

Quasi-particle resonance near threshold in neutron drip-line nuclei



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Contents

- Introduction and motivation
- HFB equation with scattering boundary condition
- Results (elastic cross section, resonance width, etc)
- Conclusion

Resonance caused by pairing

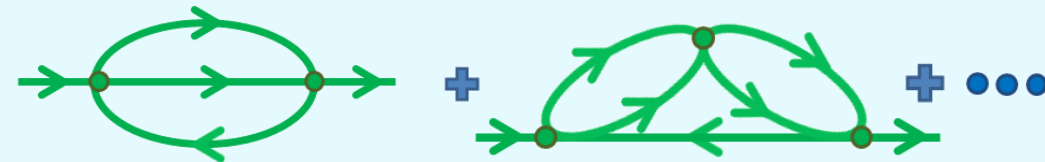
- **Quasi-particle resonance** is predicted to emerge in superfluid nuclei.

S. T. Belyaev et al., Sov. J. Nucl. Phys, 45 783 (1987)

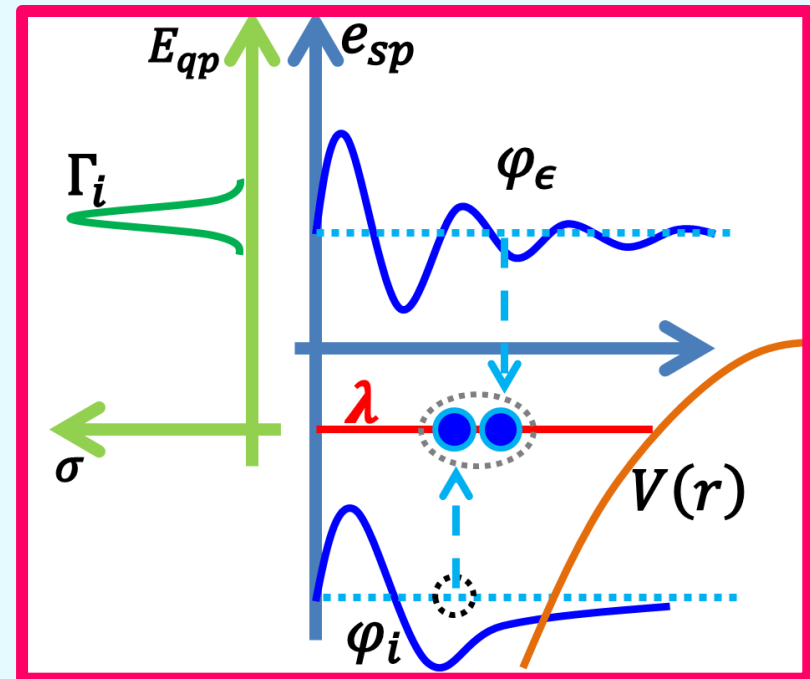
A. Bulgac, Preprint(1980); nucl-th/9907088

J. Dobaczewski et al., Phys. Rev. C 53 2809 (1996)

- In Hartree-Fock-Bogoliubov (HFB) theory, an unbound nucleon can couple to a hole state by creating a Cooper pair, and resulting in a quasi-particle resonance.



Single-particle orbit	Quasi-particle resonance emerge
Deep-hole orbit in stable nuclei	in high energy region
Weakly bound orbit in drip-line nuclei	in low energy region



We expect novel property in weakly bound system.

In stable nuclei

$$\lambda \cong -8.0\text{MeV}$$

The q.p. resonance can emerge above $E_x \geq 8.0\text{MeV}$
(Level density is very high.)

$$(\epsilon_i - \lambda)^2 \gg \Delta^2$$

Resonance width

$$\Gamma_i = 2\pi \left| \int d^3r \varphi_i(\vec{r}) \Delta(\vec{r}) \varphi_\epsilon(\vec{r}) \right|^2 \propto |\Delta|^2$$

S. T. Belyaev et al., Sov. J. Nucl. Phys, 45 783 (1987)
A. Bulgac, Preprint(1980); nucl-th/9907088
J. Dobaczewski et al., Phys. Rev. C 53 2809 (1996)

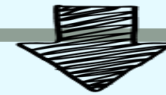
$$\Delta \nearrow \quad \Gamma_i \nearrow$$

In drip-line nuclei

$$\lambda \cong 0.0 \sim -1.0\text{MeV}$$

The q.p. resonance can emerge in **low energy region**. $E_x \leq 2.0\text{MeV}$
→ maybe observe experimentally.

$$(\epsilon_i - \lambda)^2 \leq \Delta^2$$



Pairing influence weakly
bound orbit and low-lying
resonance strongly.

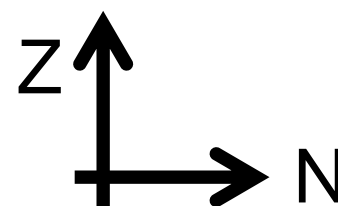
Resonance width

The perturbative equation
may not be applied.

We focus on low-lying p- and s- resonance.

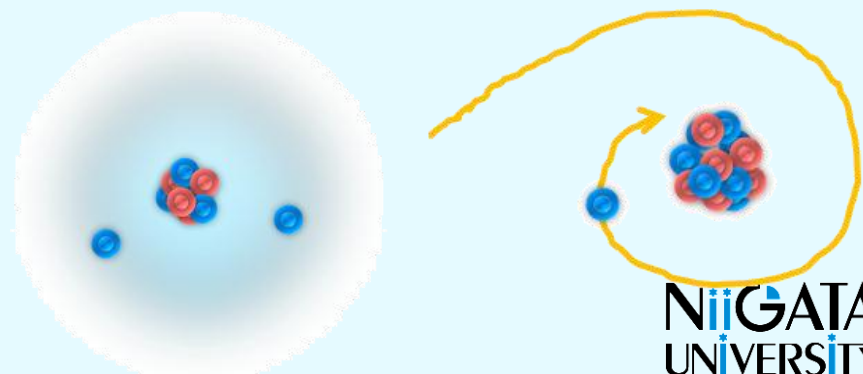
		Sc 30 0.0132s	Sc 37 0.0326s	Sc 38 0.0513s	Sc 39 0.121s	Sc 40 0.1823s	Sc 41 0.5963s	Sc 42 1.033m	Sc 43 3.891h	Sc 44 2.442d	Sc 45 100	Sc 46 83.79d	Sc 47 3.349d	Sc 48 1.82d	Sc 49 57.18m	Sc 50 1.708m	Sc 51 12.4s	Sc 52 8.2s	Sc 53 3s	Sc 54 0.292s
	Ca 34 0.0108s	Ca 35 0.05s	Ca 36 0.1s	Ca 37 0.175s	Ca 38 0.44s	Ca 39 0.8596s	Ca 40 96.941	Ca 41 1.02e+05y	Ca 42 0.647	Ca 43 0.135	Ca 44 2.086	Ca 45 162.6d	Ca 46 0.004	Ca 47 4.536d	Ca 48 0.187	Ca 49 8.718m	Ca 50 13.9s	Ca 51 10s	Ca 52 4.6s	Ca 53 0.09s
	K 33 0.0386s	K 34 0.0702s	K 35 0.19s	K 36 0.342s	K 37 1.226s	K 38 7.636m	K 39 93.2581	K 40 0.0117	K 41 6.7302	K 42 12.36h	K 43 22.3h	K 44 22.13m	K 45 17.81m	K 46 1.75m	K 47 17.5s	K 48 6.8s	K 49 1.26s	K 50 0.472s	K 51 0.365s	K 52 0.105s
Ar 31 0.015s	Ar 32 0.098s	Ar 33 0.173s	Ar 34 0.8445s	Ar 35 1.775s	Ar 36 0.3365	Ar 37 35.04d	Ar 38 0.0632	Ar 39 269y	Ar 40 99.6003	Ar 41 1.827h	Ar 42 32.9y	Ar 43 5.37m	Ar 44 11.87m	Ar 45 21.48s	Ar 46 8.4s	Ar 47 1.23s	Ar 48 0.475s	Ar 49 0.17s	Ar 50 0.085s	Ar 51 0.0141s
Cl 30 0.0415s	Cl 31 0.15s	Cl 32 0.298s	Cl 33 2.511s	Cl 34 32m	Cl 35 75.78	Cl 36 3.01e+05y	Cl 37 24.22	Cl 38 37.23m	Cl 39 55.6m	Cl 40 1.35m	Cl 41 38.4s	Cl 42 6.9s	Cl 43 3.07s	Cl 44 0.56s	Cl 45 0.4s	Cl 46 0.223s	Cl 47 0.101s	Cl 48 0.0175s	Cl 49 0.0103s	Cl 50 0.00644s
S 29 0.187s	S 30 1.178s	S 31 2.572s	S 32 94.93	S 33 0.76	S 34 4.29	S 35 87.51d	S 36 0.02	S 37 5.05m	S 38 2.838h	S 39 11.5s	S 40 8.8s	S 41 1.99s	S 42 1.013s	S 43 0.22s	S 44 0.1s	S 45 0.068s	S 46 0.05s	S 47 0.0109s	S 48 0.00632s	
P 28 0.2703s	P 29 4.142s	P 30 2.498m	P 31 100	P 32 14.26d	P 33 25.34d	P 34 12.43s	P 35 47.3s	P 36 5.6s	P 37 2.31s	P 38 0.64s	P 39 0.28s	P 40 0.153s	P 41 0.15s	P 42 0.11s	P 43 0.033s	P 44 0.0185s	P 45 0.0061s	P 46 0.00472s	P 47 0.00305s	
Si 27 4.16s	Si 28 92.2297	Si 29 4.6832	Si 30 3.0872	Si 31 2.622h	Si 32 153y	Si 33 6.18s	Si 34 2.77s	Si 35 0.78s	Si 36 0.45s	Si 37 0.09s	Si 38 0.102s	Si 39 0.0475s	Si 40 0.033s	Si 41 0.02s	Si 42 0.0125s	Si 43 0.0056s	Si 44 0.00352s		Si 46 0.00195s	
Al 26 7.17e+05y	Al 27 100	Al 28 2.241m	Al 29 6.56m	Al 30 3.6s	Al 31 0.644s	Al 32 0.033s	Al 33 0.0417s	Al 34 0.0563s	Al 35 0.15s	Al 36 0.09s	Al 37 0.0107s	Al 38 0.0076s	Al 39 0.0076s	Al 40 0.00374s	Al 41 0.0022s	Al 42 0.00146s	Al 43 0.000989s			
Mg 25 10	Mg 26 11.01	Mg 27 9.458m	Mg 28 20.92h	Mg 29 1.3s	Mg 30 0.335s	Mg 31 0.23s	Mg 32 0.086s	Mg 33 0.0905s	Mg 34 0.02s	Mg 35 0.07s	Mg 36 0.0039s	Mg 37 0.00483s	Mg 38 0.00259s	Mg 39 0.00197s	Mg 40 0.00122s					
Na 24 15h	Na 25 59.1s	Na 26 1.077s	Na 27 0.301s	Na 28 0.0305s	Na 29 0.0449s	Na 30 0.048s	Na 31 0.017s	Na 32 0.0132s	Na 33 0.008s	Na 34 0.0055s	Na 35 0.0015s									
Ne 23 37.24s	Ne 24 3.38m	Ne 25 0.602s	Ne 26 0.197s	Ne 27 0.032s	Ne 28 0.019s	Ne 29 0.0148s	Ne 30 0.0073s	Ne 31 0.0034s	Ne 32 0.0035s		Ne 34 0.0017s									
F 22 4.23s	F 23 2.3s	F 24 0.39s	F 25 0.05s	F 26 0.0096s	F 27 0.0052s		F 29 0.0024s		F 31 0.00156s											
O 21 3.42s	O 22 2.3s	O 23 0.097s	O 24 0.061s		O 26 0.00535s															
N 20 0.13s	N 21 0.095s	N 22 0.024s	N 23 0.0141s																	
C 19 0.049s	C 20 0.014s		C 22 0.0061s																	
	B 10 0.00292s																			

**Weakly bound
2p-orbit
(This study)**



WWW Chart of the Nuclides 2010

- Low-angular momentum wave play an important role in weakly bound nuclei.
Ex.) halo, capture etc...



How the width is governed by the pairing ?

- Previous works of the quasi-particle resonance in drip-line nuclei.

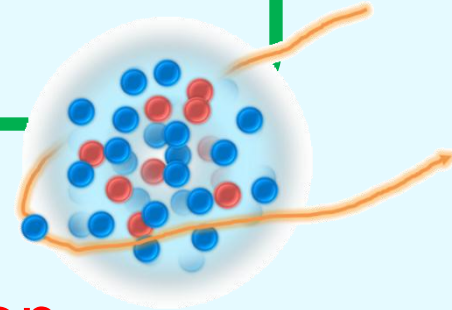
M. Grasso et al., Phys. Rev. C 64 064321 (2001)

I. Hamamoto et al., Phys. Rev. C 68 034312 (2003)

J. C. Pei et al., Phys. Rev. C 84 024311 (2011)

- Our study:

- ✓ How the resonance width is governed by the pairing in neutron drip-line nuclei.
- ✓ What is different point from the resonance associate with deep-hole state.

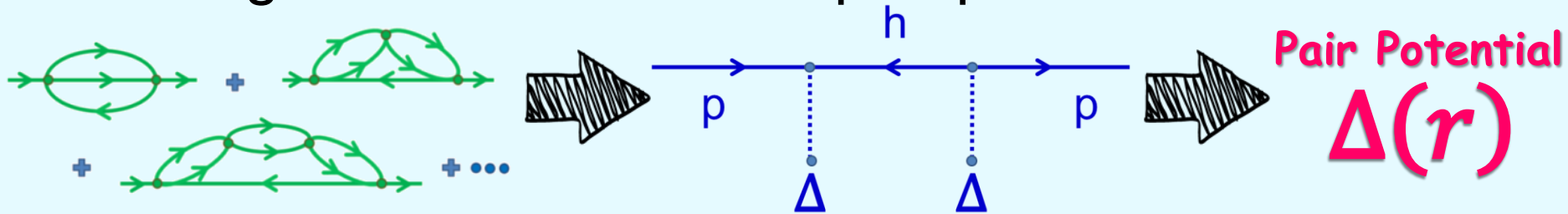


- We consider,
a neutron drip-line nucleus + one neutron
with the HFB theory in the coordinate space (cHFB).

J. Dobaczewski et al., Nucl. Phys. A 422 103 (1984)

The cHFB equation for resonance

- Pairing effects describe as pair potential in cHFB.



- The HFB equation in the coordinate space

$$\begin{pmatrix} -\frac{\hbar^2}{2m} \frac{d^2}{dr^2} + U_{ij}(r) - \lambda & \Delta(r) \\ \Delta(r) & \frac{\hbar^2}{2m} \frac{d^2}{dr^2} - U_{ij}(r) + \lambda \end{pmatrix} \begin{pmatrix} u_{ij}(r) \\ v_{ij}(r) \end{pmatrix} = E \begin{pmatrix} u_{ij}(r) \\ v_{ij}(r) \end{pmatrix} \quad \psi(\vec{r}, \sigma) = \frac{1}{r} \begin{pmatrix} u_{ij}(r) \\ v_{ij}(r) \end{pmatrix} [Y_l(\theta, \phi) \chi_{\frac{1}{2}}(s_z)]_{jm}$$

J. Dobaczewski et al., Nucl. Phys. A 422 103 (1984)

- Scattering boundary condition of quasi-particle

$$\frac{1}{r} \begin{pmatrix} u_{ij}(r) \\ v_{ij}(r) \end{pmatrix} \xrightarrow{r \gg R} \begin{pmatrix} \cos \delta_{lj} j_l(k_1 r) - \sin \delta_{lj} n_l(k_1 r) \\ Dh_l^{(1)}(ik_2 r) \end{pmatrix} \quad k_1 = \sqrt{\frac{2m(\lambda + E)}{\hbar^2}}, \quad \kappa_2 = \sqrt{-\frac{2m(\lambda - E)}{\hbar^2}}$$

→ phase shift
elastic cross section

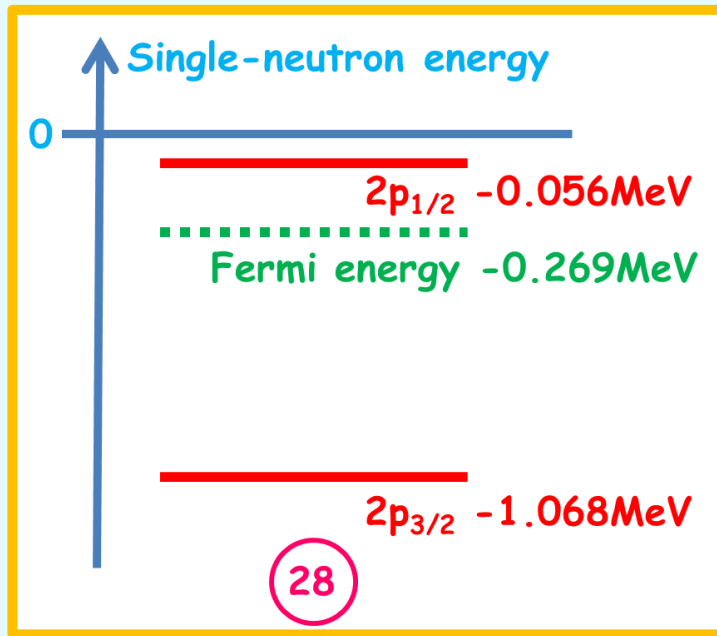
S. T. Belyaev et al., Sov. J. Nucl. Phys, 45 783 (1987)

M. Grasso et al., Phys. Rev. C 64 064321 (2001)

I. Hamamoto et al., Phys. Rev. C 68 034312 (2003)

Numerical analysis with ($^{46}\text{Si} + n$)

- We consider the ($^{46}\text{Si} + n$) system.



✓ drip-line nucleus for Si isotopes.

✓ deformation is small.

Ex) M. V. Stoitsov et al. PRC68. 054312 (2003)

← Energy levels of ^{46}Si
(with Woods-Saxon potential)

• Weakly bound 2p orbits

✓ Fermi energy -0.269MeV

☆ Nuclear potential & pair potential = Woods-Saxon type

$$U_{ij}(r) = \left[V_0 + (\vec{l} \cdot \vec{s}) V_{so} \frac{r_0^2}{r} \frac{d}{dr} \right] f_{ws}(r)$$

$$\Delta(r) = \Delta_0 f_{ws}(r)$$

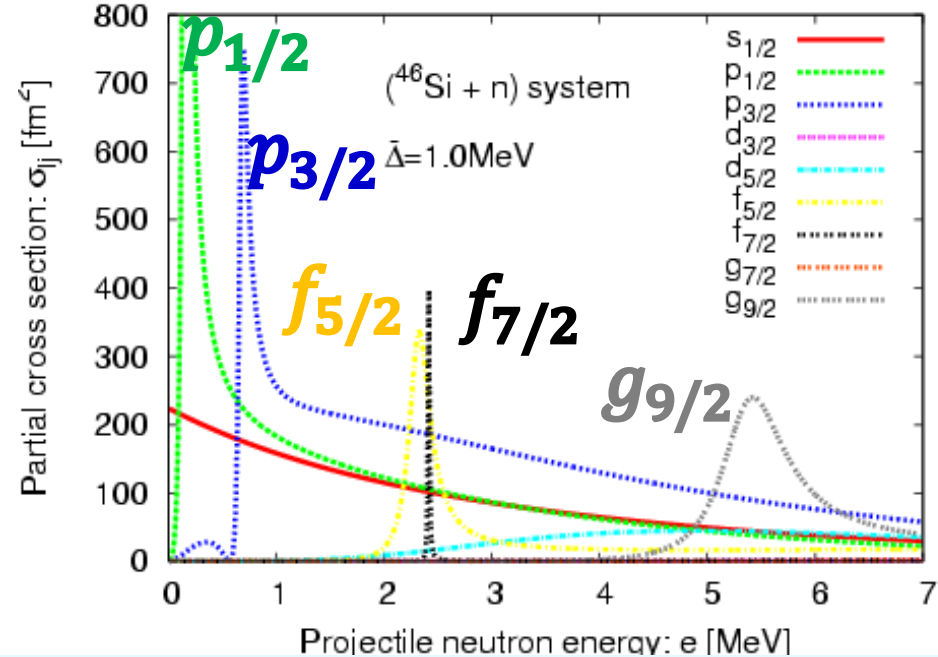
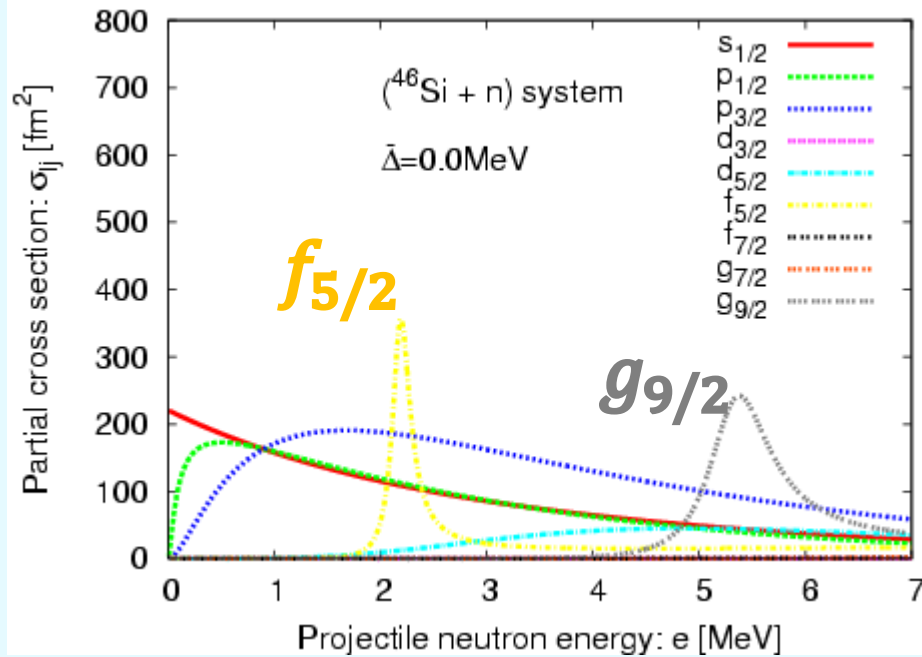
$$f_{ws}(r) = \left[1 + \exp\left(\frac{r-R}{a}\right) \right]^{-1}$$

The averaged strength of the pair field

$$\bar{\Delta} = \frac{\int d\vec{r} r^2 \Delta(r) f_{ws}(r)}{\int d\vec{r} r^2 f_{ws}(r)}$$

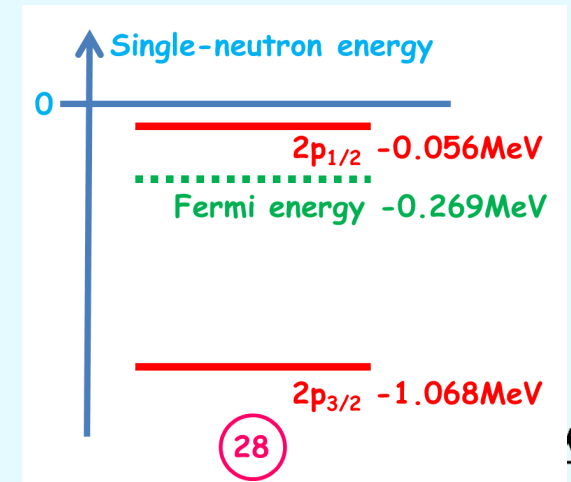
I. Hamamoto, B. R. Mottelson, Phys. Rev. C 68 034312 (2003)

Elastic cross section

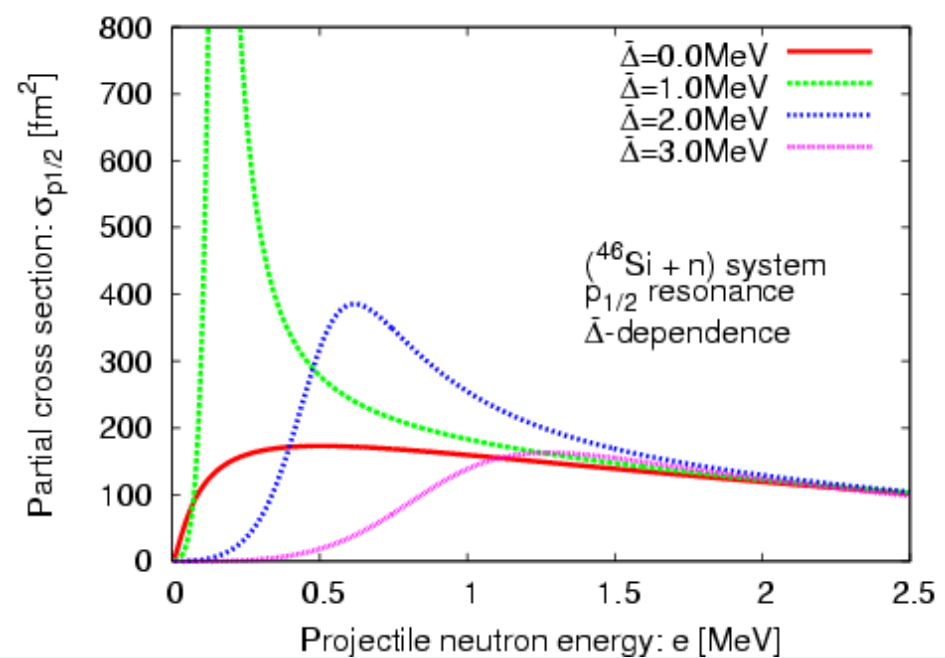
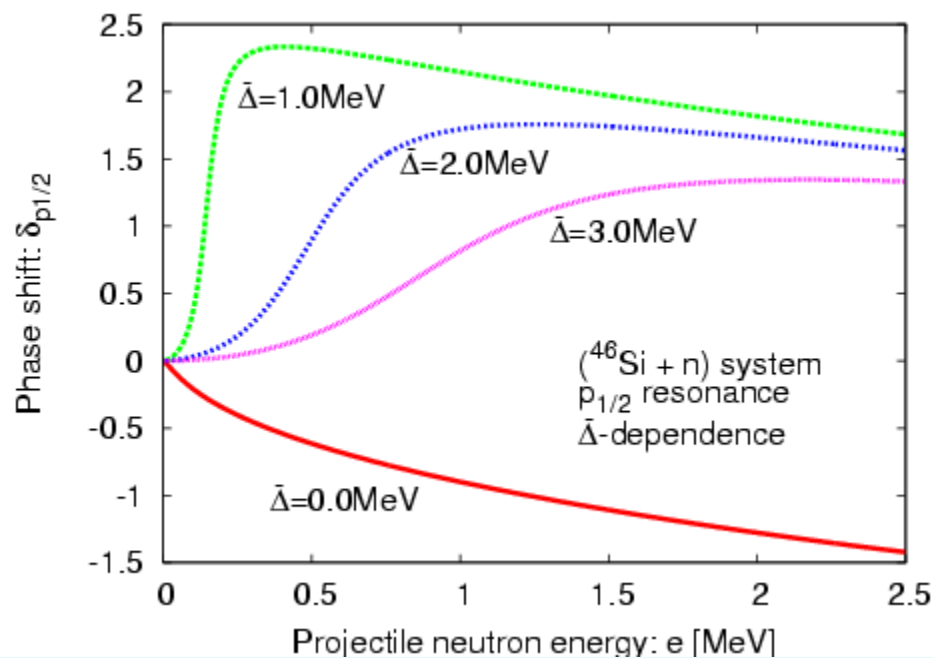


○ When pairing is switched on, the $2p_{1/2}$ ($2p_{3/2}$) orbits emerges as a resonance

→ weakly bound orbits emerge as resonances by the pairing effect



We focus on $p_{1/2}$ resonance



- The strength of pairing.

$$\bar{\Delta} = 0.0 \sim 3.0 \text{ MeV}$$

- Resonance energy and width depend on strength of the pairing

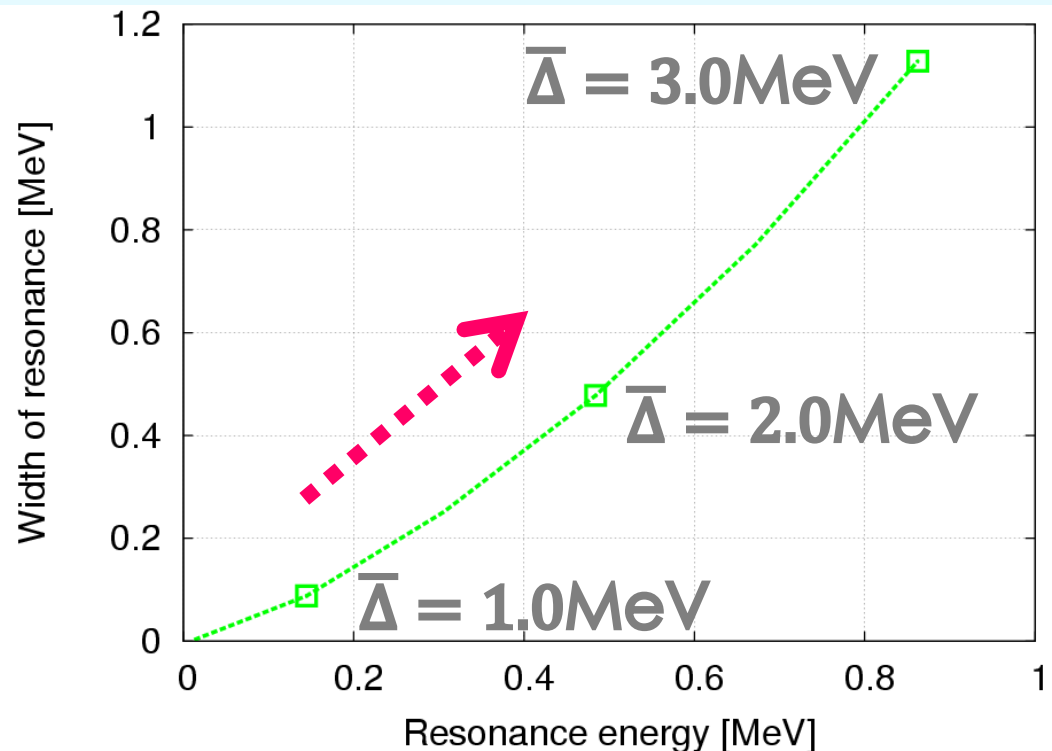
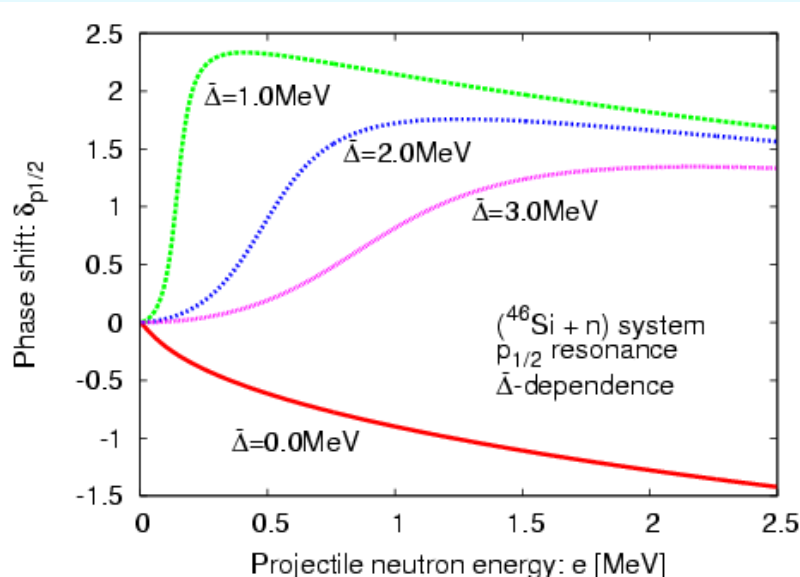
Cf. the empirical value for ^{46}Si

$$\Delta = \frac{12.0}{\sqrt{A}} \text{ MeV} = \frac{12.0}{\sqrt{46}} \cong 1.7 \text{ MeV}$$

We extract the resonance width and energy

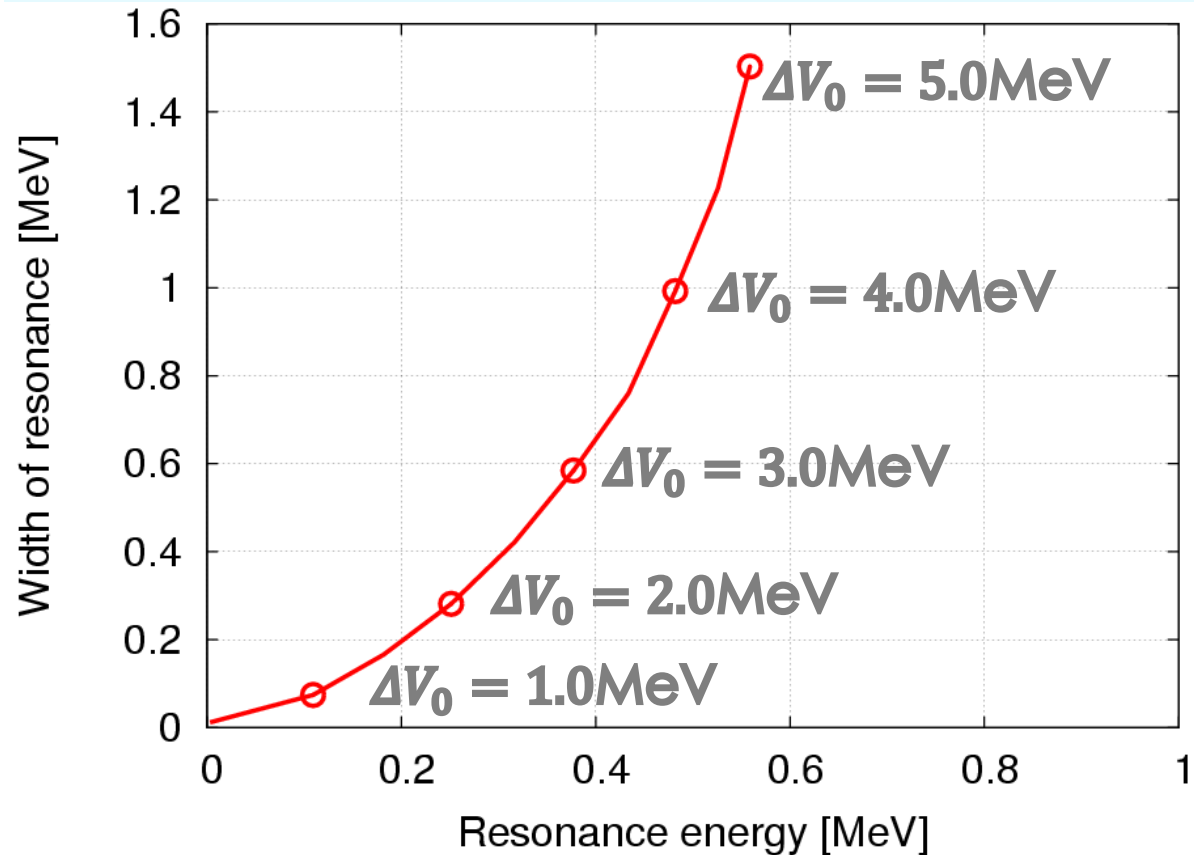
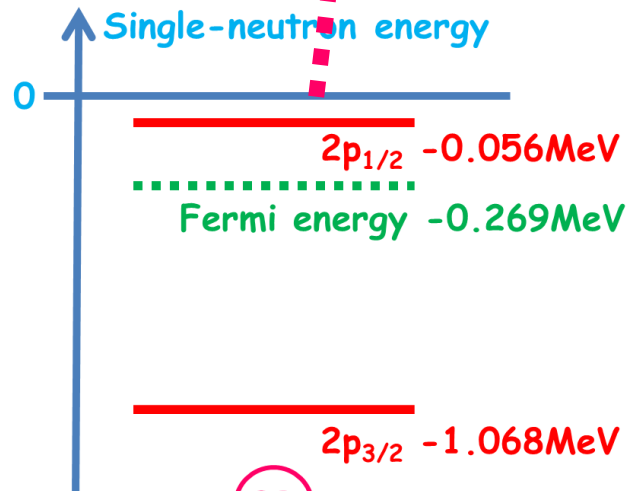
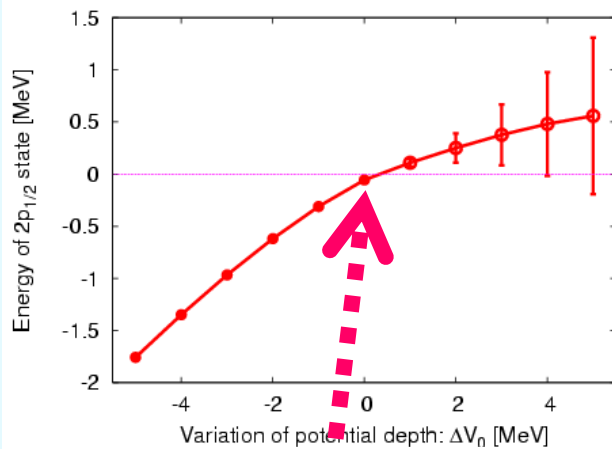
- We calculate the width and energy from phase shifts using fitting method.

$$\delta(e) = \arctan\left(\frac{2(e - e_R)}{\Gamma}\right) + a(e - e_R) + b$$

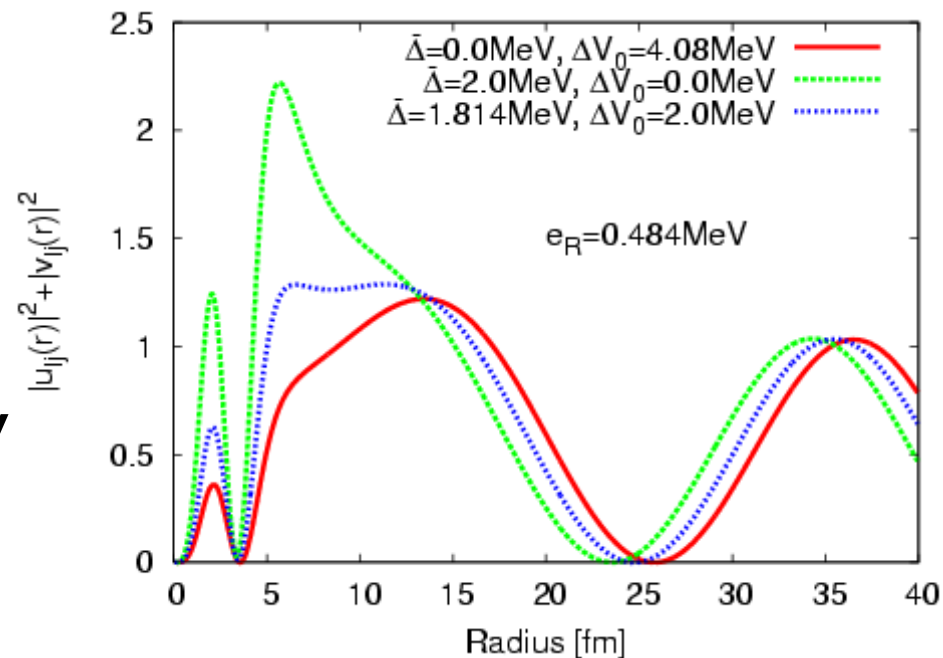
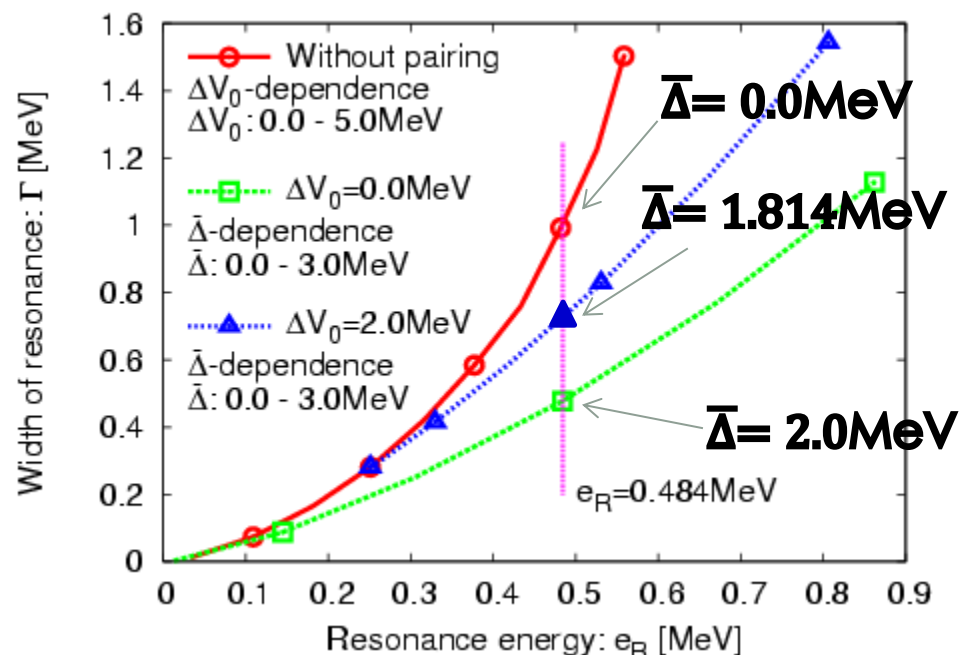


We also consider single-particle resonance

- We also calculate the width and energy for $p_{1/2}$ single-particle potential resonance.



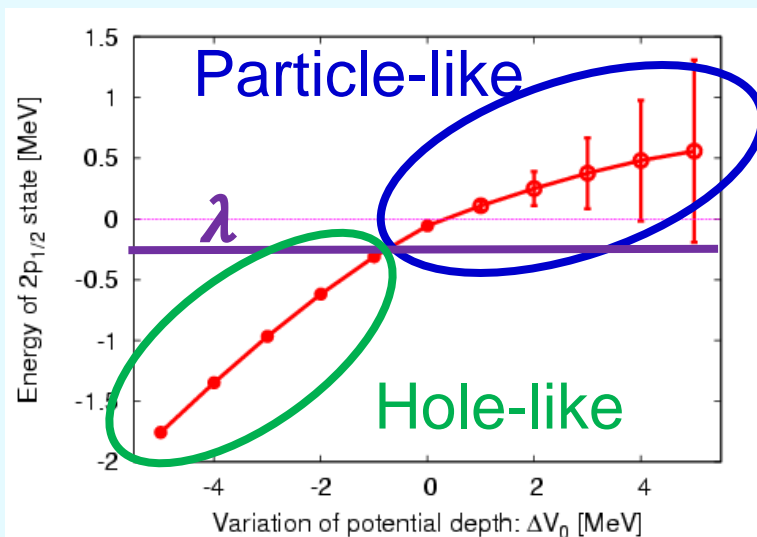
q.p. resonance vs s.p. resonance



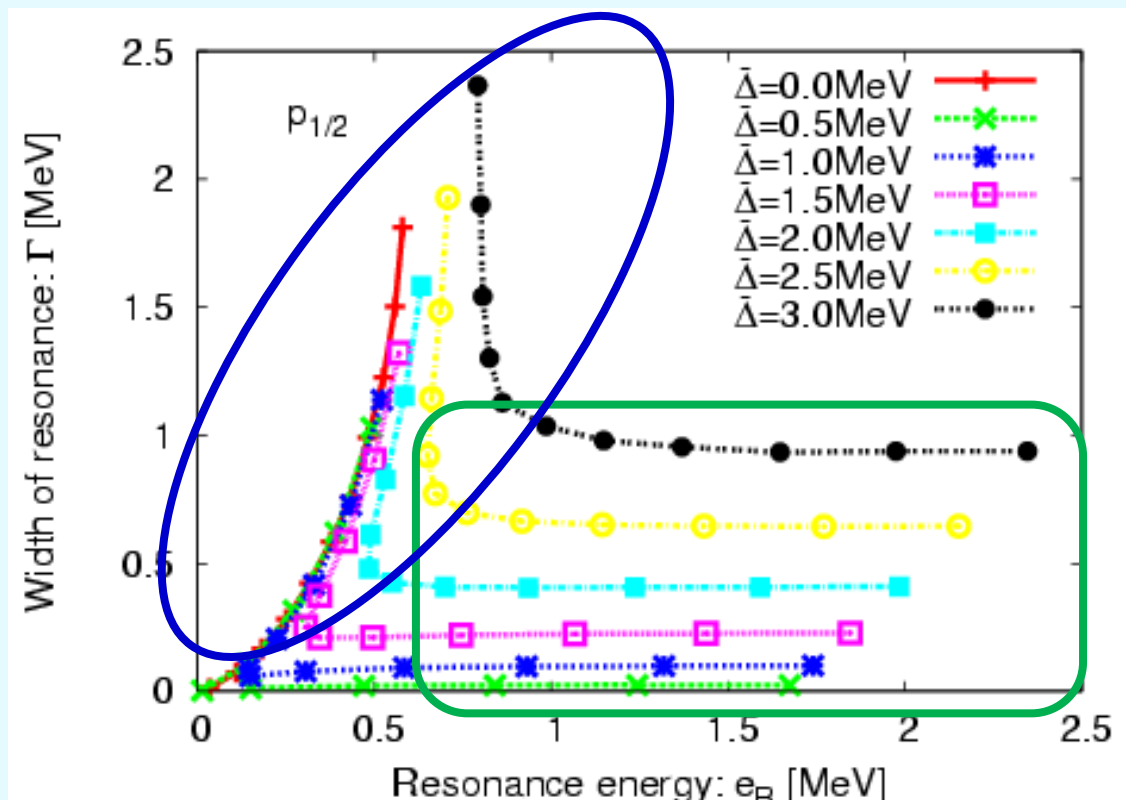
- If we compare these results at the same resonance energy, we found that the width of quasi-particle resonance is narrower than the width of single-particle potential resonance.
- Pairing decrease the resonance width.

The width with various condition

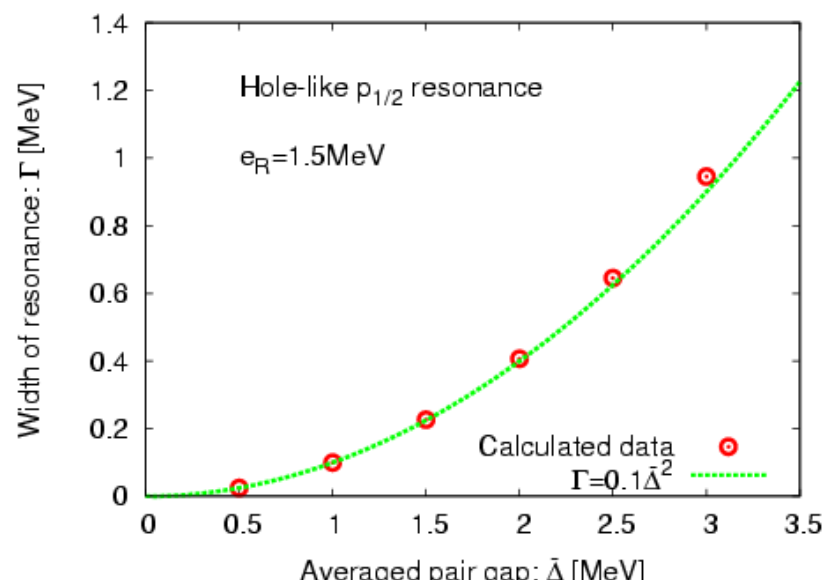
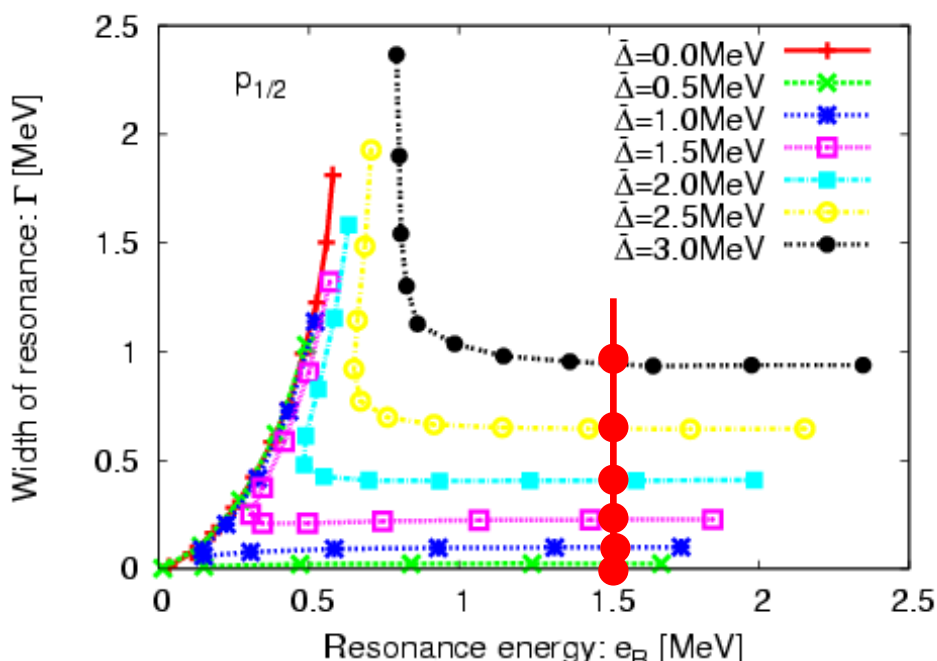
- We change
- the strength of pairing $\bar{\Delta} = 0.0 \sim 3.0 \text{ MeV}$
- the depth of nuclear potential $\Delta V_0 = -6.0 \sim 4.0 \text{ MeV}$
- ✓ We fix the Fermi energy ($= -0.269 \text{ MeV}$)



Single-particle energy
of $2p_{1/2}$ and ΔV_0



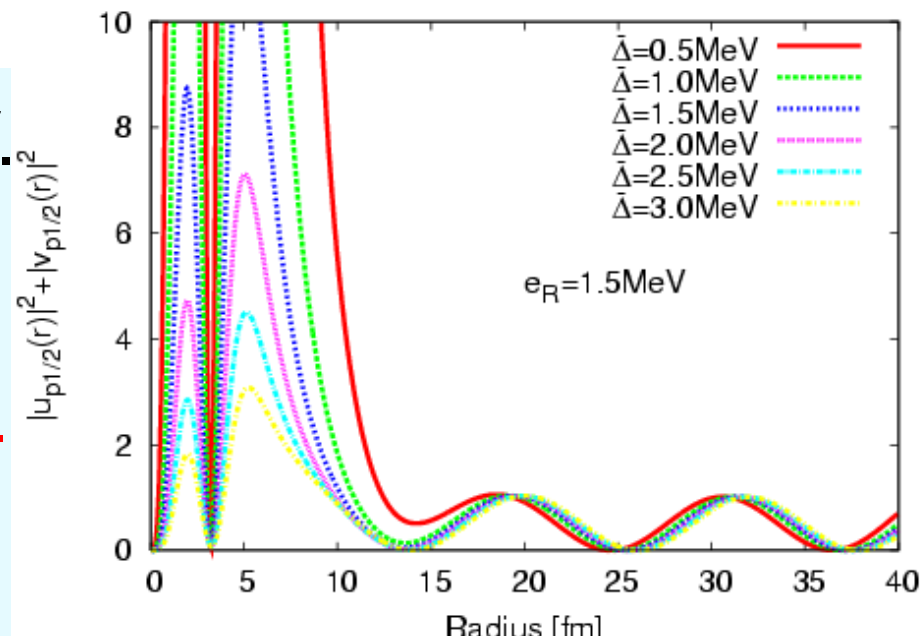
The width of hole-like quasi-particle resonance



● We focus on $e_R = 1.5$ MeV.

● The width $\propto \bar{\Delta}^2$.

→ Pairing increase the width for hole-like quasi-particle resonance.

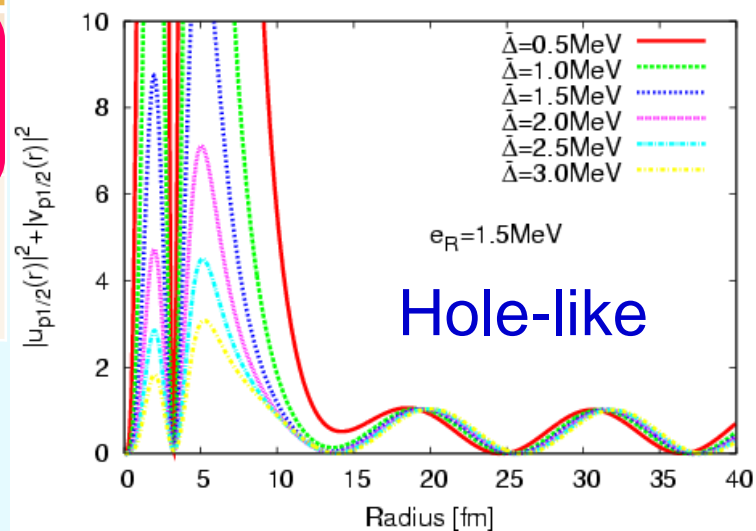
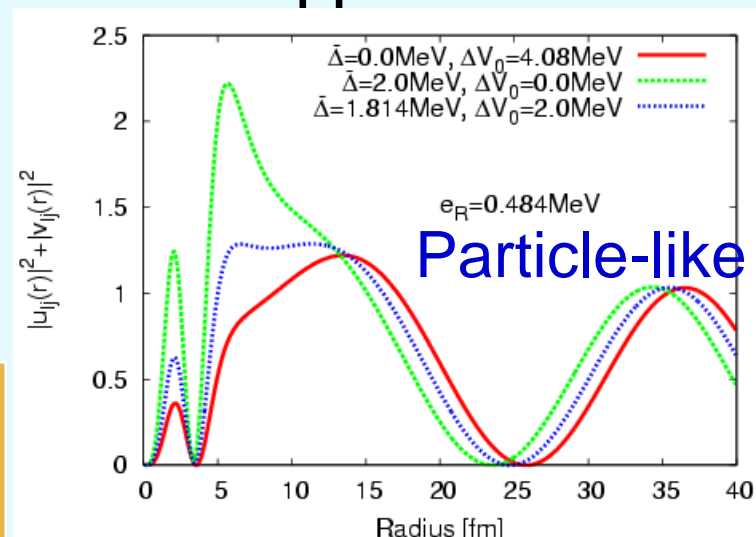


Particle-like vs hole-like

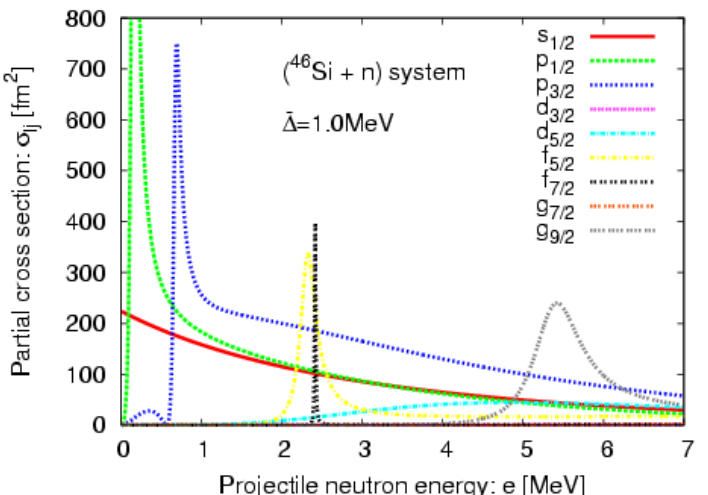
- We analyze the different point between particle-like q.p resonance & hole-like q.p. resonance with BCS formula.

$$v_f^2 = \frac{1}{2} \left(1 - \frac{\epsilon_f - \lambda}{\sqrt{(\epsilon_f - \lambda)^2 + \Delta^2}} \right) \quad u_f^2 = \frac{1}{2} \left(1 + \frac{\epsilon_f - \lambda}{\sqrt{(\epsilon_f - \lambda)^2 + \Delta^2}} \right)$$

Original $p_{1/2}$ orbit	When pairing increase	v^2 and u^2	Width of resonance
Particle-like	$\Delta \nearrow$	$v^2 \nearrow$	$\Gamma \searrow$
Hole-like	$\Delta \nearrow$	$u^2 \nearrow$	$\Gamma \nearrow$

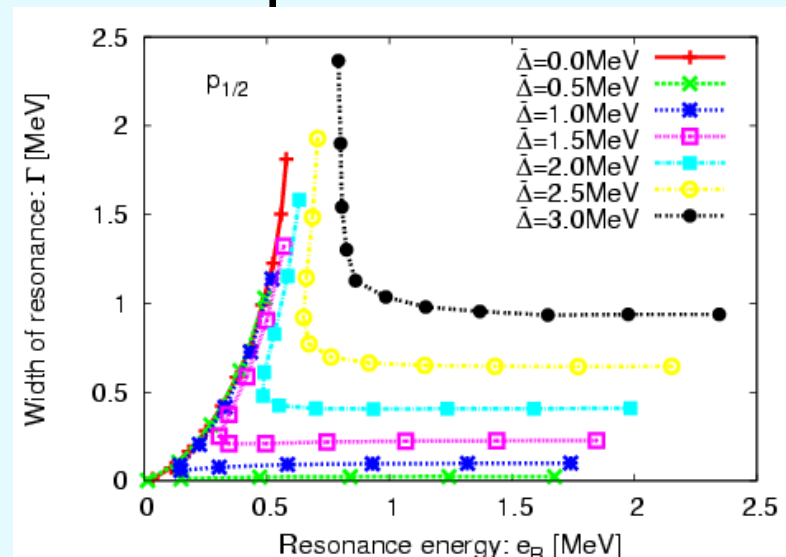


Conclusion



- Weakly bound orbits emerge as low-lying quasi-particle resonances when pairing is switched on in drip-line nuclei.

- The width of quasi-particle resonance is influenced by the pairing.



The pairing effect on the width of quasi-particle resonance

- For particle-like : decrease the width
- For hole-like : increase the width