## ICNT workshop@RIKEN

Skyrme-EDF for charge-changing excitation modes

Nigata Univ.
Kenichi Yoshida

## Outline

$\sqrt{ }$ Self-consistent Skyrme-pnQRPA approach
$\checkmark$ GT strengths in neutron-rich deformed Zr isotopes deformation
$\mathrm{T}=0$ pairing
$\checkmark$ pn-pair transfer strengths in $\mathrm{N}=\mathrm{Z}$ nuclei in fp-shell collectivity of the pn-pairing vibration

## Skyrme energy-density functional (EDF) approach

Energy functional: $\mathcal{E}=\int d \boldsymbol{r} \mathcal{H}(\boldsymbol{r})$
Energy density: $\mathcal{H}=\mathcal{H}_{\text {kin }}+\mathcal{H}_{\text {skyrme }}+\mathcal{H}_{\mathrm{em}}$
Skyrme energy density: $\mathcal{H}_{\text {Skyrme }}=\sum_{t=0,1} \sum_{t_{3}=-t}^{t}\left(\mathcal{H}_{t t_{3}}^{\text {even }}+\mathcal{H}_{t t_{3}}^{\text {odd }}\right)$

$$
\mathcal{H}_{t t_{3}}^{\text {even }}=C_{t}^{\rho} \rho_{t t_{3}}^{2}+C_{t}^{\Delta \rho} \rho_{t t_{3}} \Delta \rho_{t t_{3}}+C_{t}^{\tau} \rho_{t t_{3}} \tau_{t t_{3}}+C_{t}^{\nabla J} \rho_{t t_{3}} \nabla \cdot \mathbf{J}_{t t_{3}}+C_{t}^{J} \overleftrightarrow{J}_{t t_{3}}^{2}
$$

$$
\mathcal{H}_{t t_{3}}^{\mathrm{odd}}=C_{t}^{s} \mathbf{s}_{t t_{3}}^{2}+C_{t}^{\Delta s} \mathbf{s}_{t t_{3}} \cdot \Delta \mathbf{s}_{t t_{3}}+C_{t}^{T} \mathbf{s}_{t t_{3}} \cdot \mathbf{T}_{t t_{3}}+C_{t}^{\nabla s}\left(\nabla \cdot \mathbf{s}_{t t_{3}}\right)^{2}+C_{t}^{j} \mathbf{j}_{t t_{3}}^{2}+C_{t}^{\nabla j} \mathbf{s}_{t t_{3}} \cdot \nabla \times \mathbf{j}_{t t_{3}}
$$

T-odd densities vanish in g.s of e-e nuclei
T-odd Skyrme energy density is not well constrained,
but plays a role in studying compact star
EoS of polarized neutron matter:
T-odd components

$$
\frac{\mathcal{H}}{\rho_{0}}=2^{4 / 3}\left(\frac{3 \pi^{2}}{2}\right)^{2 / 3}\left[\frac{\hbar^{2}}{2 m}+\left(C_{0}^{\tau}+\overline{C_{1}^{\tau}}+C_{0}^{T}+C_{1}^{T}\right) \rho_{0}\right] \rho_{0}^{2 / 3}+\left(C_{0}^{\rho}+\overline{C_{1}^{\rho}}+C_{0}^{s}+\overline{C_{1}^{s}}\right) \rho_{0}
$$

## T-odd energy density seen in nuclear responses

Collective motion is generated by the residual interaction

$$
v_{\mathrm{res}}\left(\boldsymbol{r}_{1}, \boldsymbol{r}_{2}\right) \equiv \frac{\delta^{2} \mathcal{E}}{\delta \rho\left(\boldsymbol{r}_{1}\right) \delta \rho\left(\boldsymbol{r}_{2}\right)}: \text { self-consistency }
$$

spin density spin excitation modes

Nuclear response to the IV external field: $\int d \boldsymbol{r} r^{L} Y_{L} \hat{s}_{1 t}$
IV spin density: $\hat{\boldsymbol{s}}_{1 t}=\psi^{\dagger}(\boldsymbol{r} \sigma \tau)\langle\sigma| \boldsymbol{\sigma}\left|\sigma^{\prime}\right\rangle\langle\tau| \boldsymbol{\tau}_{t}\left|\tau^{\prime}\right\rangle \psi\left(\boldsymbol{r} \sigma^{\prime} \tau^{\prime}\right)$

Gamow-Teller, Spin dipole,...

## Self-consistent Skyrme-EDF approach for spin-isospin excitations in superfluid systems

North Carolina + Oak Ridge group

PHYSICAL REVIEW C, VOLUME 60, 014302
$\boldsymbol{\beta}$ decay rates of $r$-process waiting-point nuclei in a self-consistent approach
J. Engel, ${ }^{1}$ M. Bender, ${ }^{1,2}$ J. Dobaczewski, ${ }^{2,3,4}$ W. Nazarewicz, ${ }^{2,3,5}$ and R. Surman ${ }^{1}$

Skyrme-pnQRPA (1999) coordinate-space ID-HFB + QRPA in canonical basis

Application to fixing the coupling constants

Large-scale calculations of the double- $\beta$ decay of ${ }^{76} \mathrm{Ge},{ }^{130} \mathrm{Te},{ }^{136} \mathrm{Xe}$, and ${ }^{150} \mathrm{Nd}$ in the deformed self-consistent Skyrme quasiparticle random-phase approximation
M. T. Mustonen ${ }^{1,2, *}$ and J. Engel ${ }^{1, \dagger}$

## Milano group

The first deformed Skyrme-pnQRPA coordinate-space 2D-HFB + QRPA in canonical basis

PHYSICAL REVIEW C 76, 044307 (2007)
Spin-isospin nuclear response using the existing microscopic Skyrme functionals
S. Fracasso and G. Colò

ID-HFBCS + QRPA

## Sichuan + Wako group

Physics Letters B 719 (2013) 116-120
Role of $T=0$ pairing in Gamow-Teller states in $N=Z$ nuclei
C.L. Bai ${ }^{\text {a }}$, H. Sagawa ${ }^{\text {b,c, },}$, M. Sasano ${ }^{\text {c }}$, T. Uesaka ${ }^{\text {c }}$, K. Hagino ${ }^{\text {d }}$, H.Q. Zhang ${ }^{e}$, X.Z. Zhang ${ }^{e}$, F.R. Xu ${ }^{\text {f }}$

## Intermezya -constrained by ab-initio calculations-

Skyrme-EDF for matter

$$
\mathcal{H}=\mathcal{H}_{\mathrm{kin}}+\sum_{t=0,1}\left(C_{t}^{\rho} \rho_{t}^{2}+C_{t}^{\tau} \rho_{t} \tau_{t}+C_{t}^{s} s_{t}^{2}+C_{t}^{T} s_{t} \cdot \boldsymbol{T}_{t}\right)
$$

w/ density dependence

$$
\begin{aligned}
& C_{t}^{\rho}=A_{t}^{\rho}+B_{t}^{\rho} \rho_{0}^{\alpha} \\
& C_{s}^{\rho}=A_{s}^{\rho}+B_{s}^{\rho} \rho_{0}^{\alpha}
\end{aligned}
$$

Configuration Interaction QMC cal. by Trento group
A. Roggero et al., arXiv: | 406 . 163 I


Polarized neutron matter w/ impurity
"impurity energy"
spin-up $N$ neutrons, and a spinup (-down) proton

$$
\frac{\varepsilon_{p \uparrow}-\varepsilon_{p \downarrow}}{E_{F}}=\frac{4 m\left(C_{0}^{s}-C_{1}^{s}\right)}{3 \pi^{2} \hbar^{2}} k_{F}-\frac{2 m\left(C_{0}^{T}-C_{1}^{T}\right)}{5 \pi^{2} \hbar^{2}} k_{F}^{3}
$$

purely T-odd

## Self-consistent pnQRPA for spin-isospin excitations in deformed nuclei <br> starting point: Skyrme EDF $\mathcal{E}[\rho(\boldsymbol{r}), \tilde{\rho}(\boldsymbol{r})]$

variation w.r.t densities
The coordinate-space Hartree-Fock-Bogoliubov eq. for ground states
J. Dobaczewski et al., NPA422(1984)103

$$
\left(\begin{array}{cc}
h^{q}(\boldsymbol{r}, \sigma)-\lambda^{q} & \tilde{h}^{q}(\boldsymbol{r}, \sigma) \\
\tilde{h}^{q}(\boldsymbol{r}, \sigma) & -\left(h^{q}(\boldsymbol{r}, \sigma)-\lambda^{q}\right)
\end{array}\right)\binom{\varphi_{1, \alpha}^{q}(\boldsymbol{r}, \sigma)}{\varphi_{2, \alpha}^{q}(\boldsymbol{r}, \sigma)}=E_{\alpha}\binom{\varphi_{1, \alpha}^{q}(\boldsymbol{r}, \sigma)}{\varphi_{2, \alpha}^{q}(\boldsymbol{r}, \sigma)}
$$

©sp" $\quad \delta \mathcal{E} \quad q=\frac{\delta \mathcal{E}}{q} \quad q, \pi$
"s.p." hamiltonian and pair potential: $\quad h^{q}=\frac{\delta \mathcal{C}}{\delta \rho^{q}}, \quad \tilde{h}^{q}=\frac{\delta \mathcal{E}}{\delta \tilde{\rho}^{q}}$ quasiparticle basis $\alpha, \beta \ldots$

The proton-neutron quasiparticle RPA eq. for excited states $\left[\hat{H}, \hat{O}_{\lambda}^{\dagger}\right]\left|\Psi_{\lambda}\right\rangle=\omega_{\lambda} \hat{O}_{\lambda}^{\dagger}\left|\Psi_{\lambda}\right\rangle$
Collective excitation $=$ coherent superposition of 2qp excitations:

$$
\hat{O}_{\lambda}^{\dagger}=\sum_{\alpha \beta} X_{\alpha \beta}^{\lambda} \hat{a}_{\alpha, \nu}^{\dagger} \hat{a}_{\beta, \pi}^{\dagger}-Y_{\alpha \beta}^{\lambda} \hat{a}_{\bar{\beta}, \pi} \hat{a}_{\bar{\alpha}, \nu}
$$

residual interactions derived self-consistently :

$$
v_{\mathrm{res}}\left(\boldsymbol{r}_{1}, \boldsymbol{r}_{2}\right)=\frac{\delta^{2} \mathcal{E}}{\delta \rho_{1 t_{3}}\left(\boldsymbol{r}_{1}\right) \delta \rho_{1 t_{3}}\left(\boldsymbol{r}_{2}\right)} \boldsymbol{\tau}_{1} \cdot \boldsymbol{\tau}_{2}+\frac{\delta^{2} \mathcal{E}}{\delta s_{1 t_{3}}\left(\boldsymbol{r}_{1}\right) \delta s_{1 t_{3}}\left(\boldsymbol{r}_{2}\right)} \boldsymbol{\sigma}_{1} \cdot \boldsymbol{\sigma}_{2} \boldsymbol{\tau}_{1} \cdot \boldsymbol{\tau}_{2}
$$

## Canonical basis and quasiparticle basis

J. Engel et al., PRC60 (1999) 014302

$$
\begin{aligned}
A_{p n, p^{\prime} n^{\prime}}= & E_{p, p^{\prime}} \delta_{n, n^{\prime}}+E_{n, n^{\prime}} \delta_{p, p^{\prime}}+\widetilde{V}_{p n, p^{\prime} n^{\prime}}\left(u_{p} v_{n} u_{p^{\prime}} v_{n^{\prime}}\right. \\
& \left.+v_{p} u_{n} v_{p^{\prime}} u_{n^{\prime}}\right)+V_{p n, p^{\prime} n^{\prime}}\left(u_{p} u_{n} u_{p^{\prime}} u_{n^{\prime}}\right. \\
& \left.+v_{p} v_{n} v_{p^{\prime}} v_{n^{\prime}}\right) \\
B_{p n, p^{\prime} n^{\prime}}= & \widetilde{V}_{p n, p^{\prime} n^{\prime}}\left(v_{p} u_{n} u_{p^{\prime}} v_{n^{\prime}}+u_{p} v_{n} v_{p^{\prime}} u_{n^{\prime}}\right) \\
& -V_{p n, p^{\prime} n^{\prime}}\left(u_{p} u_{n} v_{p^{\prime}} v_{n^{\prime}}+v_{p} v_{n} u_{p^{\prime}} u_{n^{\prime}}\right)
\end{aligned}
$$

## calculation cost ~ BCS-QRPA

 introduction of cutoff occupation probability
## calculation cost $\sim$ 4*BCS-QRPA

introduction of cutoff
simply by the 2qp energy

$\mathrm{pp}(\mathrm{hh})$ and ph excitations are treated on the same footing

$$
\begin{align*}
& A_{\alpha \beta \alpha^{\prime} \beta^{\prime}}=\left(E_{\alpha}+E_{\beta}\right) \delta_{\alpha \alpha^{\prime}} \delta_{\beta \beta^{\prime}} \\
& +\int d 1 d 2 d 1^{\prime} d 2^{\prime}\left\{\varphi_{1, \alpha}^{\nu}\left(\boldsymbol{r}_{1} \bar{\sigma}_{1}\right) \varphi_{1, \beta}^{\pi}\left(\boldsymbol{r}_{2} \bar{\sigma}_{2}\right) \bar{v}_{\mathrm{pp}}\left(12 ; 1^{\prime} 2^{\prime}\right) \varphi_{1, \alpha^{\prime}}^{\nu *}\left(\boldsymbol{r}_{1}^{\prime} \overline{\sigma_{1}^{\prime}}\right) \varphi_{1, \beta^{\prime}}^{\pi *}\left(\boldsymbol{r}_{2}^{\prime} \bar{\sigma}_{2}^{\prime}\right)\right. \\
& +\varphi_{2, \alpha}^{\nu}\left(\boldsymbol{r}_{1} \sigma_{1}\right) \varphi_{2, \beta}^{\pi}\left(\boldsymbol{r}_{2} \sigma_{2}\right) \bar{v}_{\mathrm{pp}}\left(12 ; 1^{\prime} 2^{\prime}\right) \varphi_{2, \alpha^{\prime}}^{\nu *}\left(\boldsymbol{r}_{1}^{\prime} \sigma_{1}^{\prime}\right) \varphi_{2, \beta^{\prime}}^{\pi *}\left(\boldsymbol{r}_{2}^{\prime} \sigma_{2}^{\prime}\right) \\
& +\varphi_{1, \alpha}^{v}\left(\boldsymbol{r}_{1} \bar{\sigma}_{1}\right) \varphi_{2, \beta^{\prime}}^{\pi *}\left(\boldsymbol{r}_{2} \sigma_{2}\right) \bar{v}_{\mathrm{ph}}\left(12 ; 1^{\prime} 2^{\prime}\right) \varphi_{2, \beta}^{\pi}\left(\boldsymbol{r}_{1}^{\prime} \sigma_{1}^{\prime}\right) \varphi_{1, \alpha^{\prime}}^{\nu *}\left(\boldsymbol{r}_{2}^{\prime} \bar{\sigma}_{2}^{\prime}\right) \\
& \left.+\varphi_{1, \beta}^{\pi}\left(\boldsymbol{r}_{1} \overline{\sigma_{1}}\right) \varphi_{2, \alpha^{\prime}}^{\nu *}\left(\boldsymbol{r}_{2} \sigma_{2}\right) \bar{v}_{\mathrm{ph}}\left(12 ; 1^{\prime} 2^{\prime}\right) \varphi_{2, \alpha}^{\nu}\left(\boldsymbol{r}_{1}^{\prime} \sigma_{1}^{\prime}\right) \varphi_{1, \beta^{\prime}}^{\pi *}\left(\boldsymbol{r}_{2}^{\prime} \overline{\sigma_{2}^{\prime}}\right)\right\},  \tag{7}\\
& B_{\alpha \beta \alpha^{\prime} \beta^{\prime}}=\int d 1 d 2 d 1^{\prime} d 2^{\prime}\left\{-\varphi_{1, \alpha}^{v}\left(\boldsymbol{r}_{1} \bar{\sigma}_{1}\right) \varphi_{1, \beta}^{\pi}\left(\boldsymbol{r}_{2} \bar{\sigma}_{2}\right) \bar{v}_{\mathrm{pp}}\left(12 ; 1^{\prime} 2^{\prime}\right) \varphi_{2, \alpha^{\prime}}^{\nu}\left(\boldsymbol{r}_{1}^{\prime} \sigma_{1}^{\prime}\right) \varphi_{2, \bar{\beta}^{\prime}}^{\pi}\left(\boldsymbol{r}_{2}^{\prime} \sigma_{2}^{\prime}\right)\right. \\
& -\varphi_{2, \alpha}^{\nu}\left(\boldsymbol{r}_{1} \sigma_{1}\right) \varphi_{2, \beta}^{\pi}\left(\boldsymbol{r}_{2} \sigma_{2}\right) \bar{v}_{\mathrm{pp}}\left(12 ; 1^{\prime} 2^{\prime}\right) \varphi_{1, \alpha^{\prime}}^{\nu}\left(\boldsymbol{r}_{1}^{\prime} \bar{\sigma}_{1}^{\prime}\right) \varphi_{1, \bar{\beta}^{\prime}}^{\pi}\left(\boldsymbol{r}_{2}^{\prime} \bar{\sigma}_{2}^{\prime}\right) \\
& -\varphi_{1, \alpha}^{\nu}\left(\boldsymbol{r}_{1} \bar{\sigma}_{1}\right) \varphi_{1, \bar{\beta}^{\prime}}^{\pi}\left(\boldsymbol{r}_{2} \bar{\sigma}_{2}\right) \bar{v}_{\mathrm{ph}}\left(12 ; 1^{\prime} 2^{\prime}\right) \varphi_{2, \beta}^{\pi}\left(\boldsymbol{r}_{1}^{\prime} \sigma_{1}^{\prime}\right) \varphi_{2, \bar{\alpha}^{\prime}}^{\nu}\left(\boldsymbol{r}_{2}^{\prime} \sigma_{2}^{\prime}\right) \\
& \left.-\varphi_{1, \beta}^{\pi}\left(\boldsymbol{r}_{1} \bar{\sigma}_{1}\right) \varphi_{1, \alpha^{\prime}}^{\nu}\left(\boldsymbol{r}_{2} \bar{\sigma}_{2}\right) \bar{v}_{\mathrm{ph}}\left(12 ; 1^{\prime} 2^{\prime}\right) \varphi_{2, \alpha}^{\nu}\left(\boldsymbol{r}_{1}^{\prime} \sigma_{1}^{\prime}\right) \varphi_{2, \bar{\beta}^{\prime}}^{\pi}\left(\boldsymbol{r}_{2}^{\prime} \sigma_{2}^{\prime}\right)\right\} .
\end{align*}
$$

## QRPA for heavy axially-deformed nuclei with HPC

```
HFB cal. (64 CPUs)
    Box size: }14.7\textrm{fm}\times14.4\textrm{fm
    Cut-off: }\Omega\leqq\frac{31}{2},\mp@subsup{E}{\alpha}{}\leqq60\textrm{MeV
```

HFB Hamiltonian: block diagonal in $\Omega^{\pi}$ distributed to cores

```
QRPA cal. (512 CPUs)
```

QRPA cal. (512 CPUs)
Cut-off: }\mp@subsup{E}{\alpha}{}+\mp@subsup{E}{\beta}{}\leqq60\textrm{MeV

```
    Cut-off: }\mp@subsup{E}{\alpha}{}+\mp@subsup{E}{\beta}{}\leqq60\textrm{MeV
```

Matrix elements of A and B : 2D-numerical integration independent of each 2 qp configuration

```
For each K}\mp@subsup{}{}{\pi
    # of 2qp excitation: ~50,000
```

matrix elements: 590 core hours
diagonalization: 330 core hours
$\checkmark$ GT strengths and beta-decay properties of neutron-rich Zr isotopes

## $\beta$-decay half-lives in neutron-rich Zr isotopes

S. Nishimura et al., PRL106(2011)052502


## $\beta$-decay half-lives in neutron-rich Zr isotopes

S. Nishimura et al., PRL106(2011)052502


## Deformed Zr isotopes on the r-process path


T. Sumikama et al., PRL106(2011)202501

## GT giant resonance

$$
\hat{F}_{K}^{t_{3}}=\sum_{\sigma, \sigma^{\prime}} \sum_{\tau, \tau^{\prime}} \int d \boldsymbol{r} \hat{\psi}^{\dagger}(\boldsymbol{r} \sigma \tau)\langle\sigma| \boldsymbol{\sigma}_{K}\left|\sigma^{\prime}\right\rangle\langle\tau| \boldsymbol{\tau}_{t_{3}}\left|\tau^{\prime}\right\rangle \hat{\psi}\left(\boldsymbol{r} \sigma^{\prime} \tau^{\prime}\right)
$$



## $\sqrt{ }$ sudden onset of deformation at $N=60$

SLy4: A. Blazkiewicz et al., PRC71(2005)054321
$\checkmark$ fragmentation of strength distribution due to deformation
separable pnRPA:
P. Urkedal et al., PRC64(2001)054304
$\sqrt{ } \mathrm{SkM}^{*}$ and SLy4 give almost the same ( $\Delta \mathrm{E}<\mathrm{I} \mathrm{MeV}$ ) excitation energies of GTGR

$$
\mathrm{go}^{\prime}=0.94\left(\mathrm{SkM}^{*}\right), 0.90 \text { (SLy4) }
$$


excitation energy w.r.t. the g.s of daughter
MeV smearing width

## GTGR: the need of self-consistency


$\sqrt{ }$ a repulsive character of the residual interaction raises the peak energy
$\sqrt{ }$ low-lying strengths are absorbed to the highenergy peak
strong collectivity of the GTGR
$\sqrt{ }$ the collectivity generated by the Landau-Migdal approximation is weak

$$
v_{\mathrm{ph}}\left(\boldsymbol{r}_{1} \boldsymbol{r}_{2}\right)=N_{0}^{-1}\left[f_{0}^{\prime} \tau_{1} \cdot \tau_{2}+g_{0}^{\prime} \sigma_{1} \cdot \sigma_{2} \tau_{1} \cdot \tau_{2}\right] \delta\left(\boldsymbol{r}_{1}-\boldsymbol{r}_{2}\right)
$$

LM parameter:
M. Bender et al., PRC65(2002)054322

self-consistency is required for a quantitative description of the GTGR

## IV giant dipole resonance:Anti-analog GDR

$$
\hat{F}_{1 K}^{t_{3}}=\sum_{\sigma} \sum_{\tau, \tau^{\prime}} \int d \boldsymbol{r} r Y_{1 K} \hat{\psi}^{\dagger}(\boldsymbol{r} \sigma \tau)\langle\tau| \boldsymbol{\tau}_{t_{3}}\left|\tau^{\prime}\right\rangle \hat{\psi}\left(\boldsymbol{r} \sigma \tau^{\prime}\right)
$$



SLy4 + mixed-type pairing
$\sqrt{ }$ deformation splitting (Ksplitting) is clearly seen as in GDR

## AGDR and neutron skin thickness

linear relationship between the neutron-skin thickness and the AGDR energy

A. Krasznahorkay, N. Paar, D. Vretenar, M.N.Harakeh, Phys. Scr. T154(2013)4018

J.Yasuda, T.Wakasa et al., PTEP2013, 063D02

## $\mathrm{T}=0(\mathrm{~S}=\mathrm{I})$ pairing

$\sqrt{ }$ affects the GT response

## if we have (a) $\mathrm{T}=\mathrm{I}$ pairing condensate(s)

due to the coupling between the p-h and p-p excitations
we may see the effect in the low-lying states that are generated by 2qp excitations around the Fermi levels
$\checkmark$ does not affect the gs properties in $N>Z$ nuclei
a form of the interaction or an np-pairing EDF is seldom known

## Take the simplest one;

$$
v_{\mathrm{pp}}^{T=0}\left(\boldsymbol{r}, \boldsymbol{r}^{\prime}\right)=\frac{1+P_{\sigma}}{2} \frac{1-P_{\tau}}{2} V_{0} \delta\left(\boldsymbol{r}-\boldsymbol{r}^{\prime}\right)
$$

the pairing strength determined to reproduce the $\beta$-decay half-life of ${ }^{100} \mathrm{Zr}$ (7.1 s)

## Low-lying GT states

SLy4+Mixed-type pairing
0.1 MeV smearing width

0.1 MeV smearing width

## Low-lying GT states



## selection rule for GT-

$\left.\left|\left\langle\pi\left[N n_{3} \Lambda\right] \Omega=\Lambda+1 / 2\right| t_{-} \sigma_{+1}\right| \nu\left[N n_{3} \Lambda\right] \Omega=\Lambda-1 / 2\right\rangle \mid=\sqrt{2}$
${ }^{106} Z r$
constructed dominantly by $\pi[413] 7 / 2 \otimes \nu[413] 5 / 2$ particle-like particle-like $\sqrt{ } \mathrm{T}=0$ pairing effective

108, $110 Z r$
$\pi[413] 7 / 2 \otimes \nu[413] 5 / 2$ particle-like hole-like $\checkmark T=0$ pairing ineffective

## Beta-decay half-lives


$\sqrt{ }$ Fermi's golden rule
N. B. Gove, M. J. Martin,

At. Data Nucl. Data Tables 10(1971)205
$\checkmark$ Fermi and Gamow-Teller strengths included
$\sqrt{ } \mathrm{SkM}^{*}$ produces longer half-lives primarily due to a large Q -value
approximate Q -value

$$
\begin{aligned}
Q_{\beta^{-}} & =\Delta M_{\mathrm{n}-\mathrm{H}}+B(A, Z+1)-B(A, Z) \\
& \simeq \Delta M_{\mathrm{n}-\mathrm{H}}+\lambda_{\nu}-\lambda_{\pi}-E_{0} \\
E_{0} & =\min \left[E_{\nu}+E_{\pi}\right]
\end{aligned}
$$

cf. J. Engel et al., PRC60(1999)014302

## Beta-decay half-lives with $\mathrm{T}=0$ pairing


$\sqrt{ }$ Strength of $\mathrm{T}=0$ pairing determined at $\mathrm{N}=60$

## SLy4

$\sqrt{ }$ reproduces well the observed isotopic dependence with $\mathrm{T}=0$ pairing
$\checkmark$ Effect of the $\mathrm{T}=0$ pairing is small beyond N=68

## SkM*

$\sqrt{\text { gives a strong deformed gap at } N=64}$
$\sqrt{ }$ pn-pair transfer strengths in $\mathrm{N}=\mathrm{Z}$ nuclei in fp-shell

## Proton-neutron pair excitations

$$
\begin{aligned}
& \hat{P}_{T=0, S=1, S_{z}=0}^{\dagger} \\
& =\frac{1}{2} \sum_{\sigma, \sigma^{\prime}} \sum_{\tau} \int d \boldsymbol{r} \hat{\psi}^{\dagger}(\boldsymbol{r} \sigma \tau)\langle\sigma| \boldsymbol{\sigma}_{0}\left|\sigma^{\prime}\right\rangle \hat{\psi}^{\dagger}\left(\boldsymbol{r} \tilde{\sigma}^{\prime} \tilde{\tau}\right)
\end{aligned}
$$

IS
spin triplet

$$
\begin{aligned}
& \hat{P}_{T=0, S=1, S_{z}= \pm 1}^{\dagger} \\
& =\frac{1}{2} \sum_{\sigma, \sigma^{\prime}} \sum_{\tau} \int d \boldsymbol{r} \hat{\psi}^{\dagger}(\boldsymbol{r} \sigma \tau)\langle\sigma| \sigma_{ \pm 1}\left|\sigma^{\prime}\right\rangle \hat{\psi}^{\dagger}\left(\boldsymbol{r} \tilde{\sigma}^{\prime} \tilde{\tau}\right)
\end{aligned}
$$

IV
spin singlet

$$
\begin{aligned}
& \hat{P}_{T=1, T_{z}=0, S=0}^{\dagger} \\
& =\frac{1}{2} \sum_{\sigma} \sum_{\tau, \tau^{\prime}} \int d \boldsymbol{r} \hat{\psi}^{\dagger}(\boldsymbol{r} \sigma \tau)\langle\tau| \boldsymbol{\tau}_{0}\left|\tau^{\prime}\right\rangle \hat{\psi}^{\dagger}\left(\boldsymbol{r} \tilde{\sigma} \tilde{\tau}^{\prime}\right)
\end{aligned}
$$

Interactions employed for pn-pairing vibration in fp-shell nuclei

## HFB eq:

SGII + surface pairing

$$
\mathrm{V}_{0}=-520 \mathrm{MeV} \mathrm{fm} 3
$$

$$
{ }^{44} \mathrm{Ti}
$$

$$
\Delta \mathrm{n}=1.82 \mathrm{MeV}
$$

$$
\Delta p=1.87 \mathrm{MeV}
$$

## pnQRPA eq:

p-h channel: SGII
p-p channel:

$$
\begin{aligned}
& v_{\mathrm{pp}}^{T=0}\left(\boldsymbol{r} \sigma \tau, \boldsymbol{r}^{\prime} \sigma^{\prime} \tau^{\prime}\right)=f \times V_{0} \frac{1+P_{\sigma}}{2} \frac{1-P_{\tau}}{2}\left[1-\frac{\rho(\boldsymbol{r})}{\rho_{0}}\right] \delta\left(\boldsymbol{r}-\boldsymbol{r}^{\prime}\right) \\
& v_{\mathrm{pp}}^{T=1}\left(\boldsymbol{r} \sigma \tau, \boldsymbol{r}^{\prime} \sigma^{\prime} \tau^{\prime}\right)=V_{0} \frac{1-P_{\sigma}}{2} \frac{1+P_{\tau}}{2}\left[1-\frac{\rho(\boldsymbol{r})}{\rho_{0}}\right] \delta\left(\boldsymbol{r}-\boldsymbol{r}^{\prime}\right)
\end{aligned}
$$

cf. C. Bai et al., PLB719(2013)116

$\mathrm{f}=1.3$

| ${ }^{42} \mathrm{Sc}$ |  | $J^{\pi}=1^{+}$ | $J^{\pi}=0^{+}$ |
| :---: | :---: | :---: | :---: |
| configuration | $E_{\alpha}+E_{\beta}$ | $M_{\alpha \beta}^{S=1, S_{z}=0}$ | $M_{\alpha \beta}^{S=0}$ |
| $\pi 1 f_{7 / 2} \otimes \nu 1 f_{7 / 2}$ | 7.5 | 1.70 | 2.85 |
| $\pi 1 f_{7 / 2} \otimes \nu 1 f_{5 / 2}$ | 15.2 | 0.62 |  |
| $\pi 1 f_{5 / 2} \otimes \nu 1 f_{7 / 2}$ | 14.7 | 0.51 |  |
| $\pi 2 p_{3 / 2} \otimes \nu 2 p_{3 / 2}$ | 16.1 | 0.17 | 0.22 |
| $\pi 1 d_{3 / 2} \otimes \nu 1 d_{3 / 2}$ | 4.2 | 0.25 | 0.48 |
| $\pi 2 s_{1 / 2} \otimes \nu 2 s_{1 / 2}$ | 6.6 | 0.25 |  |
| $\pi 1 d_{3 / 2} \otimes \nu 1 d_{5 / 2}$ | 10.1 | 0.32 |  |
| $\pi 1 d_{5 / 2} \otimes \nu 1 d_{3 / 2}$ | 10.2 | 0.32 |  |
| $\pi 1 d_{5 / 2} \otimes \nu 1 d_{5 / 2}$ | 16.1 | 0.16 | 0.31 |

Transition matrix element

$$
\langle\lambda| \hat{P}_{T, S}^{\dagger}|0\rangle=\sum_{\alpha \beta} M_{\alpha \beta}^{T, S}
$$



## Collective pn-pairing vibration mode precursory to the $\mathrm{T}=0$ pairing condensation

$$
\Delta E=\omega_{1+}-\omega_{0+}
$$


approaching the critical point to the $\mathrm{T}=0$ pairing condensation

$$
\mathrm{f}_{\mathrm{c}}=\mathrm{I} .53
$$

## Enhancement of the transfer strengths

$\frac{\left.\left|\langle\lambda| \hat{P}_{T=0}^{\dagger}\right| 0\right\rangle\left.\right|^{2}}{\left.\left|\langle\lambda| \hat{P}_{T=1}^{\dagger}\right| 0\right\rangle\left.\right|^{2}}$

$$
\frac{\left.\left|\langle\lambda| \hat{P}_{T=0}^{\dagger}\right| 0\right\rangle\left.\right|^{2}}{\mid\left.\langle\text { unp. }| \hat{P}_{T=0}^{\dagger}|0\rangle\right|^{2}}
$$



d-tranfer: ${ }^{40} \mathrm{Ca}\left({ }^{3} \mathrm{He}, \mathrm{p}\right)^{42} \mathrm{Sc}$

$$
\frac{\sigma\left(1^{+}\right)_{\exp }}{\sigma\left(1^{+}\right)_{\mathrm{unp}}}=23.9
$$

F. Pühlhofer, NPA116 (1968) 516


## Summary

Fully-selfconsistent deformed pnQRPA is developed in a Skyrme EDF framework
Microscopic and quantitative description of spin-isospin excitations in nuclei with arbitrary mass number whichever they are spherical or deformed, located around the stability line or close to the drip line
provide a microscopic input to the astrophysical simulation

Deformation effects on spin-isospin responses
Tiny deformation splitting in Gamow-Teller excitation
Fragmentation of GTGR
Clear deformation splitting in Anti-analog GDR as seen in IVGDR
Effects of $\mathrm{T}=0$ pairing
Low-lying GT states are sensitive to the location of the Fermi levels, and the beta-decay half-lives are shortened

Proton-neutron pair-transfer strengths
Strong collectivity due to the $\mathrm{T}=0$ pairing suggests emergence of a soft mode toward the $\mathrm{T}=0$ pairing condensation

## Restoration of the isospin symmetry breaking (ISB)

Even w/o the Coulomb int, the ISB occurs in $\mathrm{N}>\mathrm{Z}$ nuclei in a MFA

$$
\left[H_{\mathrm{MF}}, T_{-}\right] \neq 0 \quad \text { C. A. Engelbrecht and R. H. Lemmer, PRL24(1970)607 }
$$

$\triangle$ IAS appears as a NG mode in the pnRPA
Ex. ${ }^{90} \mathrm{Zr}(\mathrm{N}-\mathrm{Z}=10)$ w/o Coulomb


$$
\begin{aligned}
& \rho_{\max } \times z_{\max }=14.7 \mathrm{fm} \times 14.4 \mathrm{fm} \\
& \Delta \rho=\Delta z=0.6 \mathrm{fm} \\
& E_{2 q p} \leq 60 \mathrm{MeV}
\end{aligned}
$$

## Restoration of the isospin symmetry breaking (ISB)

Ex. ${ }^{90} \mathrm{Zr}(\mathrm{N}-\mathrm{Z}=10)$ w/o Coulomb
protons are paired

$$
\text { SkM* + mixed-type pairing } \quad \Delta_{\nu}=0.00 \mathrm{MeV}
$$

## Restoration of the isospin symmetry breaking (ISB)

Ex. ${ }^{90} \mathrm{Zr}$ (N-Z=10) w/o Coulomb inclusion of the $\mathrm{T}=\mathrm{I}(\mathrm{S}=0)$ pairing interaction in the pnQRPA

$$
v_{\mathrm{pp}}^{T=1}\left(\boldsymbol{r}, \boldsymbol{r}^{\prime}\right)=\frac{1-P_{\sigma}}{2} \frac{1+P_{\tau}}{2} V_{0}\left[1-\frac{1}{2} \frac{\rho_{00}(\boldsymbol{r})}{\rho_{0}}\right] \delta\left(\boldsymbol{r}-\boldsymbol{r}^{\prime}\right)
$$



