Local chiral Effective Field Theory interactions and applications

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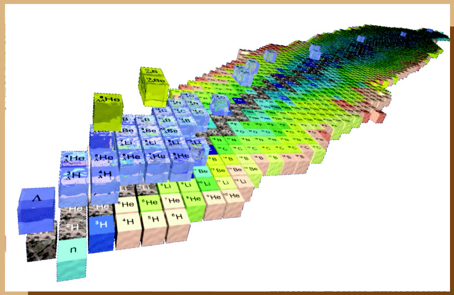
UNIVERSITY of GUELPH

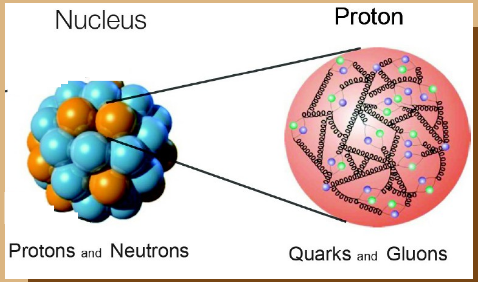
ICNT Workshop on “Physics of exotic nuclei”
RIKEN Wako Campus
June 9, 2014

Outline



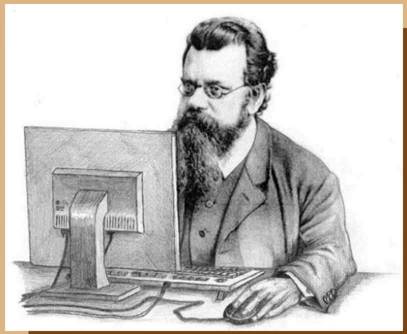
Many-nucleon problem

- Chart of nuclides
- Neutron stars
- Perturbative vs non-perturbative



Nuclear forces

- Chiral Effective Field Theory (EFT)
- Local chiral EFT



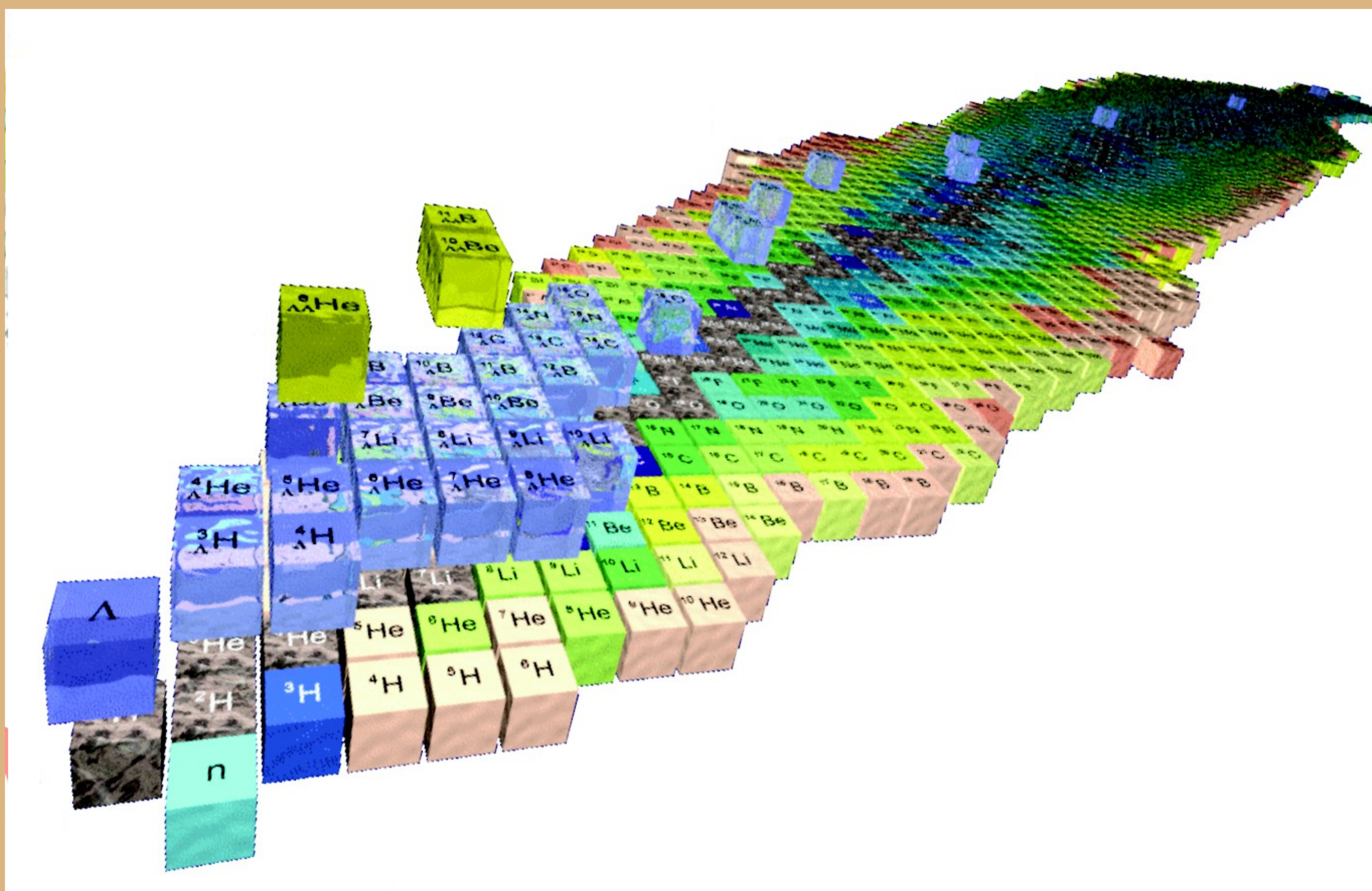
Credit: Bernhard Reischl

Few- and many-body results

- Quantum Monte Carlo (QMC)
- Many-Body Perturbation Theory (MBPT)
- No-Core Shell Model (NCSM)
- Hyperspherical Harmonics (HH)

Many-nucleon problem: nuclei

Chart of nuclides



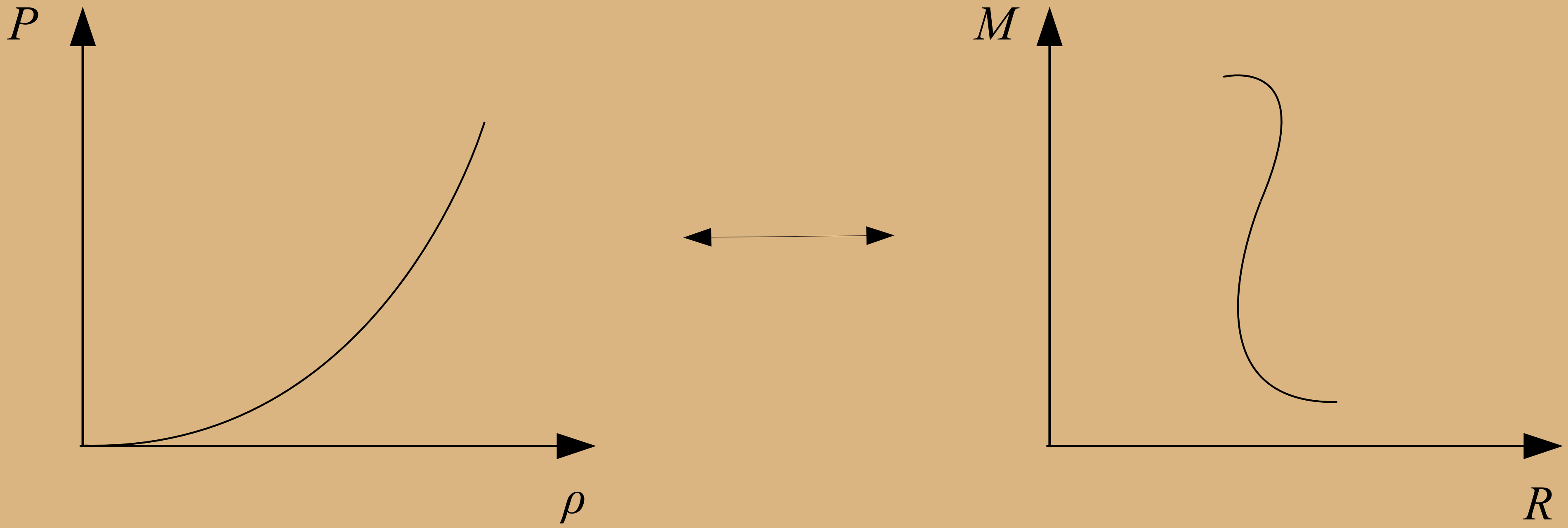
- Nuclear physics is developed and tested on earth
- Using complicated many-body methods we can try to “build nuclei from scratch”
- We then extrapolate that knowledge to more exotic systems

Neutron stars: micro-macro

TOV equations
(or Hartle-Thorne, etc)

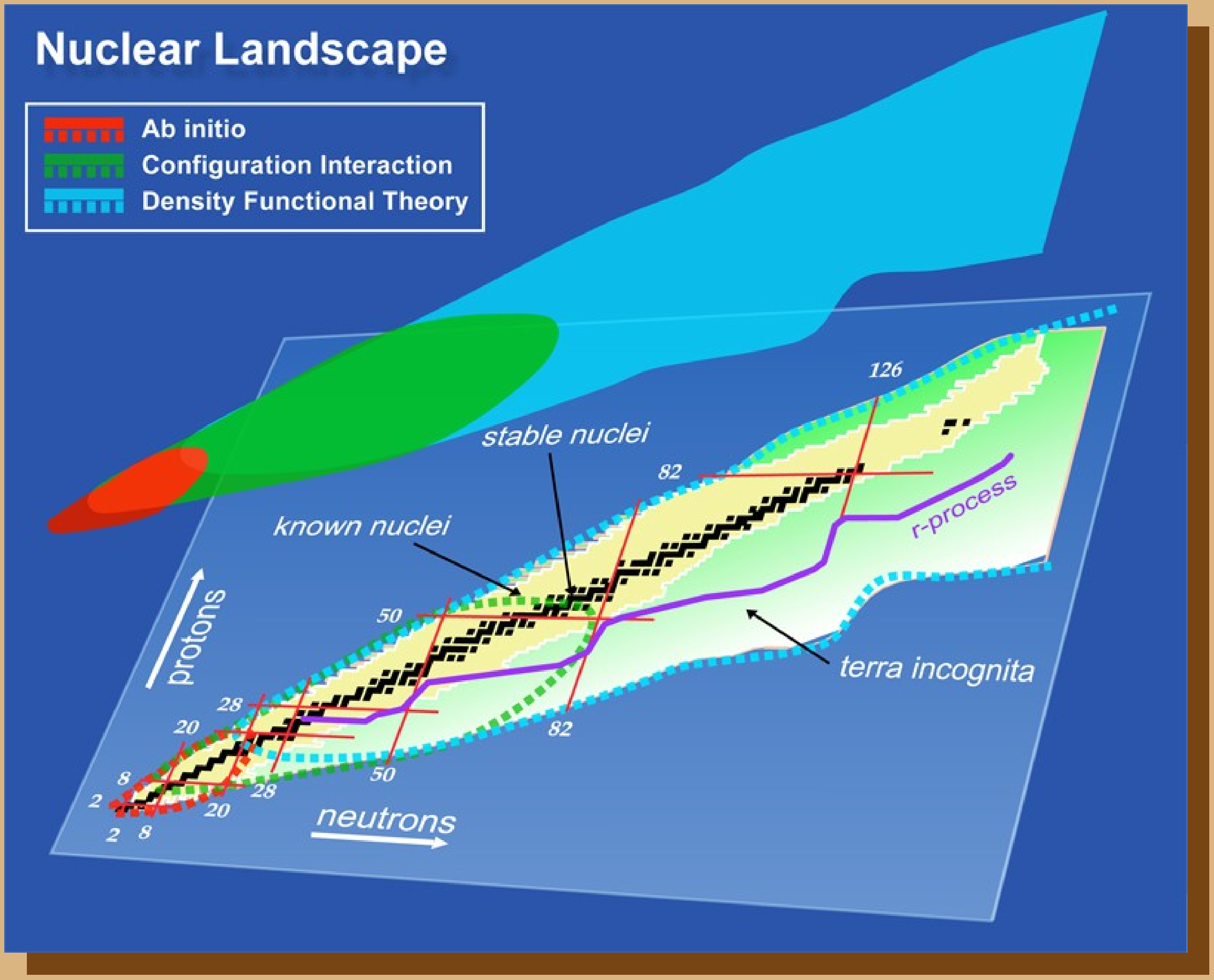
Pressure vs density

Mass vs radius



Modern goal: systematic theoretical error bars

Many-nucleon problem: methods



- No universal method exists (yet?)
- A lot to be learned if the degrees of freedom are actual particles and there are no free parameters
- Regions of overlap between different methods are crucial
- Is it possible to work at the level of nucleons & pions but still connect to the underlying level?

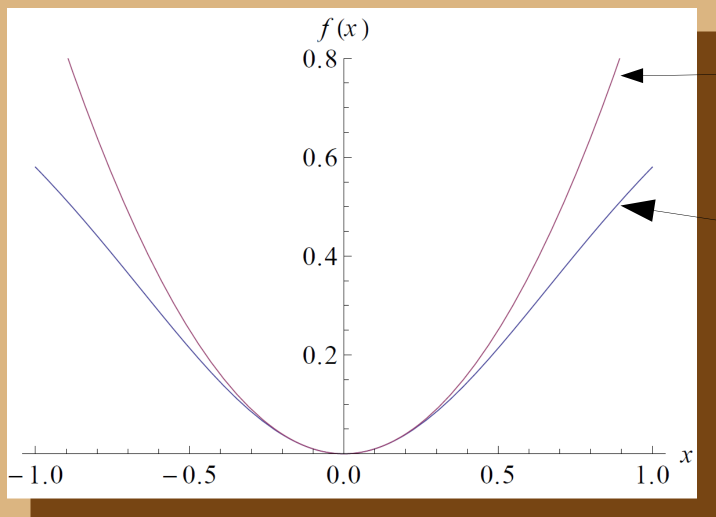
Many-nucleon problem

Lesson from history of physics:
Find a small expansion parameter α and use it to organize the different contributions:

$$\alpha + \alpha^2 + \alpha^3 + \cdots$$

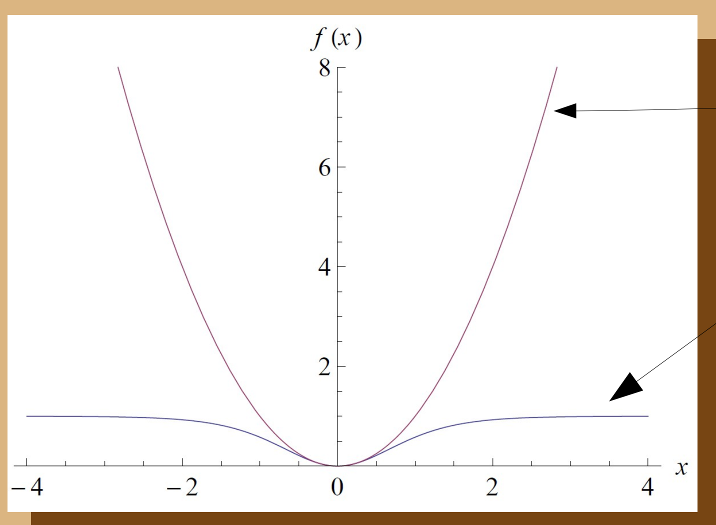
However, some problems are non-perturbative.

Perturbative vs non-perturbative



$$1-\left(\frac{1}{\cosh(x)}\right)^2 = x^2 + \text{something small} \quad ?$$

Maybe



$$1-\left(\frac{1}{\cosh(x)}\right)^2 = x^2 + \text{something small} \quad ?$$

Maybe not

Many-body problem: QMC

Quantum Monte Carlo: stochastically solve the many-body Schrödinger equation in a fully non-perturbative manner

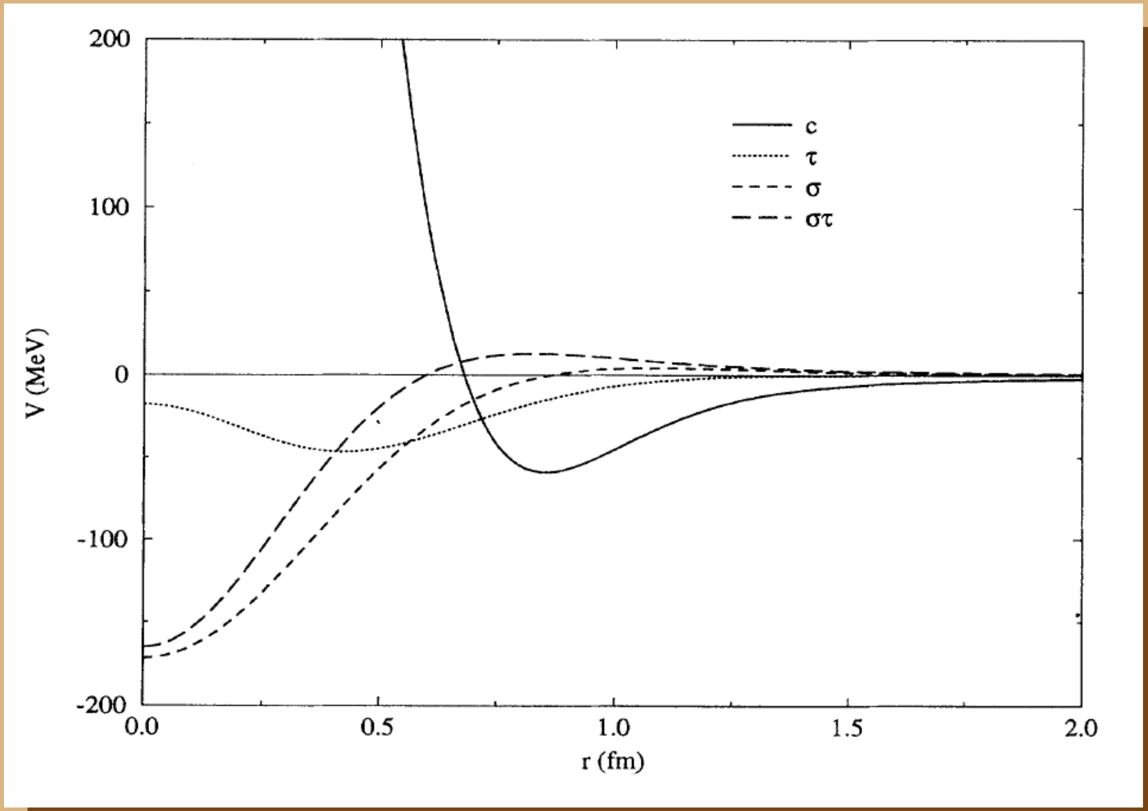
Rudiments of Diffusion Monte Carlo:

$$\Psi(\tau \to \infty) = \lim_{\tau \to \infty} e^{-(\mathcal{H} - E_T)\tau} \Psi_V$$

$$\to \alpha_0 e^{-(E_0 - E_T)\tau} \Psi_0$$

Many-body problem: QMC

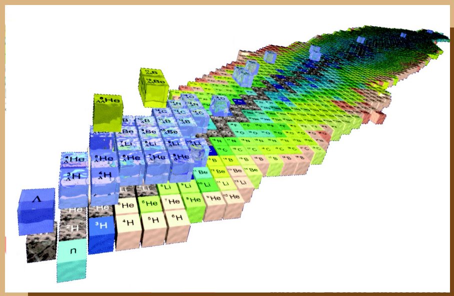
Quantum Monte Carlo was, however, limited to the use of (local) phenomenological nucleon-nucleon potentials.



Credit: Bob Wiringa

Such potentials are hard, making them non-perturbative at the many-body level (which isn't a problem for QMC, but is one for almost every other method).

Outline



Many-nucleon problem

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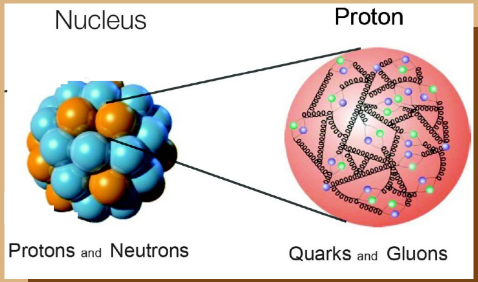
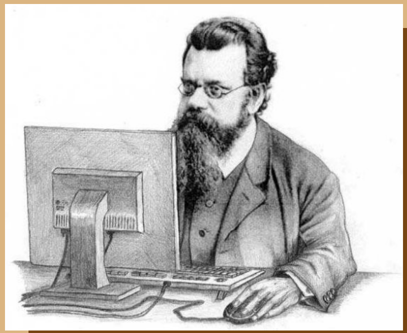


Nuclear forces

- Chiral Effective Field Theory (EFT)
- Local chiral EFT



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Few- and many-body results

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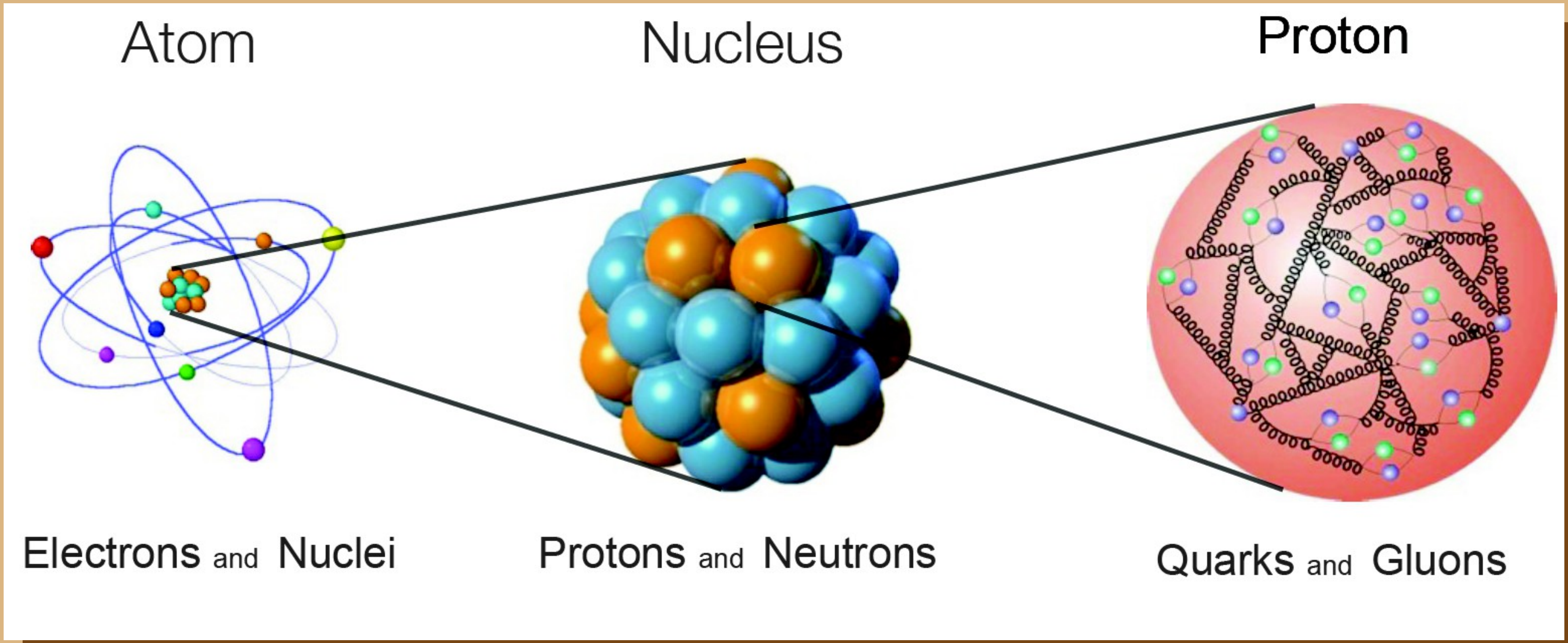
Nuclear forces: degrees of freedom



Lesson from history of physics

Find a small expansion parameter α and use it to organize the different contributions:

$$\alpha + \alpha^2 + \alpha^3 + \cdots$$

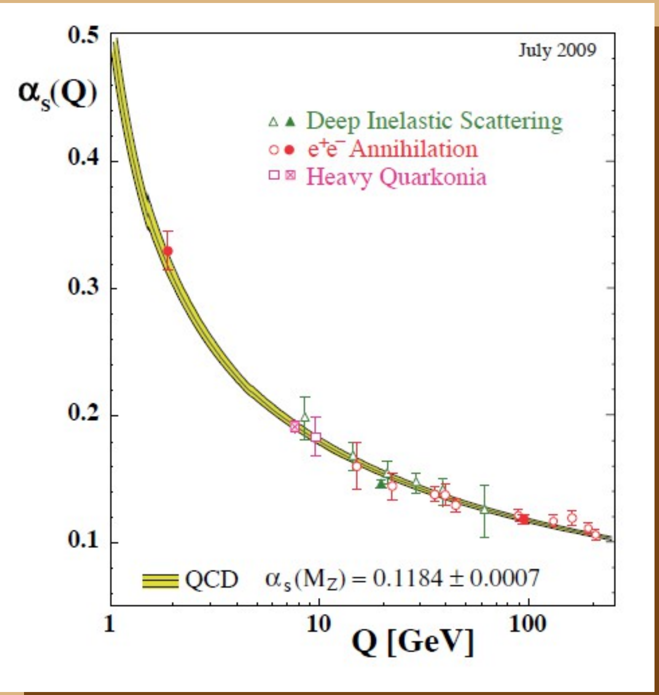
Quantum Electrodynamics (QED)

$$\alpha = \frac{1}{137.036}$$

for most practical purposes

(see Gell-Mann & Low paper with 770 citations, not Gell-Mann & Low paper with 630 citations)

Quantum Chromodynamics (QCD)



Credit: Particle Data Group

Perturbative and non-perturbative

Faced with an inherent non-perturbativeness of the many-nucleon problem we turned to a non-perturbative method (Quantum Monte Carlo).

Perhaps a non-perturbative method should also be used for nuclear forces (lattice QCD).

For now, many-nucleon studies are limited to nuclear forces that follow from nucleons and pions (but it would be nice not to ignore the existence of the underlying level).

Nuclear Hamiltonian: chiral EFT

How to build on QCD in a systematic manner?

Exploit separation of scales: $a_{^1S_0} = (11 \text{ MeV})^{-1}$

$$m_\pi = 140 \text{ MeV}$$

$$\Lambda_\chi \approx m_\rho \approx \ 800 \text{ MeV}$$

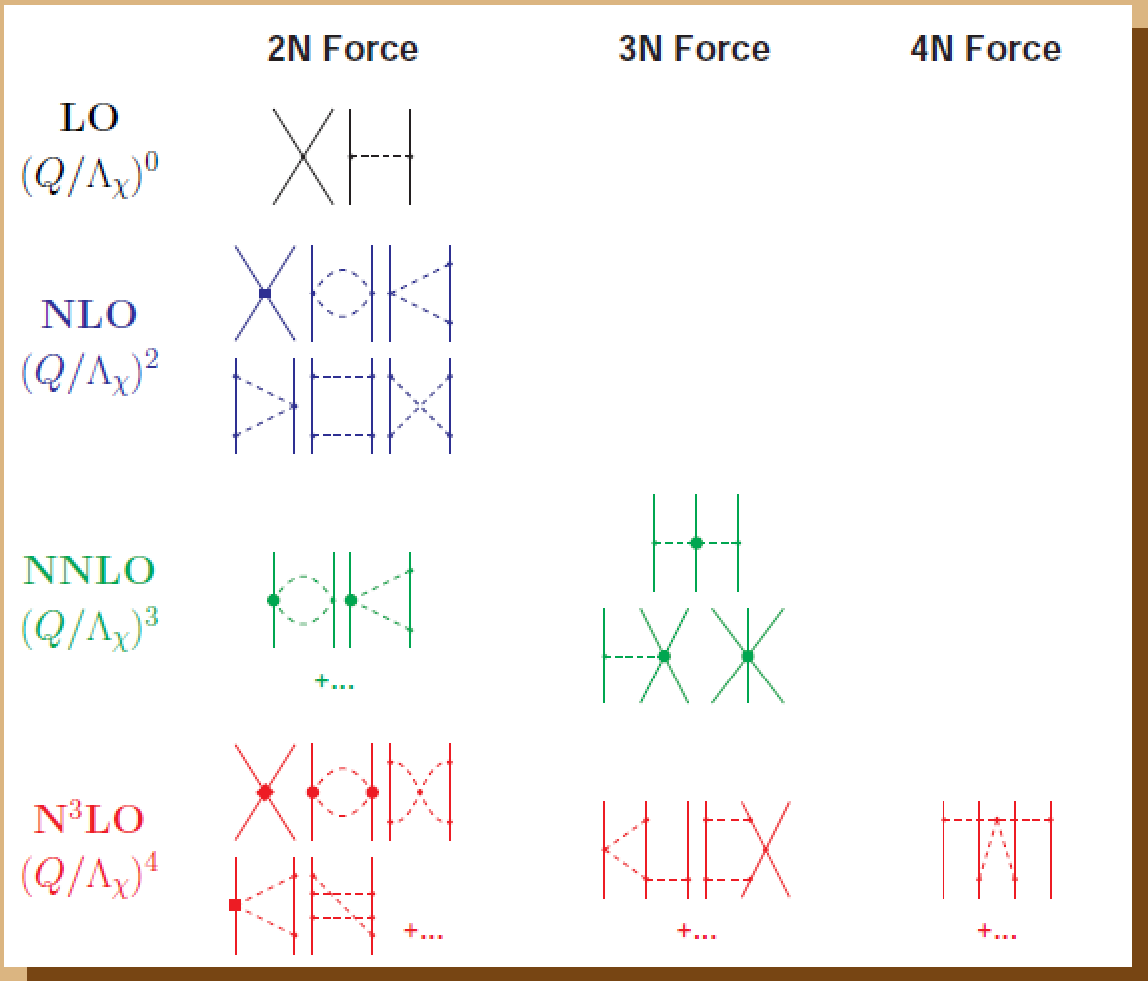
Chiral Effective Field Theory approach:

Use nucleons and pions as degrees of freedom

Systematically expand in $\dfrac{Q}{\Lambda_\chi}$

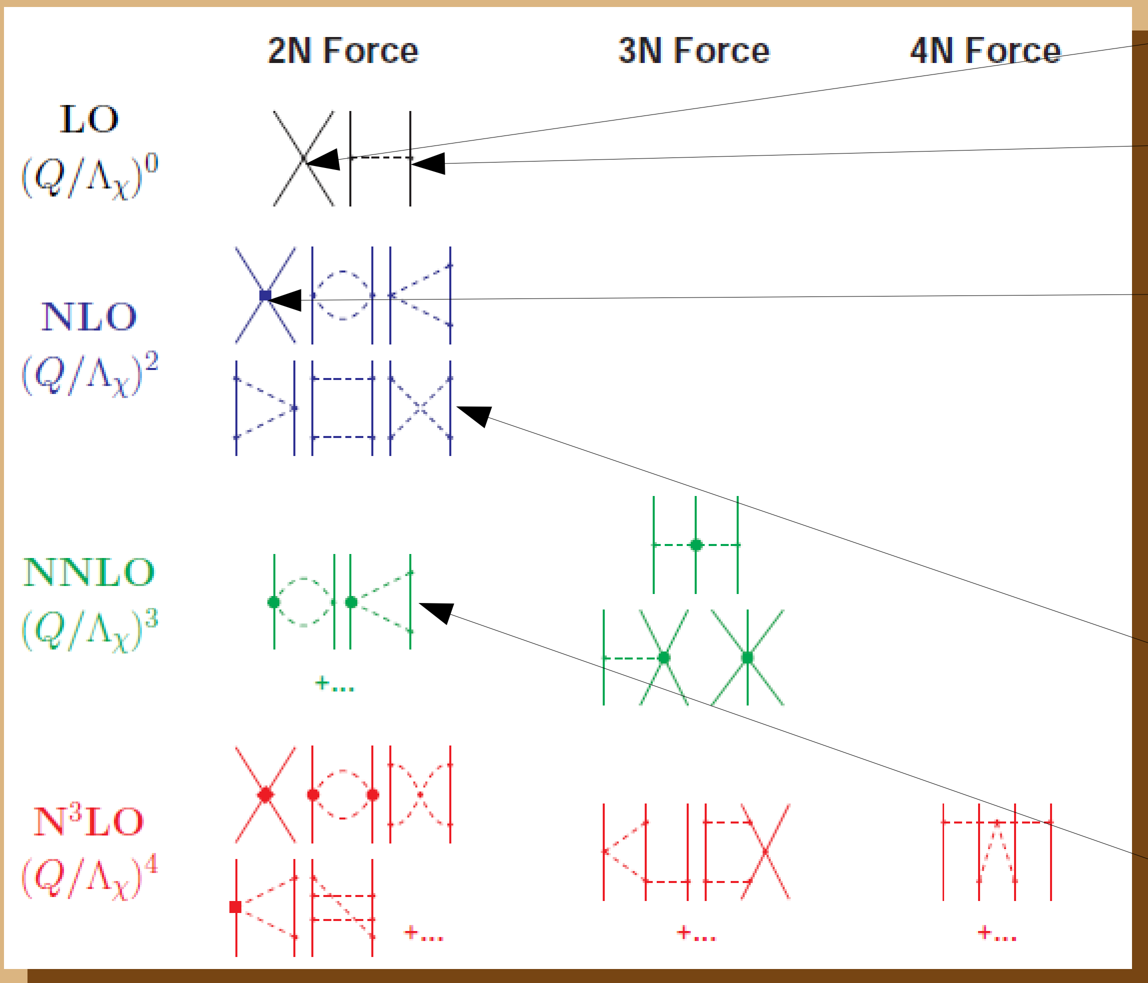
Program introduced by S. Weinberg, now taken over by the nuclear community

Nuclear Hamiltonian: chiral EFT



- Attempts to connect with underlying theory (QCD)
- Systematic low-momentum expansion
- Consistent many-body forces
- Low-energy constants from experiment or lattice QCD
- Until now non-local in coordinate space, so unused in continuum QMC
- Power counting's relation to renormalization still an open question

Nuclear Hamiltonian: chiral EFT



$$V_{\rm ct}^{(0)} = C_S + C_T\ \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$$

$$V_{1\pi}^{(0)} = -\left(\frac{g_A}{2f_\pi}\right)^2\ \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{(\boldsymbol{\sigma}_1 \cdot \mathbf{q})(\boldsymbol{\sigma}_2 \cdot \mathbf{q})}{q^2 + m_\pi^2}$$

$$\begin{aligned} V_{\rm ct}^{(2)} = {} & C_1\ q^2 + C_2\ k^2 \\ & + \left(C_3\ q^2 + C_4\ k^2\right) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \\ & + i\frac{C_5}{2}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot (\mathbf{q} \times \mathbf{k}) \\ & + C_6(\boldsymbol{\sigma}_1 \cdot \mathbf{q})(\boldsymbol{\sigma}_2 \cdot \mathbf{q}) \\ & + C_7(\boldsymbol{\sigma}_1 \cdot \mathbf{k})(\boldsymbol{\sigma}_2 \cdot \mathbf{k}) \end{aligned}$$

Long-studied
two-pion exchange

Contains couplings
from πN scattering

$\mathbf{q}$ means local

$\mathbf{k}$ means non-local

Nuclear forces: summary

Local high-quality phenomenology is hard

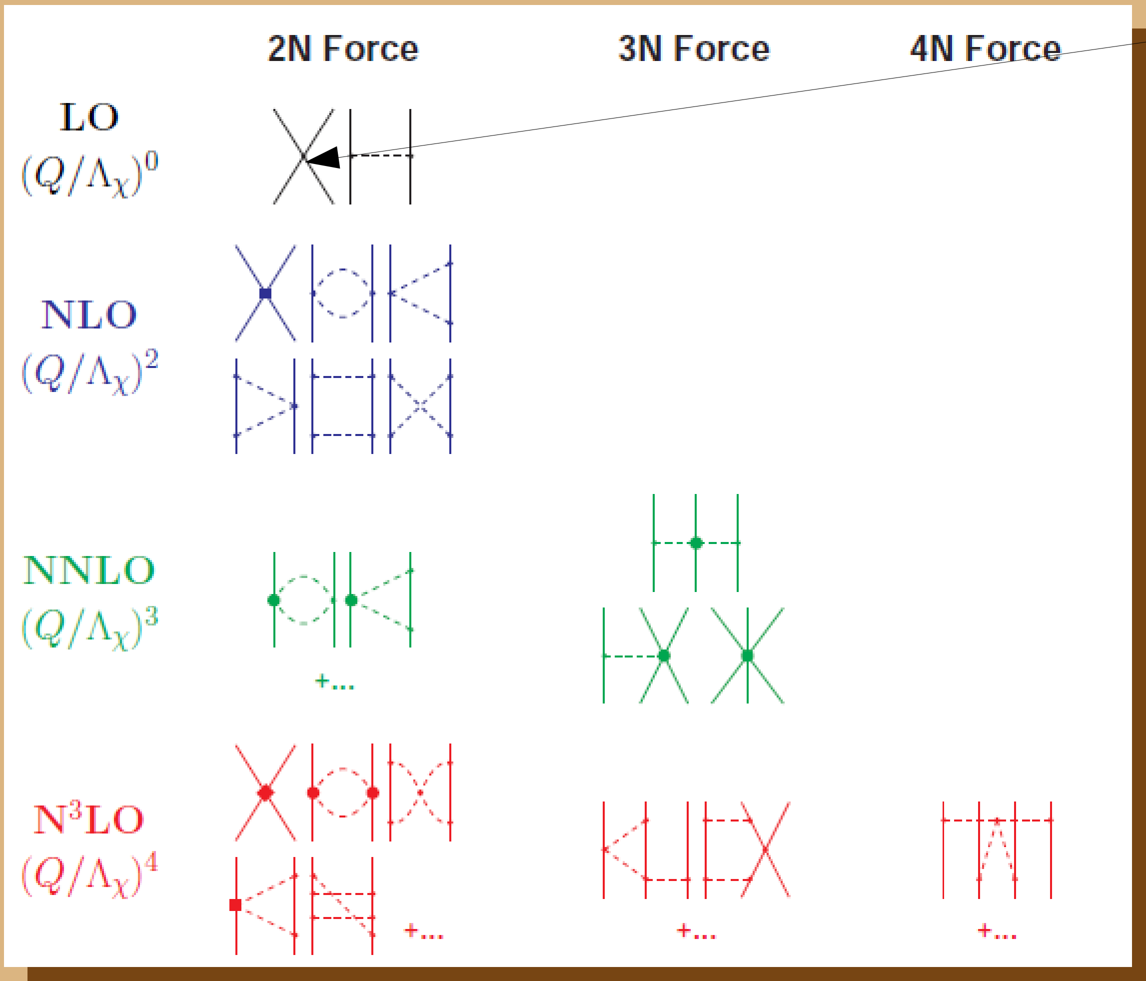
Consubstantial with the successes of nuclear QMC,
difficult to use in most other many-body methods

Chiral EFT
a) is connected to symmetries of QCD
b) has consistent many-body forces, and
c) allows us to produce systematic uncertainty bands
also happens to be non-local (such are the *sumbebekota*)

Heavily used in other methods, but not used in nuclear QMC

Turning to the resolution

Nuclear Hamiltonian: chiral EFT



$$V_{\rm ct}^{(0)} = C_S + C_T\ \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$$

Merely the standard choice.

Actually 4 terms in full set consistent with the symmetries of QCD

$$V_{\rm ct}^{(0)} = C_1 + C_2\ \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + C_3\ \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + C_4\ \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2\ \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$$

Pick 2 and antisymmetrize

How to go beyond?

Combine power of Quantum Monte Carlo with consistency of chiral Effective Field Theory

Write down a local energy-independent NN potential

- Use local pion-exchange regulator $f_{\text{long}}(r) = 1 - e^{-(r/R_0)^4}$
- Pick 7 different contacts at NLO, just make sure that when antisymmetrized they lead to a set obeying the required symmetry principles

$$
\begin{aligned}
V_{\text{ct}}^{(2)} &= C_1\, q^2 + C_2\, q^2\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\
&+ \left(C_3\, q^2 + C_4\, q^2\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2\right) \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \\
&+ i\, \frac{C_5}{2}\, (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \mathbf{q} \times \mathbf{k} \\
&+ C_6\, (\boldsymbol{\sigma}_1 \cdot \mathbf{q})(\boldsymbol{\sigma}_2 \cdot \mathbf{q}) \\
&+ C_7\, (\boldsymbol{\sigma}_1 \cdot \mathbf{q})(\boldsymbol{\sigma}_2 \cdot \mathbf{q})\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2
\end{aligned}
$$

cf.

$$
\begin{aligned}
V_{\text{ct}}^{(2)} &= C_1\, q^2 + C_2\, k^2 \\
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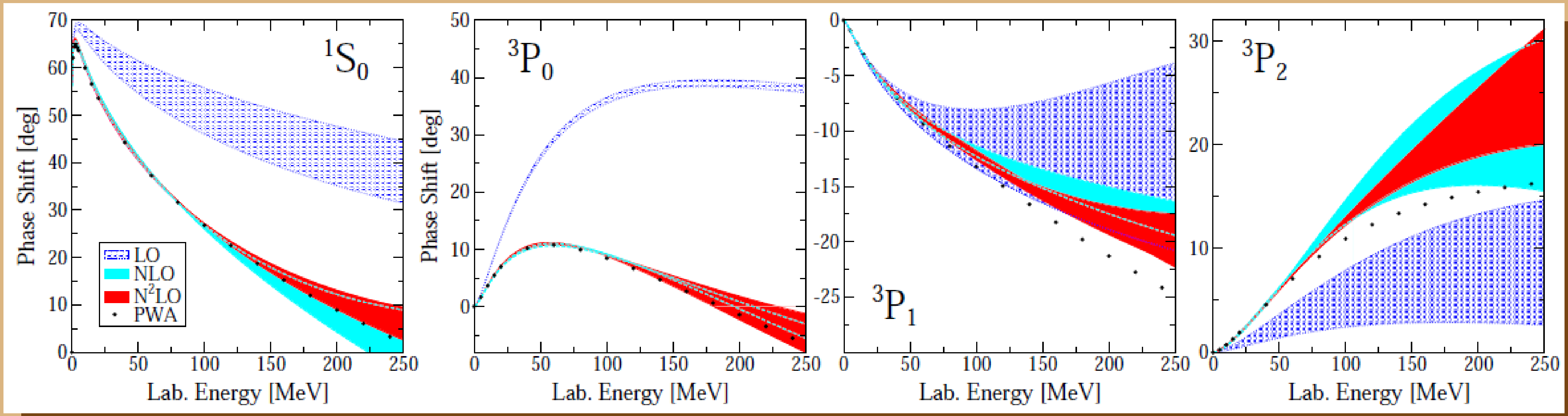
\end{aligned}
$$

How to go beyond?

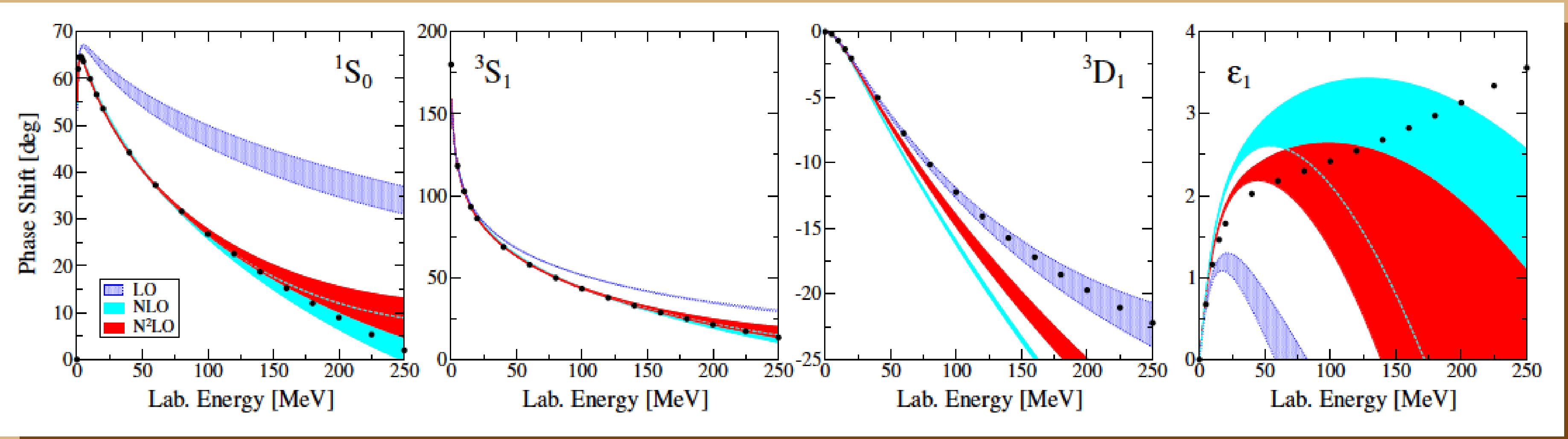
Combine power of Quantum Monte Carlo with consistency of chiral Effective Field Theory

- Write down a local energy-independent NN potential
- Before doing many-body calculations, fit to NN phase shifts



A. Gezerlis, I. Tews, E. Epelbaum, S. Gandolfi, K. Hebeler, A. Nogga, A. Schwenk, Phys. Rev. Lett. **111**, 032501 (2013).

Updated phase shifts



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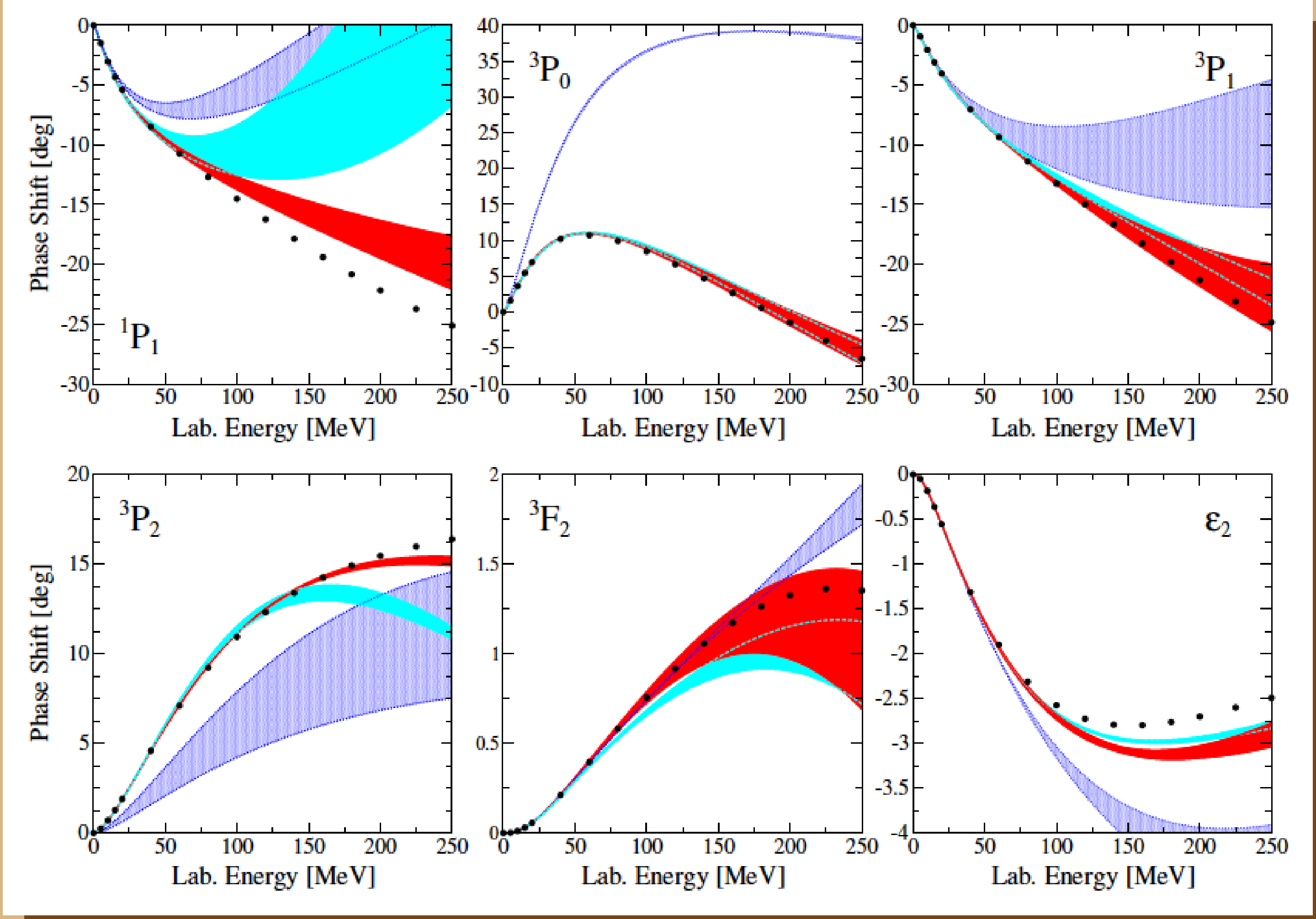

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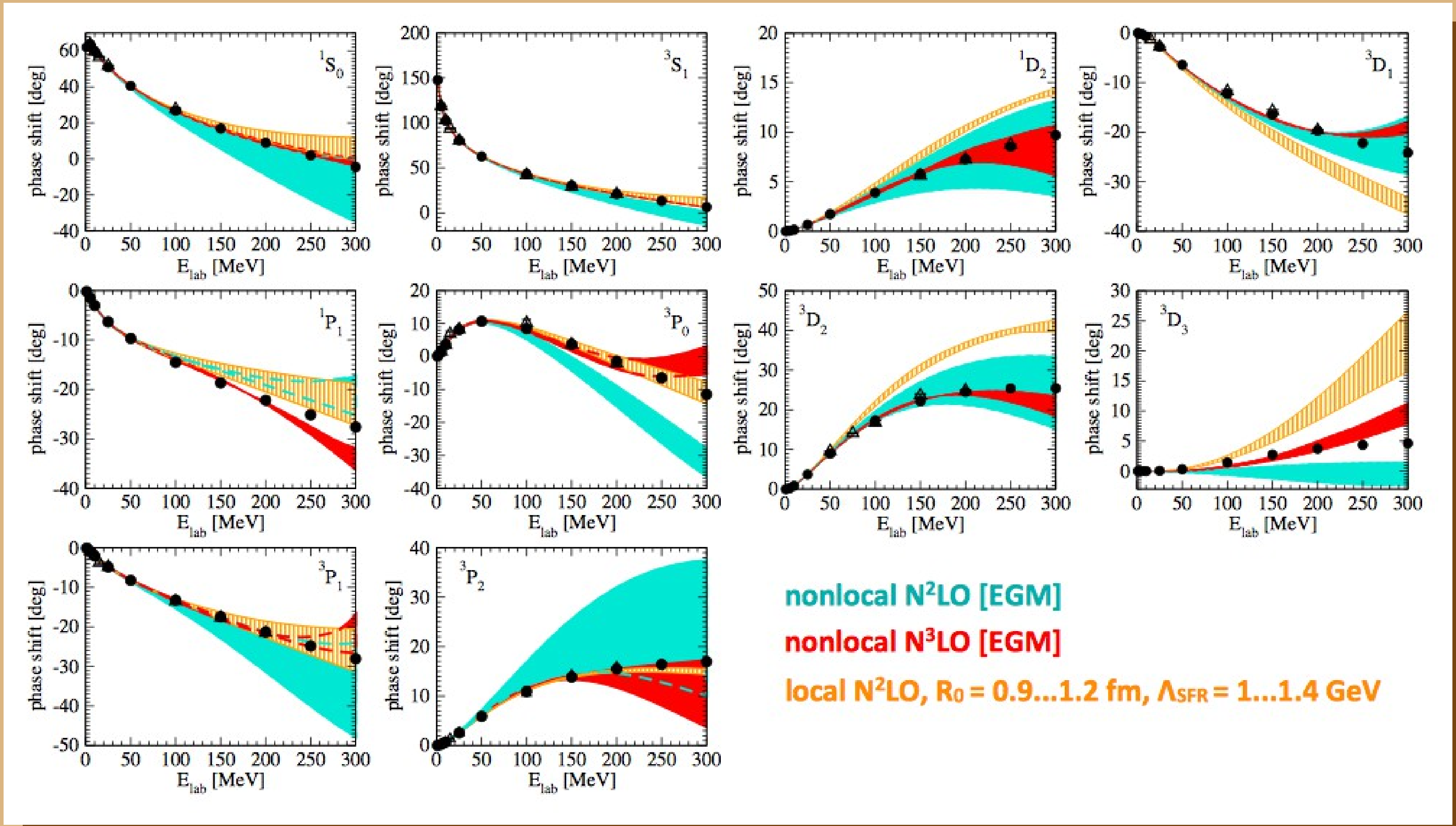
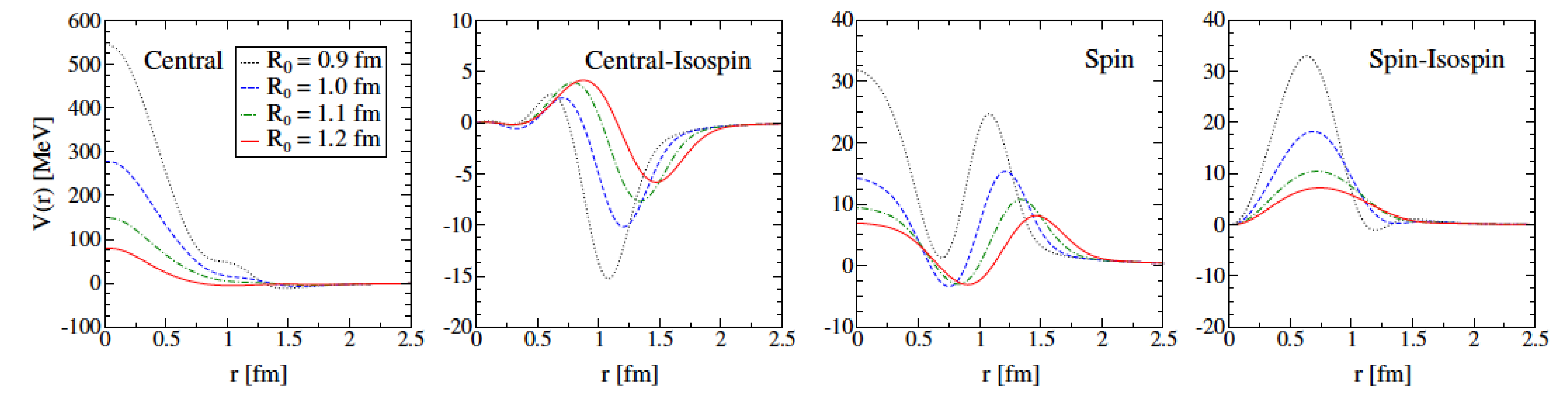
Compare to non-local EGM

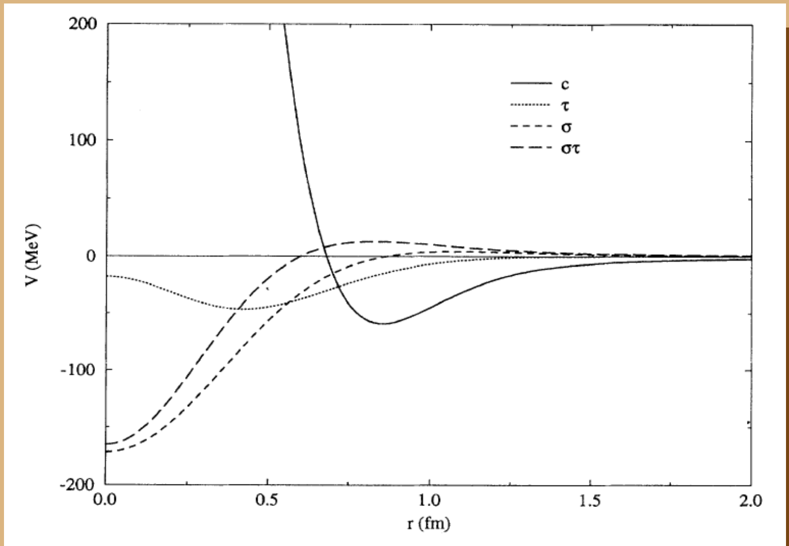


Credit: Hermann Krebs

Since it's local, let's plot it (N^{2}LO)



Compare with AV18



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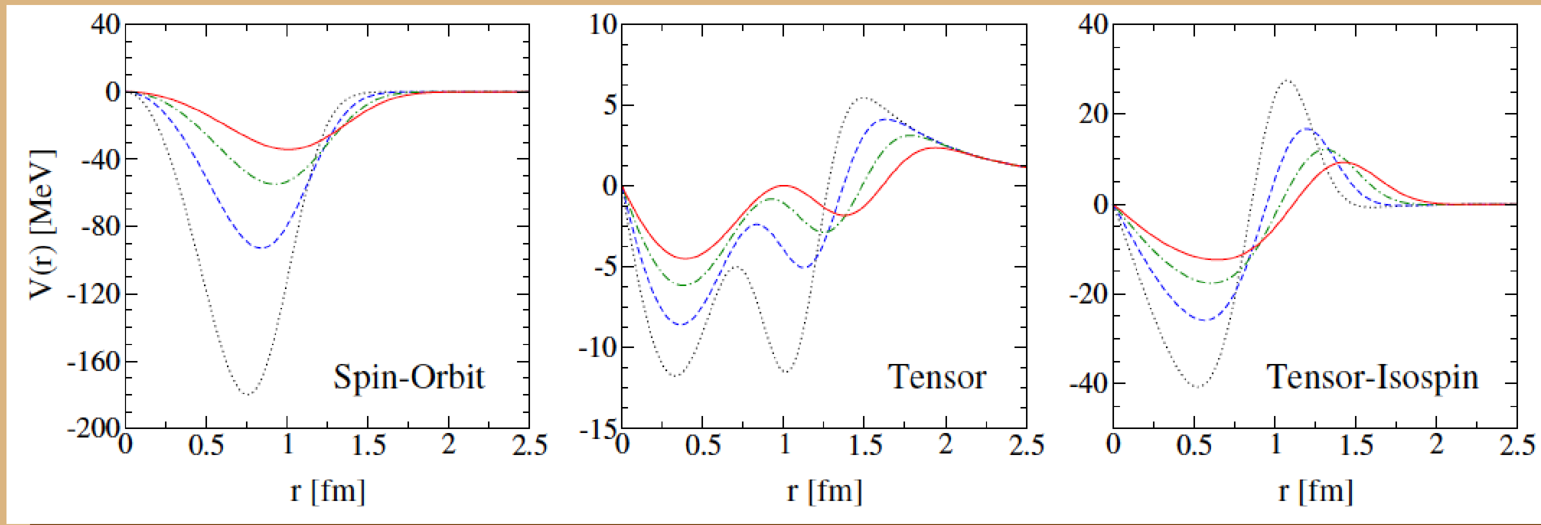

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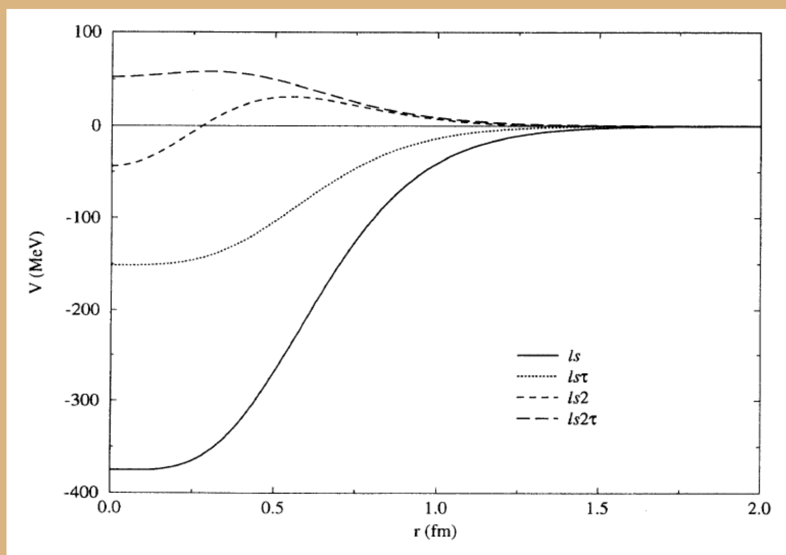

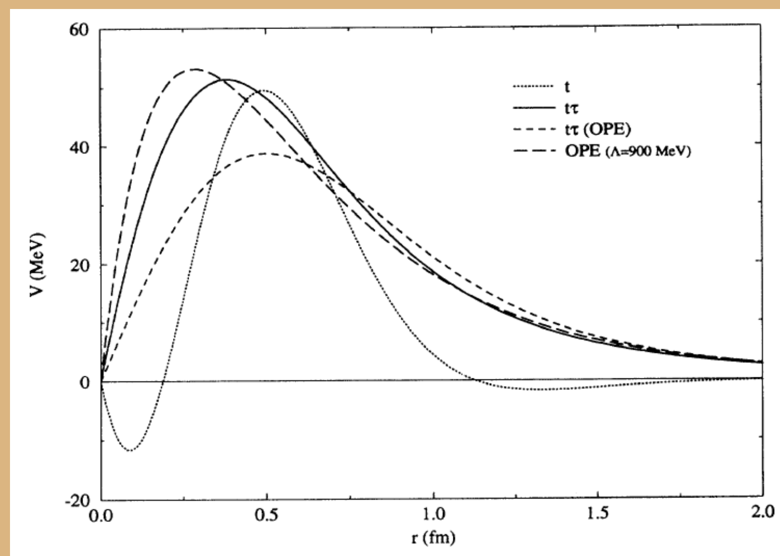

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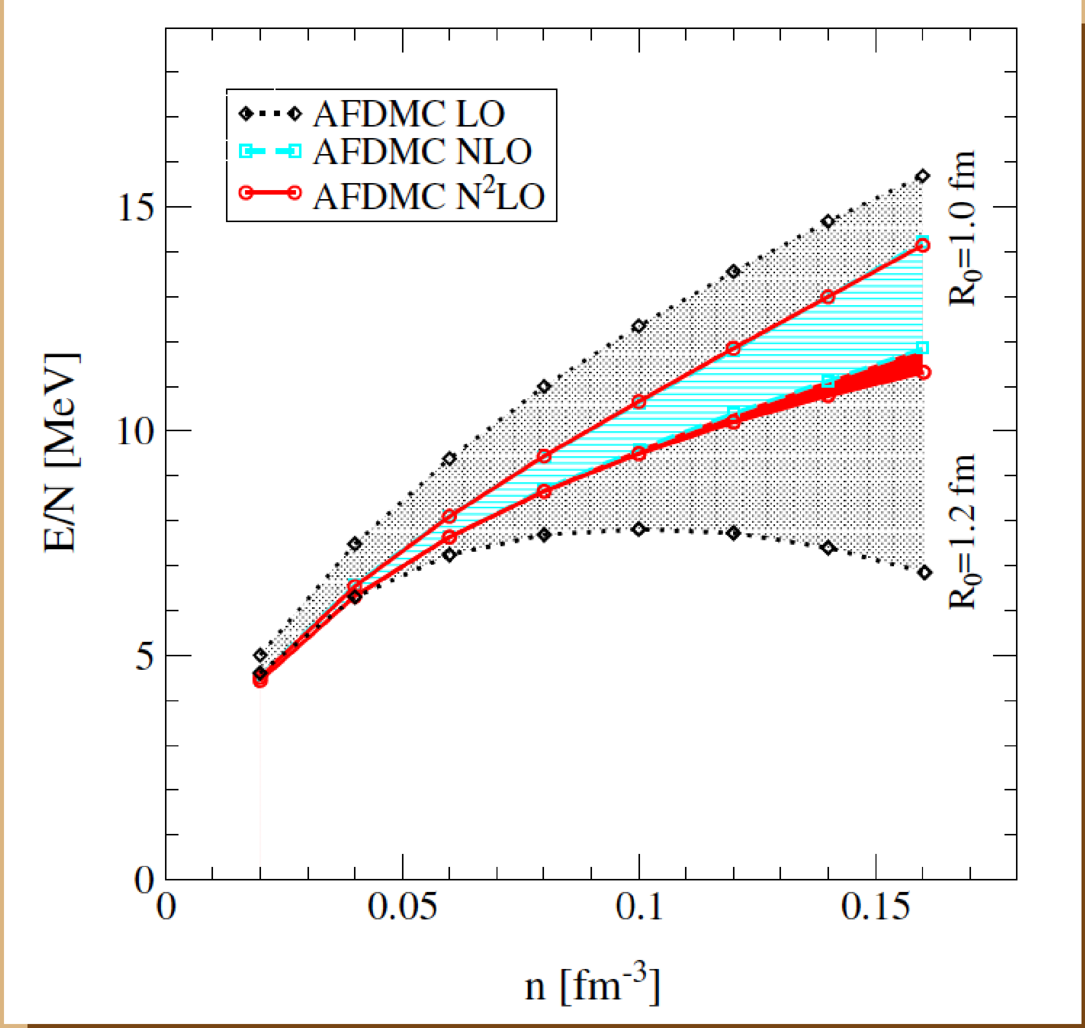
Chiral EFT in QMC

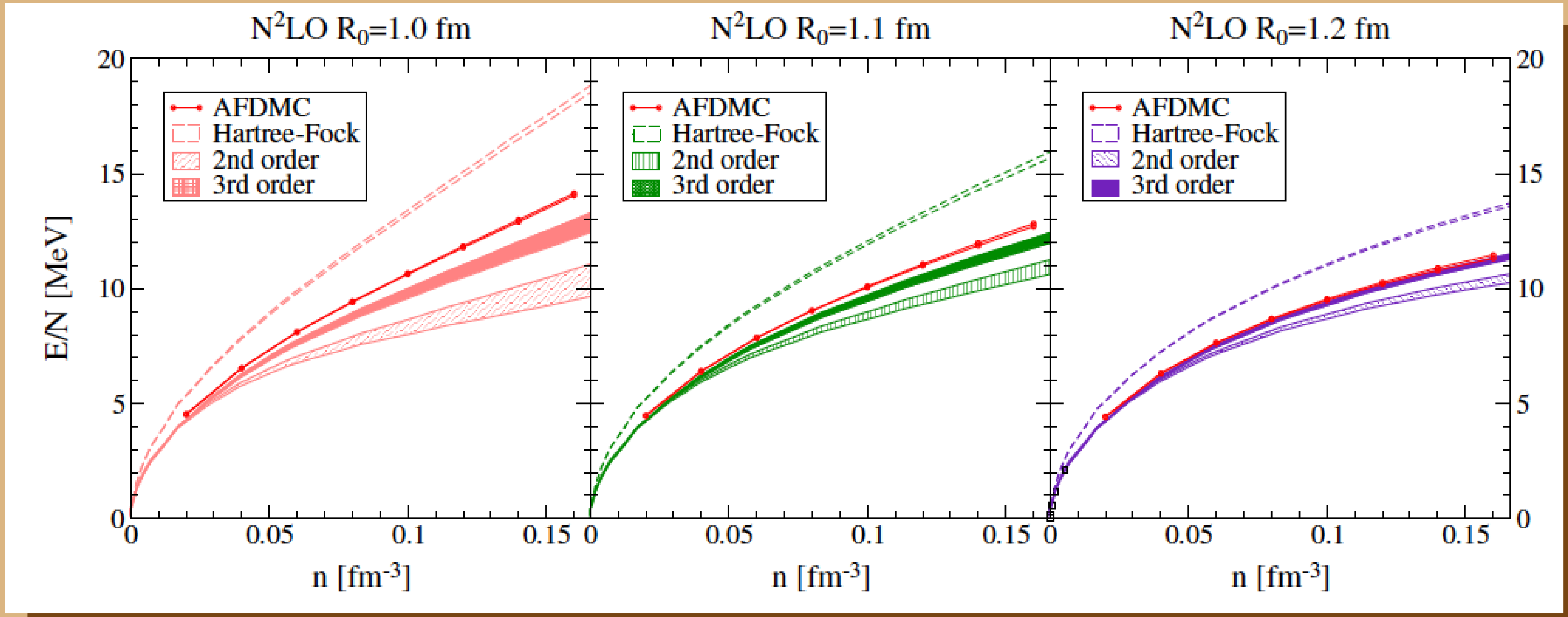


- Use Auxiliary-Field Diffusion Monte Carlo to handle the full interaction
- First ever non-perturbative systematic error bands
- Band sizes to be expected
- Many-body forces will emerge systematically

NEUTRONS

A. Gezerlis, I. Tews, E. Epelbaum, M. Freunek, S. Gandolfi, K. Hebeler, A. Nogga, A. Schwenk, arxiv:1406.0454

QMC vs MBPT

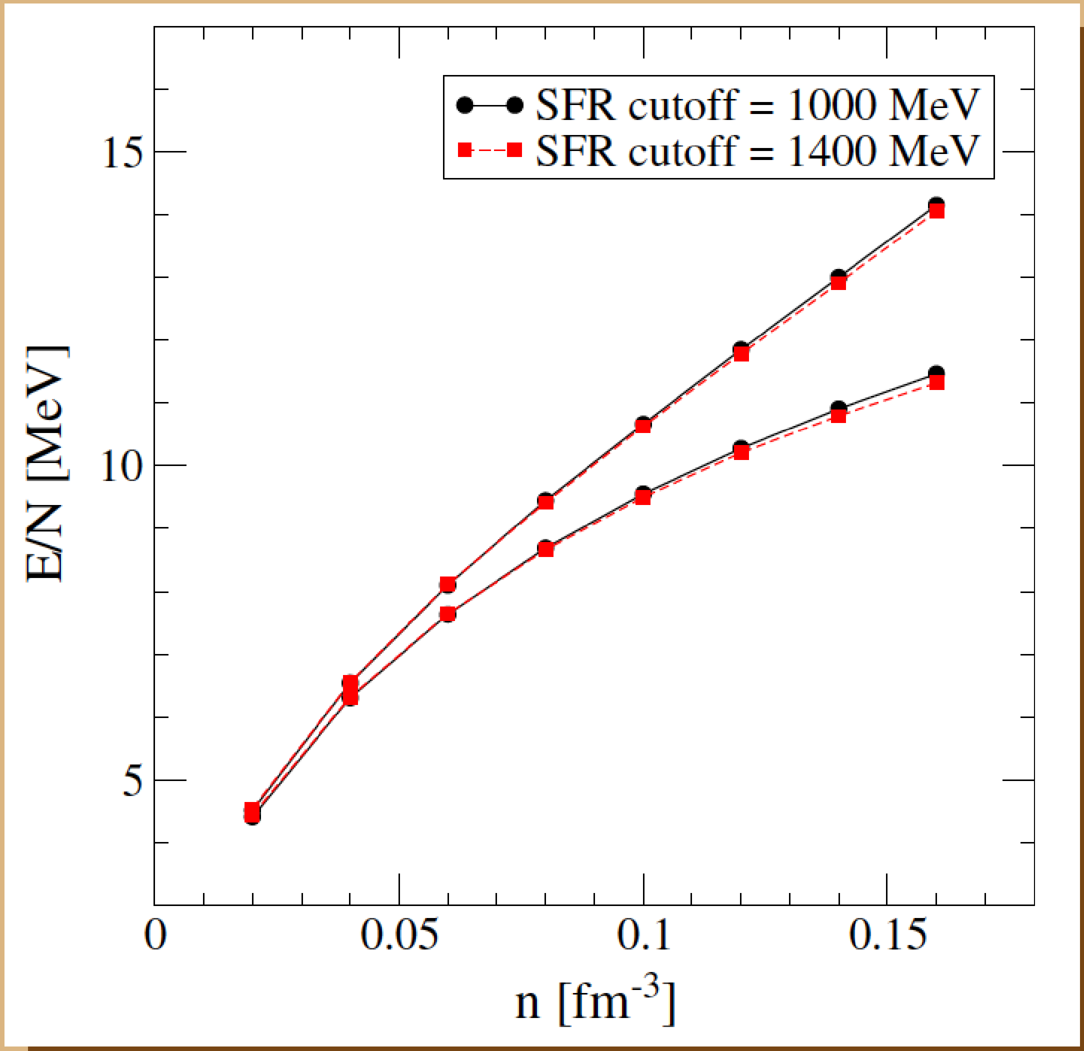


- MBPT bands come from diff. single-particle spectra
- Soft potential in excellent agreement with AFDMC

NEUTRONS

A. Gezerlis, I. Tews, E. Epelbaum, M. Freunek, S. Gandolfi, K. Hebeler, A. Nogga, A. Schwenk, arxiv:1406.0454

Varying the SFR cutoff



- SFR cutoff expected not to matter
- Spectral-function regularization cutoff (kept fixed before) now varied
- Clearly shows lack of dependence

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A. Gezerlis, I. Tews, E. Epelbaum, M. Freunek, S. Gandolfi, K. Hebeler, A. Nogga, A. Schwenk, arxiv:1406.0454

Newer results: perturbativeness

First-ever opportunity to non-perturbatively test pertubativeness

- Use one chiral order in the propagator (many-body evolution), another chiral order when evaluating observables

- As for all observables that do not commute with the Hamiltonian, we need to explicitly extrapolate:

$$\langle H \rangle_{NLO+N^2LO} = \langle T + V \rangle_{NLO} - \langle V \rangle^{\text{ex}}_{NLO} + \langle V \rangle^{\text{ex}}_{N^2LO}$$

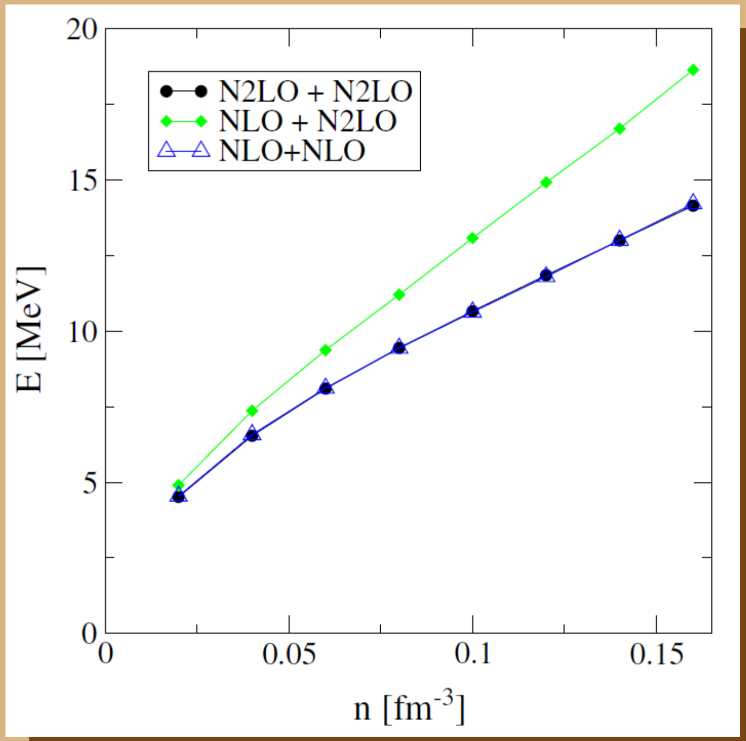
where

$$\langle \Phi | \hat{V} | \Phi \rangle^{\text{ex}} = 2 \langle \Phi | \hat{V} | \Psi_V \rangle - \langle \Psi_V | \hat{V} | \Psi_V \rangle$$

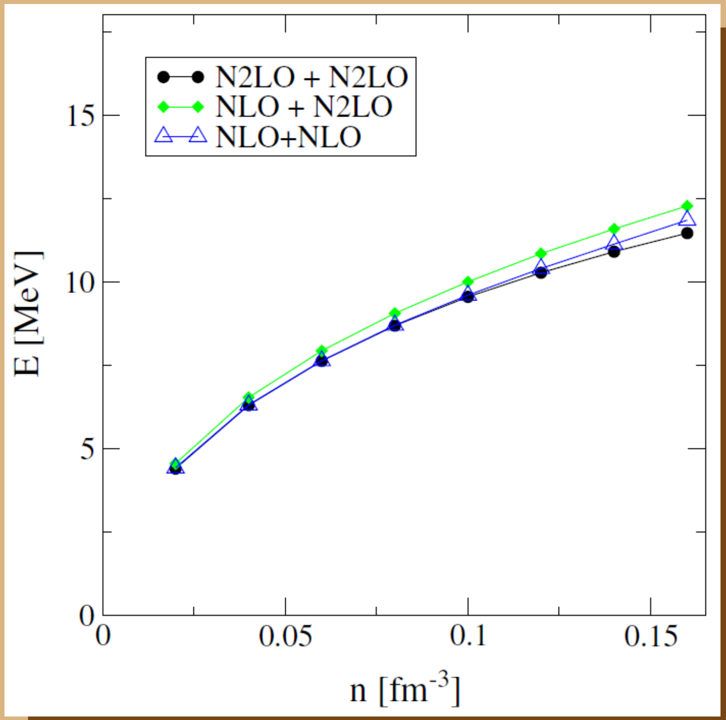
- If VMC is poor, extrapolation will be poor.

Newer results: perturbativeness

Hard(ish)
***R_0* = 1.0 fm**
***Λ* = 500 MeV**



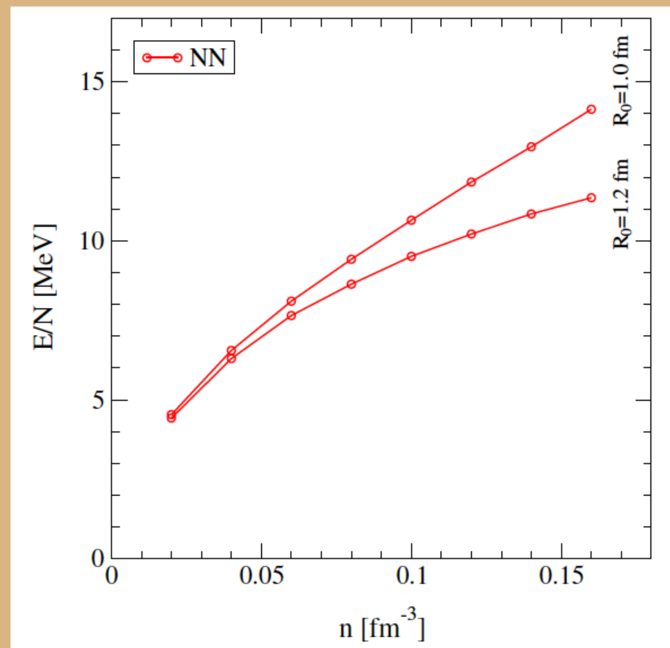
Soft
***R_0* = 1.2 fm**
***Λ* = 400 MeV**

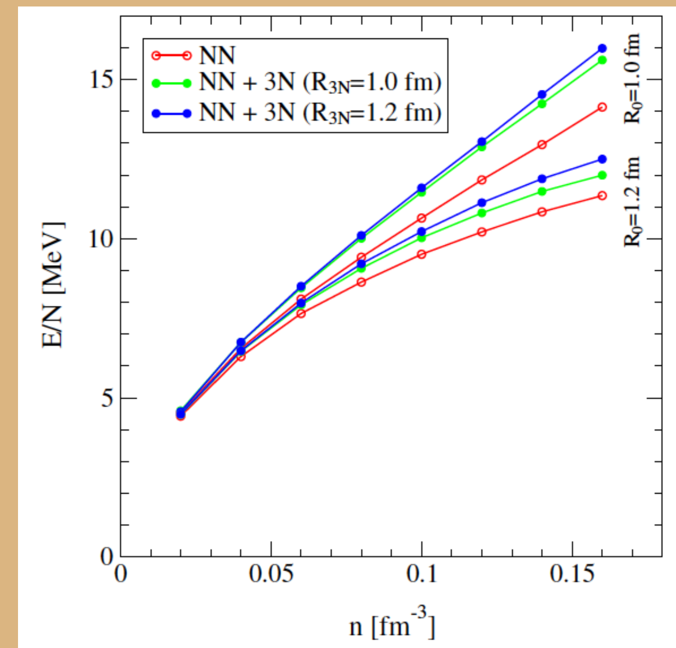


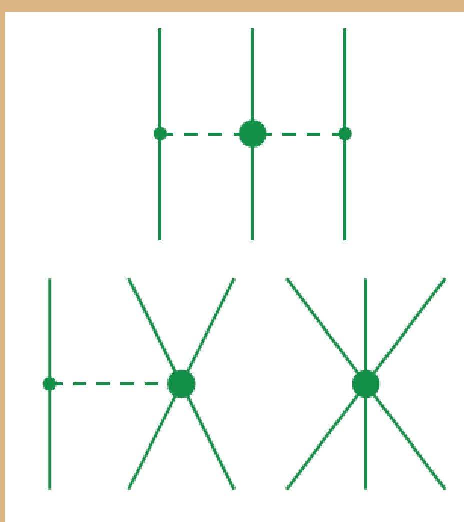
NEUTRONS

What about three-nucleon forces?

3NF: neutron matter





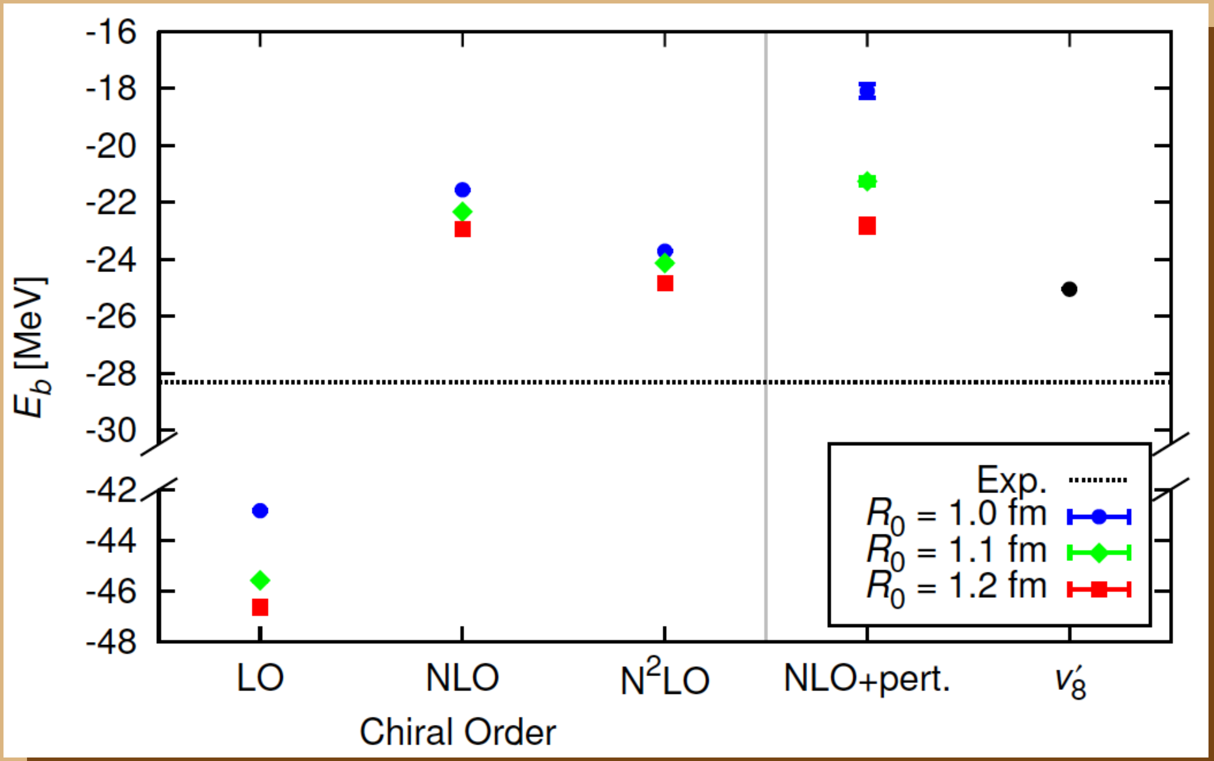


Haven't fit c_D and c_E yet, so this shows only two-pion exchange (1-2 MeV overall)

NEUTRONS

Now turn to finite nuclei

Nuclear GFMC

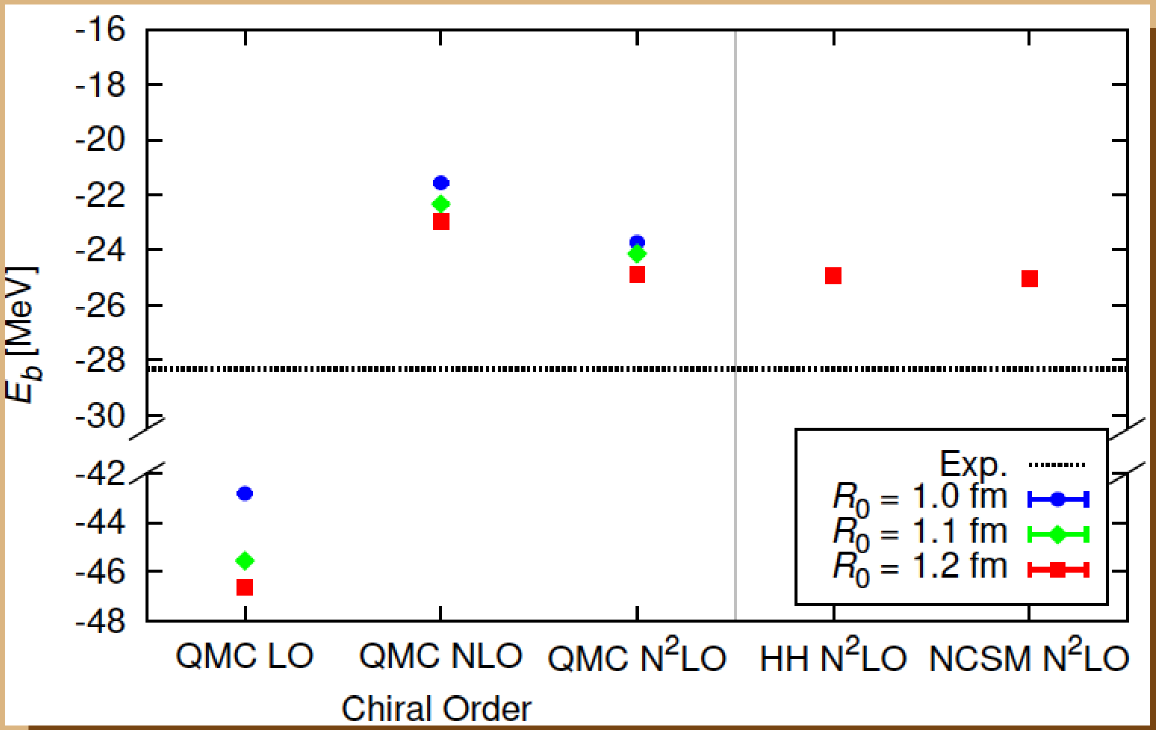


- Binding energy of ^4He
- Non-perturbative systematic error bands
- All results are strong force + Coulomb, no NNN

NUCLEONS

J. E. Lynn, J. Carlson, E. Epelbaum, S. Gandolfi, A. Gezerlis, A. Schwenk, in preparation

Local chiral EFT in other methods

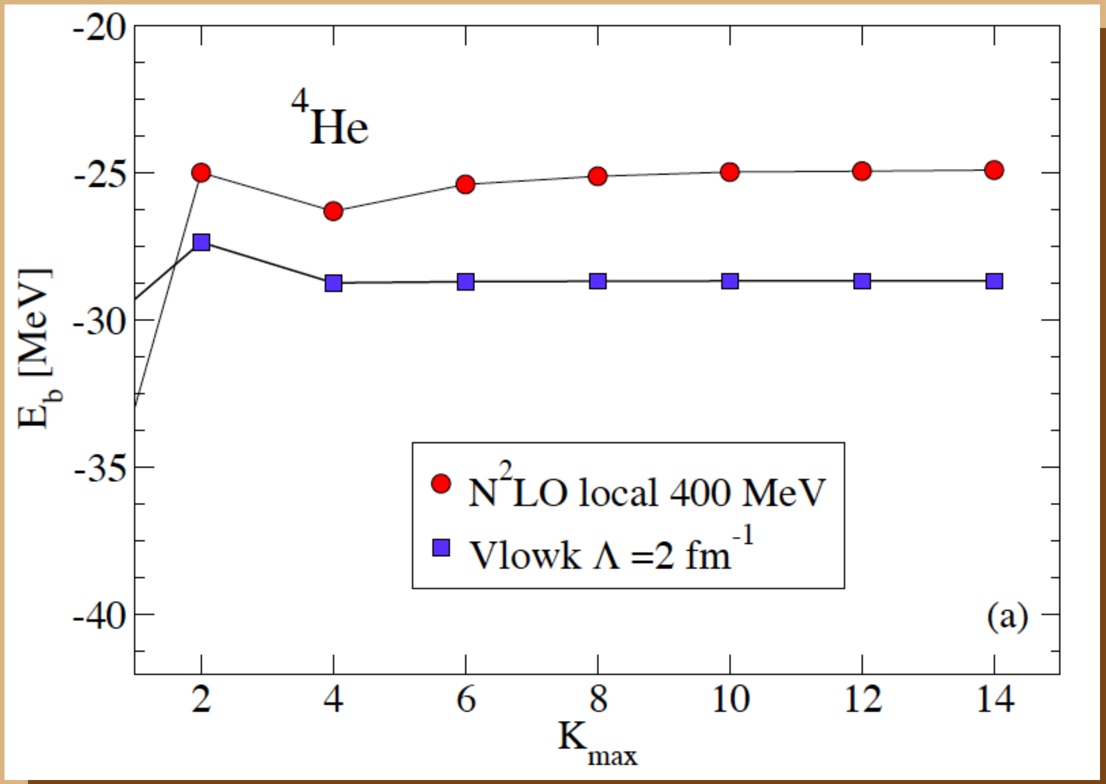


- Binding energy of ^{4}He
- Detailed benchmarking with hyperspherical harmonics (HH) and no-core shell model (NCSM)
- Probing perturbativeness

NUCLEONS

In collaboration with S. Bacca and P. Navrátil

Local chiral EFT in other methods



- Binding energy of ^{4}He
- Hyperspherical harmonics convergence vs max grandangular momentum
- Softest local potential slightly more difficult

NUCLEONS

In collaboration with S. Bacca and P. Navrátil

Conclusions

- Chiral EFT can now be used in continuum Quantum Monte Carlo methods
- Non-perturbative systematic error bands can be produced
- Back to TOV as soon as three-neutron forces have been incorporated
- “Jt is ywrite that every thing Hymself sheweth in the tastyng”

Acknowledgments

Funding

NSERC CRSNG

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Collaborators

- Sonia Bacca (TRIUMF)
- Joe Carlson (LANL)
- Evgeny Epelbaum (Bochum)
- Stefano Gandolfi (LANL)
- Joel Lynn (LANL)
- Petr Navrátil (TRIUMF)
- Achim Schwenk (Darmstadt)
- Ingo Tews (Darmstadt)