June 2014, RIKEN

Interacting boson model and nuclear mean field

Kosuke Nomura (GANIL)

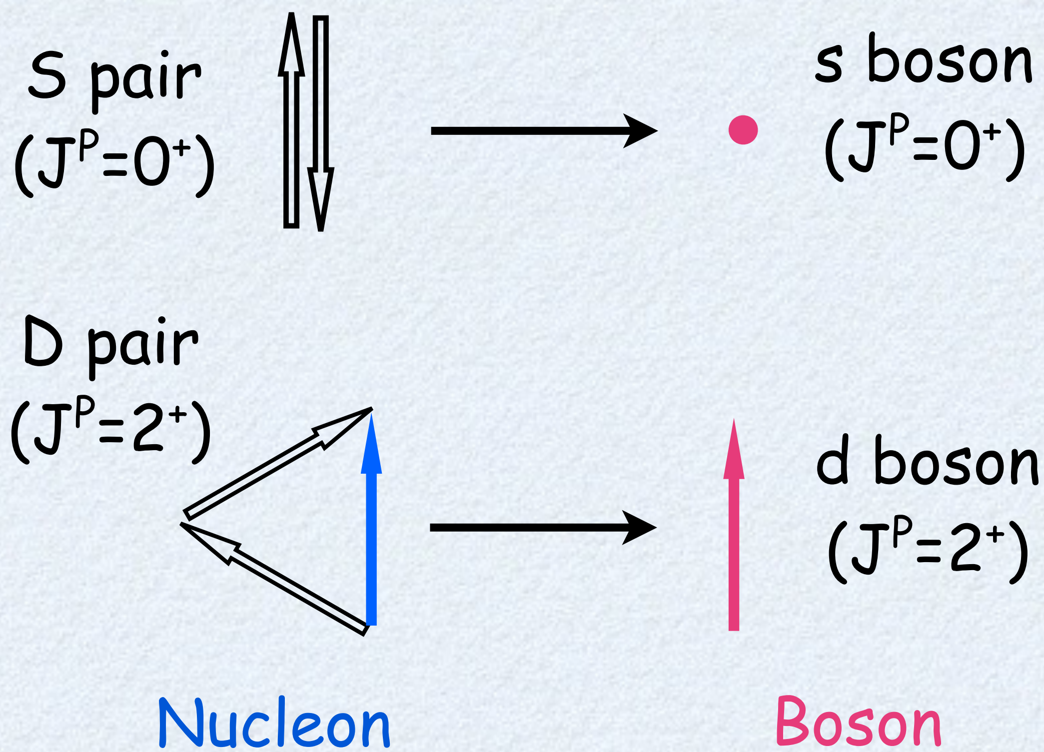
Interacting boson model (IBM)

- Ingredients: collective pairs of valence nucleons
- shell-model derivation. Valid for vibrational and γ-soft nuclei

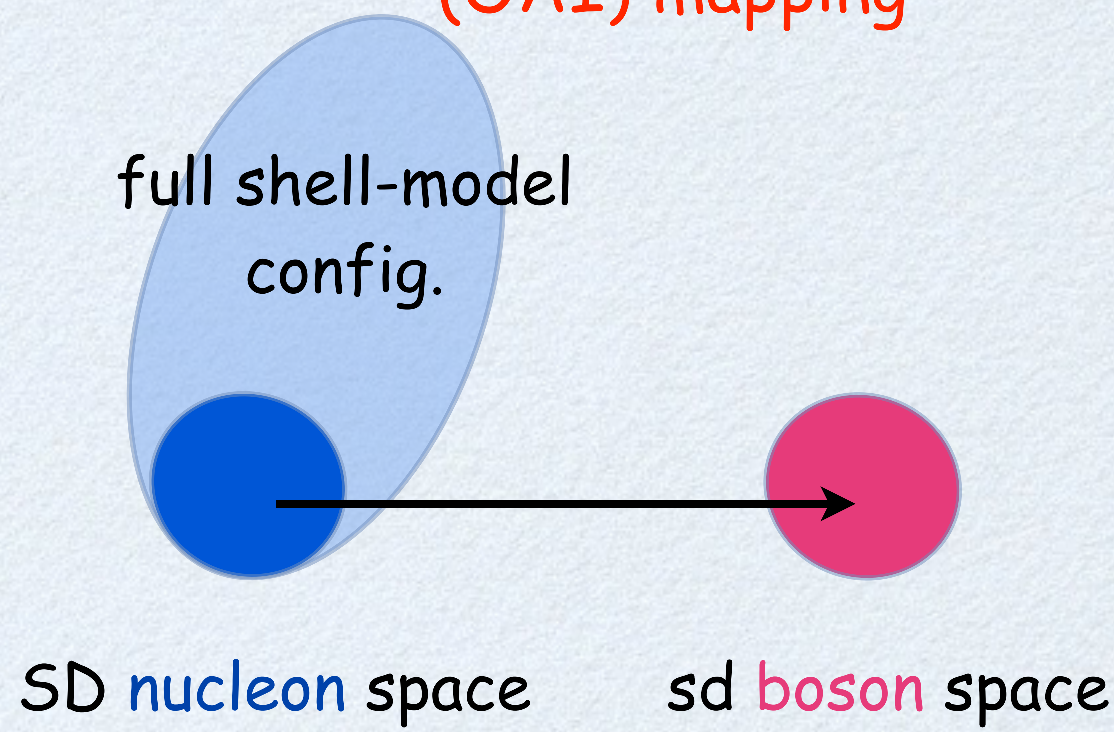
Refs:

- A. Arima, F. Iachello (1974)
- T. Otsuka, A. Arima, F. Iachello (1978)
- T. Mizusaki, T. Otsuka (1996)
- …

Microscopic basis



Otsuka-Arima-Iachello (OAI) mapping



Q. Can we derive IBM Hamiltonian in some unified way ?

Long-standing problem

Physica Scripta, Vol. 22, 468–474, 1980

Features of Nuclear Deformations Produced by the Alignment of Individual Particles or Pairs

Aage Bohr and Ben R. Mottelson

The Niels Bohr Institute and Nordita, Blegdamsvej 15, DK-2100 Copenhagen, Denmark

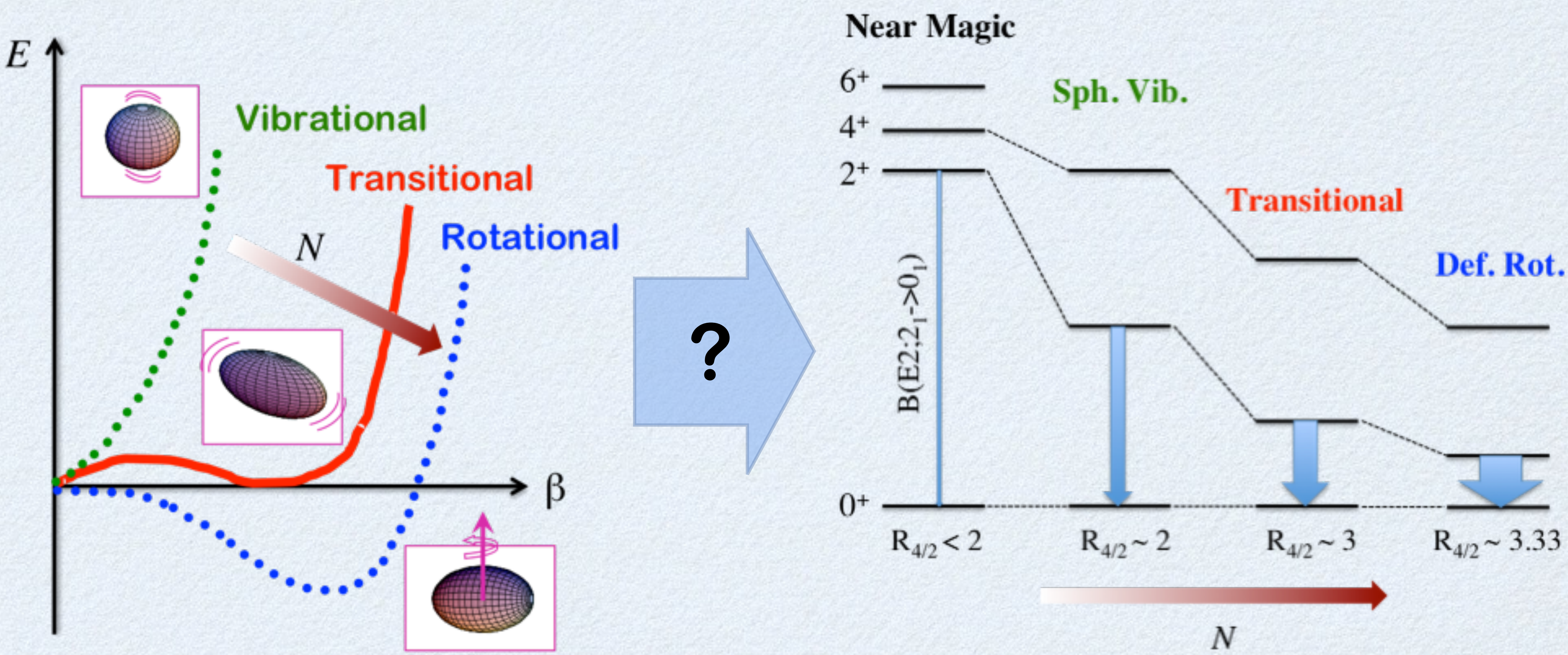
"IBM may not be sufficient to describe some properties of strongly deformed nuclei because of the SD truncation."

Many debates over the validity of the IBM. Concrete answer still missing.

- Nilsson-BCS model (T. Otsuka et al., 1982; D. Bes et al., 1982)
- Renormalization of G pair or inclusion of g boson (e.g., T. Otsuka, Ginocchio, 1985; T. Otsuka, M. Sugita, 1988)
- Conventional boson mapping (e.g., M. R. Zirnbauer, 1984)
- J projection on the intrinsic wave function (N. Yoshinaga et al., 1984)
- … Many others

This work - Principal idea -

- A given mean-field model (Skyrme, Gogny, RMF, etc) can be a good starting point for computing spectra of the intrinsic state of interest.
- We construct an IBM(-2) Hamiltonian by mapping the mean-field solution onto the interacting-boson state.



This talk

A unified way of "deriving" IBM Hamiltonian for all known cases of basic low-lying collective states:

- Vibrational (weakly deformed) nuclei [1,2]
- rotational (strongly deformed) nuclei [3]
- Triaxial (or γ-soft) nuclei [4]
- … (Other relevant cases)

Relevant publications:

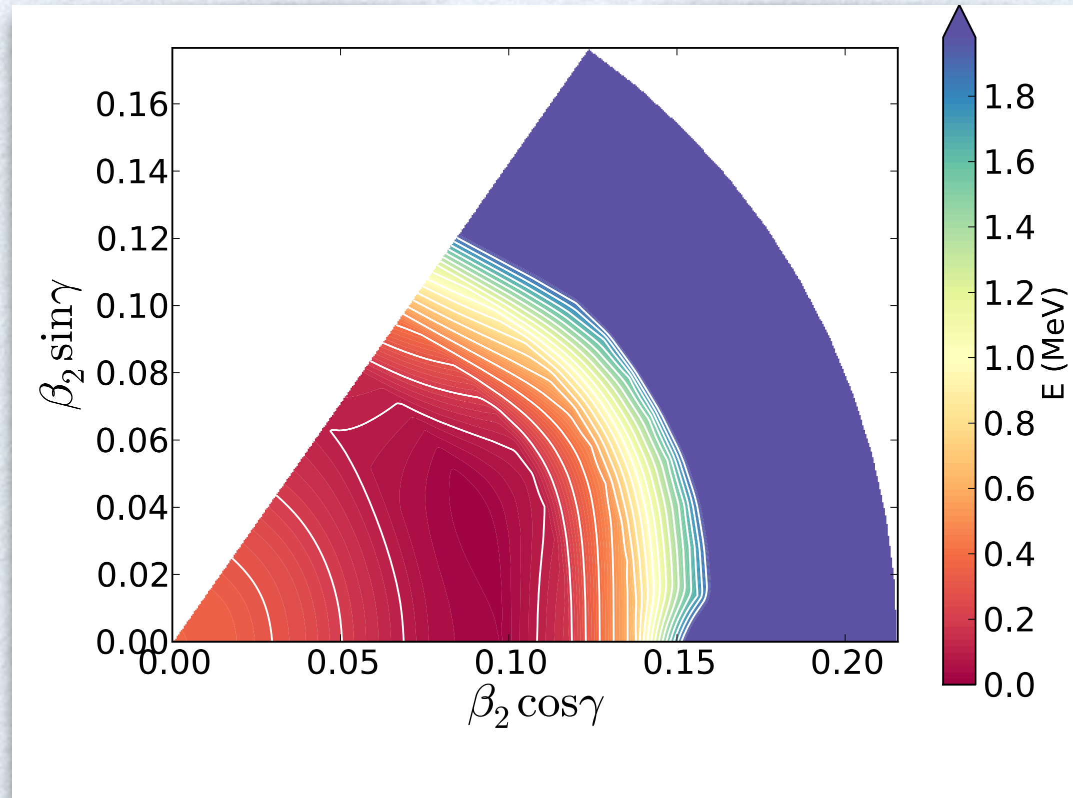
[1] K.N., N. Shimizu, T. Otsuka, PRL101, 142501 (2008)

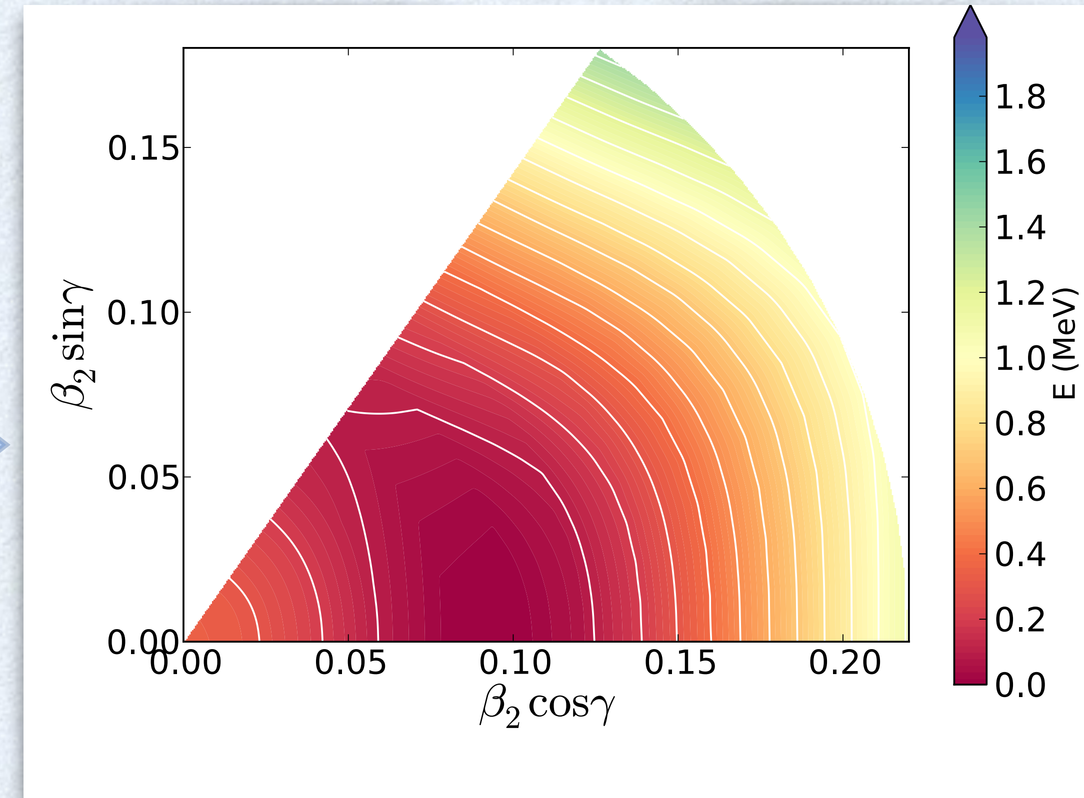
[2] K.N., N. Shimizu, T. Otsuka, PRC81, 044307 (2010)

[3] K.N., T. Otsuka, N. Shimizu, L. Guo, PRC83 041302(R) (2011)

[4] K.N., N. Shimizu, D. Vretenar, T. Nikšić, T. Otsuka, PRL108, 132501 (2012)

Basic case: mapping energy surface





Total energy of constrained HFB within a given mean-field model

Total energy for a boson condensation (energy expectation value of boson system)

An IBM-2 Hamiltonian is determined by the equality $E_{HFB} \approx E_{IBM}$; energy levels and wave functions with good spin, particle number, parity, etc.

Boson condensate

$$|\Phi^B\rangle = \frac{1}{\sqrt{N!}}(\lambda^\dagger)^N|0\rangle \qquad \lambda^\dagger = \sum_{LM} x_M^{(L)} b_{LM}^\dagger$$

- N: [# of bosons]=$(N_n+N_p)/2$
- |0>: inert core
- $x^{(L)}{}_M$: collective coordinate. For sd (L=0,2) system,

$$x_0^{(0)} = 1, \quad x_{\pm 2}^{(2)} = \frac{1}{\sqrt{2}}\beta^B \sin\gamma^B, \quad x_{\pm 1}^{(2)} = 0, \quad x_0^{(2)} = \beta^B \cos\gamma^B$$

Energy expectation value (energy surface)

$$\langle\Phi^B(\beta\gamma)|\hat{H}^B|\Phi^B(\beta\gamma)\rangle$$

γ=0 for prolate, γ=π/3 for oblate

Approximate equality to fix H^B

$$\langle\Phi^F(\beta^F\gamma^F)|\hat{H}^F|\Phi^F(\beta^F\gamma^F)\rangle \approx \langle\Phi^B(\beta^B\gamma^B)|\hat{H}^B|\Phi^B(\beta^B\gamma^B)\rangle$$

Relation with geometrical model: $\beta^B=C\beta^F$ (C>1), $\gamma^B=\gamma^F$

PES fit by wavelet

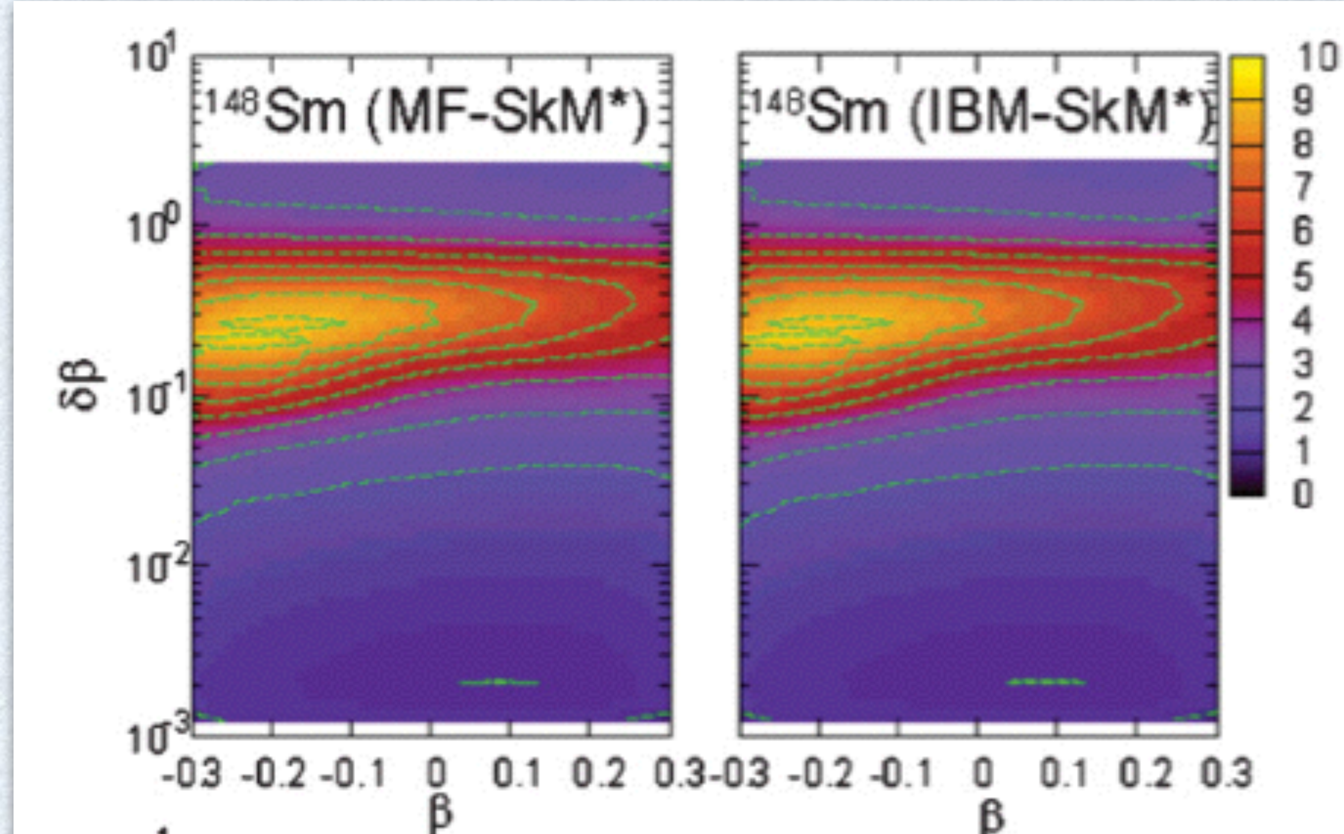
scale (frequency)　PES

$$\tilde{E}(\delta\beta,\beta) = \frac{1}{\sqrt{\delta\beta}}\int E(\beta')\Psi^*\left(\frac{\beta-\beta'}{\delta\beta}\right)d\beta'$$

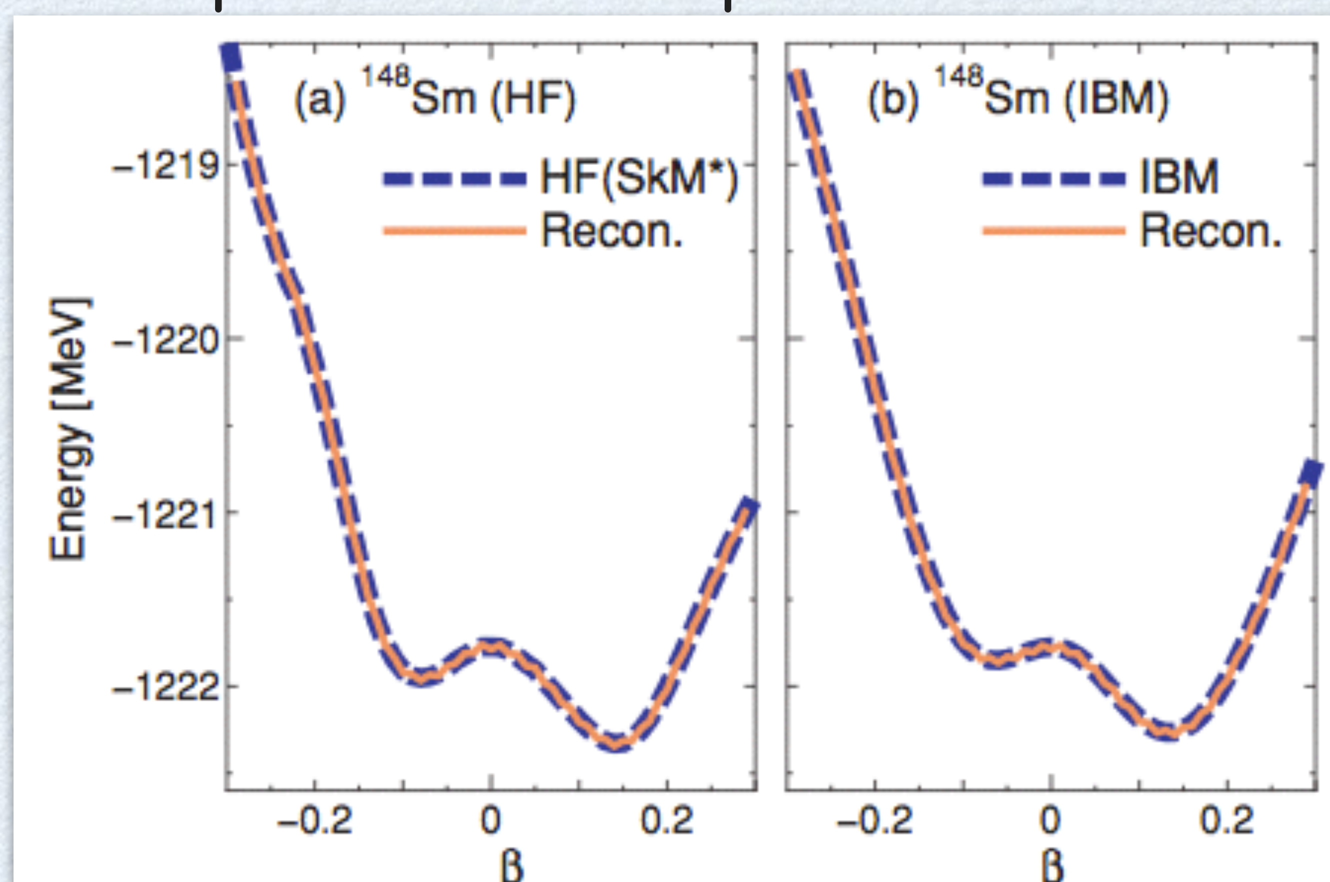
Coordinate　Basis (wavelet)

- Captures characteristic feature of the PES: curvature, minimum etc. to extract IBM parameters unambiguously.
- General rule: Fitting range should be limited to $\beta < \beta_{min}+\Delta\beta$ (do not try to fit $\beta \gg \beta_{min}$)

χ^2 fit for $\tilde{E}(\delta\beta,\beta)$



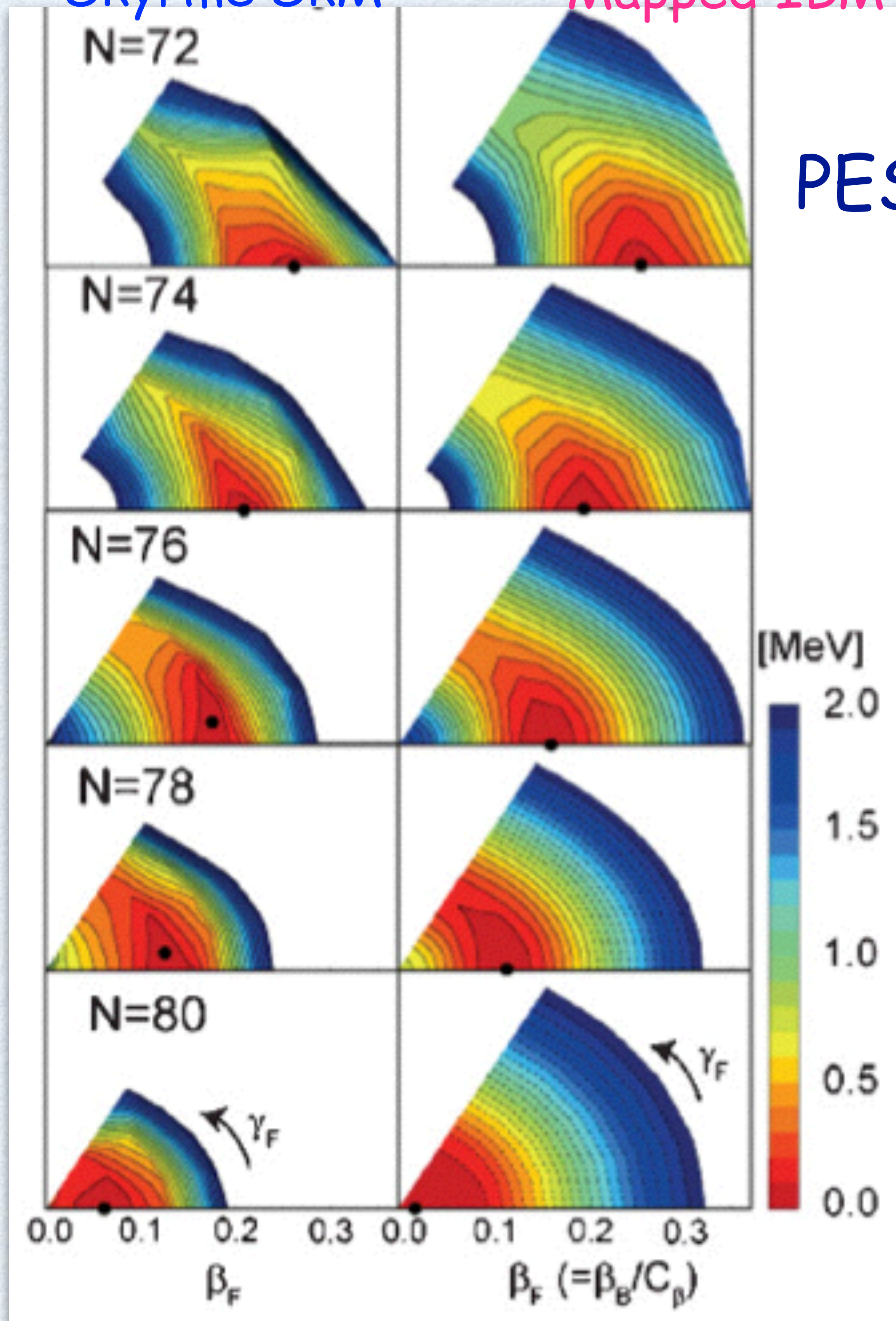
comparison in real space



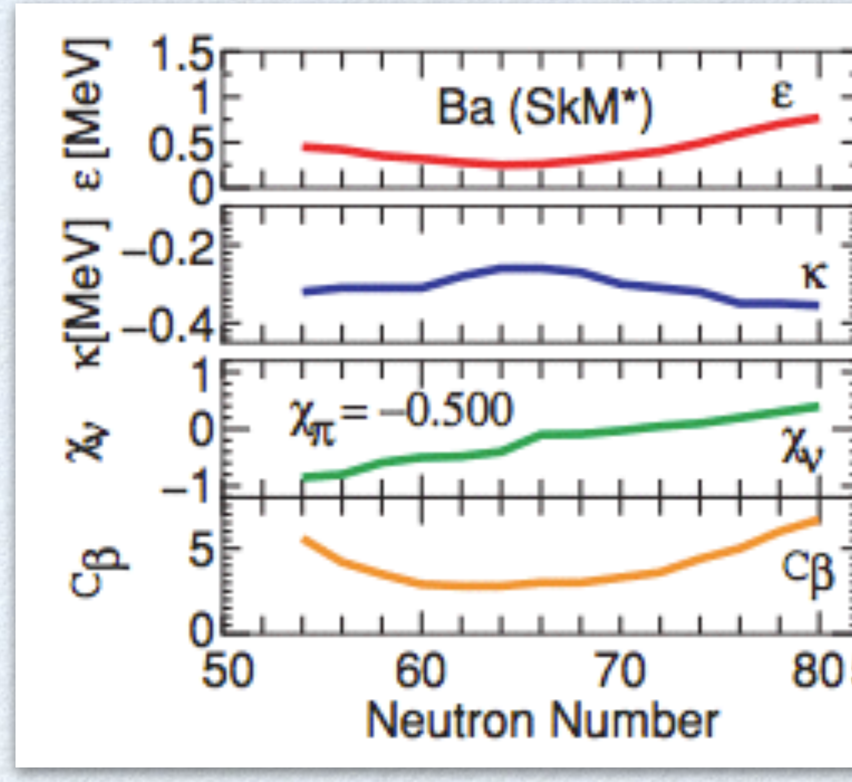
Example: Ba

Skyrme SkM*

Mapped IBM

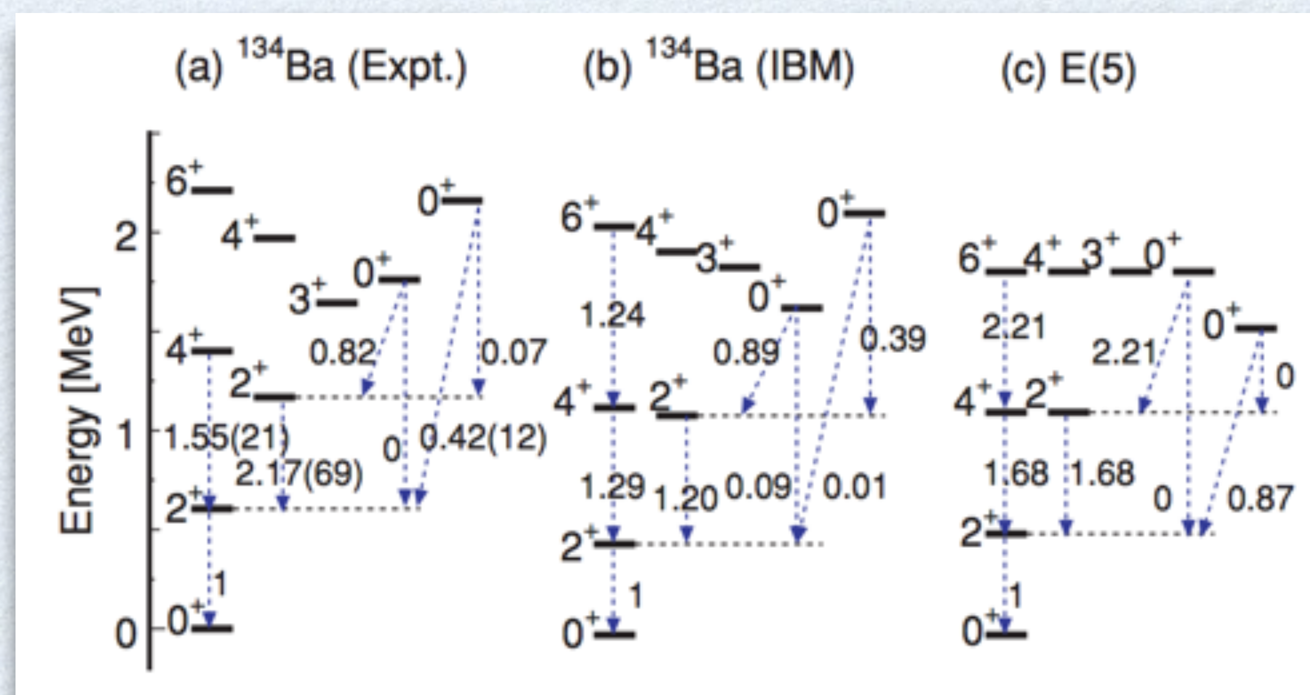


PES



Derived parameters

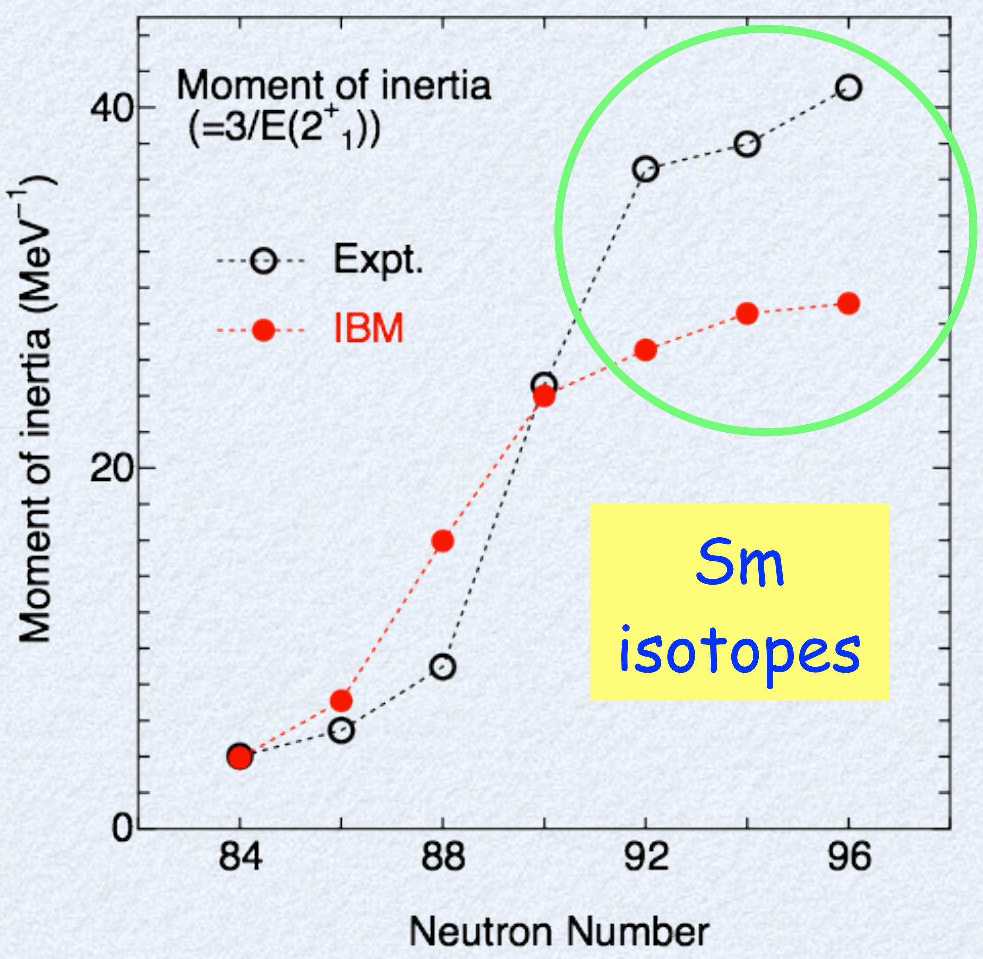
E(5) nucleus ^{134}Ba

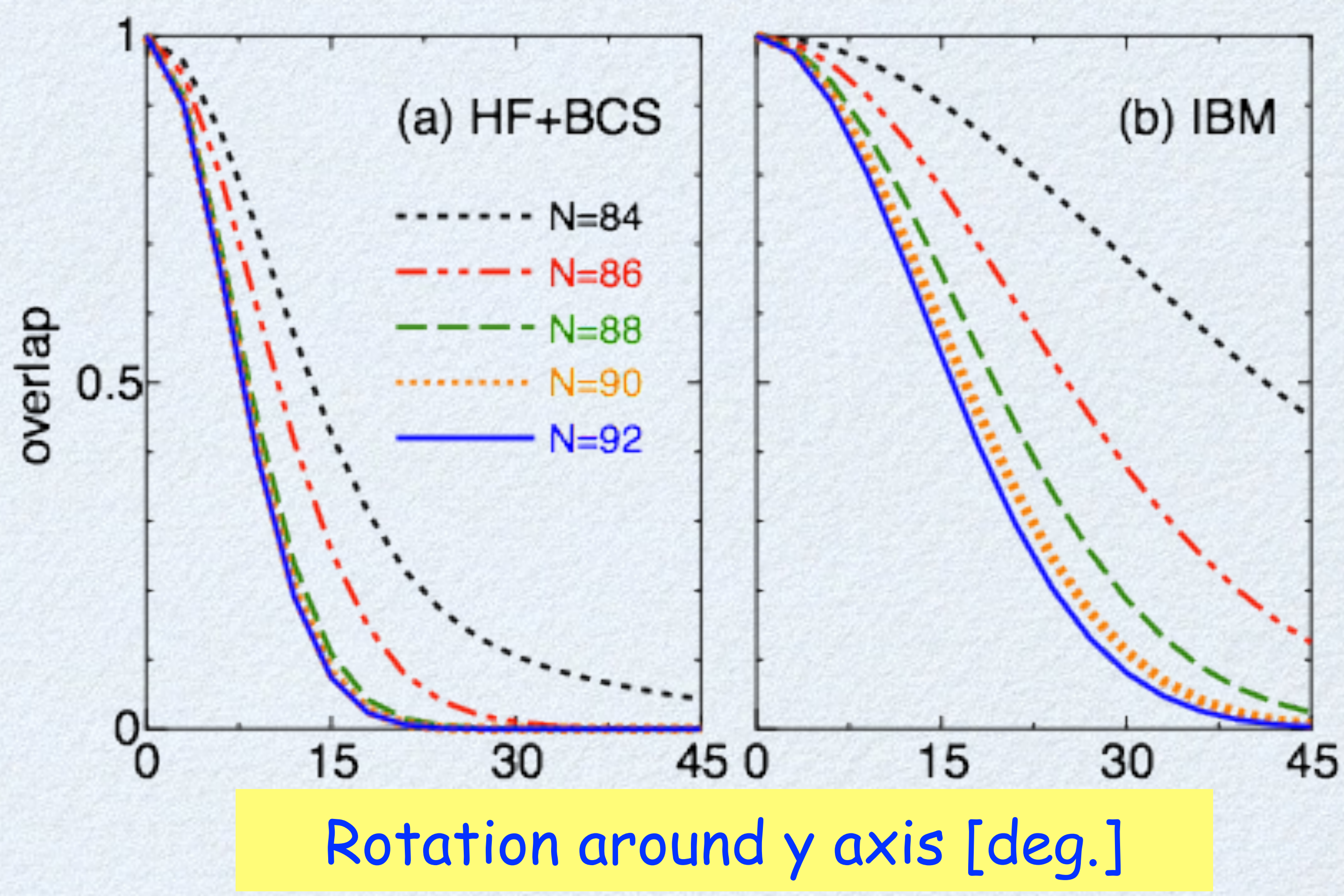


K.N., N. Shimizu, T. Otsuka, PRC81, 044307 (2010)

Problem with deformed nuclei

- Too small moment of inertia for strongly deformed nuclei
- The reason is the difference in the rotational property of intrinsic wave function between nucleon and boson systems.

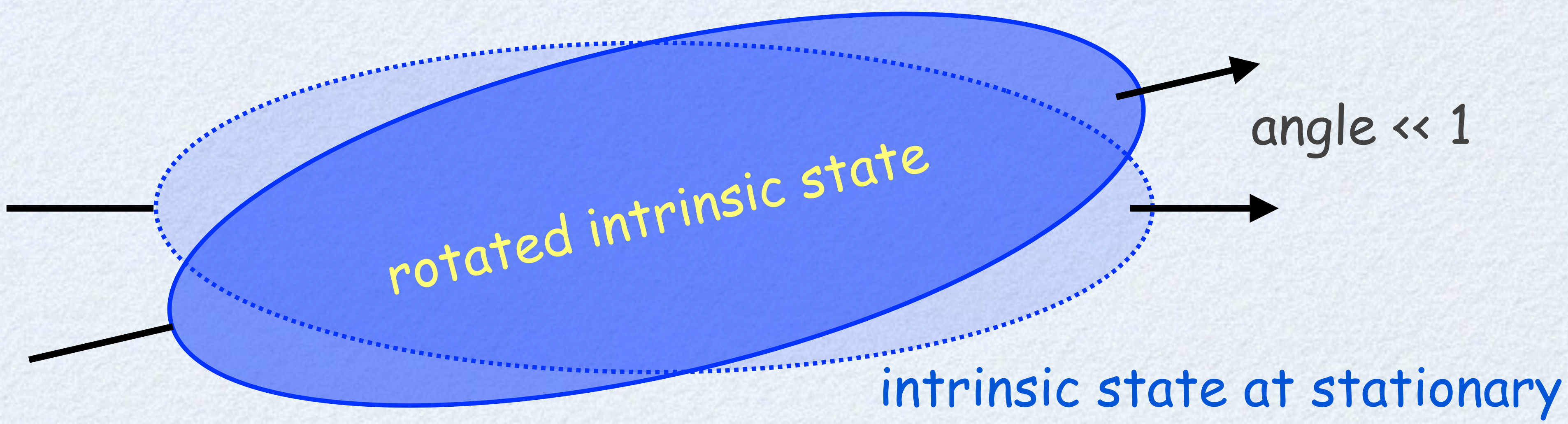




Fermion-boson mapping for deformed nuclei

- A basic rotational property should be also reproduced by bosons. We consider rotational response (= energy shift due to infinitesimal rotation of nucleon intrinsic state).
- The missing piece is the rotational LL term in the IBM, giving correct moment of inertia.

- Intuitive picture -



"Full" boson Hamiltonian

$$\hat{H}^B = \underbrace{\epsilon(\hat{n}_{d\pi} + \hat{n}_{d\nu}) + \kappa \hat{Q}_\pi \cdot \hat{Q}_\nu}_{} + \underbrace{\kappa' \hat{L} \cdot \hat{L}}_{}$$

Step 1) "Basic part": mapping of the PES. Valid for near-spherical and γ-unstable cases

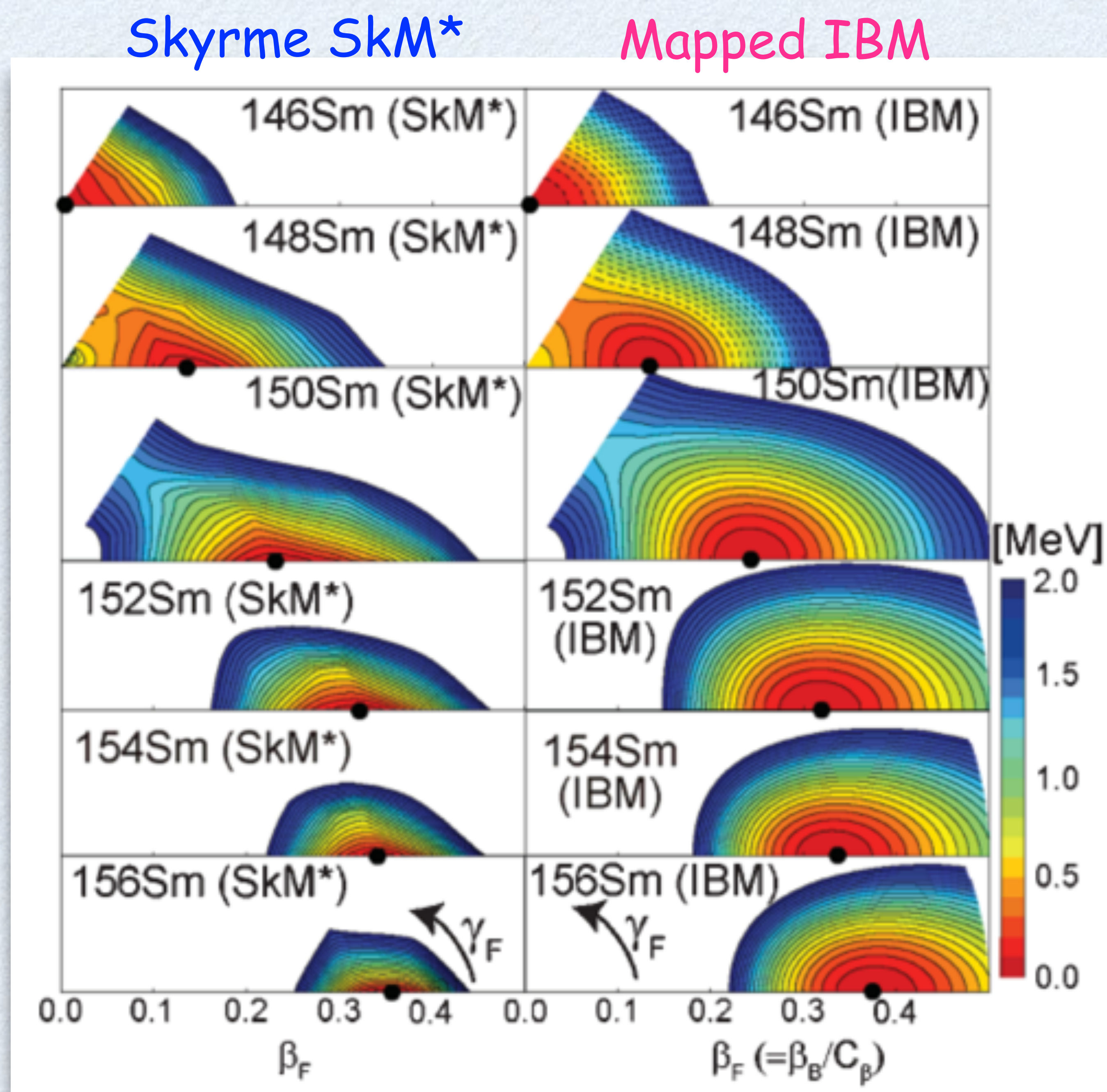
$$\langle \phi^F | \hat{H}^F | \phi^F \rangle \sim \langle \phi^B | \hat{H}^B | \phi^B \rangle$$

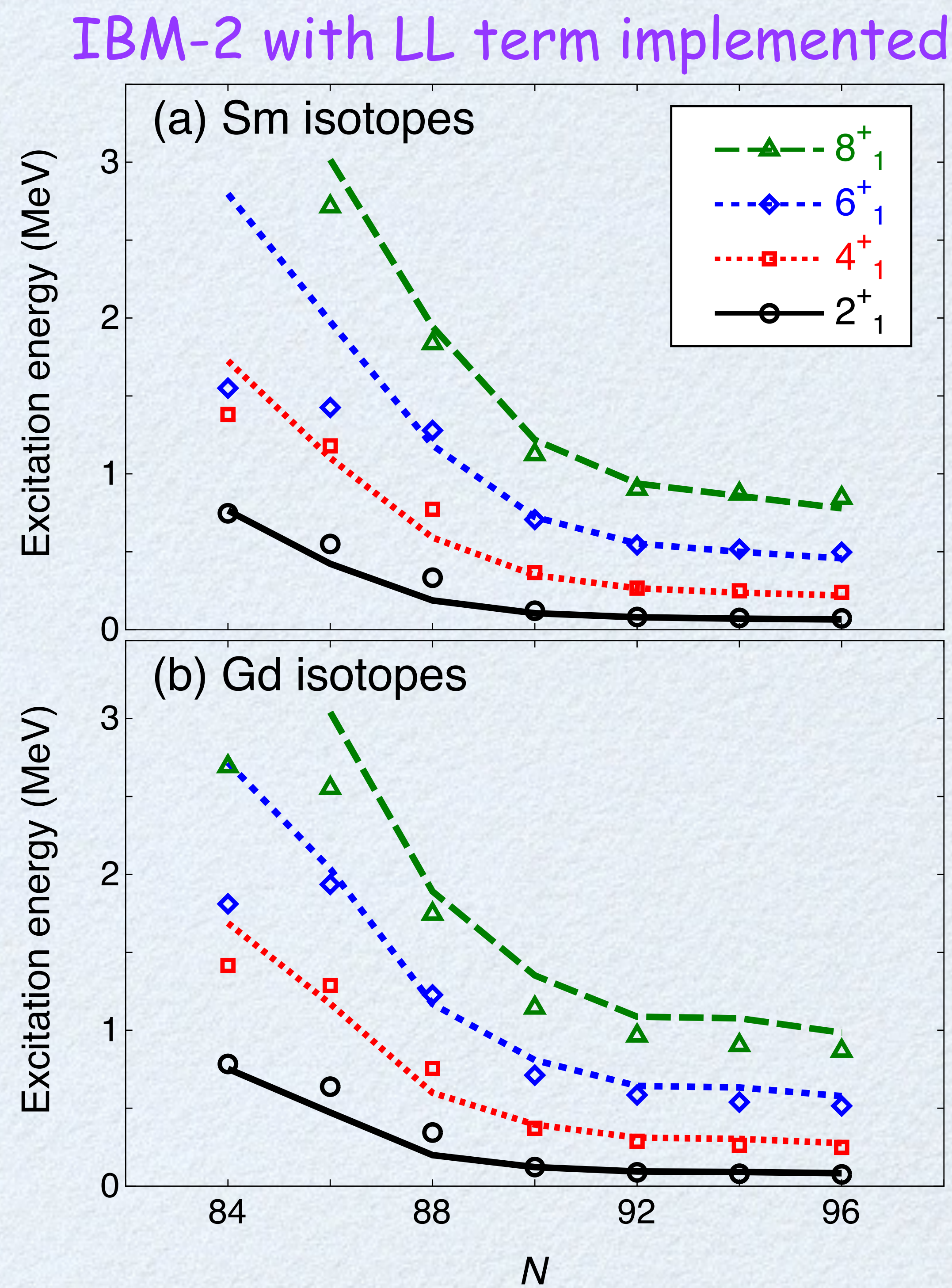
Step 2) LL term: rotational response (cranking) at a mean-field minimum. "Basic part" is kept unchanged. Necessary for strongly deformed nuclei.

$$\delta \langle \phi^F | \hat{H}^F e^{-i\hat{J}_y \delta\theta} | \phi^F \rangle \sim \delta \langle \phi^B | \hat{H}^B e^{-i\hat{J}_y \delta\theta} | \phi^B \rangle$$

angle $\delta\theta \ll 1$

Spherical-to-deformed shape transition

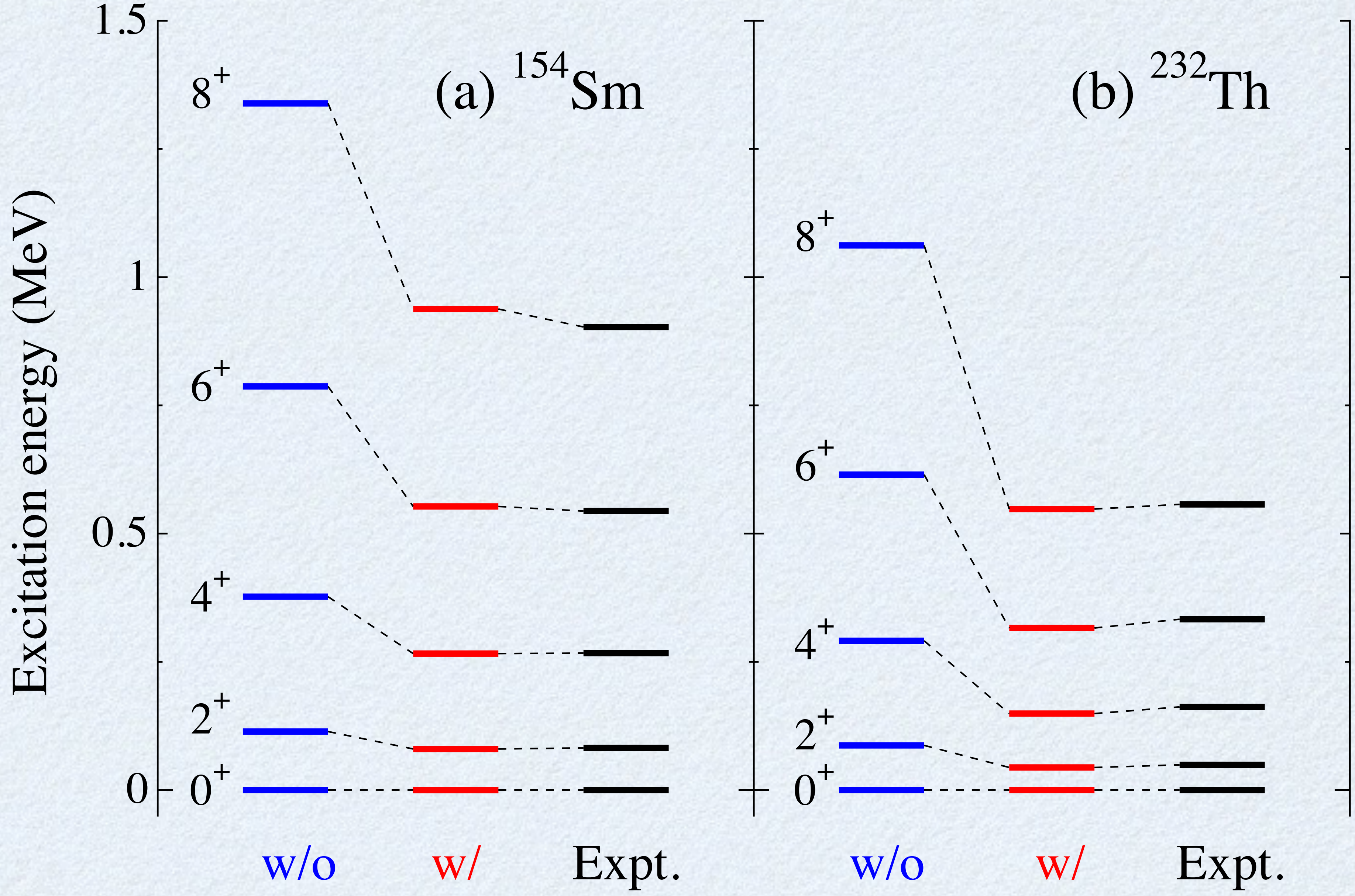




K.N., N. Shimizu, T. Otsuka, PRC81, 044307 (2010)

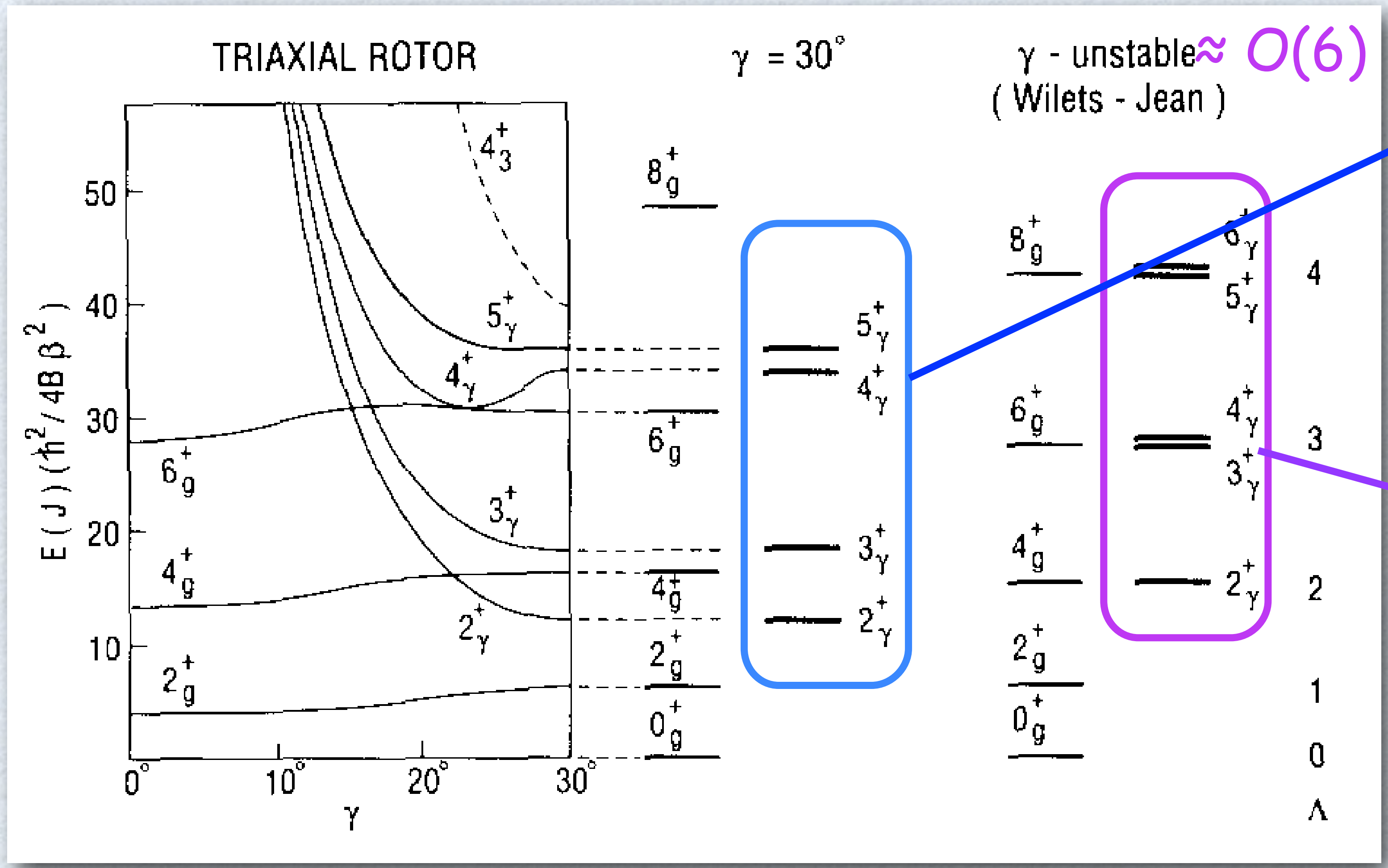
K.N., T. Otsuka, N. Shimizu, L. Guo, PRC83, 041302(R) (2011)

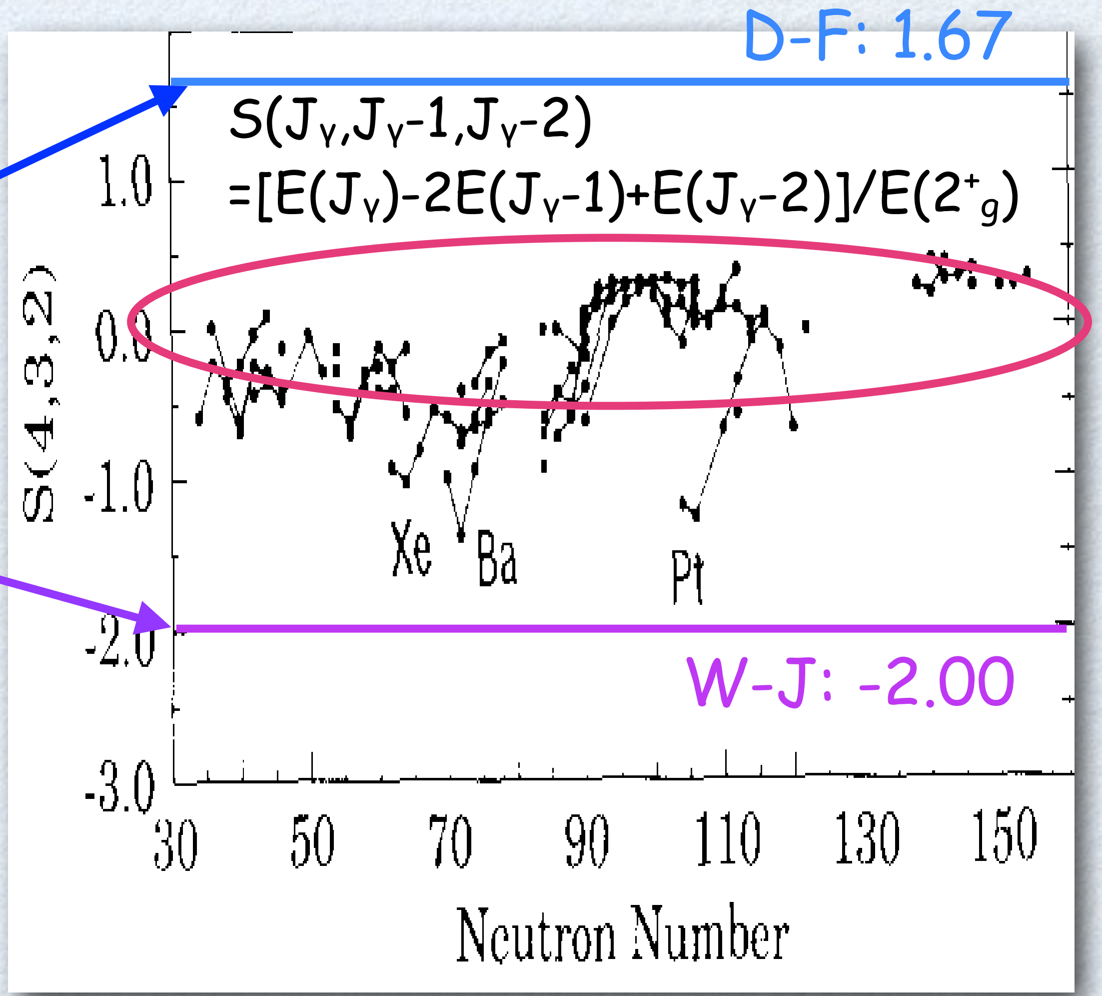
Significance of the rotational LL term



K.N., T. Otsuka, N. Shimizu, L. Guo, PRC83, 041302(R) (2011)

Is a triaxial nucleus γ rigid or unstable ?





Majority of observed soft nuclei are halfway. Empirical models cannot explain it. Microscopic realization?

- Rigid triaxial rotor model (Davydov & Filippov, 1958)
- γ-unstable rotor model (Wilets & Jean, 1956)
- Equivalence between W-J and O(6) in IBM (Ginocchio & Kirson, 1980)

Boson's "three-body force"

Van Isacker, Chen (1981);
Heyde et al (1984)

A solution would be to produce a **triaxial minimum**. O(6)-like 2-body Hamiltonian does not work out. Then, why not 3-body?

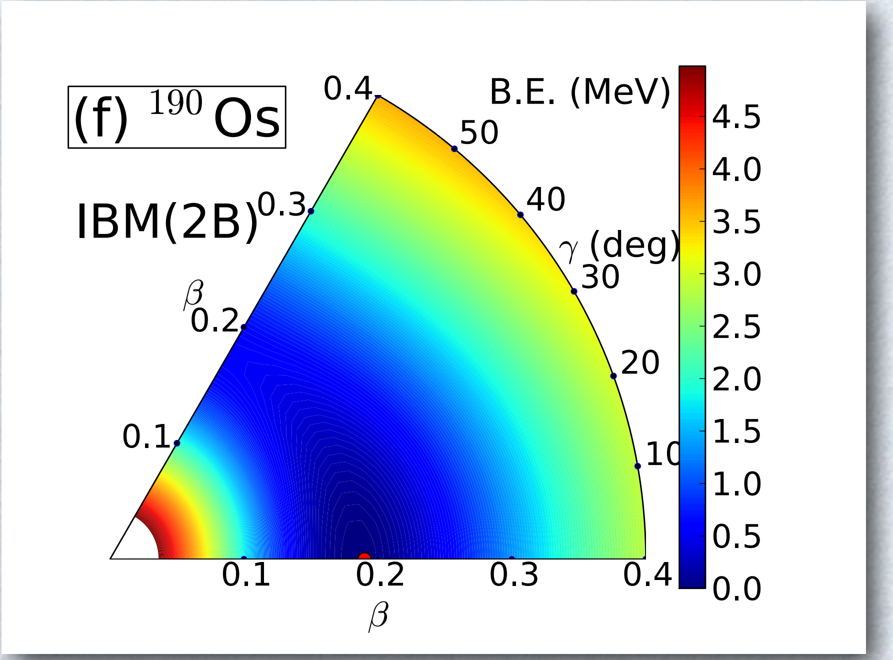
3B Hamiltonian:

$$\hat{H}_{3B} = \sum_{\rho' \neq \rho} \kappa''_{\rho} [d^{\dagger}_{\rho} \times d^{\dagger}_{\rho} \times d^{\dagger}_{\rho'}]^{(3)} \cdot [\tilde{d}_{\rho'} \times \tilde{d}_{\rho} \times \tilde{d}_{\rho}]^{(3)}$$

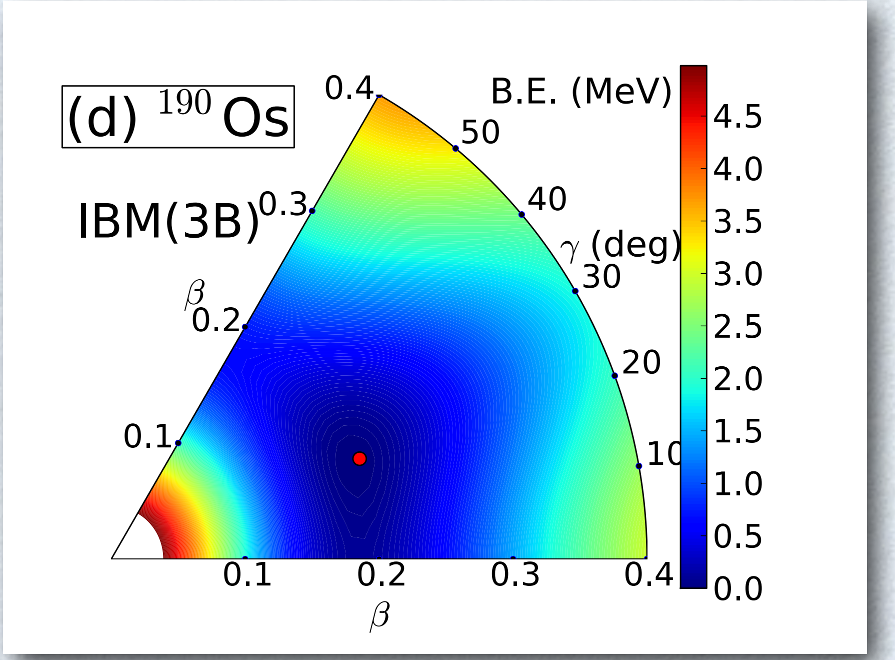
classical limit:

$$\langle \hat{H}_{1B} + \hat{H}_{2B} \rangle \propto \cos 3\gamma \qquad \langle \hat{H}_{3B} \rangle \propto \cos^2 3\gamma$$

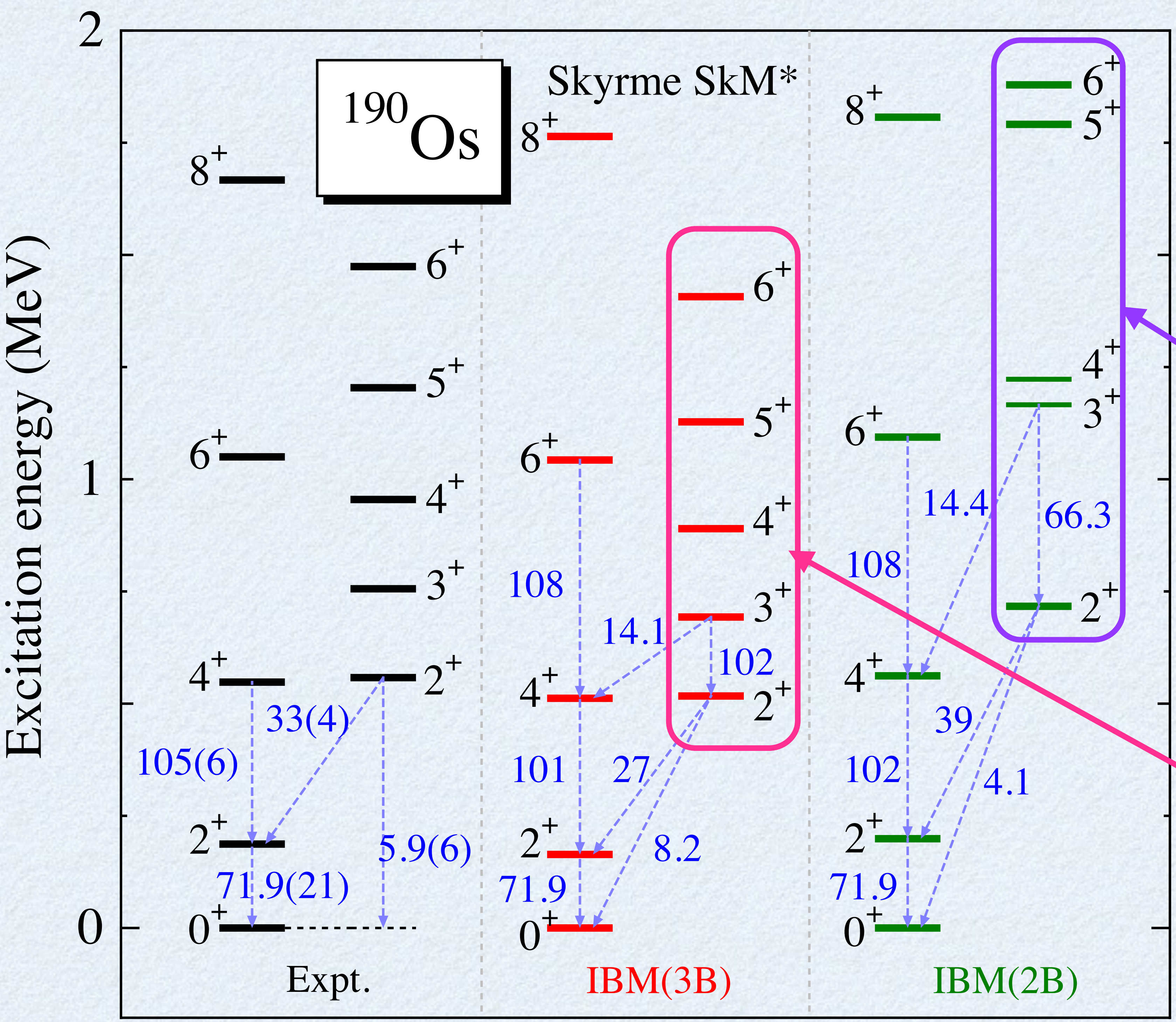
minimum at γ=0 or π/3

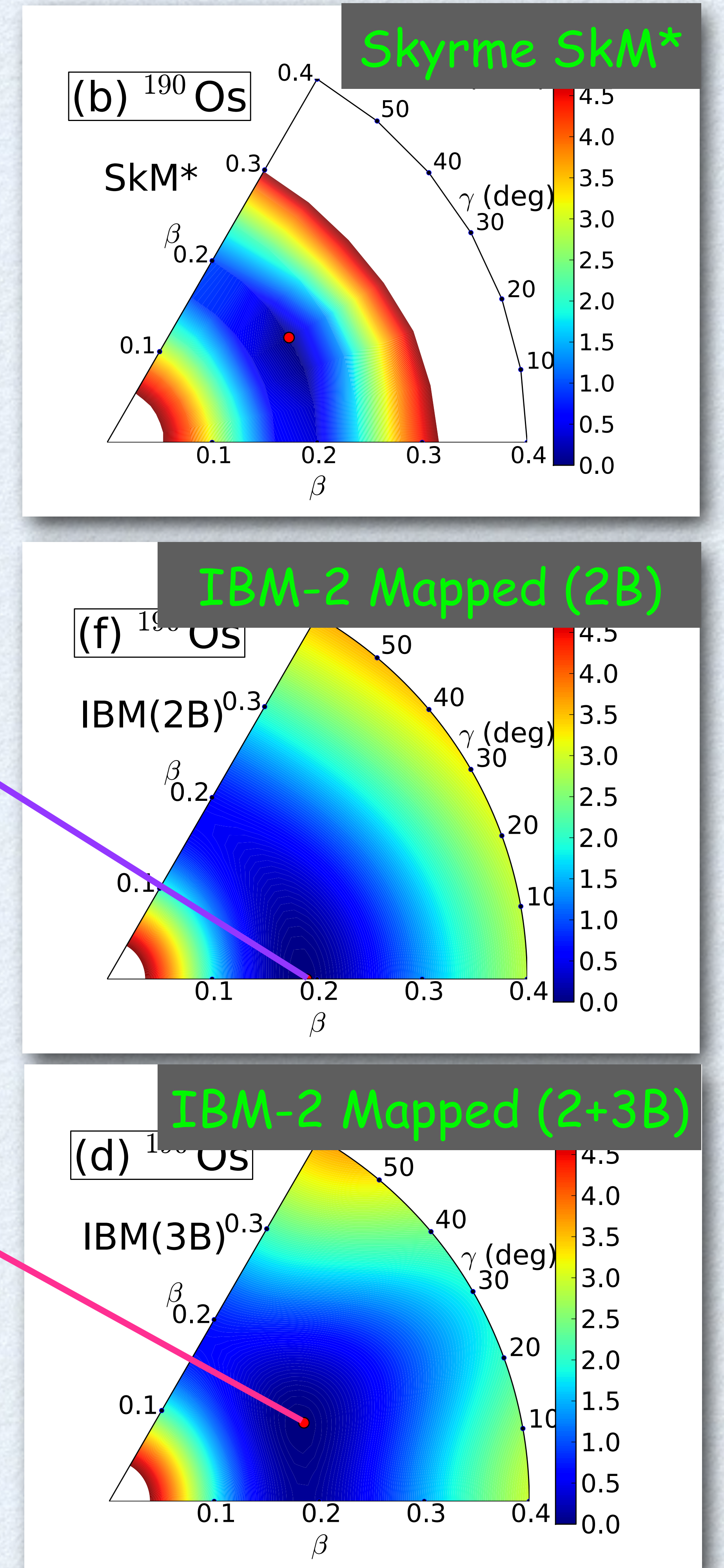


minimum at γ=π/6



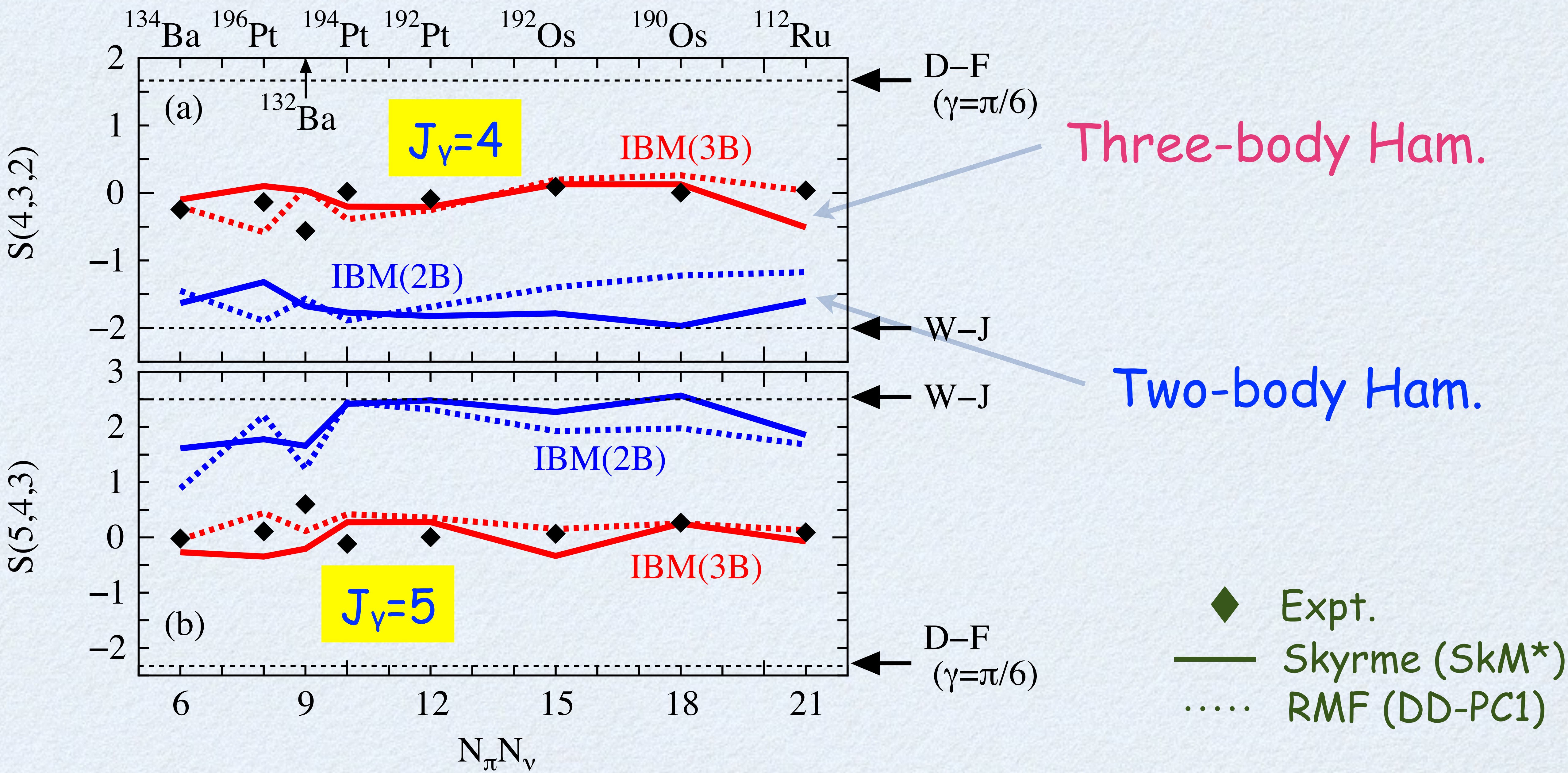
The "harmonic" structure of quasi-γ band is reproduced **only if** the IBM-2 Hamiltonian is comprise of up to **three-body** boson terms.





Robustness

- Independently of EDFs, neither W-J nor D-F picture is realized in presumably all triaxial nuclei. New symmetry/phase?
- In the IBM, 3B term **must be** included.



Configuration mixing in IBM

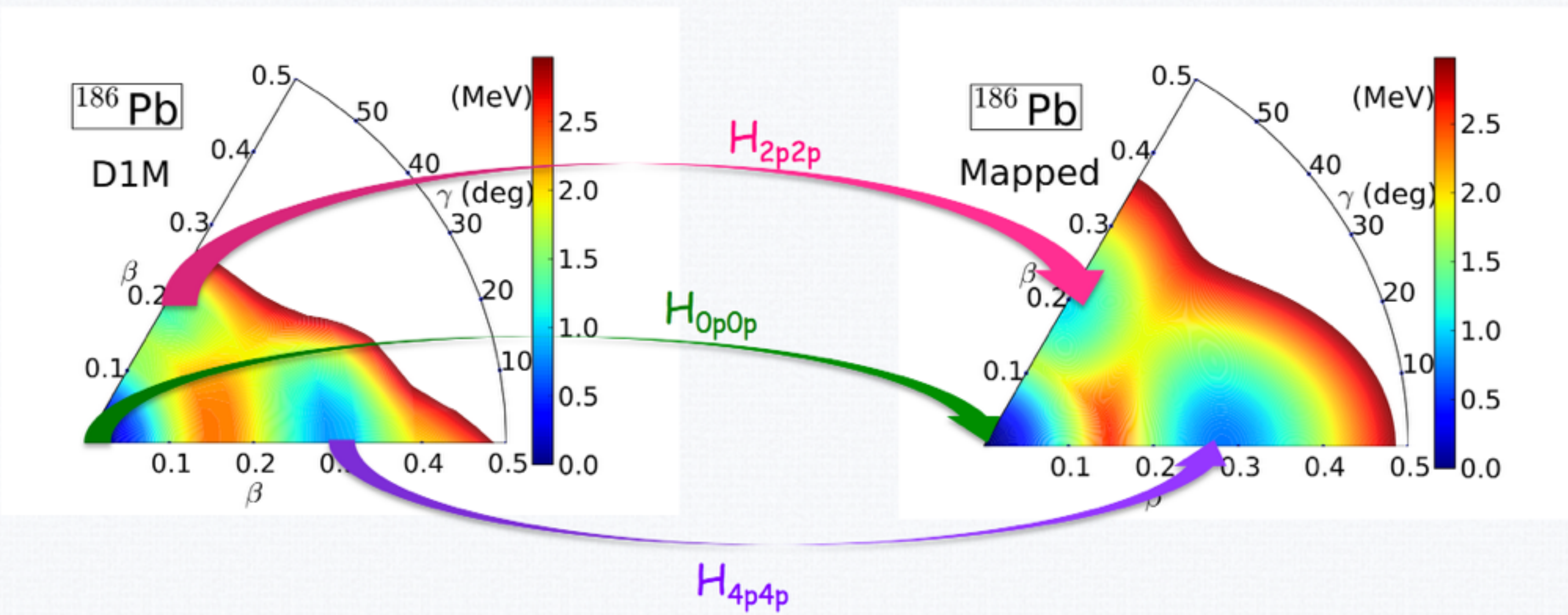
cf. Duval, Barrett (1981)

Consider the mixing of different Hamiltonians for ph excitations, each associated to a mean-field minimum

$$H = H_{0p0h} + (H_{2p2h} + \Delta_{2p2h}) + (H_{4p4h} + \Delta_{4p4h}) + V_{02} + V_{24}$$

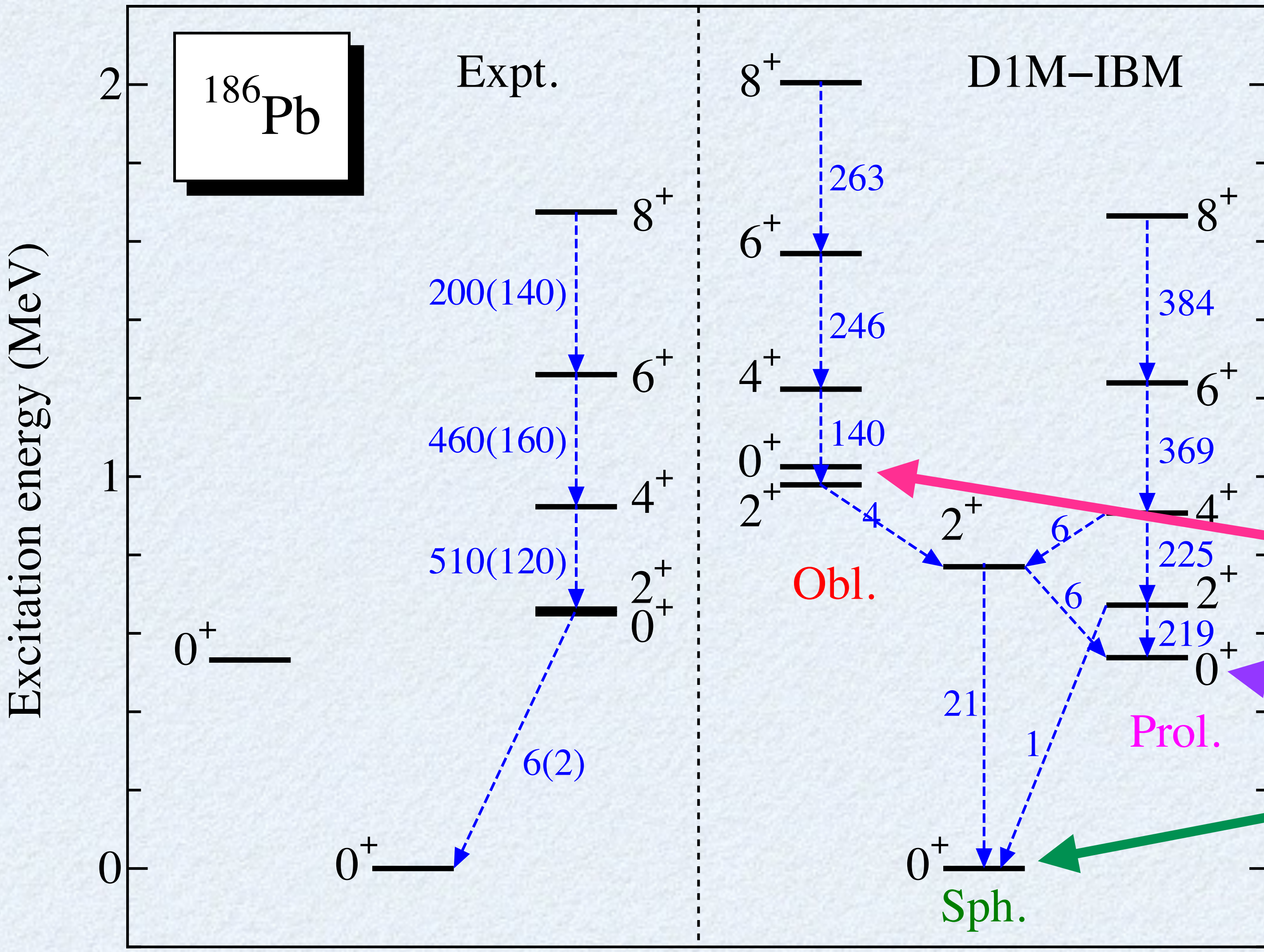
energy difference between minima

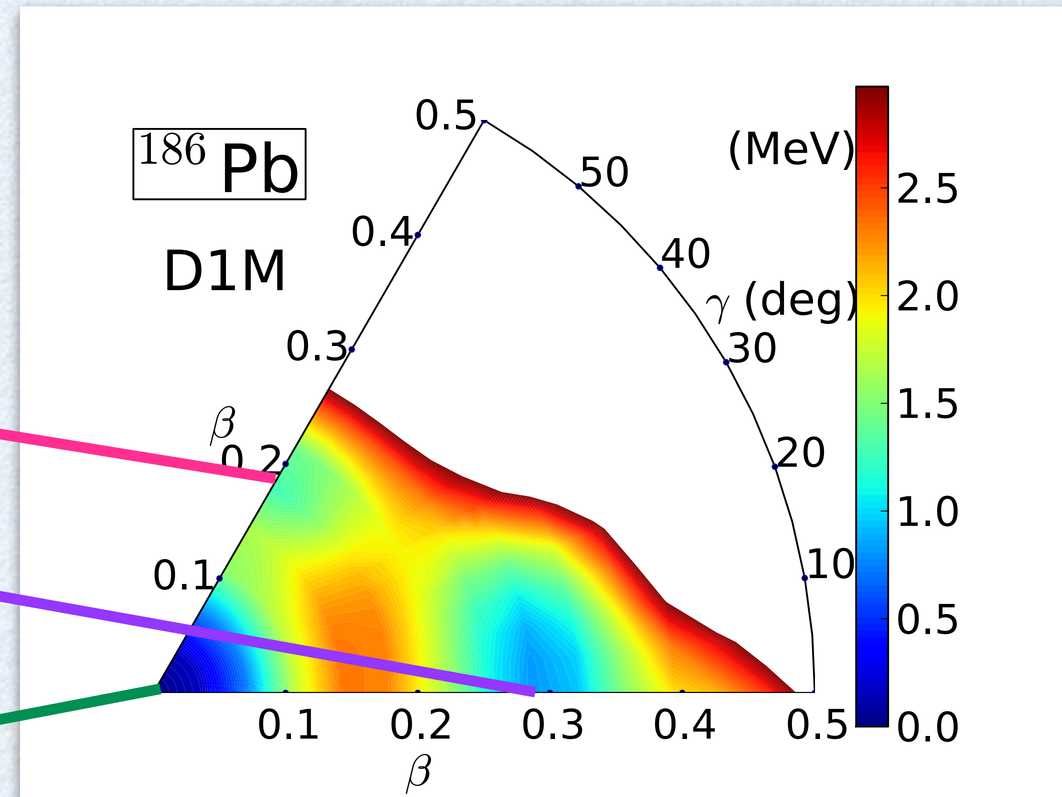
topology of barrier



Level scheme: ^{186}Pb

From Gogny-D1M set

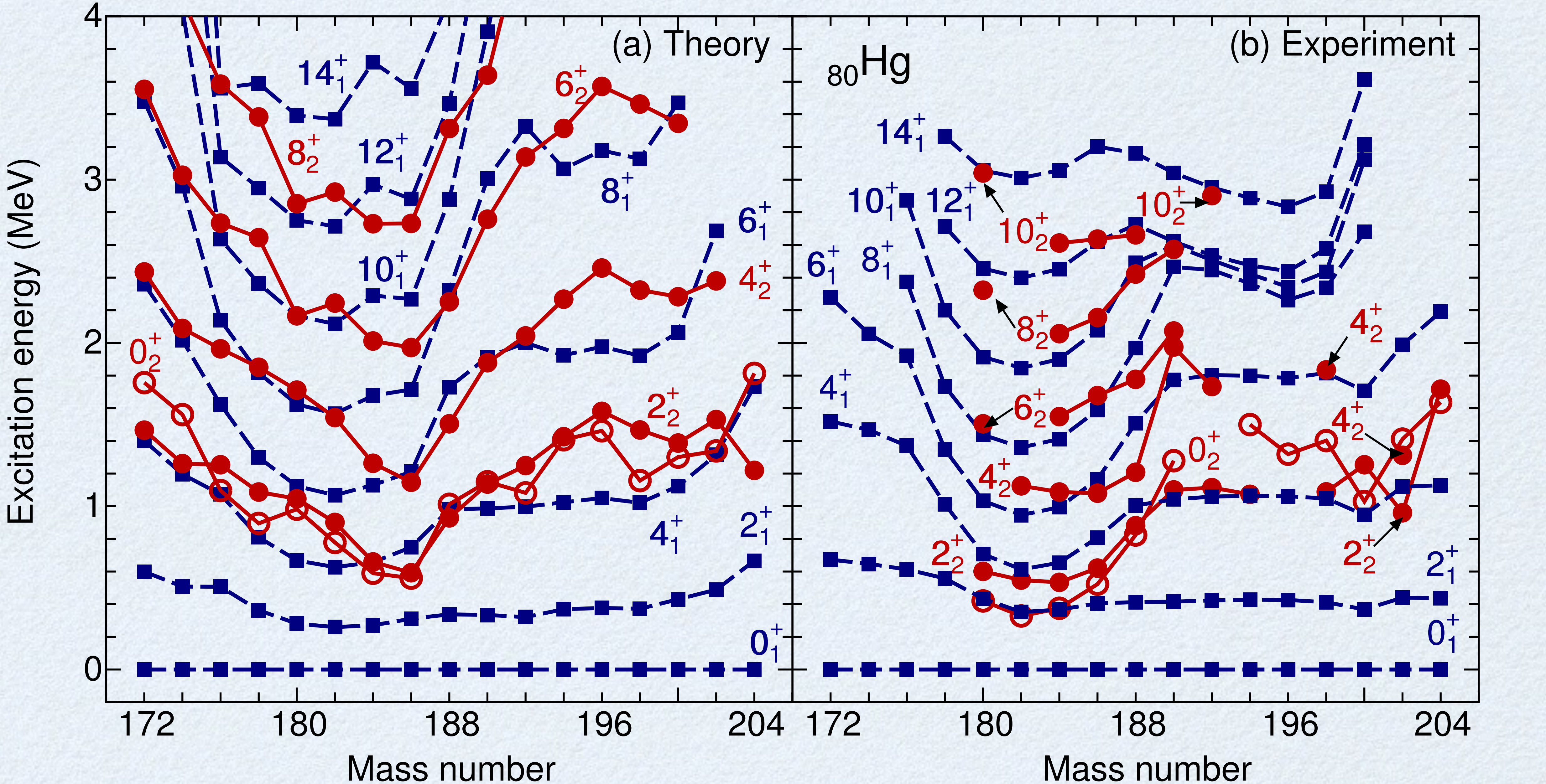




K.N., R. Rodríguez-Guzmán, L. M. Robledo, N. Shimizu, PRC86, 034322 (2012)

Systematics: Hg chain

K.N., R. Rodríguez-Guzmán, L. M. Robledo, PRC87, 064313 (2013)



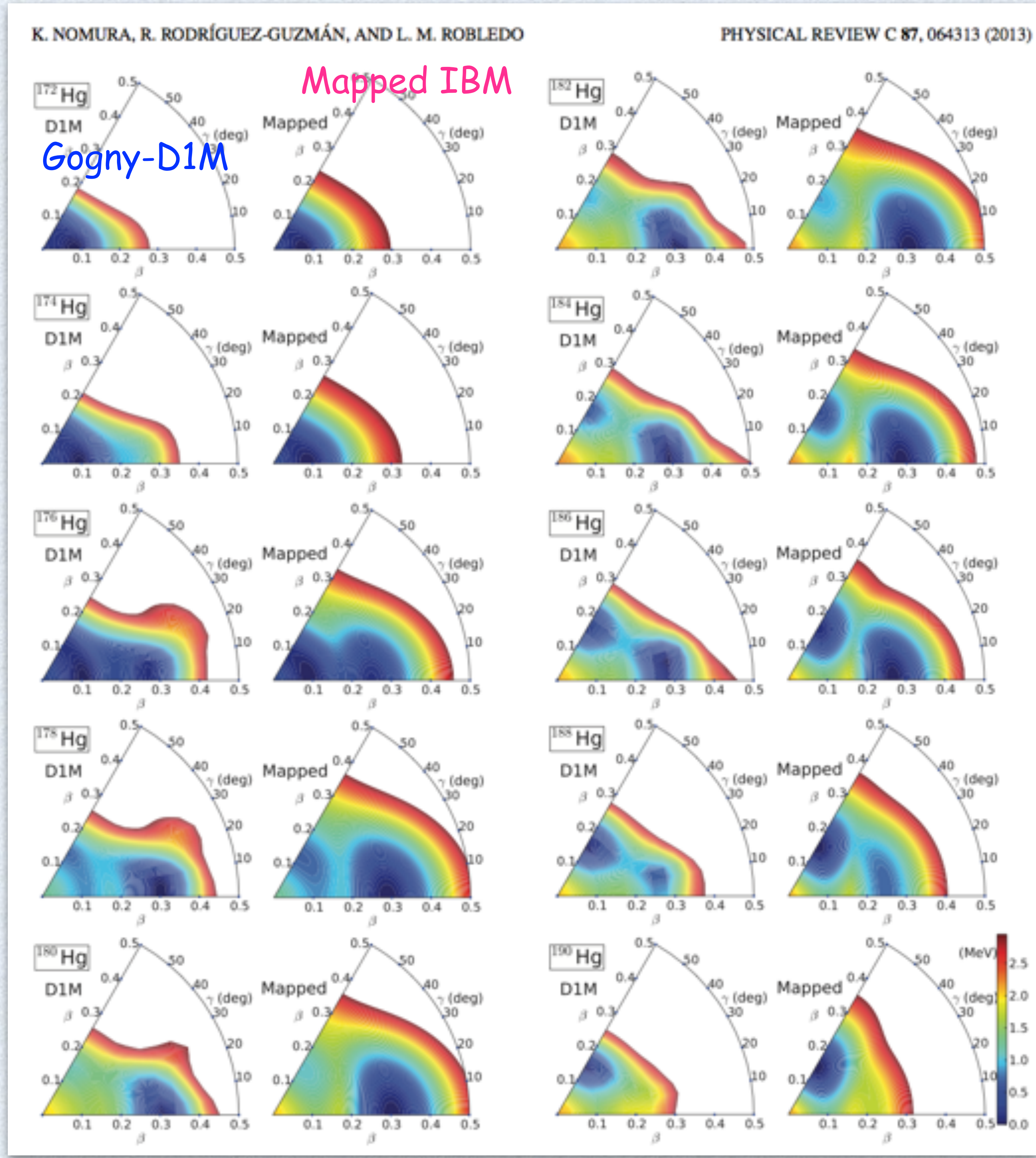
0^+_{gs} Pro. Obl.

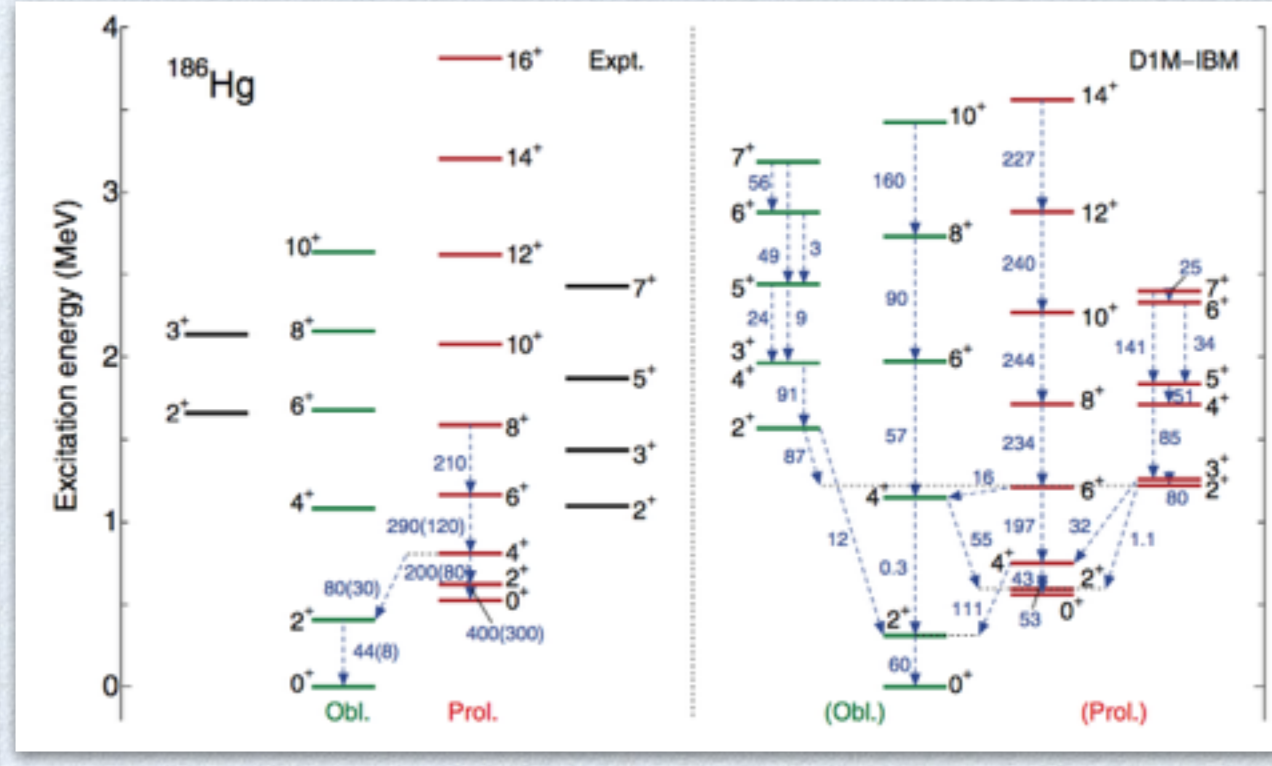
0^+_{exc} Obl. Pro.

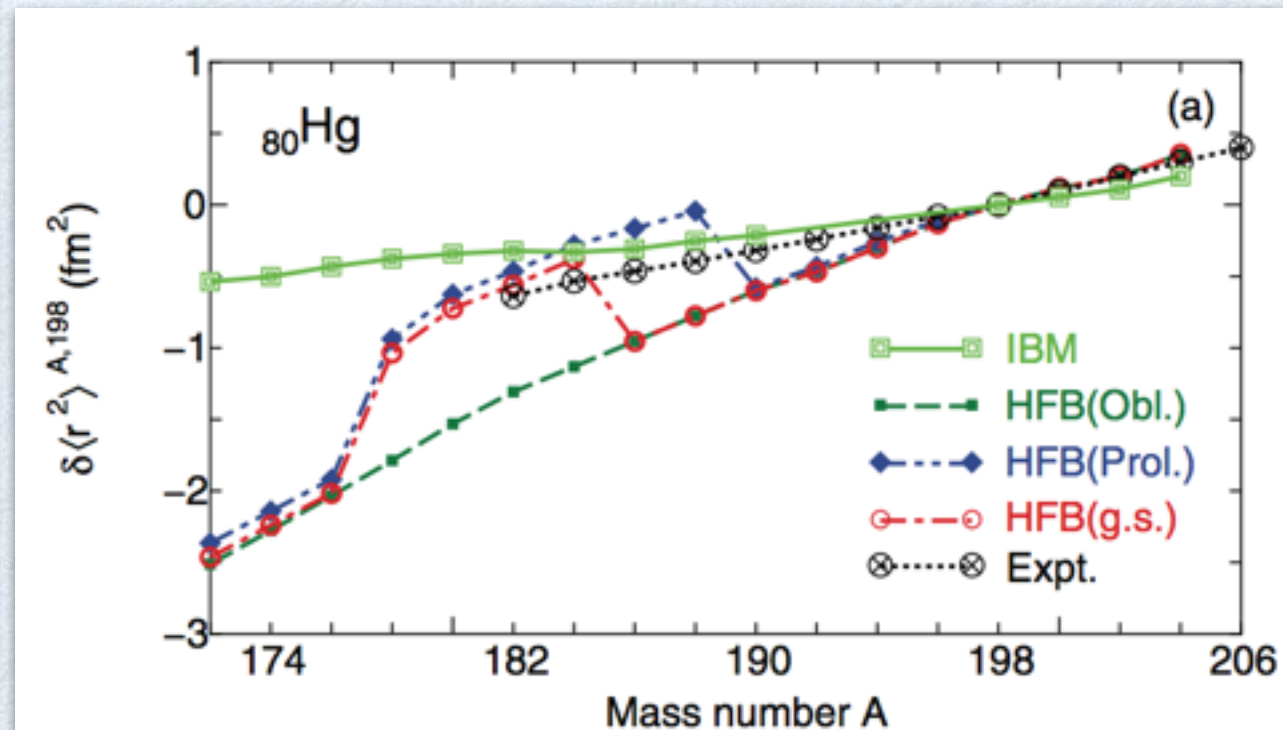
0^+_{gs} Obl.

0^+_{exc} Pro.

Systematics: Hg chain

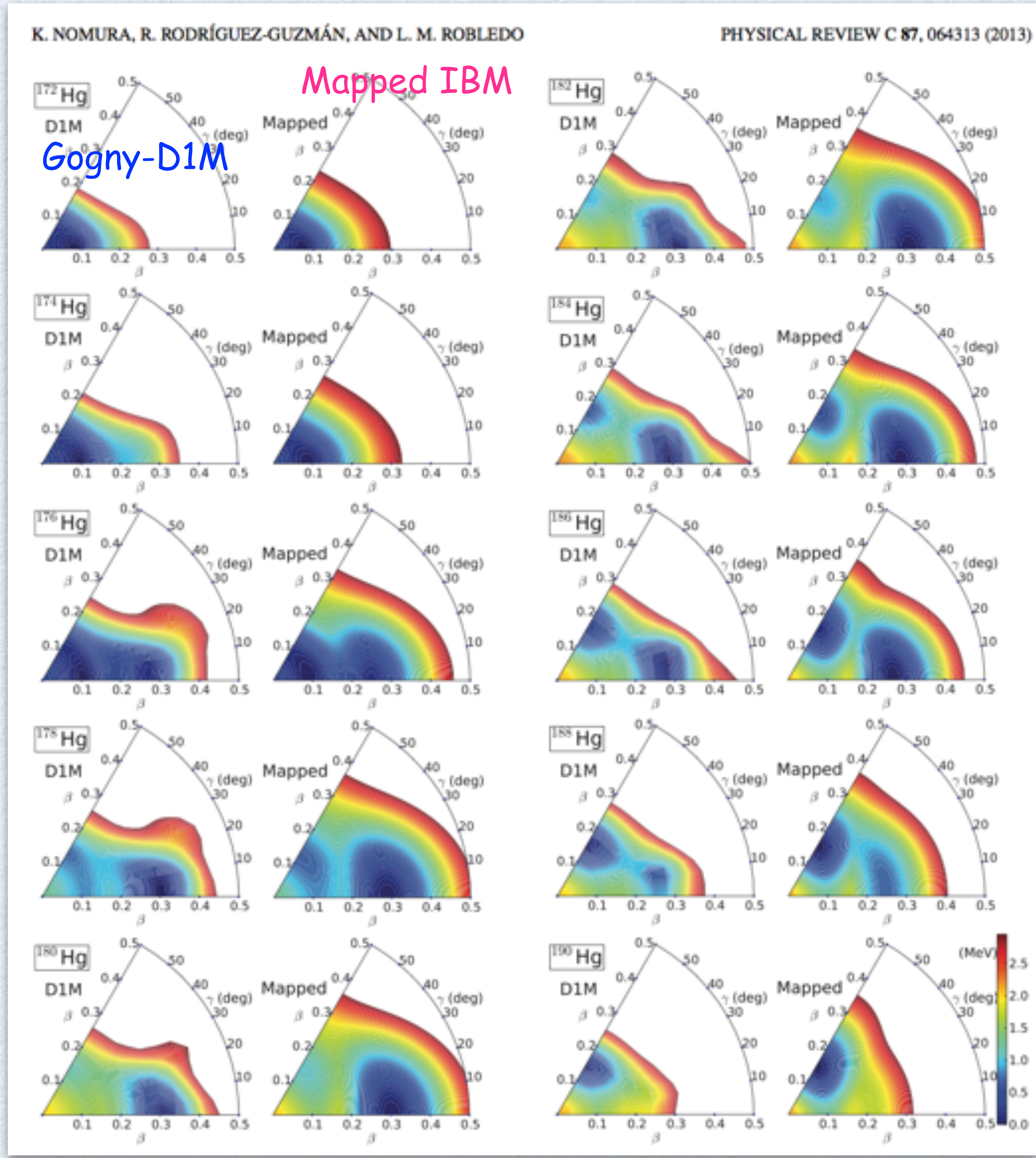






- IBM-2 with config. mixing (0p-0h + 2p-2h) using Duval-Barrett's technique
- Spherical-Prolate-Oblate transition
- Reasonable spectroscopic and intrinsic properties

How good is the chosen interaction?



Gogny-D1M triaxial map predicts prolate ground state for A<184.

This seems to be a common problem for many of the global EDFs

- Gogny D1S (Hilaire et al.)
- Skyrme SLy6 (Yao, Bender, Heenen, PRC (2013))
- NL3 (Niksic et al. PRC (2012))

Application to “pear-shaped” nuclei

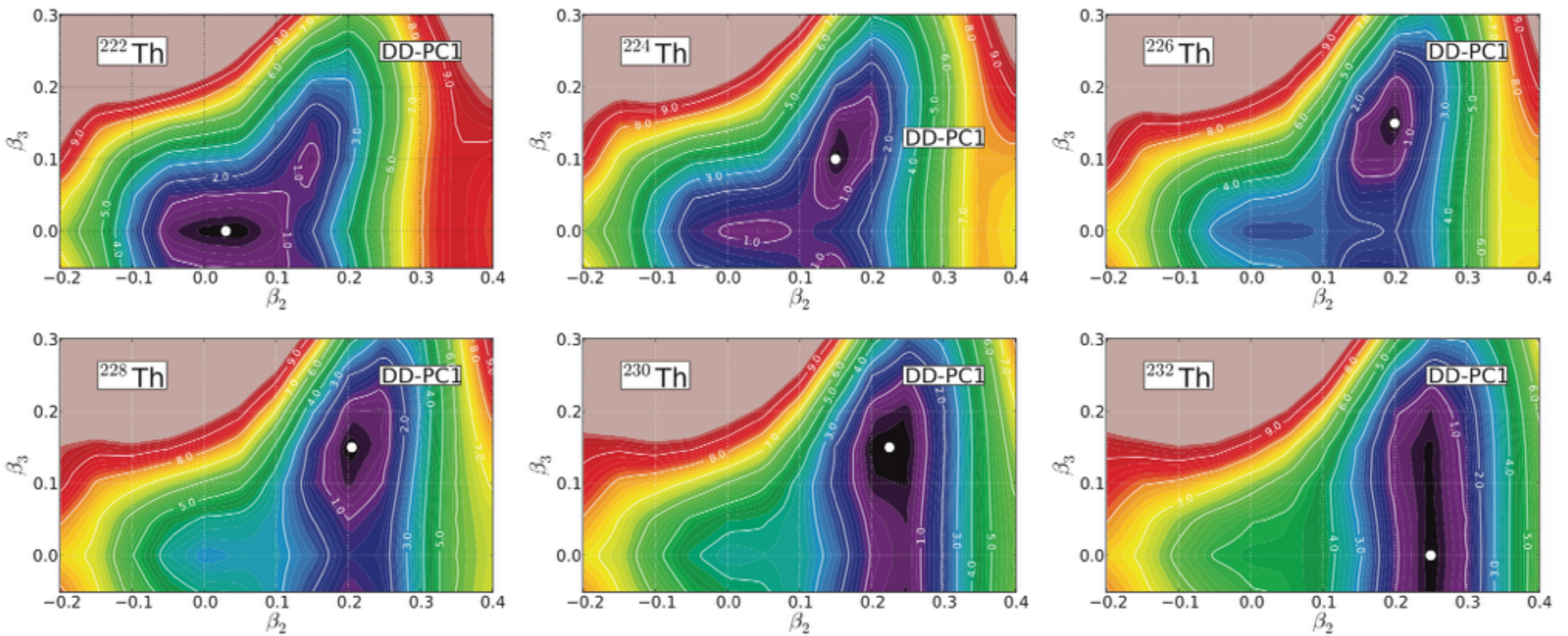
PRC Editors' Suggestion

Microscopic description of octupole shape-phase transitions in light actinide and rare-earth nuclei

K. Nomura, D. Vretenar, T. Nikšić, and Bing-Nan Lu

Phys. Rev. C **89**, 024312 (2014) – Published 24 February 2014

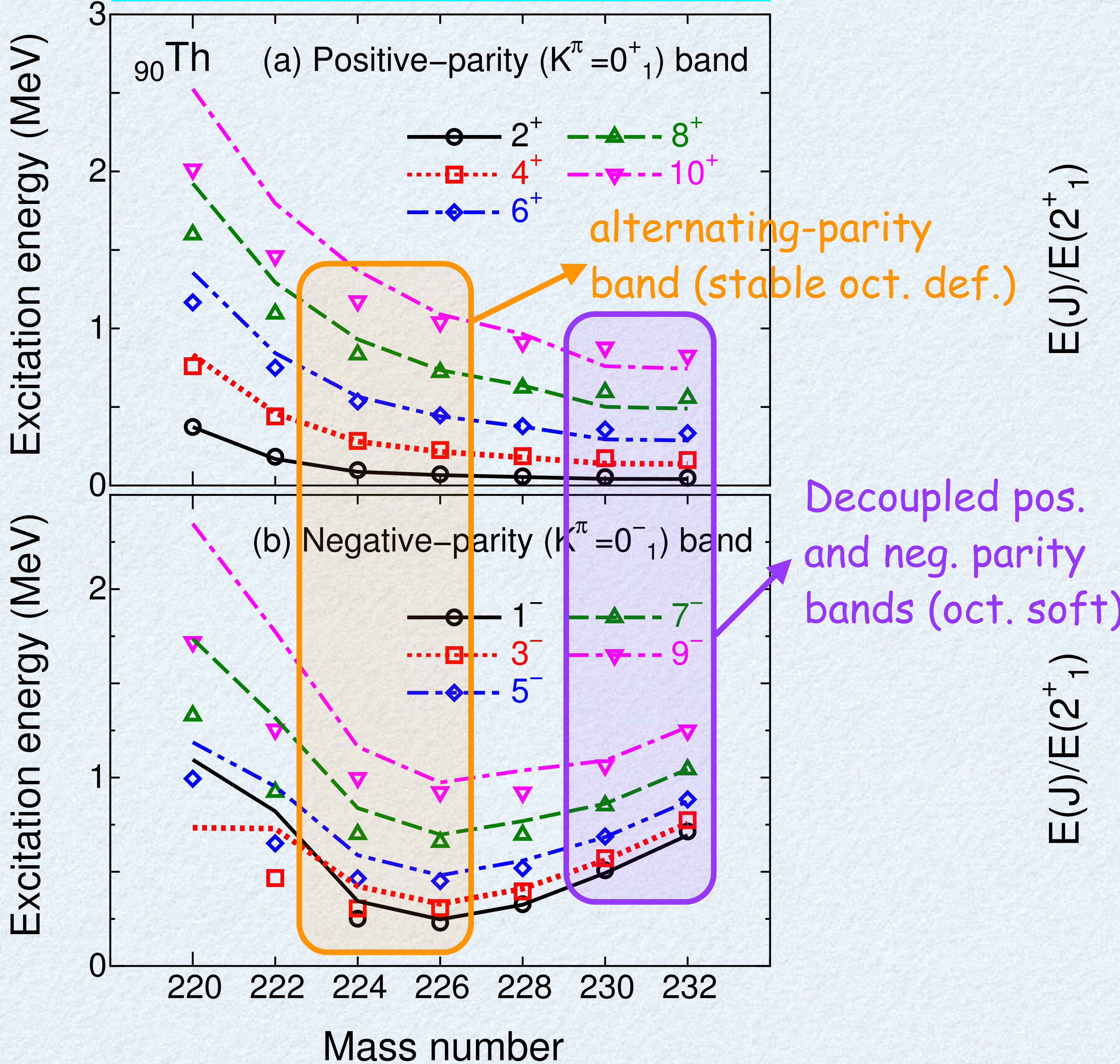
β_2-β_3 potential energy surface of 222-232Th (from DD-PC1 functional)



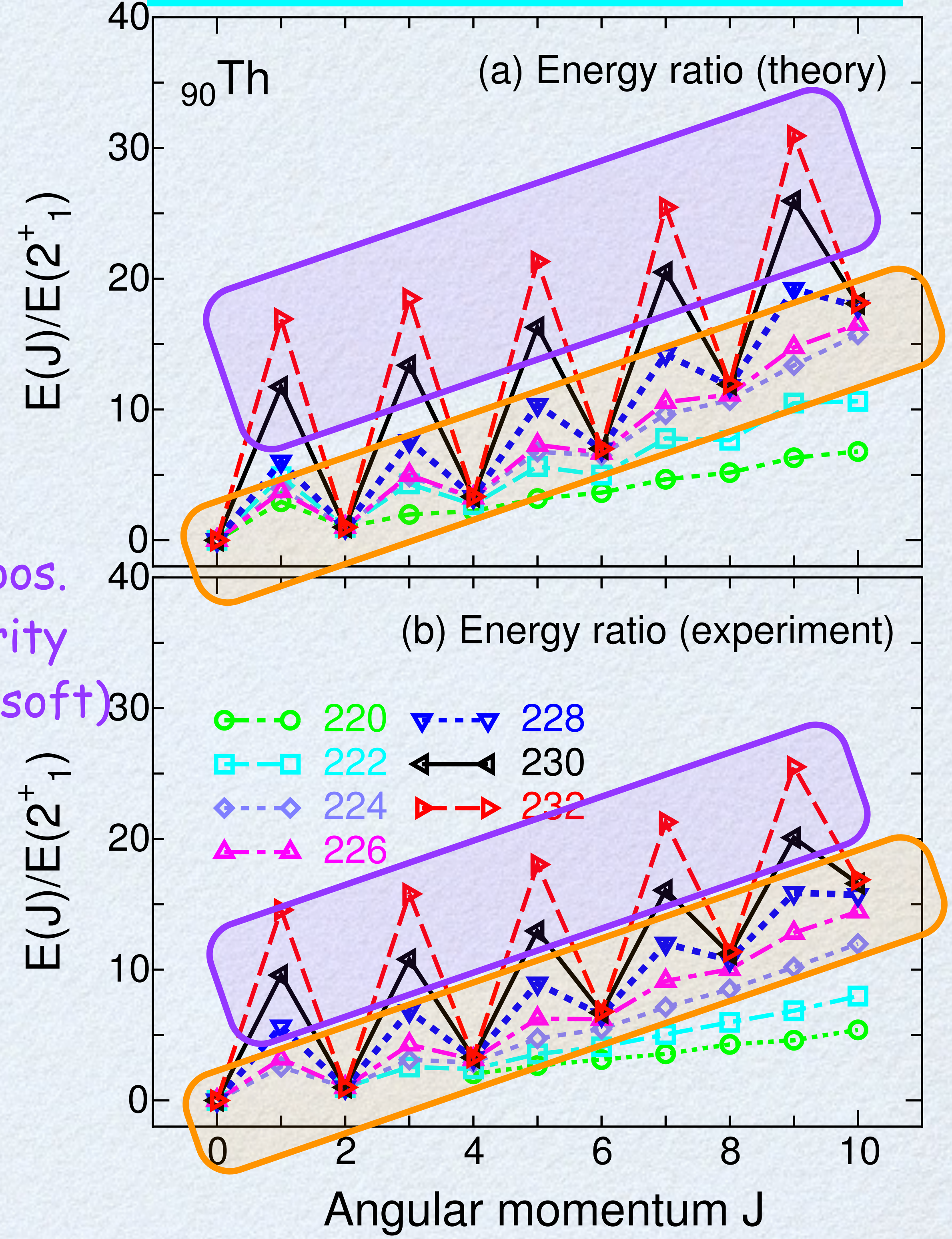
K.N., D. Vretenar, T. Niksic, B.-N. Lu, PRC88, 024312 (2014)

Octupole shape transition in Th

energy spectra from sdf IBM

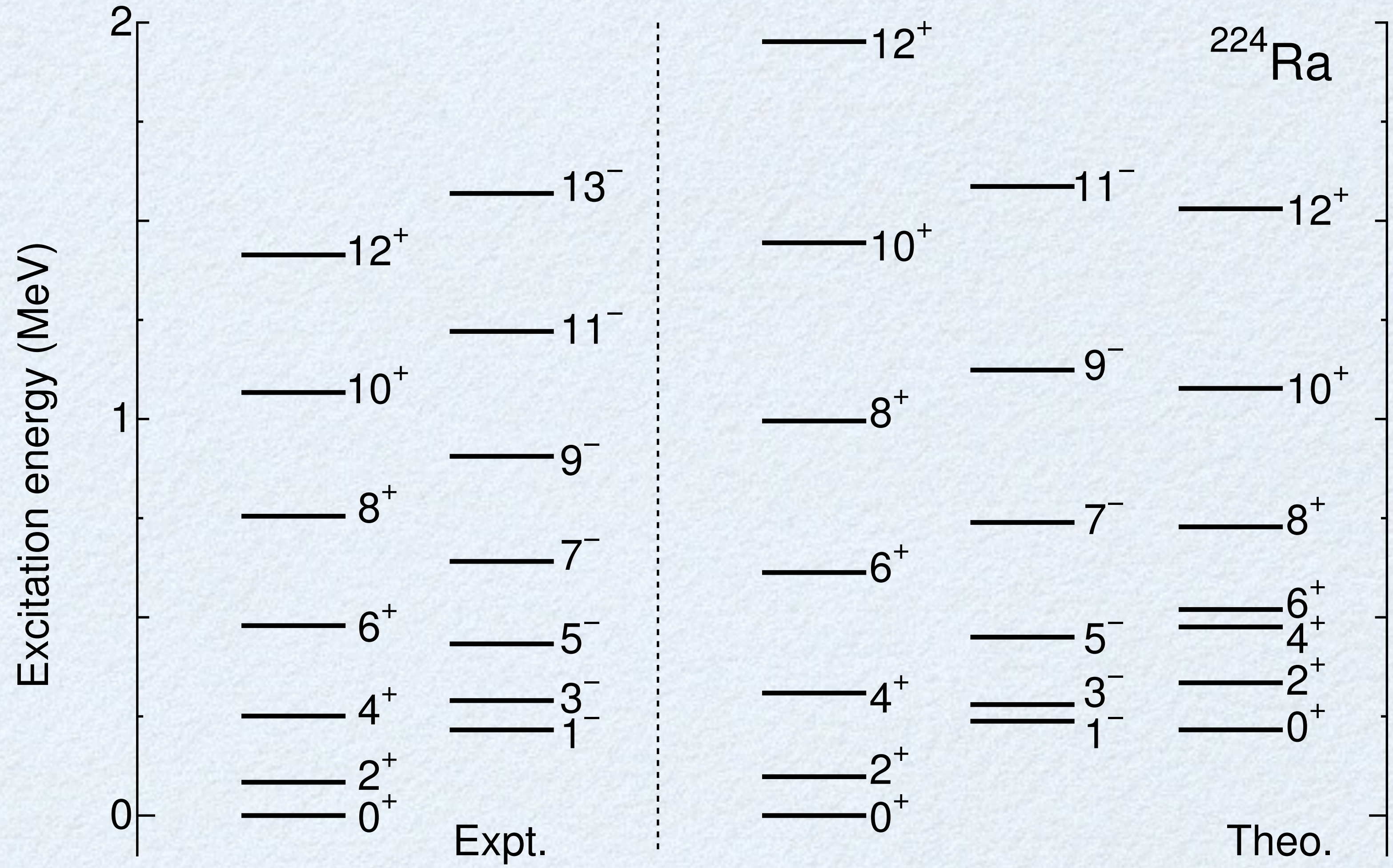


staggering in E(J)/E(2⁺) ratio



K.N., D. Vretenar, B.-N. Lu, PRC88, 021303(R) (2013)

“octupole nucleus” ^{224}Ra



Data from L. P. Gaffney, P. A. Butler et al., Nature 497, 199 (2013)

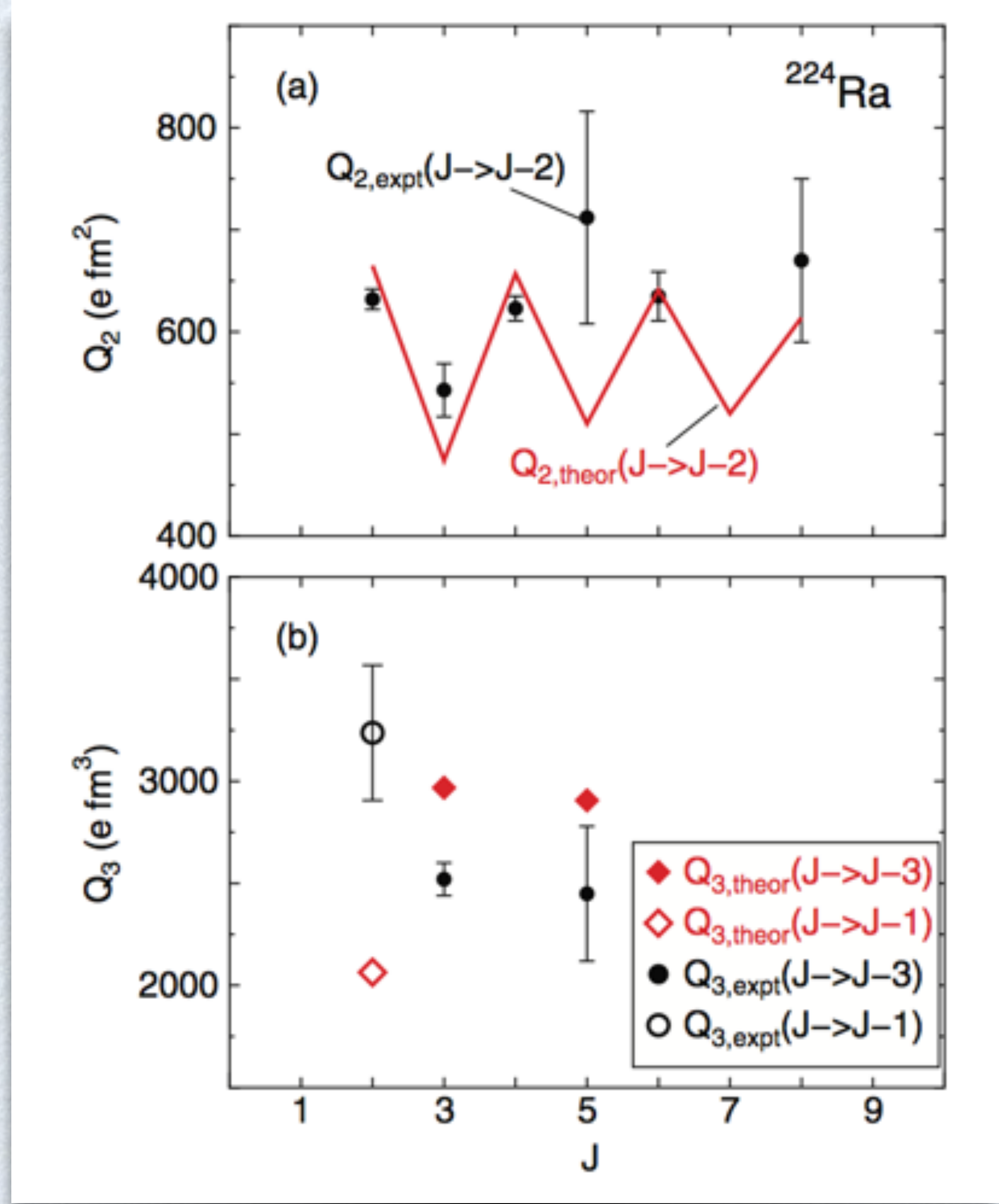
"octupole nucleus" ^{224}Ra

B(Eλ)

TABLE I. Comparison between experimental [6] and theoretical $B(E\lambda)$ values for transitions in ^{224}Ra (in Weisskopf units). All transitions shown in the table, except for the $E2$ transition from the band head of the band built on the 2_2^+ state to the 0^+ ground state, are between yrast states.

	Expt. (W.u.)	Theor. (W.u.)
$B(E2; 2^+ \to 0^+)$	98 ± 3	109
$B(E2; 3^- \to 1^-)$	93 ± 9	71
$B(E2; 4^+ \to 2^+)$	137 ± 5	152
$B(E2; 5^- \to 3^-)$	190 ± 60	97
$B(E2; 6^+ \to 4^+)$	156 ± 12	159
$B(E2; 8^+ \to 6^+)$	180 ± 60	153
$B(E2; 2_2^+ \to 0^+)$	1.3 ± 0.5	0
$B(E3; 3^- \to 0^+)$	42 ± 3	42
$B(E3; 1^- \to 2^+)$	210 ± 40	85
$B(E3; 3^- \to 2^+)$	<600	46
$B(E3; 5^- \to 2^+)$	61 ± 17	61
$B(E1; 1^- \to 0^+)$	$<5 \times 10^{-5}$	2.0×10^{-3}
$B(E1; 1^- \to 2^+)$	$<1.3 \times 10^{-4}$	1.1×10^{-3}
$B(E1; 3^- \to 2^+)$	$3.9^{+1.7}_{-1.4} \times 10^{-5}$	3.7×10^{-3}
$B(E1; 5^- \to 4^+)$	$4^{+3}_{-2} \times 10^{-5}$	5.0×10^{-3}
$B(E1; 7^- \to 6^+)$	$<3 \times 10^{-4}$	5.8×10^{-3}

intrinsic Q2 and Q3 momenta



Data from L. P. Gaffney, P. A. Butler et al., Nature 497, 199 (2013)

Summary

New formulation of the IBM has been developed. Bridge the gap between IBM and nuclear mean-field model.

- Physical observables in lab frame
- Valid for (in principle) any situation:

  • "Basic" part: mapping of energy surface. Static problem.
  • LL term: mapping of rotational response. Dynamical problem.
  • 3-B term: necessary for a triaxial minimum

- Real prediction for heavy unknown nuclei
- Remaining issues: mixed symmetry (in progress), super deformation, ... etc.

Thanks to

Main collaborators:

T. Otsuka (Tokyo)

N. Shimizu (Tokyo)

L. Guo (Tokyo, now in Beijing)

D. Vretenar (Zagreb)

L. M. Robledo (Madrid)

R. Rodríguez-Guzmán (Rice U)

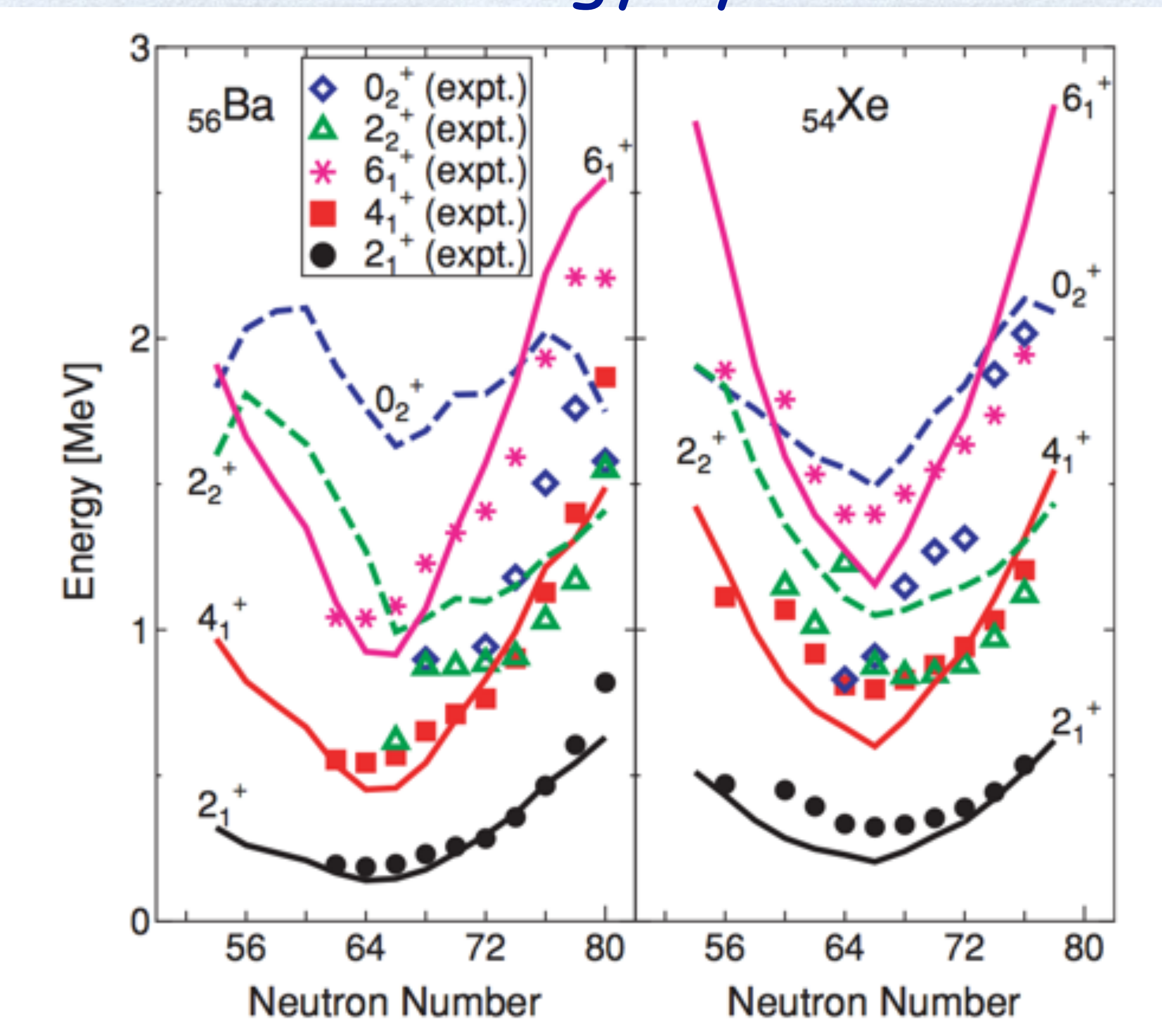
European Commission for financial aid

Thank you for your attention !

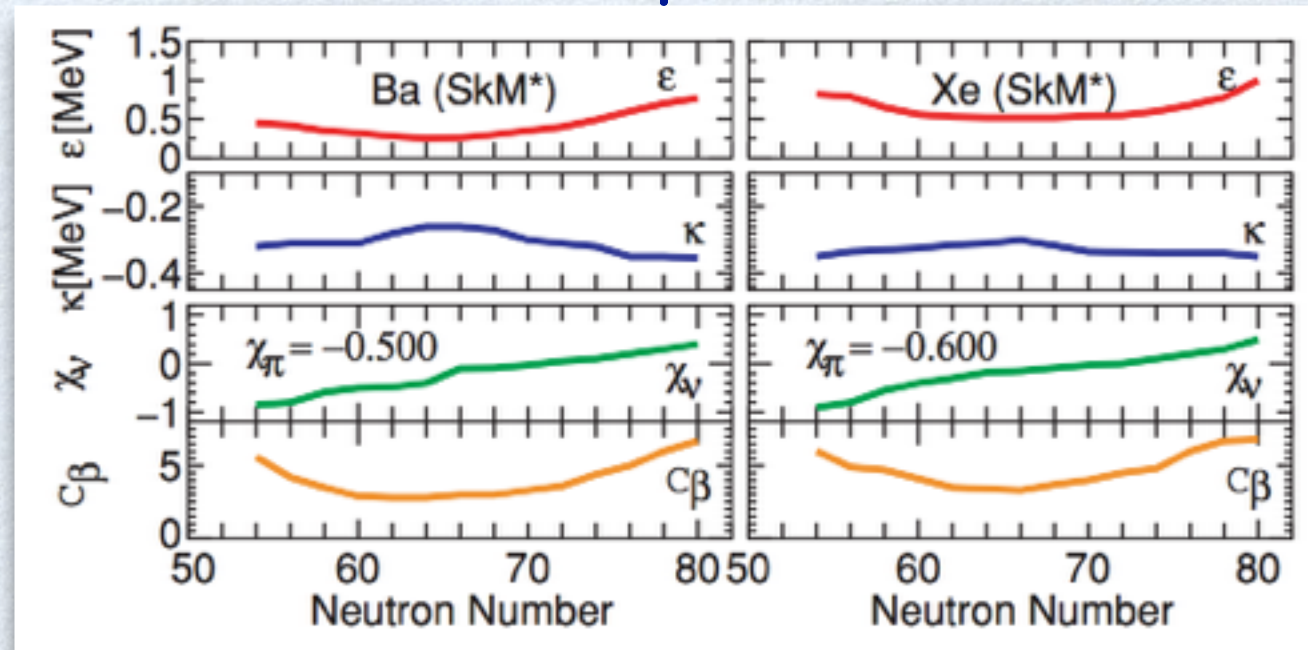
Merci beaucoup

Proof of principle

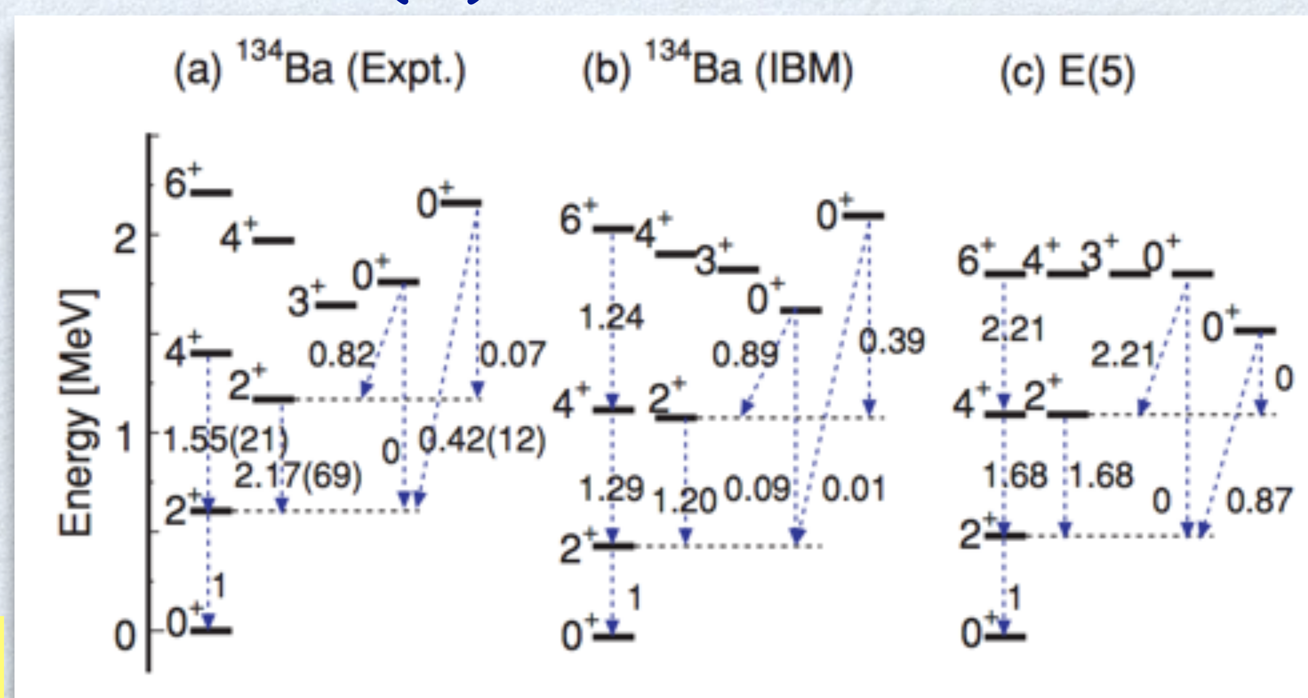
Level-energy systematics



Derived parameters



E(5) nucleus ^{134}Ba



- Boson Hamiltonian of P+QQ type
- Reasonable description of **modestly deformed nuclei** without fitting to data
- Good description of “critical” nucleus ^{134}Ba

K.N., N. Shimizu, T. Otsuka, PRC81, 044307 (2010)