原子核、冷却原子、物性物理をつなぐ普遍性とエフィモフ効果

西田 祐介（東工大）

2013年12月26日

中性子星核物質ウィンタースクール@理研

Plan of this talk

1. Universality in physics
2. What is the Efimov effect ?

   Keywords: universality
   scale invariance
   quantum anomaly
   RG limit cycle

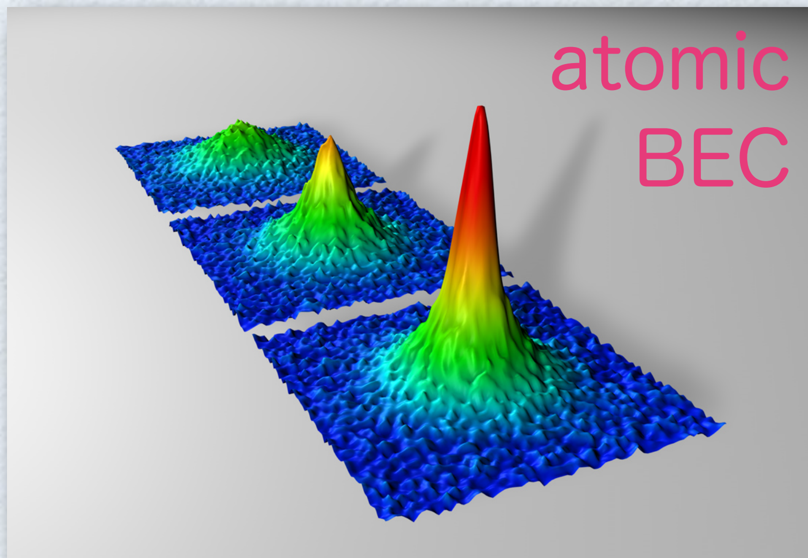
3. Efimov effect beyond cold atoms
4. New progress: Super Efimov effect

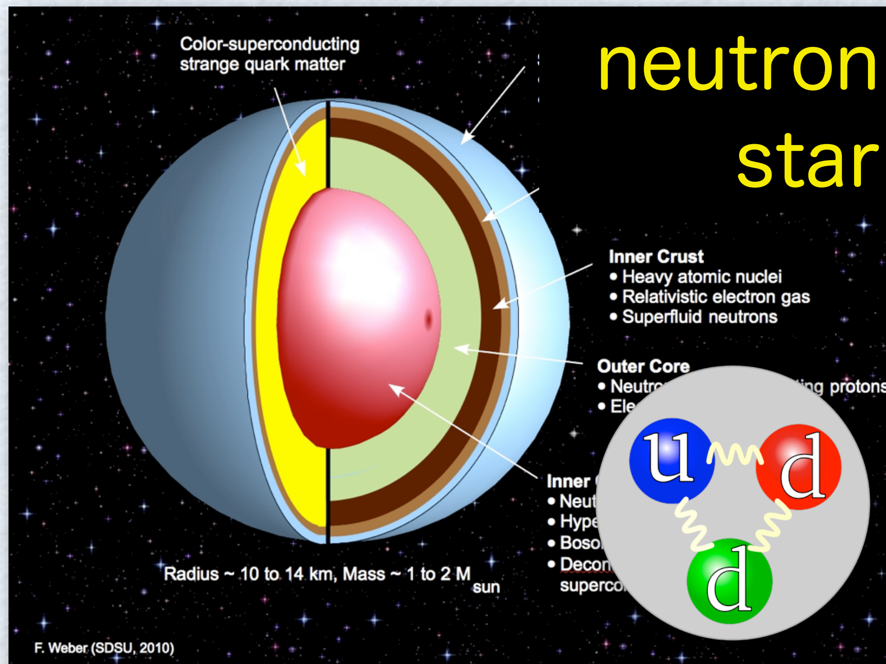
Introduction

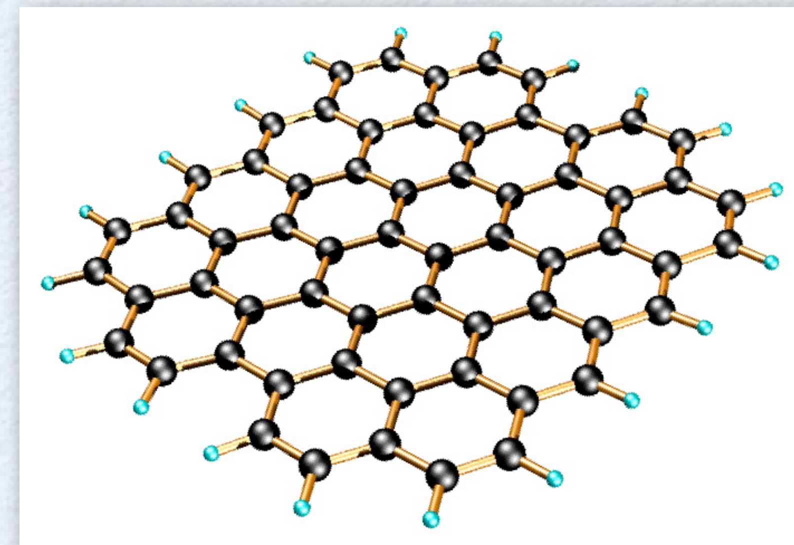
1. **Universality in physics**
2. What is the Efimov effect?
3. Efimov effect beyond cold atoms
4. New progress: Super Efimov effect

(ultimate) Goal of research

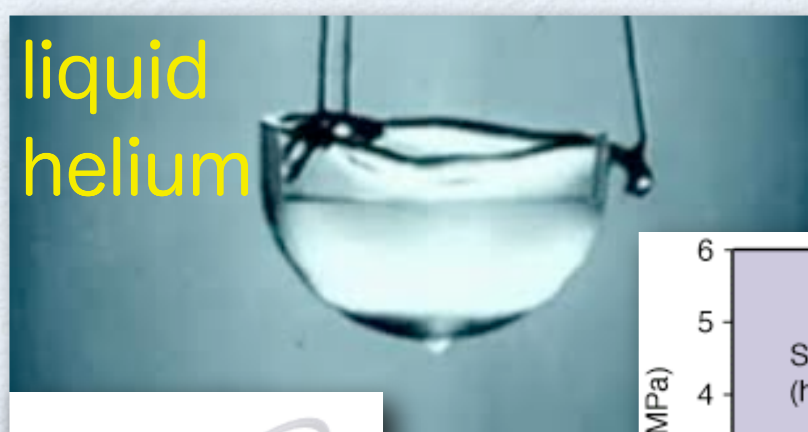
Understand physics of few and many particles governed by quantum mechanics

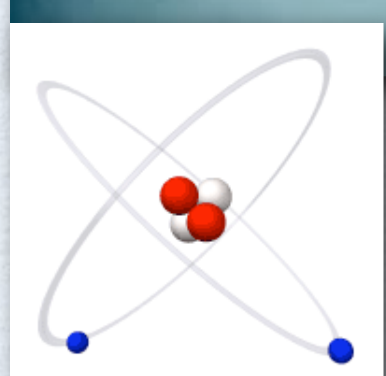


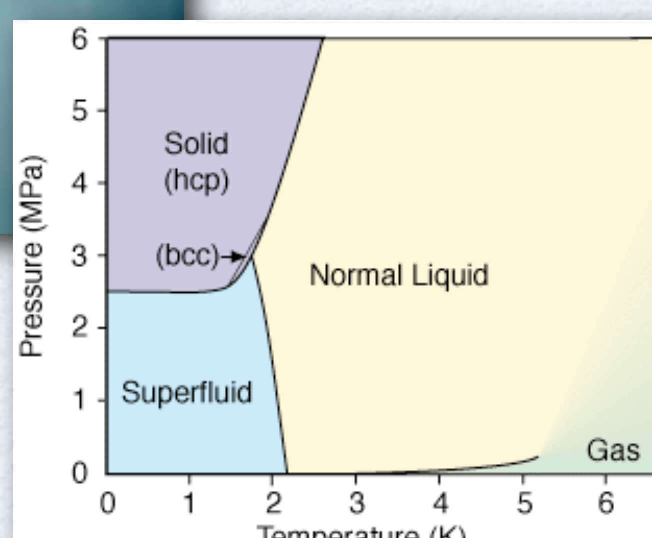




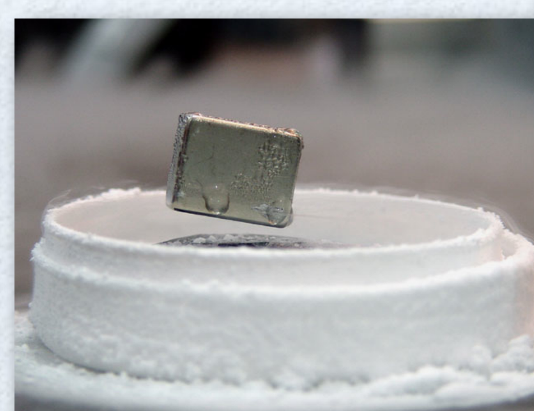
graphene

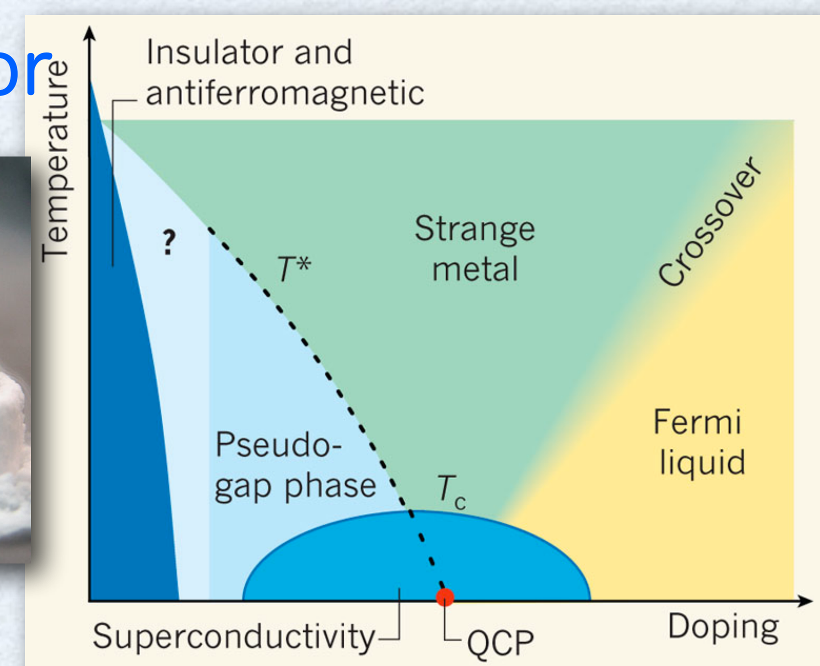






superconductor

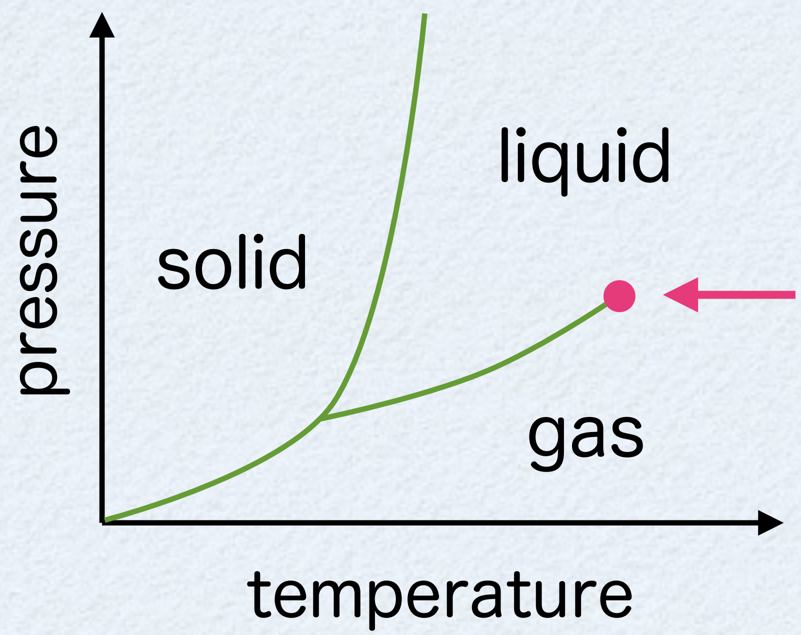


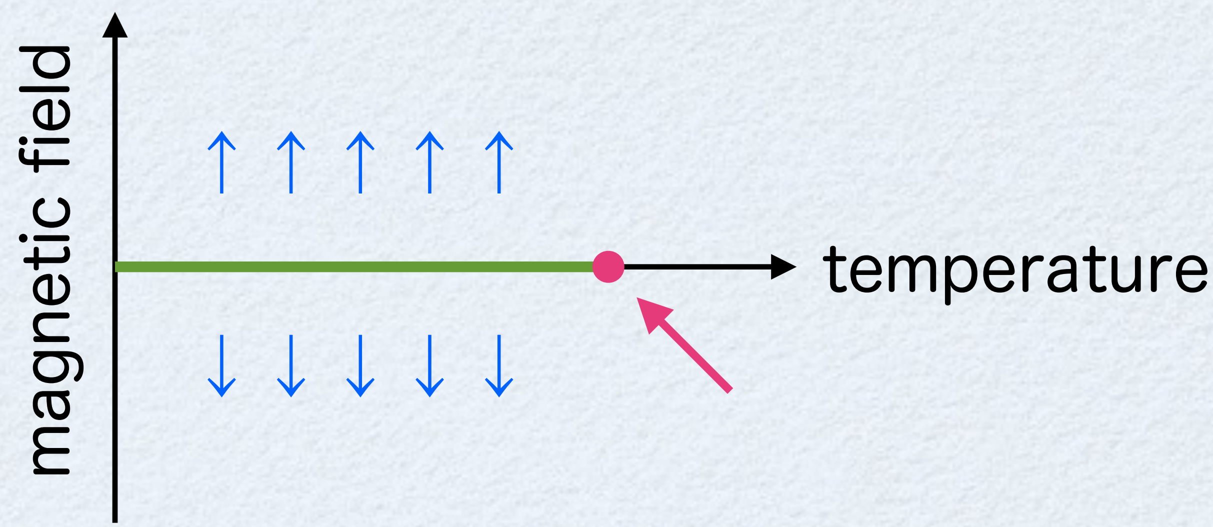


When physics is universal ?

A1. Continuous phase transitions $\Leftrightarrow$ $\xi / r_0 \to \infty$

E.g. Water  vs. Magnet 



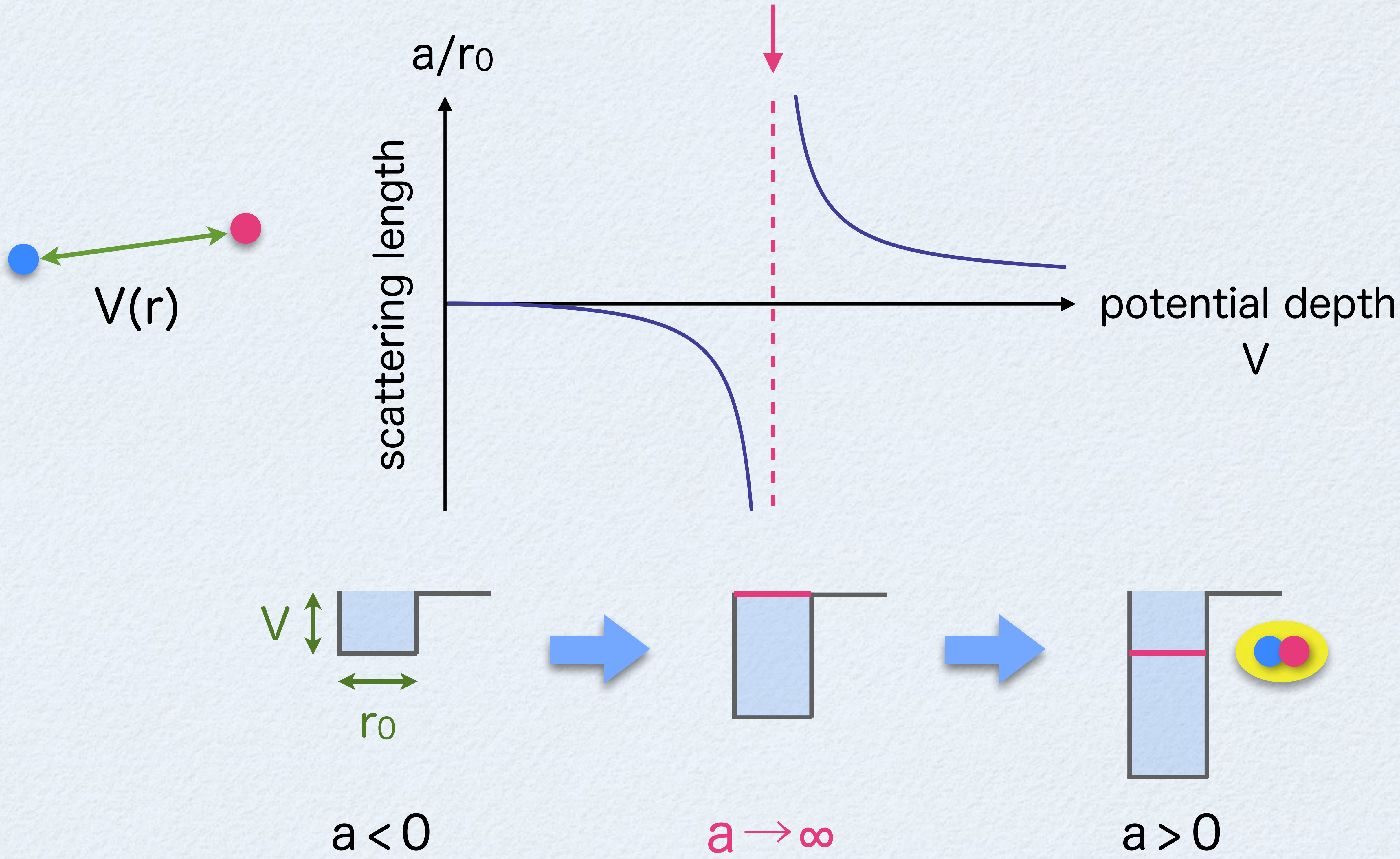


Water and magnet have the same exponent $\beta \approx 0.325$

$$\rho_{\mathrm{liq}} - \rho_{\mathrm{gas}} \sim (T_{\mathrm{c}} - T)^{\beta} \qquad M_{\uparrow} - M_{\downarrow} \sim (T_{\mathrm{c}} - T)^{\beta}$$

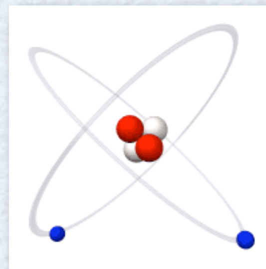
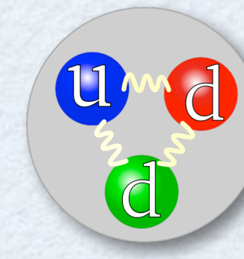
When physics is universal ?

A2. Scattering resonances $\Leftrightarrow$ $a/r_0 \to \infty$



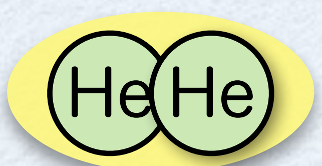
When physics is universal ?

A2. Scattering resonances $\Leftrightarrow$ $a/r_0 \to \infty$

E.g. ^{4}He atoms  vs. proton/neutron 

van der Waals force:

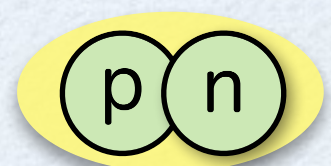
$a \approx 1\times10^{-8}$ m $\approx 20\ r_0$



$E_{binding} \approx 1.3\times10^{-3}$ K

nuclear force:

$a \approx 5\times10^{-15}$ m $\approx 4\ r_0$



$E_{binding} \approx 2.6\times10^{10}$ K

Atoms and nucleons have the same form of binding energy

$$E_{\mathrm{binding}} \to -\frac{\hbar^2}{m\,a^2} \qquad (a/r_0 \to \infty)$$

Physics only depends on the scattering length "a"

Efimov effect

1. Universality in physics
2. **What is the Efimov effect?**
3. Efimov effect beyond cold atoms
4. New progress: Super Efimov effect

Efimov effect

Efimov (1970)

Volume 33B, number 8 PHYSICS LETTERS 21 December

ENERGY LEVELS ARISING FROM RESONANT TWO-BODY FORCES IN A THREE-BODY SYSTEM

V. EFIMOV
A.F.Ioffe Physico-Technical Institute, Leningrad, USSR

Received 20 October 1970

Resonant two-body forces are shown to give rise to a series of levels in three-particle systems. The number of such levels may be very large. Possibility of the existence of such levels in systems of three α-particles (^{12}C nucleus) and three nucleons (^3H) is discussed.

The range of nucleon-nucleon forces r_0 is known to be considerably smaller than the scattering lengts a. This fact is a consequence of the resonant character of nucleon-nucleon forces. Apart from this, many other forces in nuclear physics are resonant. The aim of this letter is to expose an interesting effect of resonant forces in a three-body system. Namely, for $a \gg r_0$ a series of bound levels appears. In a certain case, the number of levels may become infinite.

Let us explicitly formulate this result in the simplest case. Consider three spinless neutral

ticle bound states emerge one after the other. At $g = g_0$ (infinite scattering length) their number is infinite. As g grows on beyond g_0, levels leave into continuum one after the other (see fig. 1).

The number of levels is given by the equation

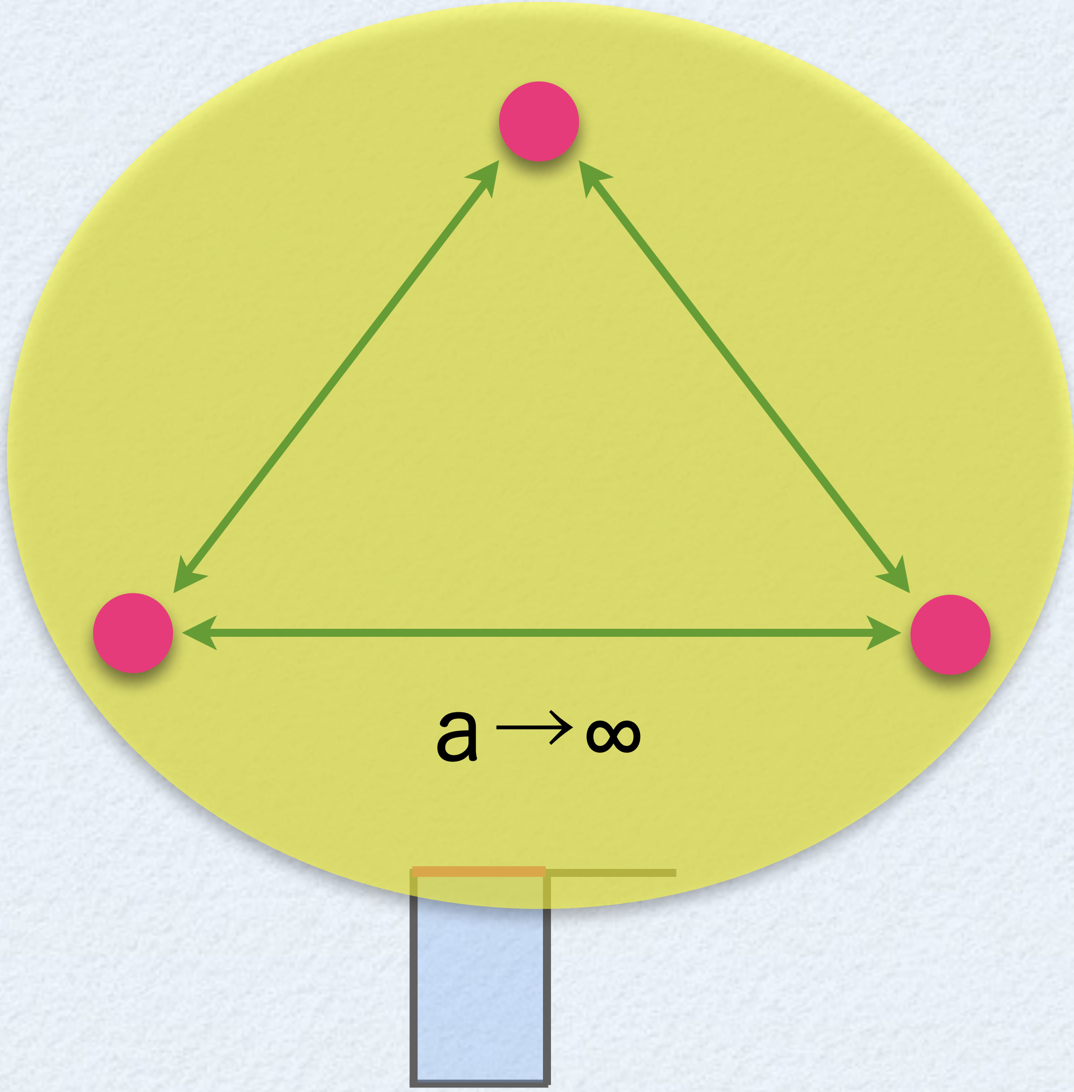
$$N \approx \frac{1}{\pi} \ln(|a|/r_0) \qquad (1)$$

All the levels are of the 0^+ kind; corresponding wave functions are symmetric; the energies $E_N \ll 1/r_0^2$ (we use $\hbar = m = 1$); the range of these bound states is much larger than r_0.

Efimov effect

When 2 bosons interact with infinite “a”,
3 bosons always form a series of bound states

Efimov (1970)



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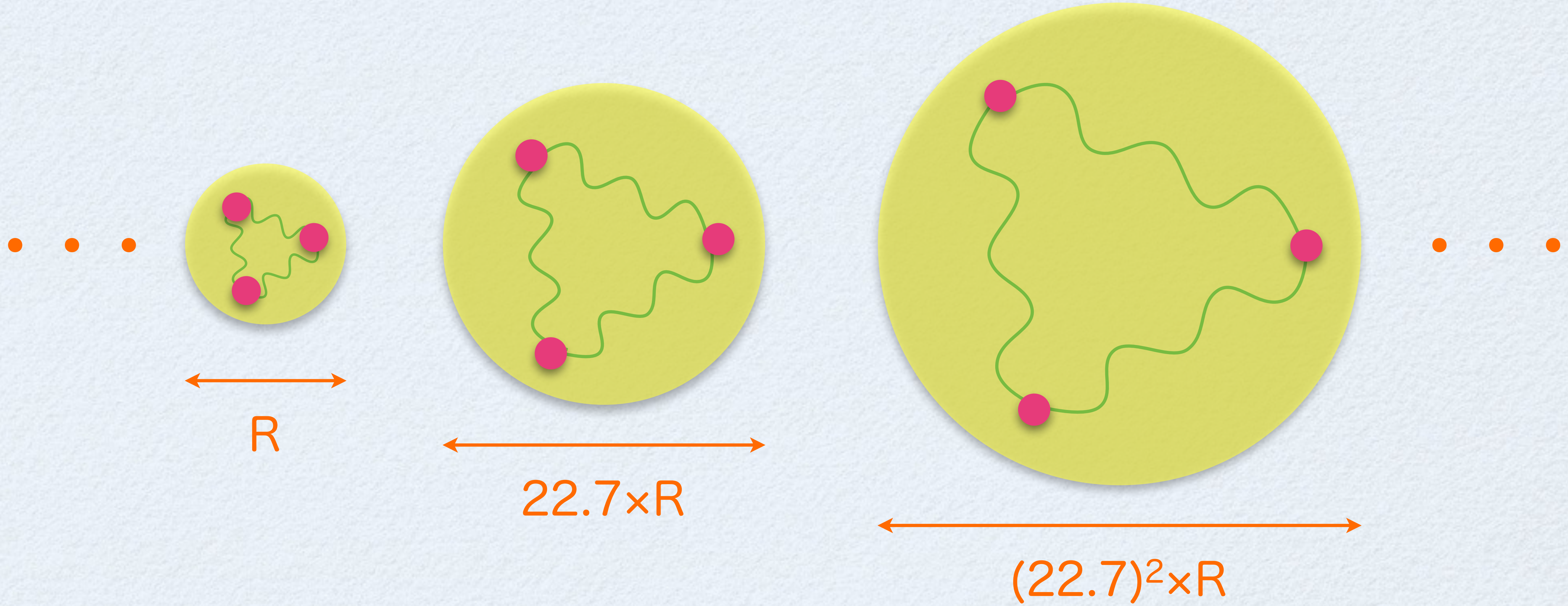


Discrete scaling symmetry

Efimov effect

When 2 bosons interact with infinite “a”,
3 bosons always form a series of bound states

Efimov (1970)

• • •

• • •

Discrete scaling symmetry

Why Efimov effect happens ?

Keywords

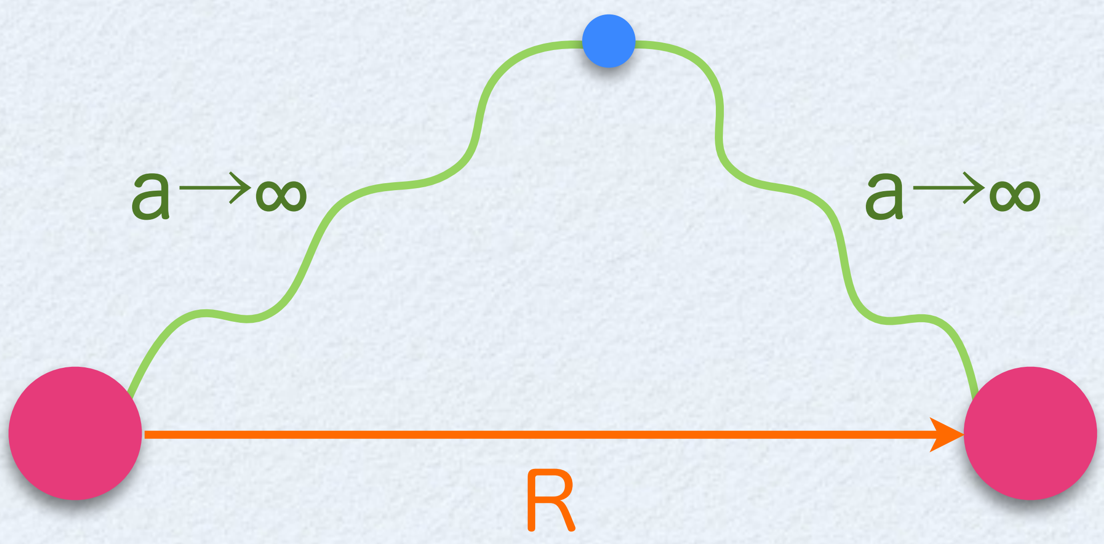
✓ Universality

- Scale invariance
- Quantum anomaly
- RG limit cycle

Why Efimov effect happens ?

Two heavy (M) and one light (m) particles

➡ Born-Oppenheimer approximation



Binding energy of a light particle

$$E_b(R) = -\boxed{\frac{\hbar^2}{2mR^2}} \times (0.5671\ldots)^2$$

Scale invariance at a→∞

Schrödinger equation of two heavy particles :

$$\left[-\frac{\hbar^2}{M}\frac{\partial^2}{\partial \boldsymbol{R}^2} + V(R)\right]\psi(\boldsymbol{R}) = -\frac{\hbar^2\kappa^2}{M}\psi(\boldsymbol{R}) \qquad V(R) \equiv E_b(R)$$

Why Efimov effect happens ?

Schrödinger equation of two heavy particles :

$$\left[-\frac{\hbar^2}{M}\left(\frac{\partial^2}{\partial R^2}+\frac{2}{R}\frac{\partial}{\partial R}\right)-\frac{\hbar^2}{2mR^2}(0.5671\ldots)^2\right]\psi(R)=-\frac{\hbar^2\kappa^2}{M}\psi(R)$$

$$\psi(R)=R^{-1/2}K_{i\alpha}(\kappa R) \qquad \alpha^2\equiv\frac{M}{2m}(0.5671\ldots)^2-\frac{1}{4}$$

$$\to R^{-1/2}\sin[\alpha\ln(\kappa R)+\delta] \qquad (R\to 0)$$

ψ'/ψ has to be fixed by short-range physics

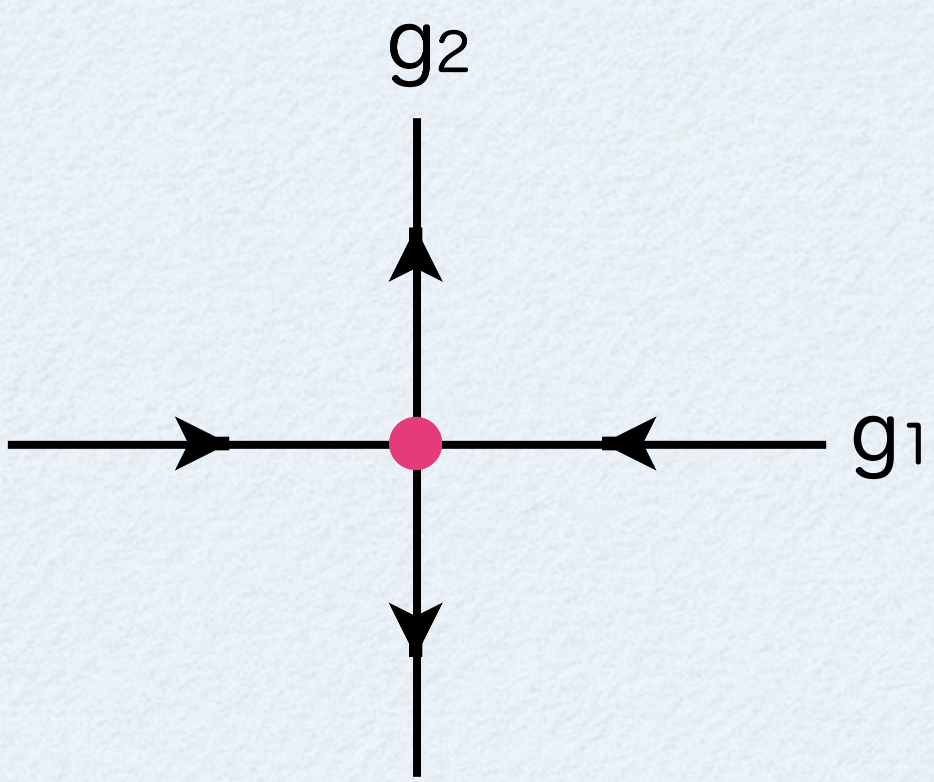
➡ If $\kappa=\kappa_*$ is a solution, $\kappa=(e^{\pi/\alpha})^n\kappa_*$ are solutions!

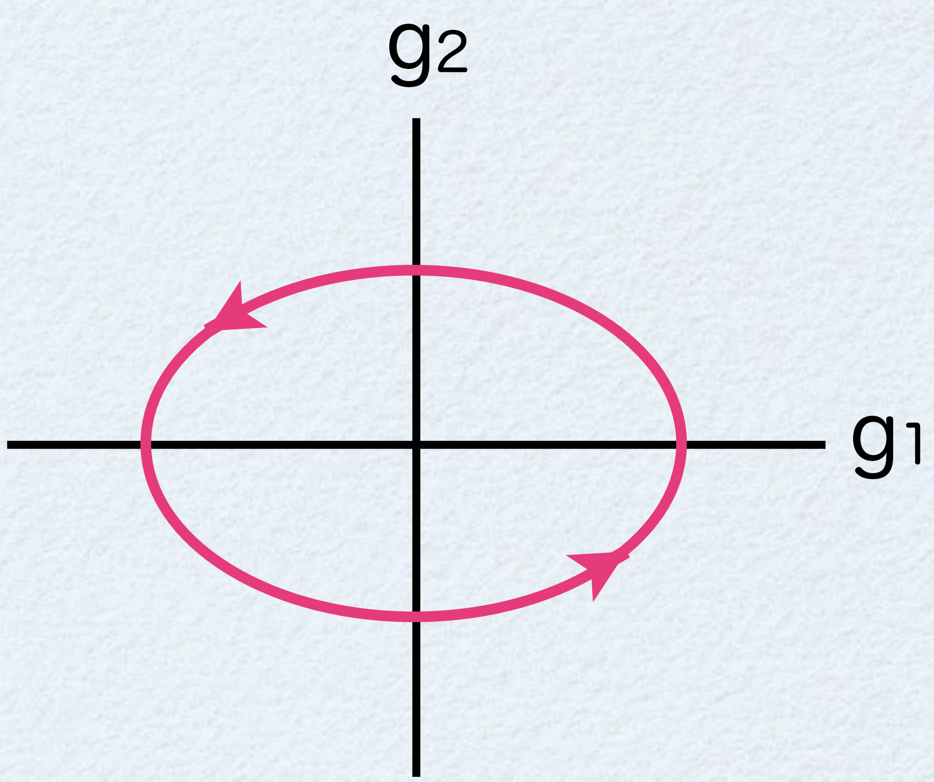
Classical scale invariance is broken by κ_*

= Quantum anomaly

Renormalization group limit cycle

Renormalization group flow diagram in coupling space





RG fixed point

⇒ Scale invariance

E.g. critical phenomena

RG limit cycle

⇒ Discrete scale invariance

E.g. E????v effect

Renormalization group limit cycle

K. Wilson (1971) considered for strong interactions

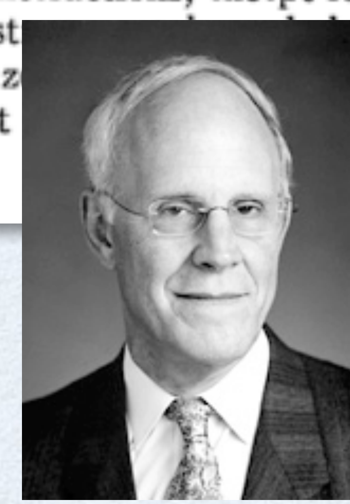
L REVIEW D VOLUME 3, NUMBER 8 15 APRIL 1971

Renormalization Group and Strong Interactions*

KENNETH G. WILSON
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
and
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850†

(Received 30 November 1970)

The renormalization-group method of Gell-Mann and Low is applied to field theories of strong interactions. It is assumed that renormalization-group equations exist for strong interactions which involve one or several momentum-dependent coupling constants. The further assumption that these coupling constants approach fixed values as the momentum goes to infinity is discussed in detail. However, an alternative is suggested, namely, that these coupling constants approach a limit cycle in the limit of large momenta. Some results of this paper are: (1) The e^+-e^- annihilation experiments above 1-GeV energy may distinguish a fixed point from a limit cycle or other asymptotic behavior. (2) If electrodynamics or weak interactions become strong above some large momentum Λ, then the renormalization group can be used (in principle) to determine the renormalized coupling constants of strong interactions, except for $U(3)\times U(3)$ symmetry-breaking parameters. (3) Mass terms in the Lagrangian of st...
must break a symmetry of the combined interactions with z...
weak interactions can be understood assuming only that ...
interactions.



QCD is asymptotic free

(2004 Nobel prize)

Renormalization group limit cycle

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Efimov effect (1970) is its rare manifestation!

Effective field theory

PHYSICAL REVIEW LETTERS

VOLUME 82 | 18 JANUARY 1999 | NUMBER 3

Renormalization of the Three-Body System with Short-Range Interactions

P. F. Bedaque,[1,*] H.-W. Hammer,[2,†] and U. van Kolck[3,4,‡]

[1] *Institute for Nuclear Theory, University of Washington, Seattle, Washington 98195*

[2] *TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, Canada V6T 2A3*

[3] *Kellogg Radiation Laboratory, 106-38, California Institute of Technology, Pasadena, California 91125*

[4] *Department of Physics, University of Washington, Seattle, Washington 98195*

(Received 9 September 1998)

We discuss renormalization of the nonrelativistic three-body problem with short-range forces. The problem becomes nonperturbative at momenta of the order of the inverse of the two-body scattering length, and an infinite number of graphs must be summed. This summation leads to a cutoff dependence that does not appear in any order in perturbation theory. We argue that this cutoff dependence can be absorbed in a single three-body counterterm and compute the running of the three-body force with the cutoff. We comment on the relevance of this result for the effective field theory program in nuclear and molecular physics. [S0031-9007(98)08276-3]

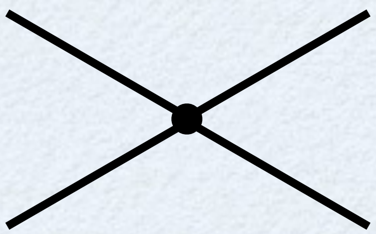
PACS numbers: 03.65.Nk, 11.80.Jy, 21.45.+v, 34.20.Gj

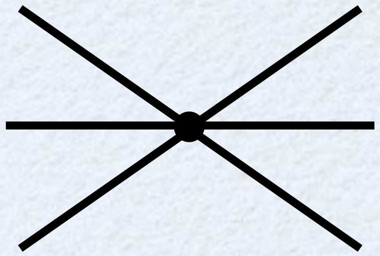
Systems composed of particles with momenta k much

dence can be absorbed in the coefficients of the leading-

Effective field theory

$$\mathcal{L} = \psi^\dagger \left(i\partial_t + \frac{\nabla^2}{2m} \right) \psi + g_2(\psi^\dagger\psi)^2 + g_3(\psi^\dagger\psi)^3$$

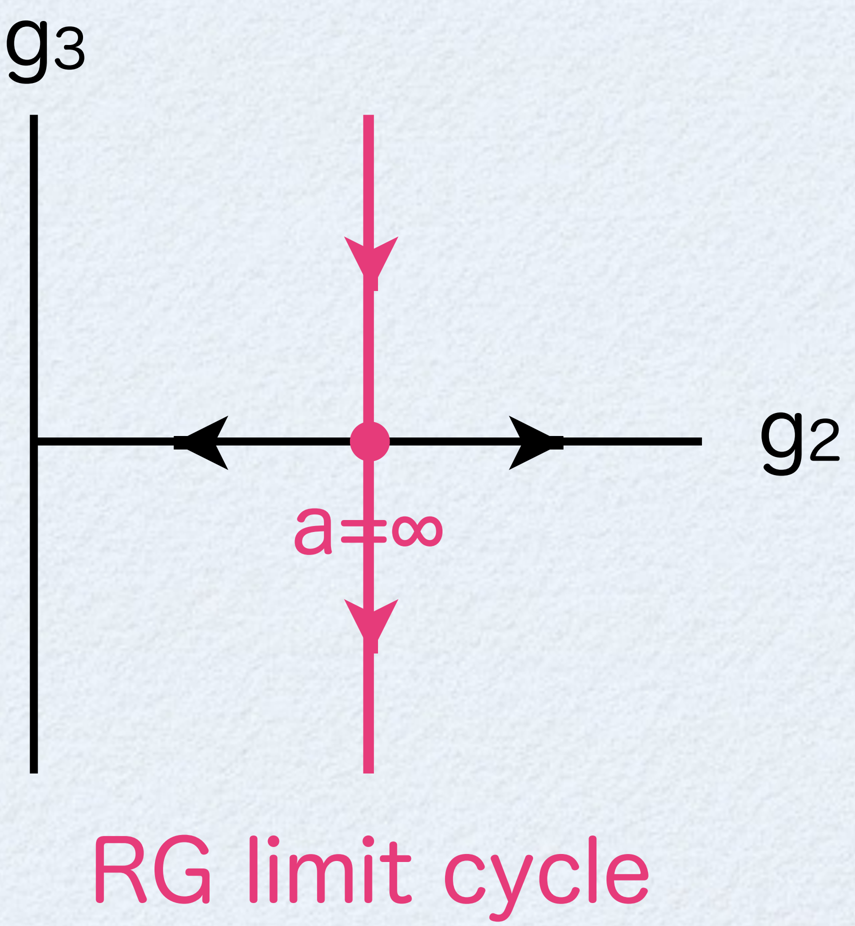


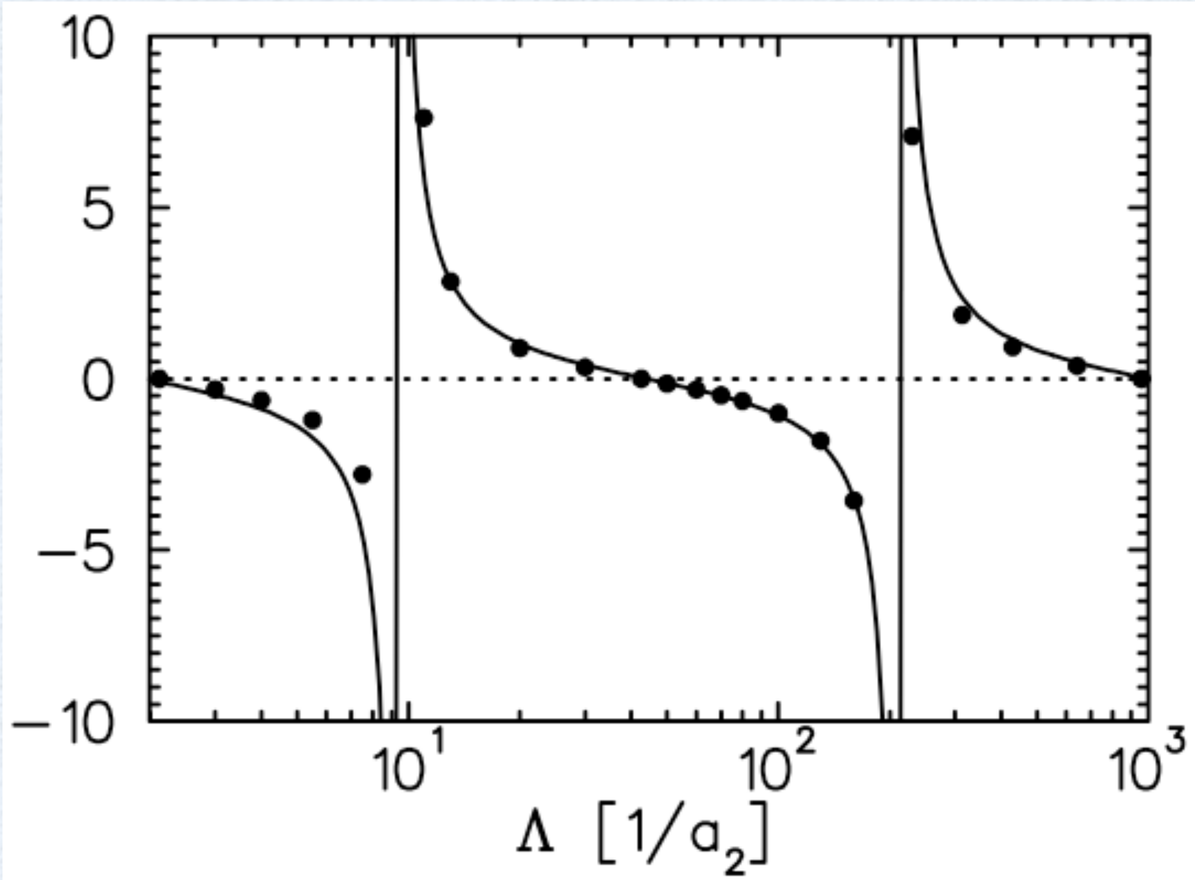


g_2 has a fixed point corresponding to a=∞

What is flow of g_3?

$$g_3(\Lambda) = -\frac{\sin[s_0 \ln(\Lambda/\Lambda_*) - \arctan(1/s_0)]}{\sin[s_0 \ln(\Lambda/\Lambda_*) + \arctan(1/s_0)]}$$





Effective field theory

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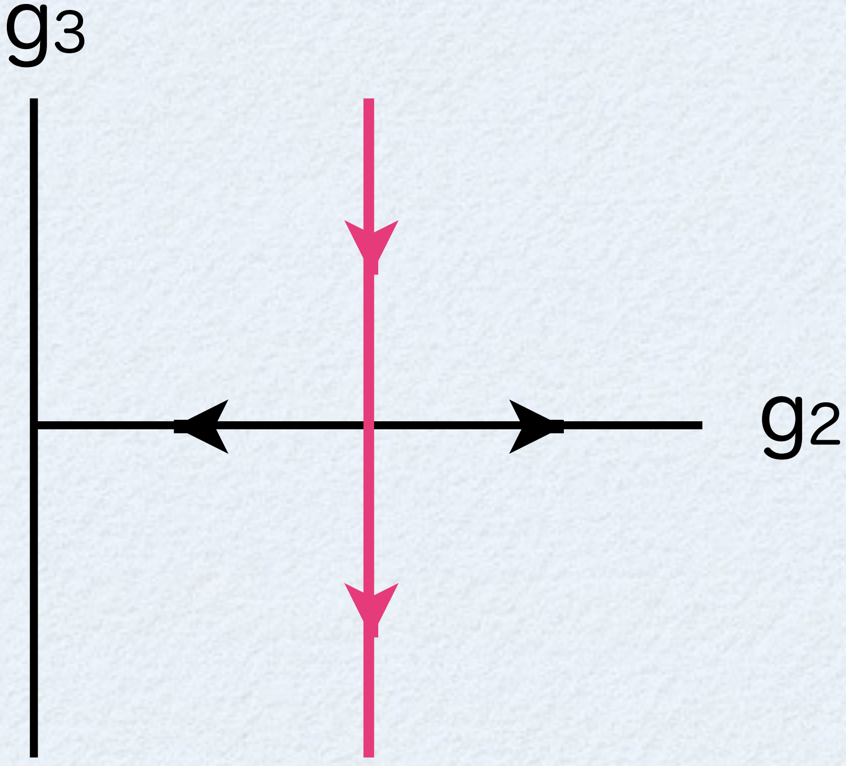
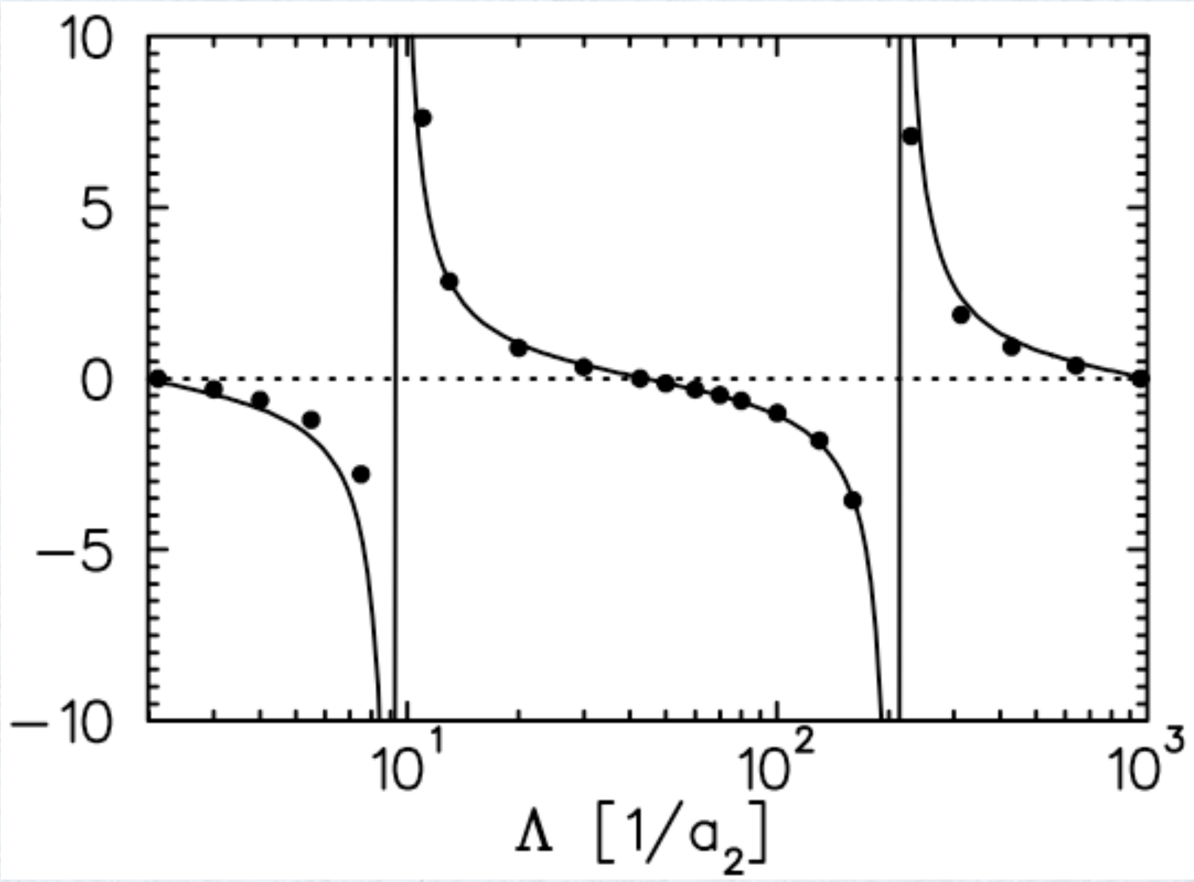
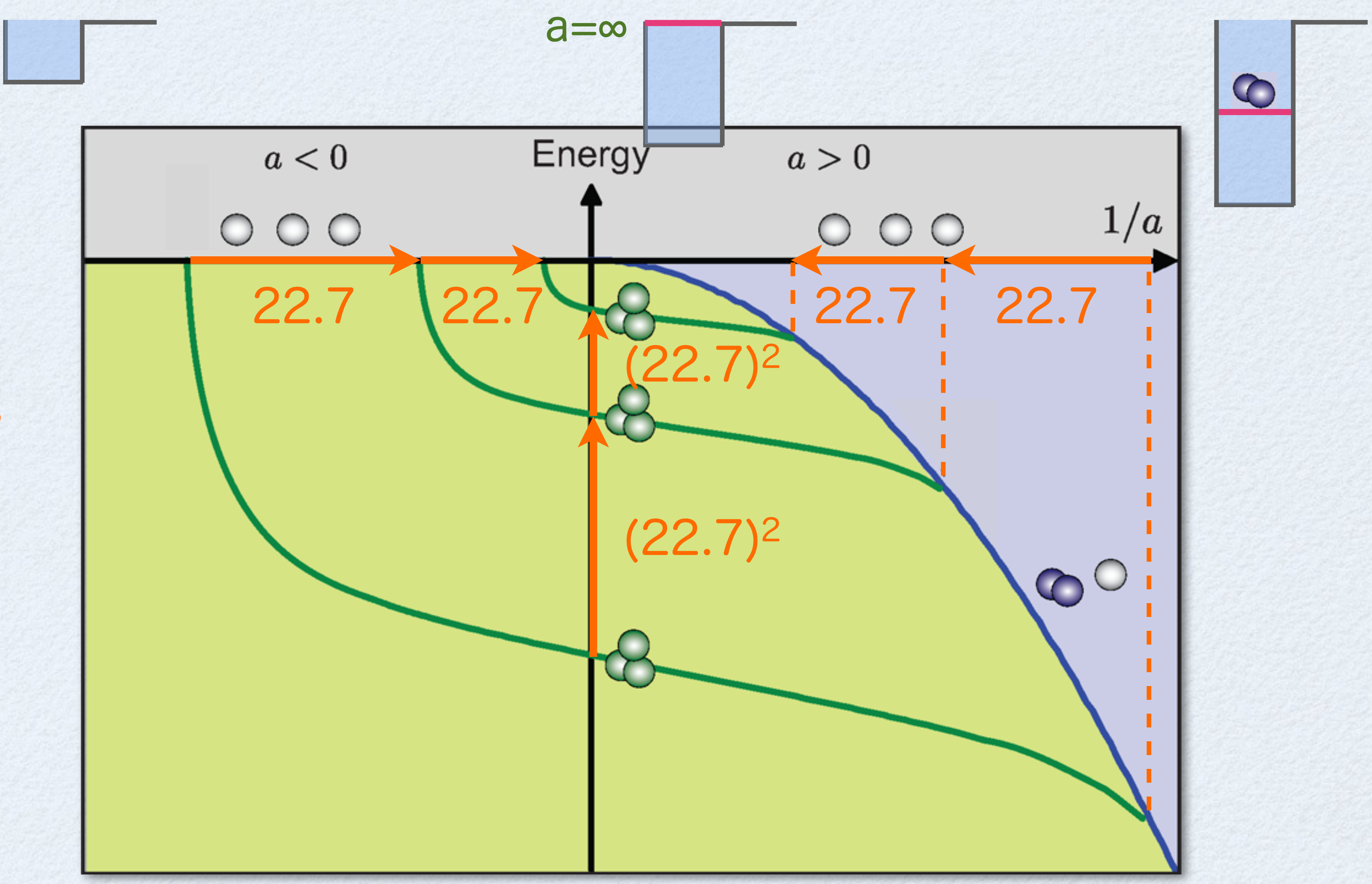




Efimov effect at a≠∞



Ferlaino & Grimm, Physics (2010)

Discrete scaling symmetry

Why 22.7 ?

Just a numerical number given by

22.6943825953666951928602171369...

log(22.6943825953666951928602171369...)

= 3.122211743110421968073091732438...

= π / 1.00623782510278148906406681234...

= π / s_0

$$\frac{2\pi}{s_0}\frac{\sinh(\frac{\pi}{6}s_0)}{\cosh(\frac{\pi}{2}s_0)} = \frac{\sqrt{3}\pi}{4}$$

22.7 = exp(π / 1.006...)

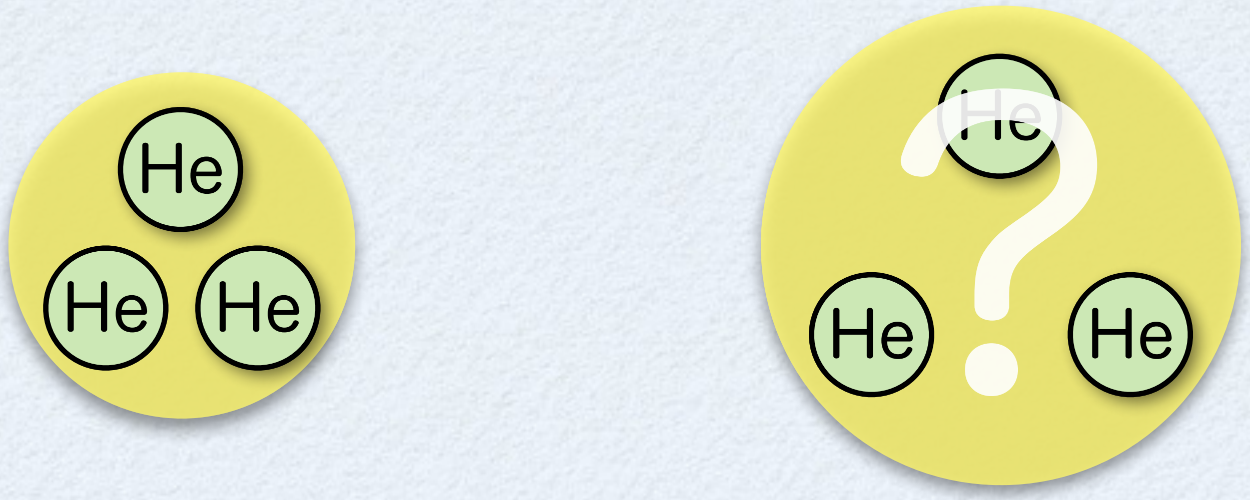
Where Efimov effect appears ?

× Originally, Efimov considered
^{3}H nucleus (≈3n) and ^{12}C nucleus (≈3α)

△ ^{4}He atoms ($a \approx 1\times10^{-8}$ m $\approx 20 r_0$) ?

2 trimer states were predicted

1 was observed (1994)

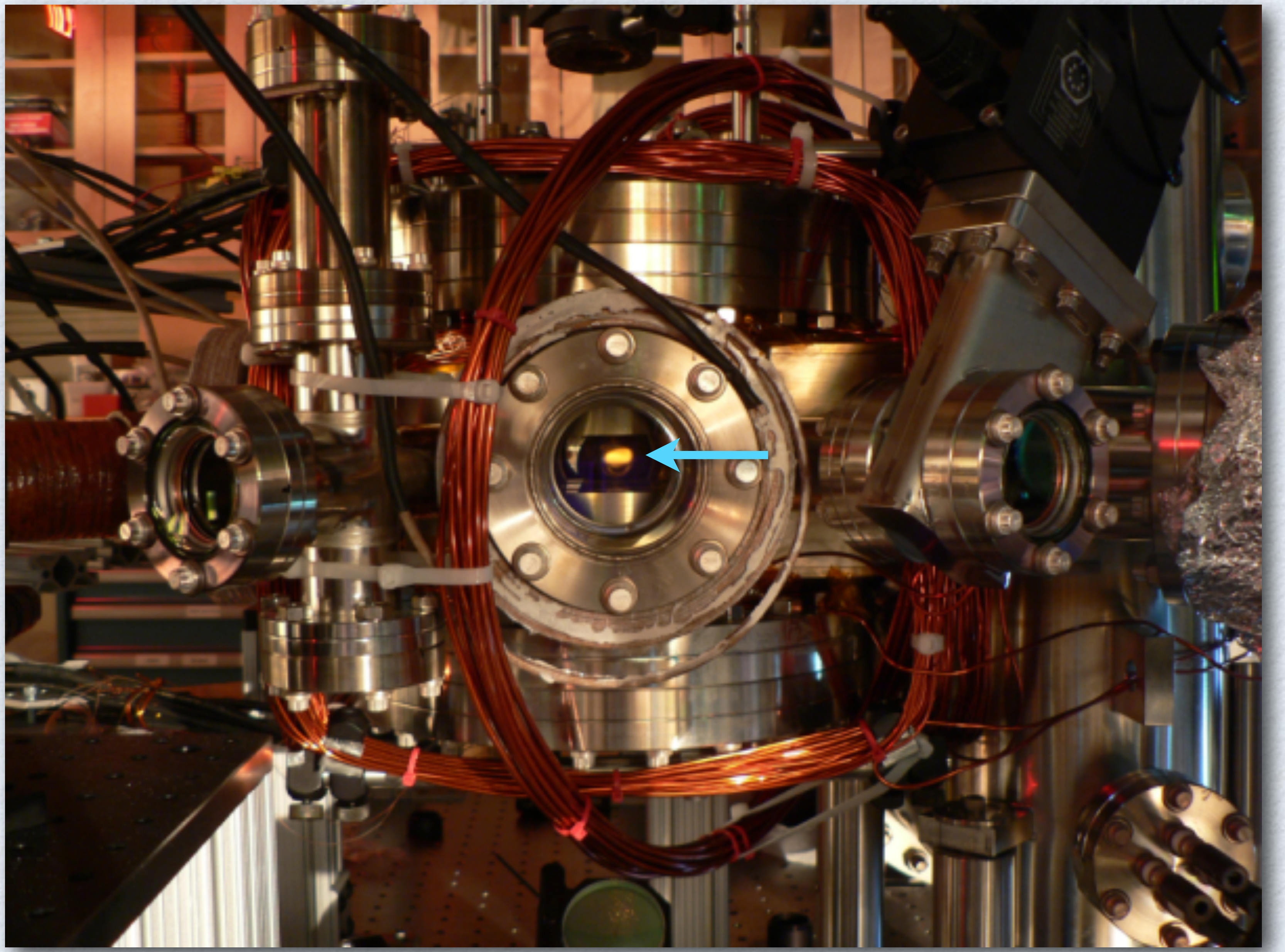


E_b = 125.8 mK   (E_b = 2.28 mK)

➡ Ultracold atoms !

Ultracold atom experiments

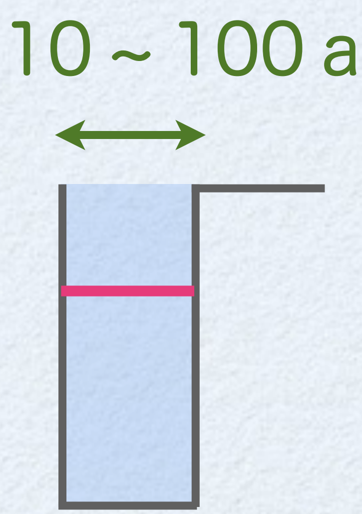
Ultracold atoms are ideal to study universal quantum physics because of the ability to design and control systems at will

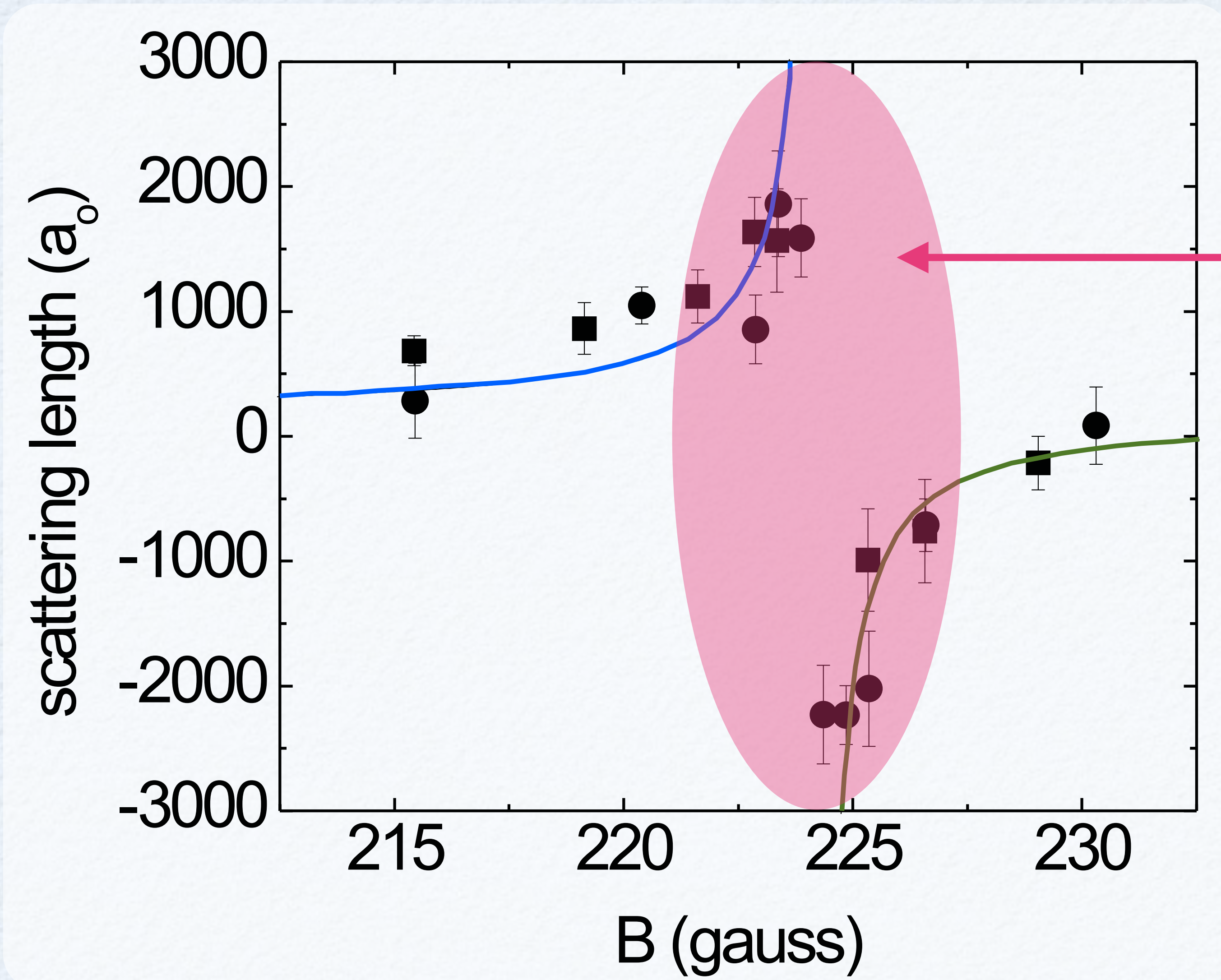


Ultracold atom experiments

Ultracold atoms are ideal to study universal quantum physics because of the ability to design and control systems at will

✓ Interaction strength by Feshbach resonances

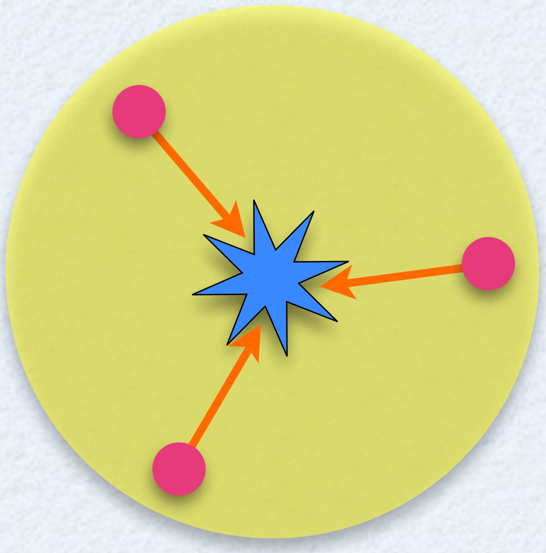




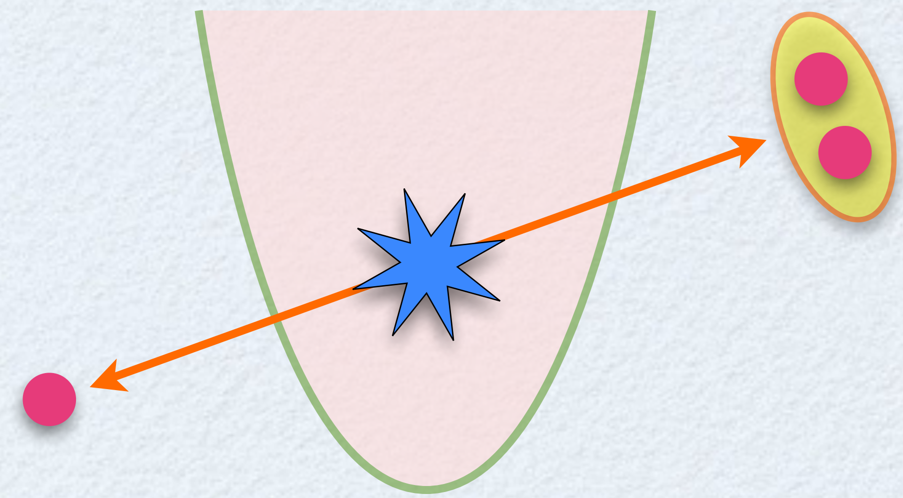
C.A. Regal & D.S. Jin PRL90 (2003)

Ultracold atom experiments

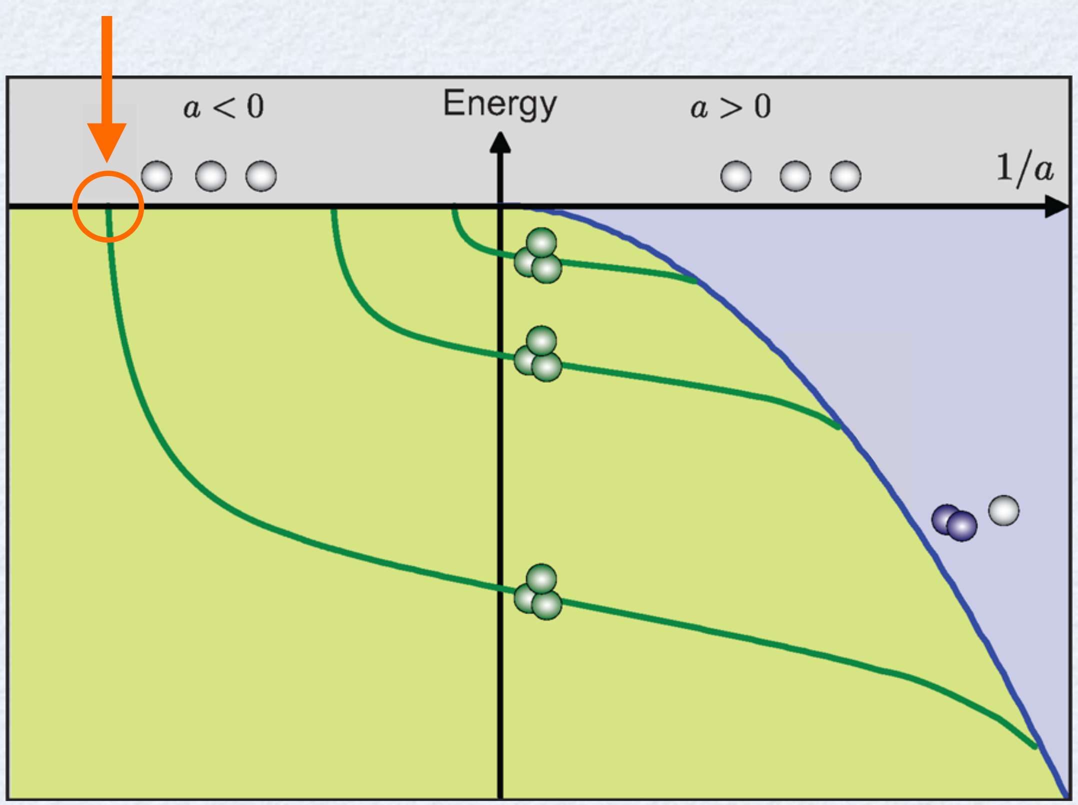
First experiment by Innsbruck group for ^{133}Cs (2006)

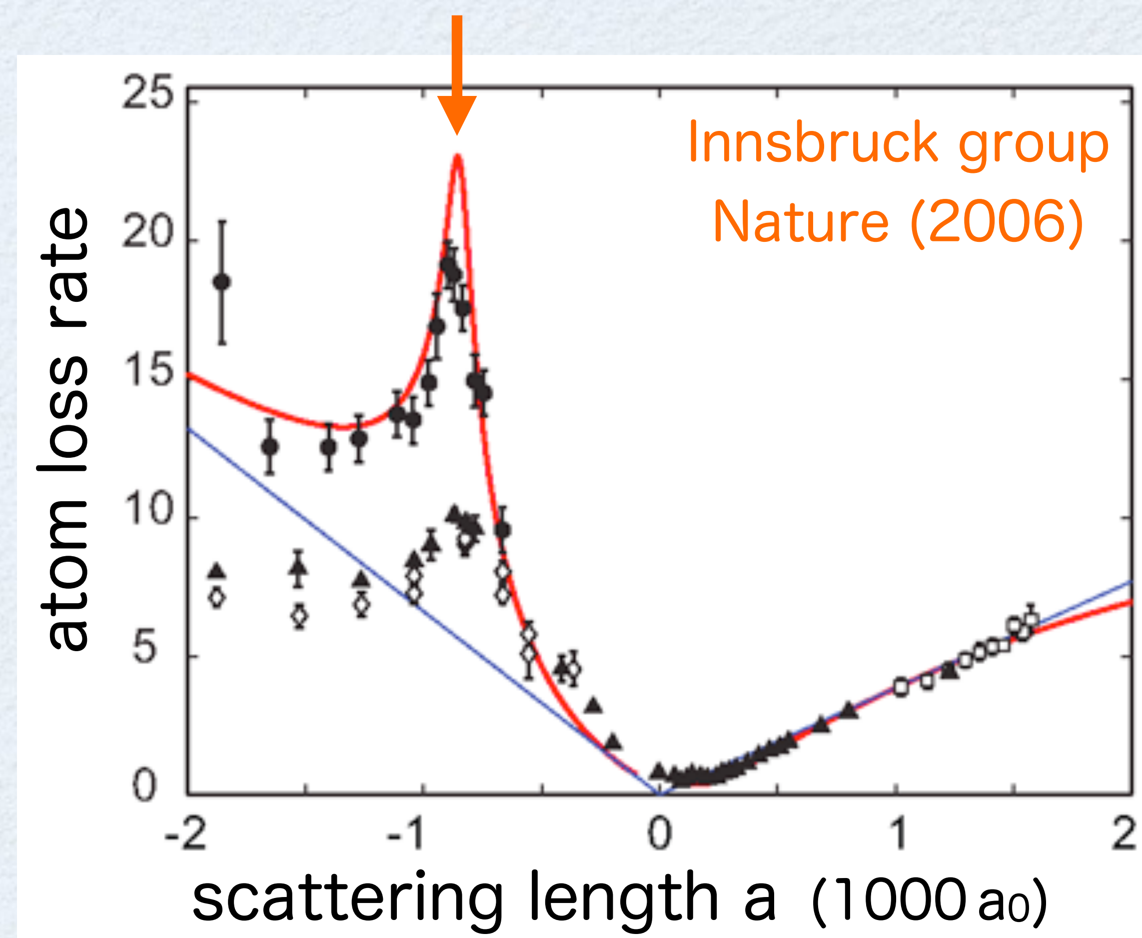


Trimer is unstable → atom loss

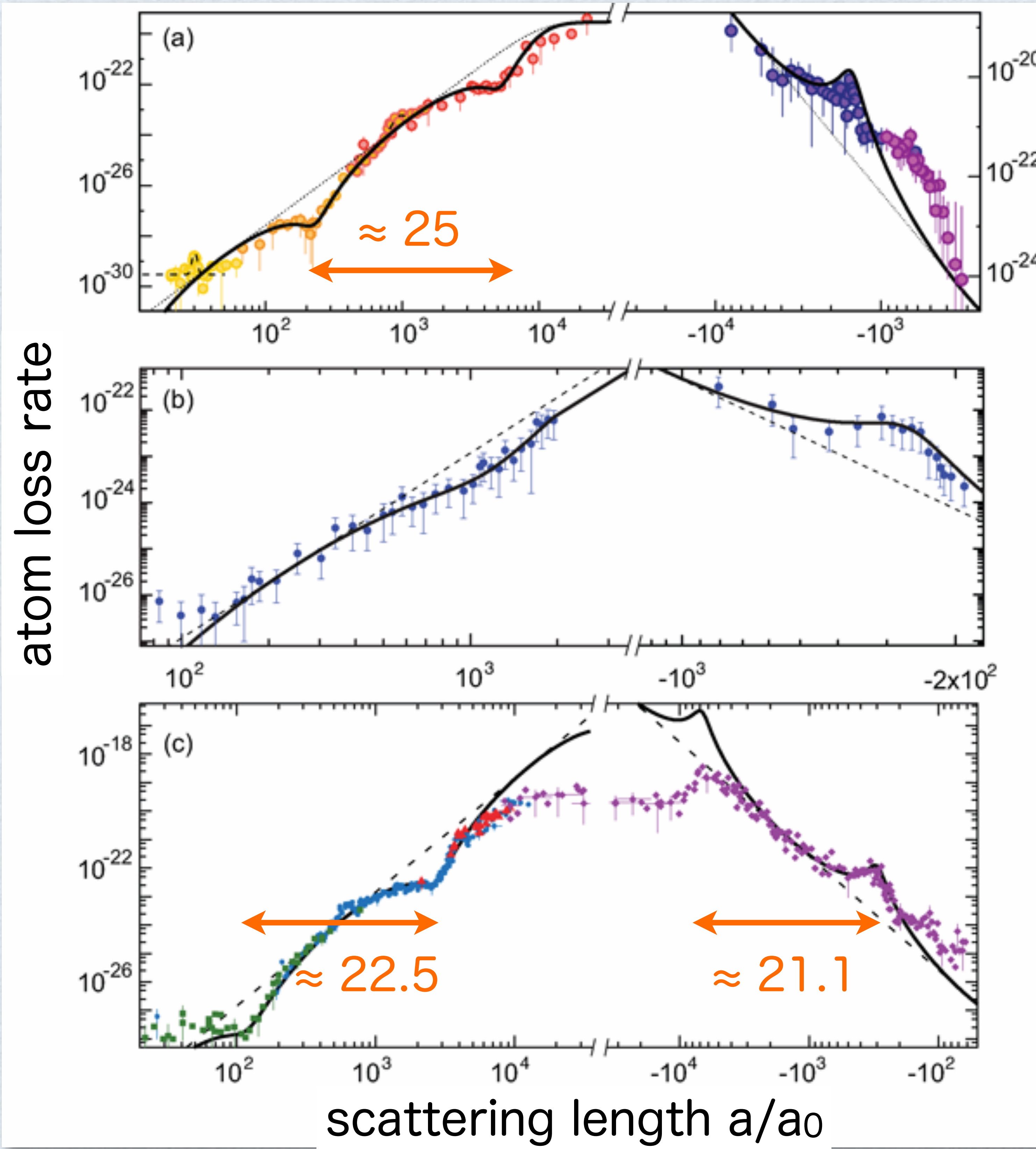


signature of trimer formation





Ultracold atom experiments



Florence group
for ^{39}K (2009)

Bar-Ilan University
for ^{7}Li (2009)

Rice University
for ^{7}Li (2009)

Discrete scaling
& Universality !

Efimov effect is "universal" ?

- Efimov effect is "universal"

  = appears regardless of microscopic details

  (physics technical term)

- Efimov effect is not "universal"

  universal = present or occurring everywhere

  (Merriam-Webster Online)

→ Can we find the Efimov effect in other physical systems ?

Efimov effect beyond cold atoms

1. Universality in physics
2. What is the Efimov effect?
3. **Efimov effect beyond cold atoms**
4. New progress: Super Efimov effect

Quantum magnet

nature physics

ARTICLES

PUBLISHED ONLINE: 13 JANUARY 2013 | DOI: 10.1038/NPHYS2523

Cristian Batista

Efimov effect in quantum magnets

Yusuke Nishida*, Yasuyuki Kato and Cristian D. Batista

Physics is said to be universal when it emerges regardless of the underlying microscopic details. A prominent example is the Efimov effect, which predicts the emergence of an infinite tower of three-body bound states obeying discrete scale invariance when the particles interact resonantly. Because of its universality and peculiarity, the Efimov effect has been the subject of extensive research in chemical, atomic, nuclear and particle physics for decades. Here we employ an anisotropic Heisenberg model to show that collective excitations in quantum magnets (magnons) also exhibit the Efimov effect. We locate anisotropy-induced two-magnon resonances, compute binding energies of three magnons and find that they fit into the universal scaling law. We propose several approaches to experimentally realize the Efimov effect in quantum magnets, where the emergent Efimov states of magnons can be observed with commonly used spectroscopic measurements. Our study thus opens up new avenues for universal few-body physics in condensed matter systems.

Sometimes we observe that completely different systems exhibit the same physics. Such physics is said to be universal and its most famous example is the critical phenomena[1]. In the vicinity of second-order phase transitions where the correlation length diverges, microscopic details become unimportant and the critical phenomena are characterized by only a few ingredients; dimensionality, interaction range and symmetry of the order parameter. Accordingly, fluids and magnets exhibit the same critical exponents. The universality in critical phenomena has been one of the central themes in condensed matter physics.

Similarly, we can also observe universal physics in the vicinity of scattering resonances where the *s*-wave scattering length diverges. Here low-energy physics is characterized solely by the *s*-wave scattering length and does not depend on other microscopic details.

emergent Efimov states of magnons. Our study thus opens up new avenues for universal few-body physics in condensed matter systems. Also, in addition to the Bose–Einstein condensation of magnons[24], the Efimov effect provides a novel connection between atomic and magnetic systems.

Anisotropic Heisenberg model

To demonstrate the Efimov effect in quantum magnets, we consider an anisotropic Heisenberg model on a simple cubic lattice:

$$H = -\frac{1}{2}\sum_{\mathbf{r}}\sum_{\hat{\mathbf{e}}}(J\, S^{+}_{\mathbf{r}} S^{-}_{\mathbf{r}+\hat{\mathbf{e}}} + J_z\, S^{z}_{\mathbf{r}} S^{z}_{\mathbf{r}+\hat{\mathbf{e}}}) - D\sum_{\mathbf{r}}(S^{z}_{\mathbf{r}})^2 - B\sum_{\mathbf{r}} S^{z}_{\mathbf{r}} \qquad (2)$$

where $\sum_{\hat{\mathbf{e}}}$ is a sum over six unit vectors; $\sum_{\hat{\mathbf{e}}=\pm\hat{\mathbf{x}},\pm\hat{\mathbf{y}},\pm\hat{\mathbf{z}}}$. Two types

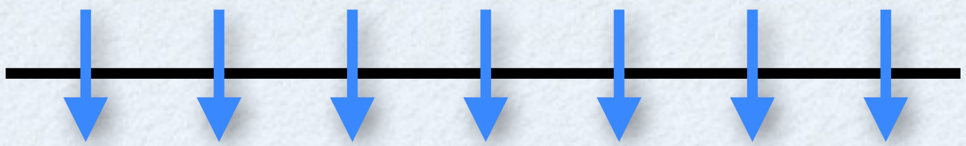
Quantum magnet

Anisotropic Heisenberg model on a 3D lattice

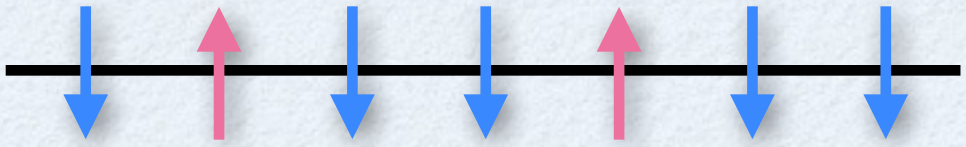
$$H = -\sum_{r}\Big[\sum_{\hat{e}}(J S_r^+ S_{r+\hat{e}}^- + J_z S_r^z S_{r+\hat{e}}^z) + D(S_r^z)^2 - B S_r^z\Big]$$

exchange anisotropy   single-ion anisotropy

Spin-boson correspondence

 ⇔

fully polarized state (B→∞)   No boson = vacuum

 ⇔ 

N spin-flips   N bosons = magnons

Quantum magnet

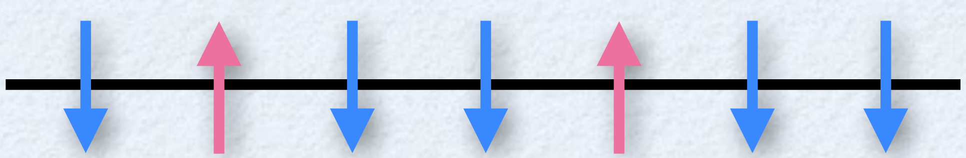
Anisotropic Heisenberg model on a 3D lattice

$$H = -\sum_r \left[\sum_{\hat{e}} \left(J S_r^+ S_{r+\hat{e}}^- + J_z S_r^z S_{r+\hat{e}}^z \right) + D (S_r^z)^2 - B S_r^z \right]$$

xy-exchange coupling
⇔ hopping

single-ion anisotropy
⇔ on-site attraction

z-exchange coupling
⇔ neighbor attraction



⇔

N spin-flips

N bosons = magnons

Quantum magnet

Anisotropic Heisenberg model on a 3D lattice

$$H = -\sum_{r}\Big[\sum_{\hat{e}}(J S_r^+ S_{r+\hat{e}}^- + J_z S_r^z S_{r+\hat{e}}^z) + D(S_r^z)^2 - B S_r^z\Big]$$

xy-exchange coupling
⇔ hopping

single-ion anisotropy
⇔ on-site attraction

z-exchange coupling
⇔ neighbor attraction

Tune these couplings to induce
scattering resonance between two magnons
⇒ Three magnons show the Efimov effect

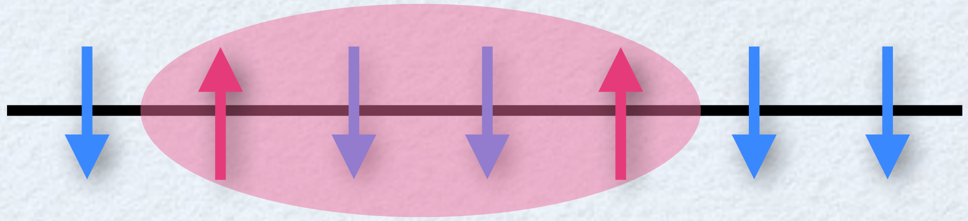
Two-magnon resonance

Scattering length between two magnons

$$\frac{a_s}{a} = \frac{\frac{3}{2\pi}\left[1 - \frac{D}{3J} - \frac{J_z}{J}\left(1 - \frac{D}{6SJ}\right)\right]}{2S - 1 + \frac{J_z}{J}\left(1 - \frac{D}{6SJ}\right) + 1.52\left[1 - \frac{D}{3J} - \frac{J_z}{J}\left(1 - \frac{D}{6SJ}\right)\right]}$$

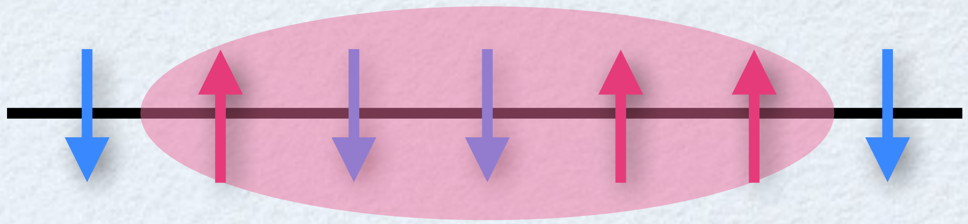
→ Two-magnon resonance ($a_s \to \infty$)

- $J_z/J = 2.94$ (spin-1/2)
- $J_z/J = 4.87$ (spin-1, D=0)
- $D/J = 4.77$ (spin-1, ferro $J_z=J>0$)
- $D/J = 5.13$ (spin-1, antiferro $J_z=J<0$)
- …



Three-magnon spectrum

At the resonance, three magnons form bound states with binding energies E_n



- **Spin-1/2**

n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-2.09×10^{-1}	—
1	-4.15×10^{-4}	22.4
2	-8.08×10^{-7}	22.7

- **Spin-1, D=0**

n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-5.16×10^{-1}	—
1	-1.02×10^{-3}	22.4
2	-2.00×10^{-6}	22.7

- **Spin-1, J_z=J>0**

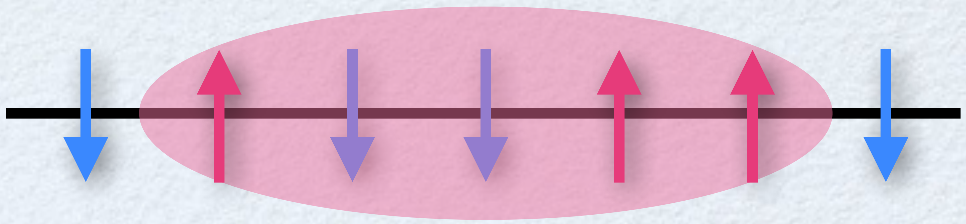
n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-5.50×10^{-2}	—
1	-1.16×10^{-4}	21.8

- **Spin-1, J_z=J<0**

n	E_n/J	$\sqrt{E_{n-1}/E_n}$
0	-4.36×10^{-3}	—
1	-8.88×10^{-6}	22.2

Three-magnon spectrum

At the resonance, three magnons form bound states with binding energies E_n



- Spin-1/2

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0	-5.16×10^{-1}	—
1	-1.02×10^{-3}	22.4
2	-2.00×10^{-6}	22.7

→ **Universal scaling law by ~ 22.7 confirms they are Efimov states !**

Nucleons

VOLUME 91, NUMBER 10 PHYSICAL REVIEW LETTERS week ending 5 SEPTEMBER 2003

An Infrared Renormalization Group Limit Cycle in QCD

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(Received 19 March 2003; published 4 September 2003)

We use effective field theories to show that small increases in the up and down quark masses would move QCD very close to the critical renormalization group trajectory for an infrared limit cycle in the three-nucleon system. We conjecture that QCD can be tuned to the critical trajectory by adjusting the quark masses independently. At the critical values of the quark masses, the binding energies of the deuteron and its spin-singlet partner would be tuned to zero and the triton would have infinitely many excited states with an accumulation point at the 3-nucleon threshold. The ratio of the binding energies of successive states would approach a universal constant that is close to 515.

DOI: 10.1103/PhysRevLett.91.102002 PACS numbers: 12.38.Aw, 11.10.Hi, 21.45.+v

The development of the renormalization group (RG) has had a profound effect on many branches of physics. Its successes range from explaining the universality of critical phenomena in condensed matter physics to the nonperturbative formulation of quantum field theories that describe elementary particles [1]. The RG can be reduced to a set of differential equations that define a flow in the space of coupling constants. Scale-invariant behavior at long distances, as in critical phenomena, can be explained by RG flow to an infrared fixed point. Scale-invariant behavior at short distances, as in asymptotically free field theories, can be explained by RG flow to an ultraviolet fixed point. However, a fixed point is only the simplest topological feature that can be exhibited by a RG flow. dom while leaving the long-distance physics invariant define a *RG flow* on the multidimensional space of coupling constants $\mathbf{g}$ for operators in the Hamiltonian:

$$\Lambda \frac{d}{d\Lambda} \mathbf{g} = \boldsymbol{\beta}(\mathbf{g}), \qquad (1)$$

where Λ is an ultraviolet momentum cutoff. Standard critical phenomena are associated with *infrared fixed points* $\mathbf{g}_*$ of the RG flow, which satisfy $\boldsymbol{\beta}(\mathbf{g}_*) = 0$. The tuning of macroscopic variables to reach a critical point corresponds to the tuning of the coupling constants $\mathbf{g}$ to a *critical trajectory* that flows to the fixed point $\mathbf{g}_*$ in the infrared limit $\Lambda \to 0$. One of the signatures of an RG fixed point is *scale invariance:* symmetry with respect to

Pions

Universal physics of three bosons with isospin

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(Dated: November 26, 2013)

We show that there exist two types of universal phenomena for three-boson systems with isospin degrees of freedom. In the isospin symmetric limit, there is only one universal three-boson bound state with the total isospin one, whose binding energy is proportional to that of the two-boson bound state. With large isospin symmetry breaking, the standard Efimov states of three identical bosons appear at low energies. Both phenomena can be realized by three pions with the pion mass appropriately tuned in lattice QCD simulations, or by spin-one bosons in cold atom experiments. Implication to the in-medium softening of multi-pion states is also discussed.

PACS numbers: 03.65.Ge, 11.30.Rd, 21.65.Jk, 67.85.Fg

Introduction.— The properties of particles interacting with a large scattering length are universal, *i.e.*, they are determined irrespective of the short range behavior of the interaction. In particular, three-particle systems with a large two-body scattering length lead to the emergence of the Efimov states [1], which has been extensively studied in cold atom physics [2]. Moreover, in condensed matter physics, collective excitations in quantum magnets are shown to exhibit the Efimov effect [3].

Since the intrinsic energy scale of the system is not relevant for such universal phenomena, they could be also realized in strong interaction governed by quantum chromodynamics (QCD) as far as suitable conditions are met. In fact, a theoretical possibility of achieving the

the lattice by changing the quark mass. From the point of view of the statistical noise, three pions with heavy quark mass is much less costly than the three nucleons with light quark mass [10]. In this sense, the three-pion system is an ideal testing ground for the universal physics in QCD.

Universal physics with the isospin symmetry.— Let us first consider the three-pion system with exact isospin symmetry. We assume that, by an appropriate tuning of the quark mass, only the s-wave $\pi\pi$ scattering length in the $I = 0$ channel, $|a_{I=0}|$, becomes much larger than the typical length scale R characterized by the interaction range. In addition, we consider momentum p sufficiently smaller than $1/R$, so that the pions can be treated

Halo nuclei

PRL **111,** 132501 (2013) PHYSICAL REVIEW LETTERS week ending 27 SEPTEMBER 2013

Efimov Physics Around the Neutron-Rich ^{60}Ca Isotope

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We calculate the neutron-^{60}Ca S-wave scattering phase shifts using state of the art coupled-cluster theory combined with modern *ab initio* interactions derived from chiral effective theory. Effects of three-nucleon forces are included schematically as density dependent nucleon-nucleon interactions. This information is combined with halo effective field theory in order to investigate the ^{60}Ca-neutron-neutron system. We predict correlations between different three-body observables and the two-neutron separation energy of ^{62}Ca. This provides evidence of Efimov physics along the calcium isotope chain. Experimental key observables that facilitate a test of our findings are discussed.

DOI: 10.1103/PhysRevLett.111.132501 PACS numbers: 21.10.Gv, 21.60.−n, 27.50.+e

Introduction.– ... dom is one of t... along the neutro... characterized by... valence nucleon... effective degree... an extremely larg... one- or two-nucleon separation energy along an isotope chain. The features of these halos are universal if the small separation energy of the valence nucleons is associated with

...st is still an open ... interest, both ...rmining precise ...olution and the ...alcium isotopes ...eutron rich cal- ... the scattering continuum and schematic three-nucleon forces, suggested that there is an inversion of the *gds* shell-model orbitals in ^{53,55,61}Ca. In particular it was suggested that a large S-wave

Other possible systems :

^{11}Li = ^{9}Li+n+n ^{20}C = ^{18}C+n+n

DNA chains

PRL **110,** 028105 (2013) PHYSICAL REVIEW LETTERS week ending 11 JANUARY 2013

Renormalization Group Limit Cycle for Three-Stranded DNA

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We show that there exists an Efimov-like three strand DNA bound state at the duplex melting point and it is described by a renormalization group limit cycle. A nonperturbative renormalization group is used to obtain this result in a model involving short range pairing only. Our results suggest that Efimov physics can be tested in polymeric systems.

DOI: 10.1103/PhysRevLett.110.028105 PACS numbers: 87.14.gk, 64.60.ae, 87.15.Zg

Consider a three-particle quantum system with a pair-wise short-range potential. Apart from the occurrence of the usual three-body bound state, a very special phenomenon occurs at the critical two-body zero-energy state. An infinite number of three-body bound states appear though the corresponding potential is not appropriate to bind any two of them; the removal of any one of them destroys the bound state. This phenomenon, valid for any short-range interaction, is known as the Efimov effect. The size of the three-body bound states, or Efimov trimers, is large compared to the potential range, and so it is a purely quantum effect [1]. Although it was predicted in the context of nuclear physics [2,3], it has now been detected in cold atoms [4].

An ideal DNA consisting of two Gaussian polymers interacting with native base pairing undergoes a critical melting transition where the two strands get detached. Maji *et al.* recently showed that if, to a double-stranded DNA at point is of a different type. This "few-chain problem" is actually described by a renormalization group "limit cycle"[2,8]. The appearance of a limit cycle invokes log periodicity in the corresponding three-body coupling in the polymer problem. So they break the continuous scale invariance around the two-body fixed point imposing a discrete scaling symmetry, the hallmark of the Efimov states.

Another motivation of this Letter is to emphasize that a three-chain polymer model, a three-stranded DNA in particular, by virtue of mathematical similarities, provides an alternative system for Efimov physics. Triplex DNA is known to occur in nature. The possibility of recognizing the bound base pairs of a duplex without opening it, by forming Hoogsteen pairs, has the potentiality of designing new types of antibiotics. In addition, H-DNA is a common motif formed during many DNA activities where there is a stretch of triplex DNA via a strand exchange mechanism

New progress

1. Universality in physics
2. What is the Efimov effect?
3. Efimov effect beyond cold atoms
4. **New progress: Super Efimov effect**

Super Efimov effect

PRL **110,** 235301 (2013) PHYSICAL REVIEW LETTERS week ending 7 JUNE 2013

Super Efimov Effect of Resonantly Interacting Fermions in Two Dimensions

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We study a system of spinless fermions in two dimensions with a short-range interaction fine-tuned to a p-wave resonance. We show that three such fermions form an infinite tower of bound states of orbital angular momentum $\ell = \pm 1$ and their binding energies obey a universal doubly exponential scaling $E_3^{(n)} \propto \exp(-2e^{3\pi n/4+\theta})$ at large n. This "super Efimov effect" is found by a renormalization group analysis and confirmed by solving the bound state problem. We also provide an indication that there are $\ell = \pm 2$ four-body resonances associated with every three-body bound state at $E_4^{(n)} \propto \exp(-2e^{3\pi n/4+\theta-0.188})$. These universal few-body states may be observed in ultracold atom experiments and should be taken into account in future many-body studies of the system.

DOI: 10.1103/PhysRevLett.110.235301 PACS numbers: 67.85.Lm, 03.65.Ge, 05.30.Fk, 11.10.Hi

Introduction.—Recently topological superconductors have attracted great interest across many subfields in physics [1,2]. This is partially because vortices in topological superconductors bind zero-energy Majorana fermions and obey non-Abelian statistics, which can be of potential use for fault-tolerance topological quantum computation [3,4]. A canonical example of such topological superconductors is a p-wave paired state of spinless fermions in two dimensions [5], which is believed to be realized in Sr_2RuO_4 [6]. Previous mean-field studies revealed that a topological quantum phase transition takes place across a

of resonantly interacting fermions in two dimensions should be taken into account in future many-body studies beyond the mean-field approximation.

Renormalization group analysis.—The above predictions can be derived most conveniently by a renormalization group (RG) analysis. The most general Lagrangian density that includes up to marginal couplings consistent with rotation and parity symmetries is

$$\mathcal{L} = \psi^\dagger\left(i\partial_t + \frac{\nabla^2}{2}\right)\psi + \phi_a^\dagger\left(i\partial_t + \frac{\nabla^2}{4} - \varepsilon_0\right)\phi_a$$

Efimov vs. Super Efimov

Efimov effect

(Efimov in 1970)

- 3 bosons
- 3 dimensions
- s-wave resonance
- exponential scaling

$$\lambda_n \sim e^{\pi n}$$

Super Efimov effect

(Y.N, S.M, D.T.S in 2013)

- 3 fermions
- 2 dimensions
- p-wave resonance
- “doubly” exponential

$$\lambda_n \sim e^{e^{3\pi n/4}}$$

Are there other phenomena
with doubly-exponential scaling ?

Hyper-inflation (economics)

東工大・高安研究室HP

2. ２重指数関数成長

２重指数関数的な変動は、ハイパーインフレーションと呼ばれる現象において一般的に観測される事が示されています。 これは、上述の指数関数成長時のXが一定値ではなく、時間の指数関数に従っている事を示しており、指数関数の肩に指数関数が乗っている事から、２重指数関数と呼ばれます。

Wikipedia

ハンガリー

ハンガリーでは第二次世界大戦後に激しいハイパーインフレが発生した。このときのインフレーションでは16年間で貨幣価値が1垓3000京分の1になったが、20桁以上のインフレーションは1946年前半の半年間に起きたものである。大戦後、1945年末まではインフレ率がほぼ一定であり、対ドルレートは指数関数的増大にとどまっていたが、1946年初頭からはインフレ率そのものが指数関数的に増大した。別の表現でいえば、物価が2倍になるのにかかる時間が、1か月、1週間、3日とだんだんと短くなっていったということである。当時を知るハンガリー人によると、一日で物価が2倍になる状況でも市場では紙幣が流通しており、現金を入手したものは皆、すぐに使ったという[26]。

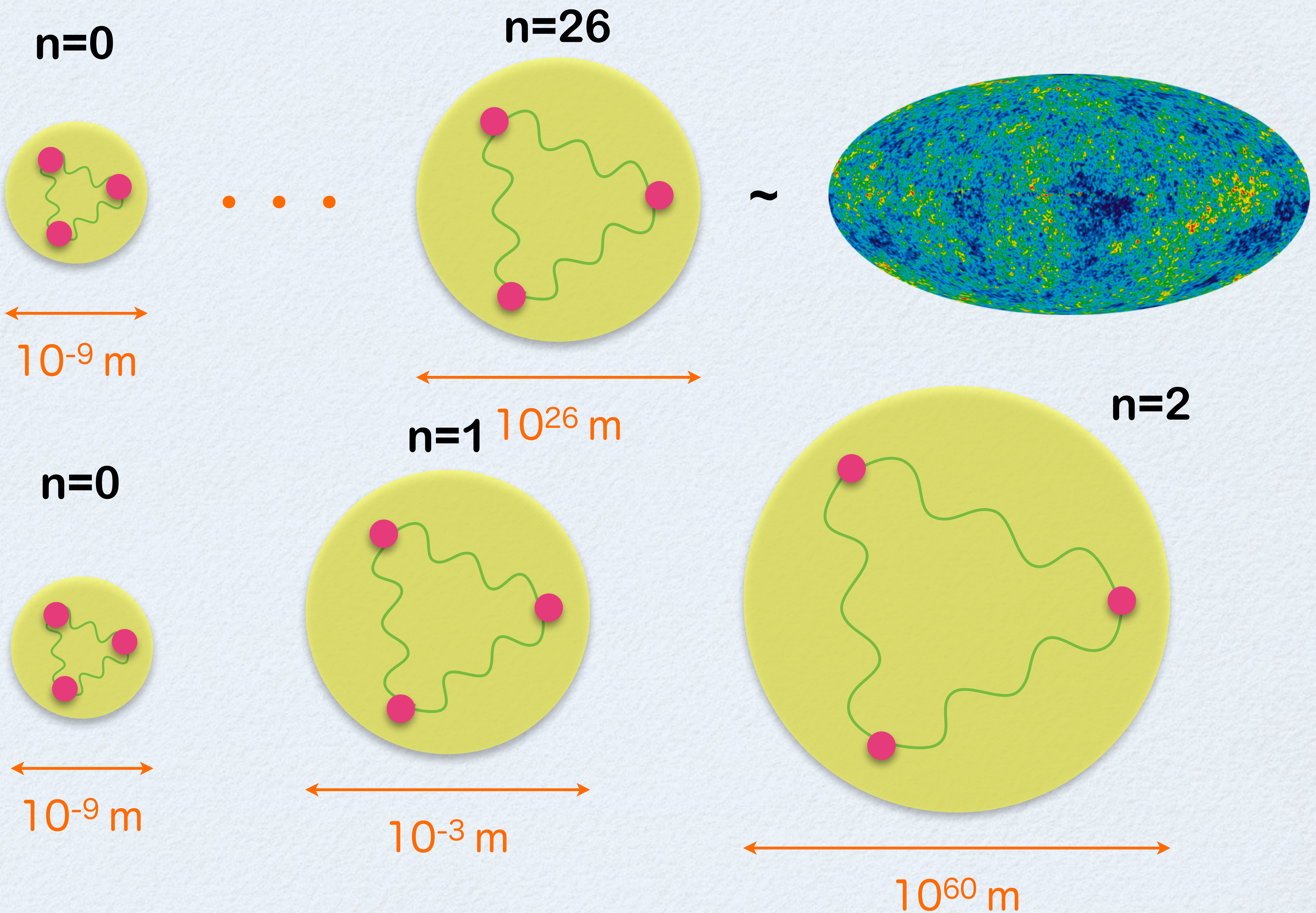
1946年に印刷された10垓ペンゲー紙幣（紙幣には10億兆と書かれている）が歴史上の最高額面紙幣であり（ただし、発行はされていない）、最悪のインフレーションとしてギネスブックに記録されている。

なお、実際に発行された最高額面紙幣は1垓ペンゲー紙幣（紙幣には1億兆と書かれている）である。

※1京は1兆の1万倍（10の16乗）、1垓は1京の1万倍（10の20乗）。

Are there other “physics” phenomena with doubly-exponential scaling ?

Efimov vs. Super Efimov



Summary

Efimov effect: universality, discrete scale inv, quantum anomaly, RG limit cycle

atomic physics

nuclear physics

condensed matter

✓ Efimov effect in quantum magnets

Y.N, Y.K, C.D.B, Nature Physics 9, 93-97 (2013)

✓ New progress: Super Efimov effect

Y.N, S.M, D.T.S, Phys.Rev.Lett.110, 235301 (2013)