Pseudospin symmetry in single particle resonant states

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iTHES mini-workshop

Exploration of hidden symmetries in atomic nuclei

Jul 27, 2013, RIKEN, Japan

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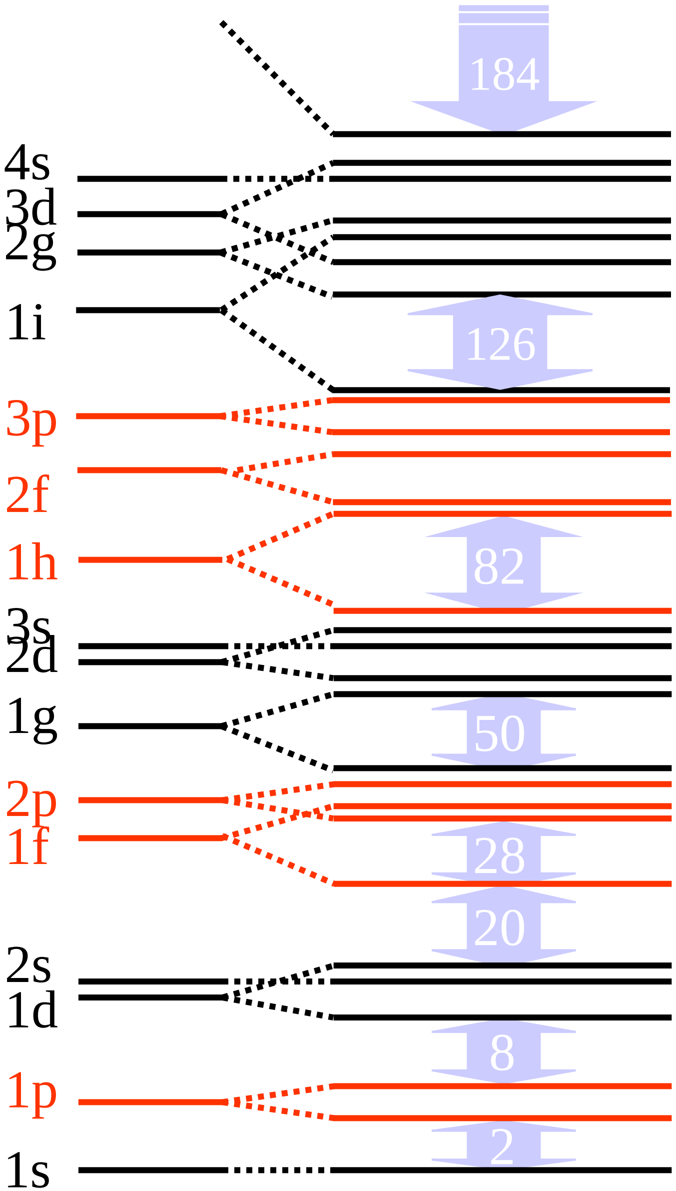
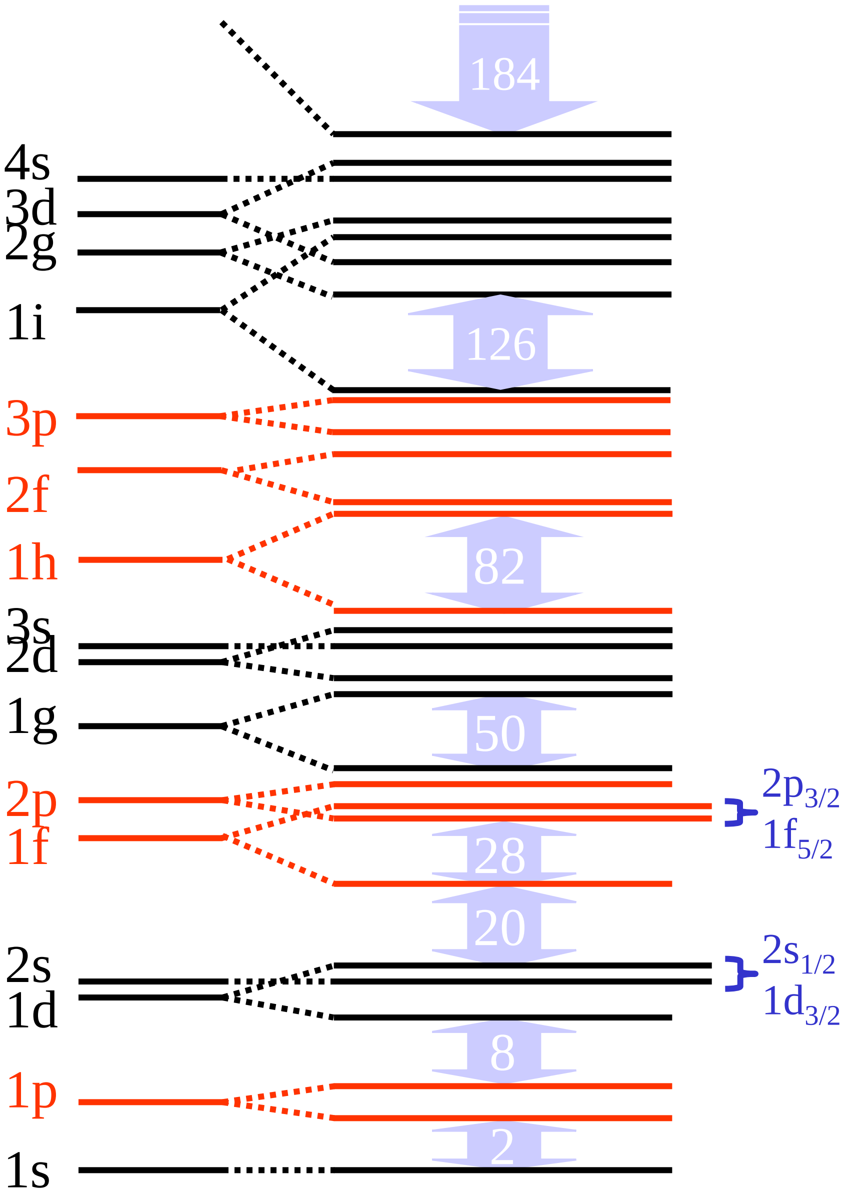
Nuclear single particle level scheme



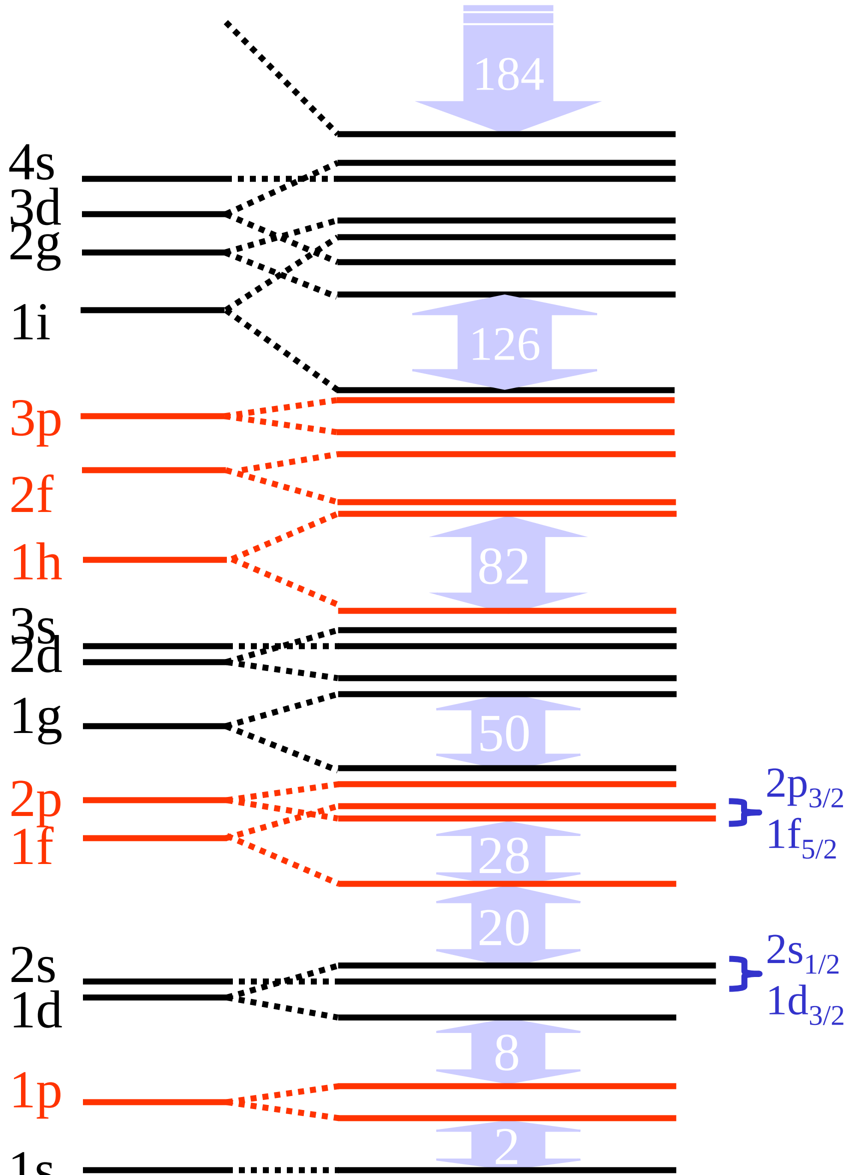
- Large spin-orbit splitting

Nuclear single particle level scheme



- Large spin-orbit splitting
- Near degeneracy of some doublets

Nuclear single particle level scheme



- Large spin-orbit splitting
- Near degeneracy of some doublets
- Pseudo-orbital a.m. & spin

$(n+1, l, \quad j = l+1/2)$

$(n \quad , l+2, j = l+3/2)$

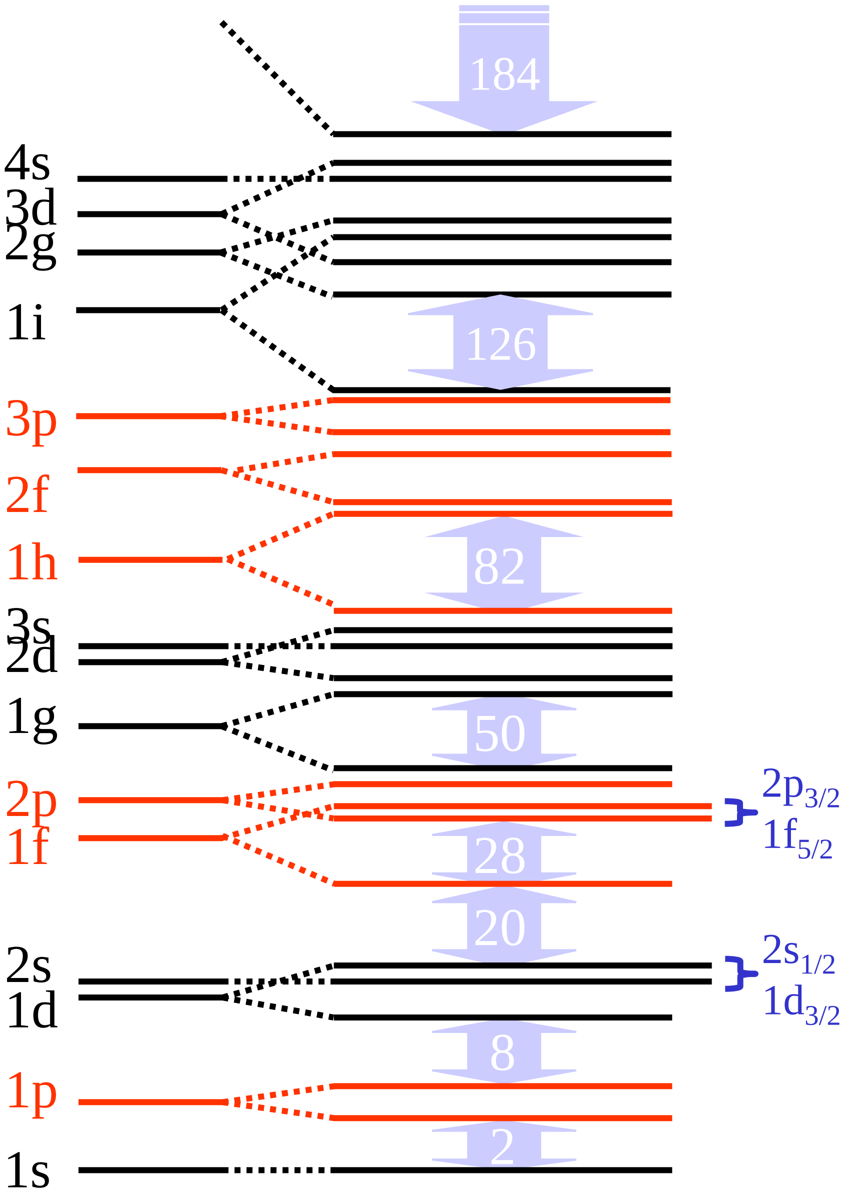
$$\tilde{l} = l + 1$$

$$\tilde{s} = 1/2$$

Arima_Harvey_Shimizu 1969_PLB30-517

Hecht_Adler 1969_NPA137-129

Nuclear single particle level scheme



- Large spin-orbit splitting
- Near degeneracy of some doublets
- Pseudo-orbital a.m. & spin
- Accidental or of symmetry, i.e., pseudospin symmetry (PSS)?

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Arima_Harvey_Shimizu 1969_PLB30-517
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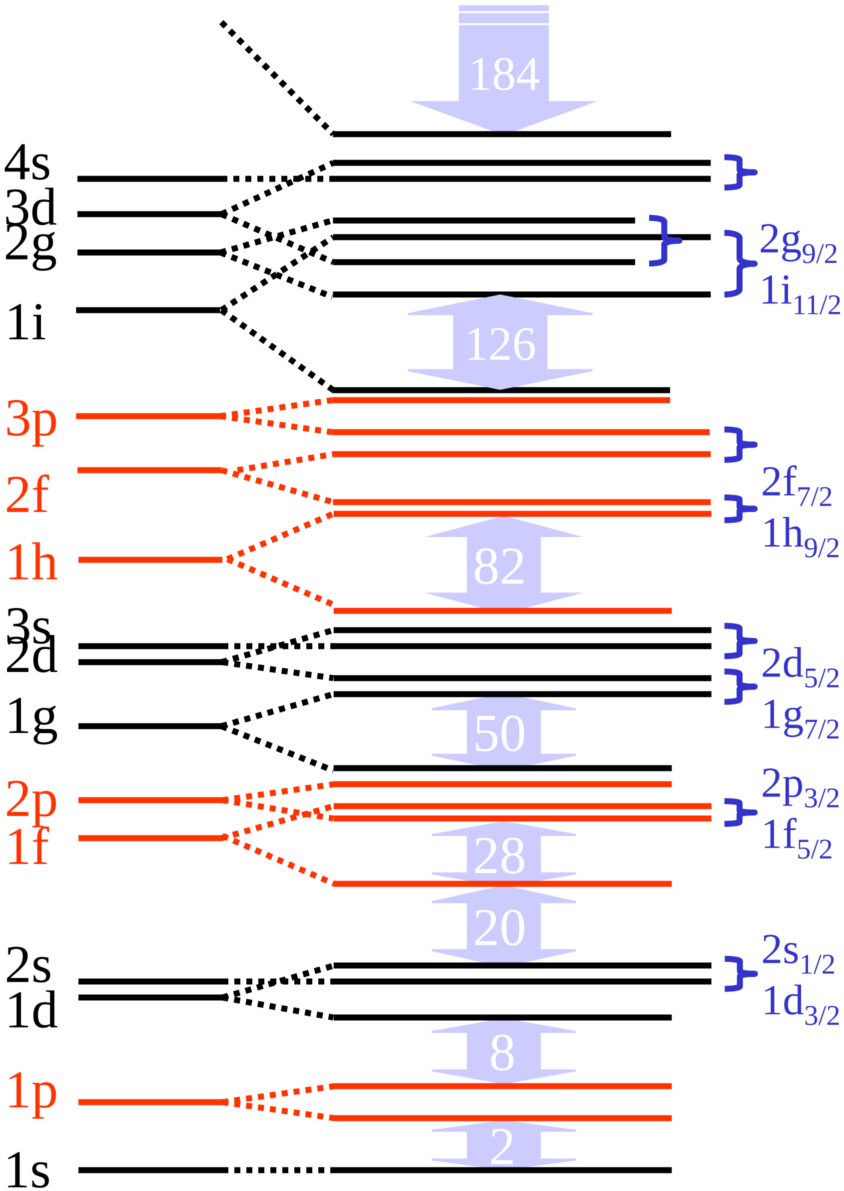

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Arima_Harvey_Shimizu 1969_PLB30-517
Hecht_Adler 1969_NPA137-129

Relativistic mean field (RMF) model

$$
\begin{aligned}
\mathcal{L} \;=\; & \bar{\psi}\left(i\partial\!\!\!/ - M\right)\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma) - g_\sigma\bar{\psi}\sigma\psi \\
& - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - g_\omega\bar{\psi}\omega\!\!\!/\psi \\
& - \frac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\vec{\rho}^\mu - g_\rho\bar{\psi}\vec{\rho}\!\!\!/\vec{\tau}\psi \\
& - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - e\bar{\psi}\frac{1-\tau_3}{2}A\!\!\!/\psi
\end{aligned}
$$

Serot_Walecka1986_ANP16-1
Reinhard1989_RPP52-439
Ring1996_PPNP37-193
Vretenar_Afanasjev_Lalazissis_Ring2005_PR409-101
Meng_Toki_Zhou_Zhang_Long_Geng2006_PPNP57-470

For nucleons: $\boldsymbol{\alpha}\cdot\boldsymbol{p} + V(\boldsymbol{r}) + \beta(M + S(\boldsymbol{r}))\psi(\boldsymbol{r}) = \epsilon\psi(\boldsymbol{r})$

Scalar & vector potentials:

$$
\begin{cases}
S(\boldsymbol{r}) = g_\sigma\sigma(\boldsymbol{r}) \\
V(\boldsymbol{r}) = g_\omega\omega^0(\boldsymbol{r}) + g_\rho\tau_3\rho^0(\boldsymbol{r}) + e\dfrac{1-\tau_3}{2}A^0(\boldsymbol{r})
\end{cases}
$$

Pseudo quantum numbers & lower component

Dirac spinor: $\psi(\boldsymbol{r}) = \frac{1}{r}\left(\begin{array}{c} iF_{n\kappa}(r)Y^{l}_{jm}(\theta,\phi) \\ -G_{\tilde{n}\kappa}(r)Y^{\tilde{l}}_{jm}(\theta,\phi) \end{array}\right)$

Relations of quantum numbers: $\left\{\begin{array}{l} j = l \pm 1/2 \\ \kappa = (-1)^{j+l+1/2}(j+1/2) \\ \tilde{l} = l - \mathrm{sign}(\kappa) \end{array}\right.$

Pseudo quantum numbers & lower component

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$$\begin{cases} j = l \pm 1/2 \\ \kappa = (-1)^{j+l+1/2}(j+1/2) \\ \tilde{l} = l - \mathrm{sign}(\kappa) \end{cases}$$

$$2s_{1/2}: \begin{pmatrix} n=2, l=0, j=l+1/2 \\ \tilde{n}=2, \tilde{l}=1, j=\tilde{l}-1/2 \end{pmatrix} \qquad 1d_{3/2}: \begin{pmatrix} n=1, l=2, j=l-1/2 \\ \tilde{n}=2, \tilde{l}=1, j=\tilde{l}+1/2 \end{pmatrix}$$

$$(2s_{1/2}, 1d_{3/2}) \Rightarrow (2\tilde{p}_{1/2,3/2})$$

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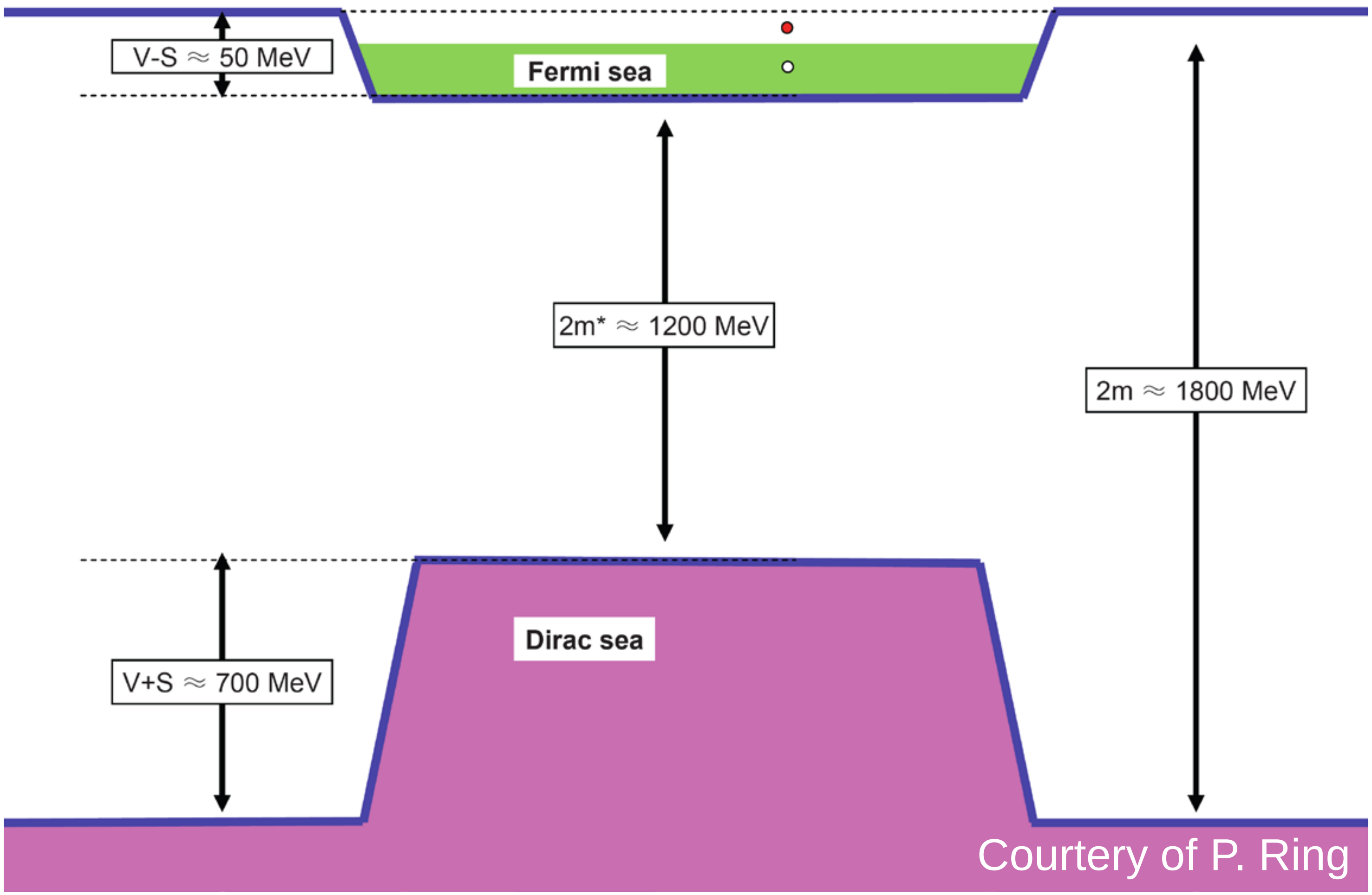
$$(2s_{1/2}, 1d_{3/2}) \Rightarrow (2\tilde{p}_{1/2,3/2})$$

Pseudo quantum numbers are nothing but quantum numbers of the lower component.

Ginocchio1997_PRL78-436

Nuclear potentials in relativistic models

- The scalar & vector potentials are both very big in amplitude, but with opposite sign: $S(r) < 0$ & $V(r) > 0$
- This results in a shallow Fermi sea & a deep Dirac sea, the latter is responsible for the large spin-orbit coupling



Origin of pseudospin symmetry

Dirac equation: $\boldsymbol{\alpha}\cdot\boldsymbol{p} + V(\boldsymbol{r}) + \beta(M + S(\boldsymbol{r}))\psi(\boldsymbol{r}) = \epsilon\psi(\boldsymbol{r})$

For spherical nuclei:
$$\begin{pmatrix} M+\Sigma(r) & -\dfrac{d}{dr}+\dfrac{\kappa}{r} \\ \dfrac{d}{dr}+\dfrac{\kappa}{r} & -M+\Delta(r) \end{pmatrix} \begin{pmatrix} F(r) \\ G(r) \end{pmatrix} = \epsilon \begin{pmatrix} F(r) \\ G(r) \end{pmatrix}$$

$$\Delta(r) \equiv V(r) - S(r),\ \ \Sigma(r) \equiv V(r) + S(r)$$

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$$\Delta(r)\equiv V(r)-S(r),\ \ \Sigma(r)\equiv V(r)+S(r)$$

For the lower component:

$$\left[\frac{d^2}{dr^2}-\frac{1}{M_-(r)}\frac{d\Sigma(r)}{dr}\frac{d}{dr}-\frac{\tilde{l}(\tilde{l}+1)}{r^2}+\frac{1}{M_-(r)}\frac{\kappa}{r}\frac{d\Sigma(r)}{dr}-M_+(r)M_-(r)\right]G(r)=0$$

$$M_+(r)\equiv M+\epsilon-\Delta(r),\ \ M_-(r)\equiv M-\epsilon+\Sigma(r)$$

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For the lower component (exact):

Zhang_Liang_Meng2009_ChinPhysLett26-092401
Zhang_Liang_Meng2010_IJMPE19-55

$$\left[\frac{d^2}{dr^2}-\frac{1}{M_-(r)}\frac{d\Sigma(r)}{dr}\frac{d}{dr}-\frac{\tilde{l}(\tilde{l}+1)}{r^2}+\frac{1}{M_-(r)}\frac{\kappa}{r}\frac{d\Sigma(r)}{dr}-M_+(r)M_-(r)\right]G(r)=0$$
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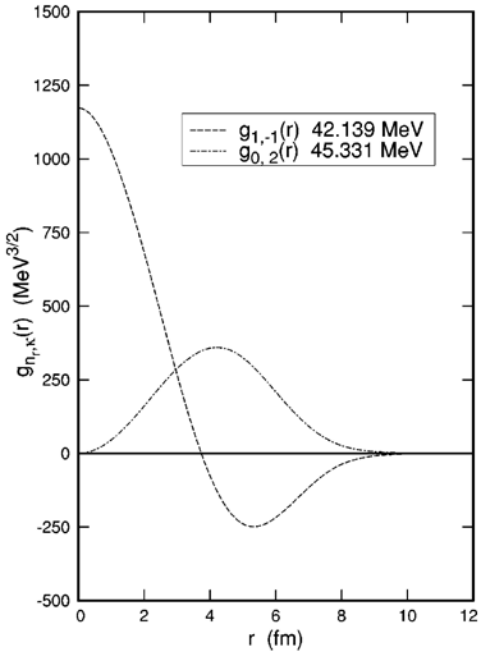
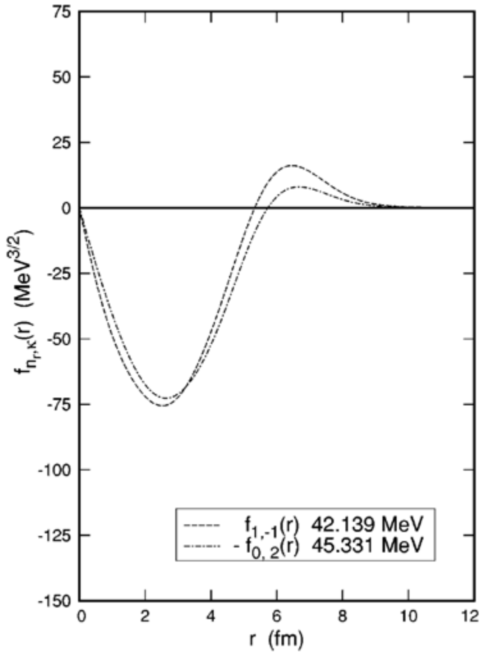
If $\Sigma(r)=0$ or $d\Sigma(r)/dr=0$, the pseudospin symmetry is conserved exactly, i.e., PSS doublets with the same pseudo quantum numbers are degenerate

Ginocchio1997_PRL78-436
Meng_Sugawara-Tanabe_Yamaji_Ring_Arima1998_PRC58-628R

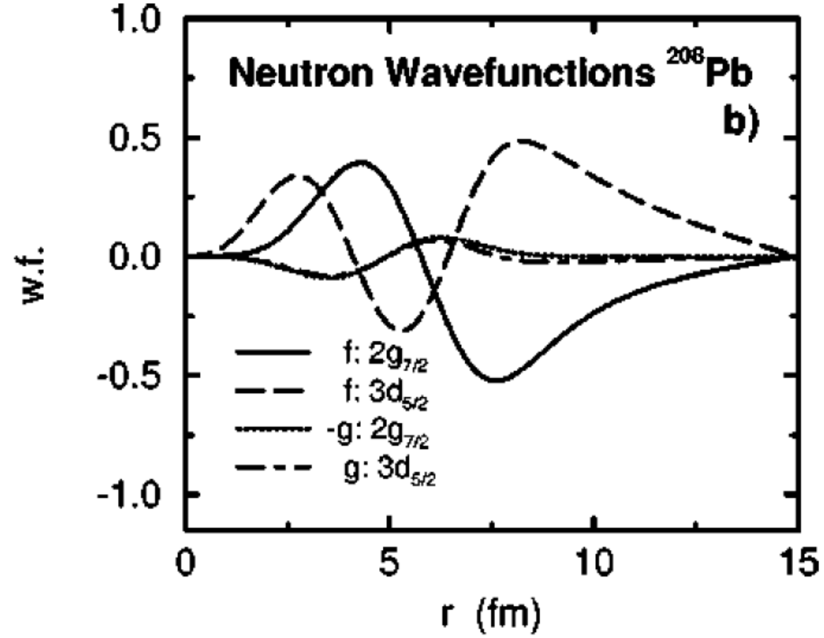
Same (similar) wave functions for PSS doublets

$$\left[\frac{d^2}{dr^2} - \frac{1}{M_-(r)}\frac{d\Sigma(r)}{dr}\frac{d}{dr} - \frac{\tilde{l}(\tilde{l}+1)}{r^2} + \frac{1}{M_-(r)}\frac{\kappa}{r}\frac{d\Sigma(r)}{dr} - M_+(r)M_-(r)\right]G(r) = 0$$





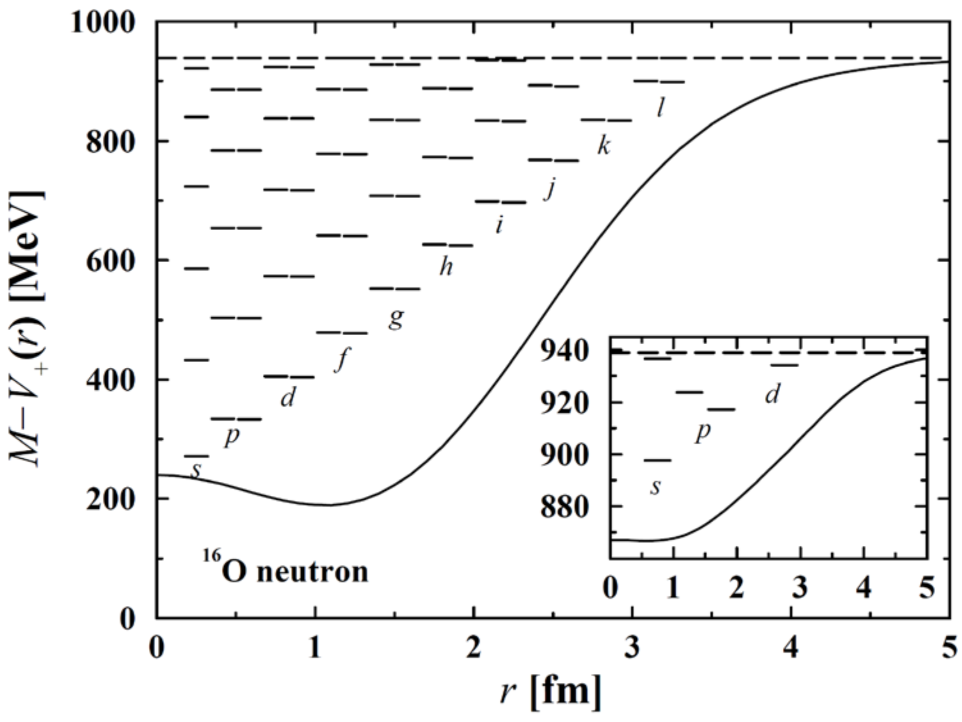
Ginocchio_Madland1998_PRC57-1167



Lalazissis_Gambhir_Maharana_Warke_Ring 1998_PRC58-45R

If $\Sigma(r) = 0$ or $d\Sigma(r)/dr = 0$, lower components of wave functions of PSS doublets should be the same

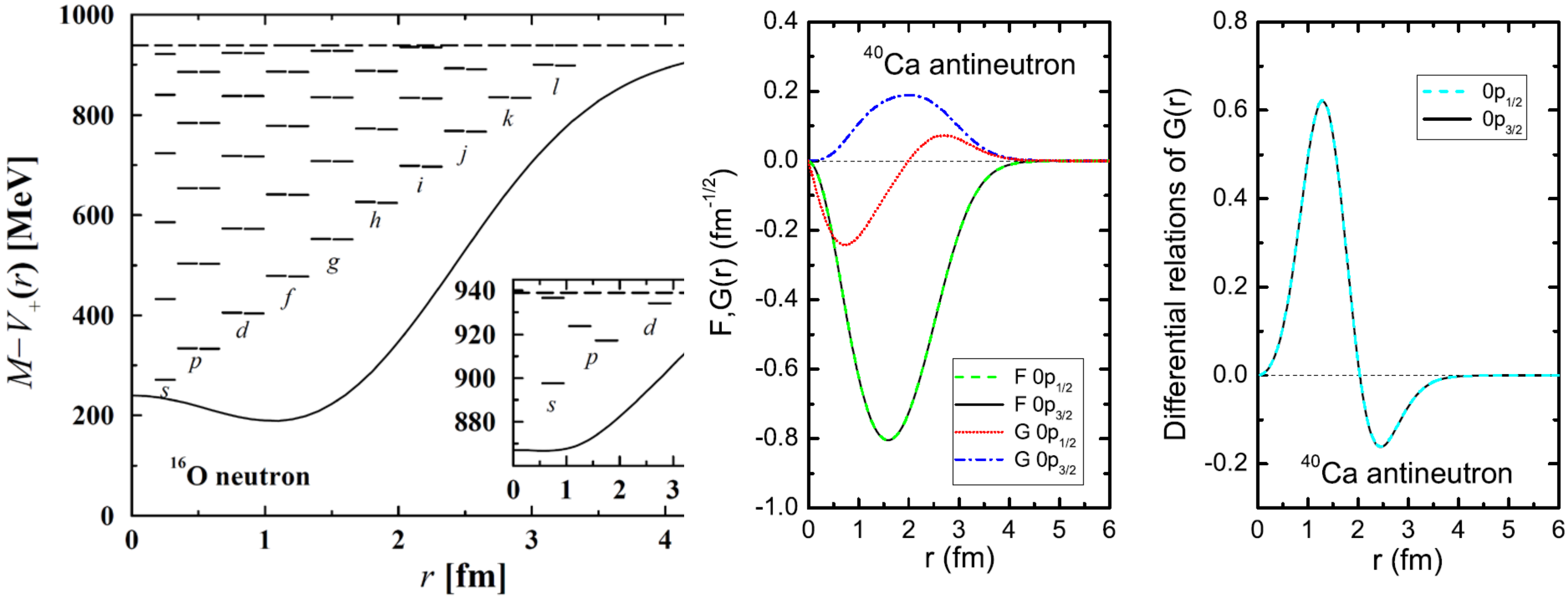
Spin symmetry in anti-particle states



- If $\Sigma(r) = 0$ or $d\Sigma(r)/dr = 0$, the spin symmetry in anti-nucleon spectra is exactly conserved
- In realistic nuclei, SS in anti-nucleon spectra is much better developed than that of PSS in single nucleon spectra

Zhou_Meng_Ring2003_PRL91-262501

Spin symmetry in anti-particle states



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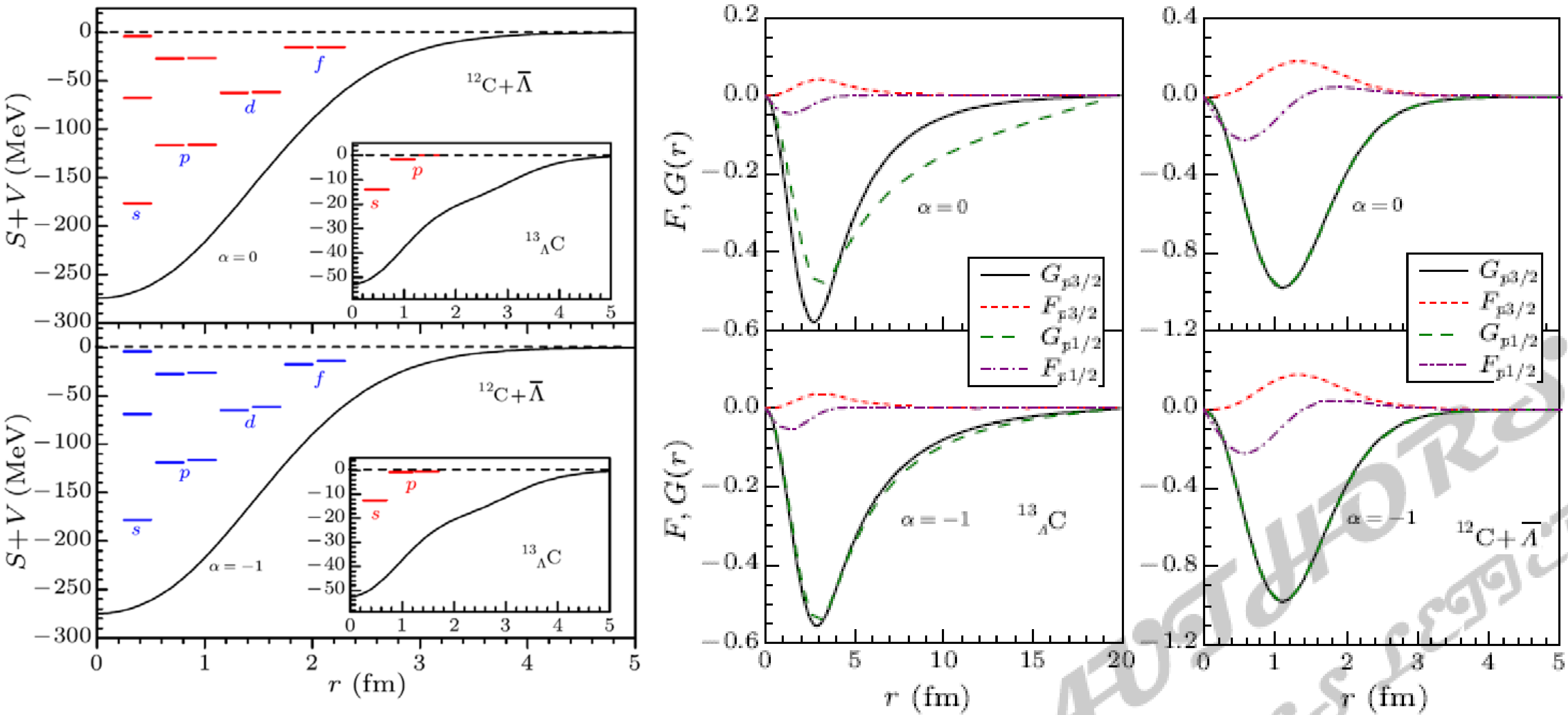
Zhou_Meng_Ring2003_PRL91-262501
He_Zhou_Meng_Zhao_Scheid2006_EPJA28-265

SS in anti-Λ spectrum of hypernuclei

CHIN. PHYS. LETT. Vol. 28, No. 9 (2011) 092101

Tensor Coupling Effects on Spin Symmetry in the Anti-Lambda Spectrum of Hypernuclei *

SONG Chun-Yan(宋春艳)[1], YAO Jiang-Ming(尧江明)[2,3], MENG Jie(孟杰)[4,3,5]**



Song_Yao_Meng2003_ChinPhysLett28-092101

Several reviews on PSS & SS

Available online at www.sciencedirect.com
SCIENCE DIRECT
Physics Reports 414 (2005) 165–261
PHYSICS REPORTS
www.elsevier.com/locate/physrep

Relativistic symmetries in nuclei and hadrons

Joseph N. Ginocchio
MS B238, Los Alamos National Laboratory, Los Alamos, NM, 87545, USA

Accepted 11 April 2005
Available online 9 June 2005
editor: G.E. Brown

Nuclear Theory & Astrophysical Applications 2005

Pseudo-spin symmetry in nuclei

R.V.Jolos
Joint Institute for Nuclear Research
Dubna

PHYSICAL REVIEW C **87**, 014334 (2013)

Pseudospin symmetry in supersymmetric quantum mechanics: Schrödinger equations

Haozhao Liang (梁豪兆),[1] Shihang Shen (申时行),[1] Pengwei Zhao (赵鹏巍),[1] and Jie Meng (孟杰)[1,2,3,*]
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(Received 26 July 2012; revised manuscript received 25 December 2012; published 24 January 2013)

Pseudospin symmetry in single particle resonances in spherical square wells

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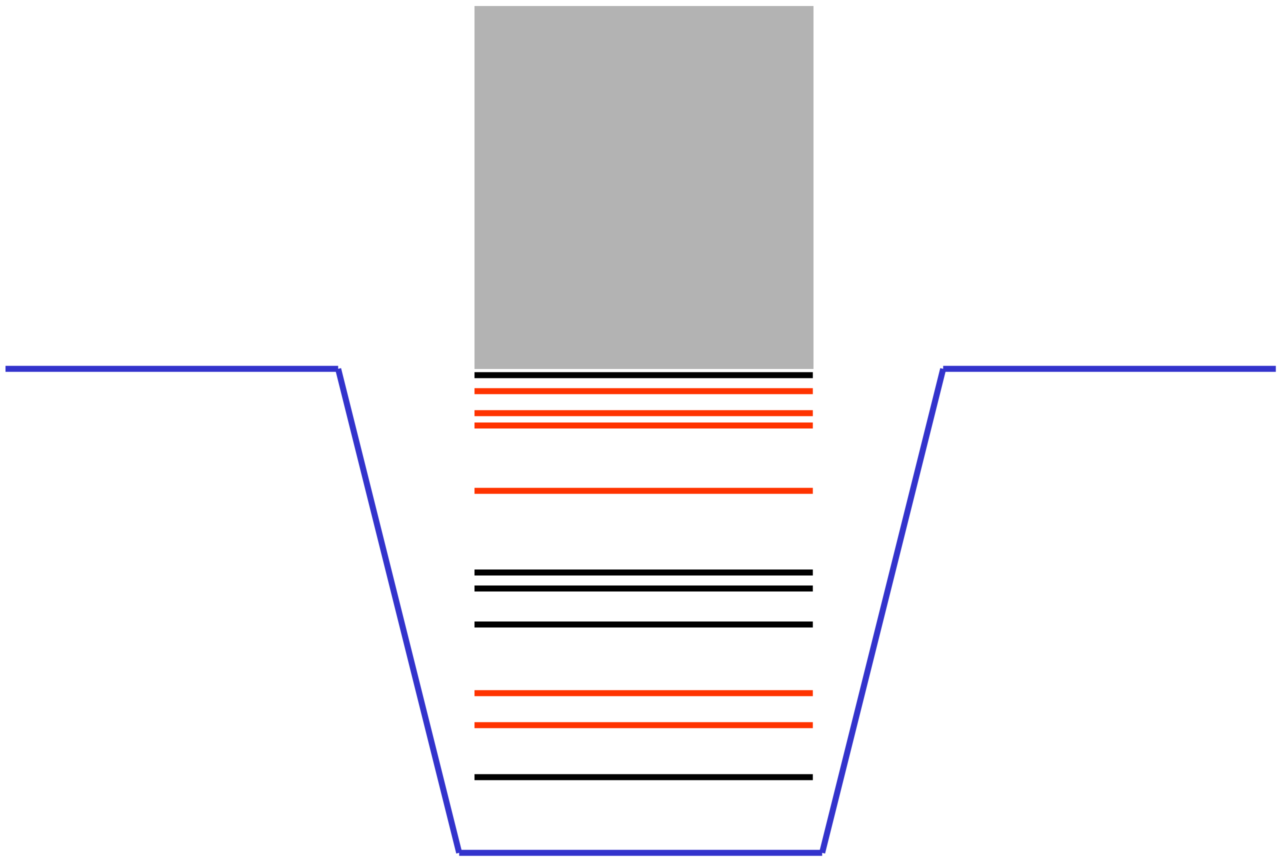
[2] *Center of Theoretical Nuclear Physics, National Laboratory of Heavy Ion A*
(Dated: May 8, 2013)
arXiv:1305.1524v1 [nucl-th]

How about single particle resonant states?



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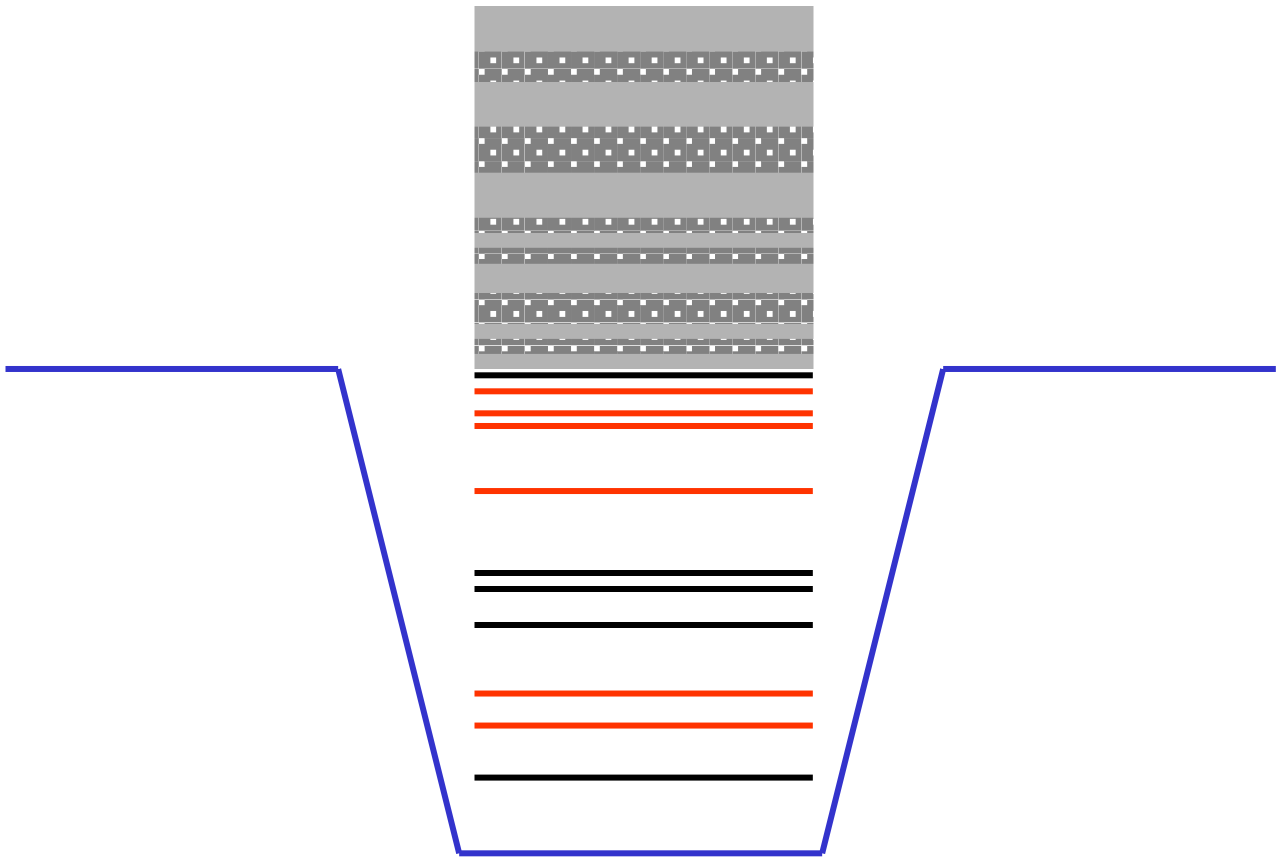

Is PSS a good symmetry also for resonances?
If yes, how about width?

PSS in s.p. resonant states: Some numerical results

PHYSICAL REVIEW C **72**, 054319 (2005)

Pseudospin symmetry in the resonant states of nuclei

Jian-You Guo,* Ruo-Dong Wang, and Xiang-Zheng Fang
School of Physics and Material Science, Anhui University, Hefei 230039, China
(Received 27 August 2005; published 30 November 2005)

PHYSICAL REVIEW C **74**, 024320 (2006)

Isospin dependence of pseudospin symmetry in nuclear resonant states

Jian You Guo* and Xiang Zheng Fang
School of Physics and Material Science, Anhui University, Hefei 230039, People's Republic of China
August 2006)

第 30 卷 增刊 II 高 能 物 理 与 核 物 理 Vol. 30, Supp. II
2006 年 12 月 HIGH ENERGY PHYSICS AND NUCLEAR PHYSICS Dec., 2006

Does Pseudo-Spin Symmetry Exist in the Continuum?*

ZHANG Shi-Sheng[1;1)] ZHANG Wei[2] SUN Bao-Hua[3] GUO Jian-You[4] ZHOU Shan-Gui[5,6]

CHIN.PHYS.LETT. Vol. 24, No. 5 (2007) 1199

Exploration of Pseudospin Symmetry in the Resonant States *

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[1]*Department of Physics, School of Science, Beihang University, Beijing 100083*
[2]*School of Physics, Peking University, Beijing 100871*
[3]*Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100080*
[4]*Center of Theoretical Nuclear Physics, National Laboratory of Hea*

(Received 8 January 2007)

Guo_Wang_Fang2005_PRC72-054319
Guo_Fang2006_PRC74-024320
Zhang_Zhang_Sun_Guo_Zhou2006_HEPNP30S2-97
Zhang_Sun_Zhou2007_ChinPhysLett24-1199

PSS in s.p. resonant states

$$\left[\frac{d^2}{dr^2} - \frac{1}{M_-(r)}\frac{d\Sigma(r)}{dr}\frac{d}{dr} - \frac{\tilde{l}(\tilde{l}+1)}{r^2} + \frac{1}{M_-(r)}\frac{\kappa}{r}\frac{d\Sigma(r)}{dr} - M_+(r)M_-(r)\right]G(r) = 0$$

$$M_+(r) \equiv M + \epsilon - \Delta(r),\ \ M_-(r) \equiv M - \epsilon + \Sigma(r)$$

$$\varepsilon \equiv \epsilon - M$$

- For bound states, $\varepsilon < 0$, one examines eigen energies

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- For bound states, $\varepsilon < 0$, one examines eigen energies
- For continuum states, $\varepsilon > 0$, one examines eigen functions – the asymptotic behavior of wave functions & phase shifts

PSS in s.p. resonant states: Jost function

$$\left[\frac{d^2}{dr^2} - \frac{1}{M_-(r)}\frac{d\Sigma(r)}{dr}\frac{d}{dr} - \frac{\tilde{l}(\tilde{l}+1)}{r^2} + \frac{1}{M_-(r)}\frac{\kappa}{r}\frac{d\Sigma(r)}{dr} - M_+(r)M_-(r)\right]G(r) = 0$$

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- For bound states, ε < 0, one examines eigen energies
- For continuum states, ε > 0, one examines eigen functions – the asymptotic behavior of wave functions & phase shifts

It is more convenient to study the Jost function !

$$G(r) = j_{\tilde{l}}(pr),\ r \to 0$$

$$G(r) = \frac{i}{2}\left[\mathcal{J}_\kappa^G(p)h_{\tilde{l}}^-(pr) - \mathcal{J}_\kappa^G(p)^* h_{\tilde{l}}^+(pr)\right],\ r \to \infty$$

$$p = \sqrt{\epsilon^2 - M^2}$$

$h^{\pm}(pr)$: Ricatti-Hankel function

Lu_Zhao_Zhou2012_PRL109-072501

PSS in s.p. resonant states: Jost function

$$\left[\frac{d^2}{dr^2} - \frac{1}{M_-(r)}\frac{d\Sigma(r)}{dr}\frac{d}{dr} - \frac{\tilde{l}(\tilde{l}+1)}{r^2} + \frac{1}{M_-(r)}\frac{\kappa}{r}\frac{d\Sigma(r)}{dr} - M_+(r)M_-(r)\right] G(r) = 0$$

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PSS in s.p. resonant states: Jost function

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$$M_+(r) \equiv M + \epsilon - \Delta(r),\ \ M_-(r) \equiv M - \epsilon + \Sigma(r)$$

$$\varepsilon \equiv \epsilon - M$$

PSS limit

$$\left[\frac{d^2}{dr^2} - \frac{\tilde{l}(\tilde{l}+1)}{r^2} + \varepsilon M_+(r)\right]G(r) = 0$$

$$G(r) = \frac{i}{2}\left[\mathcal{J}_\kappa^G(p)h_{\tilde{l}}^-(pr) - \mathcal{J}_\kappa^G(p)^* h_{\tilde{l}}^+(pr)\right],\ \ r \to \infty$$

$$p = \sqrt{\epsilon^2 - M^2}$$

For PS doublets, one always have $G_\kappa(\epsilon, r) = G_{\kappa'}(\epsilon, r)$

$$\Big\Downarrow\ r \to \infty$$

Examples: $s_{1/2}$ ($\kappa = -1$) & $d_{3/2}$ ($\kappa' = +2$)

$$\mathcal{J}_\kappa(p) = \mathcal{J}_{\kappa'}(p)$$

Lu_Zhao_Zhou2012_PRL109-072501

Two ways to justify PSS in s.p. resonant states

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Two ways to justify PSS in s.p. resonant states

$$G_\kappa(r) \propto \sin\left(pr - \frac{\tilde{l}\pi}{2} + \delta_\kappa^G(p)\right), \; r \to \infty \qquad \mathcal{J}_\kappa^G(p) = |\mathcal{J}_\kappa^G(p)| e^{-i\delta_\kappa^G(p)}$$

- ✓ $\delta(p) = n\pi + \pi/2$ corresponds to a resonant state
- ✓ The width is determined by the tangent of $\delta(p)$

$$\boxed{\delta_\kappa^G(p) = \delta_{\kappa'}^G(p)}$$

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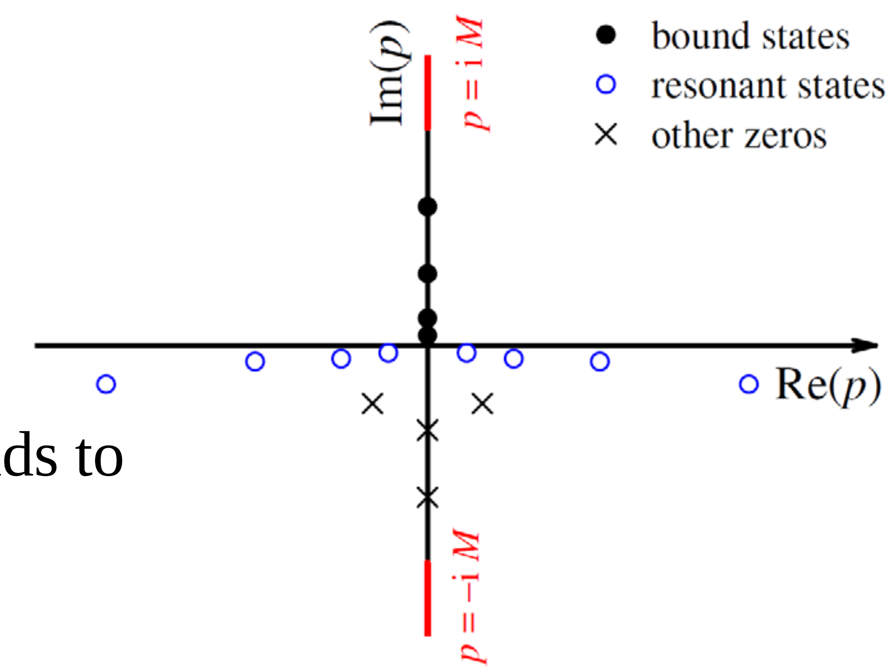
Zeros are the same for $\mathcal{J}_{\kappa'}^G(p)$ and $\mathcal{J}_\kappa^G(p)$



✓ Zeros of the Jost function corresponds to bound states, resonant states, ...

✓ $E = E_{\text{res}} - i\Gamma_{\text{res}}/2 = \sqrt{p^2 + M^2}$

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An example: Square-well potentials

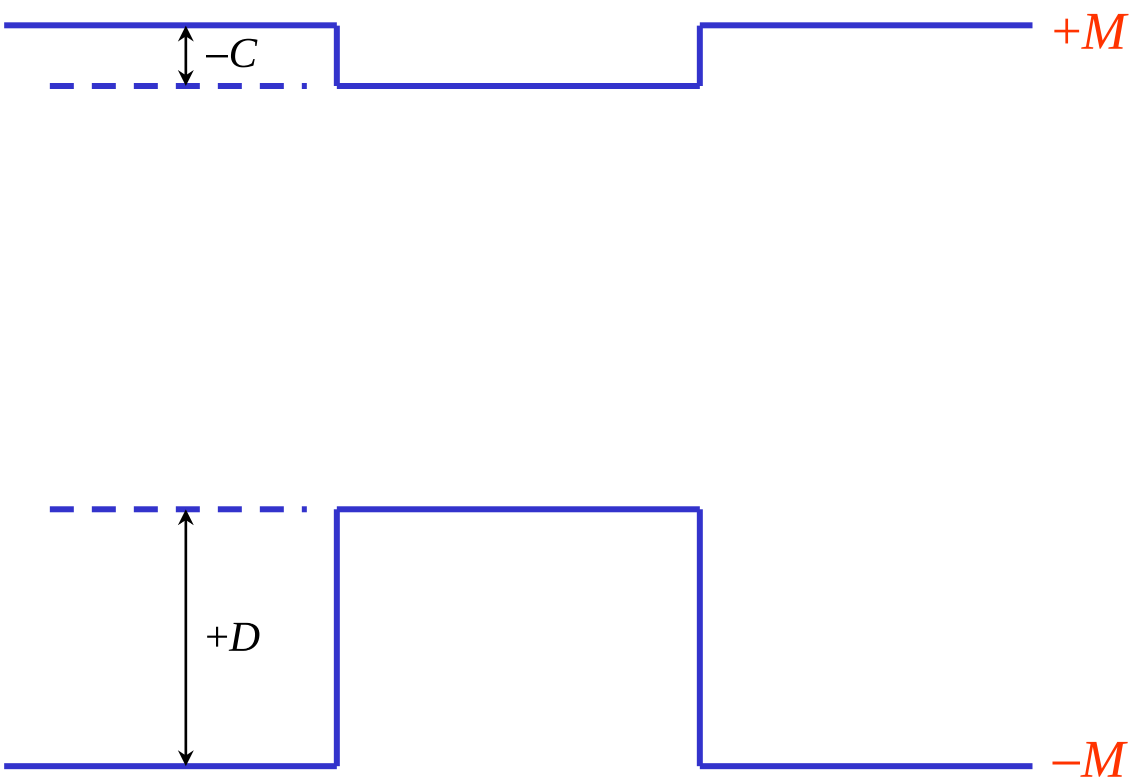
$$\Sigma(r) = \begin{cases} C, & r < R, \\ 0, & r \geq R, \end{cases}$$

$$\Delta(r) = \begin{cases} D, & r < R, \\ 0, & r \geq R. \end{cases}$$

C = 0, –66 MeV

D = 650 MeV

R = 7 fm



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An example: Square-well potentials

$$\mathcal{J}^G_\kappa(p) \quad = \quad -\frac{p^{\tilde{l}}}{2ik^{\tilde{l}+1}}\Big[j_{\tilde{l}}(kR)ph^{+\prime}_{\tilde{l}}(pR) - kj'_{\tilde{l}}(kR)h^+_{\tilde{l}}(pR) \\ -\frac{C}{\epsilon - M - C}\left(kj'_{\tilde{l}}(kR) - \frac{\kappa}{R}j_{\tilde{l}}(kR)\right)h^+_{\tilde{l}}(pR)\Big]$$

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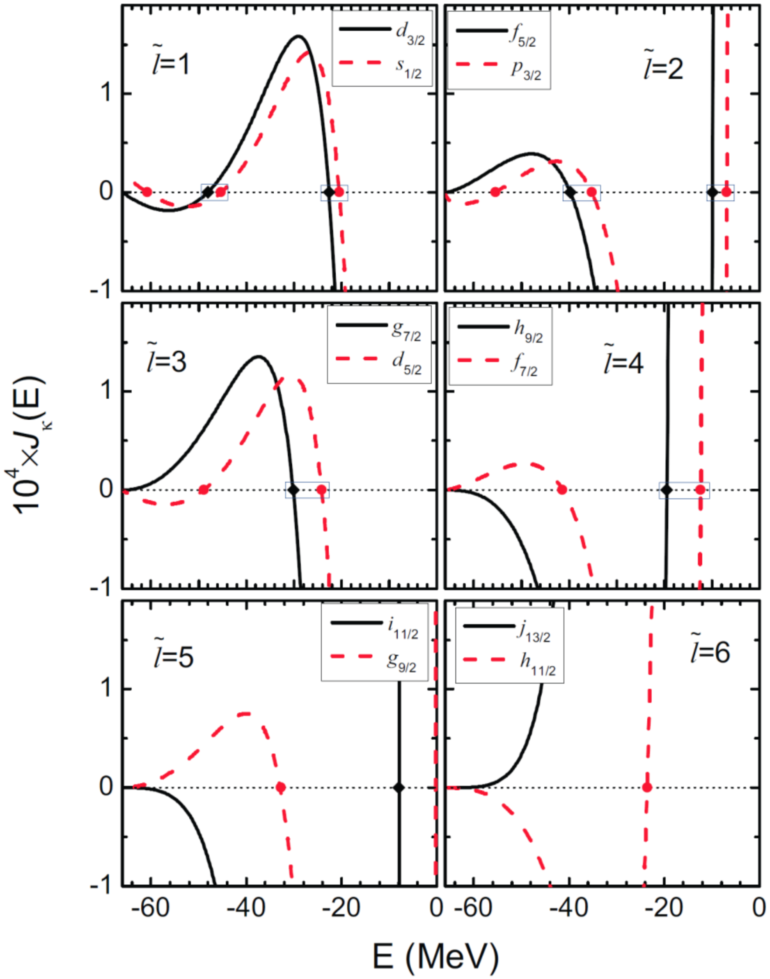

Lu_Zhao_Zhou2013_arXiv1305-1524

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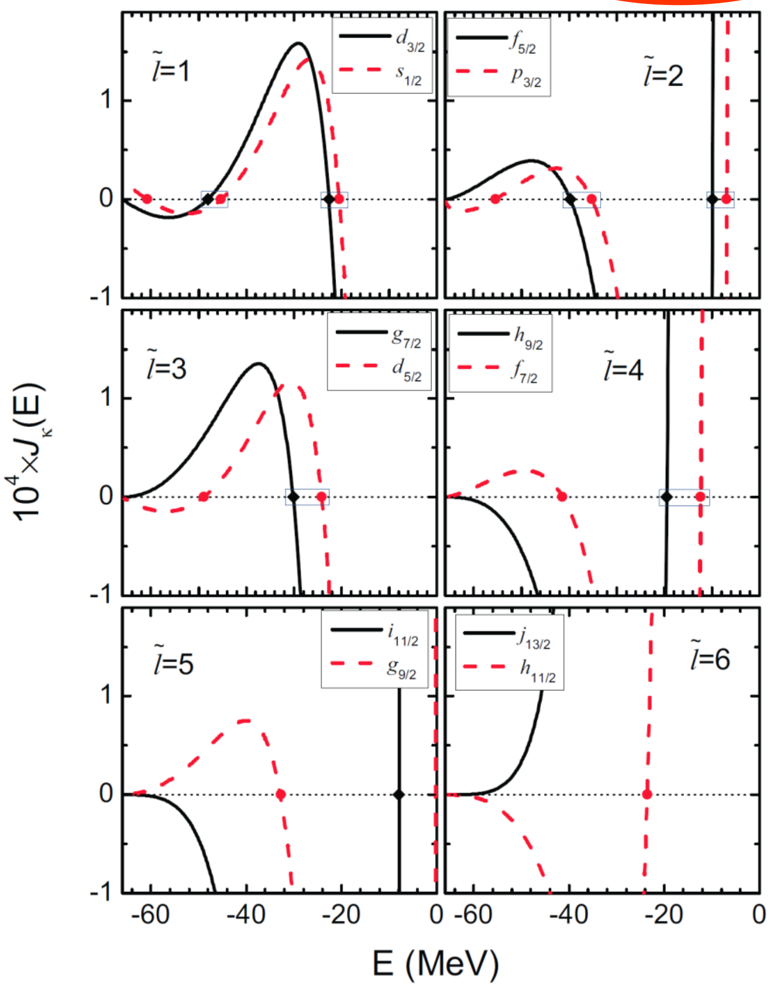

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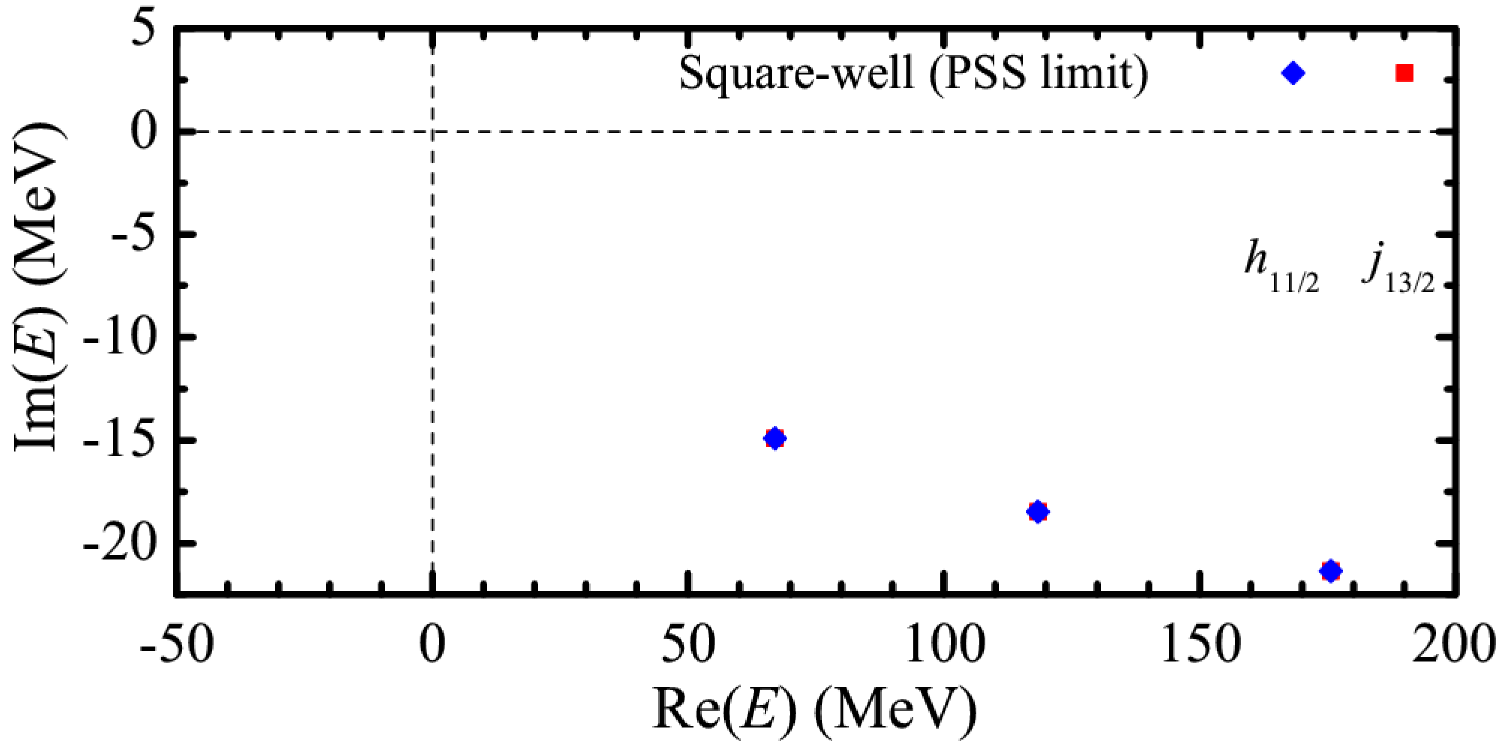
An example: Square-well potentials



- Square-well potentials w/ C = 0: PS doublets are degenerate

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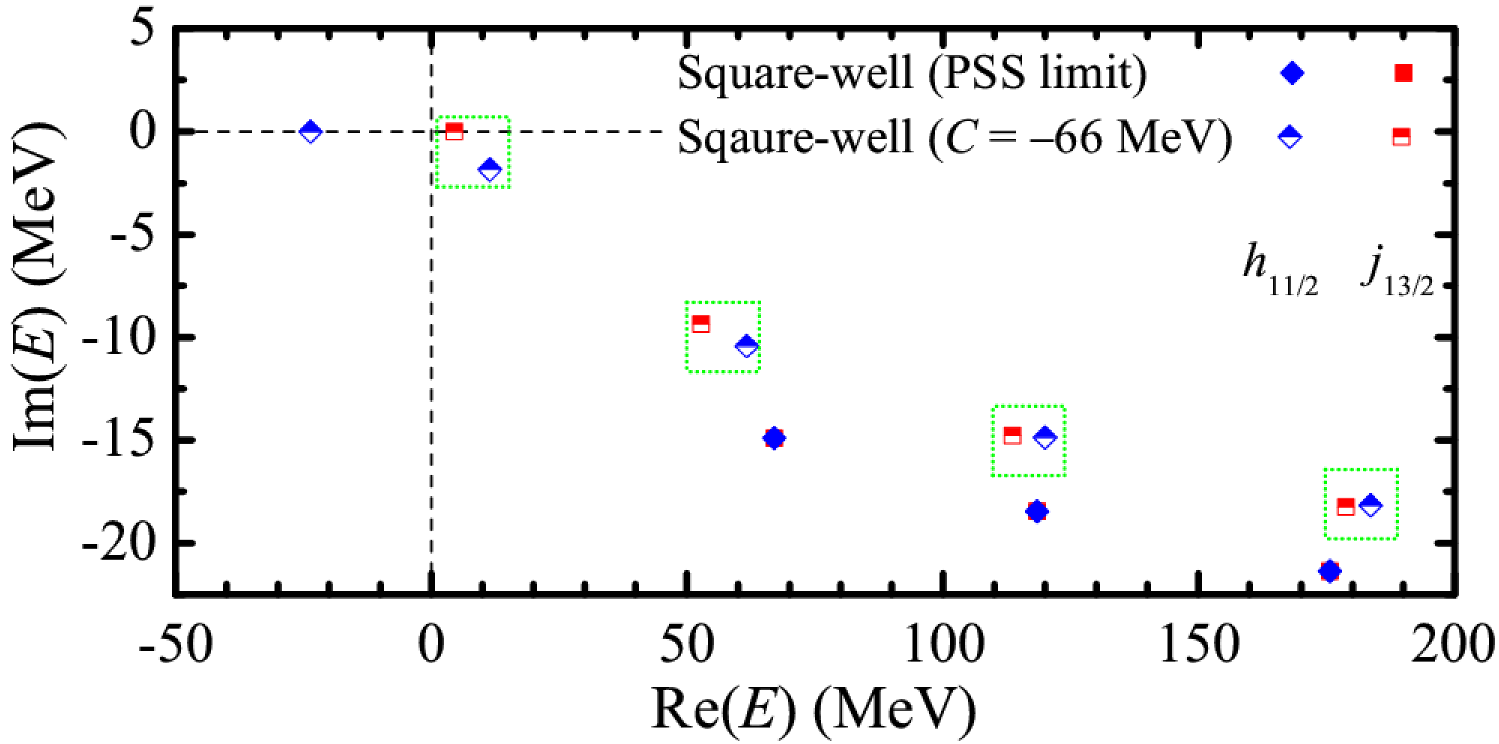
An example: Square-well potentials



- Square-well potentials w/ $C = 0$: PS doublets are degenerate
- Sqaure-well potentials w/ C < 0: PSS approximately conserved

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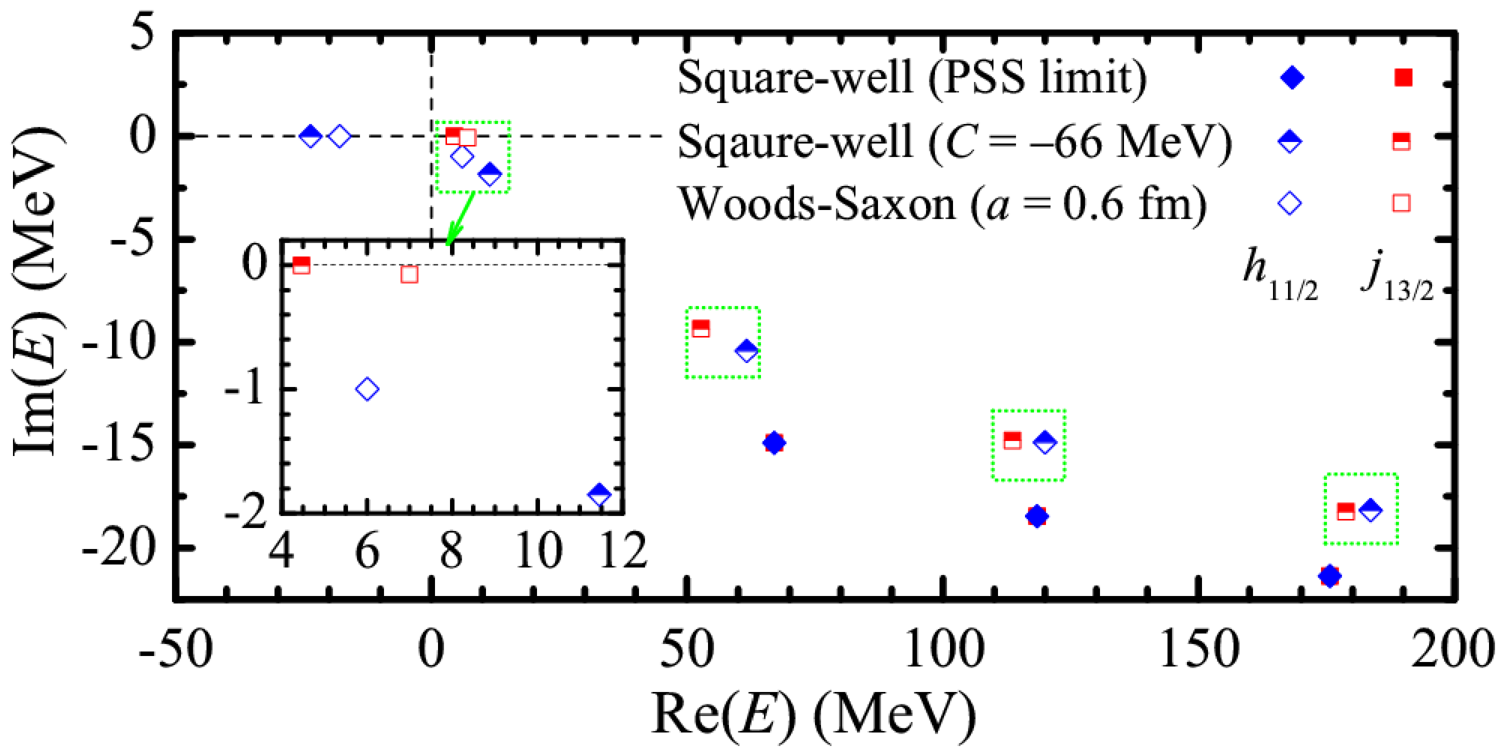
An example: Square-well & Woods-Saxon potentials



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- Sqaure-well potentials w/ C < 0: PSS approximately conserved
- Woods-Saxon potentials: PSS approximately conserved

Real Stabilization Method for resonances:
Zhang_Zhou_Meng_Zhao_PRC77-014312
Zhou_Meng_Zhao_JPB42-245001

Lu_Zhao_Zhou2012_PRL109-072501

The story of justifying PSS in s. p. resonances

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1. Numerical evidence in square well potentials

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The story of justifying PSS in s. p. resonances

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   Good! But why?

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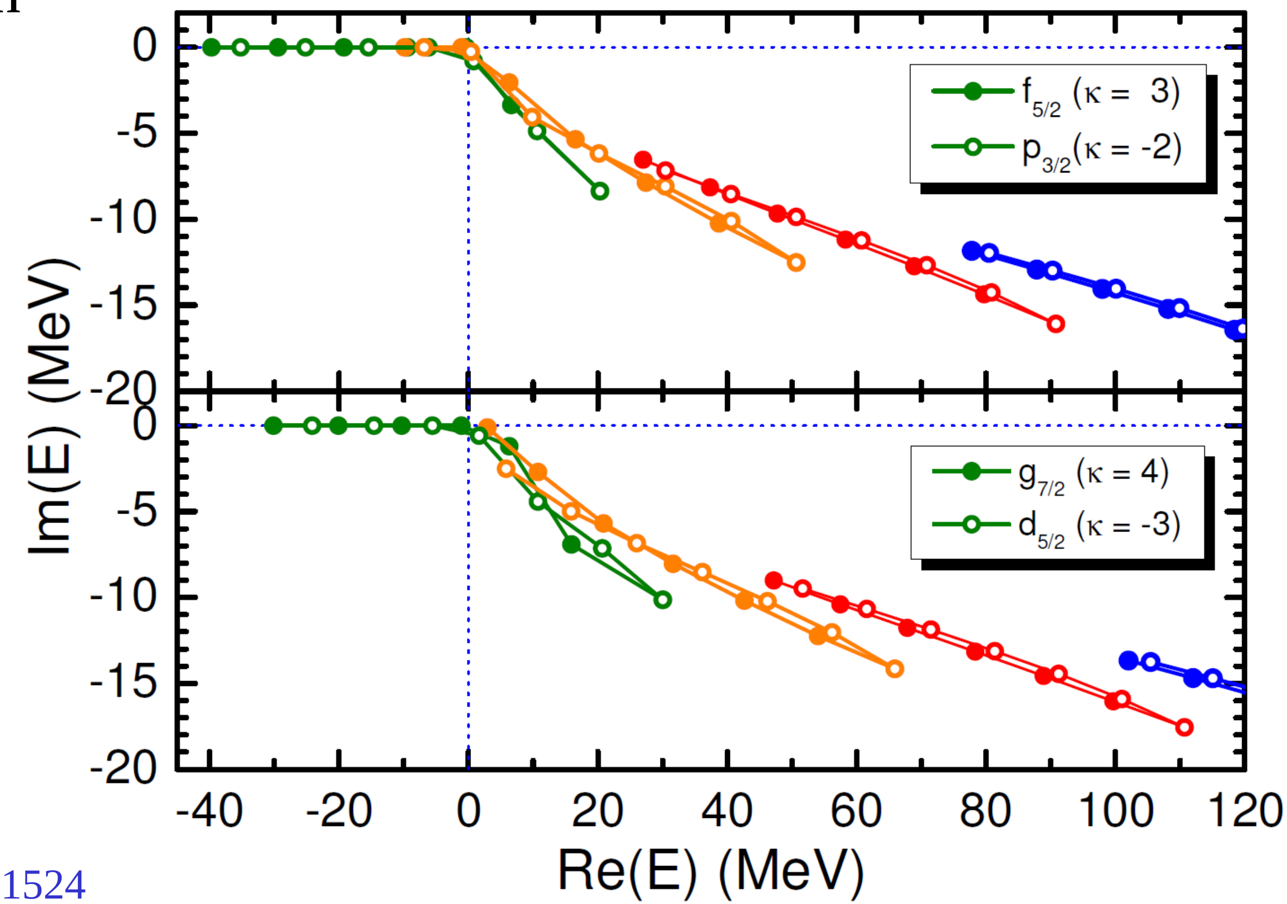
   Perfect! But why not check it numerically?

3. Numerical check in square well potentials

   Excellent! Are there more?

Dependence of PSS on the depth of potentials

- In the PSS limit, that is, $C = V + S = 0$, the zeros are paired off and each pair of states coincide with each other.
- When the potential depth increases, the zeros move gradually to the up left corner; the paired pseudospin partners separate from each other and the PSS is broken



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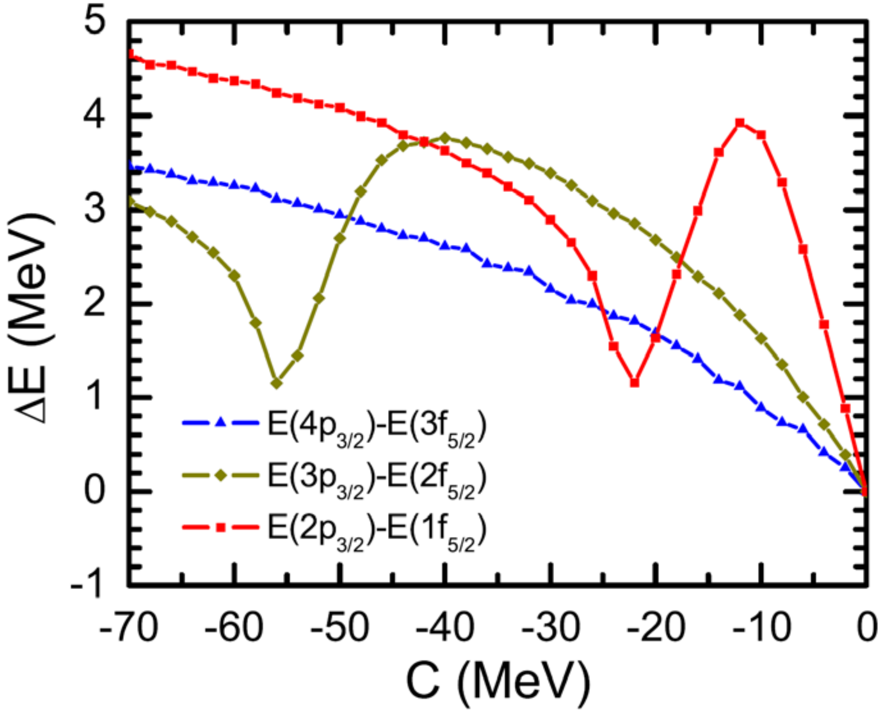
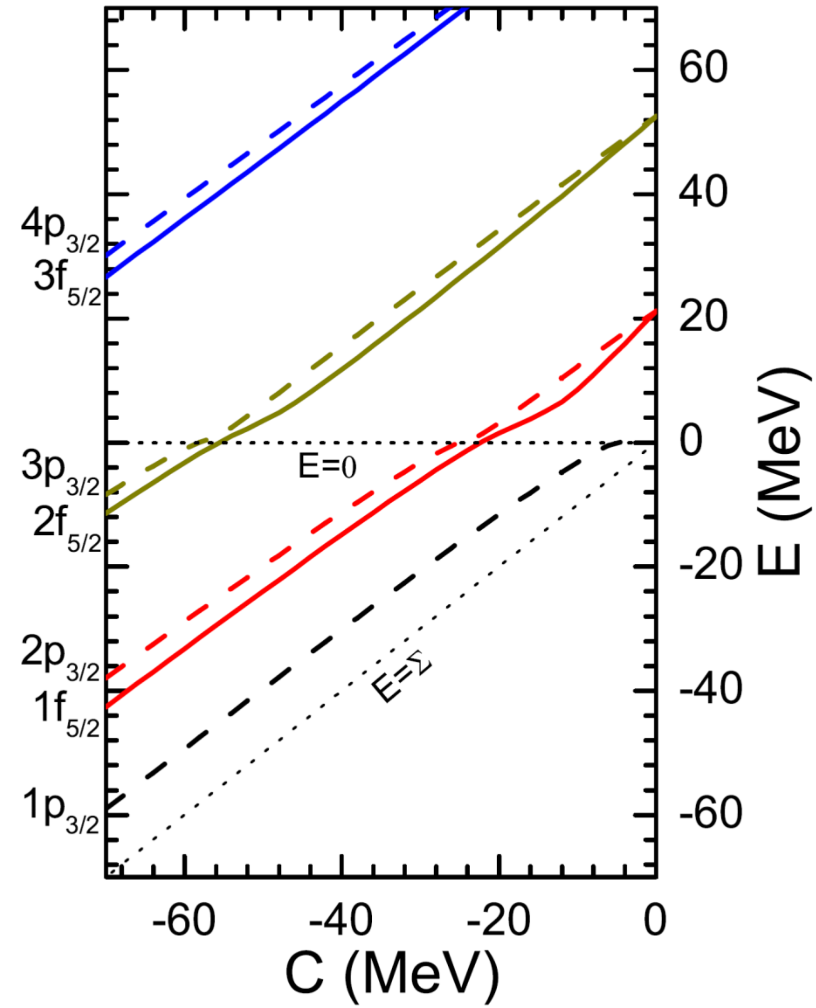
Threshold effect in energy splitting

- When the potential depth increases from 0, the energy splitting first increases then decreases until they encounter the threshold at a critical value of *C* where one of the levels becomes a bound state and the splitting takes a minimum value.
- When the potential becomes even deeper, the splitting increases again





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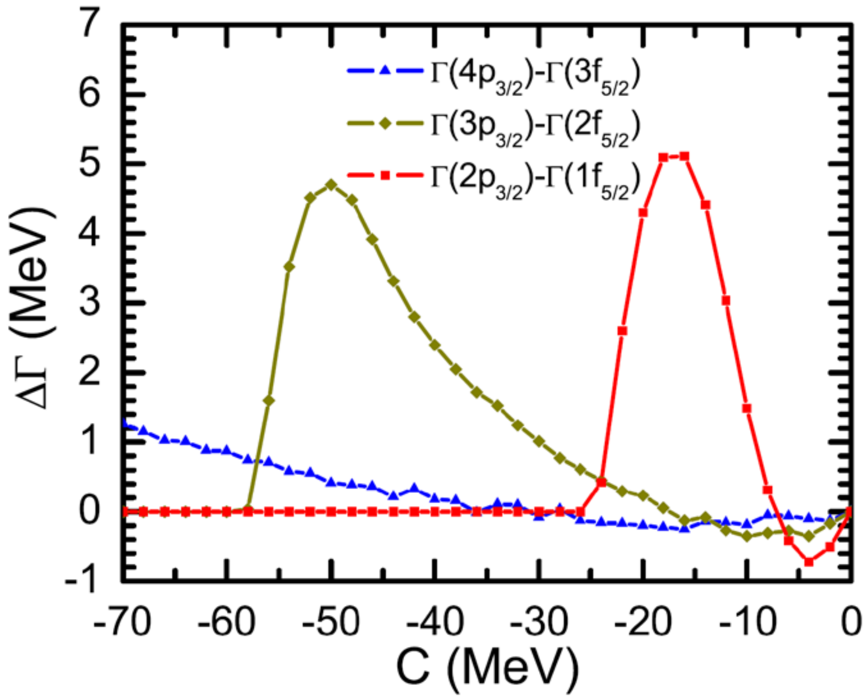
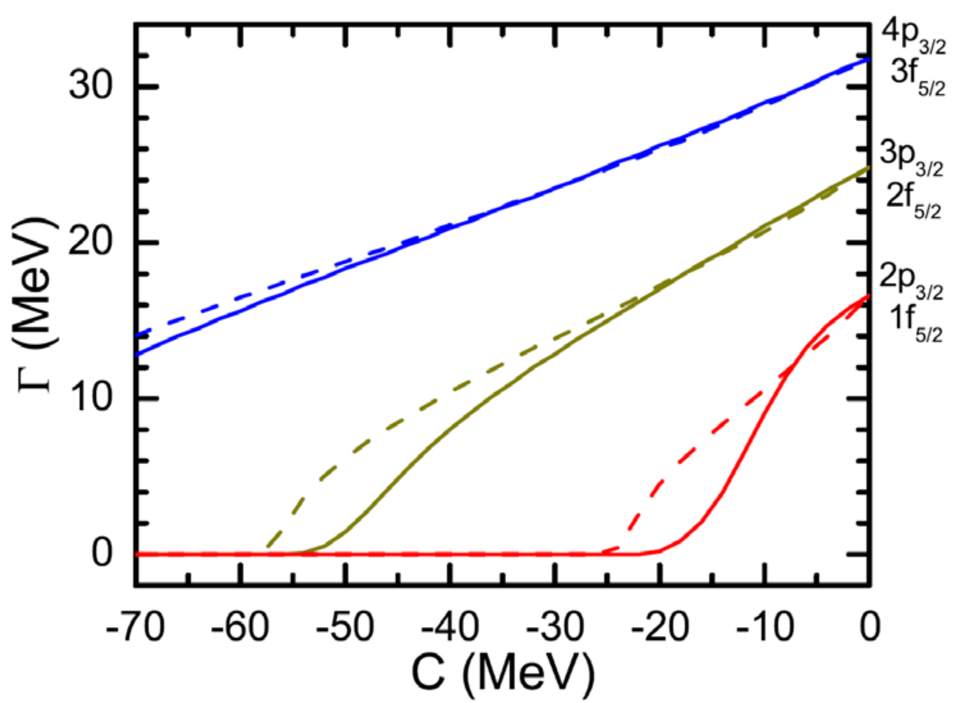
Anomaly in width splitting

- When the depth of the single particle potential increases from zero, the width splitting first decreases from zero to a maximum value with negative sign, then increases and becomes zero again.
- After the inversion of the width splitting, the splitting increases and reaches a maximum and positive value, then it becomes smaller and finally reaches zero.





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Summary

- By examining the asymptotic behavior of radial Dirac wave functions, namely, phase shifts & zeros of the Jost function, the PSS in single particle resonant states is justified to be exactly conserved under the PSS limit, i.e., when $V(r) + S(r)$ or its 1st derivative vanish
- The conservation & breaking of PSS are studied systematically

Summary & perspectives

- By examining the asymptotic behavior of radial Dirac wave functions, namely, phase shifts & zeros of the Jost function, the PSS in single particle resonant states is justified to be exactly conserved under the PSS limit, i.e., when $V(r) + S(r)$ or its 1st derivative vanish
- The conservation & breaking of PSS are studied systematically
    - Experimental evidence ?
    - Why $V(r) + S(r)$ is small ?
    - Why widths are the same for resonant PS doublets ?
    - Relations of wave functions of resonant doublet states ?
    - PSS in resonances in anti-nucleon & anti-hyperon spectra ?
    - PSS in resonances in deformed systems ?
    - Effects of Coulomb interaction ?
    - Origin of the threshold effect in the energy splitting?
    - Origin of the anomaly in the width splitting?
    - …

Zhou, Shan-Gui
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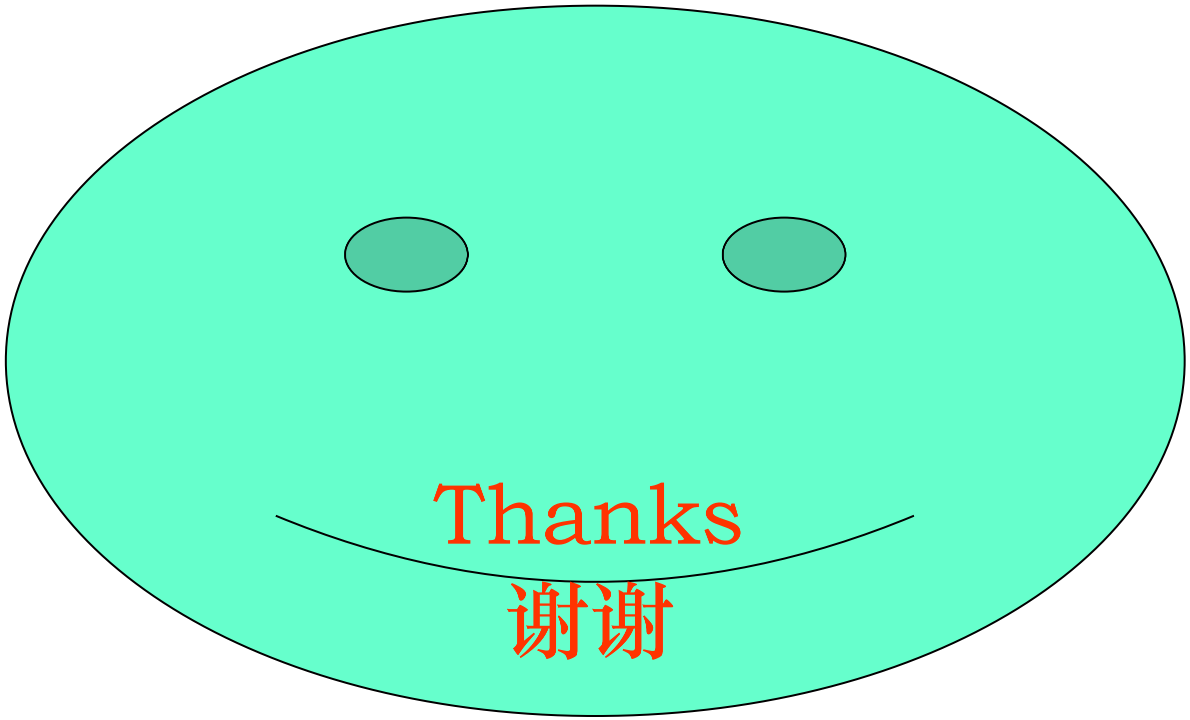

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