Microscopic description of nuclear structure

– shell structure and nuclear effective interaction

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1. Magic numbers and shell structure in nuclei
2. One- and two-particle nuclei
3. Effective interaction between identical particles
4. Proton-neutron interaction and isomers

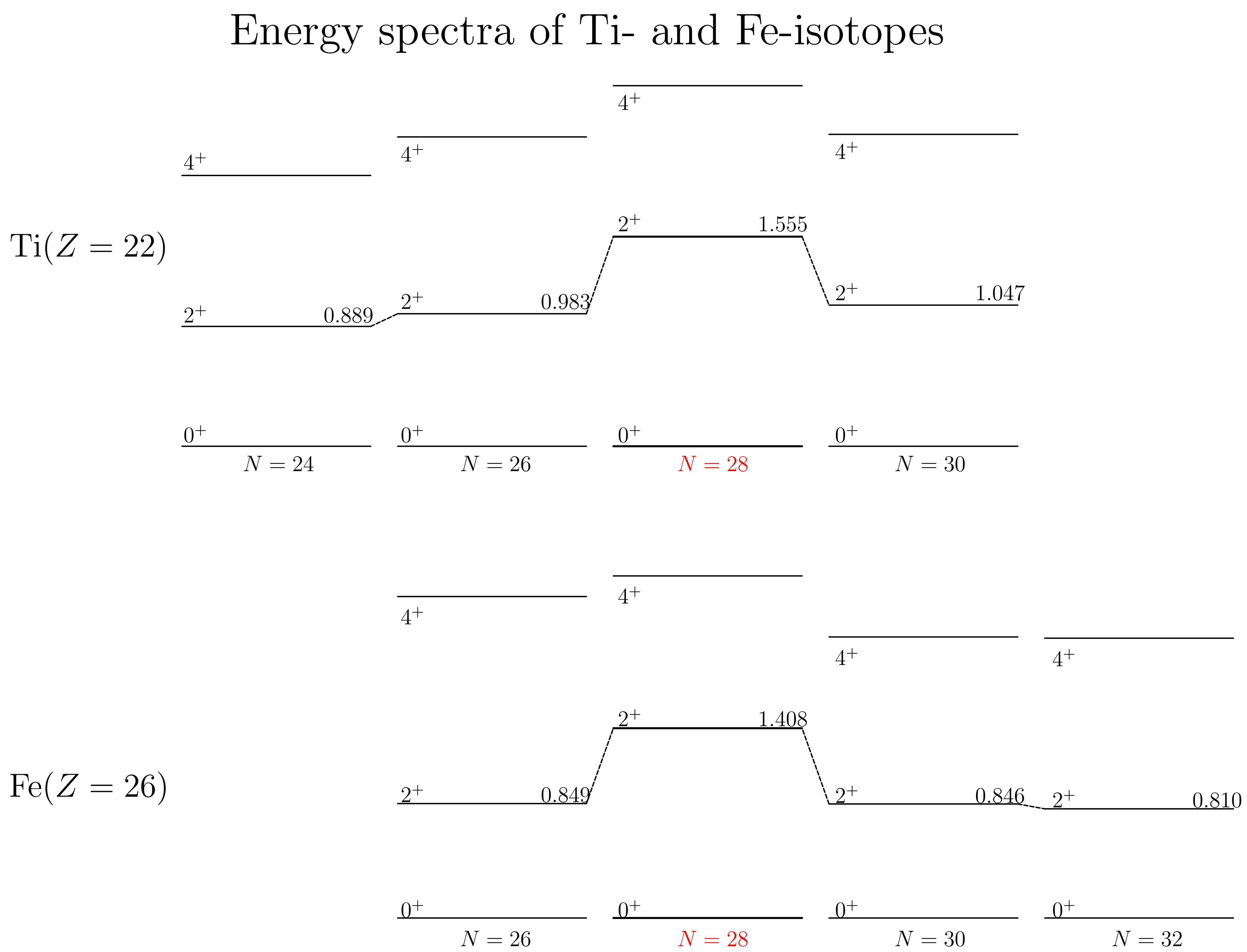
1. Magic numbers and shell structure in nuclei

Nuclear Hamiltonian

$$\mathcal{H} = \Sigma_{i=1}^{A} T_i \ + \ \Sigma_{i<j=A}^{A} V_{ij}$$

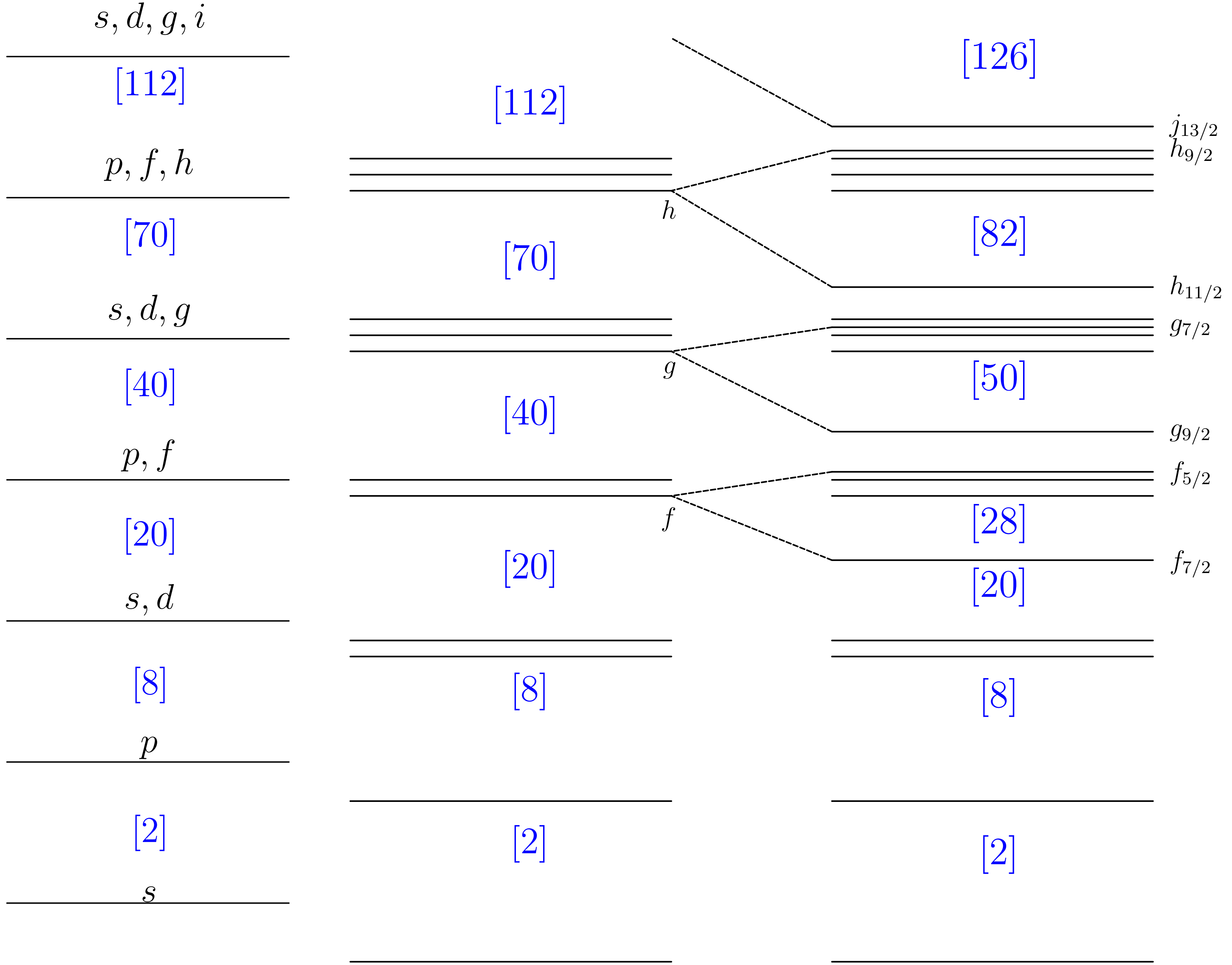
$\leftarrow$ Magic numbers

$$\mathcal{H} = \Sigma_{i=1}^{A} (T_i \ + \ U_i) \setminus \Sigma_{i<j=A}^{A} \mathcal{V}_{ij}$$


Energy spectra of Ti- and Fe-isotopes
Ti(Z = 22)
Fe(Z = 26)
4+
2+
0+
0.889
0.983
1.555
1.047
1.408
0.849
0.846
0.810
N = 24
N = 26
N = 28
N = 30
N = 32

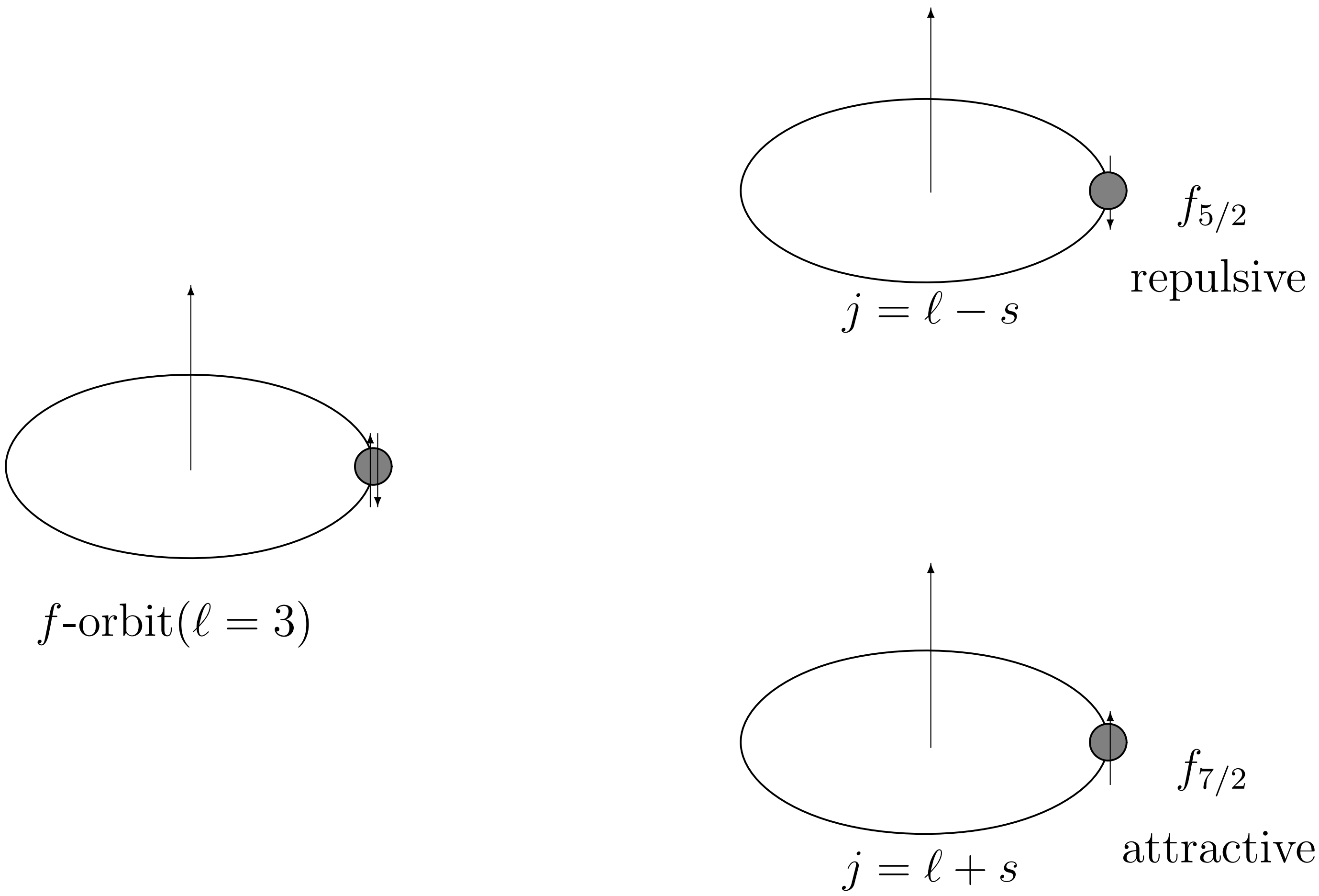
Nuclear Binding Energy

Magic numbers in nuclei



Spin-orbit Splitting

$$-\xi(\boldsymbol{\ell} \cdot \boldsymbol{s})$$



Magic Number

$$Z = 2,\ 8,\ 20,\ 28,\ 50,\ 82$$

$$N = 2,\ 8,\ 20,\ 28,\ 50,\ 82,\ 126$$

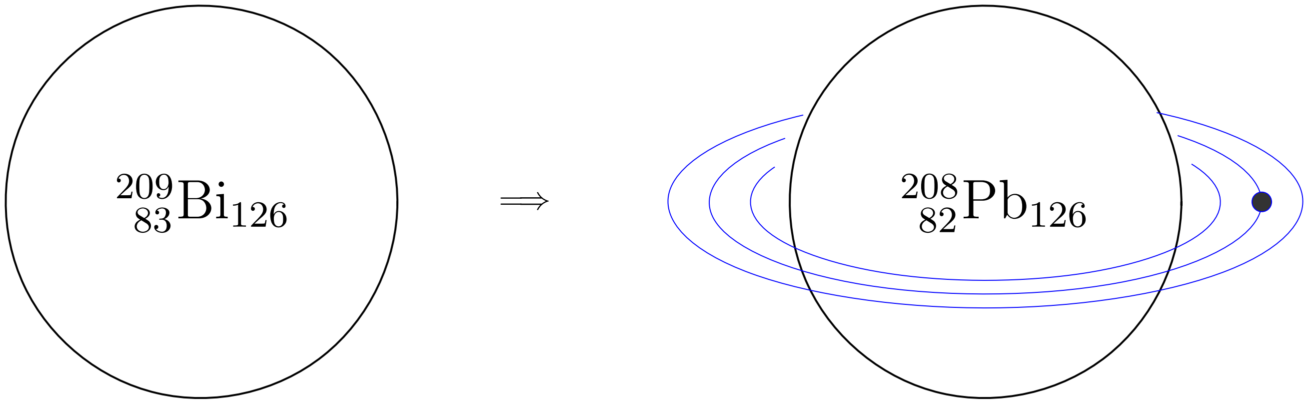
Magic numbers are explained by introducing strong spin-orbit force $-\xi(\boldsymbol{\ell} \cdot \boldsymbol{s})$

2. One- and Two-particle nuclei

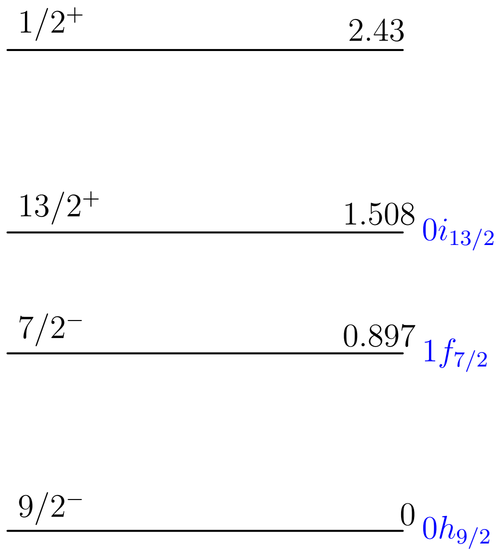
Magic numbers provide us inert core nuclei,

$$\text{e.g. } {}^{4}_{2}\mathrm{He}_{2},\ {}^{16}_{8}\mathrm{O}_{8},\ {}^{40}_{20}\mathrm{Ca}_{20},\ {}^{48}_{20}\mathrm{Ca}_{28},\ {}^{208}_{82}\mathrm{Pb}_{126}$$

$${}^{209}_{83}\mathrm{Bi}_{126} \Longrightarrow {}^{208}_{82}\mathrm{Pb}_{126} + p$$

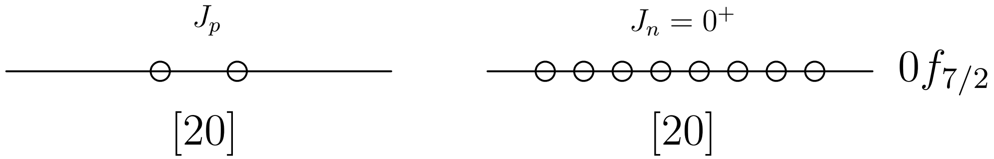


single-particle states



2-particle nuclei

$$^{50}_{22}\mathrm{Ti}_{28} \Longrightarrow \, ^{48}_{20}\mathrm{Ca}_{28} + 2p$$



Possible spin-states J in $(0f_{7/2})^2$-configuration $\Longrightarrow$ $J_p = 0^+,\ 2^+,\ 4^+,\ 6^+$

m-scheme for fermion system

Example $(d_{3/2})^2$

m=3/2	1/2	-1/2	-3/2	M
×	×			2
×		×		1
×			×	0
	×	×		0
	×		×	-1
		×	×	-2

$$M = 2,\ 1,\ 0,\ -1,\ -2 \quad \Longrightarrow \quad J = 0^+$$

$$M = 0 \quad \Longrightarrow \quad J = 0^+$$

m-scheme for boson system

Example $(d)^2$

m=2	1	0	-1	-2	M
××					4
×	×				3
×		×			2
×			×		1
×				×	0
	××				2
	×	×			1
	×		×		0
	×			×	-1
		××			0
		×	×		-1
		×		×	-2
			××		-2
			×	×	-3
				××	-4

Problem: Derive all possible J in the $f^2_{7/2}$-configuration.

m=7/2	5/2	3/2	1/2	-1/2	-3/2	-5/2	-7/2	M

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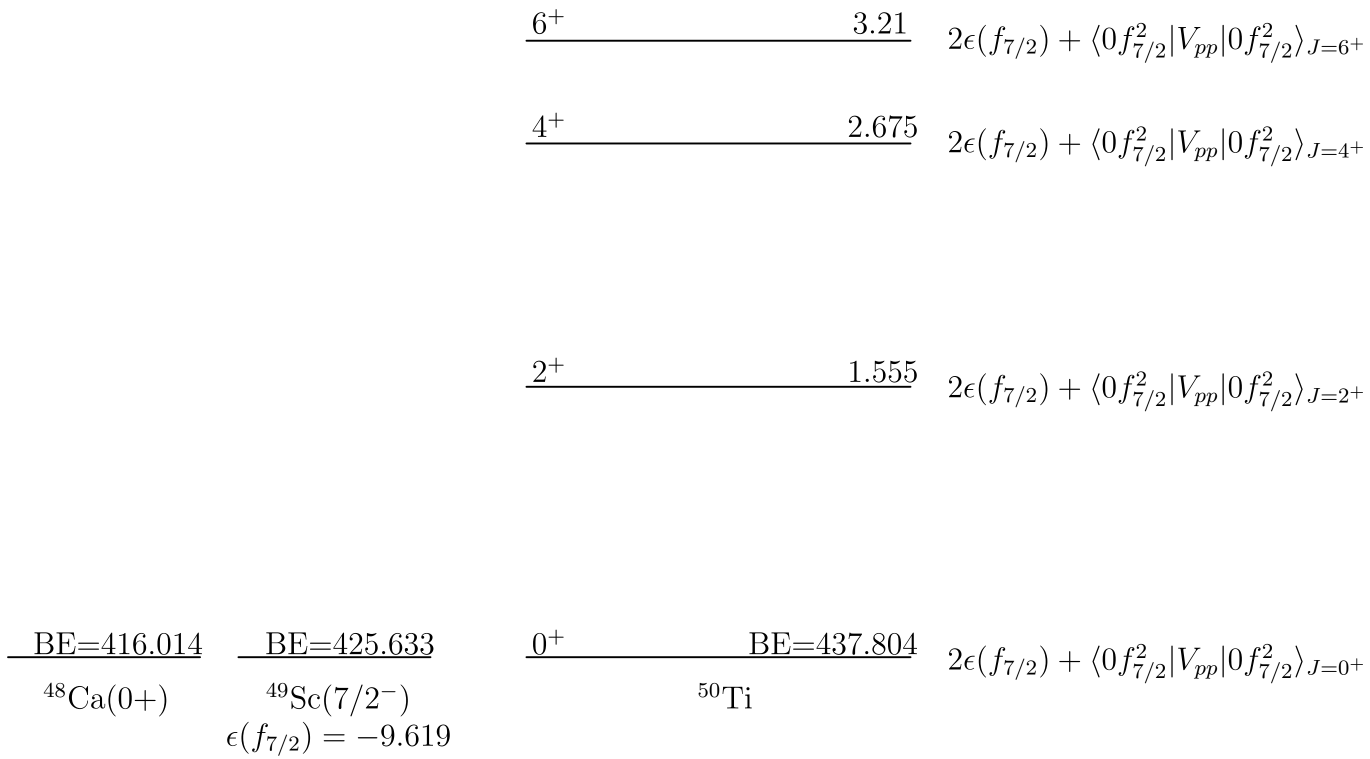
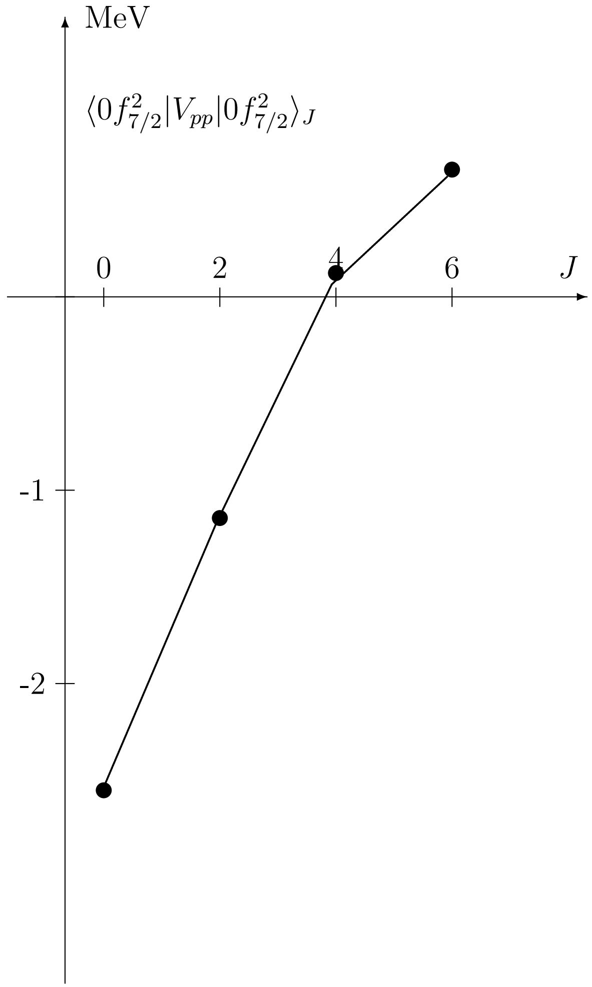
m=7/2	5/2	3/2	1/2	-1/2	-3/2	-5/2	-7/2	M

$$^{50}_{22}\mathrm{Ti}_{28} \Longrightarrow {}^{48}_{20}\mathrm{Ca}_{28} + 2p$$





Problem: Derive experimental values of $\langle 0f^2_{7/2}|V_{pp}|0f^2_{7/2}\rangle_J$ for $J = 0^+,\ 2^+,\ 4^+,\ 6^+$

$$\langle 0f^2_{7/2}|V_{pp}|0f^2_{7/2}\rangle_{J=0^+} =$$

$$\langle 0f^2_{7/2}|V_{pp}|0f^2_{7/2}\rangle_{J=2^+} =$$

$$\langle 0f^2_{7/2}|V_{pp}|0f^2_{7/2}\rangle_{J=4^+} =$$

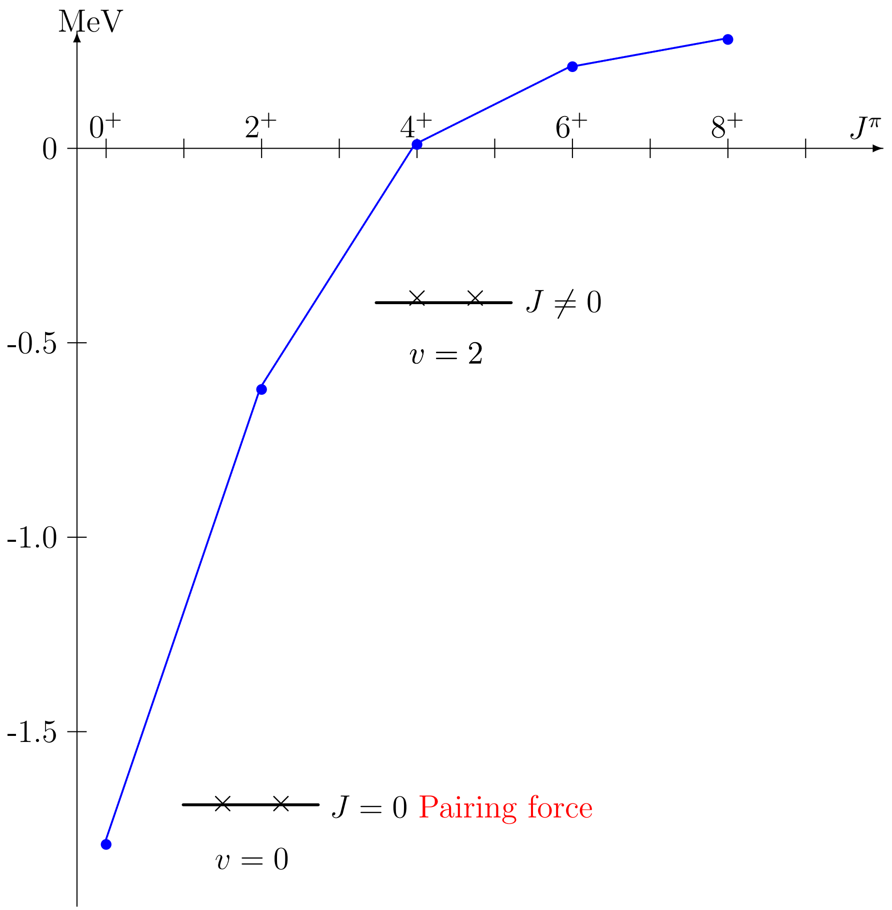
$$\langle 0f^2_{7/2}|V_{pp}|0f^2_{7/2}\rangle_{J=6^+} =$$

3. Effective two-body interaction between identical particles

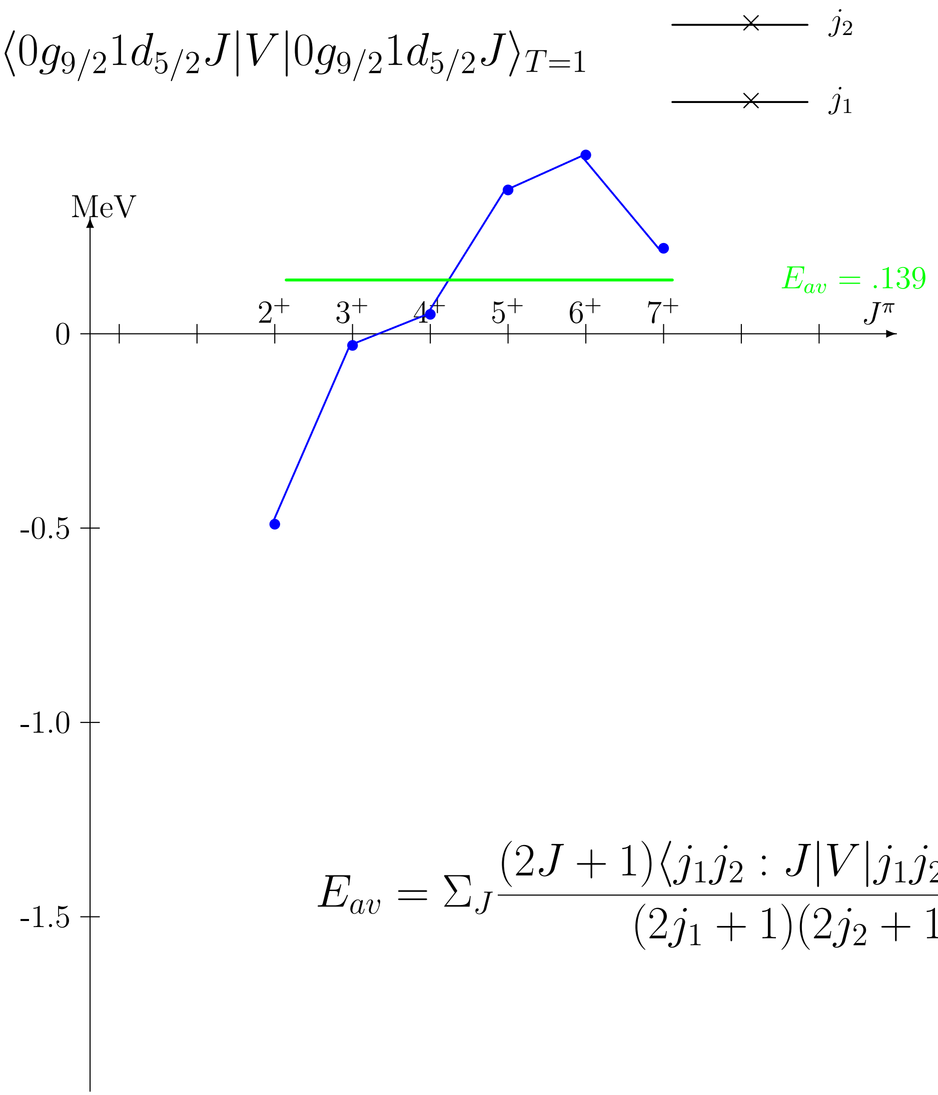
—(proton-proton or neutron-neutron interactions)

$\langle (0g_{9/2})^2 J | V | (0g_{9/2})^2 J \rangle_{T=1}$

v:seniority



$\langle 0g_{9/2} 1d_{5/2} J | V | 0g_{9/2} 1d_{5/2} J \rangle_{T=1}$



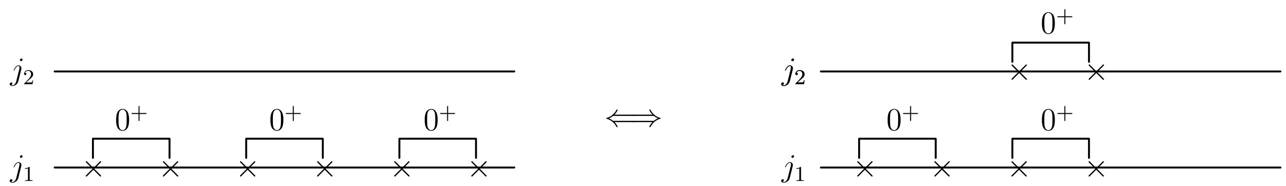
$$E_{av} = \Sigma_J \frac{(2J+1)\langle j_1 j_2 : J | V | j_1 j_2 : J \rangle_{T=1}}{(2j_1+1)(2j_2+1)}$$

Pairing property of effective interaction between identical particles

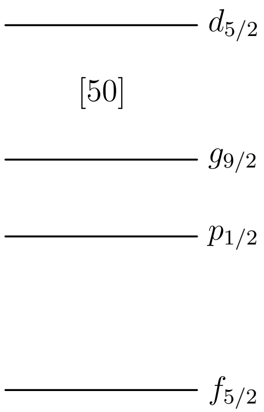
Strong attractive force $\langle j^2 J = 0^+ | V | j^2 J = 0^+ \rangle_{T=1}$

Large matrix element $\langle j^2 J = 0^+ | V | j'^2 J = 0^+ \rangle_{T=1}$

- Such property is reproduced by short-range force like $-V_0 \delta(r)$
- Application of BCS theory to nuclear system



Structure of ^{90}Zr



$$^{90}_{40}\mathrm{Zr}_{50}$$

Proton: $Z = 40 \quad (g_{9/2}, p_{1/2})_p^{-10} \;=\; (g_{9/2}, p_{1/2})_p^{2}$

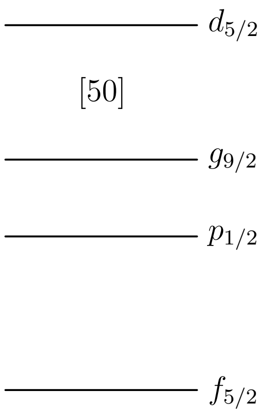
Neutron: $N = 50 \quad$ (closed shell) $\quad J_n = 0^+$

$$(g_{9/2})^2 \quad \rightarrow \quad J =$$

$$(g_{9/2}p_{1/2}) \quad \rightarrow \quad J =$$

$$(p_{1/2})^2 \quad \rightarrow \quad J =$$

Structure of ^{90}Zr



$^{90}_{40}\mathrm{Zr}_{50}$

Proton: $Z = 40 \quad (g_{9/2}, p_{1/2})_p^{-10} \quad = \quad (g_{9/2}, p_{1/2})_p^2$

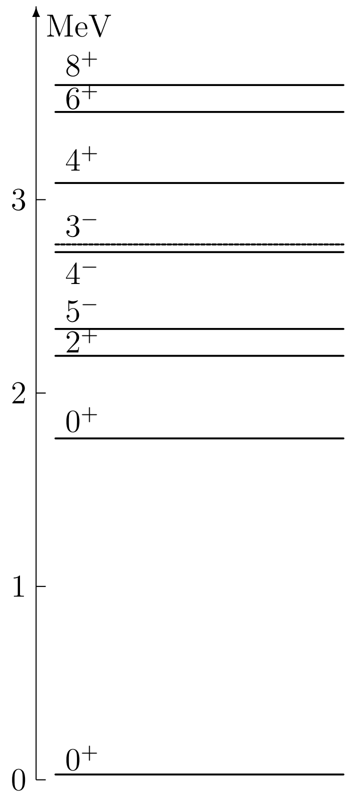
Neutron: $N = 50 \quad$ (closed shell) $\quad J_n = 0^+$

$$(g_{9/2})^2 \qquad \rightarrow \quad J = 0^+,\ 2^+,\ 4^-+,\ 6^+,\ 8^+$$

$$(g_{9/2}p_{1/2}) \quad \rightarrow \quad J = 4^-,\ 5^-$$

$$(p_{1/2})^2 \qquad \rightarrow \quad J = 0^+$$

^{90}Zr



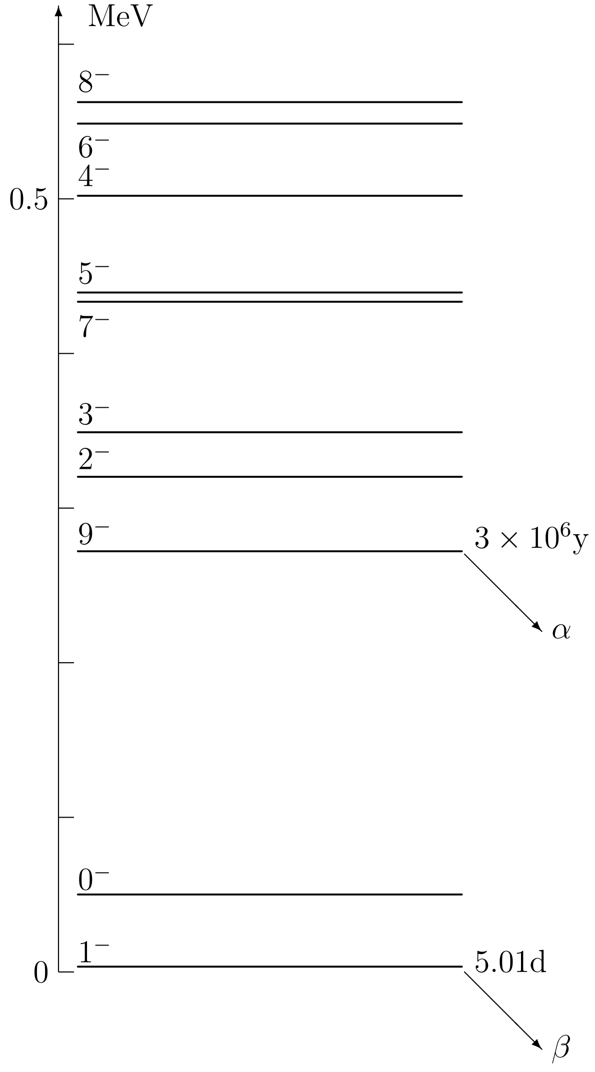
$\longleftarrow \quad 2\epsilon(g_{9/2}) + \langle g_{9/2}^2 J = 8^+ | V_{pp} | g_{9/2}^2 J = 8^+ \rangle$

$\longleftarrow \quad \epsilon(g_{9/2}) + \epsilon(p_{1/2}) + \langle g_{9/2} p_{1/2} J = 5^- | V_{pp} | g_{9/2} p_{1/2} J = 5^- \rangle$

$$\left(\begin{array}{cc} 2\epsilon(g_{9/2}) + \langle g_{9/2}^2 J = 0^+ | V | g_{9/2}^2 J = 0^+ \rangle & \langle g_{9/2}^2 J = 0^+ | V | p_{1/2}^2 J = 0^+ \rangle \\ \langle g_{9/2}^2 J = 0^+ | V | p_{1/2}^2 J = 0^+ \rangle & 2\epsilon(p_{1/2}) + \langle p_{1/2}^2 J = 0^+ | V | p_{1/2}^2 J = 0^+ \rangle \end{array} \right) \left(\begin{array}{c} \alpha \\ \beta \end{array} \right) = E \left(\begin{array}{c} \alpha \\ \beta \end{array} \right)$$

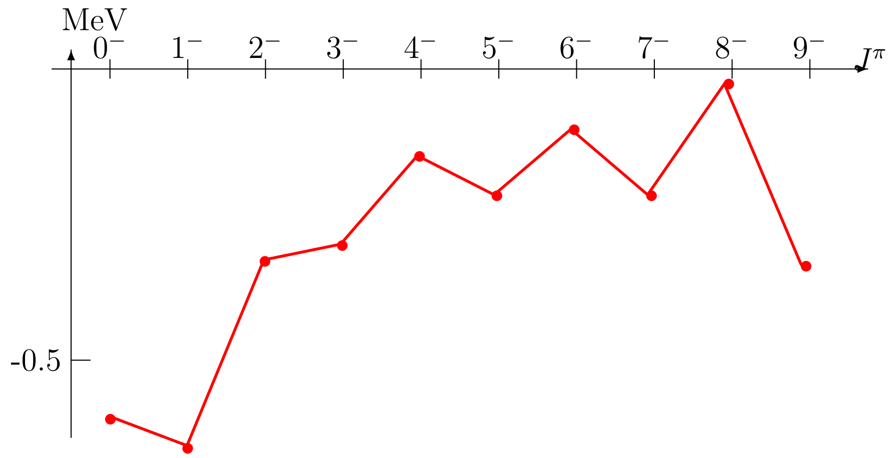
Eigen-value problem

4. Proton-neutron interaction and isomer

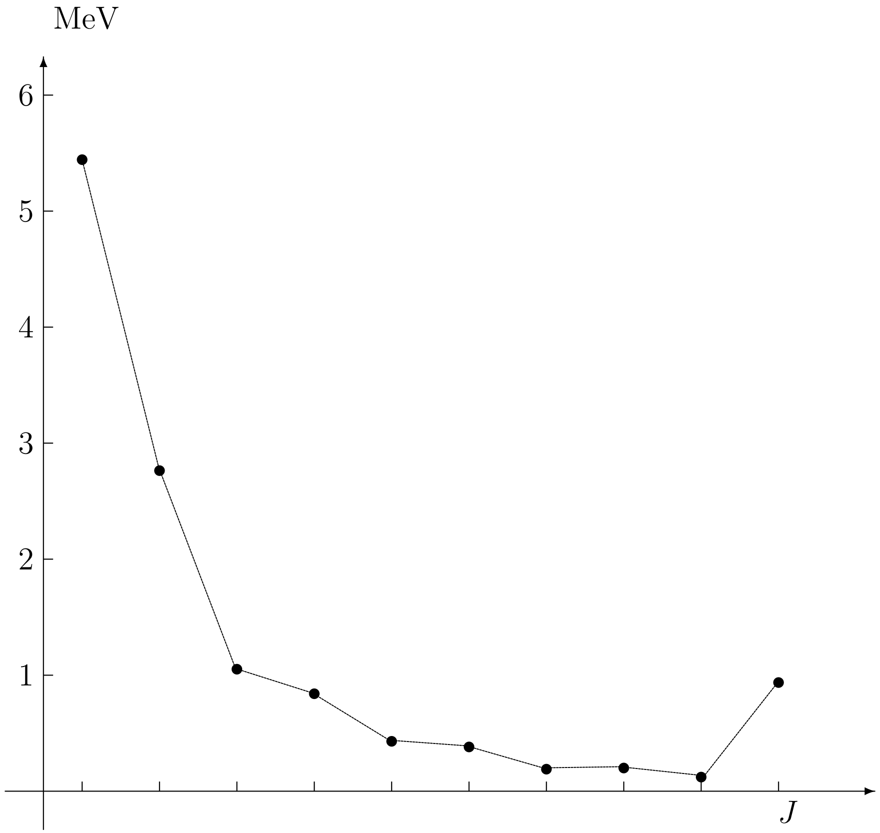


$^{210}_{83}\mathrm{Bi}_{127}$ $\qquad (0h_{9/2})_p \times (1g_{9/2})_n \; J = 0^-,\ 1^-,\ \cdots\ , 9^-$

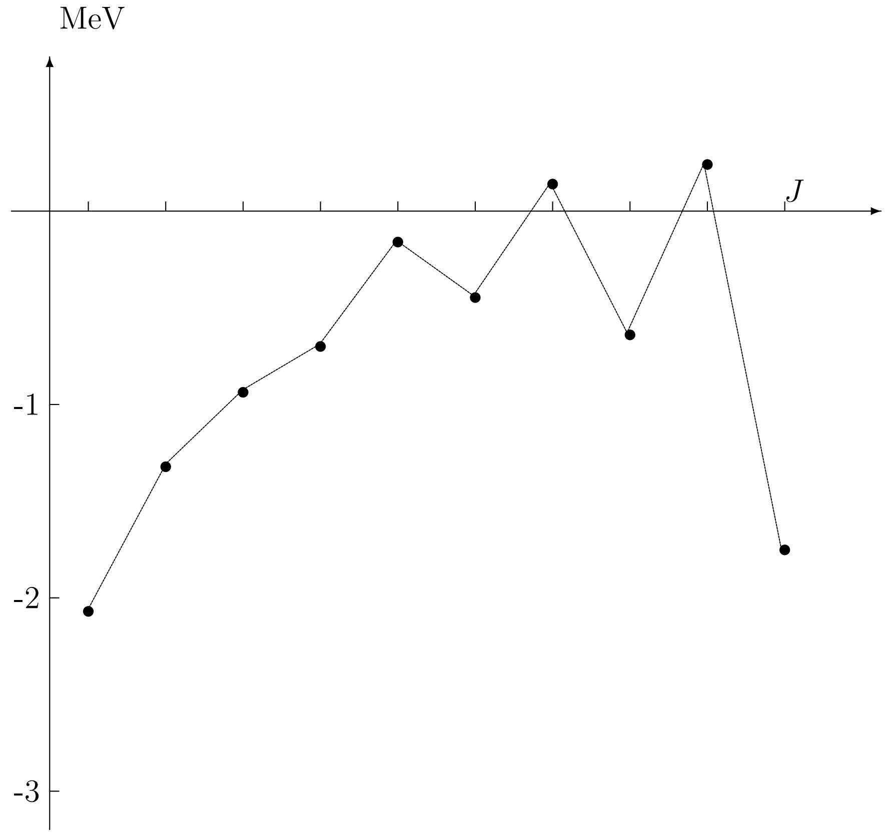
$\langle (0h_{9/2})_p (1g_{9/2})_n J | V_{pn} | (0h_{9/2})_p (1g_{9/2})_n J \rangle$



$$\langle (0g_{9/2})_p (0g_{9/2})_n^{-1} : J | V_{pn} | (0g_{9/2})_p (0g_{9/2})_n^{-1} : J \rangle$$



$$\langle (0g_{9/2})_p (0g_{9/2})_n : J | V_{pn} | (0g_{9/2})_p (0g_{9/2})_n : J \rangle$$

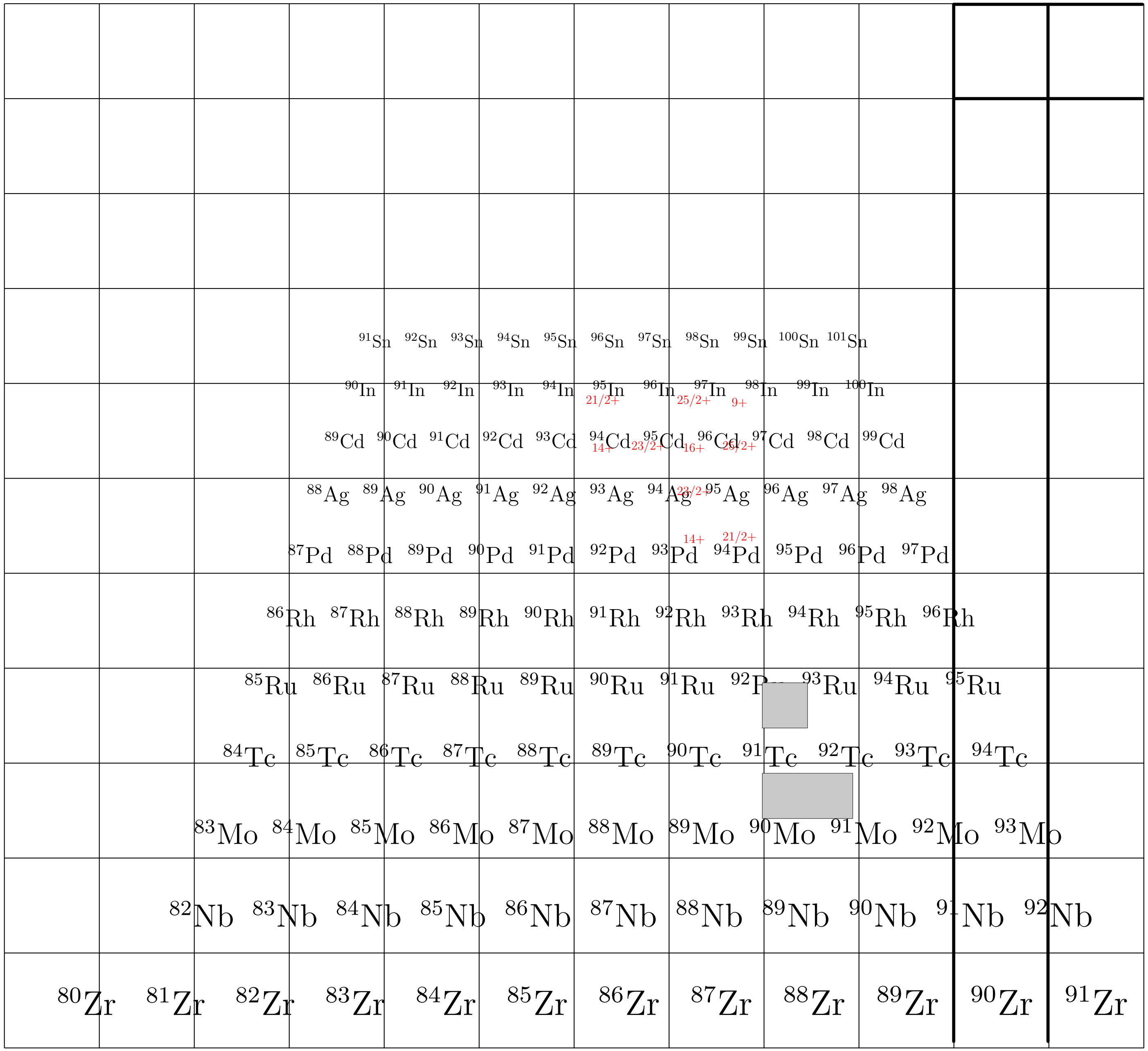


Shell-model calculations of high-spin ispmers in neutron-defficient $1g_{9/2}$-shell nuclei

K. Ogawa: Phys. Rev. C28(1983)958

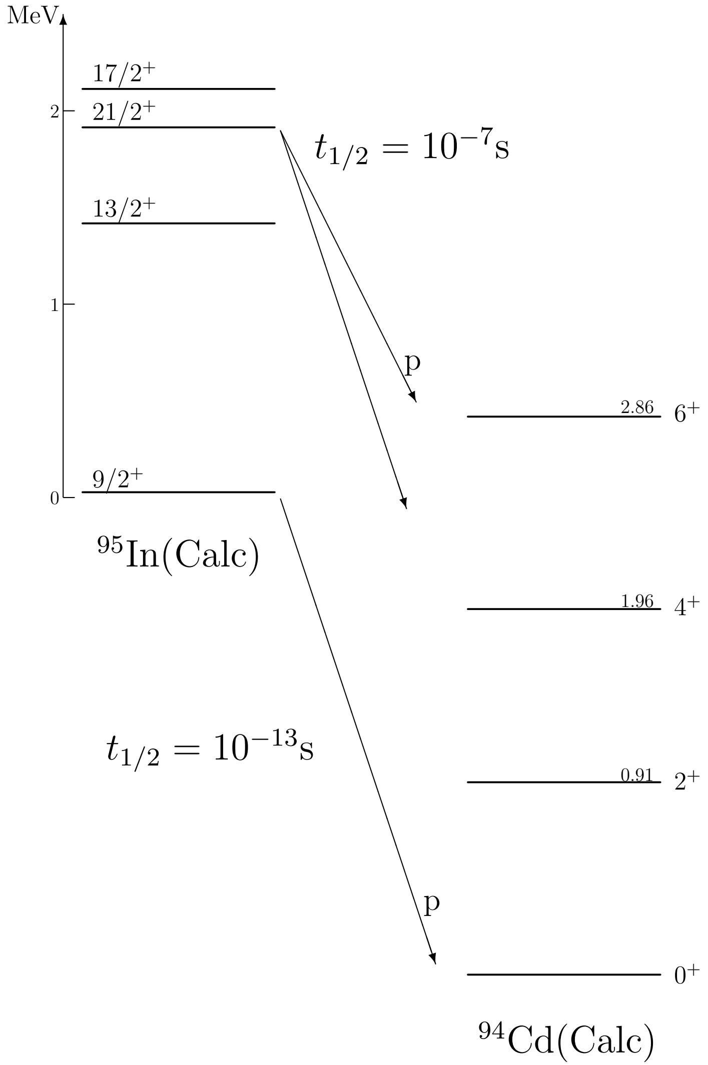
^{95}Pd

^{96}Cd



^{91}Sn ^{92}Sn ^{93}Sn ^{94}Sn ^{95}Sn ^{96}Sn ^{97}Sn ^{98}Sn ^{99}Sn ^{100}Sn ^{101}Sn
^{90}In ^{91}In ^{92}In ^{93}In ^{94}In ^{95}In ^{96}In ^{97}In ^{98}In ^{99}In ^{100}In
21/2+ 25/2+ 9+
^{89}Cd ^{90}Cd ^{91}Cd ^{92}Cd ^{93}Cd ^{94}Cd ^{95}Cd ^{96}Cd ^{97}Cd ^{98}Cd ^{99}Cd
14+ 23/2+ 16+ 25/2+
^{88}Ag ^{89}Ag ^{90}Ag ^{91}Ag ^{92}Ag ^{93}Ag ^{94}Ag ^{95}Ag ^{96}Ag ^{97}Ag ^{98}Ag
23/2+
14+ 21/2+
^{87}Pd ^{88}Pd ^{89}Pd ^{90}Pd ^{91}Pd ^{92}Pd ^{93}Pd ^{94}Pd ^{95}Pd ^{96}Pd ^{97}Pd
^{86}Rh ^{87}Rh ^{88}Rh ^{89}Rh ^{90}Rh ^{91}Rh ^{92}Rh ^{93}Rh ^{94}Rh ^{95}Rh ^{96}Rh
^{85}Ru ^{86}Ru ^{87}Ru ^{88}Ru ^{89}Ru ^{90}Ru ^{91}Ru ^{93}Ru ^{94}Ru ^{95}Ru
^{84}Tc ^{85}Tc ^{86}Tc ^{87}Tc ^{88}Tc ^{89}Tc ^{90}Tc ^{91}Tc ^{92}Tc ^{93}Tc ^{94}Tc
^{83}Mo ^{84}Mo ^{85}Mo ^{86}Mo ^{87}Mo ^{88}Mo ^{89}Mo ^{90}Mo ^{91}Mo ^{92}Mo ^{93}Mo
^{82}Nb ^{83}Nb ^{84}Nb ^{85}Nb ^{86}Nb ^{87}Nb ^{88}Nb ^{89}Nb ^{90}Nb ^{91}Nb ^{92}Nb
^{80}Zr ^{81}Zr ^{82}Zr ^{83}Zr ^{84}Zr ^{85}Zr ^{86}Zr ^{87}Zr ^{88}Zr ^{89}Zr ^{90}Zr ^{91}Zr

Stability of $^{95}_{49}$In



High-spin isomers in unstable nuclei