Light nuclei from lattice QCD

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Refs: PRD81:111504(R)(2010); PRD84:054506(2011); PRD86:074514(2012)

Advances and perspectives in computational nuclear physics

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Outline

1. Introduction
2. Problems of nuclei in lattice QCD
3. Simulation parameters
4. (Preliminary) Results
   - ^{4}He and ^{3}He channels
   - NN channels
5. Summary and future work

Introduction

$$\text{Binding force} \begin{cases} \text{protons and neutrons} & \rightarrow & \text{nuclei} \\ \text{quarks and gluons} & \rightarrow & \text{protons and neutrons} \end{cases}$$

both from strong interaction

well known in experiment

Spectrum of nuclei

success of Shell model: Jensen and Mayer (1949)

degrees of freedom of protons and neutrons

Spectrum of proton and neutron (nucleons)

should be explained by QCD

degrees of freedom of quarks and gluons

but hard to calculated due to large coupling constant

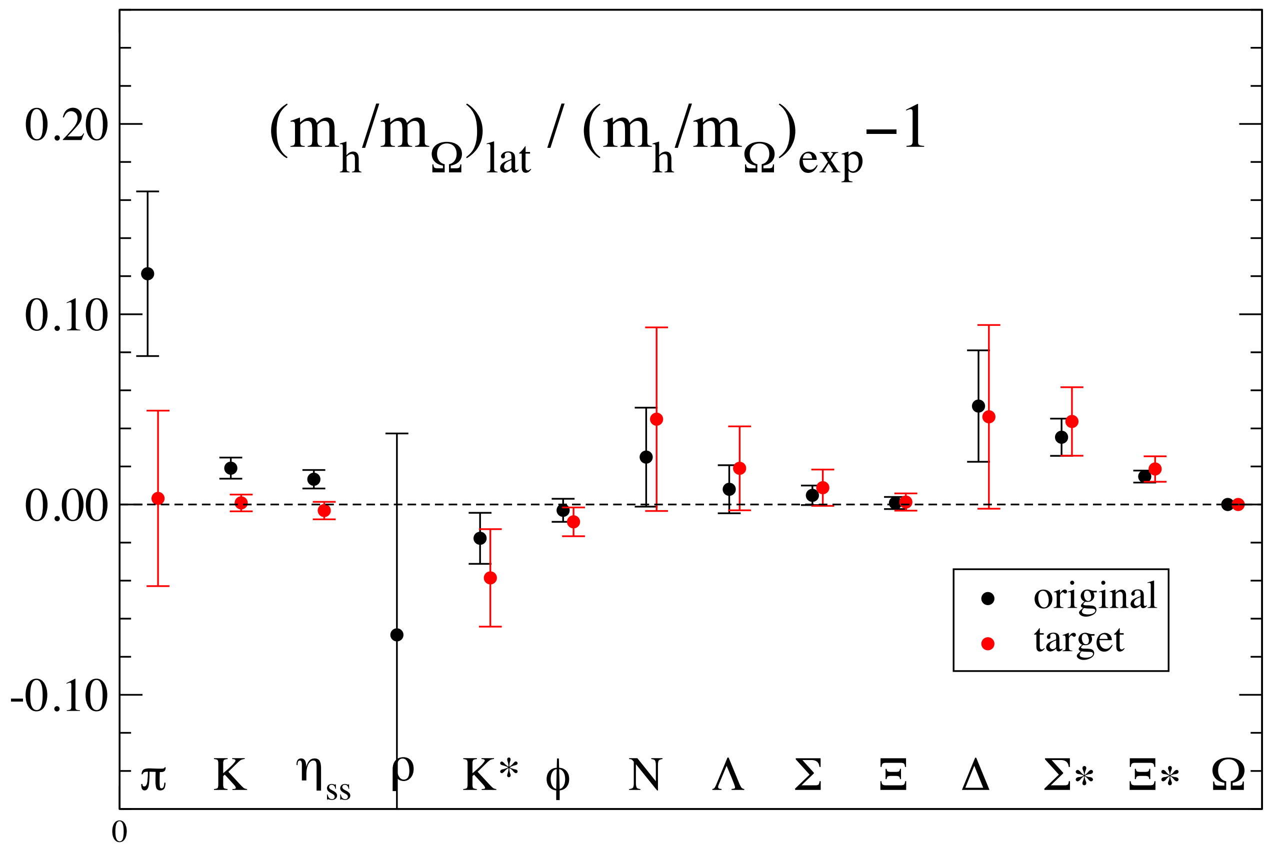
$$\text{quarks and gluons} \rightarrow \overbrace{\text{protons and neutrons} \rightarrow \text{nuclei}}^{\text{Shell model}}$$

Hadron spectrum at physical point $m_\pi = 135$ MeV

$N_f = 2+1 (m_u = m_d \neq m_s)$ '10 PACS-CS

Input parameters: g_0 and $m_l = m_u, m_d$, and m_s

target $= m_\pi = 135$ MeV



good agreement within a few%

Introduction

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success of non-perturbative calculation of QCD

such as lattice QCD

degrees of freedom of quarks and gluons

$$\underbrace{\text{quarks and gluons} \rightarrow \overbrace{\text{protons and neutrons}}}_{\text{lattice QCD}} \rightarrow \text{nuclei}$$

Shell model

Introduction

$$\text{Binding force} \begin{cases} \text{protons and neutrons} & \rightarrow & \text{nuclei} \\ \text{quarks and gluons} & \rightarrow & \text{protons and neutrons} \end{cases}$$

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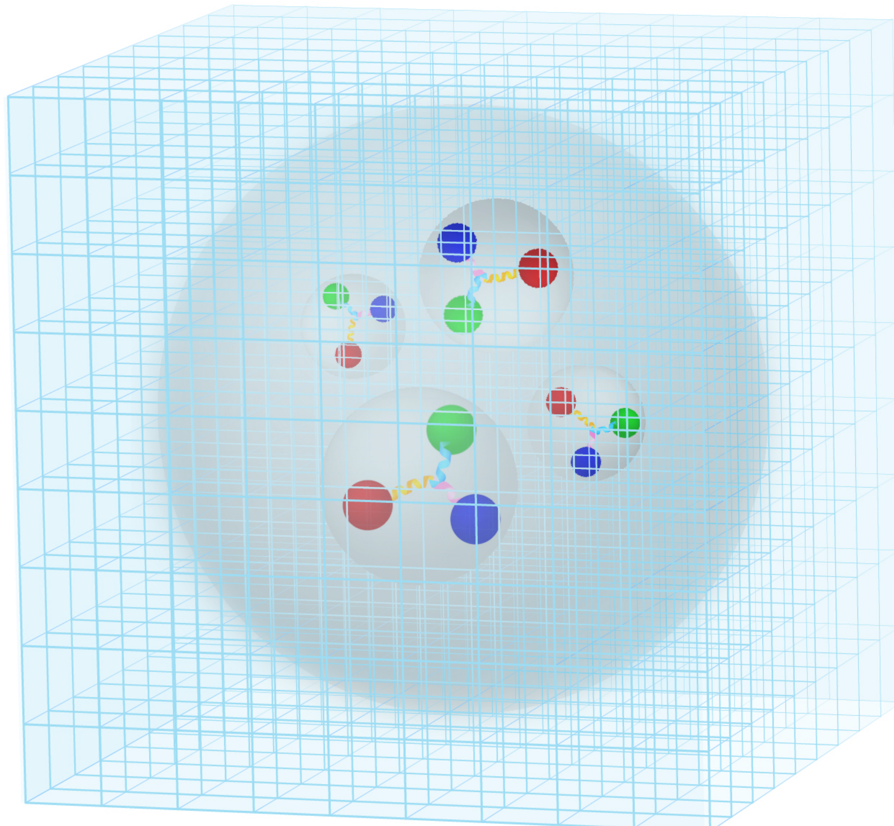
$\boxed{\text{Ultimate goal of lattice QCD: quantitatively understand property of nuclei}}$

$$\underbrace{\text{quarks and gluons} \rightarrow \overbrace{\text{protons and neutrons} \rightarrow \text{nuclei}}^{\text{Shell model}}}_{\text{lattice QCD}}$$

Ultimate goal of lattice QCD



http://www.jicfus.jp/jp/promotion/pr/mj/2014-1/

quantitatively understand property of nuclei
from first principle of strong interaction

Introduction

Motivation :

Understand property of nuclei from QCD

If we can study nuclei from QCD, we may be able to

1. reproduce spectrum of nuclei
2. predict property of nuclei hard to calculate or observe

   such as neutron rich nuclei

So far not so many works for multi-baryon bound states
Before studying such difficult problems, we should study

$\rightarrow$ Can we reproduce known binding energy in light nuclei?

Multi-baryon bound state from lattice QCD

Not observed before '09 (except H-dibaryon '88 Iwasaki *et al.*)

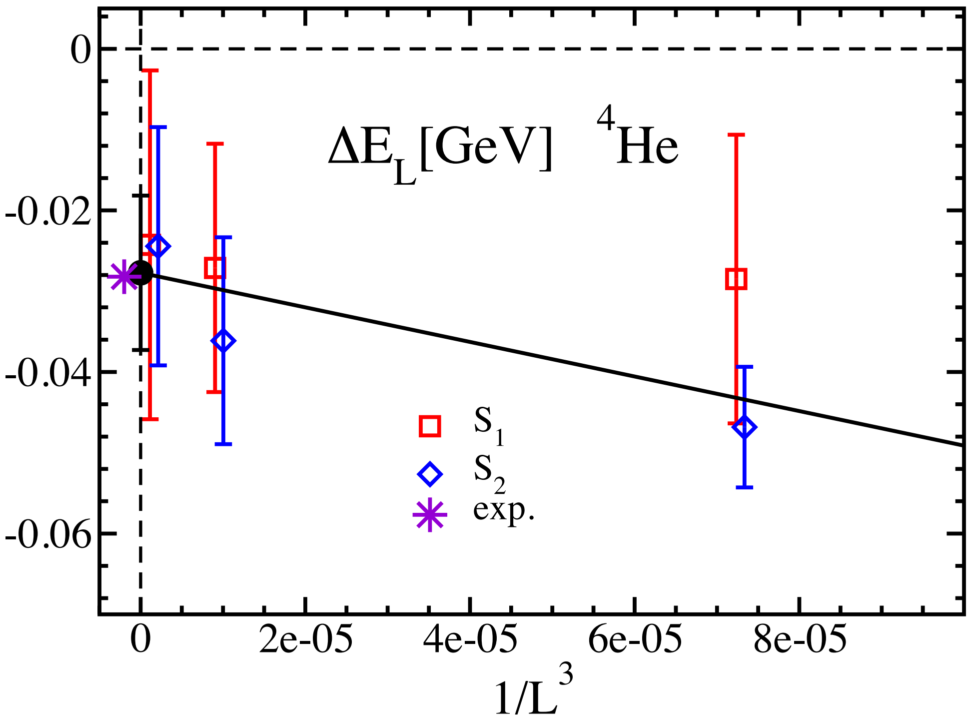
1. ^{4}He and ^{3}He

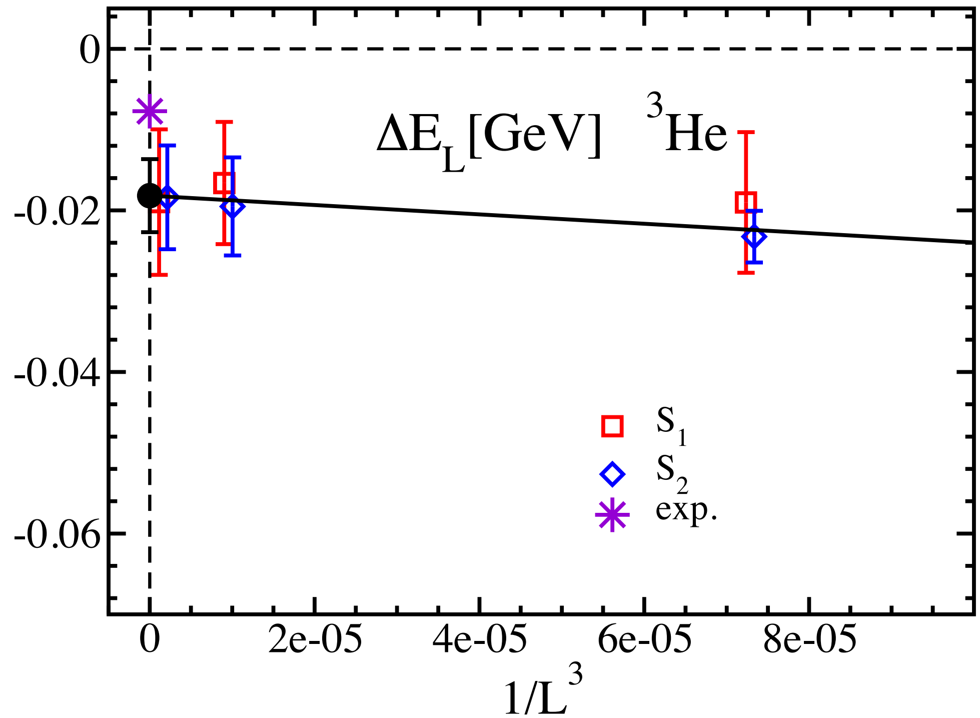
'10 PACS-CS $N_f = 0$ $m_\pi = 0.8$ GeV PRD81:111504(R)(2010)

Exploratory study of three- and four-nucleon systems

PACS-CS Collaboration, PRD81:111504(R)(2010)

Identification of bound state from volume dependence of ΔE





$\Delta E_{^4\text{He}} = 27.7(7.8)(5.5)$ MeV $\qquad$ $\Delta E_{^3\text{He}} = 18.2(3.5)(2.9)$ MeV

1. Observe bound state in both channels
2. Same order of ΔE to experiment

Several systematic errors included, e.g., $N_f = 0$, $m_\pi = 0.8$ GeV

Multi-baryon bound state from lattice QCD

Not calculated before '09 (except H-dibaryon '88 Iwasaki *et al.*)

1. ^{4}He and ^{3}He
   - '10 PACS-CS $N_f = 0$ $m_\pi = 0.8$ GeV PRD81:111504(R)(2010)
   - '12 HALQCD $N_f = 3$ $m_\pi = 0.47$ GeV, $m_\pi > 1$ GeV ^{4}He
   - '12 NPLQCD $N_f = 3$ $m_\pi = 0.81$ GeV
   - '12 TY *et al.* $N_f = 2+1$ $m_\pi = 0.51$ GeV PRD86:074514(2012)
2. H dibaryon in ΛΛ channel (S=−2, I=0)
   - '11 NPLQCD $N_f = 2+1$ $m_\pi = 0.39$ GeV
   - '11 HALQCD $N_f = 3$ $m_\pi = 0.67$–1.02 GeV
   - '11 Luo et al. $N_f = 0$ $m_\pi = 0.5$–1.3 GeV
   - '12 NPLQCD $N_f = 3$ $m_\pi = 0.81$ GeV
3. NN
   - '11 PACS-CS $N_f = 0$ $m_\pi = 0.8$ GeV PRD84:054506(2011)
   - '12 NPLQCD $N_f = 2+1$ $m_\pi = 0.39$ GeV (Possibility)
   - '12 NPLQCD $N_f = 3$ $m_\pi = 0.81$ GeV
   - '12 TY *et al.* $N_f = 2+1$ $m_\pi = 0.51$ GeV PRD86:074514(2012)

Other states: ΞΞ, '12 NPLQCD; spin-2 $N\Omega$, ^{16}O and ^{40}Ca, '14 HALQCD

$\boxed{\text{Extend our works to } N_f = 2+1 \text{ QCD with smaller } m_\pi}$

Problems of multi-nucleon bound state

Traditional method for example ^{4}He channel

$$\langle 0|O_{^4\mathrm{He}}(t)O^\dagger_{^4\mathrm{He}}(0)|0\rangle = \sum_n \langle 0|O_{^4\mathrm{He}}|n\rangle\langle n|O^\dagger_{^4\mathrm{He}}|0\rangle e^{-E_n t} \xrightarrow[t\gg 1]{} A_0\, e^{-E_0 t}$$

Difficulties for multi-nucleon calculation

1. Statistical error

Statistical error $\propto$ exp $\left(N_N\left[m_N - \frac{3}{2}m_\pi\right]t\right)$

2. Calculation cost

Wick contraction for ^{4}He $= p^2n^2 = (udu)^2(dud)^2$: 518400

proton $= p = (udu)$: 2

3. Identification of bound state on finite volume

Finite volume effect of attractive scattering state

$$\Delta E_L = E_0 - N_N m_N = O(L^{-3}) < 0 \leftrightarrow \text{binding energy}$$

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proton $= p = (udu)$: 2

Most severe problem: (every t)$\times N_{\text{meas}} \sim O(10^6)$

3. Identification of bound state on finite volume

Finite volume effect of attractive scattering state

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Problems of multi-nucleon bound state

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$$\langle 0|O_{^4\mathrm{He}}(t)O^\dagger_{^4\mathrm{He}}(0)|0\rangle = \sum_n \langle 0|O_{^4\mathrm{He}}|n\rangle\langle n|O^\dagger_{^4\mathrm{He}}|0\rangle e^{-E_n t} \xrightarrow[t\gg 1]{} A_0\, e^{-E_0 t}$$

Difficulties for multi-nucleon calculation

1. Statistical error

Statistical error $\propto \exp\left(N_N\left[m_N - \frac{3}{2}m_\pi\right]t\right)$

$\rightarrow$ heavy quark $m_\pi = 0.8$–0.3 GeV + large # of measurements

2. Calculation cost PACS-CS PRD81:111504(R)(2010)

Wick contraction for ^{4}He $= p^2n^2 = (udu)^2(dud)^2$: 518400 $\rightarrow$ 1107

$\rightarrow$ reduction using $p(n) \leftrightarrow p(n)$ $p \leftrightarrow n$, $u(d) \leftrightarrow u(d)$ in $p(n)$

Multi-meson: '10 Detmold and Savage

Multi-baryon: '12 Doi and Endres; Detmold and Orginos; '13 Günther et al.

3. Identification of bound state on finite volume

attractive scattering state $\Delta E_L = E_0 - N_N m_N = O(L^{-3}) < 0$

'86,'91 Lüscher, '07 Beane *et al.*

$\rightarrow$ Volume dependence of $\Delta E_L \rightarrow \Delta E_\infty \neq 0 \rightarrow$ bound state

Spectral weight: '04 Mathur *et al.*, Anti-PBC '05 Ishii *et al.*

Simulation parameters

$N_f = 2+1$ QCD

Iwasaki gauge action at $\beta = 1.90$

$a^{-1} = 2.194$ GeV with $m_\Omega = 1.6725$ GeV '10 PACS-CS

non-perturbative $O(a)$-improved Wilson fermion action

$m_\pi = 0.51$ GeV and $m_N = 1.32$ GeV

$m_\pi = 0.30$ GeV and $m_N = 1.05$ GeV

$m_s \sim$ physical strange quark mass

^{4}He, ^{3}He, NN(^{3}S$_1$ and ^{1}S$_0$)

		$m_\pi = 0.5$ GeV		$m_\pi = 0.3$ GeV		R
L	L [fm]	N_{conf}	N_{meas}	N_{conf}	N_{meas}	
32	2.9	200	192			
40	3.6	200	192			
48	4.3	200	192	400	1152	12
64	5.8	190	256	160	1536	5

$R = (N_{\text{conf}} \cdot N_{\text{meas}})_{0.5\text{GeV}} / (N_{\text{conf}} \cdot N_{\text{meas}})_{0.3\text{GeV}}$

Computational resources

PACS-CS, T2K-Tsukuba, HA-PACS, COMA at Univ. of Tsukuba

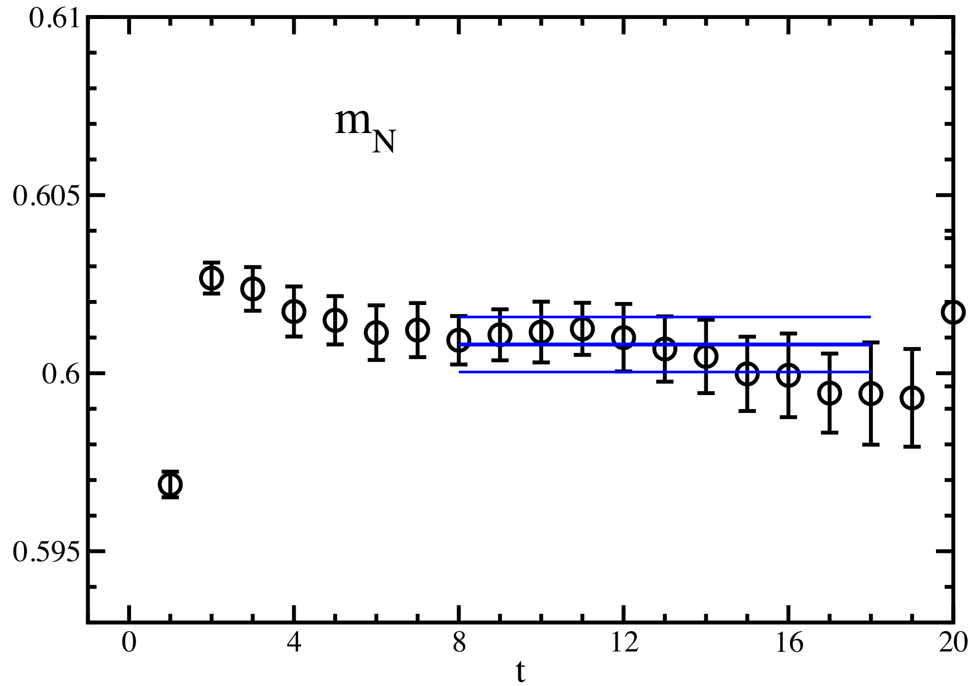
T2K-Tokyo and FX10 at Univ. of Tokyo, and K at AICS

Results

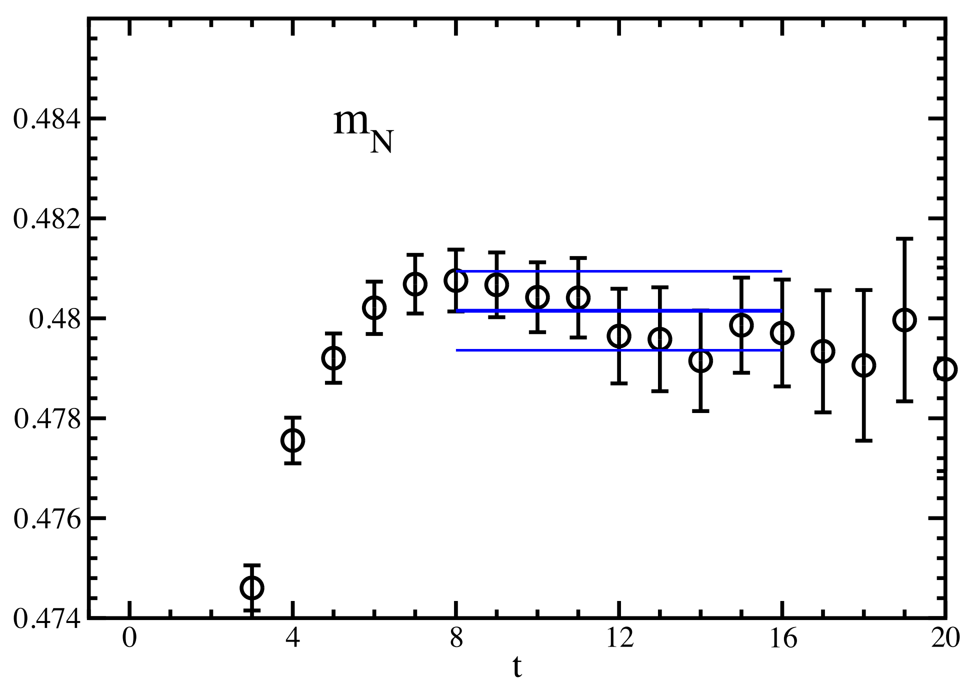
Effective mass of nucleon on $L = 5.8$ fm

$$\text{Effective } m_N = \log\left(\frac{C_N(t)}{C_N(t+1)}\right)$$

$m_\pi = 0.5$ GeV



$m_\pi = 0.3$ GeV



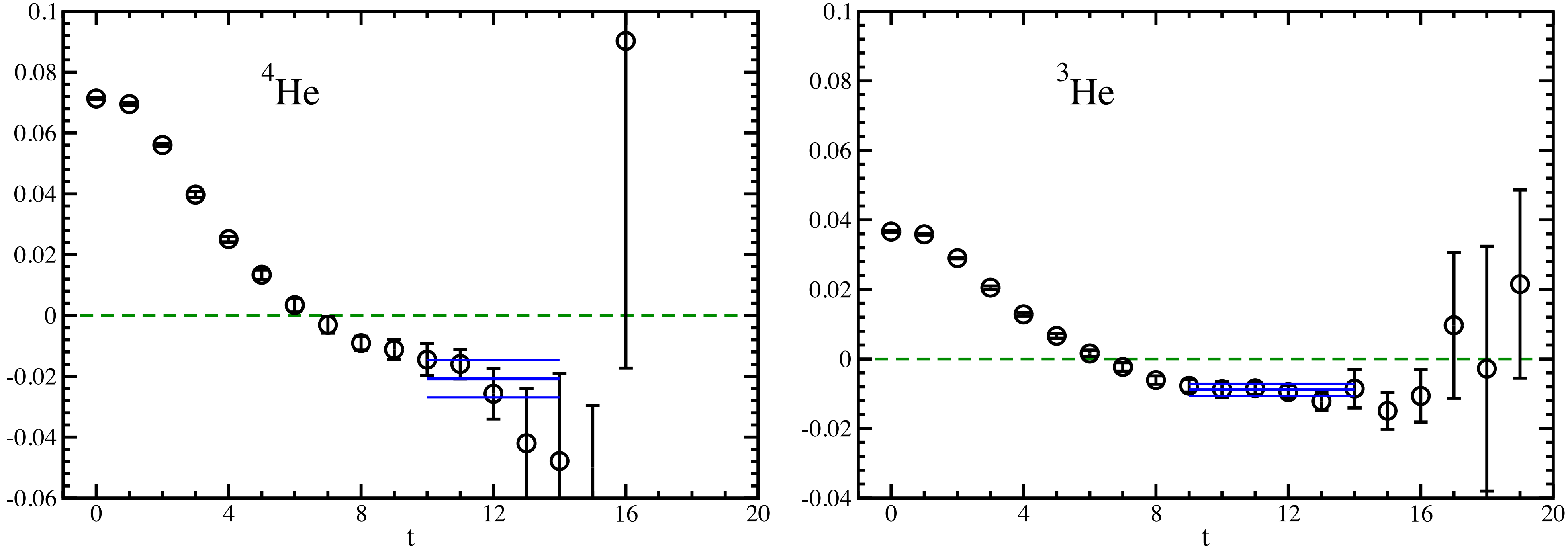
- Good plateau $t \gtrsim 7$
- Statistical error $< 0.2\%$

$\Delta E_L = E_0 - N_N m_N$ in ^{4}He and ^{3}He channels

at $m_\pi = 0.5$ GeV on $L = 5.8$ fm

TY *et al.*, PRD86:074514(2012)

$$\Delta E_L = \log\left(\frac{R_{^4\mathrm{He}}(t)}{R_{^4\mathrm{He}}(t+1)}\right) \text{ with } R_{^4\mathrm{He}}(t) = \frac{C_{^4\mathrm{He}}(t)}{(C_N(t))^4}$$

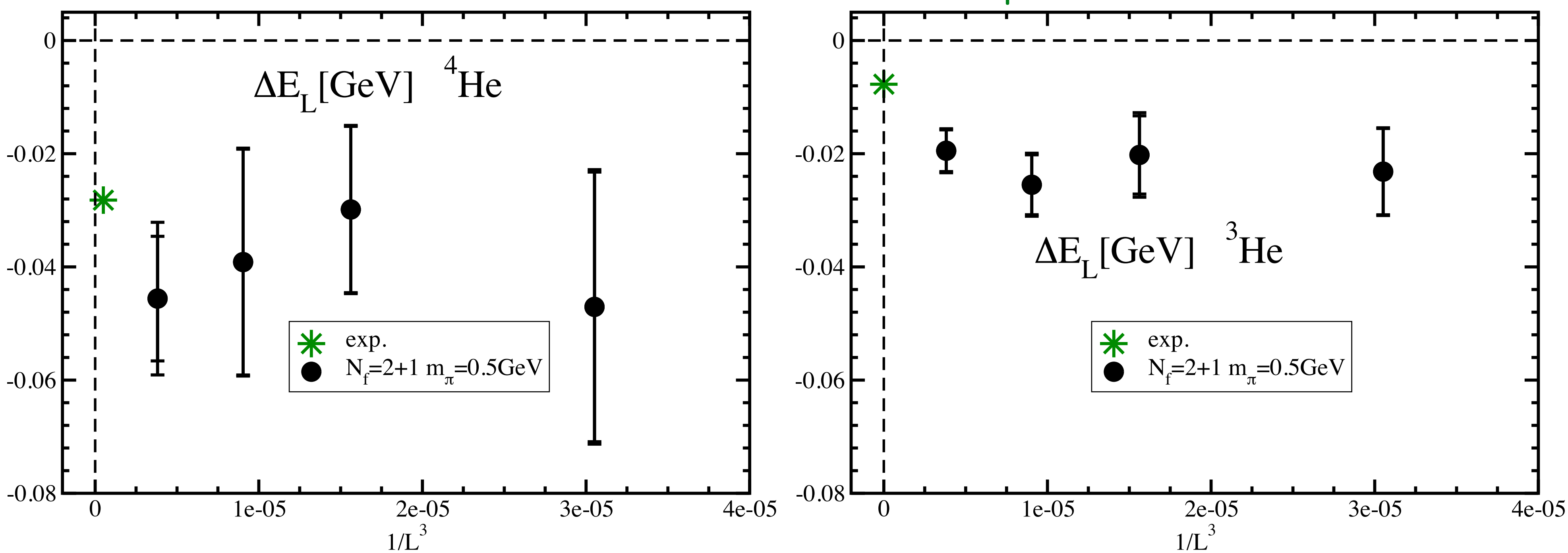


- Larger error in ^{4}He channel
- Statistical error under control in $t < 12$
- Negative ΔE_L in both channels

^{4}He and ^{3}He channels $\Delta E_L = E_0 - N_N m_N$ at $m_\pi = 0.5$ GeV

TY *et al.*, PRD86:074514(2012)

Identification of bound state from volume dependence of ΔE



- $\Delta E_L < 0$ and mild volume dependence
- Infinite volume extrapolation with $\Delta E_L = -\Delta E_{\text{bind}} + C/L^3$

^{4}He and ^{3}He channels $\Delta E_L = E_0 - N_N m_N$ at $m_\pi = 0.5$ GeV

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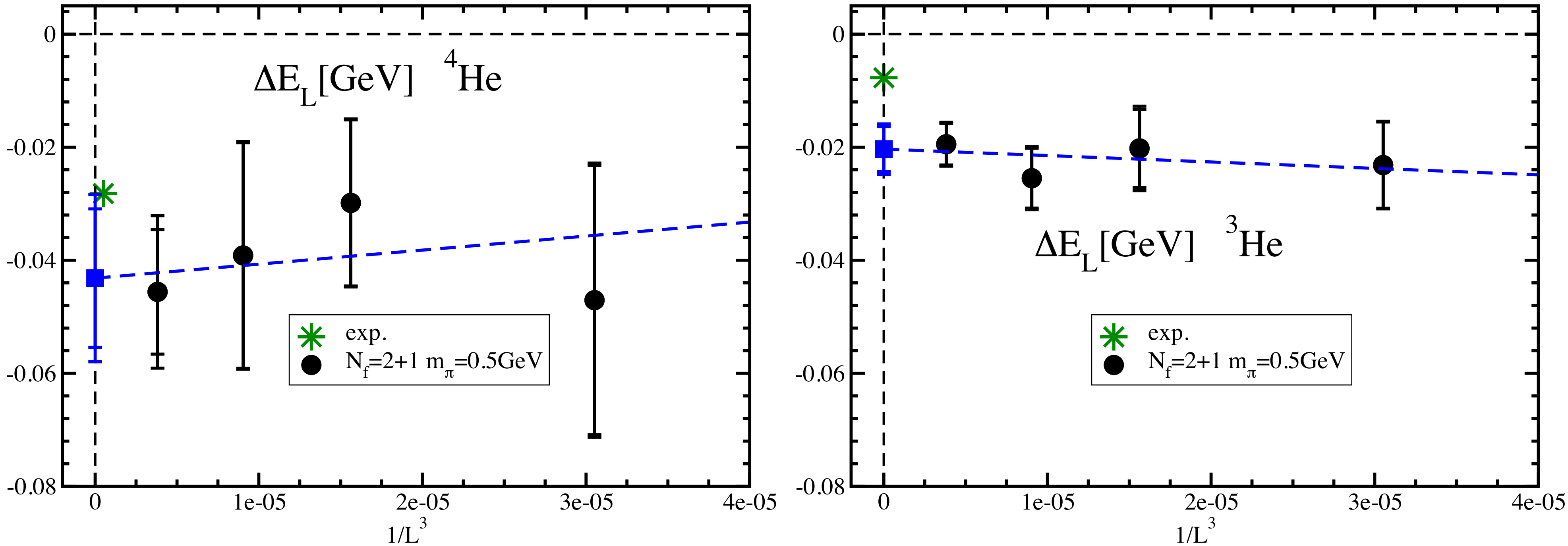
Identification of bound state from volume dependence of ΔE



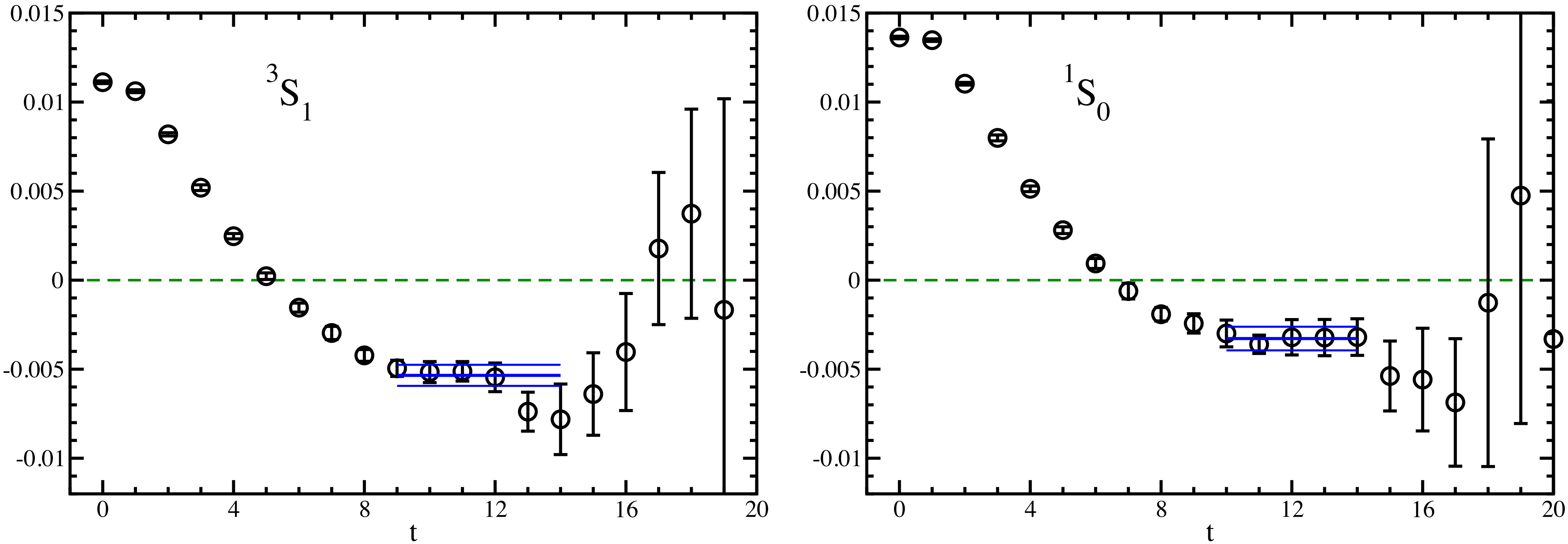
$\Delta E_{^4\text{He}} = 43(12)(8)$ MeV $\qquad \Delta E_{^3\text{He}} = 20.3(4.0)(2.0)$ MeV

Observe bound state in both channels

ΔE_L in 2-nucleon channels at $m_\pi = 0.5$ GeV on $L = 5.8$ fm

TY *et al.*, PRD86:074514(2012)

$$\Delta E_L = \log\left(\frac{R_{\rm NN}(t)}{R_{\rm NN}(t+1)}\right) \text{ with } R_{\rm NN}(t) = \frac{C_{\rm NN}(t)}{(C_N(t))^2}$$

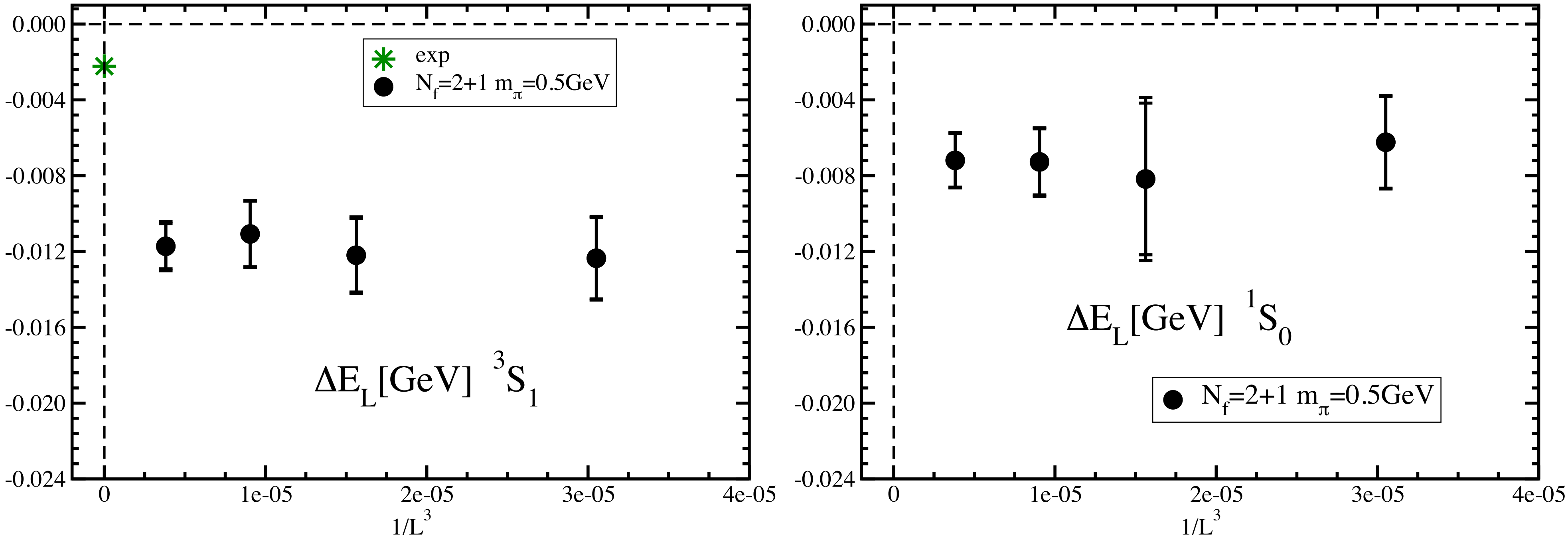


- Statistical error under control in $t \leq 12$
- Smaller error than ^{4}He and ^{3}He channels
- Negative ΔE_L in both channels

NN (^{3}S$_1$ and ^{1}S$_0$) channels $\Delta E_L = E_0 - 2m_N$ at $m_\pi = 0.5$ GeV

TY *et al.*, PRD86:074514(2012)

Identification of bound state from volume dependence of ΔE



- Negative ΔE_L
- Infinite volume extrapolation of ΔE_L '04 Beane *et al.*, '06 Sasaki & TY

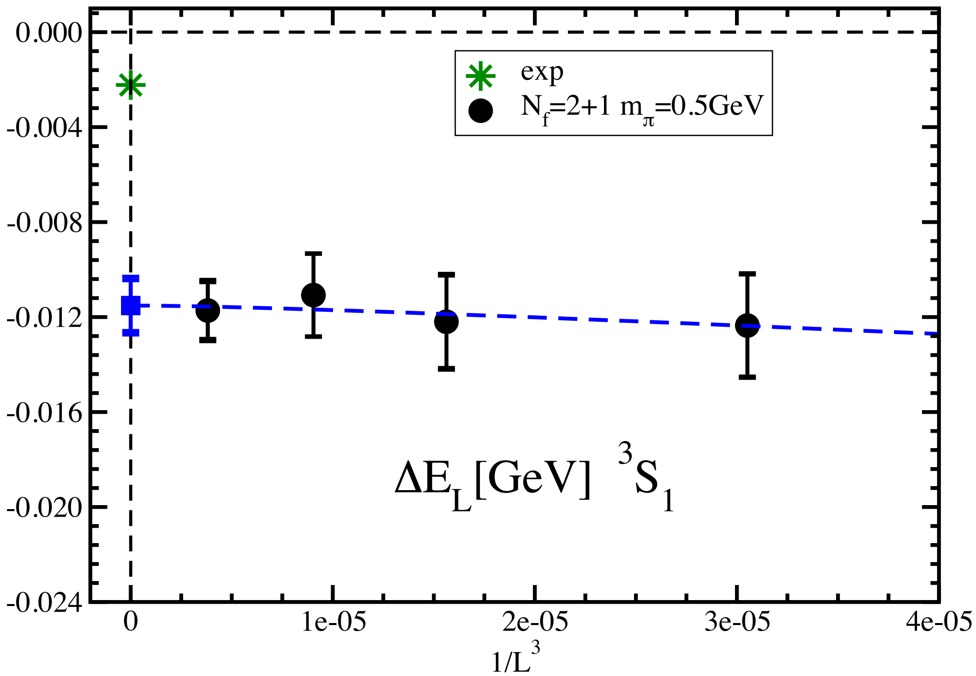
$$\Delta E_L = -\frac{\gamma^2}{m_N}\left\{1 + \frac{C_\gamma}{\gamma L}\sum_{\vec{n}}{}' \frac{\exp(-\gamma L\sqrt{\vec{n}^2})}{\sqrt{\vec{n}^2}}\right\},\ \Delta E_{\text{bind}} = \frac{\gamma^2}{m_N}$$

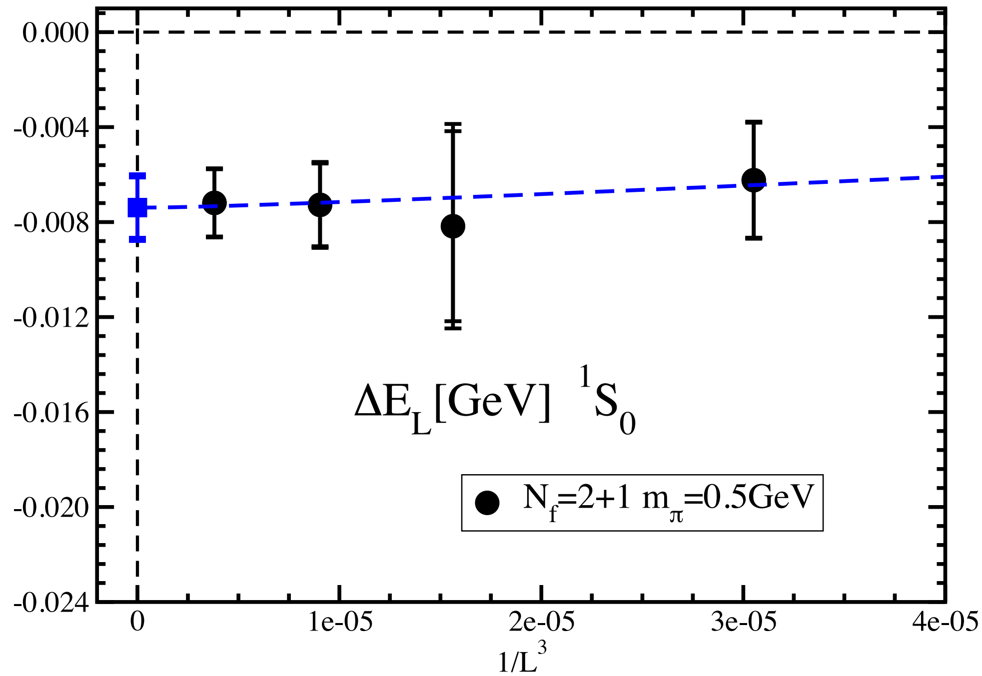
based on Lüscher's finite volume formula

NN (^{3}S$_1$ and ^{1}S$_0$) channels $\Delta E_L = E_0 - 2m_N$ at $m_\pi = 0.5$ GeV

TY *et al.*, PRD86:074514(2012)

Identification of bound state from volume dependence of ΔE



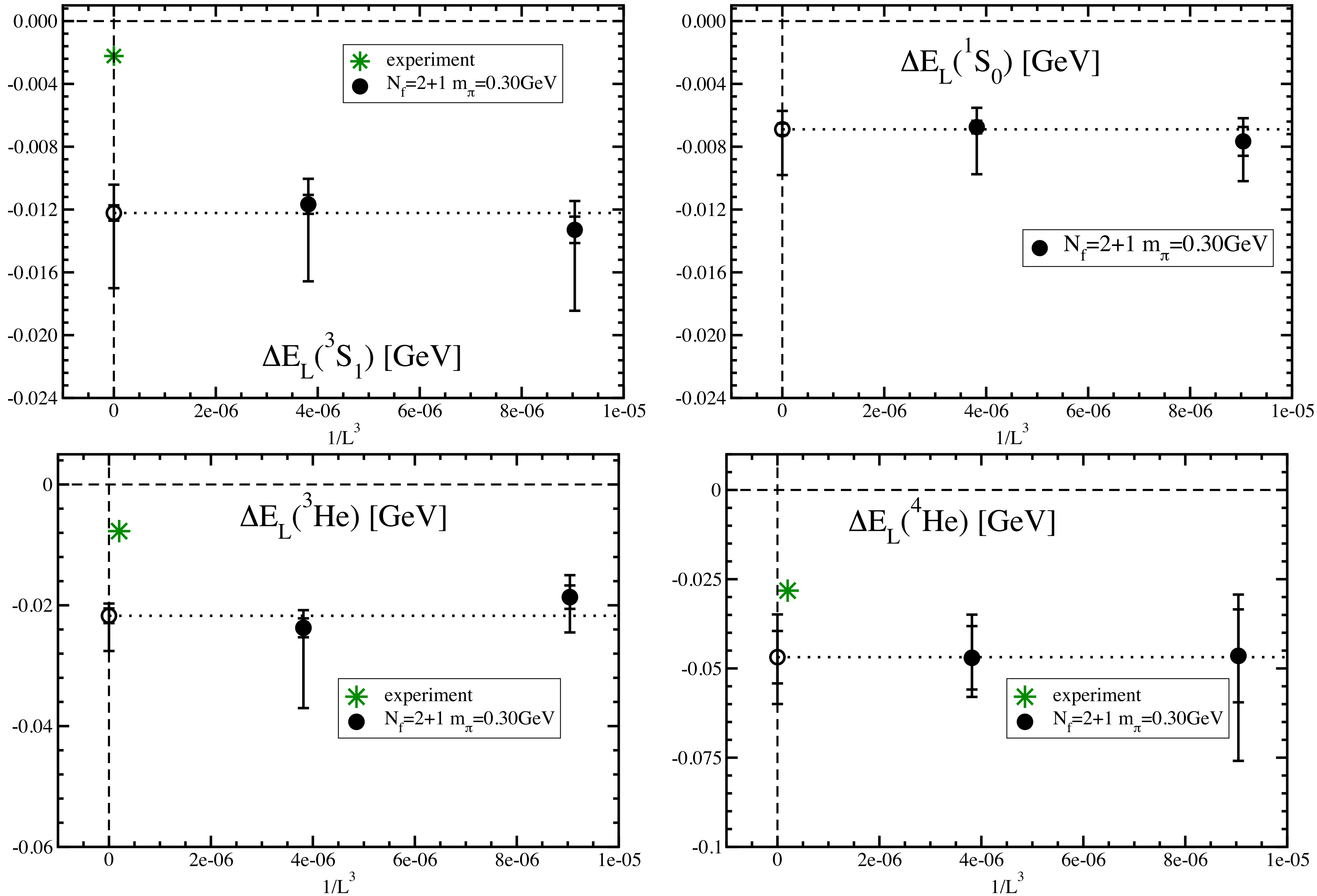


Bound state in both channels ← different from experiment

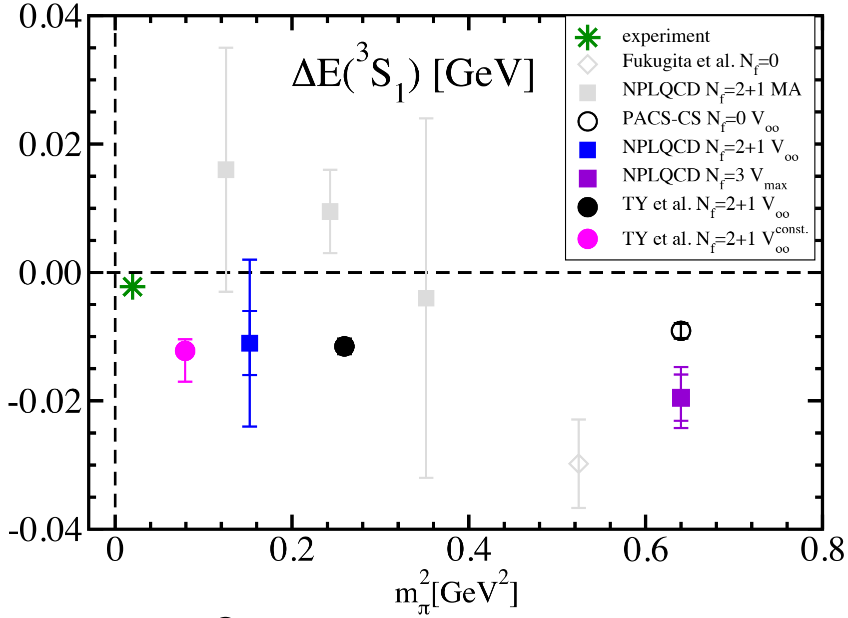
$\Delta E_{^3\mathrm{S}_1} = 11.5(1.1)(0.6)$ MeV $\qquad$ $\Delta E_{^1\mathrm{S}_0} = 7.4(1.3)(0.6)$ MeV

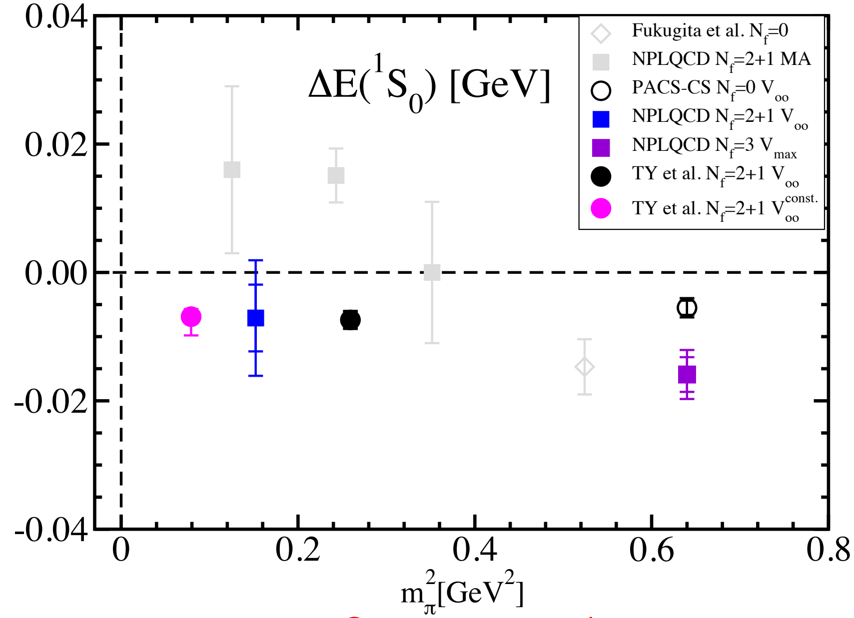
Preliminary results at $m_\pi = 0.3$GeV

Infinite volume extrapolation with two volumes



Current status of NN channels



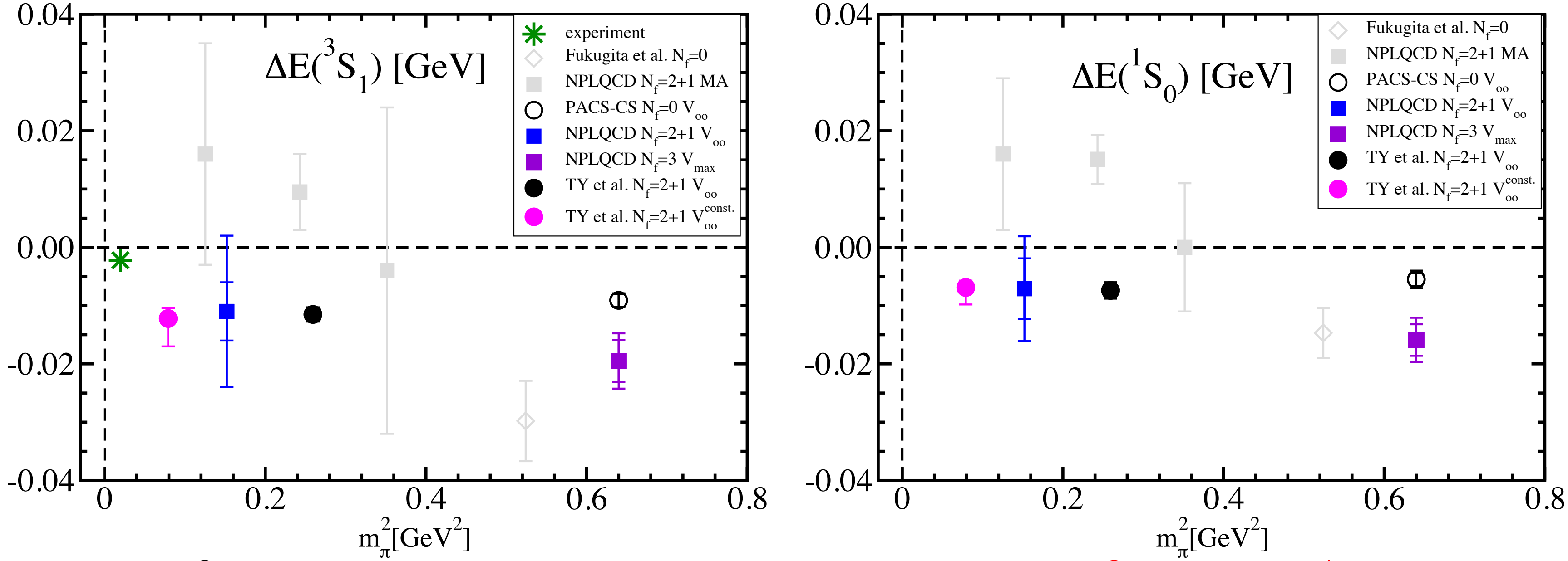


$L^3 \to \infty$: existence of bound states in $^3S_1$ and $^1S_0$

inconsistent with experiment due to larger m_π

Investigations of m_π dependence $\to$ $m_\pi = 0.14$ GeV on $L \sim 8$ fm

Current status of NN channels



$L^3 \to \infty$: existence of bound states in ^{3}S$_1$ and ^{1}S$_0$

inconsistent with experiment due to larger m_π

Investigations of m_π dependence $\to$ $m_\pi = 0.14$ GeV on $L \sim 8$ fm

Large finite volume effect expected even on $L \sim 0.8$ fm

'86 Lüscher, '04 Beane

^{3}S$_1$: $\Delta E_{\text{exp}} = 2.2$ MeV

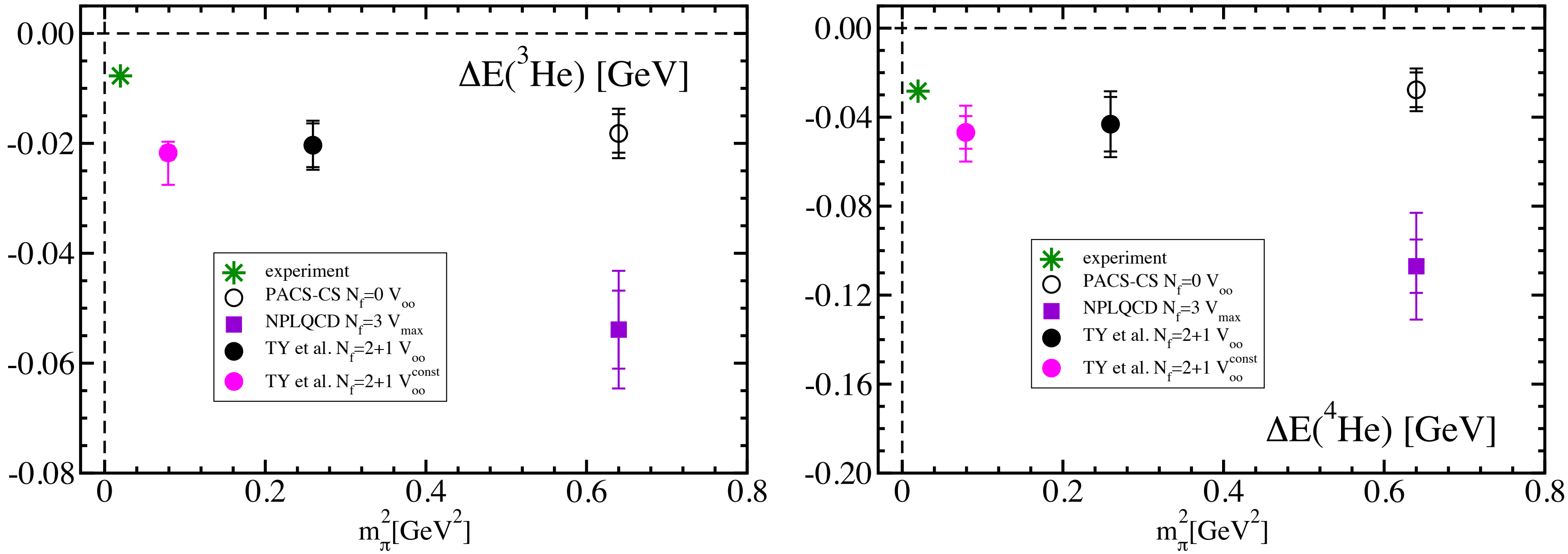
$$\Delta E_L = -\left(\Delta E_{\text{exp}} + \mathcal{O}(\exp(-L\sqrt{m_N \Delta E_{\text{exp}}}))\right) \sim -4 \text{ MeV}$$

^{1}S$_0$: $a_0^{\text{exp}} = 23.7$ fm

$$\Delta E_L = -\frac{4\pi a_0^{\text{exp}}}{m_N L^3} + \mathcal{O}(1/L^4) \sim -2 \text{ MeV}$$

Current status of ^{3}He and ^{4}He nuclei

PACS-CS, PRD81:111504(R)(2010); TY *et al.*, PRD86:074514(2012); NPLQCD, PRD87:034506(2013)



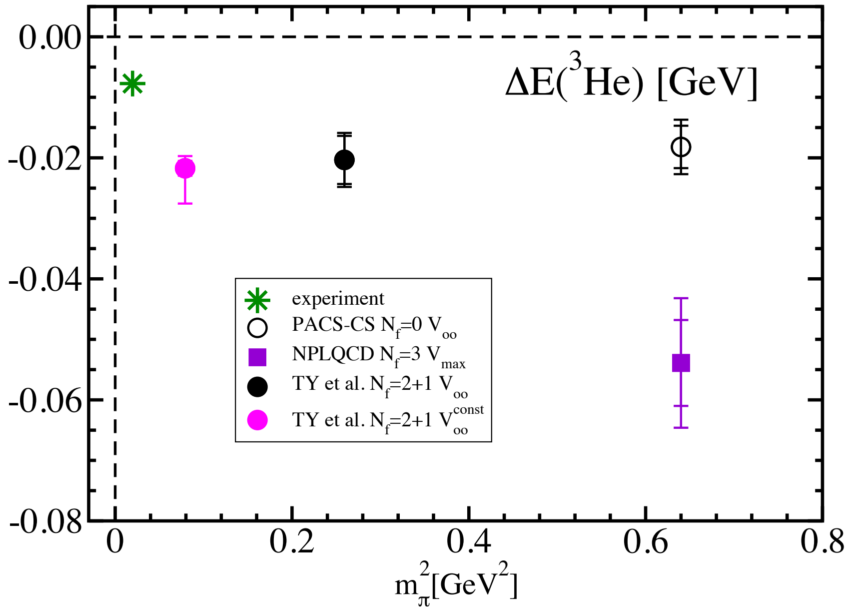
$L^3 \to \infty$ results only

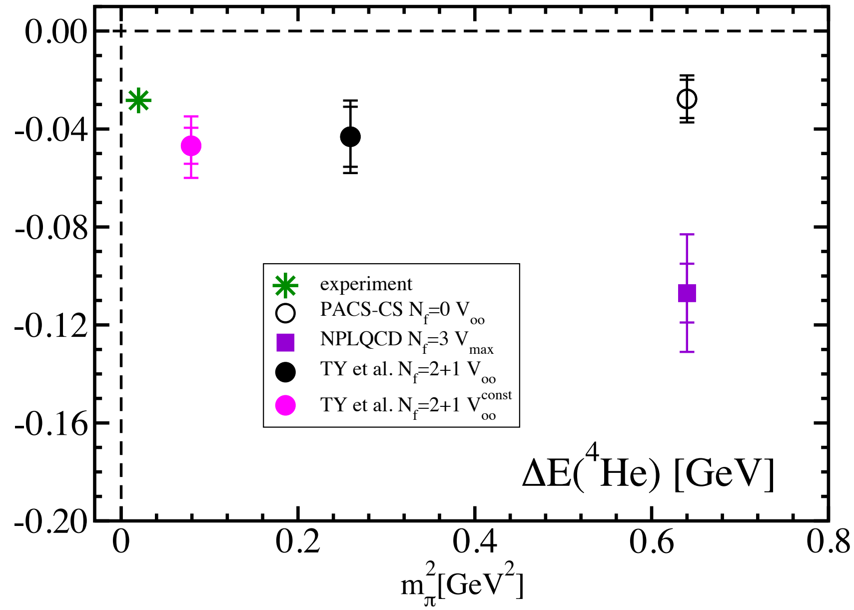
Light nuclei likely formed in 0.3 GeV $\leq m_\pi \leq$ 0.8 GeV

Same order of ΔE to experiments

Current status of ^{3}He and ^{4}He nuclei

PACS-CS, PRD81:111504(R)(2010); TY *et al.*, PRD86:074514(2012); NPLQCD, PRD87:034506(2013)





$L^3 \to \infty$ results only

Light nuclei likely formed in 0.3 GeV $\le m_\pi \le$ 0.8 GeV

Same order of ΔE to experiments $\to$ relatively easier than NN

large $|\Delta E|$ less V dependence

touchstone of quantitative understanding of nuclei from lattice QCD

Investigations of m_π dependence $\to m_\pi = 0.14$ GeV on $L \sim 8$ fm

Summary

Extend our exploratory studies to $N_f = 2+1$ lattice QCD

- $m_\pi = 0.5$ and 0.3 GeV
- Volume dependence of ΔE

$\Delta E \neq 0$ of 0th state in infinite volume limit
$\rightarrow$ bound state in ^{4}He, ^{3}He, $^3S_1$ and $^1S_0$
at $m_\pi = 0.5$ and 0.3 GeV

- ΔE larger than experiment and small m_π dependence
- Bound state in $^1S_0$ not observed in experiment
  Deep bound state in $N_f = 3$ at $m_\pi = 0.8$ GeV ('12 NPLQCD)

Need further investigations *e.g.* quark mass dependence

$N_f = 2+1$ $m_\pi = 0.14$ GeV on $L = 8$ fm calculation is on-going.