International Collaborations in Nuclear Theory (ICNT) workshop
"Physics of exotic nuclei: Theoretical advances and challenges"
9-13 June, 2014, RIKEN Wako Campus

Neutron halo in deformed relativistic Hartree-Bogoliubov theory in continuum

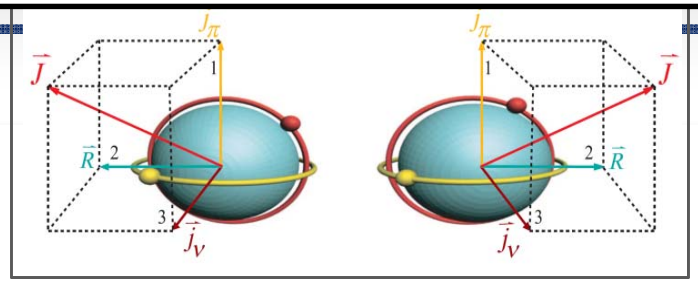
Jie MENG（孟 杰）
School of Physics， Peking University
北京大学物理学院

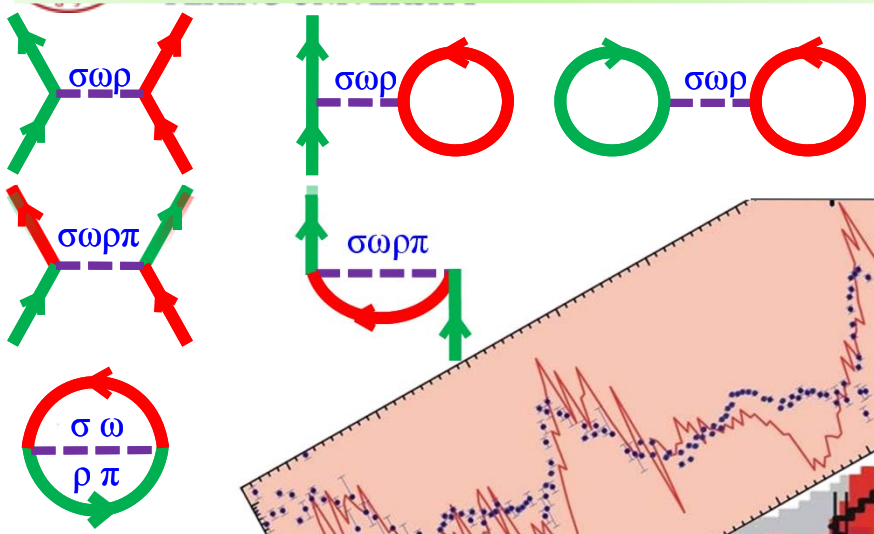
Outline

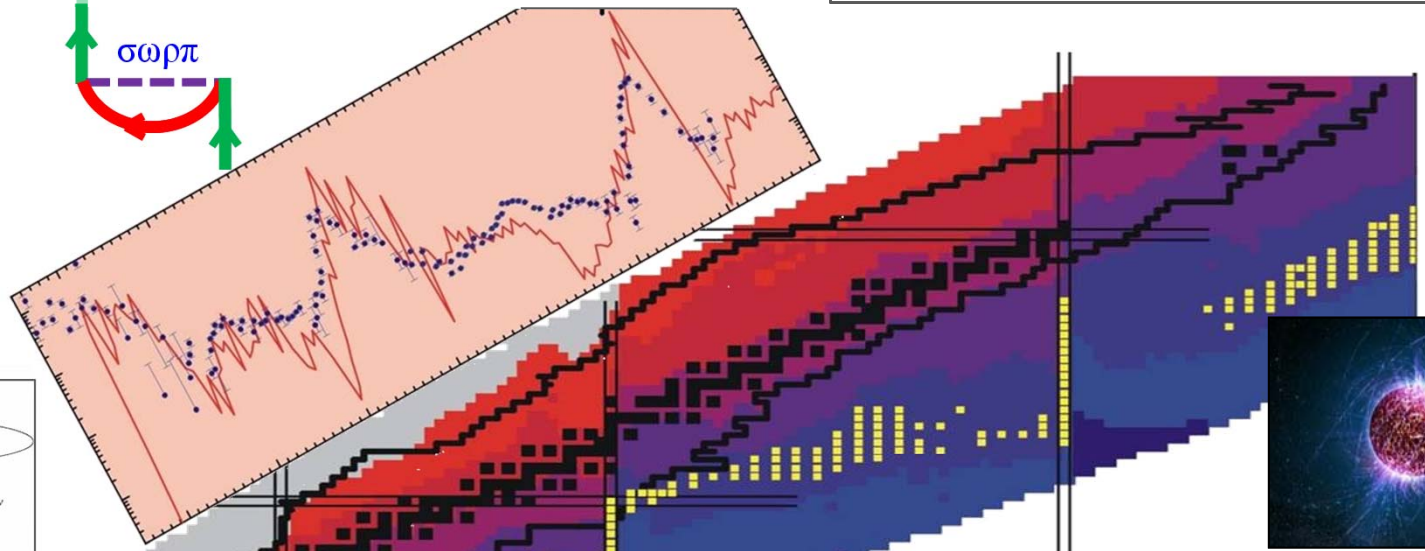
- Introduction
- Deformed RHB theory in continuum
- Magnesium isotopes
- Pairing effects on halo
- Summary & Perspectives

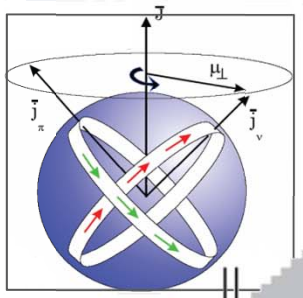
Covariant density functional

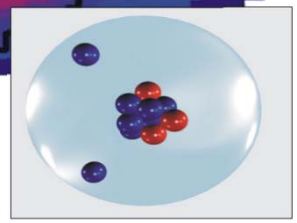
Progress in Physics. 31(2011)199-336.

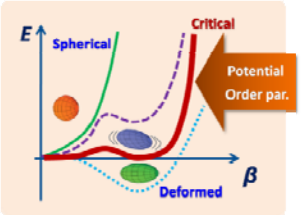


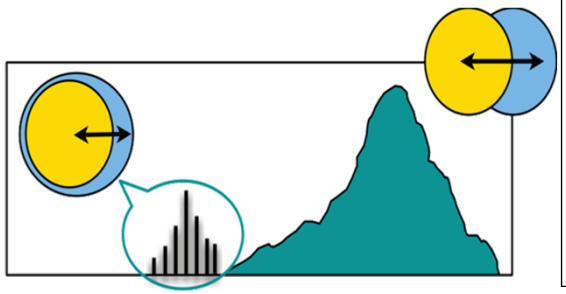


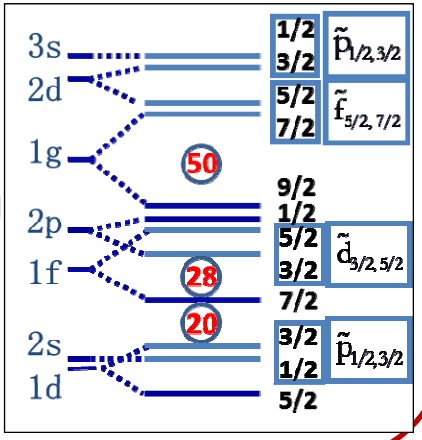






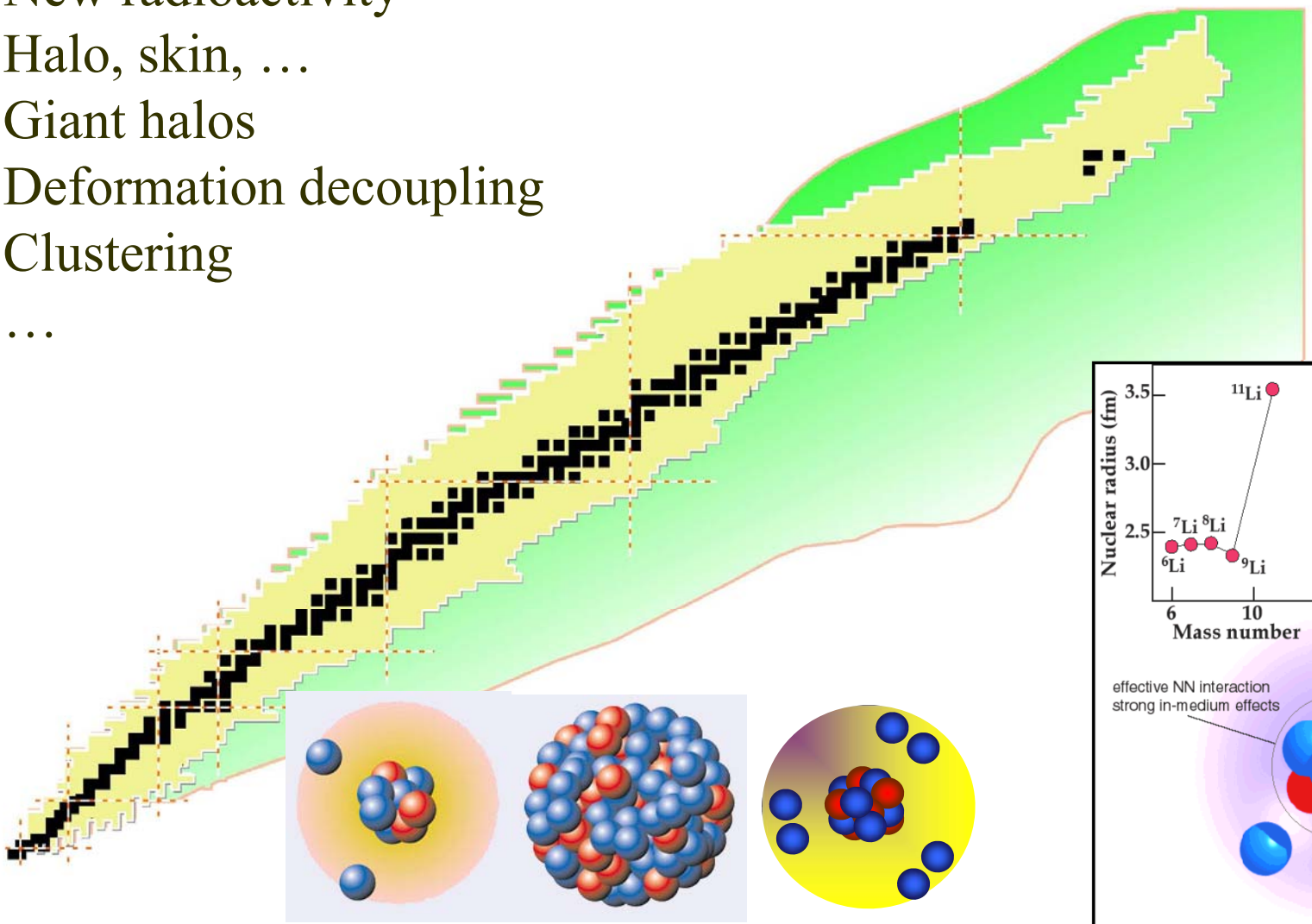


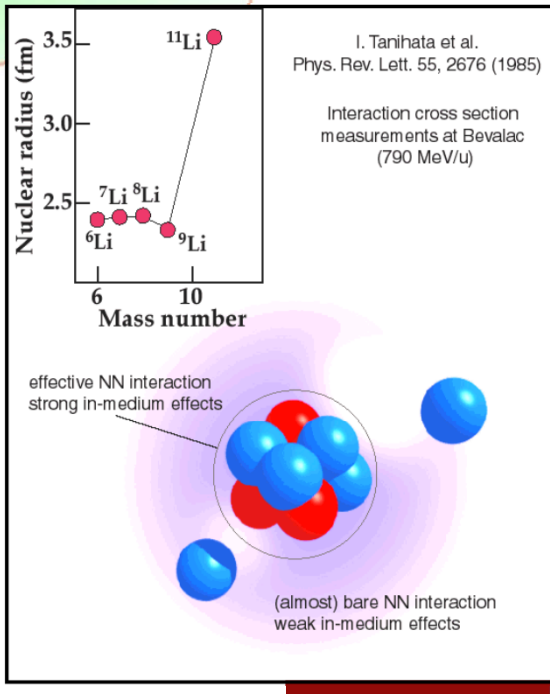




Exotic phenomena in nuclei with extreme N/Z

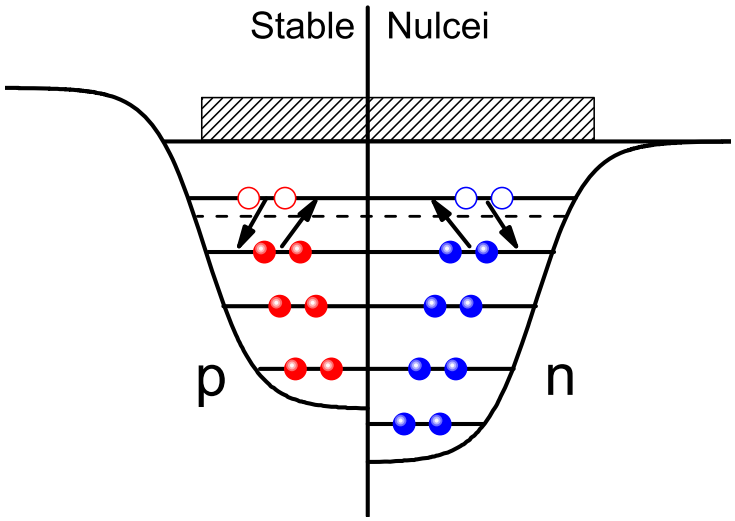
- Modifications of magic numbers
- New radioactivity
- Halo, skin, …
- Giant halos
- Deformation decoupling
- Clustering
- …

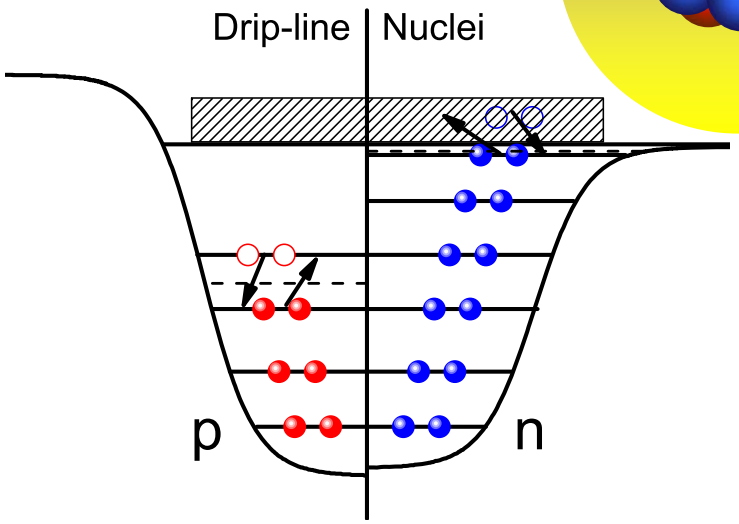


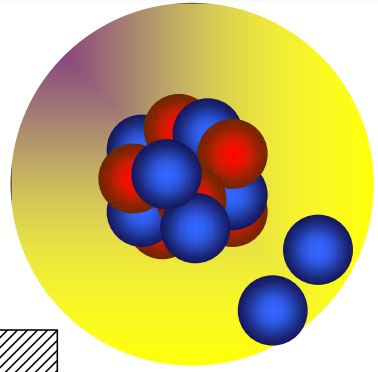


Characteristics of halo nuclei

- Weakly bound; large spatial extension
- Continuum coupling is essential







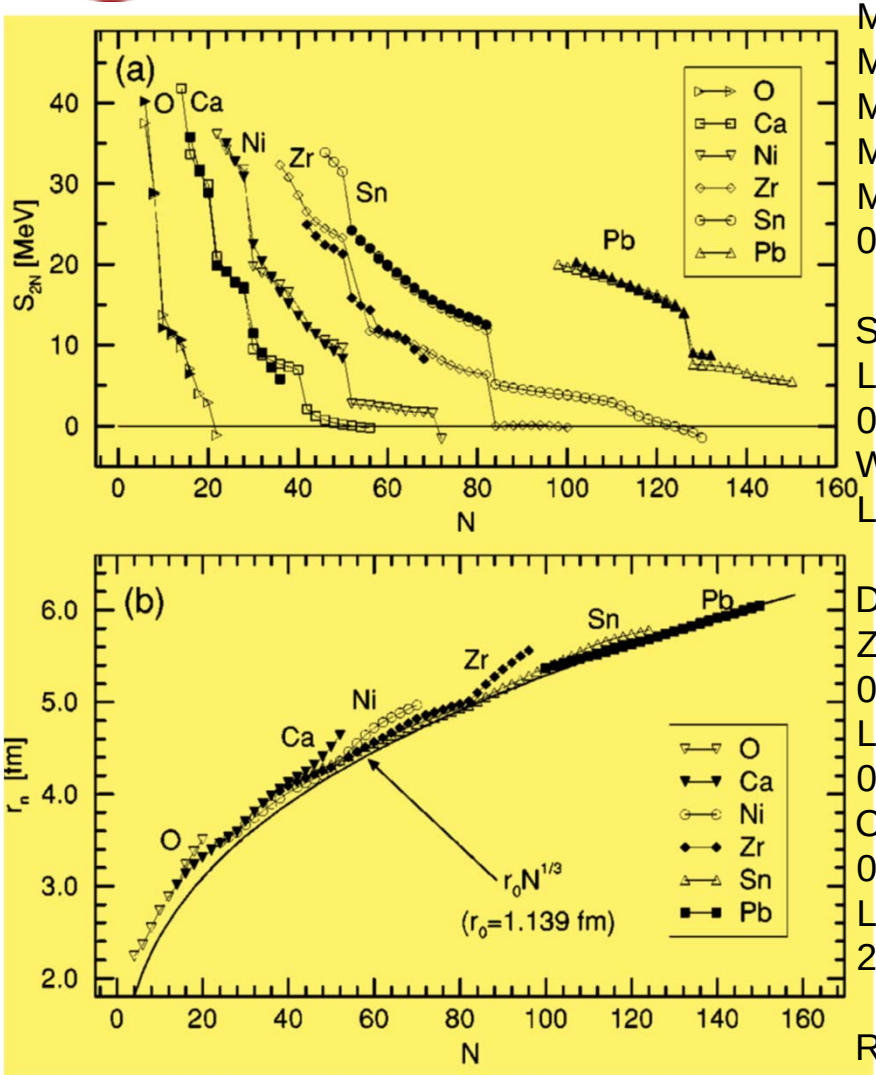
Self-consistent description:

- Weakly bound, continuum
- Large spatial distribution
- Deformation
- Couplings among …

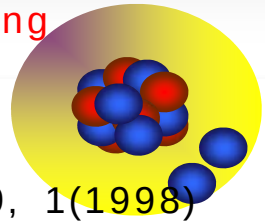
Meng_Toki_Zhou_Zhang_Long_Geng200
Prog. Part. Nucl. Phys. 57 - 470

Bulgac1980; nucl-th/9907088
Dobaczewski_Flocard_Treiner1984_NPA422-10

Halo in heavy nucleus: giant halo and deformed halo

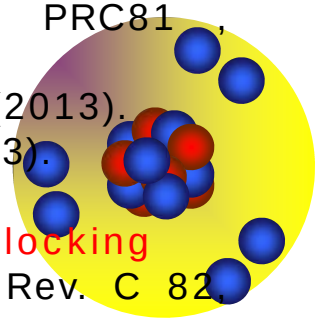


Spherical nucleus: continuum & pairing



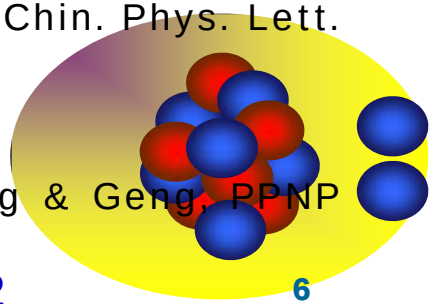
Meng & Ring, PRL77,3963 (96)
Meng & Ring, PRL80,460 (1998)
Meng, NPA 635, 3-42 (1998)
Meng, Tanihata & Yamaji, PLB 419, 1(1998)
Meng, Toki, Zeng, Zhang & Zhou, PRC65, 041302R

Spherical nucleus but in DDRHFB: Fock term



Long, Ring, Meng & Van Giai, PRC81 , 031302
Wang, Dong, Long, PRC 87, 047301(2013).
Lu, Sun, Long, PRC 87, 034311 (2013).

Deformed nucleus: deformation & blocking
Zhou, Meng, Ring & Zhao, Phys. Rev. C 82, 011301 (R)(2010)
Li, Meng, Ring, Zhao & Zhou, Phys. Rev. C 85, 024312 (2012)
Chen, Li, Liang & Meng, Phys. Rev. C 85, 067301 (2012)
Li, Meng, Ring, Zhao & Zhou, Chin. Phys. Lett. 29, 042101 (2012).



Reviews:
Meng,Toki, Zhou, Zhang, Long & Geng, PPNP 57. 460 (2006)

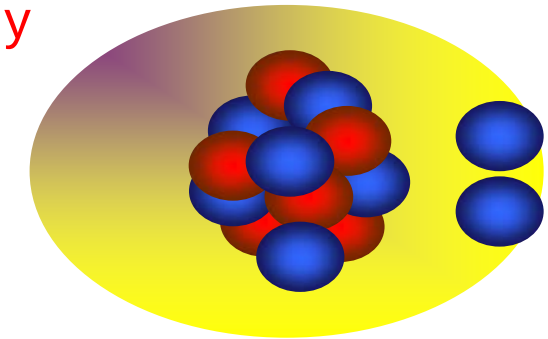
Zhang, Matsuo, Meng, PRC83:054301,2011; PRC86: 054318, 2012

Our goal

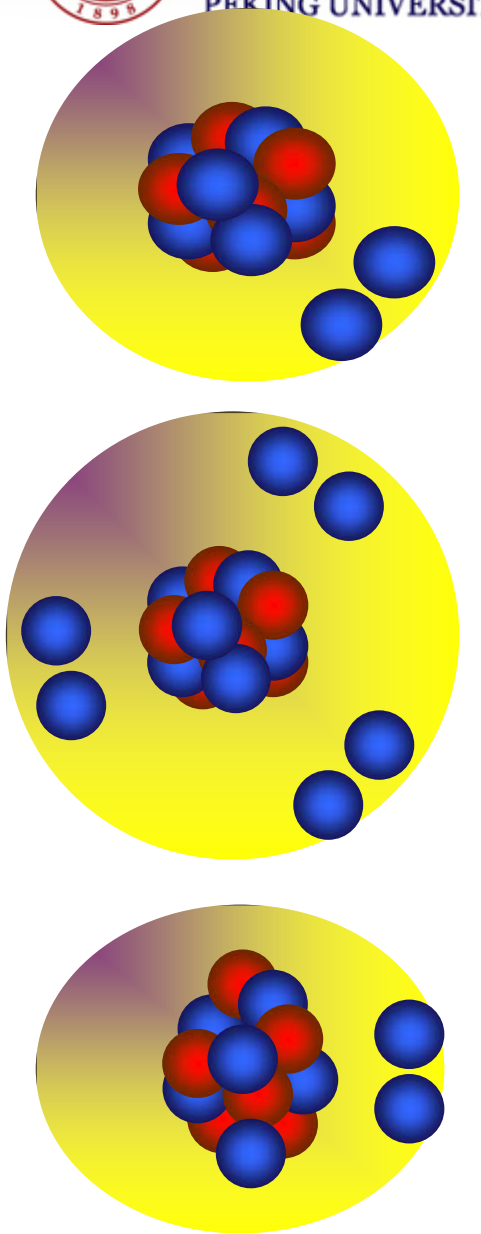
A self-consistent description of

- ✓ Deformation
- ✓ Continuum contribution
- ✓ Large spatial distribution
- ✓ Interplays among them
- ✓ Odd and even system

Extending the nucleus existence limit by continuum



Halo in deformed nuclei? deformed halo?



- Deformed single particle model: square well, spin-orbit interaction neglected

  "in the limit of very small binding energy the valence particles in specific orbitals, characterized by a very small Λ, can give rise to the halo structure which is completely decoupled from the rest of the system"

  Misu_Nazarewicz_Aberg1997_NPA614-44

- Deformed Skyrme Hartree-Fock model: by blocking the first $1/2^+$ neutron level (not ground state), a large deformation is found in ^{11}Be

  "Such a large deformation can result in a good agreement between the calculated and experimental density distributions for the nucleus"

  Pei_Xu_Stevenson2006_NPA765-29

- . . .

Outline

- Introduction
- Deformed RHB theory in continuum
- Magnesium isotopes
- Pairing effects on halo
- Summary & Perspectives

CDFT in a Woods-Saxon basis

Woods-Saxon basis is a reconciler between the HO basis and r space

Shapes	Model	Schrödinger W-S basis	Dirac W-S basis	
Spherical	Rel. Hartree Zhou_Meng_Ring2003_PRC91-262501	SRH SWS	SRH DWS	√
Axially deformed	Rel. Hartree + BCS Zhou_Meng_Ring2006_AIP Conf. Proc. 865-90		DRH DWS	√
Axially deformed	Rel. Hartree-Bogoliubov Zhou_Meng_Ring2007_ISPUN Proc. Zhou_Meng_Ring_Zhao2010_PRC82-011301R Zhou_Meng_Ring_Zhao2011_JPConfProc312-092067 Li_Meng_Ring_Zhao_Zhou2012_PRC85-024312 Li_Meng_Ring_Zhao_Zhou2012_ChinPhysLett29-042101		DRHB DWS	√

Density dependent DRHB theory in continuum
Chen_Li_Liang_Meng2012_PRC85-067301

DDRHF(B): Long, Ring, Meng & Van Giai, PRC81 , 031302

Schunck_Egido2008_PRC77-011301R; PRC78-064305

DD Relativistic Hartree-Bogoliubov equation

RHB equation

$$\begin{pmatrix} h_D - \lambda & \Delta \\ -\Delta^* & -h_D^* + \lambda \end{pmatrix} \begin{pmatrix} U_k \\ V_k \end{pmatrix} = E_k \begin{pmatrix} U_k \\ V_k \end{pmatrix}$$

where E_k is the quasiparticle energy, λ is the chemical potential

Dirac Hamiltonian

$$h_D = \alpha \cdot \mathbf{p} + V(\mathbf{r}) + \beta(m + S(\mathbf{r})) + \Sigma_{\rm rear}(\mathbf{r})$$

where the scalar and vector potentials are, respectively,

$$S(\mathbf{r}) = g_\sigma(\rho_v)\sigma(\mathbf{r}),$$

$$V(\mathbf{r}) = g_\omega(\rho_v)\omega_0(\mathbf{r}) + g_\rho(\rho_v)\tau_3\rho_0(\mathbf{r}) + e\frac{(1-\tau_3)}{2}A_0(\mathbf{r})$$

and the rearrangement term is

$$\Sigma_{\rm rear}(\mathbf{r}) = \frac{\partial g_\sigma}{\partial \rho_v}\rho_s(\mathbf{r})\sigma(\mathbf{r}) + \frac{\partial g_\omega}{\partial \rho_v}\rho_v(\mathbf{r})\omega_0(\mathbf{r}) + \frac{\partial g_\rho}{\partial \rho_v}\rho_3(\mathbf{r})\rho_0(\mathbf{r}),$$

from density-dependent meson-nucleon couplings Niksic2002PRC, Long2004PRC

$$g_\phi(\rho_v) = \begin{cases} g_\phi(\rho_{\rm sat})a_\phi\frac{1+b_\phi(x+d_\phi)^2}{1+c_\phi(x+d_\phi)^2} & \text{for } \phi = \sigma, \omega, \\ g_\phi(\rho_{\rm sat})\exp[-a_\phi(x-1)] & \text{for } \phi = \rho, \end{cases}$$

with baryonic saturation density of nuclear matter ρ_{sat}, and $x = \rho_v/\rho_{\rm sat}$

DD Relativistic Hartree-Bogoliubov equation

Equations of motion for mesons and photons read

$$(-\Delta + m_\phi)\phi(\mathbf{r}) = s_\phi(\mathbf{r}),$$

where $s_\Phi(r)$ are the corresponding source terms

$$s_\phi(\mathbf{r}) = \begin{cases} -g_\sigma(\rho_v)\rho_s(\mathbf{r}), & \text{for } \sigma, \\ g_\omega(\rho_v)\rho_v(\mathbf{r}), & \text{for } \omega, \\ g_\rho(\rho_v)\rho_3(\mathbf{r}), & \text{for } \rho, \\ e\rho_c(\mathbf{r}), & \text{for Coulomb} \end{cases}$$

with various densities

$$\begin{aligned} \rho_s(\mathbf{r}) &= \sum_{k>0} V_k^\dagger(\mathbf{r})\gamma_0 V_k(\mathbf{r}), \\ \rho_v(\mathbf{r}) &= \sum_{k>0} V_k^\dagger(\mathbf{r}) V_k(\mathbf{r}), \\ \rho_3(\mathbf{r}) &= \sum_{k>0} V_k^\dagger(\mathbf{r})\tau_3 V_k(\mathbf{r}), \\ \rho_c(\mathbf{r}) &= \sum_{k>0} V_k^\dagger(\mathbf{r})\frac{1-\tau_3}{2} V_k(\mathbf{r}), \end{aligned}$$

calculated within the no-sea approximation. The sum over k > 0 runs over the quasiparticle states corresponding to single-particle energies in and above the Fermi sea.

DD Relativistic Hartree-Bogoliubov equation

Quasiparticle wave functions U_k and V_k expended in a spherical Woods-Saxon basis

$$U_k(\boldsymbol{r}\sigma p) = \sum_{n\kappa} u^{(m)}_{k,(n\kappa)} \varphi_{n\kappa m}(\boldsymbol{r}\sigma p)$$
$$V_k(\boldsymbol{r}\sigma p) = \sum_{n\kappa} v^{(m)}_{k,(n\kappa)} \varphi_{n\kappa m}(\boldsymbol{r}\sigma p)$$

where p denotes the upper or lower component of the wavefunction.

- ✓ **Basis wave function**

$$\varphi_{n\kappa m}(\boldsymbol{r}\sigma) = \frac{1}{r}\begin{pmatrix} iG_{n\kappa}(r)\phi_{\kappa m}(\Omega\sigma) \\ -F_{n\kappa}(r)\phi_{-\kappa m}(\Omega\sigma) \end{pmatrix},$$

where the spherical spinors read

$$\phi_{\kappa m}(\Omega\sigma) = \sum_{m_l, m_s} \langle l m_l s m_s | j m \rangle Y_{l m_l}(\Omega) \chi_{\frac{1}{2} m_s}(\sigma)$$
$$\phi_{-\kappa m}(\Omega\sigma) = \sum_{m_l, m_s} \langle \tilde{l} m_l s m_s | j m \rangle Y_{\tilde{l} m_l}(\Omega) \chi_{\frac{1}{2} m_s}(\sigma)$$

$\kappa = \pm(j+1/2)$ for $j = l \mp 1/2$, $j = |\kappa| - 1/2$, $l = j + \mathrm{sign}(\kappa)/2$, $\tilde{l} = j - \mathrm{sign}(\kappa)/2$

DD Relativistic Hartree-Bogoliubov equation

For axially deformed nuclei, **densities, coupling strengths, meson fields** and **potentials** are expanded in terms of Legendre polynomials: Price1987PRC

$$f(\mathbf{r}) = \sum_{\lambda} f_\lambda(r) P_\lambda(\cos\theta), \quad \lambda = 0, 2, 4, \ldots$$

with

$$f_\lambda(r) = \frac{2\lambda+1}{2} \int_{-1}^{1} d(\cos\theta) f(\mathbf{r}) P_\lambda(\cos\theta).$$

To obtain potential energy durface, adiabatic quadrupole deformation constraint is adopted as in [Staszczak2010EPJA], i.e., **augmented Lagrangian method:**

$$\langle H' \rangle = \langle H \rangle + C\left(\langle \hat{Q}_2 \rangle - \mu_2\right)^2 + \lambda\left(\langle \hat{Q}_2 \rangle - \mu_2\right)$$

with

$$\lambda_{n+1} = \lambda_n + 2C\left(\langle \hat{Q}_2 \rangle - \mu_2\right), \qquad \lambda_1 = 0$$

where C is the stiness constant, $\langle \hat{Q}_2 \rangle$ is expectation of quadrupole moment operator, and μ_2 is the given quadrupole moment.

Variation of $\langle H' \rangle$ gives the constraint potential:

$$V_{\text{constr.}} = 2C\left(\langle \hat{Q}_2 \rangle - \mu_2\right) \cdot 2r^2 + \lambda \cdot 2r^2.$$

Parameter set for ph & pp channels

Zhou_Meng_Ring_Zhao2010_PRC82-011301R
Zhou_Meng_Ring_Zhao2011_JPConfProc312-092067
Li_Meng_Ring_Zhao_Zhou2012_PRC85-024312
Li_Meng_Ring_Zhao_Zhou2012_ChinPhysLett29-042101

NL3, PK1, ... $R_{\max} = 20$ fm, $\Delta r = 0.1$ fm

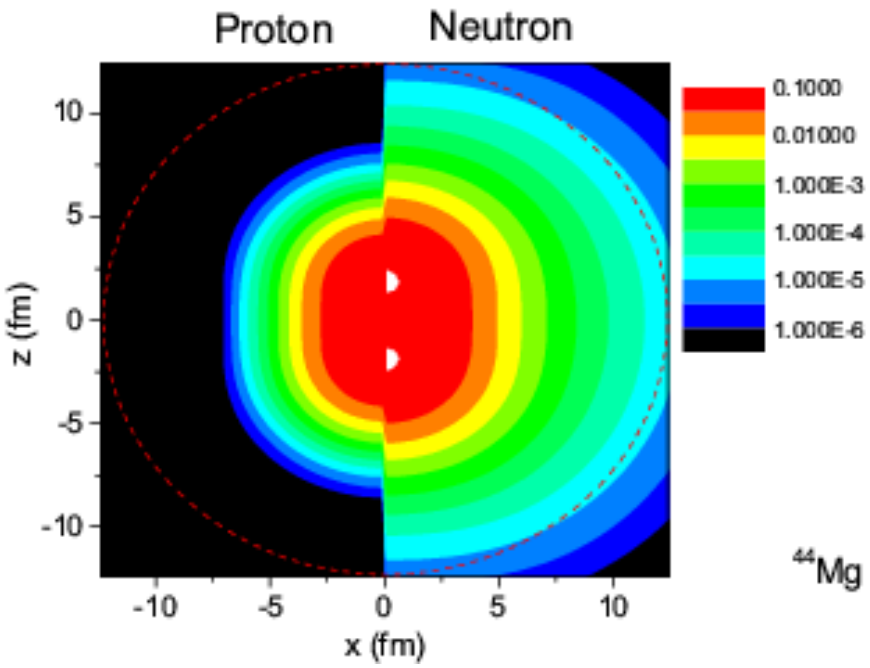
$$V^{\text{pair}} = \frac{1}{4} V_0 \delta(\mathbf{r}_1 - \mathbf{r}_2) \left(1 - \frac{\rho(\mathbf{r}_1)}{\rho_0}\right)^{\gamma}$$

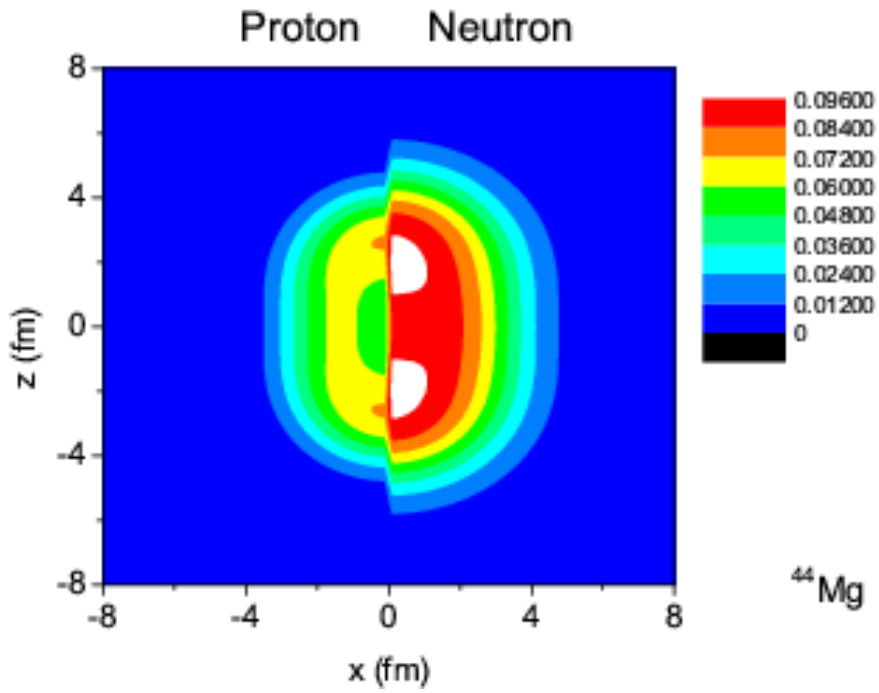
^{20}Mg: spherical from DRHBWS calculation

Model	Pairing force	Parameters	$E^{\text{p}}_{\text{pair}}$ (MeV)
SRHBHO	Gogny	D1S	-9.2382
RCHB	Surface δ	$V_0 = 374$ MeV fm^3	-9.2387
		$\rho_0 = 0.152$ fm^3	
	Sharp cutoff	$E^{\text{q.p.}}_{\text{cut}} = 60$ MeV	
DRHBWS	Surface δ	$V_0 = 380$ MeV fm^3	-9.2383
		$\rho_0 = 0.152$ fm^3	
	Smooth cutoff	$E^{\text{q.p.}}_{\text{cut}} = 60$ MeV	
		$\Gamma = 5.65$ MeV	

^{44}Mg: Density distributions

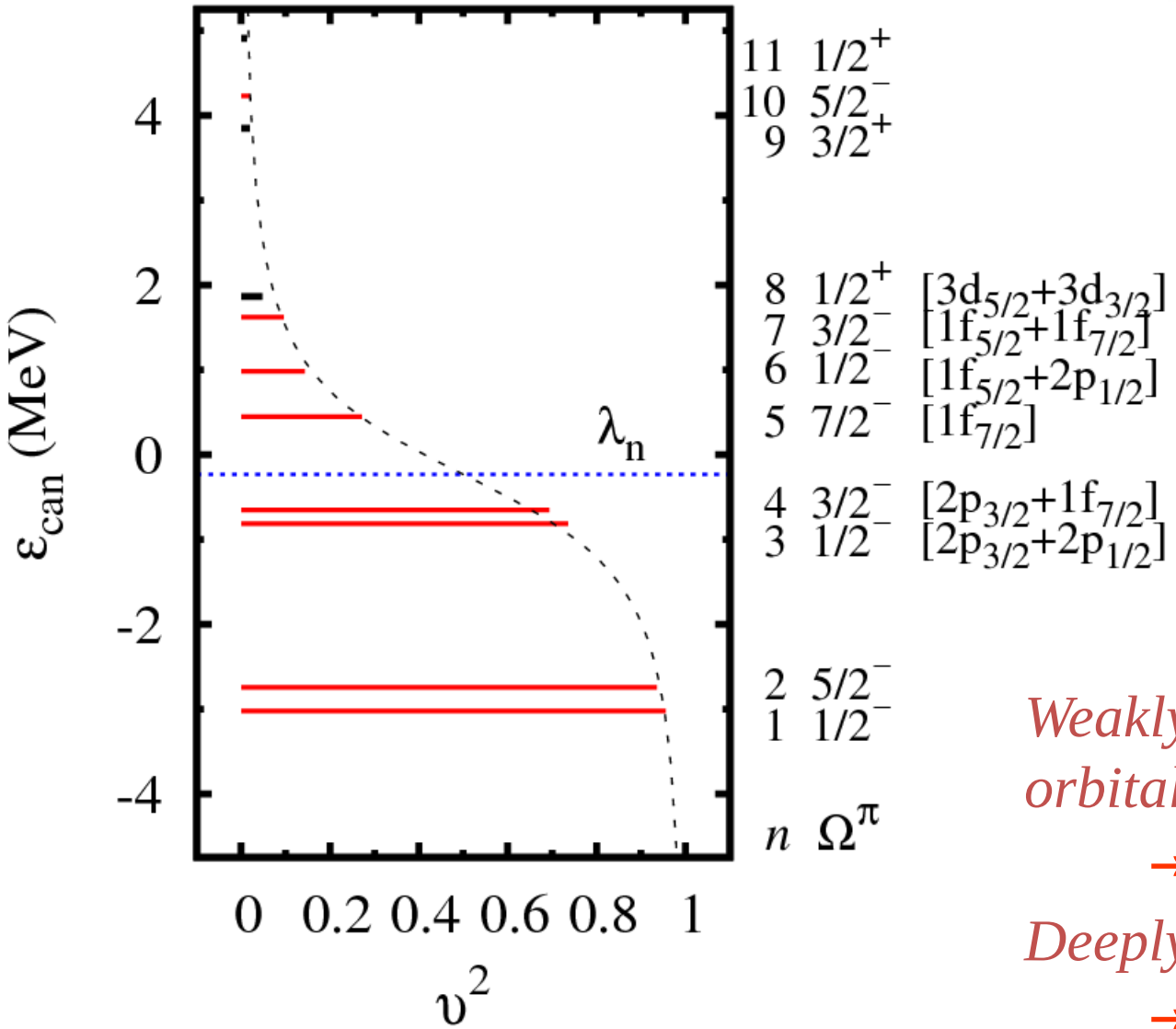
Zhou_Meng_Ring_Zhao2010_PRC82-011301R
Zhou_Meng_Ring_Zhao2011_JPConfProc312-092067
Li_Meng_Ring_Zhao_Zhou2012_PRC85-024312





- Prolate deformation
- Large spatial extension in neutron density distribution

^{44}Mg: Single neutron states in canonical basis



Weakly bound & continuum orbitals

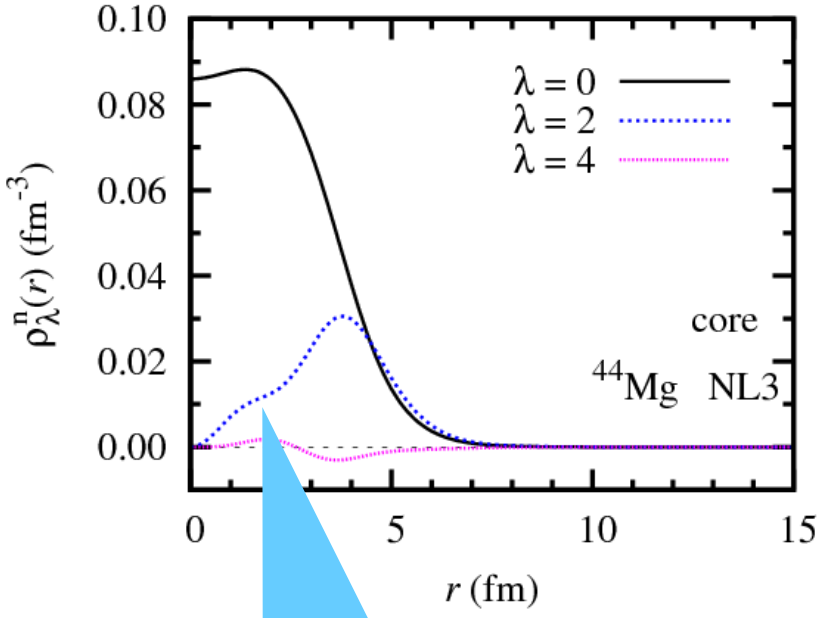
→ halo

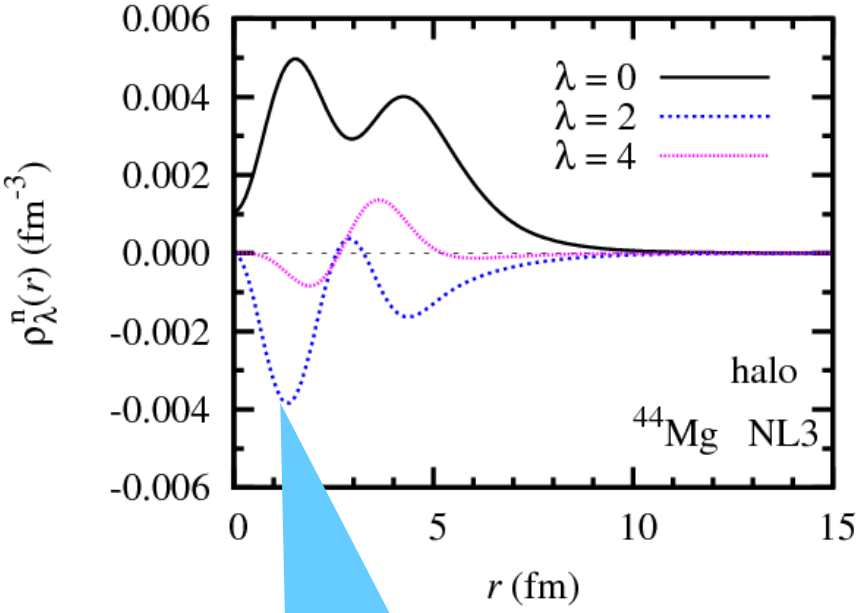
Deeply bound orbitals

→ core

^{44}Mg: Density of core & halo: shape decoupling

$$\rho(\boldsymbol{r}) = \sum_{\lambda} \rho_{\lambda}(r) P_{\lambda}(\cos\theta), \;\; \lambda = 0, 2, 4, \cdots$$

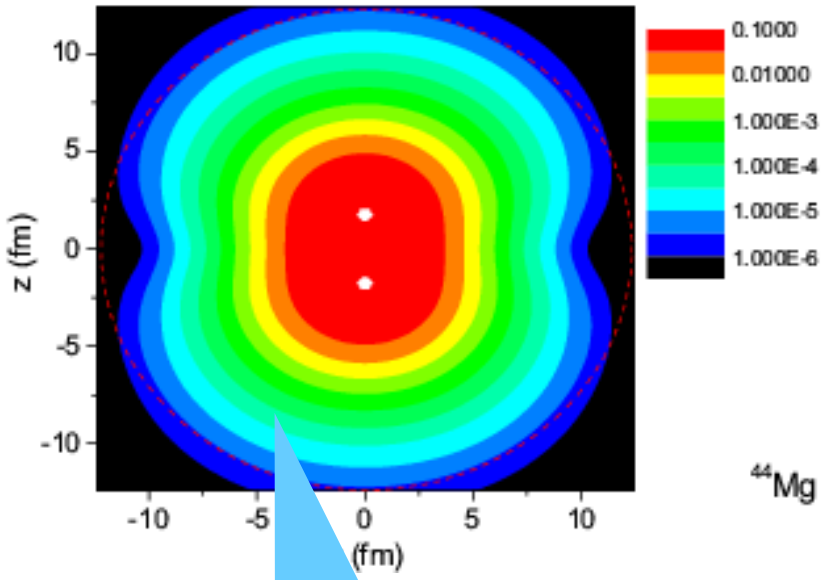


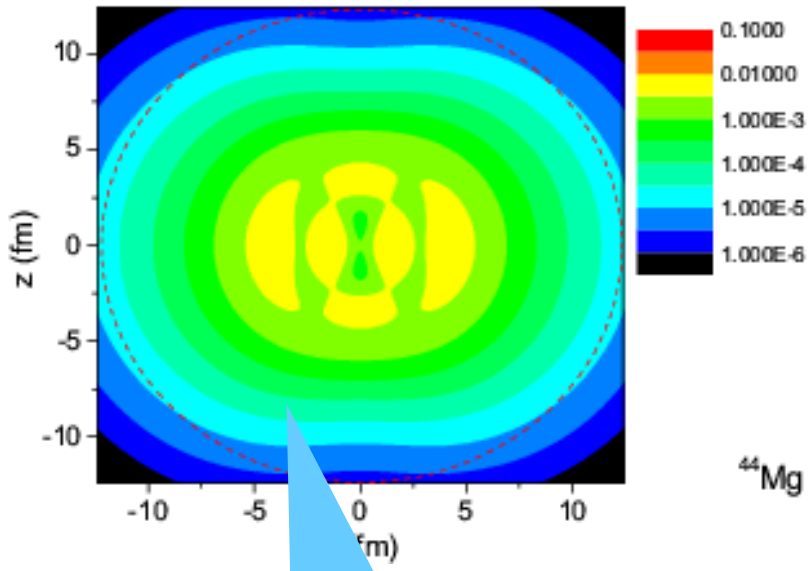


Core: prolate

halo: oblate

^{44}Mg: Density of core & halo: shape decoupling





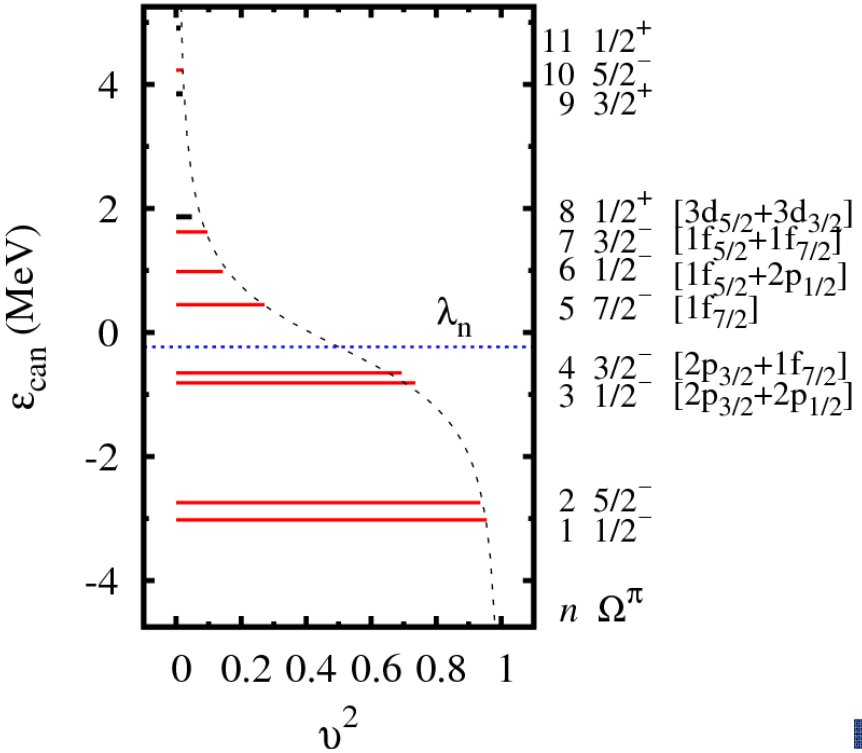
Core: prolate

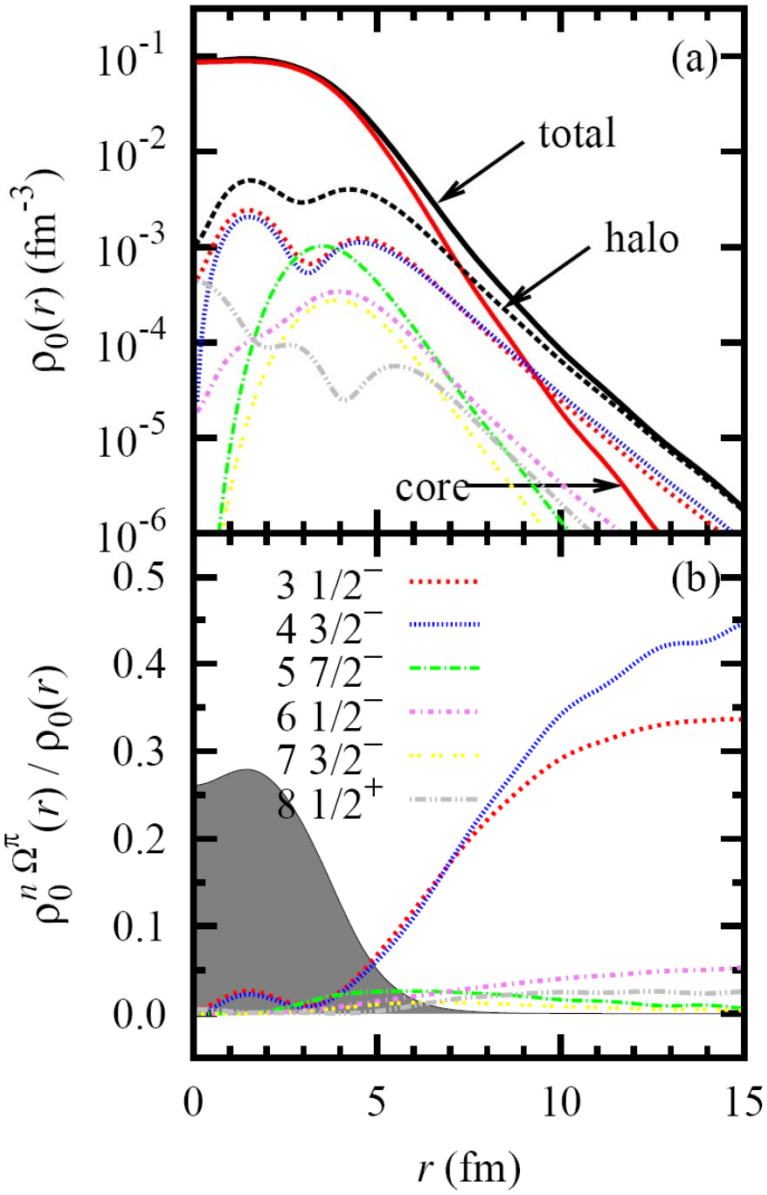
halo: oblate

^{44}Mg: Density of core & halo: shape decoupling

- The 3rd & 4th states contribute to tail part of neutron density distribution
- Main component: $2p_{3/2}$

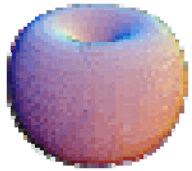
$R_{\rm core} = 3.72$ fm, $R_{\rm halo} = 5.86$ fm





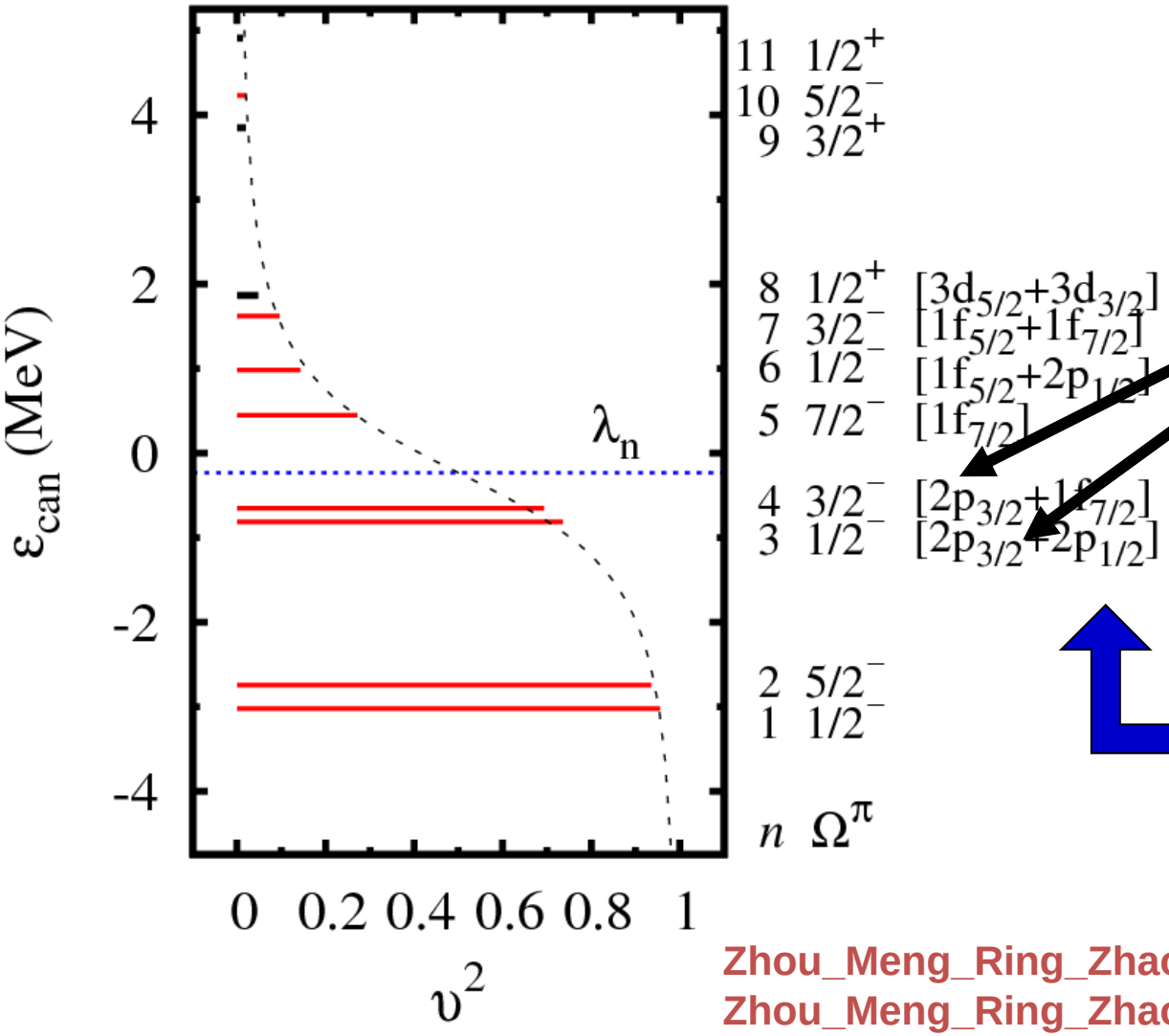
Shape of low-Λ single particle orbital

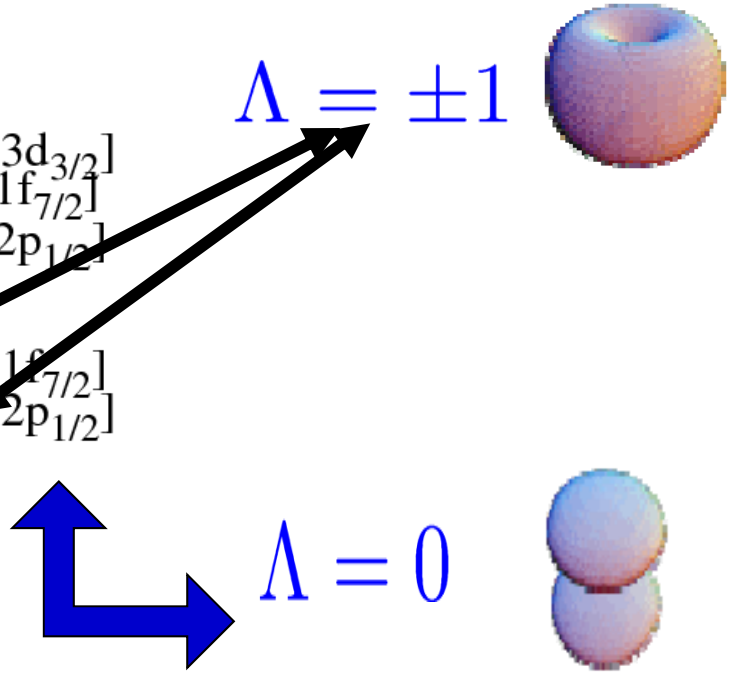
$l = 1, \Lambda = \pm 1 \quad |Y_{1\pm1}(\theta,\phi)|^2 \propto \sin^2(\theta)$



$l = 1, \Lambda = 0 \quad |Y_{10}(\theta,\phi)|^2 \propto \cos^2(\theta)$

Mechanism of shape decoupling





Zhou_Meng_Ring_Zhao2010_PRC82-011301R
Zhou_Meng_Ring_Zhao2011_JPConfProc312-092067
Li_Meng_Ring_Zhao_Zhou2012_PRC85-024312

Outline

- Introduction
- Deformed RHB theory in continuum
- Magnesium isotopes
- Pairing effects on halo
- Summary & Perspectives

Numerical details

- **Parameter: PKDD** Long2004PRC
- **Dirac Woods-Saxon basis** Zhou2003PRC, Zhou2010PRC, Li2012PRC, Chen2012PRC
  - ✓ Box size and mesh size: $R_{box} = 20$ fm, $\Delta r = 0.1$ fm;
  - ✓ Angular momentum cutoff: $j_{max} = 19/2$;
  - ✓ Energy cutoff: $E_{cut} = 150$ MeV for positive energy states; the number of negative energy state is the same as that of positive energy state in each (l,j)-block.
- **Cutoff for Legendre expansion:** Zhou2010PRC, Li2012PRC, Chen2012PRC
  - ✓ Potentials and densities: $\lambda_{v,max} = \lambda_{\rho,max} = 4$;
  - ✓ Meson fields: $\lambda_{\Phi,max} = 6$;
  - ✓ Coupling strengths: $\lambda_{g,max} = 10$.

Numerical details

- **Pairing interaction:**

$$V^{pp}(\boldsymbol{r},\boldsymbol{r}') = \frac{V_0}{2}(1-P^{\sigma})(1-\frac{\rho(\boldsymbol{r})}{\rho_{\rm sat}})\delta(\boldsymbol{r}-\boldsymbol{r}')$$

with smooth cutoff

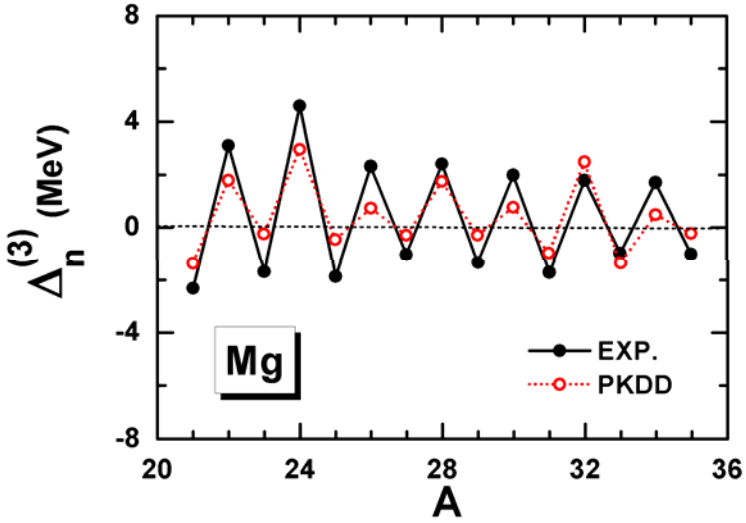
$$s(E_k) = \frac{1}{2}\left(1 - \frac{E_k - E^{\rm q.p.}_{\rm cut}}{\sqrt{(E_k - E^{q.p.}_{\rm cut})^2 + (\Gamma^{\rm q.p.}_{\rm cut})^2}}\right)$$

multiplied in occupation component $V_k(r)$ of each quasiparticle state with $v^2 < 0.5$

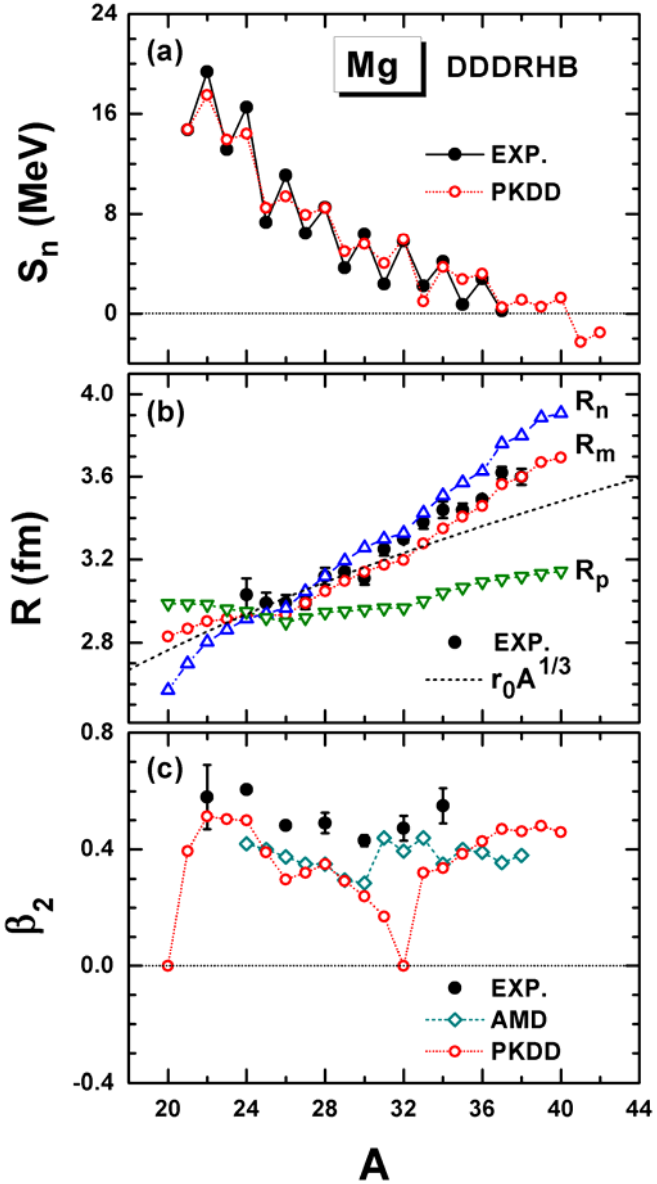
Table: Parameters of pairing correlation determined by reproducing the proton pairing energy of spherical nucleus ^{20}Mg:

Model	Pairing force	Parameters	$E^p_{\rm pair}$ (MeV)
RHB	Gogny	D1S Berger1984NPA	-5.530
DDDRHB	Surface δ force with smooth cutoff	$V_0 = 348.40$ MeV fm ^{3} $\rho_{\rm sat} = 0.152$ fm^{-3} $E^{\rm q.p.}_{\rm cut} = 60.0$ MeV $\Gamma^{\rm q.p.}_{\rm cut} = 5.65$ MeV	-5.530

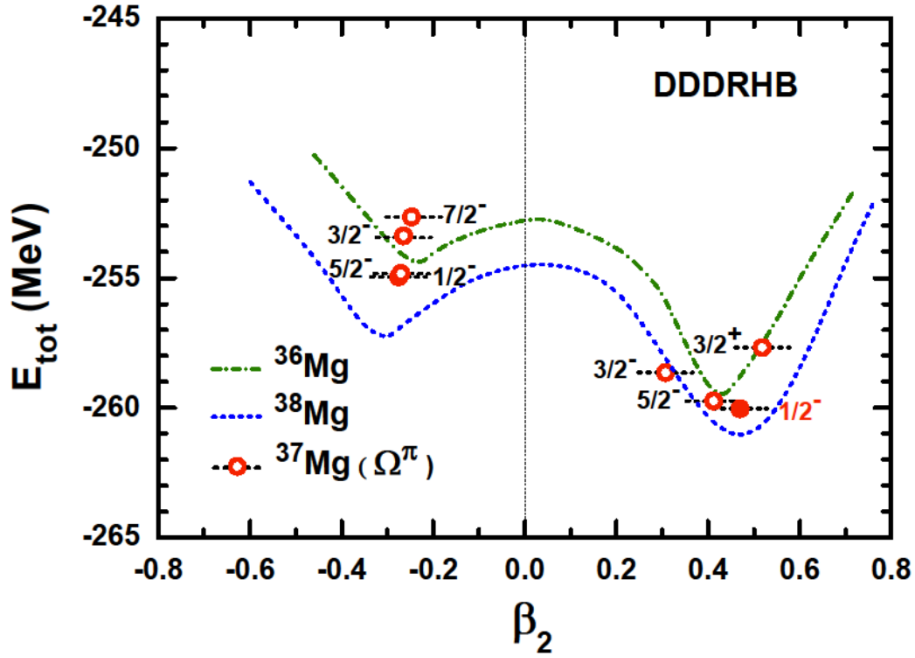
Figure: Comparison between empirical odd-even differences of nuclear masses and calculated values.



Deformed halo in ^{37}Mg and ^{38}Mg



deformed relativistic Hartree-Bogoliubov theory in continuum with blocking and quadruple deformation constrained



Chen, et al, to be published

Outline

- Introduction
- Deformed RHB theory in continuum
- Magnesium isotopes
- Pairing effects on halo
- Summary & Perspectives

Pairing effects on halo

Asymptotic behavior of wave function:

HF Calculation

$$\Psi(r) \sim \exp(-\alpha r), \ \alpha = \sqrt{2m|\varepsilon|/\hbar^2}$$

HFB Calculation

$$\Psi(r) \sim \exp(-\beta r), \ \beta = \sqrt{\frac{2m}{\hbar^2}(E-\lambda)}$$

Root mean square radius for *s* wave with $\varepsilon \to 0, \lambda \to 0$**:**

$$\langle r^2 \rangle_{\mathrm{HF}} \sim \frac{\hbar^2}{2m|\varepsilon|}$$

$$\langle r^2 \rangle_{\mathrm{HFB}} \sim \frac{\hbar^2}{2m\Delta}$$

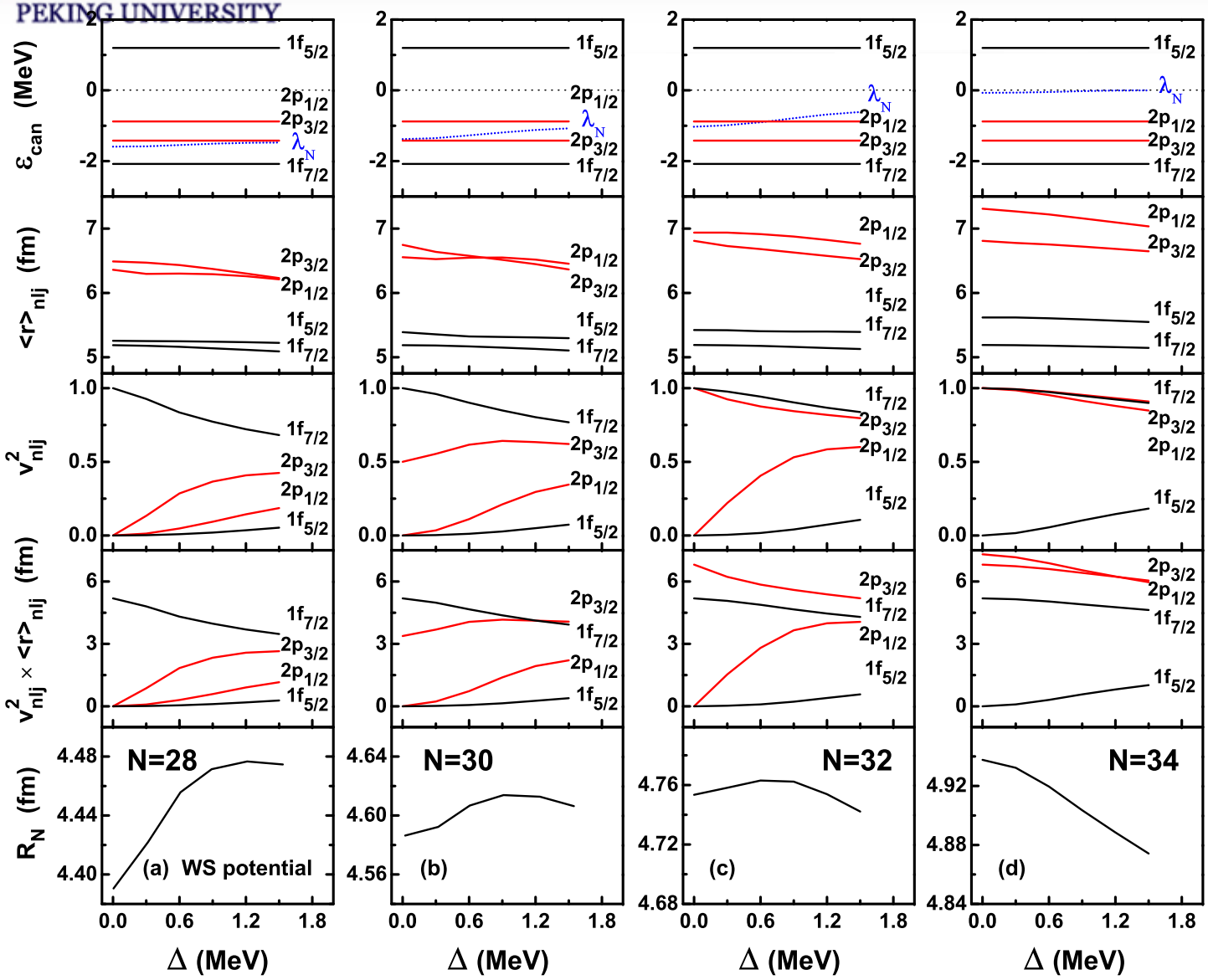
*...Therefore, in even-N nuclei, the paired densities decrease faster than the unpaired densities, which is the effect called here the "**pairing anti-halo effect**"...*

K. Bennaceur, J. Dobaczewski, M. Ploszajczak, Phys. Lett. B, 496, 154 (2000)

K. Hagino, H. Sagawa, Phys. Rev. C, 84, 011303R (2011)

K. Hagino, H. Sagawa, Phys. Rev. C, 85, 014303 (2012)

Radius as a function of pairing potential



Chen2014Phys.Rev.C14312

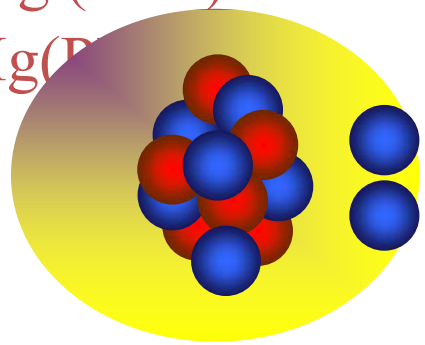
Outline

- Introduction
- Deformed RHB theory in continuum
- Magnesium isotopes
- Pairing effects on halo
- Summary & Perspectives

Summary & perspectives

- Deformed relativistic Hartree-Bogoliubov theory in continuum
  - Woods-Saxon basis --- a reconciler between HO basis & r space
  - Self-consistent description of deformation effects, continuum contribution, large spatial distributions, & possible interplays among them

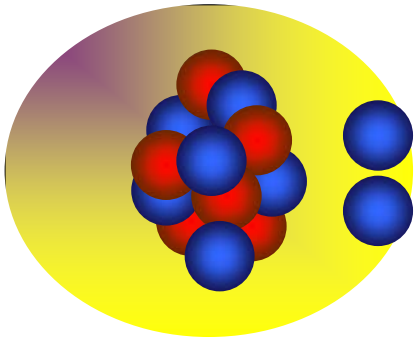
- Neutron halo in deformed nuclei
  - Occurrence of a halo in deformed nuclei depending on intrinsic structure of valence orbitals
  - Shape decoupling of the core and the halo: ^{42,44}Mg (NL3)
  - Odd-even effects in deformed halo nuclei: ^{37,38}Mg(

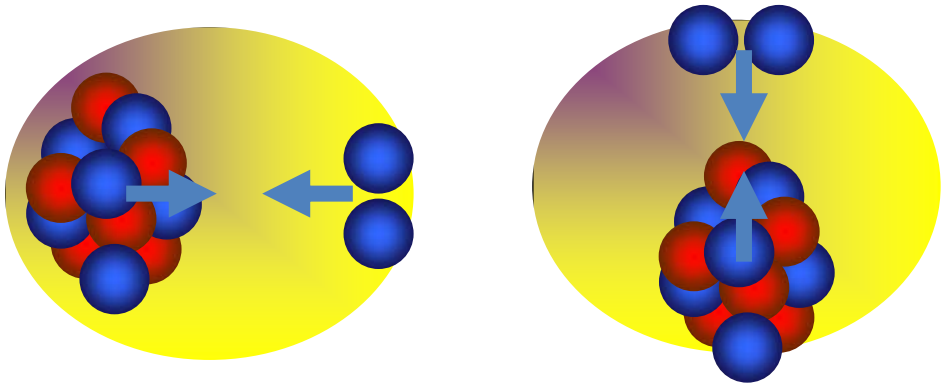


- How to probe shape decoupling ?

How to probe the shape decoupling?

- One of the most exciting frontiers: the neutron rich nuclei – halos in heavy (deformed) nucleus.
- Larger cross section
- Narrower momentum distribution
  - Double-hump ?
- New dipole modes ?
- Rotation ?
- …



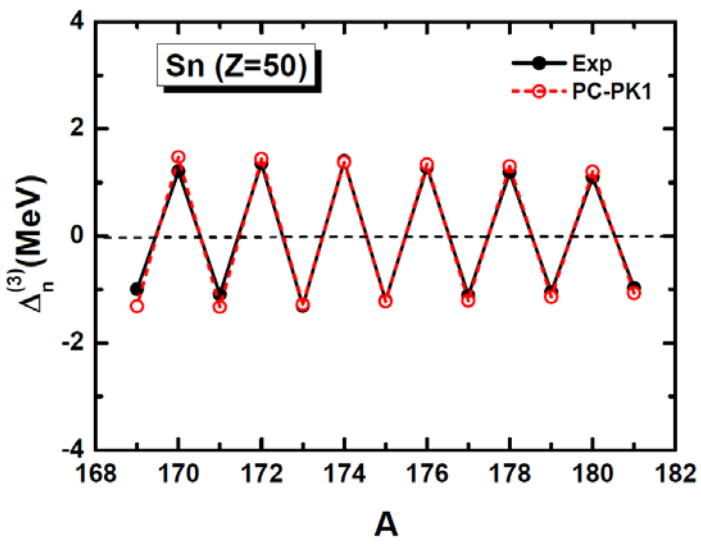


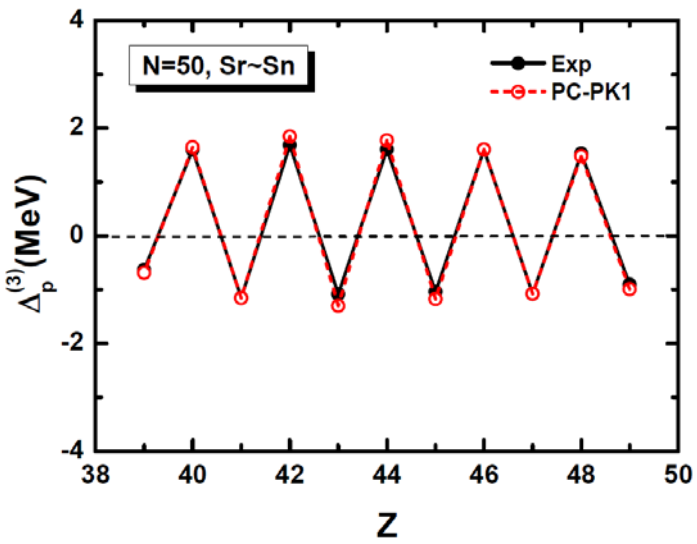
Nucleus existence limit extended by continuum

- PC-PK1: for nucleus with Z=8 to Z=130
- Box size: 20 fm; mesh size: 0.1 fm
- J_{max}=19/2, E_{cut}=100 MeV
- Density-dependent delta pairing force

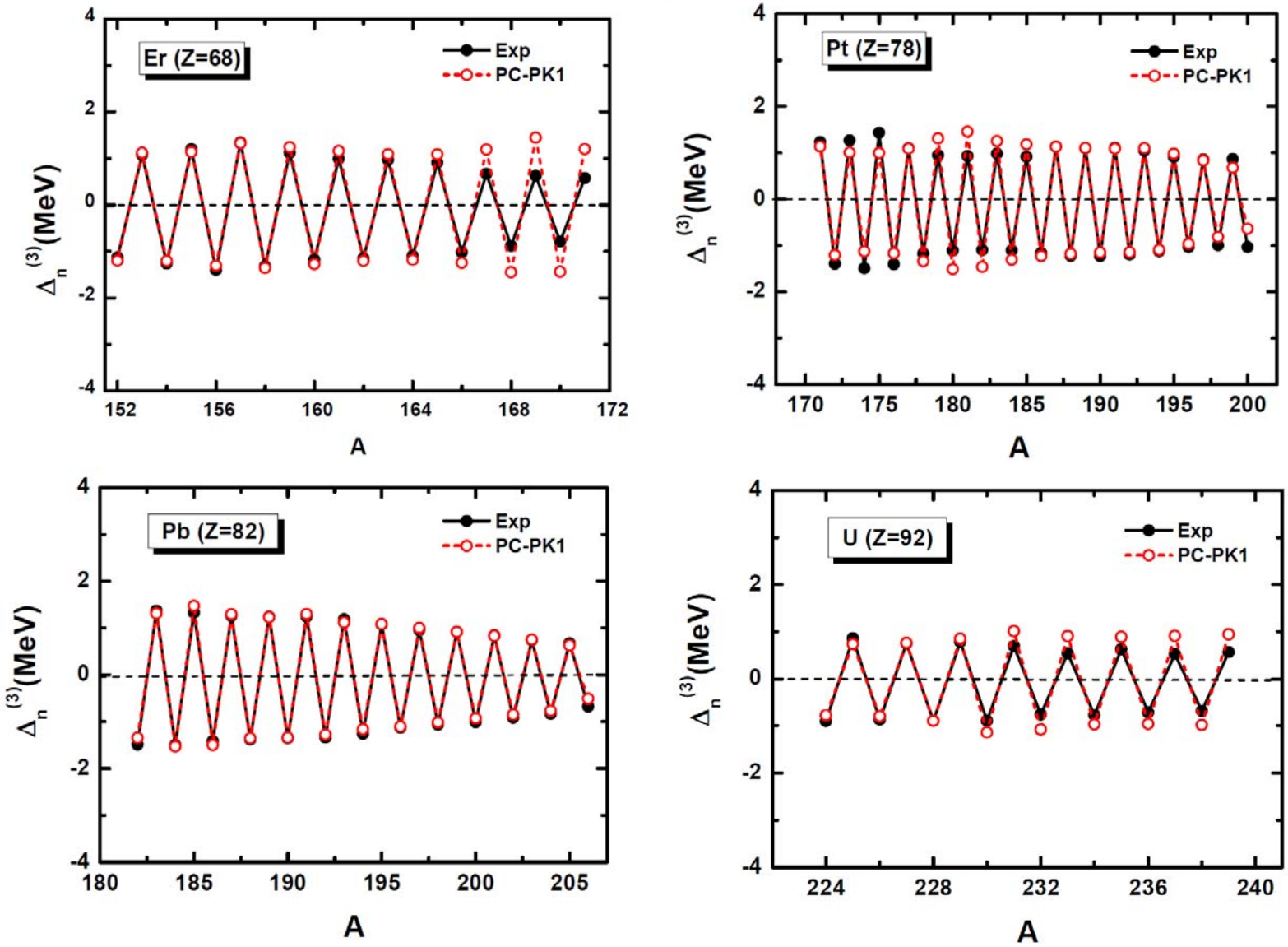
$$V^{pp}(\boldsymbol{r},\boldsymbol{r}') = \frac{V_0}{4}(1-P^{\sigma})\delta(\boldsymbol{r}-\boldsymbol{r}')(1-\frac{\rho(\boldsymbol{r})}{\rho_{sat}})$$

with the saturation density $\rho_{sat} = 0.152\ \mathrm{fm}^{-3}$, and the pairing force strength $V_0 = 685.0\ \mathrm{MeV}\cdot\mathrm{fm}^{-3}$





Pairing force and odd-even staggering



Drip-lines in variant models

PEKING UNIVERSITY

The number of bound nuclides with between 2 and 120 protons is around 7,000

28JUNE2012|VOL486|NATURE|509

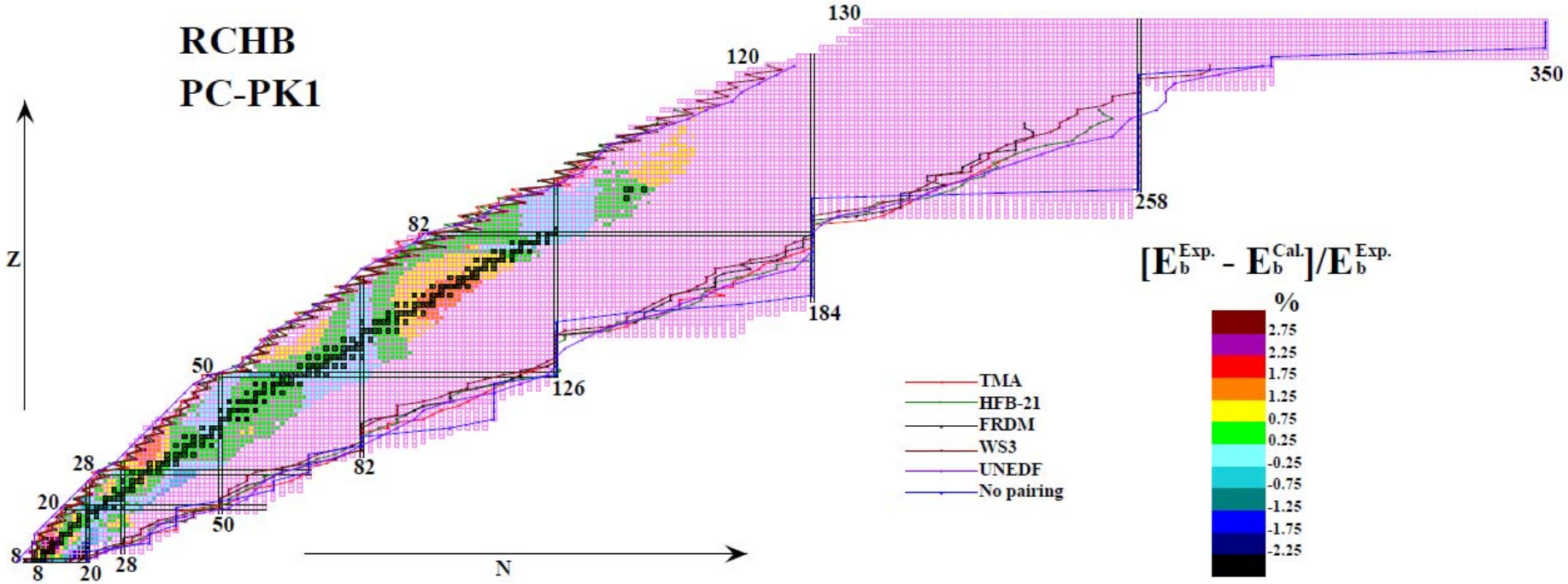


Figure: 10532 bound nuclei from Z=8 to Z=130 predicted by RCHB theory with PC-PK1. For 2227 nuclei with data, binding energy differences between data and calculated results are shown in different color. The nucleon drip-lines predicted TMA, HFB-21, WS3, FRDM , UNEDF and without pairing correlation are plotted for comparison.

Continuum contributions

Afanasjev et al PhysicsLettersB726(2013)680–684

Particle-bound e-e Z<120 nuclei is respectively 2040, 2050, 2057 and 2216 for DD-PC1, DD-ME2, DD-Meδ and NL3*

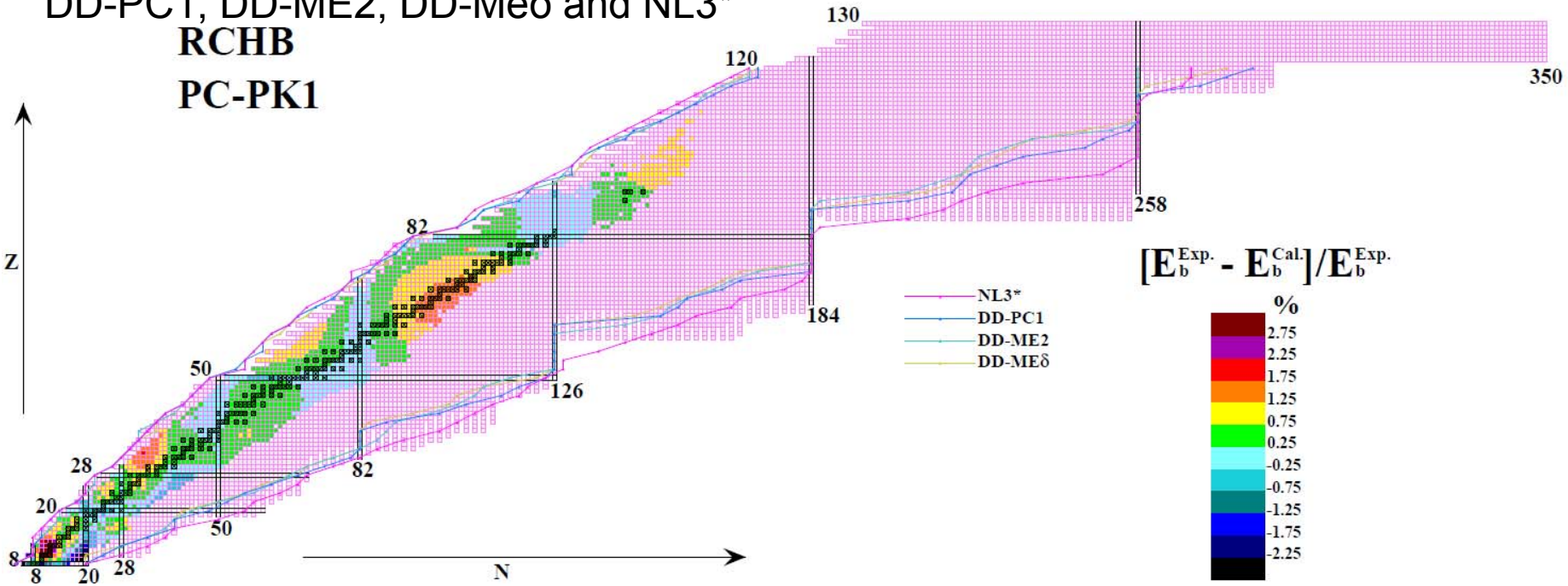
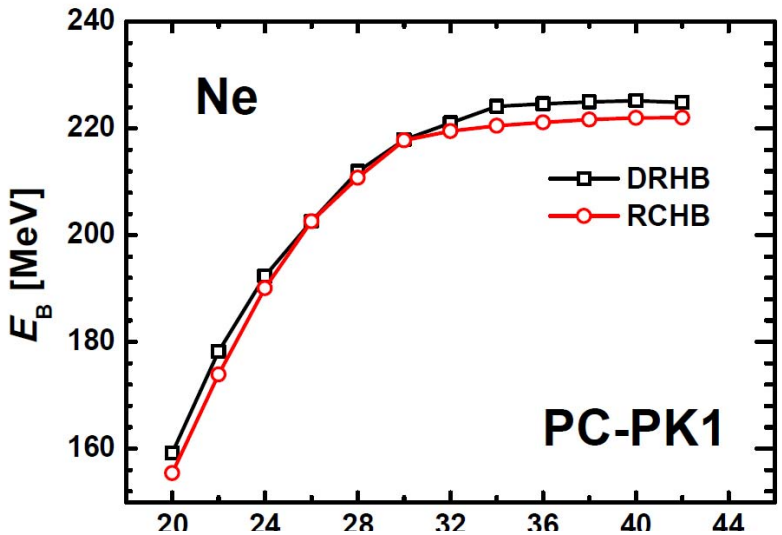
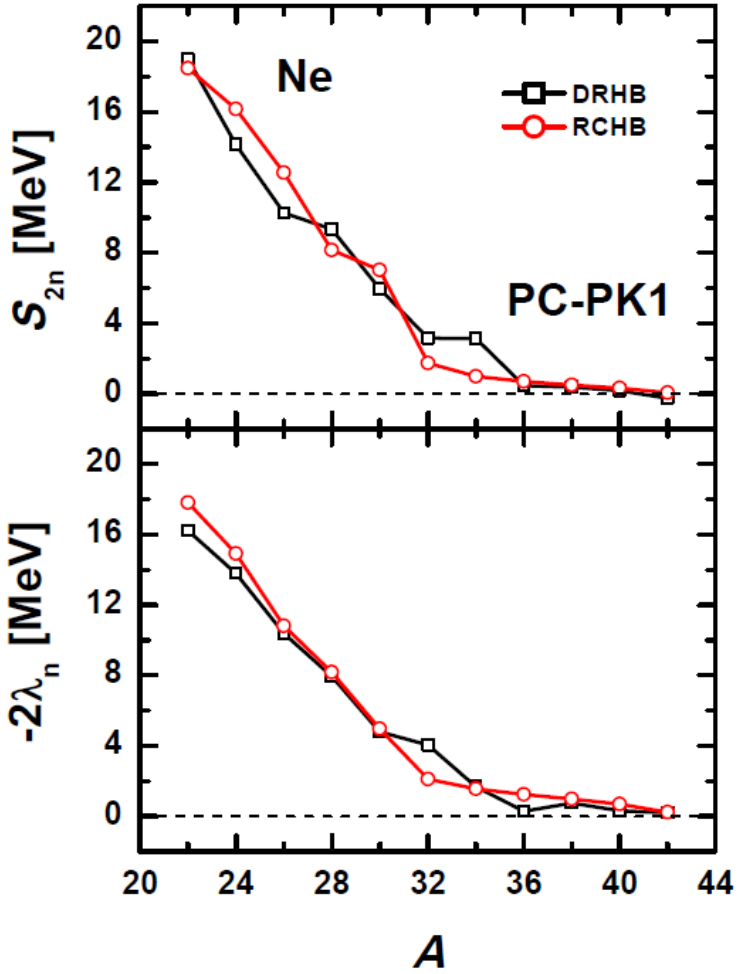
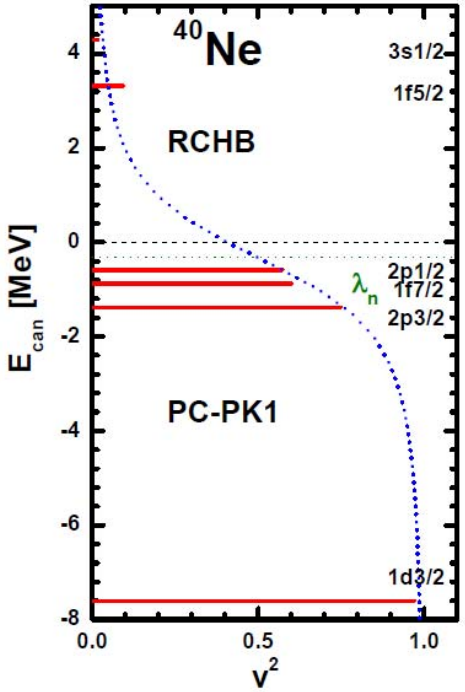


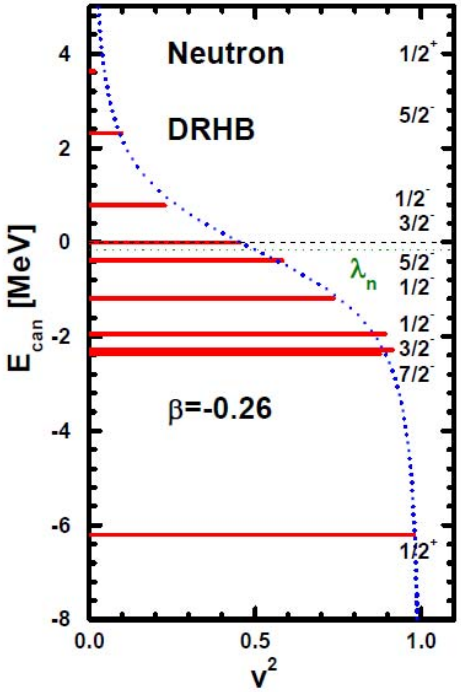
Figure: 10532 bound nuclei from Z=8 to Z=130 predicted by RCHB theory with PC-PK1. For 2227 nuclei with data, binding energy differences between data and calculated results are shown in different color. The nucleon drip-lines predicted without pairing correlation are plotted for comparison.

Deformation: Binding energy









Thank you for your attention!

北京大學
PEKING UNIVERSITY

2004年1月8日

2007-06-16

2011-12-31

2012-12-30