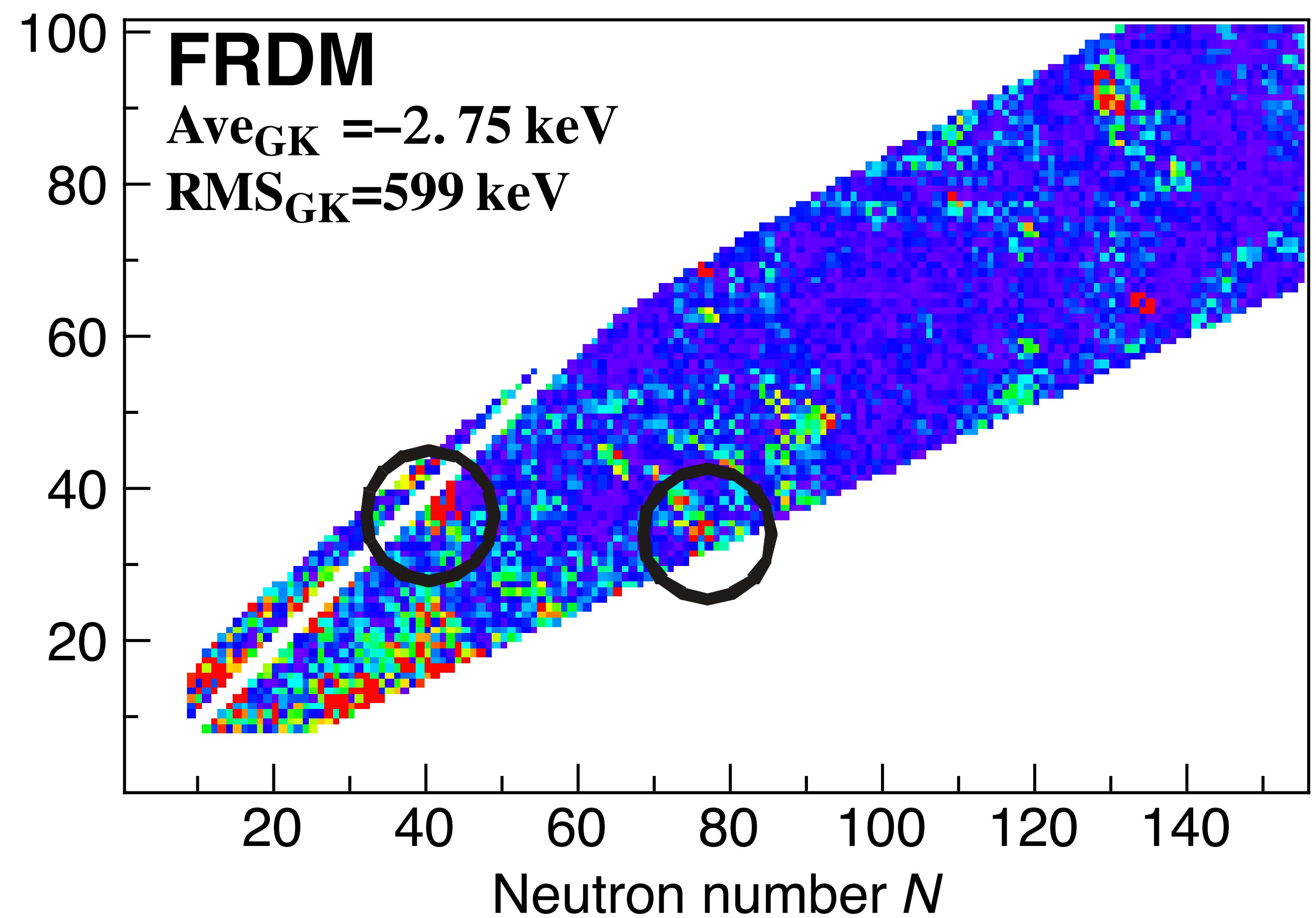
Garvey-Kelson mass relationship



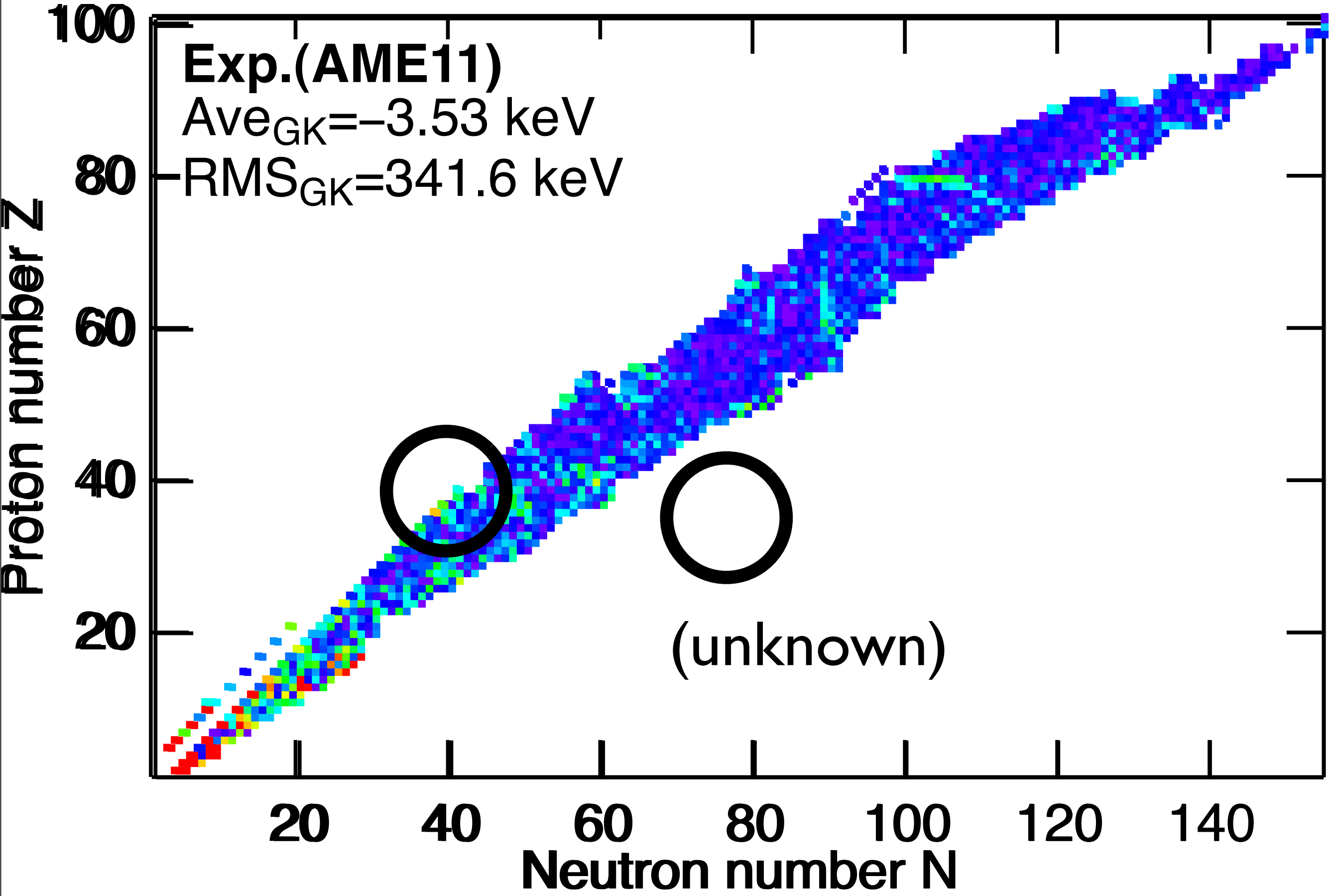


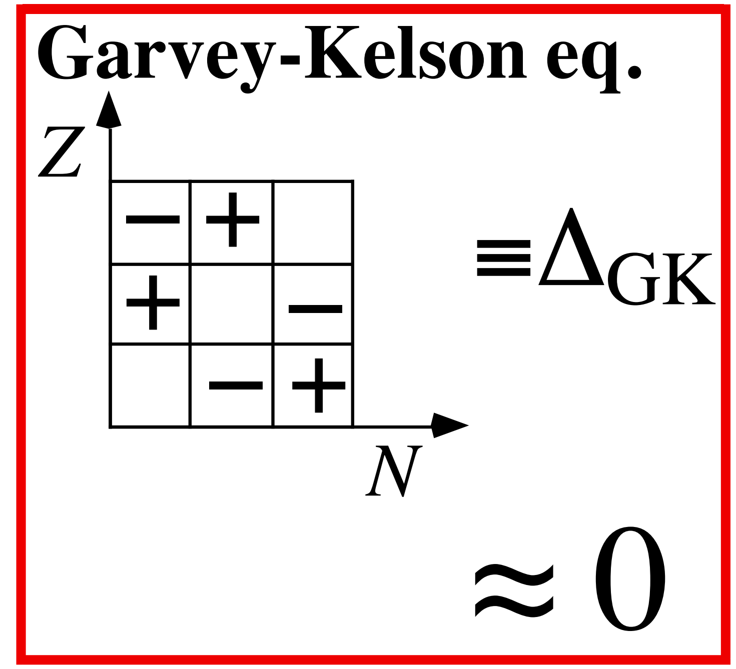


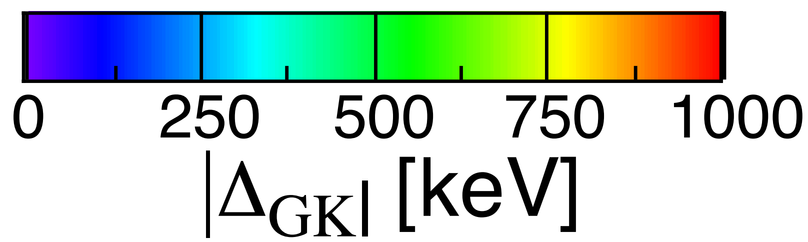


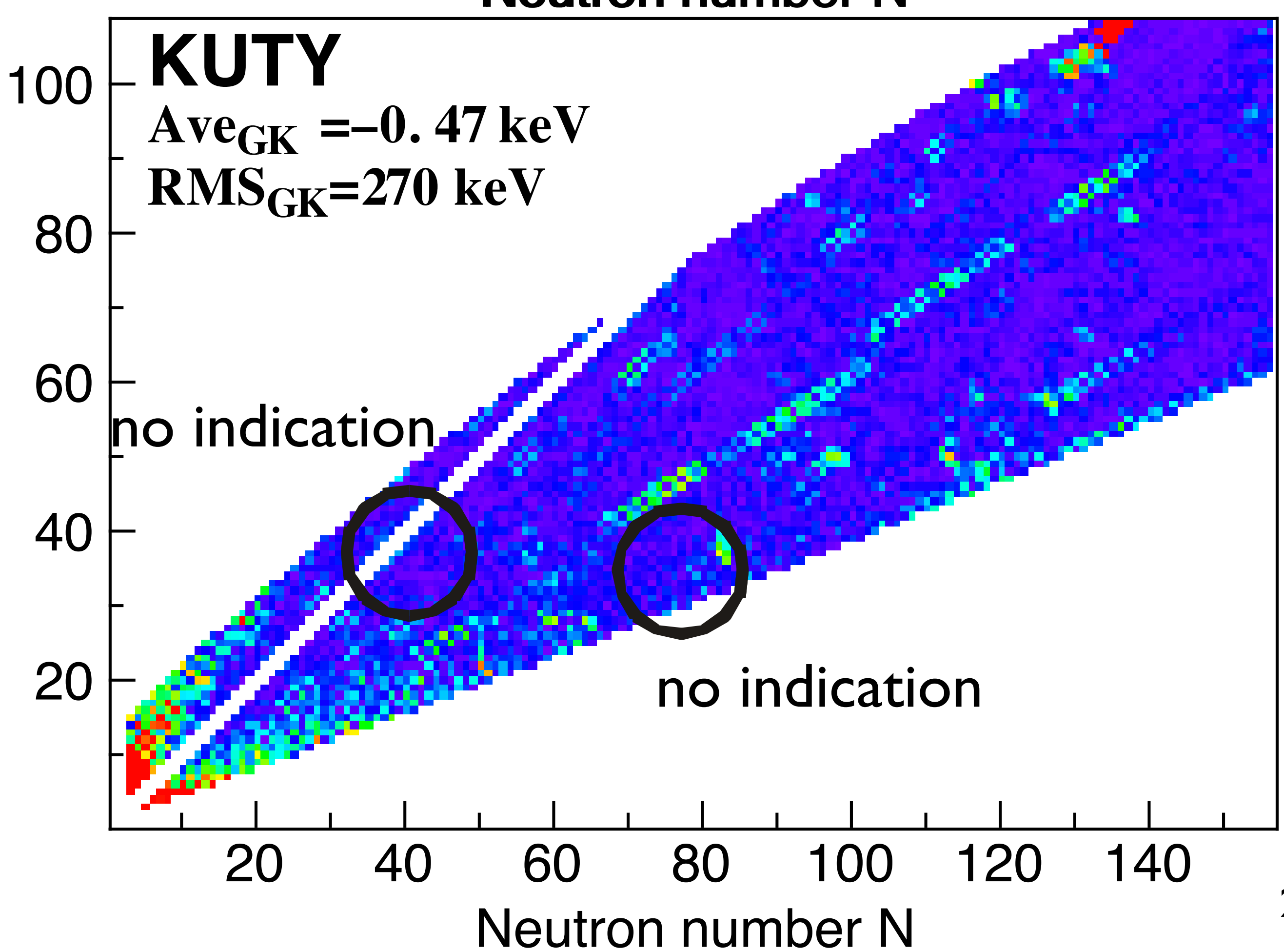


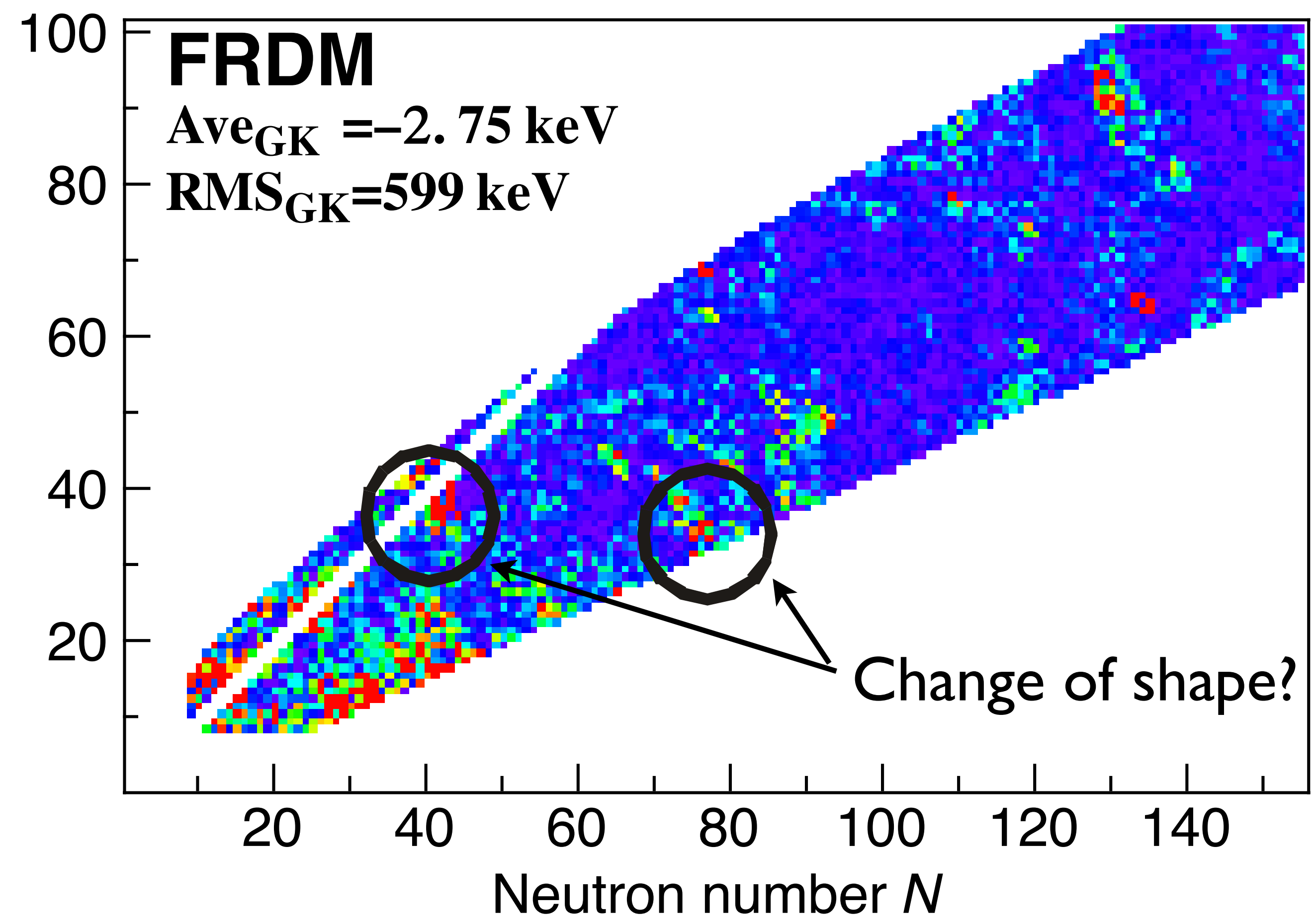
Garvey-Kelson mass relationship







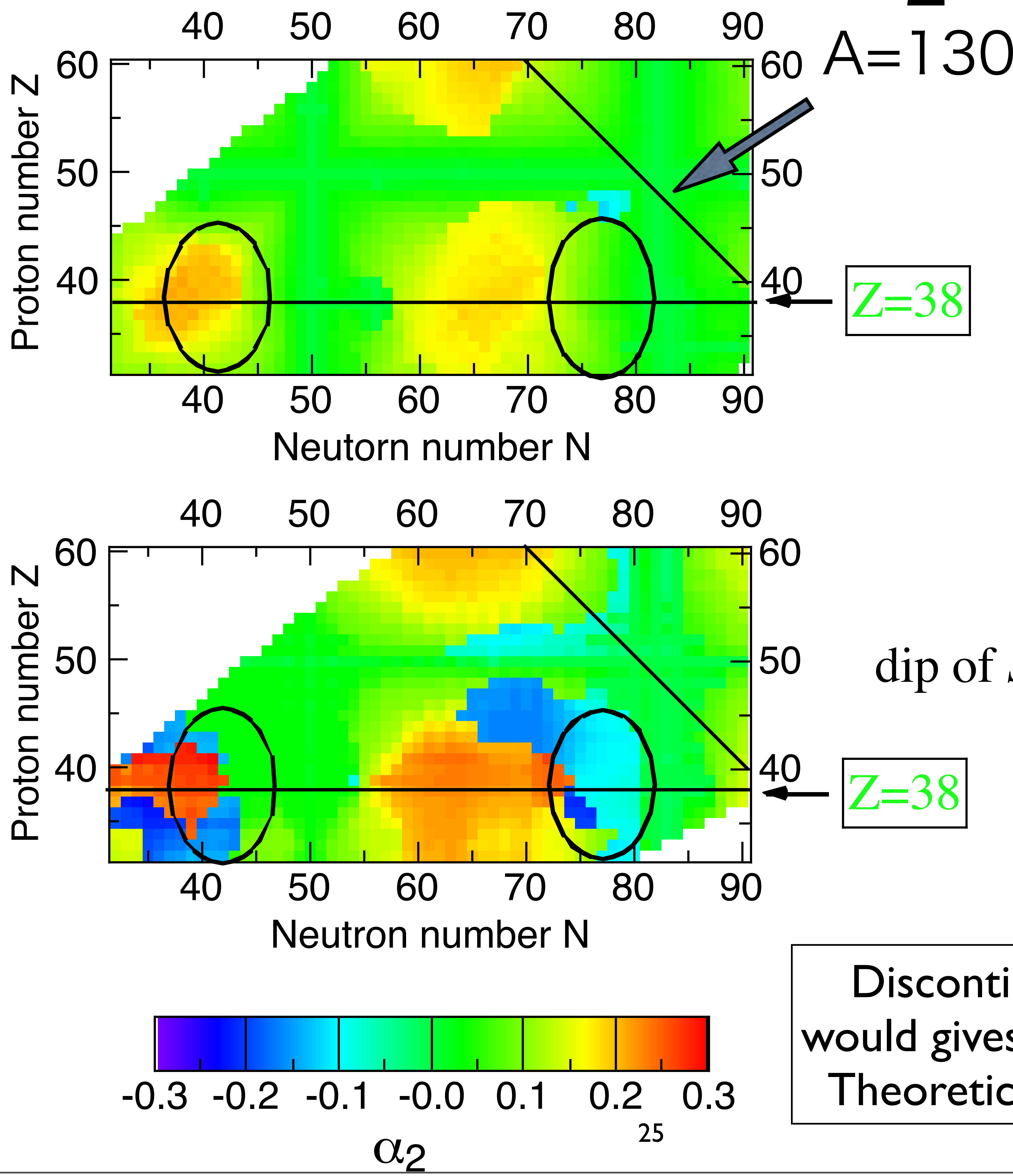




Deformation parameter α_2

KUTY

FRDM



dip of S_{2n} <=> change of α_2

(FRDM)

Discontinuous change of shape
would gives kink of Sn. (FRDM case)
Theoretical (numerical) problem?

Another possibility: Influence of shell-quenching far from stability

Chen, Dabaczewski, Kratz, Langanke, Thieleman, Vogt, Phys. Lett. **B355** (1995)37-44

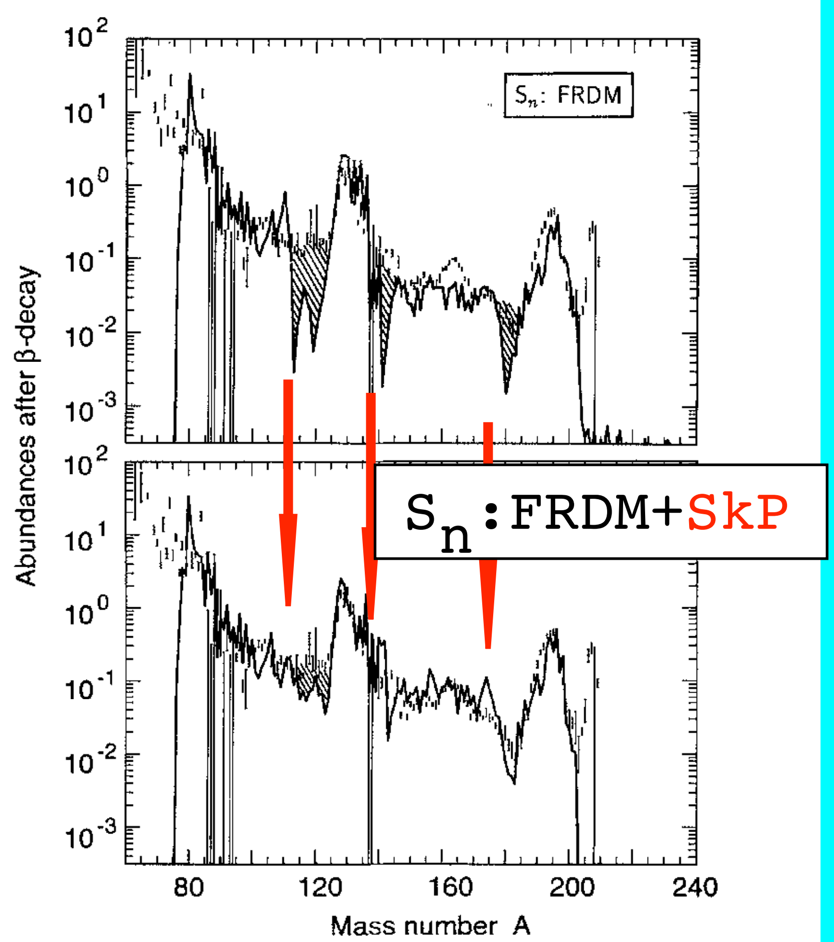


Fig. 2. r-process abundance fits obtained with ten equidistant neutron-density components from 10^{20} cm^{-3} to 3×10^{24} cm^{-3} according to Fig. 1. In the upper part, the result is presented for FRDM [10] masses with the $T_{1/2}$ and P_n values from the QRPA calculations according to Ref. [11]. In the lower part, masses of spherical nuclei around $N = 82$ have been replaced by masses from HFB calculations with the Skyrme force SkP. The quenching of the $N = 82$ shell gap (see Fig. 4) leads to a filling of the abundance troughs around $A \simeq 120$ and 140, and to a better overall reproduction of the heavy-mass region.

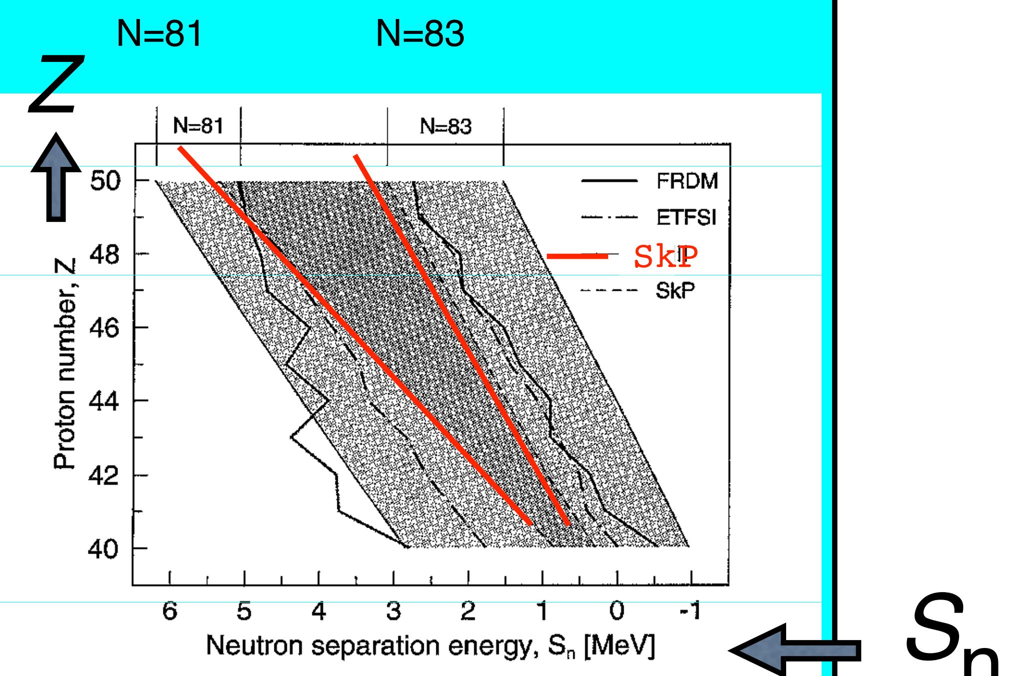


Fig. 4. Comparison of S_n values for the isotones $N = 81$ and 83 as predicted by different mass models. The difference $\delta S_n = [S_n(N = 81) - S_n(N = 83)]$ is a measure of the $N = 82$ shell strength and is shaded for SIII (light) and SkP (dark). The shell quenching with distance from stability for SkP, in contrast to SIII, can be recognized. Masses of odd-odd nuclei have not been calculated in our SIII study.

shell-quenching
=> decreasing of dips
around the peaks

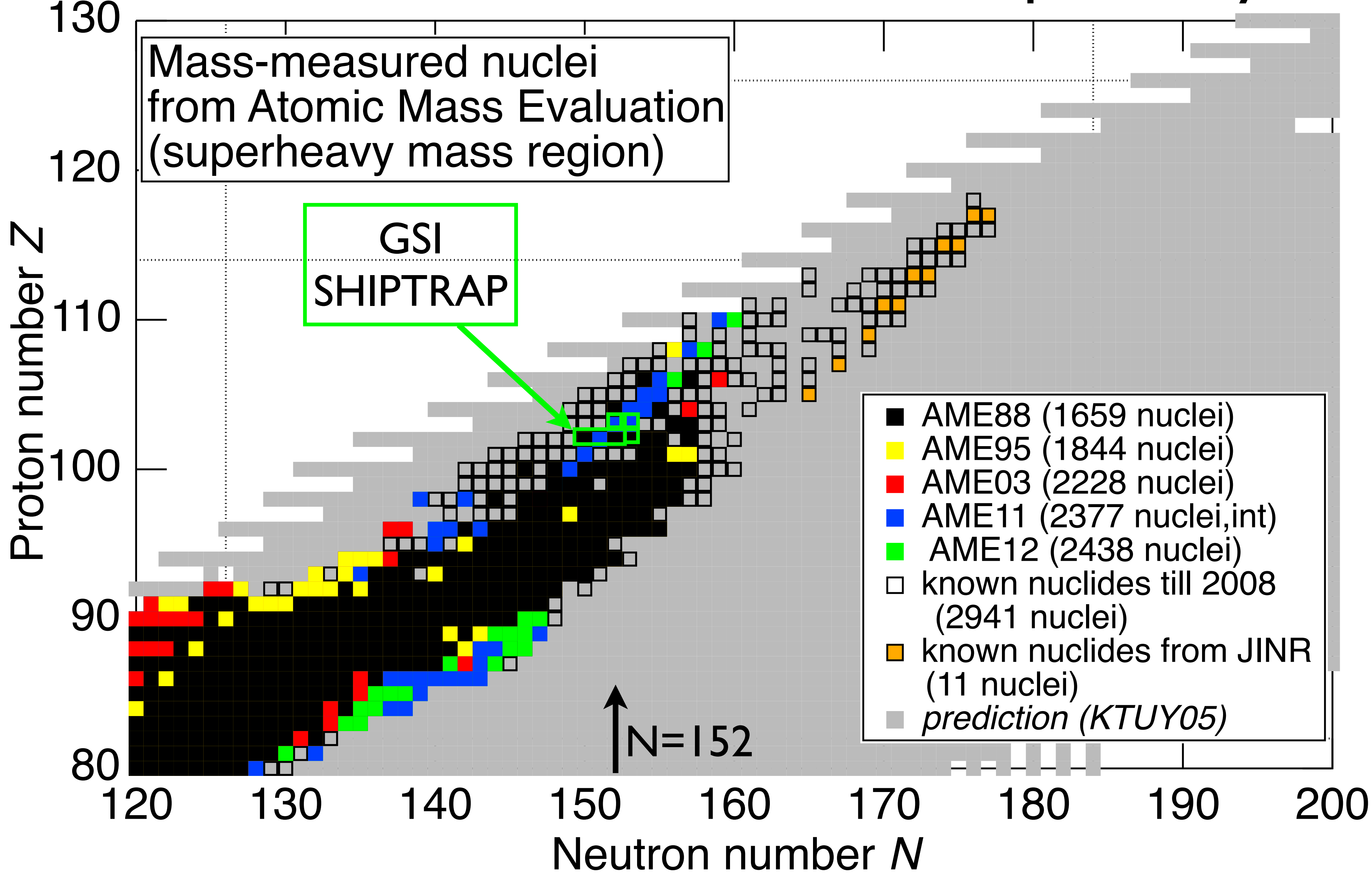
(Chen et al. 1995)

Kink of S_{2n}, or shell quenching?

It would be artificial?

Superheavy nuclei

Mass-measured nuclei in the superheavy mass region



Poor mass-measurement in the superheavy mass region

Why is direct mass measurement?

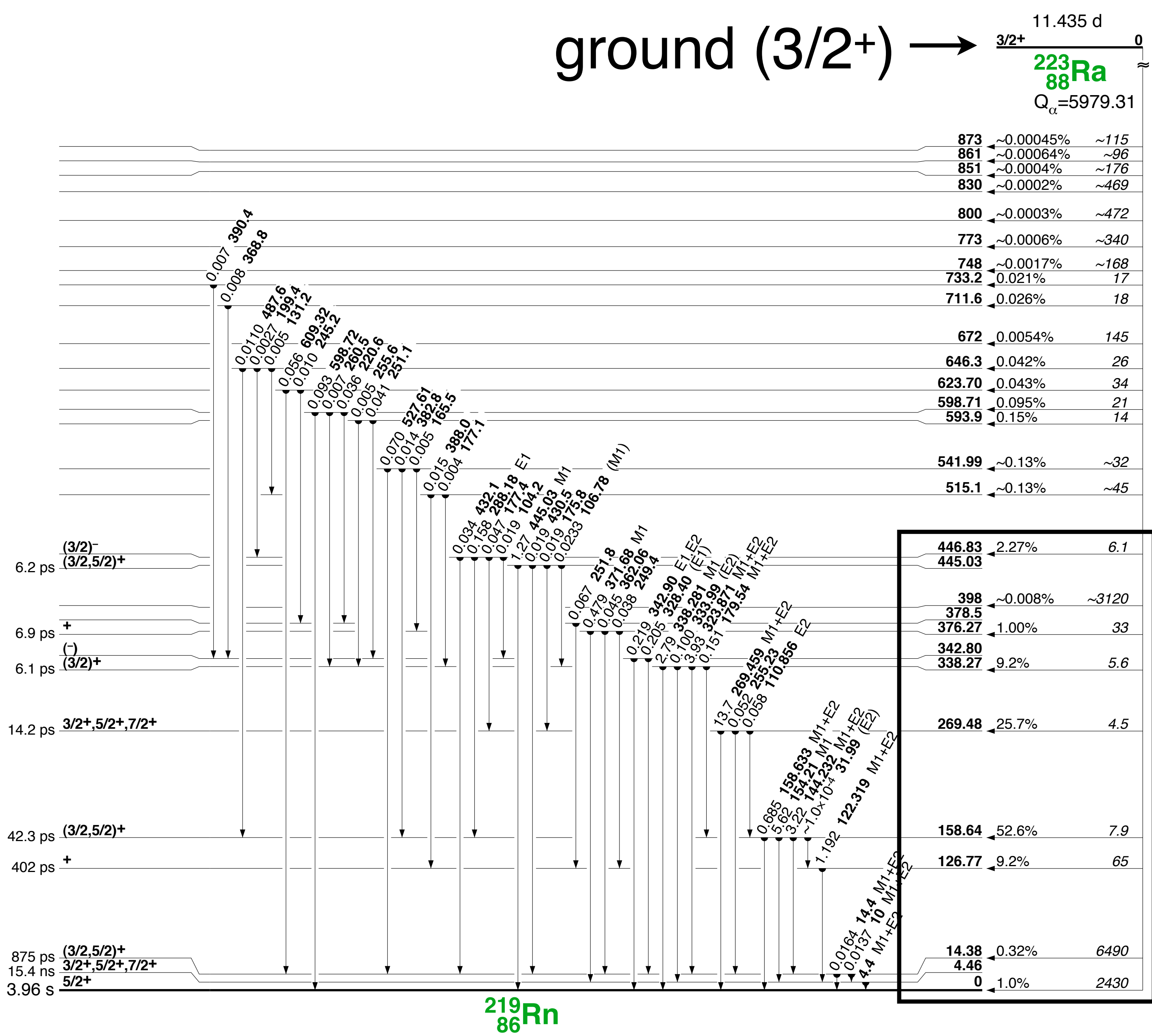
Why is direct mass measurement?

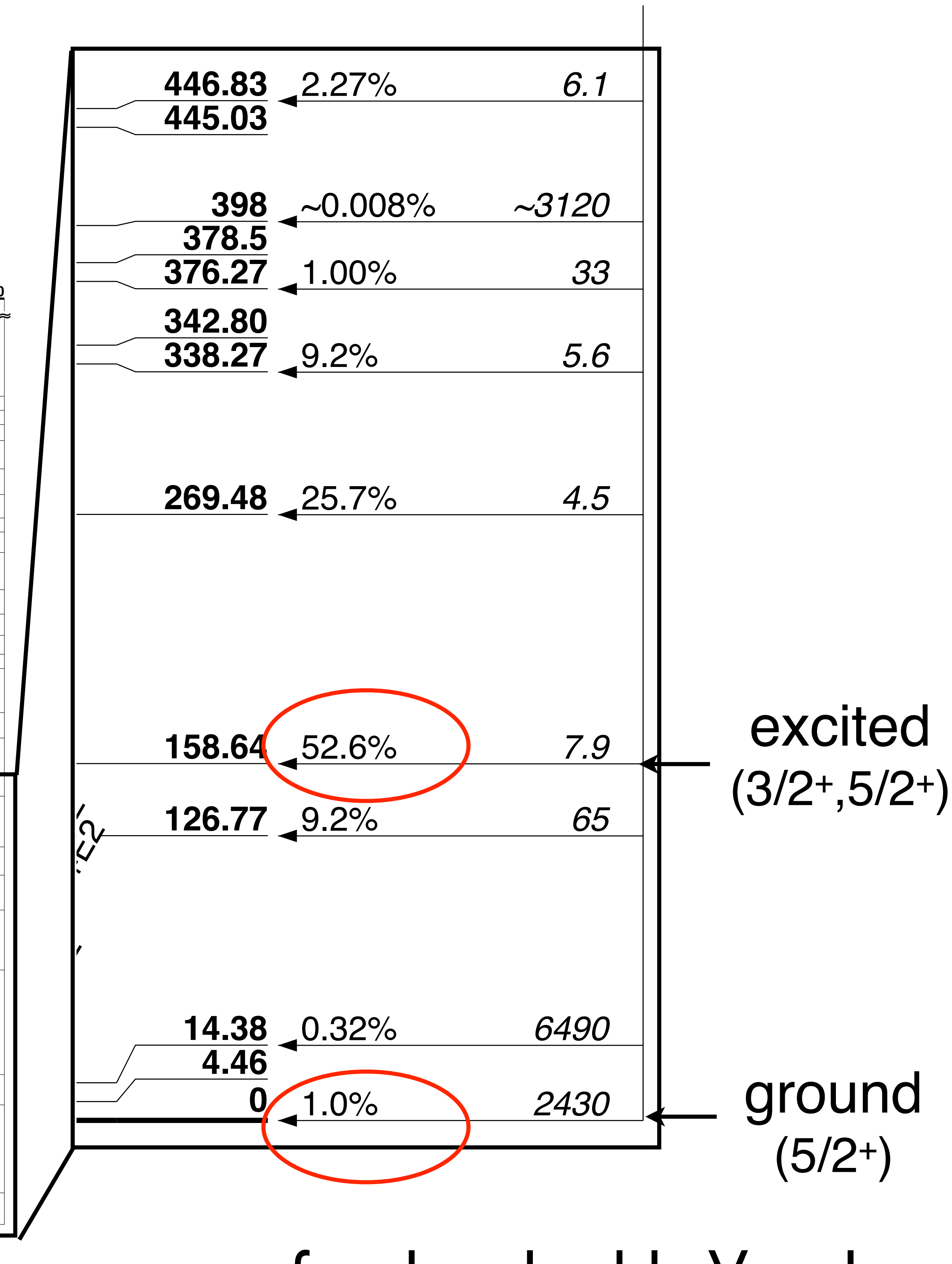
↓

To obtain ground-state energies (essentially important!)◎

Example: ground to excited states for odd-A

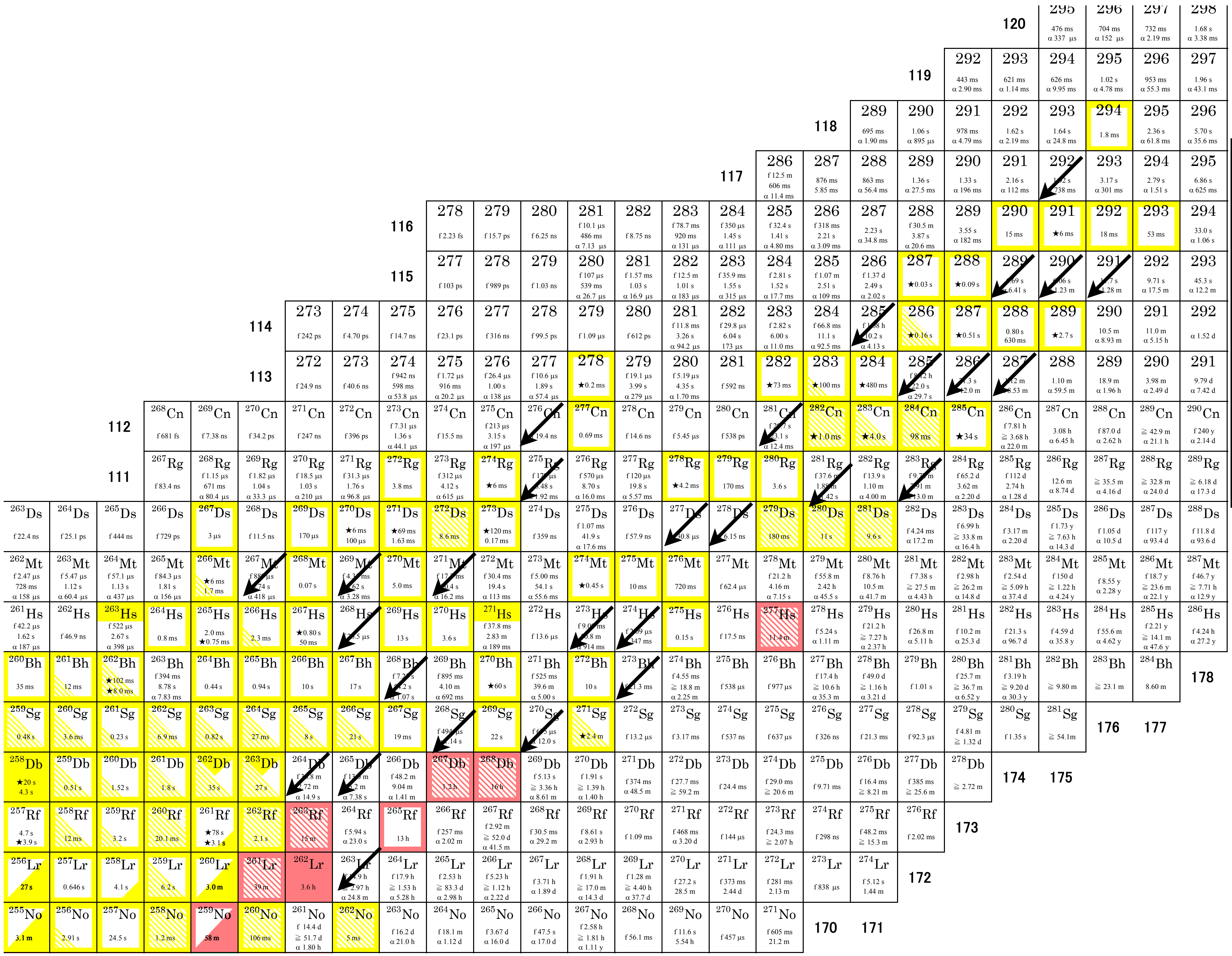
$^{223}Ra \;—(\alpha)\rightarrow\; ^{219}Rn$





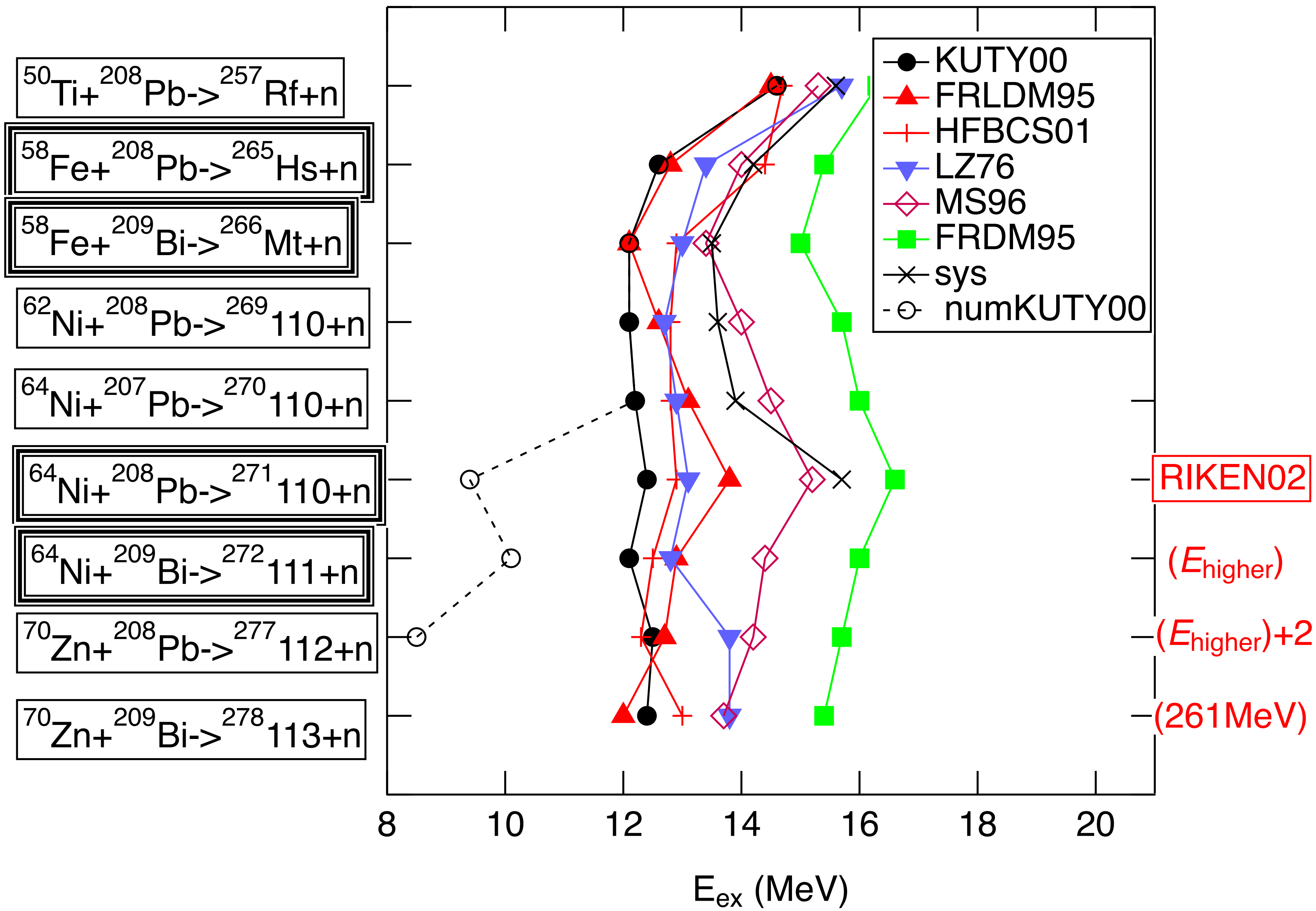
a-few-hundred-keV-order

Alpha decay chains in SHE



- Almost energies are obtained from alpha-decay chains.
- No relations of other directions are obtained.
- Direct mass-measurements are required.

Beam energy from (predicted) excitation energy



Energy at maximum cross section
(derived from injected beam energy with Q of mass formulas)

Estimation of absolute values of beam energy depends on (unknown) masses of compound nuclei

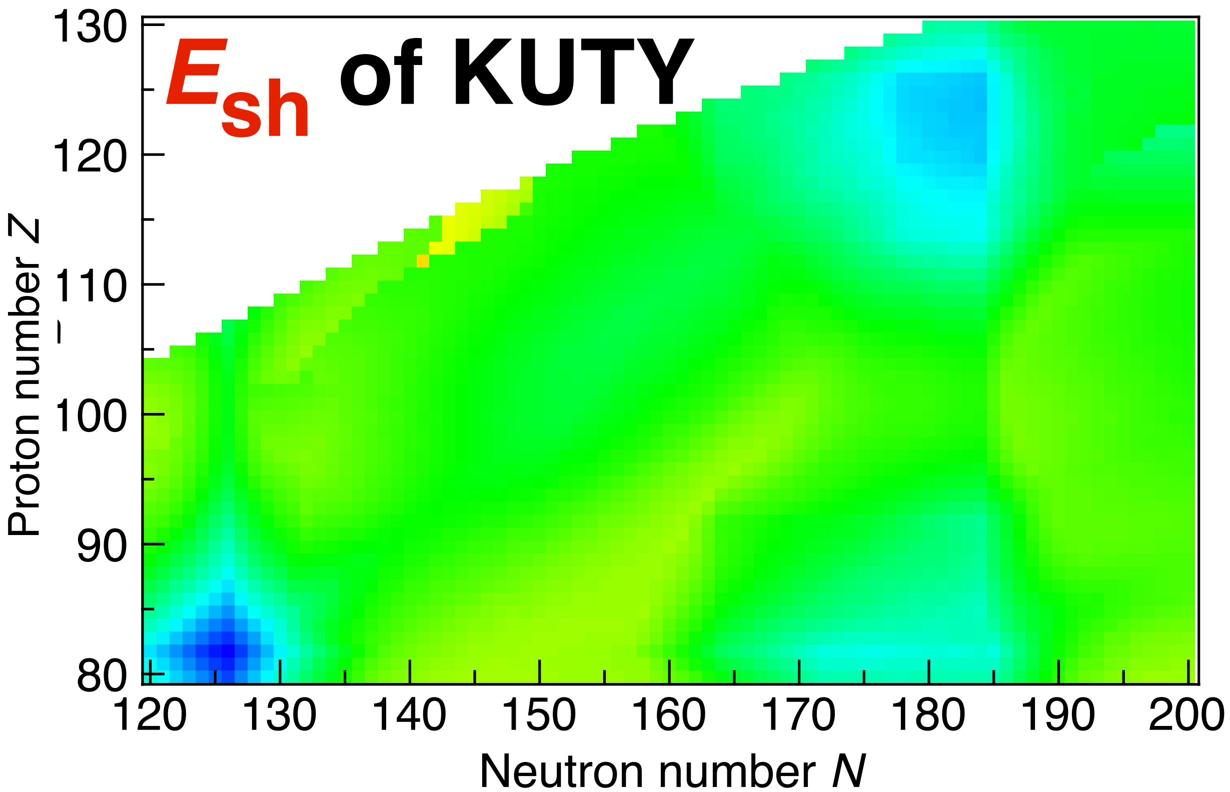
Shell energy in the superheavy mass region

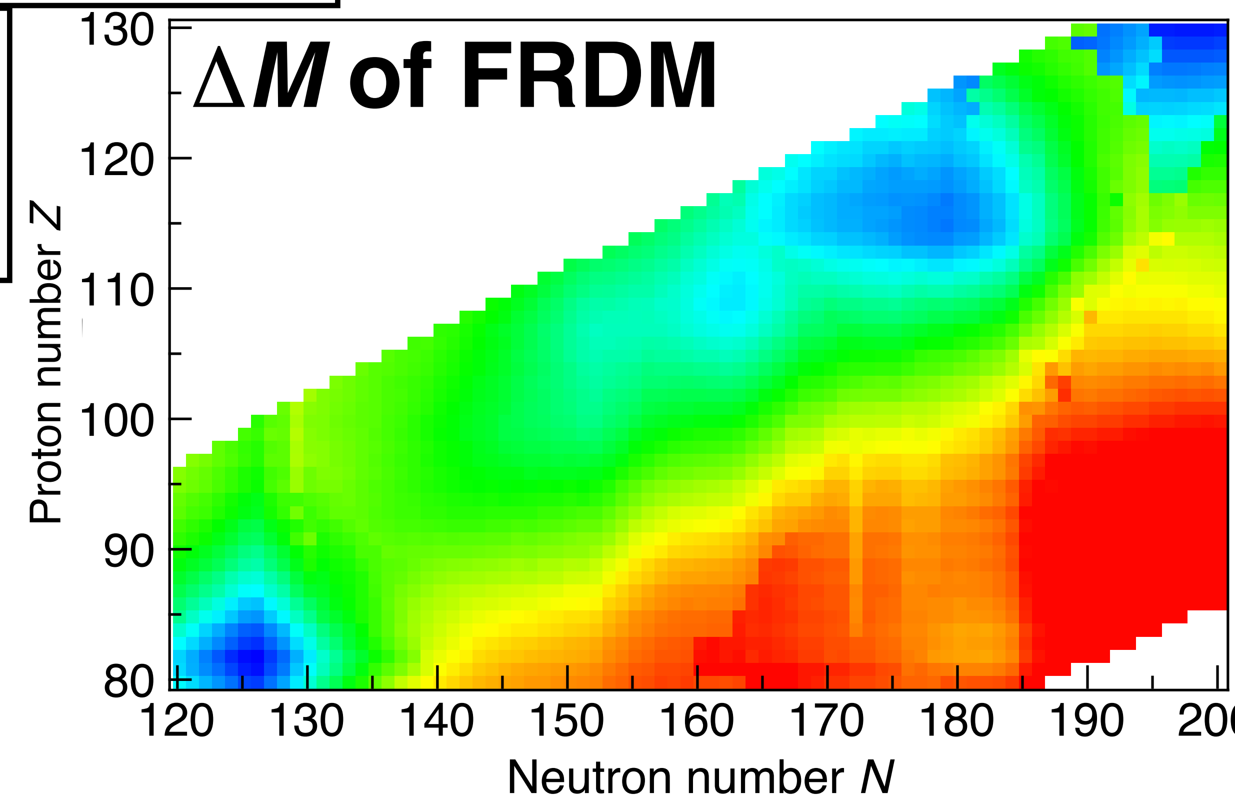
$$\Delta M(Z,N) = M_{\mathrm{FRDM}}(Z,N) - (M_{\mathrm{gross}}(Z,N) + M_{\mathrm{eo}}(Z,N))$$

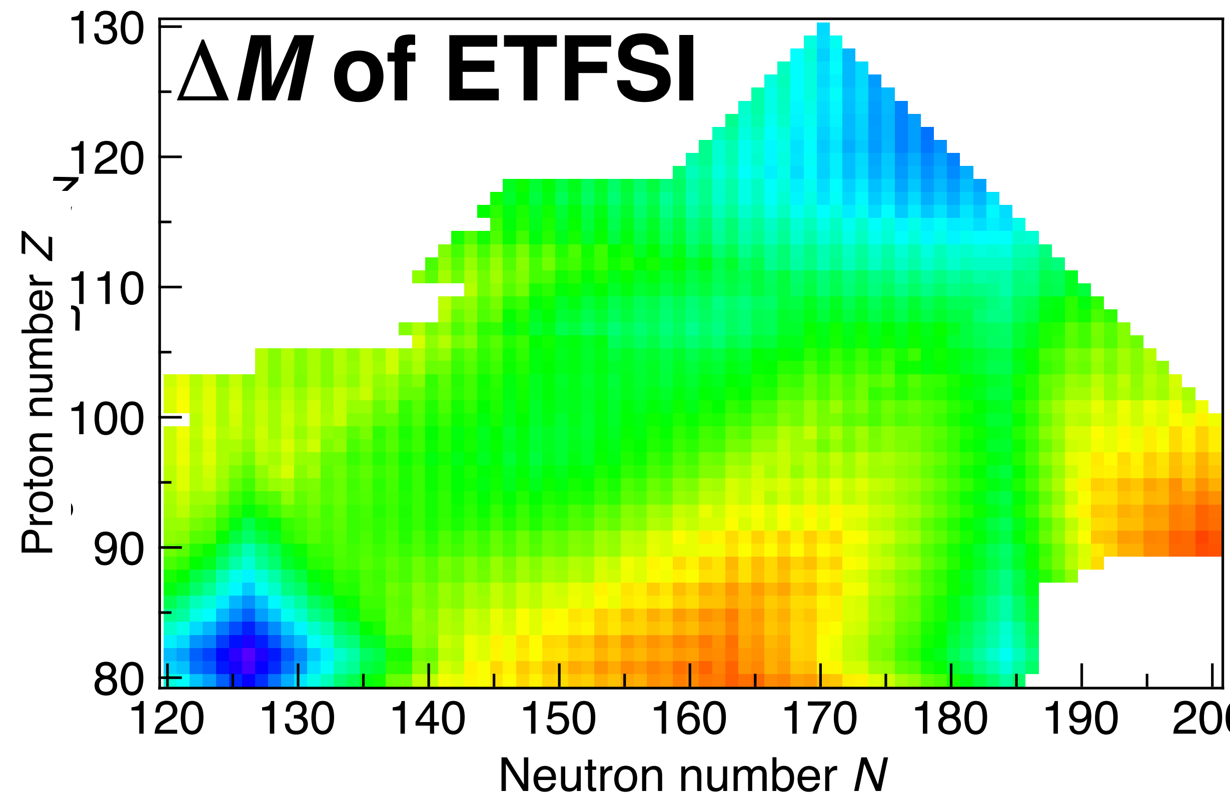
ETFSI

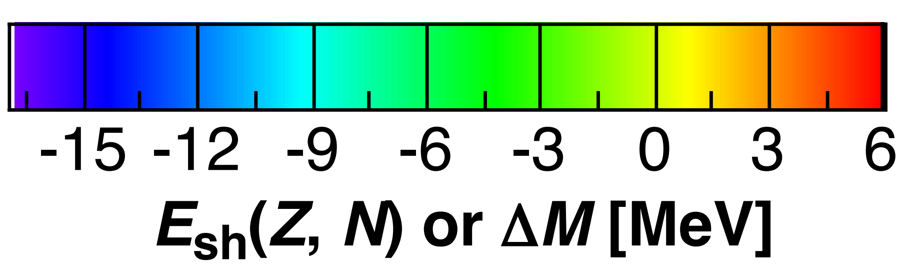
$M_{\mathrm{gross}}(Z,N)$: KUTY gross term

$M_{\mathrm{eo}}(Z,N)$: KUTY average even-odd term









Shell energy in the superheavy mass region

$$\Delta M(Z,N) = M_{\mathrm{FRDM}}(Z,N) - (M_{\mathrm{gross}}(Z,N) + M_{\mathrm{eo}}(Z,N))$$

ETFSI

$M_{\mathrm{gross}}(Z,N)$: KUTY gross term

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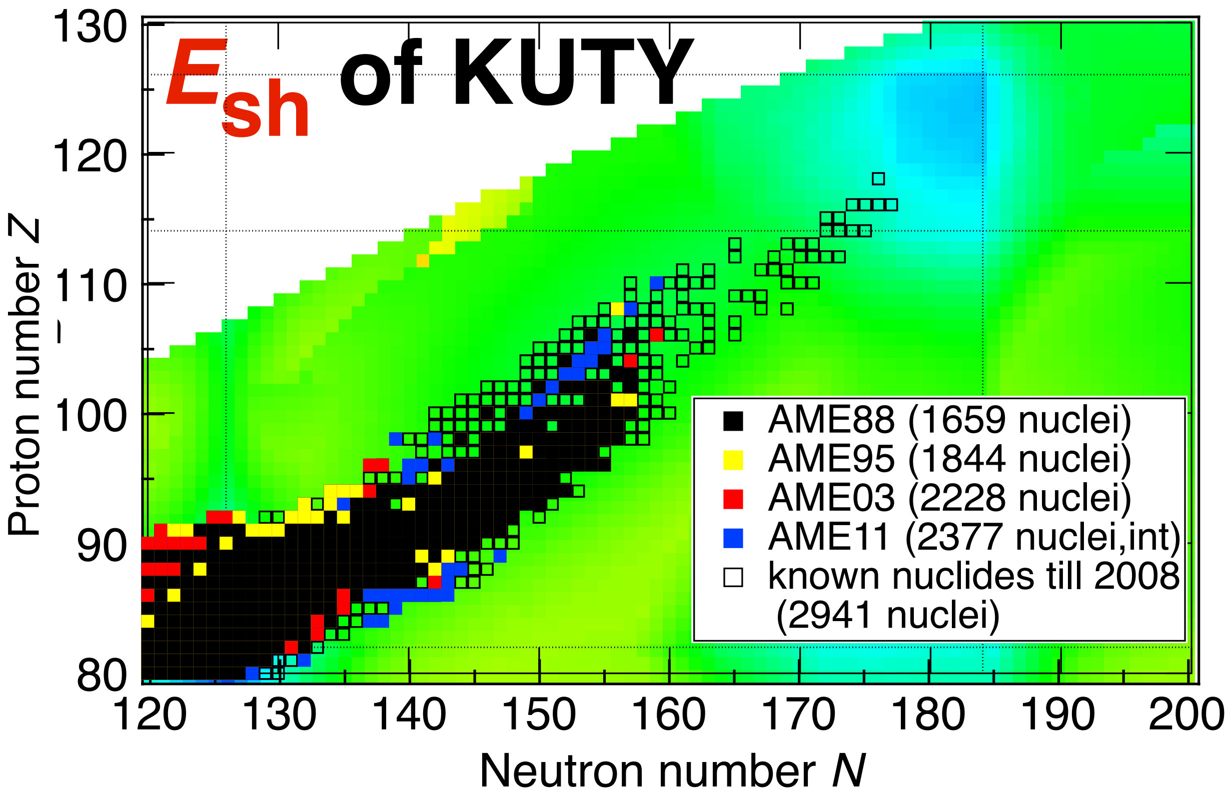
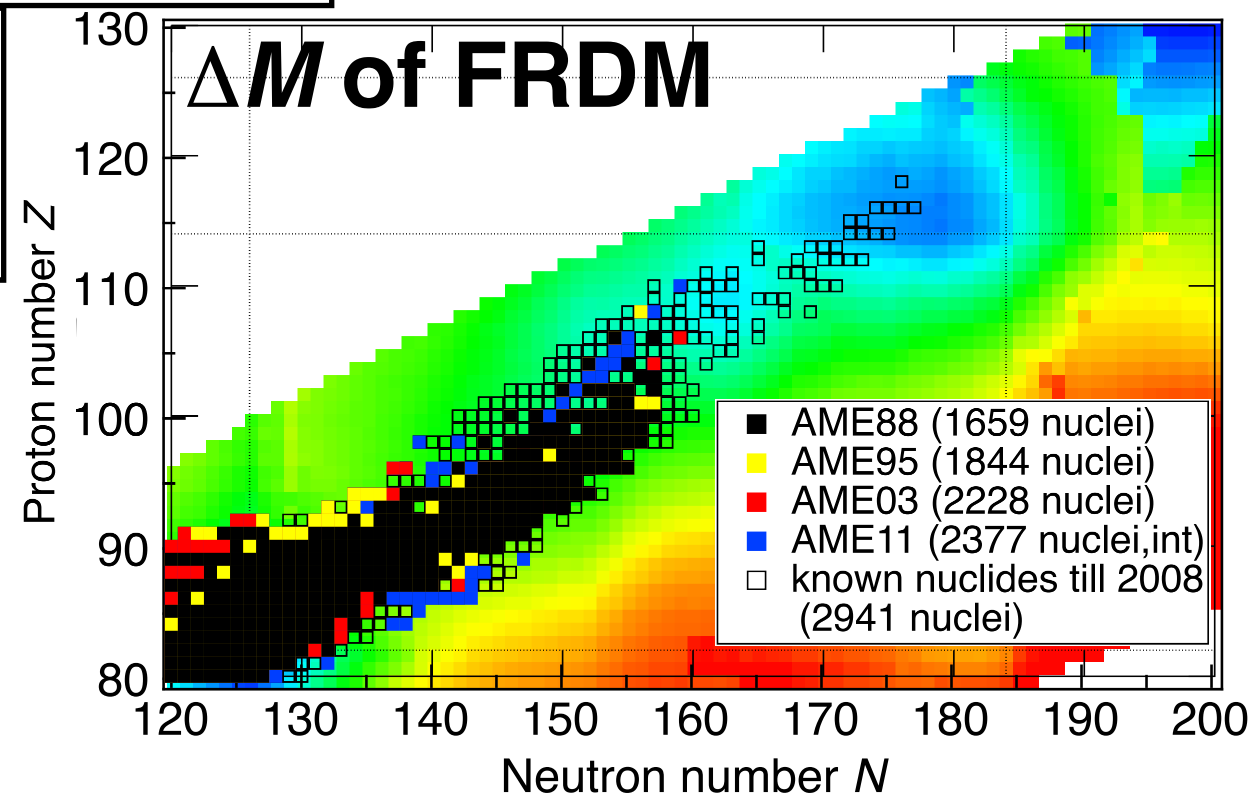
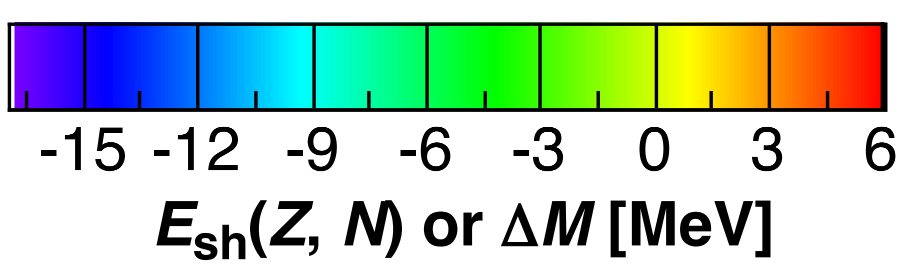
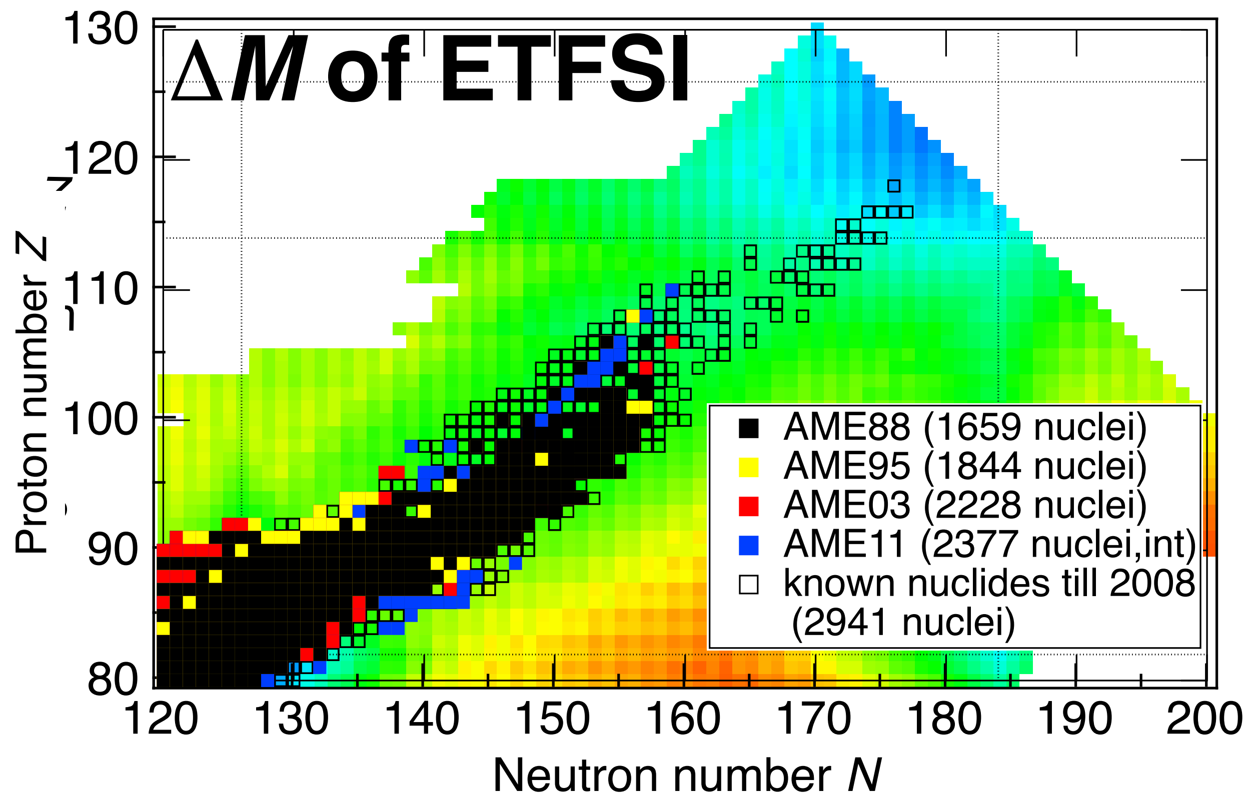






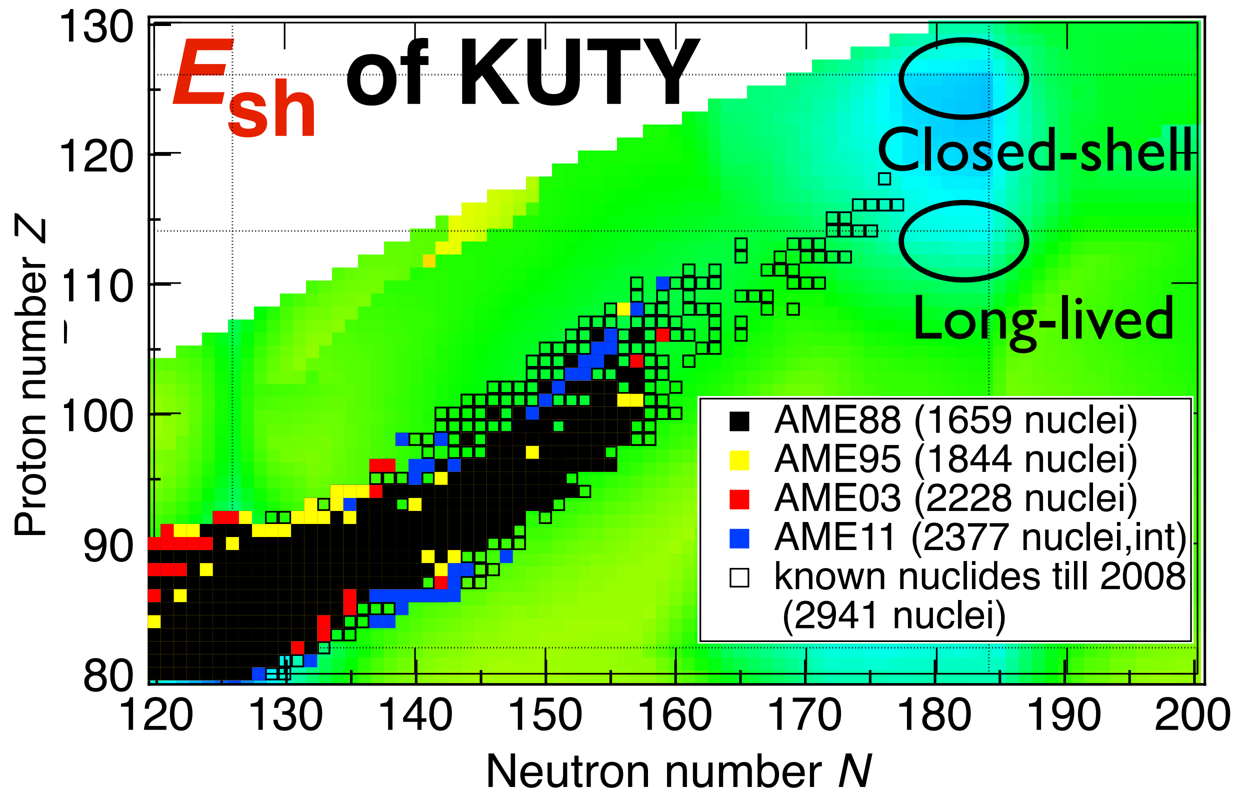


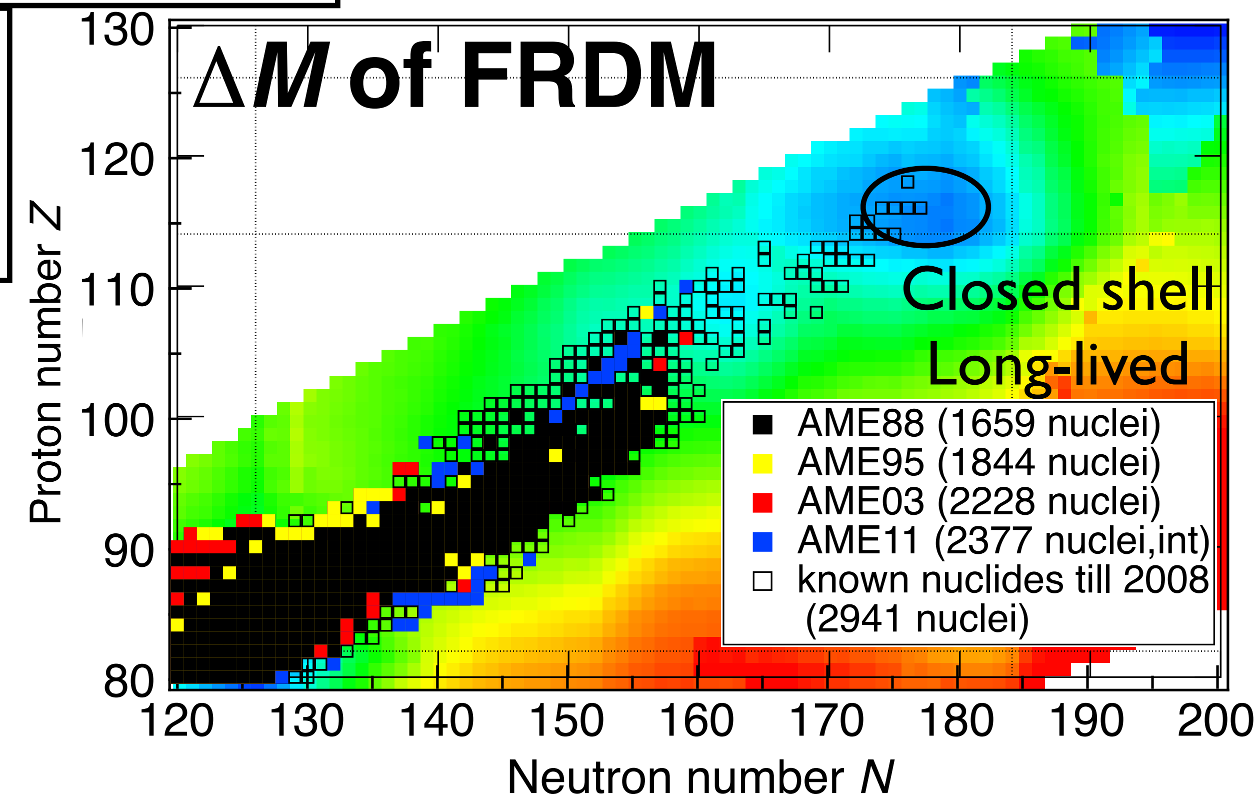
Shell energy in the superheavy mass region

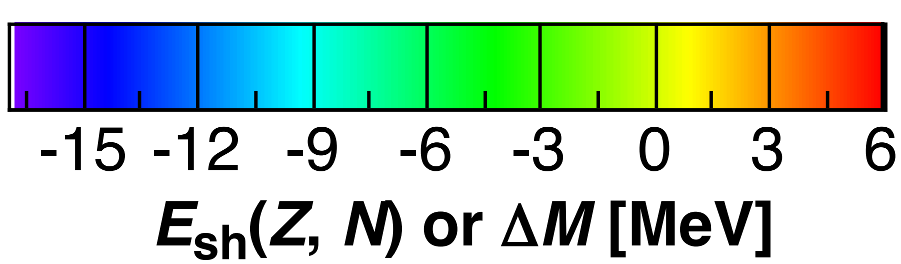
$$\Delta M(Z,N) = M_{\mathrm{FRDM} \atop \mathrm{ETFSI}}(Z,N) - \left(M_{\mathrm{gross}}(Z,N) + M_{\mathrm{eo}}(Z,N)\right)$$

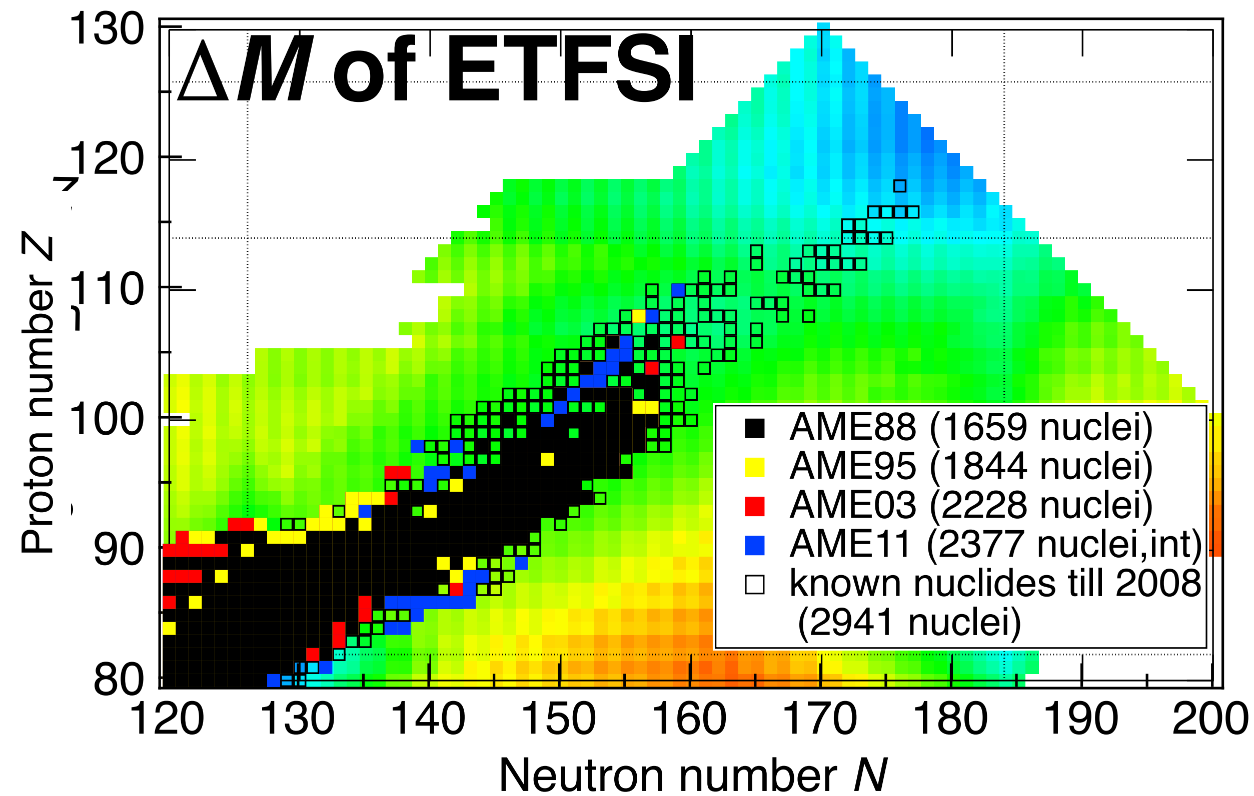
$M_{\mathrm{gross}}(Z,N)$: **KUTY gross term**

$M_{\mathrm{eo}}(Z,N)$: **KUTY average even-odd term**



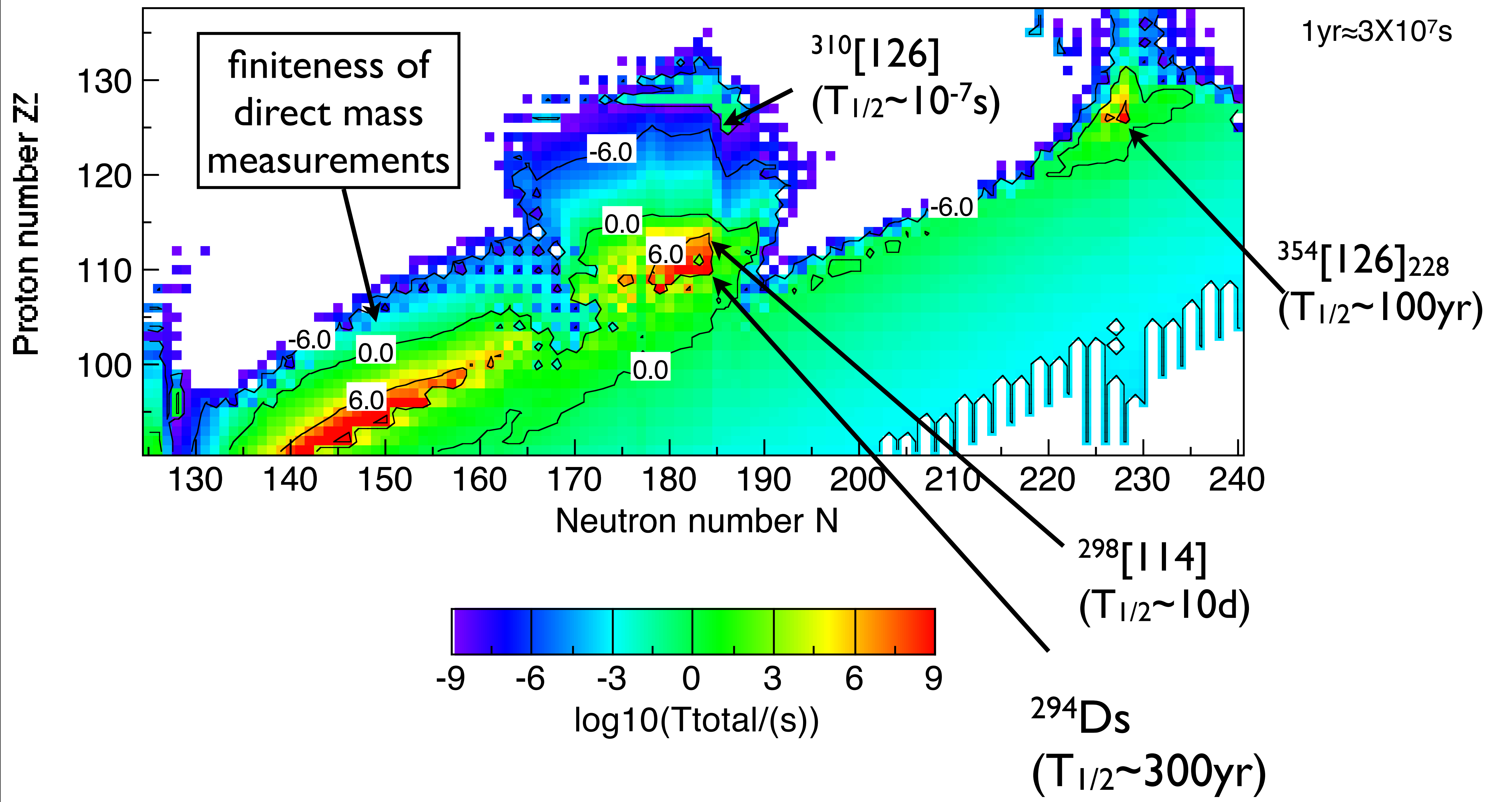






Theoretical total half-lives (α,β,p,sf)

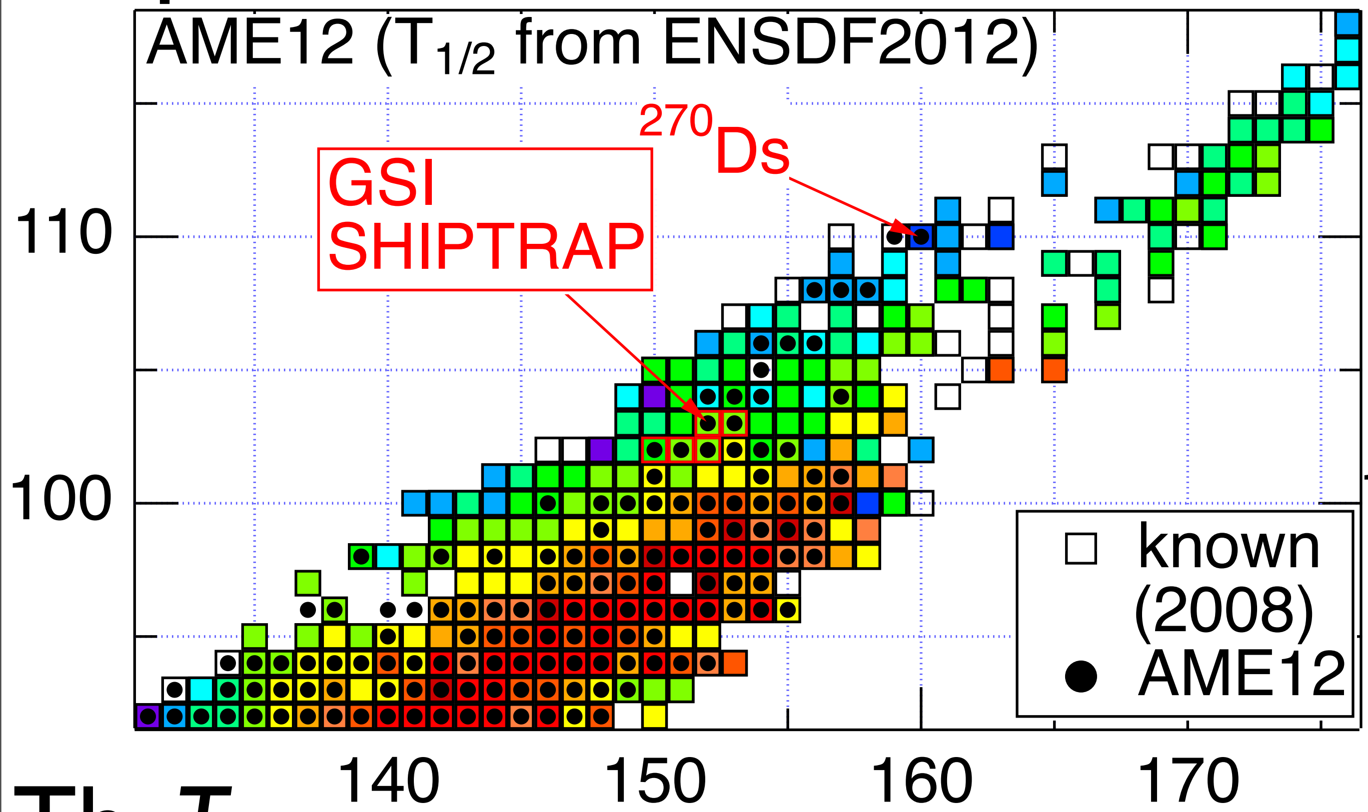
KTUY+decay models



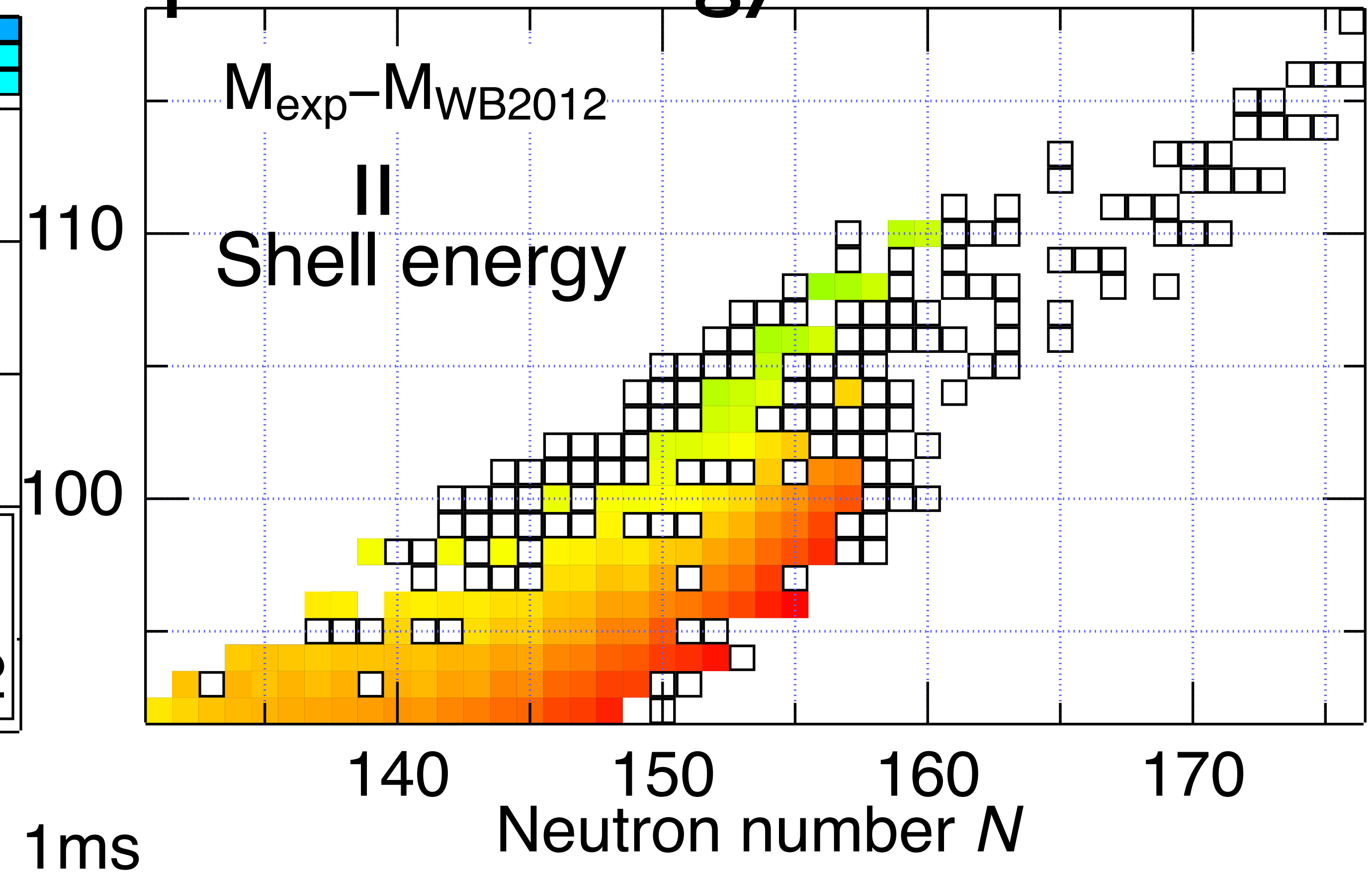
(Long-lived superheavy nuclei are located near the β-stability line)

Total half-lives (α,β,p,sf)

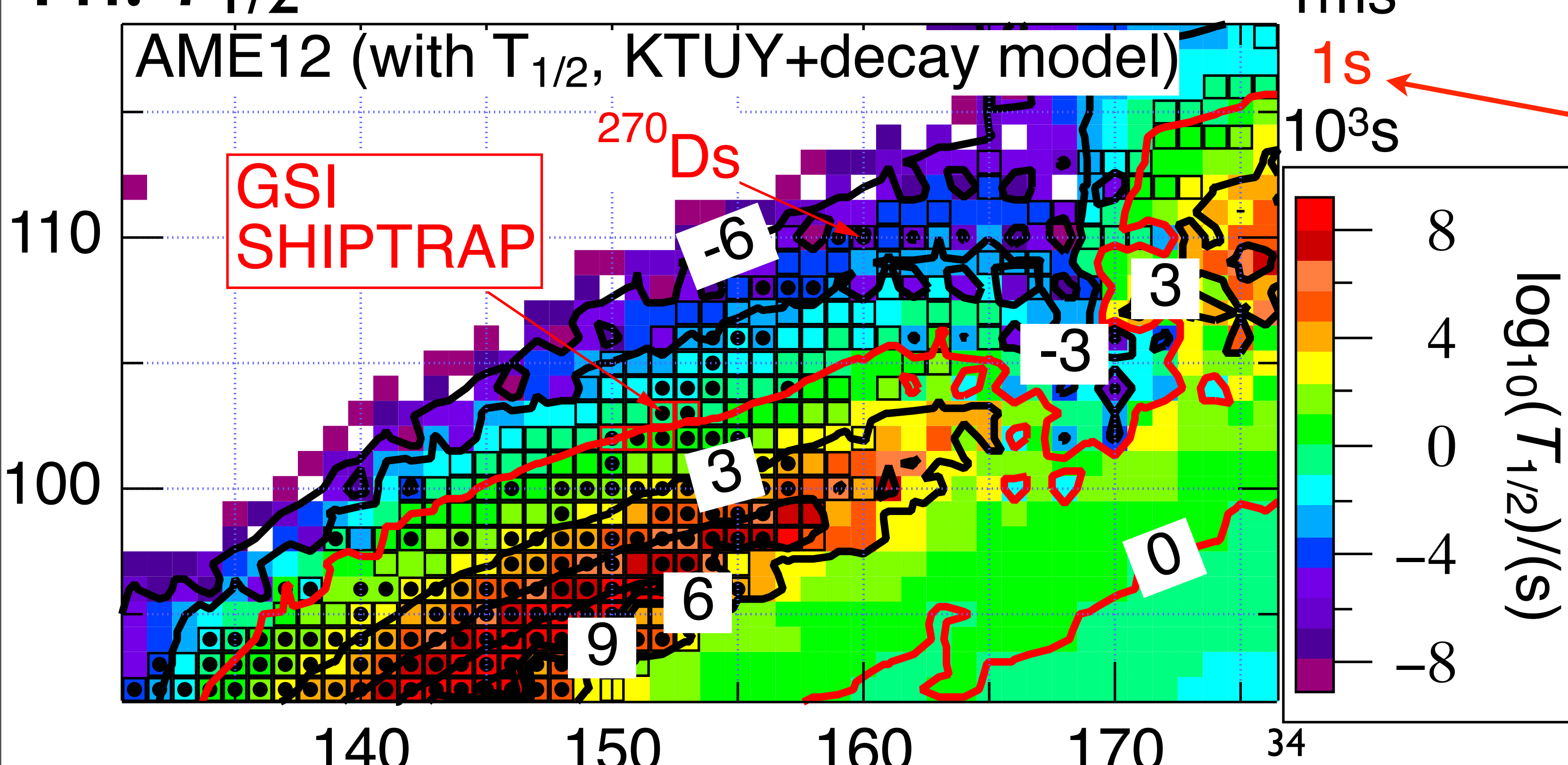
Exp. $T_{1/2}$



'Exp.' shell Energy

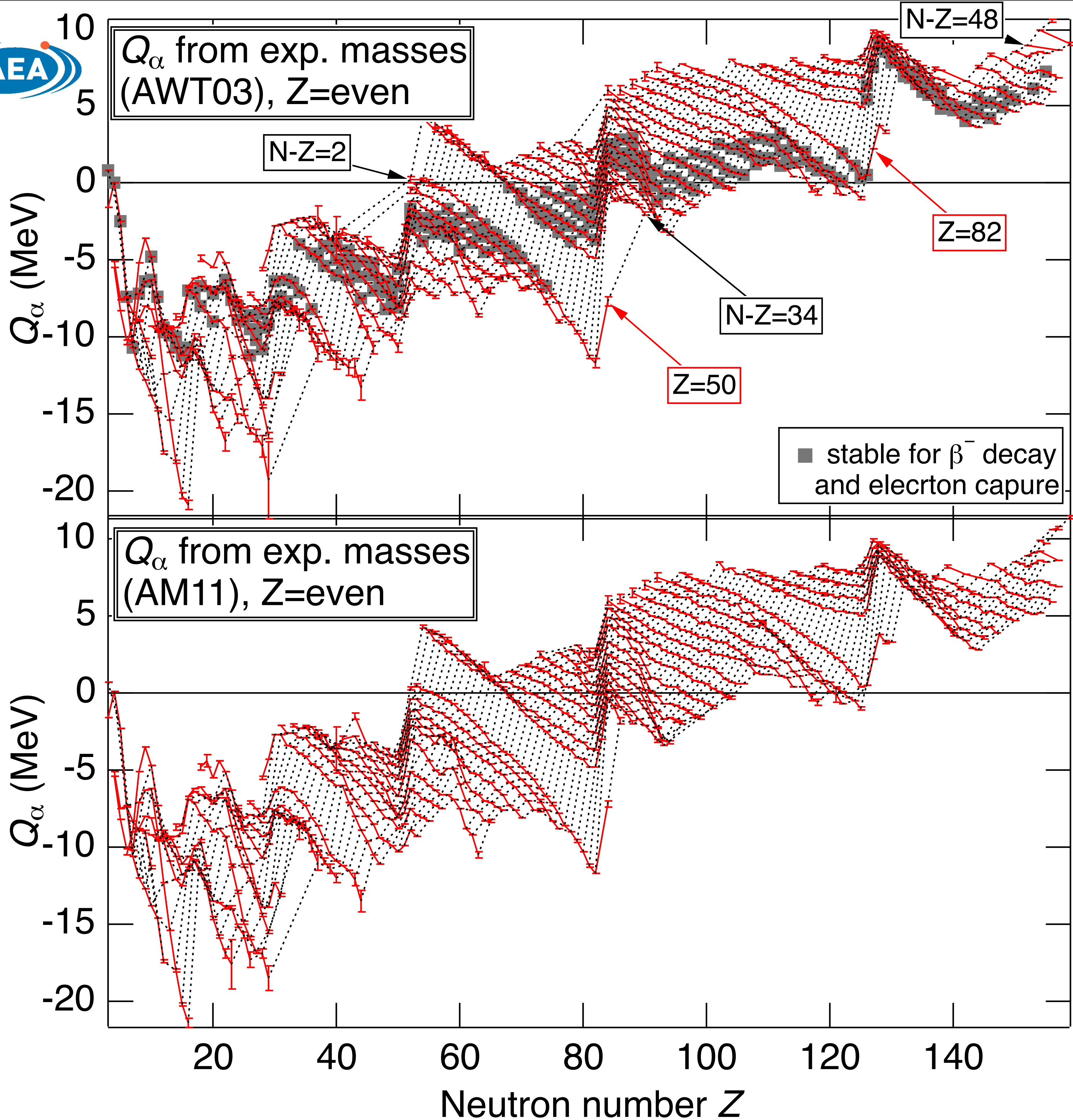


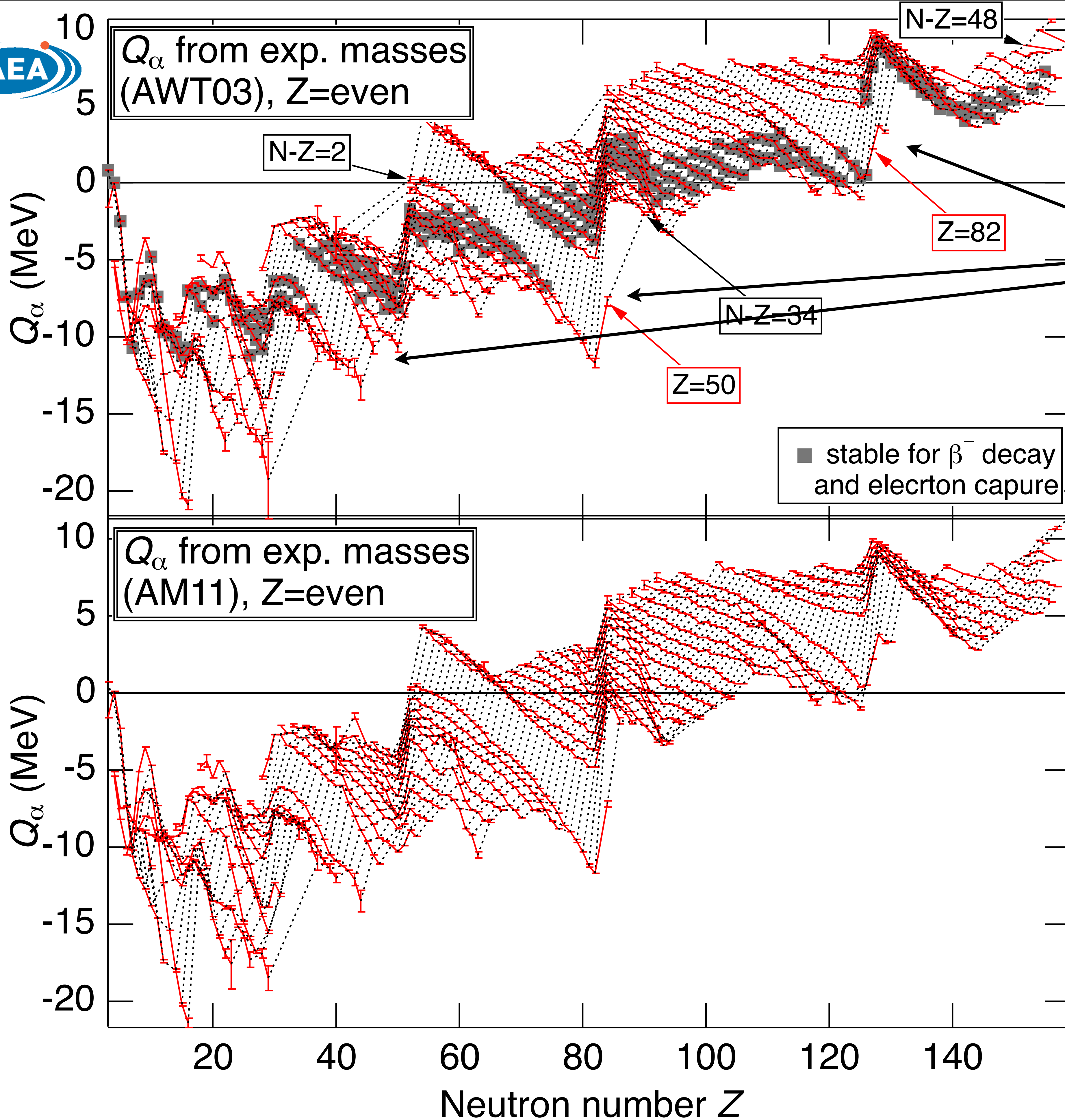
Th. $T_{1/2}$



Nuclei along the red line (1 sec) are experimentally-provable candidates for RIKENTRAP or SHIPTRAP.

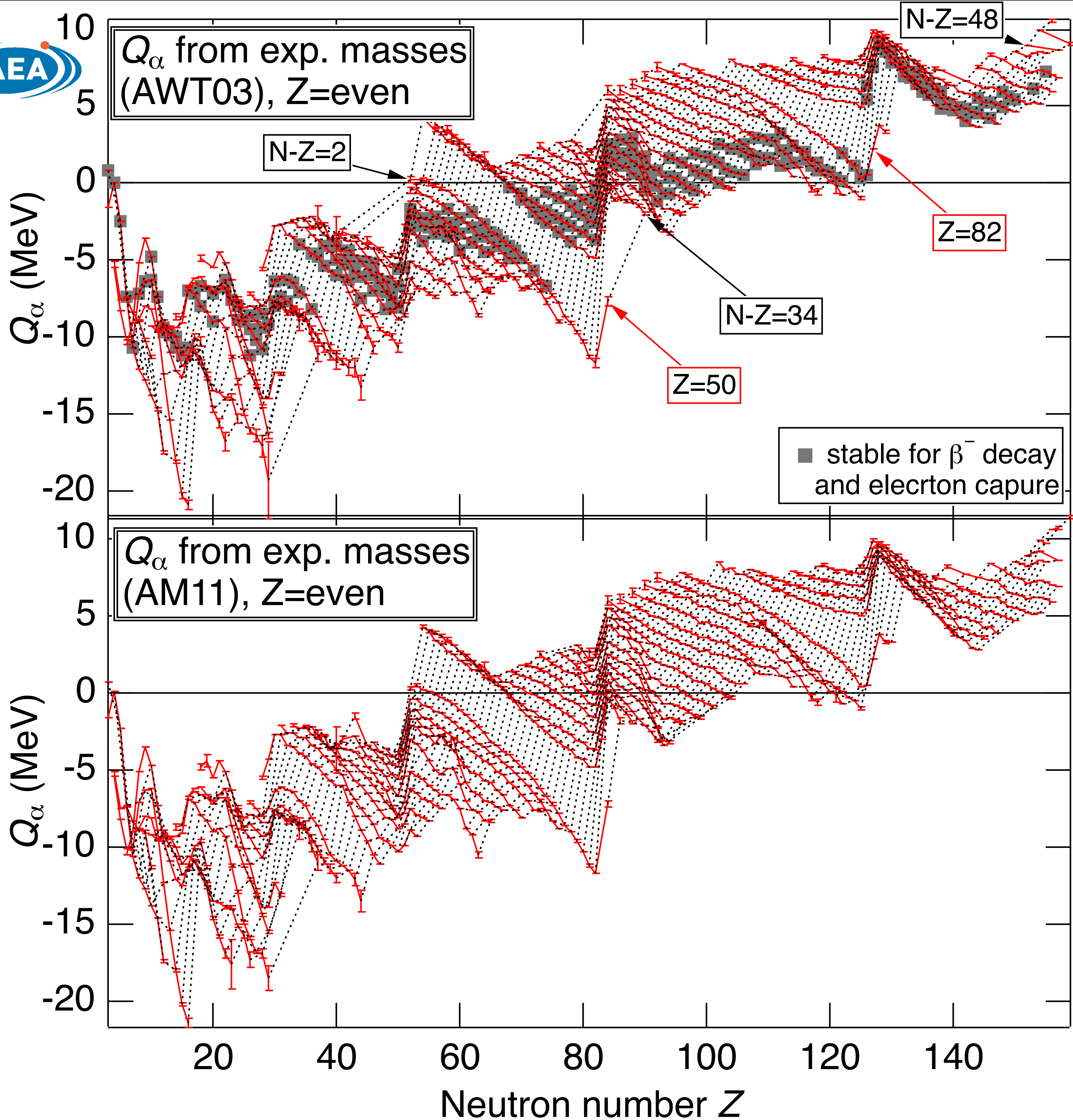
shell gaps are seen



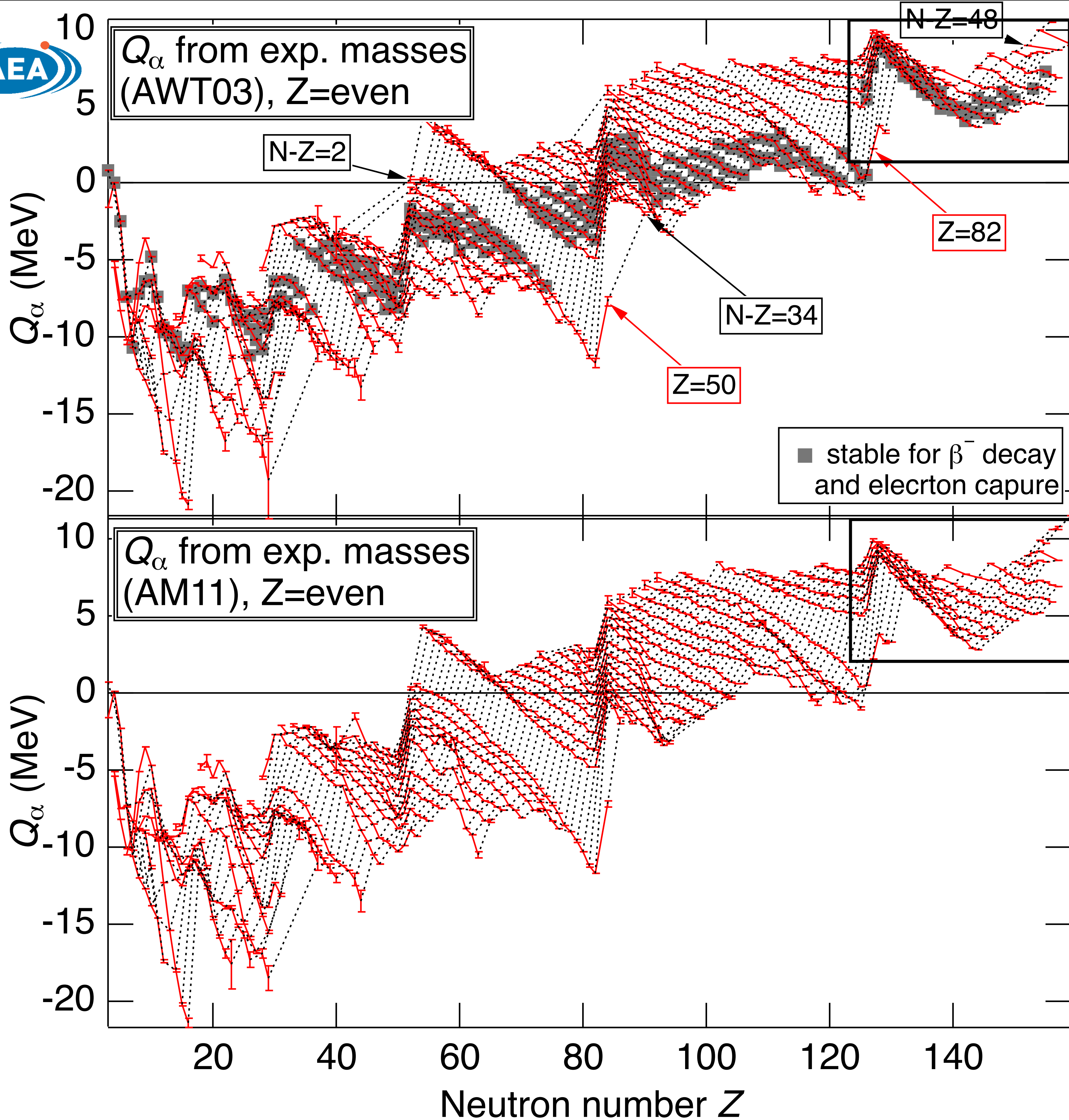


shell gaps are seen

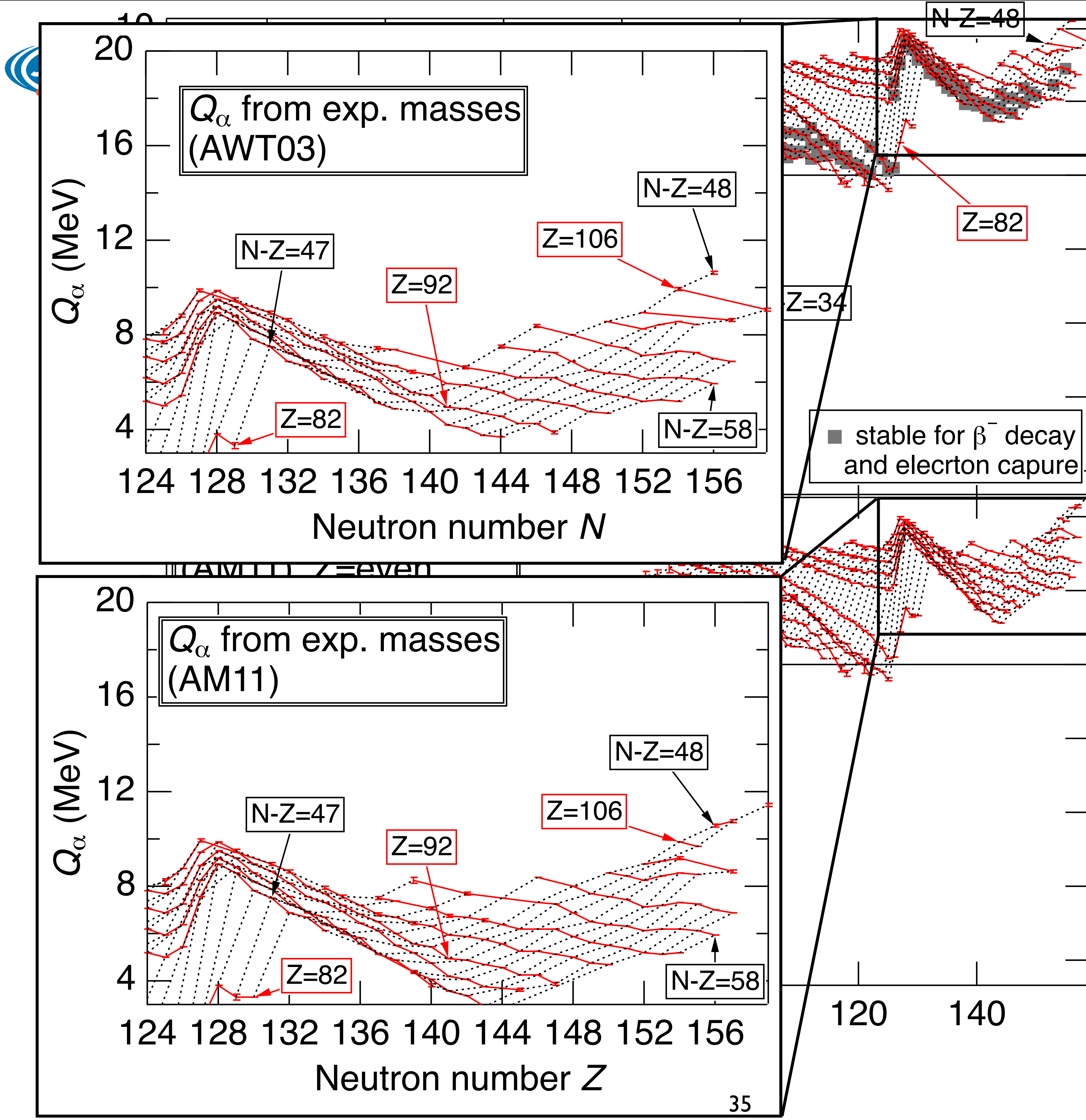
Z=28, 50, 82 shells as a gap



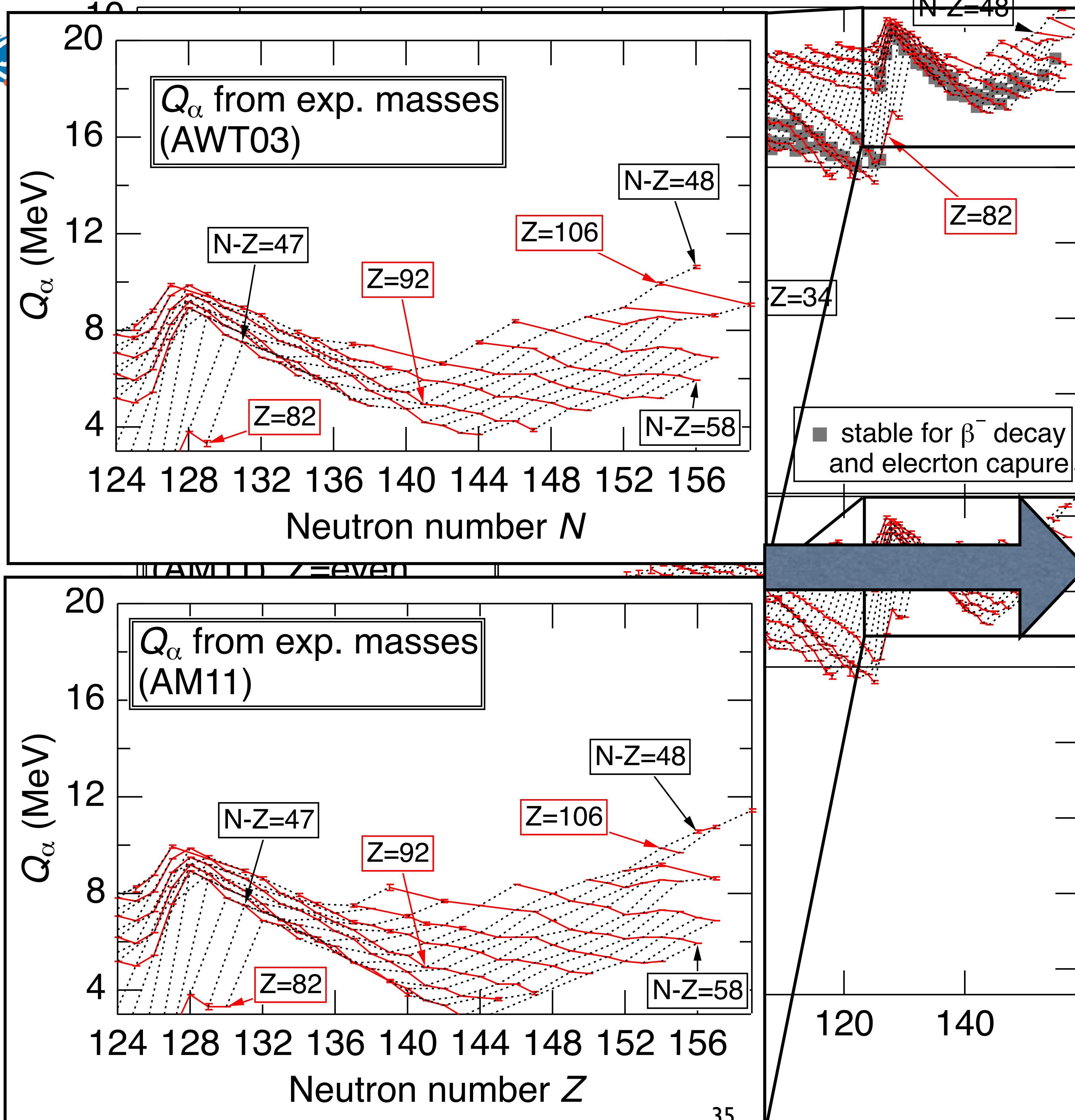
shell gaps are seen



shell gaps are seen



shell gaps are seen

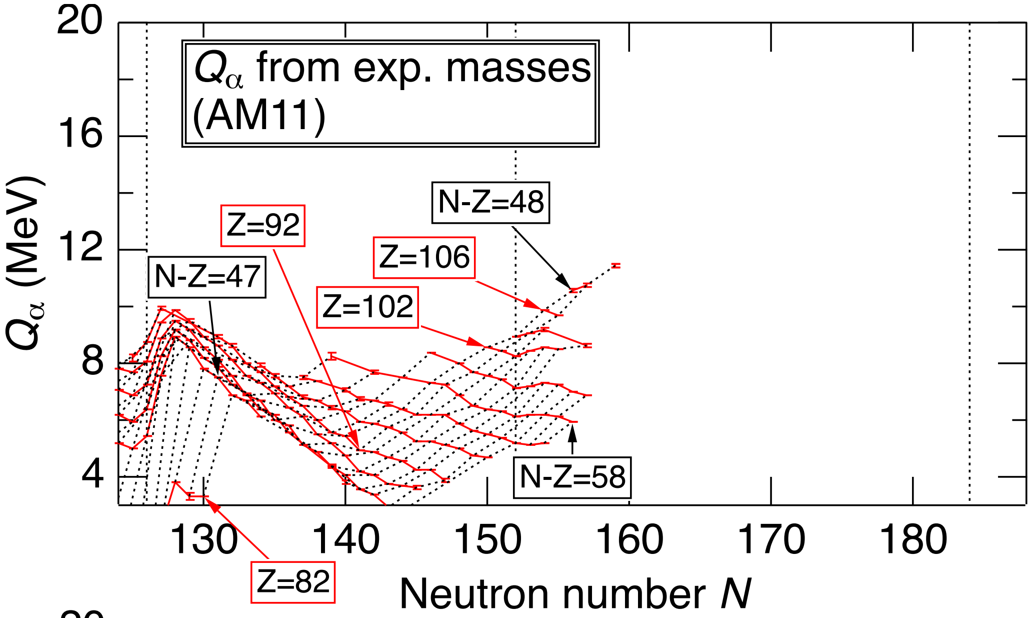


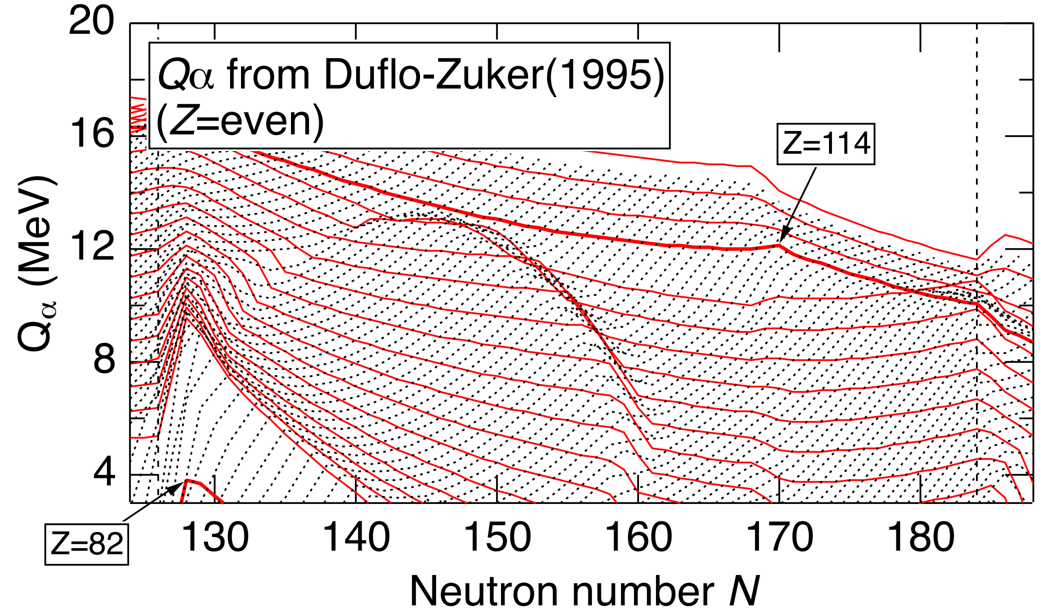
shell gaps are seen

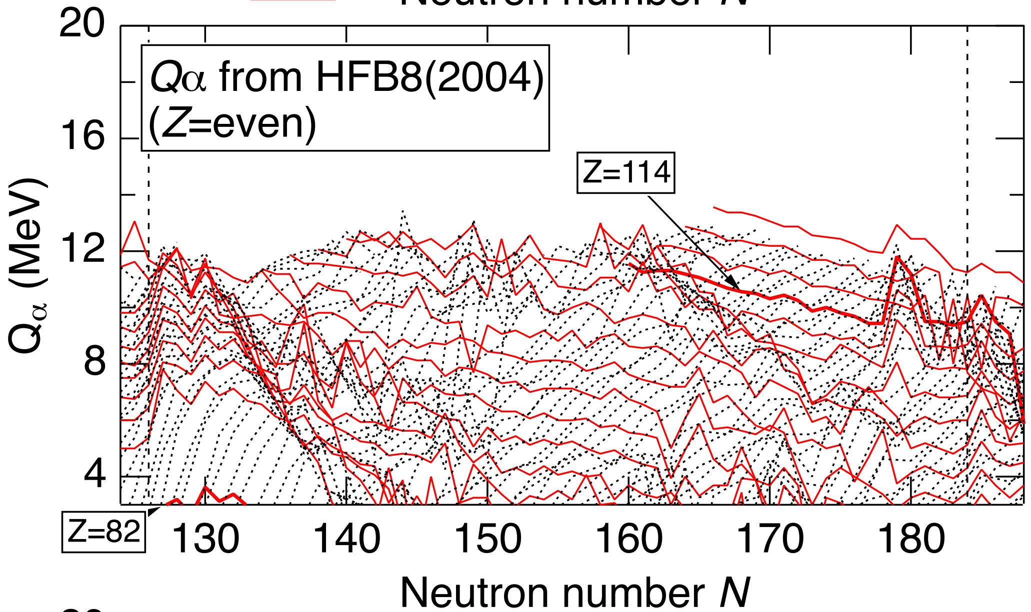
More information is required!

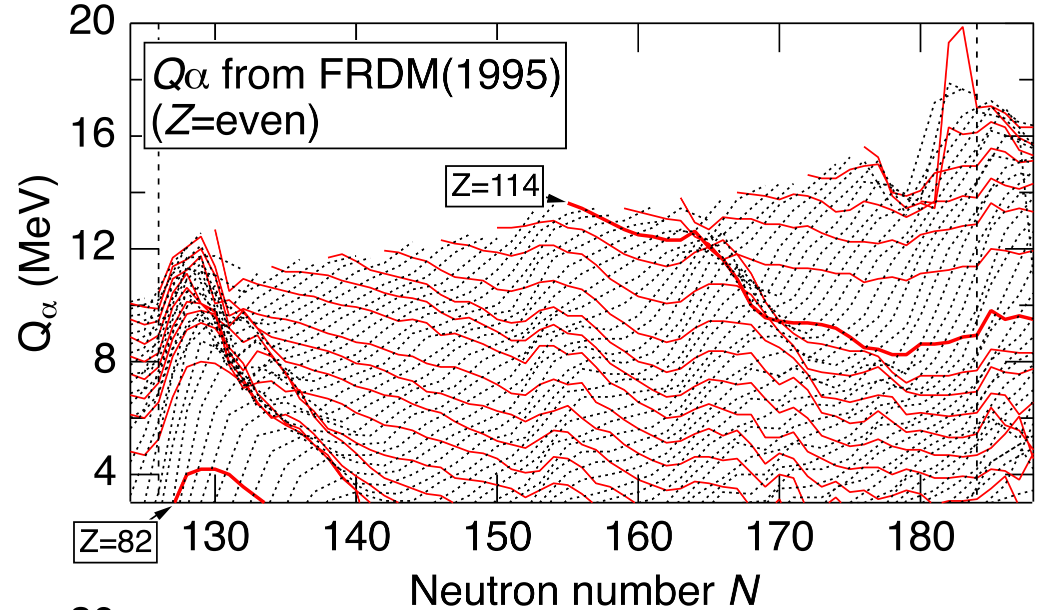
α-decay Q-value of superheavy nuclei

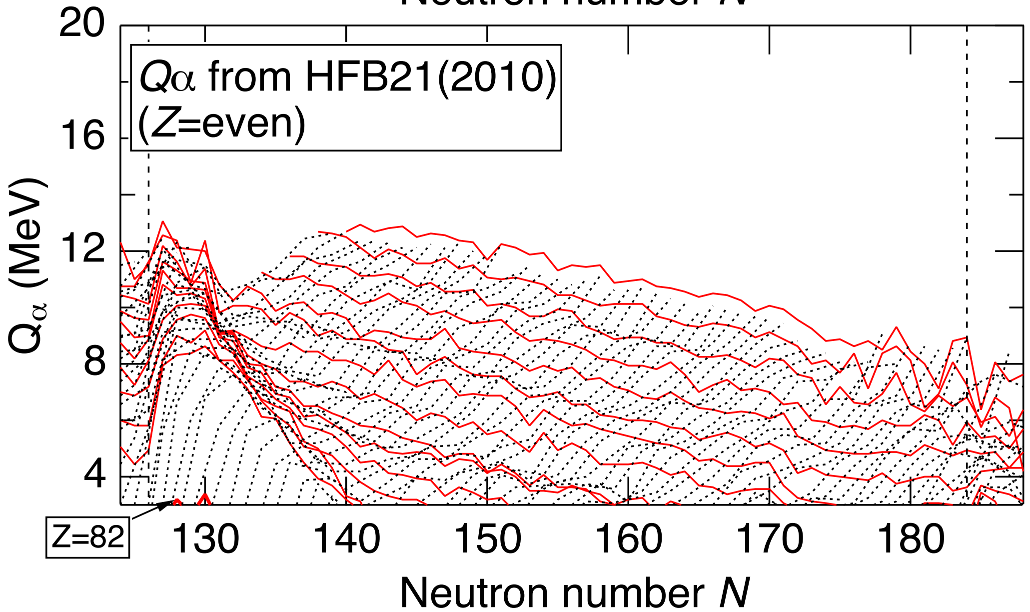
=> Prediction of structure for SHE

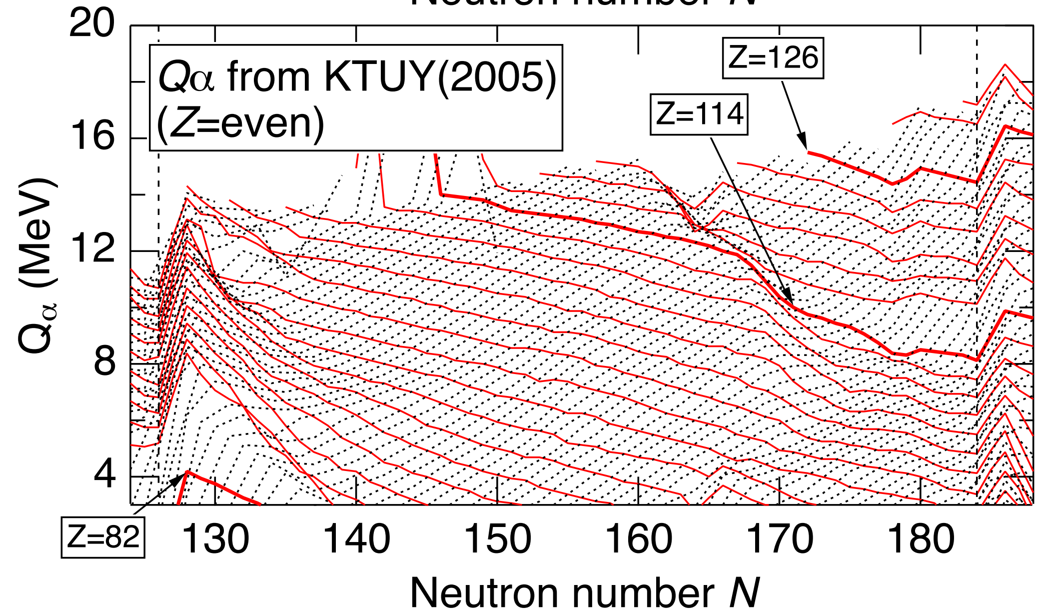






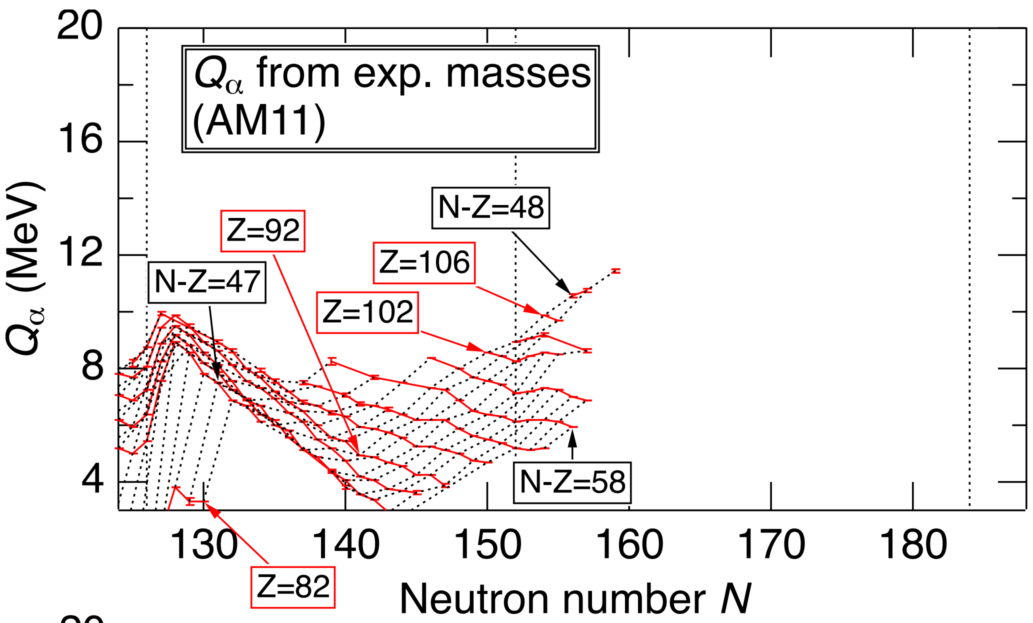


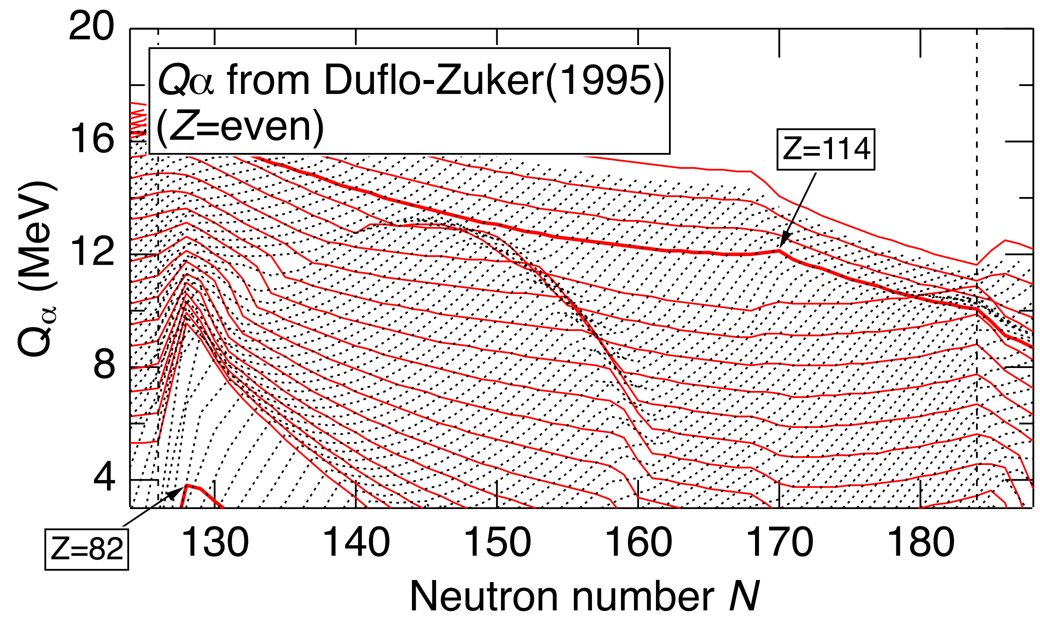


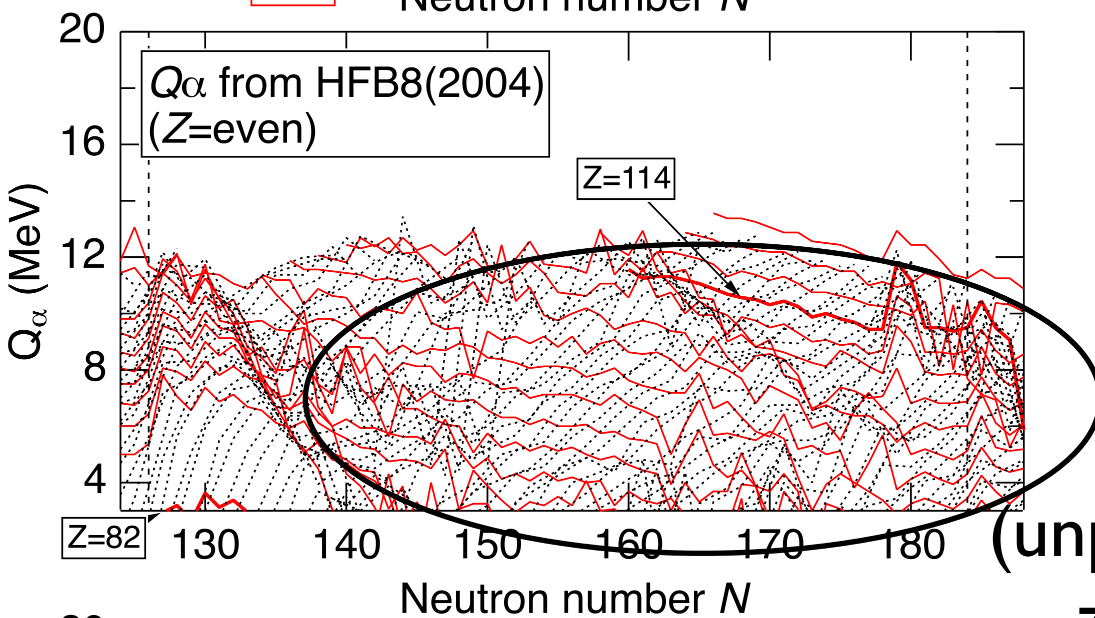


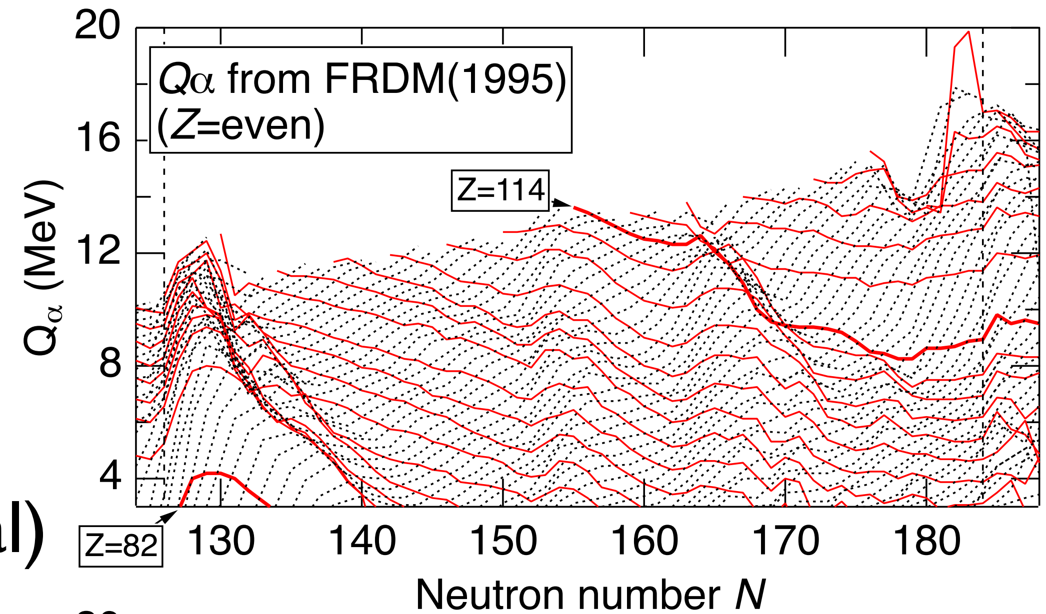
α-decay Q-value of superheavy nuclei

=> Prediction of structure for SHE

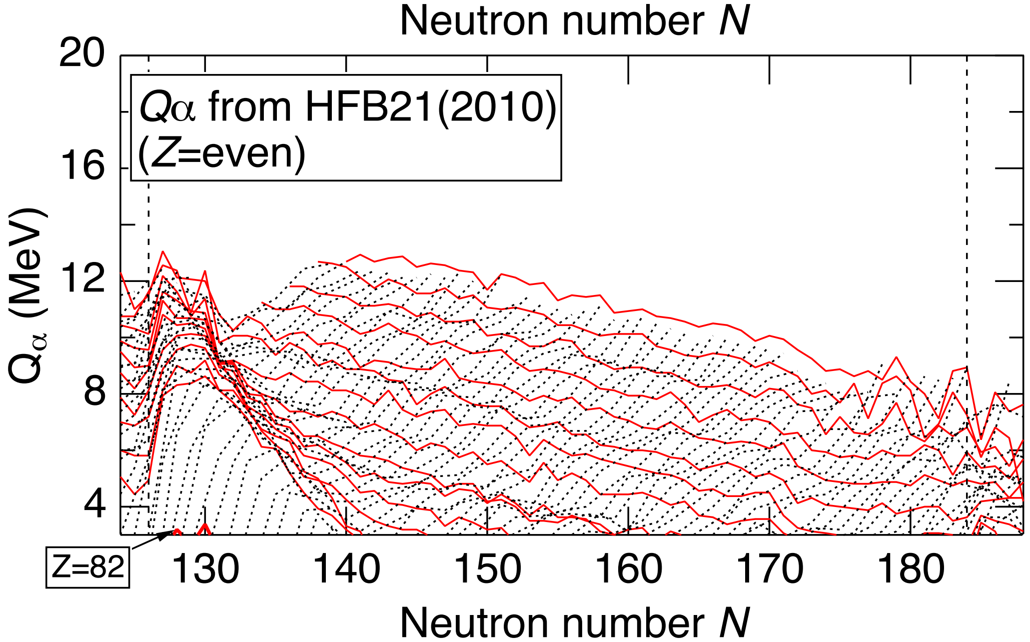


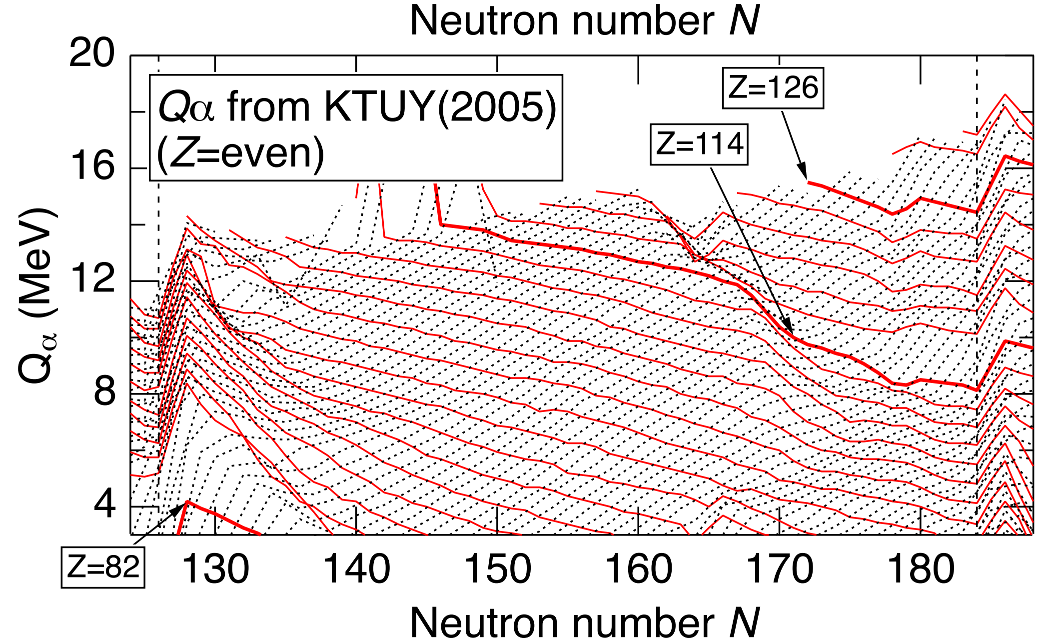






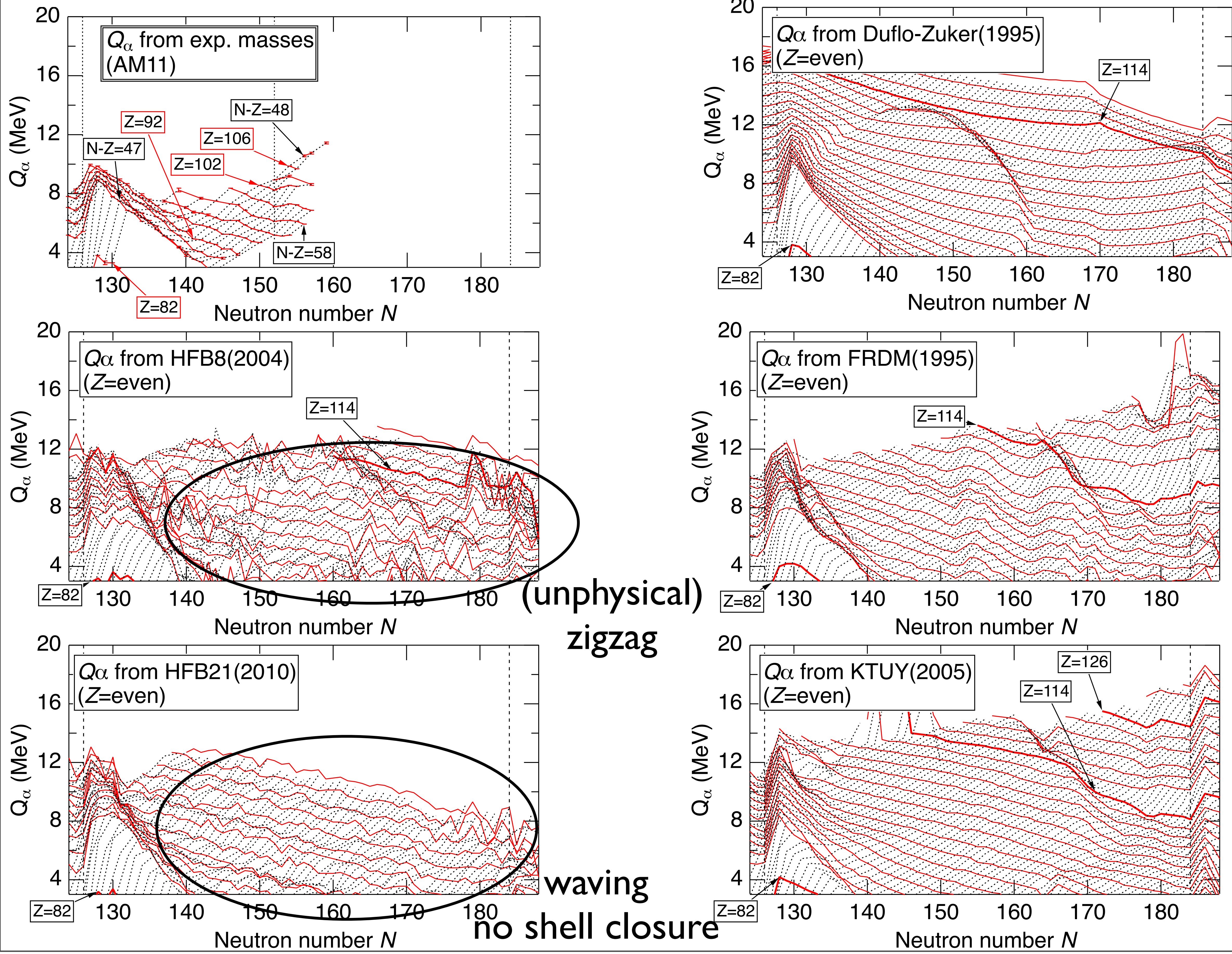
(unphysical) zigzag





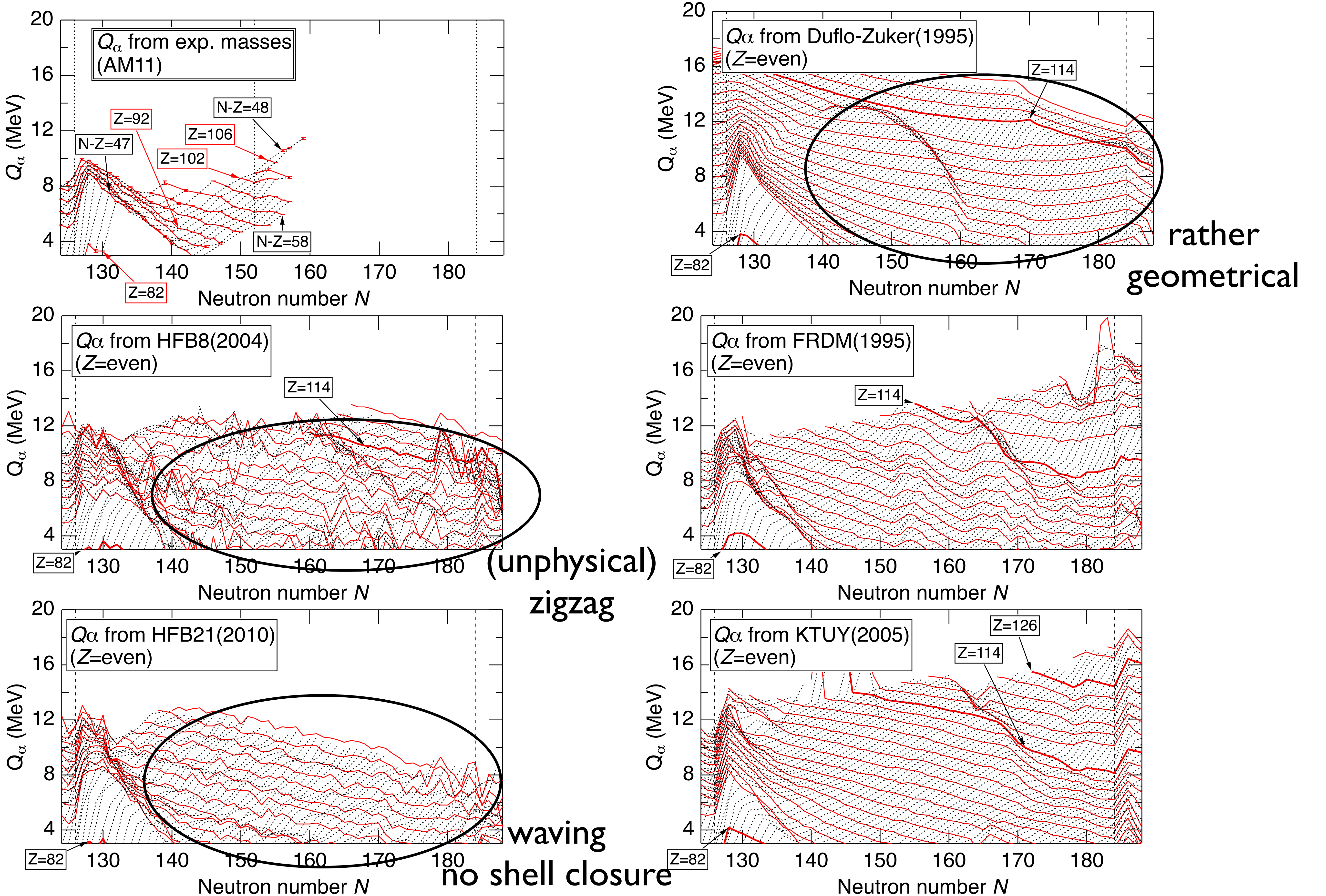
α-decay Q-value of superheavy nuclei

=> Prediction of structure for SHE



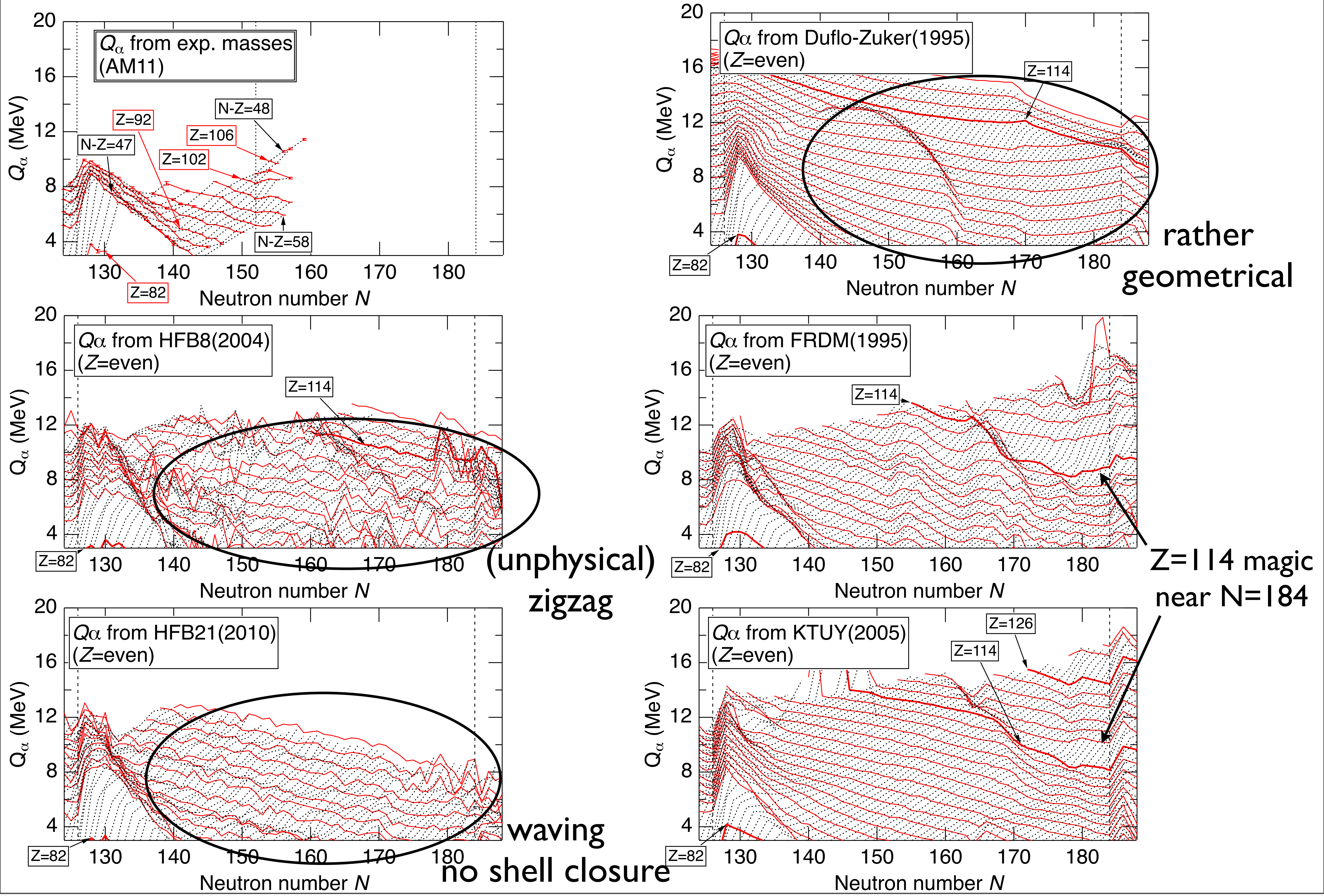
α-decay Q-value of superheavy nuclei

=> Prediction of structure for SHE



α-decay Q-value of superheavy nuclei

=> Prediction of structure for SHE



α-decay Q-value of superheavy nuclei

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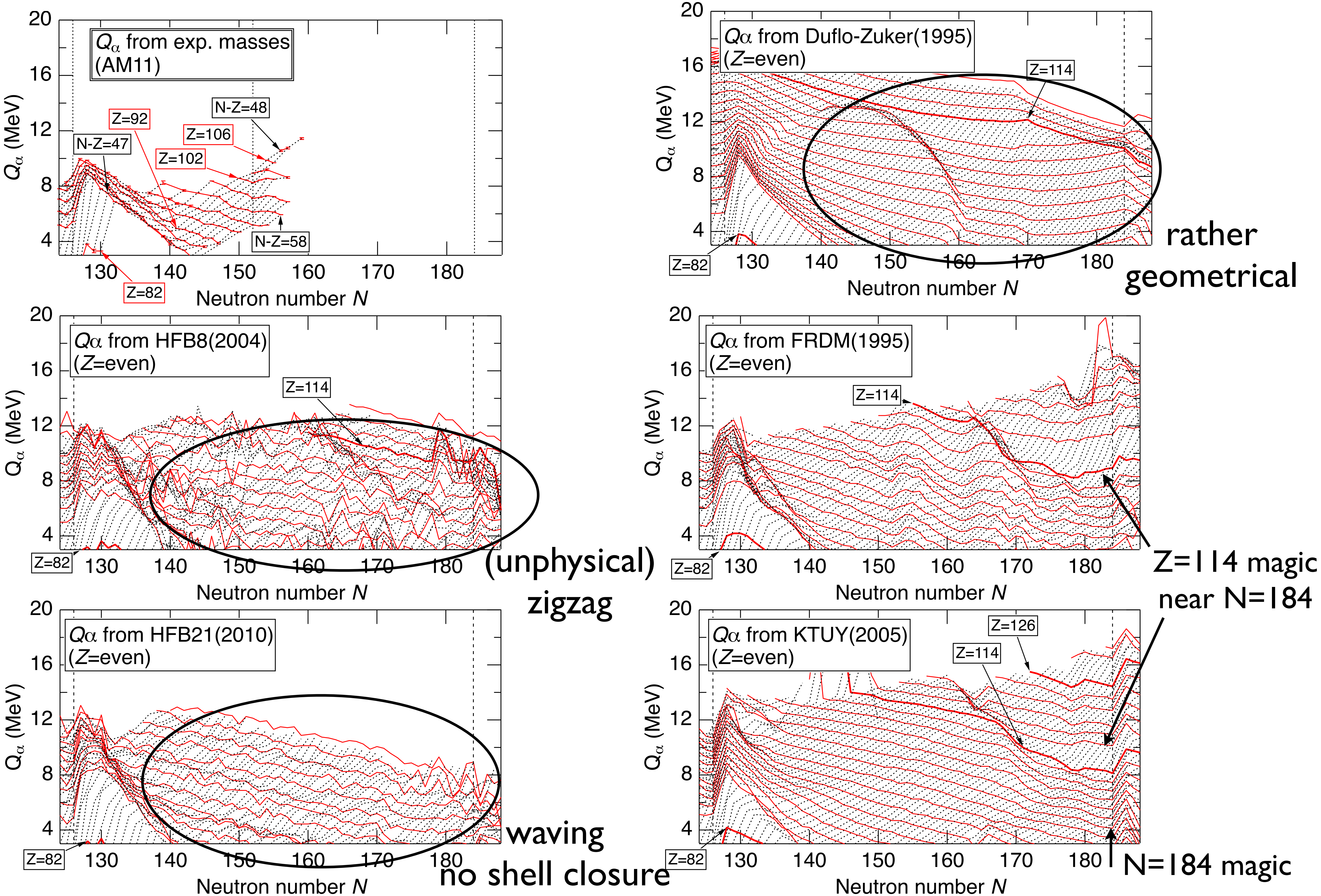
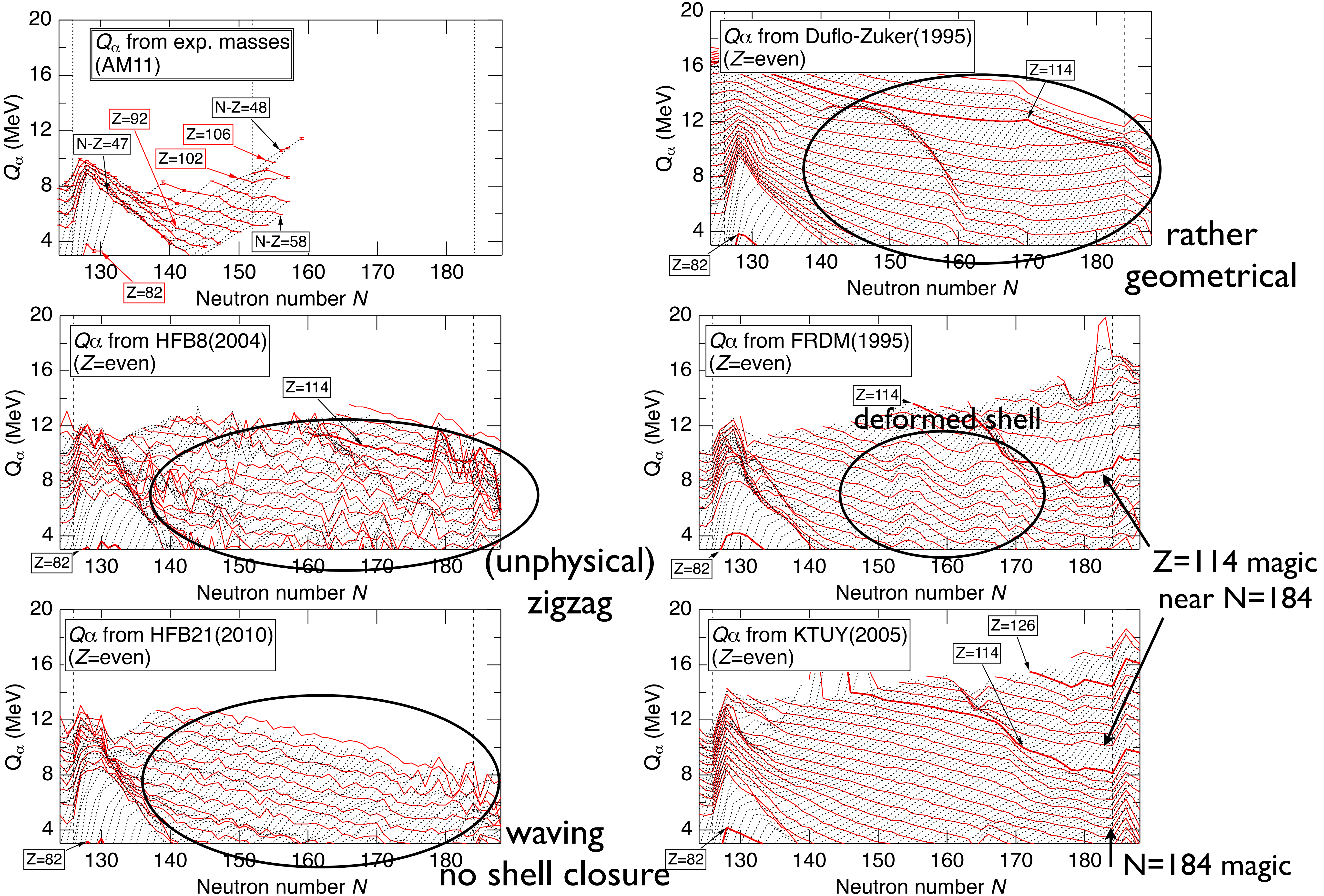


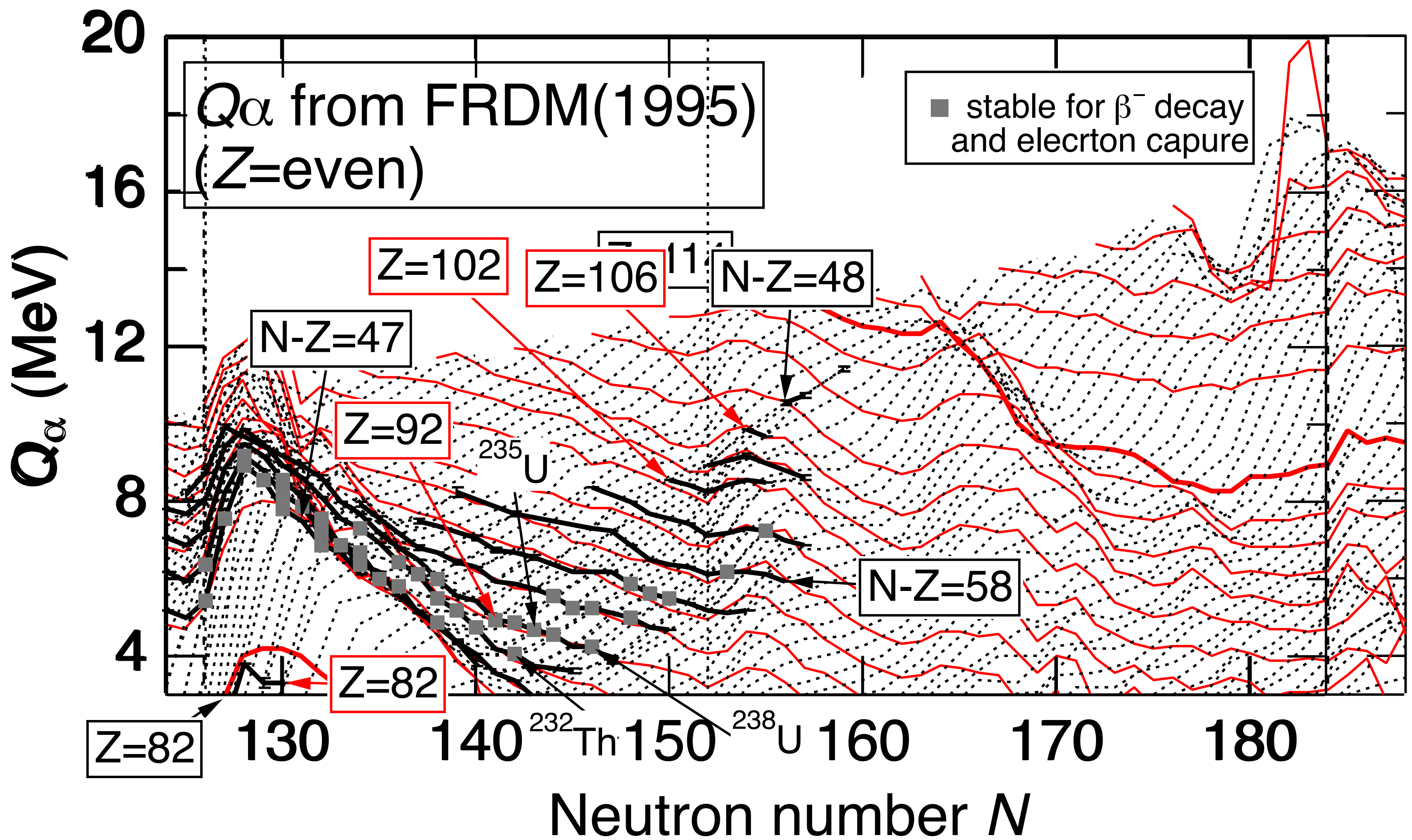
α-decay Q-value of superheavy nuclei

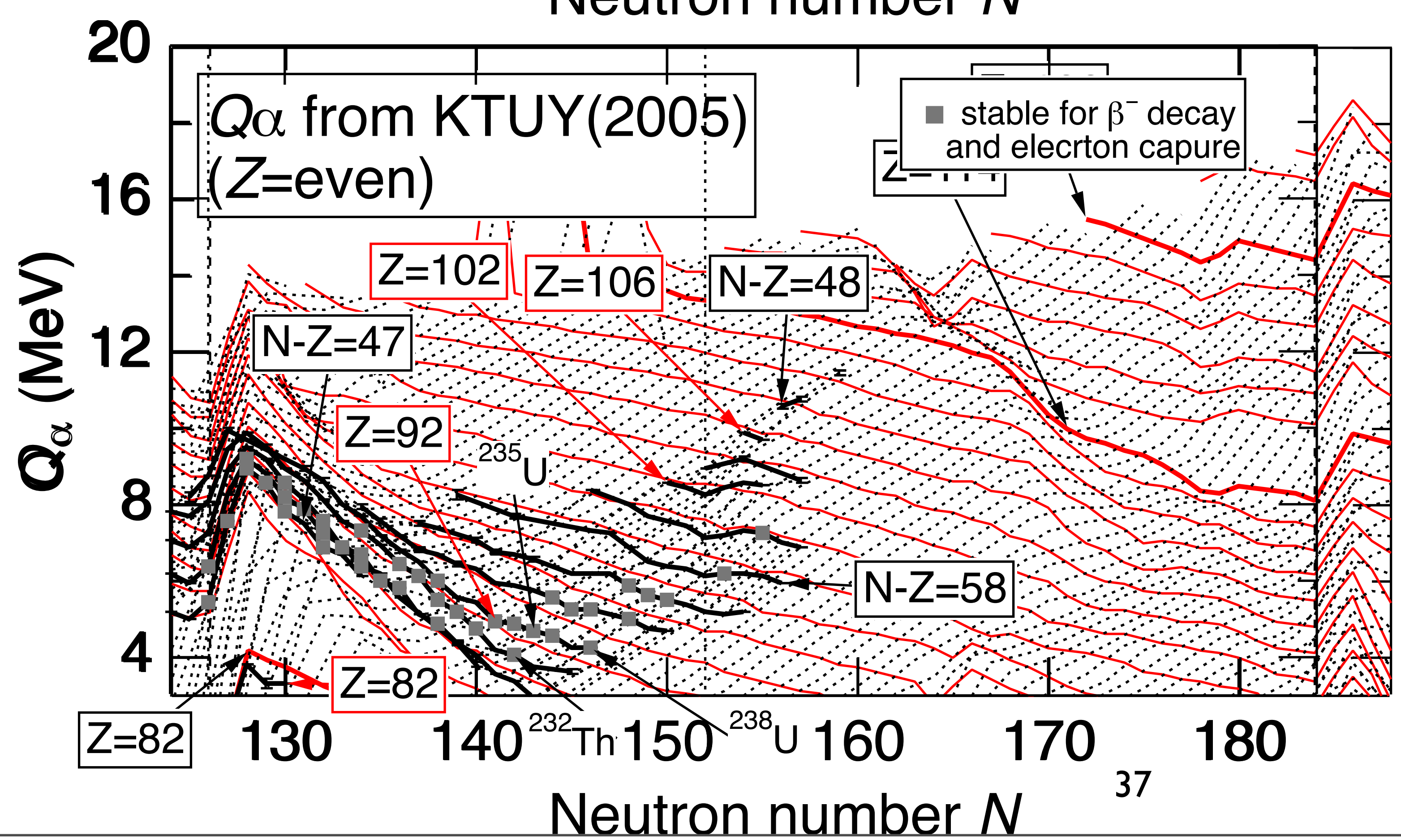
=> Prediction of structure for SHE



α-decay Q-value of superheavy nuclei

=> Prediction of structure for SHE



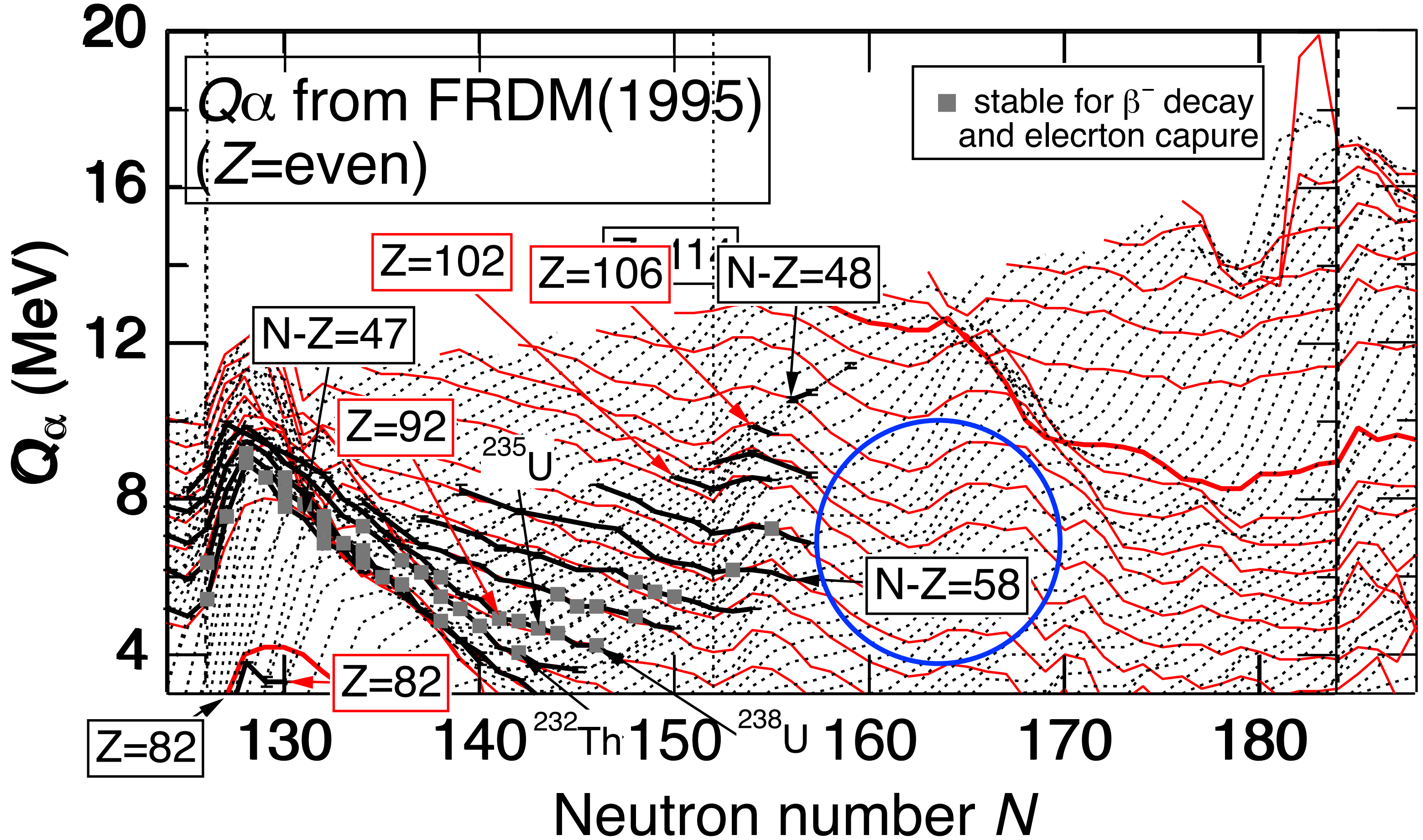


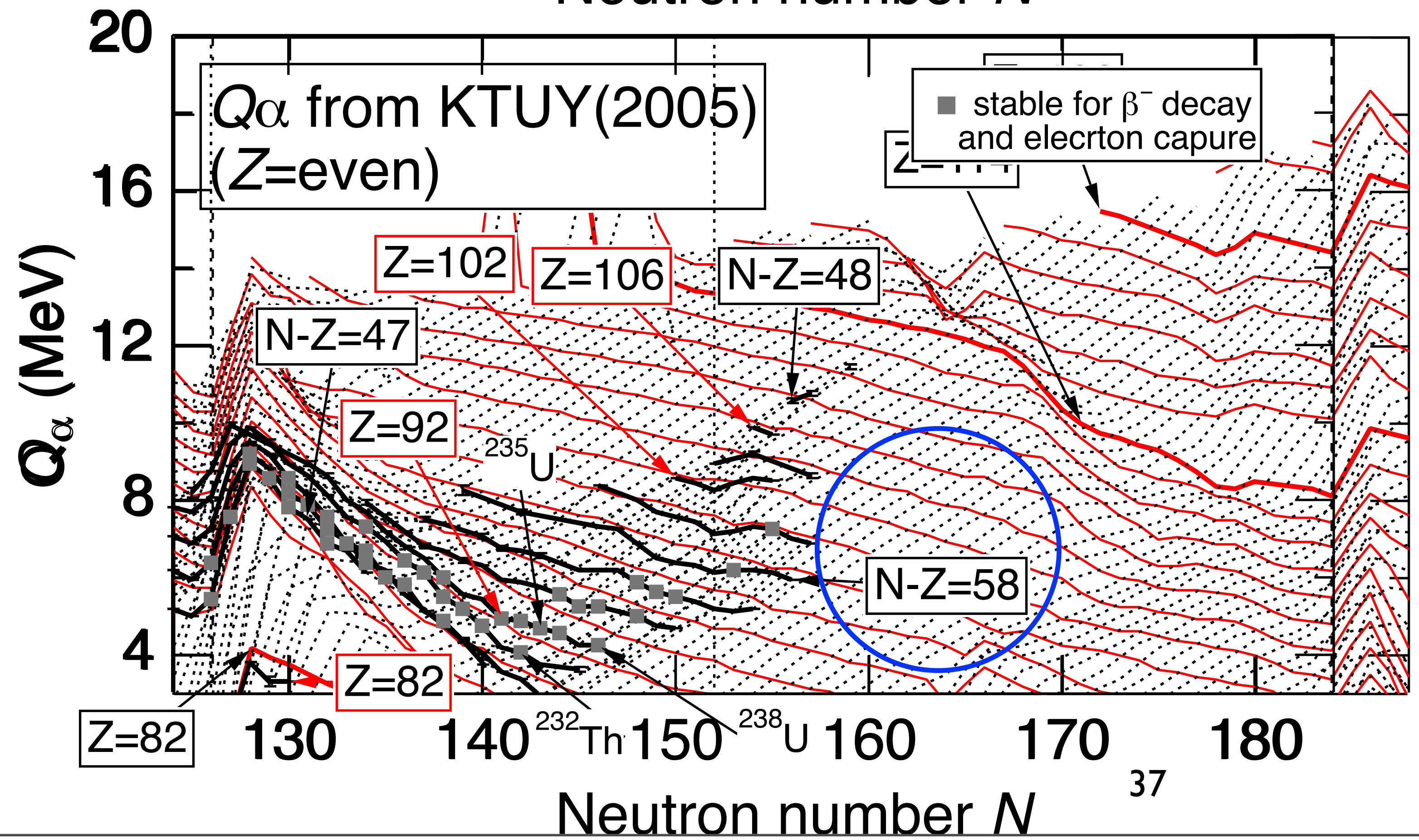
Search for N=162 deformed shell **in the ground-state** is required.

(Also for N=152)

α-decay Q-value of superheavy nuclei

=> Prediction of structure for SHE



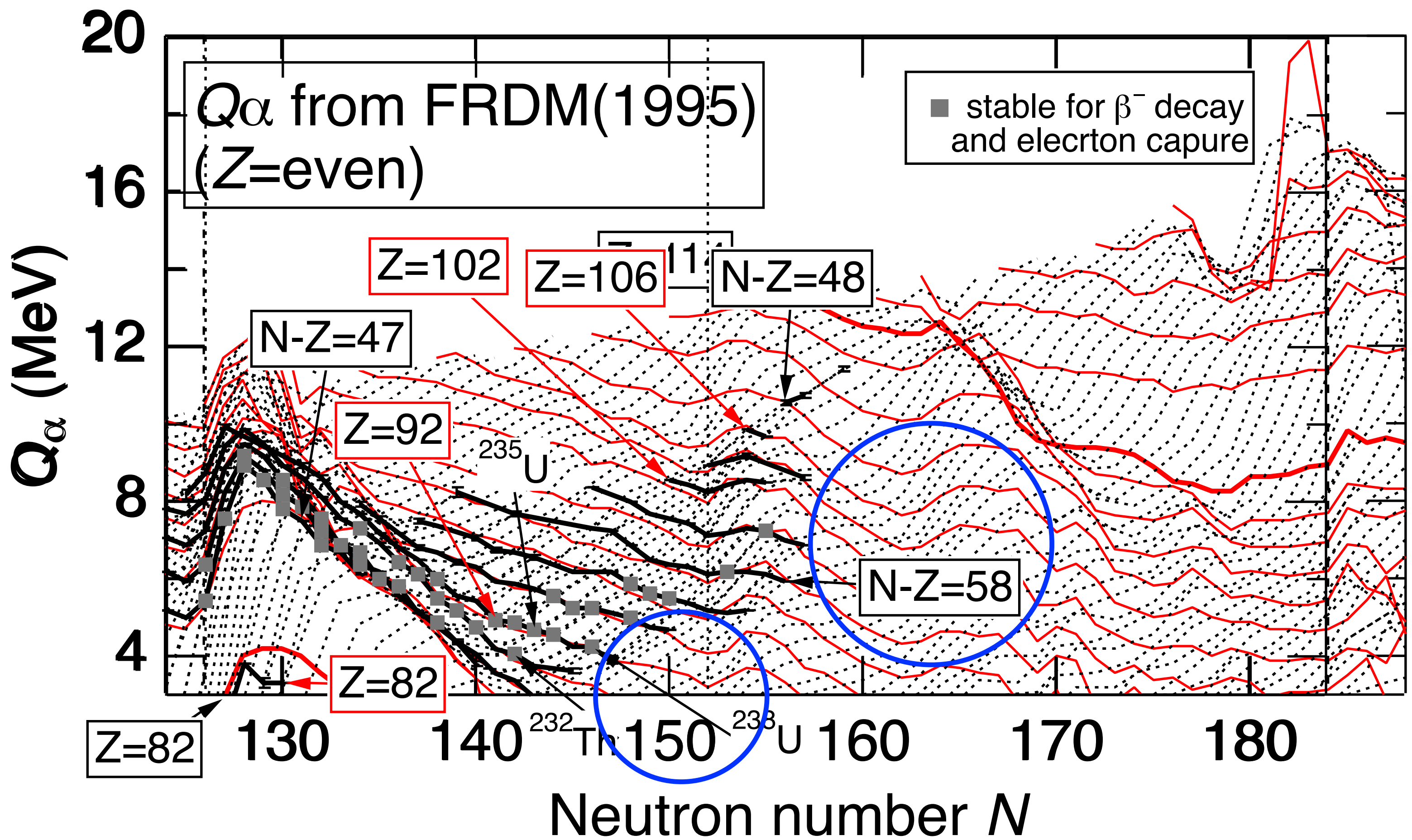


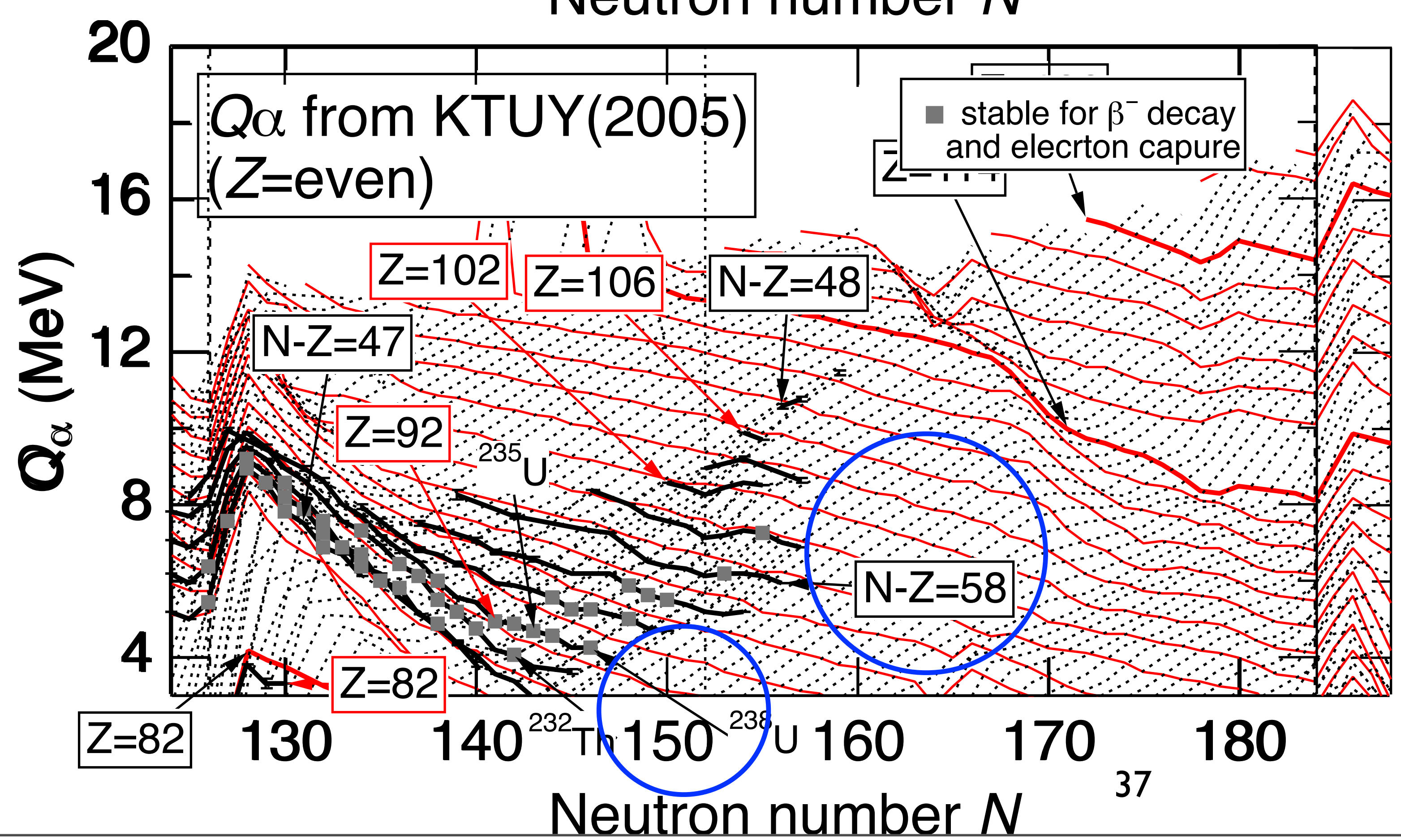
Search for N=162 deformed shell **in the ground-state** is required.

(Also for N=152)

α-decay Q-value of superheavy nuclei

=> Prediction of structure for SHE





Search for N=162 deformed shell **in the ground-state** is required.

(Also for N=152)

Conclusion

- We give a short review of systematical properties of experimental nuclear masses.
  - Mass-systematics like G-K is a good tool to check mass values.
- We survey various mass formulae:
  - Old-parametrized mass formulae (in 1976, 88) generally fail to extrapolation.
  - HFB type mass formulae sometimes give anomaly on GK-sys or alpha-chain sys.
- At the n-rich, A=130 and 195 related to the r-process, there is poor exp. mass data.
- In the neutron-rich heavy mass region and superheavy mass region, mass measurements is required for ground-state information.