

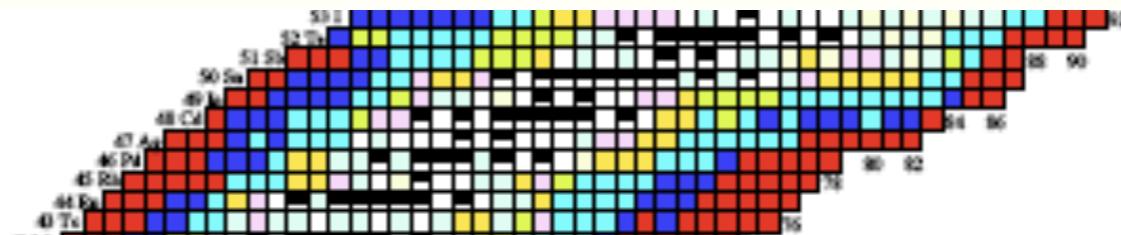
# Optimizing Energy Density Functionals for Nuclear Structure Models

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# Energy Density Functionals

- ✓ the nuclear many-body problem is effectively mapped onto a **one-body problem** without explicitly involving inter-particle interactions!



# Kohn-Sham DFT

For any interacting system, there exists a **local single-particle (Kohn-Sham) potential**, such that the exact ground-state density equals the ground-state density of a non-interacting system:

$$n(\mathbf{r}) = n_s(\mathbf{r}) \equiv \sum_i^{\text{occ}} |\phi_i(\mathbf{r})|^2$$

The single-particle orbitals are solutions of the Kohn-Sham equations:

$$[-\nabla^2/2 + v_s(\mathbf{r})] \phi_i(\mathbf{r}) = \varepsilon_i \phi_i(\mathbf{r})$$

⇒ Kohn-Sham potential:

$$v_s[n(\mathbf{r})] = v(\mathbf{r}) + \int d^3r' \frac{n(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} + v_{xc}[n(\mathbf{r})]$$

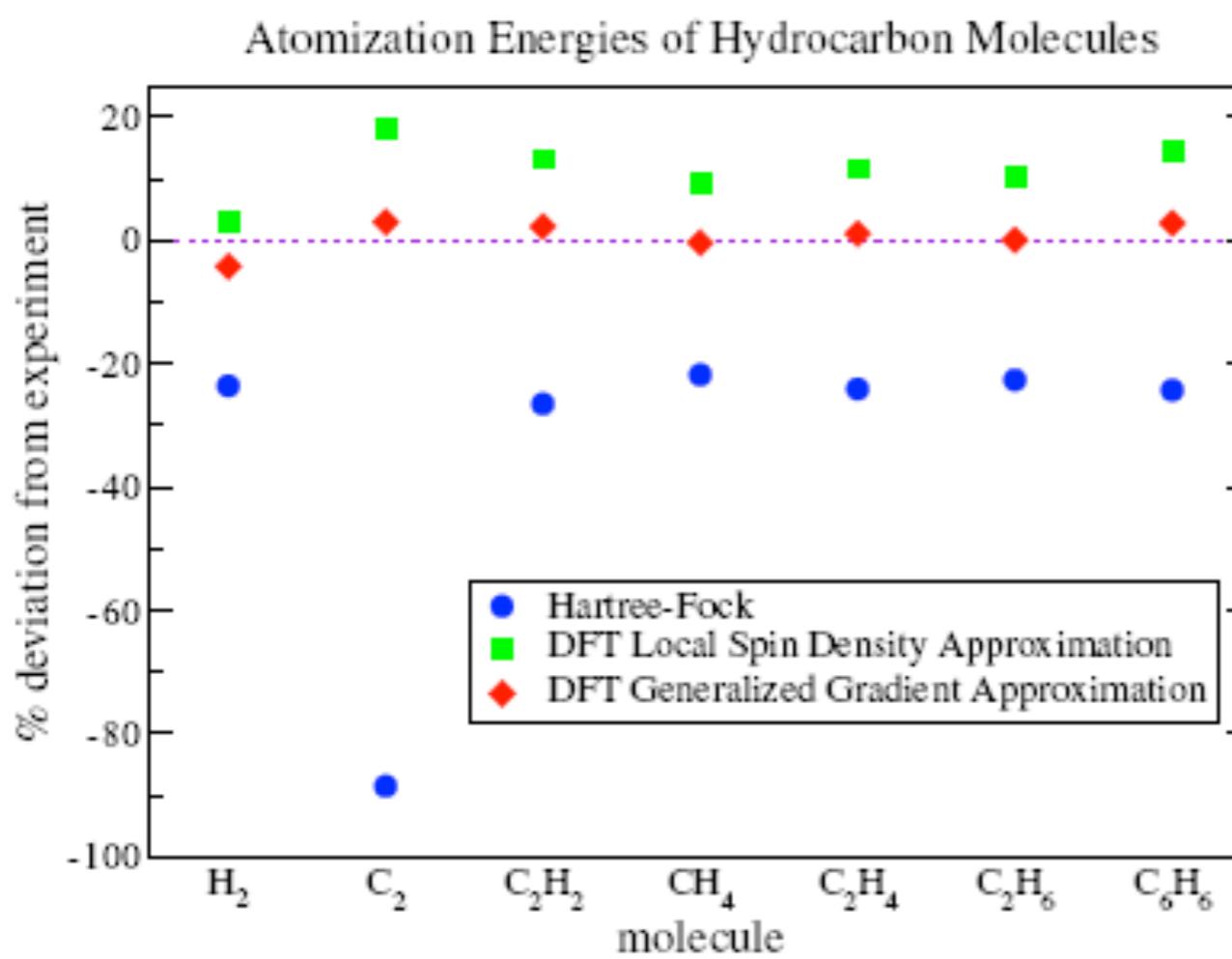
the **exchange-correlation potential** is defined by:

$$v_{xc}[n(\mathbf{r})] = \frac{\delta E_{xc}[n]}{\delta n(\mathbf{r})}$$

 self-consistent Kohn-Sham DFT: includes correlations and therefore goes beyond the Hartree-Fock. It has the advantage of being a *local scheme*.

The practical usefulness of the Kohn-Sham scheme depends entirely on whether accurate approximations for  $E_{xc}$  can be found!

# Exchange-correlation functional $\Rightarrow$ Jacob's ladder of DFT approximations for $E_{xc}$



unoccupied

$\{\Psi_i\}$

$\mathcal{E}_x$

$\mathcal{T}$  and/or

$\nabla^2 \rho$

$\nabla \rho$

$\rho$

generalized RPA

hyper-GGA

meta-GGA

GGA

LDA

Hartree world

TABLE I. Atomization energies of molecules, in kcal/mol (1 eV = 23.06 kcal/mol).  $E_{XC}$  has been evaluated on self-consistent densities at experimental geometries [33]. Nonspherical densities and Kohn-Sham potentials have been used for open-shell atoms [34]. The calculations are performed with a modified version of the CADPAC program [35]. The experimental values for  $\Delta E$  (with zero point vibration removed) are taken from Ref. [36]. PBE is the simplified GGA proposed here. UHF is unrestricted Hartree-Fock, for comparison.

System	$\Delta E^{\text{UHF}}$	$\Delta E^{\text{LSD}}$	$\Delta E^{\text{PW91}}$	$\Delta E^{\text{PBE}}$	$\Delta E^{\text{expt}}$
H <sub>2</sub>	84	113	105	105	109
LiH	33	60	53	52	58
CH <sub>4</sub>	328	462	421	420	419
NH <sub>3</sub>	201	337	303	302	297
OH	68	124	110	110	107
H <sub>2</sub> O	155	267	235	234	232
HF	97	162	143	142	141
Li <sub>2</sub>	3	23	20	19	24
LiF	89	153	137	136	139
Be <sub>2</sub>	-7	13	10	10	3
C <sub>2</sub> H <sub>2</sub>	294	460	415	415	405
C <sub>2</sub> H <sub>4</sub>	428	633	573	571	563
HCN	199	361	326	326	312
CO	174	299	269	269	259
N <sub>2</sub>	115	267	242	243	229
NO	53	199	171	172	153
O <sub>2</sub>	33	175	143	144	121
F <sub>2</sub>	-37	78	54	53	39
P <sub>2</sub>	36	142	120	120	117
Cl <sub>2</sub>	17	81	64	63	58
Mean abs. error	71.2	31.4	8.0	7.9	...

Mean absolute error of the atomization energies for 20 molecules:

Approximation	Mean abs. error (eV)
Unrestricted Hartree-Fock	3.1 (underbinding)
LDA	1.3 (overbinding)
GGA	0.3 (mostly overbinding)
Desired “chemical accuracy”	0,05

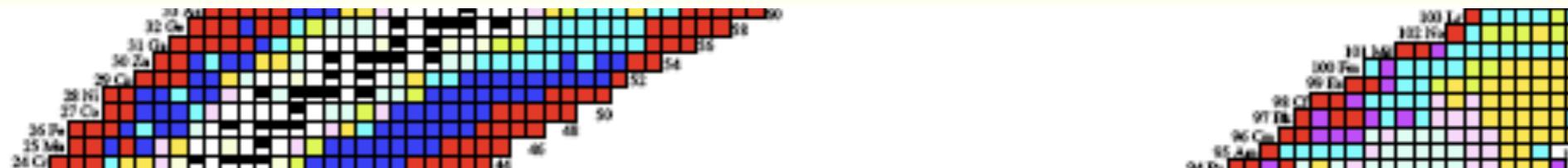
⇒ Multiply by  $10^6$  and compare with the nuclear case!

# Relativistic Energy Density Functionals

- ✓ natural inclusion of the *spin degree of freedom* (spin-orbit potential with empirical strength).



- ✓ unique parameterization of *time-odd components* (currents) of the nuclear mean-field.



- ✓ the distinction between scalar and vector self-energies leads to a natural *saturation mechanism for nuclear matter*.



## Relativistic energy density functionals:

The elementary building blocks are two-fermion terms of the general type:

$$(\bar{\psi} \mathcal{O}_\tau \Gamma \psi) \quad \mathcal{O}_\tau \in \{1, \tau_i\} \quad \Gamma \in \{1, \gamma_\mu, \gamma_5, \gamma_5 \gamma_\mu, \sigma_{\mu\nu}\}$$

... isoscalar and isovector four-currents and scalar densities:

$$j_\mu = \langle \phi_0 | \bar{\psi} \gamma_\mu \psi | \phi_0 \rangle = \sum_k \bar{\psi}_k \gamma_\mu \psi_k ,$$

$$\vec{j}_\mu = \langle \phi_0 | \bar{\psi} \gamma_\mu \vec{\tau} \psi | \phi_0 \rangle = \sum_k \bar{\psi}_k \gamma_\mu \vec{\tau} \psi_k ,$$

$$\rho_S = \langle \phi_0 | \bar{\psi} \psi | \phi_0 \rangle = \sum_k \bar{\psi}_k \psi_k ,$$

$$\vec{\rho}_S = \langle \phi_0 | \bar{\psi} \vec{\tau} \psi | \phi_0 \rangle = \sum_k \bar{\psi}_k \vec{\tau} \psi_k$$

where  $|\phi_0\rangle$  is the nuclear ground state.

⇒ build four-fermion (contact) interaction terms in the various isospace-space channels:

**isoscalar-scalar:**

$$(\bar{\psi}\psi)^2$$

**isoscalar-vector:**

$$(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma^\mu\psi)$$

**isovector-scalar:**

$$(\bar{\psi}\vec{\tau}\psi) \cdot (\bar{\psi}\vec{\tau}\psi)$$

**isovector-vector:**

$$(\bar{\psi}\vec{\tau}\gamma_\mu\psi) \cdot (\bar{\psi}\vec{\tau}\gamma^\mu\psi)$$

Empirical ground-state properties of finite nuclei can only determine a small set of parameters in the expansion of an effective Lagrangian in powers of fields and their derivatives.

Already at lowest order one finds more parameters than can be uniquely determined from data.

⇒ effective Lagrangian:

$$\begin{aligned}\mathcal{L} = & \bar{\psi}(i\gamma \cdot \partial - m)\psi \\ & -\frac{1}{2}\alpha_S(\hat{\rho})(\bar{\psi}\psi)(\bar{\psi}\psi) - \frac{1}{2}\alpha_V(\hat{\rho})(\bar{\psi}\gamma^\mu\psi)(\bar{\psi}\gamma_\mu\psi) \\ & -\frac{1}{2}\alpha_{TV}(\hat{\rho})(\bar{\psi}\vec{\tau}\gamma^\mu\psi)(\bar{\psi}\vec{\tau}\gamma_\mu\psi) \\ & -\frac{1}{2}\delta_S(\partial_\nu\bar{\psi}\psi)(\partial^\nu\bar{\psi}\psi) - e\bar{\psi}\gamma \cdot A \frac{(1-\tau_3)}{2}\psi\end{aligned}$$

$$\alpha_i(\rho) = a_i + (b_i + c_i x)e^{-d_i x} \quad (i \equiv S, V, TV) \quad x = \rho/\rho_{sat}$$



Hartree



correlations

Only one isovector term and one gradient term can be constrained by data.

# Microscopic functionals

... universal exchange-correlation functional  $E_{xc}[\rho]$

1<sup>st</sup> step: *Local Density Approximation*

$$E_{xc}^{LDA} \equiv \int \varepsilon^{ChPT}[\rho(\mathbf{r})] \rho(\mathbf{r}) d^3r$$

2<sup>nd</sup> step: *second-order gradient correction to the LDA*

EFT calculations for inhomogeneous nuclear matter:

$$\mathcal{E}(\rho, \nabla \rho) = \rho \overline{E}(k_f) + (\nabla \rho)^2 F_\nabla(k_f) + \dots$$

# Semi-phenomenological functionals

Infinite nuclear matter cannot determine the density functional on the level of accuracy that is needed for a quantitative description of structure phenomena in finite nuclei.

... start from a favorite microscopic nuclear matter EOS.

... the parameters of the functional are fine-tuned to data of finite nuclei.

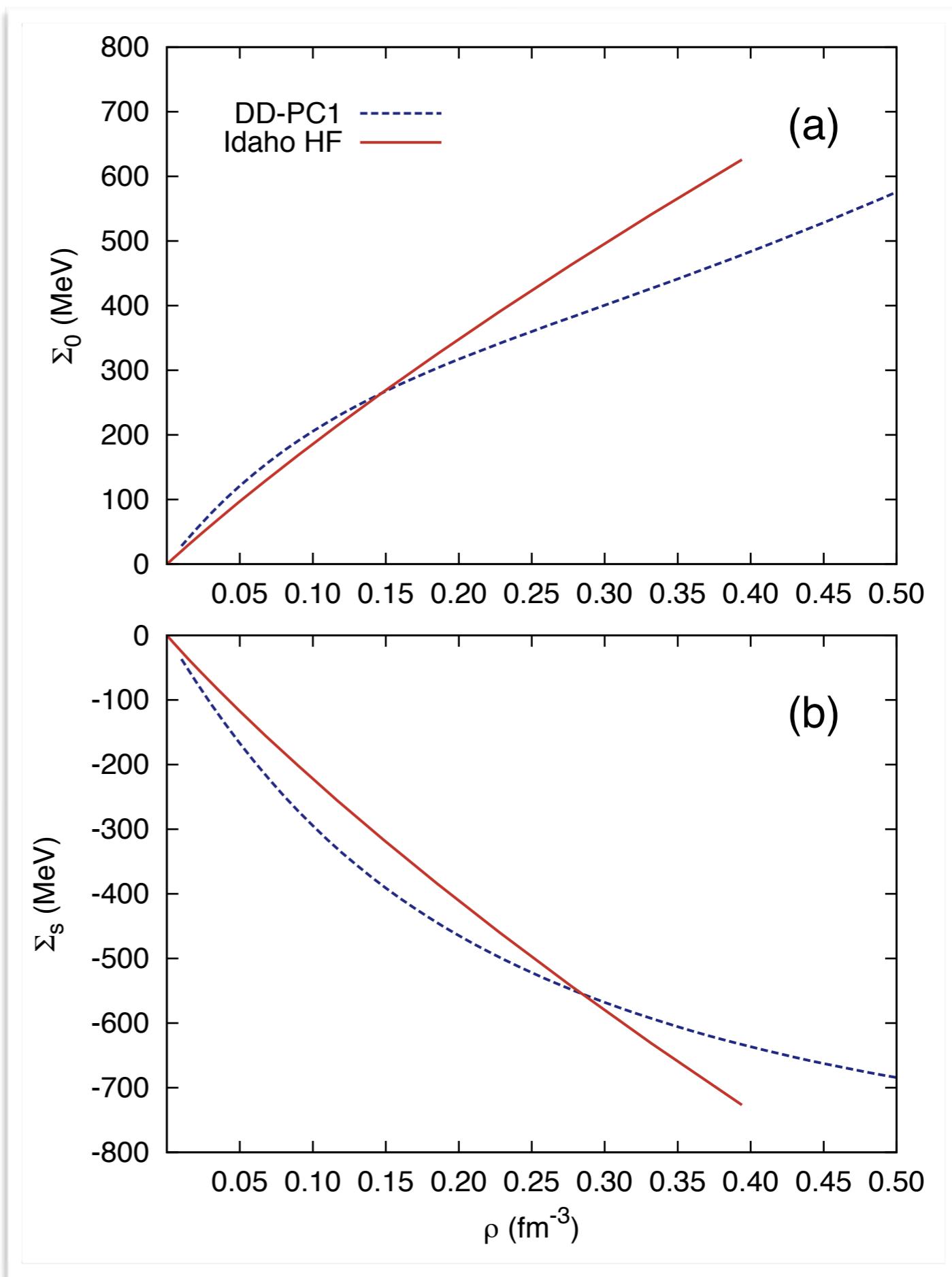
DD-PCI

... starts from microscopic nucleon self-energies in nuclear matter.

... parameters adjusted in self-consistent mean-field calculations of masses of **64** axially deformed nuclei in the mass regions  $A \sim 150\text{-}180$  and  $A \sim 230\text{-}250$ .

Density dependence of the DD-PC1 isoscalar vector and scalar nucleon self-energies in symmetric nuclear matter.

Starting approximation:  
Hartree-Fock self-energies  
calculated from the Idaho  
 $N^3LO$  NN-potential.



... calculated masses of finite nuclei are primarily sensitive to the three leading terms in the empirical mass formula:

$$E_B = a_v A + a_s A^{2/3} + a_4 \frac{(N - Z)^2}{4A} + \dots$$

... generate families of effective interactions characterized by different values of  $a_v$ ,  $a_s$  and  $a_4$ , and determine which parametrization minimizes the deviation from the empirical binding energies of a large set of deformed nuclei.

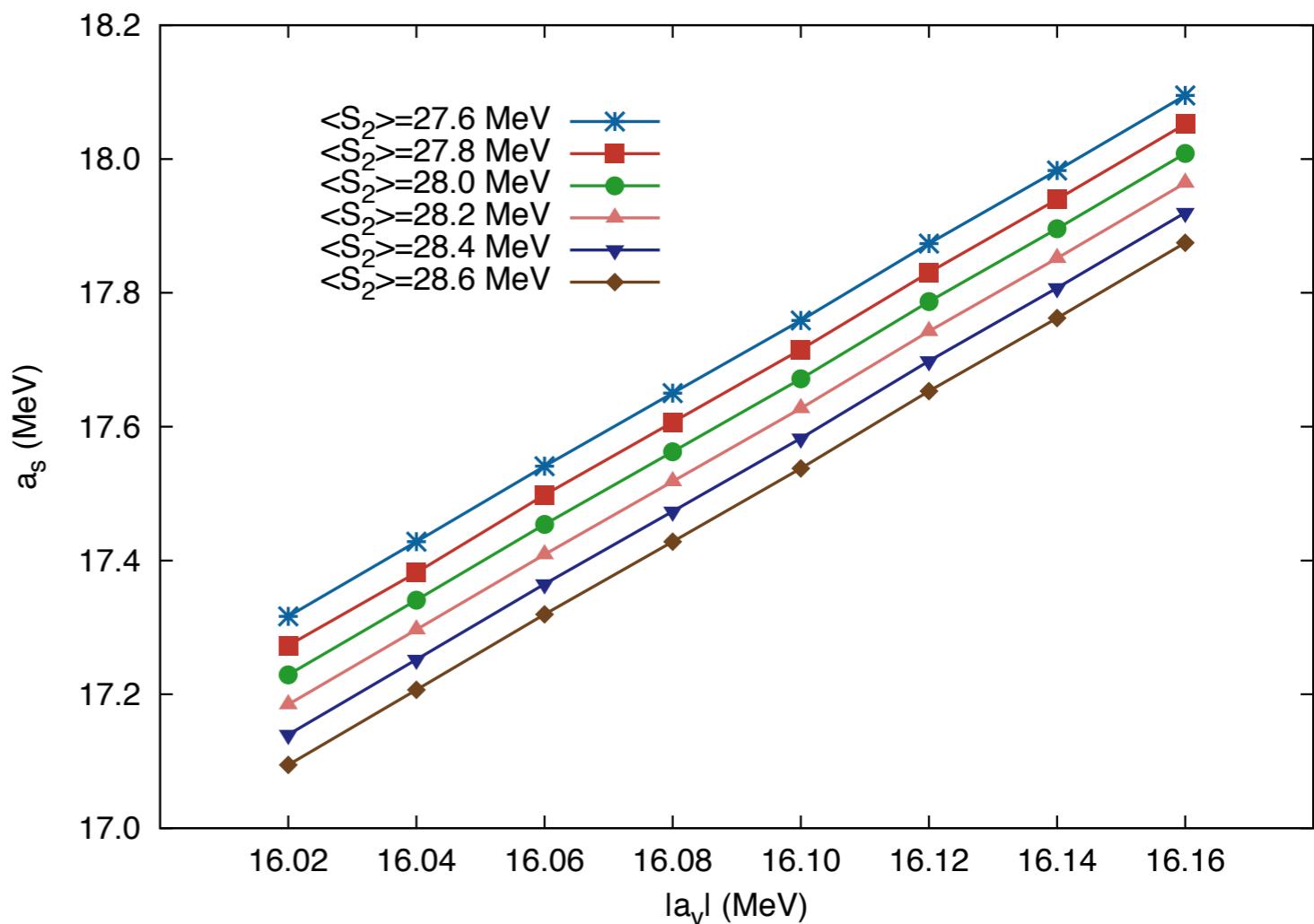
# Deformed nuclei

Binding energies used to adjust the parameters of the functional:

Z	62	64	66	68	70	72	90	92	94	96	98
$N_{min}$	92	92	92	92	92	72	140	138	138	142	144
$N_{max}$	96	98	102	104	108	110	144	148	150	152	152

Surface energies of semi-infinite nuclear matter that minimize the deviation of the calculated binding energies from data.

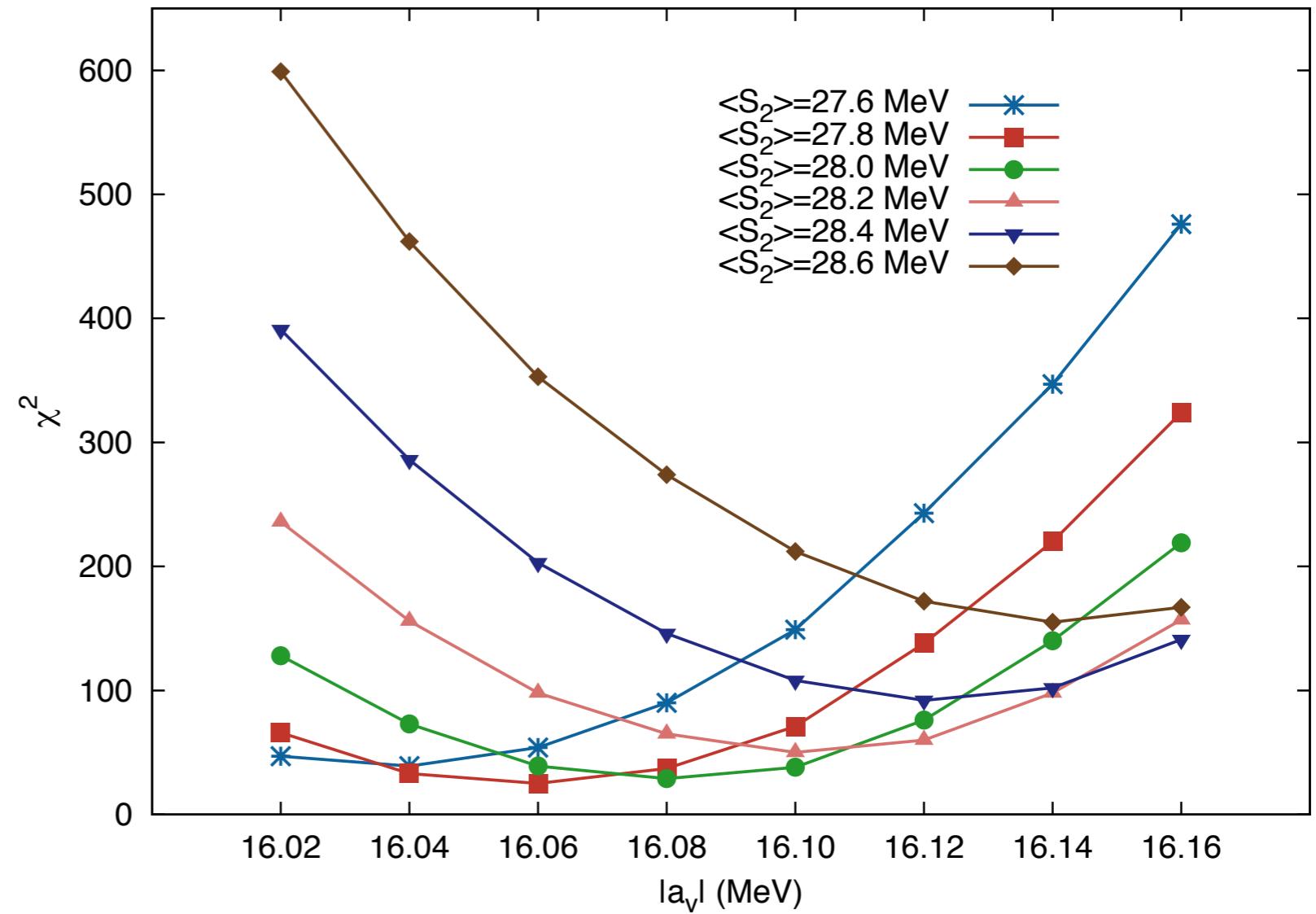
Required accuracy **0.05%**  $\Rightarrow$  absolute error of  $\pm 1$  MeV for the total binding energy



... 48 parameterizations of the energy density functional:

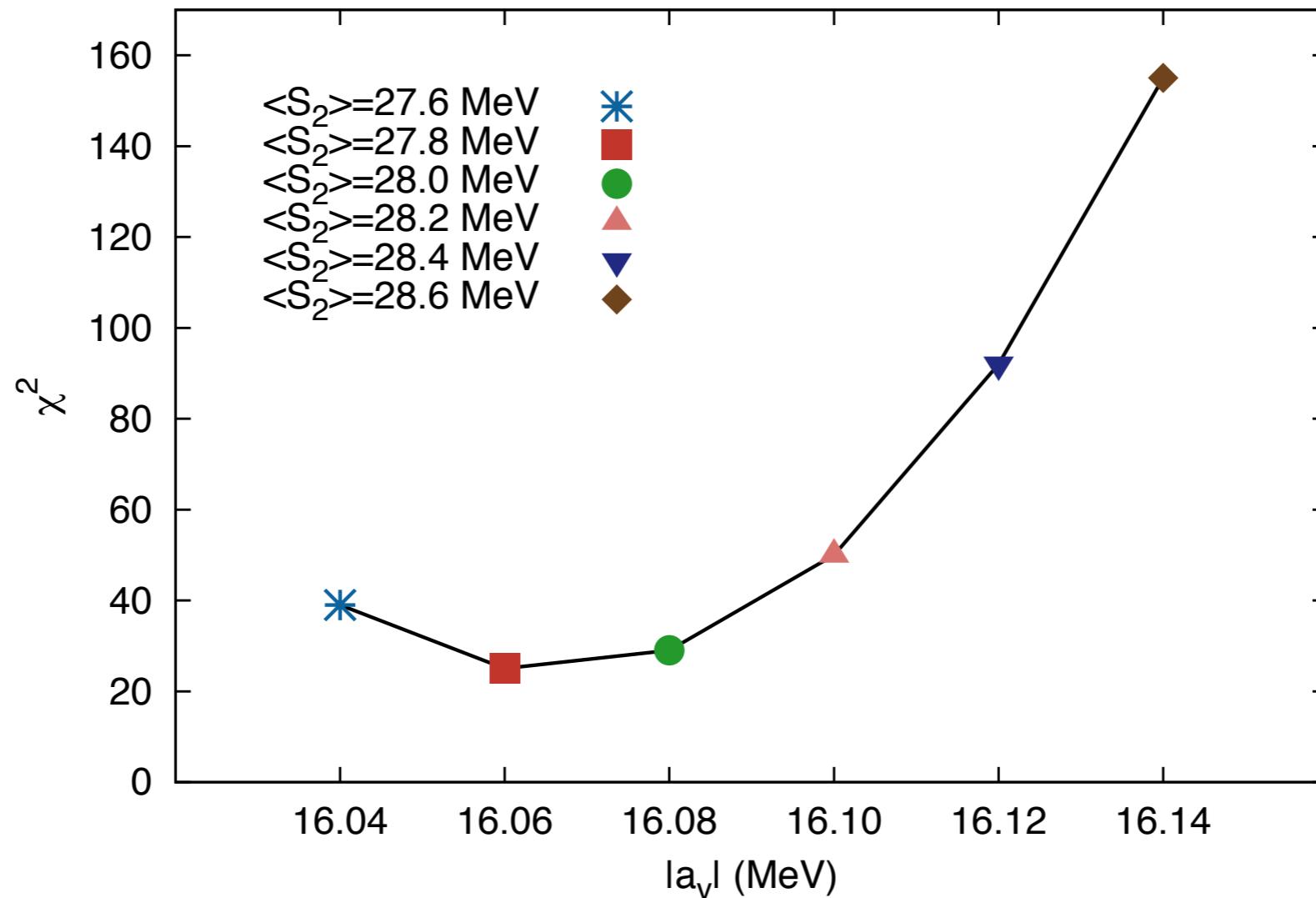
$$\chi^2 = \sum_i \left( \frac{E_B^{th}(i) - E_B^{exp}(i)}{\Delta E_B^{exp}(i)} \right)^2$$

$$\Delta E_B^{exp}(i) = 0.0005 E_B^{exp}(i)$$



For each value  $\langle S_2 \rangle$  of the symmetry energy, there is a unique combination of volume and surface energies that minimizes  $\chi^2$ !

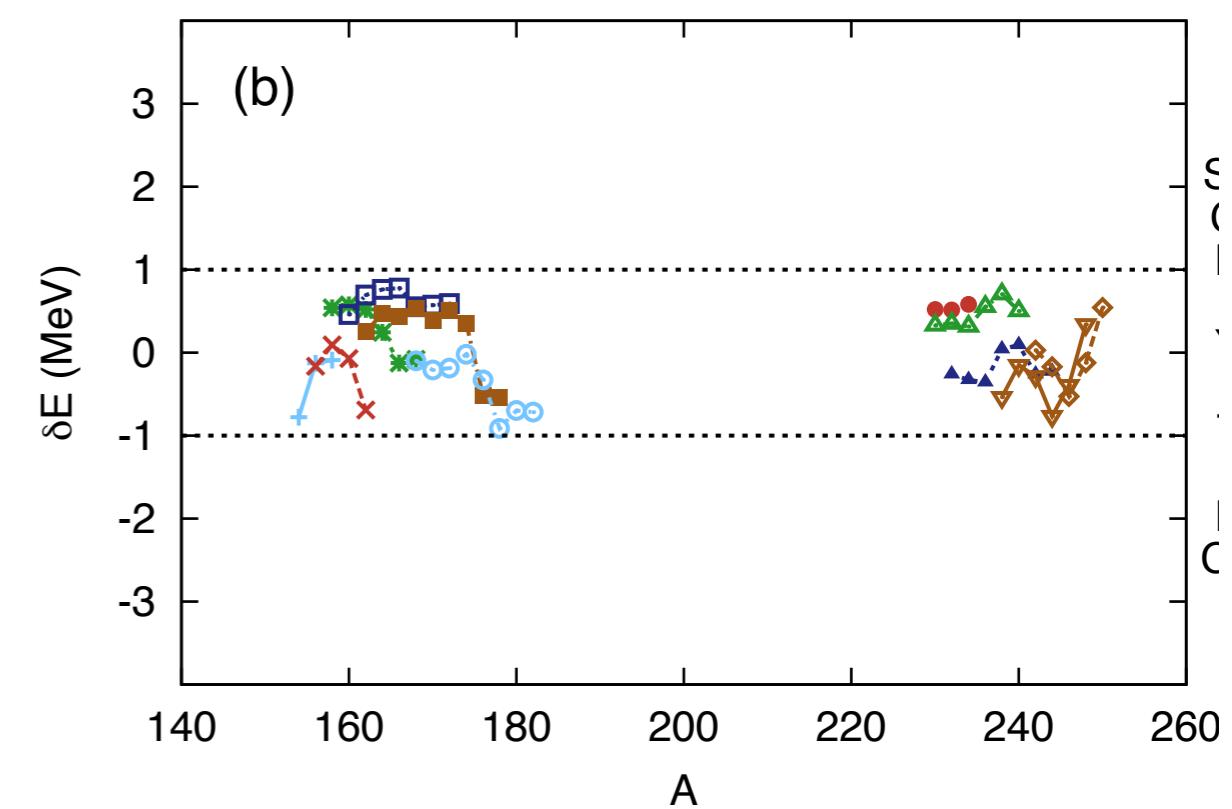
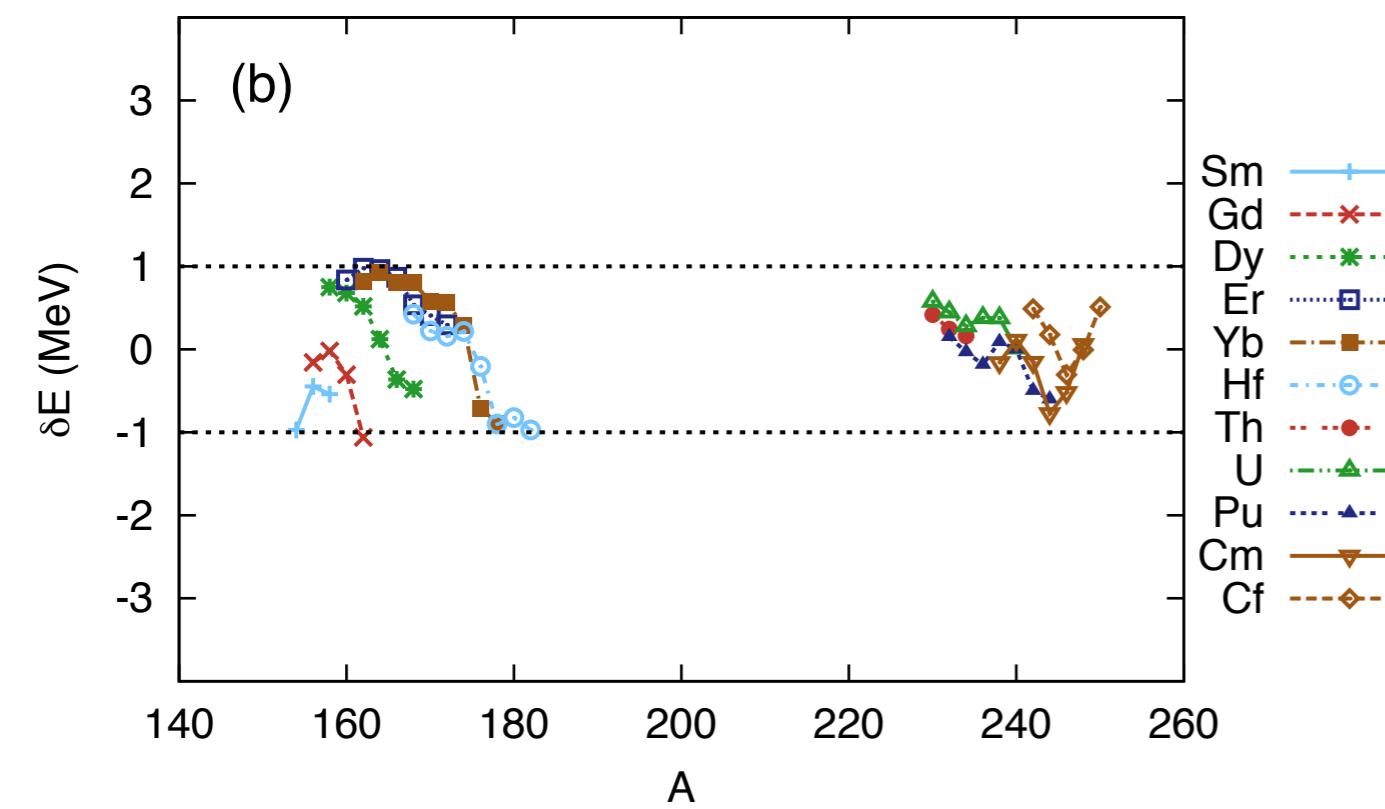
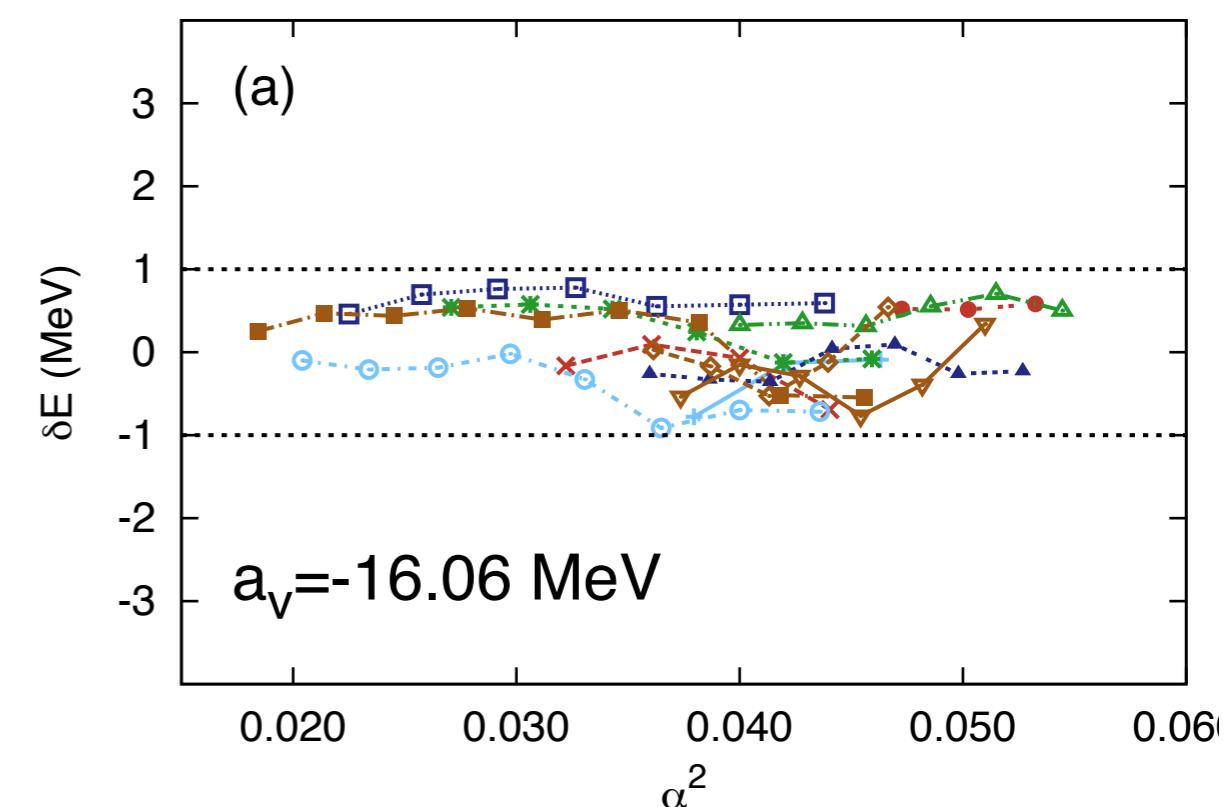
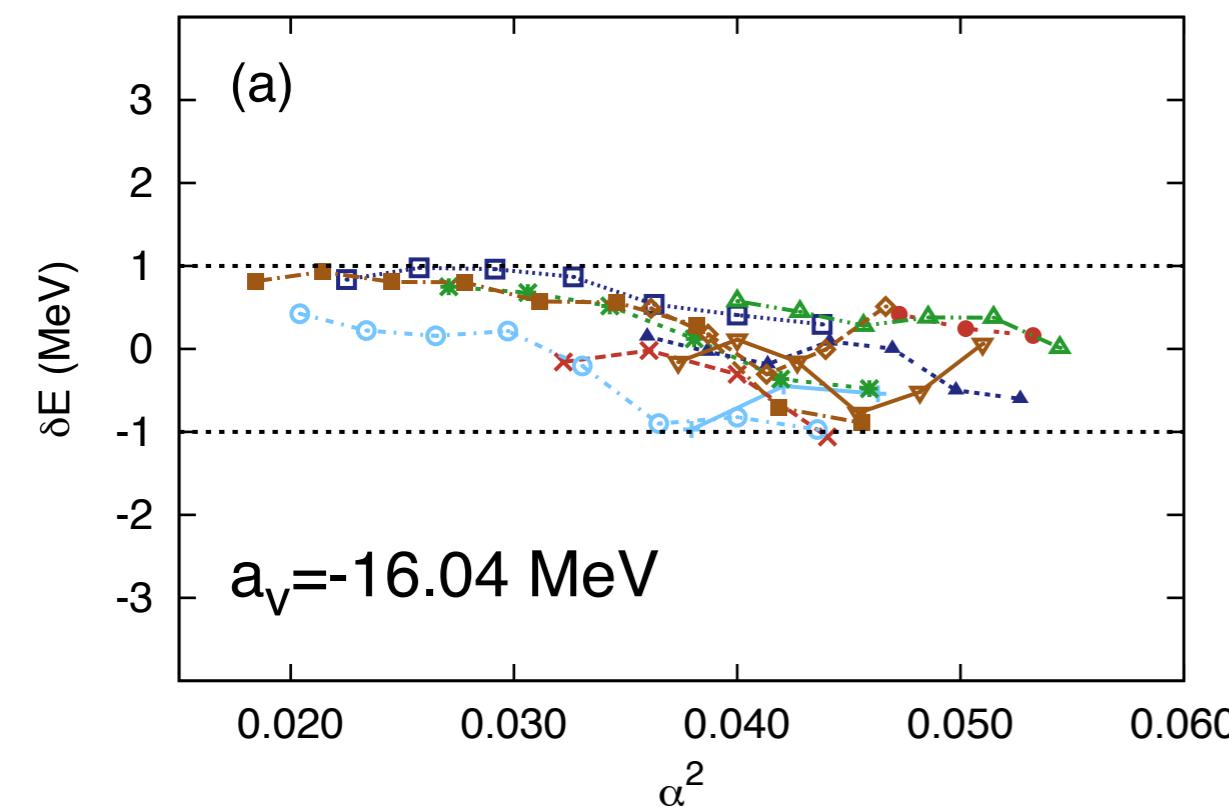
The minimum  $\chi^2$ -deviation of the theoretical binding energies from data, as a function of the volume energy coefficient:



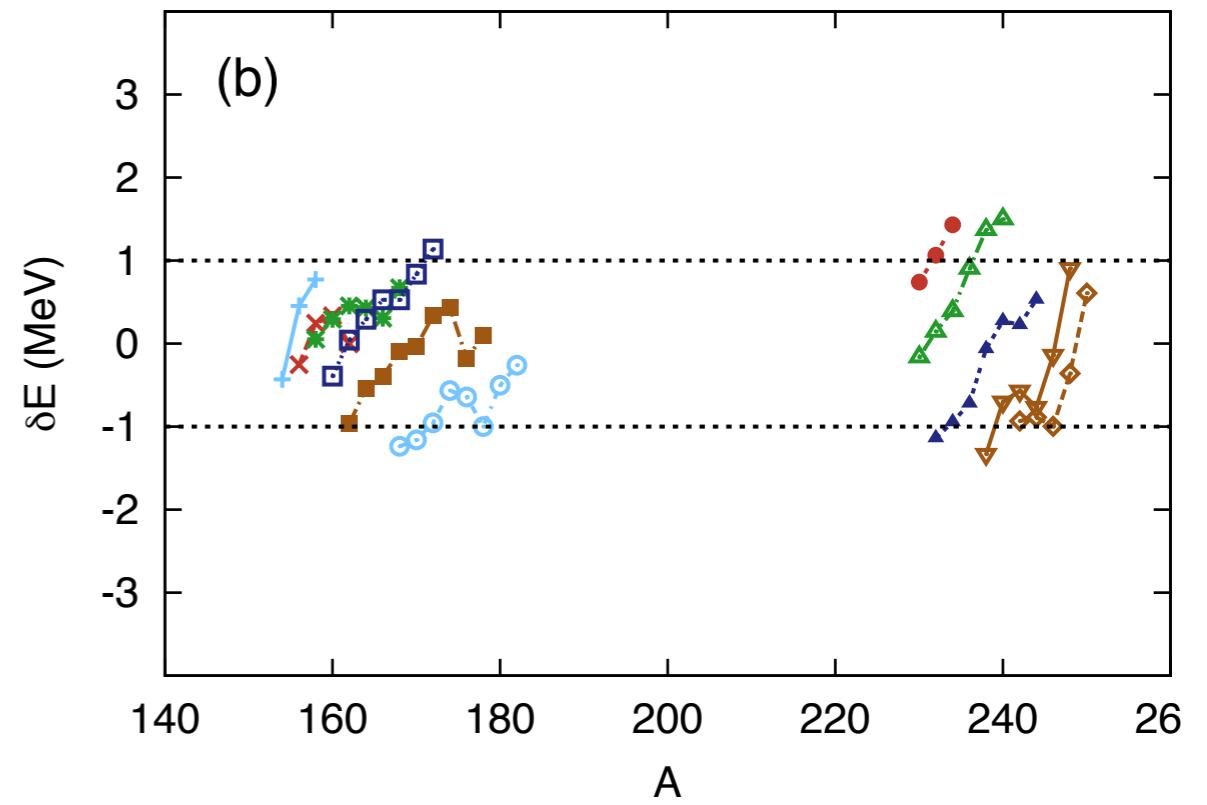
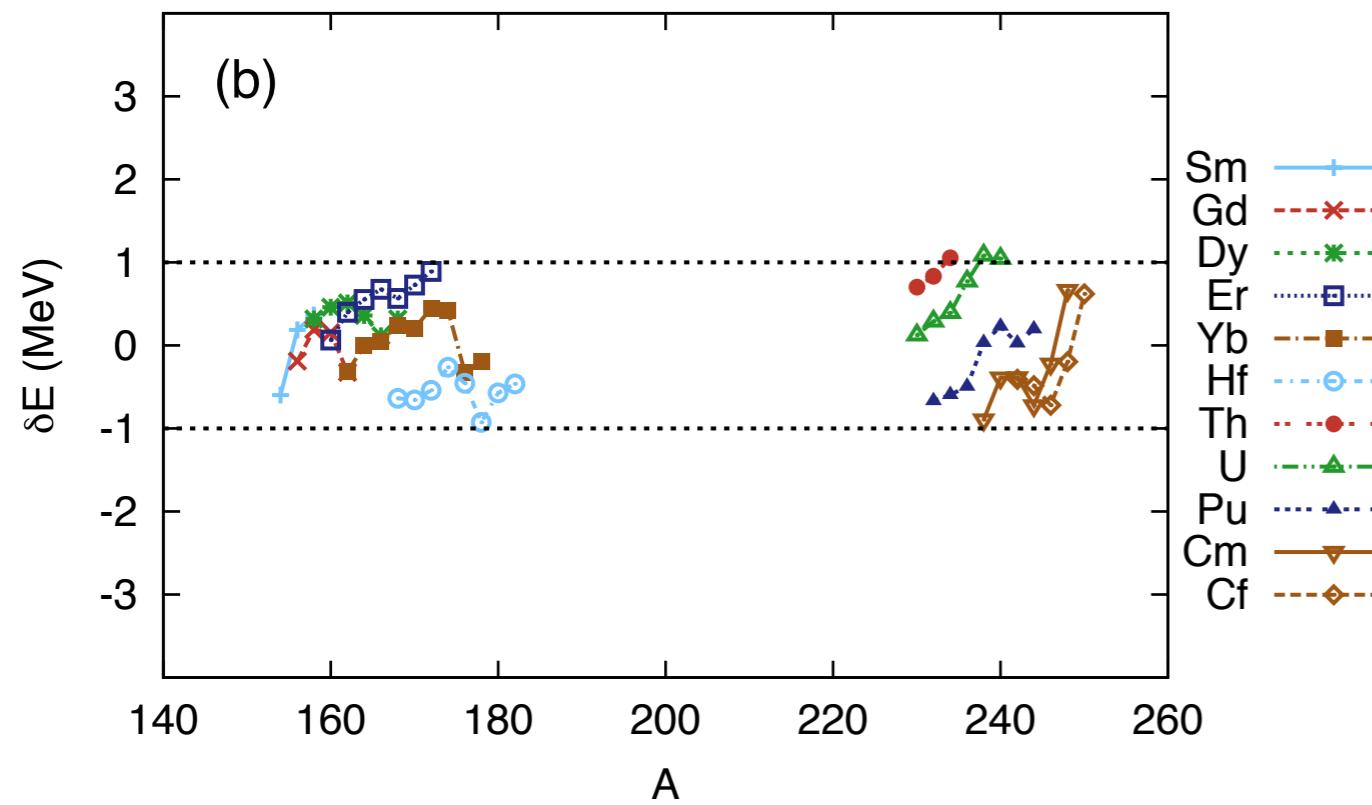
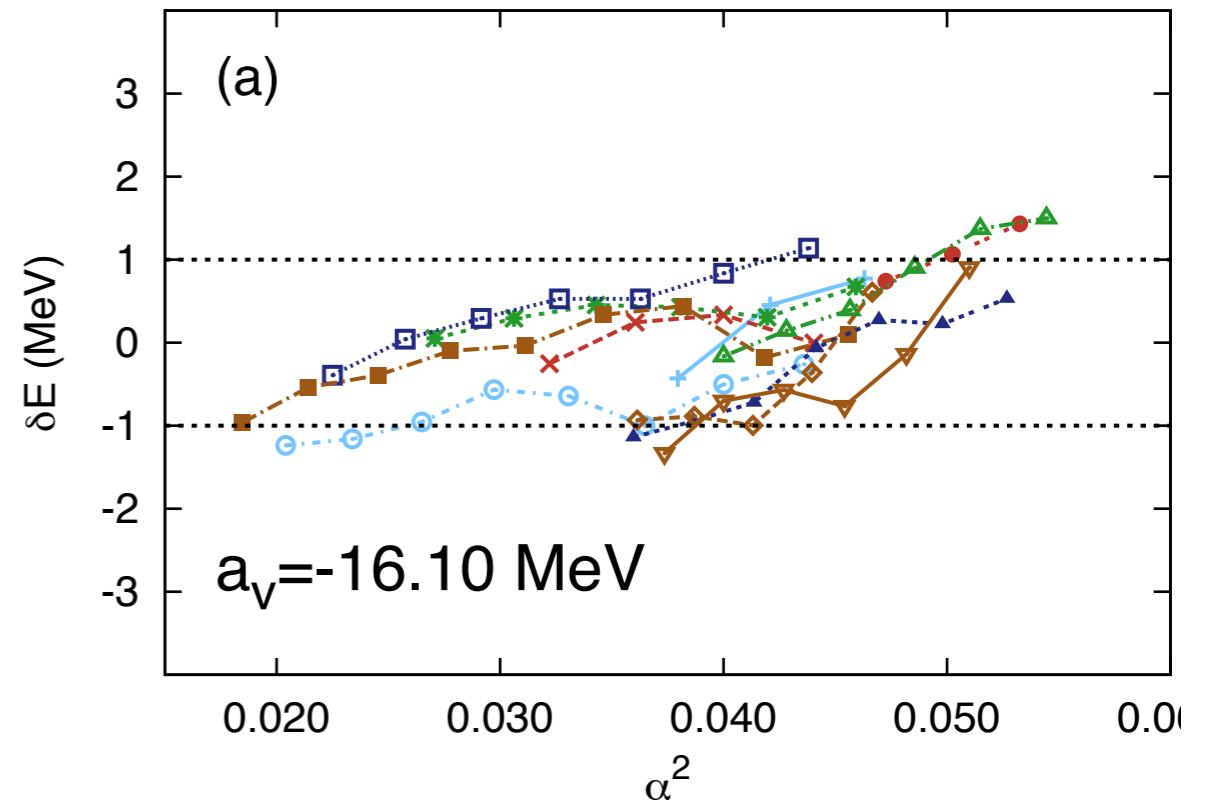
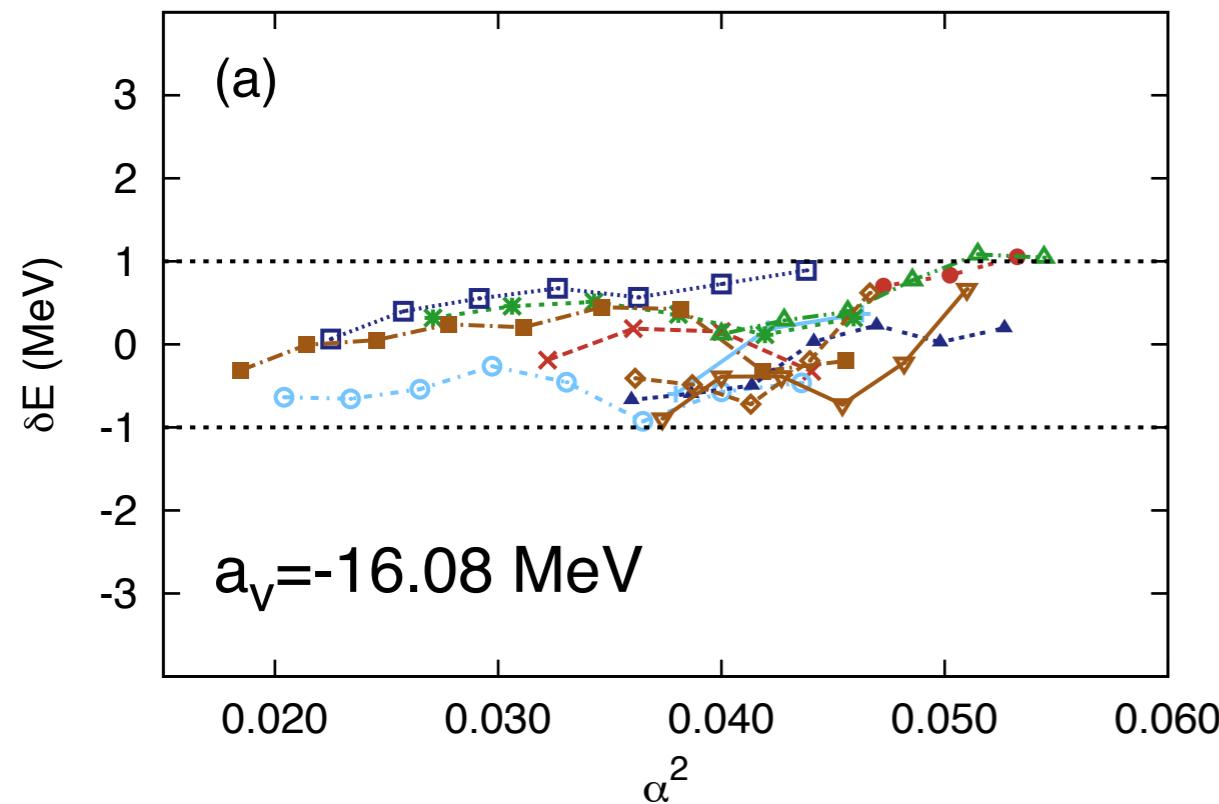
Absolute minimum:

$$a_v = -16.06 \text{ MeV} \quad \langle S_2 \rangle = 27.8 \text{ MeV} \quad a_s = 17.498 \text{ MeV}$$

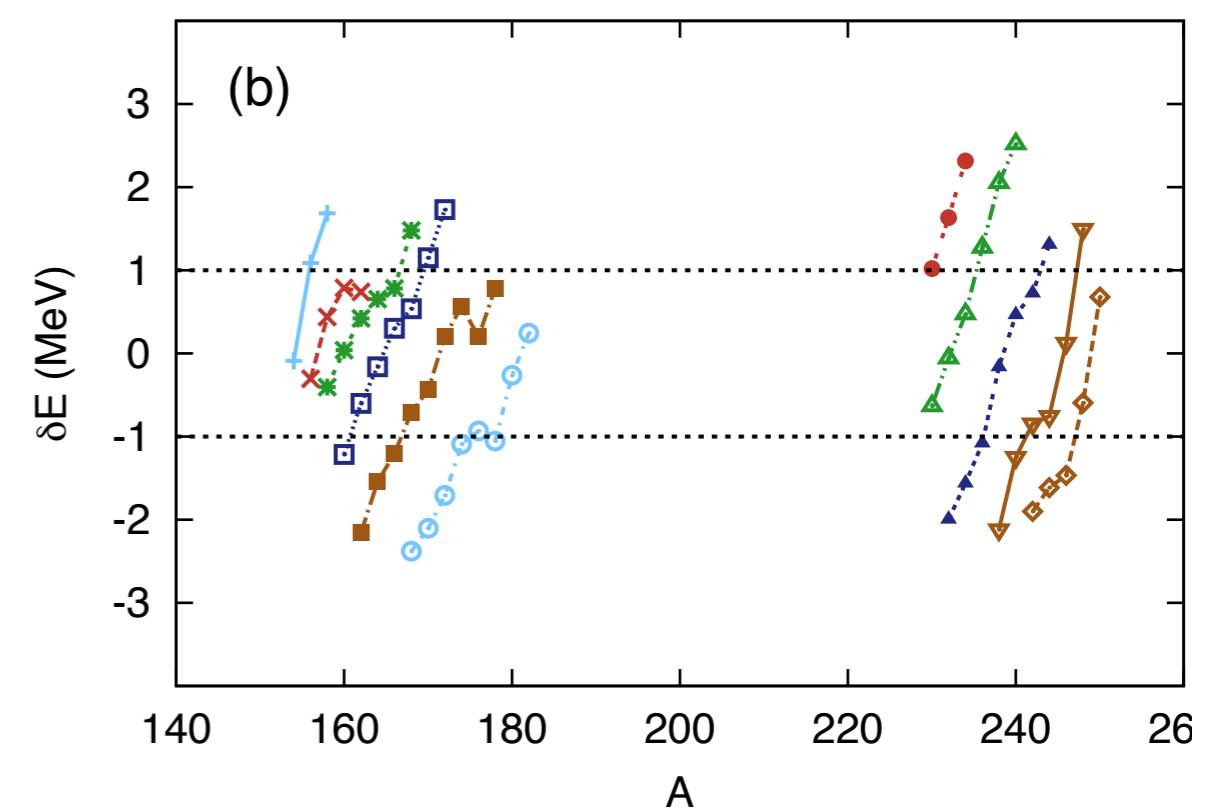
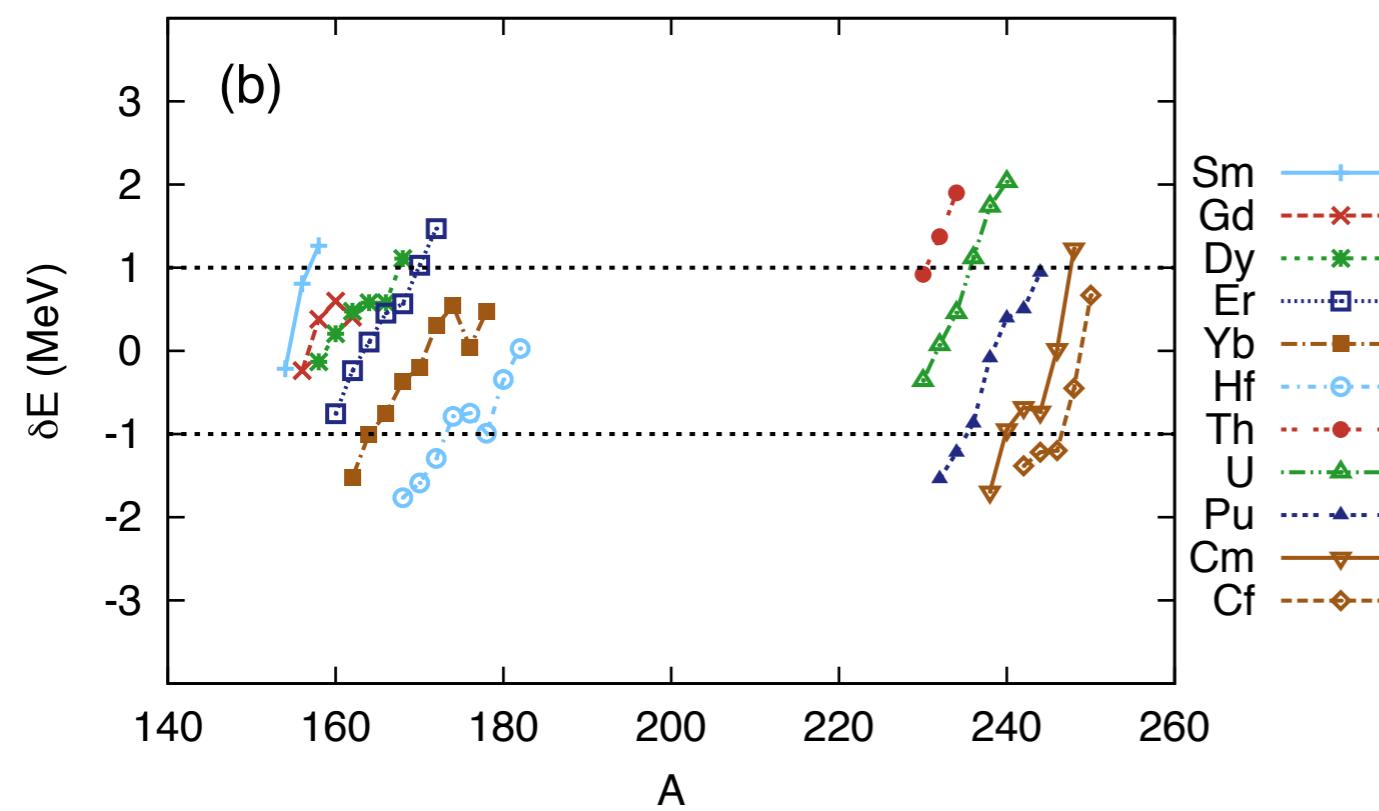
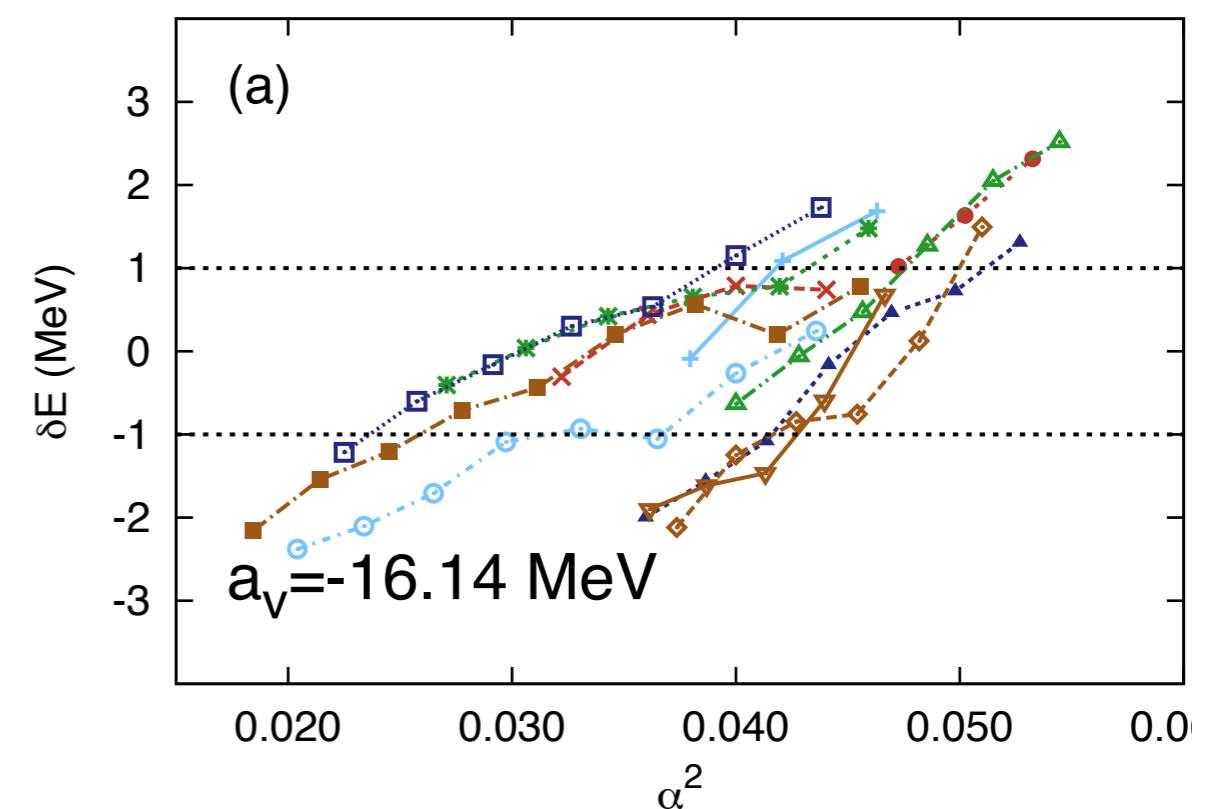
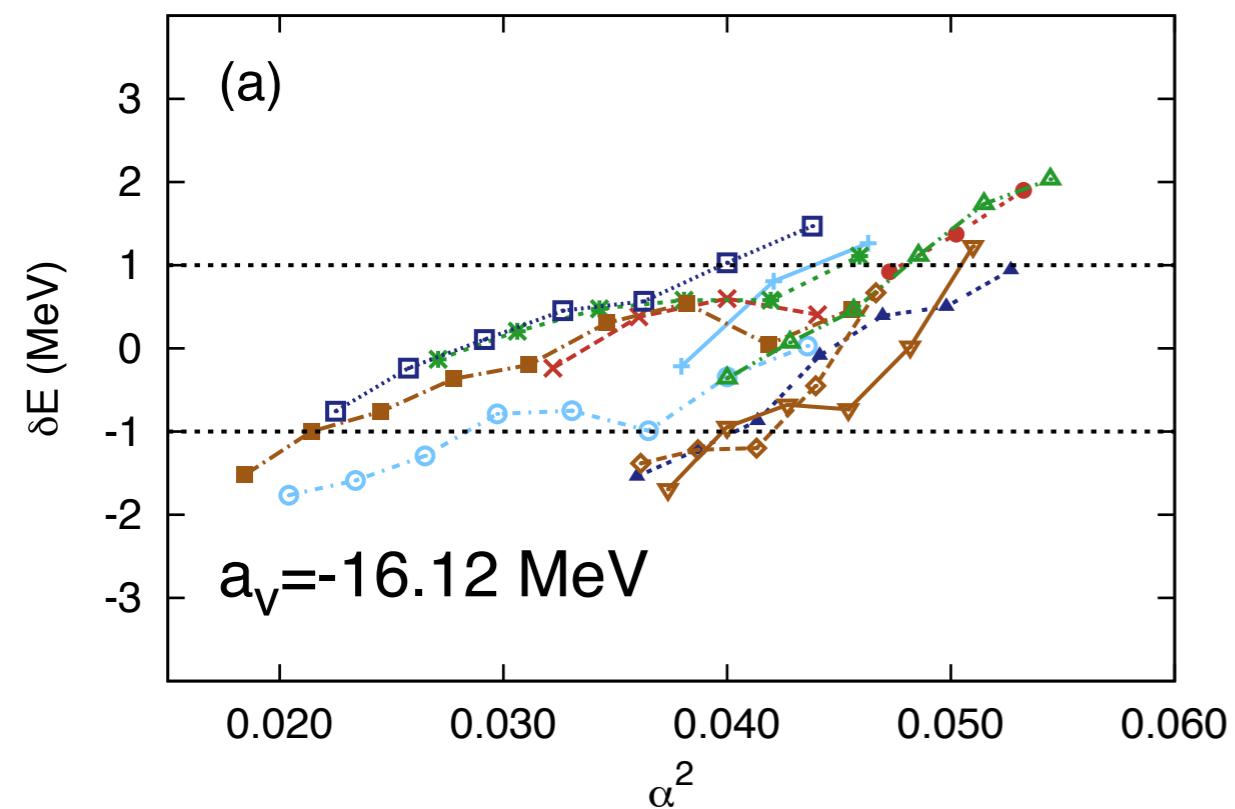
Absolute deviations of the calculated binding energies from data for 64 axially deformed nuclei:



$$\alpha^2 = \frac{(N - Z)^2}{A^2}$$

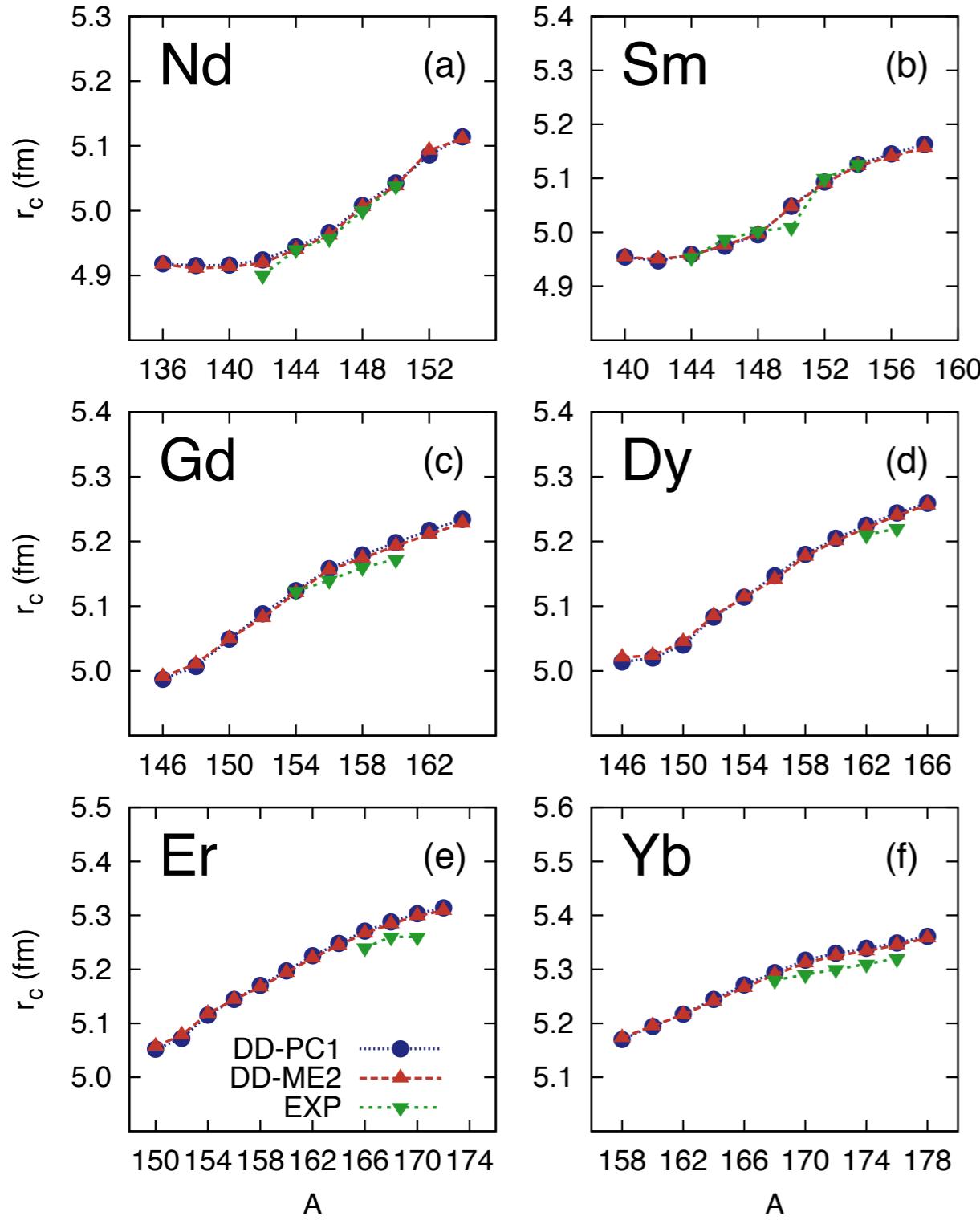


$$\alpha^2 = \frac{(N - Z)^2}{A^2}$$

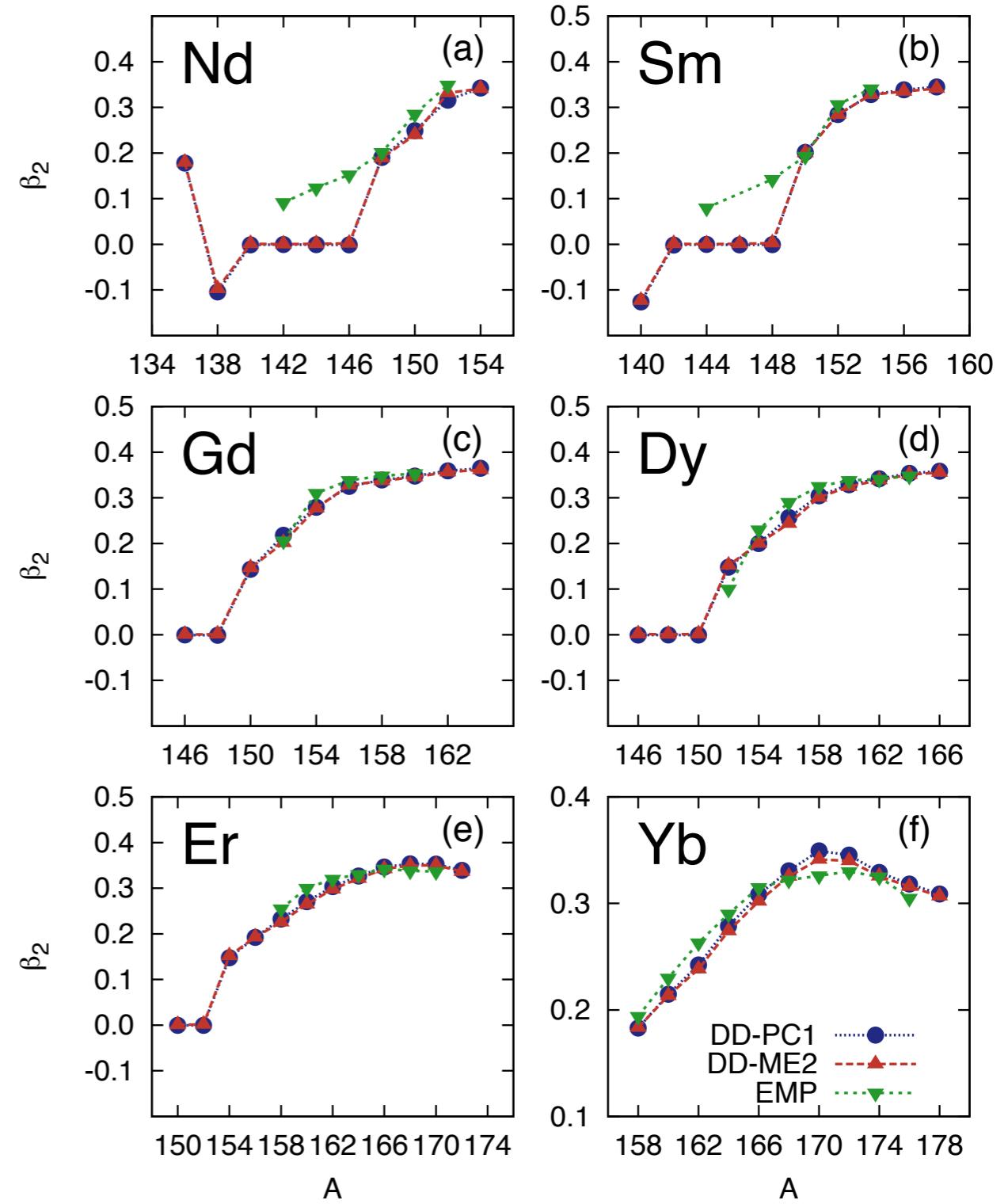


# Test: calculation of observables not included in the fitting procedure

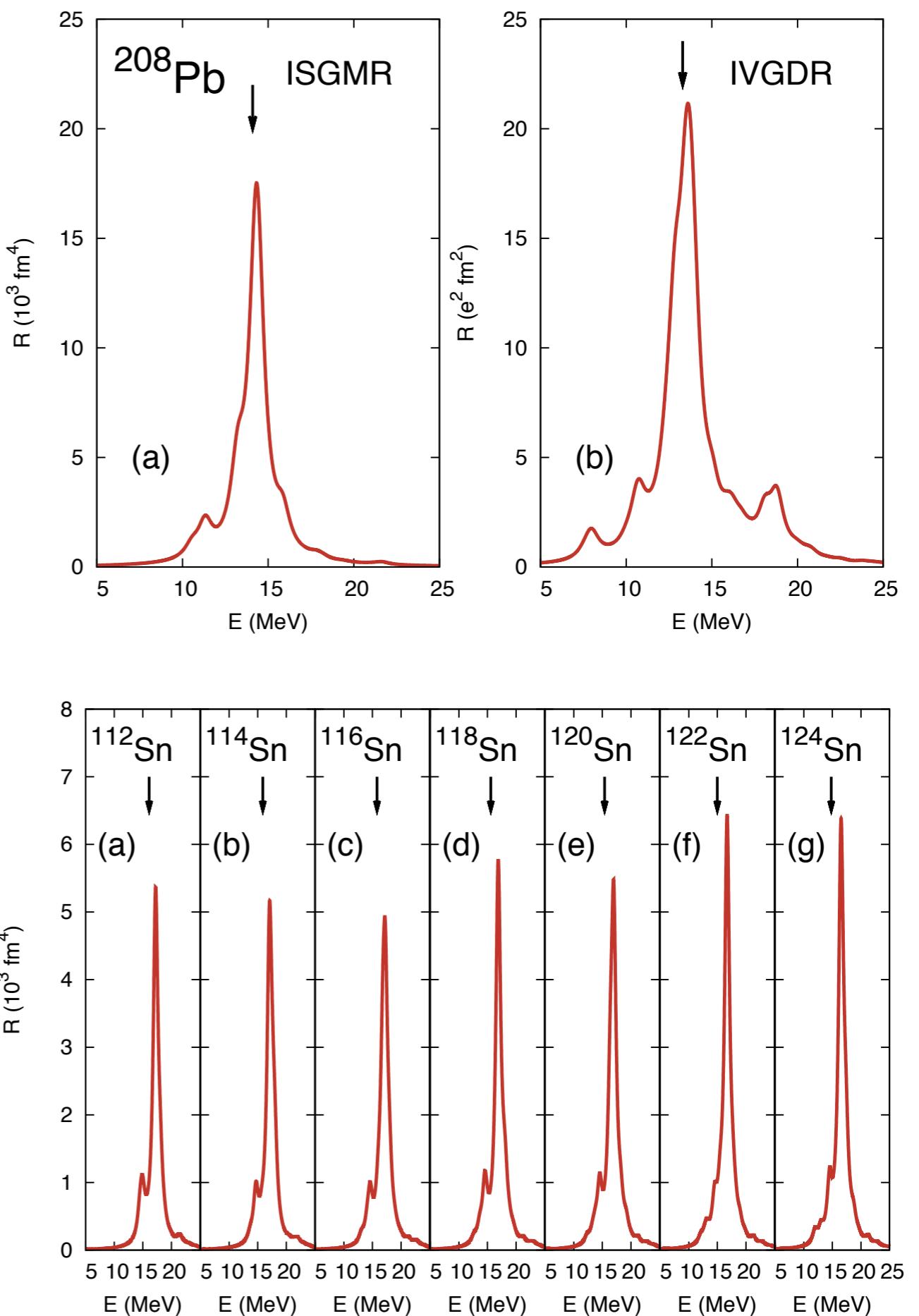
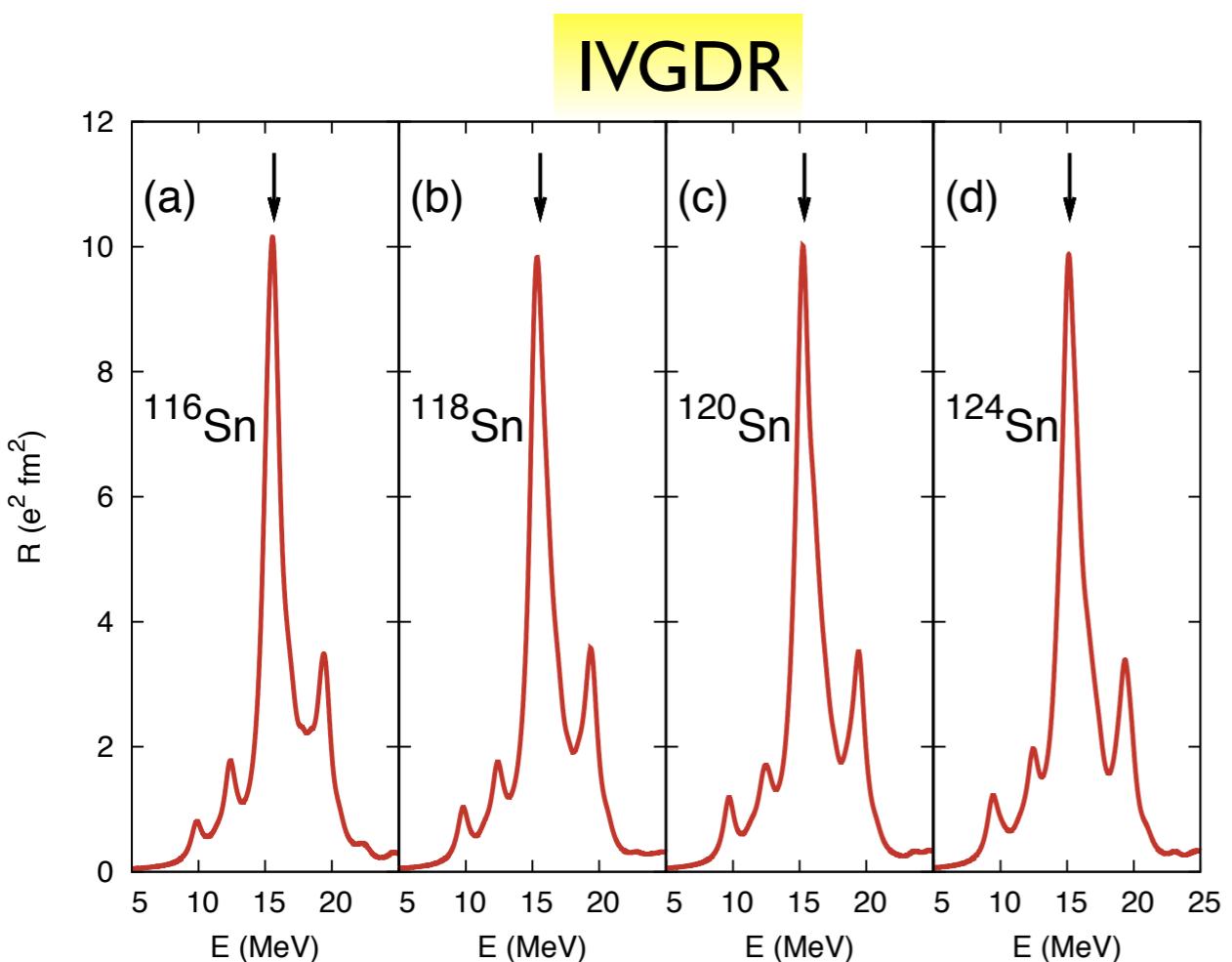
Charge radii



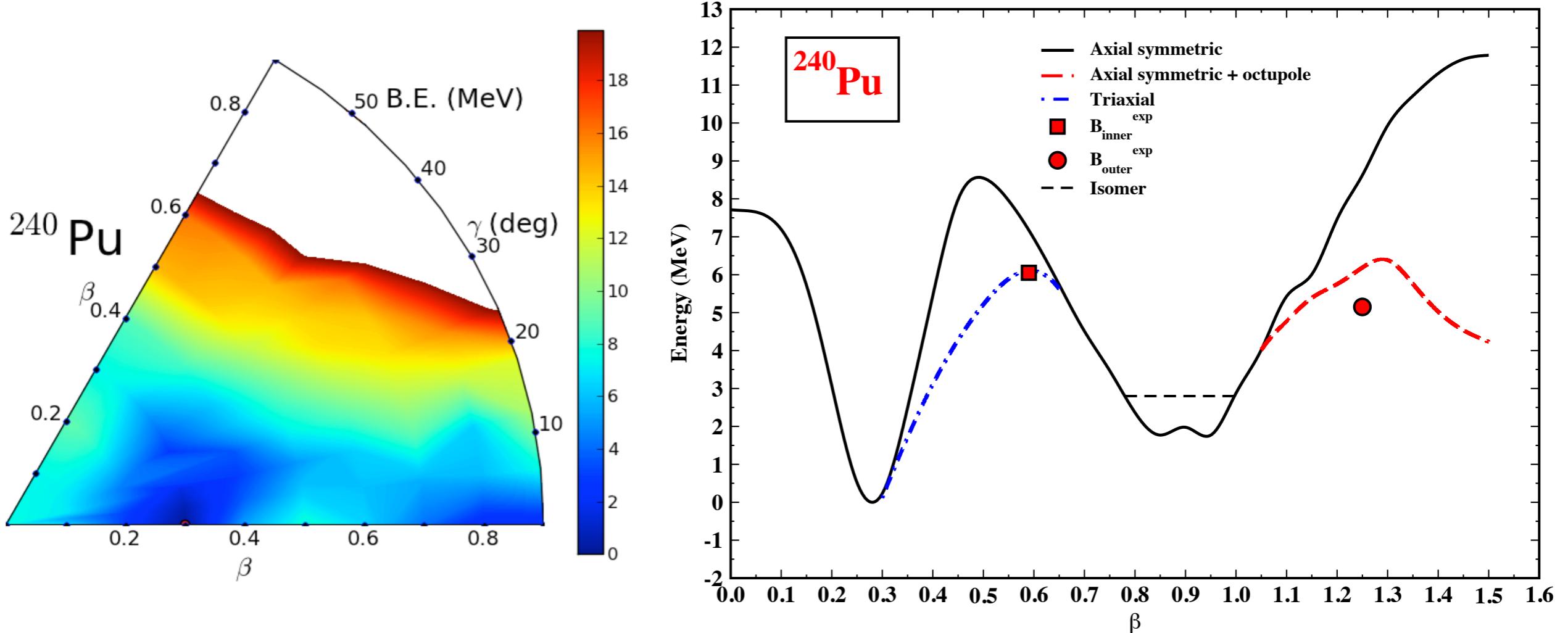
Quadrupole deformations



# Excitation energies of collective modes:



# Test: “double-humped” fission barriers of actinides



# Nuclear Many-Body Correlations



***short-range***  
(hard repulsive core of  
the NN-interaction)

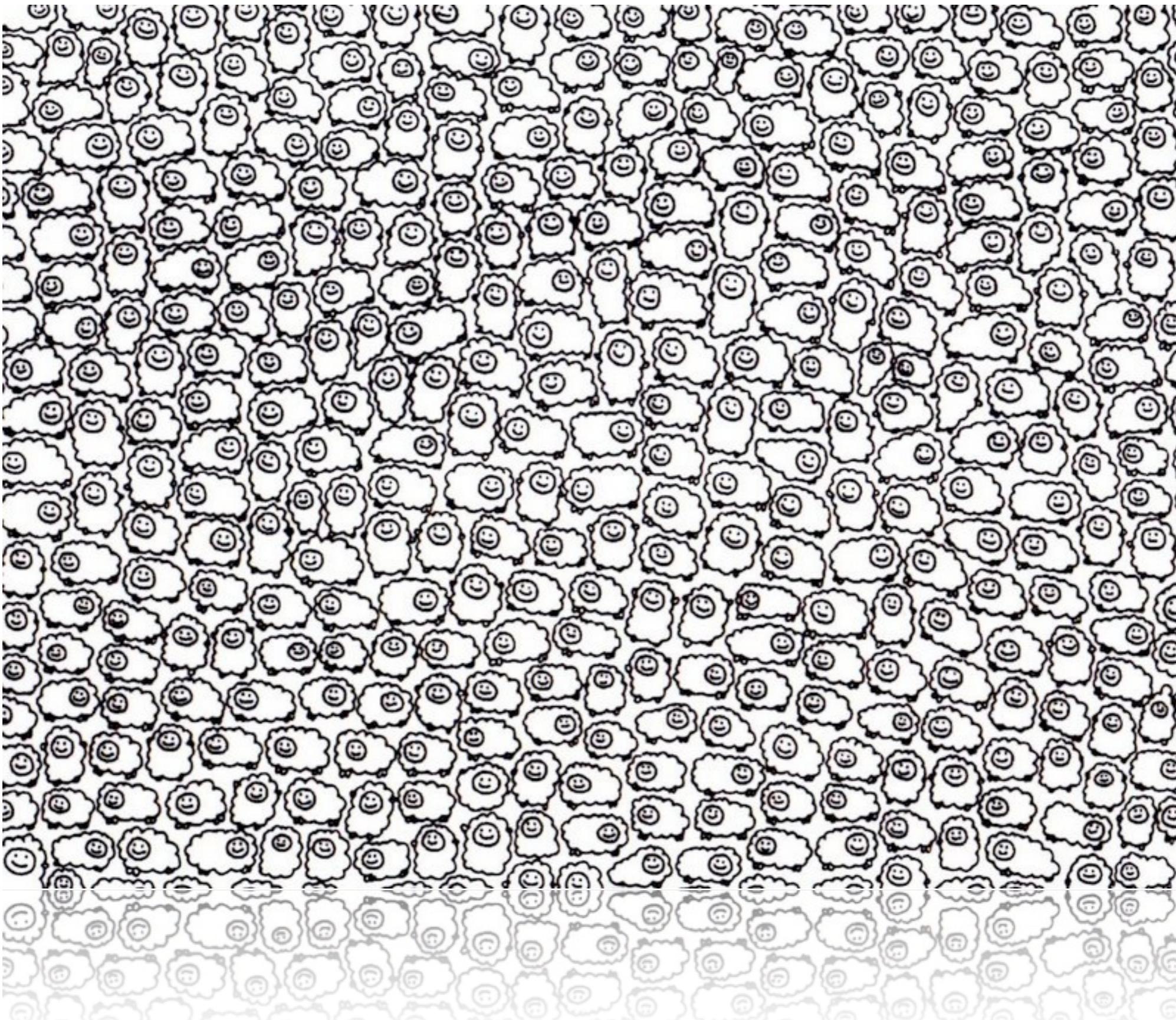
***long-range***  
nuclear resonance  
modes  
(giant resonances)

***collective correlations***  
large-amplitude soft modes:  
(center of mass motion, rotation,  
low-energy quadrupole vibrations)

...vary smoothly with nucleon number!  
Implicitly included in an effective EDF.

...sensitive to shell-effects and strong variations  
with nucleon number!  
Cannot be included in a simple Kohn-Sham EDF  
framework.

# Collective correlations



## Five-dimensional collective Hamiltonian

... nuclear excitations determined by quadrupole vibrational and rotational degrees of freedom

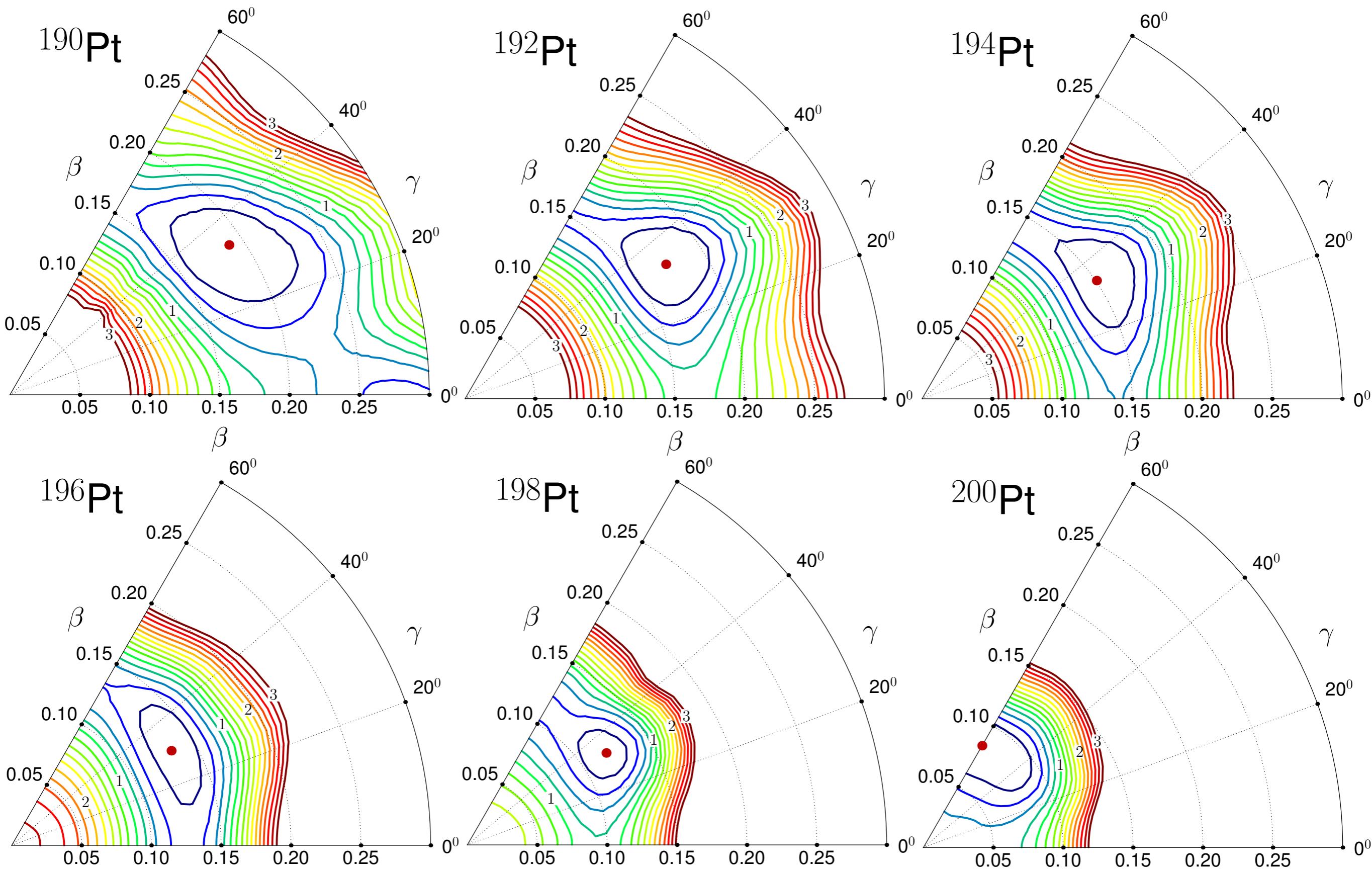
$$H_{\text{coll}} = \mathcal{T}_{\text{vib}}(\beta, \gamma) + \mathcal{T}_{\text{rot}}(\beta, \gamma, \Omega) + \mathcal{V}_{\text{coll}}(\beta, \gamma)$$

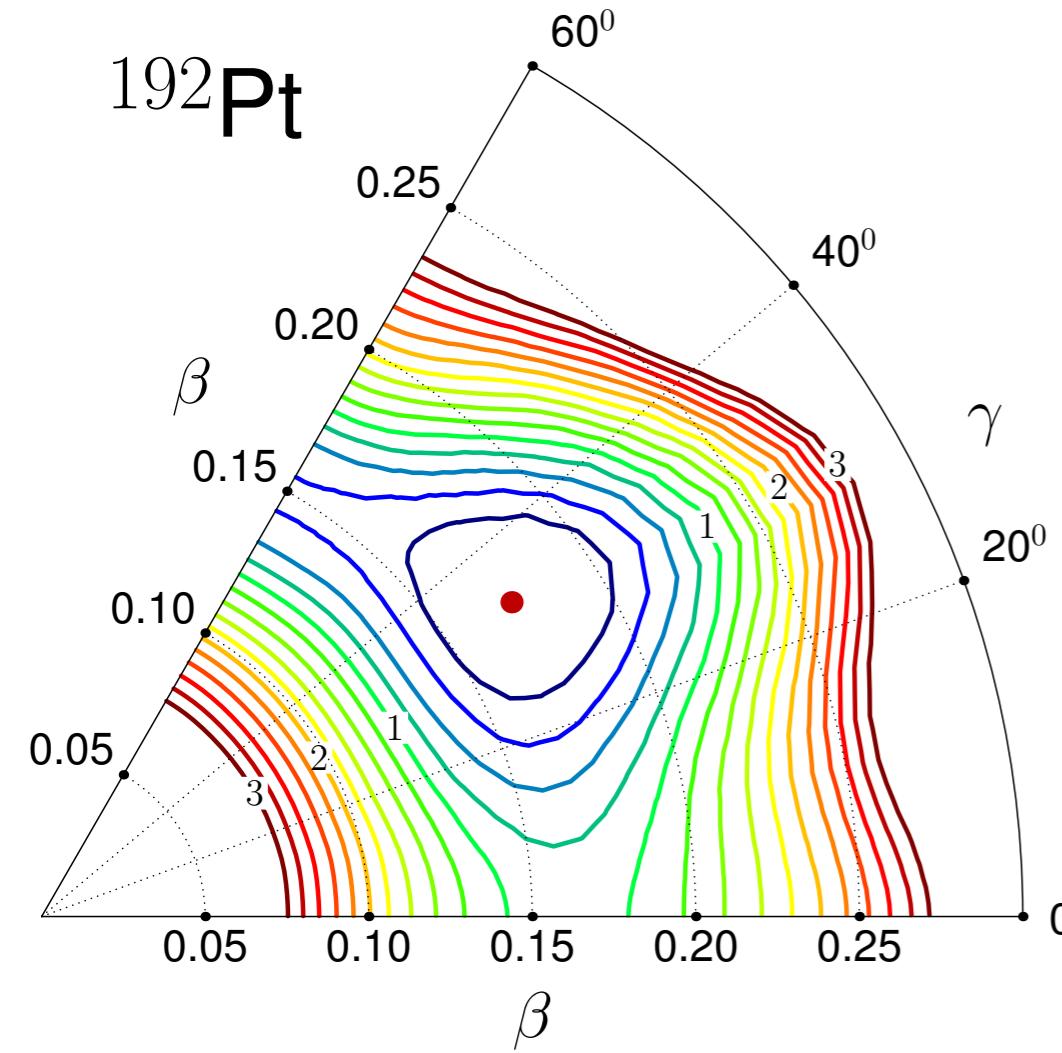
$$\mathcal{T}_{\text{vib}} = \frac{1}{2} B_{\beta\beta} \dot{\beta}^2 + \beta B_{\beta\gamma} \dot{\beta} \dot{\gamma} + \frac{1}{2} \beta^2 B_{\gamma\gamma} \dot{\gamma}^2$$

$$\mathcal{T}_{\text{rot}} = \frac{1}{2} \sum_{k=1}^3 I_k \omega_k^2$$

The entire dynamics of the collective Hamiltonian is governed by the seven functions of the intrinsic deformations  $\beta$  and  $\gamma$ : the collective potential, the three mass parameters:  $B_{\beta\beta}$ ,  $B_{\beta\gamma}$ ,  $B_{\gamma\gamma}$ , and the three moments of inertia  $I_k$ .

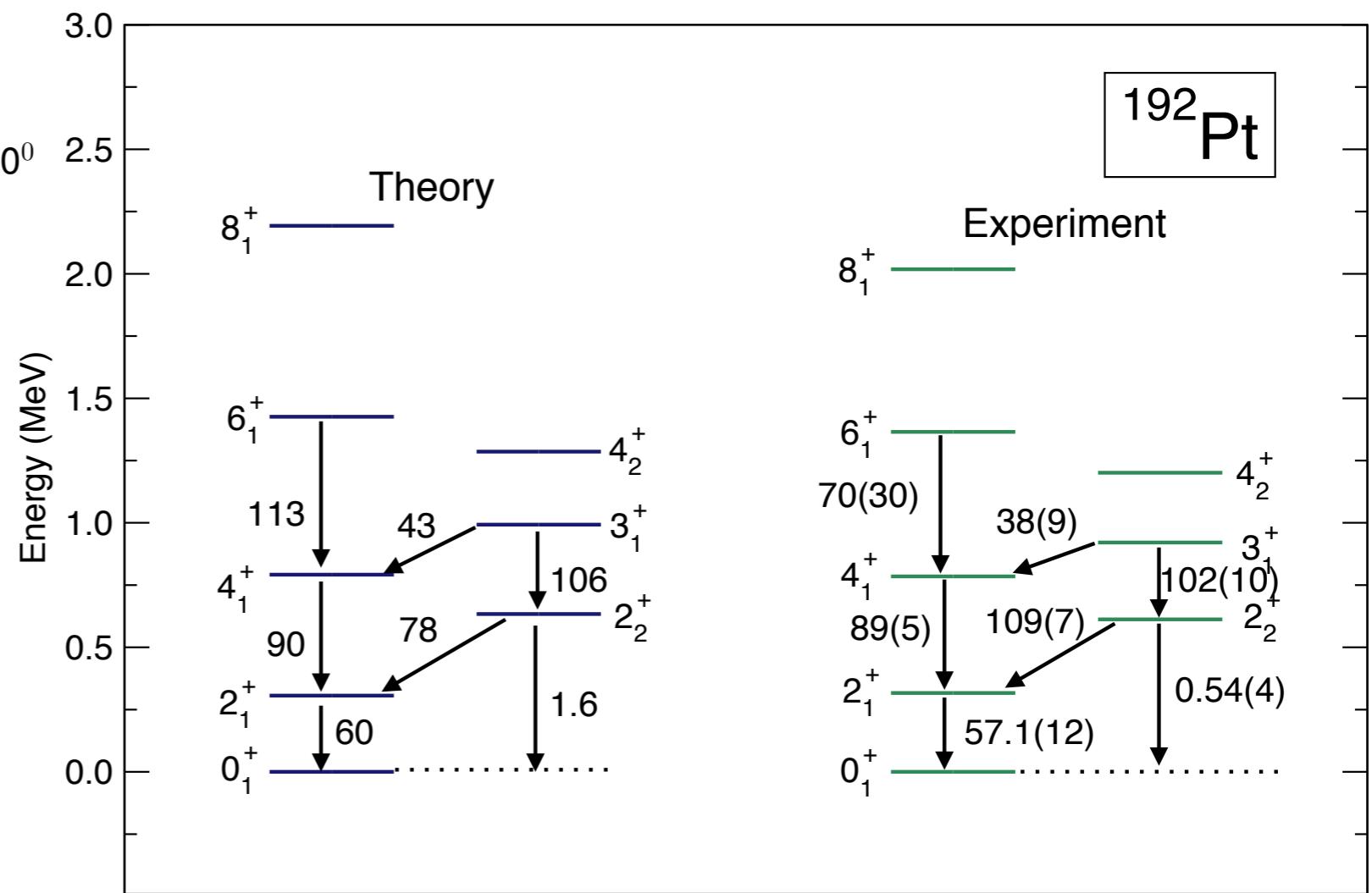
# Evolution of triaxial shapes in Pt nuclei:

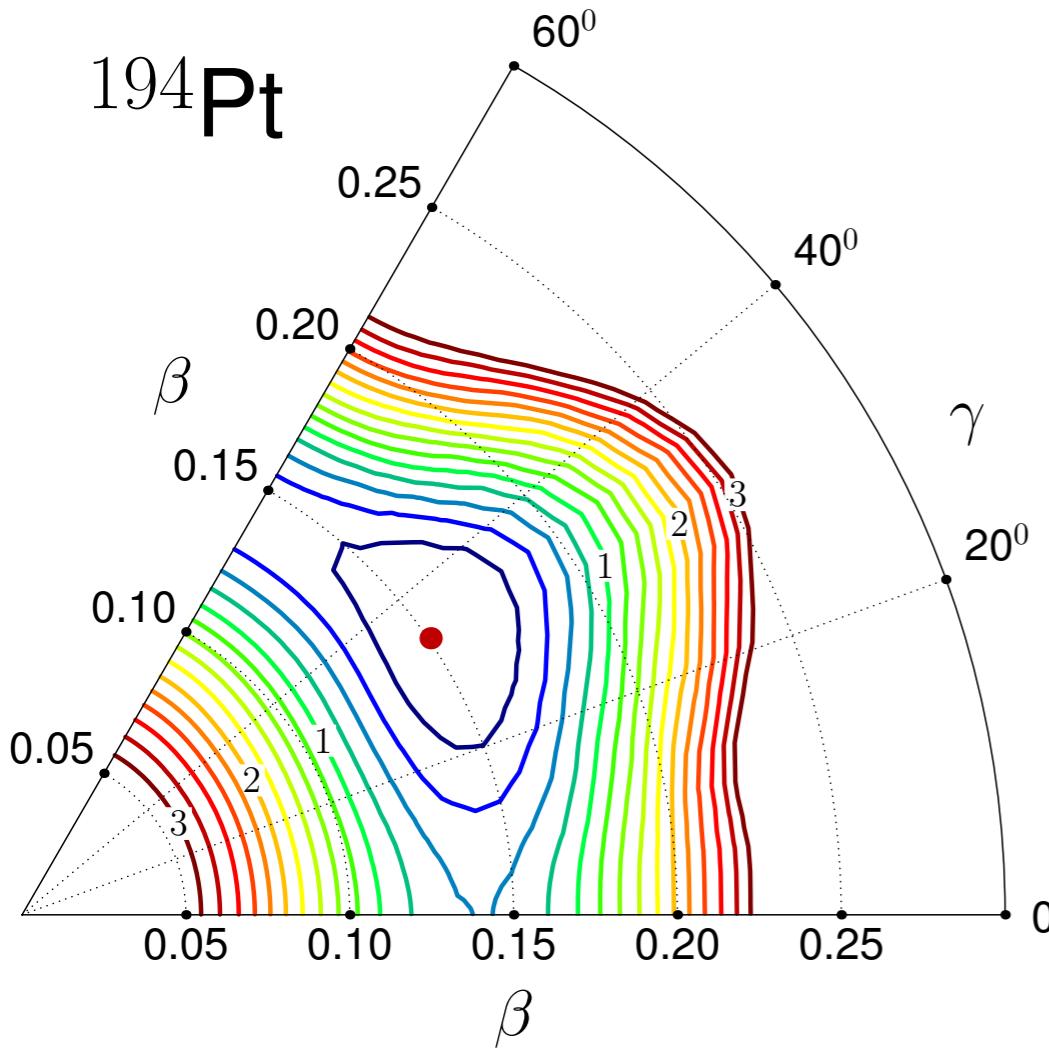




$$E_{4_1^+}^{th}/E_{2_1^+}^{th} = 2.58$$

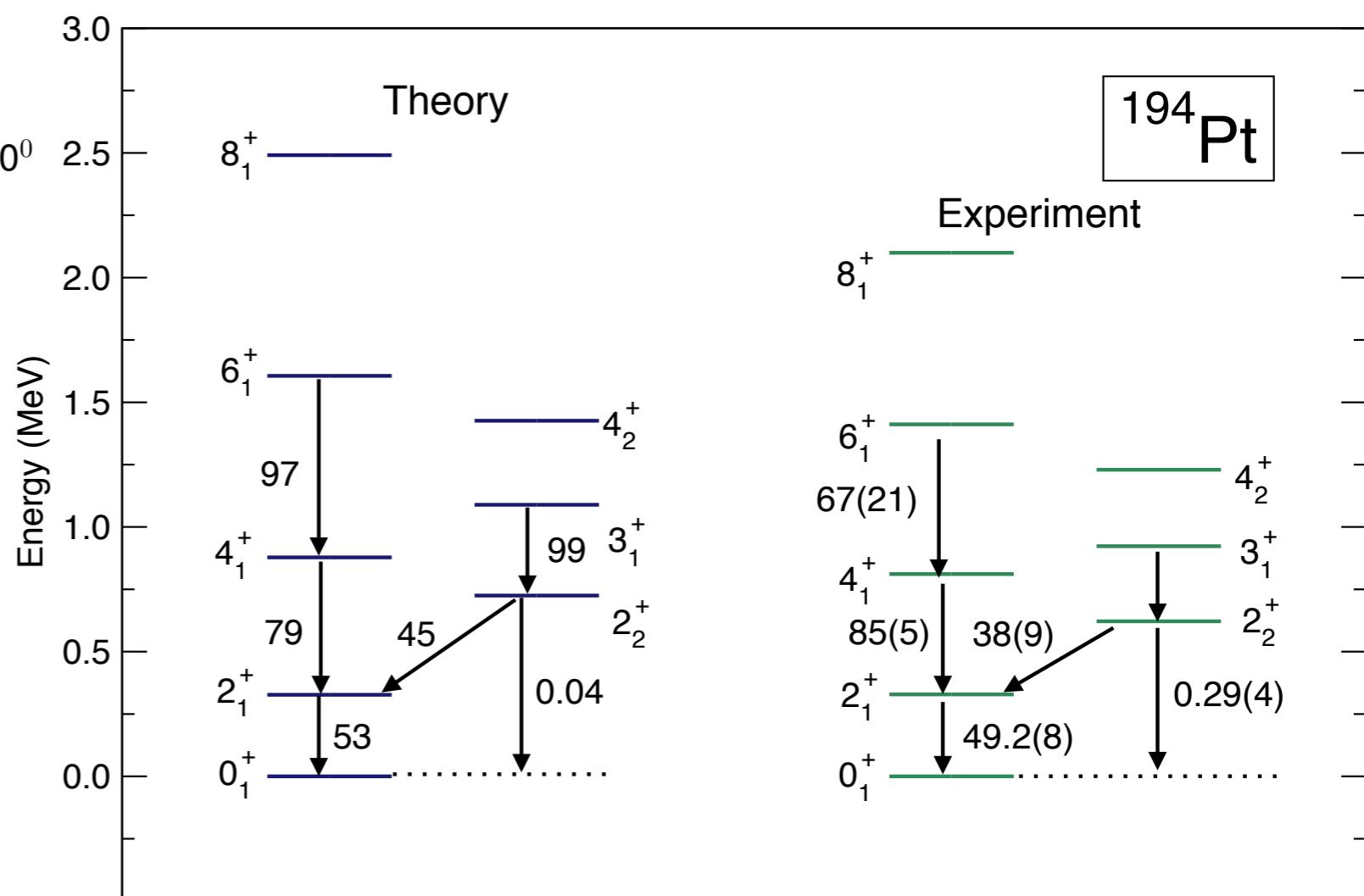
$$E_{4_1^+}^{exp}/E_{2_1^+}^{exp} = 2.48$$

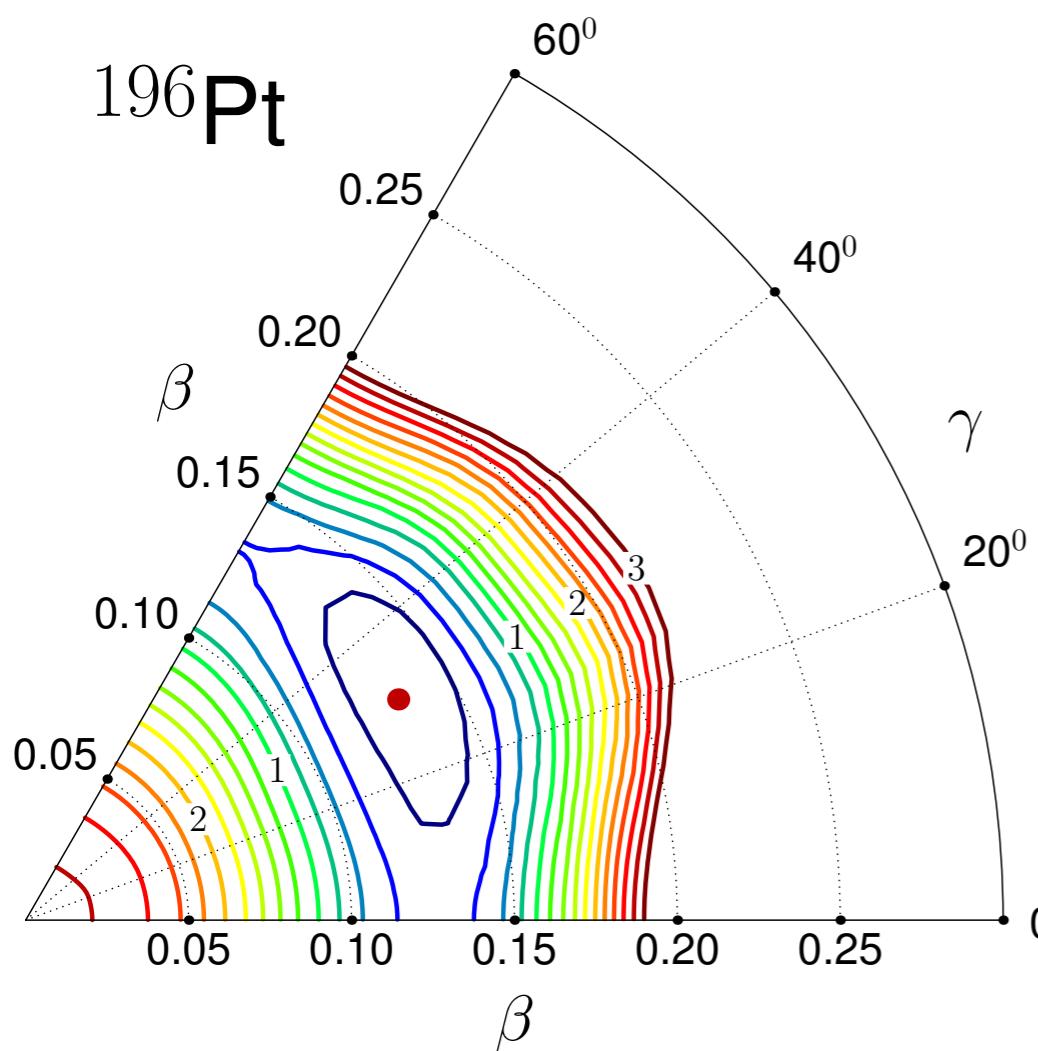




$$E_{4_1^+}^{th}/E_{2_1^+}^{th} = 2.68$$

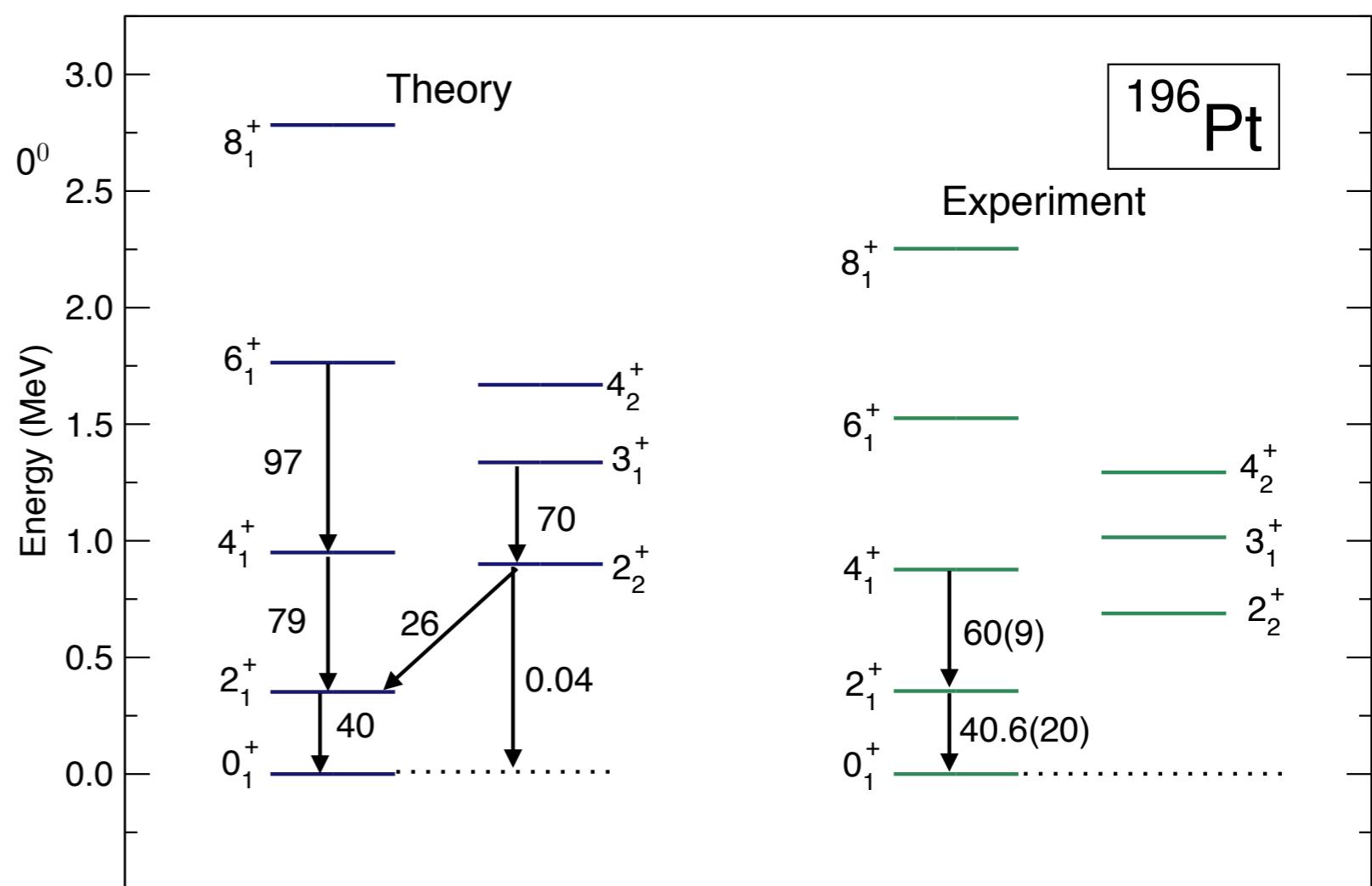
$$E_{4_1^+}^{exp}/E_{2_1^+}^{exp} = 2.47$$

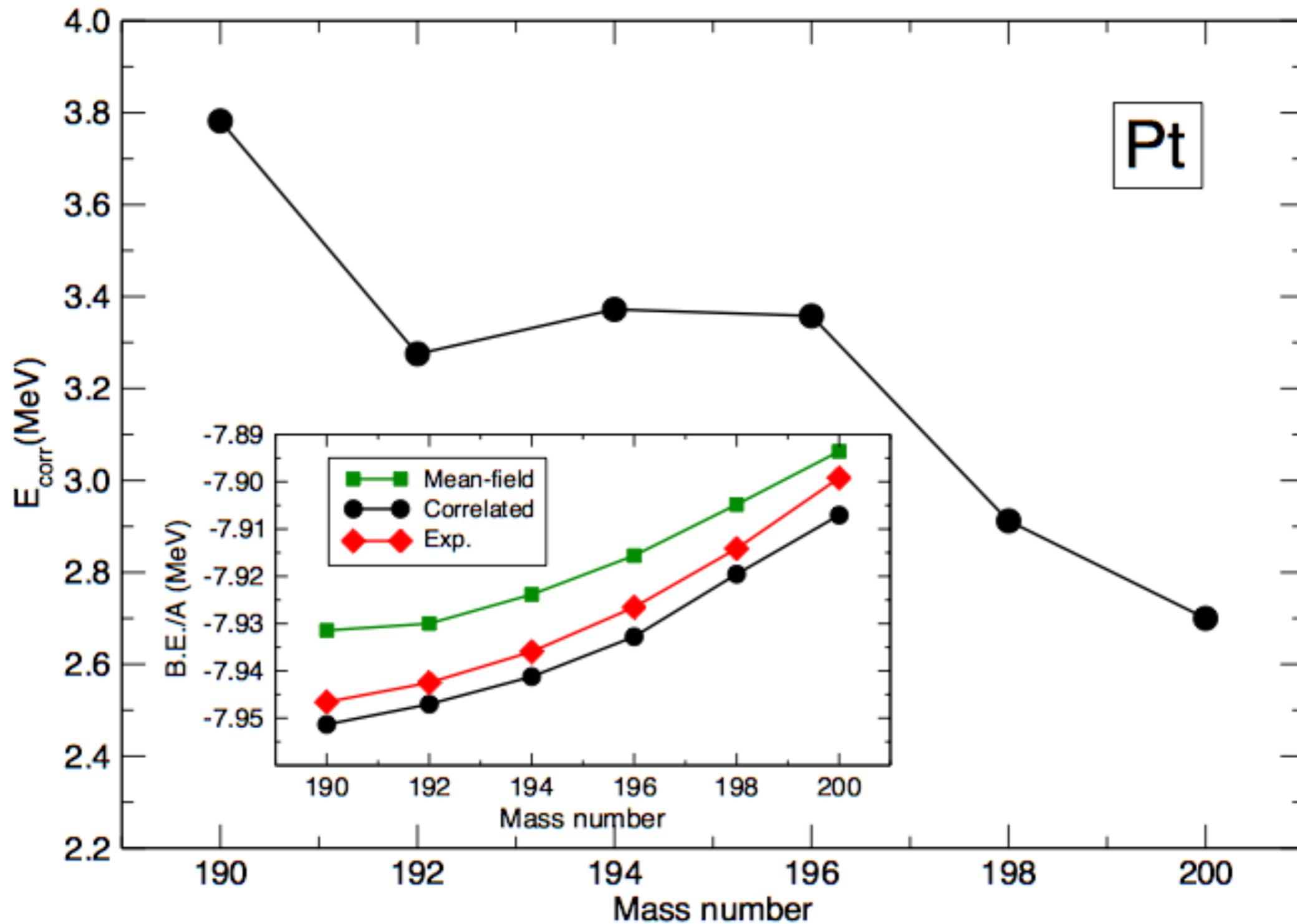




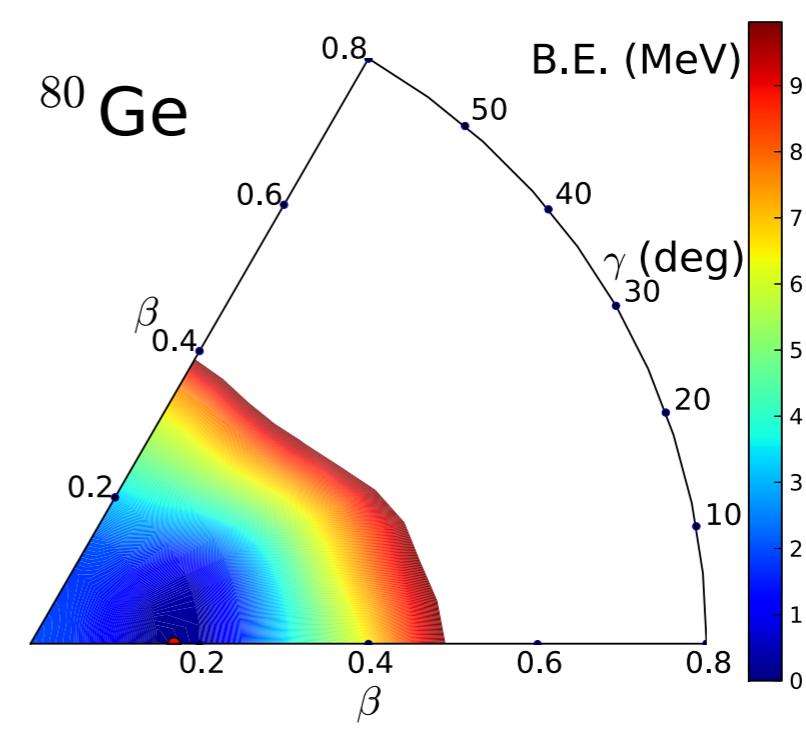
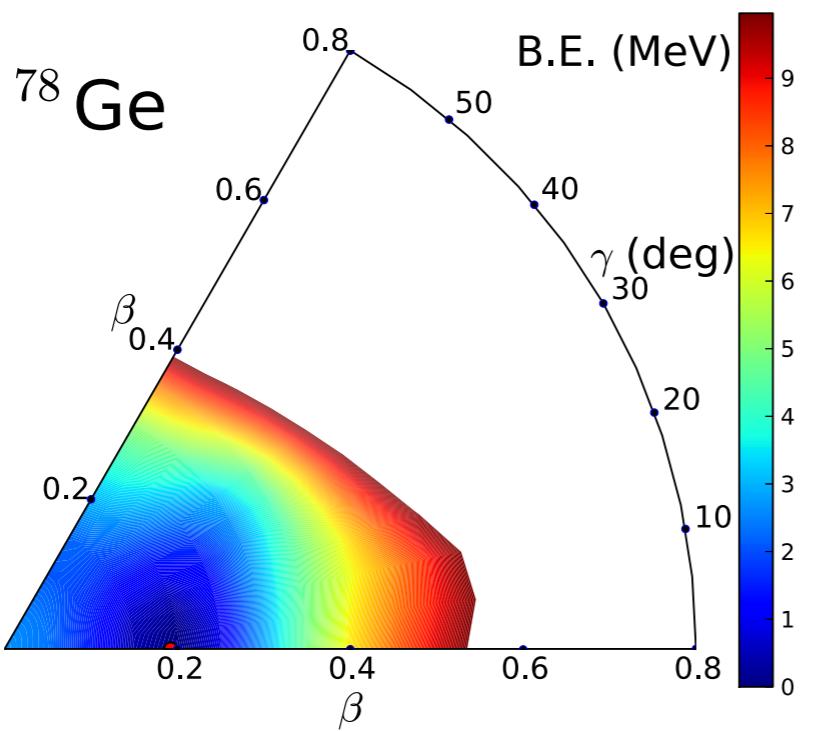
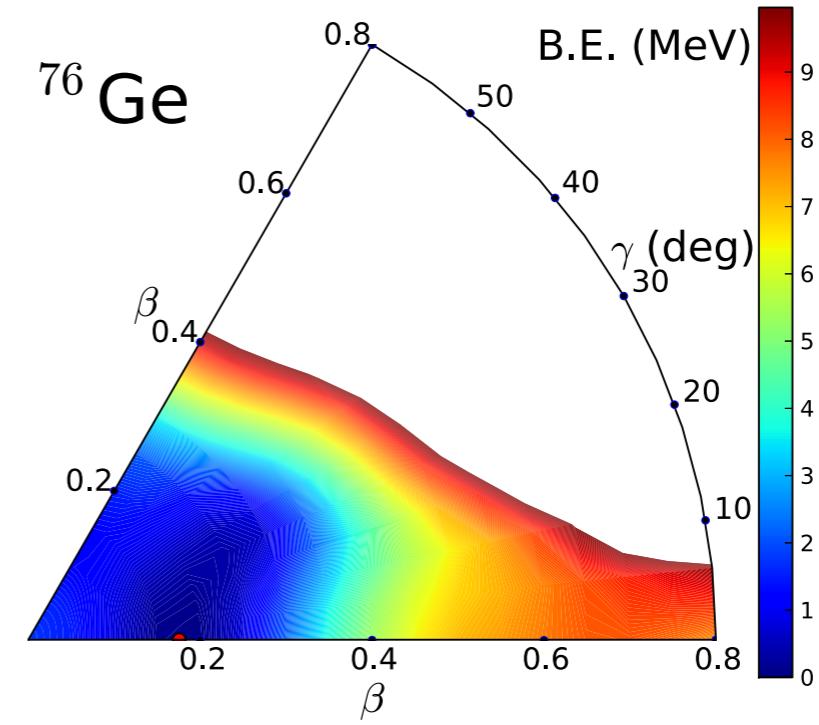
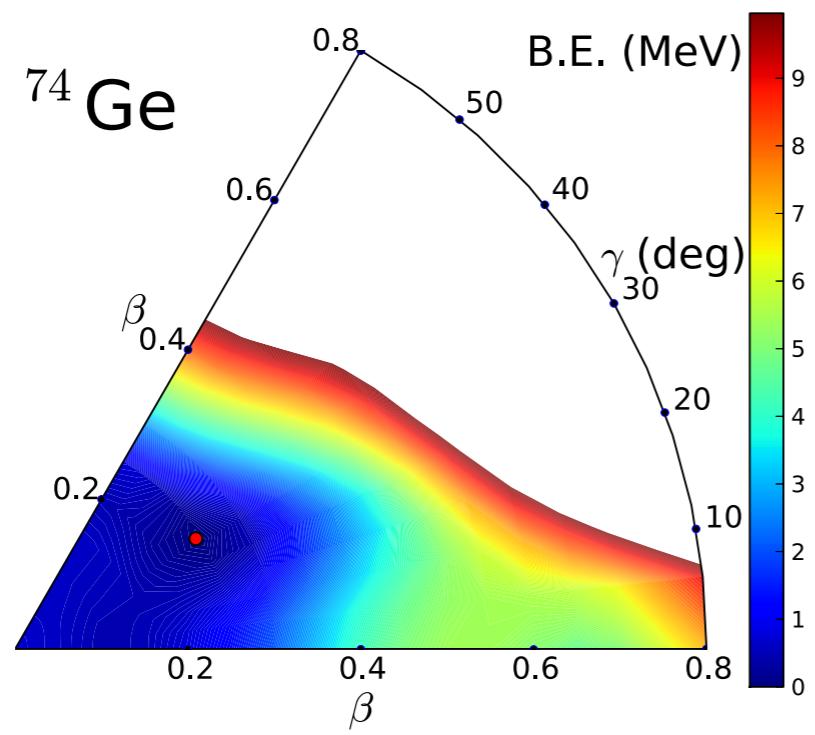
$$E_{4_1^+}^{th}/E_{2_1^+}^{th} = 2.69$$

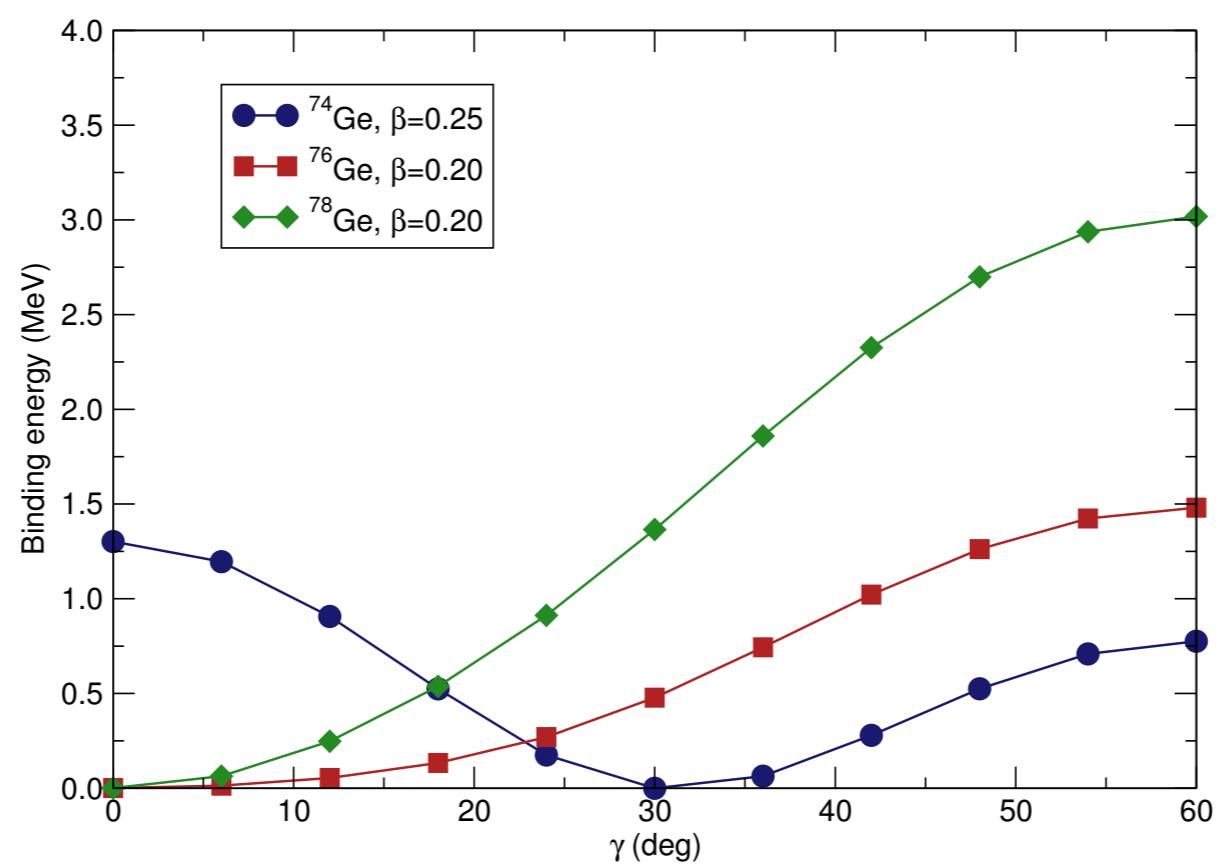
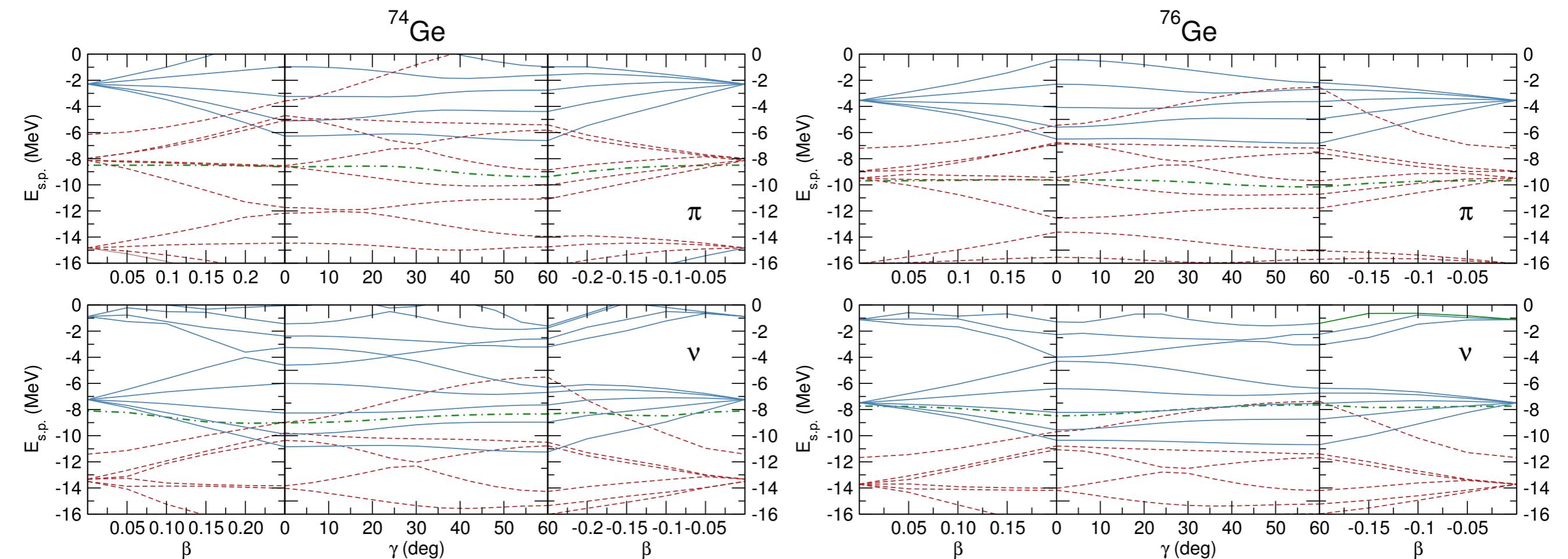
$$E_{4_1^+}^{exp}/E_{2_1^+}^{exp} = 2.47$$



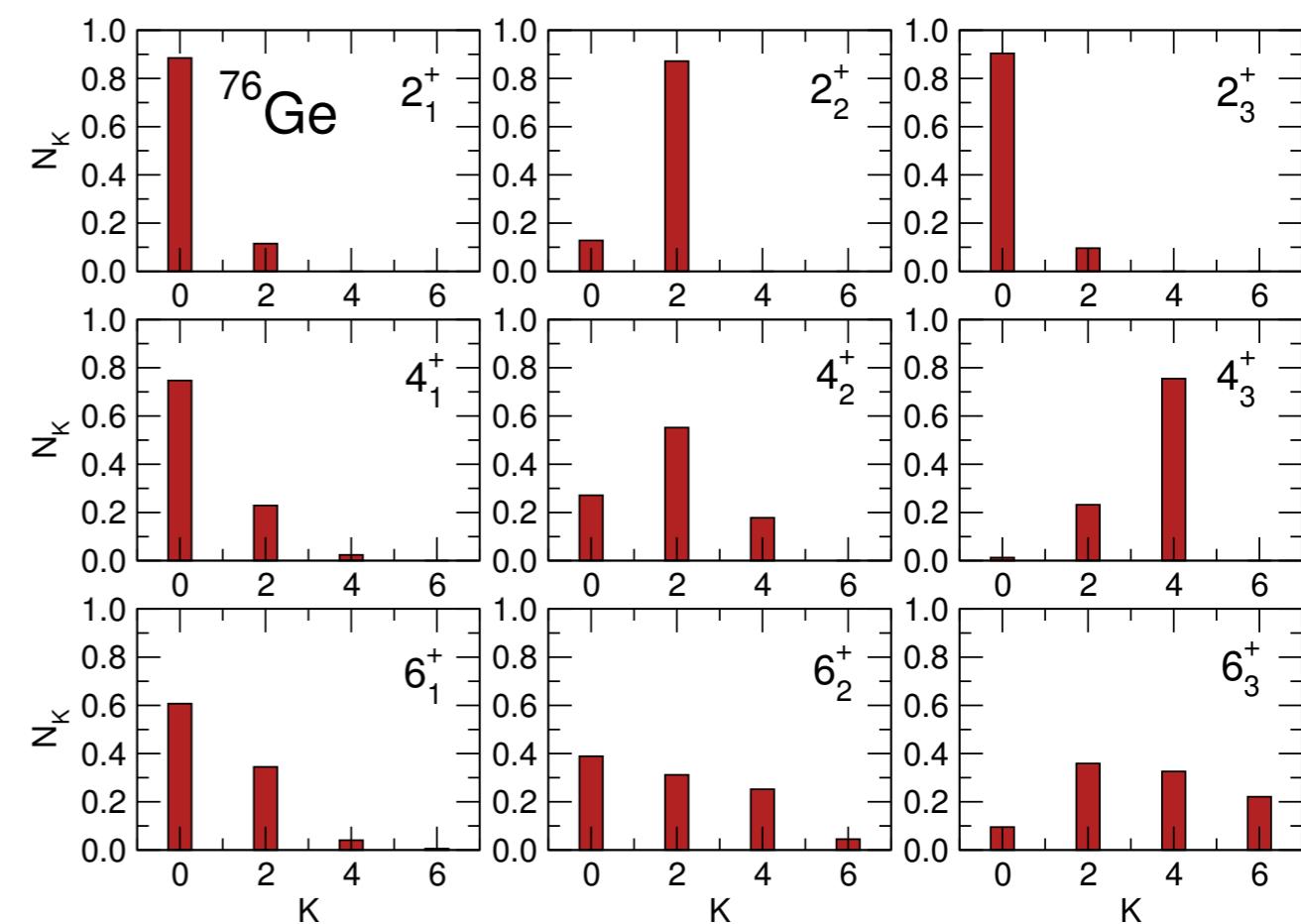
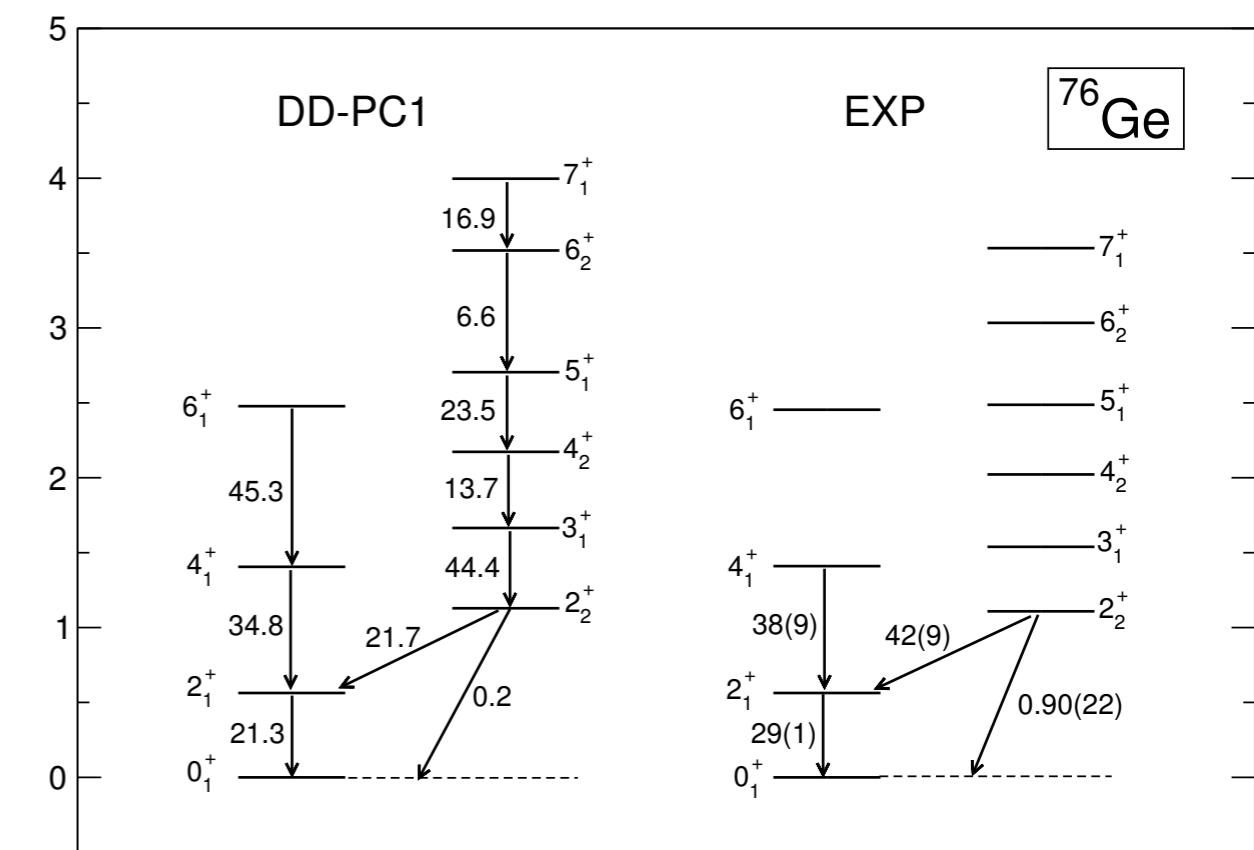
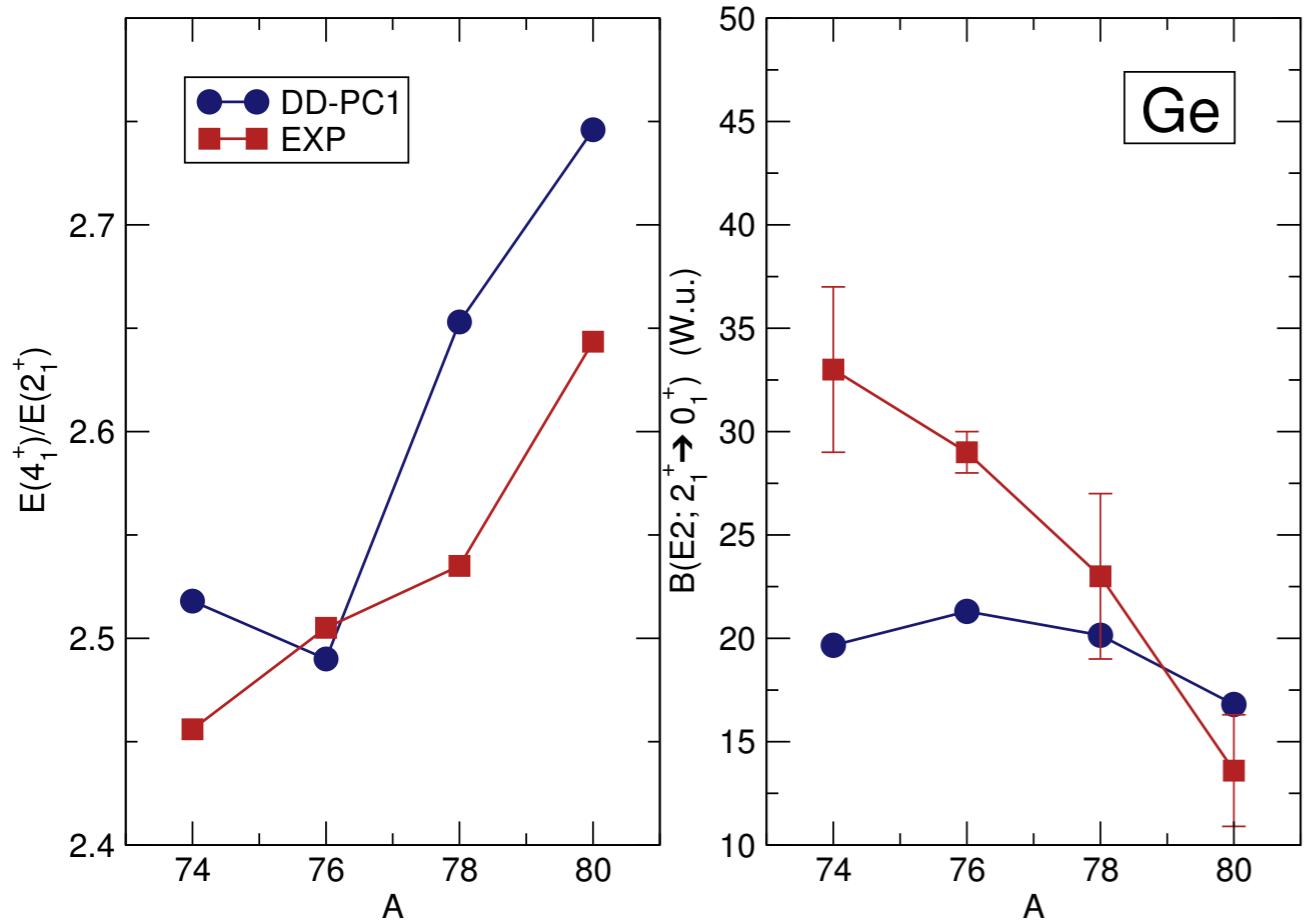


# Shape evolution and triaxiality in germanium isotopes





# Quadrupole collective Hamiltonian based on the functional DD-PC1



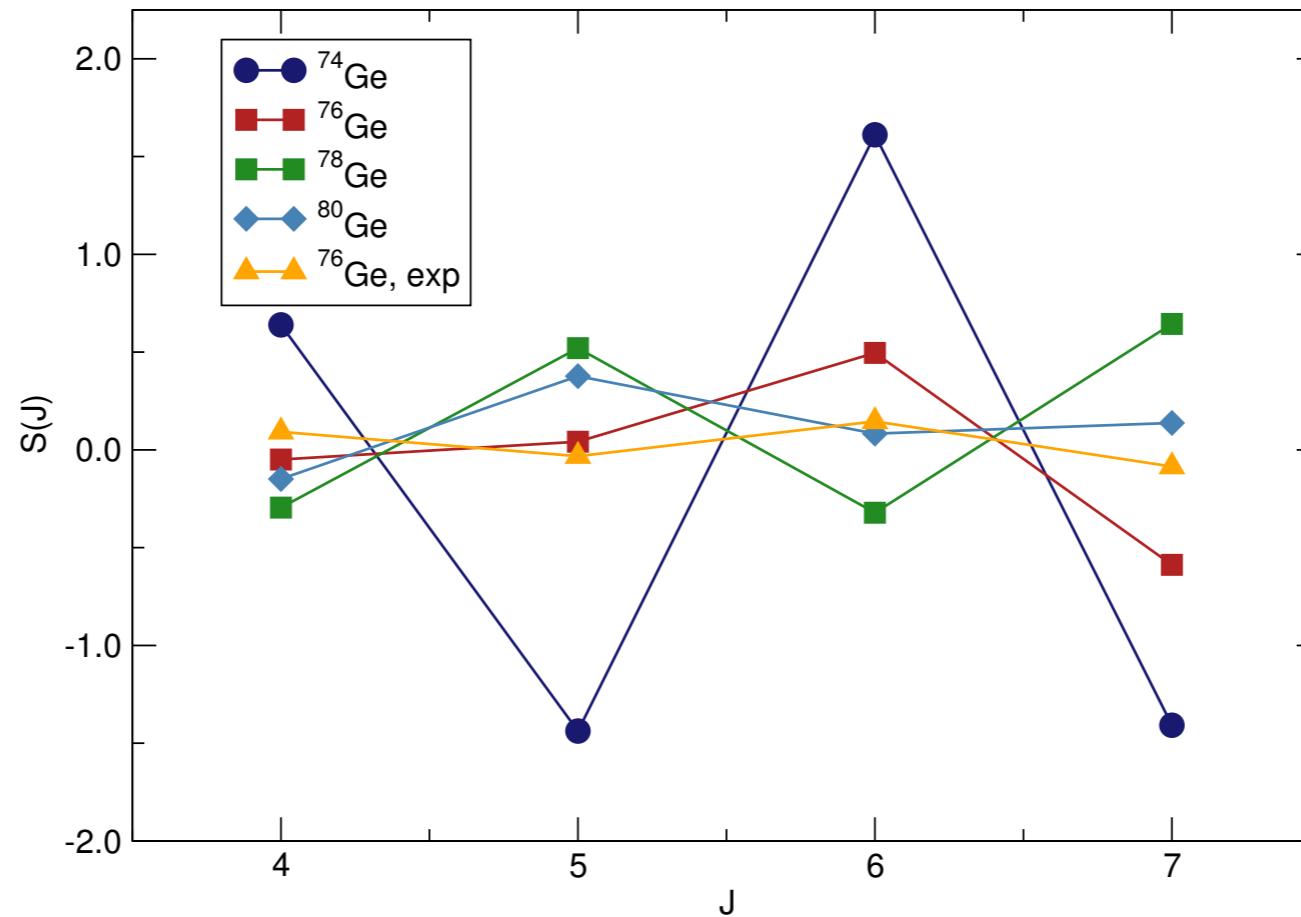
Distribution of  $K$  components (projection of the angular momentum on the body-fixed symmetry axis) in the collective wave functions of the nucleus  $^{76}\text{Ge}$ .

The level of K-mixing is reflected in the staggering in energy between odd- and even-spin states in the  $\gamma$  band:

$$S(J) = \frac{E[J_\gamma^+] - 2E[(J-1)_\gamma^+] + E[(J-2)_\gamma^+]}{E[2_1^+]}$$

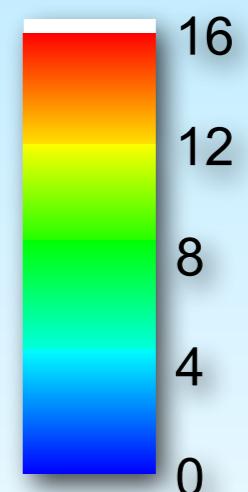
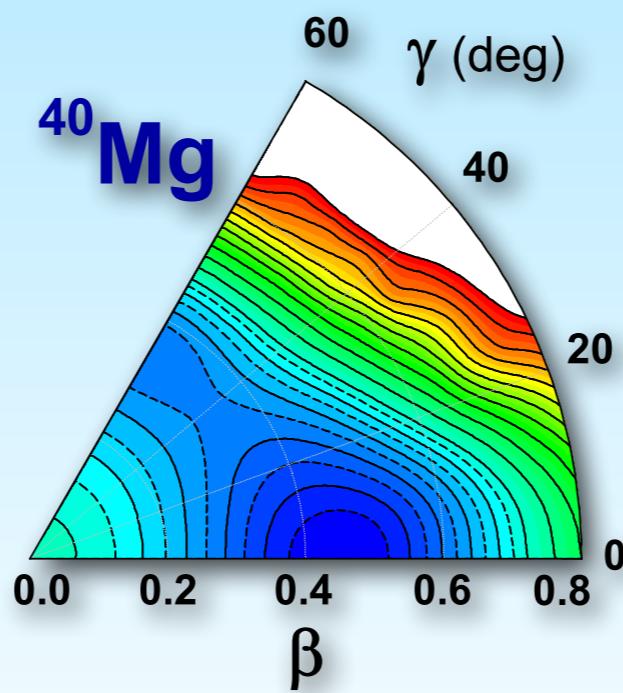
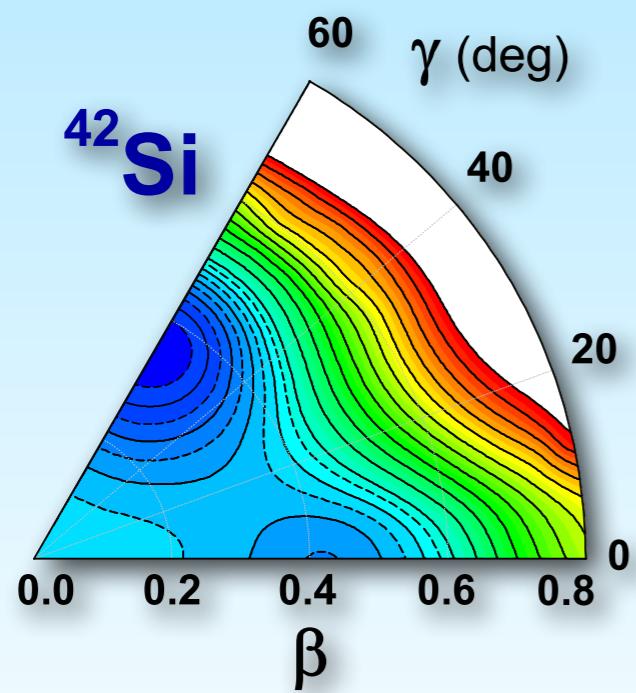
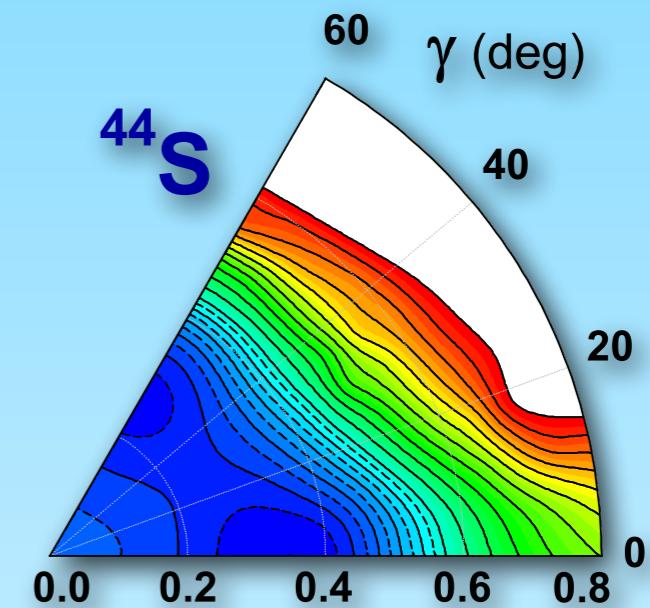
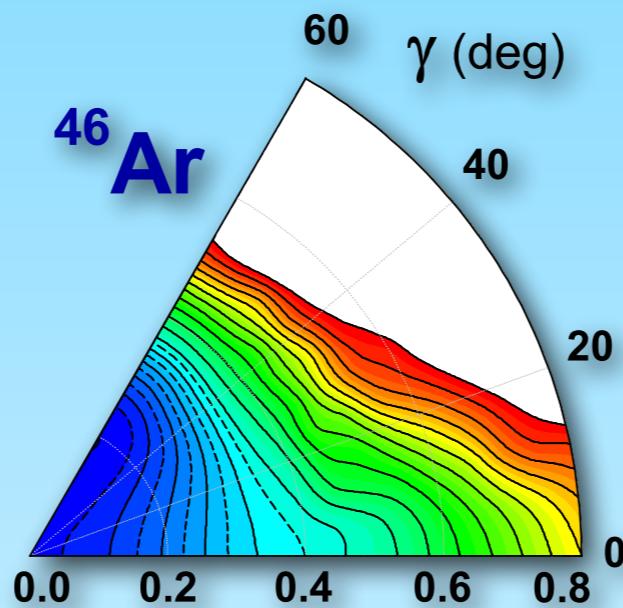
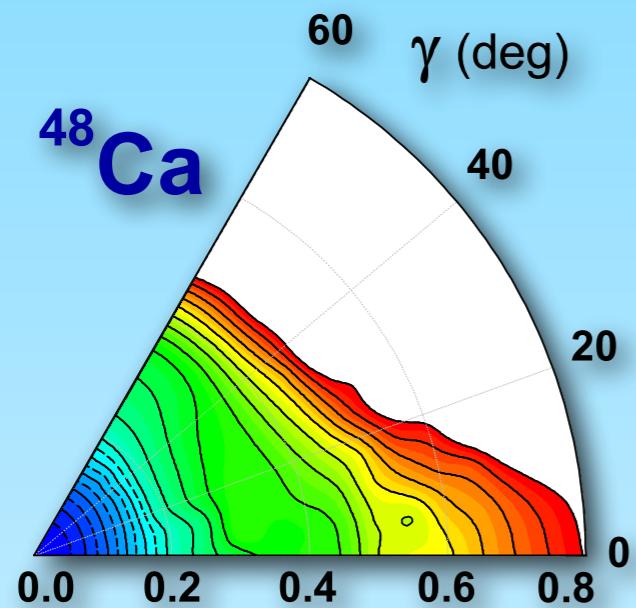
Deformed  $\gamma$ -soft potential  $\Rightarrow S(J)$  oscillates between negative values for even-spin states and positive values for odd-spin states.

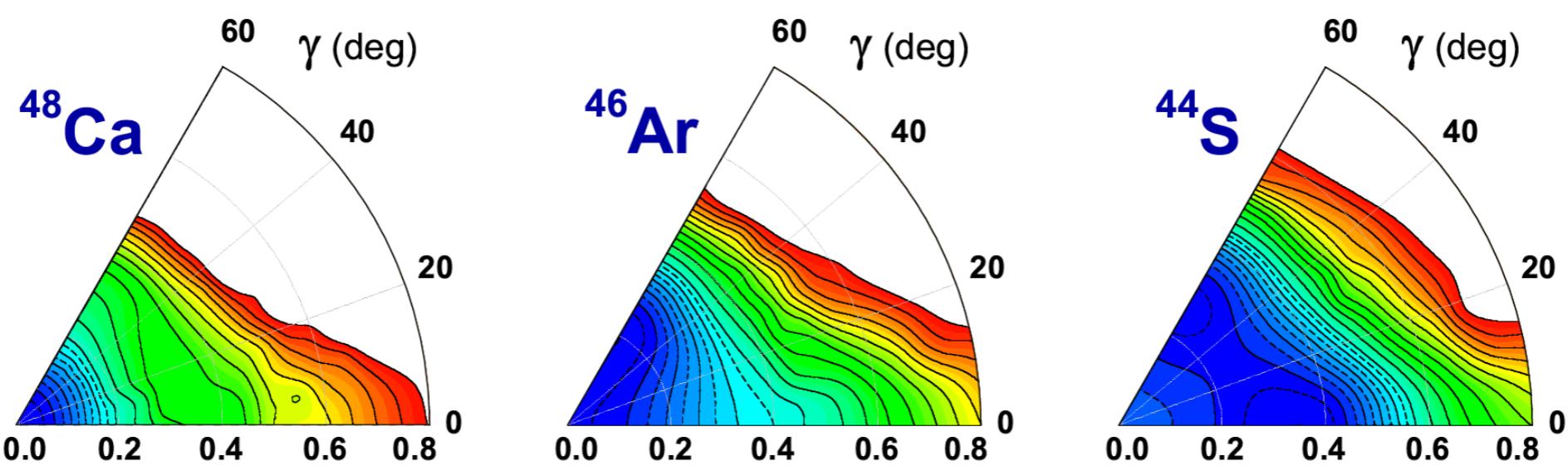
$\gamma$ -rigid triaxial potential  $\Rightarrow S(J)$  oscillates between positive values for even-spin states and negative values for odd-spin states.



The mean-field potential of  ${}^{76}\text{Ge}$  is  $\gamma$  soft. The inclusion of collective correlations (symmetry restoration and quantum fluctuations) drives the nucleus toward triaxiality, but they are not strong enough to stabilize a  $\gamma \approx 30^\circ$  triaxial shape.

# Coexisting shapes in N=28 isotones





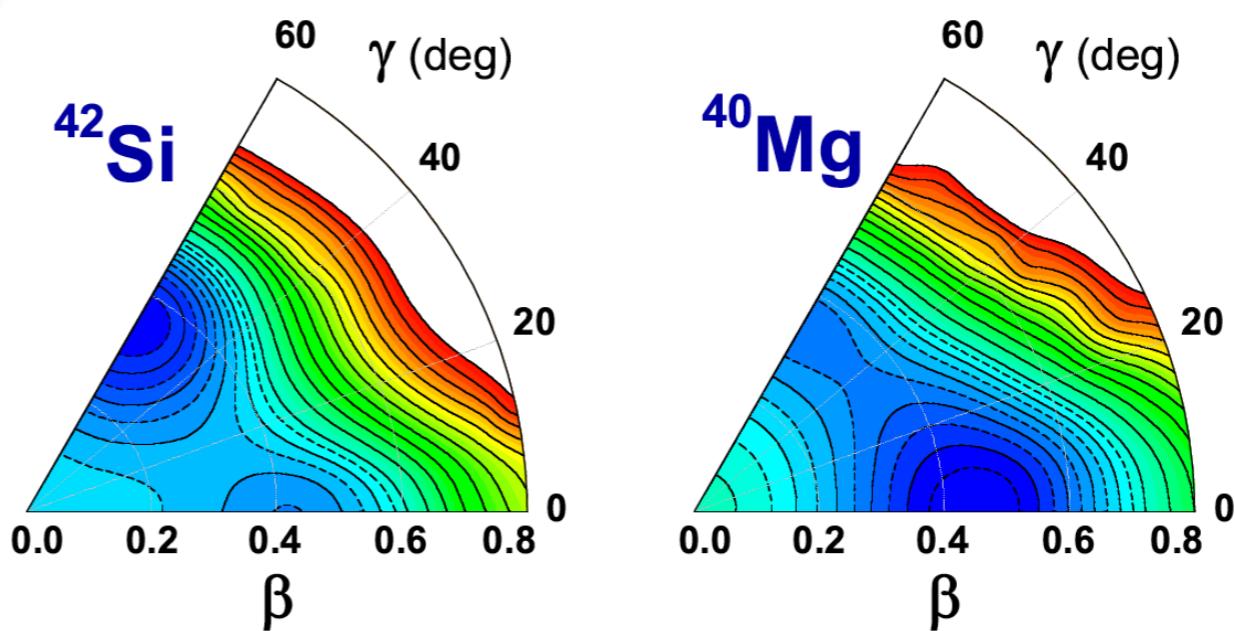
### Neutron $N=28$ spherical energy gaps

	$\Delta_{N=28}^{\text{sph.}}$	$\beta_{\min}$
$^{48}\text{Ca}$	4.73	0.00
$^{46}\text{Ar}$	4.48	-0.19
$^{44}\text{S}$	3.86	0.34
$^{42}\text{Si}$	3.13	-0.35
$^{40}\text{Mg}$	2.03	0.45

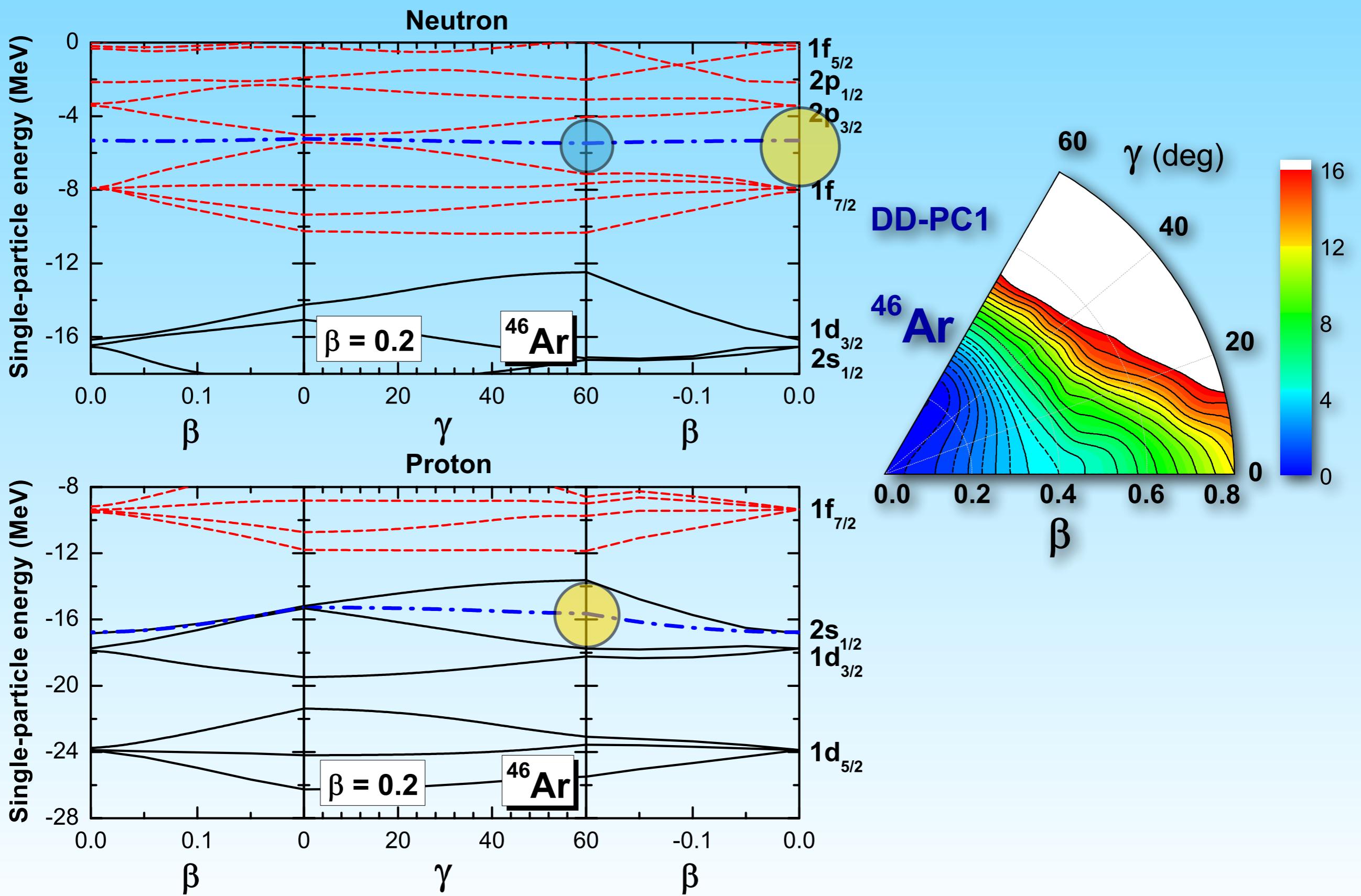
#### Experimental values:

4.80 MeV

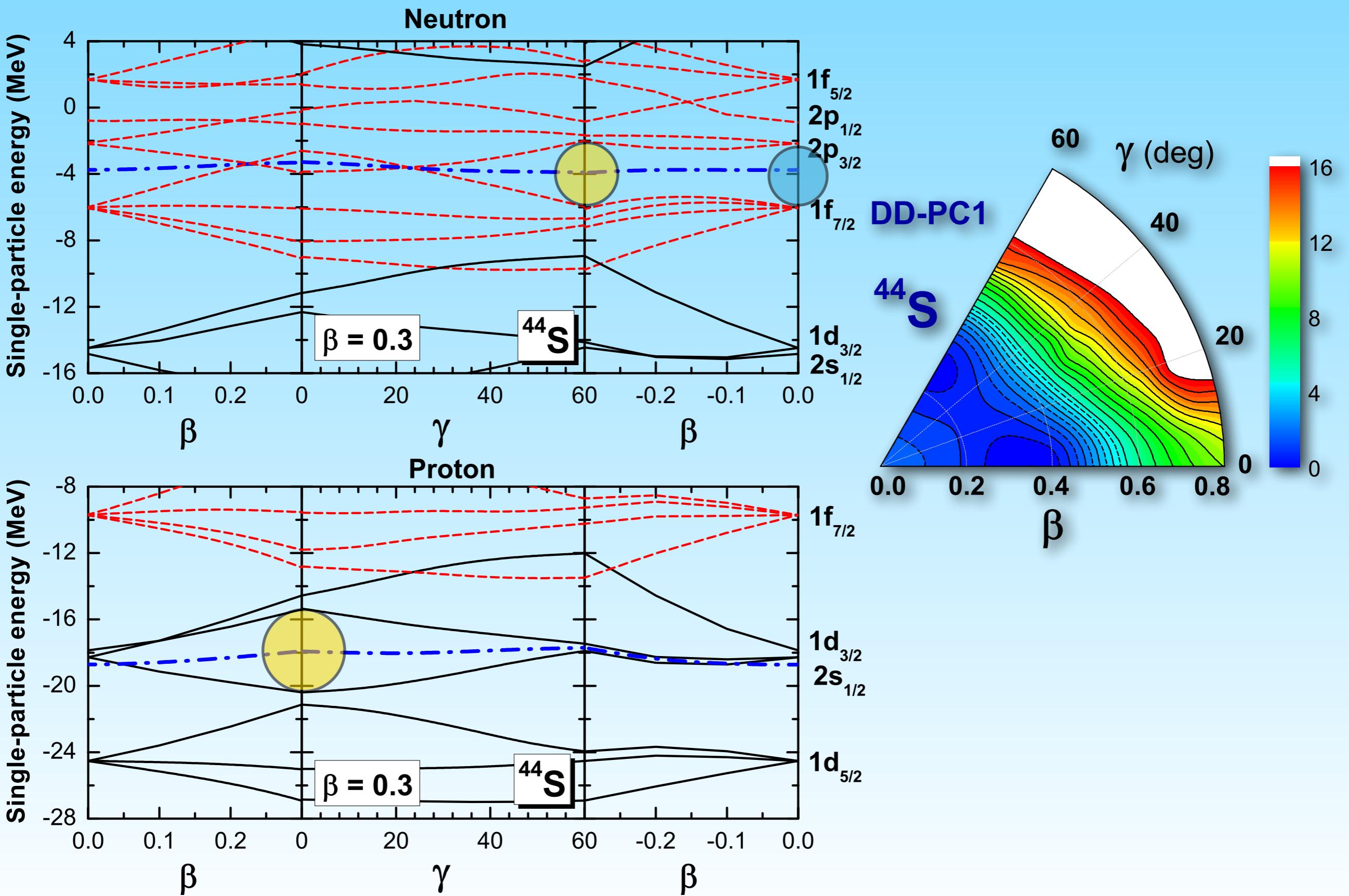
4.47 MeV



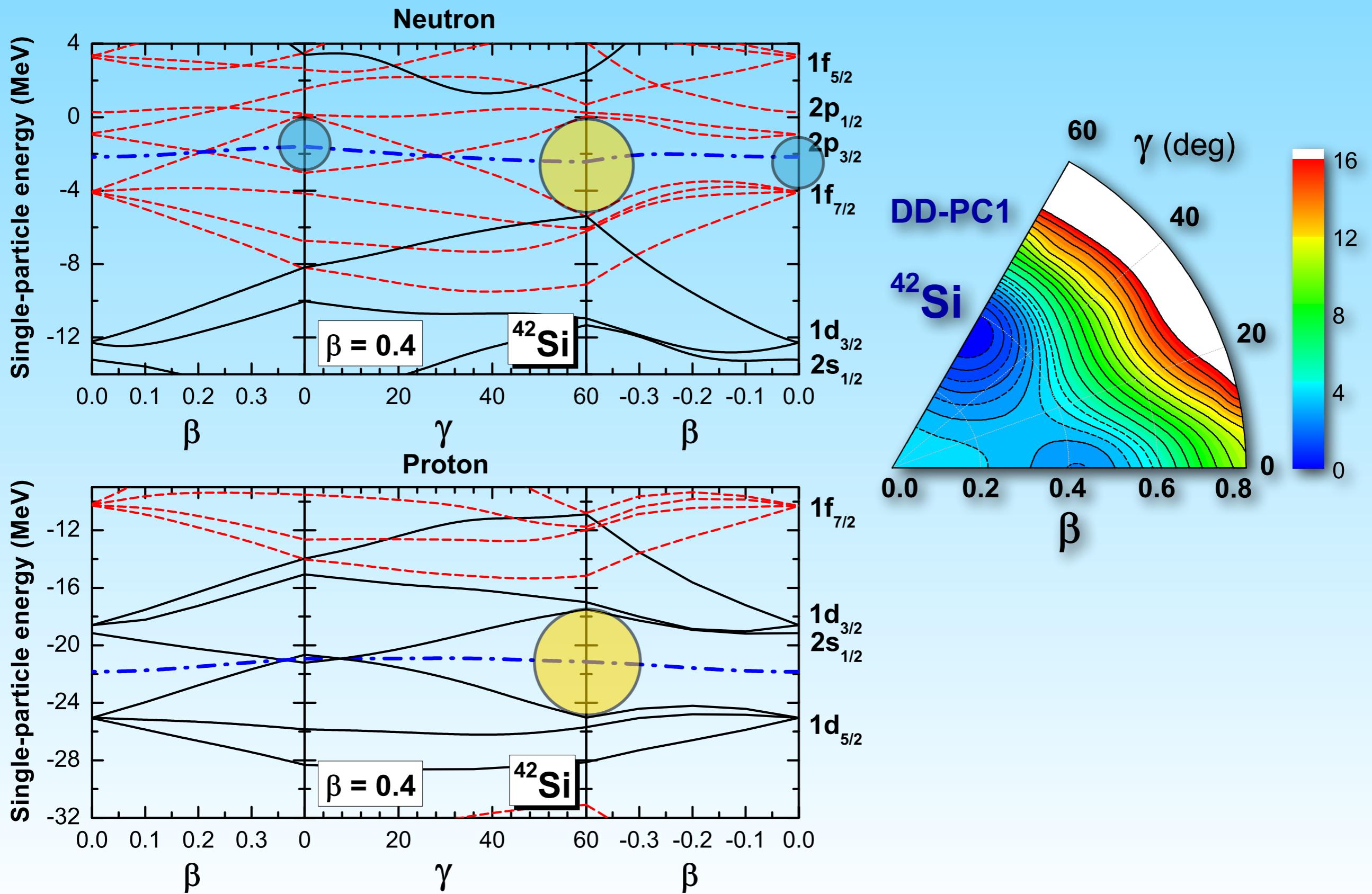
# $^{46}\text{Ar}$ : single-particle levels

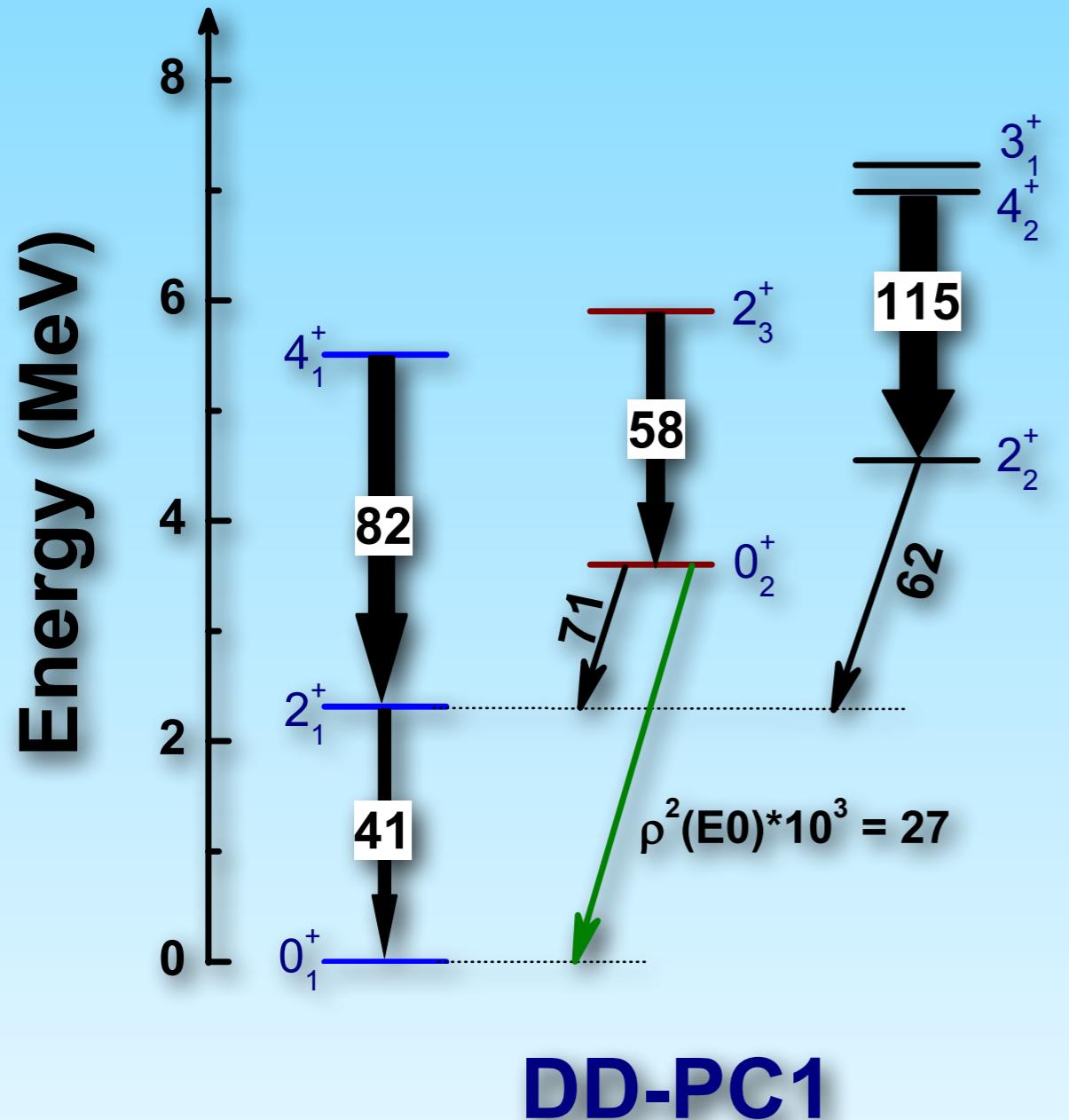


# $^{44}\text{S}$ : single-particle levels

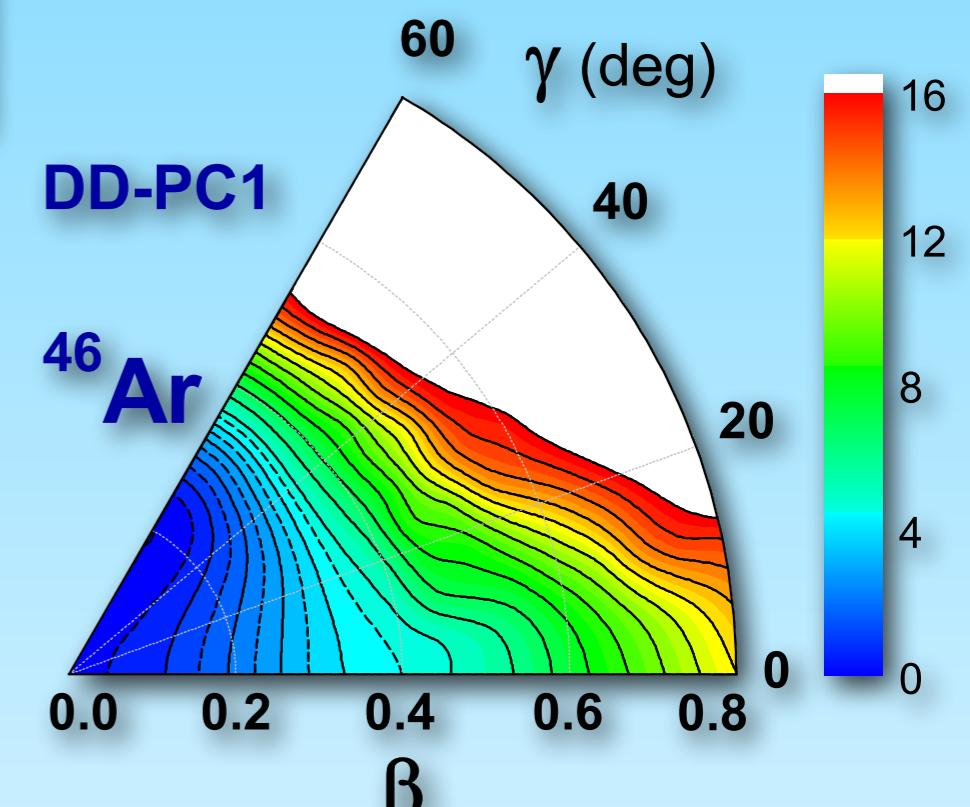
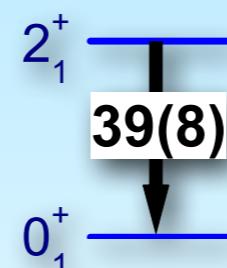


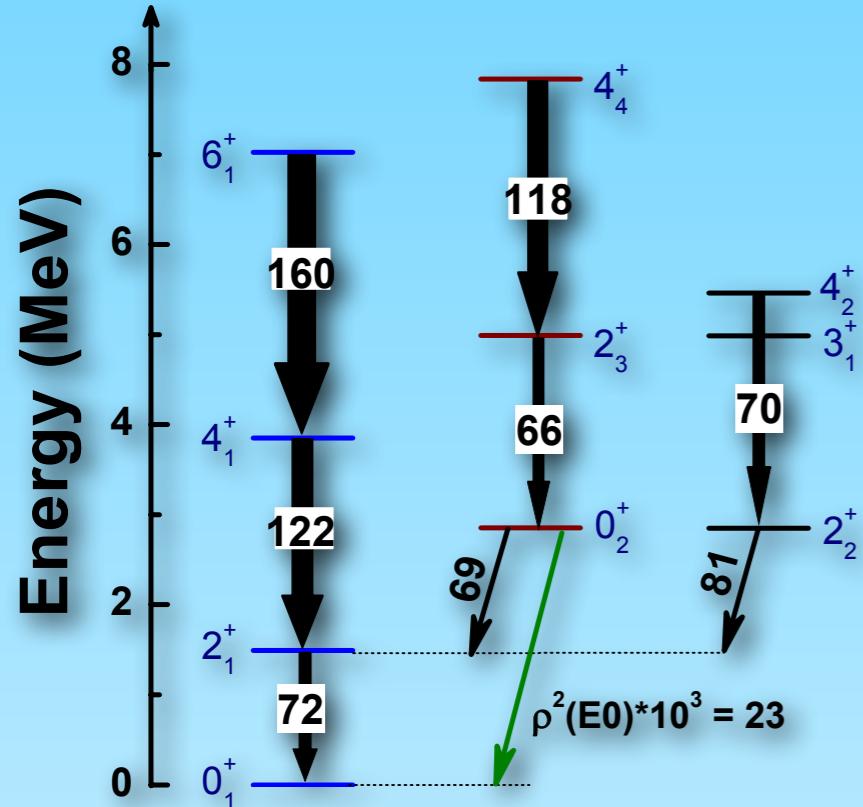
# $^{42}\text{Si}$ : single-particle levels



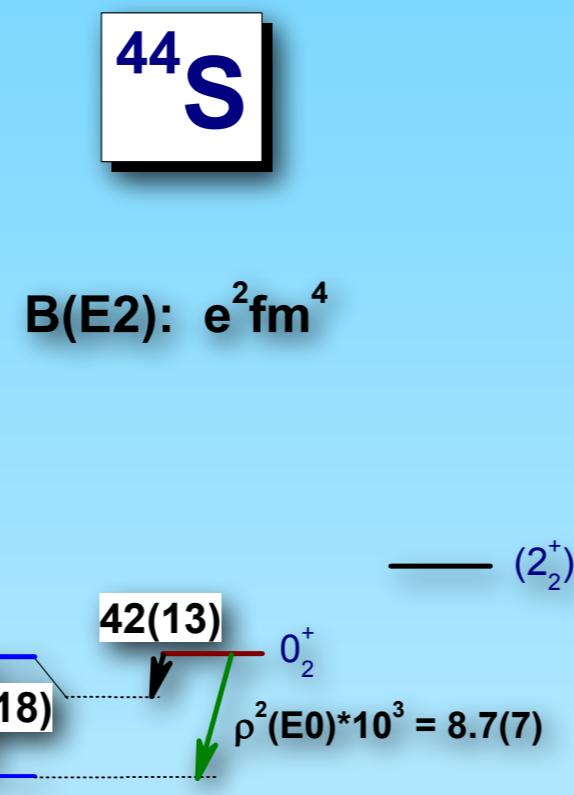


**$^{46}\text{Ar}$**

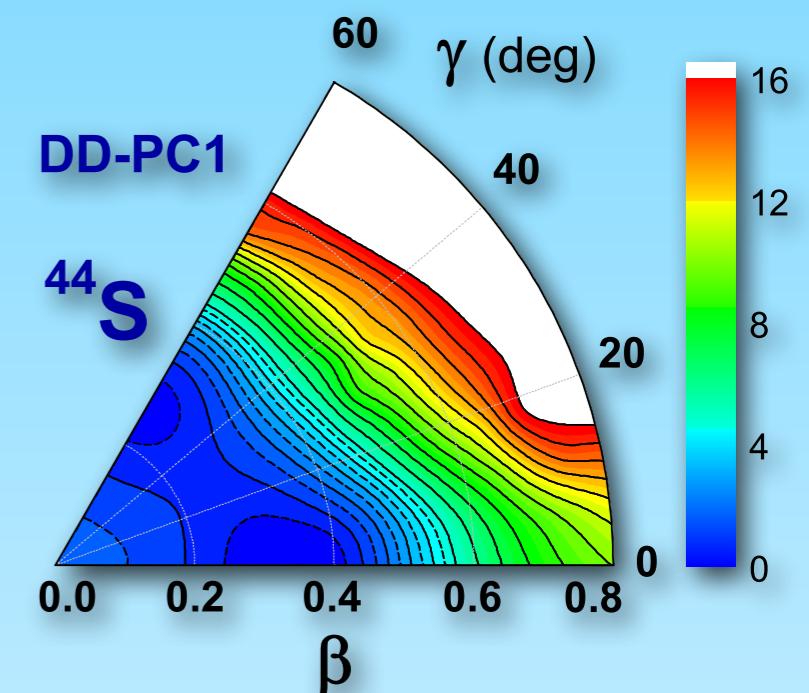




**DD-PC1**

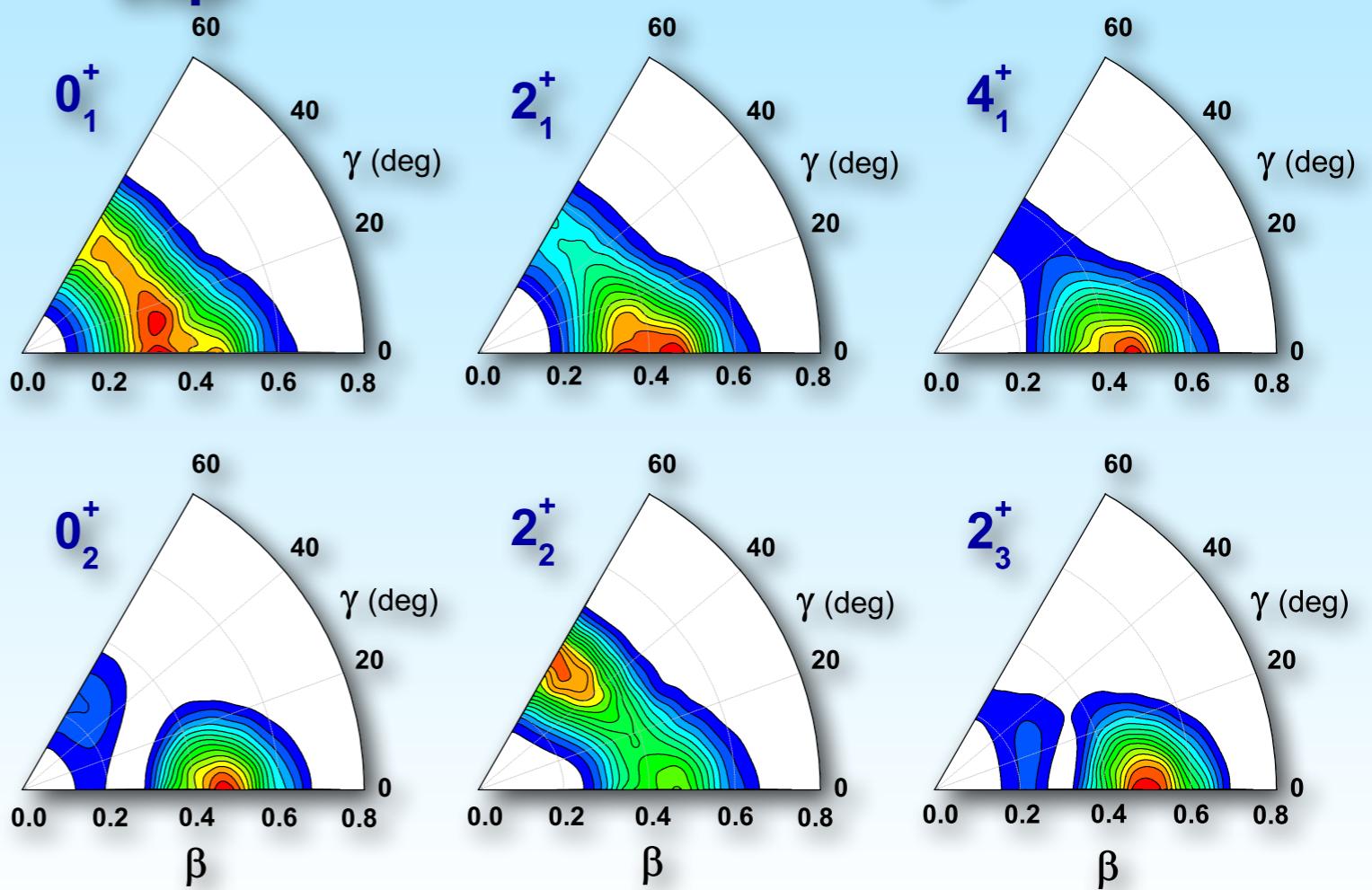


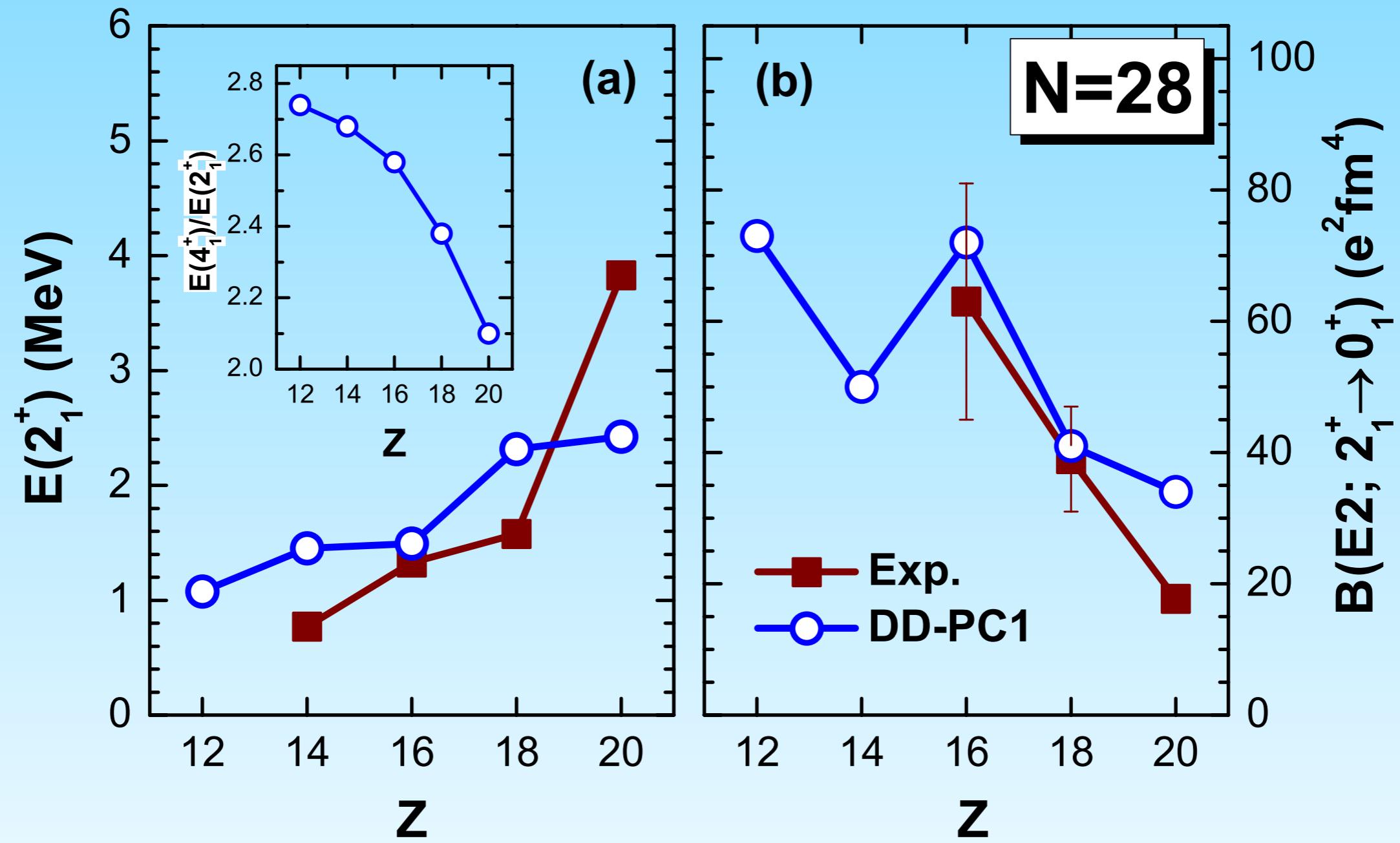
**Exp.**



Probability density distributions:

	$K = 0$	$K = 2$	$Q_{\text{spec.}}$
$2^+_1$	88.4	11.6	-10.9
$2^+_2$	21.5	78.5	7.8
$2^+_3$	80.0	20.0	-9.6





# Energy Density Functionals - Covariance Analysis

Quality measure:

$$\chi^2(\mathbf{p}) = \sum_{n=1}^N \left( \frac{\mathcal{O}_n^{(\text{th})}(\mathbf{p}) - \mathcal{O}_n^{(\text{exp})}}{\Delta \mathcal{O}_n} \right)^2$$

“Best model”  $\mathbf{p}_0 \Rightarrow$

$$\frac{\partial \chi^2(\mathbf{p})}{\partial p_i} \Big|_{\mathbf{p}=\mathbf{p}_0} \equiv \partial_i \chi^2(\mathbf{p}_0) = 0 \quad (\text{for } i = 1, \dots, F)$$

Expand the quality measure around the optimal model  $\mathbf{p}_0 \Rightarrow$

$$\chi^2(\mathbf{p}) = \chi^2(\mathbf{p}_0) + \frac{1}{2} \sum_{i,j=1}^F (\mathbf{p} - \mathbf{p}_0)_i (\mathbf{p} - \mathbf{p}_0)_j \partial_i \partial_j \chi^2(\mathbf{p}_0) + \dots$$

dimensionless variables:

$$x_i \equiv \frac{(\mathbf{p} - \mathbf{p}_0)_i}{(\mathbf{p}_0)_i}$$

→ the quadratic deviations of  $\chi^2$  from its minimum value:  $\chi^2(\mathbf{p}) - \chi^2(\mathbf{p}_0) \equiv \Delta \chi^2(\mathbf{x}) = \mathbf{x}^T \hat{\mathcal{M}} \mathbf{x}$

The symmetric  $F \times F$  matrix of second derivatives:

$$\mathcal{M}_{ij} = \frac{1}{2} \left( \frac{\partial \chi^2}{\partial x_i \partial x_j} \right)_{\mathbf{x}=0} = \frac{1}{2} (\mathbf{p}_0)_i (\mathbf{p}_0)_j \partial_i \partial_j \chi^2(\mathbf{p}_0)$$

Diagonalization  $\Rightarrow$

$$\Delta \chi^2(\mathbf{x}) = \mathbf{x}^T \left( \hat{\mathcal{A}} \hat{\mathcal{D}} \hat{\mathcal{A}}^T \right) \mathbf{x} = \xi^T \hat{\mathcal{D}} \xi = \sum_{i=1}^F \lambda_i \xi_i^2$$

The deviations of the  $\chi^2$  from its minimum value are parameterized in terms of  $F$  uncoupled harmonic oscillators  $\rightarrow$  the eigenvalues play the role of the spring constants.

**Soft direction**  $\Rightarrow$  small eigenvalue  $\lambda$ , little deterioration in  $\chi^2$ . The corresponding eigenvector  $\xi$  involves a particular linear combination of model parameters that is not constrained by the observables included in the fit.

**Stiff direction**  $\Rightarrow$  large eigenvalue  $\lambda$ ,  $\chi^2$  rapidly worsens away from minimum, the fit provides a stringent constraint on this particular linear combination of parameters.

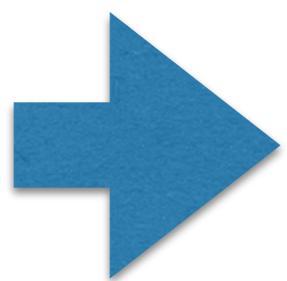
... covariance between two observables A and B:

$$\text{cov}(A, B) = \frac{1}{M} \sum_{m=1}^M \left[ (A^{(m)} - \langle A \rangle)(B^{(m)} - \langle B \rangle) \right] = \langle AB \rangle - \langle A \rangle \langle B \rangle$$

Pearson product-moment correlation coefficient:

$$\rho(A, B) = \frac{\text{cov}(A, B)}{\sqrt{\text{var}(A)\text{var}(B)}}$$

$$\text{cov}(A, B) = \sum_{i,j=1}^F \frac{\partial A}{\partial x_i} \left[ \frac{1}{M} \sum_{m=1}^M x_i^{(m)} x_j^{(m)} \right] \frac{\partial B}{\partial x_j} \equiv \sum_{i,j=1}^F \frac{\partial A}{\partial x_i} C_{ij} \frac{\partial B}{\partial x_j}$$



$$\text{cov}(A, B) = \sum_{i,j=1}^F \frac{\partial A}{\partial x_i} (\hat{\mathcal{M}}^{-1})_{ij} \frac{\partial B}{\partial x_j} = \sum_{i=1}^F \frac{\partial A}{\partial \xi_i} \lambda_i^{-1} \frac{\partial B}{\partial \xi_i}$$

... relativistic energy density functional DD-PCI  $\Rightarrow$  is it “predictive” ? Agreement with experiment?  
 $\Rightarrow$  is it “unique” ? A model is unique if all the eigenvalues  $\lambda_i$  of  $\mathcal{M}$  are large.

$$\begin{aligned}\alpha_s(\rho) &= a_s + (b_s + c_s x)e^{-d_s x} \\ \alpha_v(\rho) &= a_v + b_v e^{-d_v x} \\ \alpha_{tv}(\rho) &= b_{tv} e^{-d_{tv} x}\end{aligned}$$

PARAMETER	
$a_s$ (fm <sup>2</sup> )	-10.0462
$b_s$ (fm <sup>2</sup> )	-9.1504
$c_s$ (fm <sup>2</sup> )	-6.4273
$d_s$	1.3724
<hr/>	
$a_v$ (fm <sup>2</sup> )	5.9195
$b_v$ (fm <sup>2</sup> )	8.8637
$d_v$	0.6584
<hr/>	
$b_{tv}$ (fm <sup>2</sup> )	1.8360
<hr/>	
$d_{tv}$	0.6403
<hr/>	
$\delta_s$ (fm <sup>4</sup> )	-0.8149

Correlations between the lowest-order terms in a Taylor expansion of the density-dependent coupling functions around the saturation point:

$$c_s = -\frac{\rho_{\text{sat}}}{d_s} e^{d_s} [d_s \alpha'_s(\rho_{\text{sat}}) + \rho_{\text{sat}} \alpha''_s(\rho_{\text{sat}})],$$

$$b_s = c_s \left( \frac{1}{d_s} - 1 \right) - \alpha'_s(\rho_{\text{sat}}) \rho_{\text{sat}} \frac{e^{d_s}}{d_s},$$

$$a_s = \alpha_s(\rho_{\text{sat}}) - (b_s + c_s) e^{-d_s}$$

### Nuclear matter pseudo-observables

$$d_v = -\frac{\alpha''_v(\rho_{\text{sat}})}{\alpha'_v(\rho_{\text{sat}})} \rho_{\text{sat}},$$

$$b_v = -\alpha'_v(\rho_{\text{sat}}) \frac{e^{d_v}}{d_v},$$

$$a_v = \alpha_v(\rho_{\text{sat}}) - b_v e^{-d_v}$$

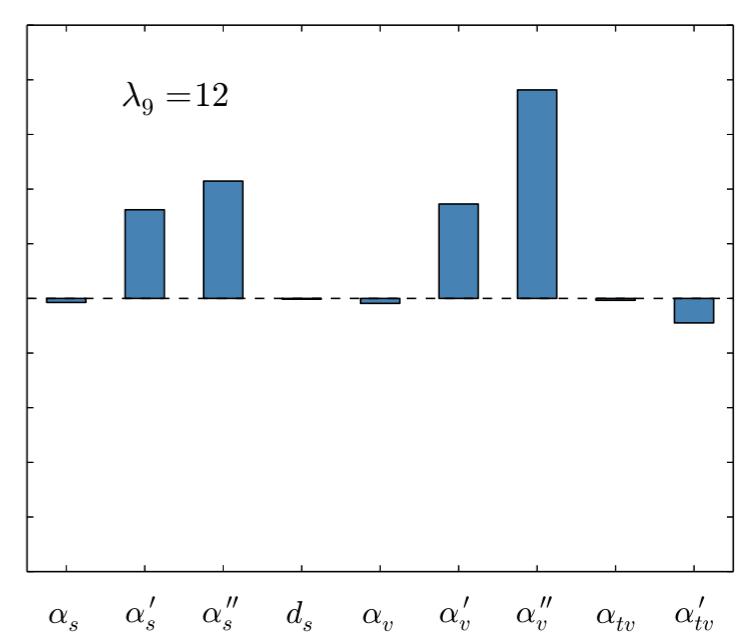
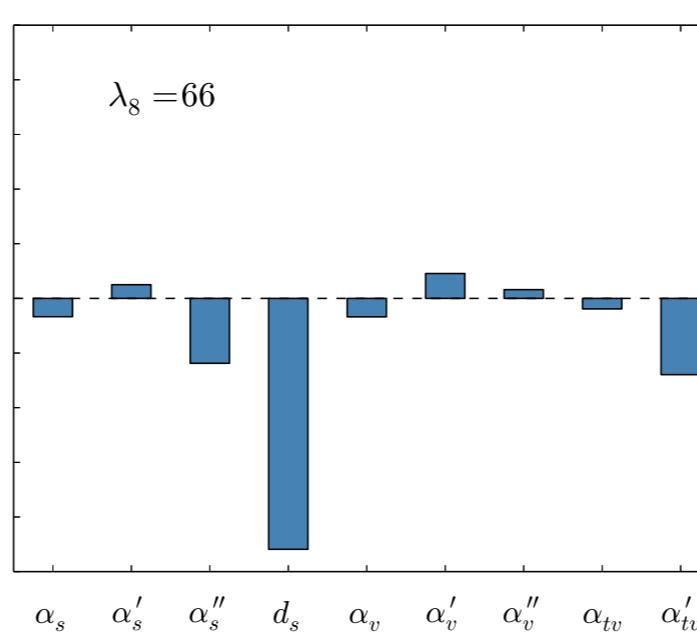
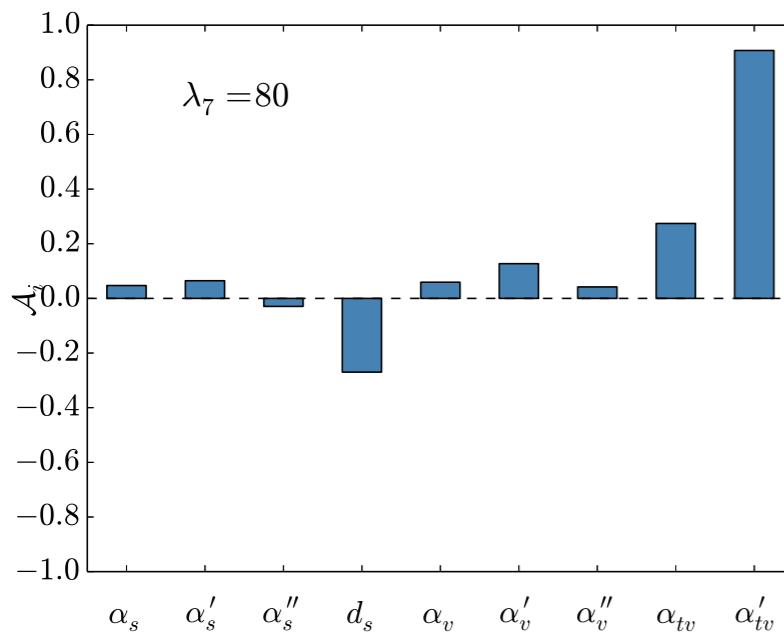
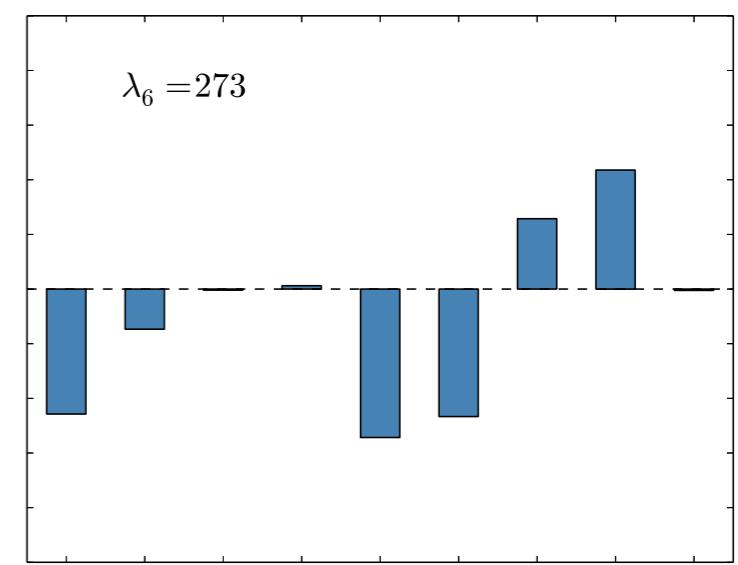
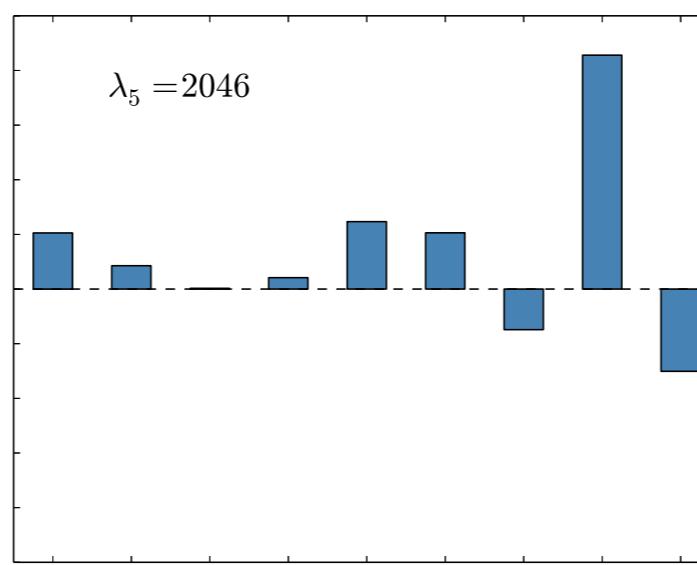
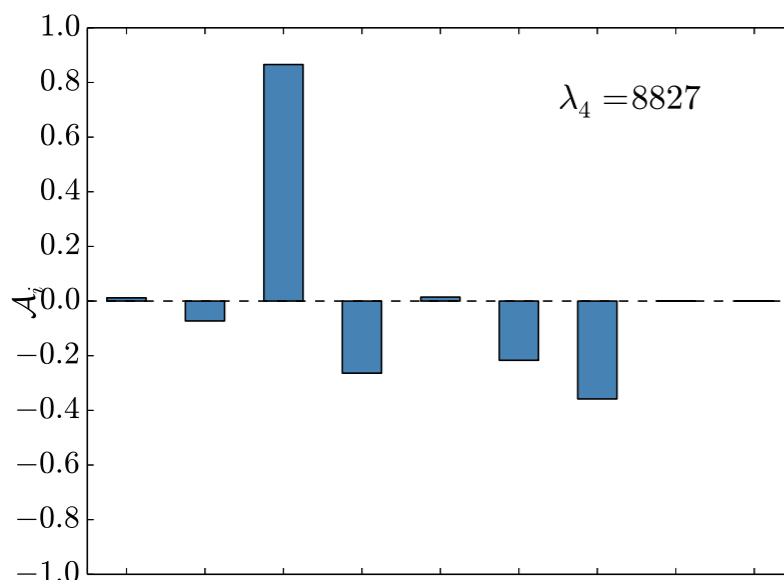
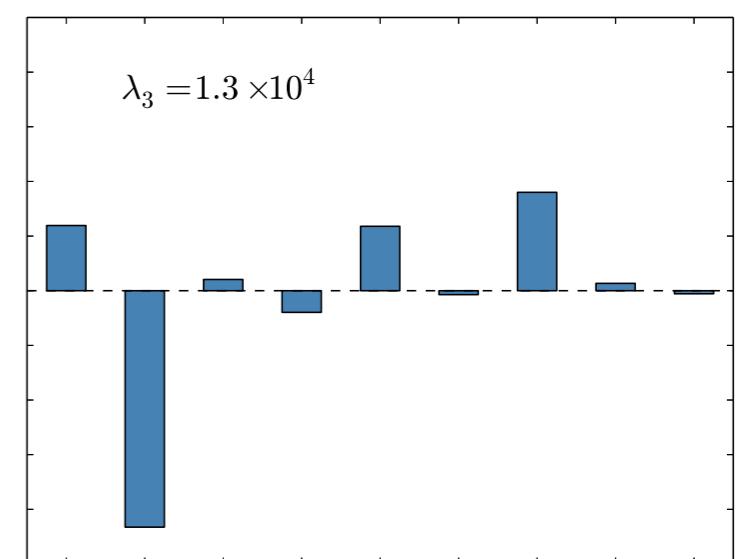
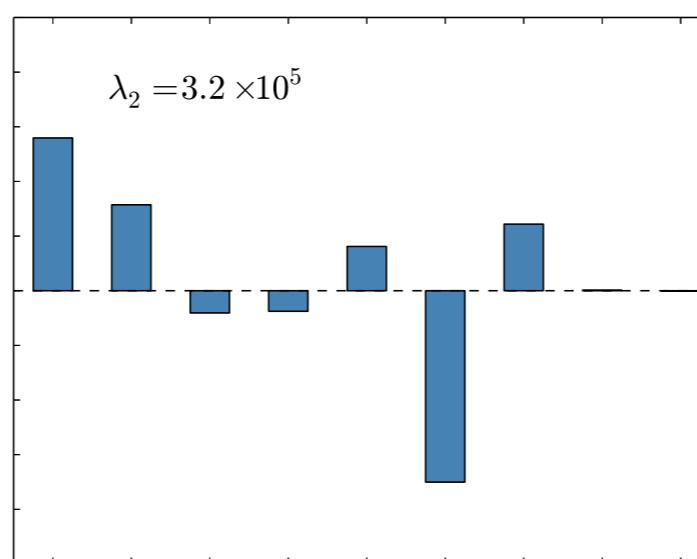
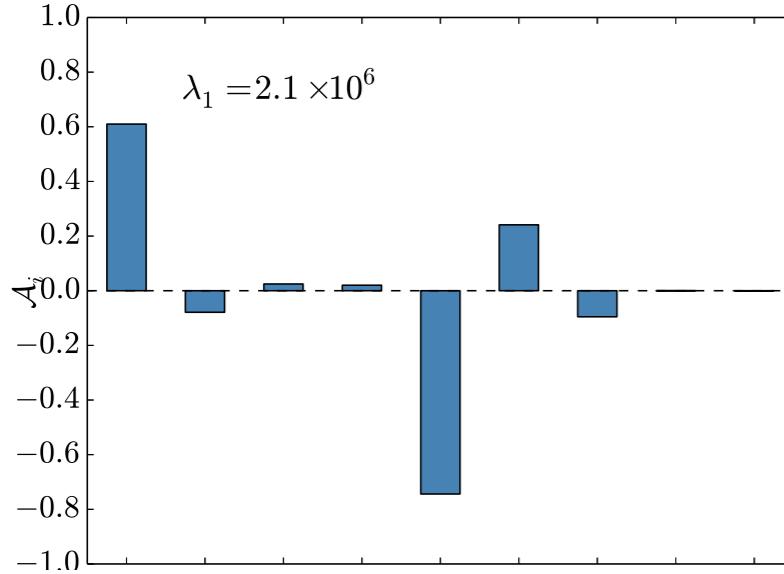
$$d_{tv} = -\rho_{\text{sat}} \frac{\alpha'_{tv}(\rho_{\text{sub}})}{\alpha_{tv}(\rho_{\text{sub}})},$$

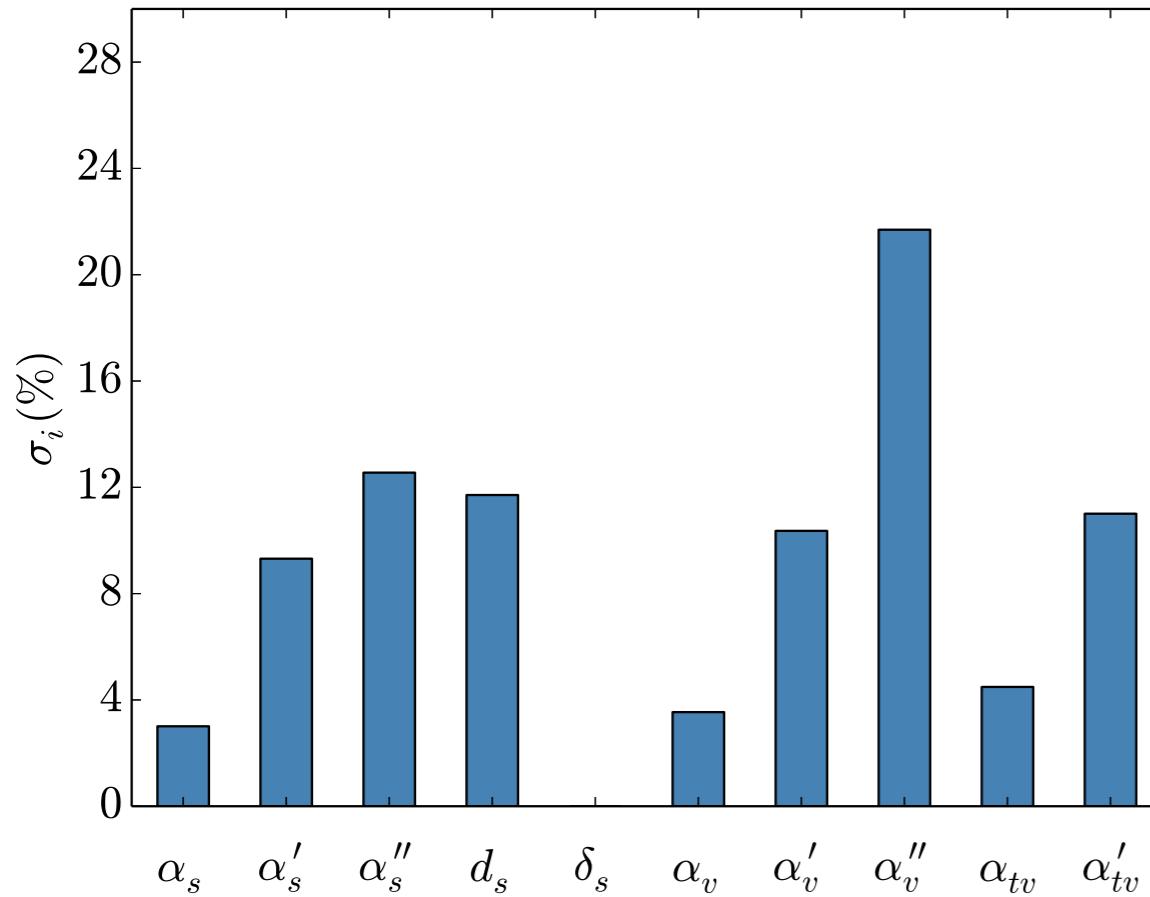
$$b_{tv} = \alpha_{tv}(\rho_{\text{sub}}) e^{d_{tv}(\rho_{\text{sub}}/\rho_{\text{sat}})}$$

$$\rho_{\text{sub}} = 0.12 \text{ fm}^{-3}$$

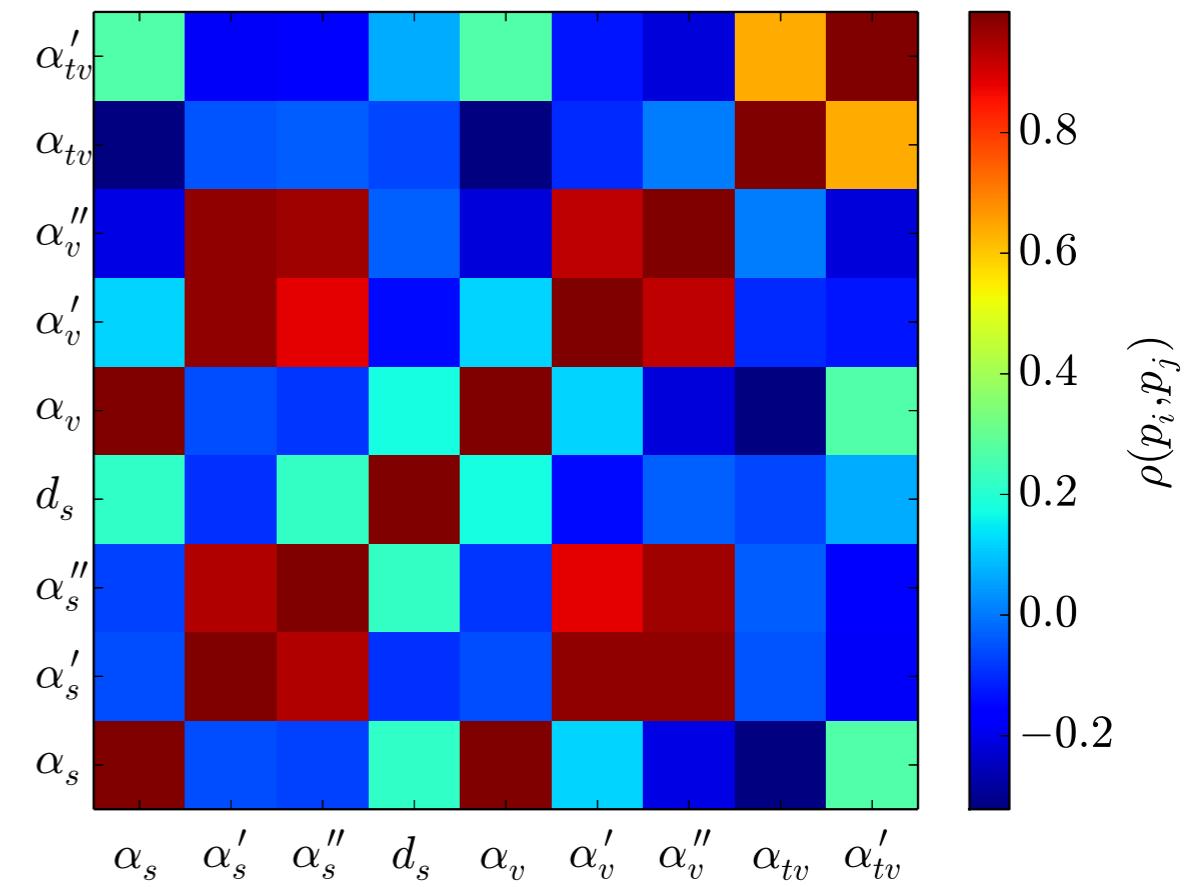
OBSERVABLE	DD-PC1
$\rho_0$	0.152 fm $^{-3}$
$\epsilon(\rho_0)$	-16.06 MeV
$\epsilon(\rho_{low})$	-6.48 MeV
$\epsilon(\rho_{high})$	34.38 MeV
$K_0$	230 MeV
$m_D$	0.58
$m^*$	0.66
$S_2(\rho_{\text{sub}})$	27.8 MeV
$L(\rho_{\text{sub}})$	57.2 MeV
$a_4$	33 MeV

# Eigenvalues and eigenvectors of the $9 \times 9$ matrix of second derivatives $M$ of $\chi^2(p)$ for the functional DD-PCI

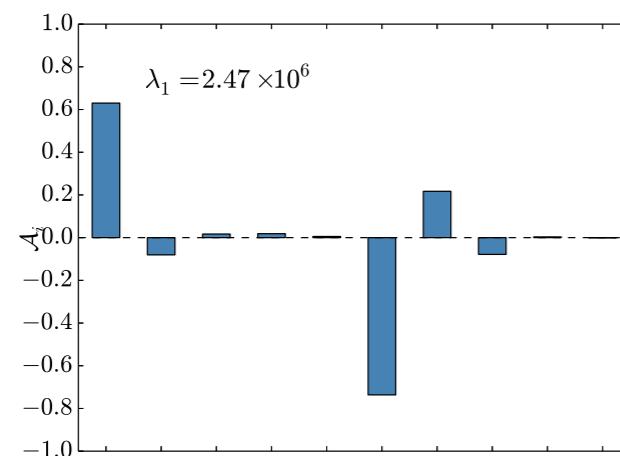




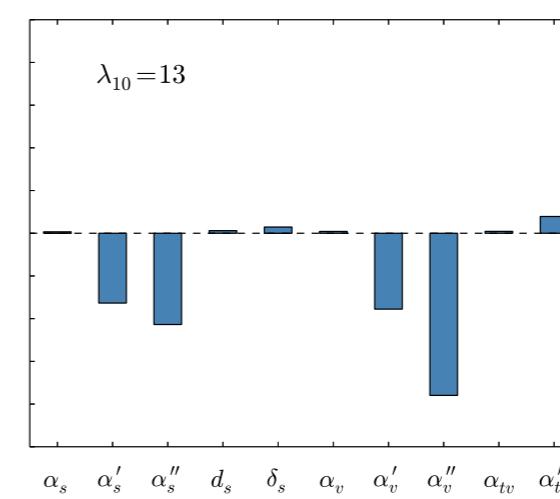
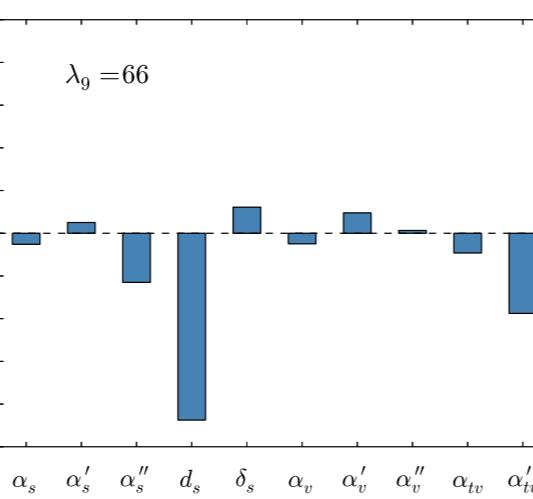
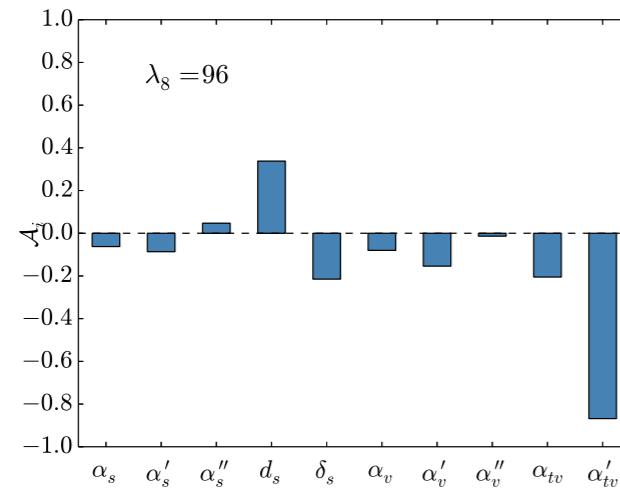
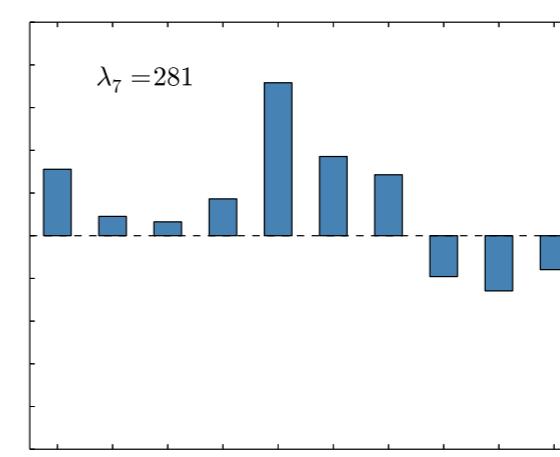
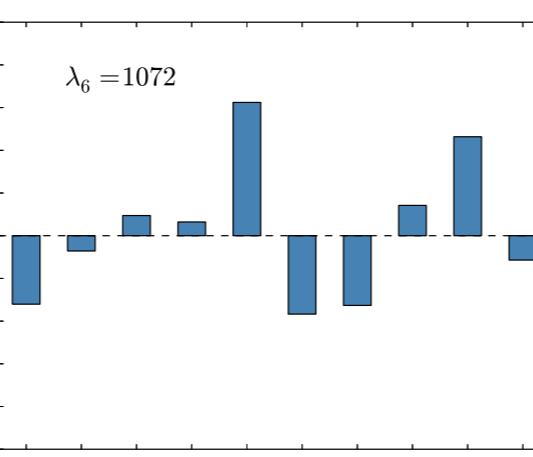
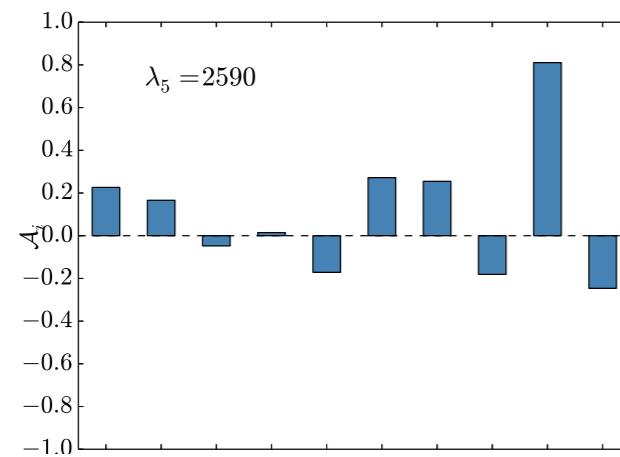
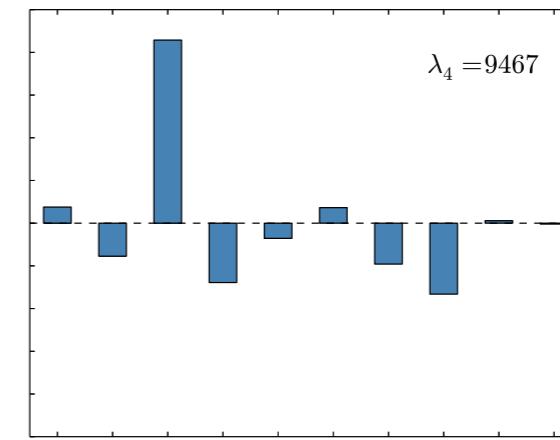
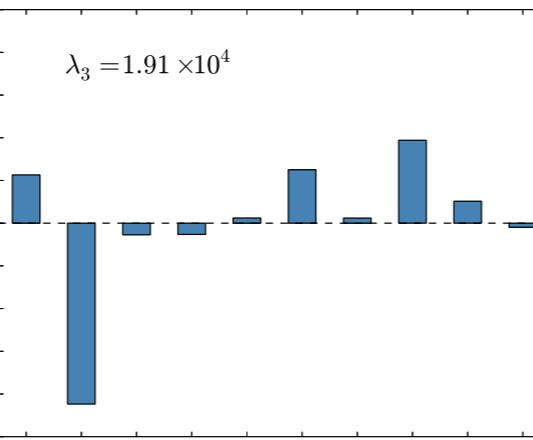
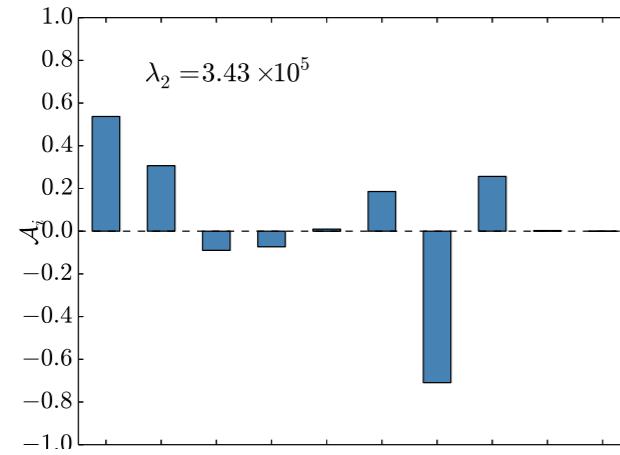
Uncertainties  $\sigma_i$  of model parameters for the functional DD-PCI.

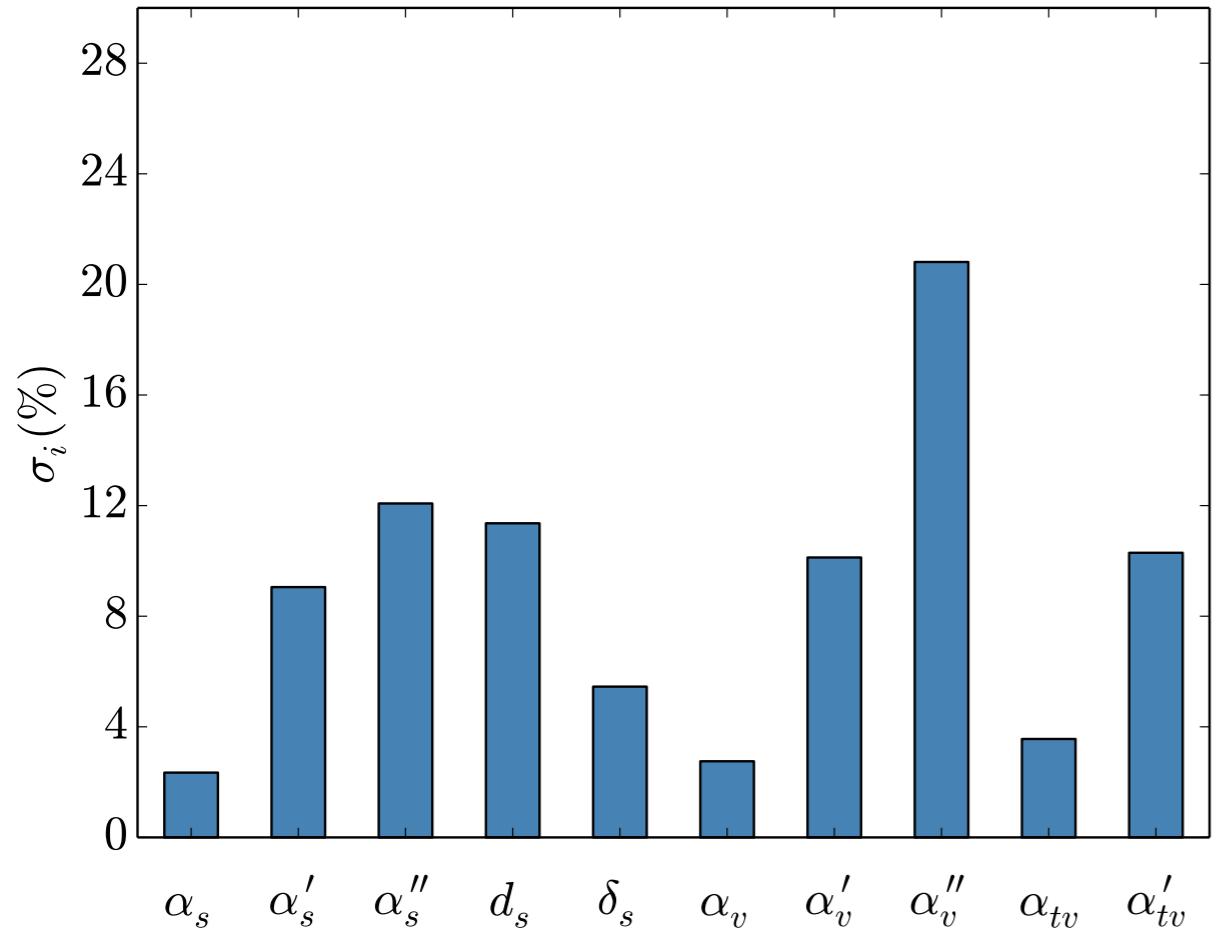


36 independent correlation coefficients between 9 model parameters that contribute to infinite nuclear matter calculations .

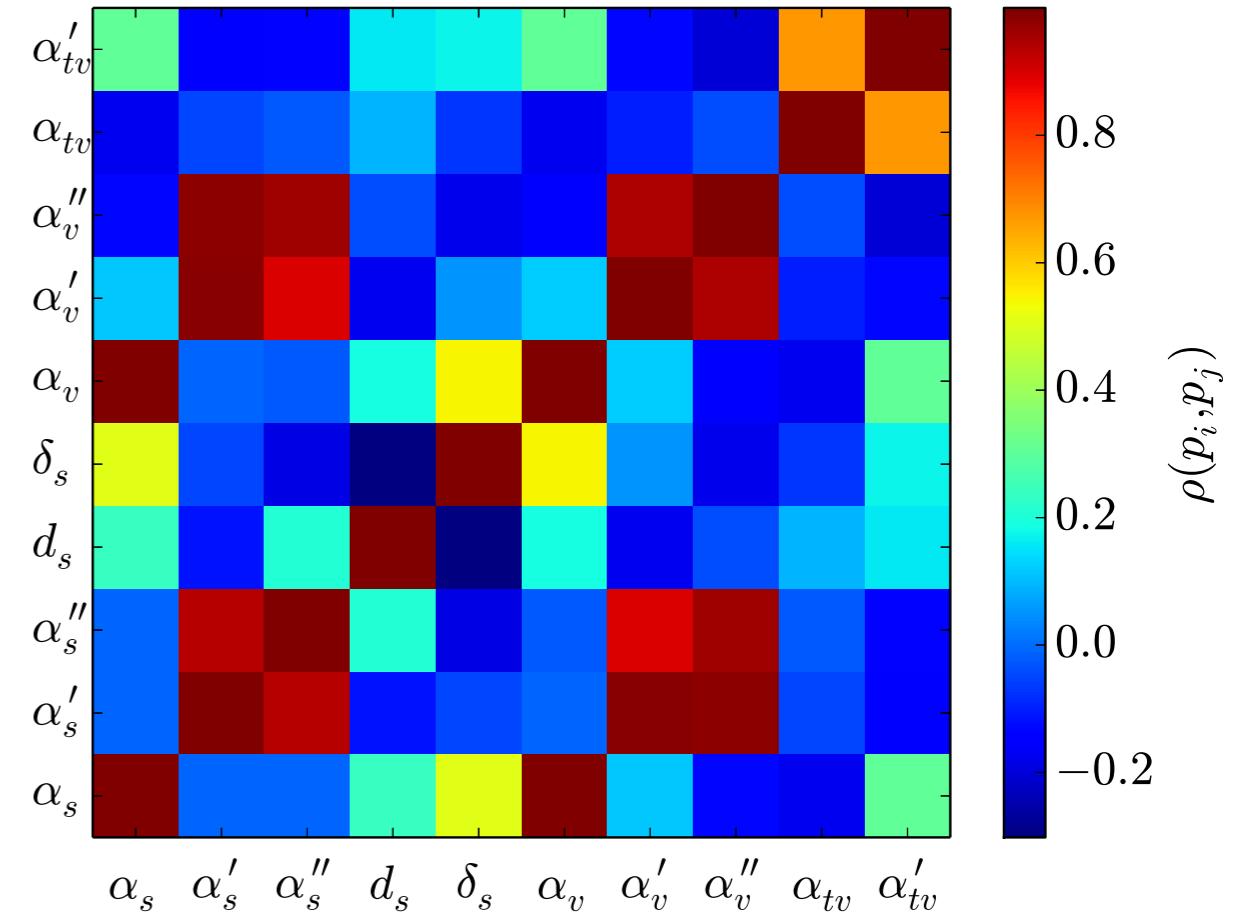


Includes semi-infinite nuclear matter with  
surface energy  $a_s = 17.5 \text{ MeV} \pm 2\%$



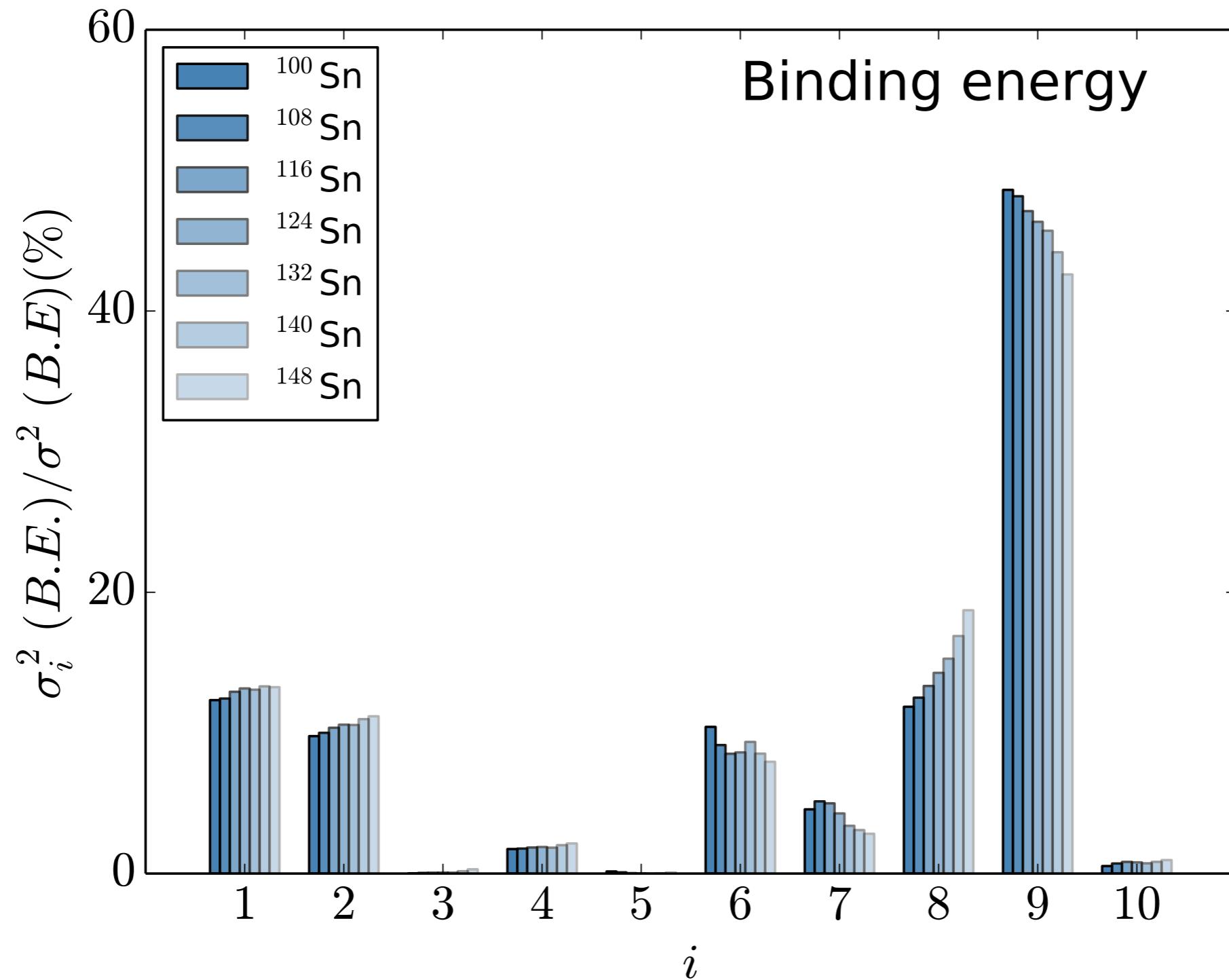


Uncertainties  $\sigma_i$  of model parameters for the functional DD-PCI.

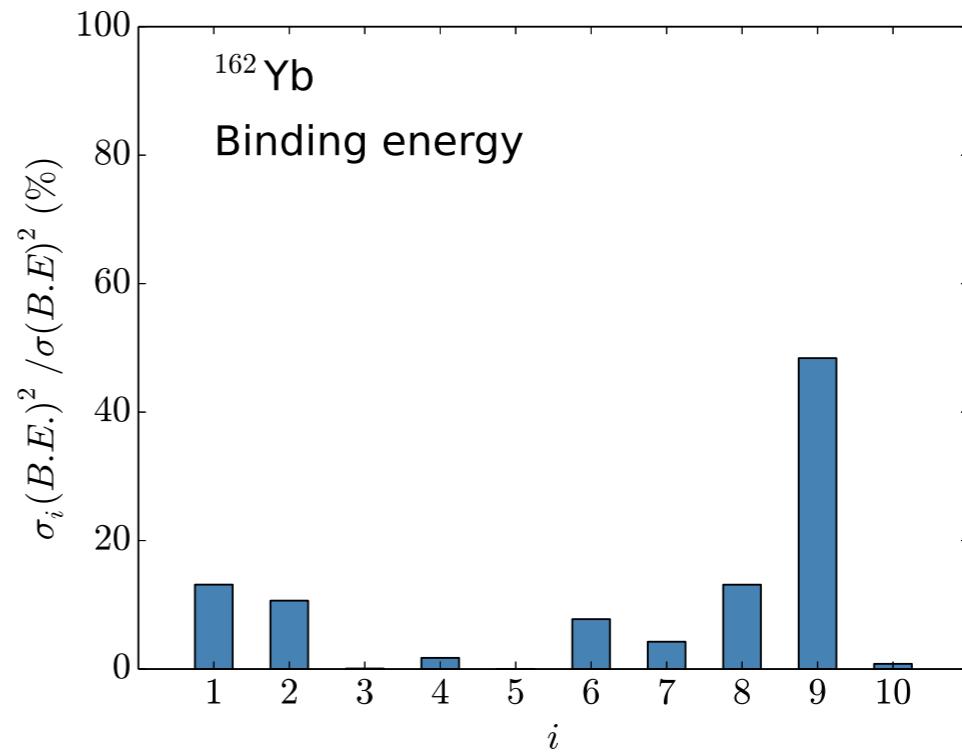
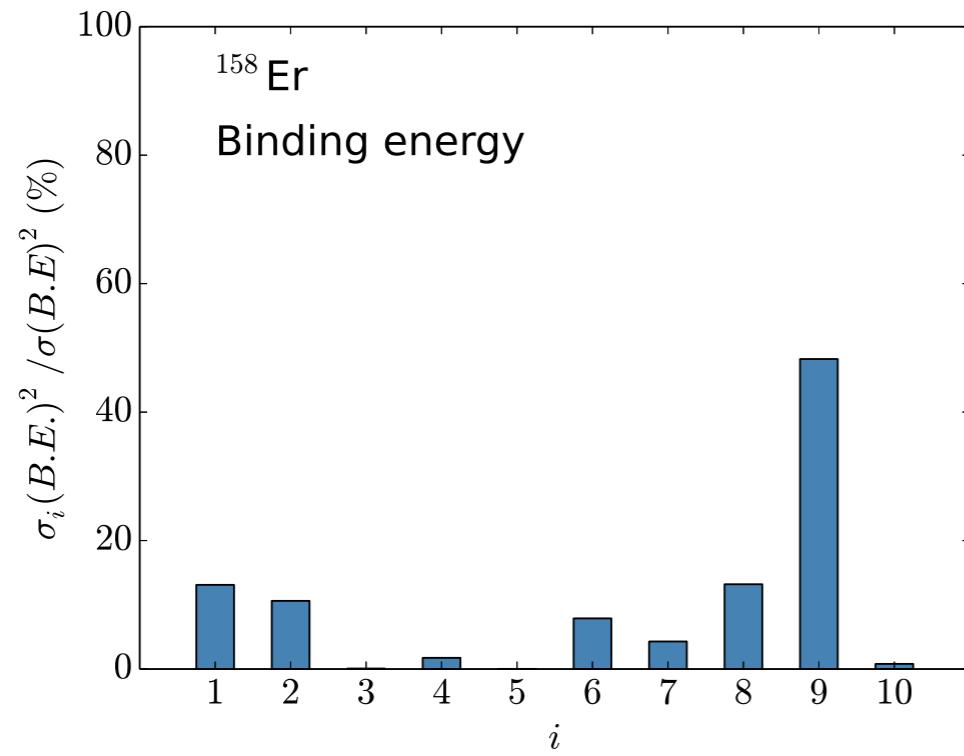
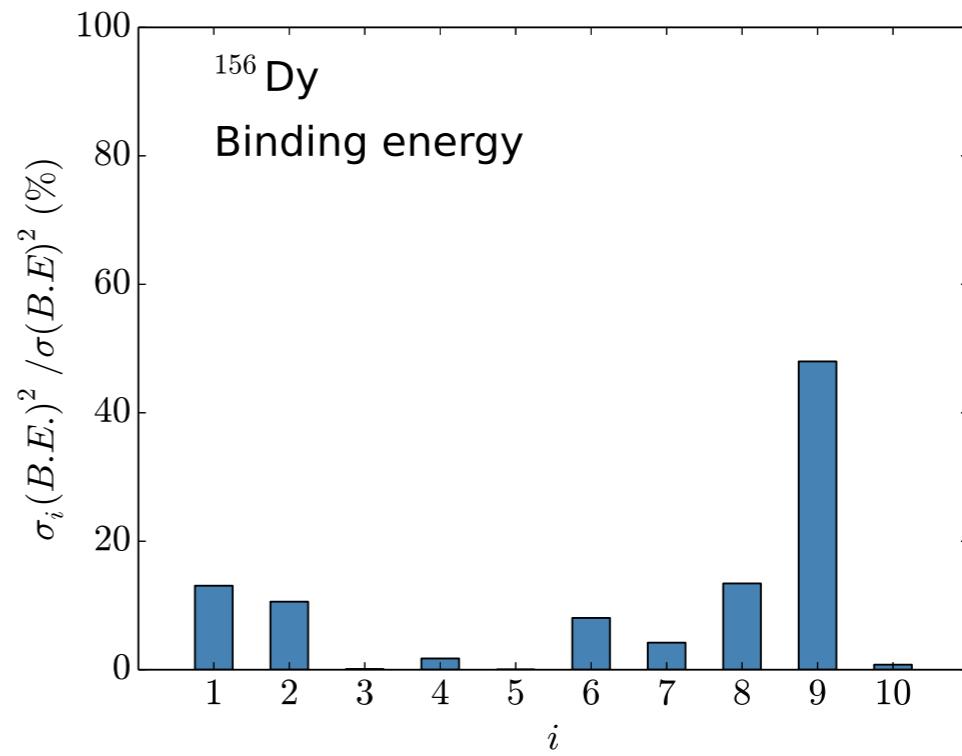
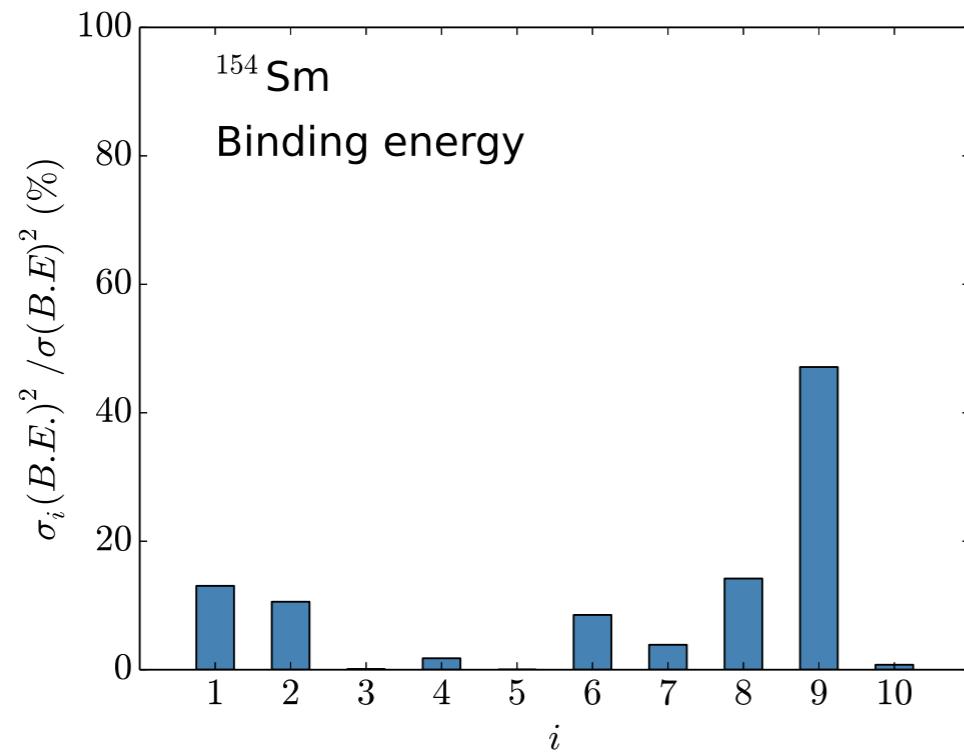


45 independent correlation coefficients between 10 model parameters that contribute to semi-infinite nuclear matter calculations .

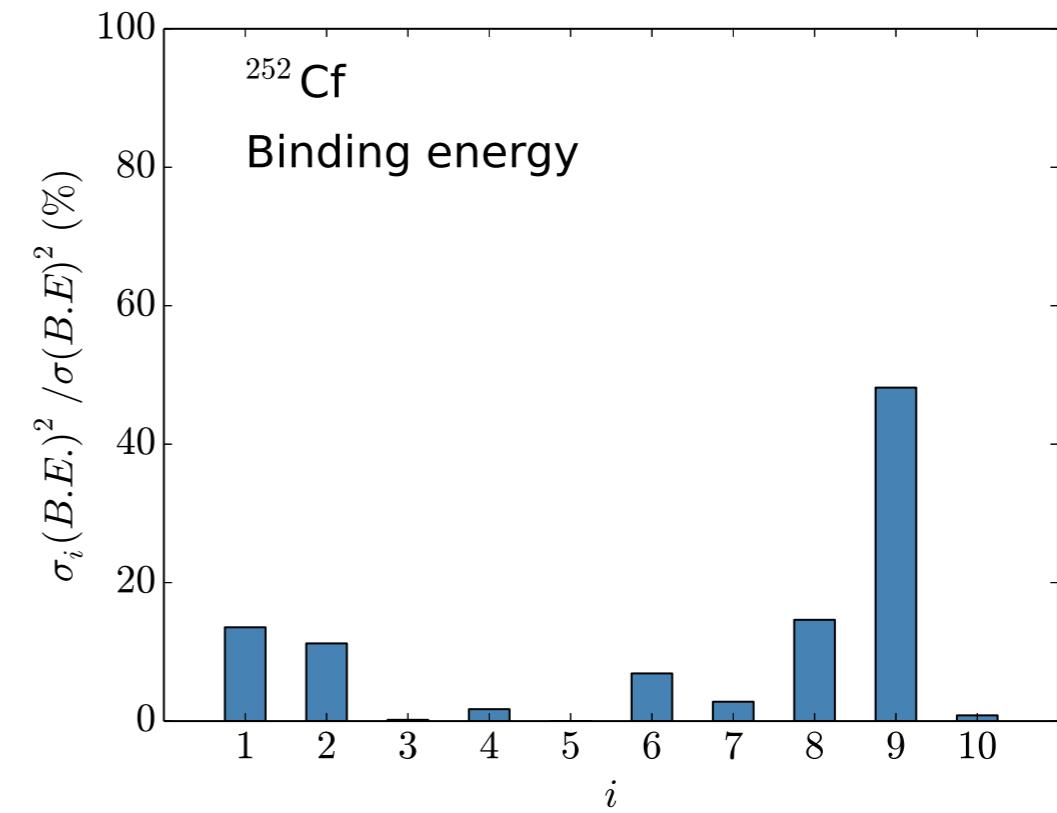
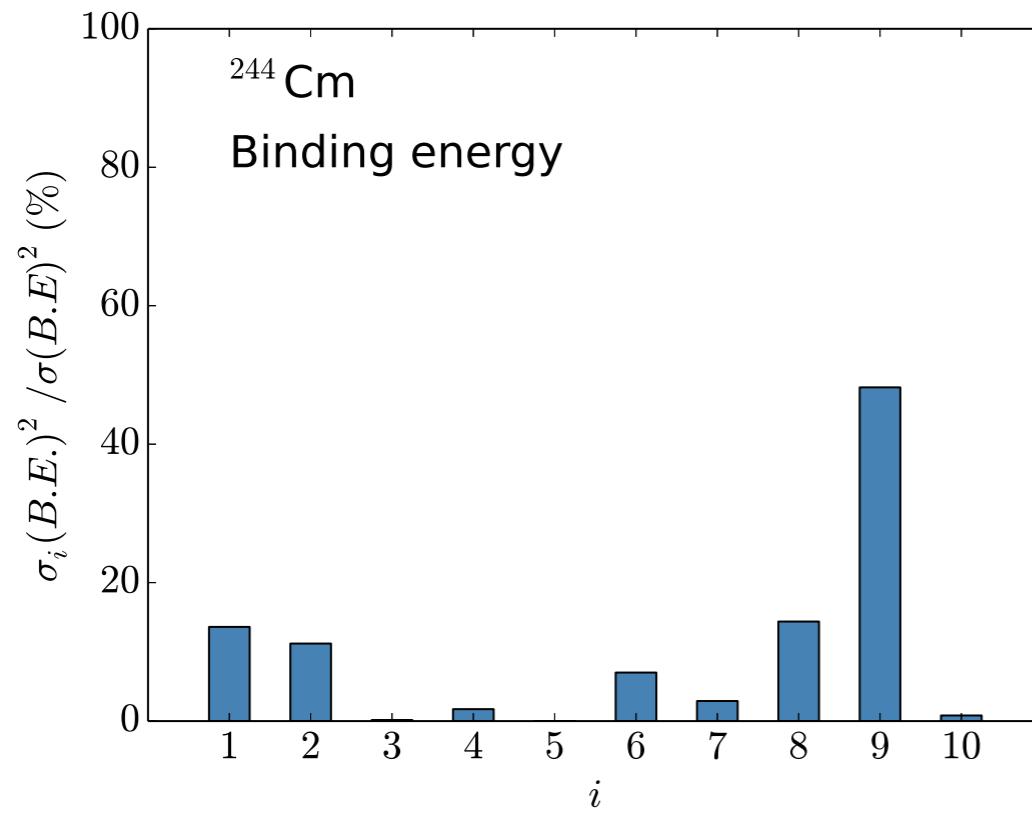
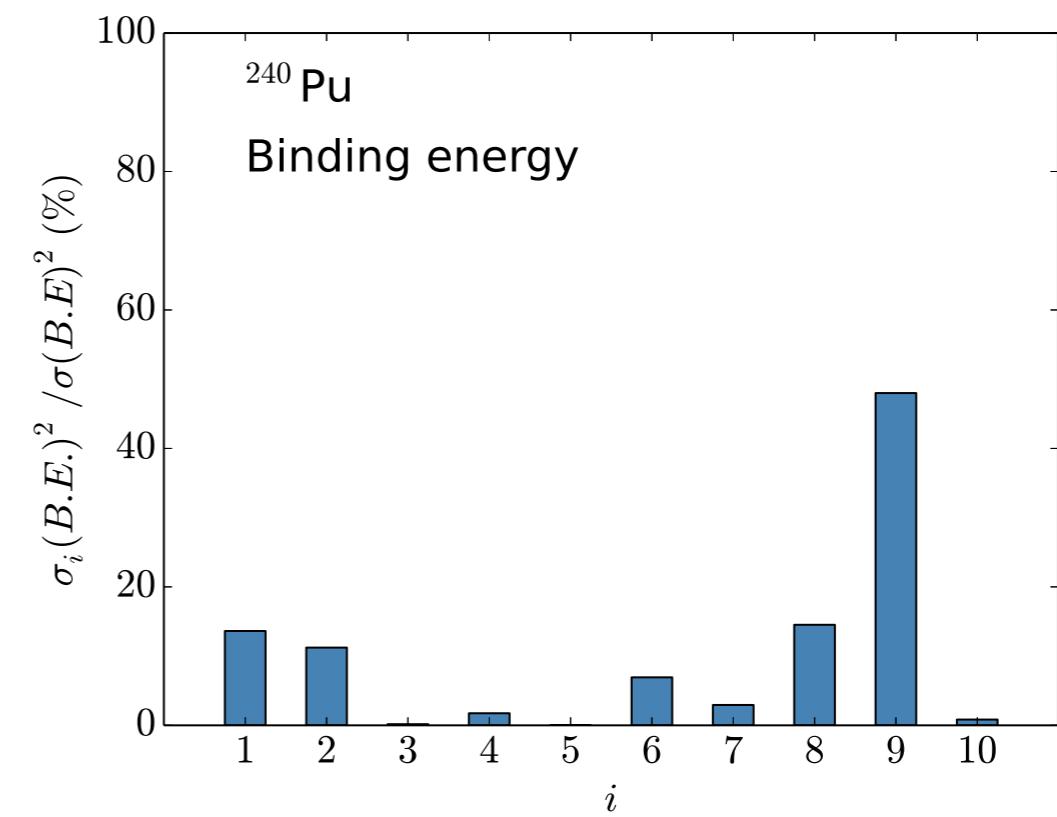
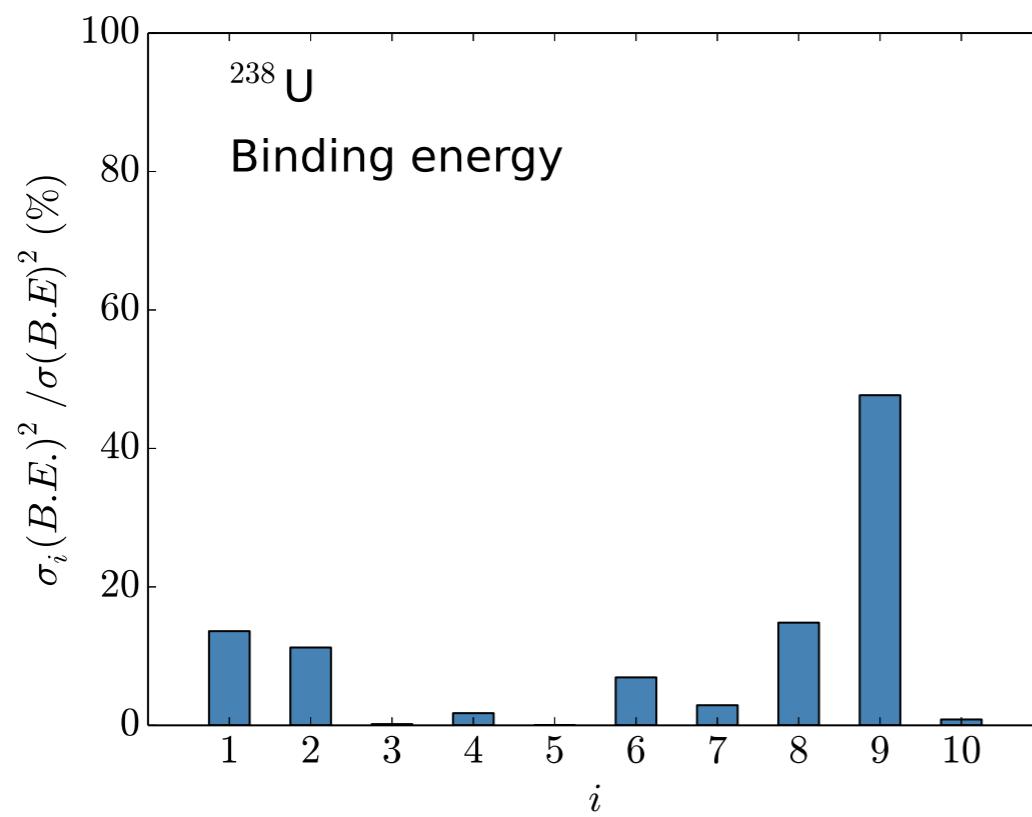
# Finite nuclei



Relative contributions in percentage of the ten linear combinations of model parameters that correspond to the eigenvectors of the matrix of second derivatives  $\mathcal{M}$  to the variances of the binding energy of tin isotopes.



Relative contributions in percentage of the ten linear combinations of model parameters that correspond to the eigenvectors of the matrix of second derivatives  $\mathcal{M}$  to the variances of the binding energy of rare-earth nuclei.



Relative contributions in percentage of the ten linear combinations of model parameters that correspond to the eigenvectors of the matrix of second derivatives  $\mathcal{M}$  to the variances of the binding energy of actinide nuclei.

Zagreb: T. Nikšić  
N. Paar  
U. Prassa  
T. Marketin  
P. Marević  
D. Uretenac

München: P. Ring  
N. Kaiser  
W. Weise

Beijing: Jie Meng  
Yifei Niu  
Chunyan Song

Chongqing: Zhipan Li  
Jiang-Ming Yao

Bologna: P. Finelli

Orsay: E. Khan  
J.-P. Ebran

Tokyo: K. Nomura  
T. Otsuka  
N. Shimizu