

Studying short-range correlations and momentum distributions with unitarily transformed interactions and operators

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Motivation

Observations

- JLab experiments found that a knocked out high-momentum proton is accompanied by a second nucleon with opposite momentum
- Cross sections for $(e, e'pn)$ and $(e, e'pp)$ reactions show strong dominance of pn - over pp -pairs

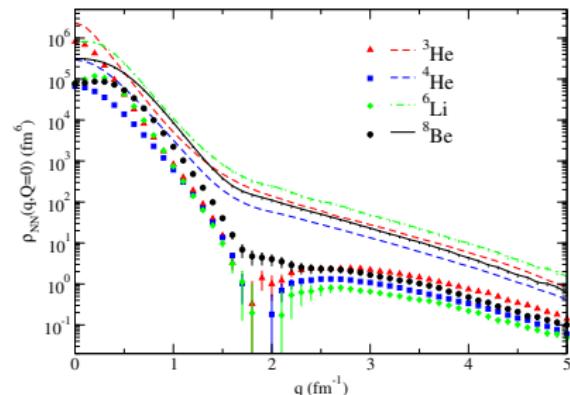
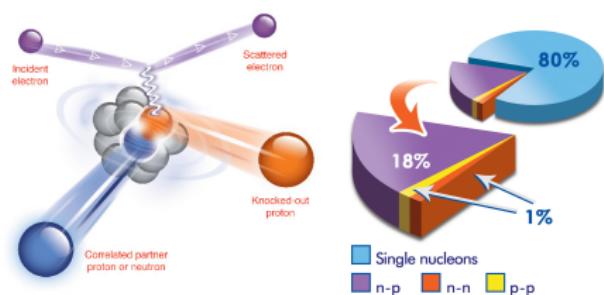
Subedi *et al.*, Science **320**, 1476 (2008)

Theoretical interpretation

- ab initio* calculations with Argonne interactions show high-momentum components
- dominance of pn - over pp -pairs due to the tensor force

Wiringa, Schiavilla, Pieper, Carlson, PRC **85**, 021001(R) (2008)

Alvioli *et al.*, Int. J. Mod. Phys. E **22**, 1330021 (2013)



Outline

Nucleon-nucleon interaction

- Properties
- Realistic interactions

Correlations in light nuclei

- One- and two-body densities with AV8' interacton

Unitary transformations

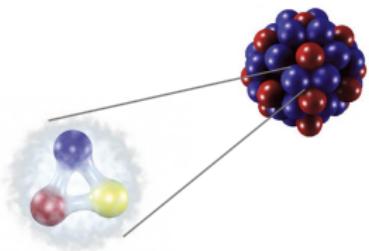
- Similarity Renormalization Group
- What happens to short-range physics ?

NCSM+SRG calculations

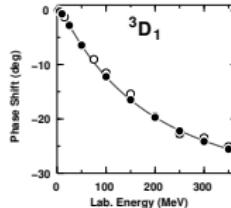
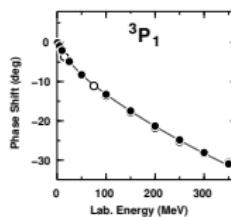
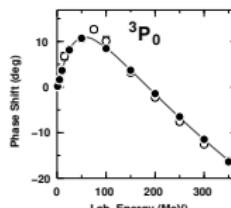
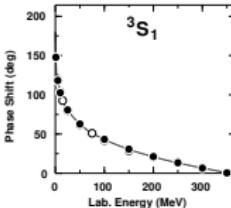
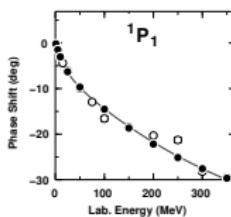
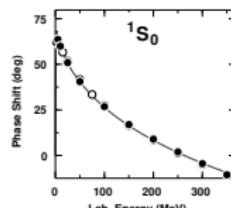
- compare AV18 and N3LO results
- ^4He two-body densities
- $^6\text{He}, ^9\text{Be}, ^{12}\text{C}$ two-body densities

Summary

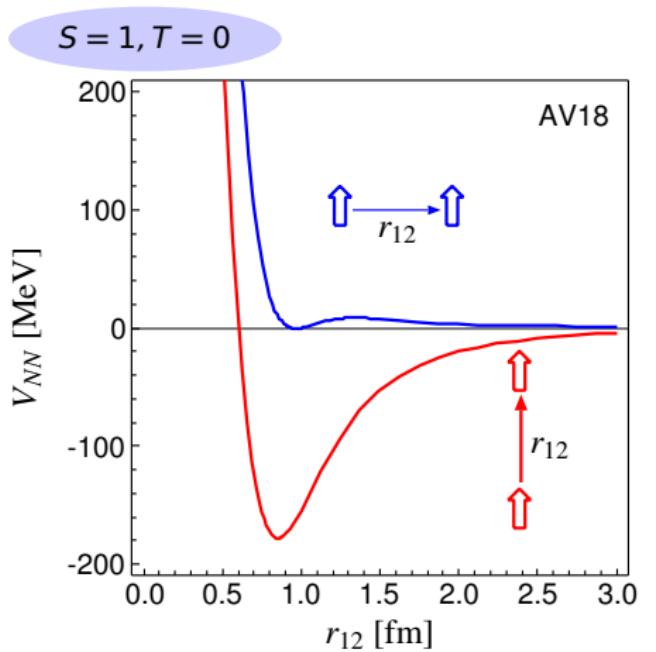
Nucleon-nucleon interaction



- Nucleons are not point-like, complicated quark and gluon sub-structure
- Nucleon-nucleon (NN) interaction: residual interaction
- Calculation within QCD not possible yet
→ construct **realistic NN potentials** describing two-nucleon properties (scattering, deuteron) with high accuracy
- Different potentials available, but some general features ...



NN interaction — Short-range and tensor correlations

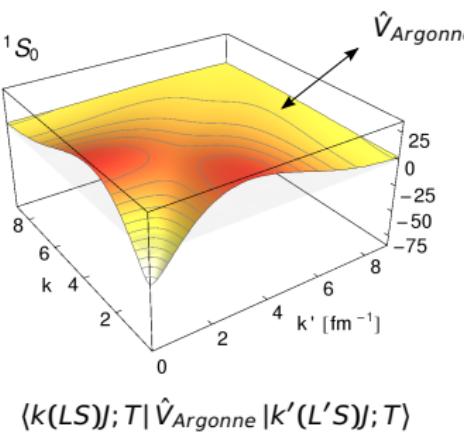


- repulsive core:
nucleons can not get closer than
 ≈ 0.5 fm → **central correlations**
- strong dependence on the
orientation of the spins due to the
tensor force (mainly from
 π -exchange) → **tensor correlations**
- the nuclear force will induce **strong
short-range correlations** in the
nuclear wave function

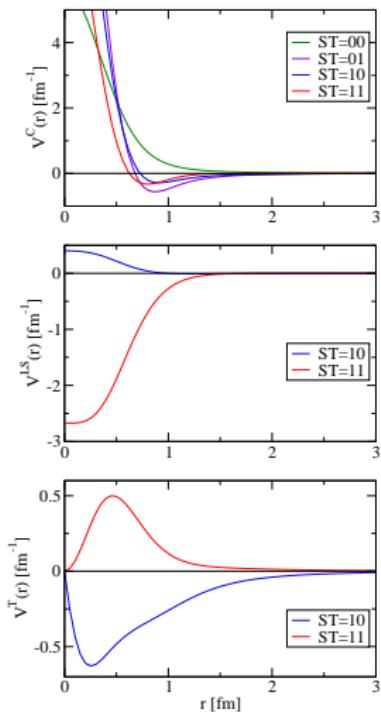
NN interaction — Argonne V18

Argonne V18

- π -exchange, phenomenological short-range
- as local as possible
- fitted to phase shifts up to 350 MeV, but describes elastic phase shifts up to 1 GeV



$$\begin{aligned}
 \hat{V}_{\text{Argonne}} = & \sum_{S,T} V_{ST}^C(\hat{r}) \hat{\Pi}_{ST} \\
 + & \sum_{S,T} V_{ST}^{L2}(\hat{r}) \hat{\mathbf{L}}^2 \hat{\Pi}_{ST} \\
 + & \sum_T V_{1T}^{LS}(\hat{r}) \hat{\mathbf{L}} \hat{\mathbf{S}} \hat{\Pi}_{1T} \\
 + & \sum_T V_{1T}^T(\hat{r}) \hat{S}_{12} \hat{\Pi}_{1T} \\
 + & \sum_T V_{1T}^{TLL}(\hat{r}) s_{12}(\hat{\mathbf{L}}, \hat{\mathbf{L}}) \hat{\Pi}_{1T} \\
 + & \dots
 \end{aligned}$$

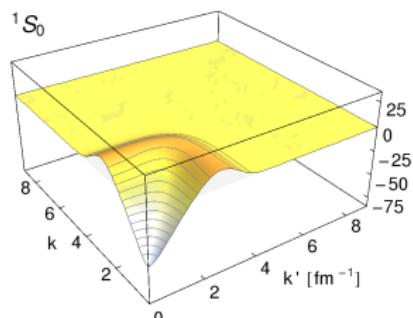


Wiringa, Stoks, Schiavilla, PRC **51**, 38 (1995)

NN interaction — Chiral effective field theory

N³LO

- potential derived using chiral EFT
- includes full π dynamics
- short-range behavior given by contact-terms
- power counting
- regulated by cut-off (500 MeV)



$$\langle k(LS)J; T | \hat{V}_{N3LO} | k'(L'S)J; T \rangle$$

Entem, Machleidt, PRC **68**, 041001 (2003)

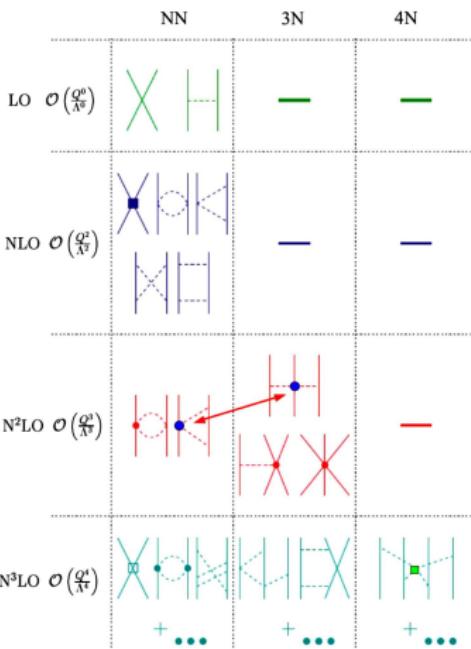


Figure taken from:

Bogner, Furnstahl, Schwenk, Part. Nucl. Phys. **65**, 94 (2010)

Solving the nuclear many-body problem

Correlated Gaussian Method

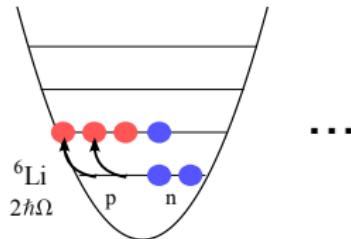
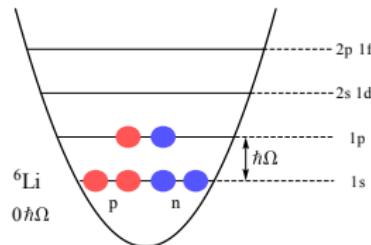
- Correlated Gaussian basis
- uses Jacobi coordinates
- exact results for $A \leq 4$

Suzuki, Horiuchi, Orabi, Arai, Few-Body Syst. **42**, 33 (2008)

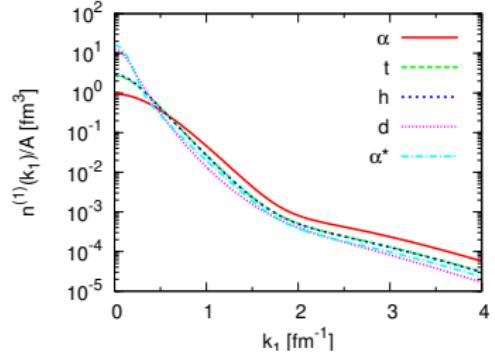
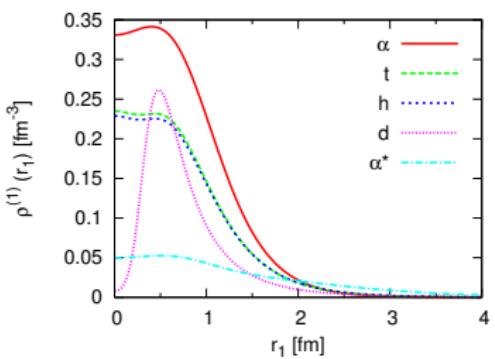
No-Core Shell Model (NCSM)

- Diagonalization of Hamiltonian in harmonic oscillator basis
- Truncation scheme
 - $0\hbar\Omega$ configuration:
lowest single particle orbits filled
 - $N\hbar\Omega$ configuration:
 N oscillator quanta above $0\hbar\Omega$ configuration
 - Truncation: use only configurations with $N \leq N_{\max}$
- Check for convergence
- Model space sizes grow rapidly with A and N_{\max}

Navrátil, Kamuntavičius, Barrett, Phys. Rev. C, **61**, 044001 (2000)



One-body densities for $A=2,3,4$ nuclei



$$\rho^{(1)}(\mathbf{r}_1) = \langle \Psi | \sum_{i=1}^A \delta^3(\hat{\mathbf{r}}_i - \mathbf{r}_1) | \Psi \rangle$$

$$n^{(1)}(\mathbf{k}_1) = \langle \Psi | \sum_{i=1}^A \delta^3(\hat{\mathbf{k}}_i - \mathbf{k}_1) | \Psi \rangle$$

- one-body densities calculated from **exact wave functions** (Correlated Gaussian Method) for AV8' interaction
- coordinate space densities reflect different sizes and densities of ^2H , ^3H , ^3He , ^4He and the 0_2^+ state in ^4He
- similar high-momentum tails in the one-body momentum distributions

Two-body densities

number of pairs in given spin-, isospin channels

$$\rho_{SM_S, TM_T}^{(2)}(\mathbf{r}_1, \mathbf{r}_2) = \langle \Psi | \sum_{i < j}^A \hat{P}_{ij}^{SM_S} \hat{P}_{ij}^{TM_T} \delta^3(\hat{\mathbf{r}}_i - \mathbf{r}_1) \delta^3(\hat{\mathbf{r}}_j - \mathbf{r}_2) | \Psi \rangle$$

$$n_{SM_S, TM_T}^{(2)}(\mathbf{k}_1, \mathbf{k}_2) = \langle \Psi | \sum_{i < j}^A \hat{P}_{ij}^{SM_S} \hat{P}_{ij}^{TM_T} \delta^3(\hat{\mathbf{k}}_i - \mathbf{k}_1) \delta^3(\hat{\mathbf{k}}_j - \mathbf{k}_2) | \Psi \rangle$$

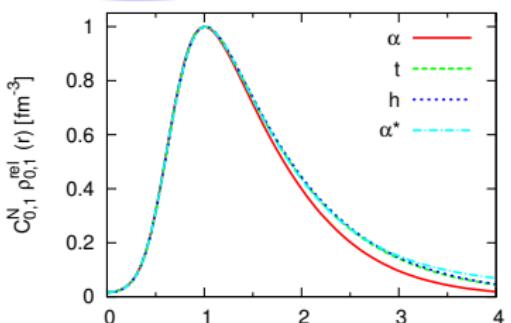
integrated over center-of-mass position $\mathbf{R} = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2)$ or the total momentum of the nucleon pair $\mathbf{K} = \mathbf{k}_1 + \mathbf{k}_2$:

$$\rho_{SM_S, TM_T}^{\text{rel}}(\mathbf{r}) = \langle \Psi | \sum_{i < j}^A \hat{P}_{ij}^{SM_S} \hat{P}_{ij}^{TM_T} \delta^3(\hat{\mathbf{r}}_i - \hat{\mathbf{r}}_j - \mathbf{r}) | \Psi \rangle$$

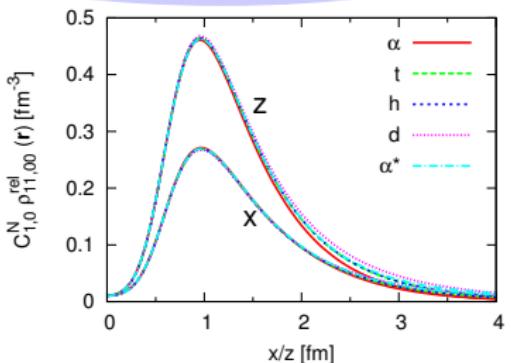
$$n_{SM_S, TM_T}^{\text{rel}}(\mathbf{k}) = \langle \Psi | \sum_{i < j}^A \hat{P}_{ij}^{SM_S} \hat{P}_{ij}^{TM_T} \delta^3\left(\frac{1}{2}(\hat{\mathbf{k}}_i - \hat{\mathbf{k}}_j) - \mathbf{k}\right) | \Psi \rangle$$

Two-body densities in coordinate space for $A=2,3,4$

$S = 0, T = 1$



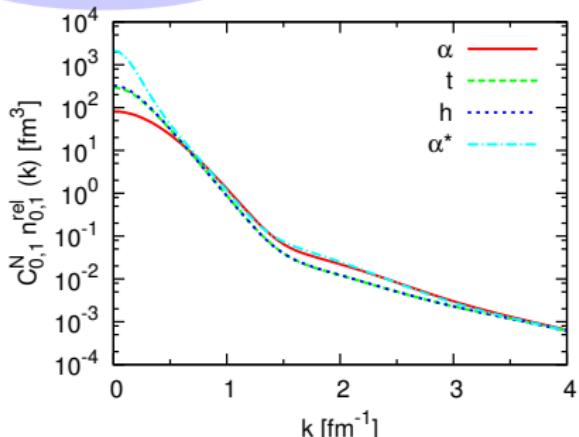
$S = 1, M_S = 1, T = 0$



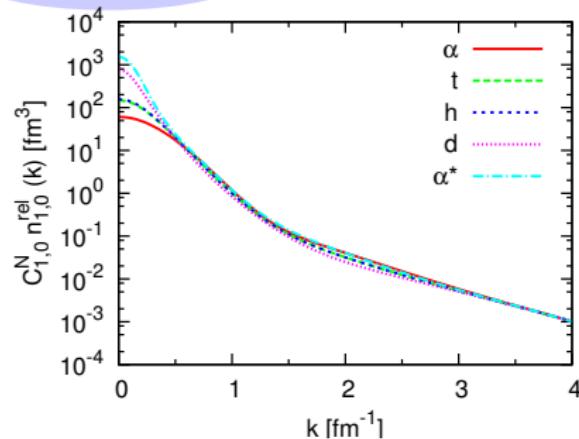
- two-body densities calculated from **exact wave functions** (Correlated Gaussian Method) for AV8' interaction
- coordinate space two-body densities reflect correlation hole and tensor correlations
- → normalize two-body density in coordinate space at $r=1.0$ fm
- → normalized two-body densities in coordinate space are **identical at short distances** for all nuclei
- also true for angular dependence in the deuteron channel

Two-body densities in momentum space for $A=2,3,4$

$S = 0, T = 1$



$S = 1, T = 0$



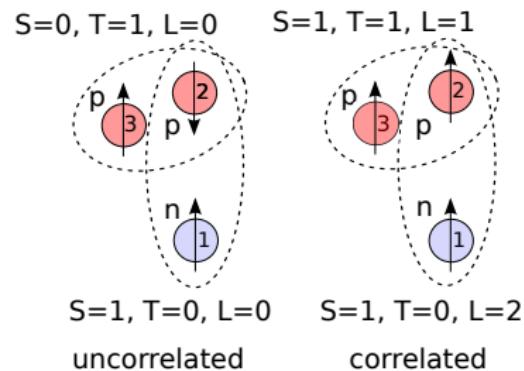
- use **normalization factors fixed in coordinate space**
- two-body densities in momentum space agree for momenta $k > 3\text{fm}^{-1}$
- moderate nucleus dependence in momentum region $1.5\text{fm}^{-1} < k < 3\text{fm}^{-1}$

Feldmeier, Horiuchi, Neff, Suzuki, PRC **84**, 054003 (2011)

Two-body densities reflect many-body correlations

number of pairs in the (ST) channels

nucleus	(00)	(01)	(10)	(11)
d	–	–	1	–
t	0.010	1.361	1.490	0.139
h	0.011	1.361	1.489	0.139
α	0.008	2.572	2.992	0.428
α^*	0.034	2.714	2.966	0.286



- occupation in $(ST)=(10)$ almost exactly as in IPM
- $(ST)=(01)$ significantly depopulated in favor of $(ST)=(11)$
- three-body correlations induced by the two-body tensor force: depopulation of $(ST)=(01)$ channel is the price one has to pay for getting the full binding from the tensor force in the $(ST)=(10)$ channel

Unitary Transformations

- Many-body problem very hard to solve with bare interaction
- Universality of SRC suggests to use unitary transformations to obtain a “soft” realistic interaction

$$\hat{H}_{\text{eff}} = \hat{U}^\dagger \hat{H} \hat{U}$$

- The transformation is done in N -body approximation

$$\hat{H}_{\text{eff}} = \hat{T} + \hat{V}_{\text{eff}}^{[2]} + \dots + \hat{V}_{\text{eff}}^{[N]}$$

and is therefore unitary only up to the N -body level

- Deuteron binding energy and NN phase shifts are conserved
- Not only the Hamiltonian, **all operators have to be transformed**

$$\hat{B}_{\text{eff}} = \hat{U}^\dagger \hat{B} \hat{U}$$

- SRG operator evolution studied for Deuteron

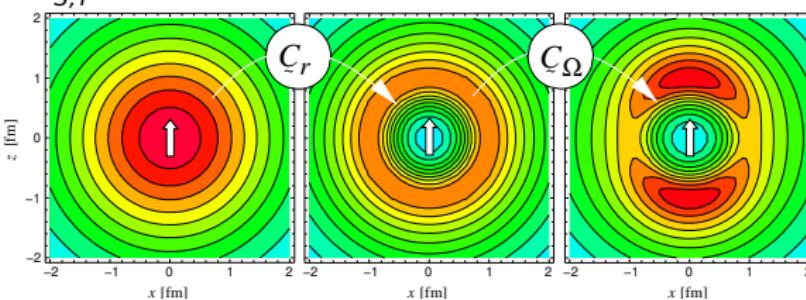
[Anderson, Bogner, Furnstahl, Perry, PRC 82, 054001 \(2010\)](#)

- SRG operator evolution for radius and Gaussian two-body operator on 3-body level

[Schuster, Quaglioni, Johnson, Jurgenson, Navrátil, arXiv:1402.7106](#)

Unitary Correlation Operator Method

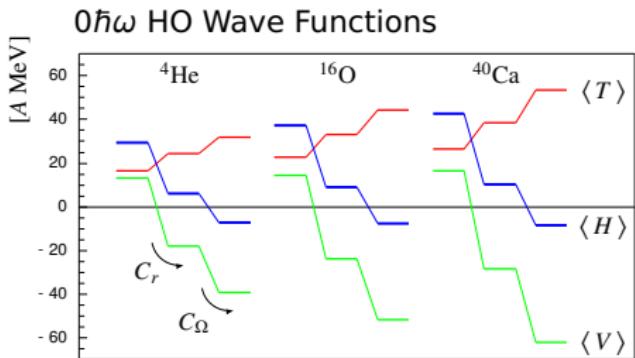
$$n_{S,T}^{\text{rel}}(\mathbf{r}) \quad S = 1, M_S = 1, T = 0$$



central correlator $\hat{\mathbf{C}}_r$ shifts density out of the repulsive core

tensor correlator $\hat{\mathbf{C}}_\Omega$ aligns density with spin orientation

both central and tensor correlations are essential for binding



Neff, Feldmeier, Nucl. Phys. **A713**, 311 (2004)

Roth, Neff, Feldmeier, Prog. Part. Nucl. Phys. **65**, 51 (2010)

Similarity Renormalization Group

- Evolve Hamiltonian and unitary transformation matrix (momentum space or HO basis)

$$\frac{d\hat{H}_\alpha}{d\alpha} = [\hat{\eta}_\alpha, \hat{H}_\alpha], \quad \frac{d\hat{U}_\alpha}{d\alpha} = -\hat{U}_\alpha \hat{\eta}_\alpha$$

- Intrinsic kinetic energy as Metagenerator

$$\hat{\eta}_\alpha = (2\mu)^2 [\hat{T}_{\text{int}}, \hat{H}_\alpha]$$

- Evolution is done on the 2-body level – α -dependence can be used to investigate missing higher-order contributions
- We will use \hat{U}_α to calculate effective operators
- Hamiltonian evolution can now be done on the 3-body level

(Jurgenson, Roth, Hebeler, ...)

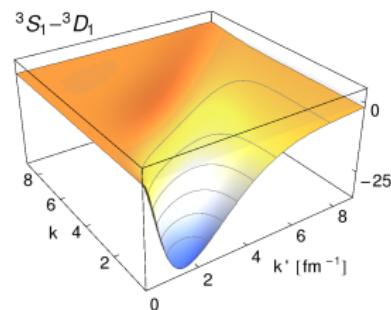
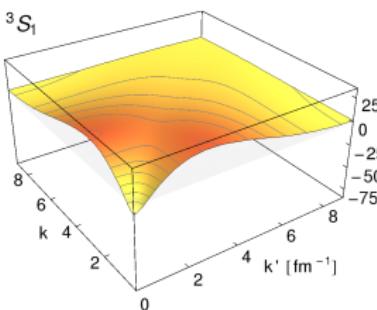
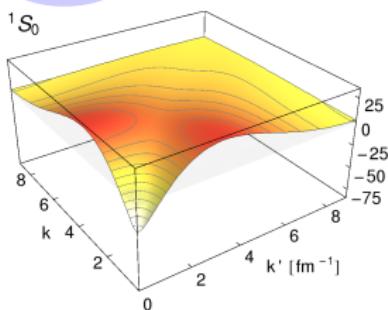
Bogner, Furnstahl, Perry, Phys. Rev. C, **75**, 061001 (2007)

Roth, Neff, Feldmeier, Prog. Part. Nucl. Phys. **65**, 50 (2010)

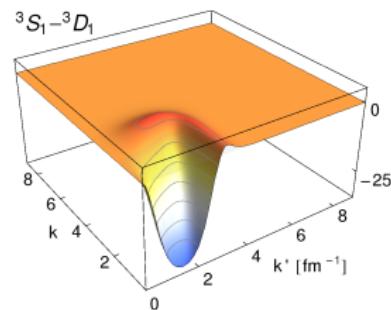
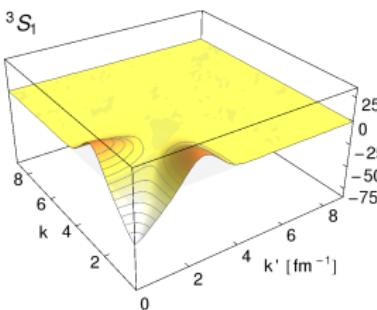
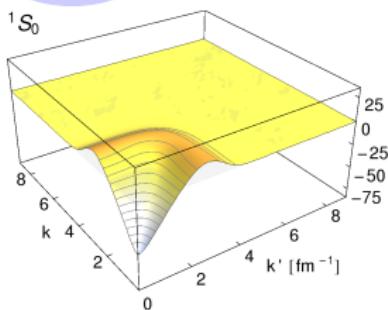
Similarity Renormalization Group

Evolution of the potential

AV8'



N³LO

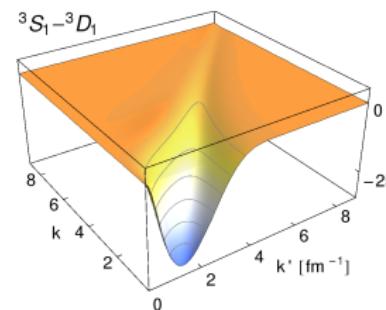
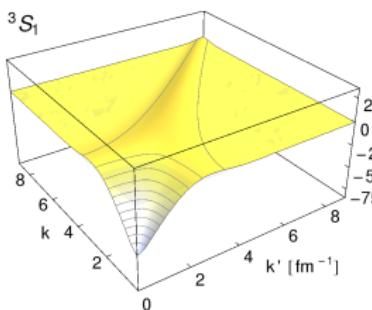
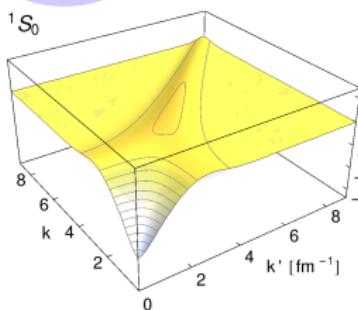


$$V(k, k') - \alpha = 0.00 \text{ (bare)}$$

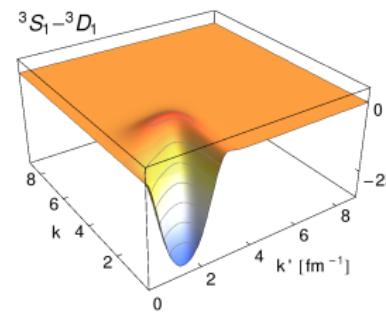
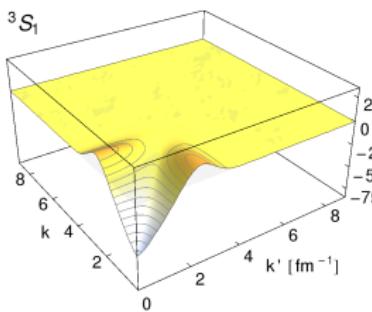
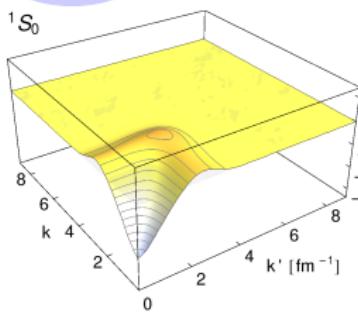
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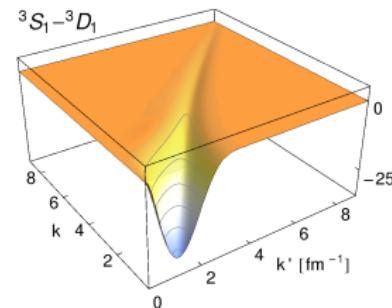
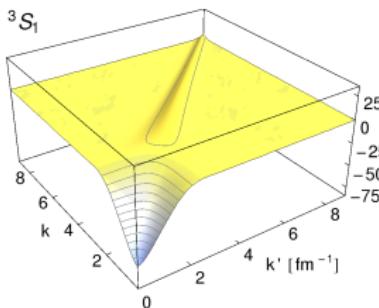
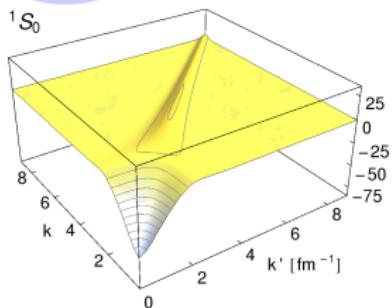


$$V(k, k') - \alpha = 0.01 \text{ fm}^4$$

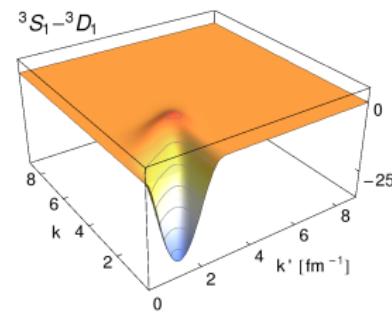
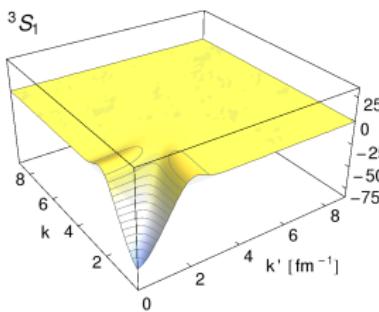
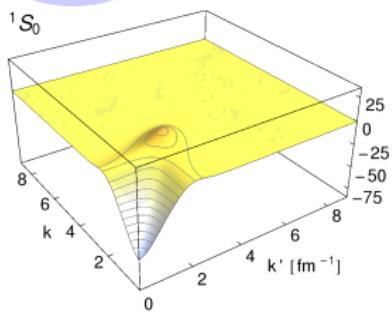
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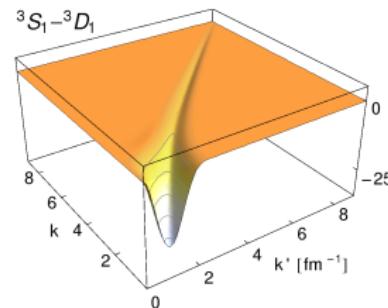
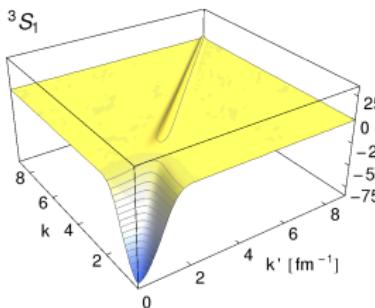
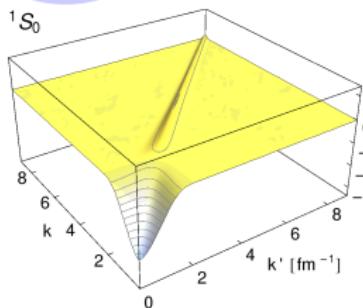


$$V(k, k') - \alpha = 0.04 \text{ fm}^4$$

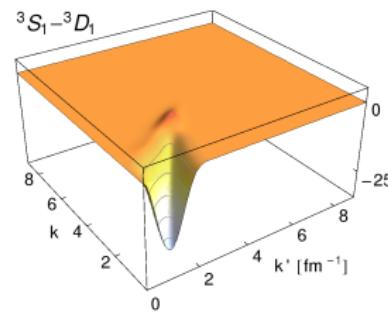
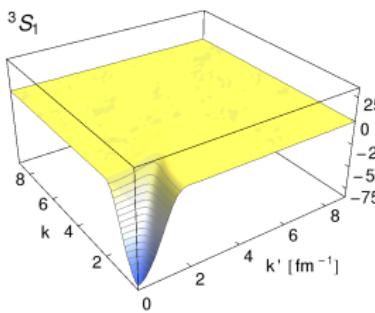
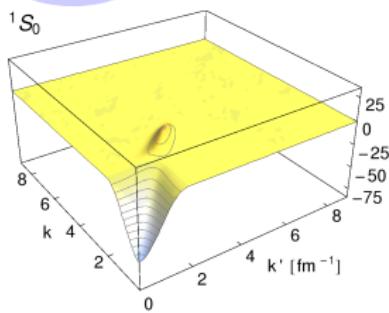
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N³LO

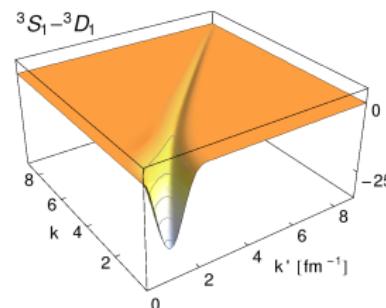
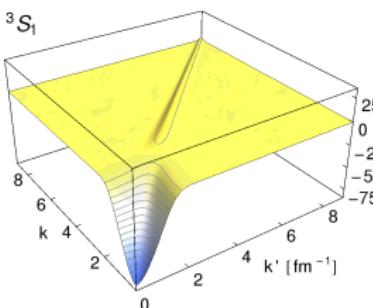
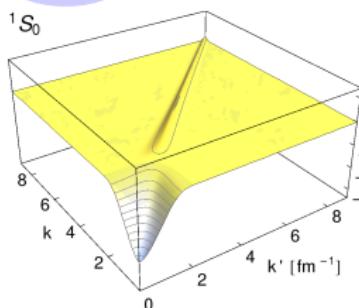


$$V(k, k') - \alpha = 0.20 \text{ fm}^4$$

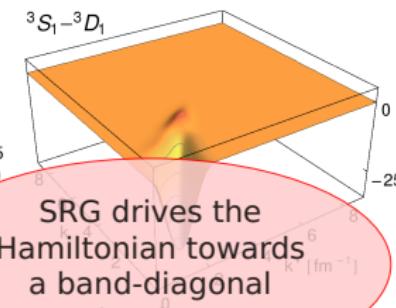
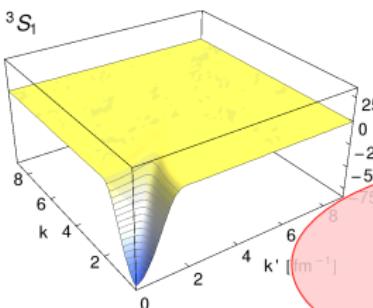
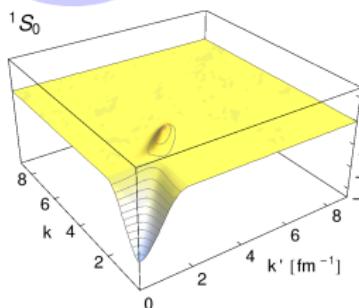
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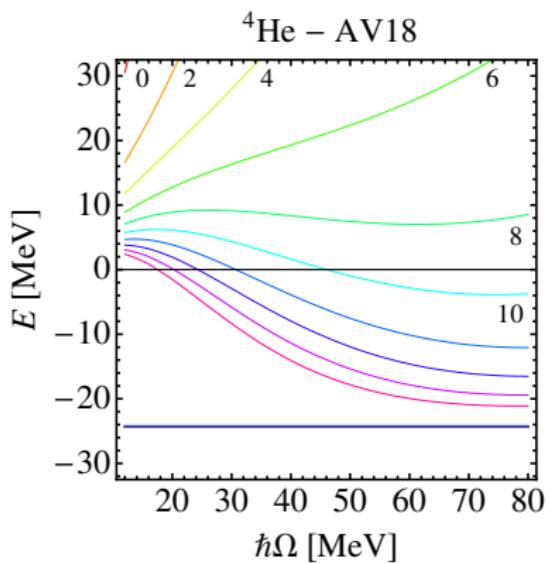
$$V(k, k') - \alpha = 0.20 \text{ fm}^4$$

SRG drives the Hamiltonian towards a band-diagonal structure

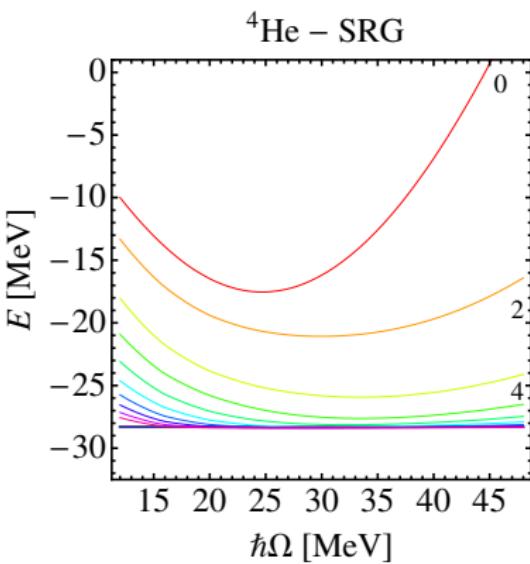
No-Core Shell Model

Convergence

bare interaction



SRG evolved interaction
($\alpha = 0.03\text{fm}^4$)

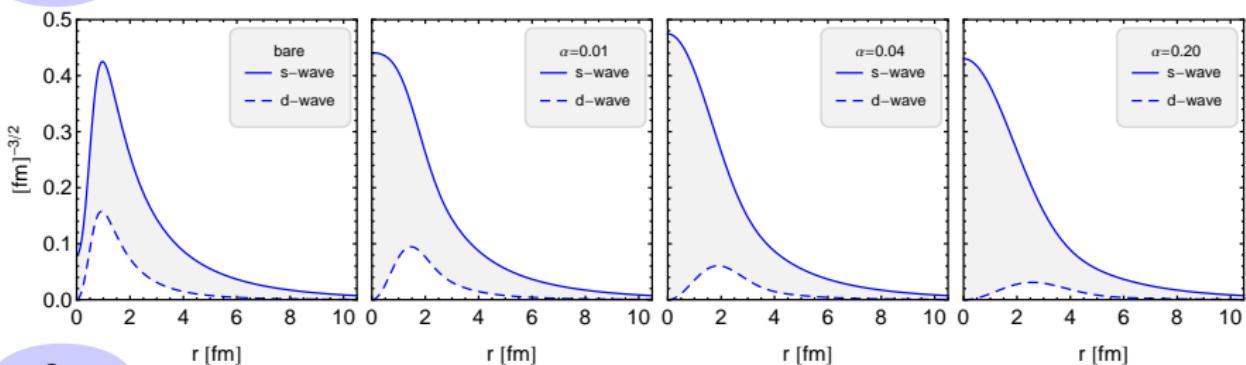


Roth, Neff, Feldmeier, Prog. Part. Nucl. Phys. **65**, 50 (2010)

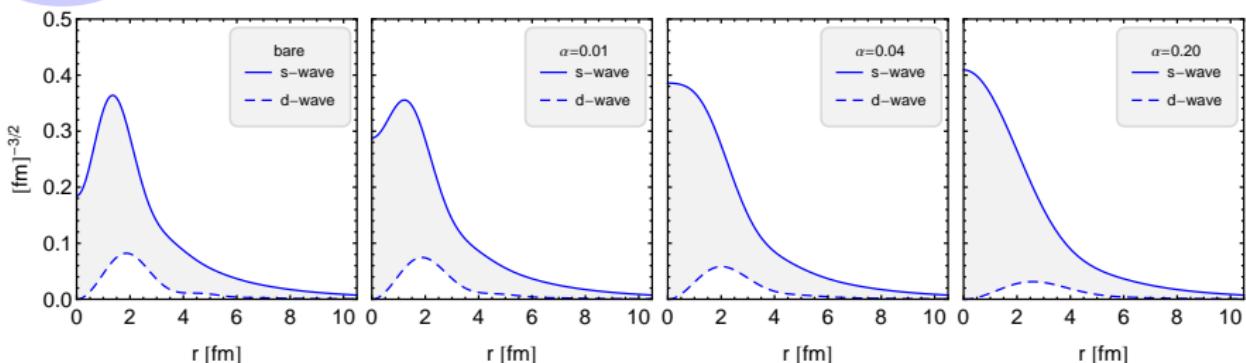
Similarity Renormalization Group

Deuteron - Coordinate Space Wave Functions

AV8'



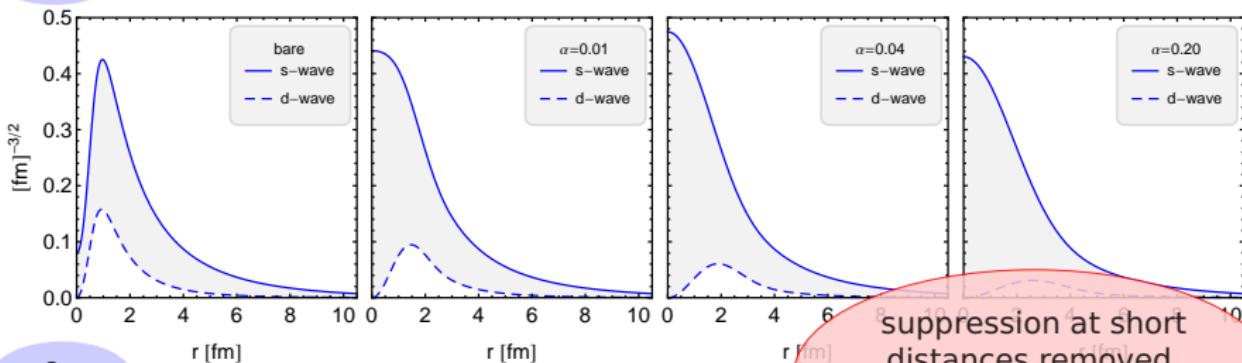
$N^3\text{LO}$



Similarity Renormalization Group

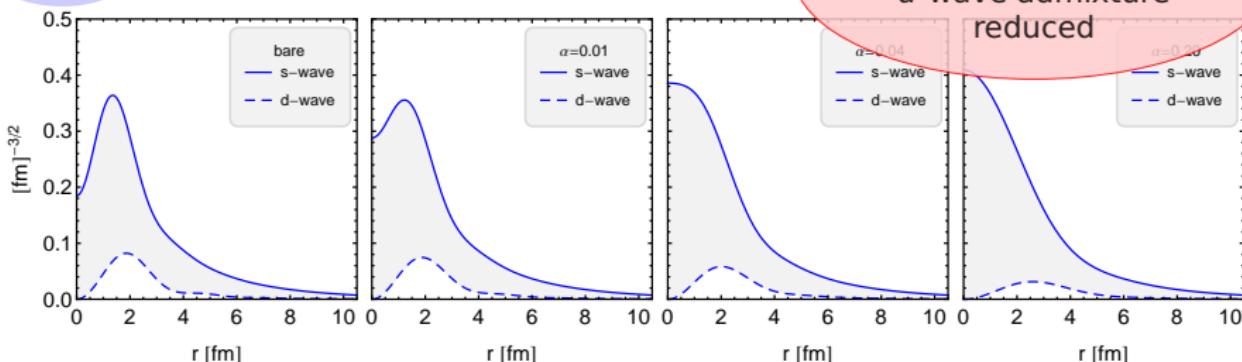
Deuteron - Coordinate Space Wave Functions

AV8'



suppression at short
distances removed,
d-wave admixture
reduced

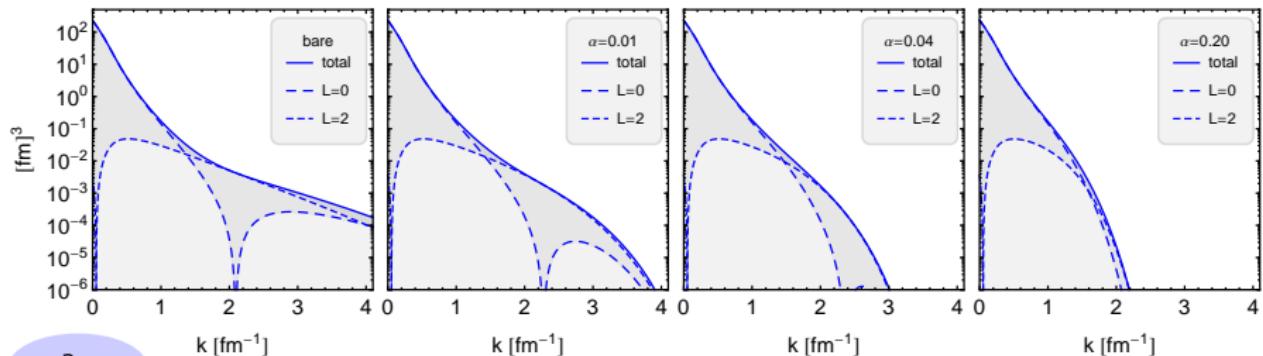
N³LO



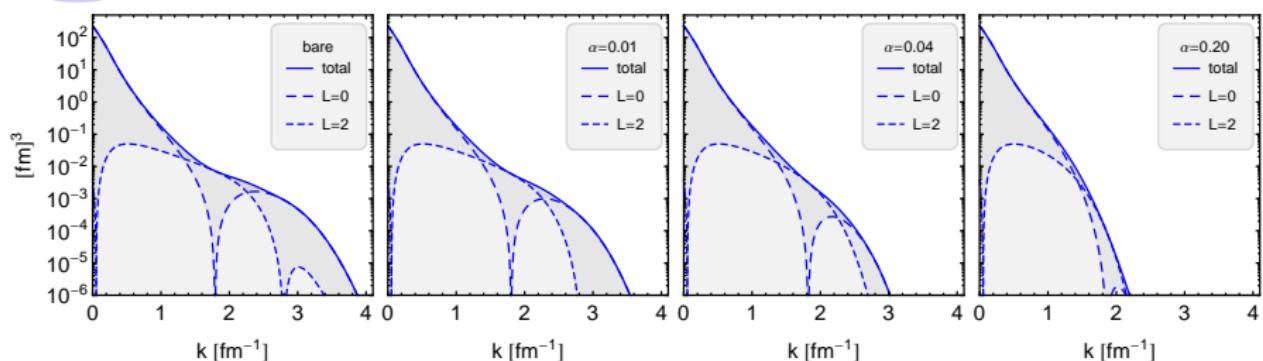
Similarity Renormalization Group

Deuteron – Momentum Space Densities

AV8'



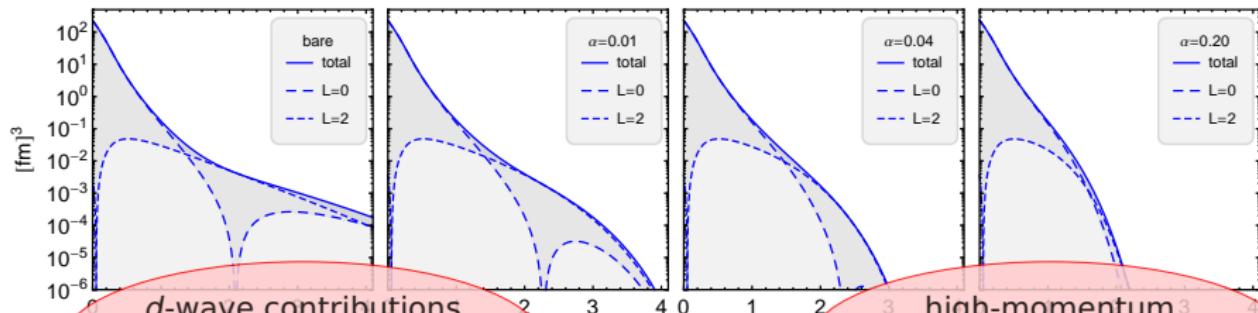
N³LO



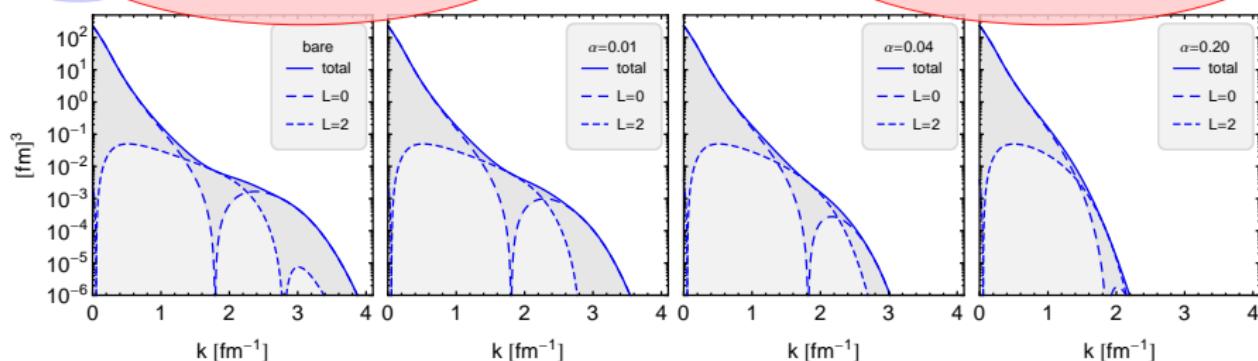
Similarity Renormalization Group

Deuteron – Momentum Space Densities

AV8'



NNLO



Two-Body Densities calculated with NCSM and SRG

Outline of Calculation

- simultaneous SRG evolution for transformed Hamiltonian and transformation matrix **on the two-body level**

$$\frac{d\hat{H}_\alpha}{d\alpha} = [\hat{\eta}_\alpha, \hat{H}_\alpha], \quad \frac{d\hat{U}_\alpha}{d\alpha} = -\hat{U}_\alpha \hat{\eta}_\alpha, \quad \hat{\eta}_\alpha = (2\mu)^2 [\hat{T}_{\text{int}}, \hat{H}_\alpha]$$

- Solve many-body problem with SRG transformed Hamiltonian in the No-Core Shell Model (ANTOINE)

$$\hat{H}_\alpha |\Psi_\alpha\rangle = E_\alpha |\Psi_\alpha\rangle$$

- Calculate two-body densities with “bare” and “effective” density operators

$$\rho_{\text{bare}} = \langle \Psi_\alpha | \hat{\rho} | \Psi_\alpha \rangle, \quad \rho_{\text{effective}} = \langle \Psi_\alpha | \hat{U}_\alpha^\dagger \hat{\rho} \hat{U}_\alpha | \Psi_\alpha \rangle$$

- Investigate **convergence** of NCSM calculations and **α -dependence of two-body densities**

Two-Body Densities calculated with NCSM and SRG

Implementation Details

- SRG evolution for \hat{H}_α and \hat{U}_α in momentum space, $k_{\max} = 15\text{fm}^{-1}$
- Unitary transformation only acts on relative coordinates and not on the center-of-mass of the pairs in the two-body density operator
- (SRG transformed) momentum space matrix elements are expanded in HO basis
- jj -coupled matrix elements are calculated using the Talmi-Moshinski brackets
- slight modification of jj -coupling procedure necessary for two-body densities that depend on the pair momentum or pair position

^4He Two-body densities

Why ^4He ?

- two-body densities available for bare AV8' interaction
- bare N³LO can be converged in NCSM

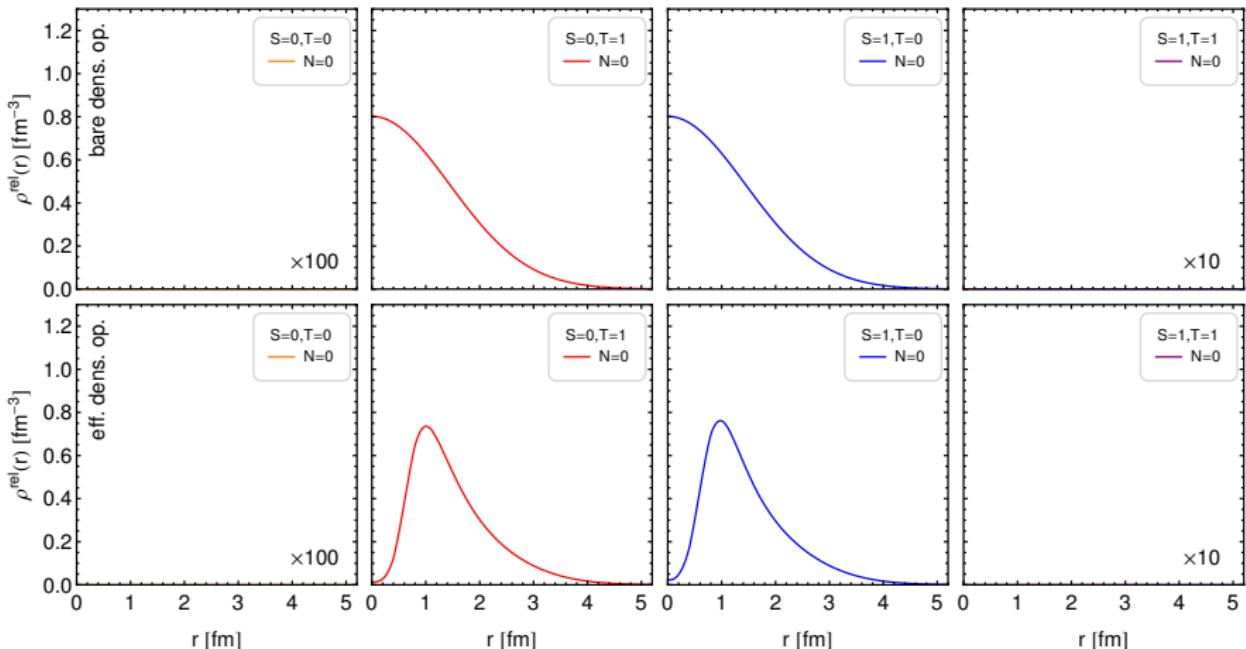
Objectives

- Compare AV8' and N³LO results
- Check for NCSM convergence
- Check flow dependence $\alpha = 0, 0.01, 0.04, 0.20 \text{fm}^4$
 $(\Lambda = \infty, 3.16, 2.24, 1.50 \text{fm}^{-1})$
- Can we see many-body effects ?

Convergence with the model space size

AV8' – $\alpha = 0.04 \text{ fm}^4$

Two-body Density in Coordinate Space

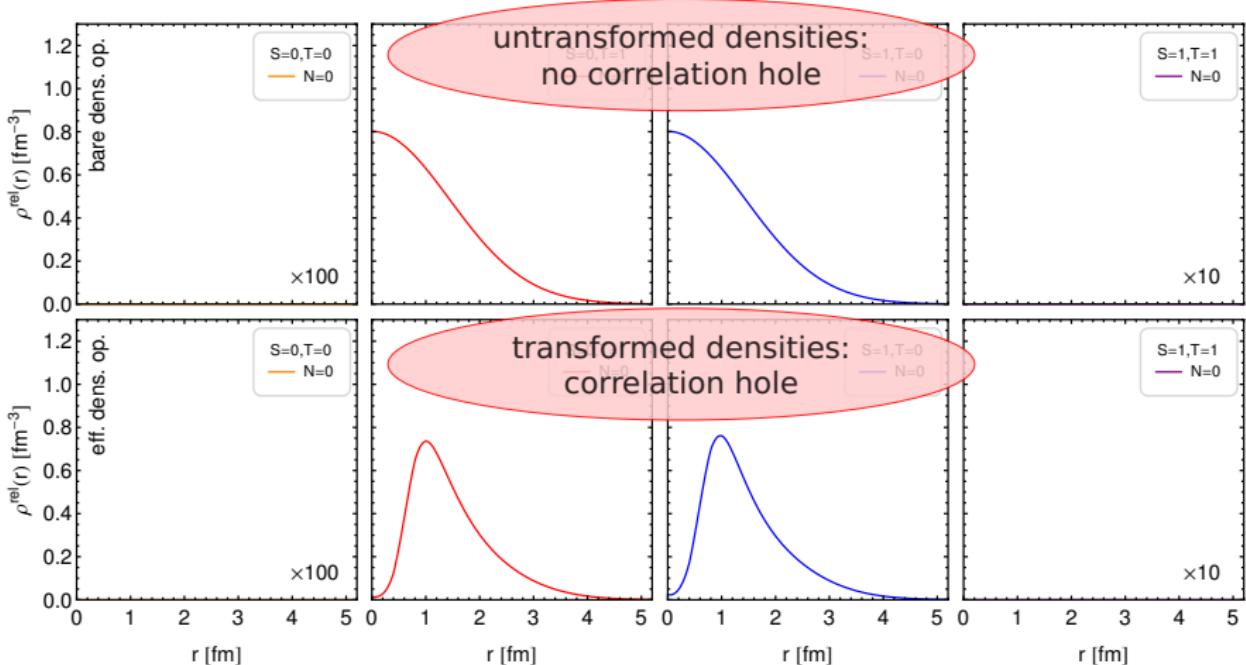


$\hbar\Omega = 20 \text{ MeV}$ corresponds roughly to the size of ⁴He

Convergence with the model space size

AV8' – $\alpha = 0.04 \text{ fm}^4$

Two-body Density in Coordinate Space

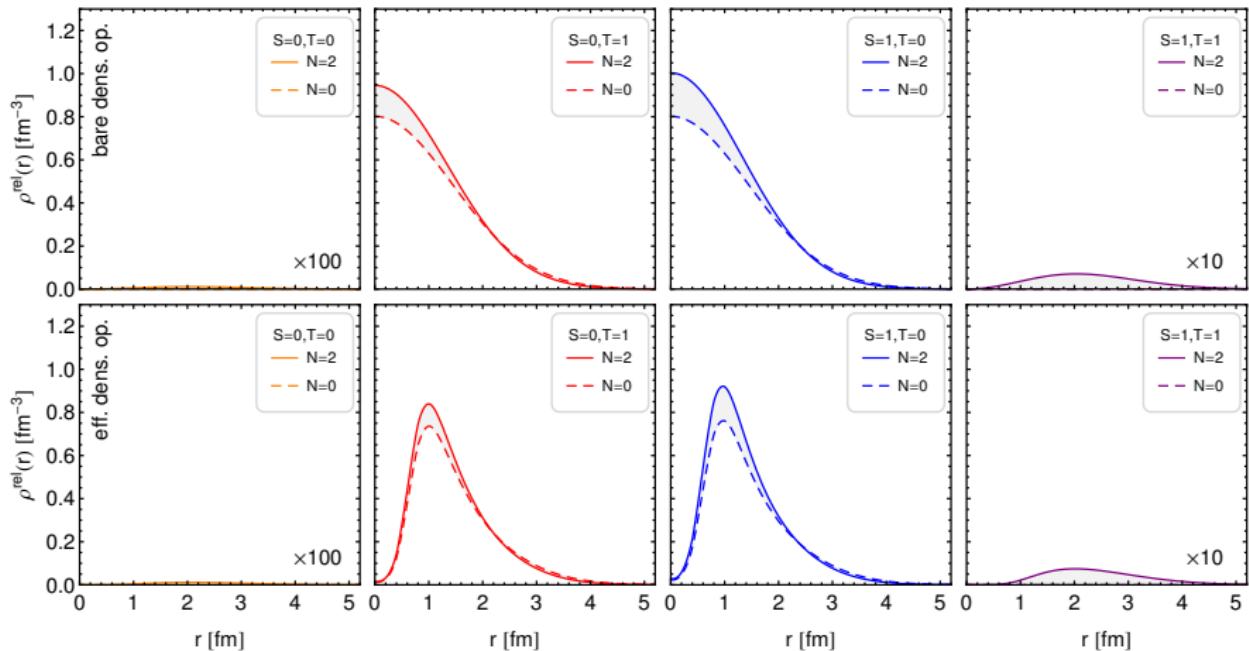


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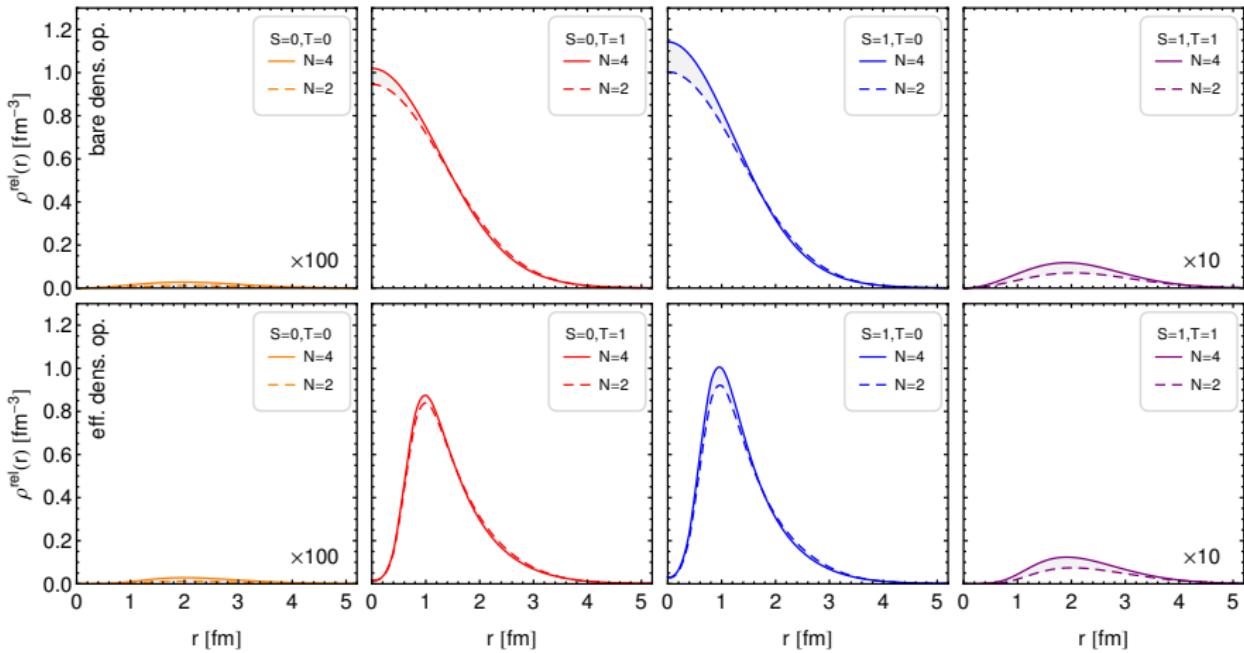


$\hbar\Omega = 20 \text{ MeV}$ corresponds roughly to the size of ${}^4\text{He}$

Convergence with the model space size

AV8' – $\alpha = 0.04 \text{ fm}^4$

Two-body Density in Coordinate Space

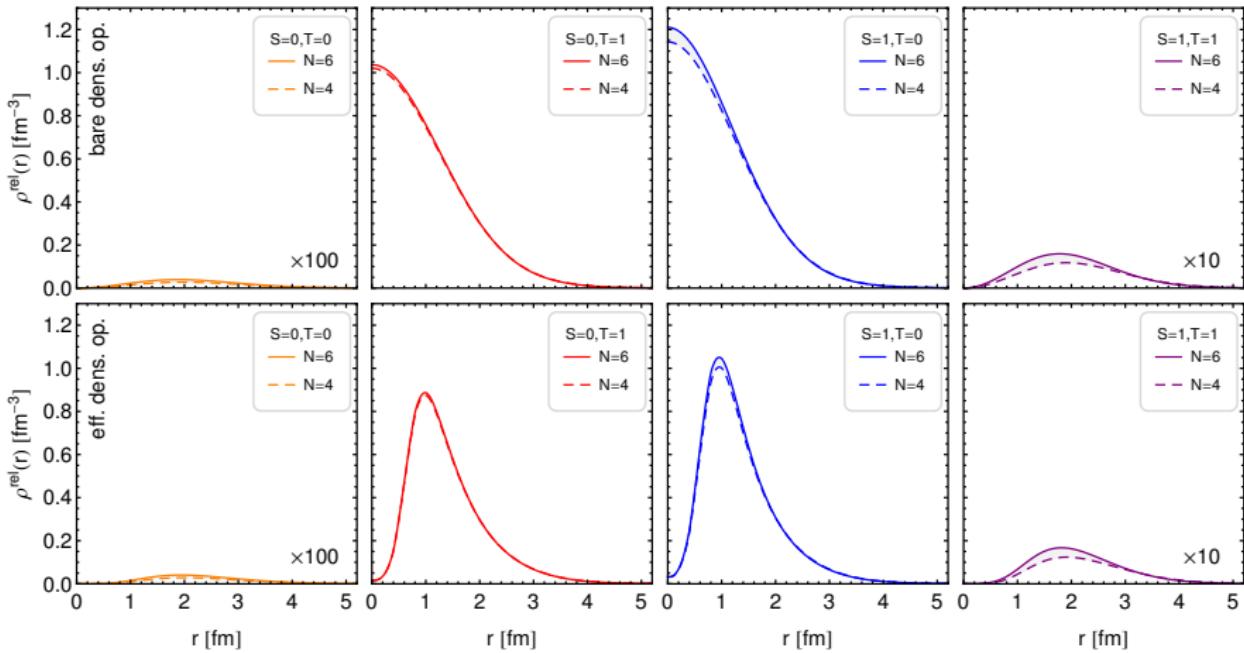


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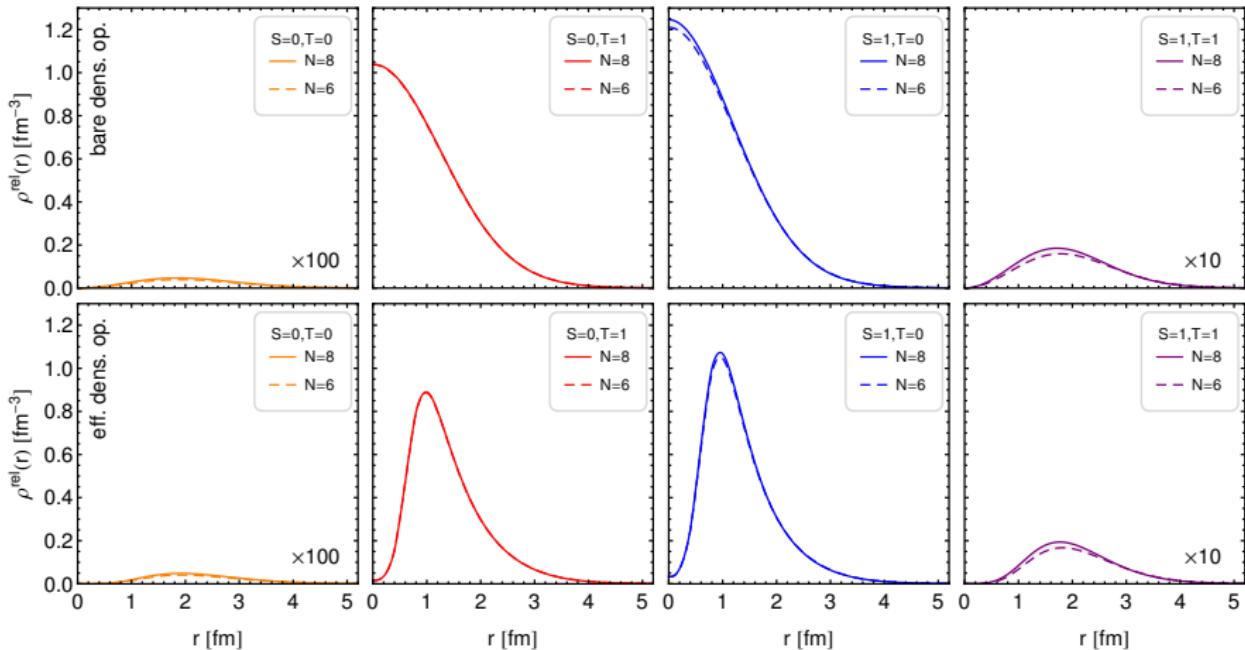


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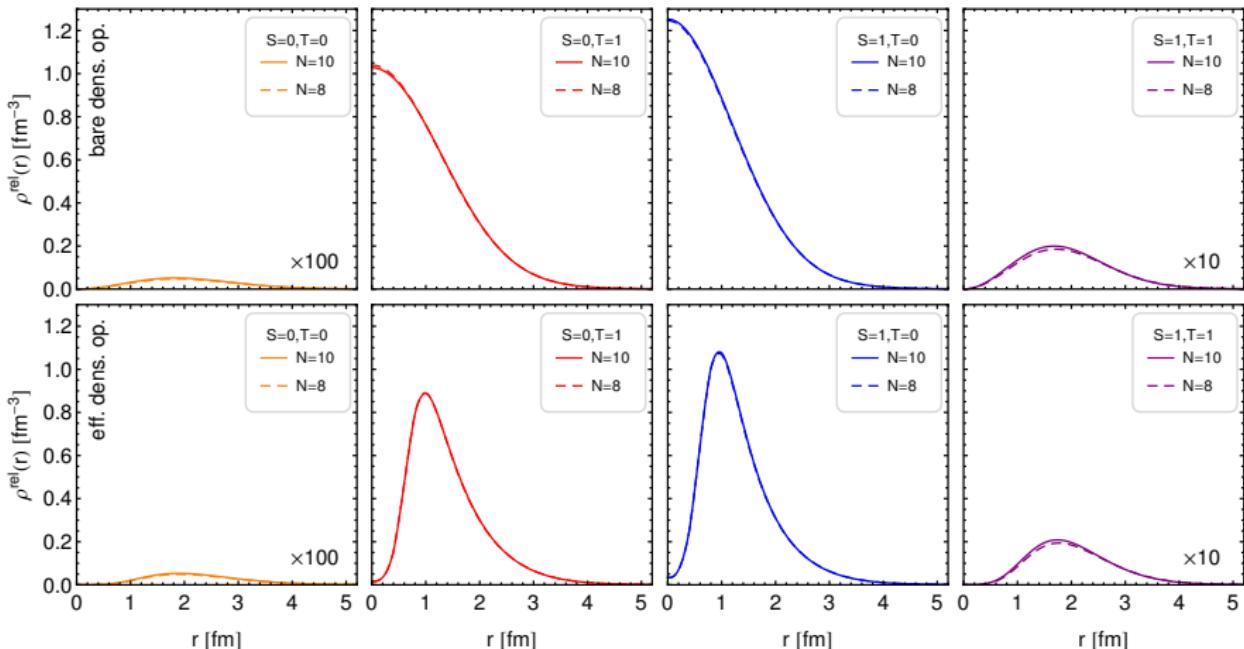


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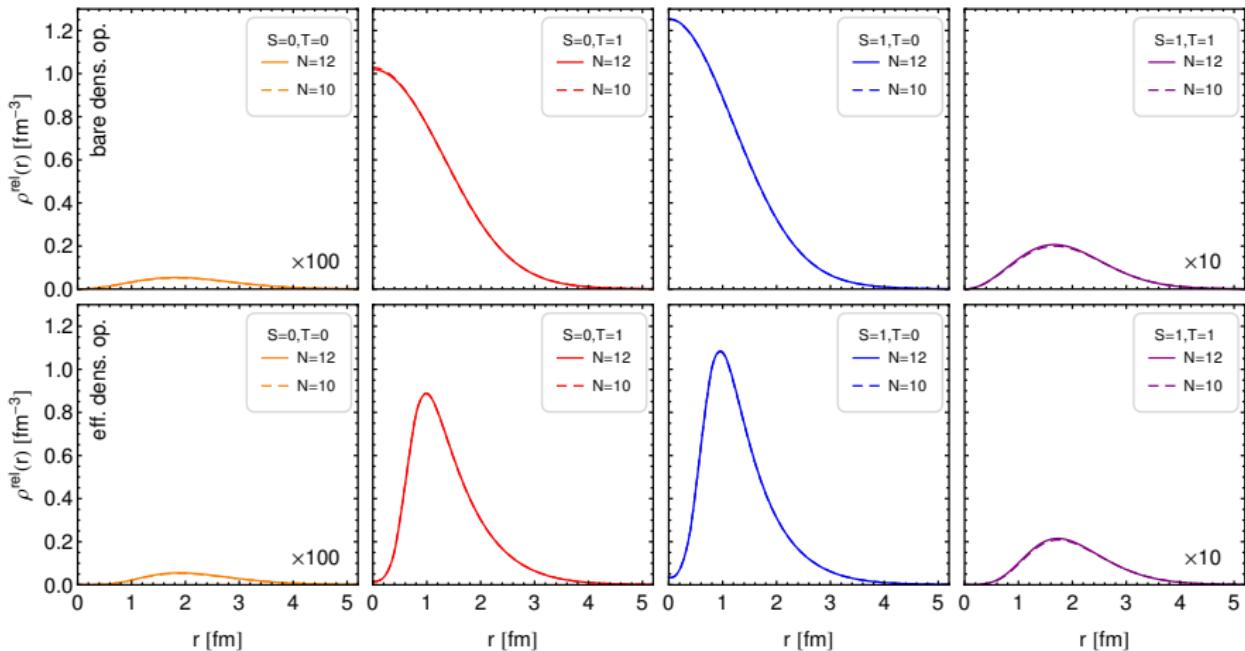


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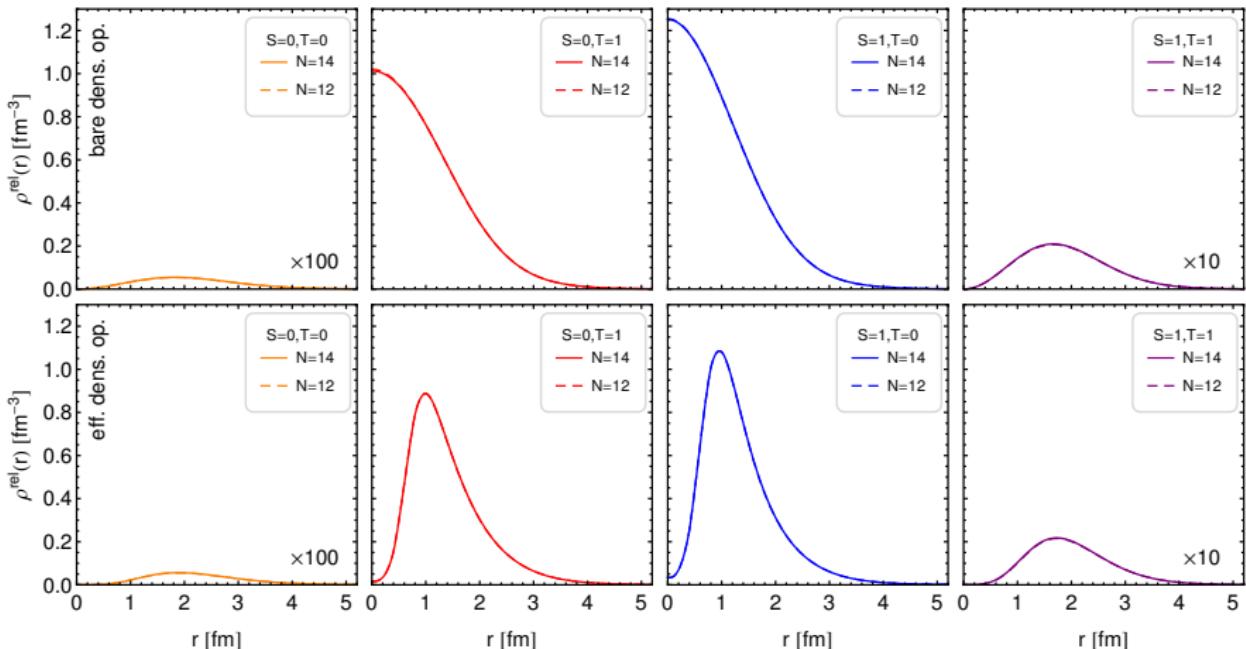


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Convergence with the model space size

AV8' – $\alpha = 0.04 \text{ fm}^4$

Two-body Density in Coordinate Space

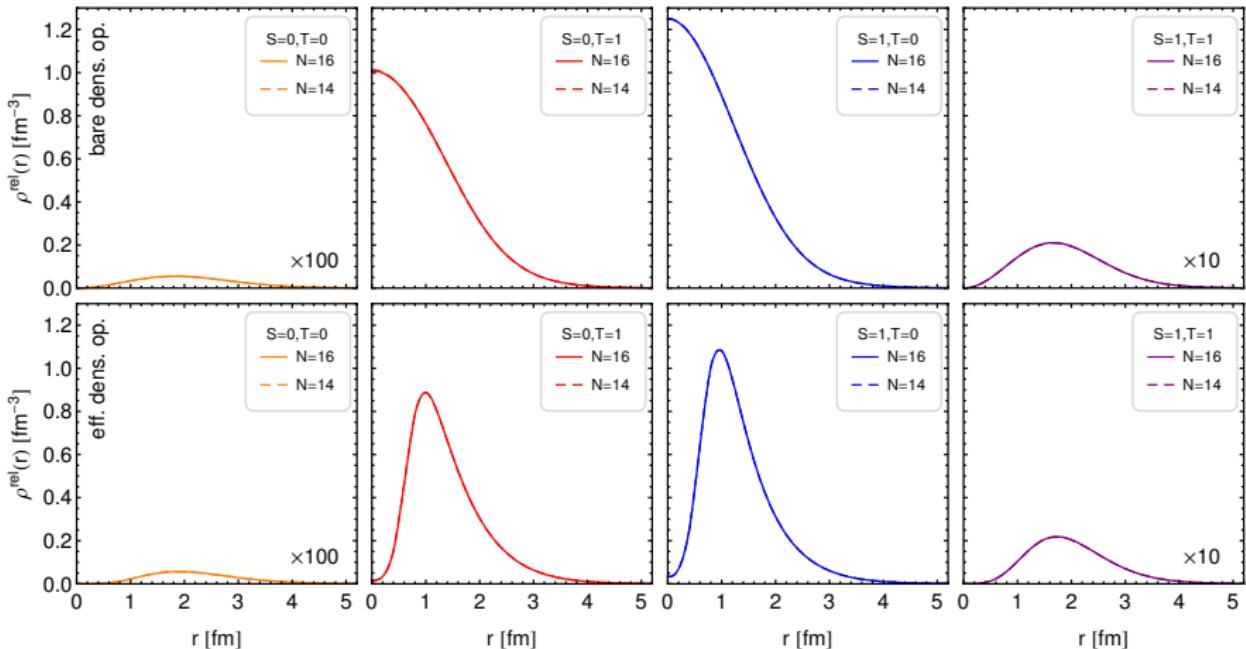


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Two-body Density in Coordinate Space

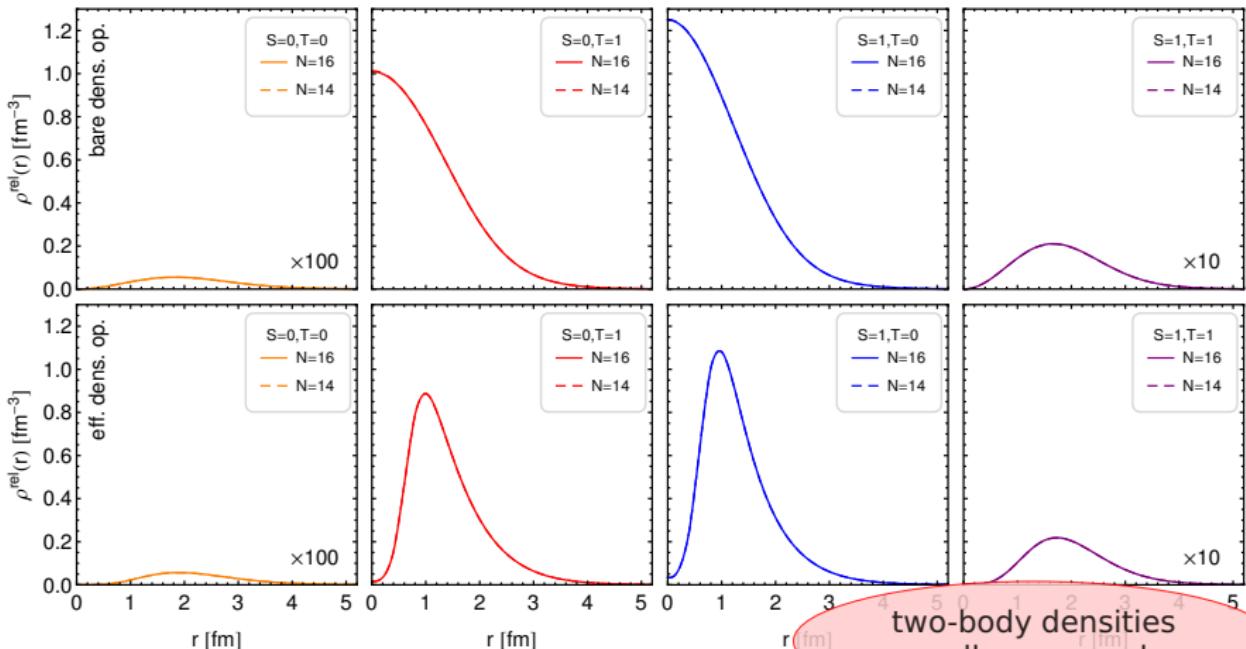


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Convergence with the model space size

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Two-body Density in Coordinate Space



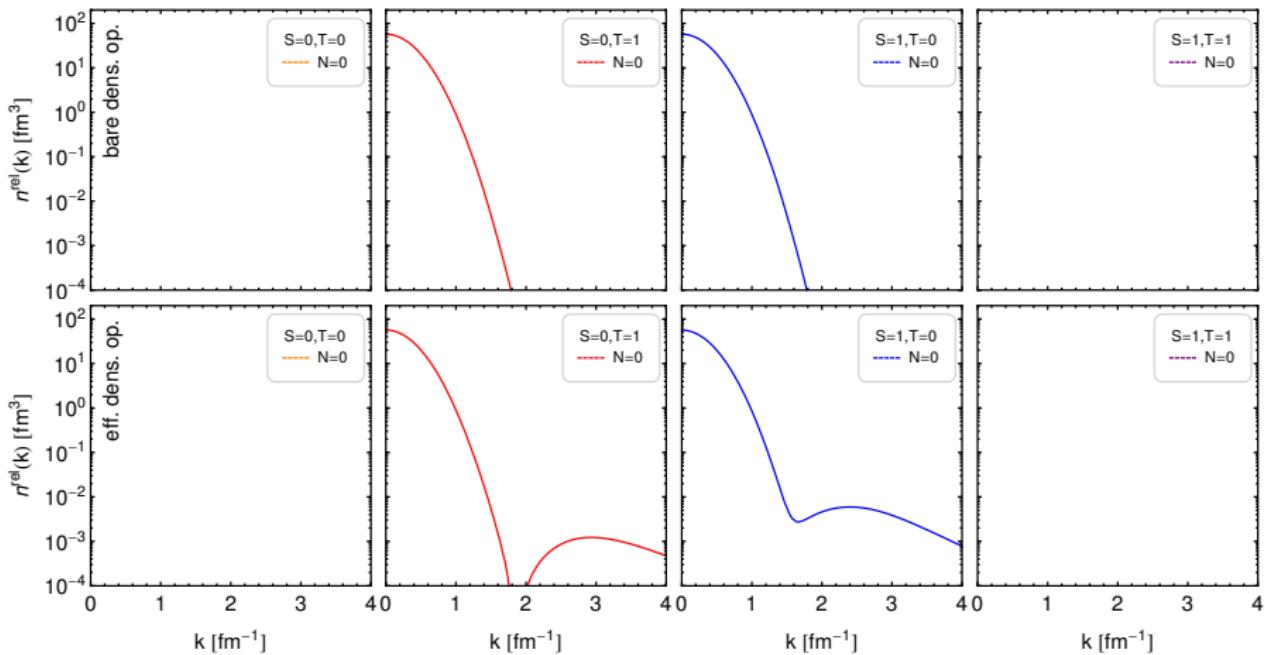
$\hbar\Omega = 20 \text{ MeV}$ corresponds roughly to the size of ${}^4\text{He}$

two-body densities
well converged

Convergence with the model space size

AV8' – $\alpha = 0.04 \text{ fm}^4$

Two-body Density in Momentum Space

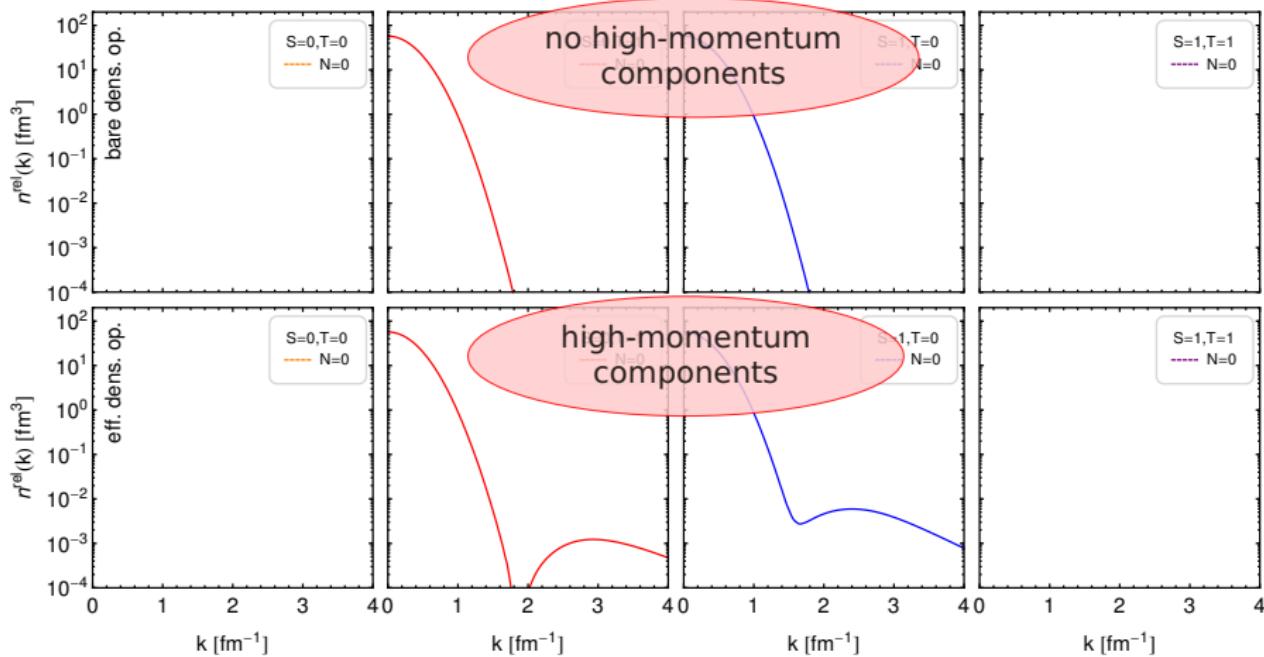


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Convergence with the model space size

AV8' – $\alpha = 0.04 \text{ fm}^4$

Two-body Density in Momentum Space

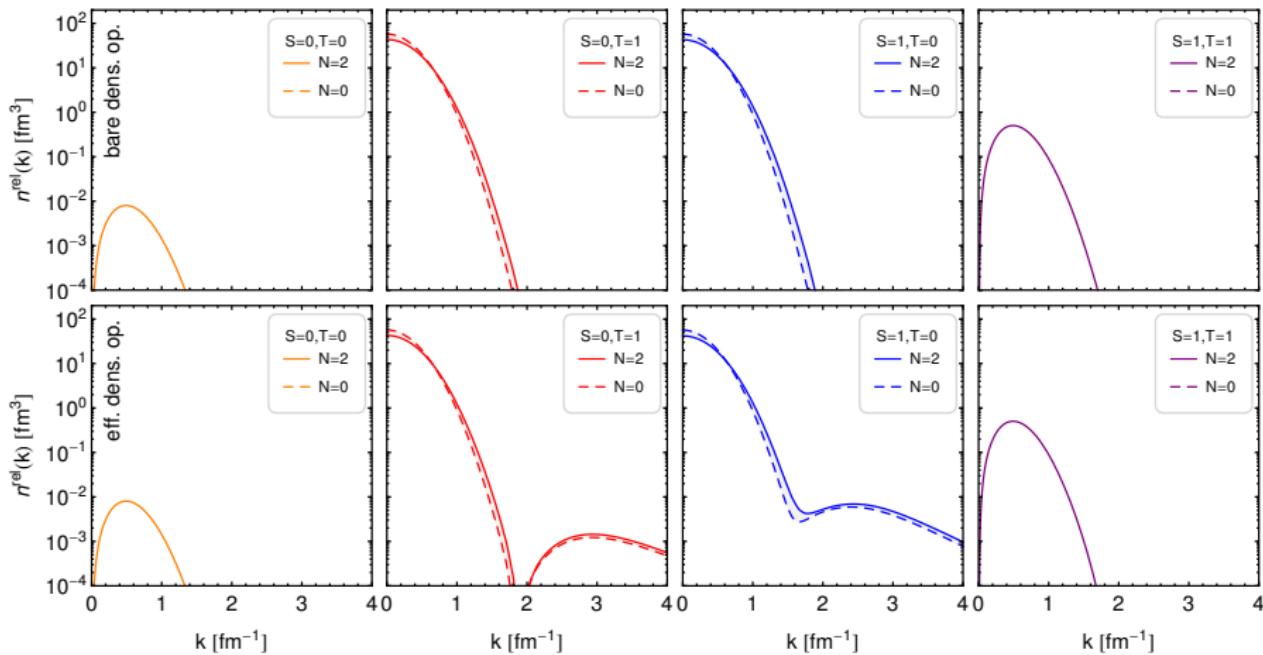


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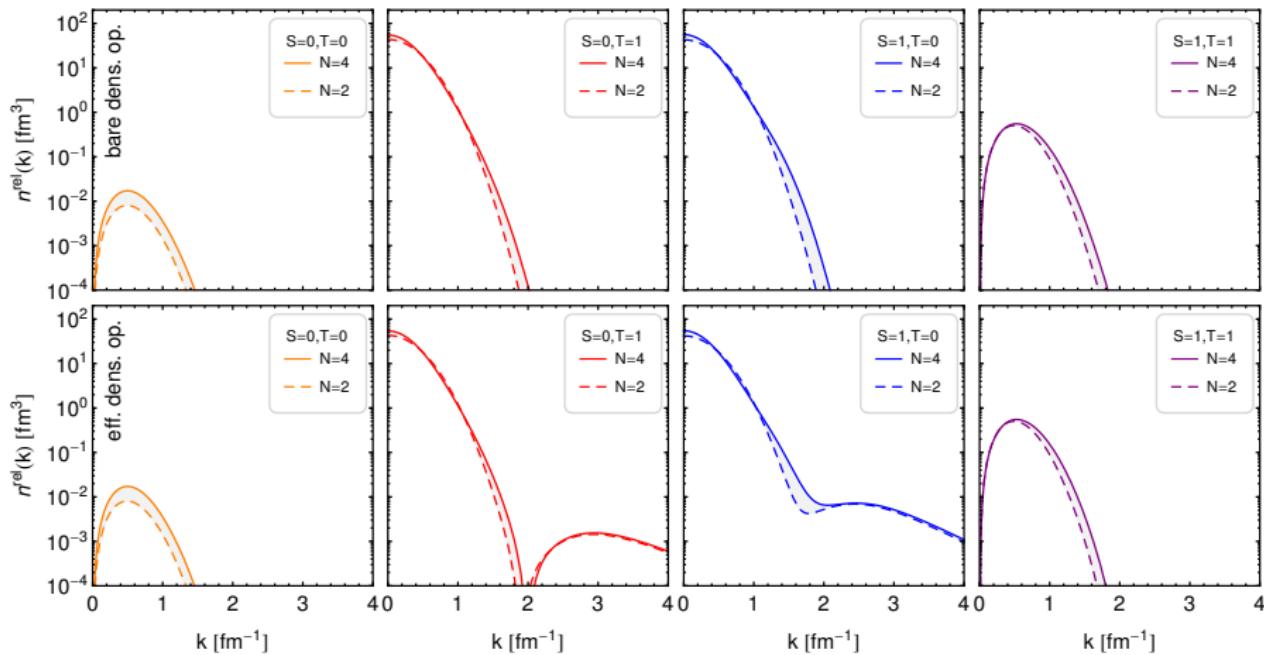


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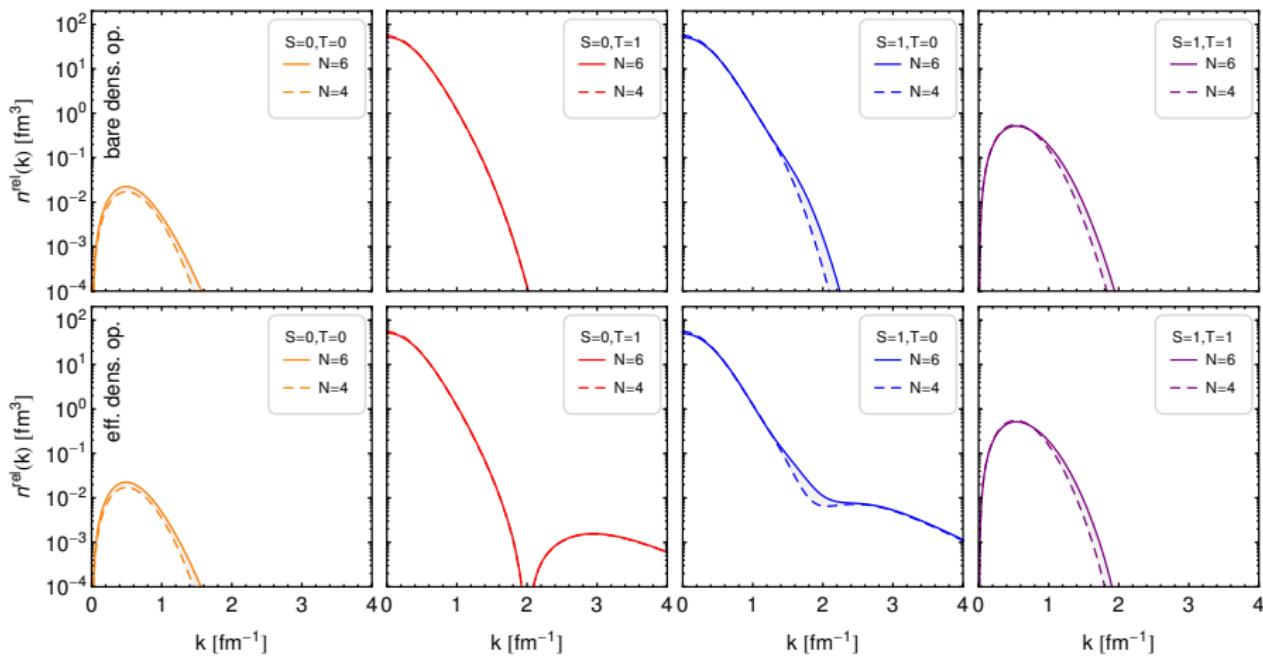


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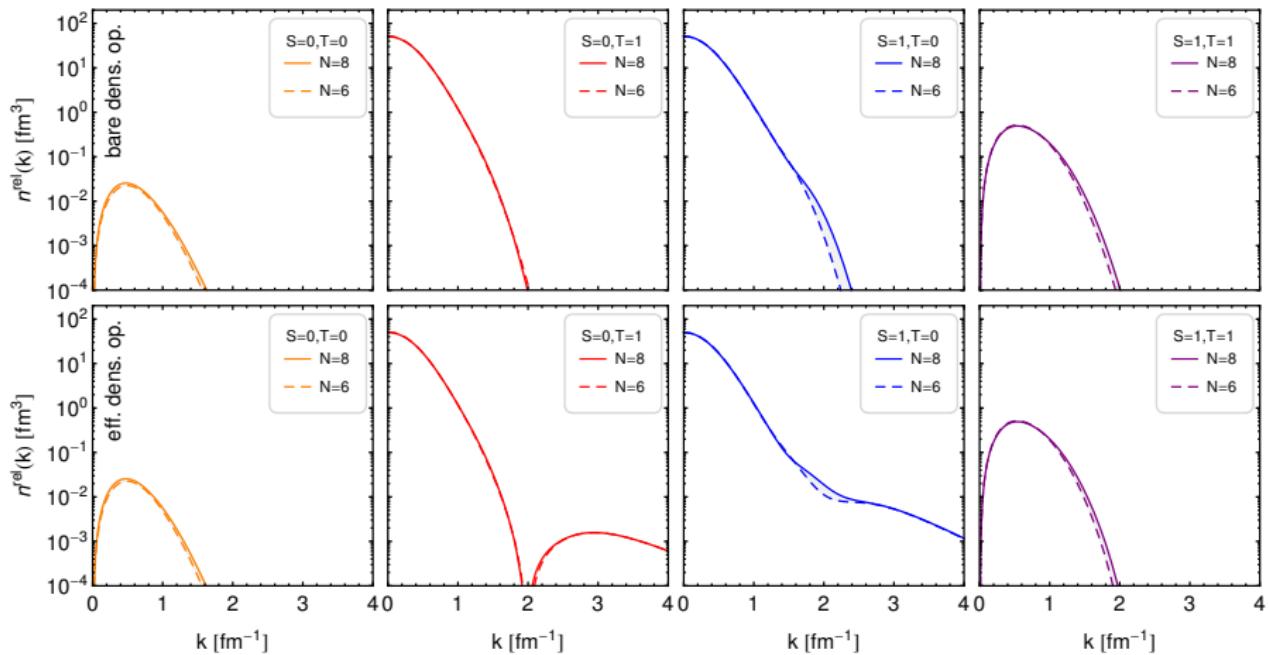


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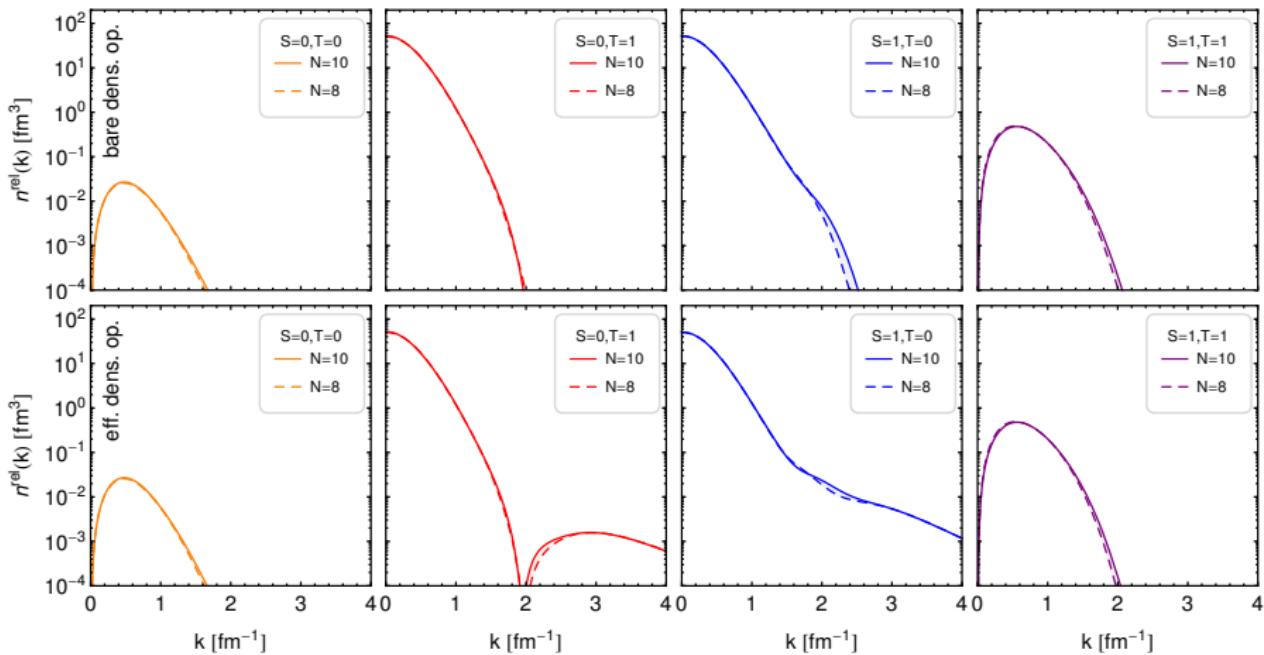


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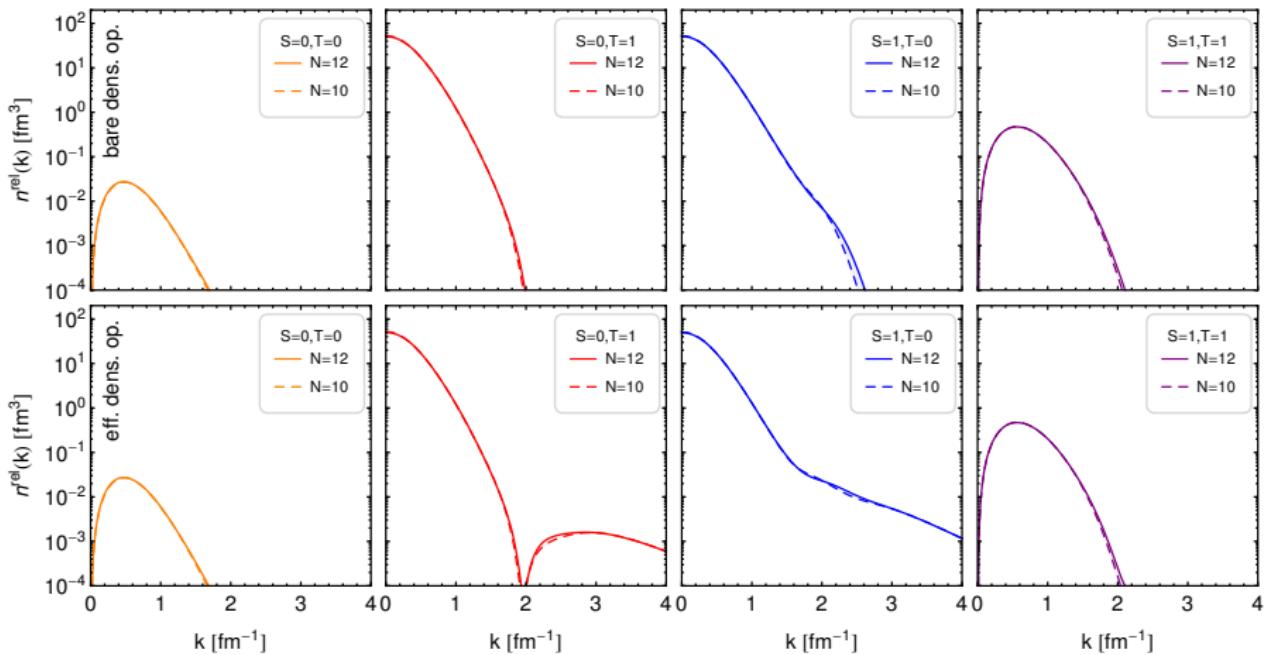


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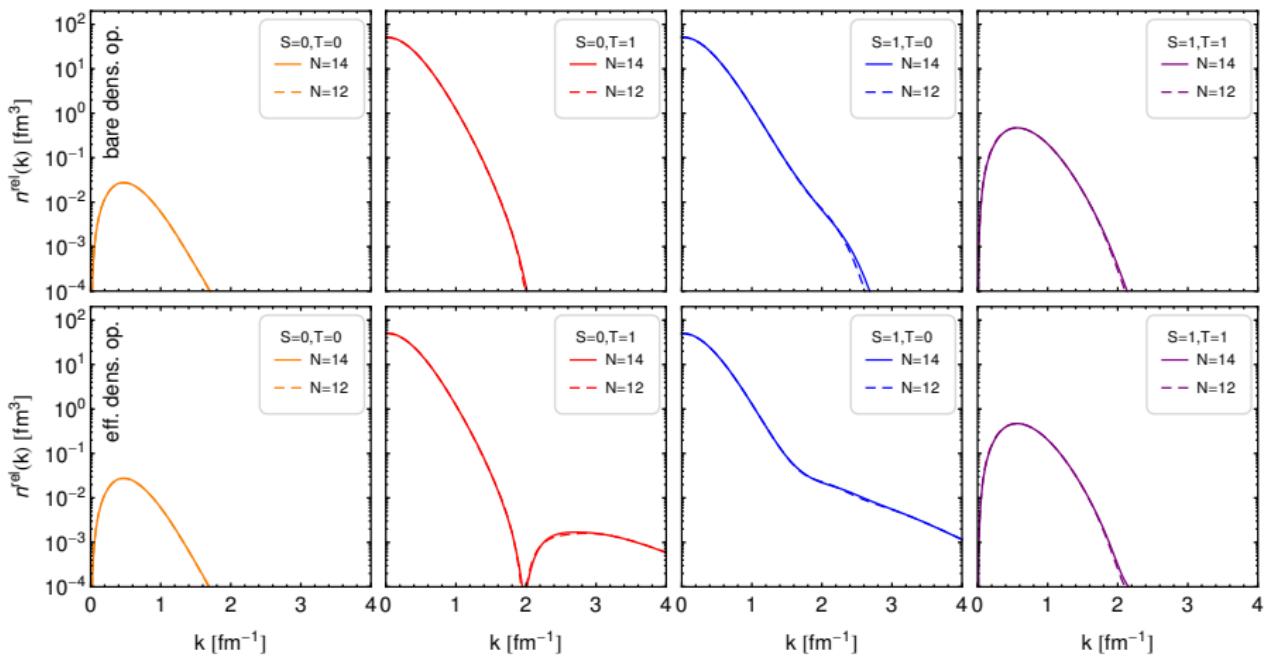


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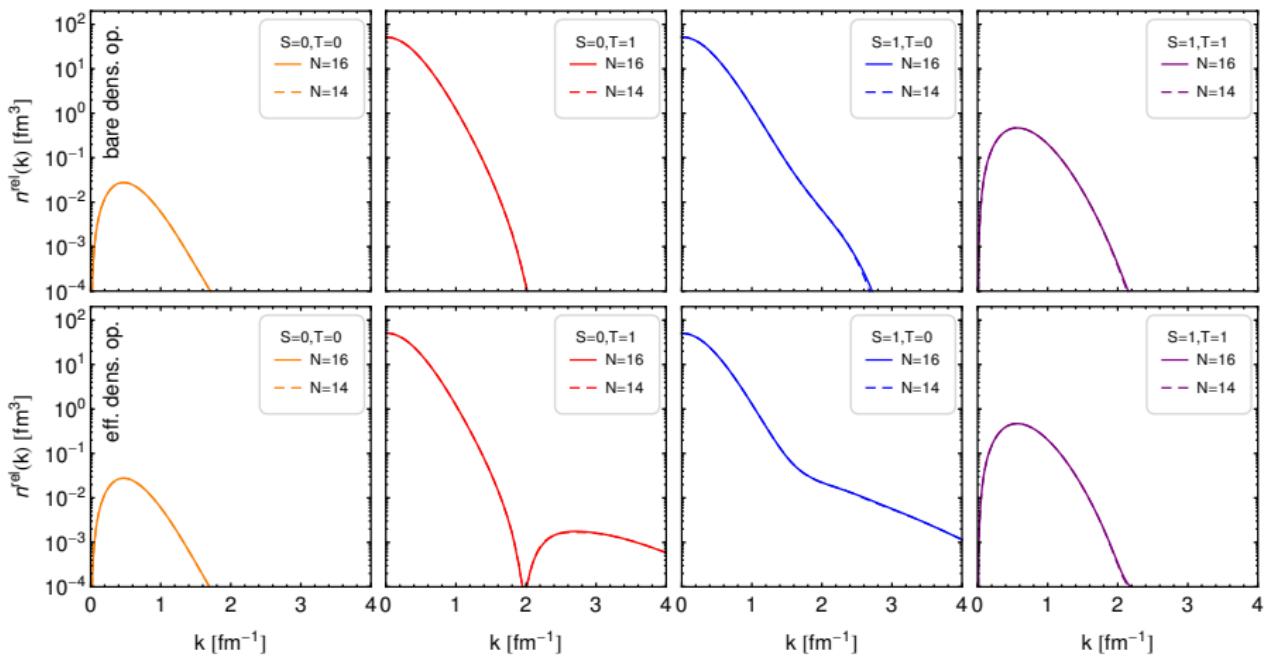


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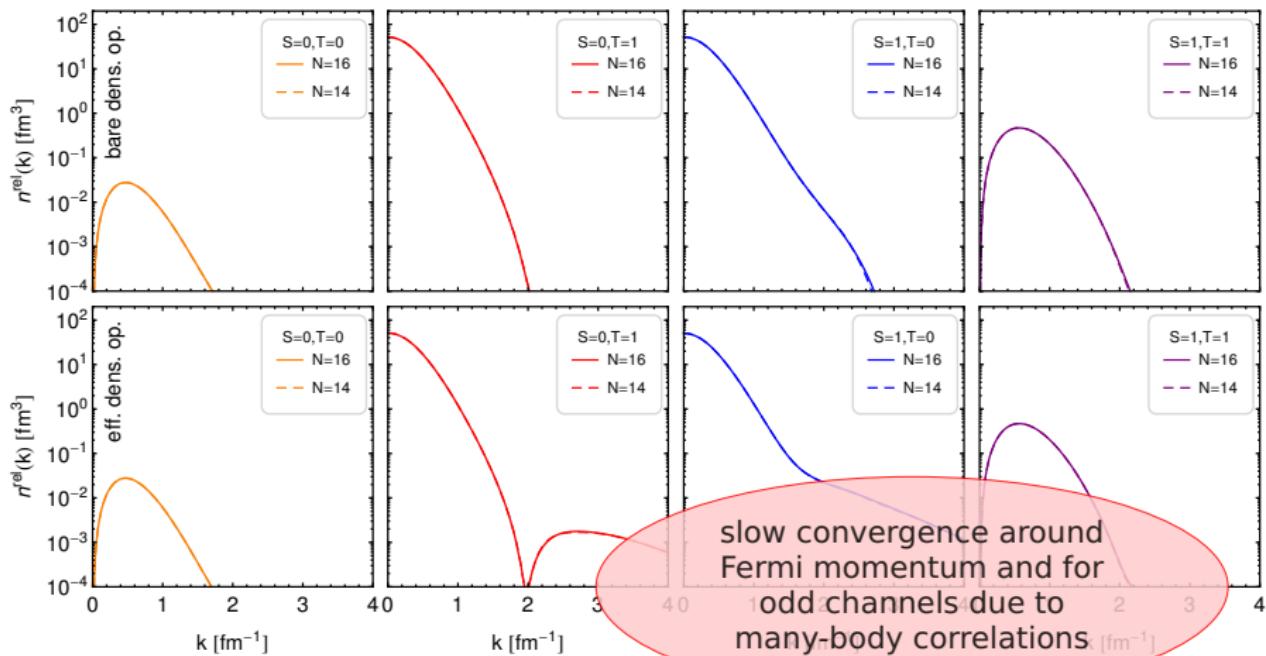


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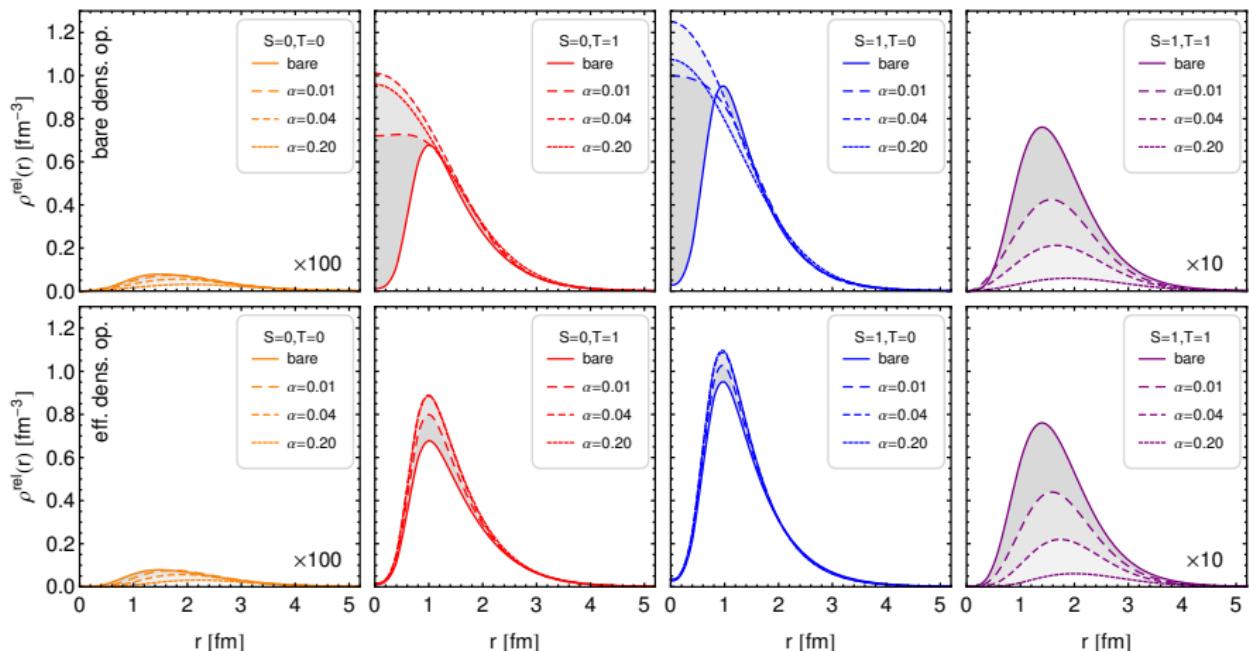


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Dependence on Flow Parameter

AV8' Interaction

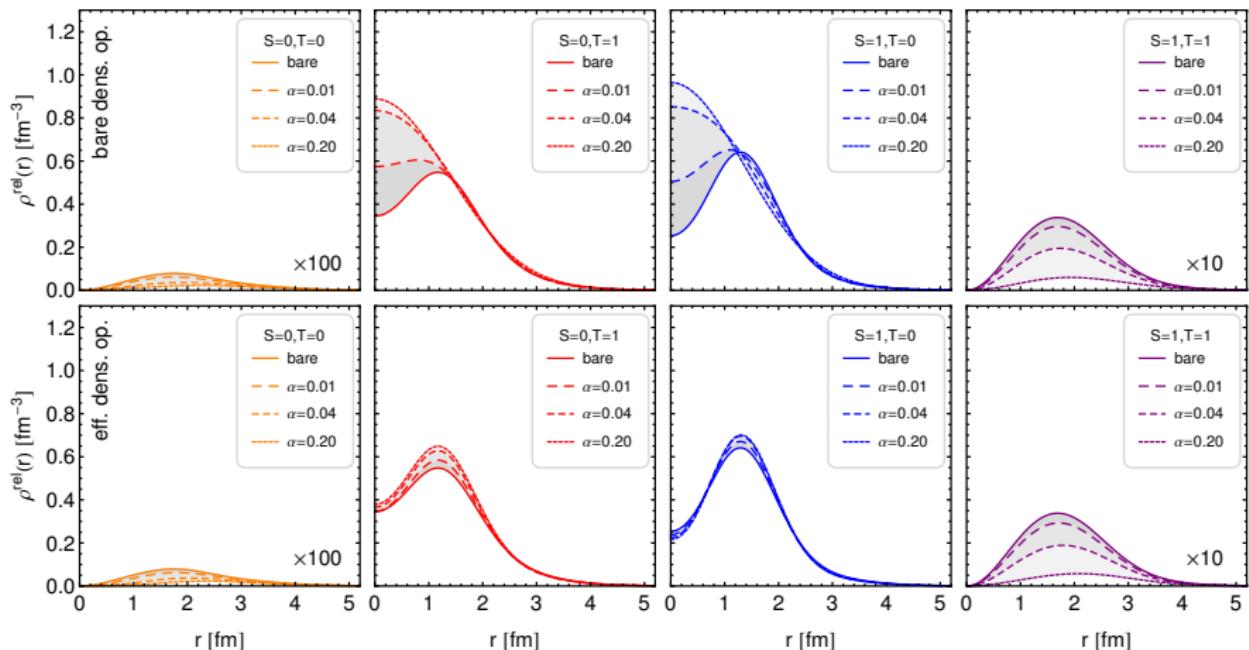
Two-body Density in Coordinate Space



Dependence on Flow Parameter

N^3LO Interaction

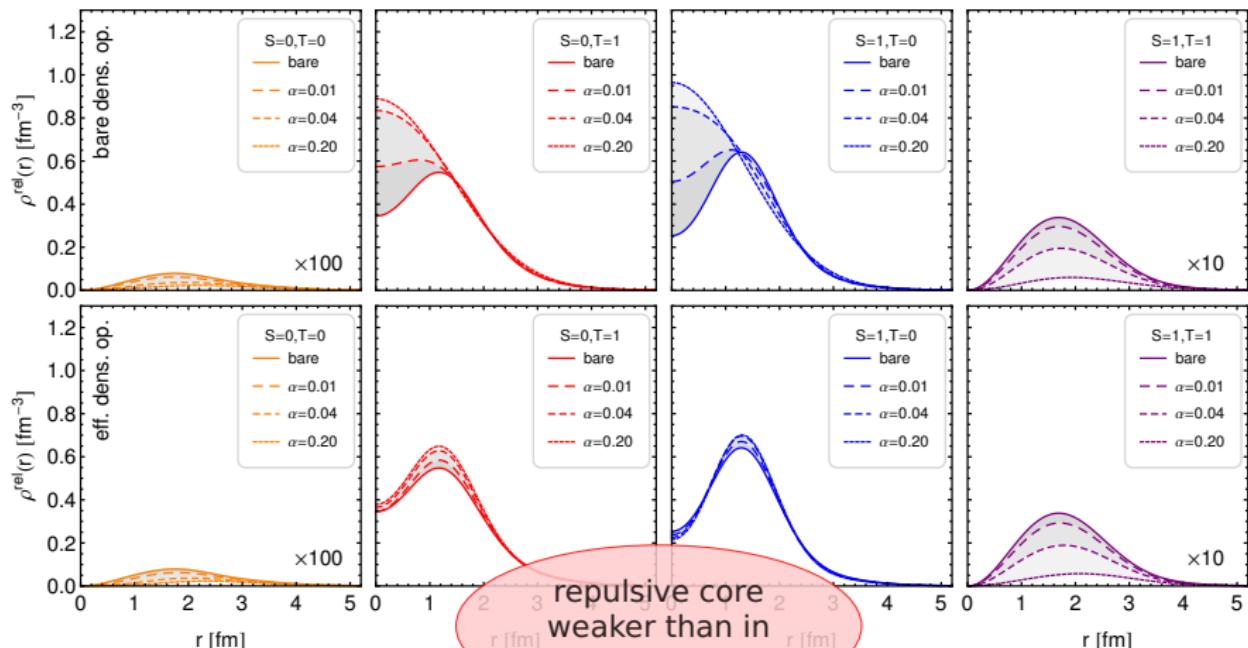
Two-body Density in Coordinate Space



Dependence on Flow Parameter

N³LO Interaction

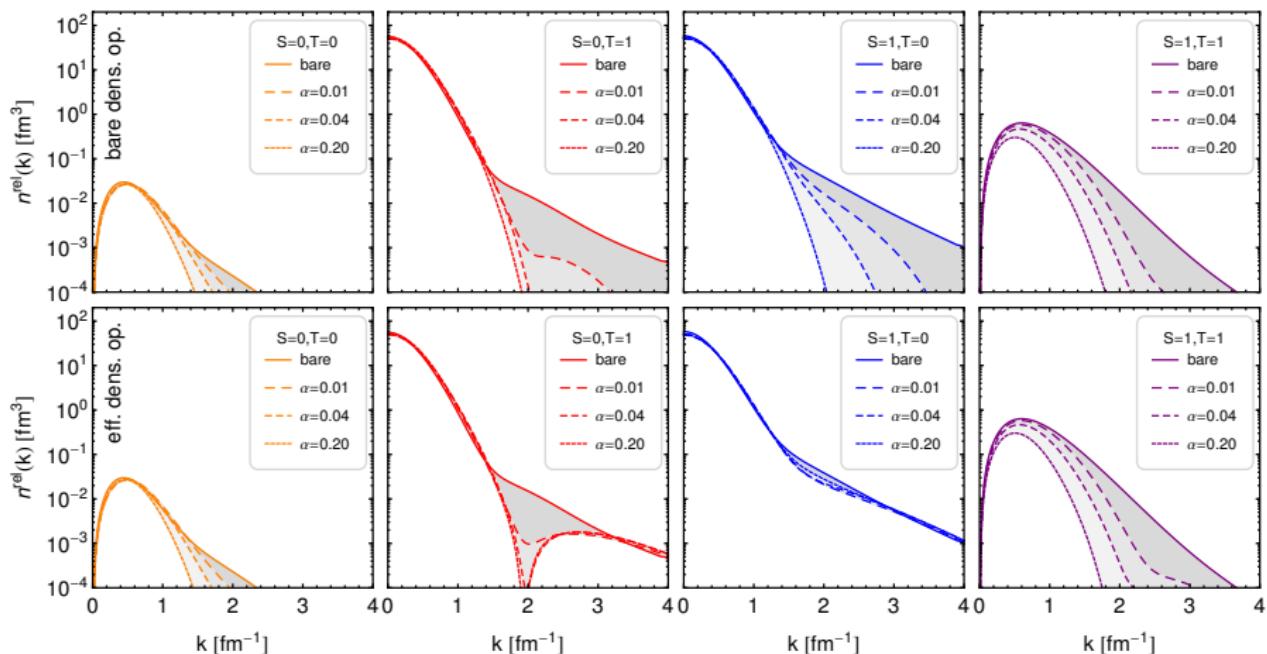
Two-body Density in Coordinate Space



Dependence on Flow Parameter

AV8' Interaction

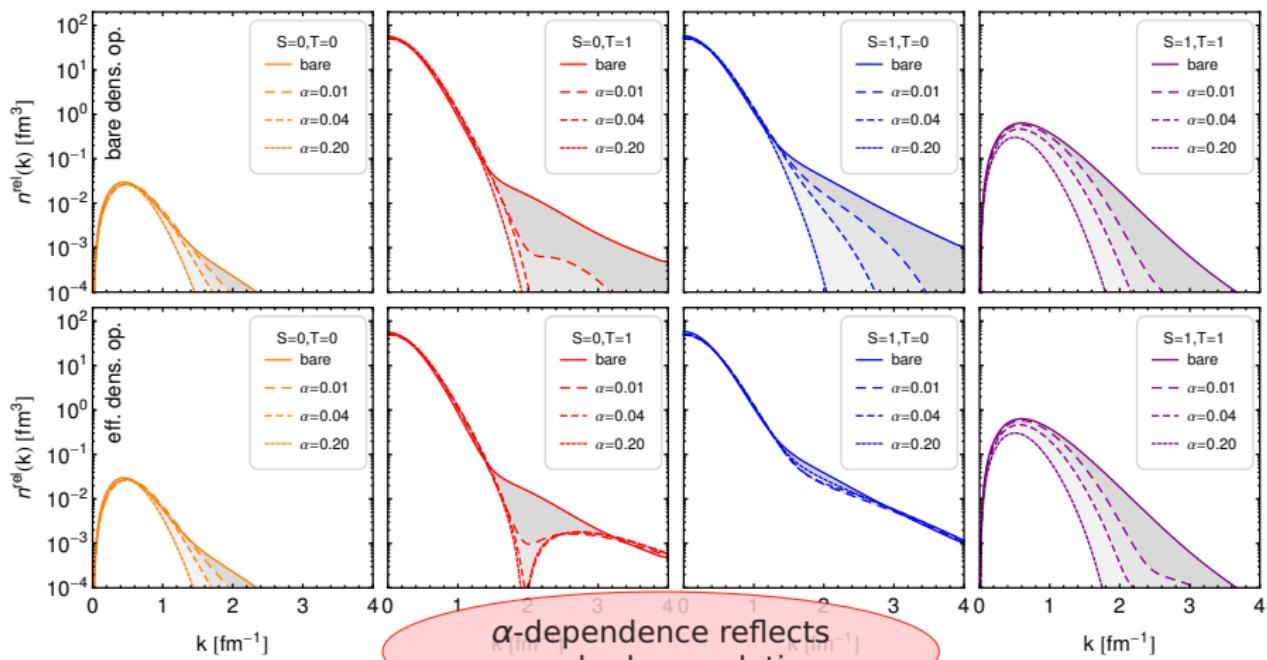
Two-body Density in Momentum Space



Dependence on Flow Parameter

AV8' Interaction

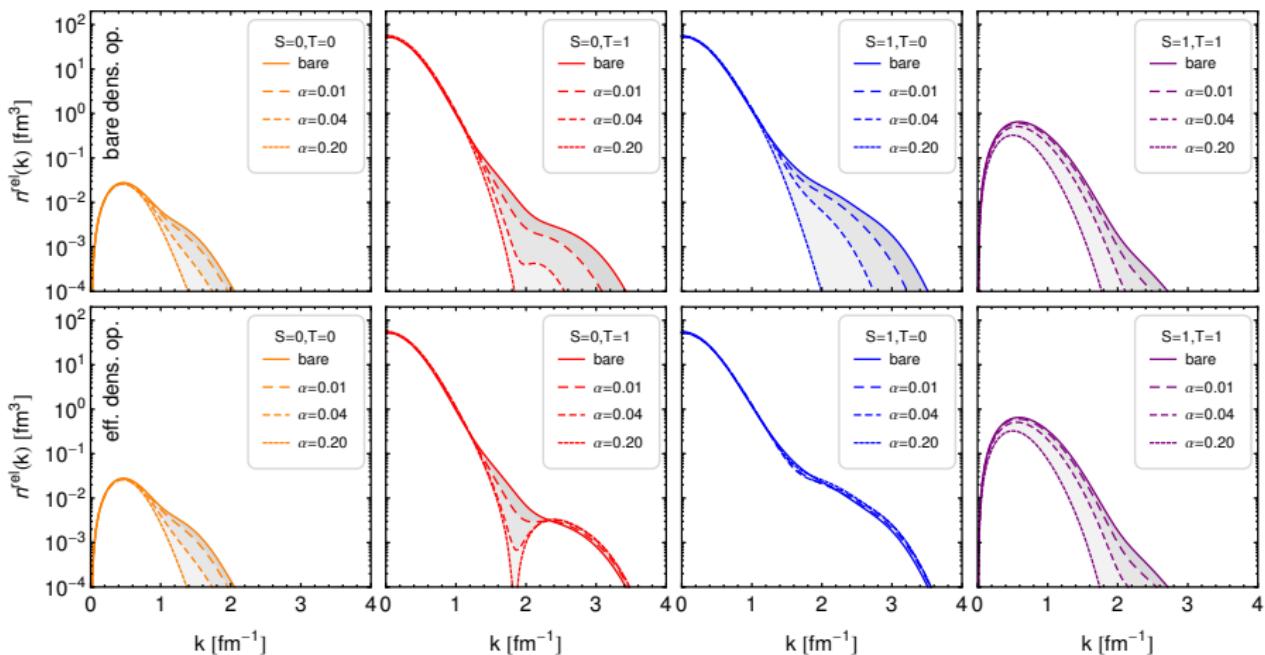
Two-body Density in Momentum Space



Flow dependence

N^3LO Interaction

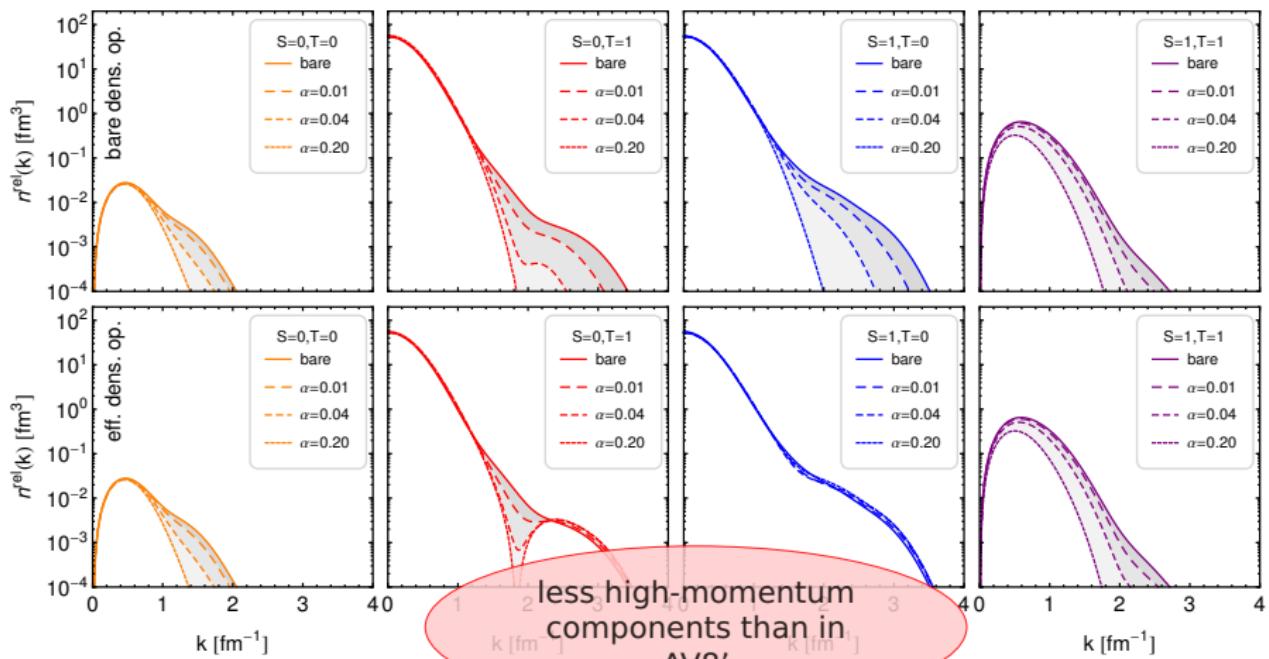
Two-body Density in Momentum Space



Flow dependence

N³LO Interaction

Two-body Density in Momentum Space



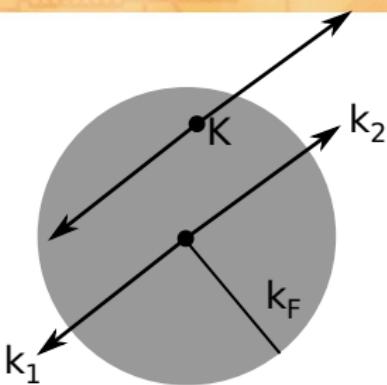
⁴He many-body correlations

number of pairs in (S, T) channels

Interaction	(0,0)	(0,1)	(1,0)	(1,1)
AV8'	0.008	2.572	2.992	0.428
AV8' $\alpha = 0.01 \text{ fm}^4$	0.008	2.708	2.992	0.292
AV8' $\alpha = 0.04 \text{ fm}^4$	0.007	2.821	2.993	0.179
AV8' $\alpha = 0.20 \text{ fm}^4$	0.005	2.925	2.995	0.075
N ³ LO	0.009	2.710	2.991	0.290
N ³ LO $\alpha = 0.01 \text{ fm}^4$	0.007	2.745	2.992	0.255
N ³ LO $\alpha = 0.04 \text{ fm}^4$	0.006	2.817	2.994	0.183
N ³ LO $\alpha = 0.20 \text{ fm}^4$	0.004	2.921	2.995	0.079

- occupation of (S, T) = (0, 1) and (S, T) = (1, 0) channels reflect many-body correlations
- AV8' induces stronger many-body correlations than N³LO
- with increasing flow parameter many-body correlations become weaker

Pair momentum $K = 0$



- for vanishing pair momentum and $k > k_F$ only high-momentum nucleons are sampled
- vanishing pair momentum implies vanishing pair angular momentum
- look only at pairs with $K = 0$:

$$n_{SM_S, TM_T}^{(2)}(\mathbf{K} = 0, \mathbf{k}) = \langle \Psi | \sum_{i < j}^A \hat{P}_{ij}^{SM_S} \hat{P}_{ij}^{TM_T} \delta^3(\hat{\mathbf{k}}_i + \hat{\mathbf{k}}_j) \delta^3\left(\frac{1}{2}(\hat{\mathbf{k}}_i - \hat{\mathbf{k}}_j) - \mathbf{k}\right) | \Psi \rangle$$

Translational invariant two-body densities

- To calculate the translational invariant two-body density the single-particle coordinates are transformed to Jacobi coordinates

$$(\mathbf{x}_1, \dots, \mathbf{x}_{A-1}, \mathbf{x}_A) \rightarrow (\xi_0, \xi_1, \dots, \xi_{A-3}, \boldsymbol{\eta}, \boldsymbol{\vartheta})$$

with

$$\boldsymbol{\eta} = \sqrt{\frac{2(A-2)}{A}} \left[\frac{1}{A-2} (\mathbf{x}_1 + \dots + \mathbf{x}_{A-2}) - \frac{1}{2} (\mathbf{x}_{A-1} + \mathbf{x}_A) \right], \quad \boldsymbol{\vartheta} = \sqrt{\frac{1}{2}} (\mathbf{x}_{A-1} - \mathbf{x}_A)$$

- The two-body density in the lab system $\rho^{(2)}$ is then given by a linear transformation of the translational invariant two-body density $\tilde{\rho}^{(2)}$

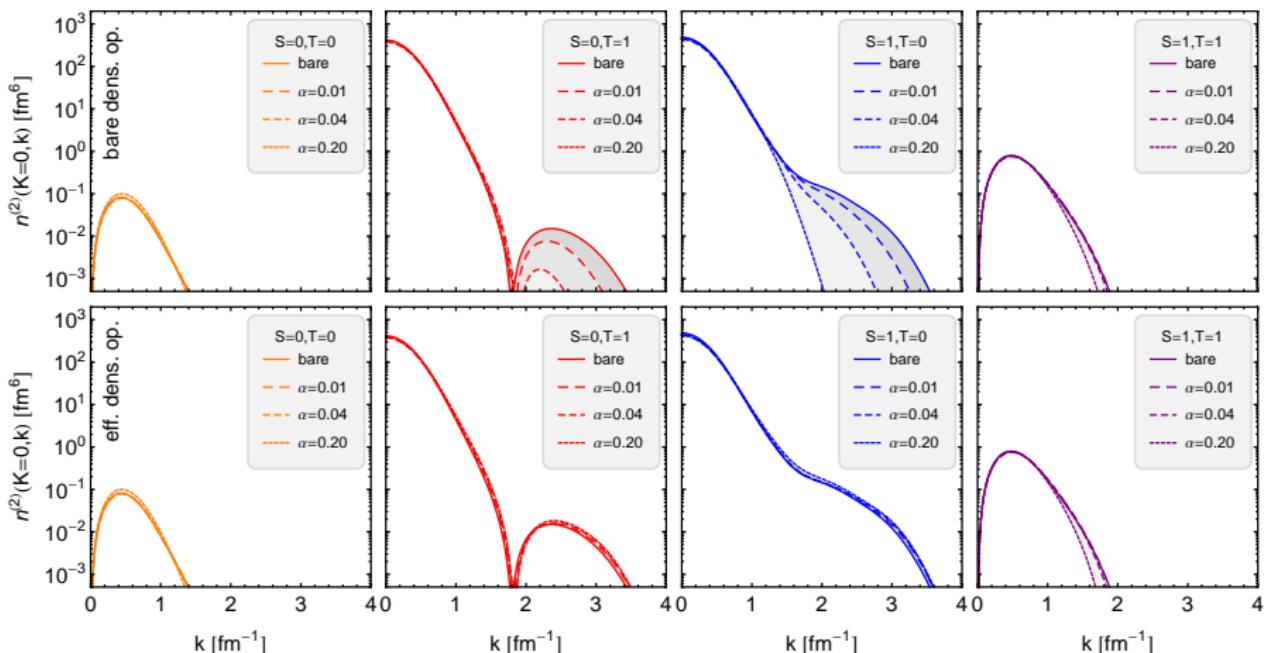
$$\sum_{M_1} \rho_{n'_1 l_1 m_1, N'_1 L_1 M_1; n_1 l_1 m_1, N_1 L_1 M_1}^{(2)} = \sum_{NN'L} M_{N'_1 L_1, N_1 L_1; N'L, NL}^{(0)} \sum_M \tilde{\rho}_{n'_1 l_1 m_1, N'L M; n_1 l_1 m_1, NLM}^{(2)}$$

- invert transformation to obtain translational invariant density from NCSM wave function

Flow dependence - $K = 0$

N^3LO Interaction

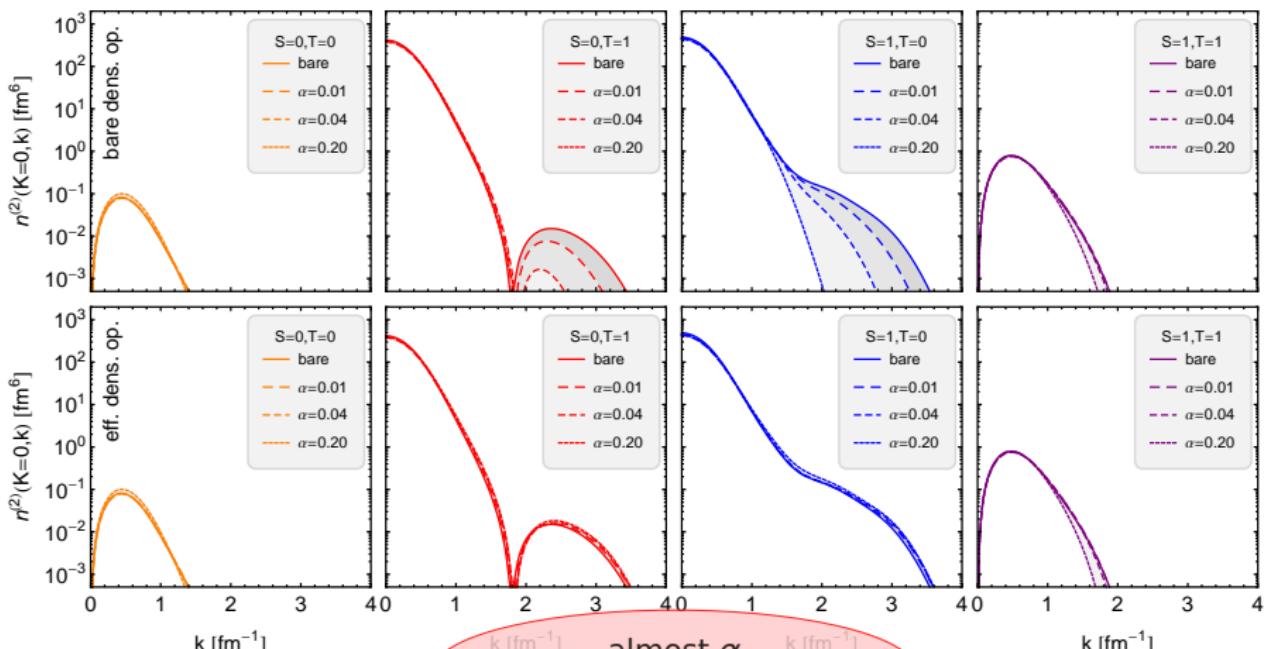
Two-body Density in Momentum Space



Flow dependence - $K = 0$

$N^3\text{LO}$ Interaction

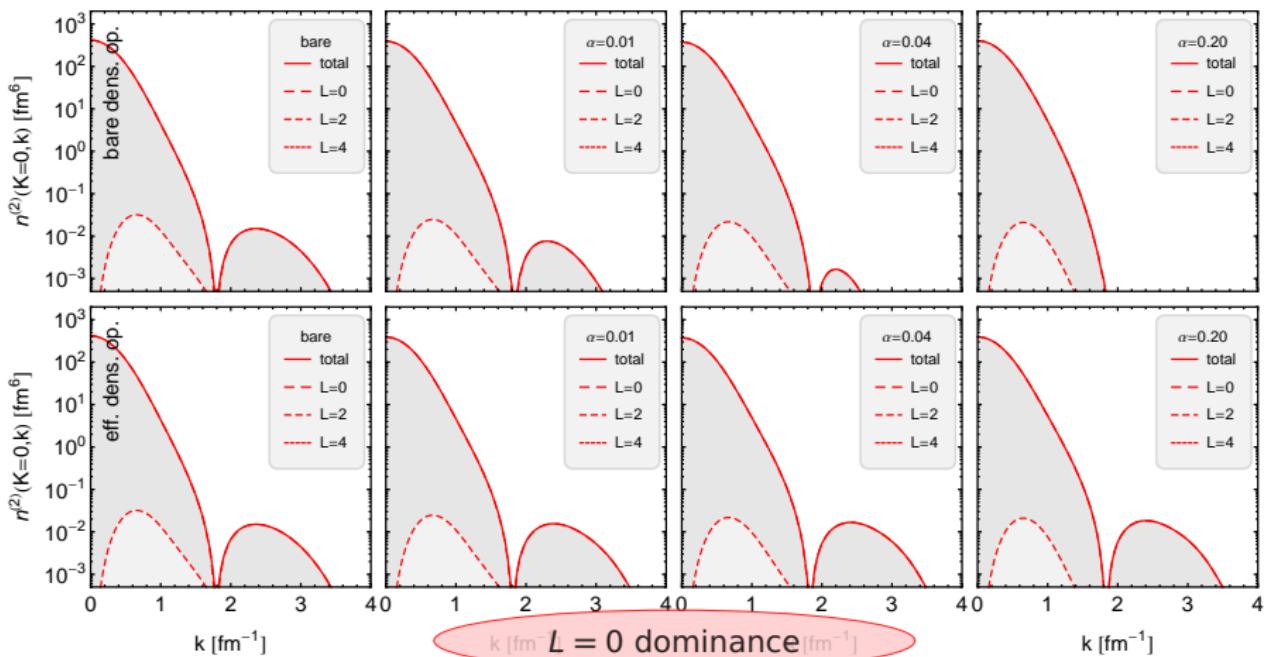
Two-body Density in Momentum Space



Contributions from different angular momenta – $K = 0$

N³LO Interaction

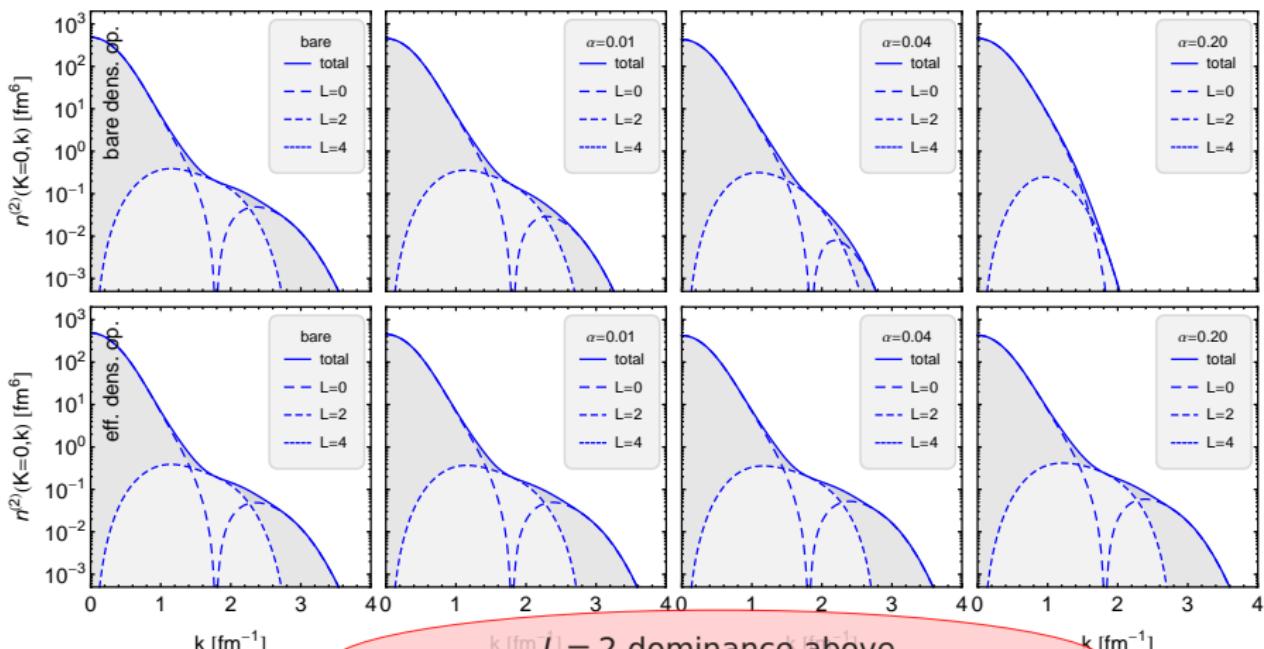
Two-body Density in Momentum Space $S = 0, T = 1$



Contributions from different angular momenta – $K = 0$

N^3LO Interaction

Two-body Density in Momentum Space $S = 1, T = 0$

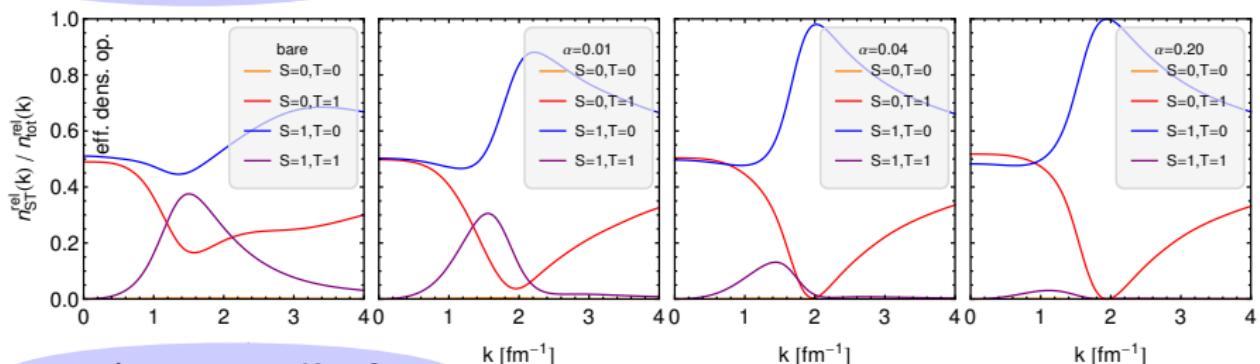


k [fm⁻¹]
 $L=2$ dominance above
Fermi momentum

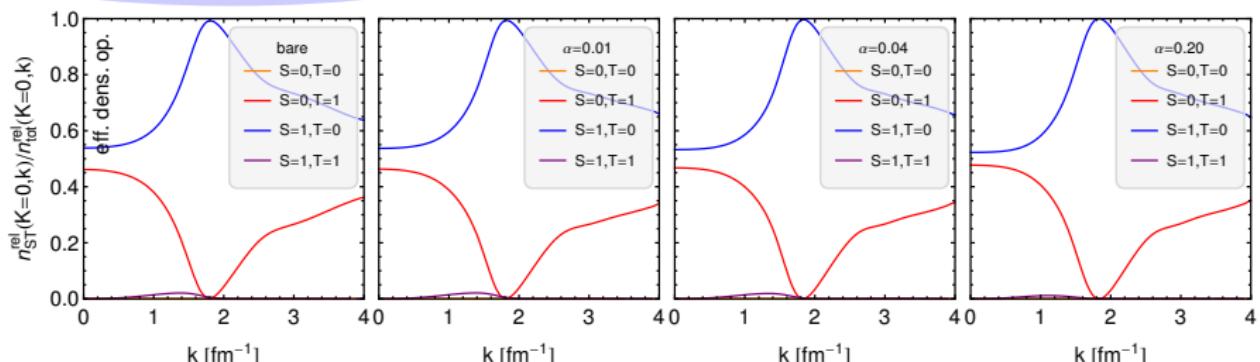
Relative contributions of ST channels

N³LO interaction

all pair momenta



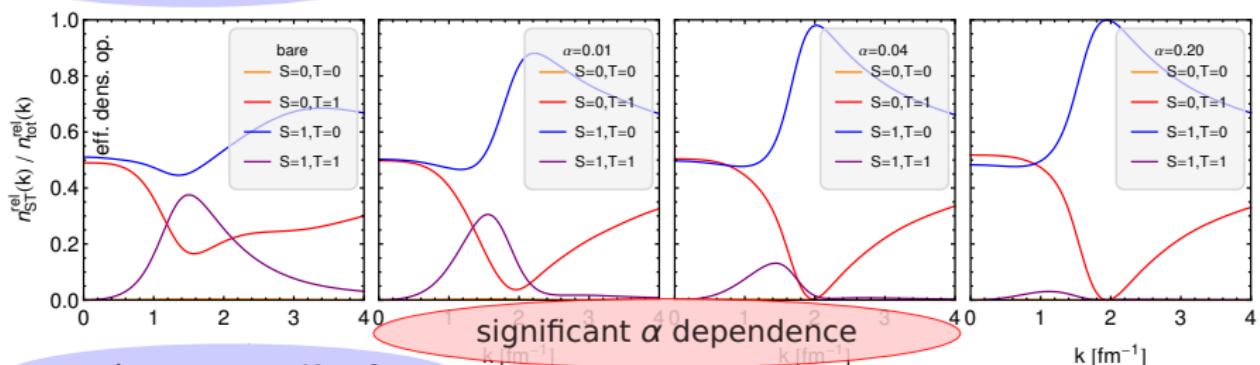
pair momenta $K = 0$



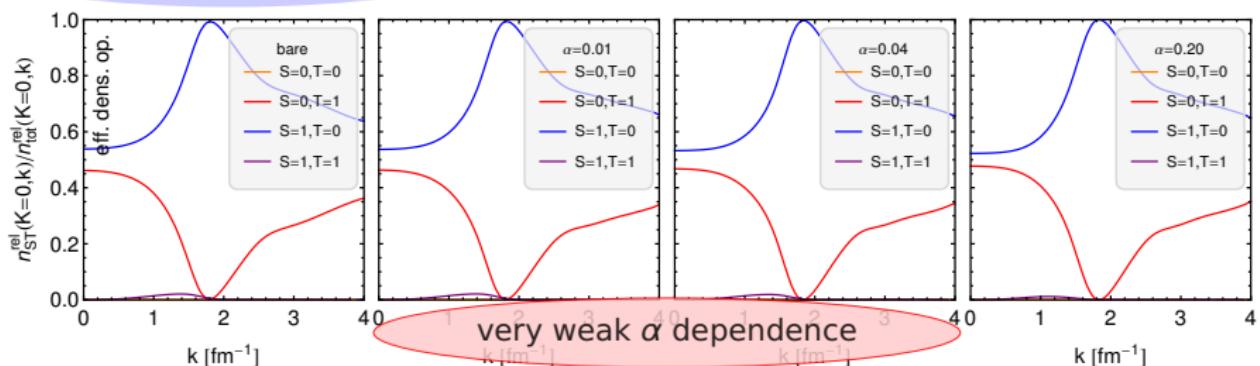
Relative contributions of ST channels

N³LO interaction

all pair momenta



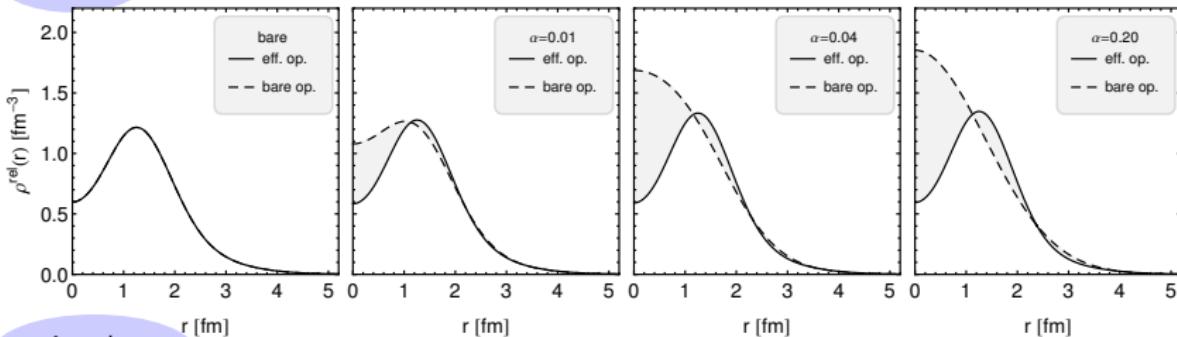
pair momenta $K = 0$



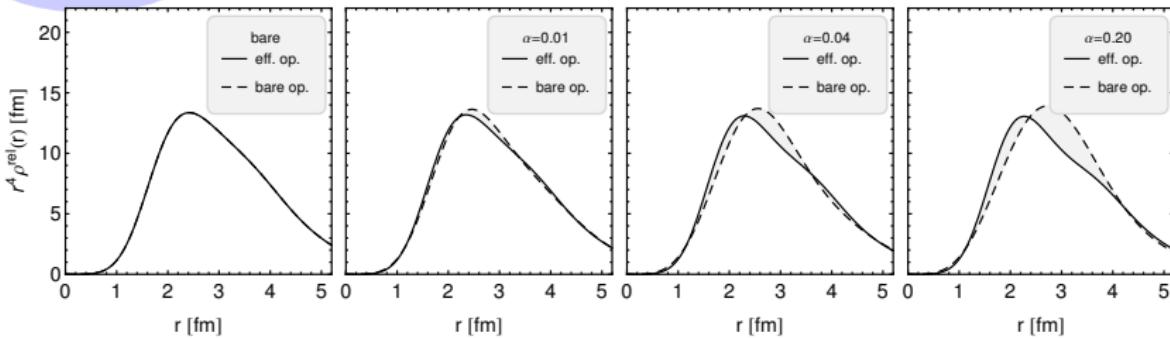
^4He Long-Range Observables

N^3LO Interaction

$\rho^{\text{rel}}(r)$



$r^4 \rho^{\text{rel}}(r)$



^4He Radii

N³LO Interaction

- point matter radius:

$$r_{\text{rms}}^2 = \frac{1}{A^2} \int dr r^4 \rho^{\text{rel}}(r)$$

- radii calculated from $N = 16$, $\hbar\Omega = 36\text{MeV}$ model space wave functions

	“bare” radius [fm]	“effective” radius [fm]
N ³ LO	1.503	
N ³ LO $\alpha = 0.01\text{ fm}^4$	1.478	1.477
N ³ LO $\alpha = 0.04\text{ fm}^4$	1.458	1.453
N ³ LO $\alpha = 0.20\text{ fm}^4$	1.482	1.451

long-range observables weakly affected
for large flow parameters

$^4\text{He}, ^6\text{He}, ^9\text{Be}, ^{12}\text{C}$ Results

Experimental efforts to study short-range correlations at JLAB, FAIR

Calculation

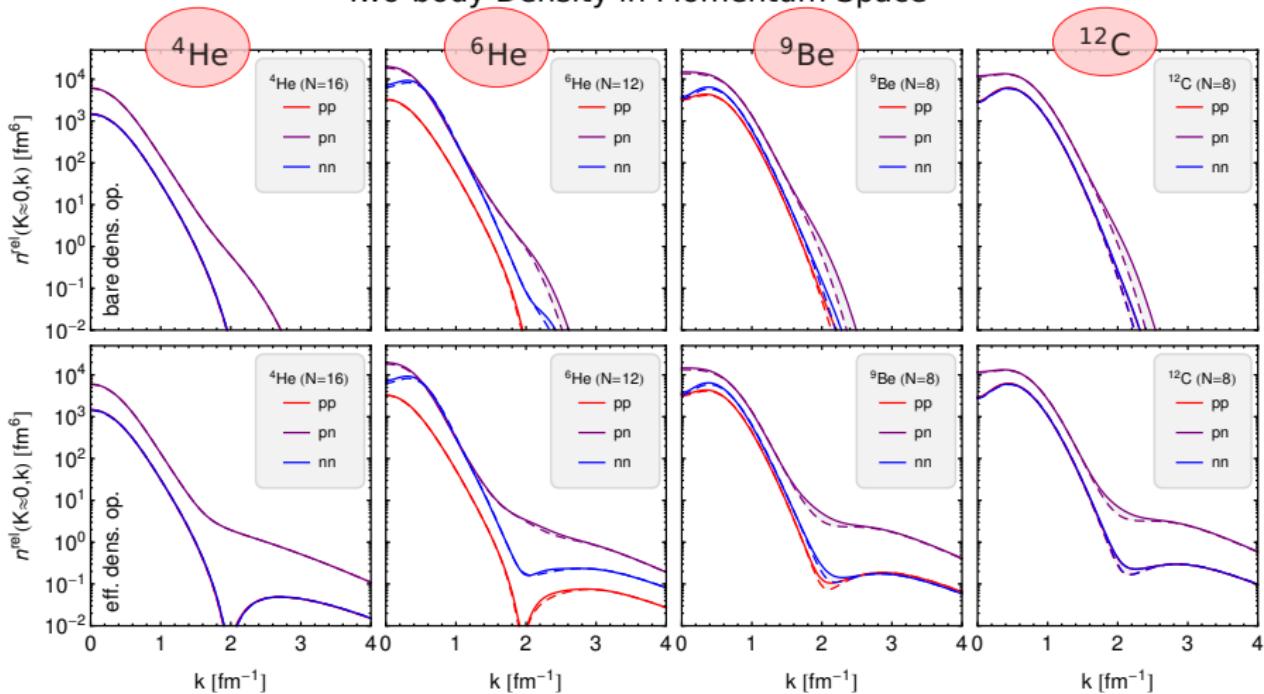
- bare AV18 and N³LO can not be converged for heavier nuclei
- NCSM converges only for larger flow parameters

Objectives

- Compare AV18 and N³LO results
- Check for NCSM convergence
- Check flow dependence $\alpha = 0.04, 0.08, 0.20 \text{fm}^4$ ($\Lambda = 2.24, 1.88, 1.50 \text{fm}^{-1}$)
- What is different from ^4He ?

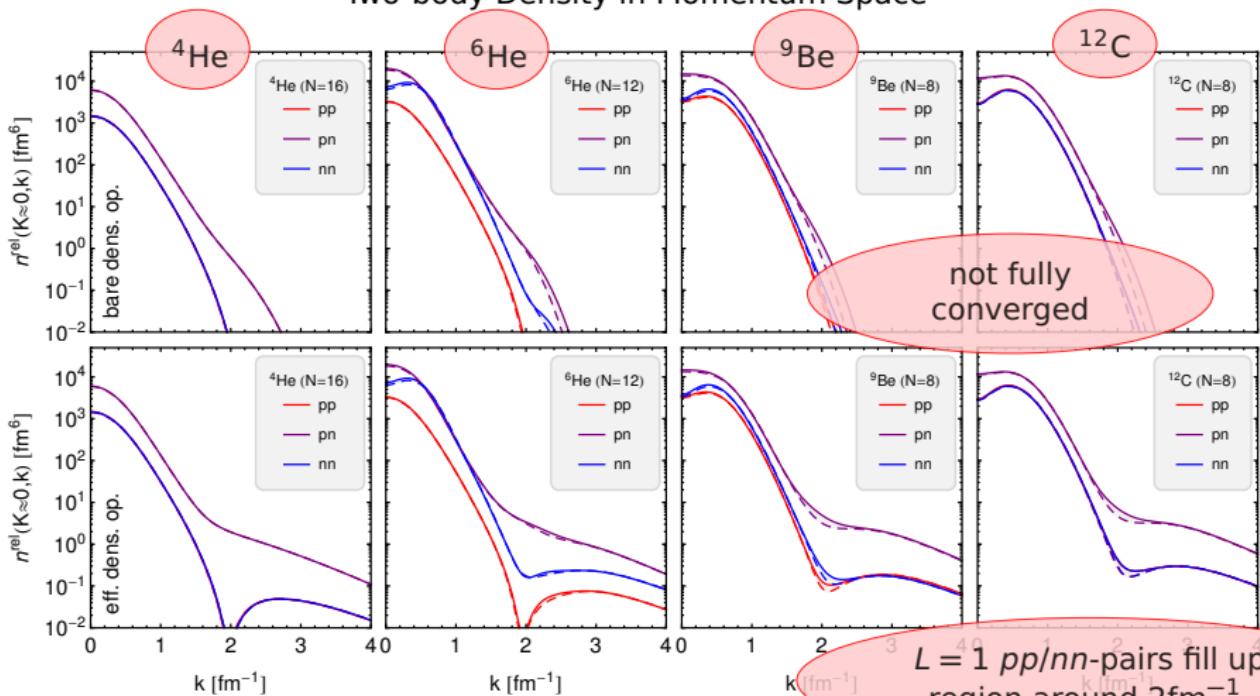
AV18, $K \approx 0$, $\alpha=0.04 \text{ fm}^4$

Two-body Density in Momentum Space



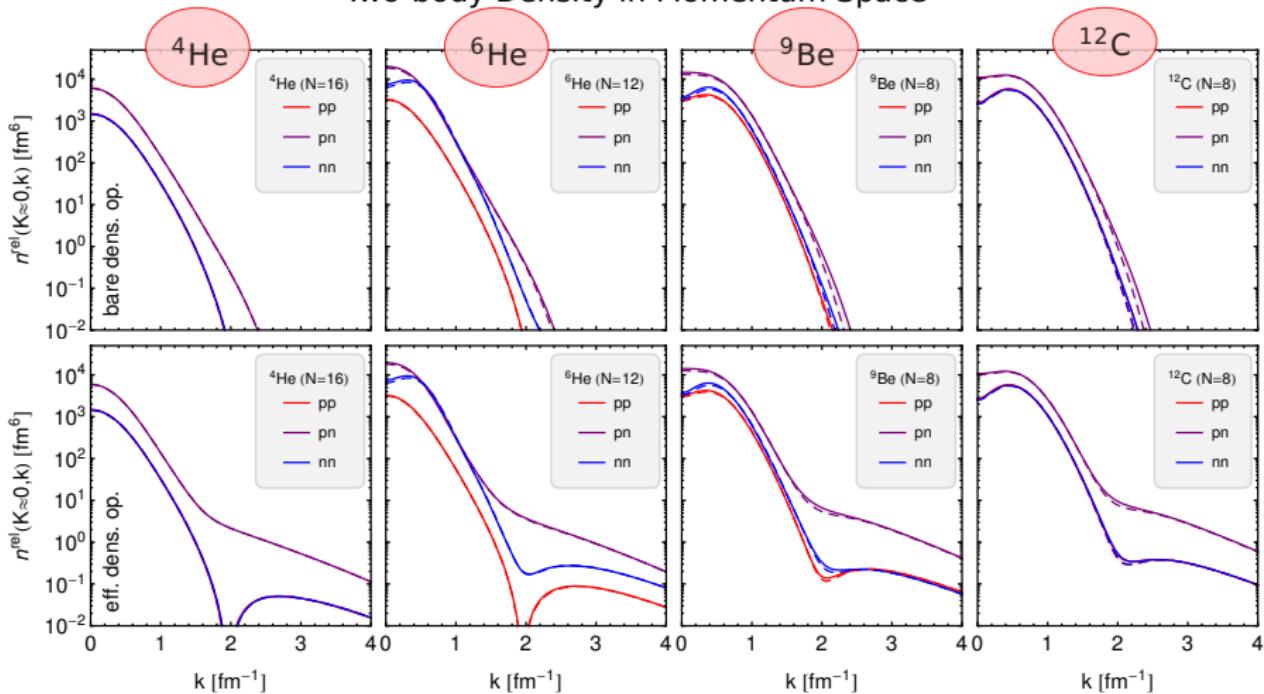
AV18, $K \approx 0$, $\alpha=0.04 \text{ fm}^4$

Two-body Density in Momentum Space



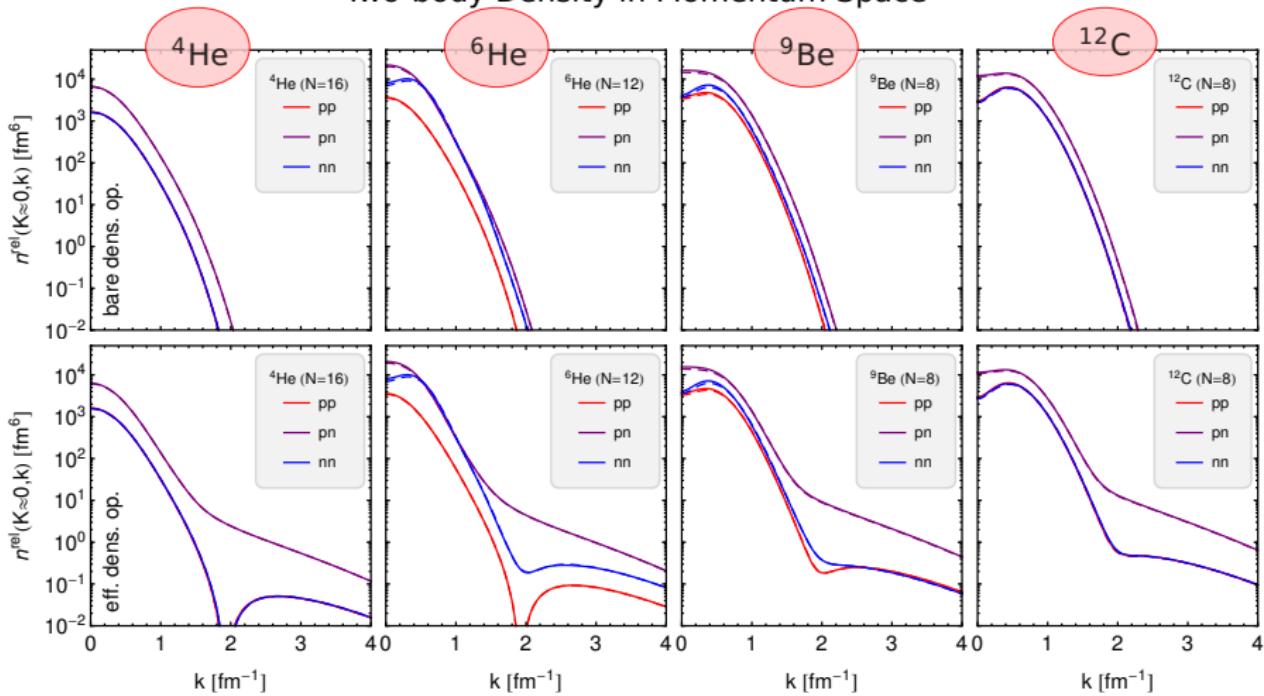
AV18, $K \approx 0$, $\alpha=0.08 \text{ fm}^4$

Two-body Density in Momentum Space



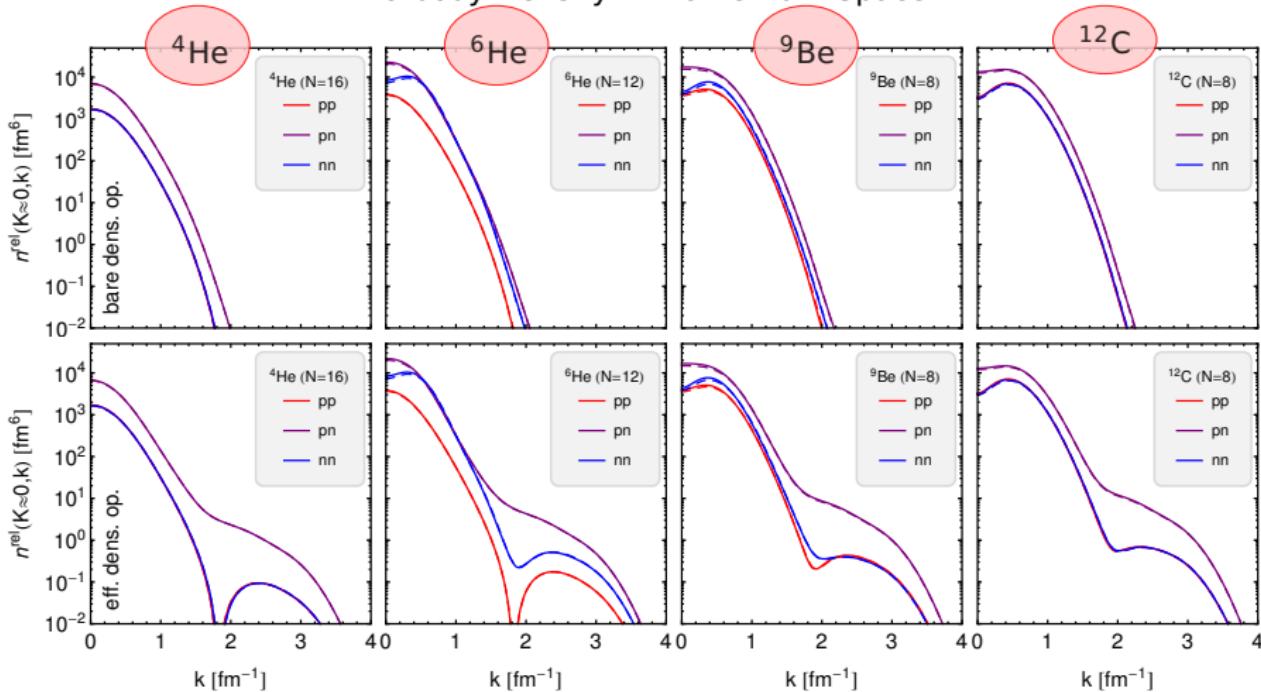
AV18, $K \approx 0$, $\alpha=0.20 \text{ fm}^4$

Two-body Density in Momentum Space



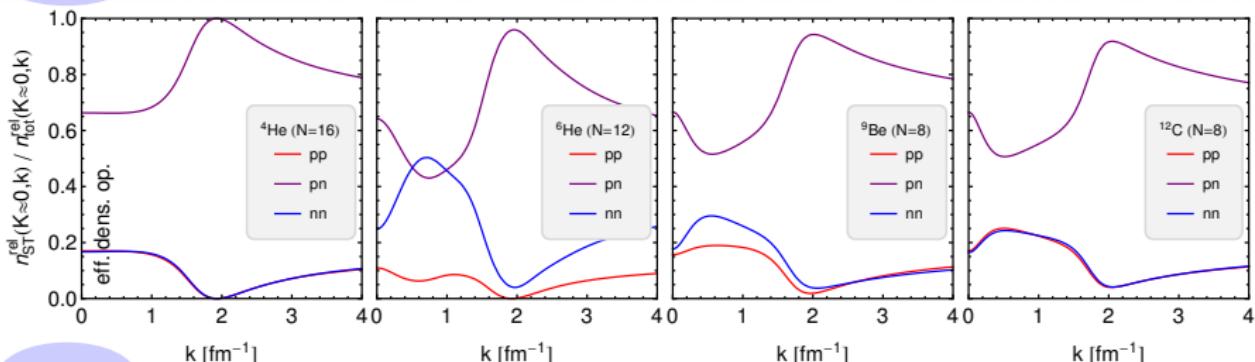
$\text{N}^3\text{LO}, K \approx 0, \alpha=0.20 \text{ fm}^4$

Two-body Density in Momentum Space

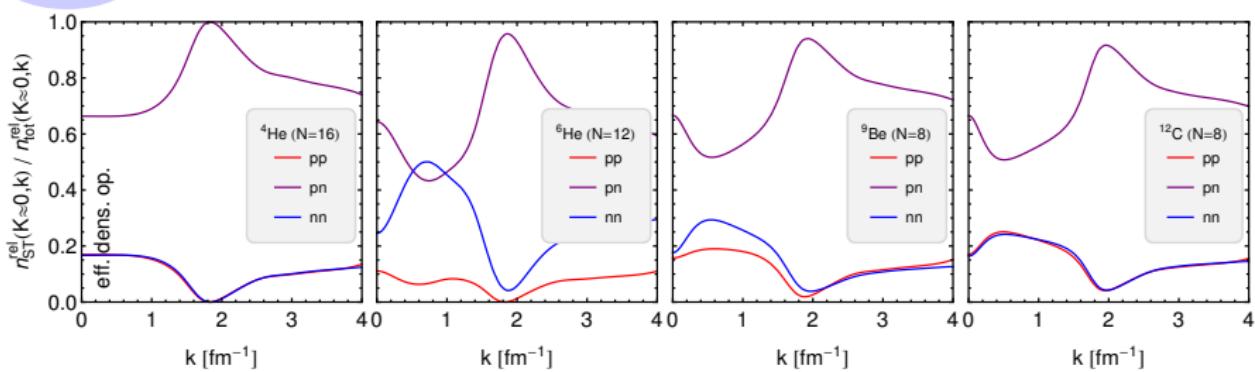


pp, pn, nn relative contributions

AV18

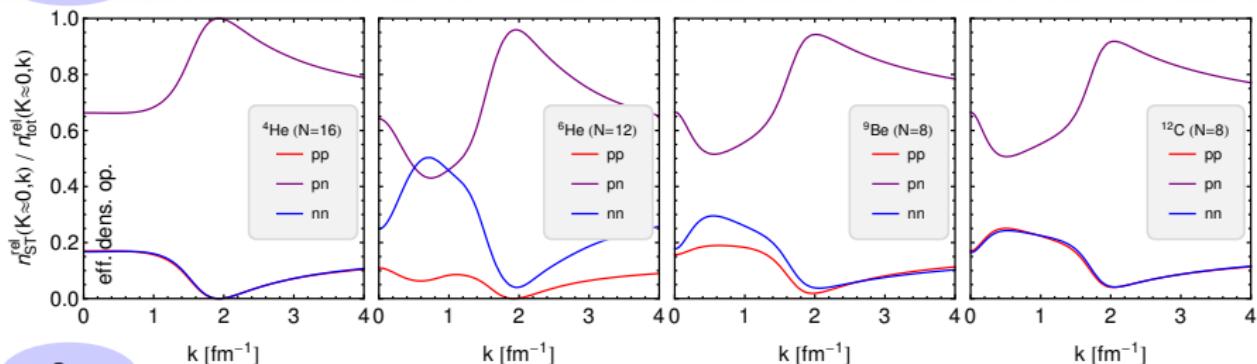


N³LO

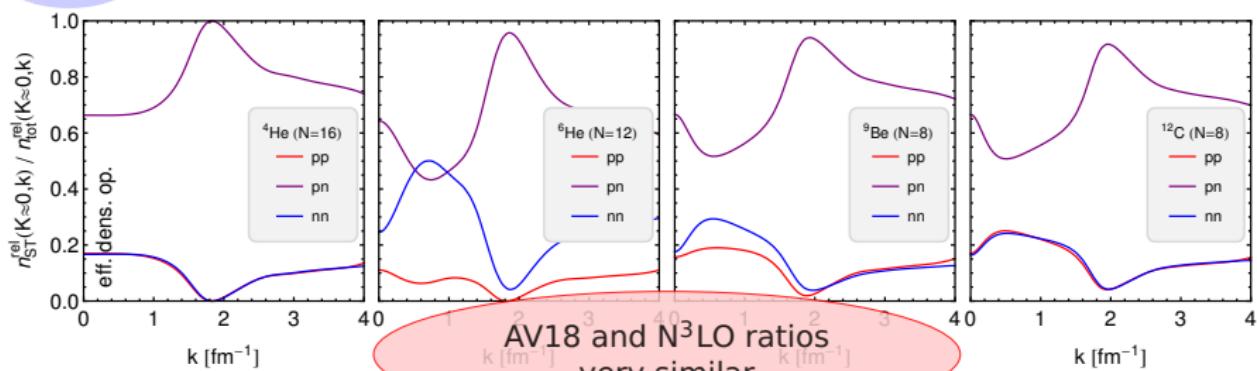


pp, pn, nn relative contributions

AV18



N³LO



AV18 and N³LO ratios
very similar

Summary

Similarity Renormalization Group

- SRG evolved Hamiltonian and transformation matrix
- “bare” and “effective” density operators

^4He two-body densities

- stronger short-range correlations for AV8' than N^3LO
- short-range and high-momentum components described by effective operators
- high-momentum components above the Fermi momentum dominated by $S = 1, T = 0, L = 2$ pairs (tensor correlations)
- α -dependence in $T = 1$ channels reflects three-body correlations induced by the tensor force
- $K = 0$ momentum distributions (almost) independent from α – preferred choice for studying two-body correlations

$^6\text{He}, ^9\text{Be}, ^{12}\text{C}$ two-body densities

- Similar behavior in all nuclei – pn pairs dominate momentum distribution
- $T = 1$ pairs with $L = 1$ fill up the pp/nn momentum distributions above the Fermi momentum
- AV18 and N^3LO provide similar results up to $k \approx 3\text{fm}^{-1}$