Exotic rotational modes of non-exotic triaxial deformation in nculei

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- How exotic is an axially asymmetric shape in nuclei at the ground state?
- What is the issue here?
  - Soft v.s. Rigid in the ground states
- Why bother?
  - How is it related to the overall physics?

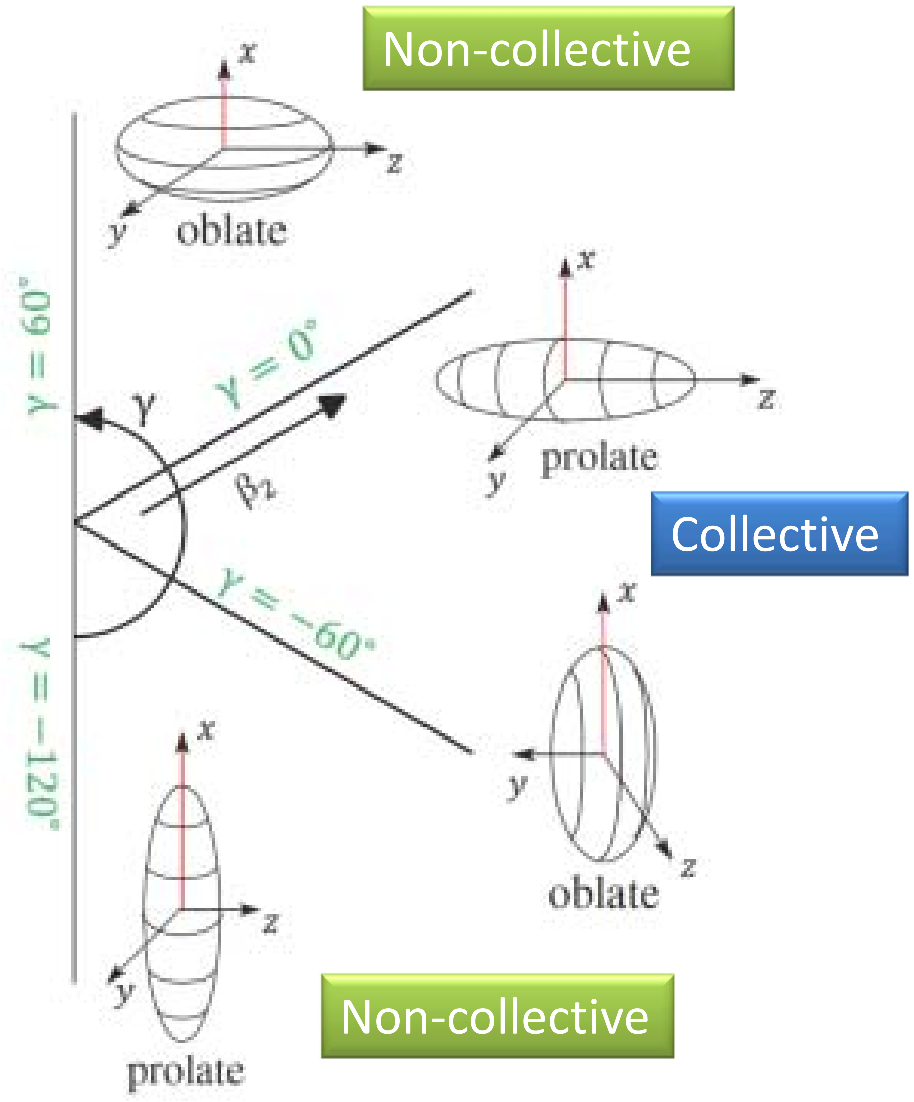
Contents

- Rigid v.s. soft triaxaility
- Exotic rotational modes related to triaxiality
  - Wobbling
  - Smooth band termination
  - Signature Inversion
  - Chiral doublets
- Possible experimental idea at J-PARC
- Summary

High spin/Cranking (PAC) regime

medium spin

Plane of Quadrupole Deformation



Quantum Mechanical rotation

Hamiltonian

$$H_{rot} = \sum_{k=1}^{3} \frac{\hbar^2}{2J_k} R_k^2$$

For an axially symmetric rotor, rotation is only possible around the axis perpendicular to the symmetry axis

$$J_x \neq 0, \text{ and } J_y = J_z = 0$$

Y. Fujioka , Master Thesis, Kyusyu University, (2012)

Triaxial rotor + irrotational flow moment of inertia

Axis length

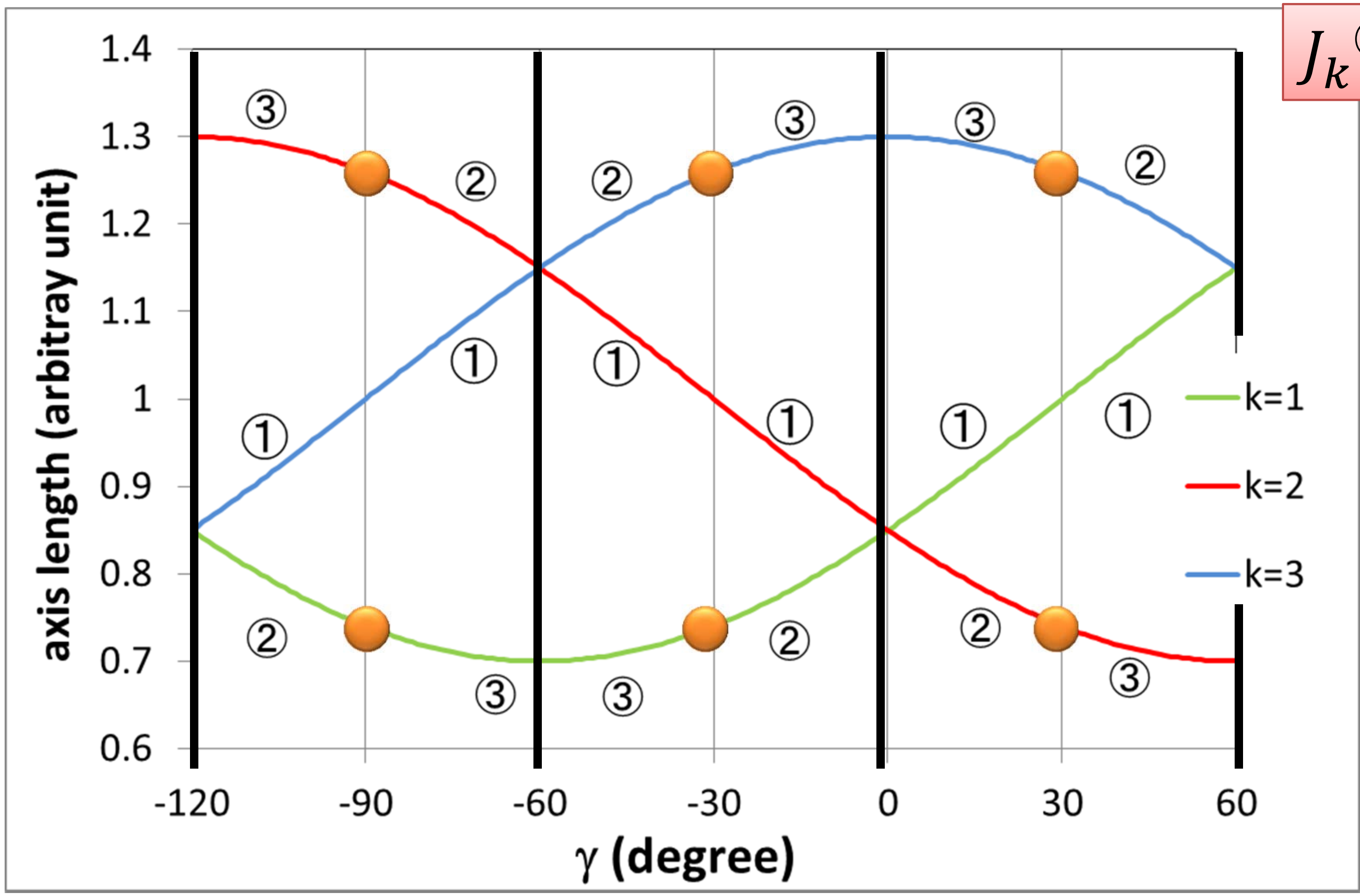
$$R_k = R_0\left[1+\sqrt{\frac{5}{4\pi}}\beta\cos\left(\gamma-\frac{2\pi}{3}k\right)\right]$$

Moment of Inertia

$$J_k = \frac{4}{3}J_0\sin^2\left(\gamma-\frac{2\pi}{3}k\right)$$

k=1,2,3

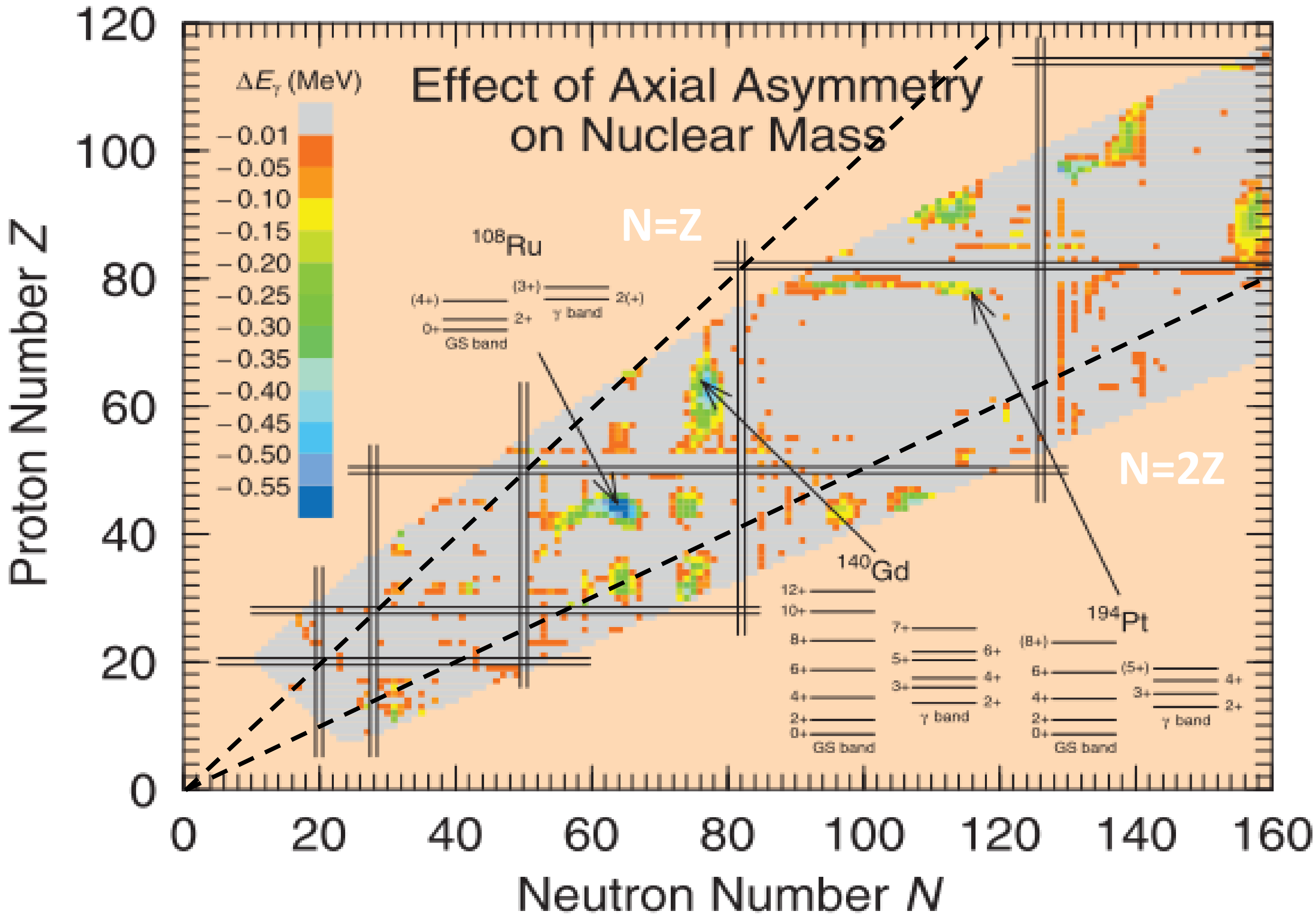
$J_k{}^{①} > J_k{}^{②} > J_k{}^{③}$



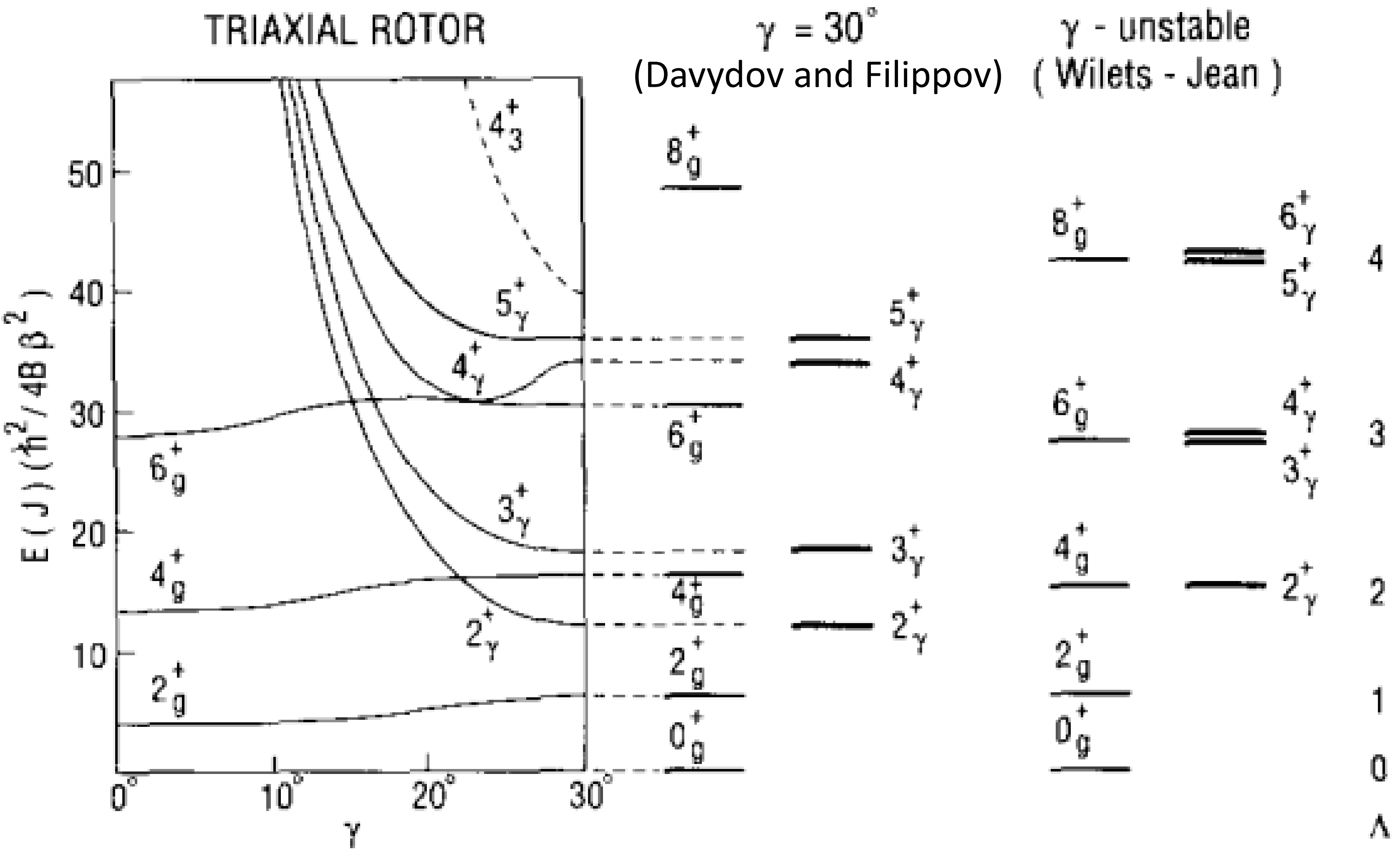
$J_k{}^{②} = J_k{}^{③}$

Global calculations of ground–state triaxiality

P. Moller et al.,**PRL97,162502 (2006)**



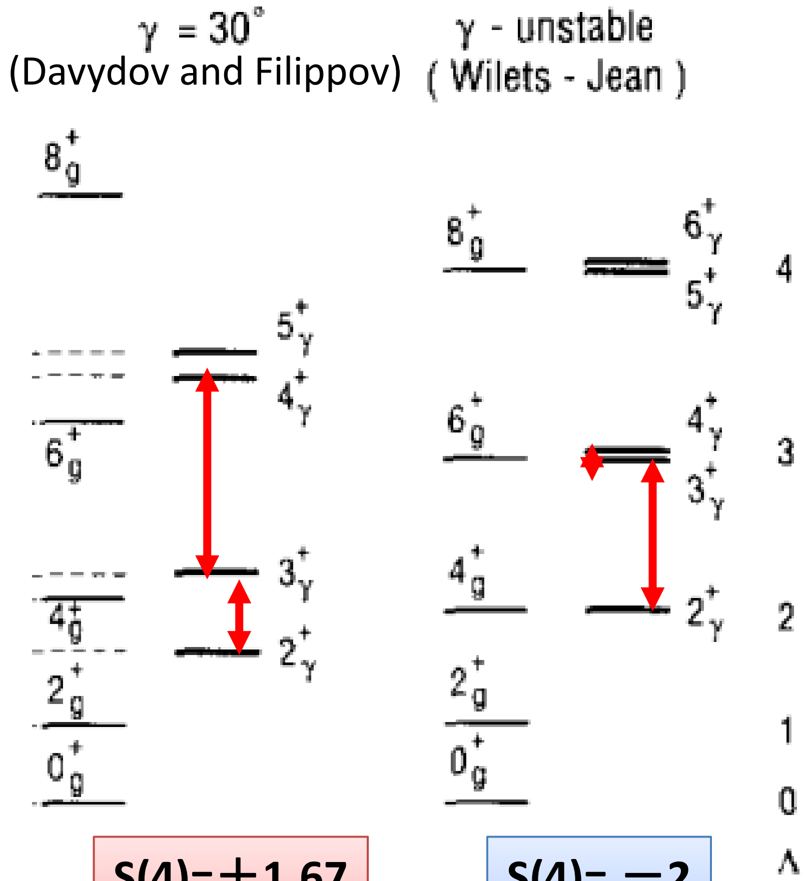
Two extreme models of axial asymmetry



N. Z. Zamfier and R. Casten, Phys. Lett. B, 260, 265 (1991)

Two extreme models of axial asymmetry

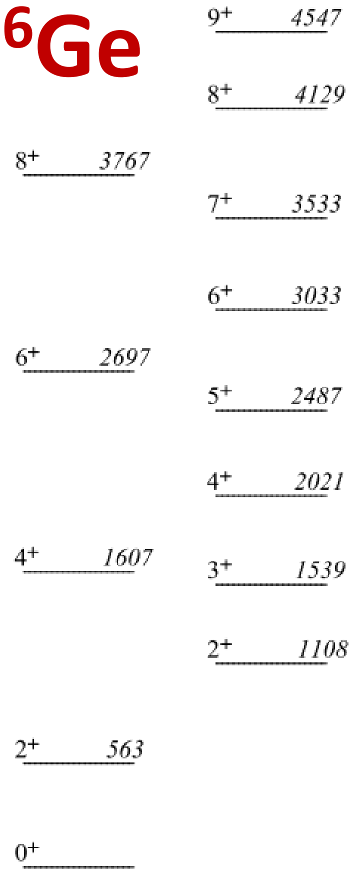
$$S(I) = \frac{[E(I) - E(I-1)] - [E(I-1) - E(I-2)]}{E(2_1^+)}$$

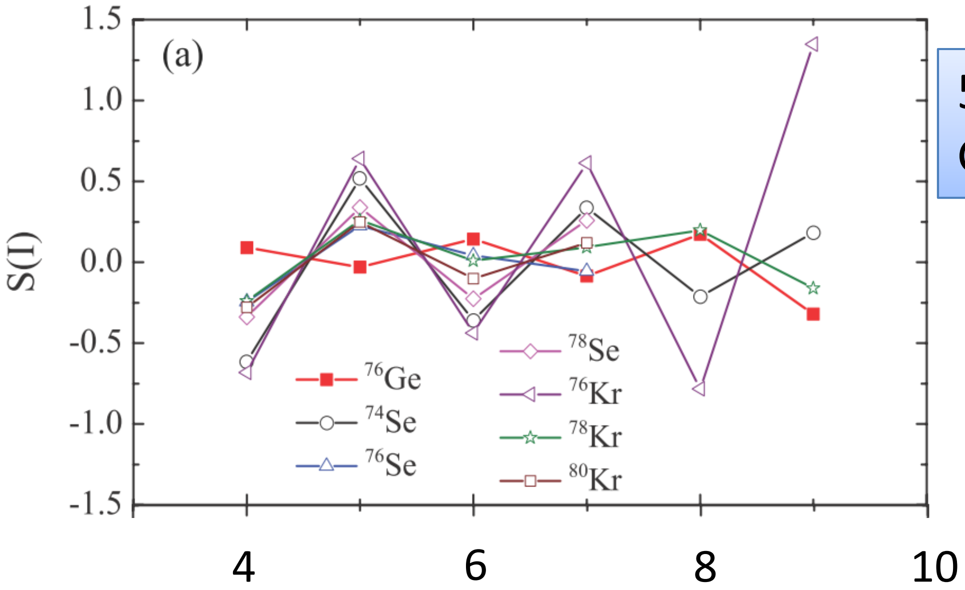


S(4)=＋1.67
S(5) < 0

S(4)= −2
S(5) > 0

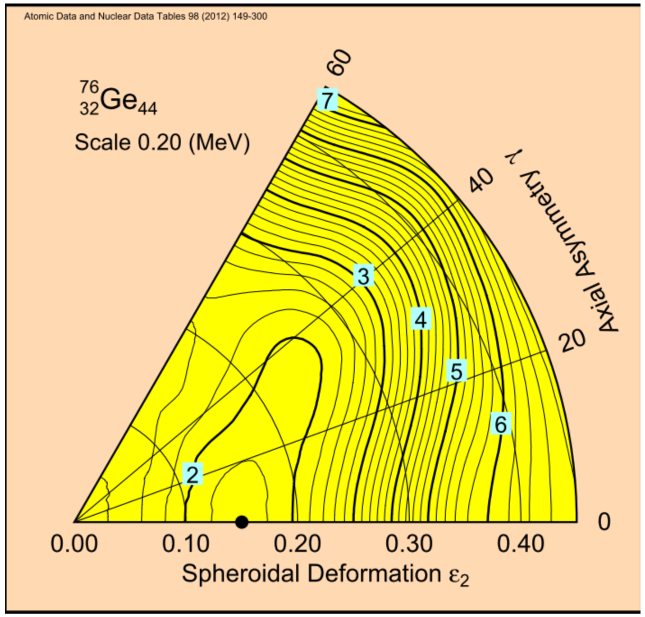
^{76}Ge





530 MeV ^{76}Ge + ^{238}U
Gammasphere@ANL

Y. Toh et al., Phys. Rev. C, 87, 041304(R) (2013)



$$r = \frac{E_{2_2^+}}{E_{2_1^+}} = \frac{1+\sqrt{1-\frac{8}{9}\sin^2(3\gamma)}}{1-\sqrt{1-\frac{8}{9}\sin^2(3\gamma)}}$$

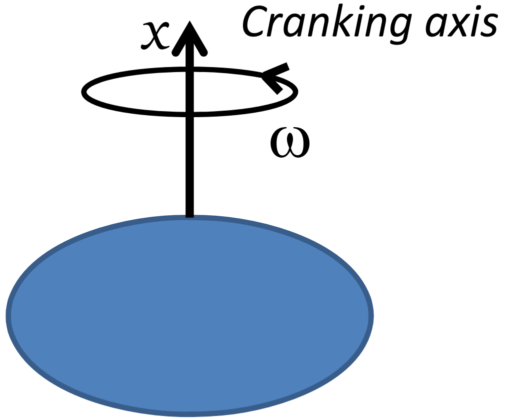
$r(^{76}\text{Ge})=1.96$
$\rightarrow \gamma=30°$

Davydov and Filippov, Nucl. Phys. 8, 237 (1958)

http://t2.lanl.gov/nis/data/astro/molnix96/peseps2gamma.html

PAC regime

- Principal Axis Cranking (PAC)
  - Rotation about the principle axis
- Labeling of a band with parity and signature (π,α)
  - α is additive
- Favoured and unfavoured signature band
  - Favoured signature: $e^{-i\pi j}$
  - Unfavoured signature: $e^{i\pi j}$



Cranking Hamiltonian

$$H^{\omega} = \sum h^{\omega} = \sum (h - \hbar\omega j_x)$$

Symmetry of Cranking Hamiltonian

$$R_x(\pi) = e^{-i\pi j_x}$$

Signature Quantum Number

$$r = e^{-i\pi\alpha}$$
$$I = \alpha \bmod 2$$

I	α	r
0,2,4,6,..	0	+1
1,3,5,7,..	1	-1
1/2,5/2,9/2,13/2,..	+1/2	-i
3/2,7/2,11/2,15/2,..	-1/2	+i

High Spin Frontier
Cranking (PAC) domain

Wobbling Modes

Smooth Band Termination

Wobbling mode

- Q.M. rotation of triaxail body
  - asymmetric top in clasical analogue
- Wobbling of triaxial body around the total angular momentum
- First observation in ^{163}Lu
  - S.W. Odegard *et al*, PRL 86 5866 (2001)
- Experimentally established stable triaxial deformation at high spin (cranking regime)
  - Lu isotopes (161,163,165, and 167)
  - Large mixing ratio of the interband transition (dominantly of E2 character)
  - Band configurations based on the cranking calculations (ultimate cranker)
  - *EM* transitions calculated by particle rotor model

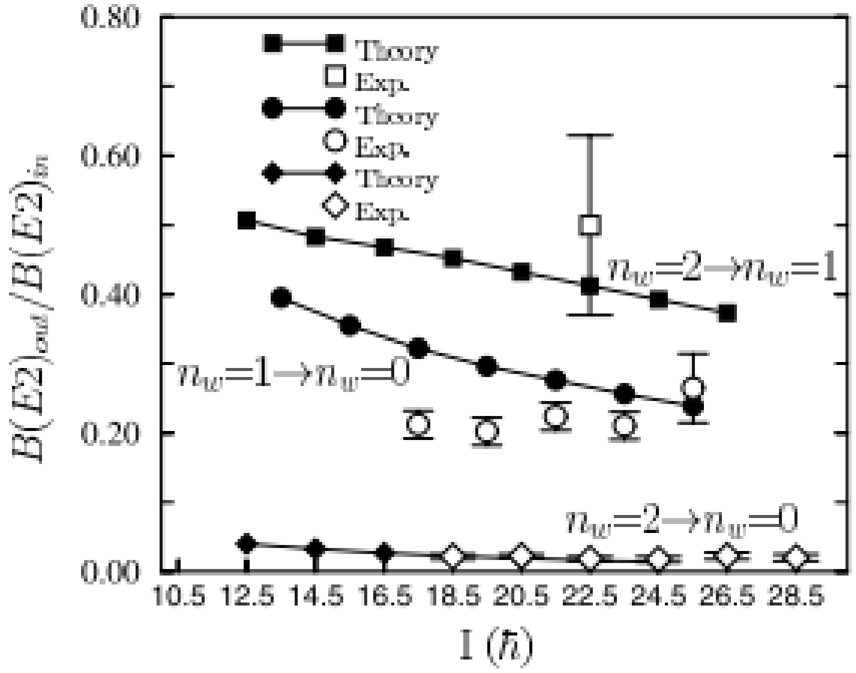
Quantum mechanically, non-zero components of collective rotation along all three principal axes are allowed.

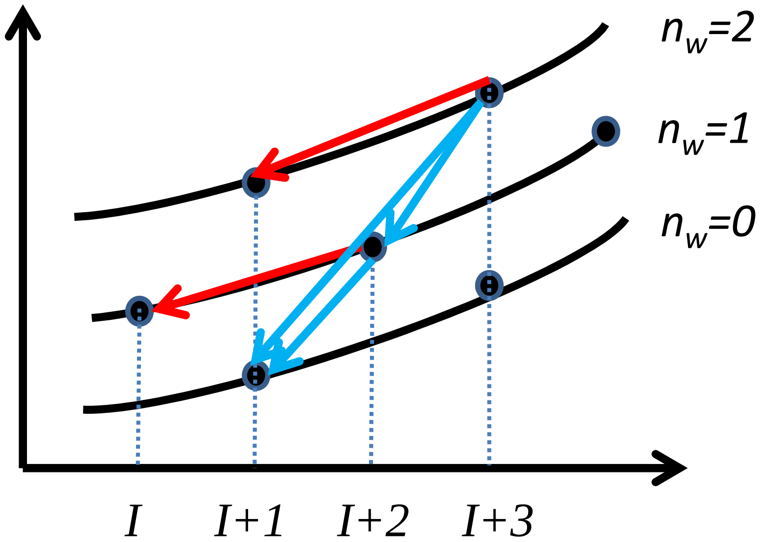
Cranking domain

$$I \approx I_x$$
$$\mathcal{J}_x \gg \mathcal{J}_y \neq \mathcal{J}_z$$

$$E(I,n) = \frac{I(I+1)}{2\mathcal{J}_x} + \hbar\omega_w(n_w + \frac{1}{2})$$

$$\hbar\omega_w = I\sqrt{\left(\frac{1}{\mathcal{J}_y} - \frac{1}{\mathcal{J}_x}\right)\left(\frac{1}{\mathcal{J}_z} - \frac{1}{\mathcal{J}_x}\right)}$$





D. R. Jense et al, PRL89, 142503 (2002)

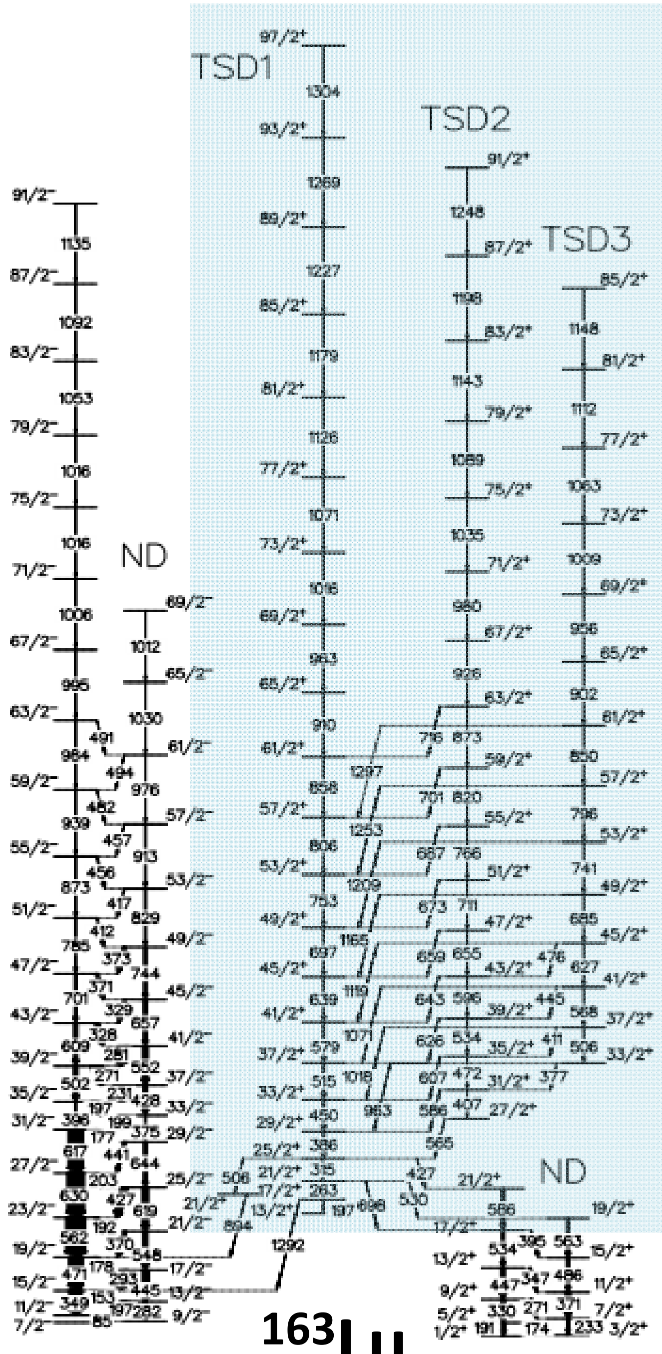
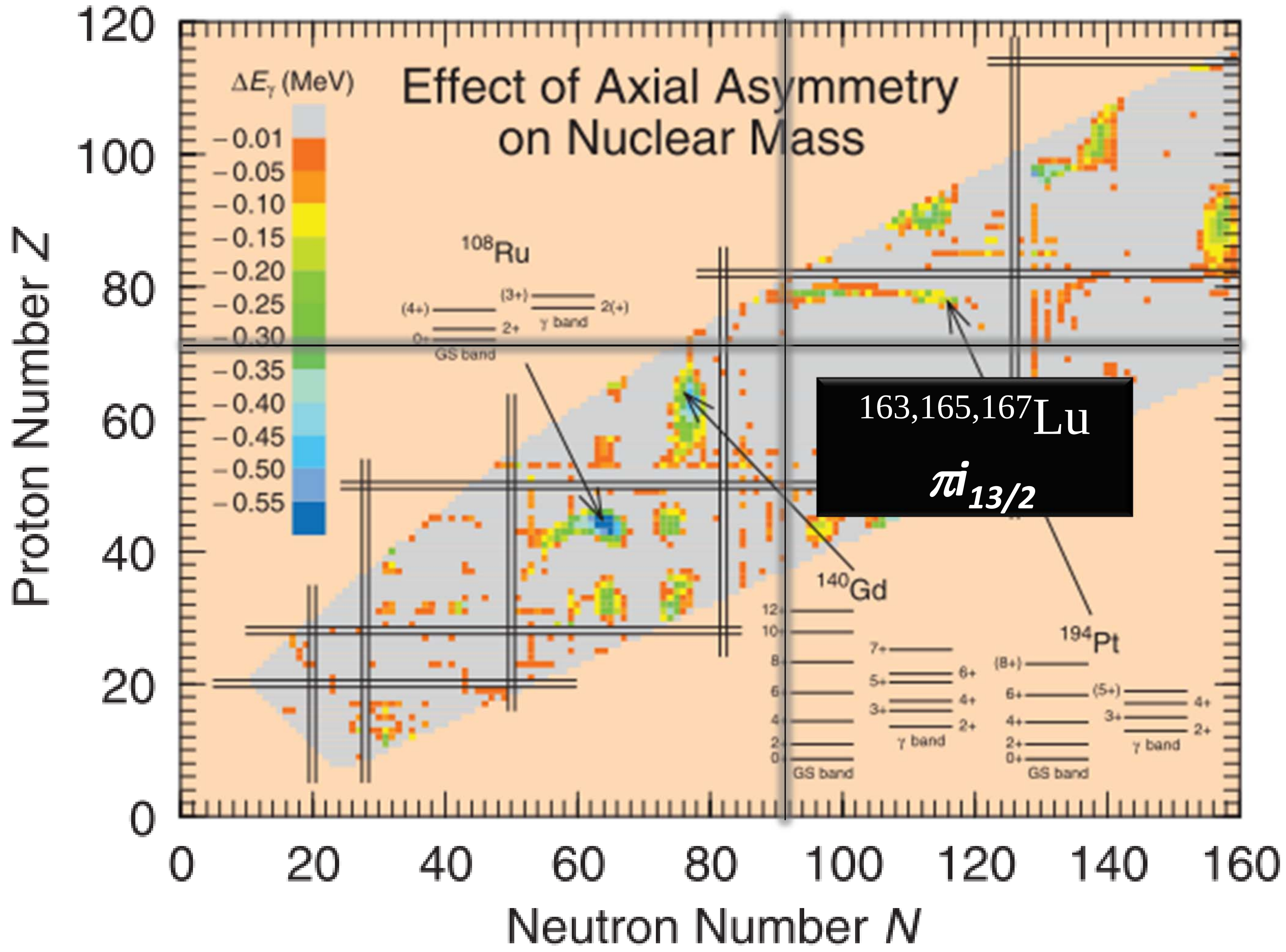


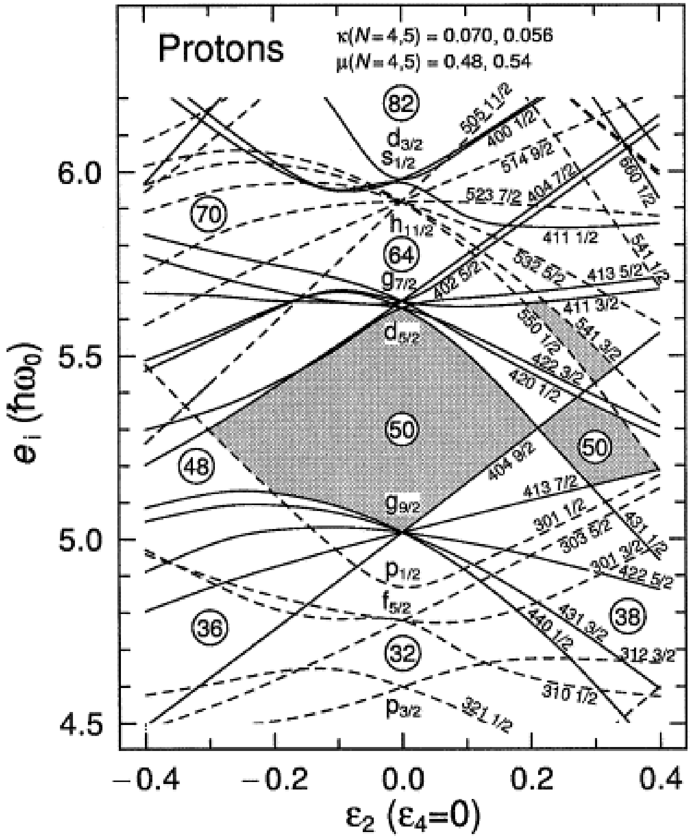
FIG. 1. Partial level scheme of ^{163}Lu.

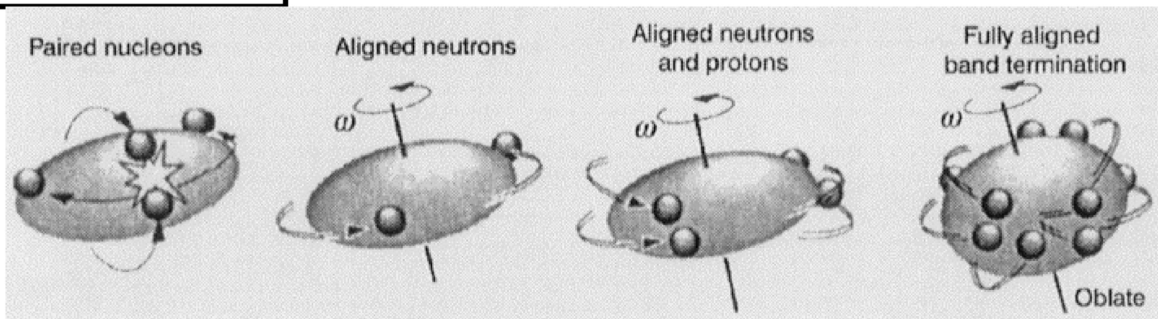


Effect of Axial Asymmetry
on Nuclear Mass
ΔEγ (MeV)
−0.01
−0.05
−0.10
−0.15
−0.20
−0.25
−0.30
−0.35
−0.40
−0.45
−0.50
−0.55
108Ru
GS band
γ band
140Gd
194Pt
163,165,167Lu
πi13/2
Proton Number Z
Neutron Number N

Smooth band termination

- Gradual (smooth) restoration of rotational symmetry
  - Collective to single particle degrees of freedom
  - Change of shape from prolate to oblate via the *triaxial* plane
- High spin extreme
  - PAC is a good approximation
  - Little influence of pairing
  - rigid body moment of inertia
- First observation in 1994
  - P.V. Janzen et al, PRL 72, 1160 (1994)
  - 20 (Ge + ACS) + 71 BGO filters at Chalk River Lab.
  - Many cases identified in the mass A ~100 region (Gammaspher, Euroball)



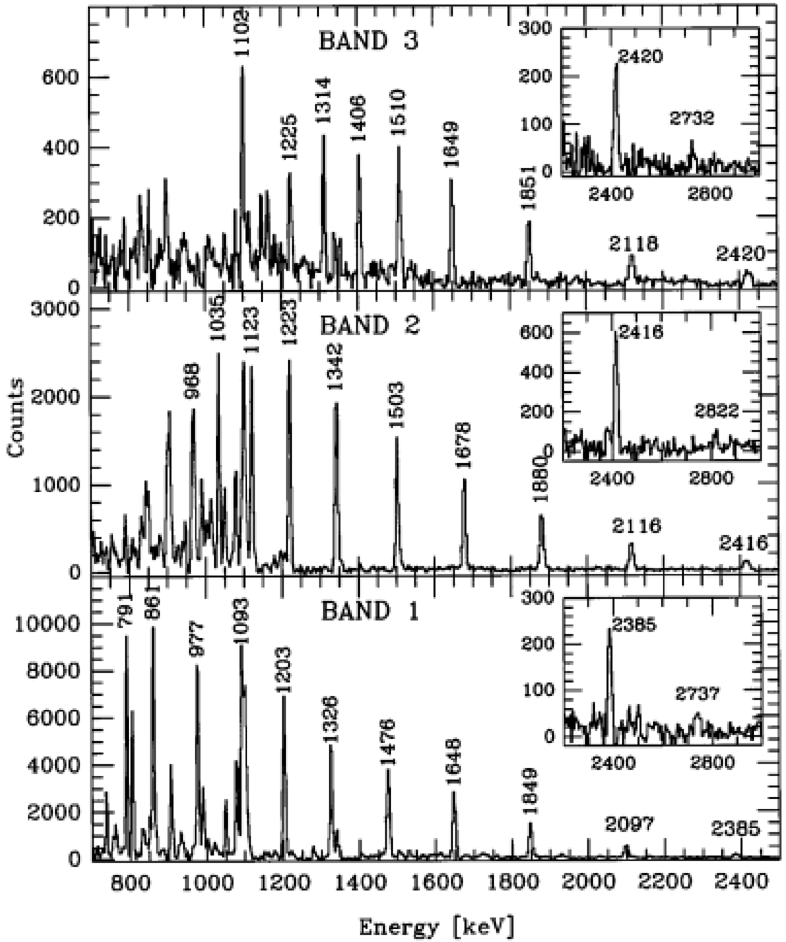


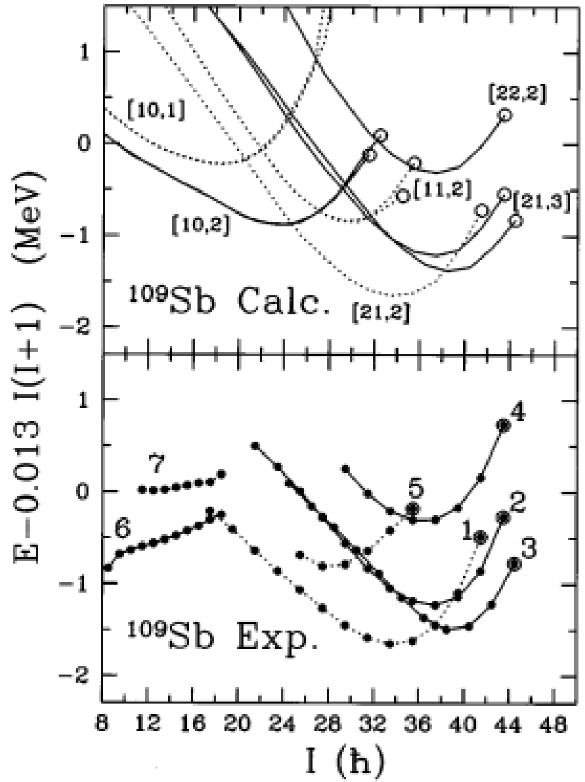
A.V. Afanasjev and D.B. Fossan et al, Physics Reports 322 (1999)

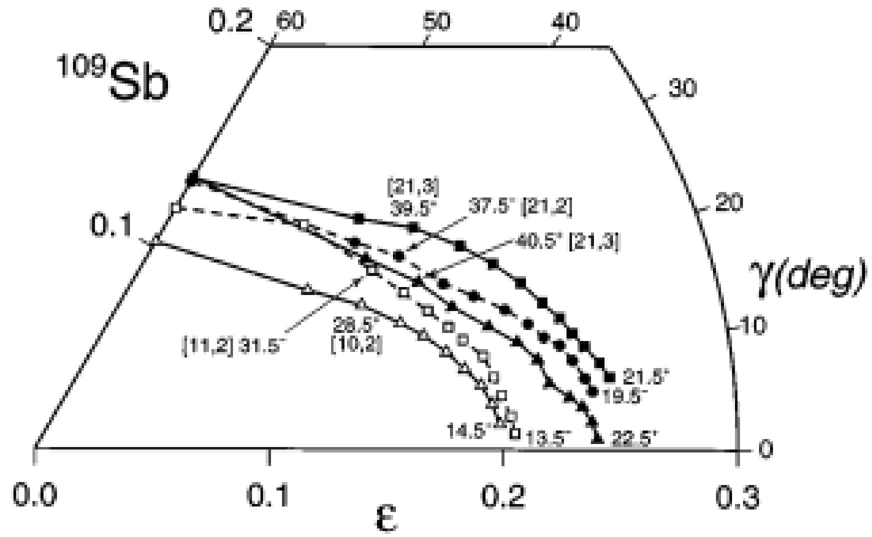
^{109}Sb

[p1,p2,n] =[# of $g_{9/2}$ p hole, $h_{11/2}$ p, $h_{11/2}$ n]

Ex. [21,3]=$\pi(g_{9/2})^{-2}(g_{7/2}d_{5/2})^{2}(h_{11/2})^{1}\nu(g_{7/2}d_{5/2})^{5}(h_{11/2})^{3} \rightarrow I^{\pi}=89/2^{+}$

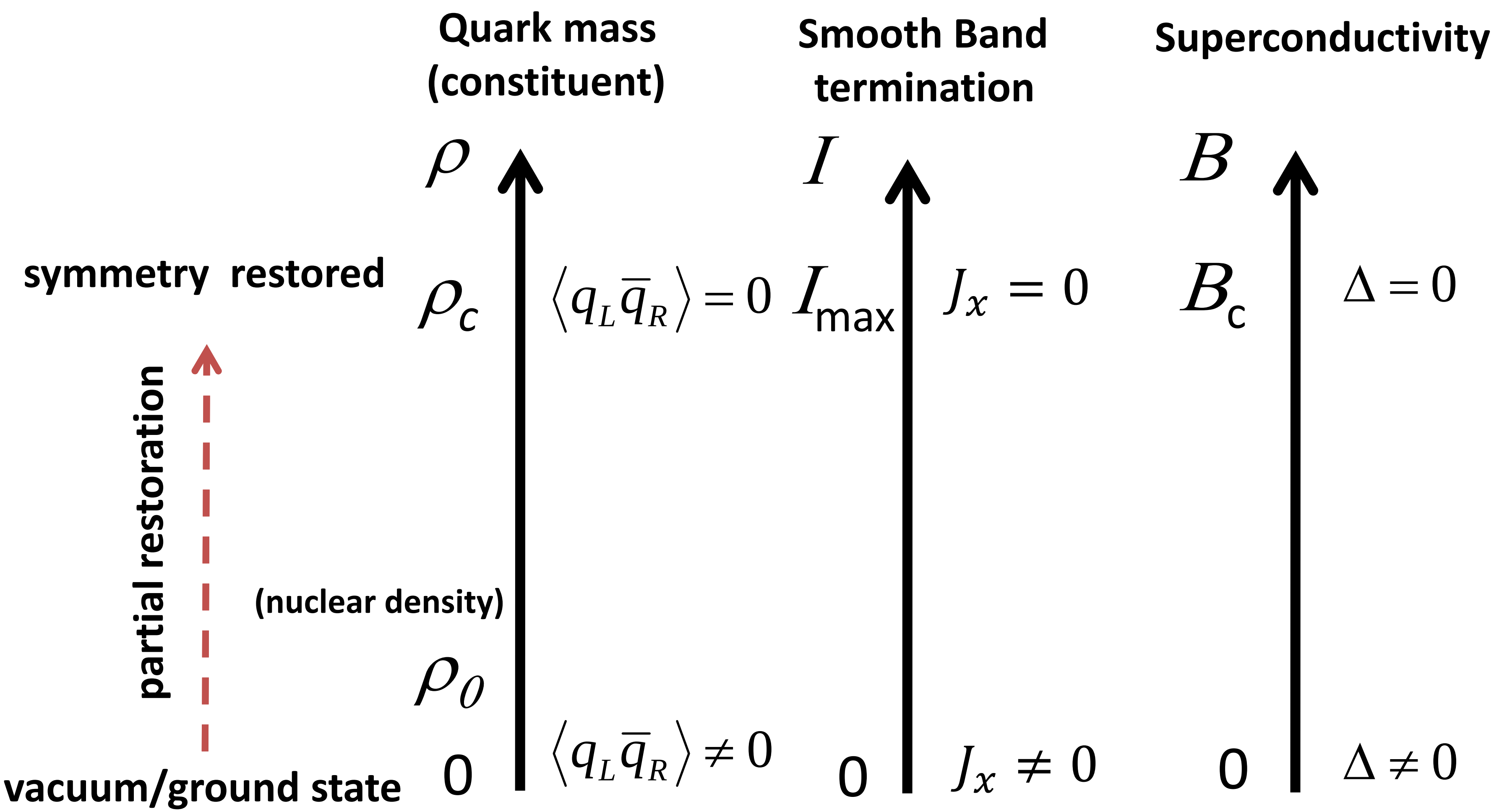






H. Schnare et al, PRC 54, 1598 (1996)

Restoration of broken symmetry



Medium spin

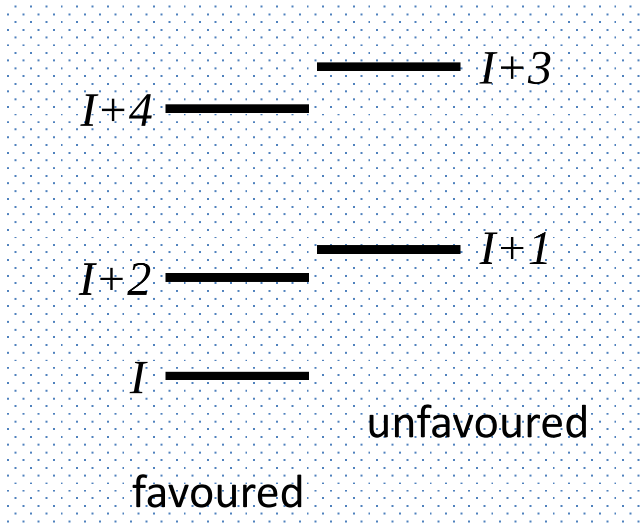
Signature Inversion

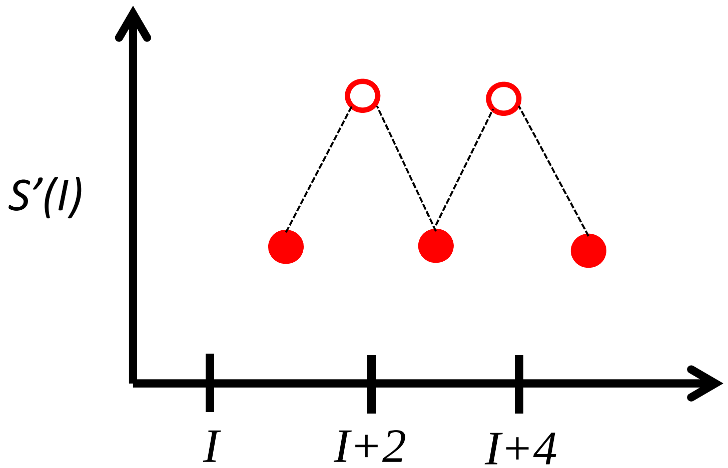
Chiral Doublet

Signature inversion

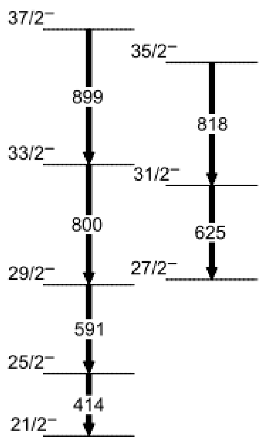
- Unfavored signature states lowered in energy than the favoured ones over some spin range
  - signature order inverted
- Systematically observed in the (transitional)mass A〜130 and 160 region
  - With q.p. configurations
- Suggested role of trixaility and *pn* interaction
  - N. Tajima, Nuclear Physics A572 (1994) 365

$$S'(I) = \frac{E(I) - E(I-1)}{2I}$$



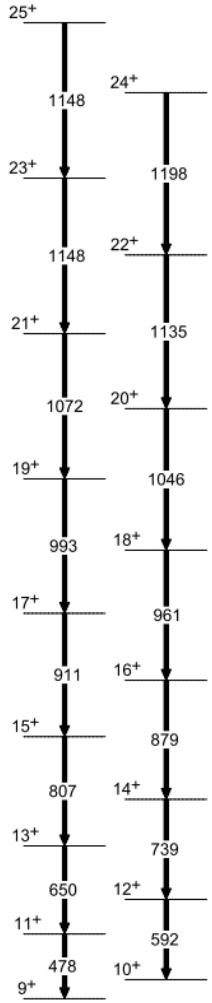


^{127}Cs



$\pi h_{11/2}$

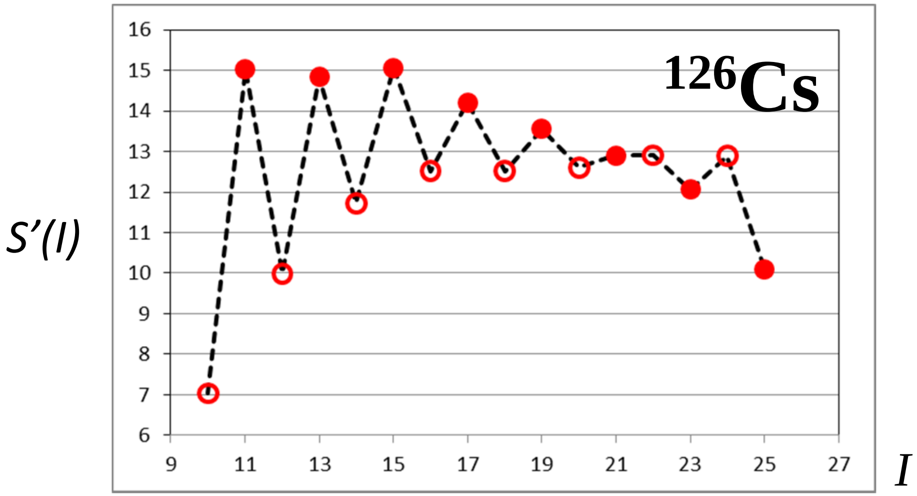
^{126}Cs

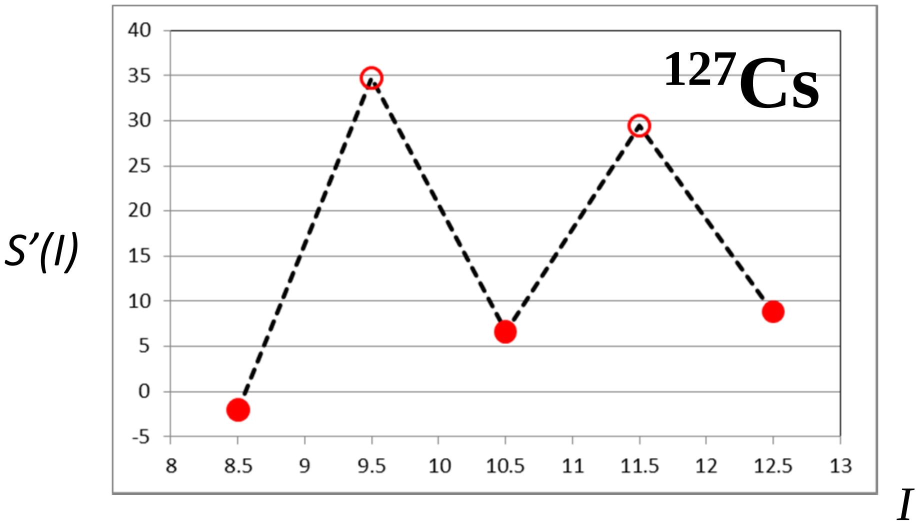


$\pi h_{11/2} \nu h_{11/2}$

● favoured

○ ubfavoured



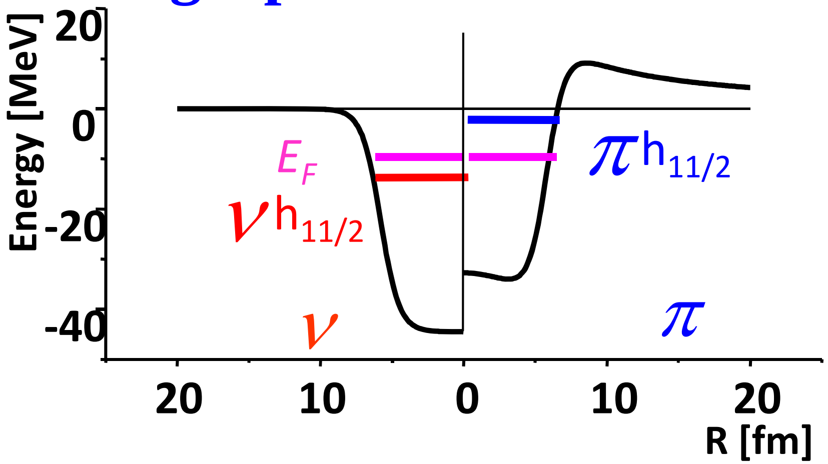


Y. Liang et al, PRC 42, 890 (1990) *S. Wang et al, PRC 74, 017302 (2006)*

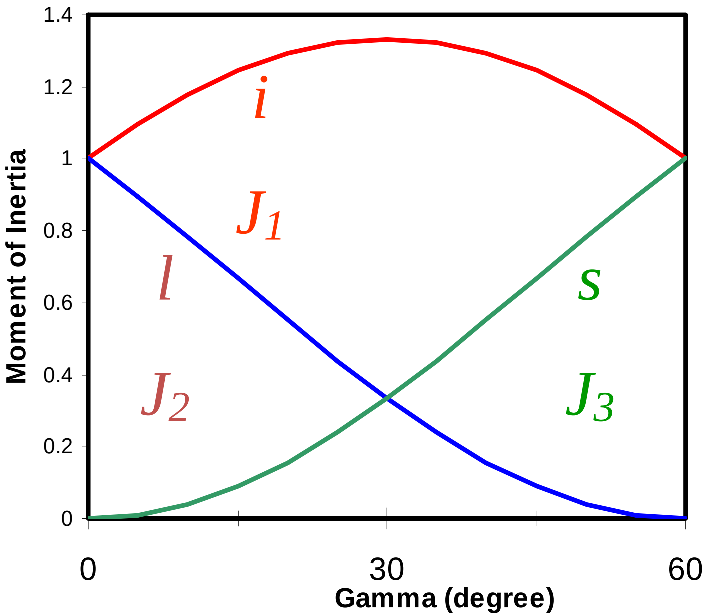
Chiral doublet bands

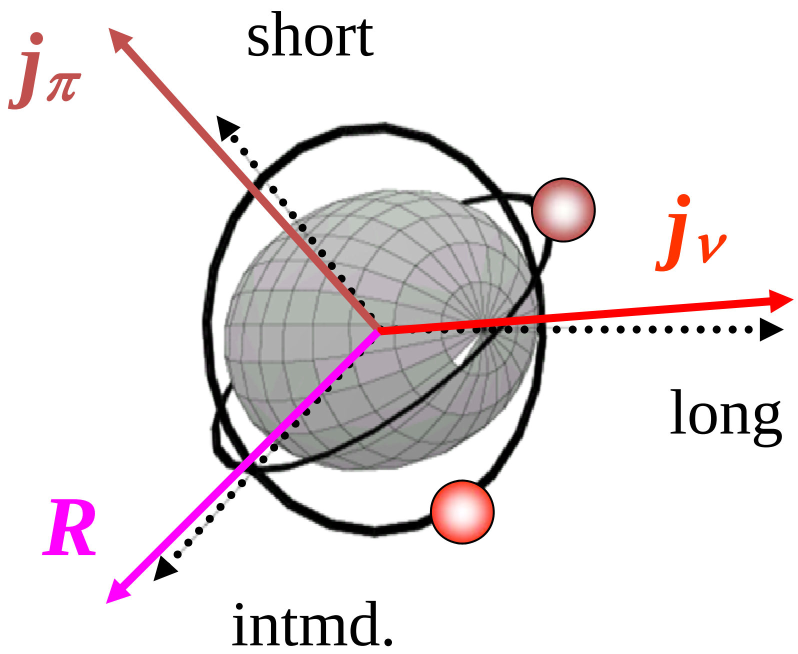
S.Frauendorf and J.Meng, NPA617, 131 (1997)

Single particle



Collective core: Moment of inertia





$$H_{rot} = \frac{R_1^{\ 2}}{2J_1} + \frac{R_2^{\ 2}}{2J_2} + \frac{R_3^{\ 2}}{2J_3}$$

$$J_k = \frac{4}{3} J_0 \sin^2(\gamma - \frac{2\pi}{3} k)$$

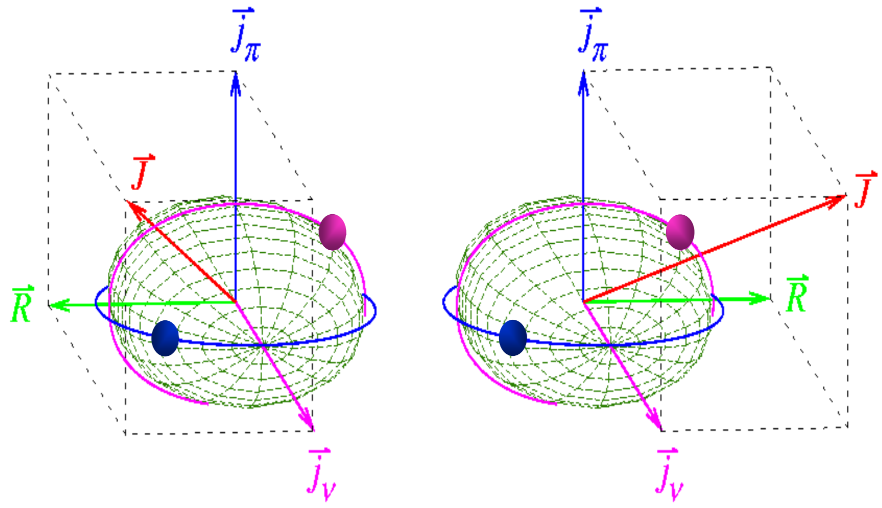
Chirality

$$[\mathsf{O}, H] = 0, \mathsf{O} = TR_y(\pi)$$

$$H|\mathsf{R}\rangle = \varepsilon_{\mathsf{R}}|\mathsf{R}\rangle, H|\mathsf{L}\rangle = \varepsilon_{\mathsf{L}}|\mathsf{L}\rangle,$$

$$|\mathsf{R}\rangle = \mathsf{O}|\mathsf{L}\rangle, |\mathsf{L}\rangle = \mathsf{O}|\mathsf{R}\rangle,$$

$$\varepsilon_{\mathsf{R}} = \varepsilon_{\mathsf{L}}$$



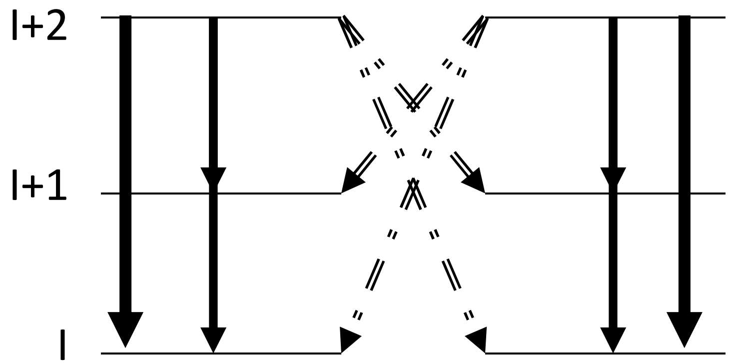
$$|IM+\rangle = \frac{1}{\sqrt{2}}(|\mathcal{R}\rangle + |\mathcal{L}\rangle),$$

$$|IM-\rangle = \frac{i}{\sqrt{2}}(|\mathcal{R}\rangle - |\mathcal{L}\rangle),$$

$$H|IM\pm\rangle = \varepsilon_{\pm}^{IM}|IM\pm\rangle,$$

$$\mathcal{O}|IM\pm\rangle = |IM\pm\rangle,$$

$$\varepsilon_{+}^{IM} = \varepsilon_{-}^{IM}.$$



Different from Parity
T: time reversal operator anti-linear

Why bother ?

Exotic collective modes in senses of

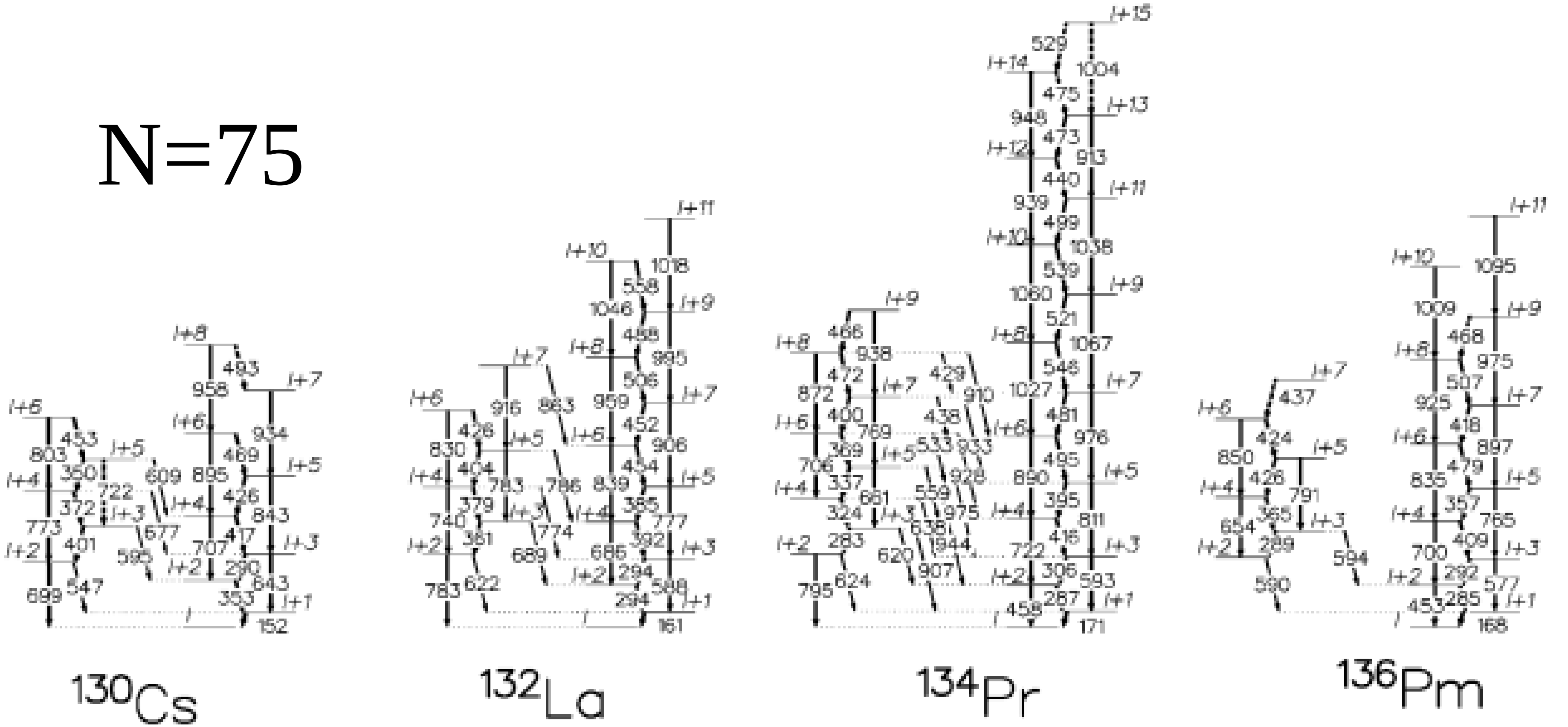
- numbers and kinds of symmetry broken spontaneously in a fermionic many-body system
- strong coupling of collective and two single-particle degrees of freedom

SSB					O.P.	ANG	spectrum
空間	$R_z(\phi)$	$R_z(\pi)$	P	$TR_y(\pi)$			
	•				β_2	回転 (帯)	$I^+, (I+2)^+, (I+4)^+$,..
	•	•			β_2, θ	回転 (帯)	$I^+, (I+1)^+, (I+2)^+$,..
	•	•	•$(R_z(\pi))$		β_2, β_3	パリティ 2 重項 (帯)	$I^+, (I+1)^-, (I+2)^+$,..
	•	•		•	β_2, γ θ, ϕ	カイラル 2 重項 (帯)	$2\times(I^+, (I+1)^+, (I+2)^+$,..)
非空間	$R_{z_g}(\phi)$	$R_{z_g}(\pi)$					
	•				Δ	対回転 (帯)	$N, N\pm2, N\pm4$,...

T. Koike, *Genshikau Kenkyu* Vol.53, Sup.3, April (2009) based on the Table II. by S. Faruendorf, Rev. Mod. phys. 73, 463, (2001)

The first systematic observation of chiral candidates

N=75



$$\pi h_{11/2} \nu h_{11/2}$$

***K. Starosta et al.*,** Phys. Rev. Lett **86,** 971 (2001)

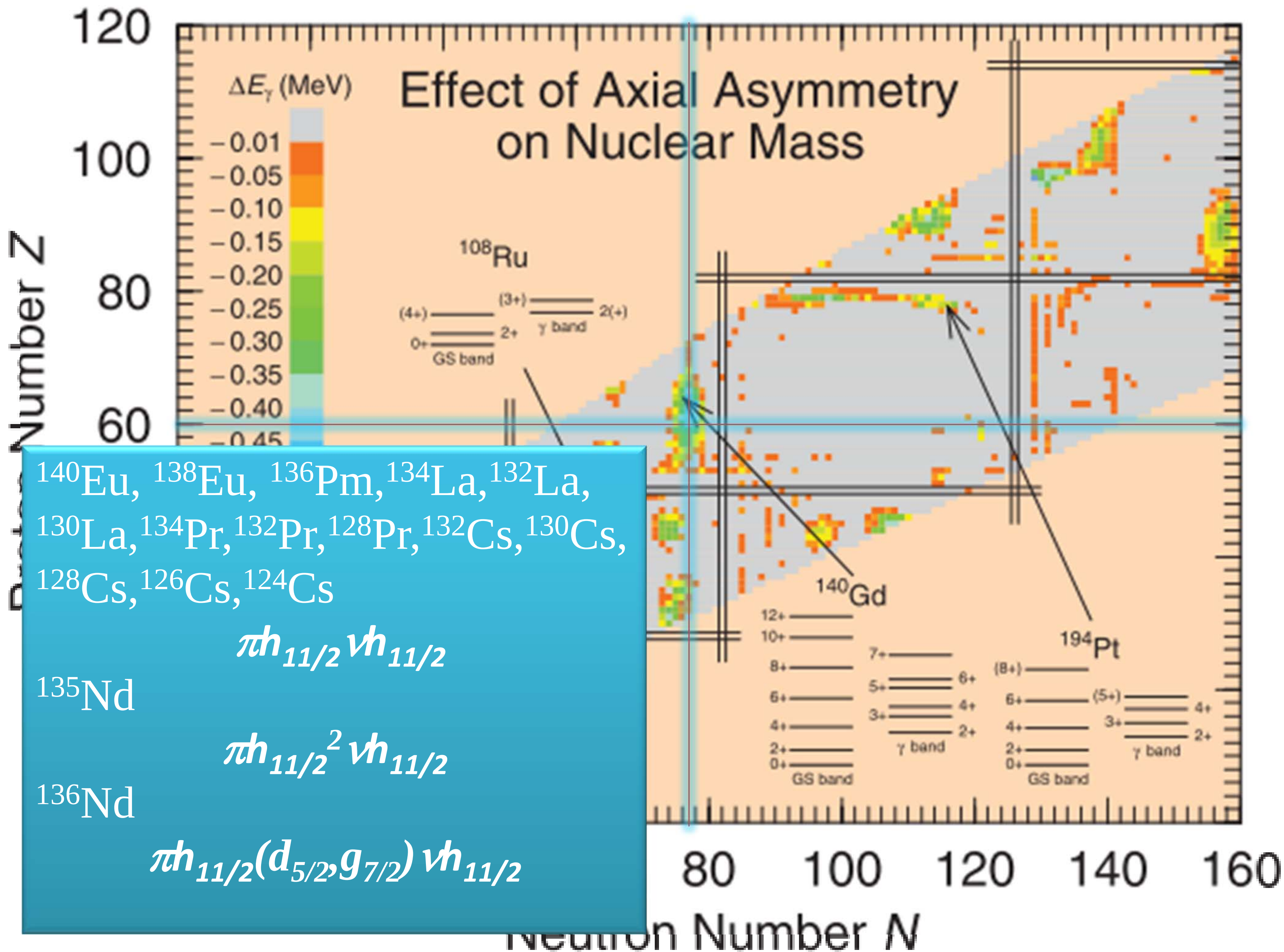
A~130			
Nucleus	s.p. config.		*E.M.*
^{140}Eu	$\pi h_{11/2}\nu h_{11/2}$	odd-odd	
^{138}Eu	$\pi h_{11/2}\nu h_{11/2}$	odd-odd	
^{138}Pm	$\pi h_{11/2}\nu h_{11/2}$	odd-odd	
^{136}Pm	$\pi h_{11/2}\nu h_{11/2}$	odd-odd	
^{135}Nd	$\pi {h_{11/2}}^2\nu h_{11/2}$	odd-A	yes[3]
^{134}La	$\pi h_{11/2}\nu h_{11/2}$	odd-odd	
^{132}La	$\pi h_{11/2}\nu h_{11/2}$	odd-odd	yes
^{134}Pr	$\pi h_{11/2}\nu h_{11/2}$	odd-odd	yes[4]
^{132}Pr	$\pi h_{11/2}\nu h_{11/2}$	odd-odd	
^{128}Pr	$\pi h_{11/2}\nu h_{11/2}$	odd-odd	
^{132}Cs	$\pi h_{11/2}\nu h_{11/2}$	odd-odd	
^{130}Cs	$\pi h_{11/2}\nu h_{11/2}$	odd-odd	
^{128}Cs	$\pi h_{11/2}\nu h_{11/2}$	odd-odd	yes[5]
^{126}Cs	$\pi h_{11/2}\nu h_{11/2}$	odd-odd	
^{124}Cs	$\pi h_{11/2}\nu h_{11/2}$	odd-odd	

A ~105			
Nucleus	s.p. config.		*E.M.*
^{106}Ag	$\pi g_{9/2}\nu h_{11/2}$	odd-odd	
^{105}Ag	$\pi g_{9/2}\nu {h_{11/2}}^2$	odd-A	
^{106}Rh	$\pi g_{9/2}\nu h_{11/2}$	odd-odd	
^{105}Rh	$\pi g_{9/2}\nu {h_{11/2}}^2$	odd-A	
^{104}Rh	$\pi g_{9/2}\nu h_{11/2}$	odd-odd	yes[6]
^{103}Rh	$\pi g_{9/2}\nu {h_{11/2}}^2$	odd-A	yes[6]
^{102}Rh	$\pi g_{9/2}\nu h_{11/2}$	odd-odd	
^{112}Ru	$\nu h_{11/2}(d_{5/2}, g_{7/2})$	even-even	
^{110}Ru	$\nu h_{11/2}(d_{5/2}, g_{7/2})$	even-even	
^{108}Ru	$\nu h_{11/2}(d_{5/2}, g_{7/2})$	even-even	
^{100}Tc	$\pi g_{9/2}\nu h_{11/2}$	odd-odd	
^{106}Mo	$\nu h_{11/2}(d_{5/2}, g_{7/2})$	even-even	

A ~190			
Nucleus	s.p. config.		*E.M.*
^{188}Ir†	$\pi h_{9/2}\nu i_{13/2}$	odd-odd	
^{198}Tl	$\pi h_{9/2}\nu i_{13/2}$	odd-odd	

A~80**			
Nucleus	s.p. config.		*E.M.*
^{80}Br	$\pi g_{9/2}\nu g_{9/2}$	odd-odd	
	$\pi {g_{9/2}}^2\nu g_{9/2}$	odd-A	

T.Koike NPA834 (2010) 36c



^{140}Eu, ^{138}Eu, ^{136}Pm, ^{134}La, ^{132}La, ^{130}La, ^{134}Pr, ^{132}Pr, ^{128}Pr, ^{132}Cs, ^{130}Cs, ^{128}Cs, ^{126}Cs, ^{124}Cs

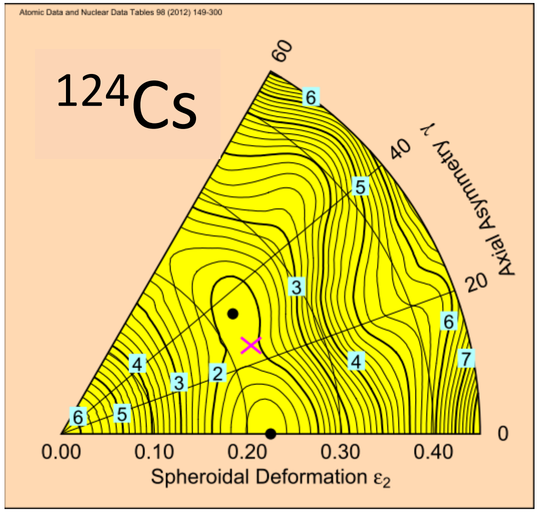
$\pi h_{11/2} \nu h_{11/2}$

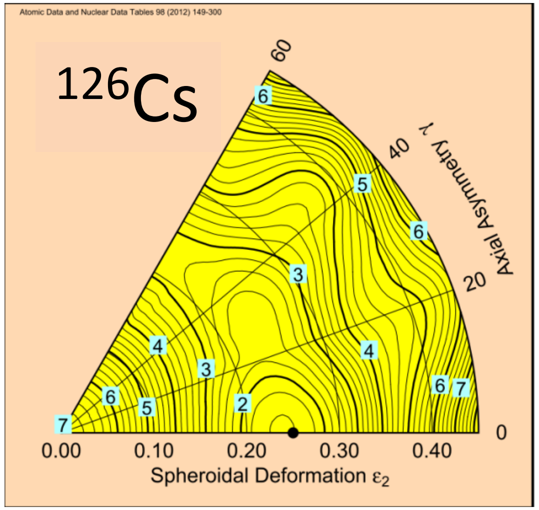
^{135}Nd

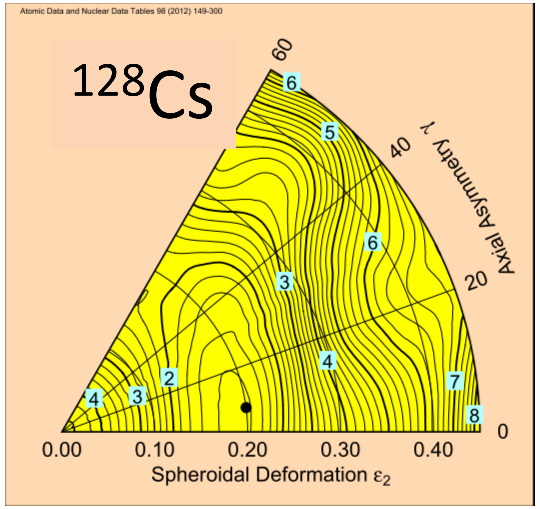
$\pi h_{11/2}^{2} \nu h_{11/2}$

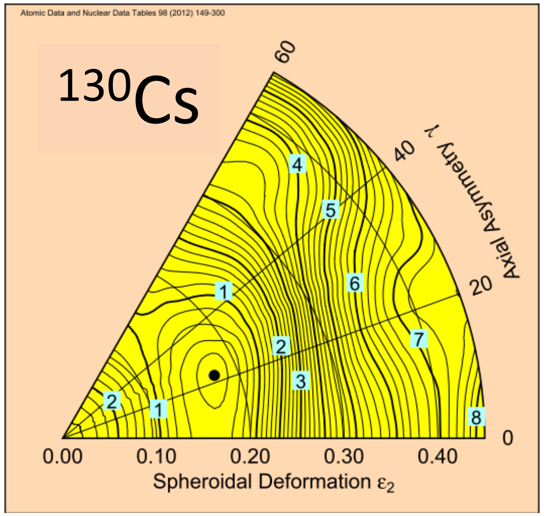
^{136}Nd

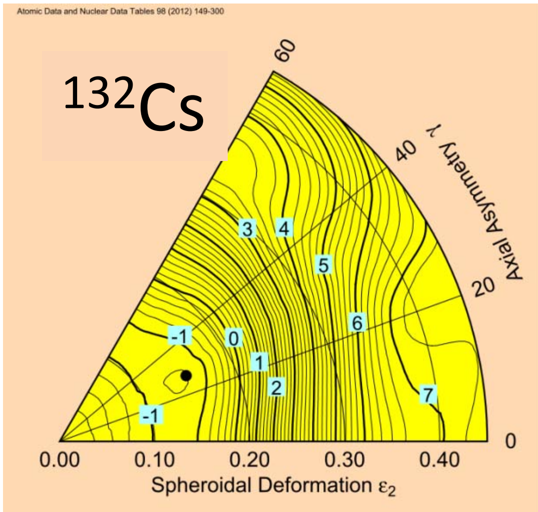
$\pi h_{11/2}(d_{5/2}, g_{7/2}) \nu h_{11/2}$

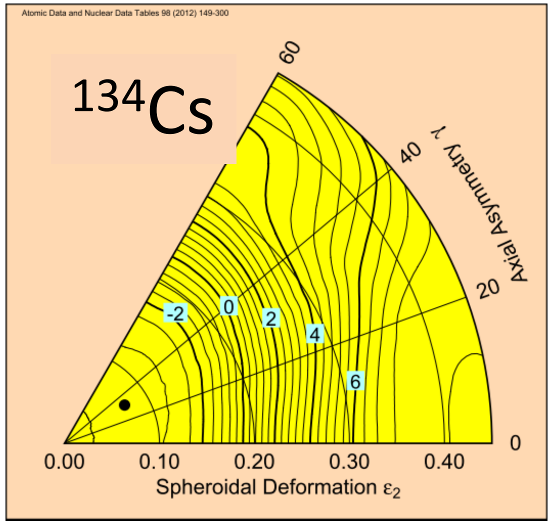

Atomic Data and Nuclear Data Tables 98 (2012) 149-300
^{124}Cs
Axial Asymmetry γ
Spheroidal Deformation ε2
0.00
0.10
0.20
0.30
0.40
0
20
40
60

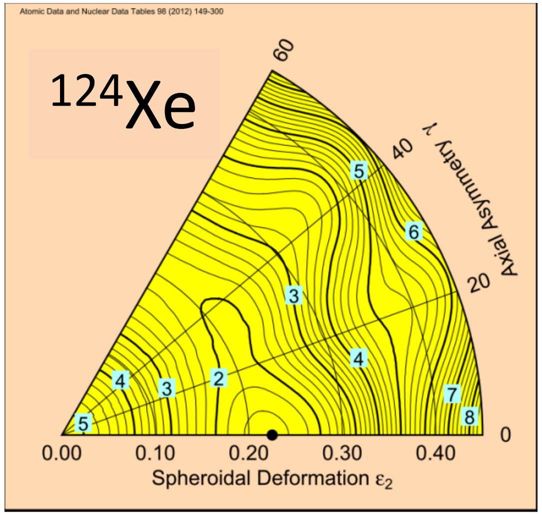

Atomic Data and Nuclear Data Tables 98 (2012) 149-300
^{126}Cs
Axial Asymmetry γ
Spheroidal Deformation ε2
0.00
0.10
0.20
0.30
0.40
0
20
40
60

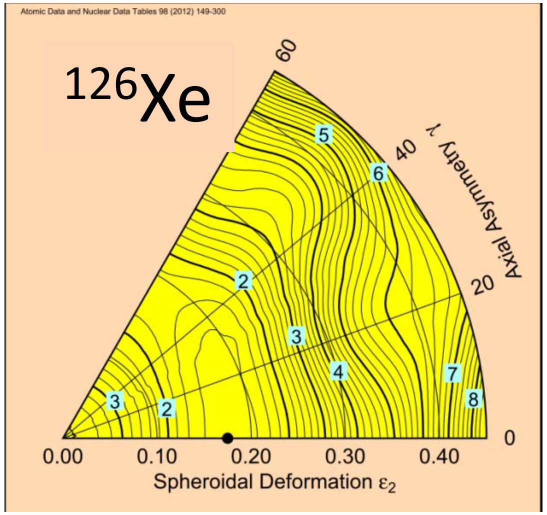

Atomic Data and Nuclear Data Tables 98 (2012) 149-300
^{128}Cs
Axial Asymmetry γ
Spheroidal Deformation ε2
0.00
0.10
0.20
0.30
0.40
0
20
40
60

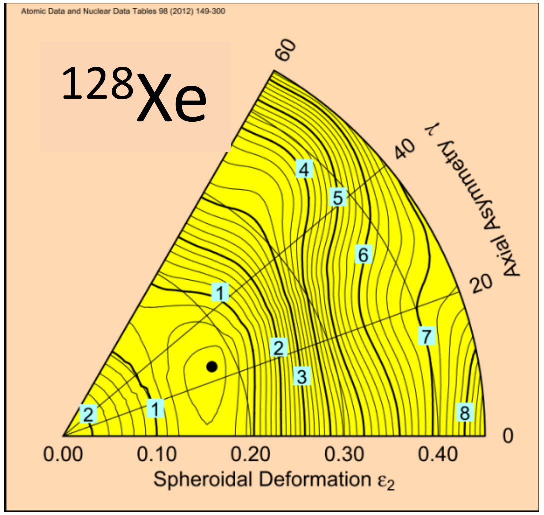

Atomic Data and Nuclear Data Tables 98 (2012) 149-300
^{130}Cs
Axial Asymmetry γ
Spheroidal Deformation ε2
0.00
0.10
0.20
0.30
0.40
0
20
40
60

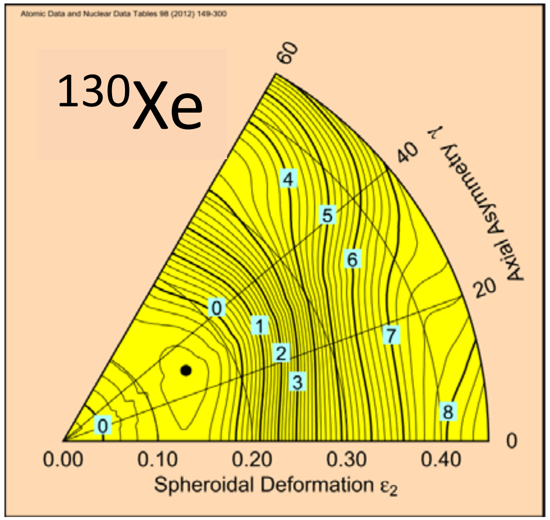

Atomic Data and Nuclear Data Tables 98 (2012) 149-300
^{132}Cs
Axial Asymmetry γ
Spheroidal Deformation ε2
0.00
0.10
0.20
0.30
0.40
0
20
40
60

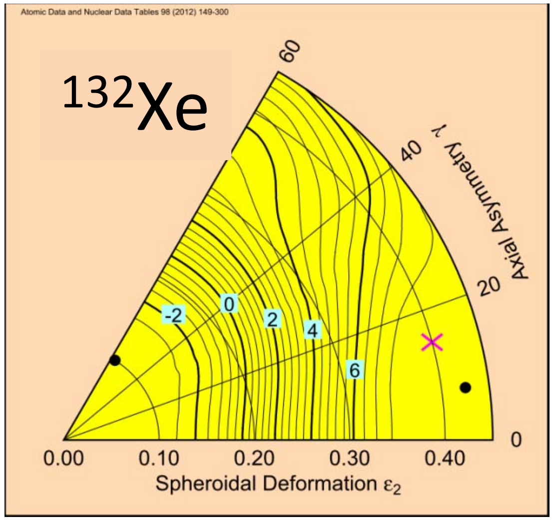

Atomic Data and Nuclear Data Tables 98 (2012) 149-300
^{134}Cs
Axial Asymmetry γ
Spheroidal Deformation ε2
0.00
0.10
0.20
0.30
0.40
0
20
40
60

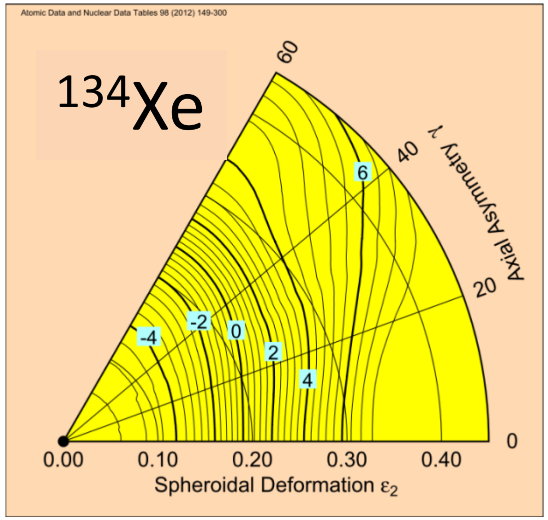

Atomic Data and Nuclear Data Tables 98 (2012) 149-300
^{124}Xe
Spheroidal Deformation ε_2
Axial Asymmetry γ
0.00 0.10 0.20 0.30 0.40
0 20 40 60

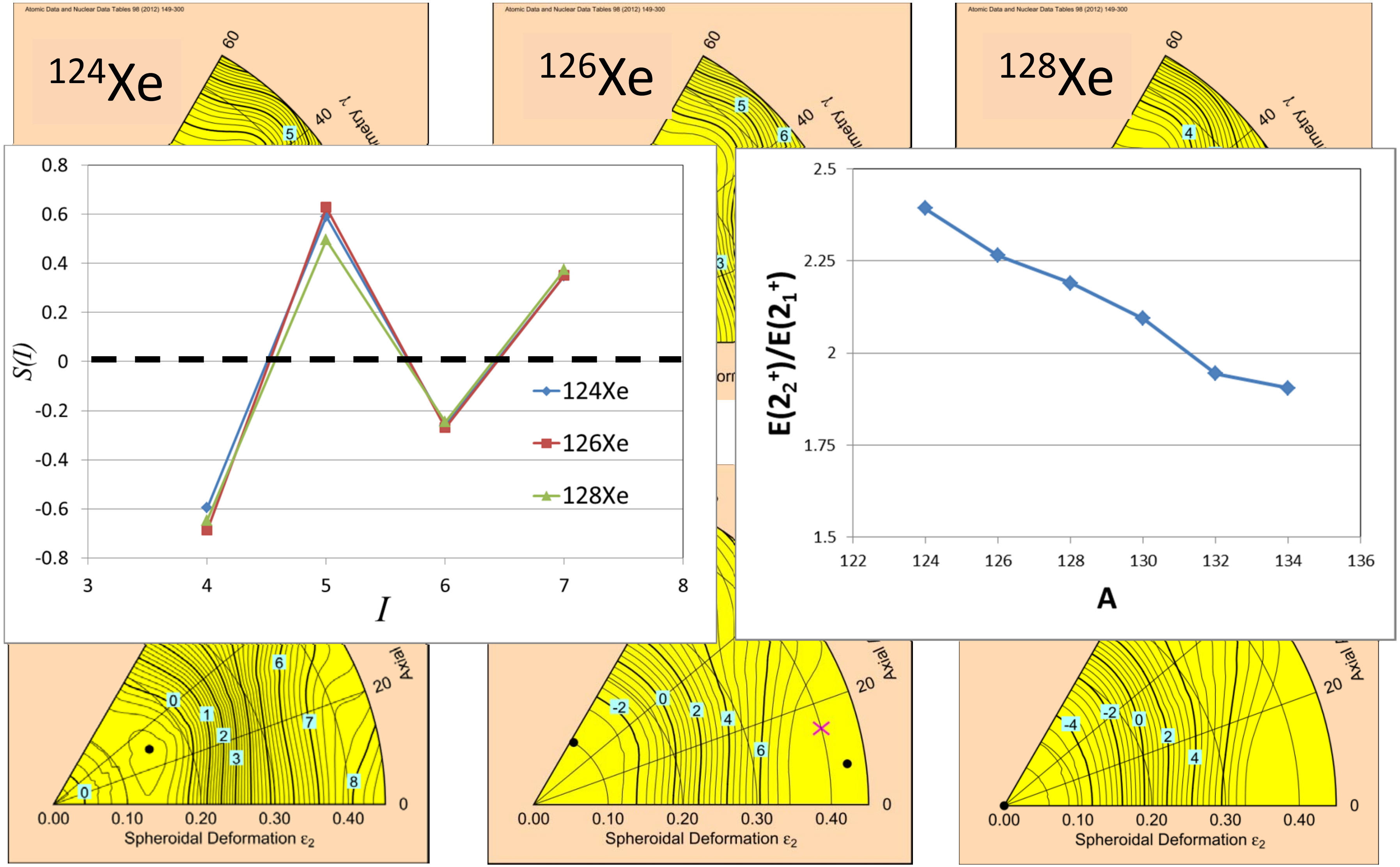

Atomic Data and Nuclear Data Tables 98 (2012) 149-300
^{126}Xe
Spheroidal Deformation ε_2
Axial Asymmetry γ
0.00 0.10 0.20 0.30 0.40
0 20 40 60


Atomic Data and Nuclear Data Tables 98 (2012) 149-300
^{128}Xe
Spheroidal Deformation ε_2
Axial Asymmetry γ
0.00 0.10 0.20 0.30 0.40
0 20 40 60

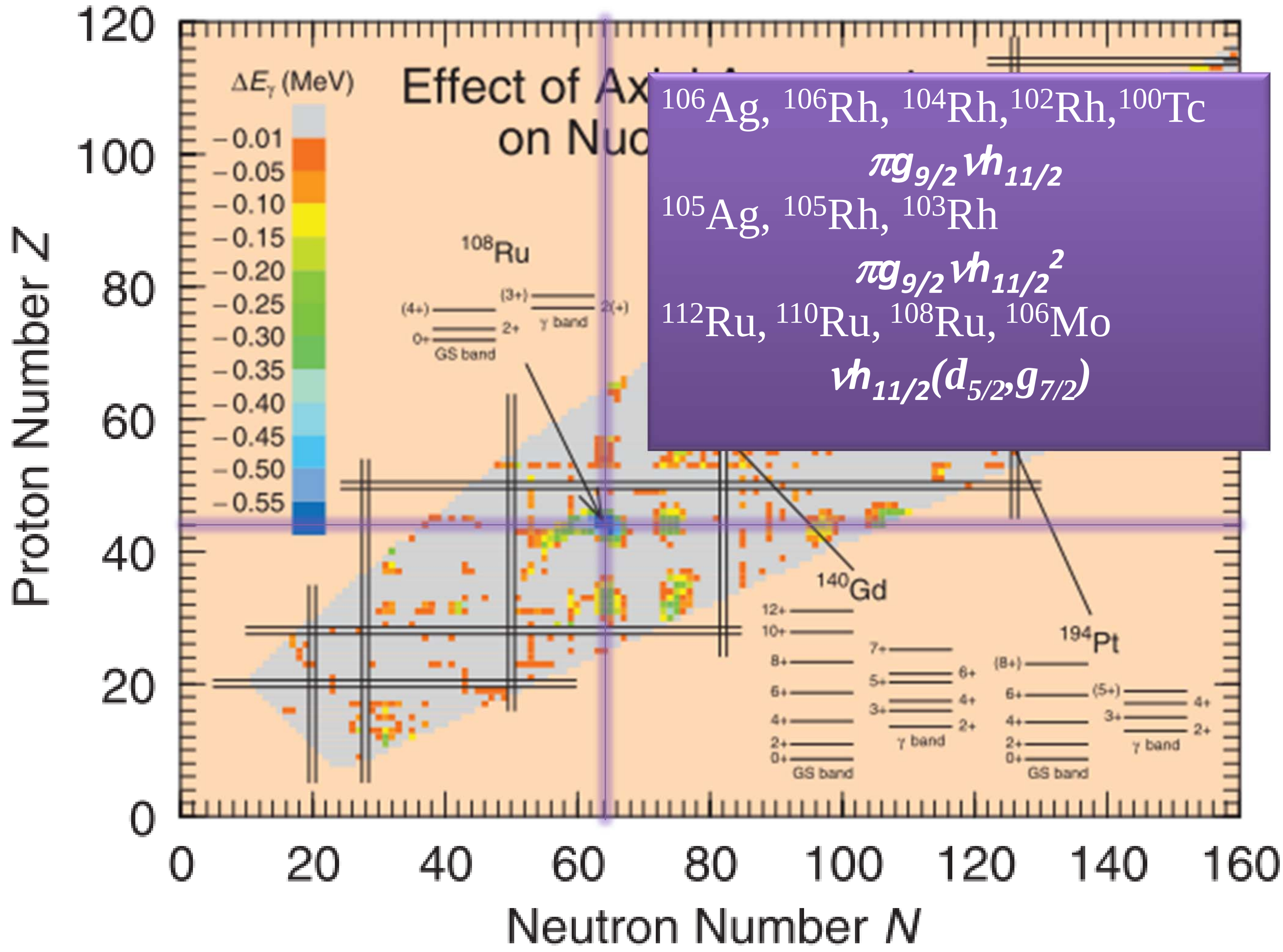

Atomic Data and Nuclear Data Tables 98 (2012) 149-300
^{130}Xe
Spheroidal Deformation ε_2
Axial Asymmetry γ
0.00 0.10 0.20 0.30 0.40
0 20 40 60


Atomic Data and Nuclear Data Tables 98 (2012) 149-300
^{132}Xe
Spheroidal Deformation ε_2
Axial Asymmetry γ
0.00 0.10 0.20 0.30 0.40
0 20 40 60


Atomic Data and Nuclear Data Tables 98 (2012) 149-300
^{134}Xe
Spheroidal Deformation ε_2
Axial Asymmetry γ
0.00 0.10 0.20 0.30 0.40
0 20 40 60



Atomic Data and Nuclear Data Tables 98 (2012) 149-300
124Xe
126Xe
128Xe
Spheroidal Deformation ε2
S(I)
I
124Xe
126Xe
128Xe
E(2_2^+)/E(2_1^+)
A



^{106}Ag, ^{106}Rh, ^{104}Rh, ^{102}Rh, ^{100}Tc

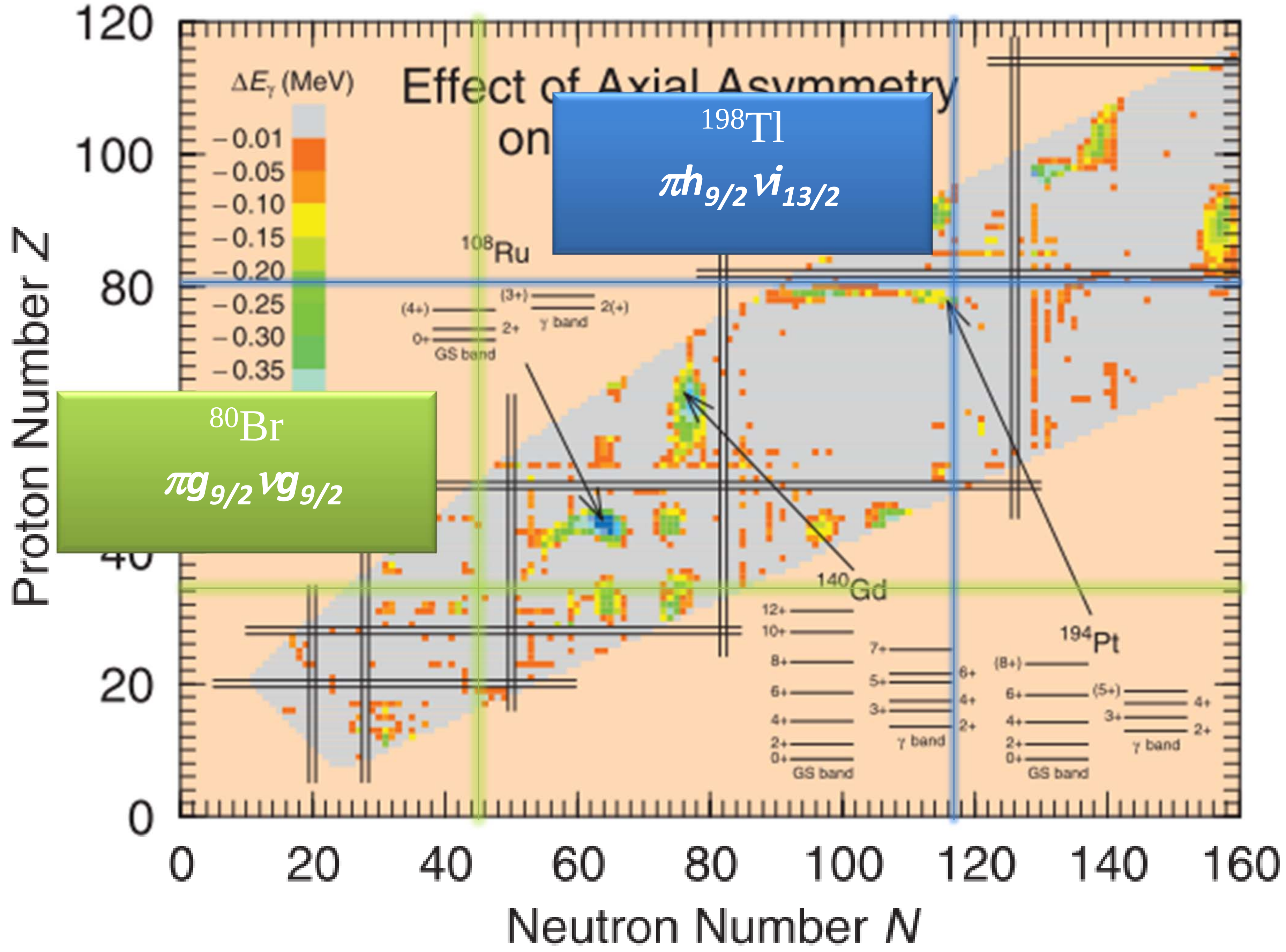
$\pi g_{9/2} \nu h_{11/2}$

^{105}Ag, ^{105}Rh, ^{103}Rh

$\pi g_{9/2} \nu h_{11/2}^{2}$

^{112}Ru, ^{110}Ru, ^{108}Ru, ^{106}Mo

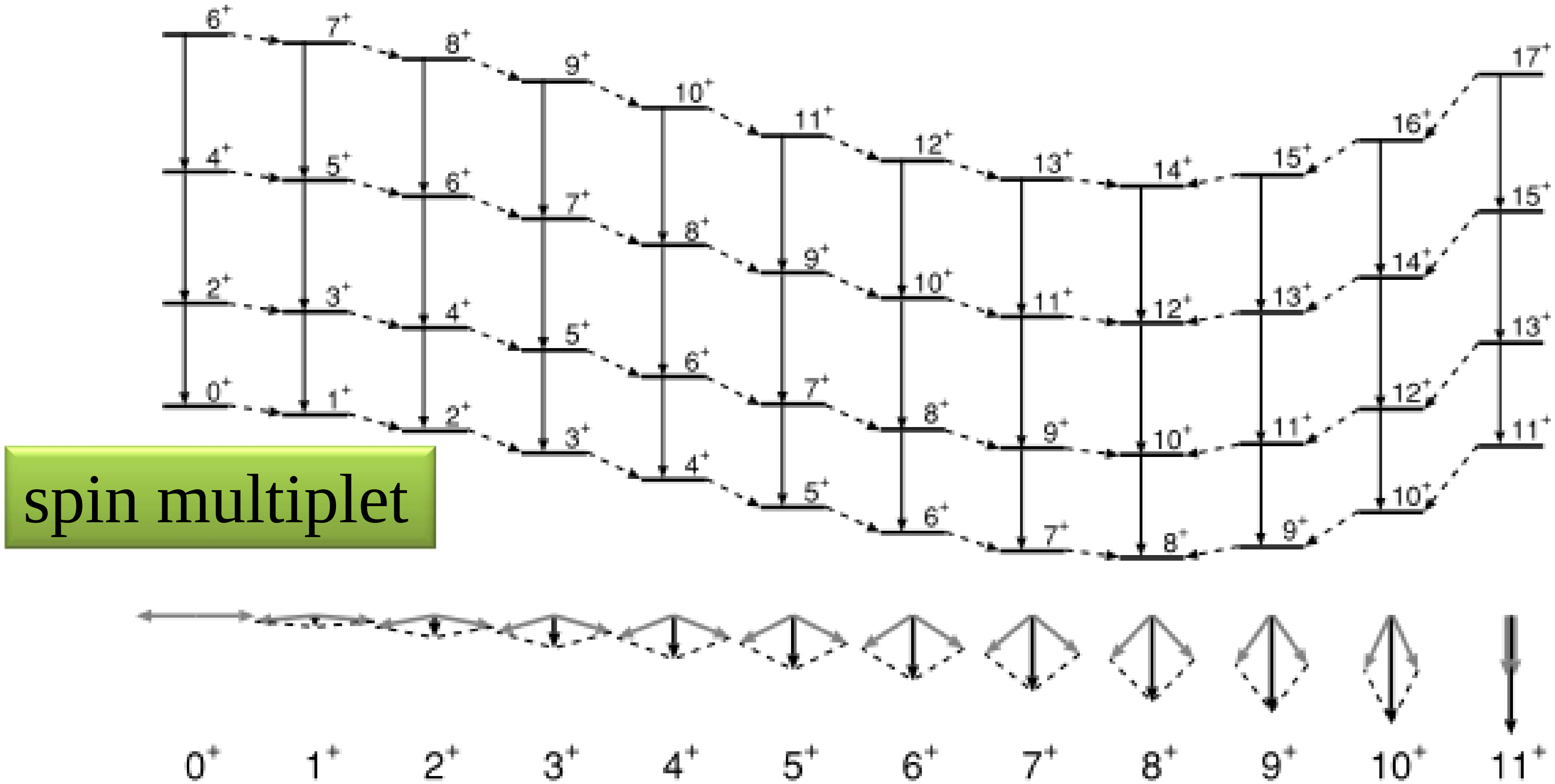
$\nu h_{11/2}(d_{5/2}, g_{7/2})$



Effect of Axial Asymmetry on
^{198}Tl
$\pi h_{9/2} \nu i_{13/2}$
^{80}Br
$\pi g_{9/2} \nu g_{9/2}$
ΔE$_\gamma$ (MeV)
−0.01
−0.05
−0.10
−0.15
−0.20
−0.25
−0.30
−0.35
^{108}Ru
^{140}Gd
^{194}Pt
GS band
γ band
Proton Number Z
Neutron Number N

Weak coupling limit: chopstick mode

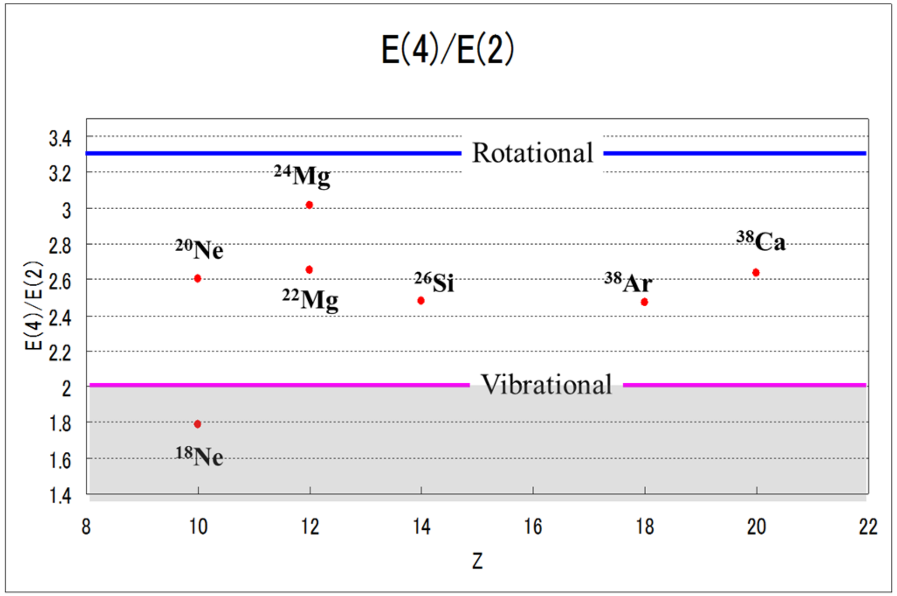
K. Higashiyama et al., PRC72, 024315 (2005)

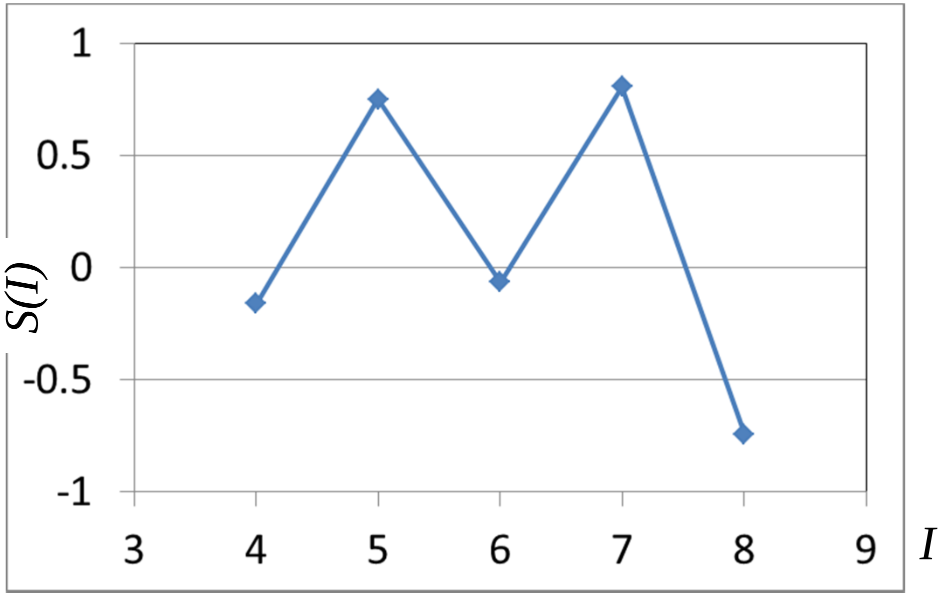


Remarks

- Coupling of single particle and collective rotation seems to enhance salient features of triaxial mass distribution, especially of shape driving unique parity high-j intruder orbitals ($h_{11/2}$,$i_{13/2}$)
- Regardless of interpretation, level degeneracy is obtained by several theoretical calculations for an ideal case of chiral geometry: almost model independent
- The systematic observation of ΔI=I doublet bands in different mass region and configurations needs to be clarified
  - Strong coupling limit interpretation: chiral geometry
  - Weak coupling limit interpretation: chopstic mode

Trixailaity in *sd*-shell: ^{24}Mg

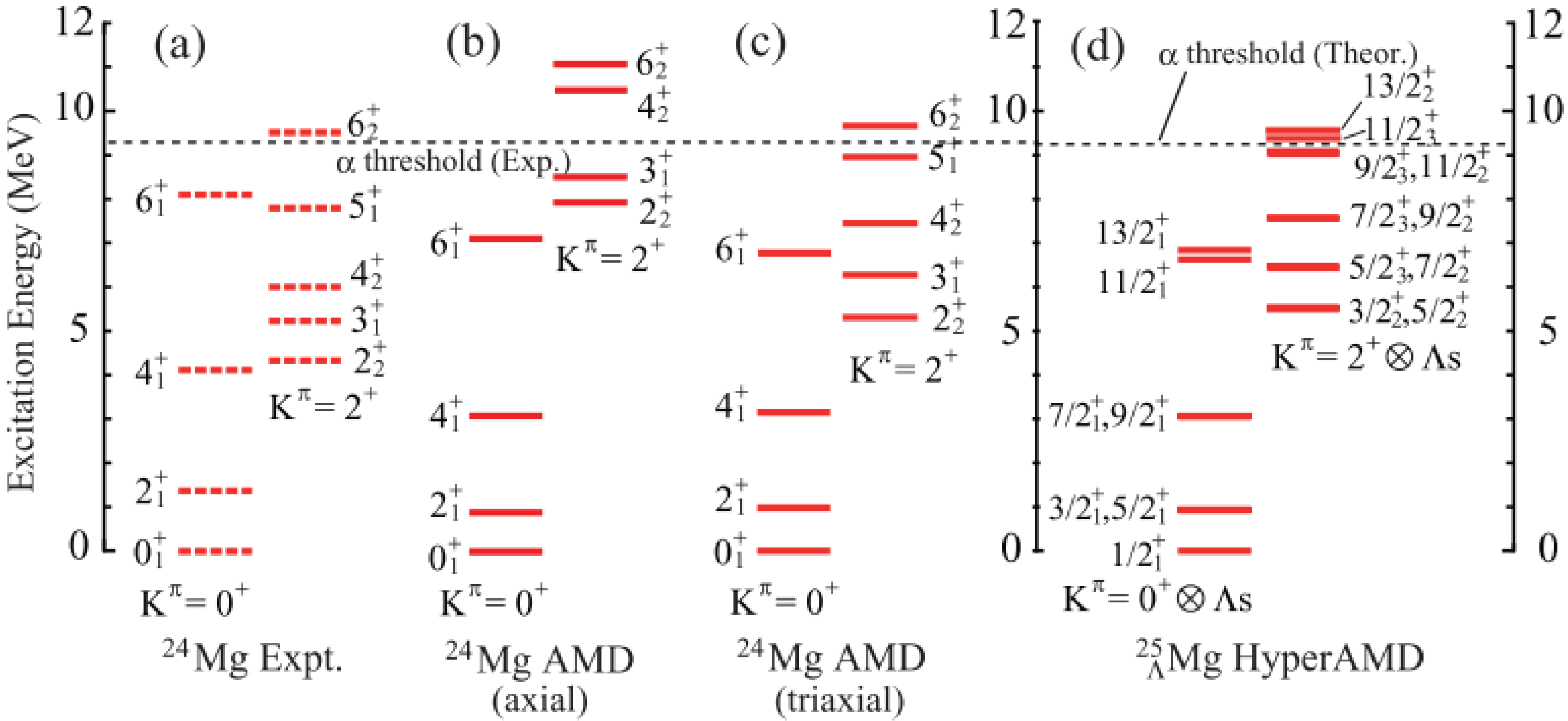




$E(2_2^+)/E(2_1^+)=3.0$

$\rightarrow \gamma=22^\circ$

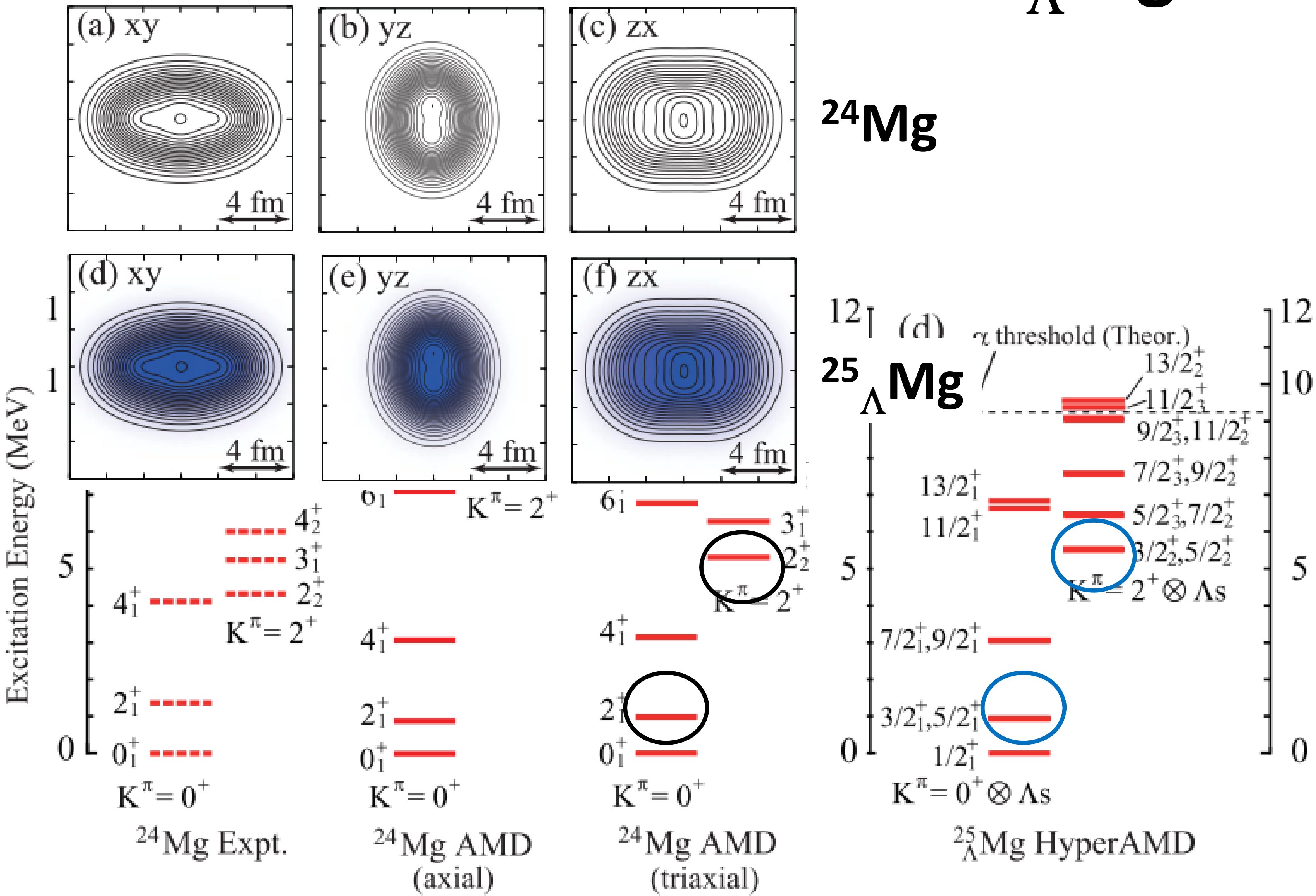
HyperAMD calculation: $^{25}{}_{\Lambda}$Mg



M.Isaka *et al*., Phys. Rev. C85,034303 (2012)

Intrinsic w.f. density distributions

...tion: $^{25}_{\Lambda}$Mg



M.Isaka *et al.*, Phys. Rev. C85,034303 (2012)

Summary

- Global calculation of the ground state indicates triaxial deformation in the ground states is NOT exotic
- Experimental verification of *static* deformation has yet to be established
  - Recent evidence: ^{76}Ge
- Exotic rotational modes involving triaxial mass distribution
  - Wobbling : strongest evidence
  - Smooth band termination: smooth transition of prolate to oblate shape via triaxial deformation
  - Signature inversion: deviation from axially symmetric limit
  - Chiral doublets: experimental verification debated, but the newest addition to triaxial rotation (fully 3D rotation); most exotic in terms of spontaneous symmetry breaking
- Examination of ^{24}Mg using Λ-hyperron as a probe at J-PARC