Nucleon Structure by Lattice QCD

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RBC Nucleon group
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RIKEN BNL Research Center

BROOKHAVEN NATIONAL LABORATORY

RIKEN

July 29 2013, sPHENIX workfest, RIKEN, Wako, Japan

Contents

Who are we ?

- RIKEN-BNL Research Center

  2 fellows, 2 PostDocs
  + visiting scientists / students

  RIKEN BNL Columbia (RBC) Collaboration (1998-)

- Columbia University
  2 faculty, 1 PostDoc,
  7+2 Students

- University of Connecticut
  1 faculty, 1 Students

- BNL HEP Theory
  3+1 scientists, 1+2 PostDocs,
  1 student (SciDAC, LDRD, JSPS)

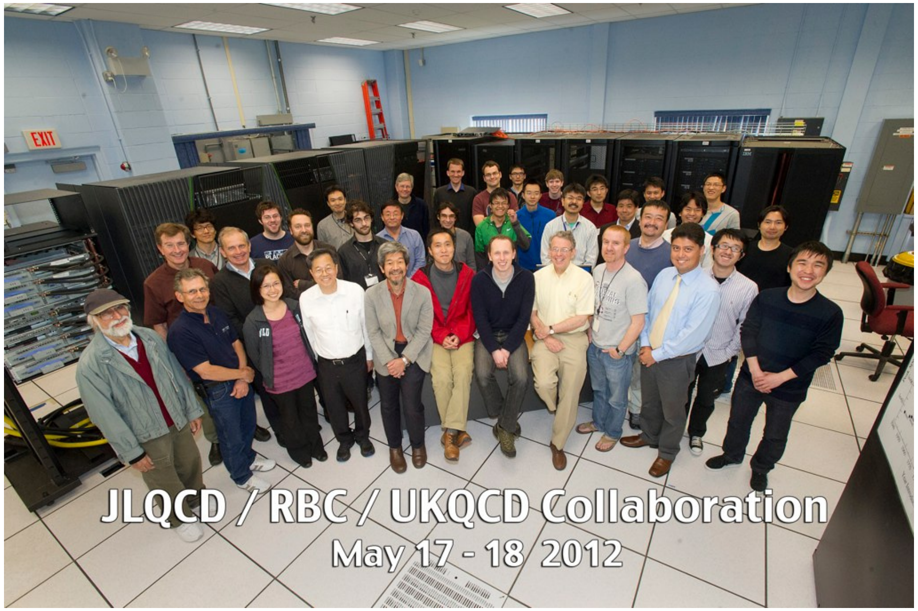
- BNL LG Theory
  3 scientists, 3+1 PostDocs (SciDAC)

14 current students,
~23 PhD theses since 2005

- + UKQCD Collaboration (2005-)
  - Univ. of Edinburgh
    1 faculty, 2 PostDocs, 2+1 students
  - Univ. of Southampton
    3 faculty, 2+1 Postdoc, 4 students

- + JLQCD (2012- , collaborating for physics measurement methods)
  - KEK, Tsukuba & Osaka Univ

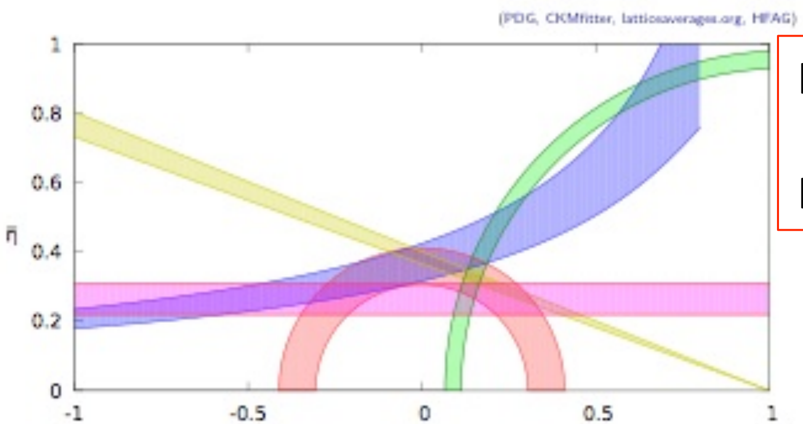
(# of personnel: accumulation of last 3 years)
(#(current) + #(just left, but still collaborating))



$K \rightarrow \pi\pi$ decay amplitude
[RBC/UKQCD]

- 40+years awaited theoretical calculation
  [1964 Cronin-Fitch, Nobelprize
  1999 NA48@CERN, KTeV@FNAL]
- Provide a new constraint on CKM Unitarity

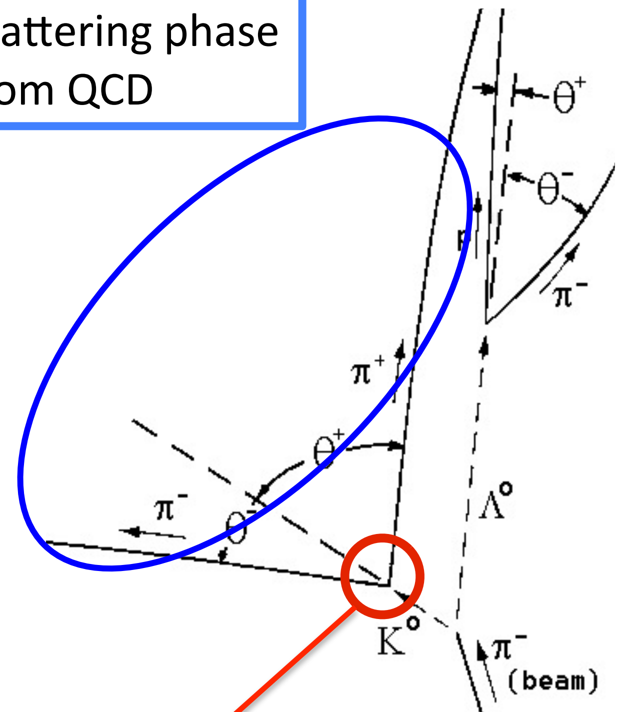
π π final state scattering phase from QCD

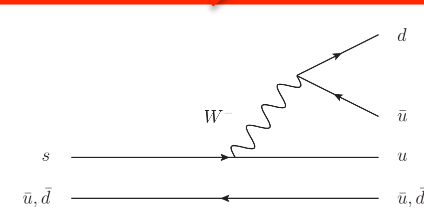


$K_L \rightarrow \pi\pi\pi$ (CP even)

$K_S \rightarrow \pi\pi$ (CP odd)

$\Delta I = 1/2$ rule
$\varepsilon' / \varepsilon$

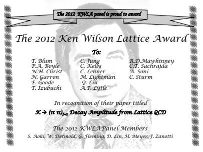




Electro-Weak (CKM) phase
Vus V*ud, Vts V*td

$K \to \pi\,\pi$ Amplitude, I=2 channel

2012 Ken Wilson Lattice Award



Lattice: $32^3 \times 64 \times 32$, 2+1 DWF, Iwasaki DSDR, $a^{-1} = 1.375(9)$ GeV, $m_{\pi^+} = 142.9(1.1)$ MeV

	ReA_2	ImA_2
Lattice artifacts	15%	15%
Finite-volume corrections	6.2%	6.8%
Partial quenching	3.5%	1.7%
Renormalization	1.7%	4.7%
Unphysical kinematics	3.0%	0.22%
Derivative of the phase-shift	0.32%	0.32%
Wilson coefficients	7.1%	8.1%
Total	18%	19%

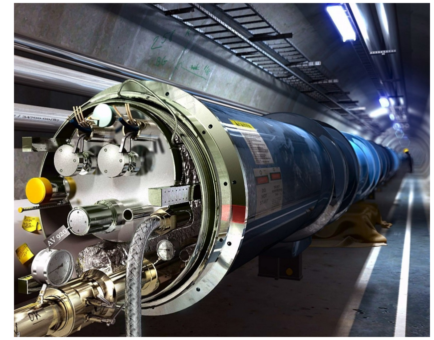
	Re$(A_2)(10^8/\mathrm{GeV})$	Im$(A_2)(10^{13}/\mathrm{GeV})$
Lat.	1.436(62)(258)	-6.83(51)(130)
Exp.	1.479(4) (K^+)	(n/a)

$\mathrm{Re}(\varepsilon'/\varepsilon)_{\mathrm{EWP}} = -6.52(49)(124) \times 10^{-4}$

Errors: (stat)(syst)

Lattice Gauge Theory Receipt

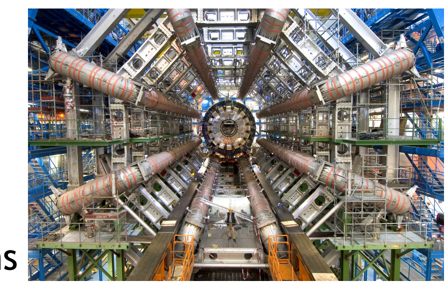
- QCD vacuum ensemble generation $\sim$ Accelerator



  - choice of gauge / sea quark actions
  - Algorithms / Machines
  - for each parameters $(a^{-1}, V, m^{\text{sea}})$

- Physical observable measurements $\sim$ Detector
  - valence quark propagators (low eigenvectors), m_f
  - Hadron n-point green's functions, matrix elements

$$\langle \mathcal{O} \rangle = \int \mathcal{D}U_\mu \mathcal{D}\bar{q}_i \mathcal{D}q_i \mathcal{O} e^{-S_{\text{LGT}}} / \langle 1 \rangle$$



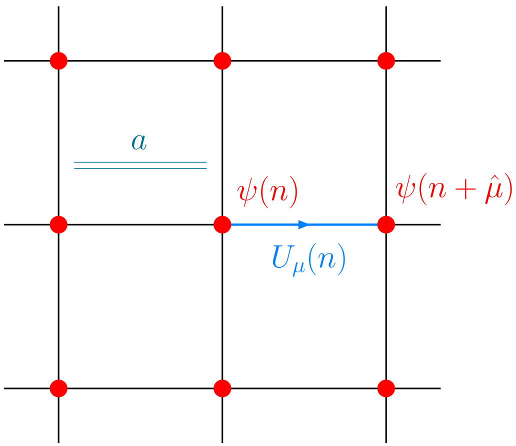
  - Renormalize and Chiral/Volume/continuum extrapolations
  - Algorithms / Machines (GPU,

$\Longrightarrow$ The final answers

Lattice Gauge Theory

- Analysis of Quantum Field Theory such as Quantum Chromo Dynamics, needs non-perturbative calculation.

  $\Psi(x), A_\mu(x), x \in \mathcal{R}^4$: *continuous infinity*
  quantum divergences: *needs regularization and renormalization*



- Discretized Euclidean space-time
- lattice spacing $a \sim 0.1$ fm
  (UV cut-off $|p| \leq \pi/a$)

- $\psi(n)$: Fermion field (Grassmann number)
- $U_\mu(n)$: Gauge field

- Feynman's path integral for Huge dimensional variables $32^3 \times 64 \sim 150$M
  Number of states (for simplest 4^4 Ising) $2^{4^4} \sim 10^{77}$ needs more than 10^{35} years !

RIKEN RICC ('09) ~ 110 Tflops peak

BG/Q('12) @Edinburgh, KEK~ 2 x 1.2 Pflops peak

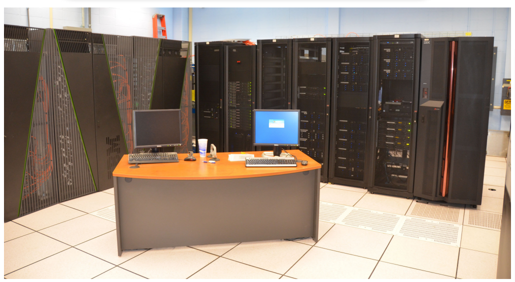
RBRC Computing

RIKEN/Nishina and iTHES

BNL
HEP/NP/LGT

US Universities
Columbia
Connecticut
CCNY (Colorado)

IBM



QCDCQ('12) ~ 600Tflops peak

RIKEN-BNL Research Center

UKQCD
Edinburgh Southampton
JLQCD
KEK, Tsukuba

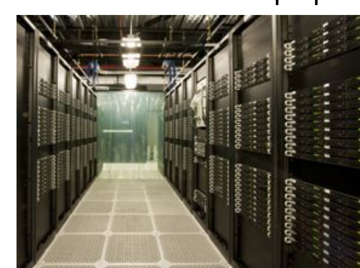
NYBlue('07)~ 130 Tflops peak

NYCCS
CCS/ITD

USQCD

ANL Mira ('12) ~10 Pflops peak

FNAL/Jlab ~ 160 Tflops peak



RBRC In-house Computing resources

2k node RBRC BGQ, 400 TFlops, 2012-
1k node BNL BGQ, 200 TFlops, 2012-

3k nodes RBRC/BNL BGQ, 600 TFlops, 2012
0.5 k nodes USQCD BGQ, 100 TFlops, 2013-2

- Rapid R&D of idea
- Try & error algorithms
- Optimize code on fully controlled **in-house machine**

- attract top-notch scientists
- define computing group

Extends / propagates to leadership computers
Kei, ANL, ONL

48 k node ALCF Mira, 10 PFlops

88 k node , AICS Kei, 11 PFlops

many breakthroughs in science & computing

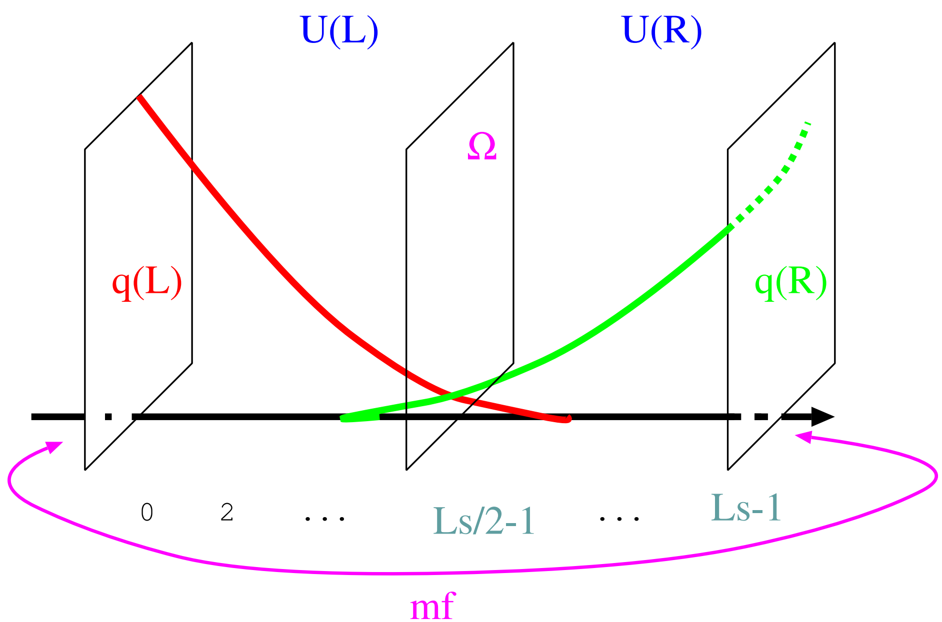
Domain Wall Quarks (for up, down, and strange)

[Kaplan, Shamir, Blum & Soni]

- 4D lattice quark utilizing an "extra dimension", L_s. (expensive)
- Almost perfect chiral symmetry

  Small unphysical mixing for the Weak Matrix Elements
  Error from discretization is small $\mathcal{O}(a^2\Lambda_{QCD}^2) \sim$ a few %.
  Chiral extrapolation is simpler, continuum like.

- Unitary theory (at long distance).



Flavor-Chiral Symmetry in QCD

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_{gauge} + i\bar{q}\,\not{D}q \qquad q = (u, d) \text{ or } = (u, d, s)$$

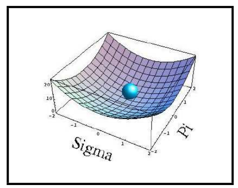
- pseudoscalar (PS) mesons are light as it would be Nambu-Goldstone bosons in the quark massless limit, where the spontaneous chiral symmetry breaking occurs,

$$SU(N_F)_V \times SU(N_F)_A \times U(1)_V \times U(1)_A \to SU(N_F)_V \times U(1)_V\text{:}$$

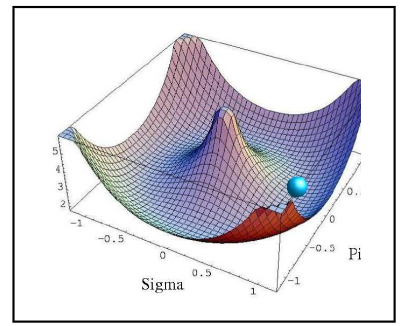
$$J_5(x) = i\bar{q}\gamma_5 q, \sigma(x) = \bar{q}q$$

$$A^a_\mu(x) = \overline{q}(x)\tau^a\gamma_5\gamma_\mu q(x)$$

$$\partial_\mu A_\mu(x) = 2mJ_5(x).$$



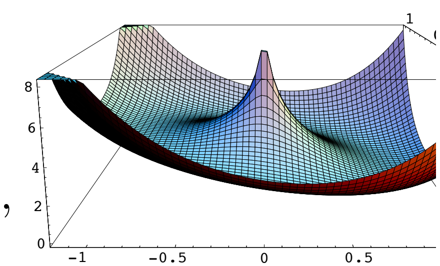
$$\langle \bar{q}q \rangle \neq 0 \quad \Longrightarrow$$



$$\partial_\mu \langle A_\mu(x) J_5(y) \rangle = 2m \langle J_5(x) J_5(y) \rangle ,$$

$$\partial_0 \langle A_0(x)|\pi\rangle \, e^{-M_\pi(x_0 - y_0)} (2E_\pi) \langle \pi | J_5(y) \rangle = 2m \langle J_5(x) J_5(y) \rangle ,$$

$$\longrightarrow M_\pi^2 \propto m \quad \text{(PCAC relation)}$$



Nucleon Structure

- **Form Factors**
  - Inelastic scattering
  - EM form factors
    Shape, Size, Current distribution of Nucleon
  - EW & BSM form factors
    Beta decay
    Electric Dipole Moment (EDM)

- **Structure Functions**
  - Moments of Parton Distribution Functions
    quark's momentum fraction <x>q
    quark's helicity fraction <x>Δq
  - Direct calculation of PDF q(x), g(x) [Xiangdong JI 2013]
  - GPD, TMD

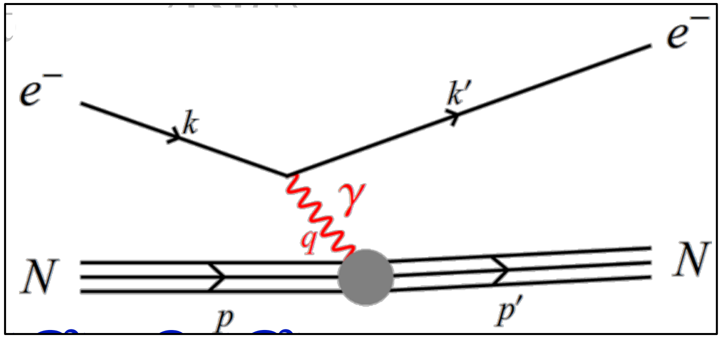
Lattice details

- QCD ensemble ("Accelerator")
  - Nf=2+1 (up, down, strange dynamical quark) Domain Wall Fermion, generated by RBC/UKQCD Collaboration
  - Iwasaki RG-improved gauge action and ~ 1/a mass quark to diminish the residual chiral symmetry breaking (lattice artifact)
  - beta = 1.75
  - Use masses of Pion, Kaon, Omega to set scale and quark masses → 1/a = 1.37 GeV,
  - two up/down quark masses (isospin symmetric)

am_l	am_s	$L^3 \times T$	L_s	m_π [MeV]	$m_\pi L$	a [fm]	$am_{\rm res}$
0.001	0.042	$32^3 \times 64$	32	170	4.0	0.146	0.0018
0.0042	0.042	$32^3 \times 64$	32	250	5.8	0.146	0.0018

EM Form Factors of Nucleon

- e-N Elastic Scattering
  - Dirac Form Factor F1(Q^2)
  - Pauli Form Factor F2(Q^2)



$$\langle \mathcal{N}(P')|J^{\mu}_{EM}|\mathcal{N}(P)\rangle = \bar{u}(P')\left[\gamma^{\mu} F_1(Q^2) + i\sigma^{\mu\nu}\frac{q_{\nu}}{2M_N}F_2(Q^2)\right]u(P)$$

- F1(Q^2=0) is EM charge of Nucleon
- F2(Q^2=0)= κ is Anomalous Magnetic Moment

- Mean square radii from slope of F_i(Q^2) at Q^2=0

$$\langle r_i^2 \rangle = -\frac{6}{F_i(0)}\frac{\partial F_i(Q^2)}{\partial Q^2}|_{Q^2=0}$$

  - Dirac radius : r1
  - Pauli radius : r2

Challenge in Nucleon calculation

- Some quantities are very sensitive to chiral symmetry → Use of expensive chiral lattice quark DWF. This has great advantages manifest in, e.g. CKM physics, K→ππ, ε'/ε
- Nucleon is more difficult : statistical Noise
- To avoid the excited state contamination, one needs to separate Nucleons and the EM current.

But signal to noise degrades exponentially

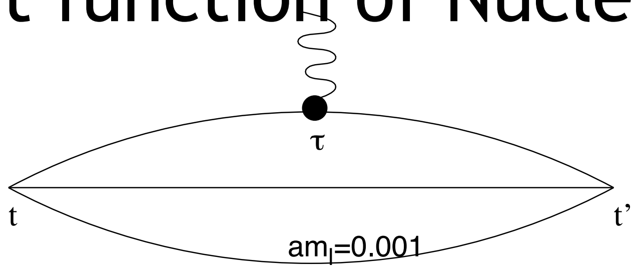
[Lepage]

$$S/N \sim C\exp[-t/\tau],$$

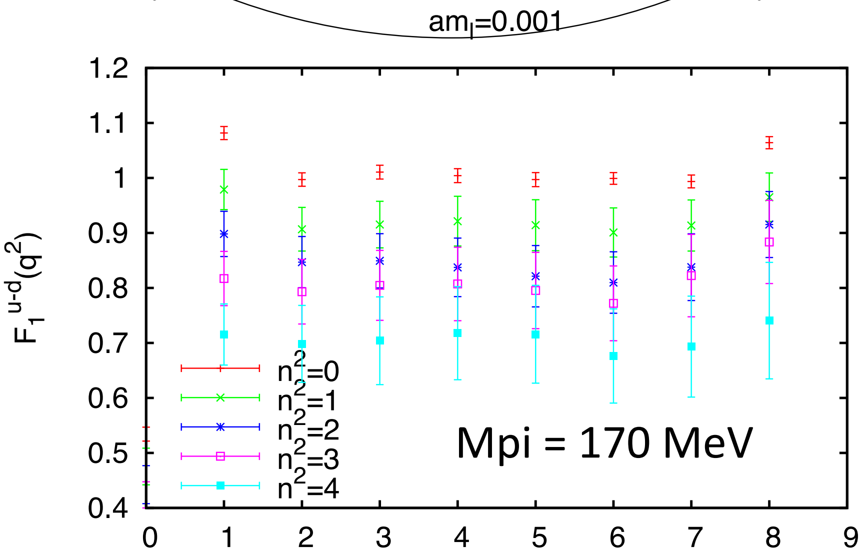
$$\tau = [m_N - 3/2m_\pi]^{-1} \approx 0.25\mathrm{fm}$$

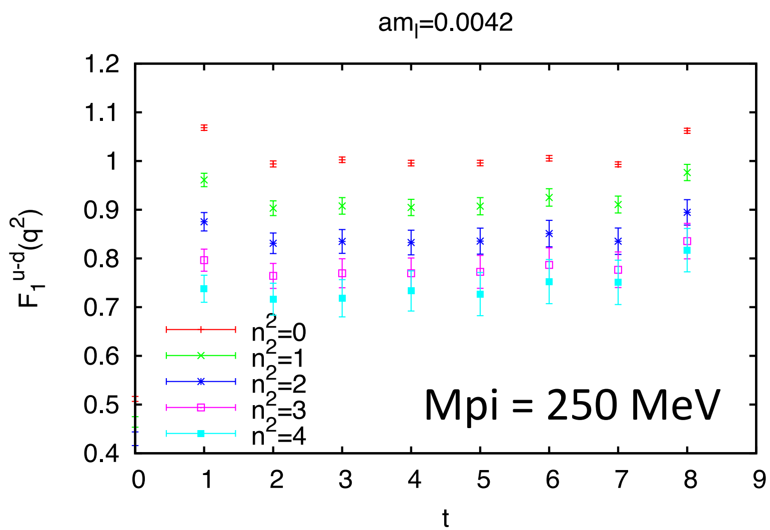
Example in F1 plateau

- 3pt function of Nucleon with momentum



$$P = \frac{2\pi}{L} n$$





- Lighter pion is more noisy
- distance b/w two N is 1.3 fm (also 1.1fm), S/N is proportional to exp(t/tau) ~ 1/ 300

[Blum, TI, Shintani arXiv:1208.4349, arXiv:1212.5542]

Covariant Approximation Averaging (CAA) a new class of Error reduction techniques

Original

$$\mathcal{O} = \mathcal{O}^{(\mathrm{appx})} - \mathcal{O}^{(\mathrm{rest})}$$

Lattice Symmetry

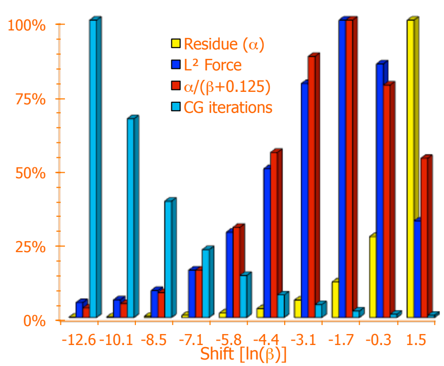
unbiased imporved

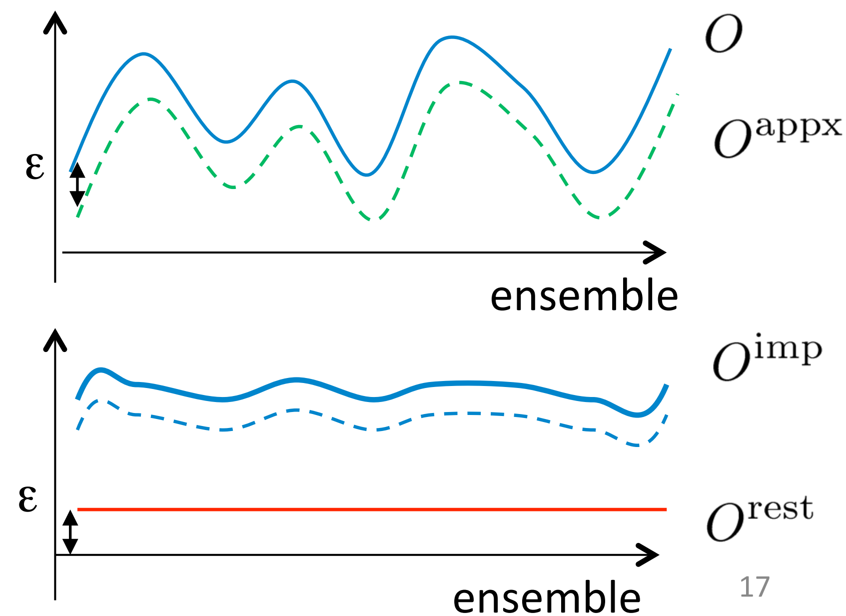
$$\mathcal{O}^{(\mathrm{imp})} = \mathcal{O}^{(\mathrm{rest})} + \frac{1}{N_G} \sum_{g \in G} \mathcal{O}^{(\mathrm{appx}),g}$$

Expensive : infrequently measured

Cheap : frequently measured

- $O^{(\mathrm{imp})}$ has smaller error
  $O^{(\mathrm{appx})}$ need to be cheap & not to be too accurate
  N_G suppresses the bulk part of noise cheaply





Examples of Covariant Approximations (contd.)

- All Mode Averaging **AMA**
  Sloppy CG or Polynomial approximations

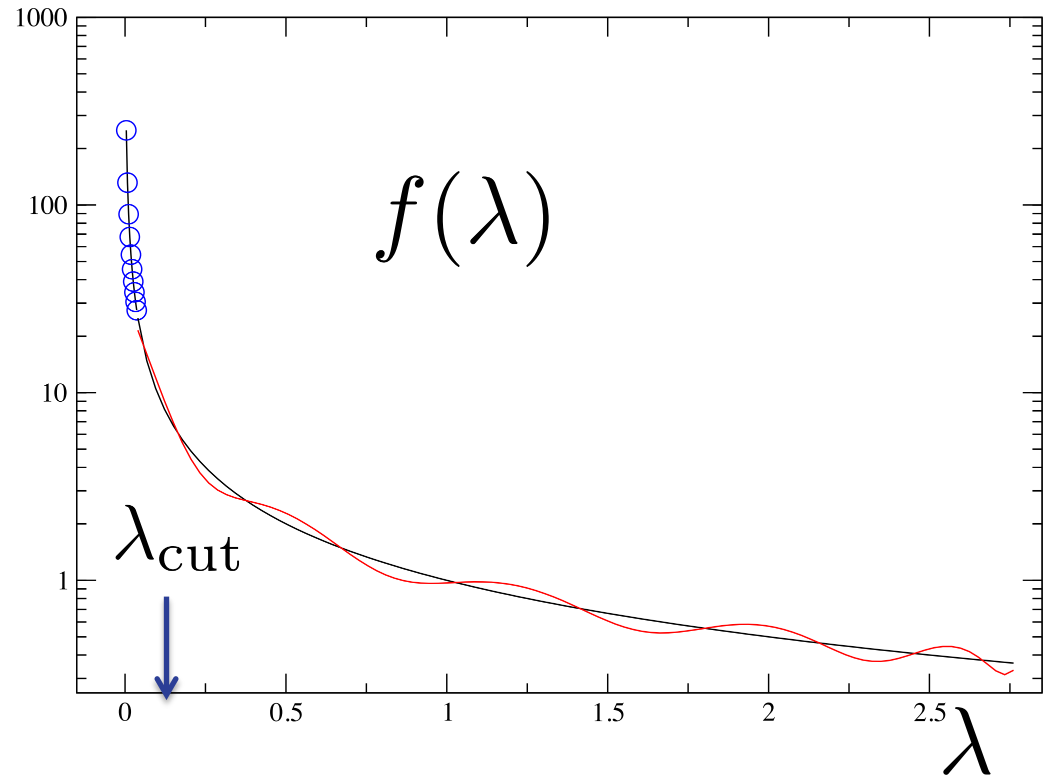
$$\mathcal{O}^{(\mathrm{appx})} = \mathcal{O}[S_l],$$

$$S_l = \sum_{\lambda} v_\lambda f(\lambda) v_\lambda^\dagger,$$

$$f(\lambda) = \begin{cases} \frac{1}{\lambda}, & |\lambda| < \lambda_{\mathrm{cut}} \\ P_n(\lambda) & |\lambda| > \lambda_{\mathrm{cut}} \end{cases}$$

$$P_n(\lambda) \approx \frac{1}{\lambda}$$

If quark mass is heavy, e.g. ~ strange, low mode isolation may be unneccesary



accuracy control :

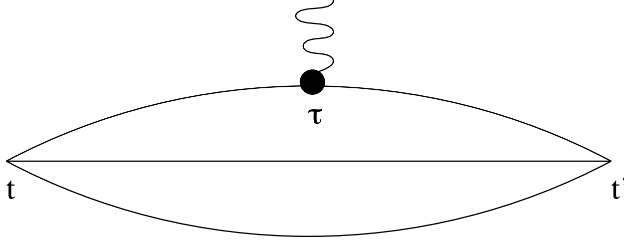
- low mode part : # of eig-mode
- mid-high mode : degree of poly.

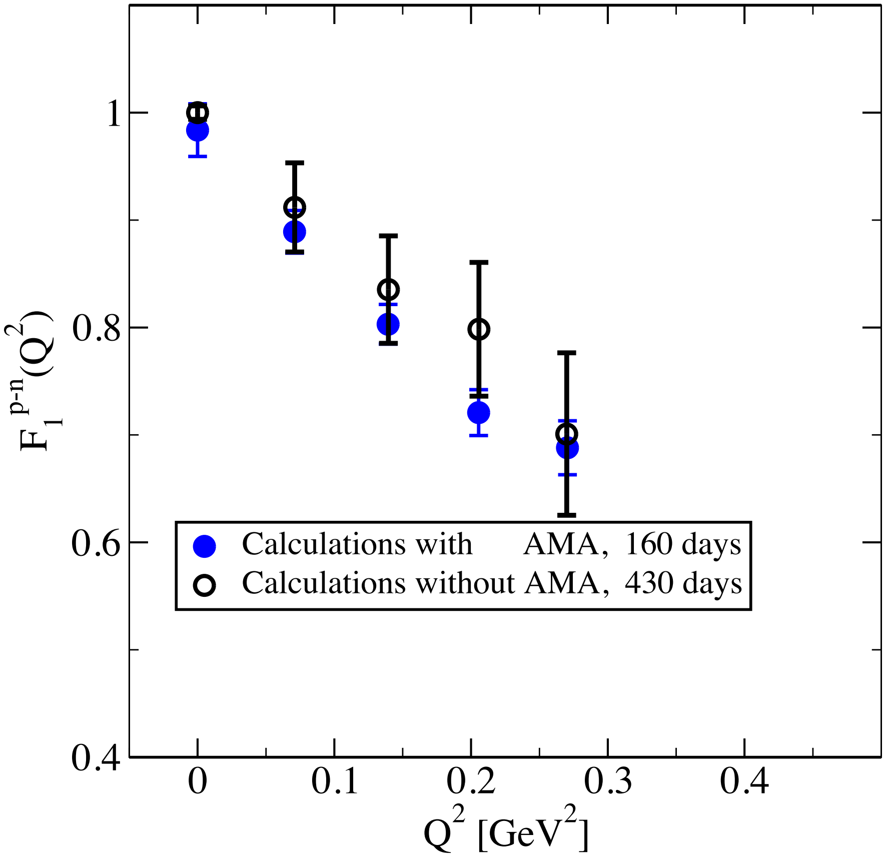
AMA at work

- Target : V=32^3 x 64 =$(4.6\text{fm})^3$x9.6fm, Ls=32 Shamir-DWF, a^{-1}=1.37 GeV, Mpi = 170 MeV
- Use Ls=16 Mobius as the approximation
  [Brower, Neff, Orginos, arXiv:1206.5214]

- quark propagator cost on SandyBridge 1024 cores (XSEDE gordon@SDSC)
  - non-deflated CG, r(stop)=1e-8 : ~9,800 iteration, 5.7 hours / prop
  - Implicitly restarting Lanczos of Chebyshev polynomials of even-odd prec operator for 1000 eigenvectors
    [Neff et al. PRD64, 114509 (2001)] : 12 hours
  - deflated CG with 1000 eigenvectors : ~700 iteration, 20 min /prop
  - deflation+sloppy CG, r(stop)=5e-3 : ~125 iteration, 3.2 min /prop
- Multiplicative Cost reduction for General hadrons could combine with {EigCG | AMG} and Distillation:
  x1.2 (Mobius) x 14 (deflation) x 7 (sloppy CG) ~ x 110

AMA at work
[M. Lin]





- $F_1(Q^2)$: tsep = 9 a ~ 1.3 fm

  1 forward + 2 (up and down) seq-props, contraction cost is ~15% of sloppy propagator

- Error bar
  x 2 – 2.7 ~ sqrt(4400/600)
- Total cost reduction upto
  (430 / 160) * (4400/600)
  ~ x19.7

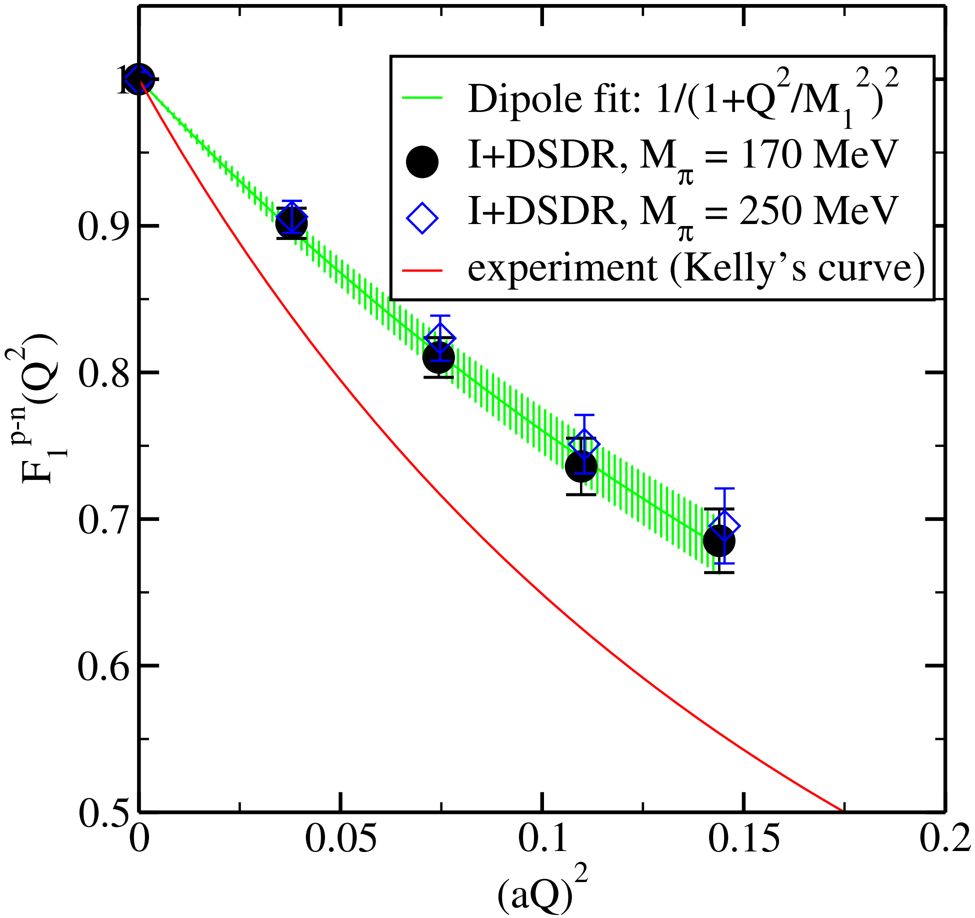
- Note this is still sub-optimal, 4 exact source and without deflation. (would be x30 for 2 exact sources)

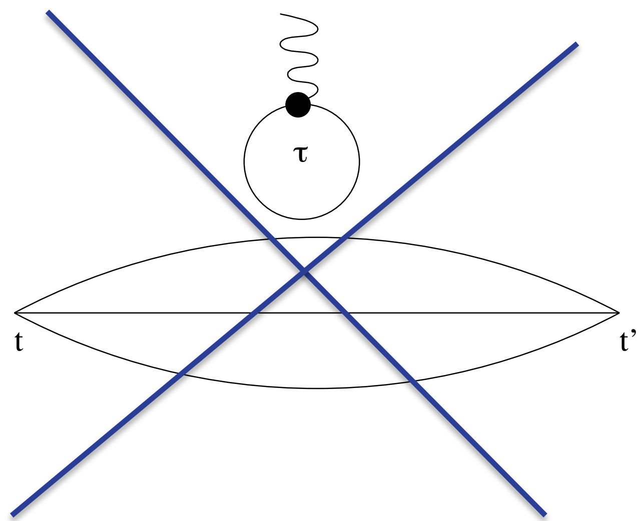
- non-deflated CG, 150 config x 4 sources = 600 measurements :
  5.7 * 3 * 4 * 150 config = 10K hours, 430 days

- AMA : 39 config, 4 exact solves / config (perhaps overkill) , N_G=112 sloppy solves
  => 39 x 112 = 4400 AMA measurements :
  (5.7 * 3 * 4 + 12 + 0.06 * 3 * 112) * 39 config = 3.9 K hours, 160 days
  4-exact (68%) + Lanczos (12%) + sloppy CG (20%)

Isovector $F_1(Q^2)$ [Meifeng Lin]
Preliminary

- Mild quark mass dependence
- Dipole fit, Dirac radius from slope at Q2 = 0

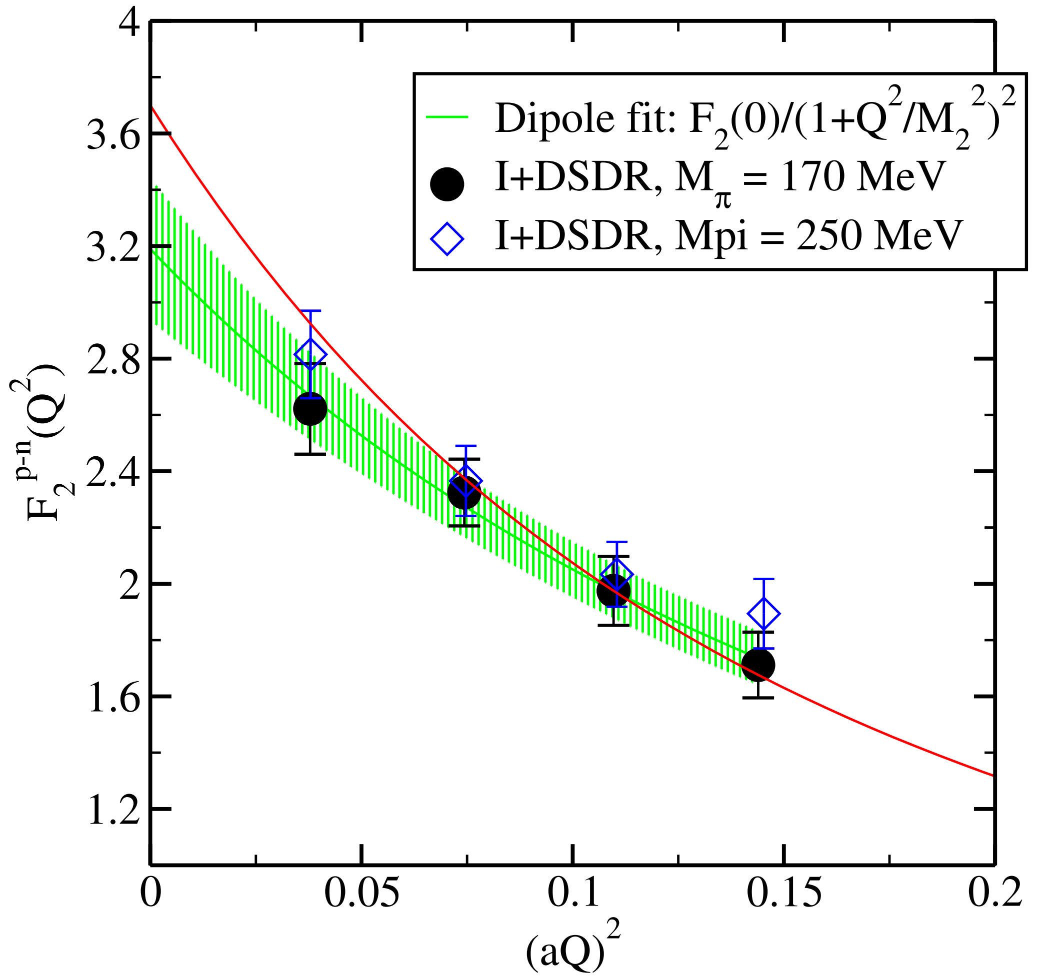




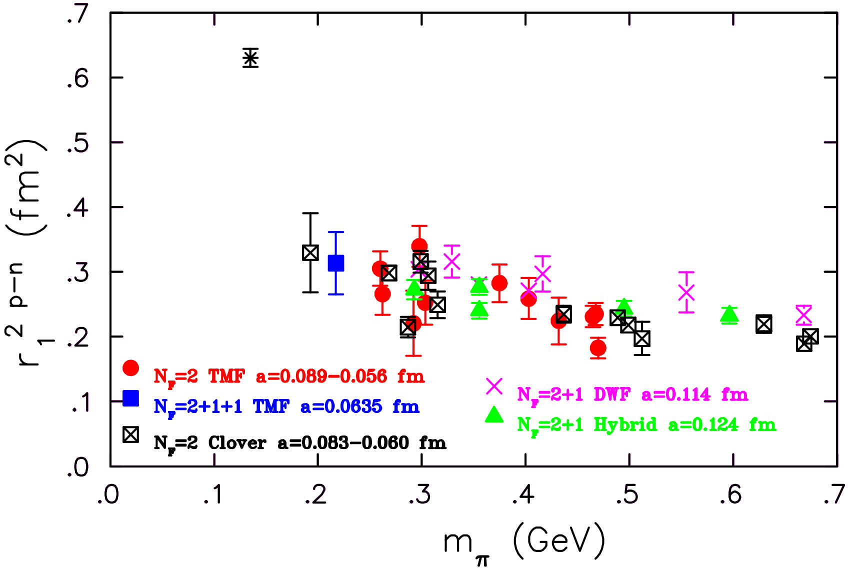
[Iso scalar needs disconnected diagram]

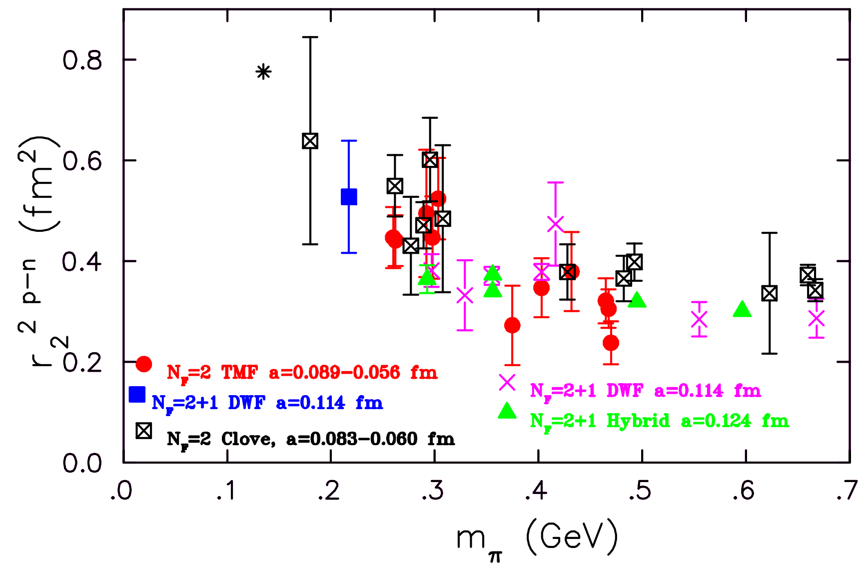
Isovector $F_2(Q^2)$ [Meifeng Lin]
Preliminary

- Mild quark mass dependence
- Much smaller statistical error with AMA



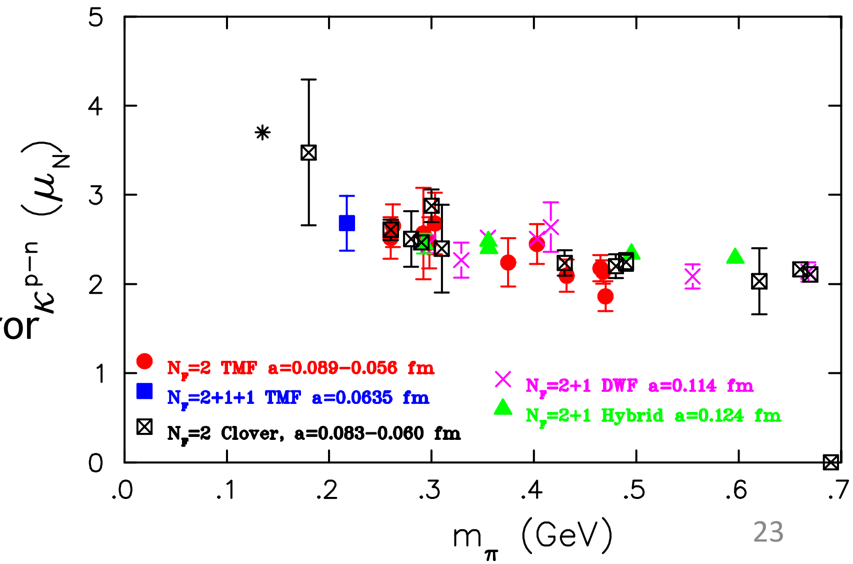
Radii, κ results by others





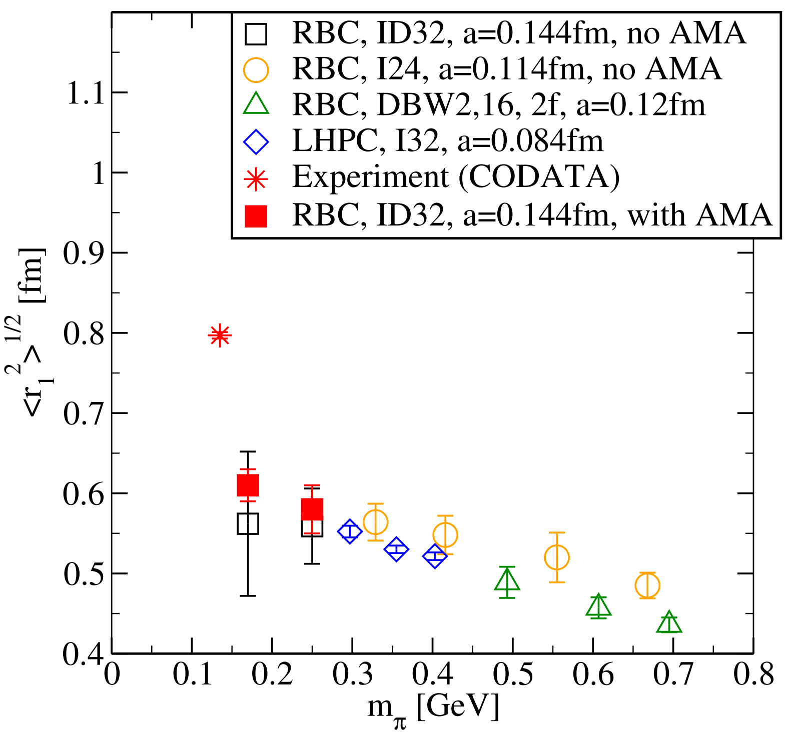
Compilation Alexandrou et al. arXiv:1303.6818

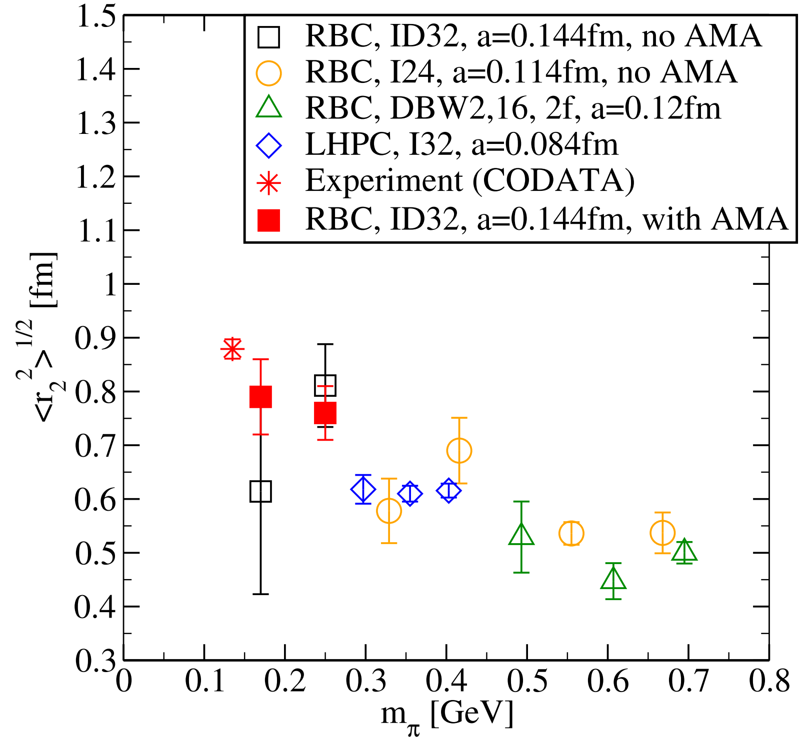
- Isovector (no disconnected quark diagrams)
- Dirac radius, r1, undershoots
- r2 also except the lightest results with 30% error
- still large extraction in quark mass
- Finite Volume Effects / Discretization Error



Dirac and Pauli radii [Meifeng Lin]
Preliminary

- Statistical error is much reduced by AMA
- r2 at Mpi=170MeV is closest result (~8 % stat err)!
  r1 still undershoots (~ 4% stat err)
- Anomalous magnetic moment κ = F2(0) = 3.2 (3) at M(pi)=170 MeV
  experiment : κ (exp) = 3.71





Axial Charge gA [S. Ohta]

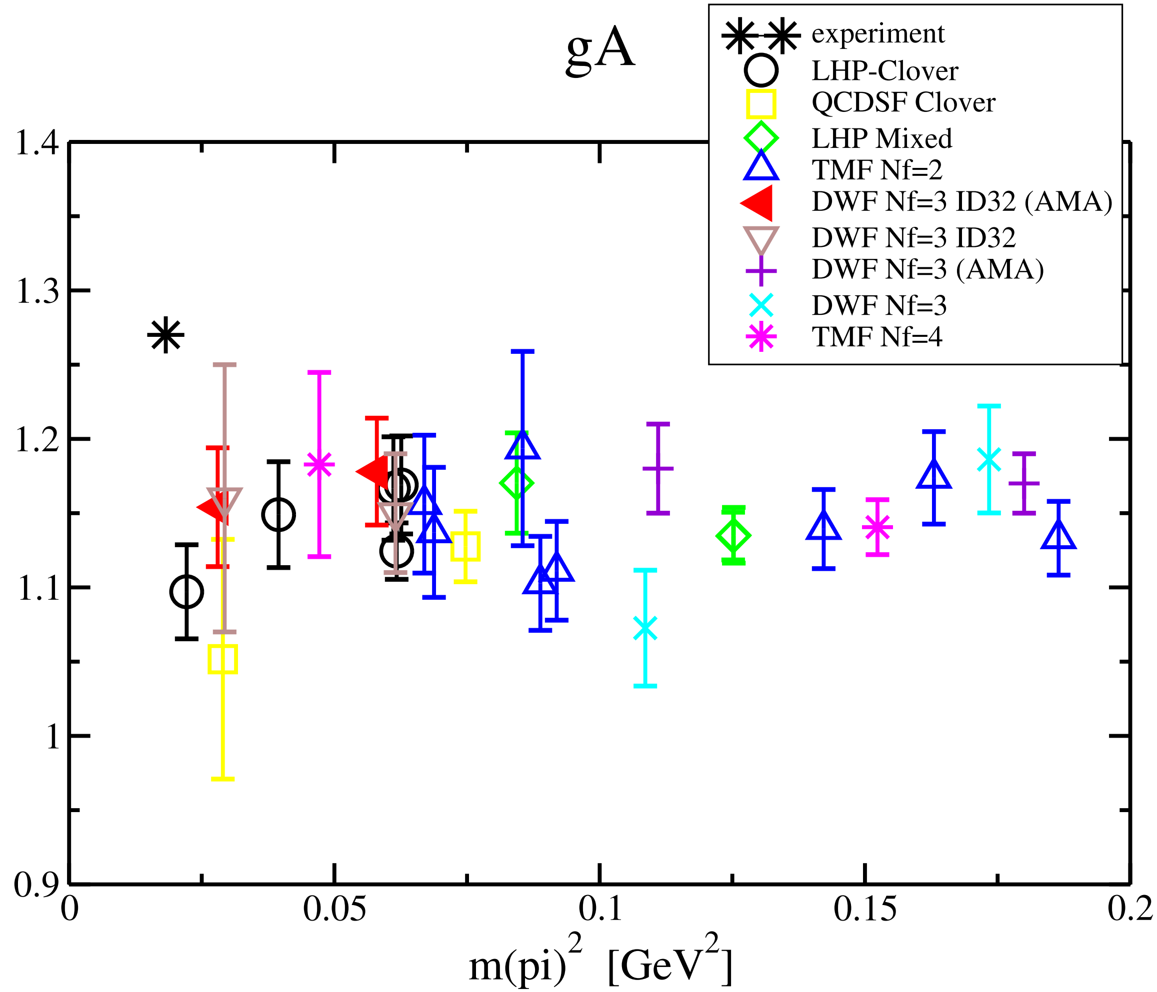
Preliminary

$$\langle p|A_\mu^+|n\rangle = \overline{u}_p\left[\gamma_\mu\gamma_5 G_A(q^2) + q_\mu\gamma_5\frac{G_P(q^2)}{2M_N}\right]u_n$$

- ~ 10%, 3 σ deficit

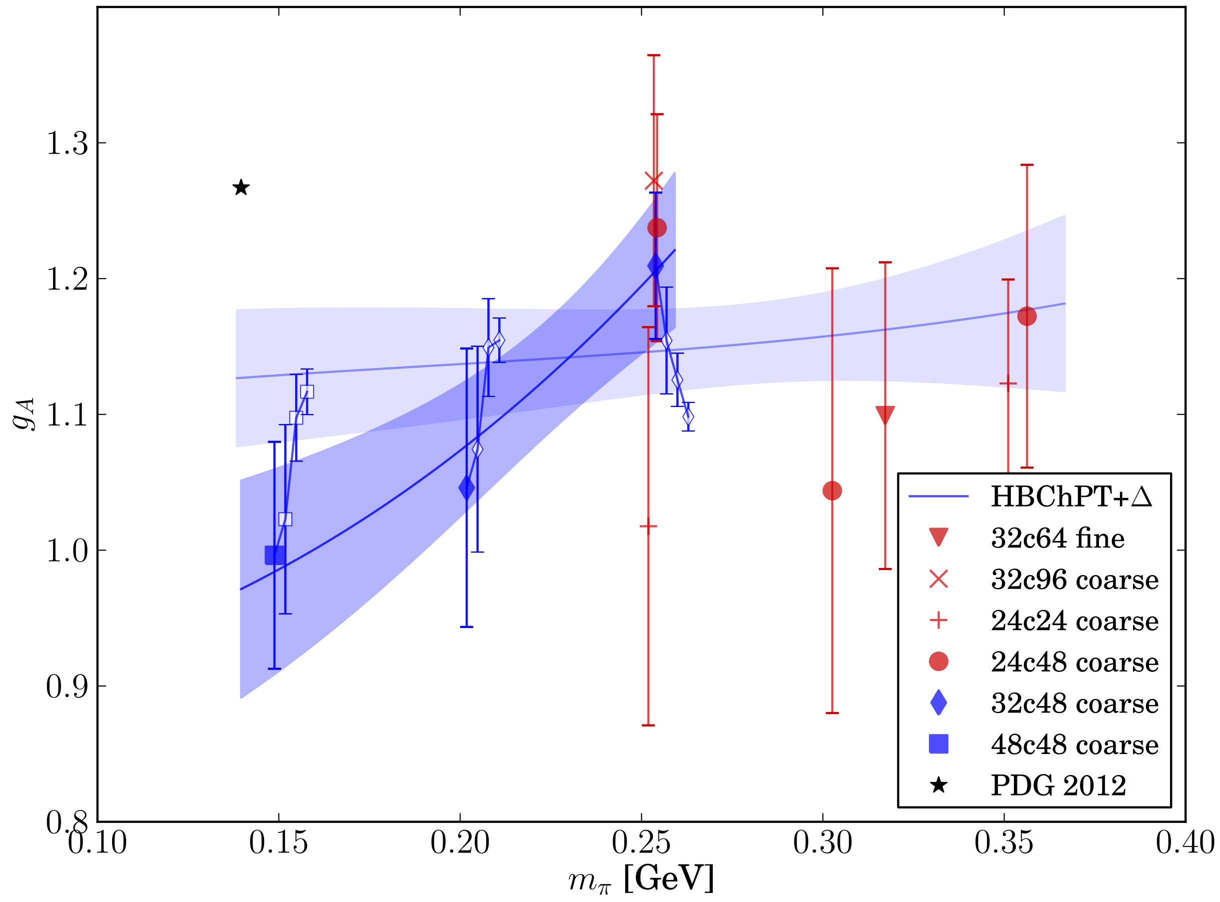
- excited state (v)
- statistics (v)
- Discretization (v)

- Finite Volume ?



Excited State Contamination in gA

- LHP, J.Green et al. (2012)
- Excited state increased gA



Structure functions

- pQCD extracts PDF from DIS   q(x), g(x)
  [Koike's talk...]
- Euclidean Lattice QCD could calculate moments

$$\langle x\rangle_q = \int_0^1 dx x[q(x)+\bar{q}(x)], \quad \langle x\rangle_{\Delta q} = \int_0^1 dx x[q(x)-\bar{q}(x)],$$

via QCD matrix elements of local operator

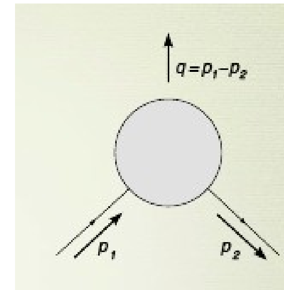
$$\mathcal{O}^{\{\mu_1\ldots\mu_n\}}_{[\gamma_5]} = \overline{\psi}(0)\gamma^{\{\mu_1}[\gamma_5]i\overleftrightarrow{D}^{\mu_2}\cdots i\overleftrightarrow{D}^{\mu_n\}}\psi(0)$$

[Direct PDF calculation is proposed by X.Ji 2013]

Renormalization : Calibration of "Detector"

New Renormalization Schemes

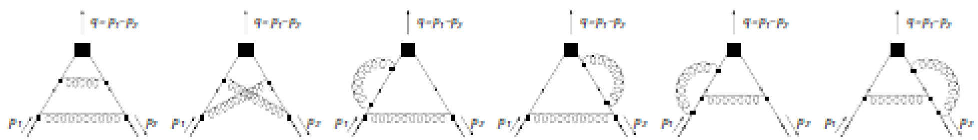
[09 C. Sturm, Y. Aoki, N. Christ, TI, C. Sachrajda, A. Soni]
[10 L. Almeida, C. Sturm]



$(q^2 = p_1^2 = p_2^2)$

- Match the normalization of operator on lattice and in continuum theory (MS) via RI/SMOM schemes

  calculate the 3 pt amplitudes for a momentum configuration (SMOM) on lattice non-perturbatively, and in continuum theories.



- We find symmetric momentum (SMOM) configuration is useful to reduce one of the dominant systematic errors due to IR effects.

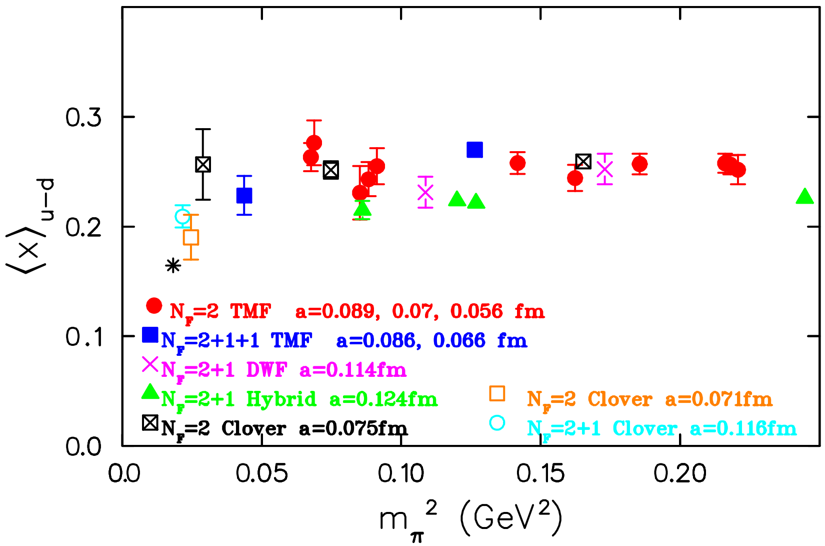
- Quark mass renormalization error

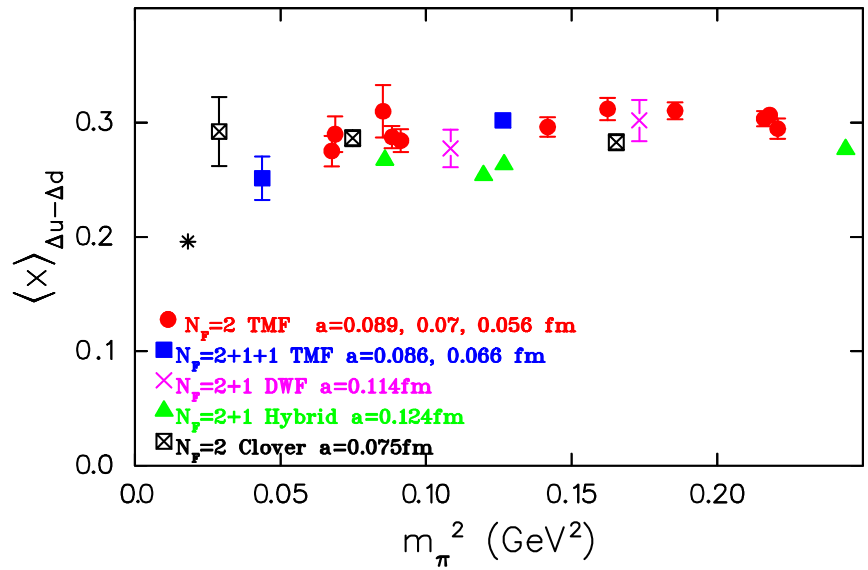
  ~10% (MOM) → ~ 5% (SMOM) → ~ 2% (SMOM 2loop) ,

- Using different RI/SMOM schemes (using various spinor projections) to check the systematic errors

Isovector Mometum/Helicity fraction

(renormalized to Msbar μ= 2GeV)

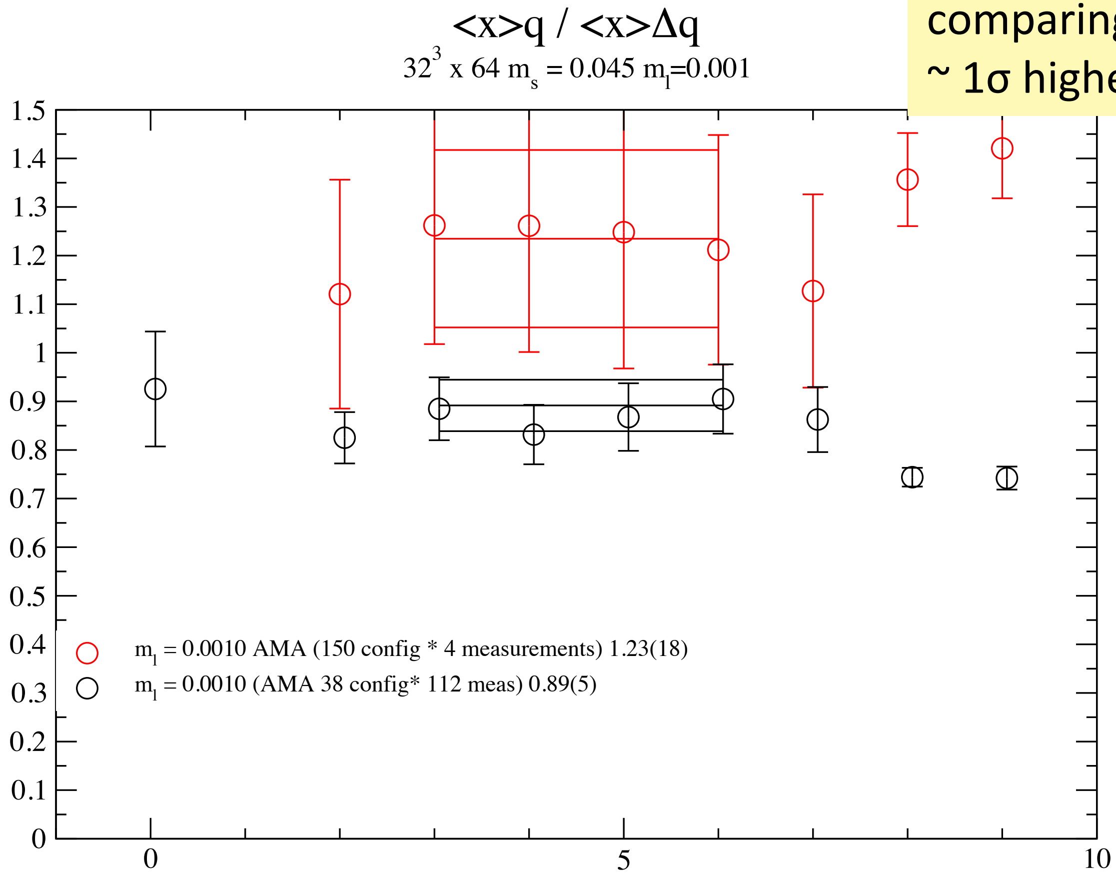




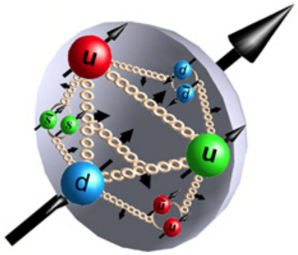
Compilation Alexandrou et al. arXiv:1303.6818

<x> q / <x>dq from DSDR Preliminary

At very light mass, Mpi=170MeV
~ 6 % statistical error from AMA
no excited state contamination by
comparing 1.3 fm & 1.1 fm separation
~ 1σ higher than experiment



Nucleon Spin
[X. Ji's talk]



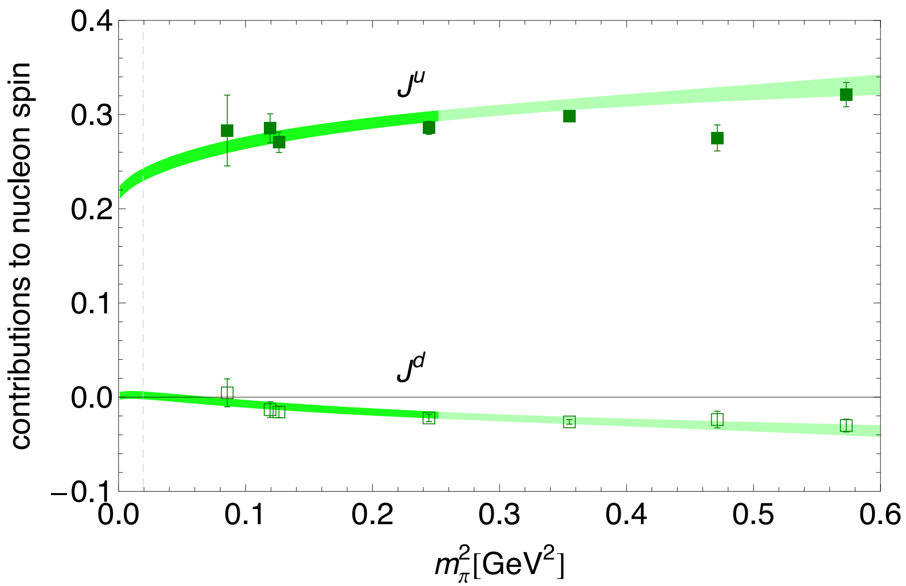
- Neucleon Spin

$$\frac{1}{2} = J^q + J^g, \quad J^q = \frac{1}{2}\Delta\Sigma^q + L^q$$

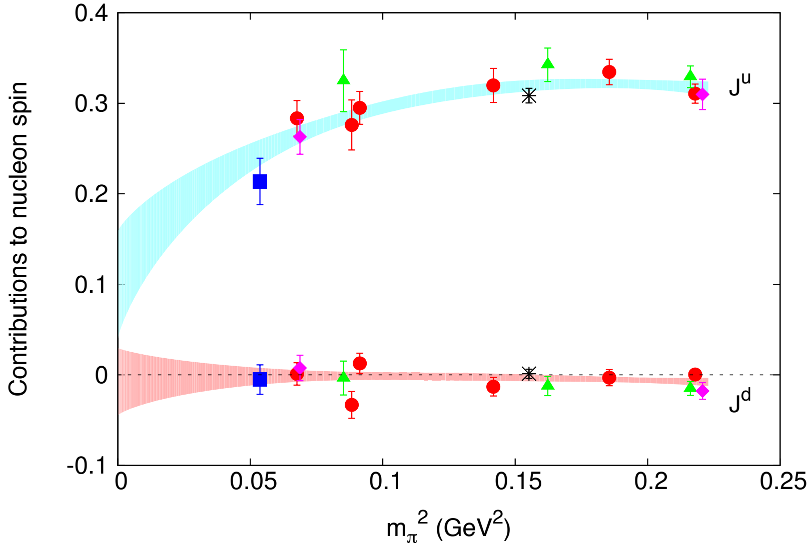
- On Lattice, quark's total ang. mom and spin is computable by GPD [X. Ji 97]

$$J^q = \frac{1}{2}[A^q_{20}(0) + B^q_{20}(0)], \quad \Delta\Sigma^q = \tilde{A}_{10}(0)$$

Quark Angular Momentum



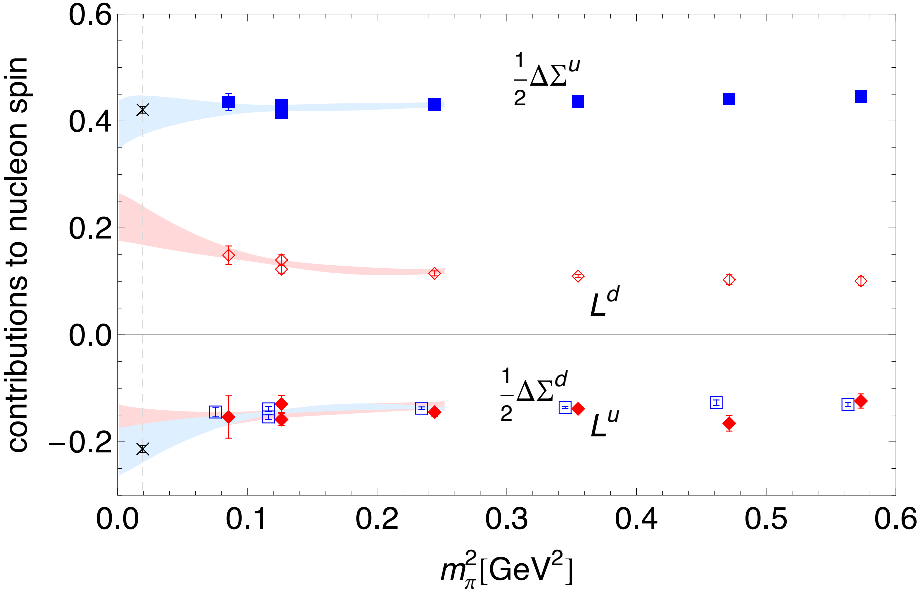
[LHPC, PRD82 (2010) 094502]

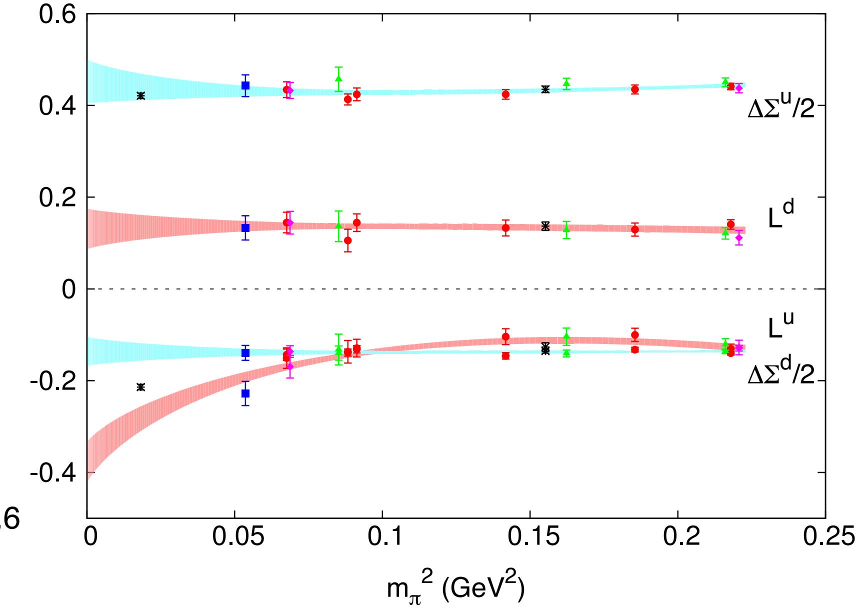


[ETMC, arXiv:1104.1600]

- About a half of Nucleon spin comes from up quark
- Down quark : ~0 contribution
- Disconnected diagram is ignored
- Lighter quark and larger Volume

$J_q = \Delta \Sigma /2 + L_q$





[LHPC, PRD82 (2010) 094502]

[ETMC, arXiv:1104.1600]

- spin and orbital angular mom has opposite signs
- up and down quarks has opposite signs
- Cancellation between down quark spin and orbital momentum : J_d ~ 0

- Disconnected diagram is ignored
- Lighter quark and larger Volume

Summary & Prospects

- Nucleon structure by Lattice QCD
- Challenges (compared to Meson)
  - Noise , exponentially growing
  - Volume → (10 fm)^3 lattice at Kei is very interesting
  - discretization error : 1/a ~ 2—3 GeV, (c.f X. Ji's direct g(x), q(x))

PLAN (RBC/UKQCD)

- On physical DWF quark mass M(pi) ~ 135 MeV
- Two lattice spacings, a ~ 0.11 fm, 0.085 fm
- Volume ~ (5.5 fm)3, M(pi) L ~ 3.8

- QCDCQ (BlueGene/Q) : x 20 speed
- Statistical error reduction by AMA :
  x5 ~ x20 smaller cost

Meson results is already very interesting on these QCD ensembles

- Meson decay constants

$$f_\pi = 130.2 \pm 2.9_{\rm stat}\ \ \text{MeV} \qquad \text{(Preliminary)}$$

$$f_K = 156.1 \pm 3.2_{\rm stat}\ \ \text{MeV} \qquad \text{(Preliminary)}$$

$$f_\pi / f_K = 1.198 \pm 0.006_{\rm stat} \qquad \text{(Preliminary)}$$

Preliminary
R. Mawhinney
J. Frison
Lattice13

- Kaon Bag parameter, Kl3 form factor (CKM)

$$B_K(\overline{\rm MS}, 3\,{\rm GeV}) = 0.533 \pm 0.003_{\rm stat} \pm 0.000_{\rm chiral} \pm 0.002_{\rm finite\ V} \pm 0.011_{\rm pert} \ \text{(Preliminary)}$$

$$\hat{B}_K = 0.754 \pm 0.004_{\rm stat} \pm 0.0015_{\rm sys} \qquad \text{(Preliminary)}$$

$$f_+^{K\pi}(0) = 0.9670(20)\left({}^{+\ 0}_{-42}\right)_{m_q}(7)_{\rm FSE}(17)_a$$

$$|V_{us}| = 0.2237\,(7)\,\left({}^{+10}_{-\ 0}\right)_{m_q}(2)_{\rm FSE}(4)_a$$

$\rightarrow \approx 0$

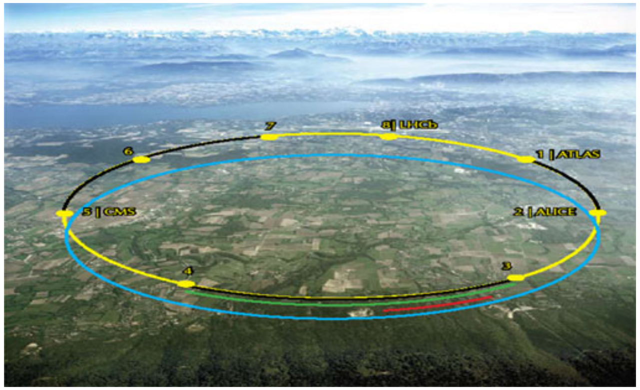
with phys. point data

precision $\lesssim 0.3\%$ feasible!

Preliminary
A. Juettner
Lattice13

Physics Highlights

- I experiment <-> [QCD corrections] ⊗ [Standard Model & Beyond]



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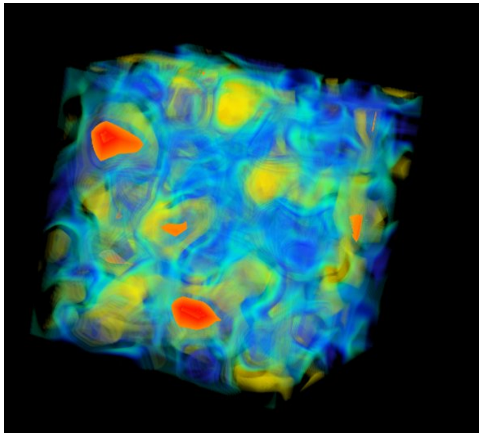

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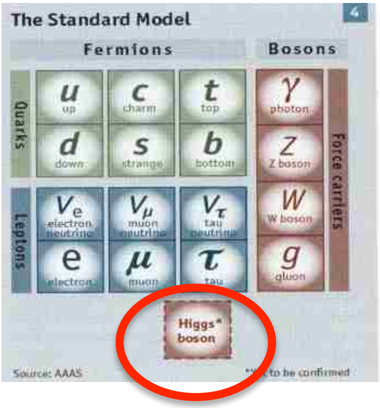
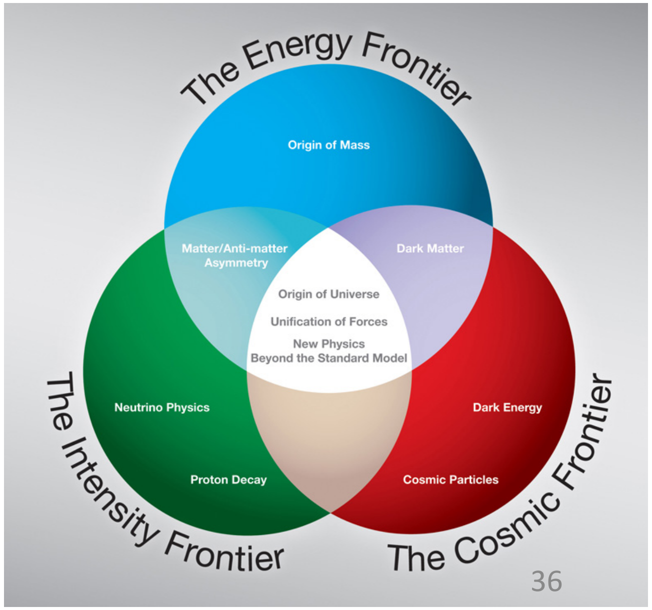


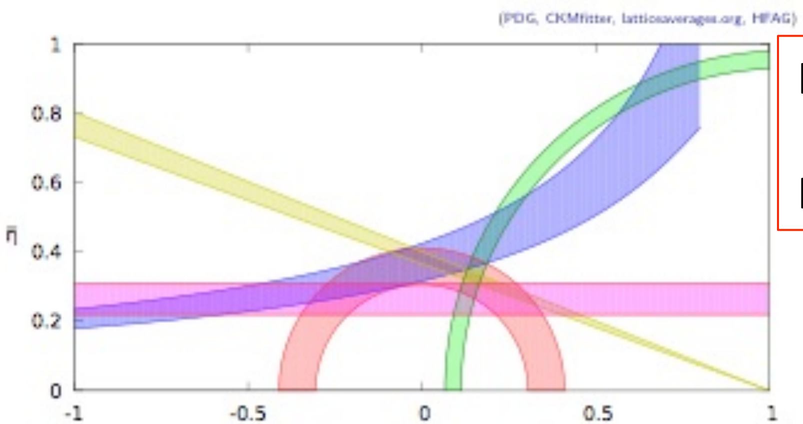
- K→ππ I=2 & I=0, $\Delta M(K_L - K_S)$
- A new class of error reduction technique (AMA)
- QCDOC → QCDCQ : on-physics point ($M\pi$=135 MeV) large volume ~$(5\ \mathrm{fm})^3$, QCD ensembles with DWF
- QCD + Electromagnetism :
  - Hadron's polarizabilities
  - QCD + dynamical QED
- Nucleon Electric Dipole Moments
- CKM (K & B), Computer Algebra System for perturbation



K → π π decay amplitude
[RBC/UKQCD]

- 40+years awaited theoretical calculation
  [1964 Cronin-Fitch, Nobelprize
  1999 NA48@CERN, KTeV@FNAL]
- Provide a new constraint on CKM Unitarity

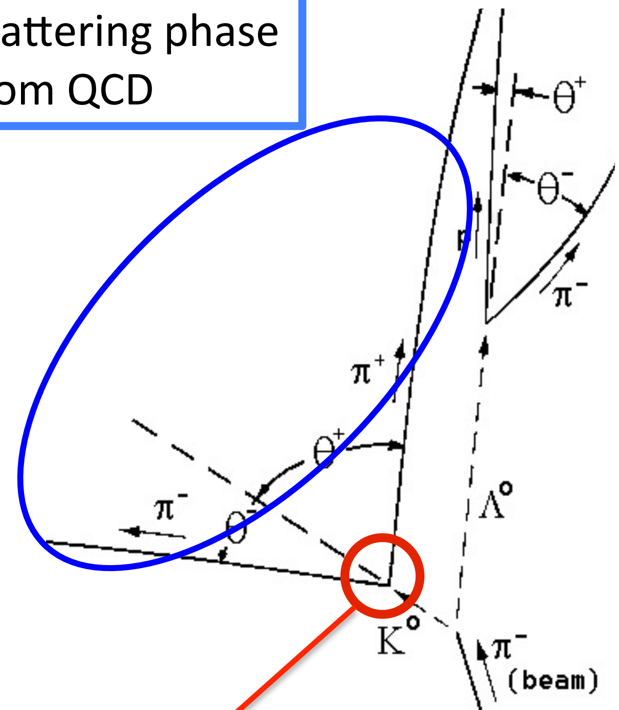
π π final state scattering phase from QCD

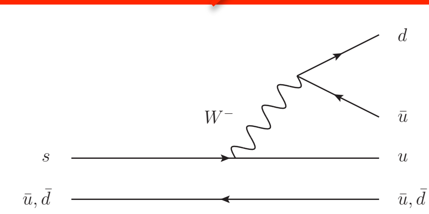


$K_L \rightarrow \pi\pi\pi$ (CP even)

$K_S \rightarrow \pi\pi$ (CP odd)

Δ I = ½ rule
ε' / ε

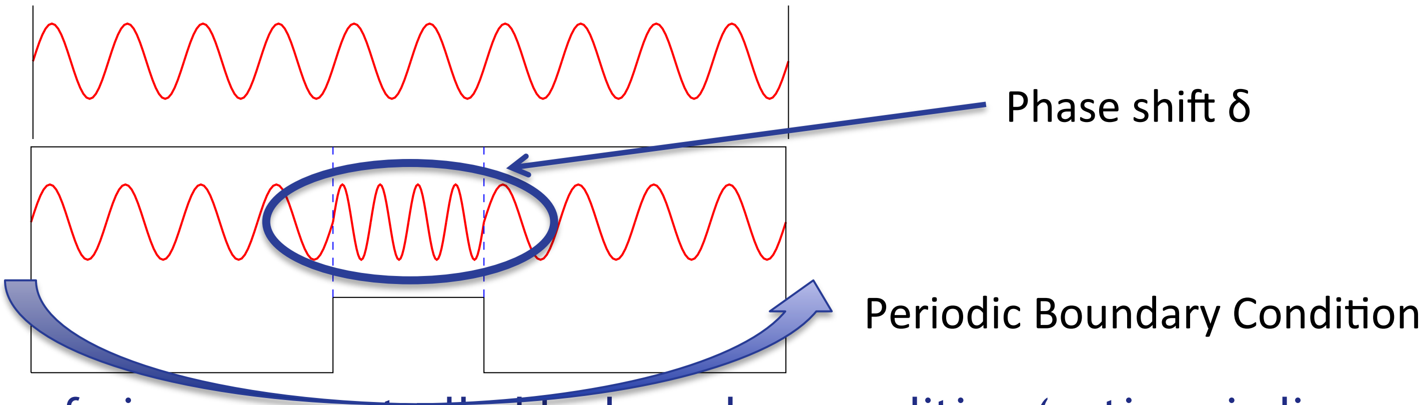




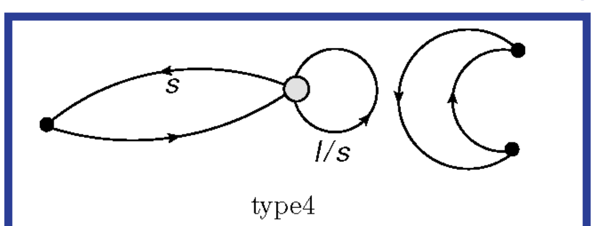
Electro-Weak (CKM) phase
Vus V*ud, Vts V*td

$K \rightarrow \pi\pi$ decay on lattice

- Relates energy on finite volume $E_{\pi\pi}(V)$ to phase shift δ to obtain complex Amp($K\rightarrow\pi\pi$) = $|A_I|\, e^{i\delta I}$ (Luscher, Lellouch-Luscher)



- Momentum of pions are controlled by boundary condition (anti-periodic or G-parity b.c.)
- Mixing and Renormalization of operator is done using non-perturbative renormalization (NPR)
- Chiral Symmetry is curtail
- I=2 channel is under control, I=0 is still a challenge due to disconnected diagrams.



$K \to \pi\,\pi$ Amplitude, I=2 channel

2012 Ken Wilson Lattice Award

PRL **108,** 141601 (2012) PHYSICAL REVIEW LETTERS week ending 6 APRIL 2012

$K \to (\pi\pi)_{I=2}$ Decay Amplitude from Lattice QCD

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(RBC and UKQCD Collaborations)

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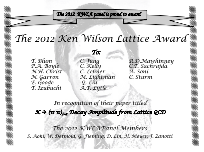
(Received 8 November 2011; published 4 April 2012)

We report on the first realistic *ab initio* calculation of a hadronic weak decay, that of the amplitude A_2 for a kaon to decay into two π mesons with isospin 2. We find $\mathrm{Re}A_2 = (1.436 \pm 0.063_{\mathrm{stat}} \pm 0.258_{\mathrm{syst}})10^{-8}$ GeV in good agreement with the experimental result and for the hitherto unknown imaginary part we find $\mathrm{Im}A_2 = -(6.83 \pm 0.51_{\mathrm{stat}} \pm 1.30_{\mathrm{syst}})10^{-13}$ GeV. Moreover combining our result for $\mathrm{Im}A_2$ with experimental values of $\mathrm{Re}A_2$, $\mathrm{Re}A_0$, and ϵ'/ϵ, we obtain the following value for the unknown ratio $\mathrm{Im}A_0/\mathrm{Re}A_0$ within the standard model: $\mathrm{Im}A_0/\mathrm{Re}A_0 = -1.63(19)_{\mathrm{stat}}(20)_{\mathrm{syst}} \times 10^{-4}$. One consequence of these results is that the contribution from $\mathrm{Im}A_2$ to the direct CP violation parameter ϵ' (the so-called Electroweak Penguin contribution) is $\mathrm{Re}(\epsilon'/\epsilon)_{\mathrm{EWP}} = -(6.52 \pm 0.49_{\mathrm{stat}} \pm 1.24_{\mathrm{syst}}) \times 10^{-4}$. We explain why this calculation of A_2 represents a major milestone for lattice QCD and discuss the exciting prospects for a full quantitative understanding of CP violation in kaon decays.

DOI: 10.1103/PhysRevLett.108.141601 PACS numbers: 12.38.Gc, 11.15.Ha, 11.30.Er, 13.25.Es



Lattice: $32^3 \times 64 \times 32$, 2+1 DWF, Iwasaki DSDR, $a^{-1} = 1.375(9)$ GeV, $m_{\pi^+} = 142.9(1.1)$ MeV

	$\mathrm{Re}A_2$	$\mathrm{Im}A_2$
Lattice artifacts	15%	15%
Finite-volume corrections	6.2%	6.8%
Partial quenching	3.5%	1.7%
Renormalization	1.7%	4.7%
Unphysical kinematics	3.0%	0.22%
Derivative of the phase-shift	0.32%	0.32%
Wilson coefficients	7.1%	8.1%
Total	18%	19%

	$\mathrm{Re}(A_2)(10^8/\mathrm{GeV})$	$\mathrm{Im}(A_2)(10^{13}/\mathrm{GeV})$
Lat.	1.436(62)(258)	-6.83(51)(130)
Exp.	1.479(4) (K^+)	(n/a)

$\mathrm{Re}(\varepsilon'/\varepsilon)_{\mathrm{EWP}} = -6.52(49)(124) \times 10^{-4}$

Errors: (stat)(syst)

Computing Group

- Group Leader : Taku Izubuchi (BNL)
- University Fellow : Brian Tiburzi (CCNY) Ethan Neil (Colorado)
- Fellow : Tomomi Ishikawa
- PostDocs : Christoph Lehner (Foreign PostDoc) → BNL from 2013
  Eigo Shintani
  (C. Kelly, S. Seryzin FPR from 2013)
- Visiting students :

  Michael Abramczyk (Connecticut)

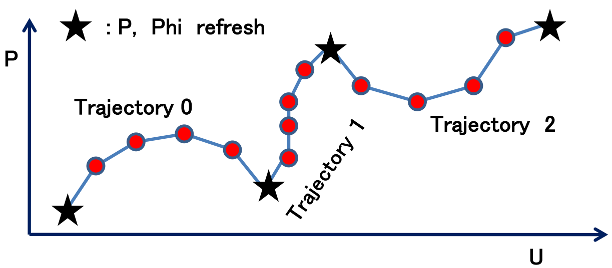
- Visiting scientists :

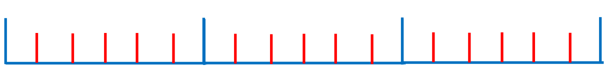
  Yasumichi Aoki (Nagoya)
  Thomas Blum (Connecticut)
  Chulwoo Jung (BNL)
  Taichi Kawanai (BNL/ RIKEN Brain PostDoc)
  Meifeng Lin (Yale → Argonne)
  Robert Mawhinney (Columbia)
  Shigemi Ohta (KEK)

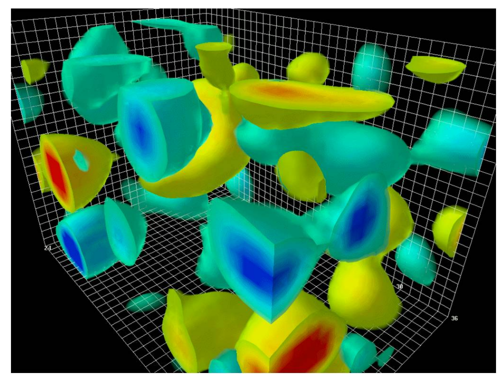
Hybrid Monte Carlo (LGT's "Accelerator")

- Monte Carlo to Sample Important configurations of QCD action $e^{-S_{\text{QCD}}}$
- Accumulate samples of QCD vacuum, typically $\mathcal{O}(100) \sim \mathcal{O}(1,000)$ files of gauge configuration $U_\mu(n)$ on disk ($1 \sim 10$ GB/conf).
- By solving a classical QCD, with an occasional stochastic ''hit'': exactly $\propto e^{-S_{\text{QCD}}}$
- Must generate sequentially $\{U_\mu^{(0)} \to U_\mu^{(1)} \to \cdots\}$, which needs capable machines.

$$\text{Prob}(U_\mu) \propto \det D_{u,d,s}[U] e^{-S_g}$$







[D. Leinweber]

- RHMC for odd flavor [Clark Kennedy]
- Solve short (long) modes more (less) frequently [Hasenbush's trick]

Physics measurements "Detectors"

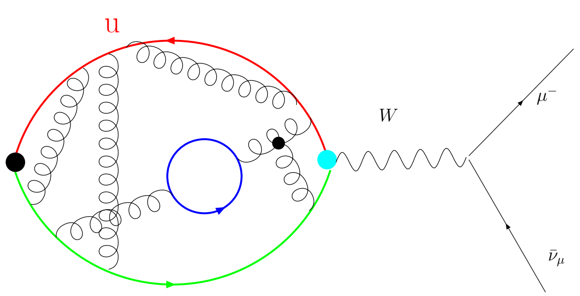
- Measurements physical observables on the vacuum ensemble.

$$\langle \mathcal{O} \rangle = \int \underline{\mathcal{D}U_\mu \, \text{Prob}[U_\mu]} \times \mathcal{O}[U_\mu]$$

- Could do Analysis on many configurations independently (trivial parallel jobs) $\longrightarrow$ could also use PC Clusters

- We made hadron operator (EW operators) from quark, and let the quark propagates on each of the generated QCD configuration (by solving the Dirac Eq)

- Obtain hadron mass or QCD matrix elements of operators



$\langle 0|\bar{d}\gamma_5 u(0)|\pi\rangle \frac{e^{ipx}}{\sqrt{2E}} \langle \pi|\bar{u}\gamma_m u\gamma_5 d|0\rangle \times G_F V_{ud} m_\mu \bar{\nu}(1-\gamma_5)\mu$

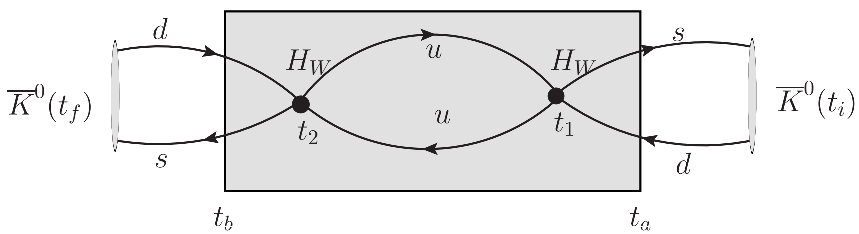
$$\mathcal{M}(\pi \to \mu\overline{\nu}) \sim i f_\pi q_\mu \times G_F V_{ud} m_\mu (\overline{\nu}\mu)_L$$

$$= \langle \pi(q)|\bar{u}\gamma_\mu\gamma_5 d(0)|0\rangle \times G_F V_{ud} m_\mu (\overline{\nu}\mu)_L$$

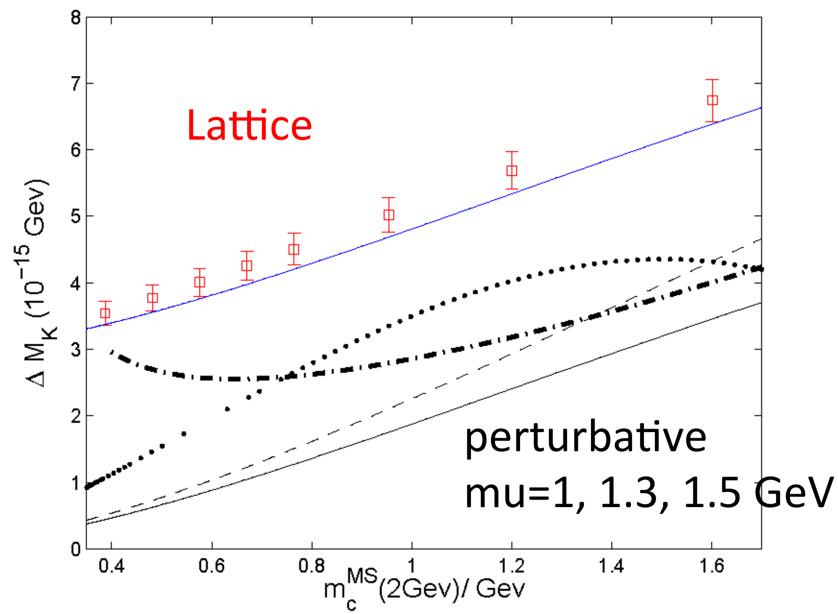
K_L-K_S mass difference

[RBC/UKQCD, N. Christ, J. Yu et al. arXiv:1212.5931]

- Evaluate K^0-K^0bar amplitude in 2nd order ElectroWeak Hamiltonian : 4pt Green's function



$$\mathscr{A} = \frac{1}{2}\sum_{t_2=t_a}^{t_b}\sum_{t_1=t_a}^{t_b}\langle 0|T\left\{\overline{K^0}(t_f)H_W(t_2)H_W(t_1)\overline{K^0}(t_i)\right\}|0\rangle.$$



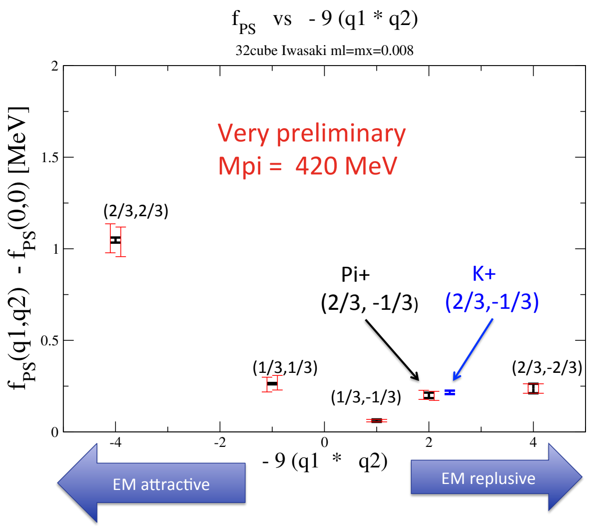
M_K (GeV)	ΔM_K (x 10^{-12} MeV)
563	5.12(24)
707	6.92(39)
918	11.12(94)
1162	20.10(312)

- ΔM_K^{expt} = 3.483(6) 10^{-12} MeV
- Omit disconnected diagrams and Unphysical kinematics, m_π = 421 MeV
- charm quark for GIM cancellation
- 4pt function is useful for the rare Kaon decay : $K \to \pi\ \nu\ \nu$

QCD+QED simulation

[T. Blum, TI et al.]   [Tomomi Ishikawa]

- EM effects on PS decay

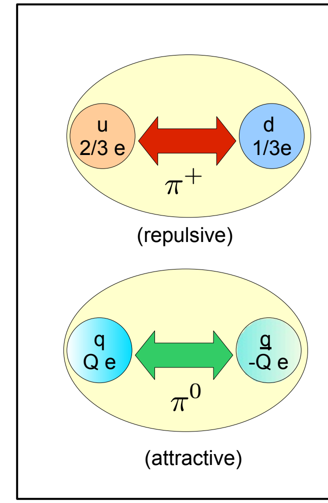


- Statistically well resolved by +e/-e averaging.
- c.f. [Bijnens Danielsson 2006]

$$f_{\pi^+,\mathrm{NLO}}/F_0 = 0.0039$$

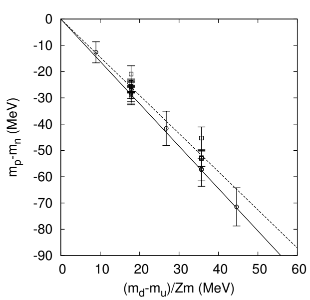
$$f_{K^+,\mathrm{NLO}}/F_0 = 0.0056$$

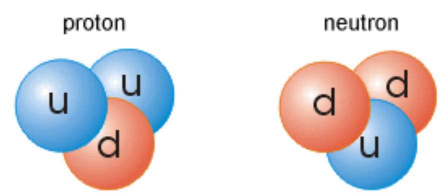
- EM turned on, but mu = md
- Iwasaki-DWF Nf=2+1,
- (2.7 fm)^3, a^{-1} ~ 2.3 GeV



- Proton / Neutron mass difference

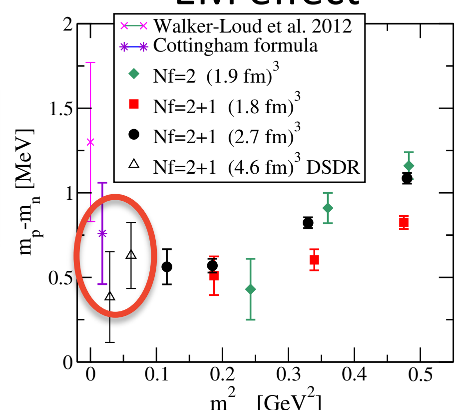
(m_u-m_d) effect





DSDR DWF Nf=2+1
(4.6 fm)^3,
a^{-1} ~ 1.4 GeV

EM effect



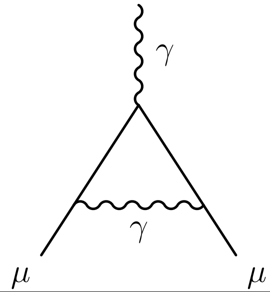
	$m_u - m_d$	EM
NPLQCD	2.26(72)	
BLUM	2.51(71)	0.54(24)
RM123	2.80(70)	
QCDSF-UKQCD	3.13(77)	
	2.68(35)	0.54(24)

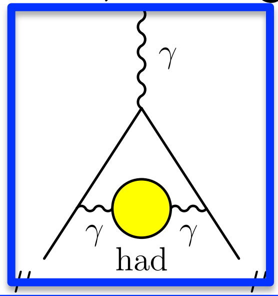
$$\Longrightarrow M_N - M_p| = 2.14(42)\ \ \mathrm{MeV}$$

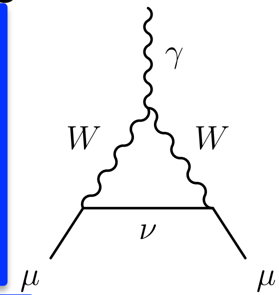
(experiment: 1.2933321(4) MeV)

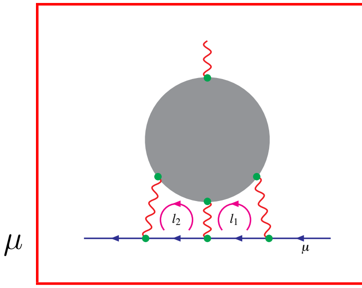
muon's anomalous magnetic moment

- One of the most precisely determined numbers, starting from the construction of QED.









Hadronic Vacuum Polarization (HVP)

Hadronic Light-by-Light (HLbL)

$$a_\mu = \frac{g-2}{2} = (116\,592\,089 \pm 54 \pm 33) \times 10^{-11}$$ BNL-E821

[Andreas Hoecker, Tau 2010, arXiv:1012.0055 [hep-ph]]

Contribution	Result ($\times 10^{-11}$).
QED (leptons)	116 584 718.09 $\pm$ 0.15
HVP (lo)	6 923. $\pm$ 42
HVP (ho)	-97.9 $\pm$ 0.9
HLBL	105. $\pm$ 26
EW	154. $\pm$ 2
Total SM	116 591 802 $\pm$ $42_{\text{HVP(lo)}}$ $\pm$ 26_{HLBL} $\pm$ 02 (49_{tot}).

BNL E821
FNAL new g-2
J-PARC

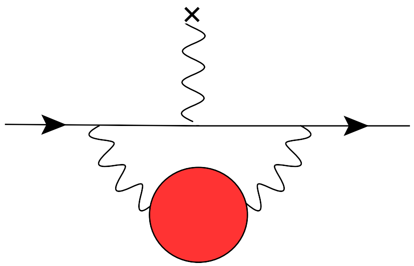
- 287 ± 80 or 3.6σ difference between experiment and SM prediction.

E989 at FNAL is to reduce the total experimental error by, at least, a factor of four over E821, or 0.14 ppm !

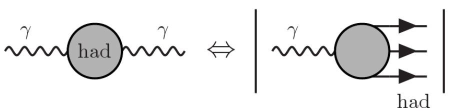
Hadronic Vacuum Polarization

- Currently estimated by σ (e+e-)
  <u>0.6 % accuracy</u>

$$a_\mu^{\mathrm{HVP}} = \frac{1}{4\pi^2}\int_{4m_\pi^2}^{\infty} ds K(s)\sigma_{\mathrm{total}}(s)$$



$$a_\mu^{\mathrm{had,LOVP}} = (\quad 694.91 \quad \pm 4.27 \quad) \times 10^{-10}$$
$$a_\mu^{\mathrm{had,HOVP}} = (\quad -9.84 \quad \pm 0.07 \quad) \times 10^{-10}$$

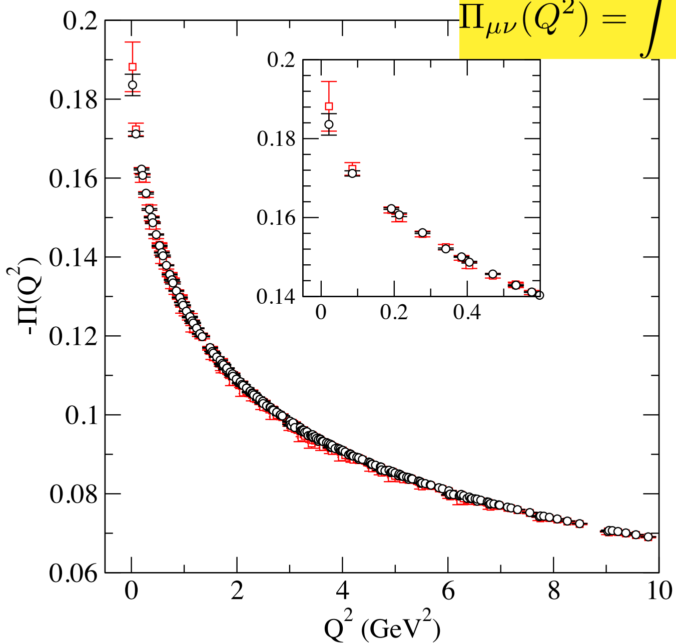


- Lattice calculation [T.Blum (2003)]

$$\Pi_{\mu\nu}(Q^2) = \int dx \langle V_\mu(x) V_\nu(0)\rangle e^{-q(x-y)}$$

$$a_\mu^{\mathrm{HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2 f(Q^2)\Pi(Q^2)$$

$$\Pi_{\mu\nu}(Q) = \left(g_{\mu\nu} - \frac{Q_\mu Q_\nu}{Q^2}\right)\Pi(Q^2)$$

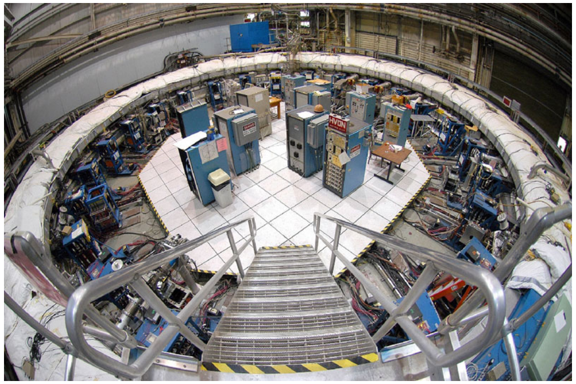


- Use new error reduction technique All Mode Averaging (AMA) <u>x 4- 20 improvements</u>
  [T.Blum, TI, E. Shintani (2012)]

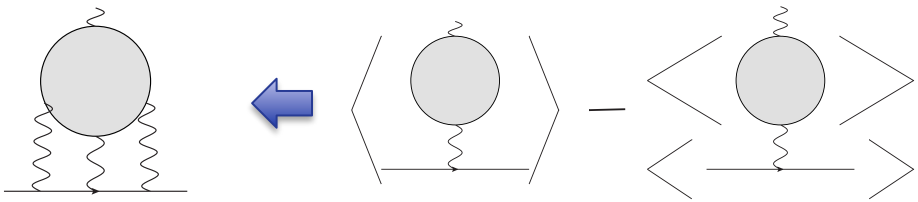
- accurate $\pi(Q^2)$ at $Q^2 \to 0$ is needed :
  twisted boundary condition and/or Analytic continuation to Minkowski momentum

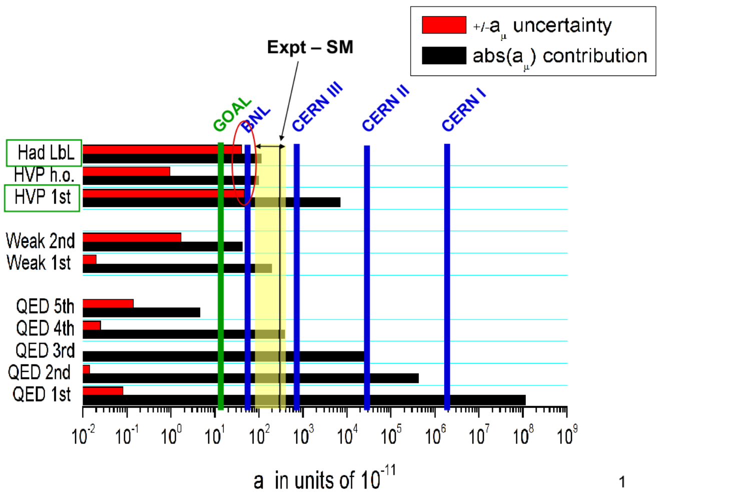
  to be competitive : O(5-10%) → < O(1%)

Hadronic Light-by-Light
[T. Blum, Hayakawa]



- Compute whole diagram using <u>lattice QCD+QED</u>
- LbL is a part of O(α 3) : need subtraction
  [M. Hayakawa, T.Blum, TI, N. Yamada (2005)]

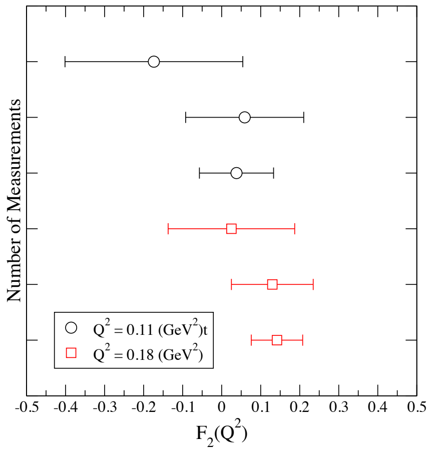




D. Hertzog, LPNHE Workshop, Paris Feb. 2010

The First signal (preliminary) using AMA

$F_2(Q^2)$ stable with additional measurements ($20 \to 40 \to 80$ configs)



24^3 lattice size

$Q^2 = 0.11$ and 0.18 GeV^2

$m_\pi \approx 329$ MeV

$m_\mu \approx 190$ MeV

- Very encouraging first results
  order of mag ~ model
- Unphysical mass / momentum
- Disconnected diagrams