Realtime Techniques for studying Dynamics in Cold Atom and Nuclear Systems

Michael McNeil Forbes

Washington State University, Pullman

University of Washington, Seattle

Outline

- The Many Body Problem

  Quantum Monte Carlo (QMC), Mean Field Theory, Density Functional Theory (DFT)

- Nuclear Dynamics

  Glitches in Neutron Stars (vortex dynamics, pinning, quantum turbulence)

  Fission in nuclei, Excitations (GDR), Reactions

- From Cold Atoms to Nuclei and Neutron Stars

  Validated Methods

  DFT, Vortex pinning, Glitches, Quantum Turbulence

- Realtime Techniques

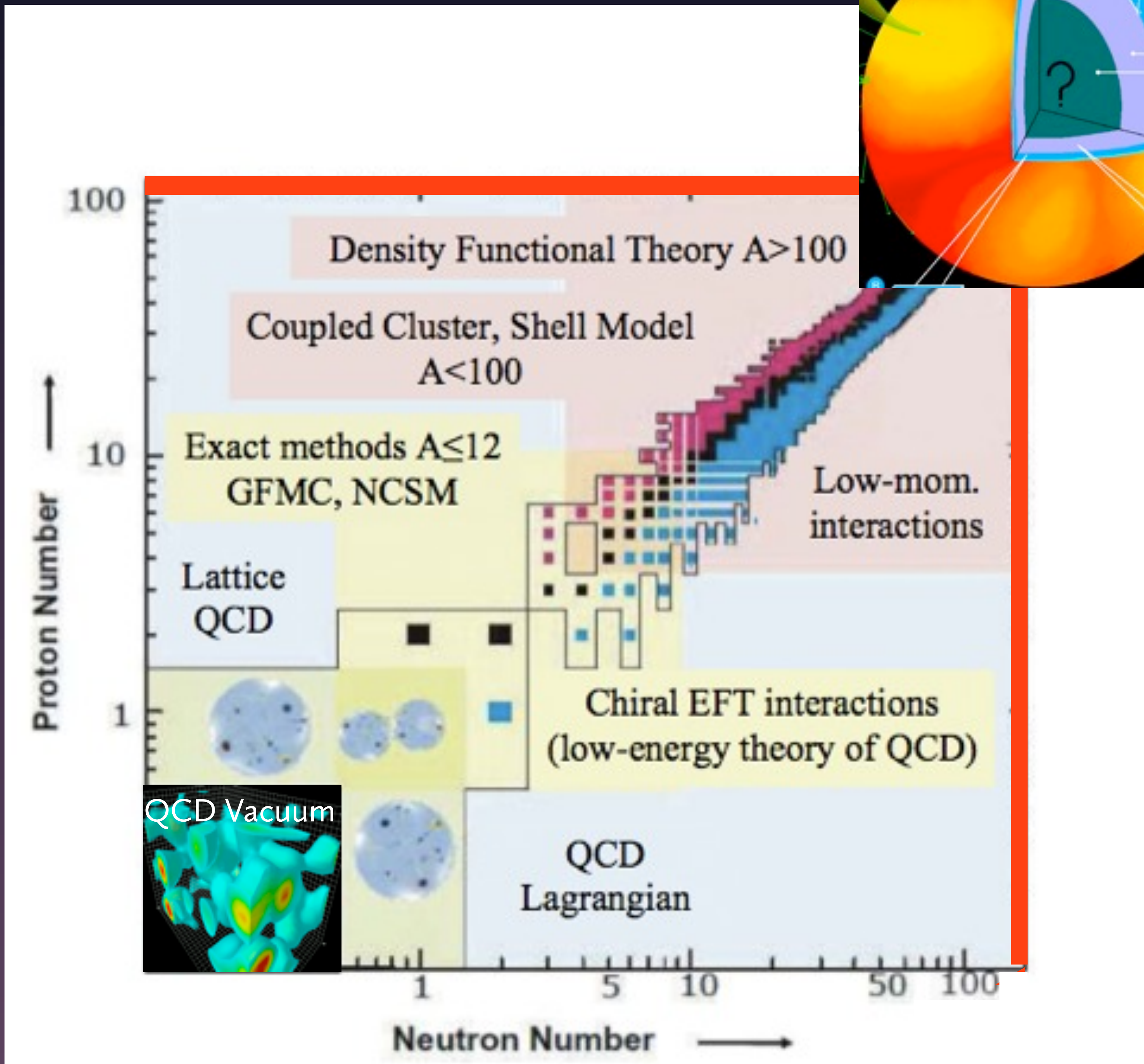
  Directly probe dynamics

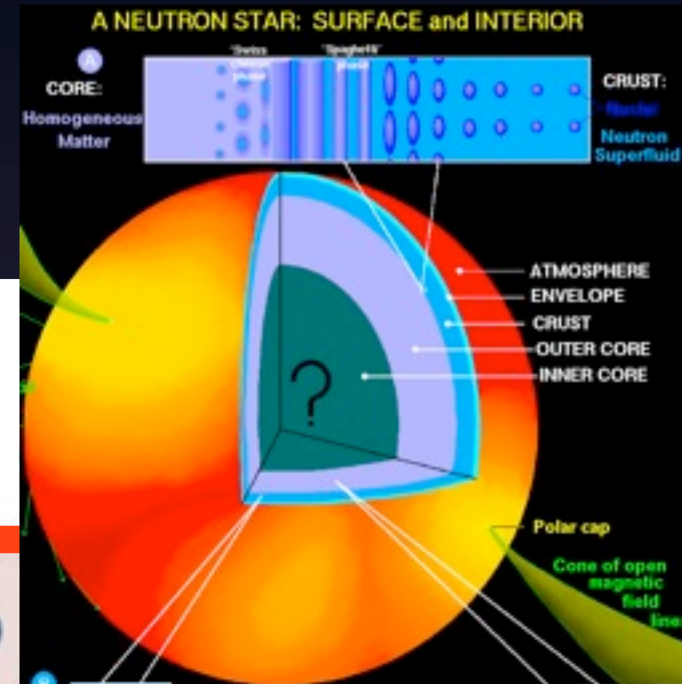
  Efficient simulation (Quantum Friction state prep., extract pinning interaction)

Topic Outline

- The Many Body Problem of Nuclear physics
  - Focus on neutron stars and fission as problems
- What is the UFG and how does it relate to nuclei?
  - Overview of crossover, polarized phases
  - Agreement of experiment and theory for UFG statics
  - FFLO states? Mention bosons for flat trap.
- Realtime methods for Nuclear Dynamics
  - Glitches in Neutron Stars (vortex dynamics, pinning, quantum turbulence)
  - Problems with static energy calculations
  - How realtime methods help: good scaling
  - Quantum Friction
- DFTs (SLDA and GPE)
  - Hydrodynamic vs Fermionic, comparison and some successes
- Vortex dynamics and MIT experiment
- Quantum Turbulence
- Conclusion: the path from Cold Atoms to Nuclei and Neutron Stars

The Nuclear Landscape

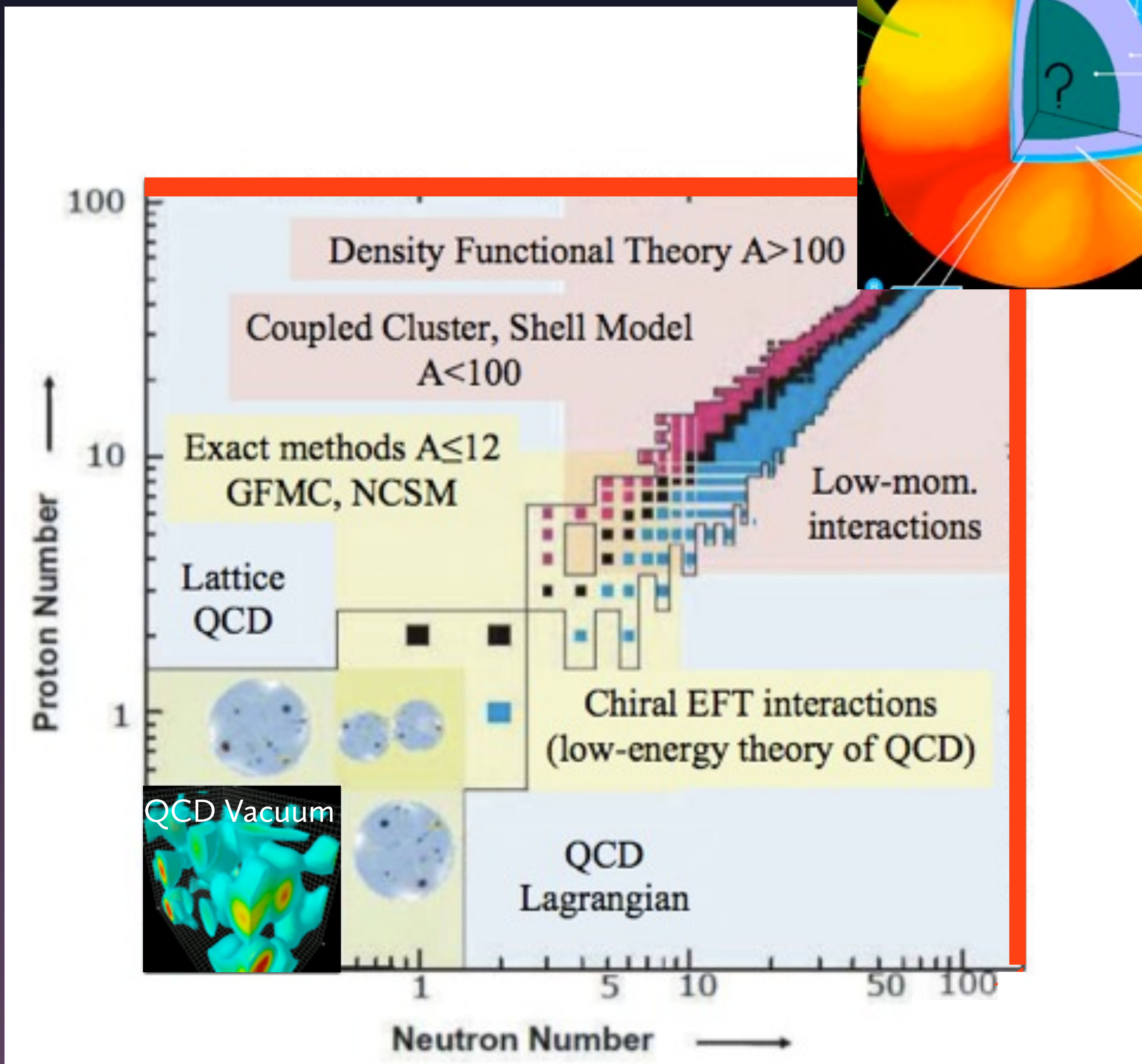


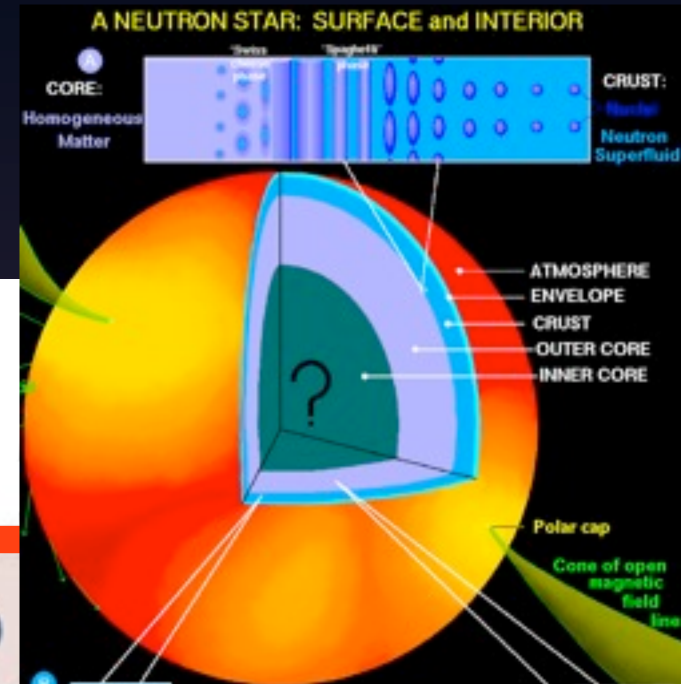


- Lattice QCD,
  nucleons, interactions
- QMC, etc.
  small to medium nuclei
- DFT,
  medium to large nuclei
- Neutron stars?
  Molecular Dynamics
  Hydrodynamics

QCD Vacuum Animation: Derek B. Leinweber (http://www.physics.adelaide.edu.au/~dleinweb/VisualQCD/Nobel/index.html)
Neutron Star Structure: (Dany Page) Landscape: (modified from a slide of A. Richter)

The Nuclear Landscape

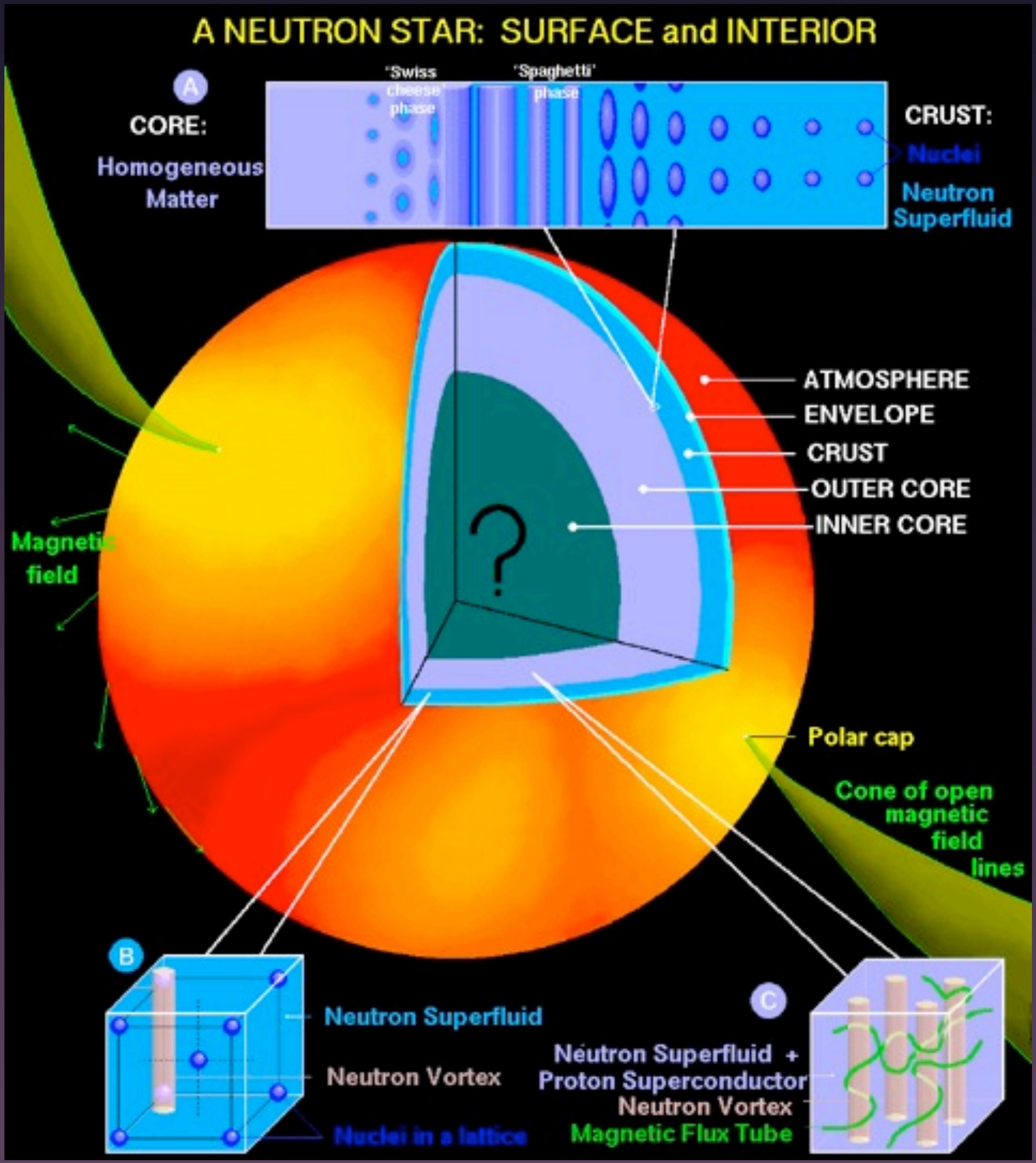




- Lattice QCD,
  nucleons, interactions
- QMC, etc.
  small to medium nuclei
- DFT,
  medium to large nuclei
- Neutron stars?
  Molecular Dynamics
  Hydrodynamics

QCD Vacuum Animation: Derek B. Leinweber (http://www.physics.adelaide.edu.au/~dleinweb/VisualQCD/Nobel/index.html)
Neutron Star Structure: (Dany Page) Landscape: (modified from a slide of A. Richter)

Neutron Stars



Neutron superfluid in Crust is almost a Unitary Fermi Gas
($a_s \sim -7r_e$, $k_F a_s \sim -10$)

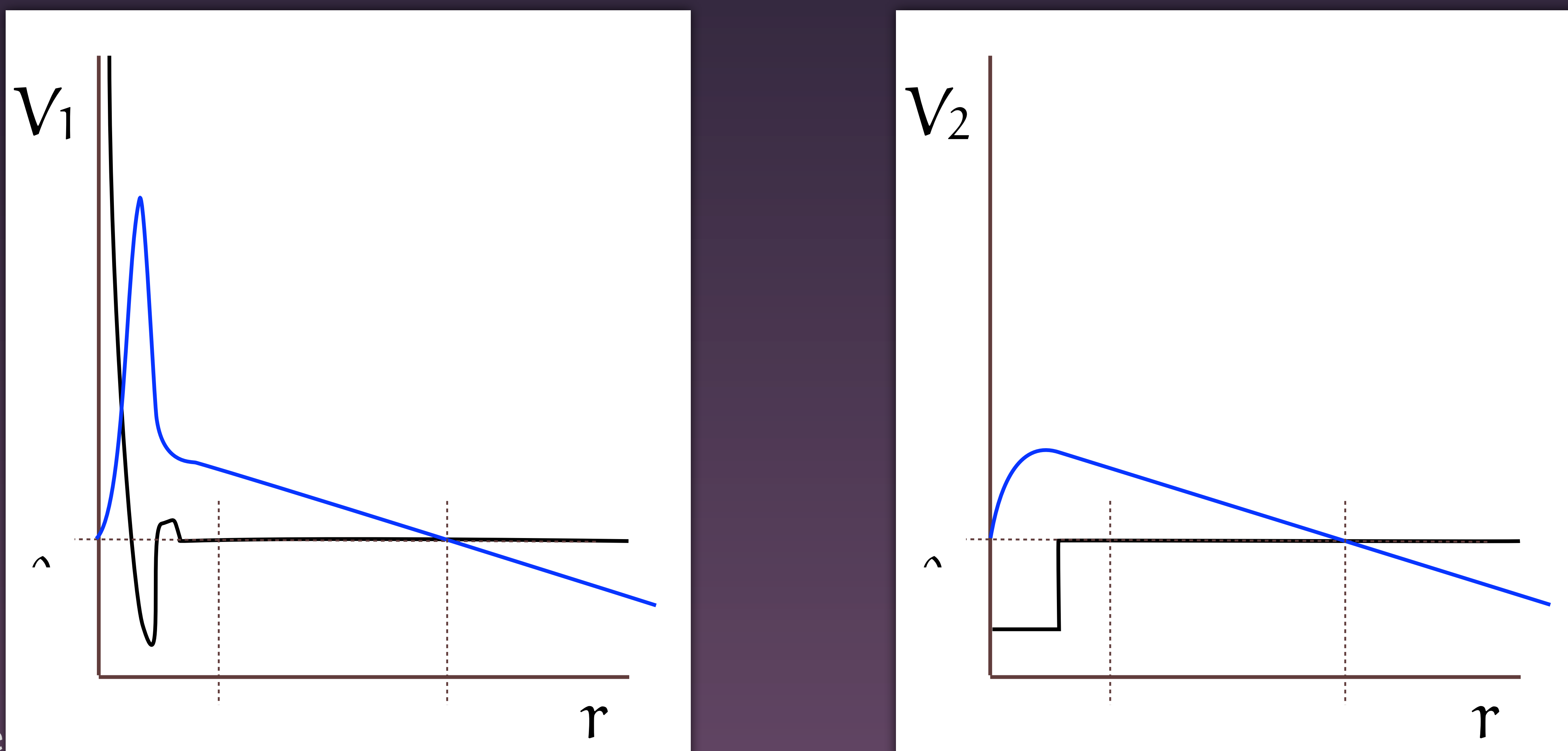
Many relevant phenomena
- Vortex pinning (glitches)
- Heat transport
- Equation of State

Can we use cold-atoms to model nuclear matter?
- More complicated interactions
  - Three-body, tensor forces etc.

Dany Page: http://www.astroscu.unam.mx/neutrones/NS-Picture/NS-Picture.html

Universality

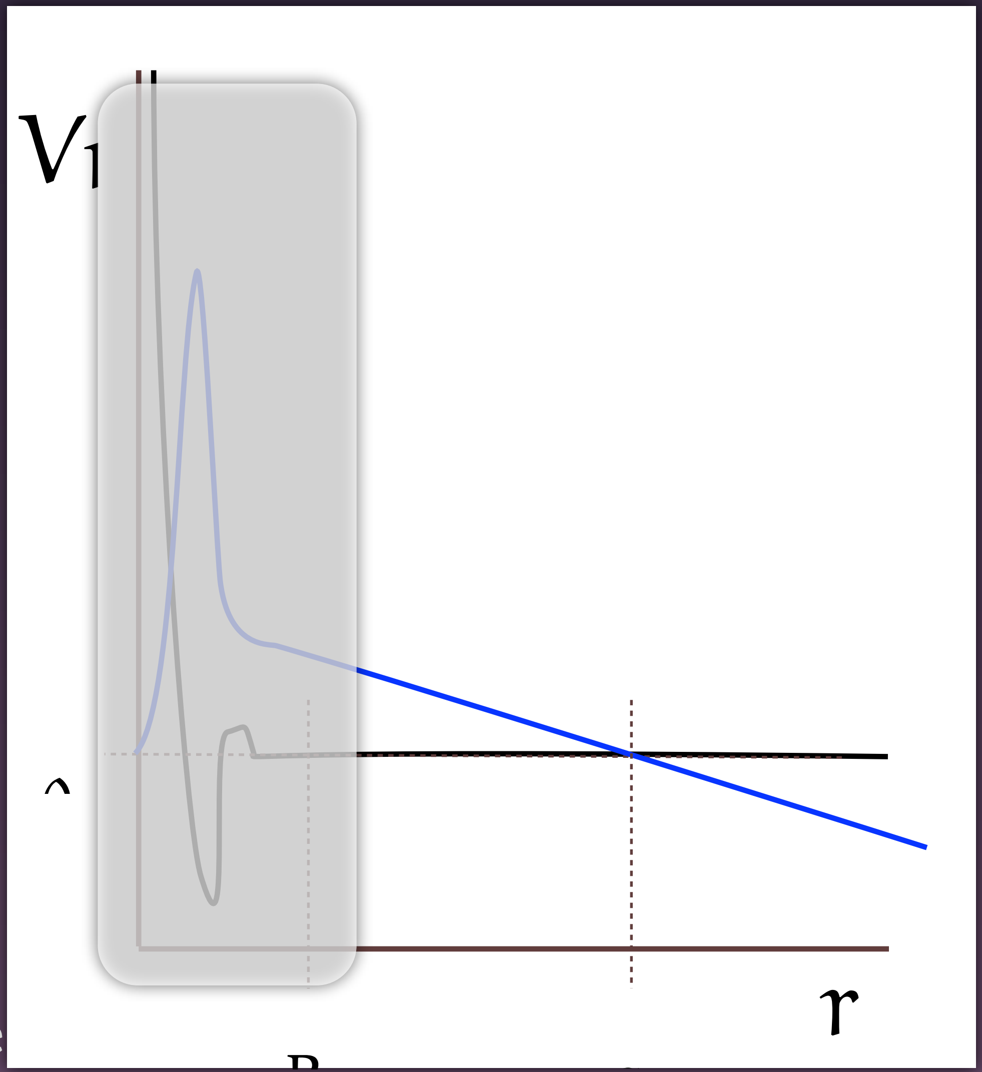
- Short distance irrelevant:
  - At long distance (r>R) potentials equivalent $V_1 \equiv V_2$
  - Characterized by scattering length a

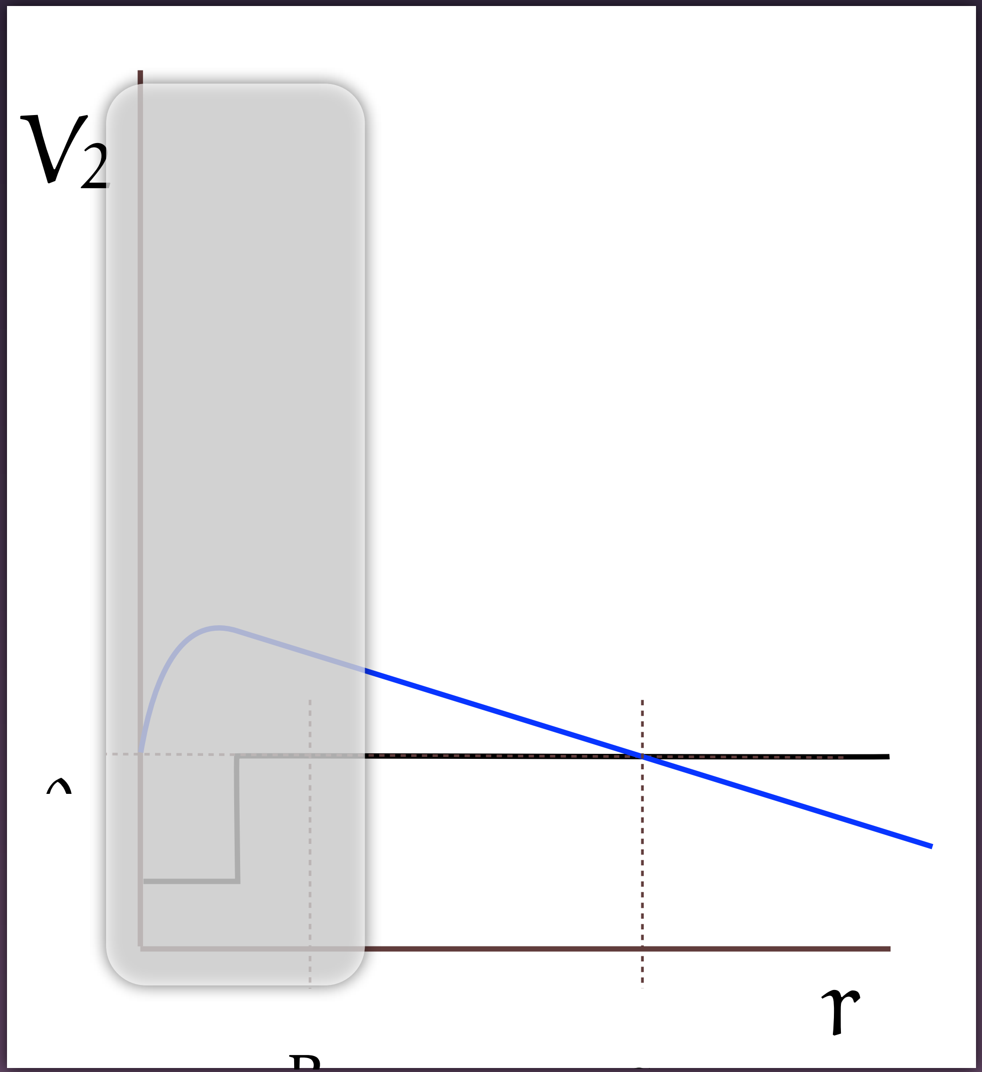


Image

Universality

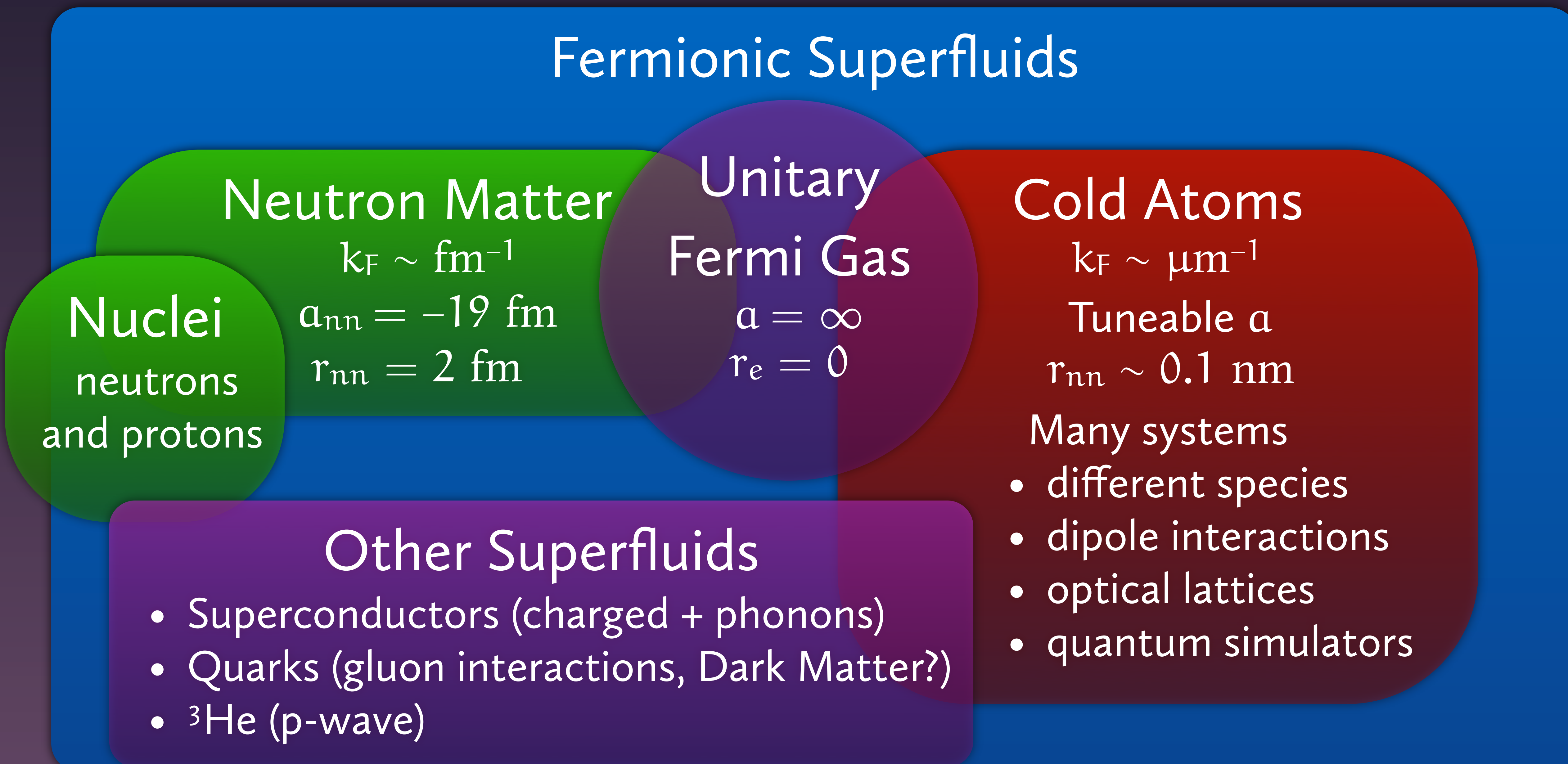
- Short distance irrelevant:
  - At long distance $(r > R)$ potentials equivalent $V_1 \equiv V_2$
  - Characterized by scattering length a





Image

Fermionic Superfluids Universality



Fermionic Superfluids Universality

Fermionic Superfluids

Neutron Matter

$k_F \sim \mathrm{fm}^{-1}$

$a_{nn} = -19\ \mathrm{fm}$

$r_{nn} = 2\ \mathrm{fm}$

Nuclei

neutrons and protons

Unitary Fermi Gas

$a = \infty$

$r_e = 0$

Cold Atoms

$k_F \sim \mu\mathrm{m}^{-1}$

Tuneable a

$r_{nn} \sim 0.1\ \mathrm{nm}$

Many systems

- different species
- dipole interactions
- optical lattices
- quantum simulators

Other Superfluids

- Superconductors (charged + phonons)
- Quarks (gluon interactions, Dark Matter?)
- ^{3}He (p-wave)

From Cold Atoms to Nuclear Physics

- Tuneable interactions

  UFG and neutron matter

  Few-body resonances, Efimov trimers

  Simulate more complicated systems

  (stimulated spin orbit couplings, polar atoms, optical lattices, boselets/fermilets)

- Benchmark for many-body theory

  Directly compare to experiment

  DFT, works well for statics at T=0 and $n_a = n_b$

  Need to test: dynamics, polarized systems, finite T

Unitary Fermi Gas (UFG)

$$\widehat{\mathcal{H}} = \int\Big(\overbrace{\widehat{a}^\dagger\widehat{a}}^{\widehat{n}_a} E_a + \overbrace{\widehat{b}^\dagger\widehat{b}}^{\widehat{n}_b} E_b\Big) - \int V\,\widehat{n}_a\widehat{n}_b$$

$$E_{a,b} = \frac{p^2}{2m} - \mu_{a,b}, \quad \mu_\pm = \frac{\mu_a \pm \mu_b}{2}$$

- Characterize interactions by single number:
  - S-wave scattering length a

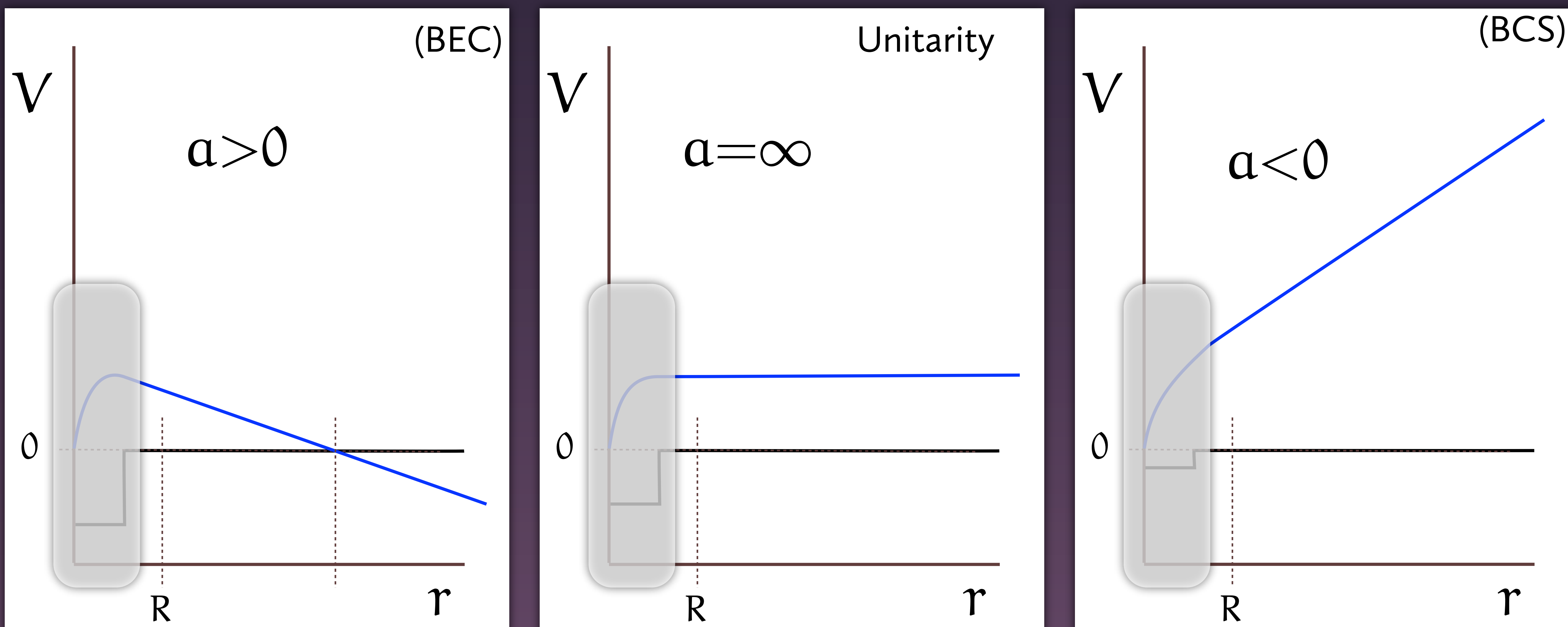
  Gas is dilute so we can ignore small-scale structure

- Tune interactions with magnetic field

  Feshbach Resonance

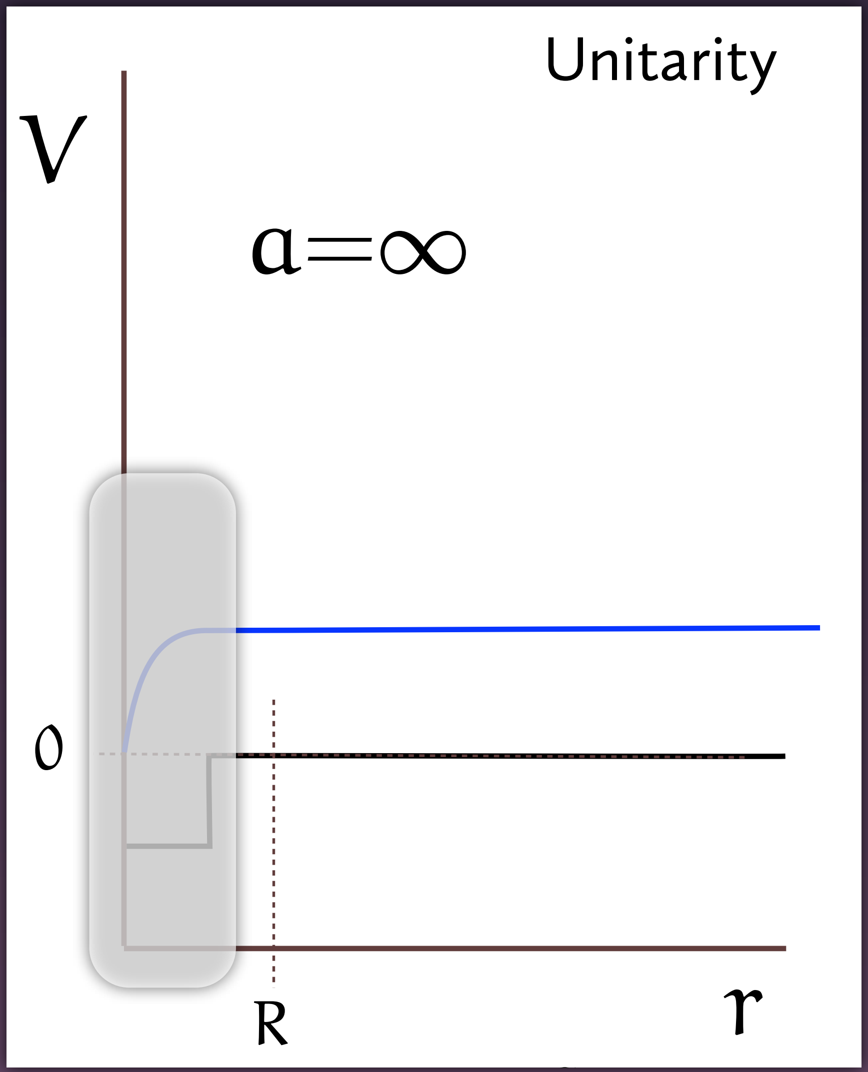
Unitary Fermi Gas

- S-wave scattering length
- BEC – Unitary – BCS crossover



Unitary Fermi Gas

- Nothing startling: bound state simply has has E=0
- Dimer becomes infinitely large



Unitary Fermi Gas (UFG)

$$\hat{\mathcal{H}} = \int \Bigl(\overbrace{\hat{\mathbf{a}}^\dagger \hat{\mathbf{a}}}^{\hat{\mathbf{n}}_a} E_a + \overbrace{\hat{\mathbf{b}}^\dagger \hat{\mathbf{b}}}^{\hat{\mathbf{n}}_b} E_b \Bigr) - \int V\, \hat{\mathbf{n}}_a \hat{\mathbf{n}}_b$$

$$E_{a,b} = \frac{p^2}{2m} - \mu_{a,b}, \qquad \mu_\pm = \frac{\mu_a \pm \mu_b}{2}$$

- Unitary limit $a=\infty$: No interaction length scale!
- Universal physics:
  - $\mathcal{E}(\rho) = \xi \mathcal{E}_{FG}(\rho) \propto \rho^{5/3}$, $\xi=0.376(5)$
- Simplest non-trivial model (dimensional analysis)

Unitary Fermi Gas (UFG)

$$\widehat{\mathcal{H}} = \int \Big(\overbrace{\widehat{a}^{\dagger}\widehat{a}}^{\widehat{n}_a} E_a + \overbrace{\widehat{b}^{\dagger}\widehat{b}}^{\widehat{n}_b} E_b \Big) - \int V\, \widehat{n}_a \widehat{n}_b$$

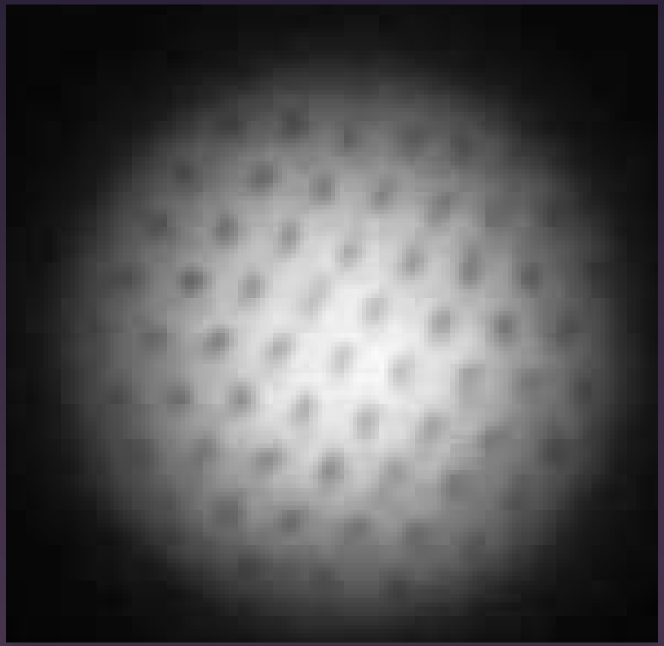
$$E_{a,b} = \frac{p^2}{2m} - \mu_{a,b}, \qquad \mu_{\pm} = \frac{\mu_a \pm \mu_b}{2}$$

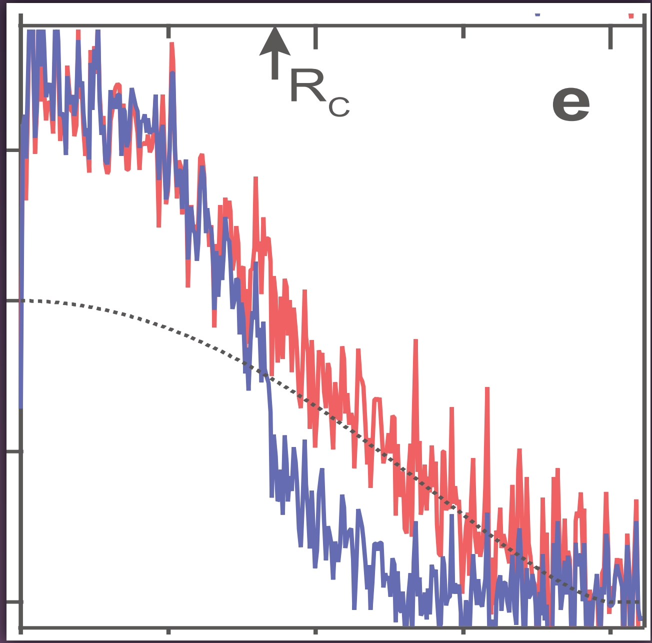
- Universal physics:
  - $\mathcal{E}(\rho) = \xi \mathcal{E}_{FG}(\rho) \propto \rho^{5/3}$, $\boxed{\xi = 0.376(5)}$
- Simple, but hard to calculate!

  Bertsch Many Body X-challenge

Unitary Fermi Gas Realized in Cold Atoms

- ^{6}Li in Feshbach Resonance
- 10^6 in harmonic traps (magneto-optical)
- Control numbers (RF transitions)
- Stir, slice, etc. with lasers
- Expansion and in-situ imaging





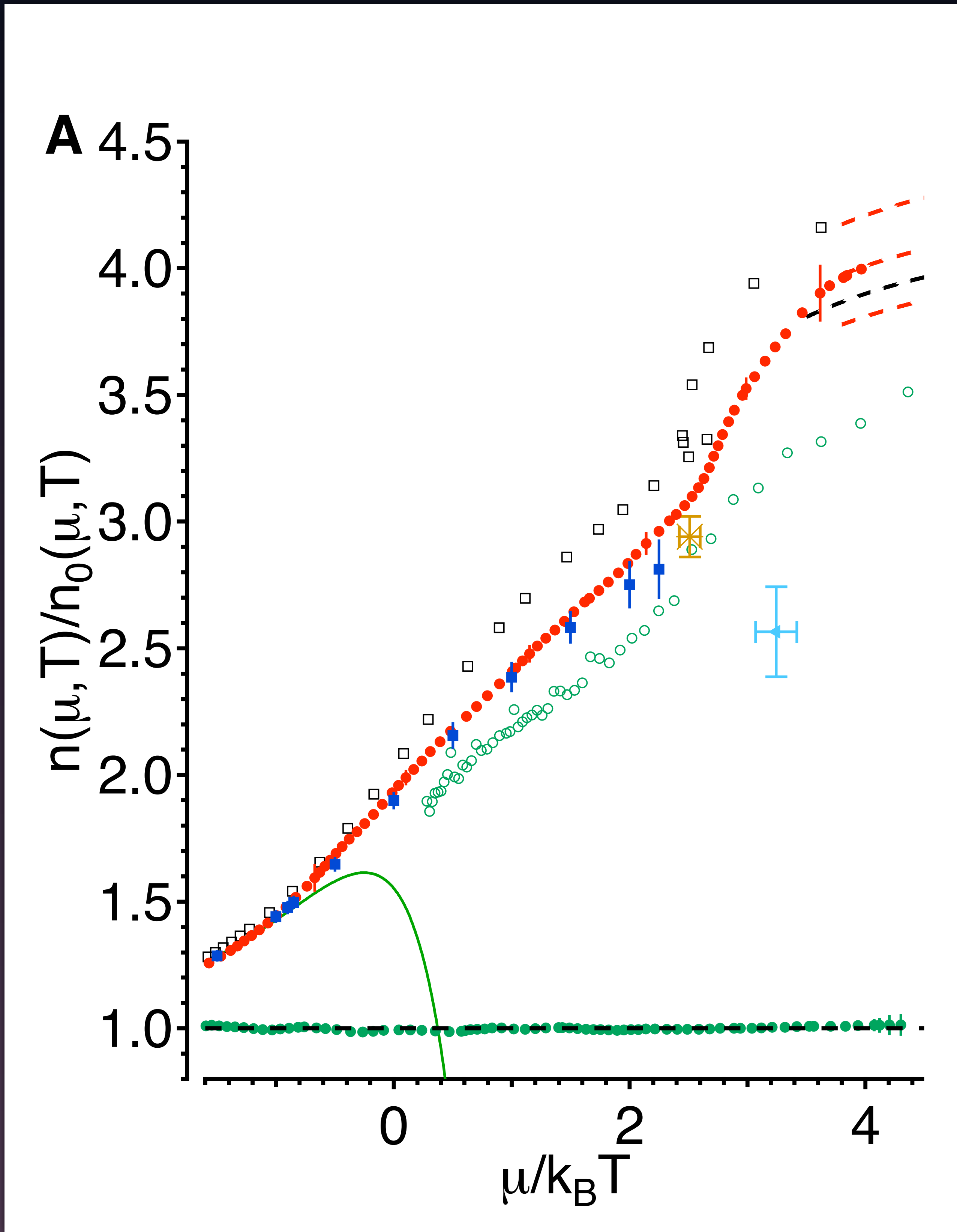
Unitary Fermi Gas (UFG)

$$\widehat{\mathcal{H}} = \int \Big(\overbrace{\widehat{a}^\dagger \widehat{a}}^{\widehat{n}_a} E_a + \overbrace{\widehat{b}^\dagger \widehat{b}}^{\widehat{n}_b} E_b \Big) - \int V\, \widehat{n}_a \widehat{n}_b$$

$$E_{a,b} = \frac{p^2}{2m} - \mu_{a,b}, \qquad \mu_\pm = \frac{\mu_a \pm \mu_b}{2}$$

- Universal physics:
  - $\mathcal{E}(\rho) = \xi \mathcal{E}_{FG}(\rho) \propto \rho^{5/3}$, $\boxed{\xi = 0.376(5)}$
- Simple, but hard to calculate!

  Bertsch Many Body X-challenge



Unitary Equation of State

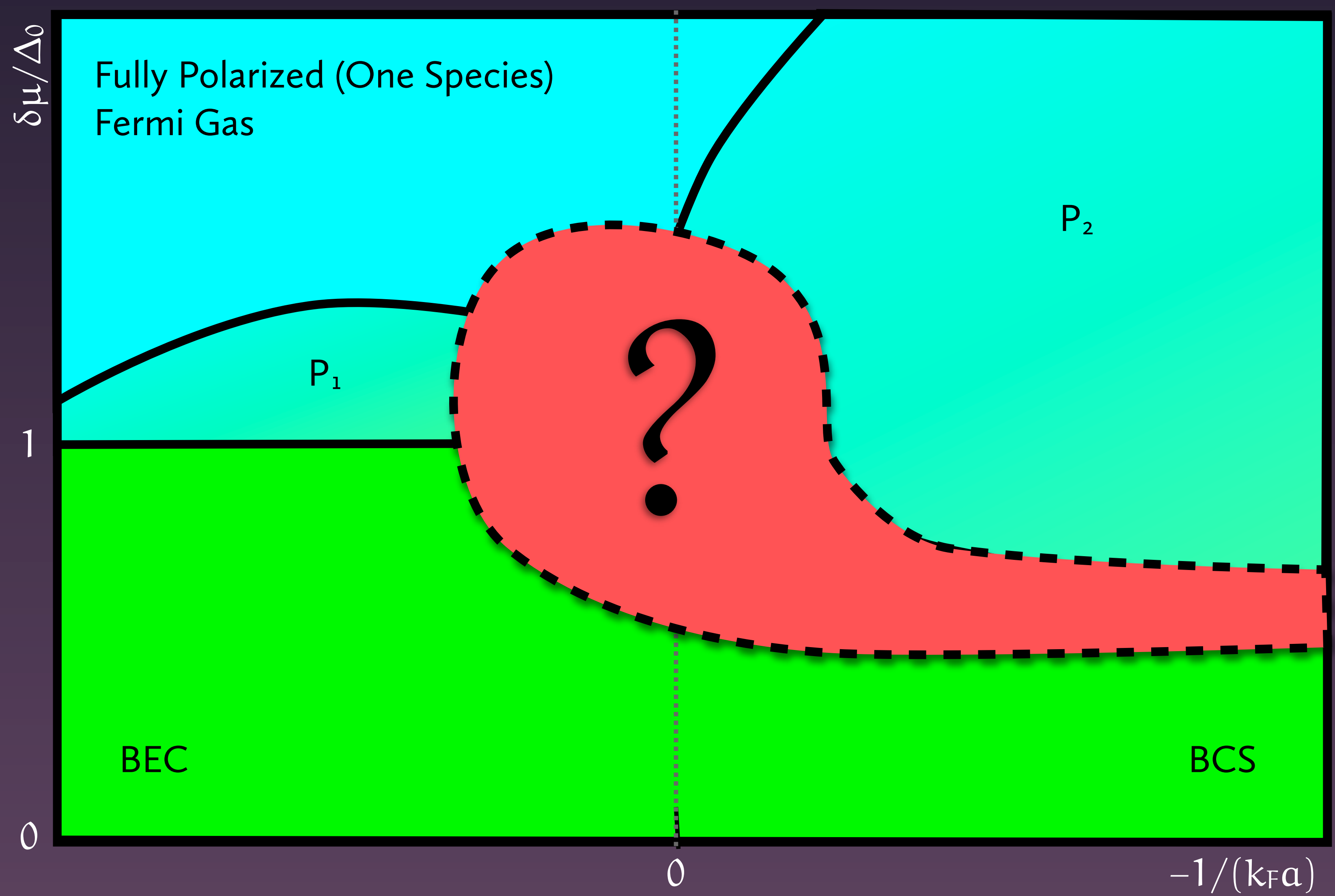
- Only scales: T and N
  - One convex dimensionless function $h_T(\mu/T)$

$$P = \left[T h_T\left(\frac{\mu}{T}\right)\right]^{5/2}$$

- Measured to percent level:
  - $\xi_{exp} = 0.370(5)(8)$

Ku, Sommer, Cheuk, and Zwierlein (2012)
Zürn, Lompe, Wenz, Jochim, Julienne, and Hutson (2013) corrected resonance

Bec-Bcs Crossover Phase Diagram (T=0)



D.T. Son and M. Stephanov (2005)
P-wave states by A.Bulgac, M.M.Forbes, A.Schwenk (PRL 2006)

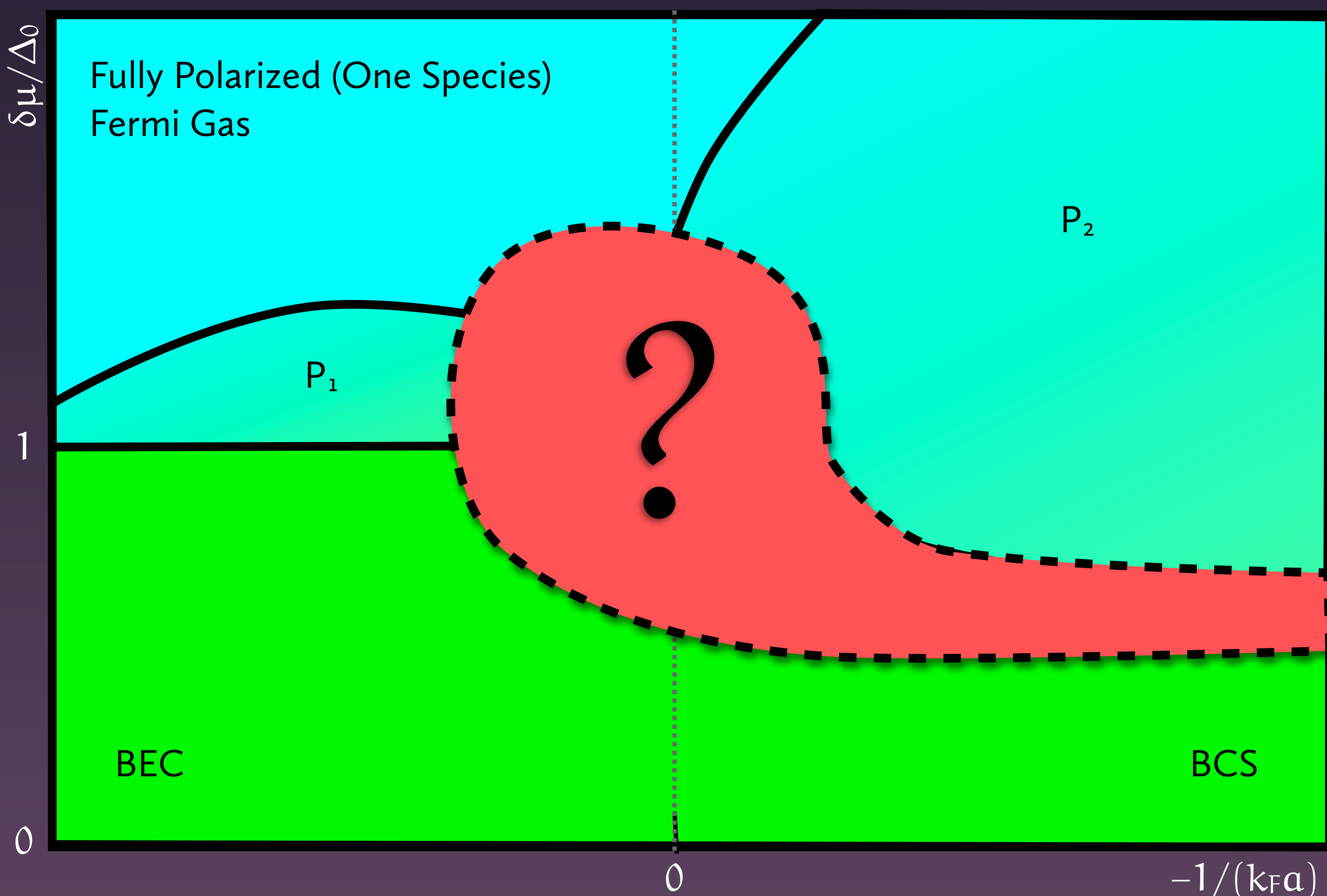
Grand canonical

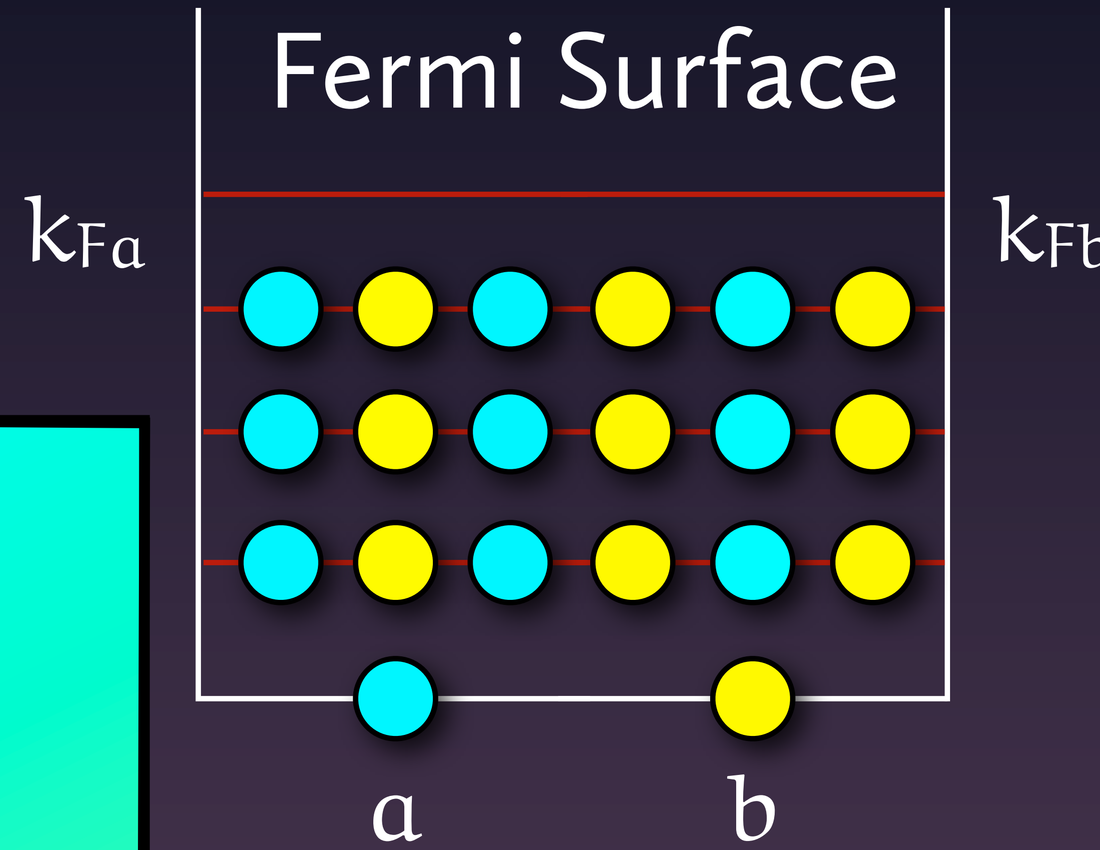
Bcs-bec Crossover

No solid evidence for what happens in the middle here

Need precision measurements

Symmetric Matter

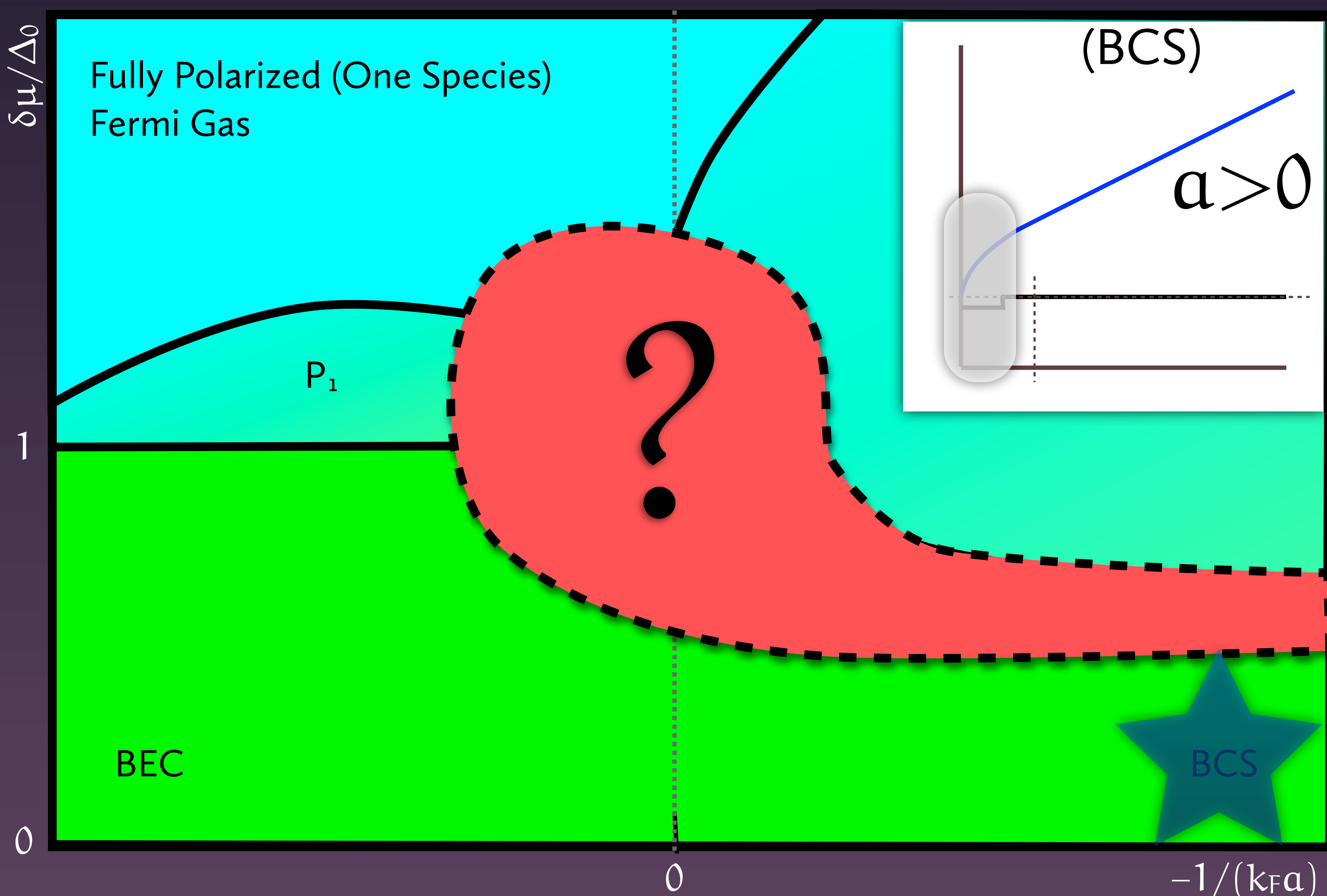


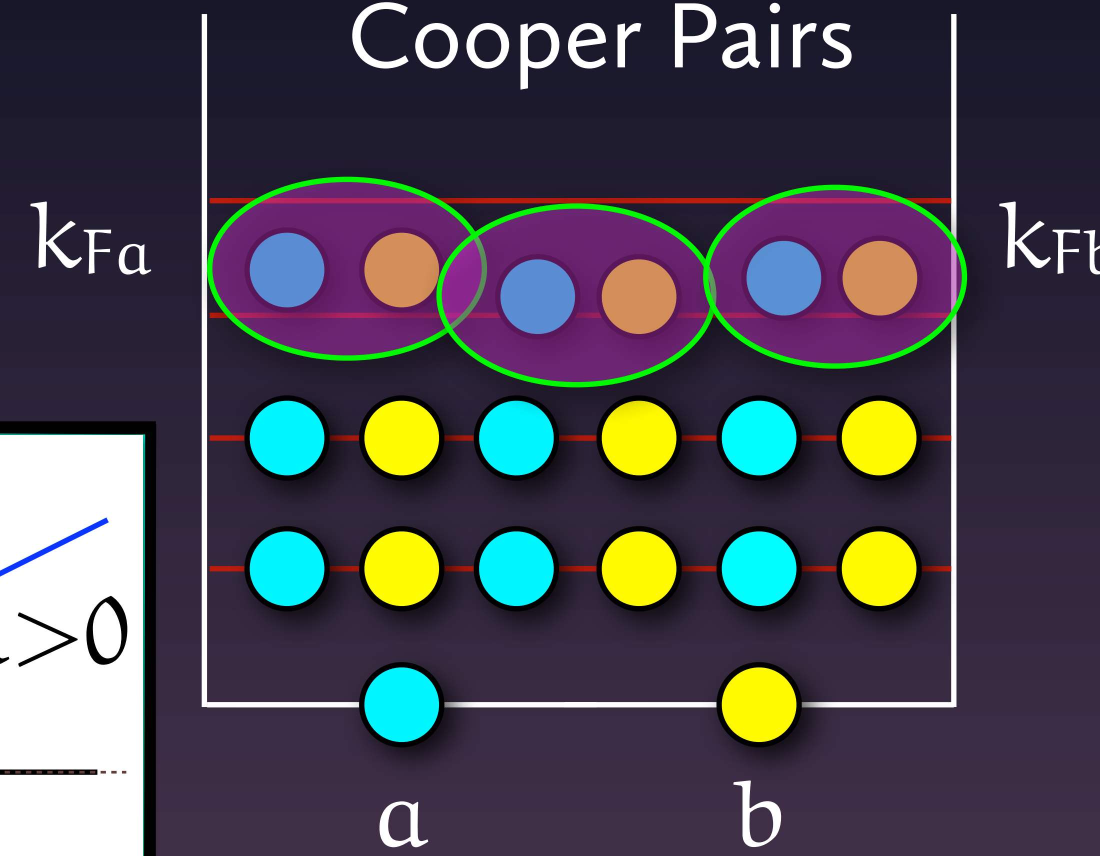


Equal Fermi surfaces

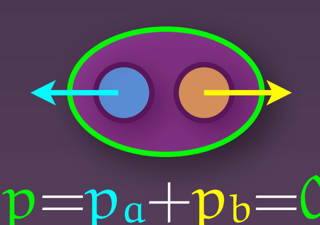
D.T. Son and M. Stephanov (2005)
P-wave states by A.Bulgac, M.M.Forbes, A.Schwenk (PRL 2006)

Symmetric BCS State





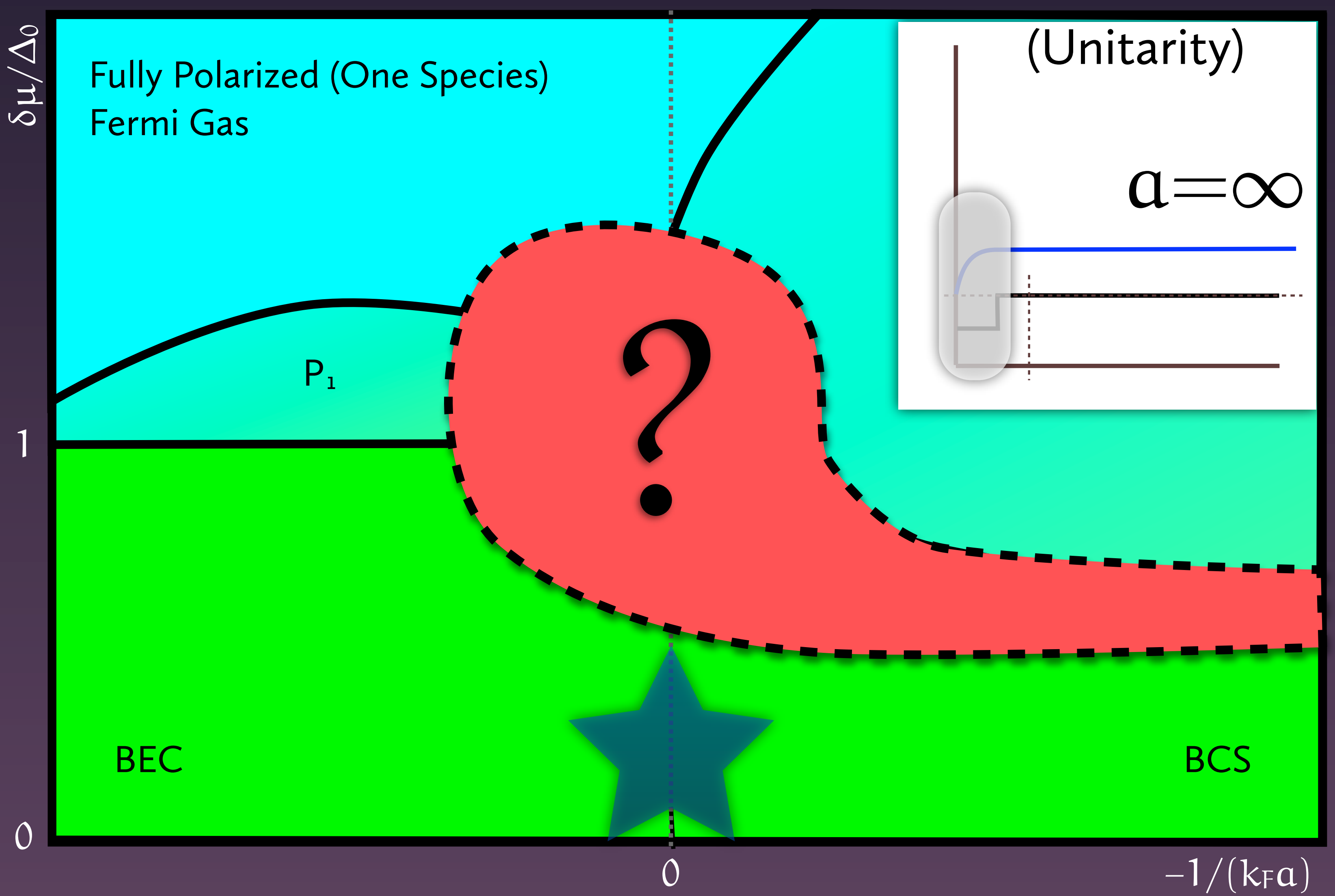
Zero momentum pairs

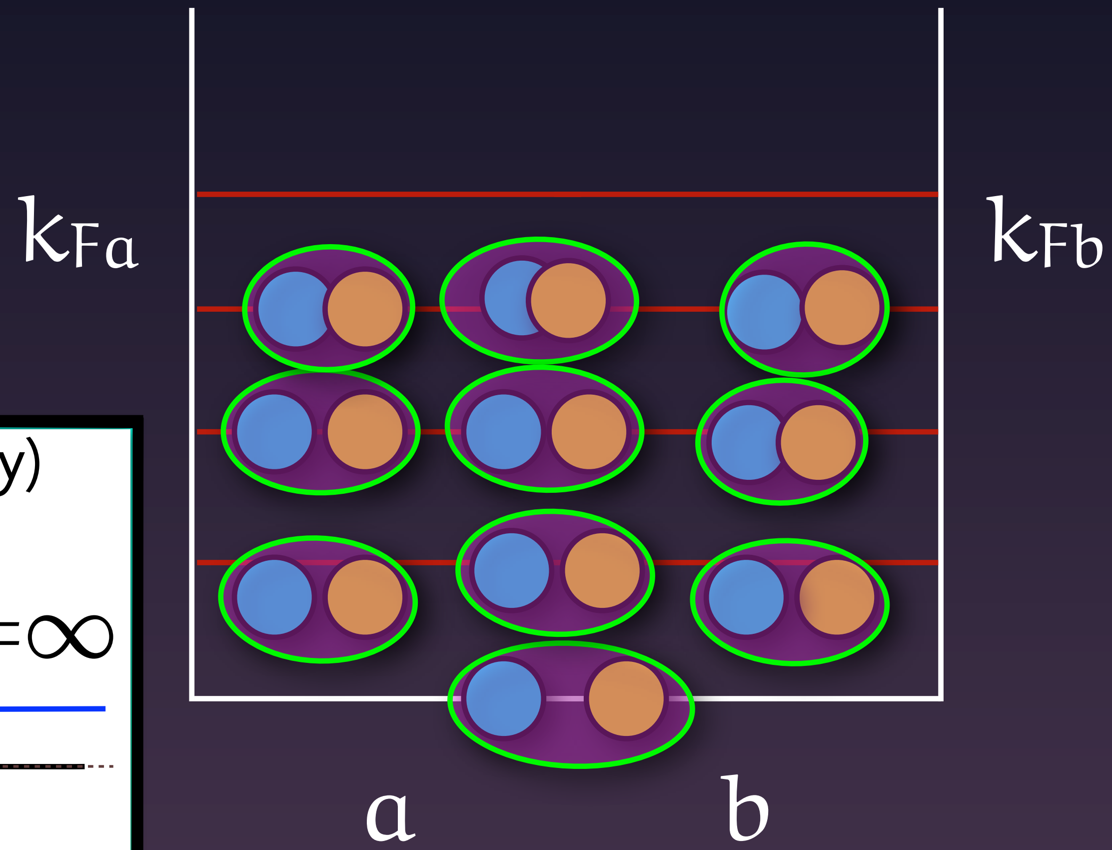


D.T. Son and M. Stephanov (2005)

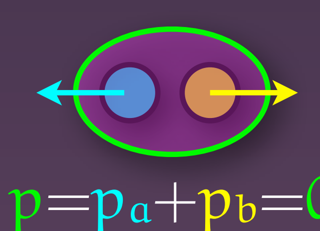
P-wave states by A.Bulgac, M.M.Forbes, A.Schwenk (PRL 2006)

Symmetric Unitary Gas





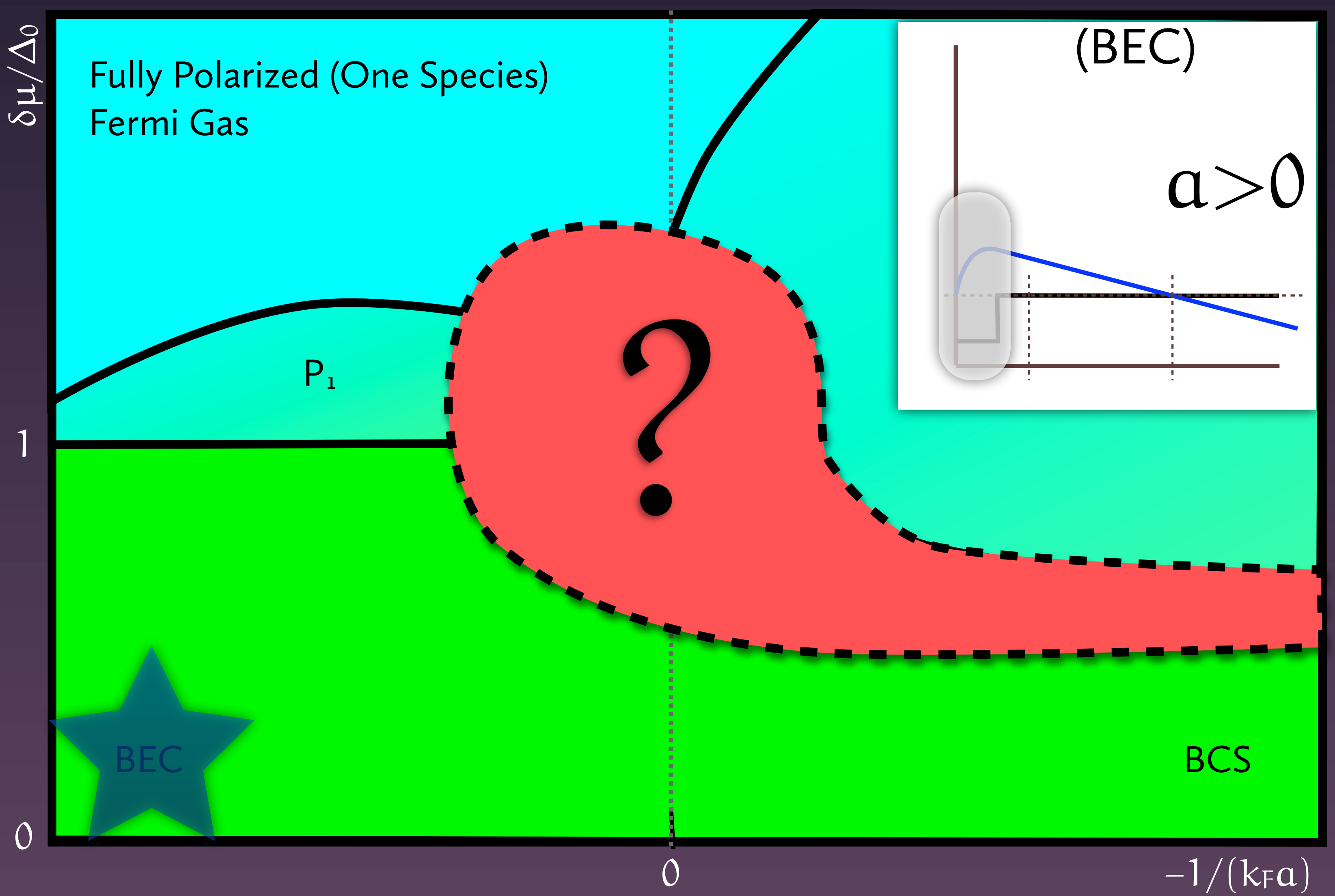
Zero momentum pairs

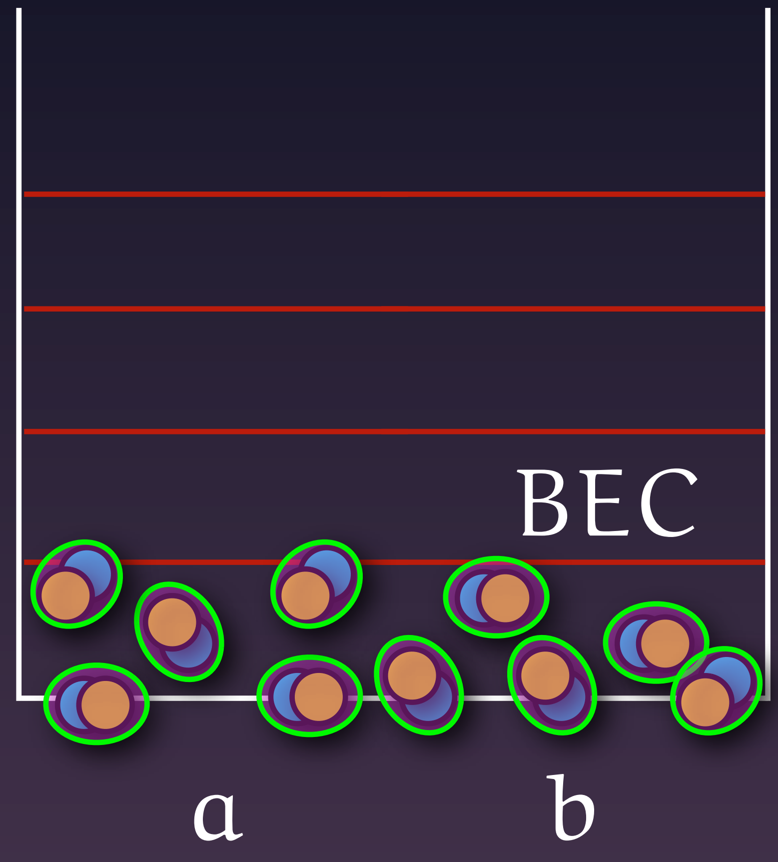


D.T. Son and M. Stephanov (2005)

P-wave states by A.Bulgac, M.M.Forbes, A.Schwenk (PRL 2006)

Symmetric BEC State



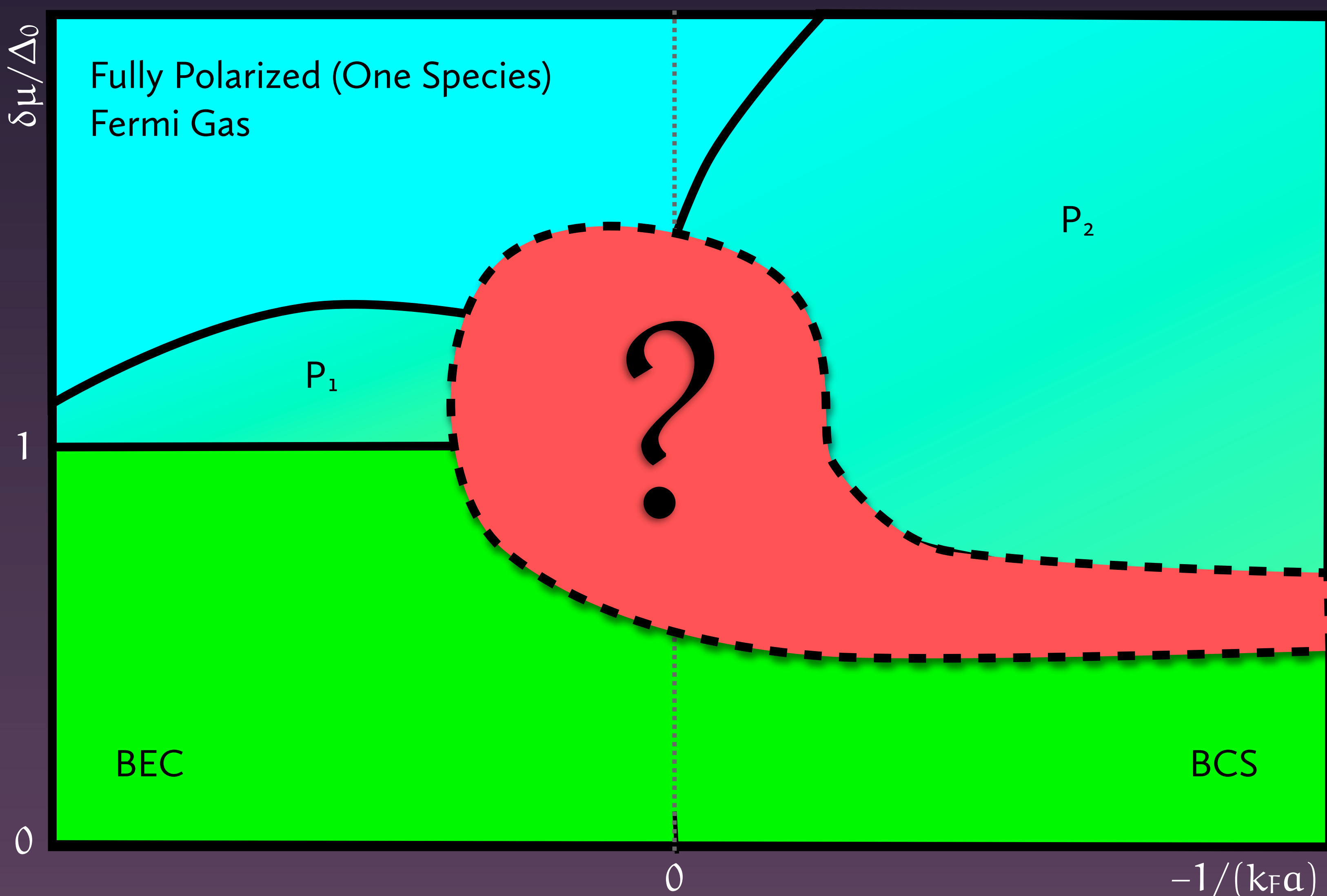


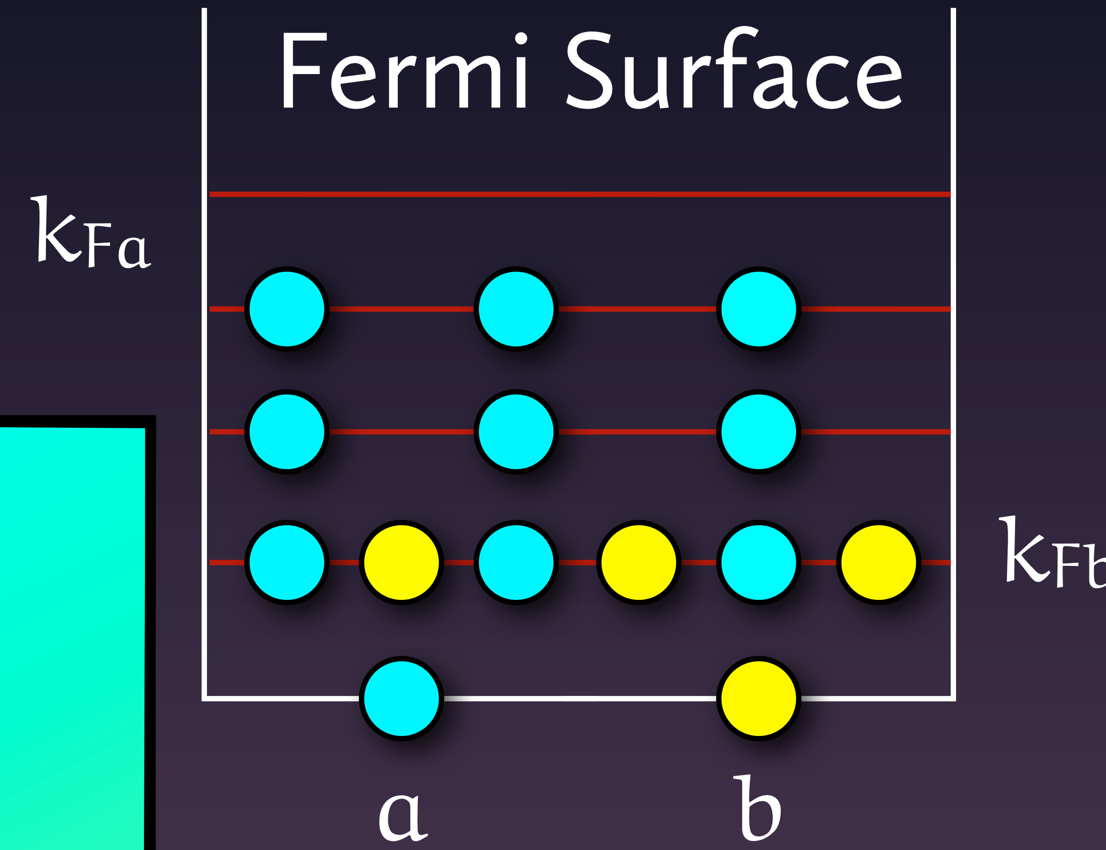
Tightly bound pairs

$p = p_a + p_b = 0$

D.T. Son and M. Stephanov (2005)
P-wave states by A.Bulgac, M.M.Forbes, A.Schwenk (PRL 2006)

Asymmetric?





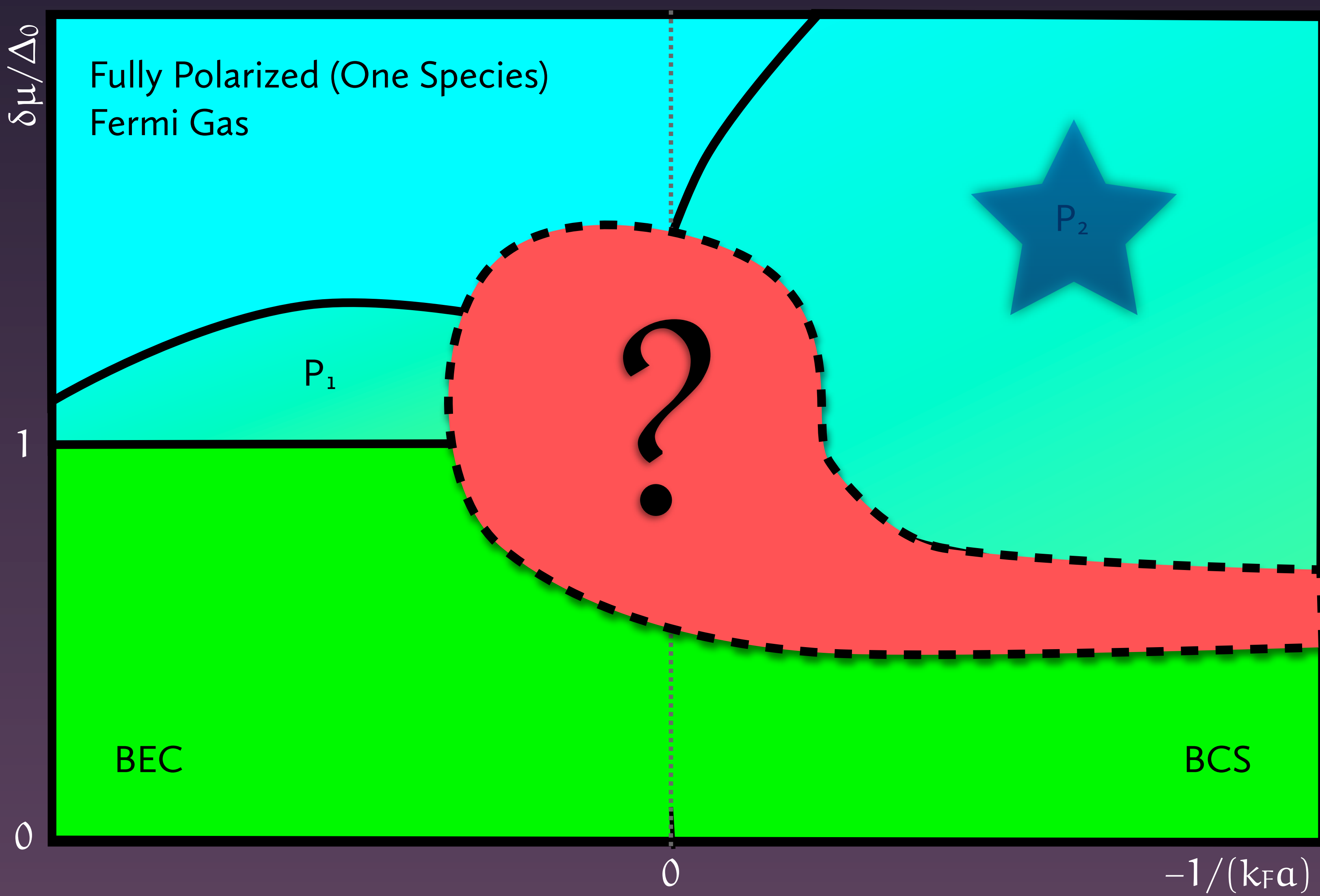
Unequal Fermi surfaces

- Frustrates pairing

D.T. Son and M. Stephanov (2005)

P-wave states by A.Bulgac, M.M.Forbes, A.Schwenk (PRL 2006)

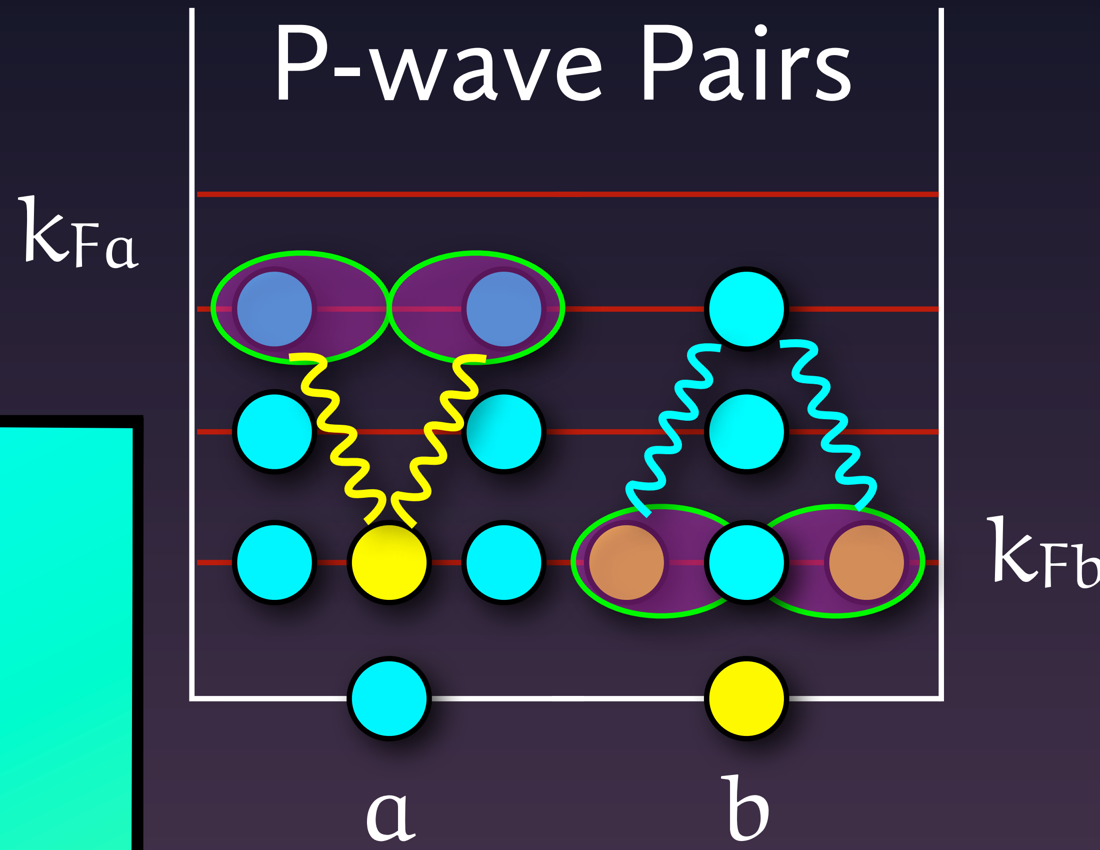
Asymmetric P-wave pairs



D.T. Son and M. Stephanov (2005)

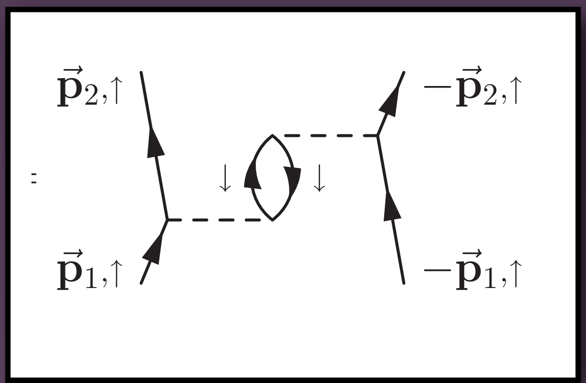
P-wave states by A.Bulgac, M.M.Forbes, A.Schwenk (PRL 2006)

Intra-species P-wave Pairs

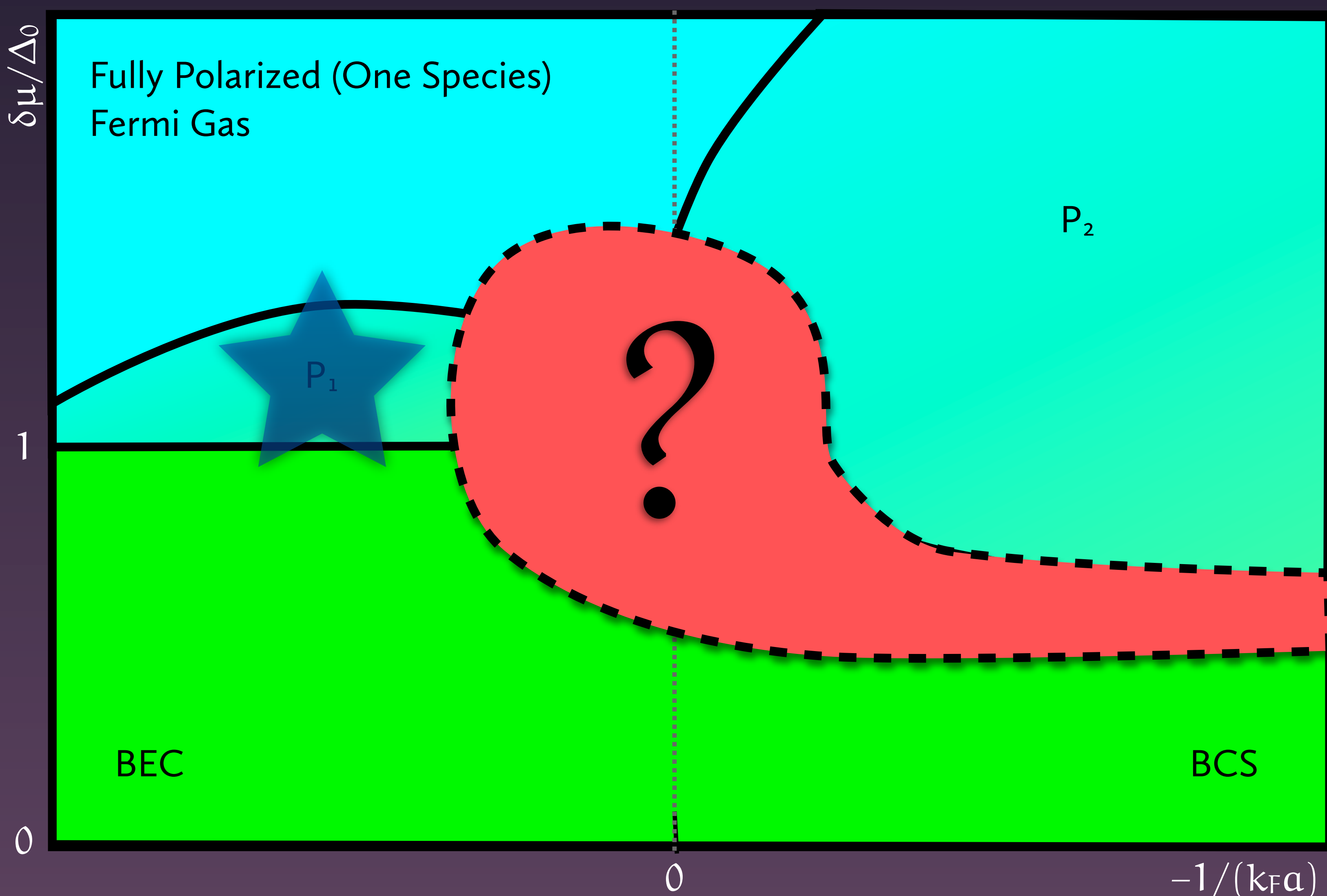


Kohn-Luttinger implies attractive at some l

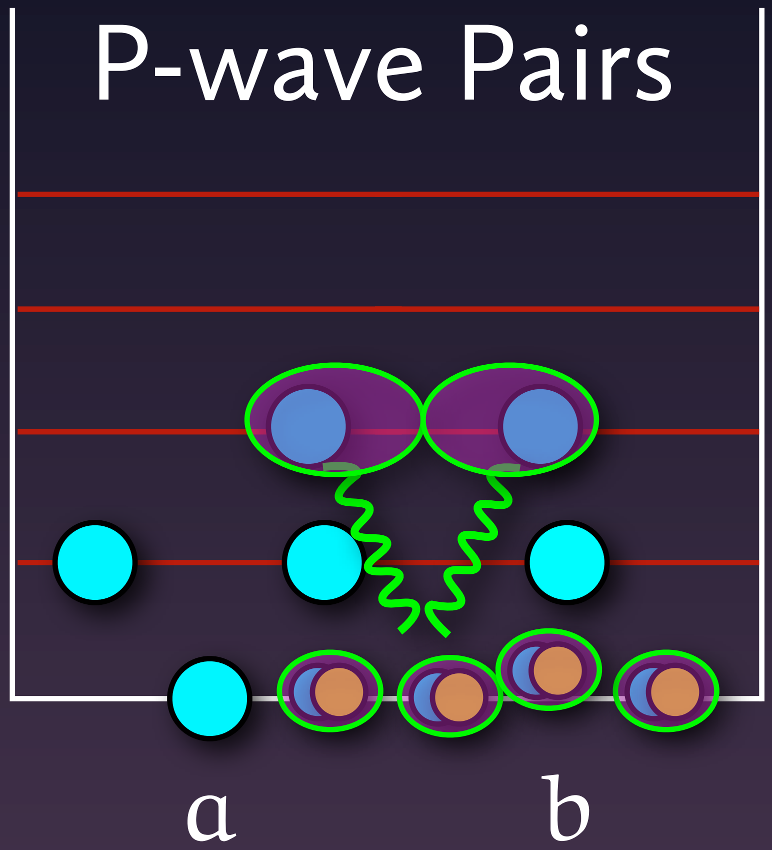
Two coexisting superfluids



Asymmetric P-wave BEC



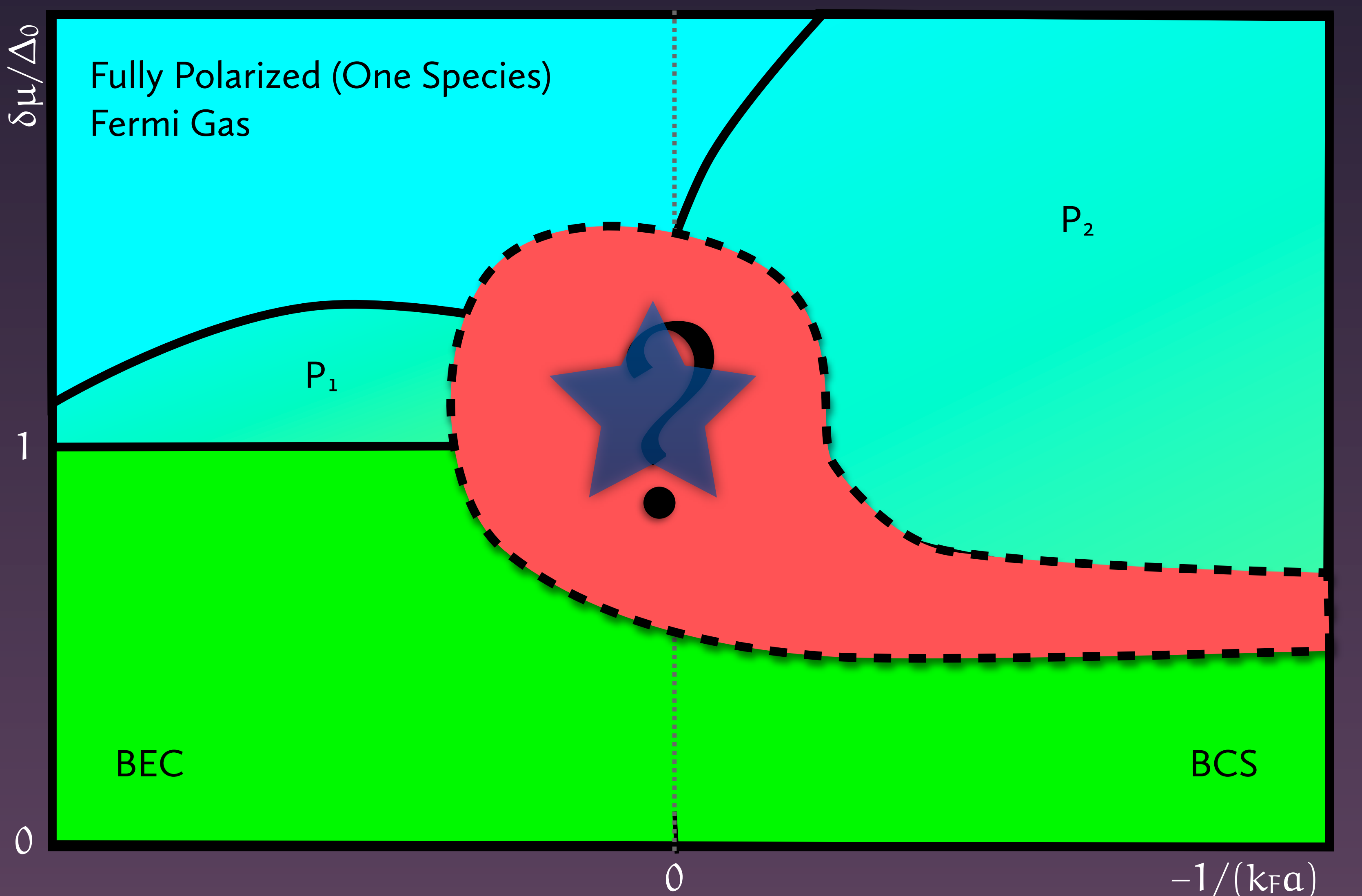
Intra-species P-wave Pairs



BEC and P-wave superfluids coexist homogeneously

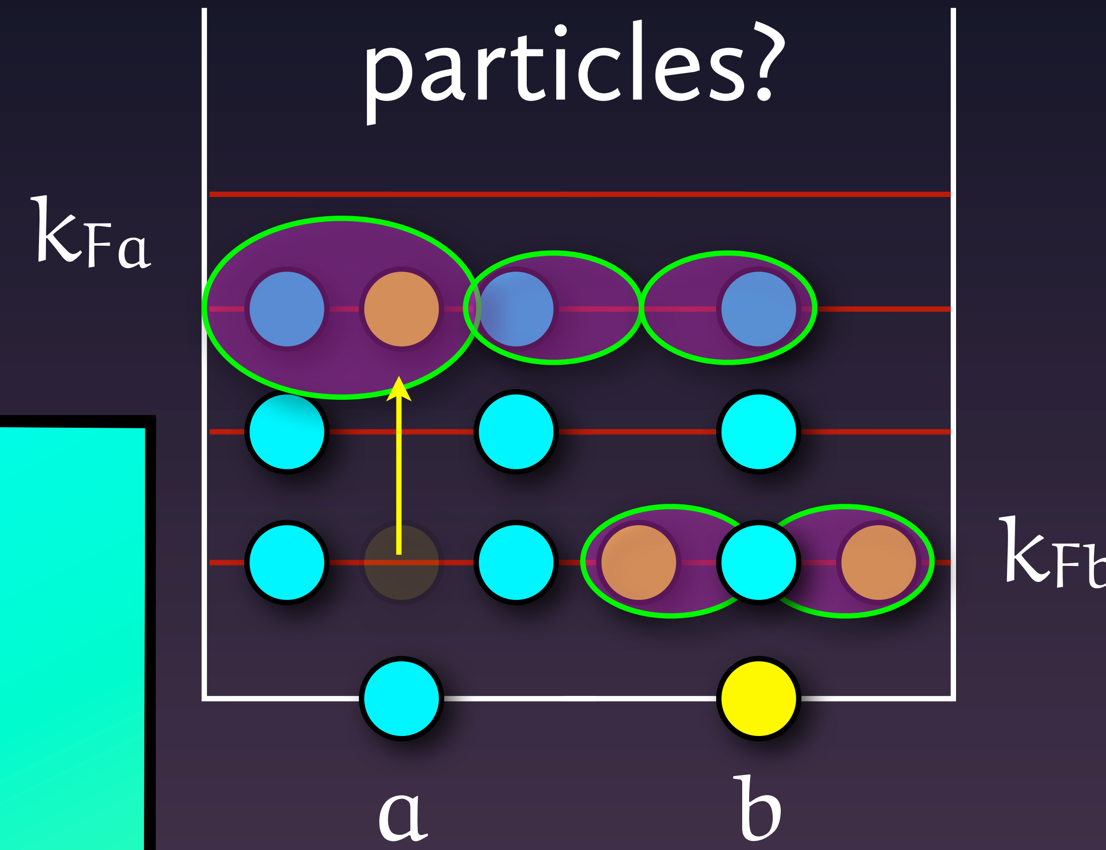
D.T. Son and M. Stephanov (2005)
P-wave states by A.Bulgac, M.M.Forbes, A.Schwenk (PRL 2006)

Asymmetric Gapless SF



D.T. Son and M. Stephanov (2005)
P-wave states by A.Bulgac, M.M.Forbes, A.Schwenk (PRL 2006)

Pairing promotes particles?

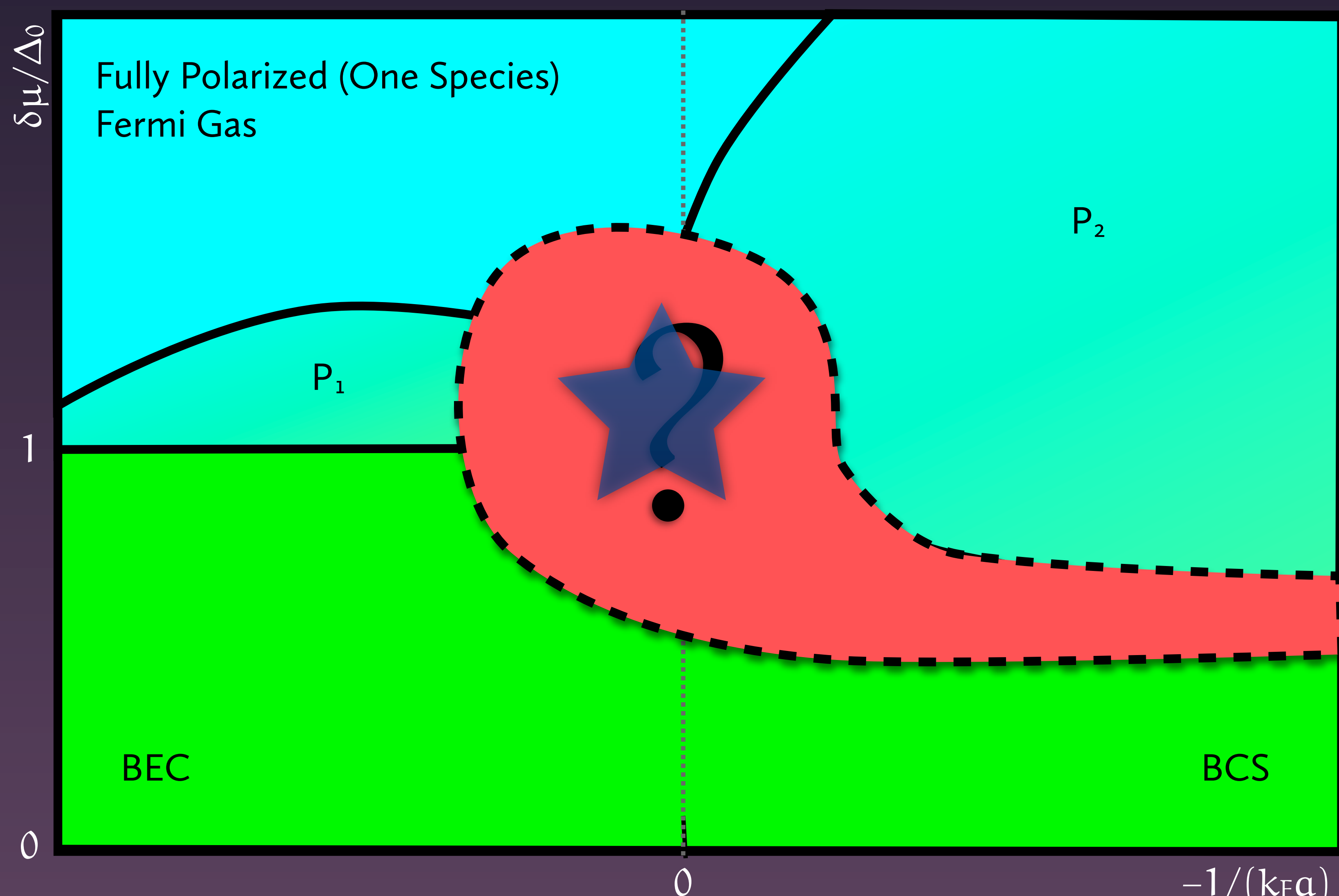


"Breach" in pairing

Still induced P-wave

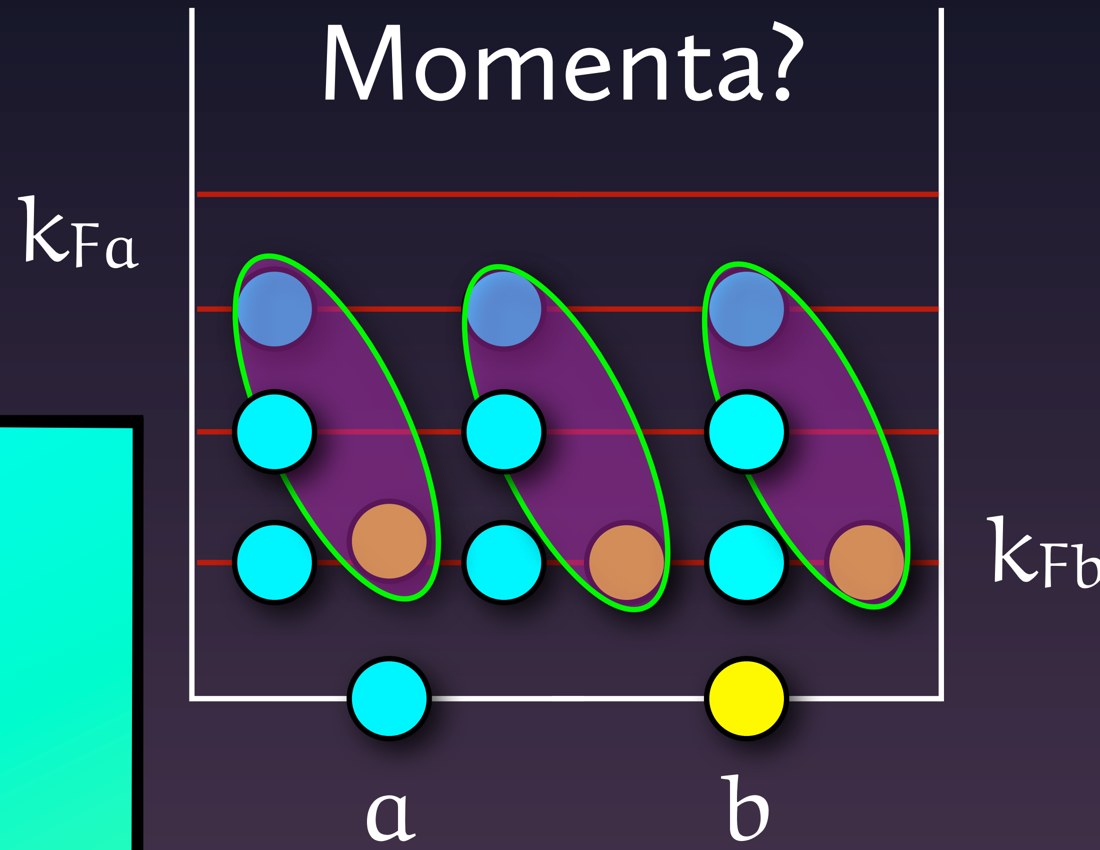
- May need large mass ratio or structured interactions (not likely at weak coupling in cold atoms)

Asymmetric FFLO



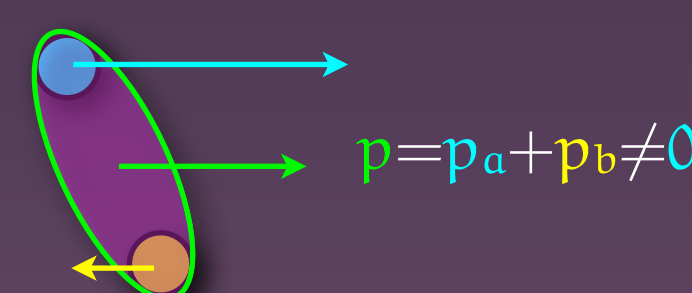
D.T. Son and M. Stephanov (2005)
P-wave states by A.Bulgac, M.M.Forbes, A.Schwenk (PRL 2006)

Pairs have Momenta?

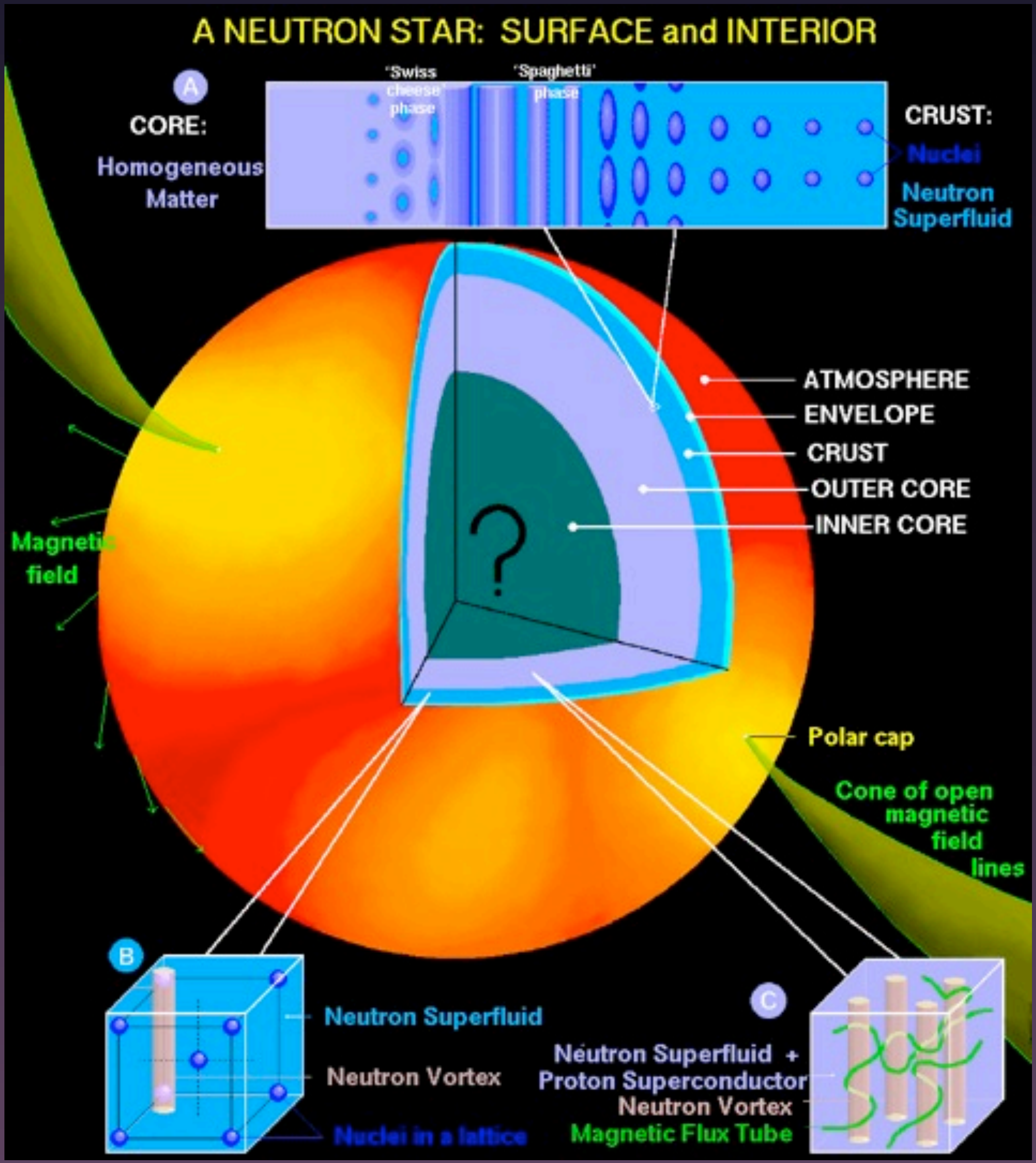


State (LO) is crystal (supersolid)

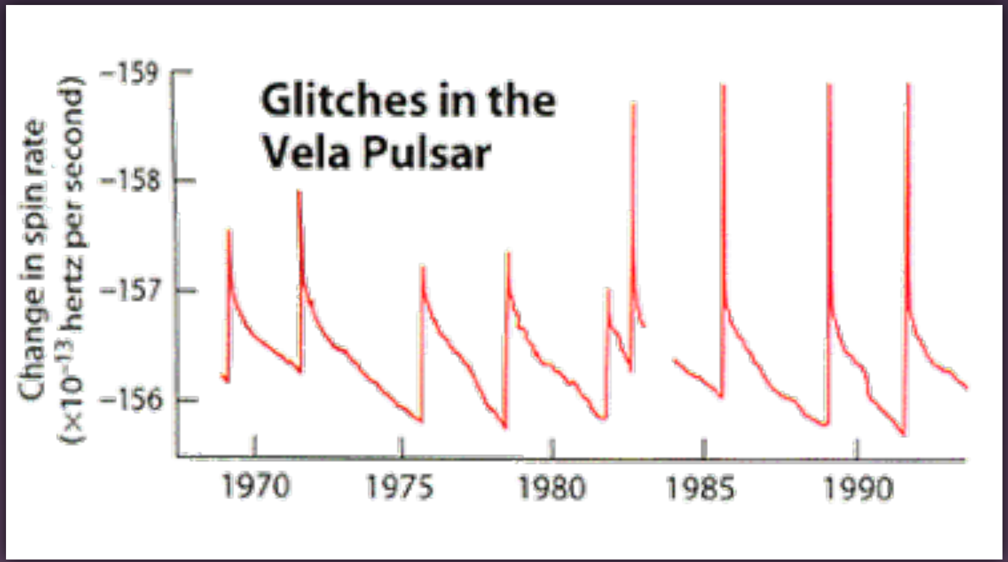
Pairs have momentum



Glitches



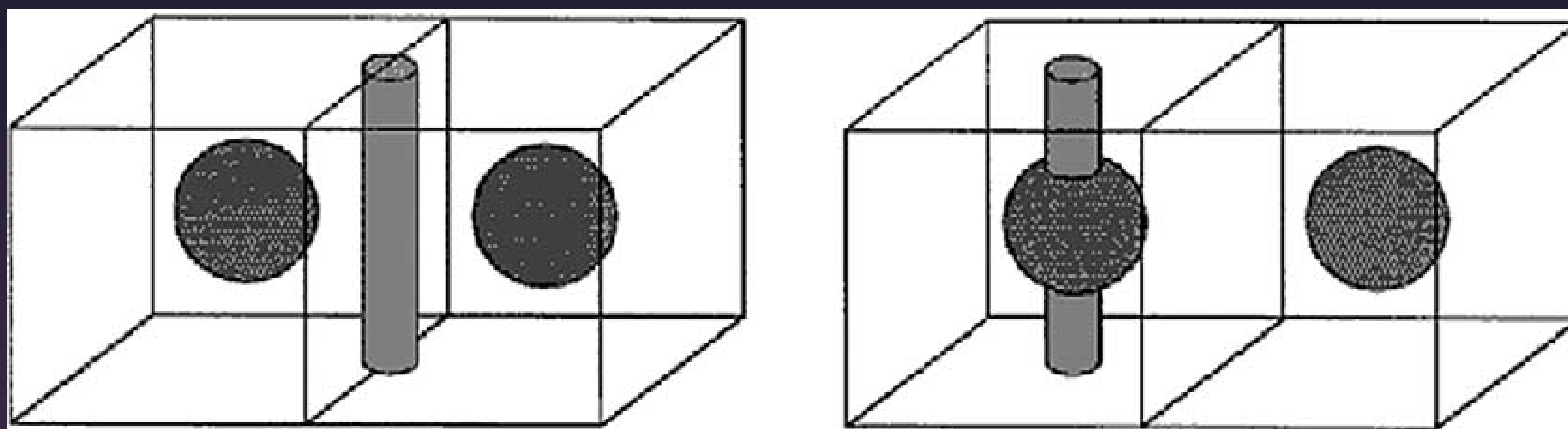
- Rapid increase in pulsation rate
- Anderson and Itoh (1975) suggested pinned superfluid vortices

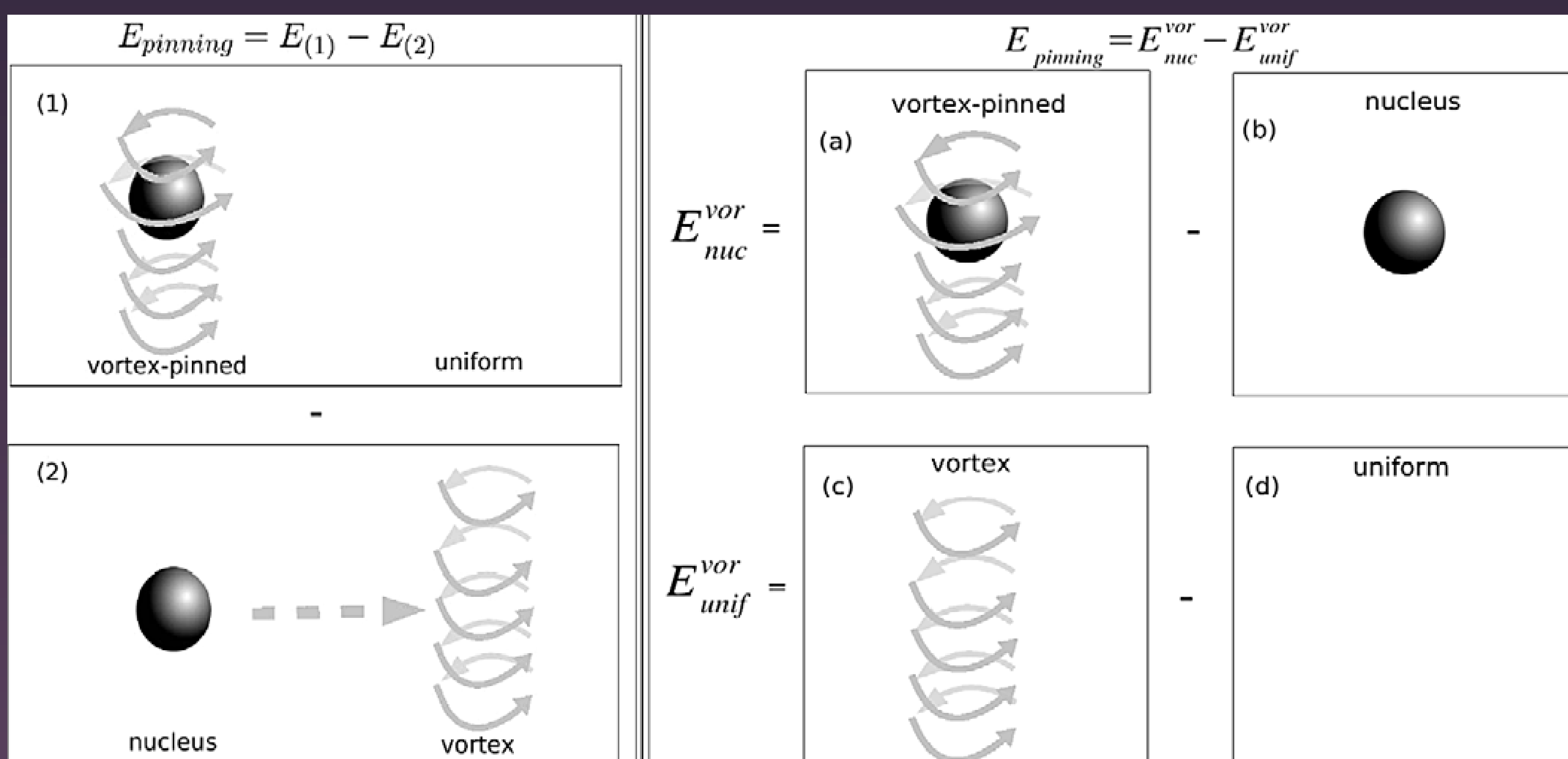


Pulsar Astronomy by Andrew G. Lyne and Francis Graham-Smith

Dany Page: http://www.astroscu.unam.mx/neutrones/NS-Picture/NS-Picture.html

Pinning from Statics





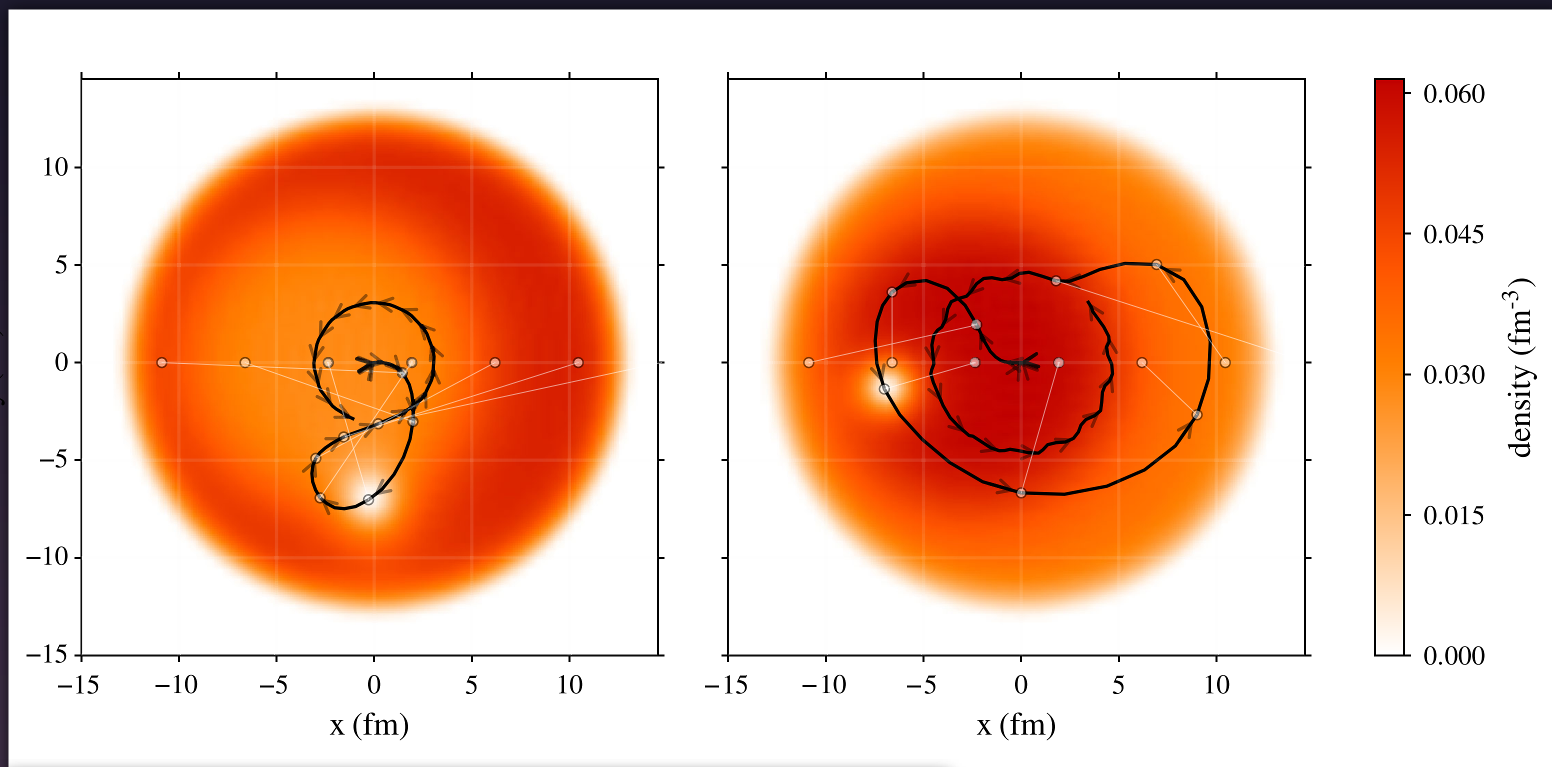
Energy calculations

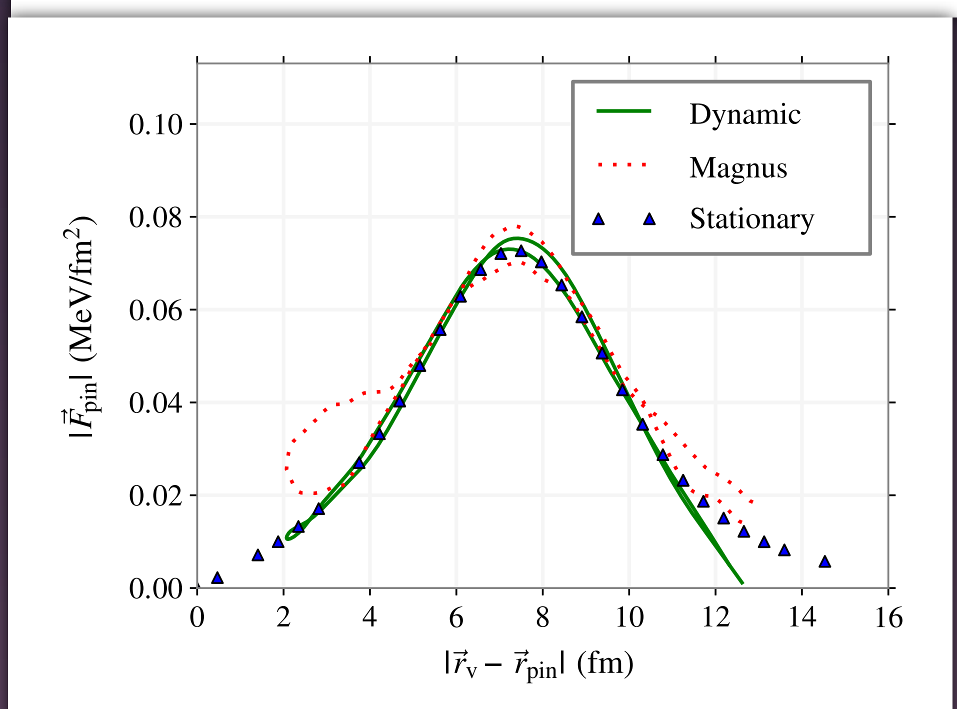
- Must diagonalize to high precision
  (subtraction involved)
- How to extract F(r)?

P. Donati, P.M. Pizzochero Nucl. Phys. A742 (2004) 363

Avogadro, F. Barranco, R. A. Broglia, and E. Vigezzi, Nucl. Phys. A811 (2008) 378

Pinning: Dynamics





Extract force with dynamical methods

- Scales well numerical:
  No diagonalization
- Extract force at any separation

Bulgac, Forbes, and Sharma: PRL 110 (2013) 241102

Density Functional Theory (DFT)

- The (exact) ground state density in any external potential V(x) minimizes a functional (Hohenberg Kohn):

$$\int d^3x \{\mathcal{E}[n(x)] + V(x)n(x)\}$$

- Functional may be complicated (non-local)
  - Need to find physically motivated approximations
- (think adjustable Mean Field Theory)

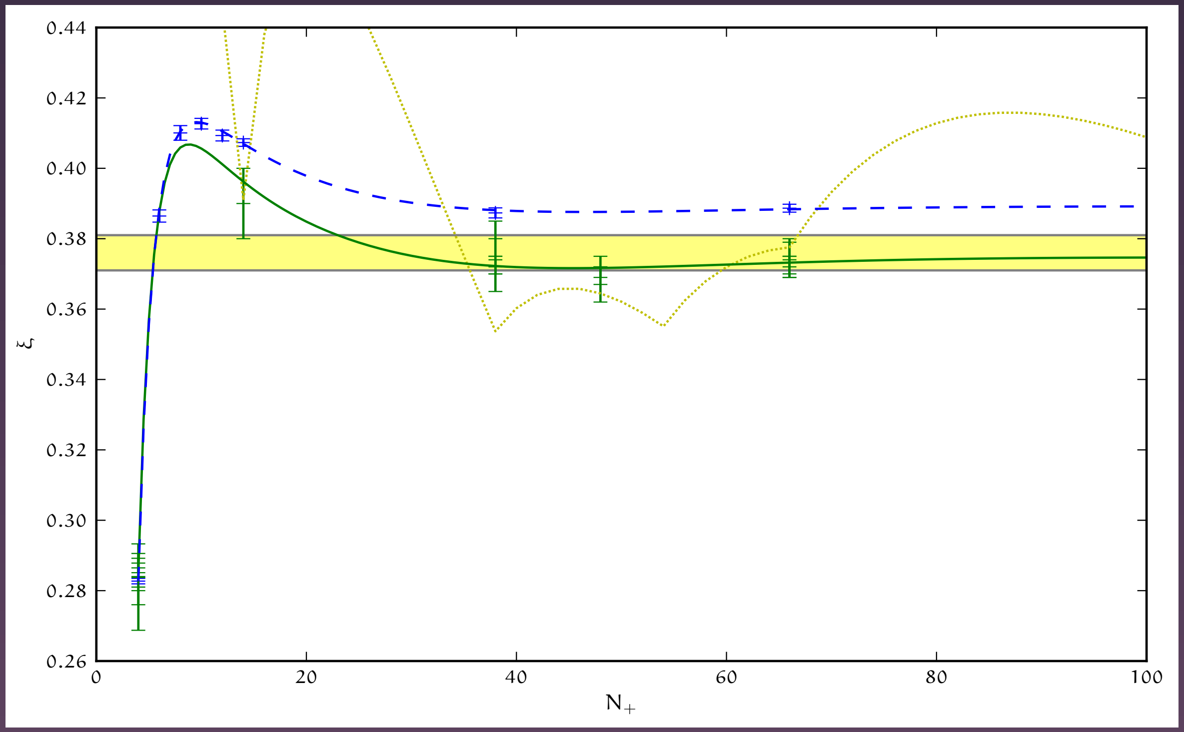
Density Functional Theory (DFT)

- Define functional with physically motivated model
- Fit parameters to experiment/QMC
- Functional extrapolates from small to large
- Seems very effective for the Unitary Fermi Gas

SLDA: Superfluid Local Density Approximation

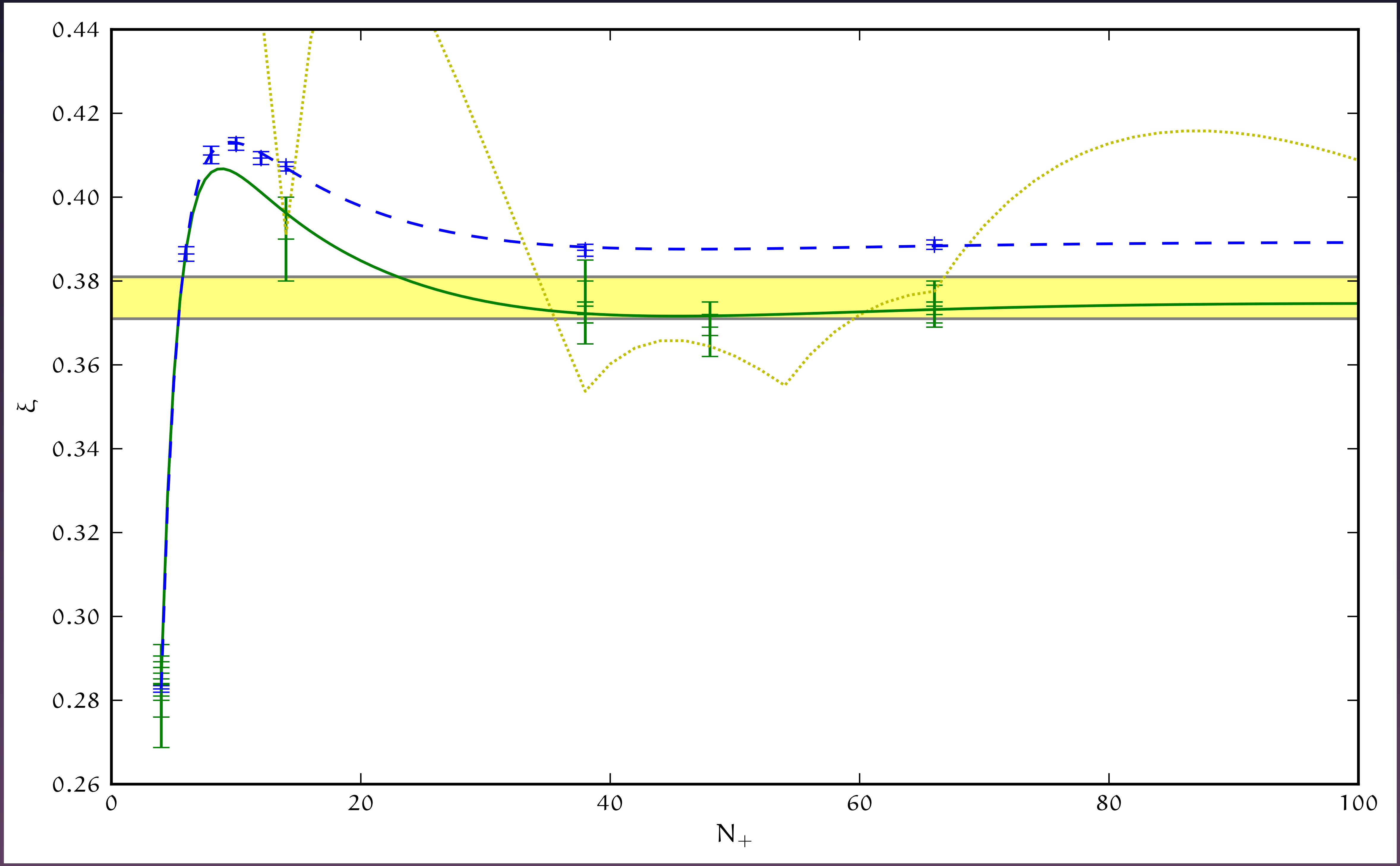
$$\mathcal{E}(n, \tau, \nu) = \alpha \frac{\tau}{m} + \beta \frac{(3\pi^2 n)^{5/3}}{10 m \pi^2} + g_{\text{eff}} \nu^\dagger \nu$$

- Three densities: $n \approx \langle a^\dagger a \rangle$, $\tau \approx \langle \nabla a^\dagger \nabla a \rangle$, $\nu \approx \langle ab \rangle$
- Three parameters:
  - Effective mass (m/α)
  - Hartree (β), Pairing (g)



Forbes, Gandolfi, Gezerlis (2012)

Slda: Superfluid Local

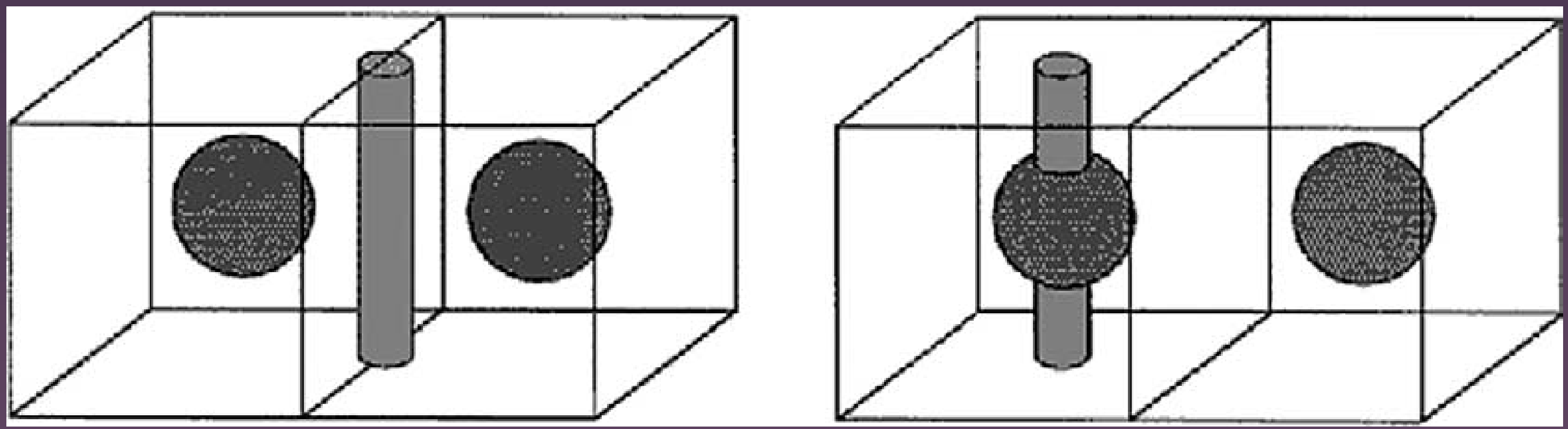


Forbes, Gandolfi, Gezerlis (2012)

TDDFT (TDSLDA)

$$\imath \partial_t \Psi_n = H[\Psi]\Psi_n = \begin{pmatrix} \frac{-\alpha\nabla^2}{2m} - \mu + U & \Delta^\dagger \\ \Delta & \frac{\alpha\nabla^2}{2m} + \mu - U \end{pmatrix} \begin{pmatrix} u_n \\ v_n \end{pmatrix}$$

- Computational challenge: Finding initial (ground) state?
  - Root-finders requires repeated diagonalization of s.p. Hamiltonian
  - Slow and does not scale well
  - Only suitable for small problems or if symmetries can be used

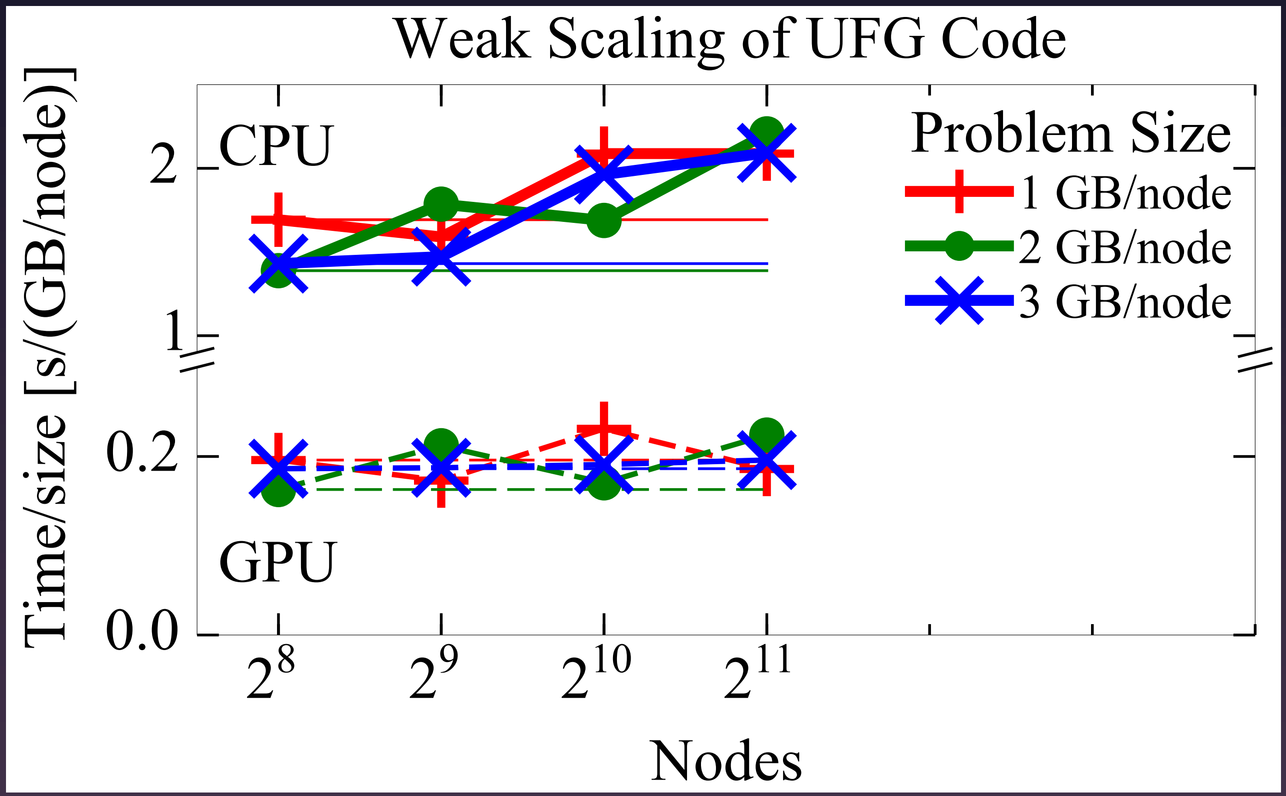


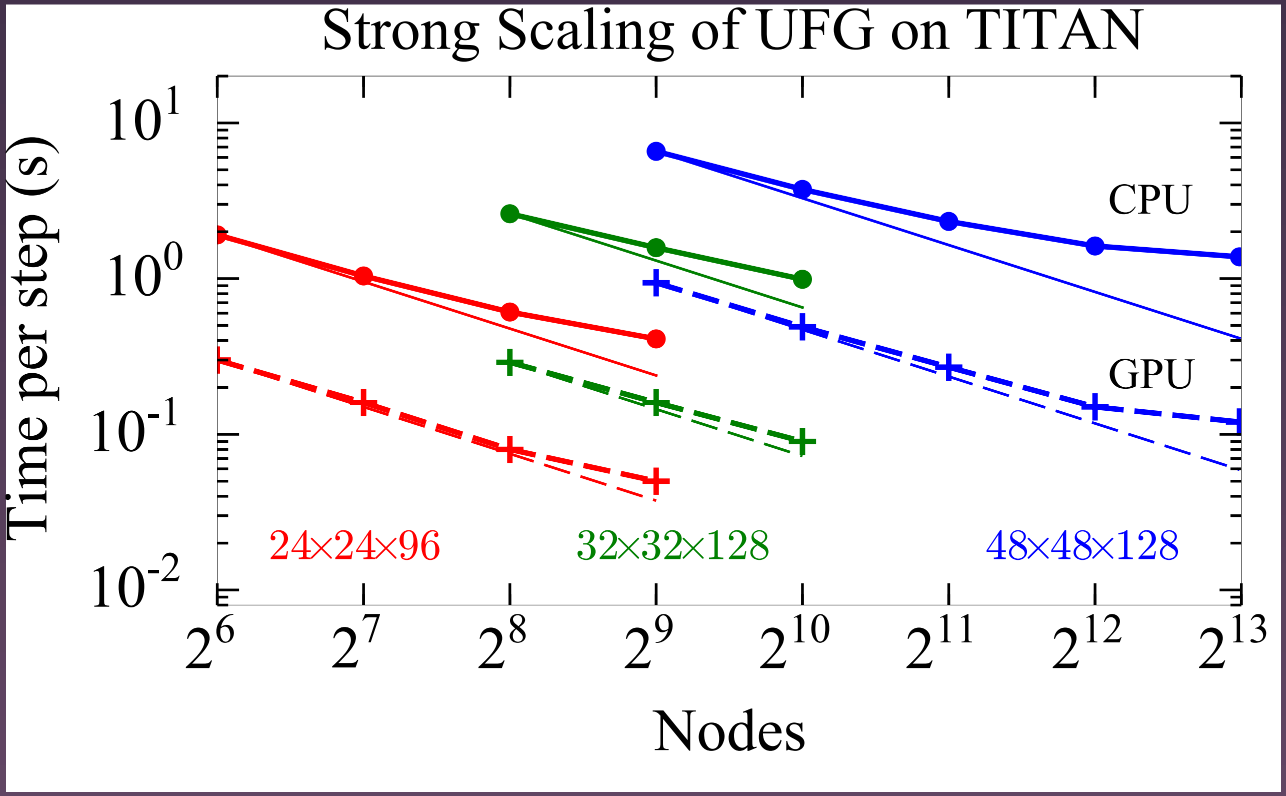
Realtime Evolution

$$\imath\partial_t\Psi_n = H[\Psi]\Psi_n = \begin{pmatrix} \frac{-\alpha\nabla^2}{2m} - \mu + U & \Delta^\dagger \\ \Delta & \frac{\alpha\nabla^2}{2m} + \mu - U \end{pmatrix} \begin{pmatrix} u_n \\ v_n \end{pmatrix}$$

- No diagonalization needed for evolution
  - Just apply Hamiltonian
  - Use FFT for kinetic term
- Efficient realtime evolution the scales well
  - Distribute wavefunctions over nodes
  - Utilize GPUs
- Split Operator or ABM evolution

Scaling Properties





SLDA realtime code

- Both Weak and Strong scaling

- Fully utilizes GPUs
  (GPUs provide 90% of TITAN's compute power)

State Preparation?

- How to find initial (ground) state?
- Root-finders repeatedly diagonalize s.p. Hamiltonian
  - Slow and does not scale well
- Imaginary time evolution?
  - Non-unitary: spoils orthogonality of wavefunctions
  - Re-orthogonalization unfeasible (communication)

Quantum Friction

$$V_t \propto -\frac{\hbar \vec{\nabla} \cdot \vec{j}_t}{\rho_t} = \frac{\hbar \dot{\rho}_t}{\rho_t} \propto \frac{-\Im(\psi_t^\dagger \nabla^2 \psi_t)}{\rho_t}$$

- Unitary evolution (preserves orthonormality)
- Easy to compute: local time-dependent potential

  Acts to remove local currents
- Couple with quasi-adiabatic state preparation

  Bulgac, Forbes, Roche, and Wlazłowski (2013) [arXiv:1305.6891]

Quantum Friction

$$V_t \propto -\frac{\hbar\vec{\nabla}\cdot\vec{j}_t}{\rho_t} = \frac{\hbar\dot{\rho}_t}{\rho_t} \propto \frac{-\Im(\psi_t^\dagger\nabla^2\psi_t)}{\rho_t}$$

- Consider evolution with potential $H+V_t$:

$$\partial_t E = -i\ \mathrm{Tr}\left([H,\rho]\cdot V_t\right)$$

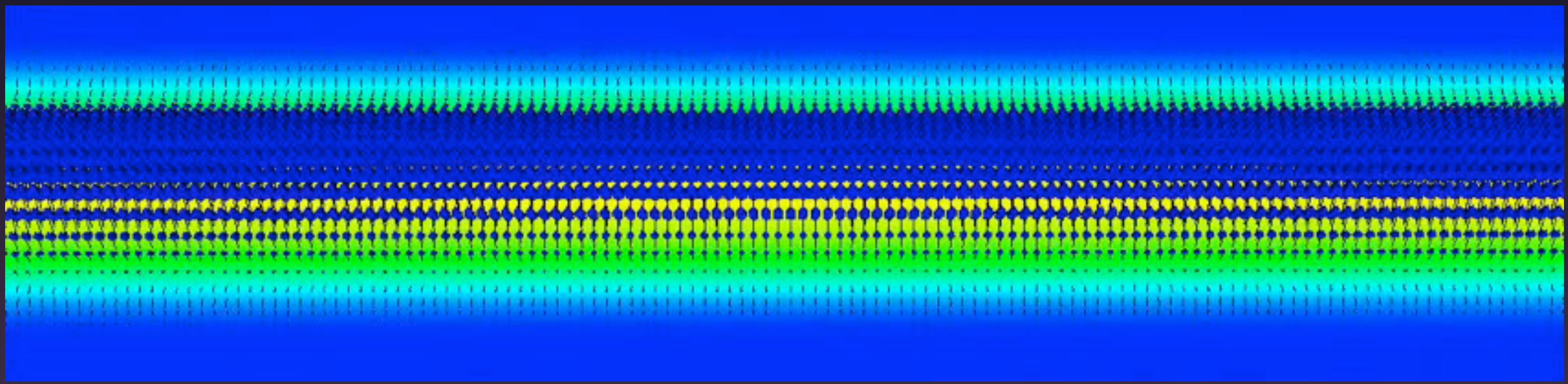
- Therefore $V_t = i[H,\rho]^\dagger$ guarantees $\partial_t E \leqslant 0$

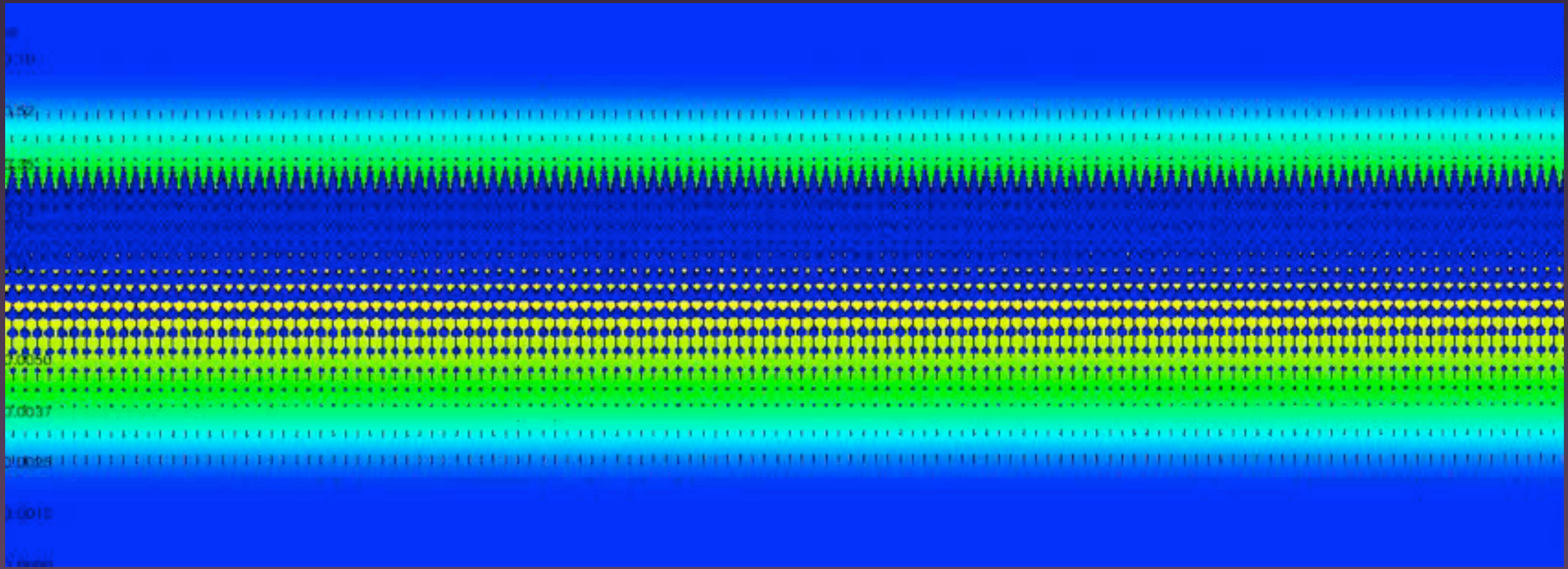
  Non-local potential equivalent to "complex time" evolution

  Not suitable for fermionic problem

- Diagonal version is a local potential: $V_t = \mathrm{diag}(i[H,\rho]^\dagger)$

State Preparation





Bulgac, Forbes, Kelley, Roche, Wlazłowski (2013) [arXiv:1306.4266]:
32x32x128

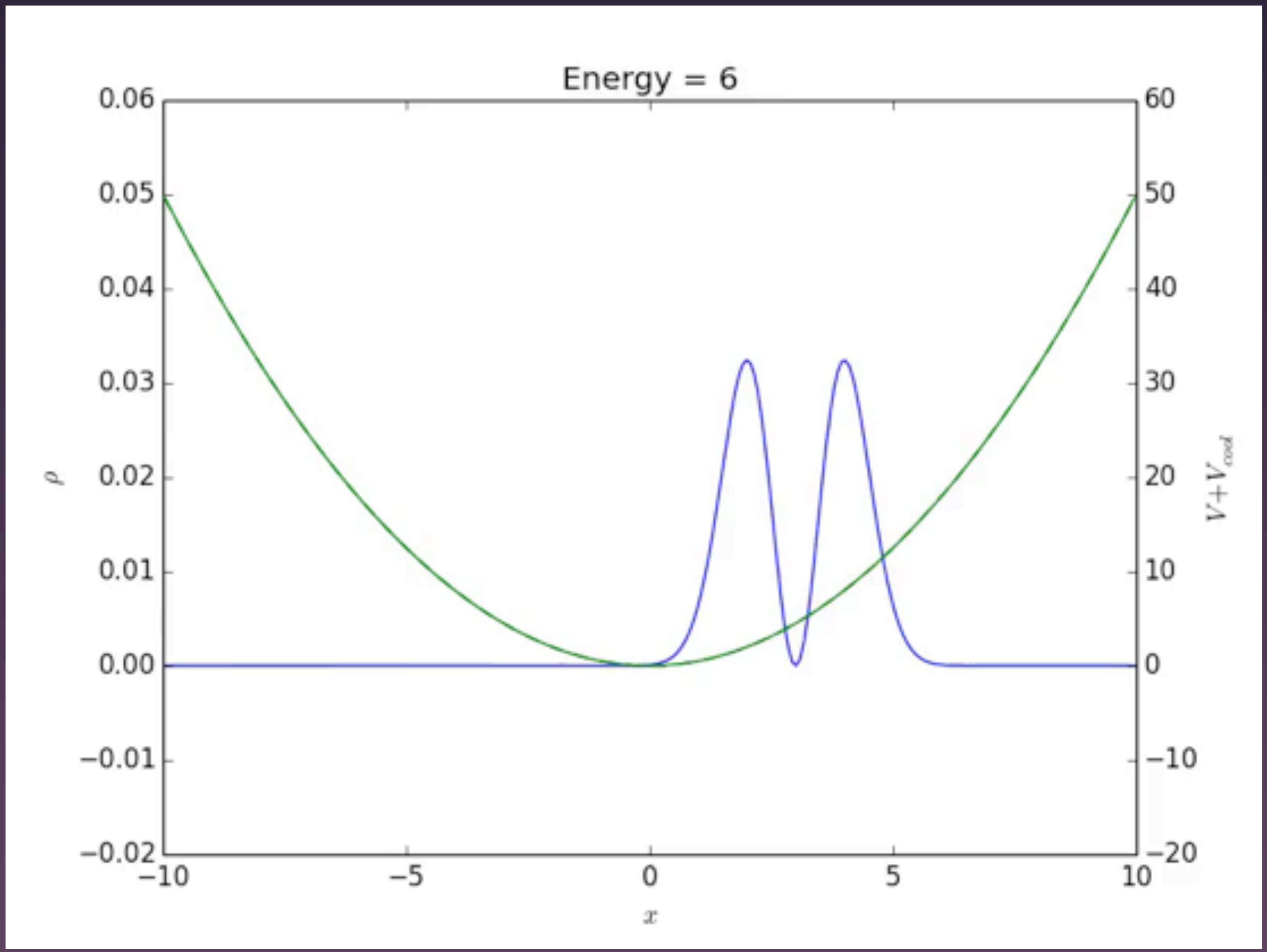
Quantum Friction

Potential counteracts currents

Use with dynamics to minimize energy

Harmonic oscillator with an excited state

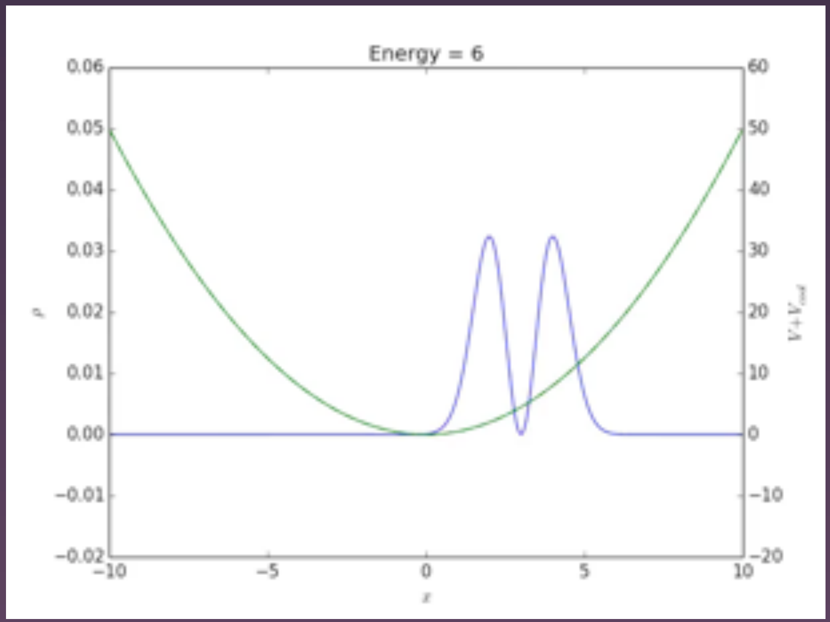
Quantum Friction



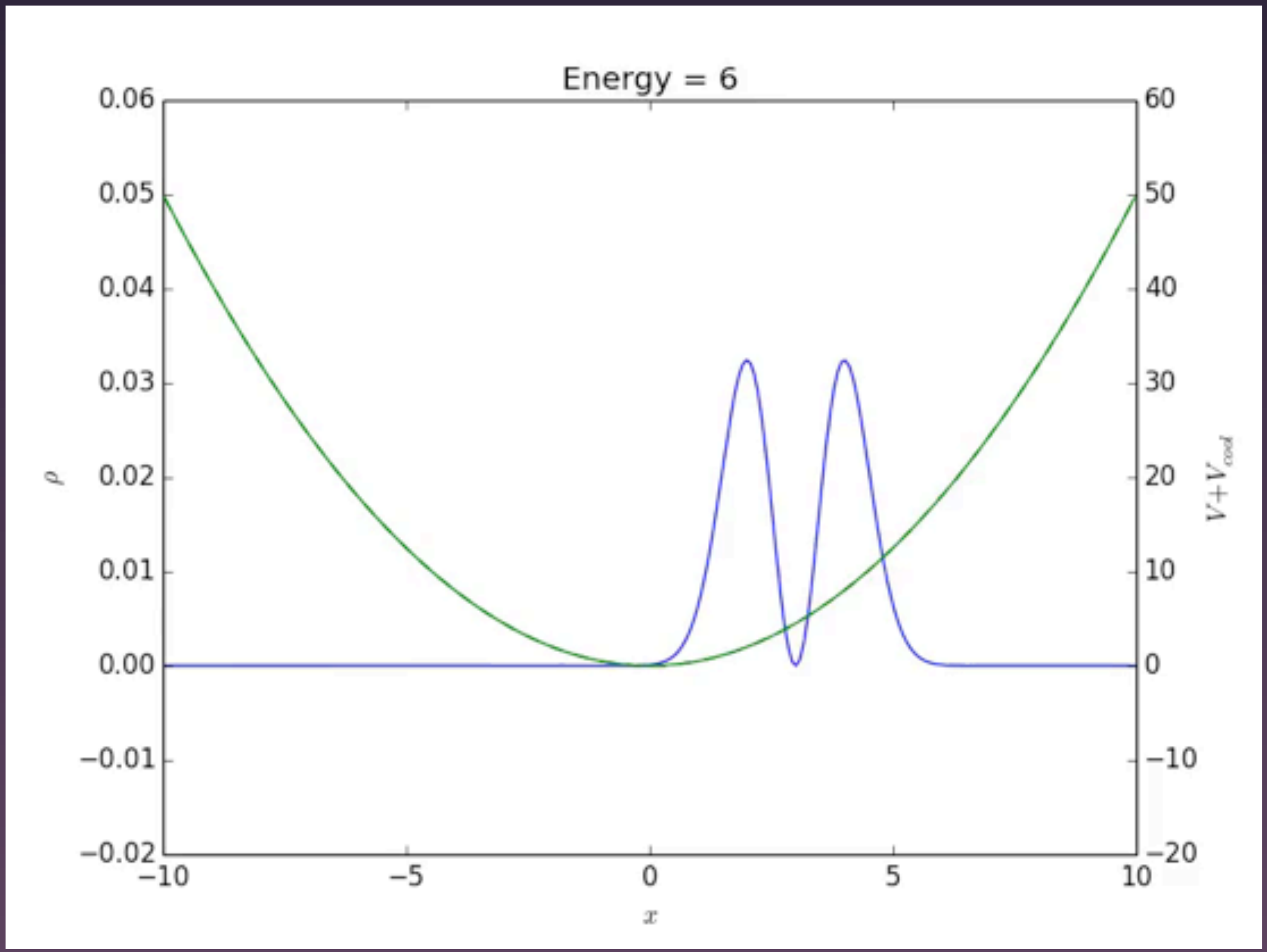
Harmonic oscillator with an excited state

Potential counteracts currents

Use with dynamics to minimize energy



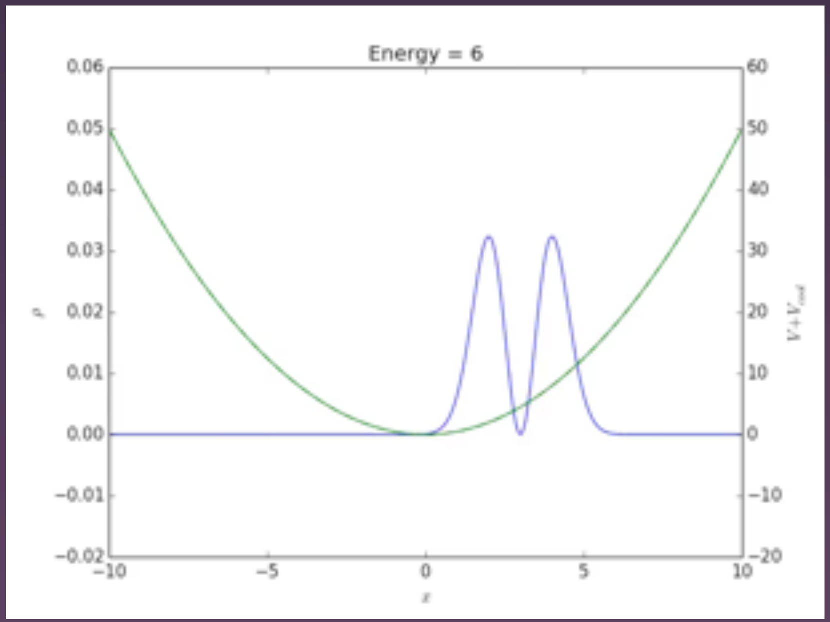
Quantum Friction



Harmonic oscillator with an excited state

Potential counteracts currents

Use with dynamics to minimize energy



Quantum Friction

$$V_t \propto -\frac{\hbar\vec{\nabla}\cdot\vec{j}_t}{\rho_t} = \frac{\hbar\dot{\rho}_t}{\rho_t} \propto \frac{-\Im(\psi_t^\dagger\nabla^2\psi_t)}{\rho_t}$$

- General method: (works for many problems)
  Needs a good initial state to ensure reasonable occupation numbers
- Easy to compute: local time-dependent potential
  Acts to remove local currents
- Couple with quasi-adiabatic state preparation
  Bulgac, Forbes,Roche, and Wlazłowski (2013) [arXiv:1305.6891]

TDDFT (TDSLDA)

$$\imath\partial_t\Psi_n = H[\Psi]\Psi_n = \begin{pmatrix} \frac{-\alpha\nabla^2}{2m} - \mu + U & \Delta^\dagger \\ \Delta & \frac{\alpha\nabla^2}{2m} + \mu - U \end{pmatrix} \begin{pmatrix} u_n \\ v_n \end{pmatrix}$$

- Still Computationally expensive:

  Need to evolve each hundreds of thousands of wavefunctions

- Possible for moderate systems (nuclei) using supercomputers, resonances, induced fission etc.

  Maybe cold atoms (if axially symmetric etc.)

  Probably not for neutron stars (glitching dynamics)

Bosons are "easy"

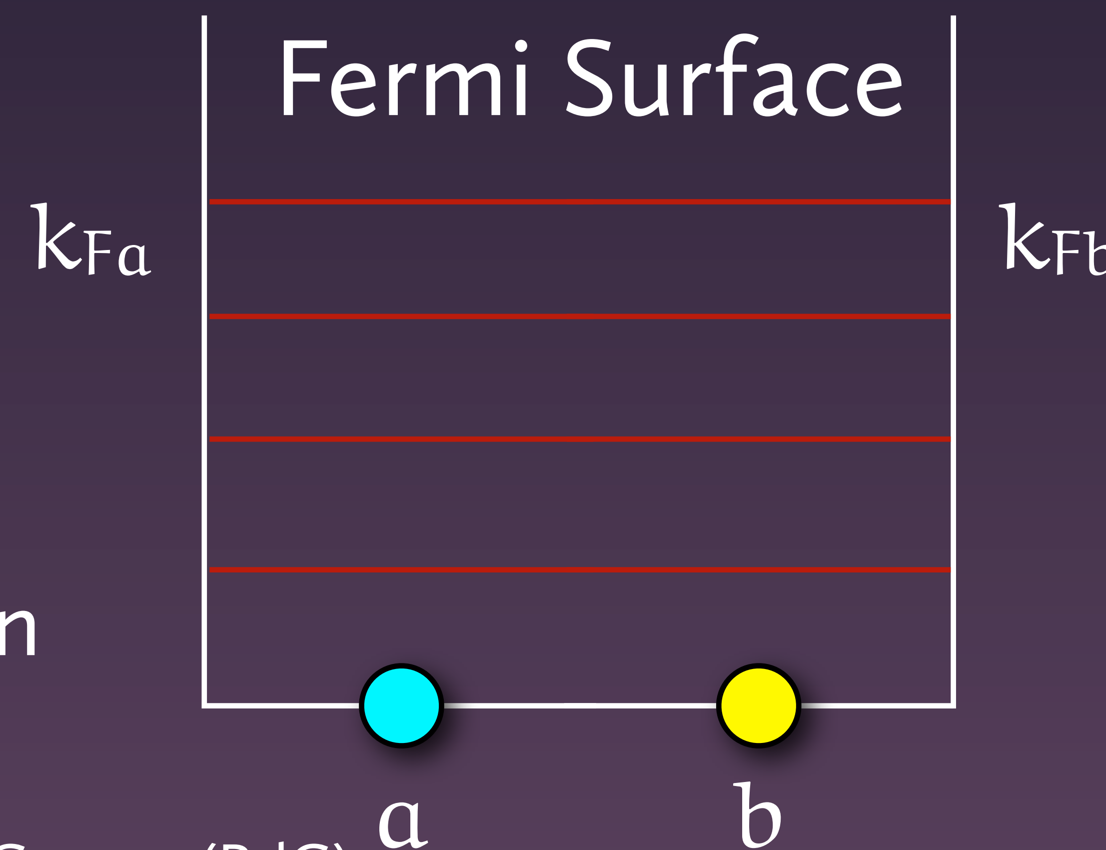
$$E[\Psi] = \int d^3\vec{x} \, \left(\frac{\hbar^2 |\nabla\Psi(\vec{x})|^2}{2m_B} + V_F(\vec{x})\rho_F + g\frac{|\Psi|^4}{2} \right)$$

$$i\partial_t\Psi = \left(-\frac{\nabla^2}{2m_B} + [V + g|\Psi|^2] \right) \Psi$$

- Gross-Pitaevskii Equation (GPE)
- (all) bosons in single ground state
  Include interactions through mean field
- Non-linear Schrödinger equation
- Only one wave function

Fermions are harder

$$\imath\partial_t \Psi_n = H[\Psi]\Psi_n = \begin{pmatrix} \frac{-\alpha\nabla^2}{2m} - \mu + U & \Delta^\dagger \\ \Delta & \frac{\alpha\nabla^2}{2m} + \mu - U \end{pmatrix}\begin{pmatrix} u_n \\ v_n \end{pmatrix}$$



- Pauli Exclusion (blocking)
  - Oarticles in different states
- Must track N wavefunctions
  - Non-linear Schrödinger equation for each wavefunction

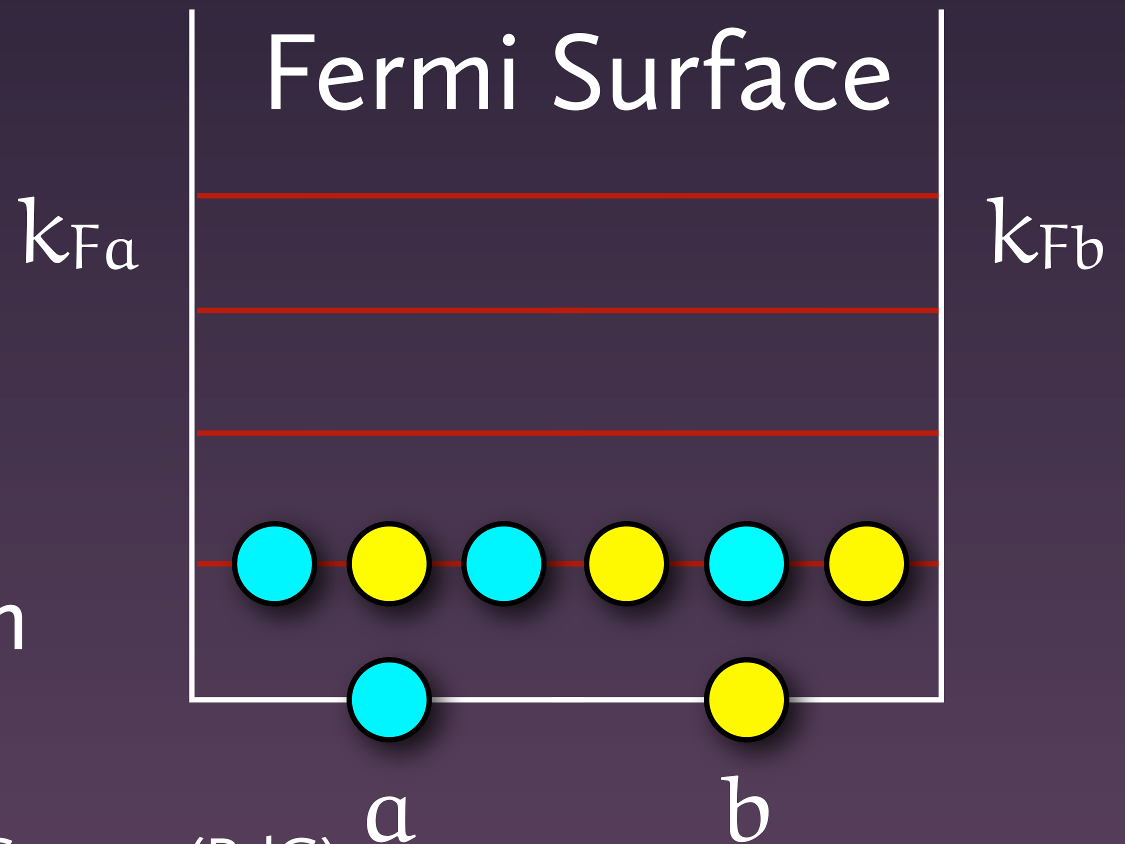
  Hartree-Fock–Bogoliubov (HFB), Bogoliubov de-Gennes (BdG)
- Must use symmetries or supercomputers

Fermions are harder

$$\imath\partial_t\Psi_n = H[\Psi]\Psi_n = \begin{pmatrix} \frac{-\alpha\nabla^2}{2m} - \mu + U & \Delta^\dagger \\ \Delta & \frac{\alpha\nabla^2}{2m} + \mu - U \end{pmatrix} \begin{pmatrix} u_n \\ v_n \end{pmatrix}$$

- Pauli Exclusion (blocking)
  - Oarticles in different states
- Must track N wavefunctions
  - Non-linear Schrödinger equation for each wavefunction

  Hartree-Fock–Bogoliubov (HFB), Bogoliubov de-Gennes (BdG)
- Must use symmetries or supercomputers

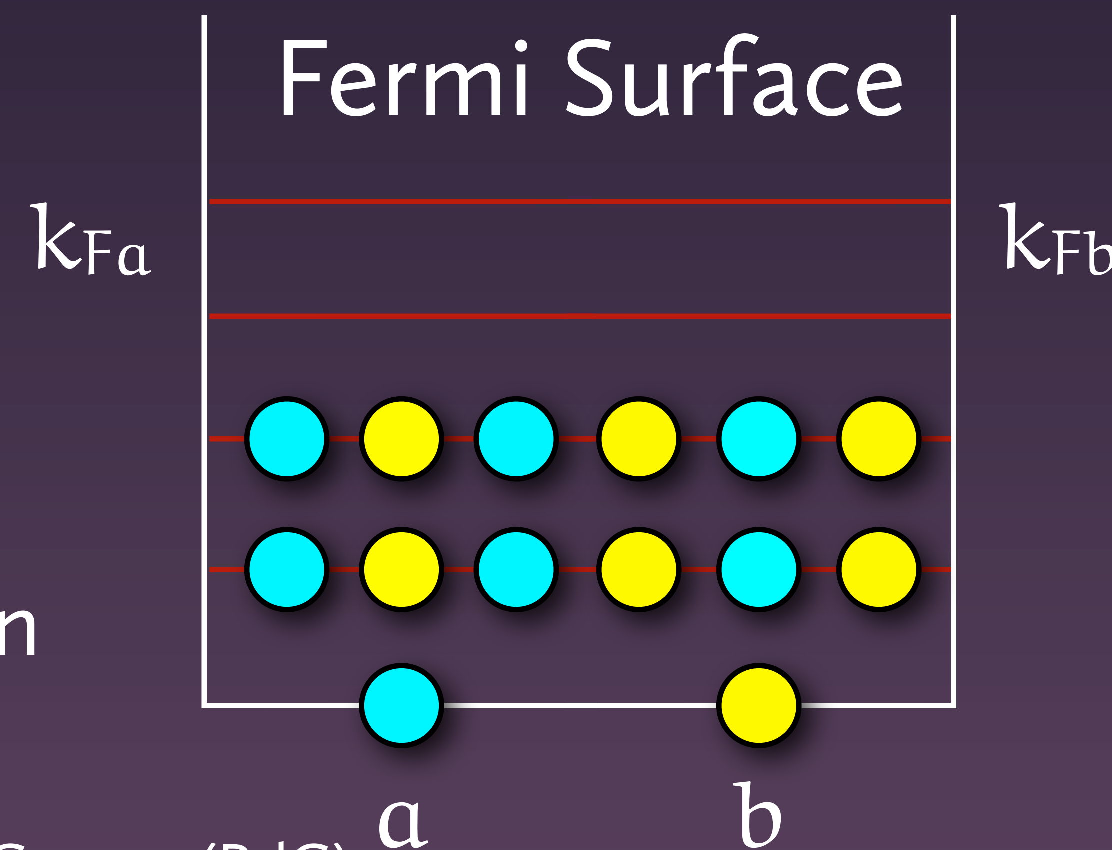


Fermions are harder

$$\imath\partial_t\Psi_n = H[\Psi]\Psi_n = \begin{pmatrix} \frac{-\alpha\nabla^2}{2m} - \mu + U & \Delta^\dagger \\ \Delta & \frac{\alpha\nabla^2}{2m} + \mu - U \end{pmatrix}\begin{pmatrix} u_n \\ v_n \end{pmatrix}$$

- Pauli Exclusion (blocking)
  - Oarticles in different states
- Must track N wavefunctions
  - Non-linear Schrödinger equation for each wavefunction

  Hartree-Fock–Bogoliubov (HFB), Bogoliubov de-Gennes (BdG)

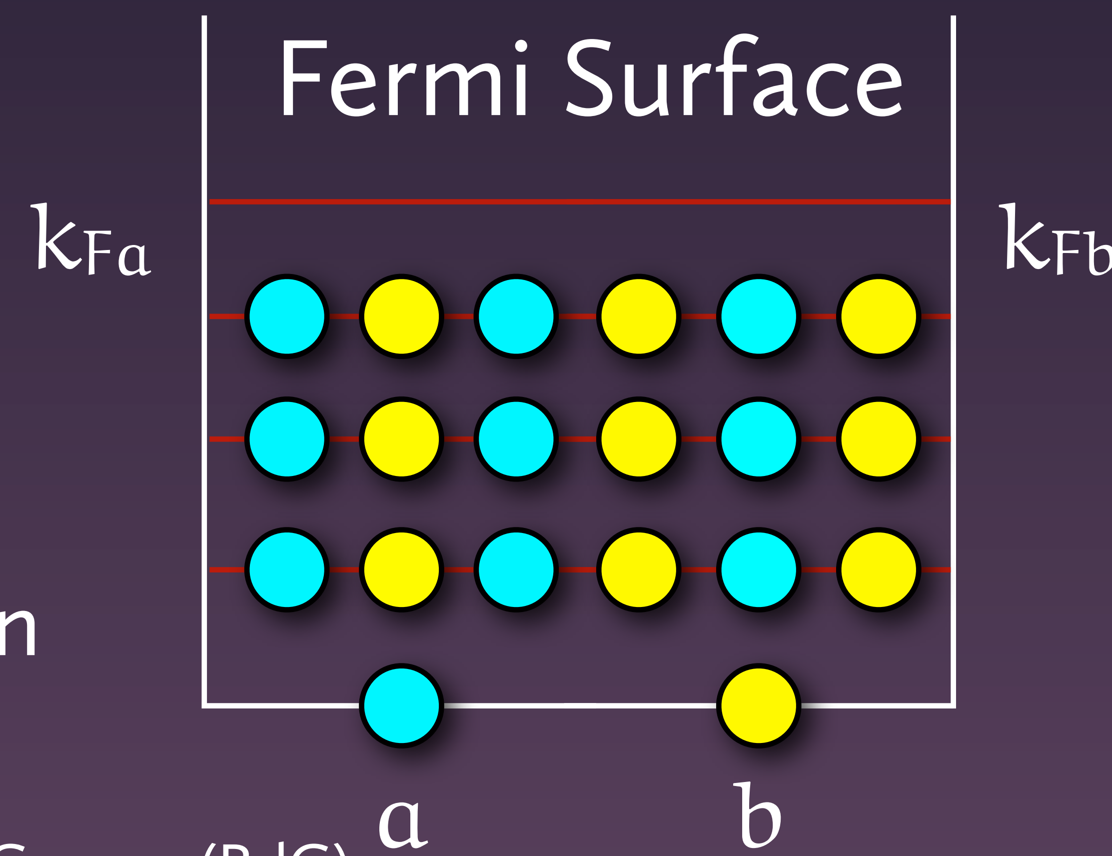


- Must use symmetries or supercomputers

Fermions are harder

$$\imath\partial_t\Psi_n = H[\Psi]\Psi_n = \begin{pmatrix} \frac{-\alpha\nabla^2}{2m} - \mu + U & \Delta^\dagger \\ \Delta & \frac{\alpha\nabla^2}{2m} + \mu - U \end{pmatrix} \begin{pmatrix} u_n \\ v_n \end{pmatrix}$$

- Pauli Exclusion (blocking)
  - Oarticles in different states
- Must track N wavefunctions
  - Non-linear Schrödinger equation for each wavefunction



Hartree-Fock–Bogoliubov (HFB), Bogoliubov de-Gennes (BdG)

- Must use symmetries or supercomputers

GPE model for UFG

$$E[\Psi] = \int d^3\vec{x} \left(\frac{|\nabla\Psi(\vec{x})|^2}{4m_F} + V_F(\vec{x})\rho_F + \xi\mathcal{E}(\rho_F) \right)$$

$$i\partial_t\Psi = \left(-\frac{\nabla^2}{4m_F} + 2\big(V_F + \xi\epsilon(\rho_F)\big) \right)\Psi$$

- Describe non-interacting particles:
  - Schrödinger Equation
- Capture interactions through “mean field” $V \propto |\Psi|^5$
  - Non-linear Schrödinger Equation
- Fermions require antisymmetrization (Pauli exclusion)
  - Can use SLDA, but...

GPE model for UFG

$$E[\Psi] = \int d^3\vec{x} \left(\frac{|\nabla\Psi(\vec{x})|^2}{4m_F} + V_F(\vec{x})\rho_F + \xi\mathcal{E}(\rho_F) \right)$$

$$i\partial_t\Psi = \left(-\frac{\nabla^2}{4m_F} + 2\big(V_F + \xi\epsilon(\rho_F)\big) \right)\Psi$$

- Bosonic model works remarkably well!
- Think:
  - Boson = Fermion pair (dimer) $\quad \rho_F = 2|\Psi|^2$
- Galilean Covariant (fixes mass) $\quad \mathcal{E}_{FG} \propto \rho_F^{5/2}$
- Match Unitary Equation of State $\quad \epsilon_F = \mathcal{E}'_{FG}(\rho_F) \propto \rho_F^{3/2}$

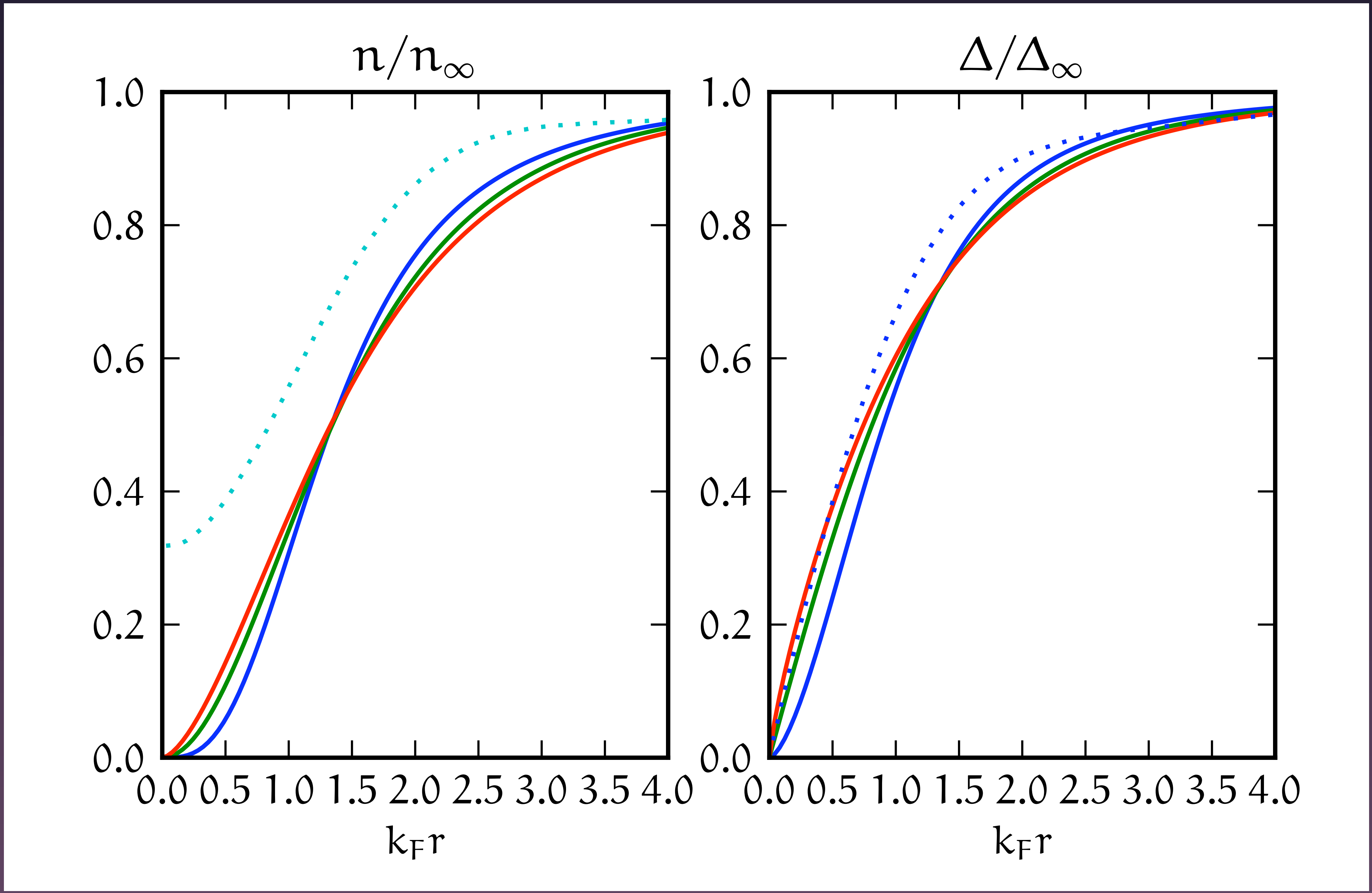
Matching Theories: The Good

- Galilean Covariance (fixes mass/density relationship)
- Equation of State
- Hydrodynamics
  - speed of sound (exact)
  - phonon dispersion (to order q^3)
  - static response (to order q^2)

Matching Theories: The Bad

- GPE has $\rho = 2|\Psi|^2$
  - Density vanishes in core of vortex
  - Implies $\int|\Psi|^2$ conserved
    - (Approximate conservation $\int|\Psi|^2$ in Fermi simulations provides measure of applicability)
- No "normal state"
  - Two fluid model needed?
  - Coarse graining (transfer to "normal" component)

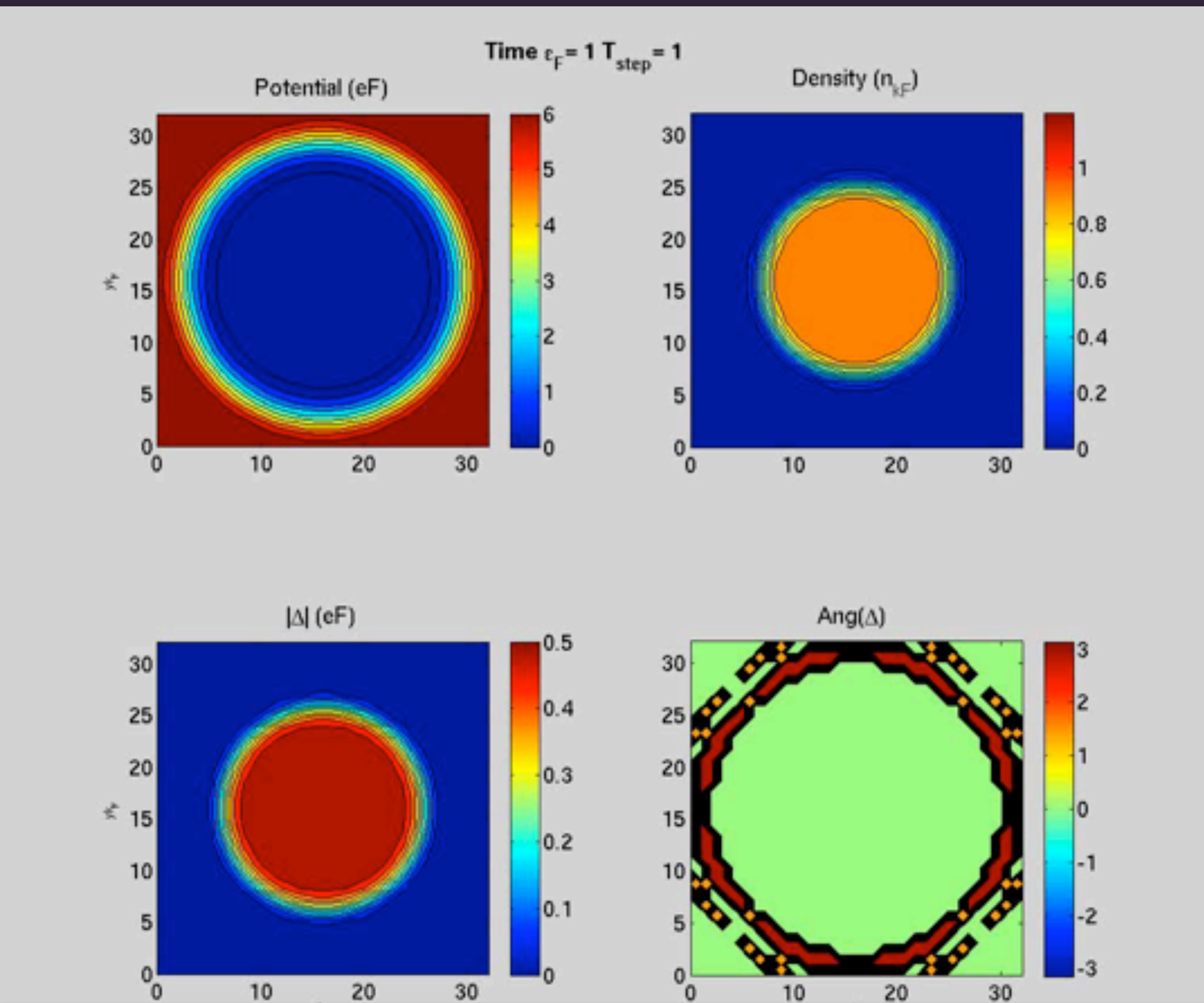
Vortex Structure



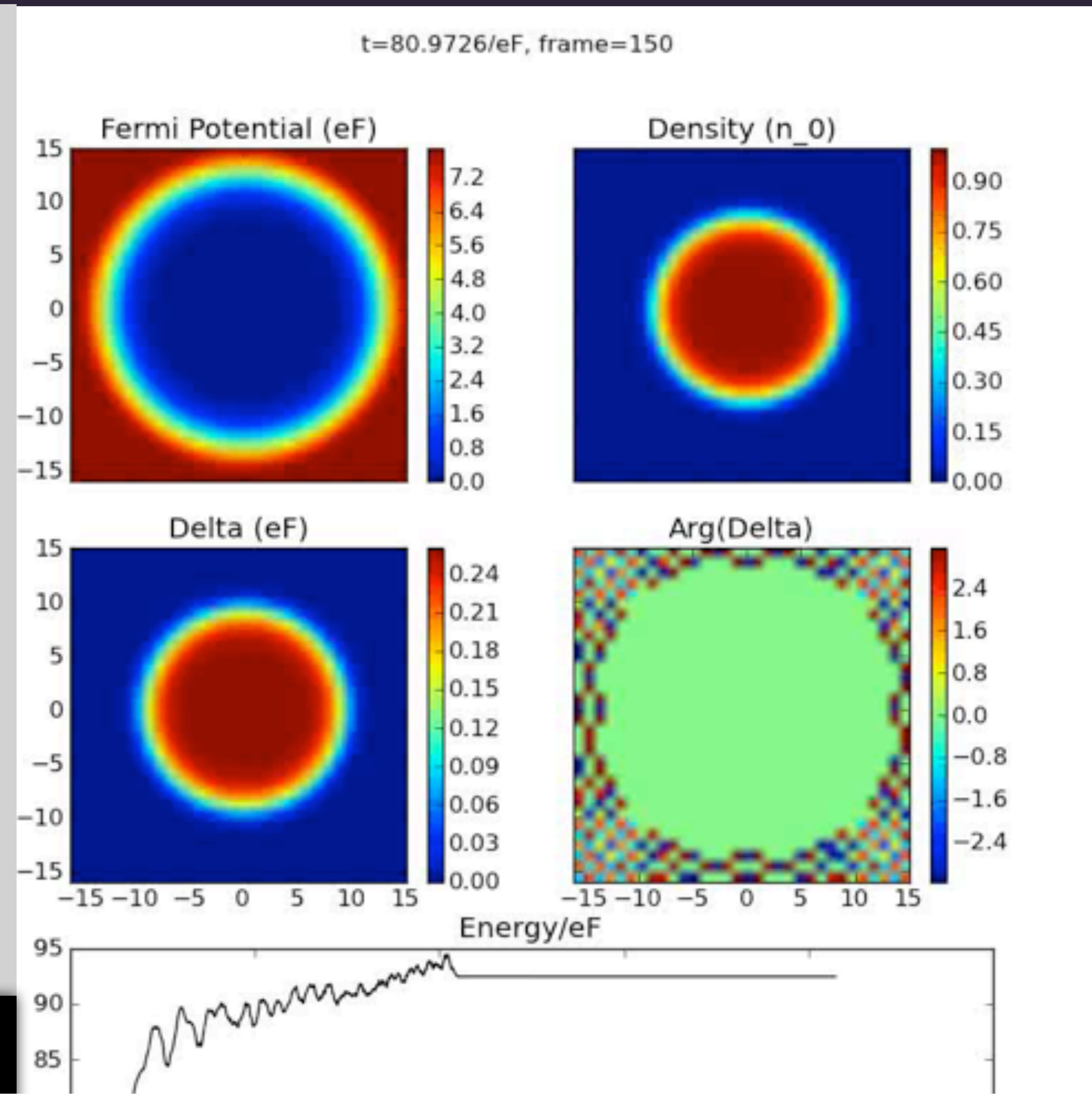
Comparison

Fermions
SLDA TDDFT

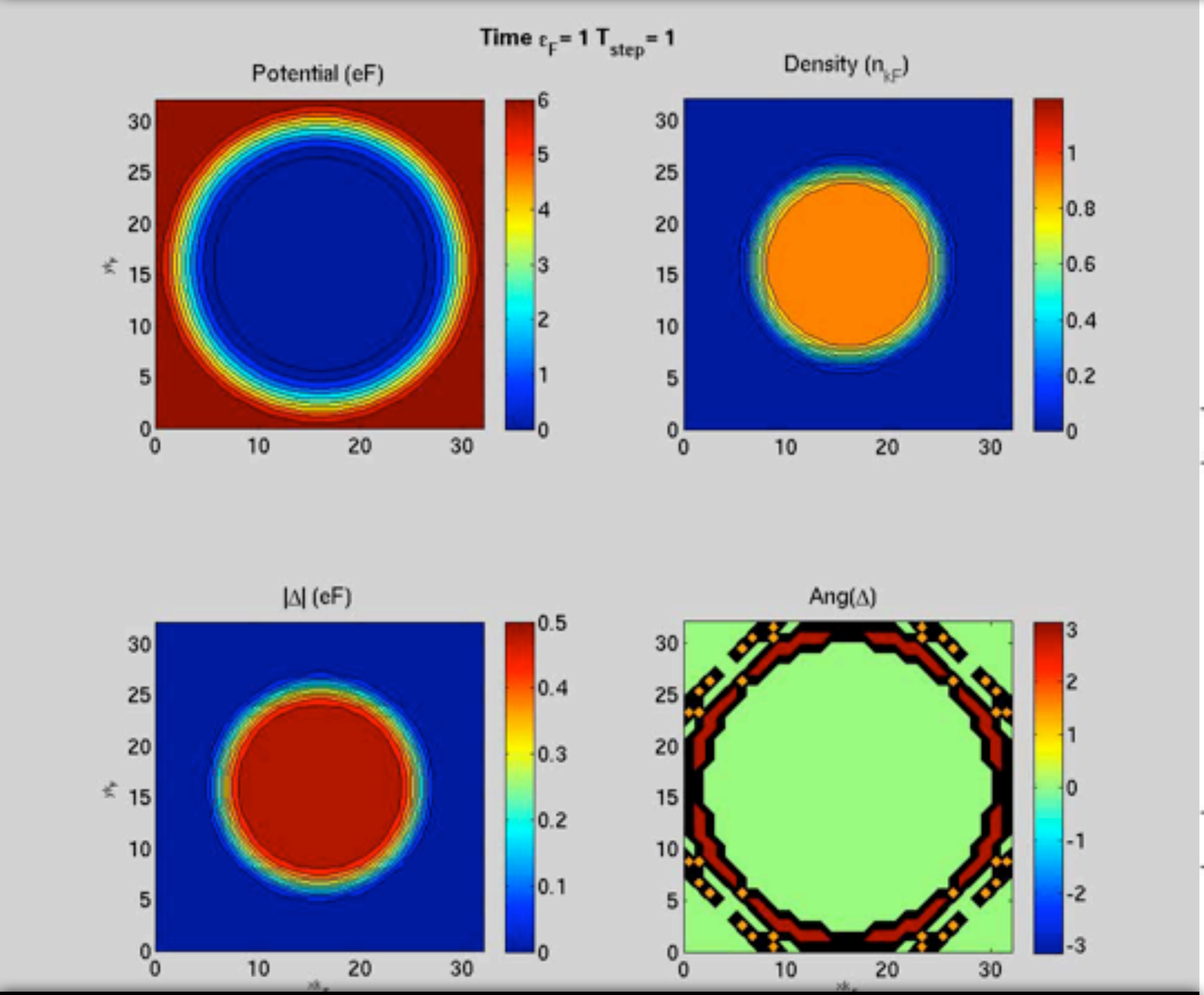
Gross Pitaevskii
model



Bulgac et al. (Science 2011)

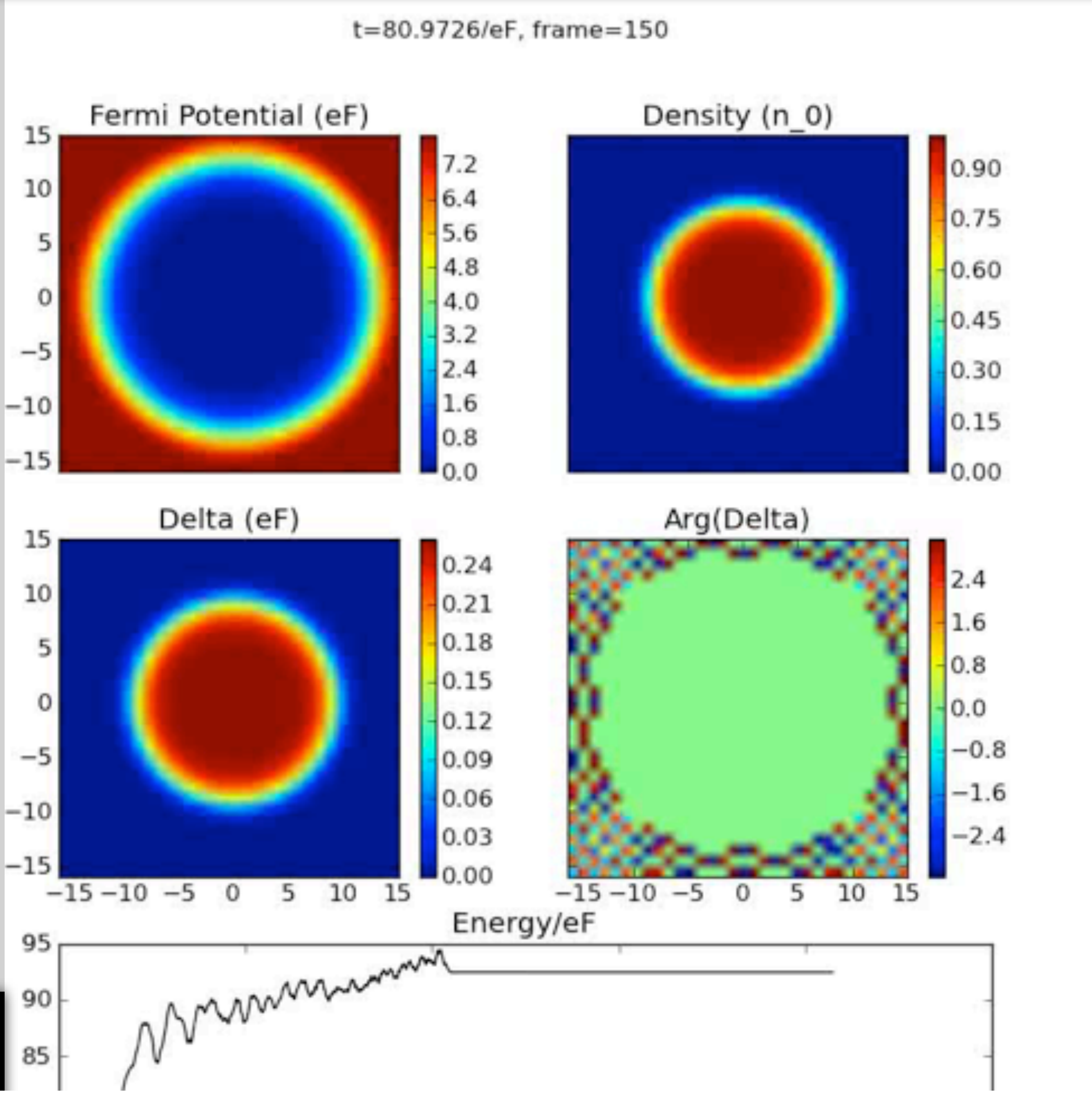


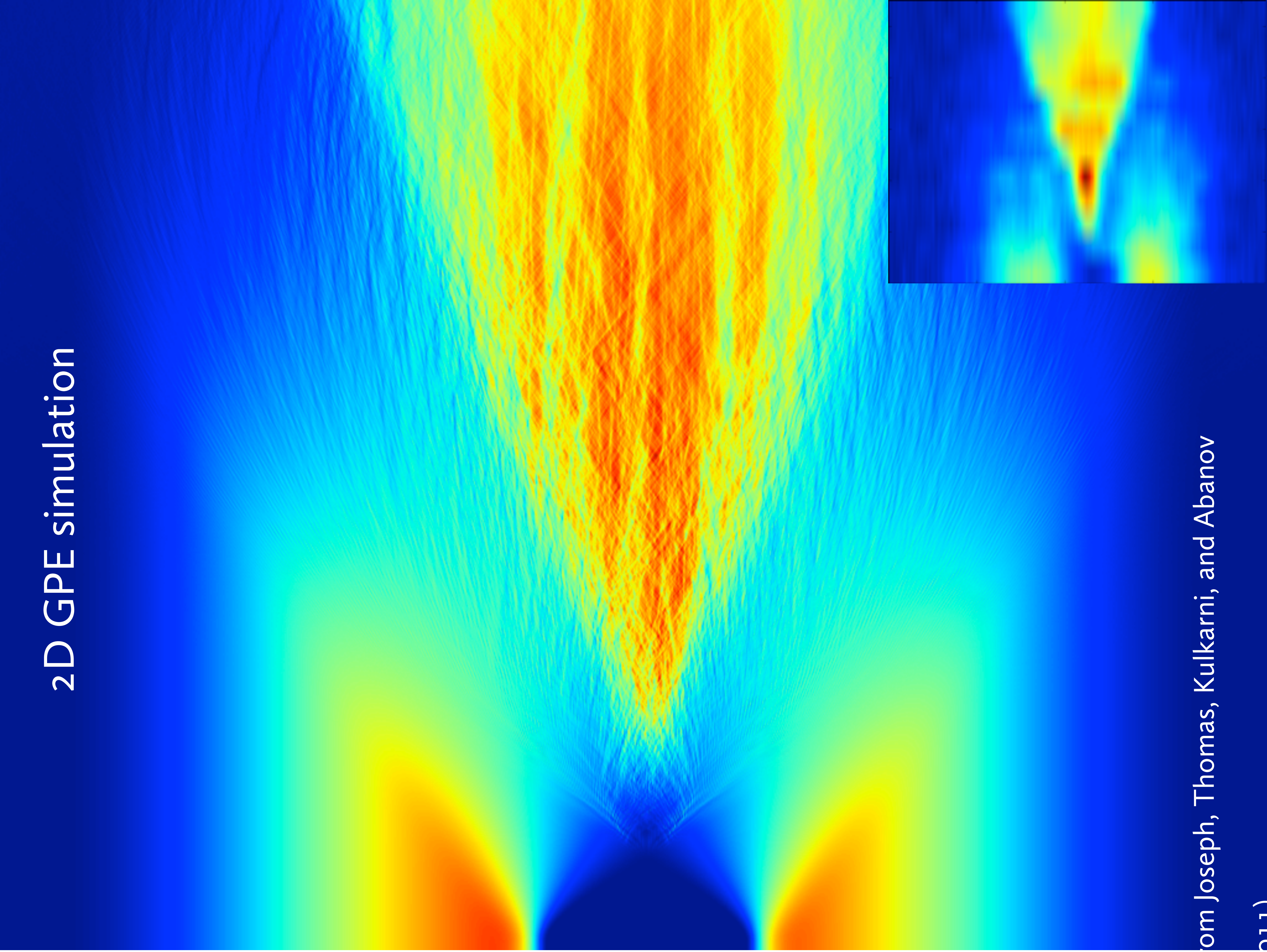
- Fermions:
  - Simulation hard!
  - Evolve 10^4–10^6 wavefunctions
  - Requires supercomputers



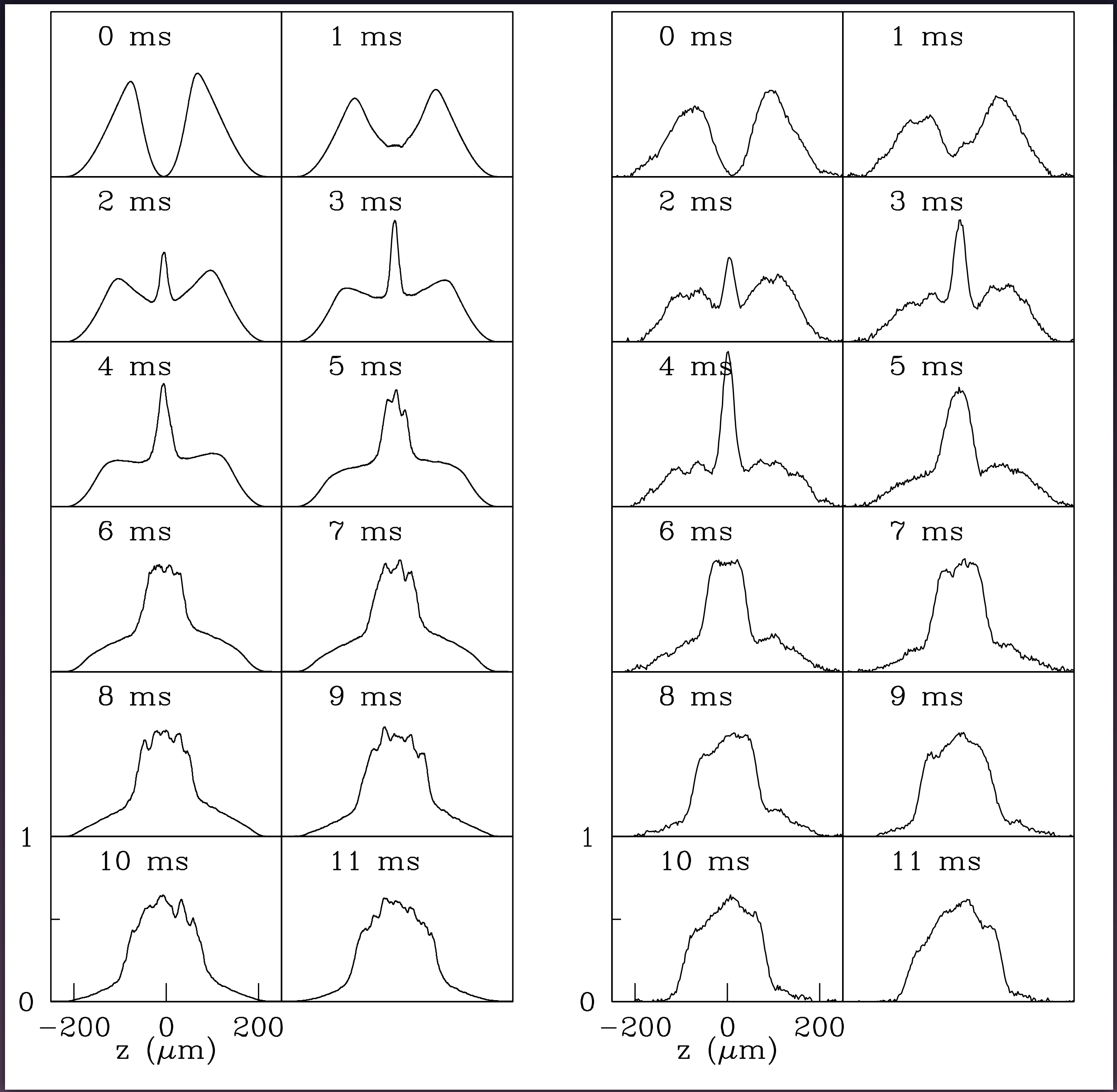
Bulgac et al. (Science 2011)

- GPE:
  - Simulation much easier!
  - Evolve 1 wavefunction
  - Use supercomputers to study large volumes




2D GPE simulation
om Joseph, Thomas, Kulkarni, and Abanov

GPE vs. Experiment



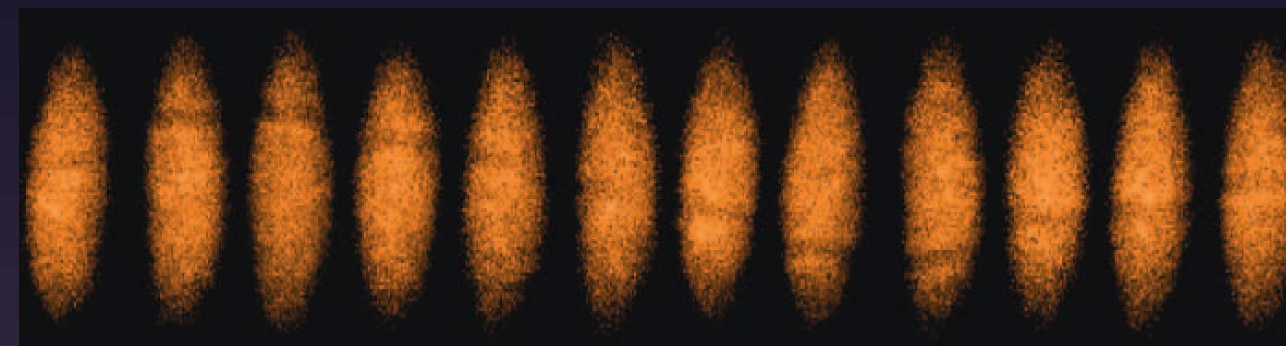
Ancilotto, L. Salasnich, and F. Toigo (2012)

From Cold Atoms to Neutron Stars

- Use (expensive) Fermi calculations to determine parameters (vortex nucleus interaction)
  - Validate with cold atoms
  - Time-dependent method scales well: Bulgac, Forbes and Sharma (2013)

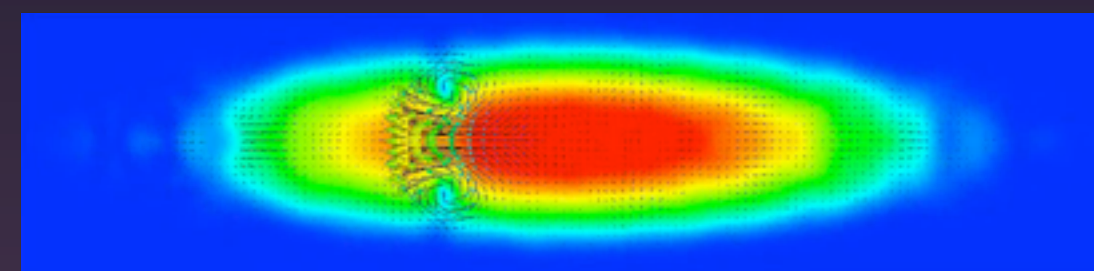
- Fit a GPE-like theory
  - Use this to model macroscopic dynamics

Vortices: an application



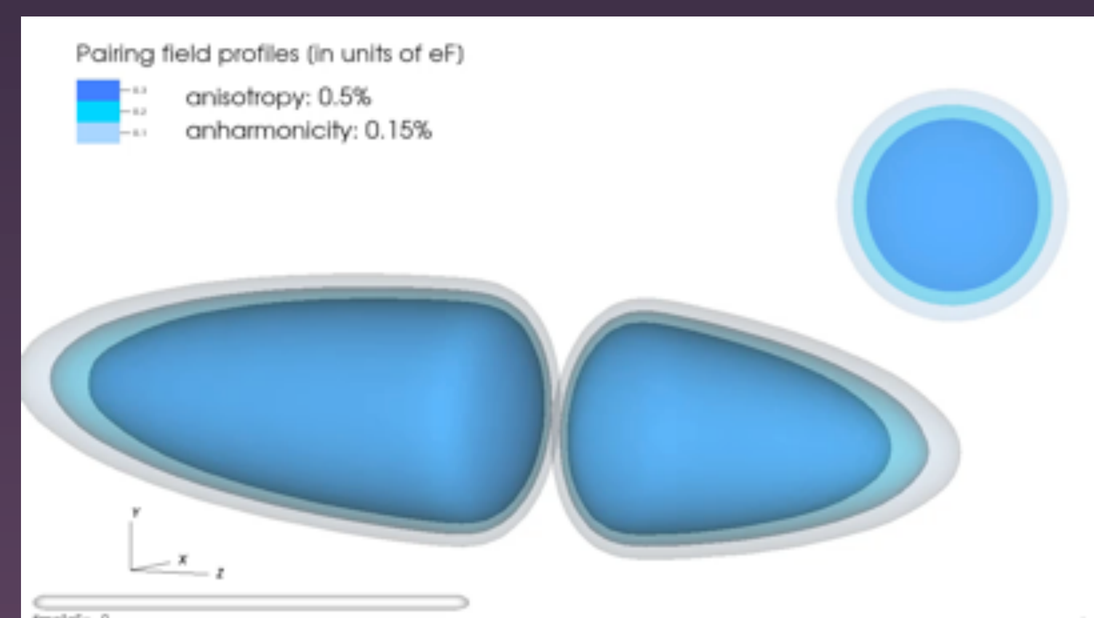
- Resolving a Mystery:
  MIT Heavy Solitons
  = Vortex Rings & Vortices

  Fermionic DFT for small systems
  validates bosonic model for realistic systems



- Vortex Reconnection

  Experimental realization of the
  mechanism behind quantum turbulence



- Apply to nuclear physics

  Time-dependent fission
  Pulsar glitches
  Quantum turbulence

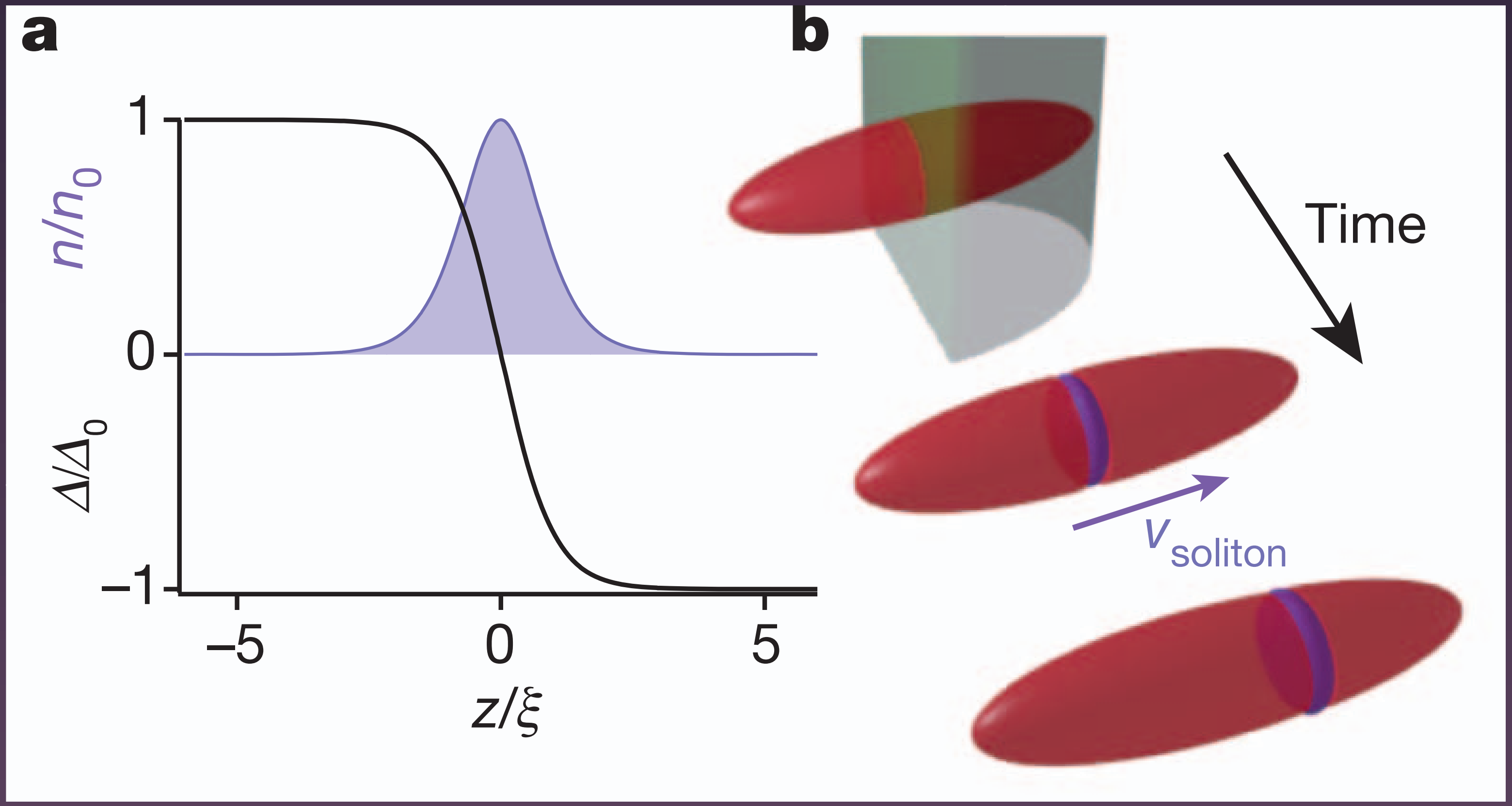
MIT Experiment

- ^{6}Li atoms ($N \approx 10^6$) cooled in harmonic trap
- Step potential used to imprint a soliton
- Let system evolve
- Image after ramping magnetic field B and expanding
- Observe an oscillating soliton with long period $T \approx 12T_z$
  - Bosonic solitons (BECS) oscillate with $T \approx \sqrt{2}T_z \approx 1.4T_z$
  - Fermionic solitons (BdG) oscillate with $T \approx 1.7T_z$
  - Interpret as "Heavy Solitons"
  - Later resolved as vortex rings and vortices

Yefsah et al. Nature 499 (2013) 426 [arXiv:1302.4736]
Ku et al. PRL 113 (2014) 065301

MIT Experiment

$$\hbar\partial_t(\delta\varphi) = \delta V$$ (phase difference on either side of trap)



Imprint soliton

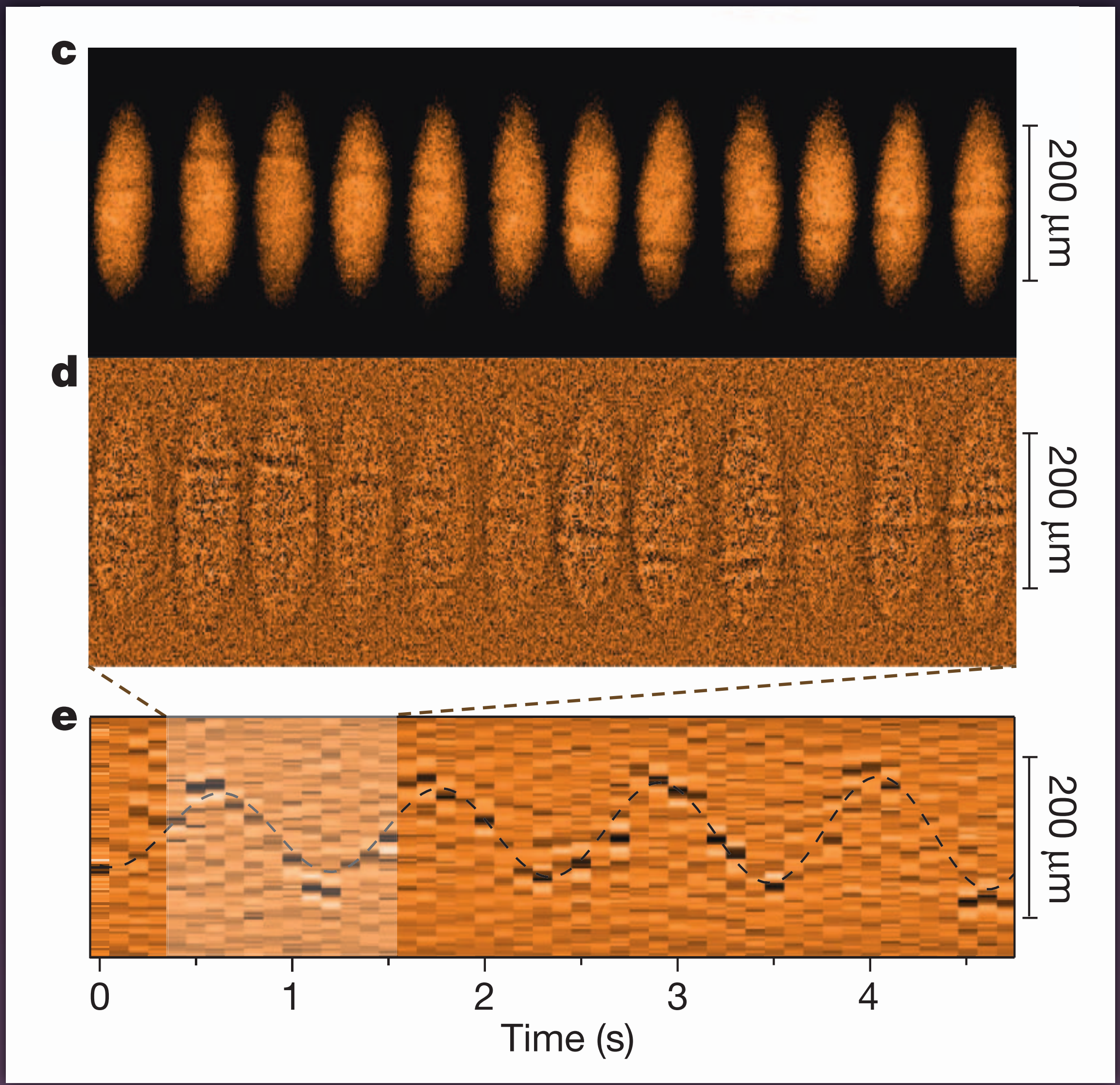
Step potential
phases evolve to
π phase shift

Flat domain wall
(dark/grey soliton)

Yefsah et al. Nature 499 (2013) 426 [arXiv:1302.4736]
Ku et al. PRL 113 (2014) 065301

MIT Experiment

(each image is a different run)



Yefsah et al. Nature 499 (426) 2013 [arXiv:1302.4736]

Thick solitons

- 10 × coherence length

Slowly moving

$T \approx 12 T_z$

Theory (Walls):

$T \sim 1.2–1.4 T_z$

Is theory wrong?

Density Functional Theory (DFT)

- Superfluid Local Density Approximation (SLDA)
  - Well tested for statical properties
  - Can we also use for dynamics
  - Expensive

    (one of the largest supercomputing calculations to date)
- Effective Thomas-Fermi (ETF) model
  - "Bosonic model" (GPE with correct EOS)
  - Not as reliable, but can be scaled up

Vortex Rings in a Trap

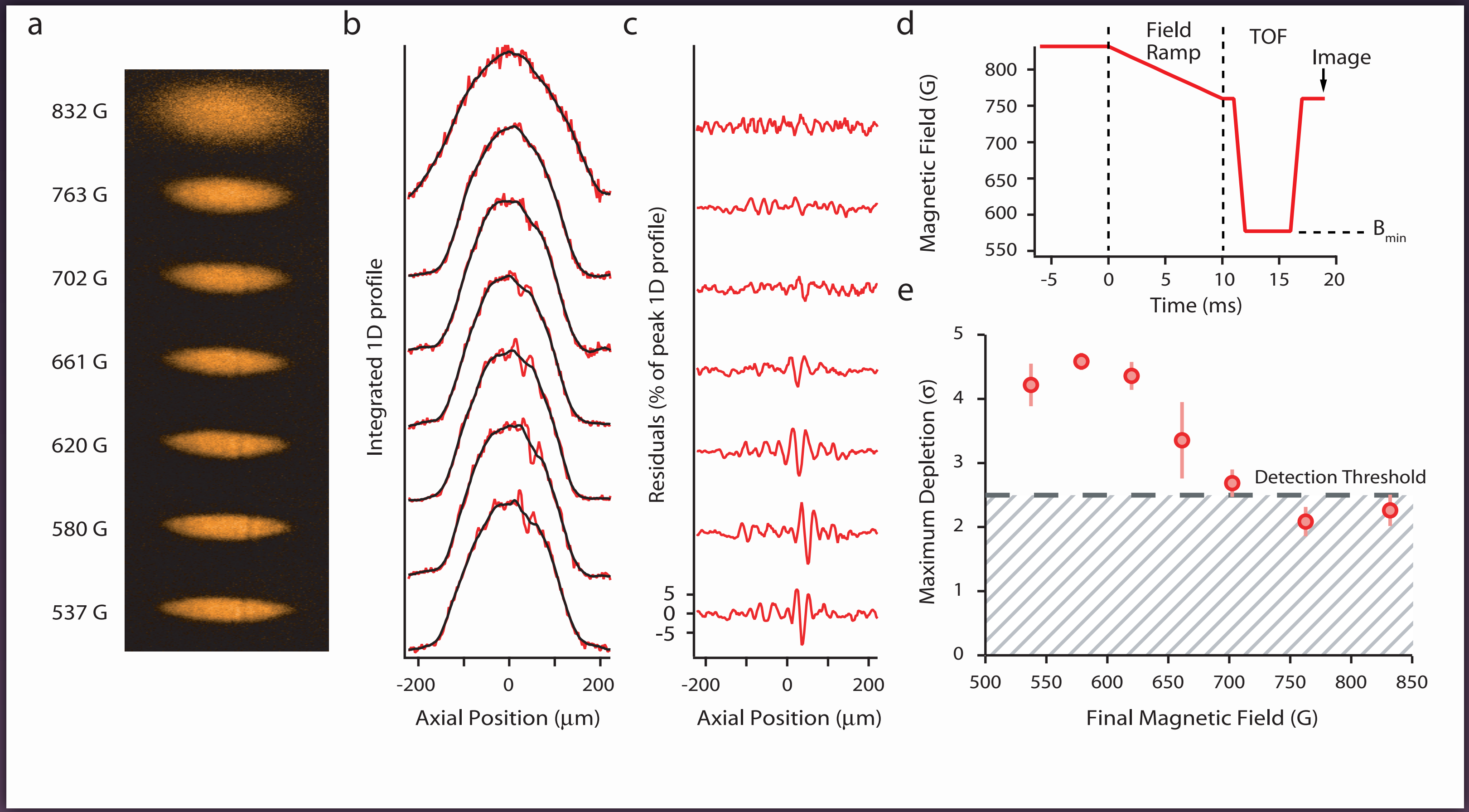
$$M_I = \frac{F}{\dot{v}} \sim 8\pi^2 mnR^3 \left(\ln \frac{R}{l_{coh}} \right)^{-1}$$

$$M_{VR} = mN_{VR} \sim mn\, 2\pi R\, \pi l_{coh}^2$$

- M_I: Inertial (kinetic mass) differs significantly from
- M_{VR}: Mass depletion
- Long periods

$$\frac{T}{T_z} \sim \sqrt{\frac{M_I}{M_{VR}}} \sim \frac{2R/l_{coh}}{\sqrt{\ln(R/l_{coh})}}$$

MIT Experiment

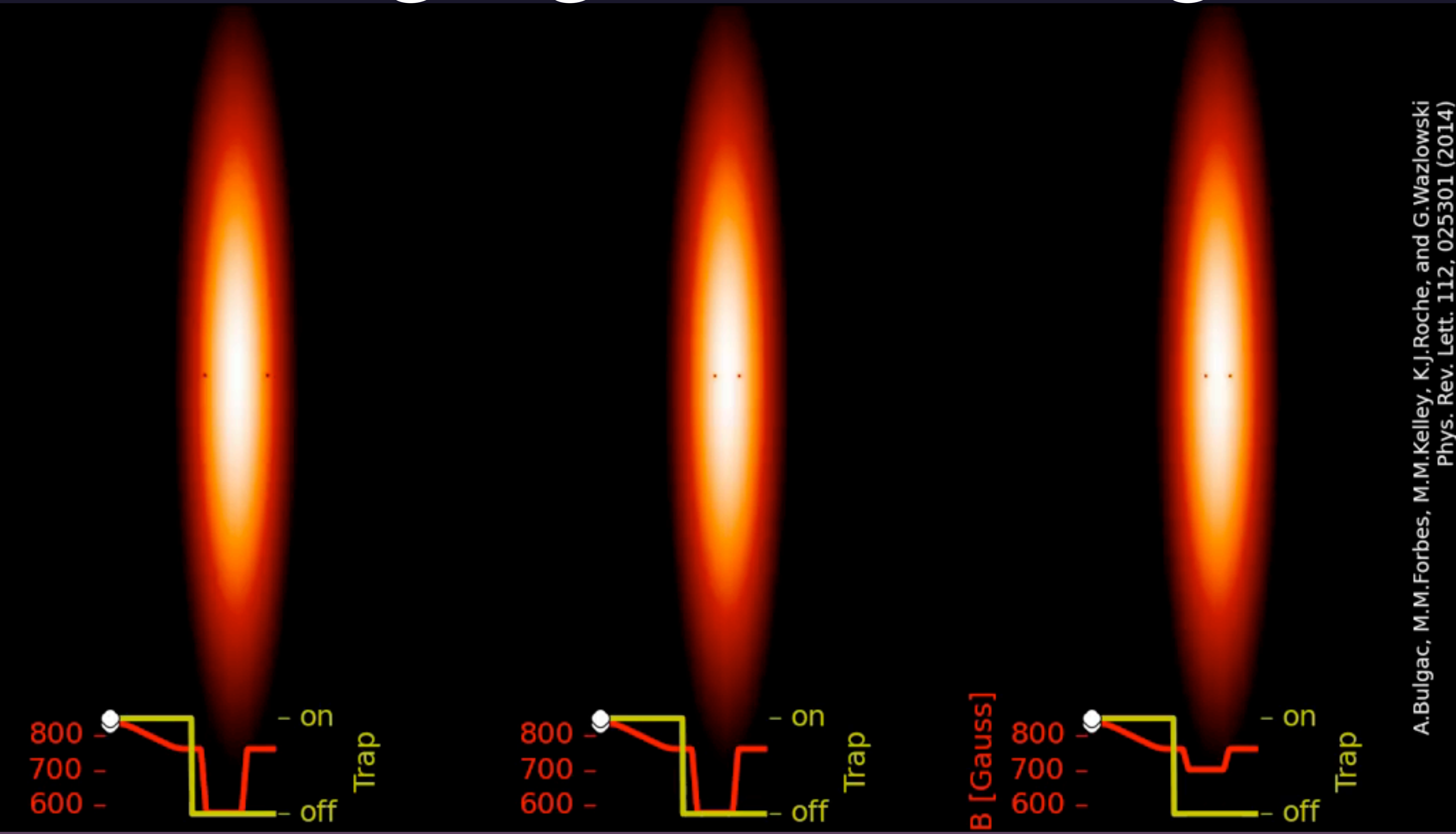


Subtle imaging:

- Need expansion (turn off trap)
- Must ramp to B<700G
- ~10% depletion

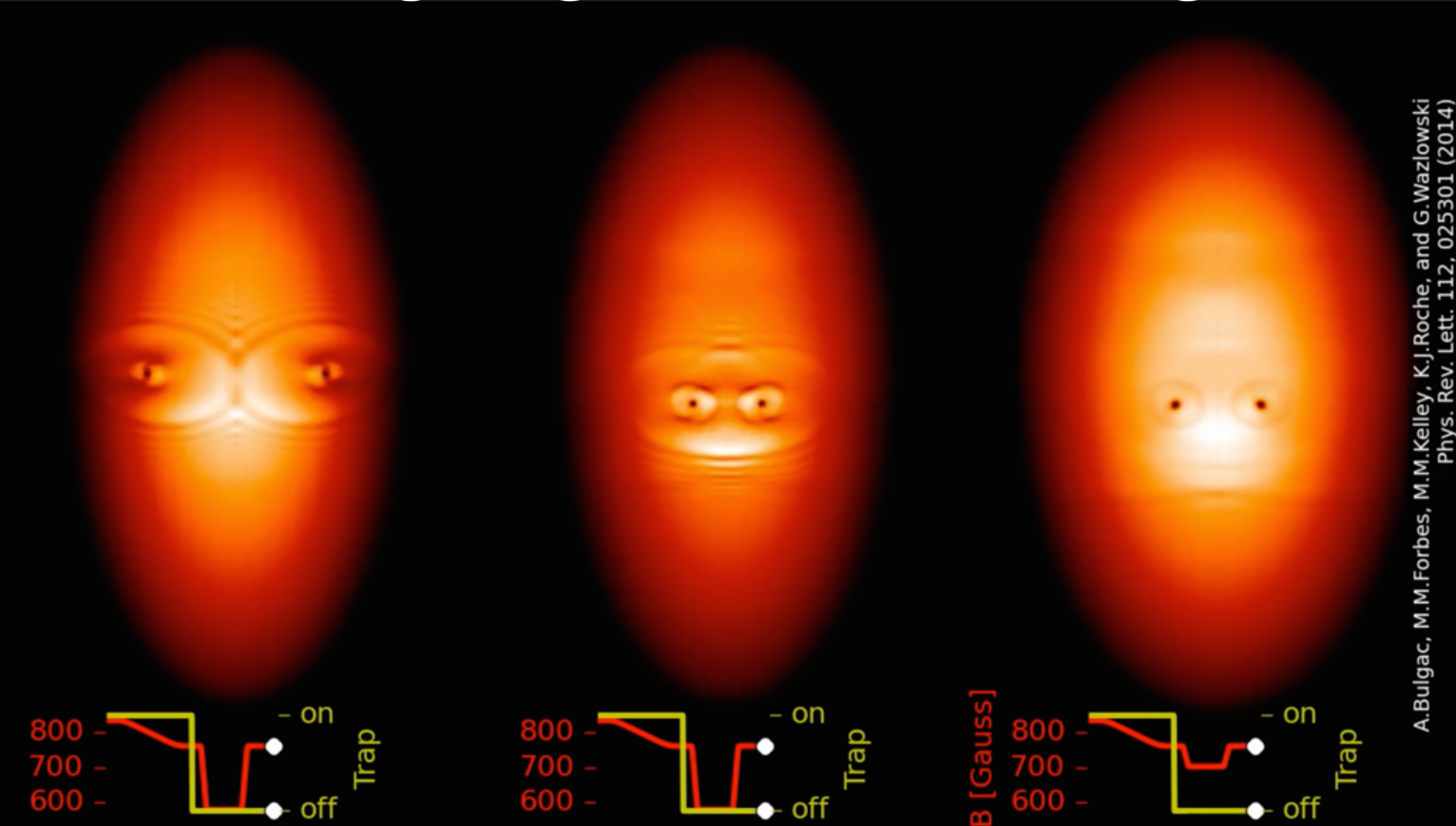
Yefsah et al. Nature 499 (426) 2013 [arXiv:1302.4736]

Imaging Vortex Rings



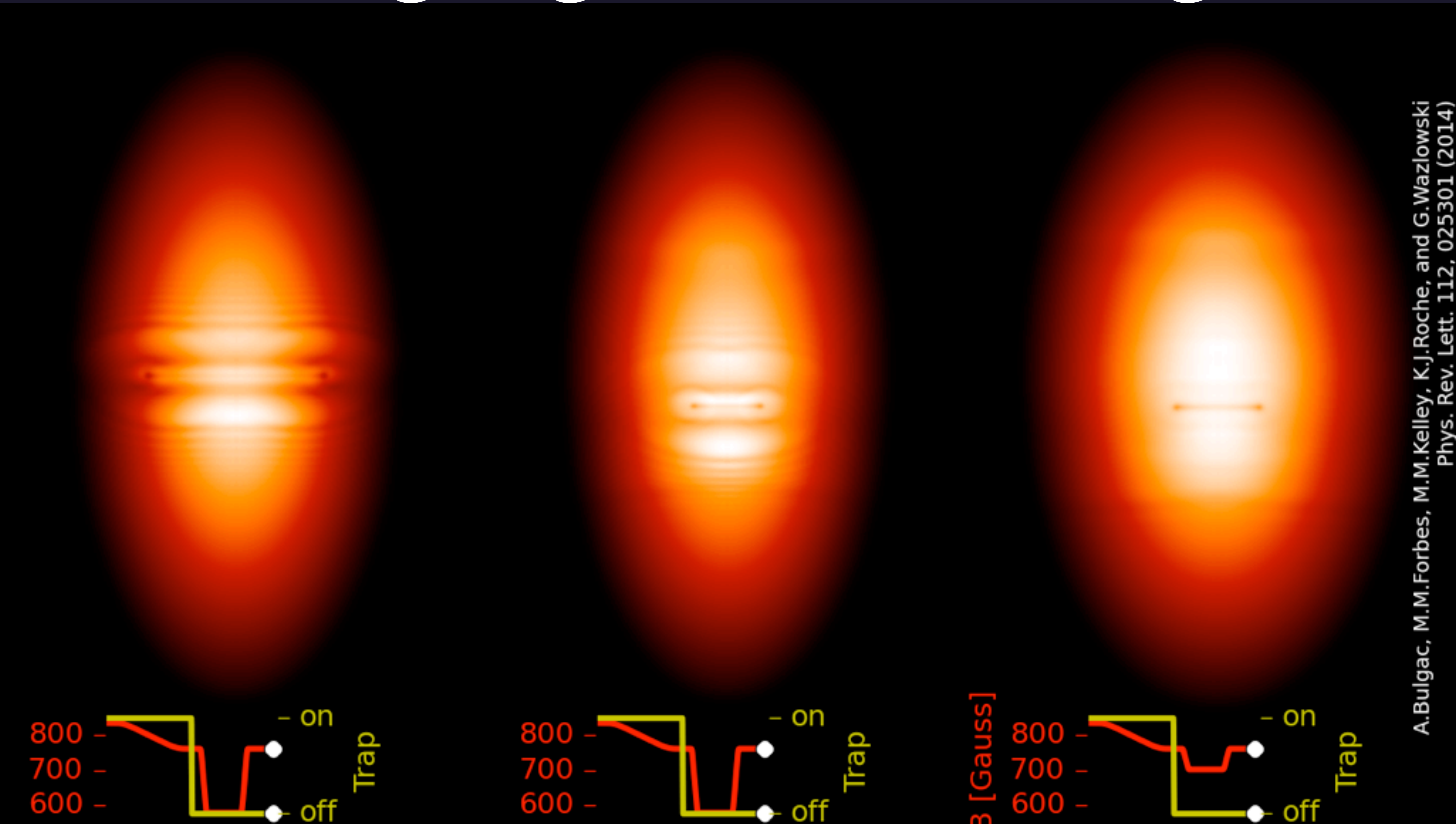
Bulgac, Forbes, Kelley, Roche, Wlazłowski (2013) [arXiv:1306.4266]

Imaging Vortex Rings



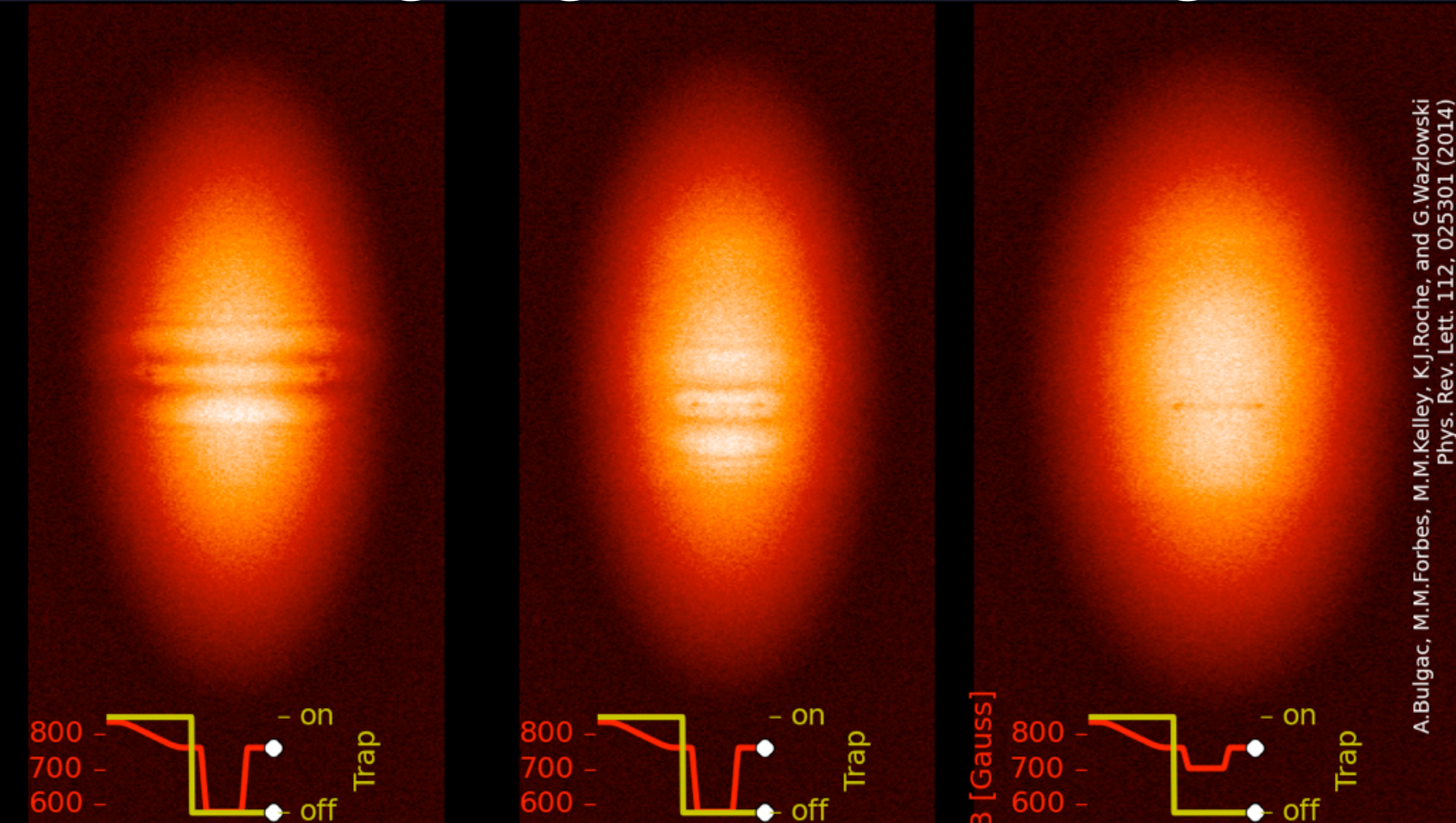
Bulgac, Forbes, Kelley, Roche, Wlazłowski (2013) [arXiv:1306.4266]

Imaging Vortex Rings



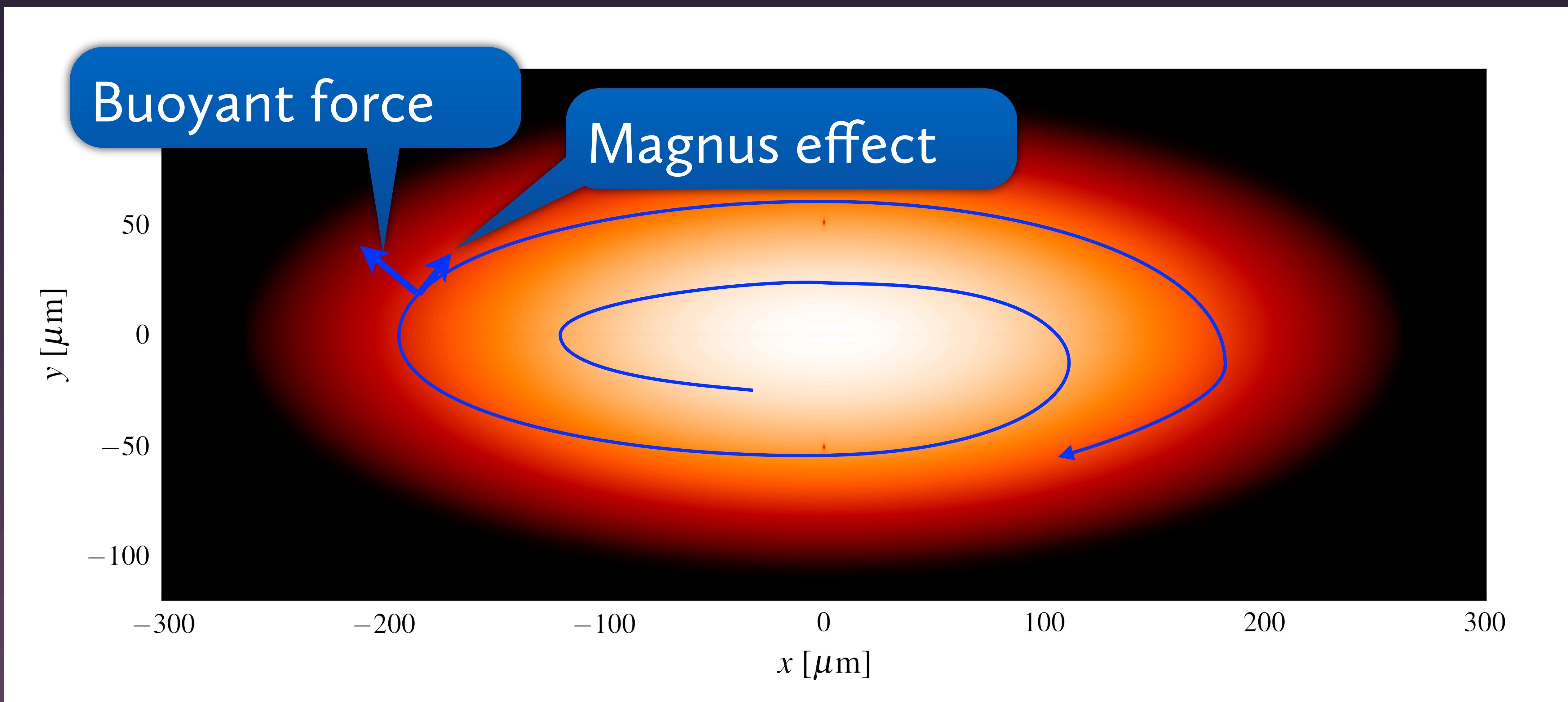
Bulgac, Forbes, Kelley, Roche, Wlazłowski (2013) [arXiv:1306.4266]

Imaging Vortex Rings

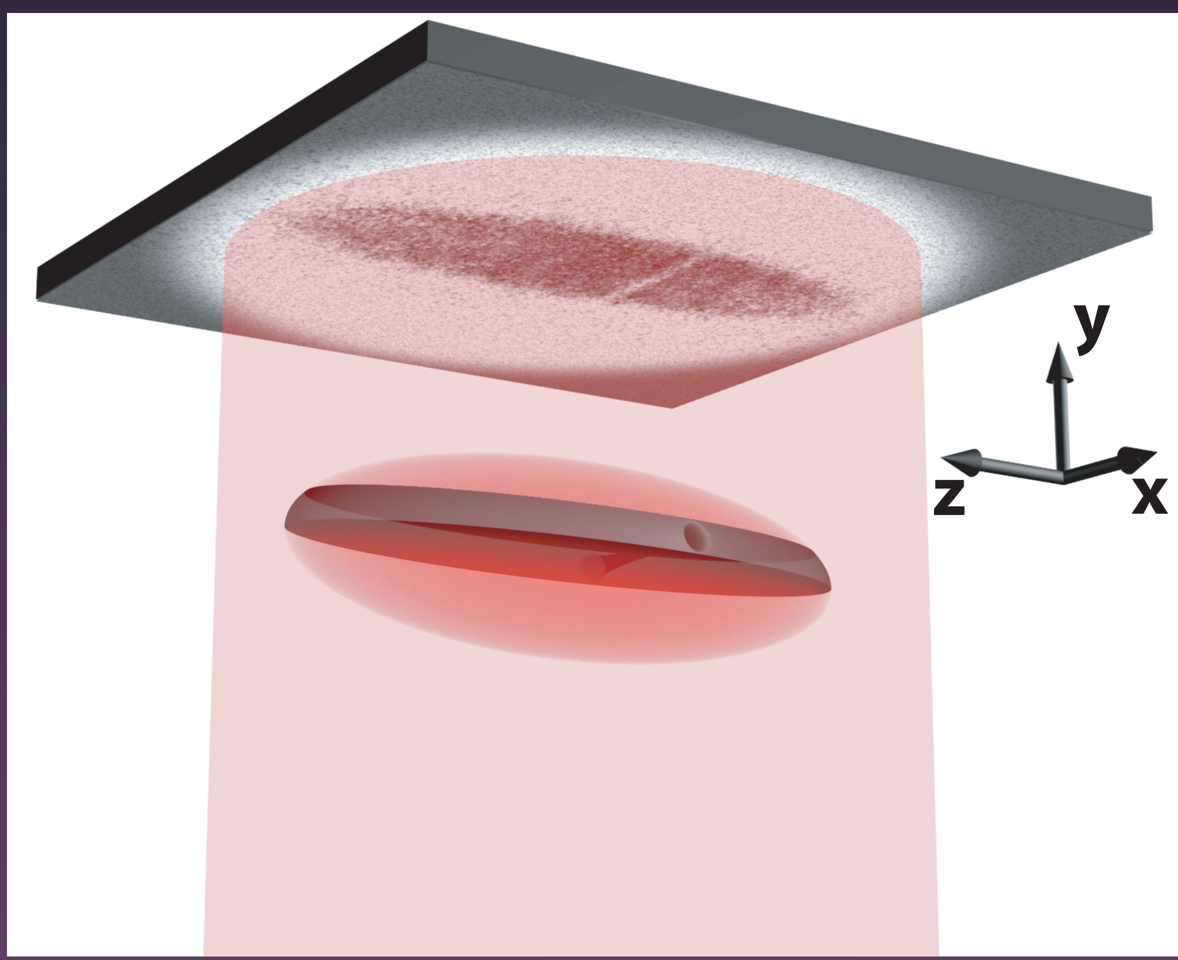


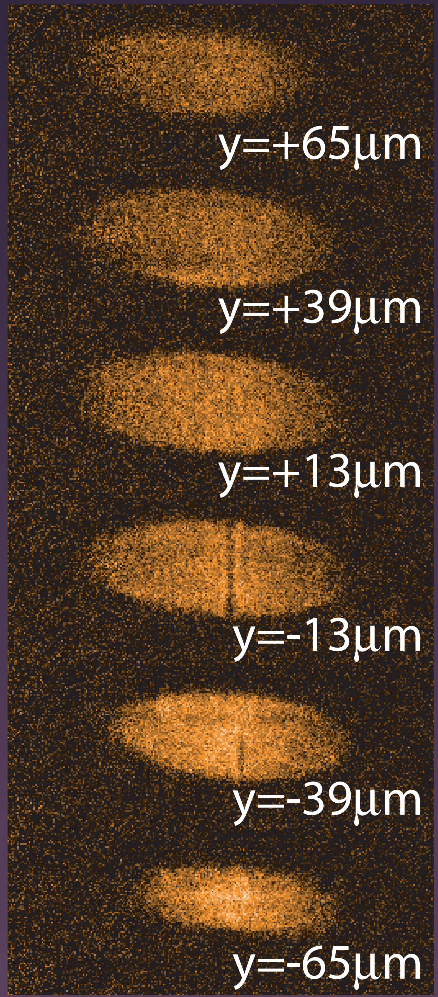
Bulgac, Forbes, Kelley, Roche, Wlazłowski (2013) [arXiv:1306.4266]

Vortex Motion



Experiment MIT 2014
No Axial Symmetry!





Better tomographic imaging reveals vortex

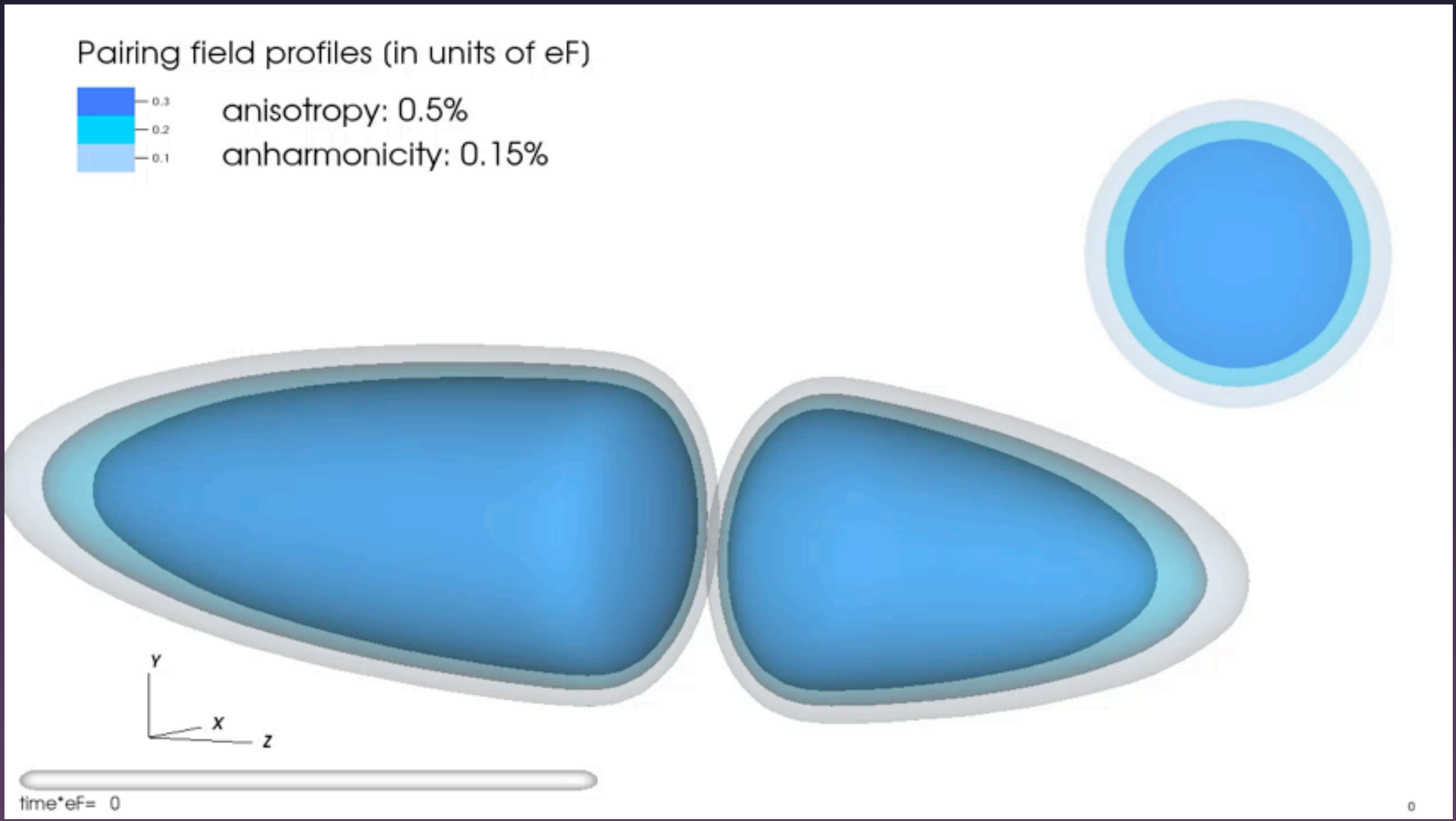
Gravity breaks trap asymmetry

Only imaged in one direction

Width consistent with a vortex core ~ l_{coh}

Ku et al. PRL 113 (2014) 065301

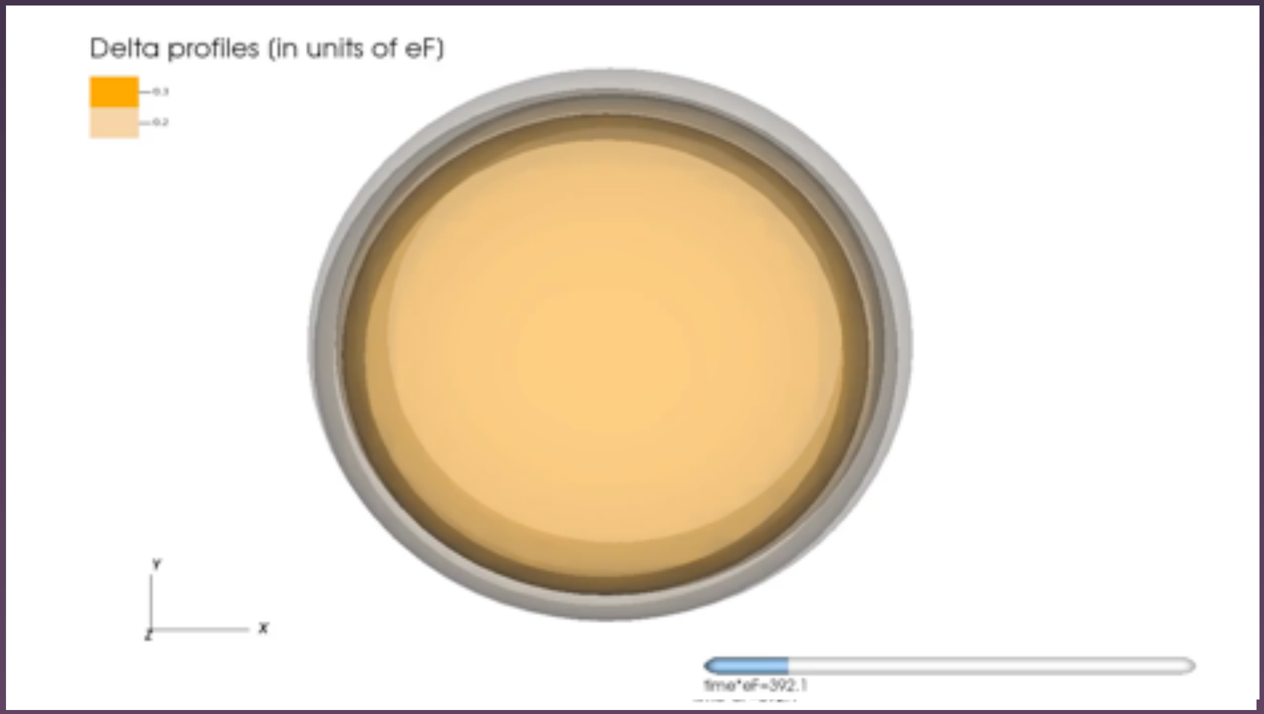
Wall, Ring, Vortex



Wlazłowski, Bulgac, Forbes, and Roche [arXiv:1404.1038]

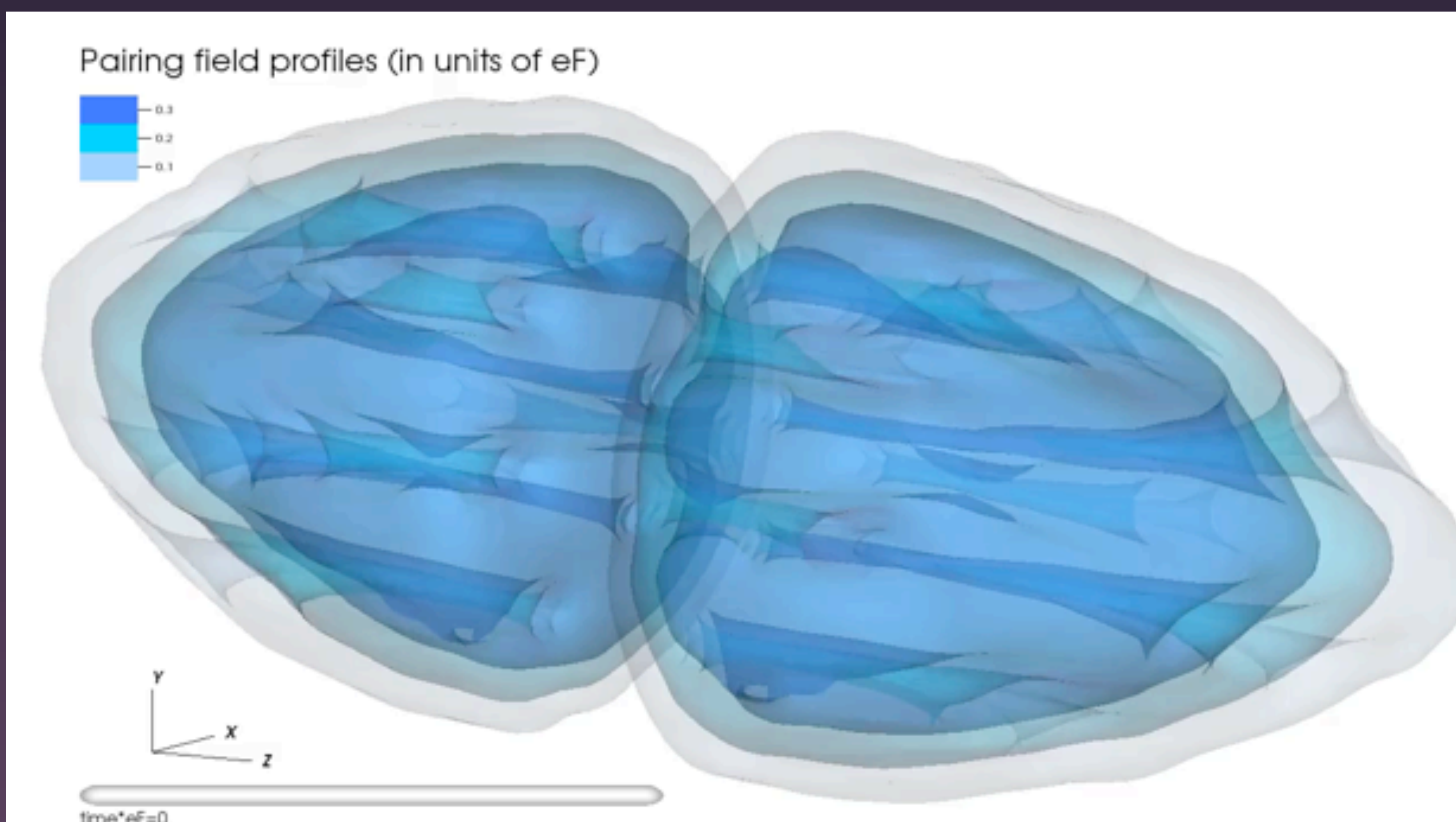
Vortex Reconnection
Quantum Turbulence

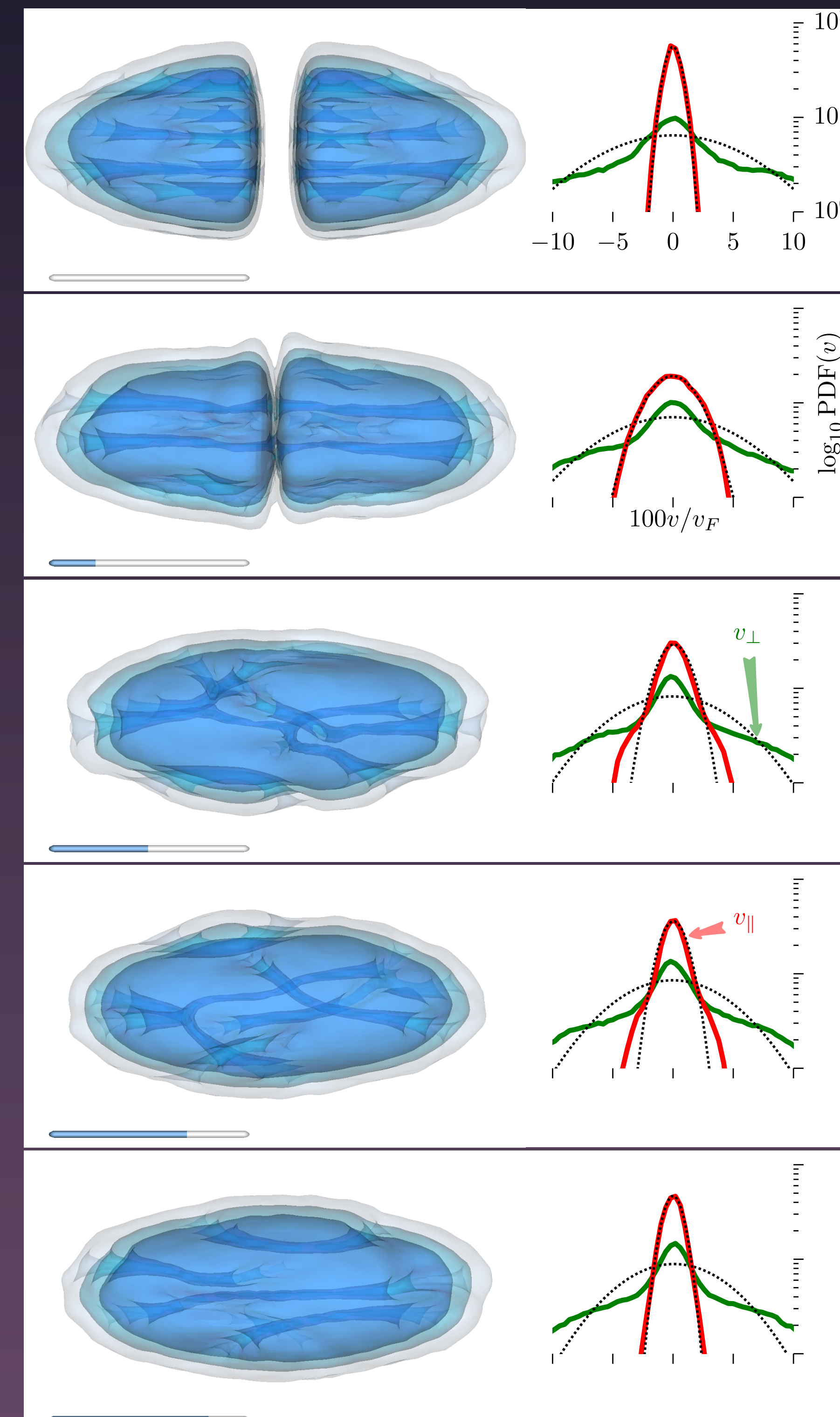
- Vortex reconnection: the origin of quantum turbulence
  - Feynman 1955
  - Very few experimental realizations



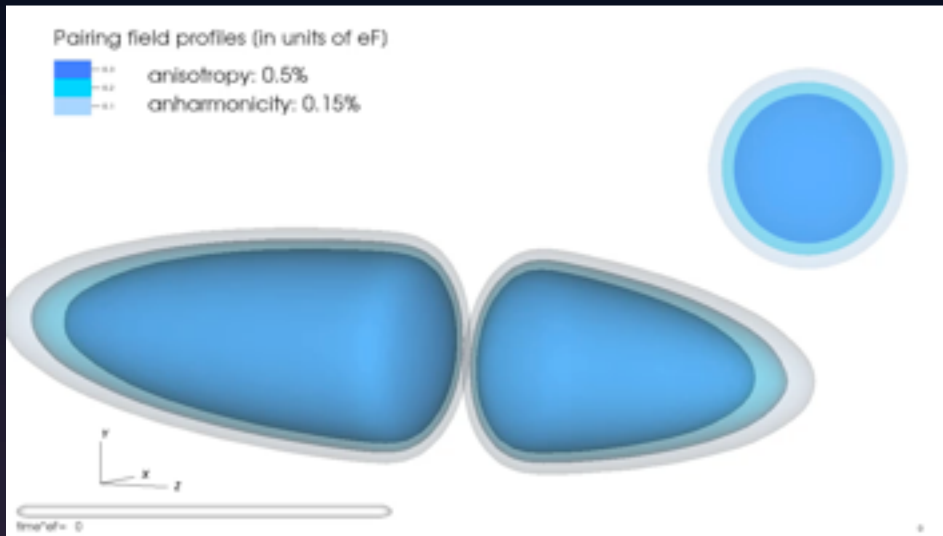
Paoletti, Fisher, Sreenivasan, and Lathrop, PRL 101, 154501 (2008)

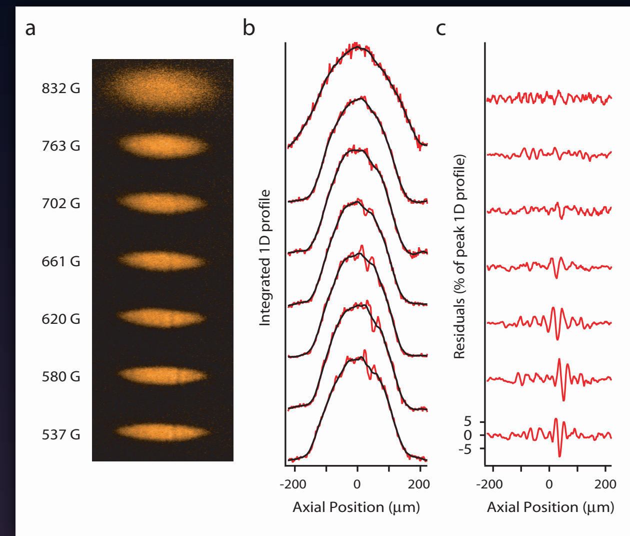
Quantum Turbulence with Fermions

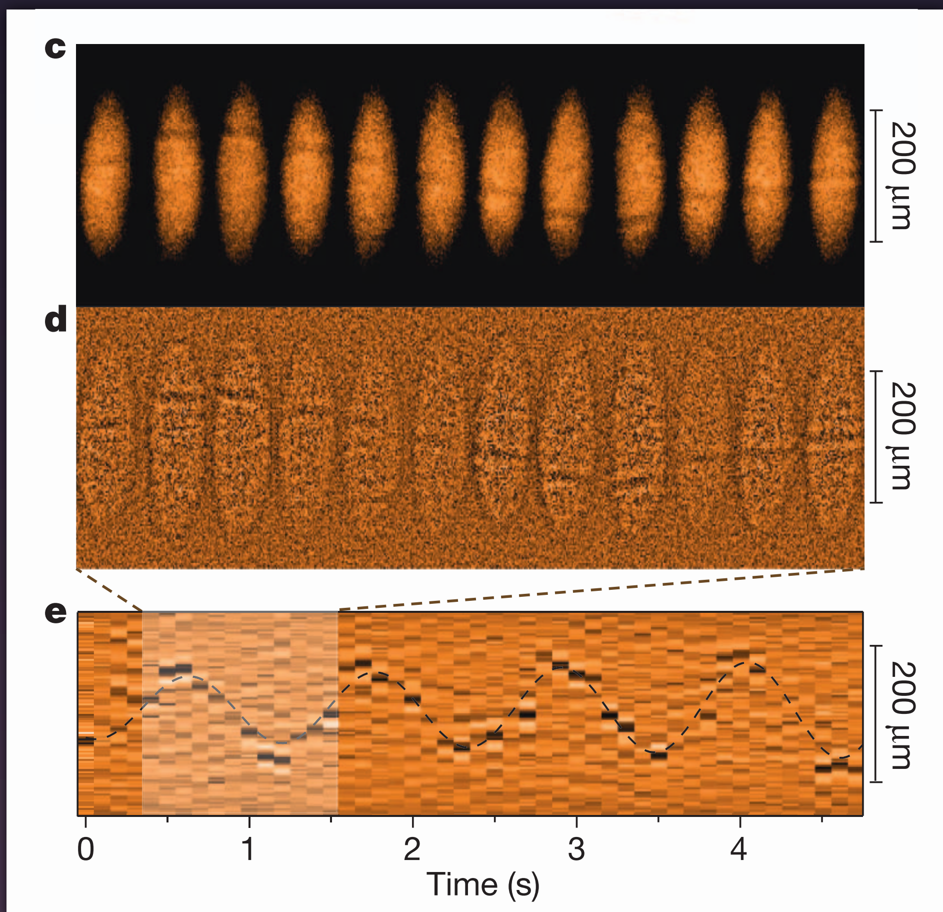


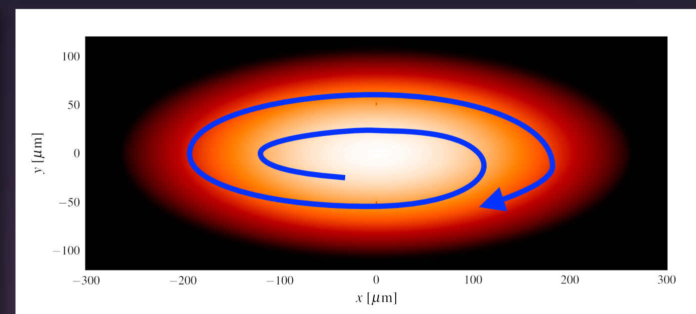


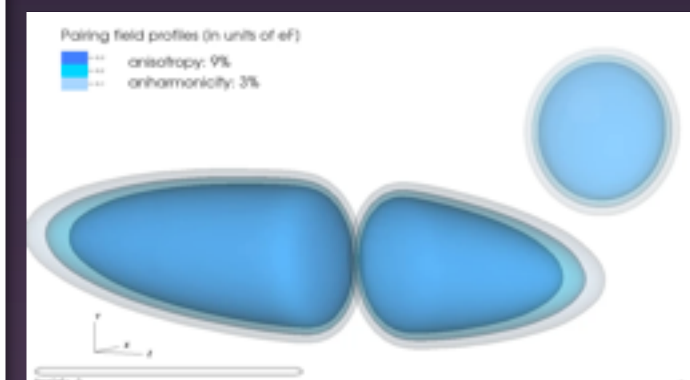
Wlazłowski, Bulgac, Forbes, and Roche [arXiv:1404.1038]

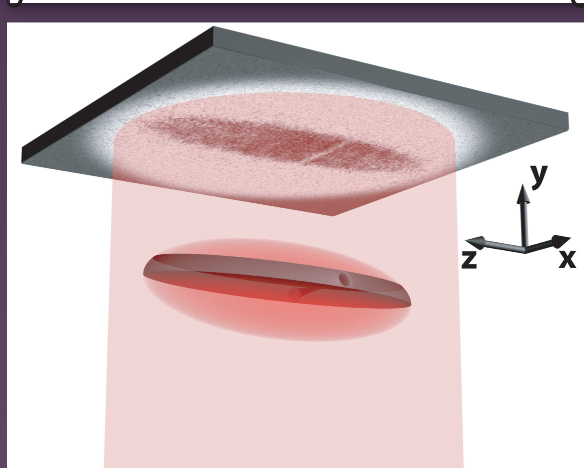


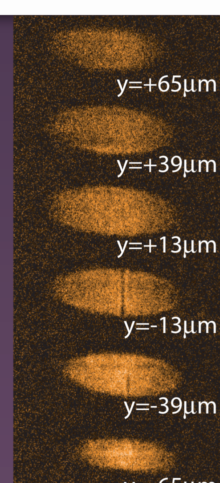






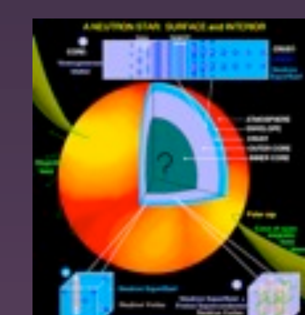






Solitons? Vortices!

- MIT sees vortices
  - Long periods
  - Dependence on aspect ratio and interaction
  - Imaging limitations
- Validates DFTS
  - Nuclear dynamics
  - Neutron stars



- New arena to study Quantum Turbulence

Conclusion

- Cold Atoms
  - Interaction understood and controllable
  - Test many-body part of theory

    Static and dynamical theories, quantitative tests on percent level
  - Directly simulate physics of interest

    Vortex dynamics, quantum turbulence, few-body interaction and resonance
  - Quantum simulators?

    Can we simulate gauge theories? (Try to simulate lattice models)

    Boselet and Femilet models of nuclei?

- Realtime Dynamics

  Powerful tool, scales well, probes interesting physics

  Hydrodynamical models of nuclei? (DNP talk on Friday)

Future Work?

- Finite T
- Polarized systems
- Large scale dynamics
- Applications
- Stochastic DFT