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Ab-initio study of nuclear four-body reactions

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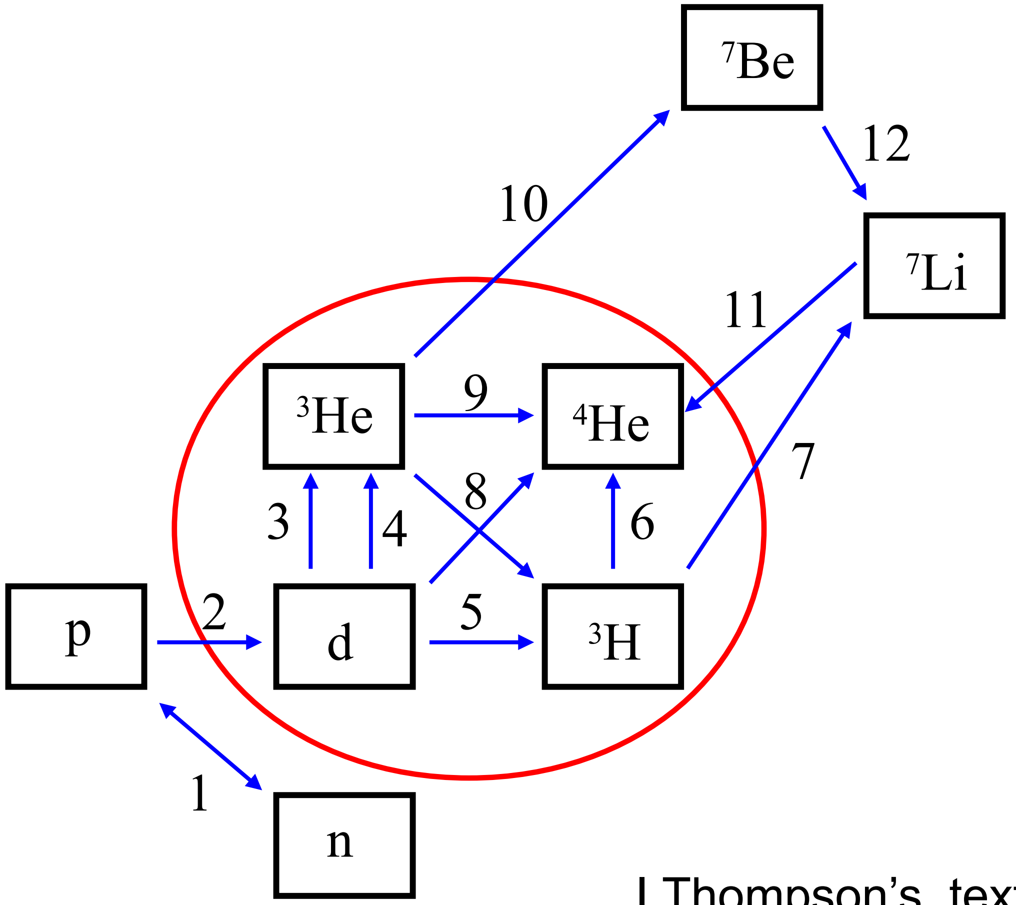
Corrlated Gaussian + Microscopic R-matrix Method

P. Descouvemont, D. Baye, Y. Suzuki, S. Aoyama, K. Arai, AIP ADVANCES, (2014)041011.
S. Aoyama, K. Arai, Y. Suzuki, P. Descouvemont, D. Baye, FBS52, (2012)97.
K. Arai, S. Aoyama, Y. Suzuki, P. Descouvemont, D. Baye, PRL107 (2011) 132502.

Dominant reactions in primordial nucleosynthesis

Normally, the primordial nucleosyntesis is explained by the reaction chain calculation which is based on a simple nuclear model or a mere extrapolation from experiments.



I.Thompson's textbook

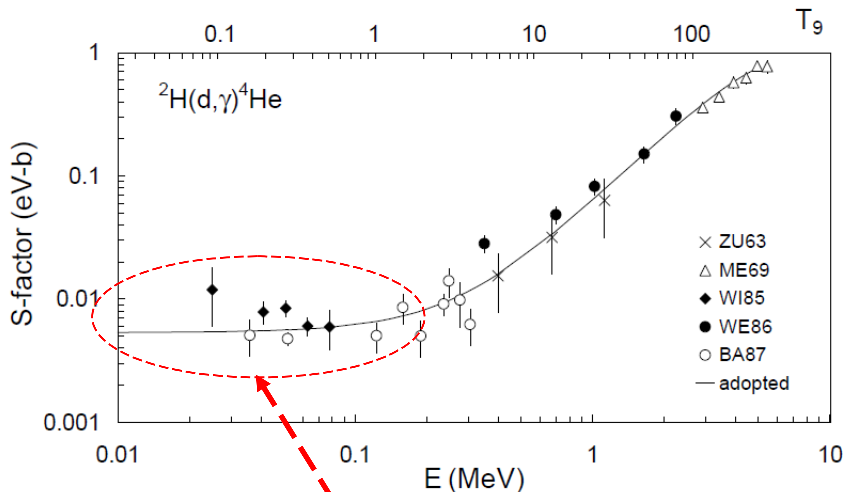
1: $n \leftrightarrow p$
2: $p(n,\gamma)d$
3: $d(p,\gamma)^3\mathrm{He}$
4: $d(d,n)^3\mathrm{He}$
5: $d(d,p)^3\mathrm{H}$
6: $^3\mathrm{H}(d,n)^4\mathrm{He}$
7: $^3\mathrm{H}(^4\mathrm{He},\gamma)^3\mathrm{H}$
8: $^3\mathrm{He}(n,p)^3\mathrm{H}$
9: $^3\mathrm{He}(d,p)^4\mathrm{He}$
10: $^3\mathrm{He}(^4\mathrm{He},\gamma)^7\mathrm{Be}$
11: $^7\mathrm{Li}(p,^4\mathrm{He})^4\mathrm{He}$
12: $^7\mathrm{Be}(n,p)^7\mathrm{Li}$

$d(d,\gamma)^4\mathrm{He}$

Can we understand these reactions in *ab-initio* way?

Is there some effects of tensor interaction in Big-Bang?

Effect of tensor interaction in low energy region

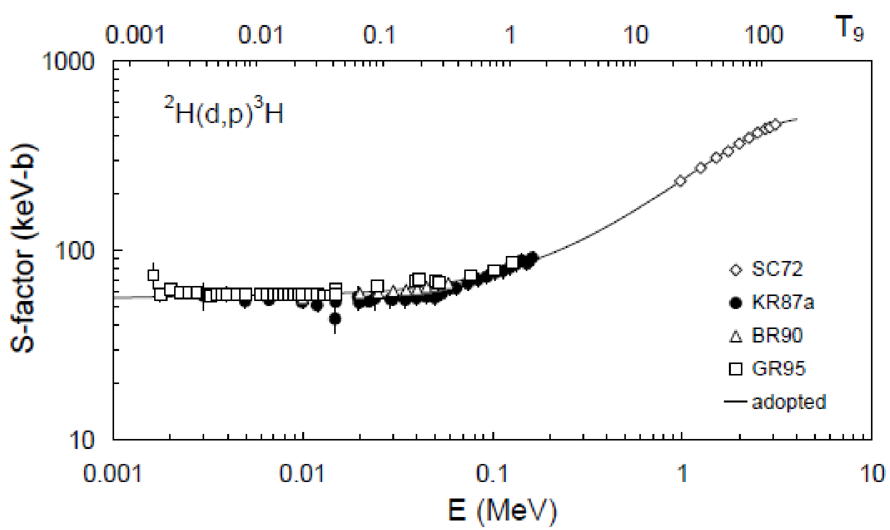


Nacre compilation
(C.Angulo et al.NPA656(1999)3)

^{2}H(d,γ)^{4}He Astrophysical S-factor

d +d S-wave → D-state in the 0^+ g.s. of ^{4}He

H. J. Assenbaum and K. Langanke, PRC36(1987)17



^{2}H(d, p)^{3}H, ^{2}H(d, n)^{3}He

Effect of Tensor Interaction?

Hamiltonian(4-body case)

$$H = \sum_{i=1}^{4} T_i - T_{\rm cm} + \sum_{i<j}^{4} V_{ij} + \sum_{i<j<k}^{4} V_{ijk},$$

Realistic Interaction: AV8' (+Coulomb+3NF)

V_{ij}: Central+LS+Tensor+Coulomb

Pudliner, Pandharipande, Carlson , Pieper, Wiringa: PRC56(1997)1720

V_{ijk}: Effective three nucleon force

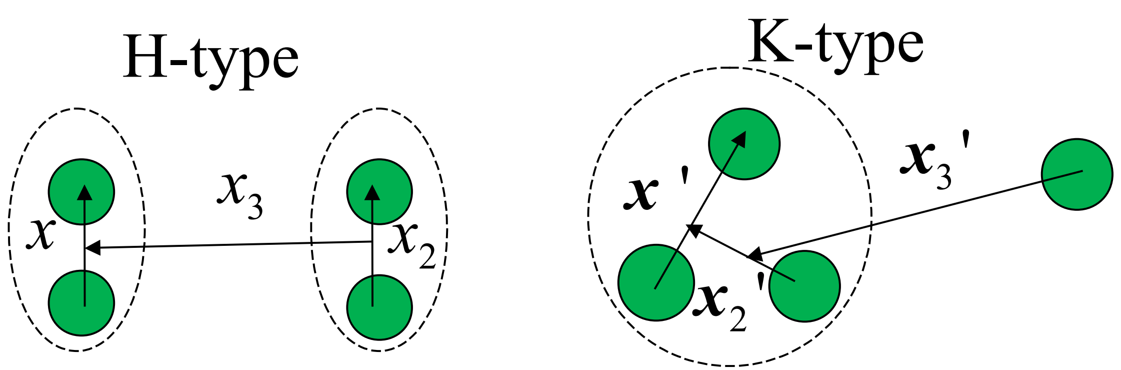
Hiyama, Gibson, Kamimura, PRC 70(2003)031001

Effective Interaction: MN (+Coulomb)

V_{ij}: Central+Coulomb

Thompson, LeMere, Tang, NPA(1977)286

Gaussian Basis Function



Correlated Gaussian function with triple global vectors for four nucleon system

Unnatural parity 0^-

L1=L2=L12=L3=1

$$F_{L_1L_2(L_{12})L_3LM}(u_1,u_2,u_3,A,x)$$
$$= \exp\left(-\frac{1}{2}\tilde{x}Ax\right)[[\mathcal{Y}_{L_1}(\widetilde{u_1}x)\mathcal{Y}_{L_2}(\widetilde{u_2}x)]_{L_{12}}\mathcal{Y}_{L_3}(\widetilde{u_3}x)]_{LM}$$

Single gloval vector

Double global vector

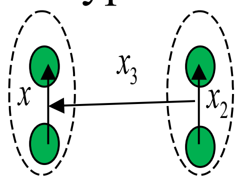
New extension(triple)

$$\mathcal{Y}_{L_iM_i}(\widetilde{u_i}x) = |\widetilde{u_i}x|^{L_i}Y_{L_iM_i}(\widehat{\widetilde{u_i}x}) \qquad \widetilde{u_i}x = \sum_{j=1}^{N-1}(u_i)_j x_j$$

For H-type, we can choose, $\widetilde{u_1}=(1,0,0)$, $\widetilde{u_2}=(0,1,0)$ and $\widetilde{u_3}=(0,0,1)$

We also write the K-type basis function in the same form.

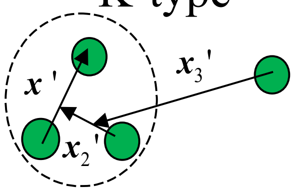
H-type



$$\exp\left(-\frac{1}{2}\widetilde{x'}A_Kx'\right)[[\mathcal{Y}_{L_1}(x'_1)\mathcal{Y}_{L_2}(x'_2)]_{L_{12}}\mathcal{Y}_{L_3}(x'_3)]_{LM}$$

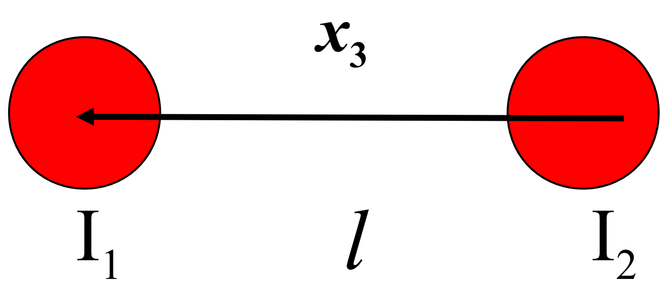
$$x' = U_{KH}x \qquad \widetilde{u_1}=(1,0,0),\ \widetilde{u_2}=(0,-\tfrac{1}{2},1)\text{ and }\widetilde{u_3}=(0,\tfrac{2}{3},\tfrac{2}{3})$$

K-type



$$A = (u_1u_2u_3)A_K\begin{pmatrix}\widetilde{u_1}\\ \widetilde{u_2}\\ \widetilde{u_3}\end{pmatrix} = \widetilde{U_{KH}}A_KU_{KH}$$

Microscopic R-matrix Method



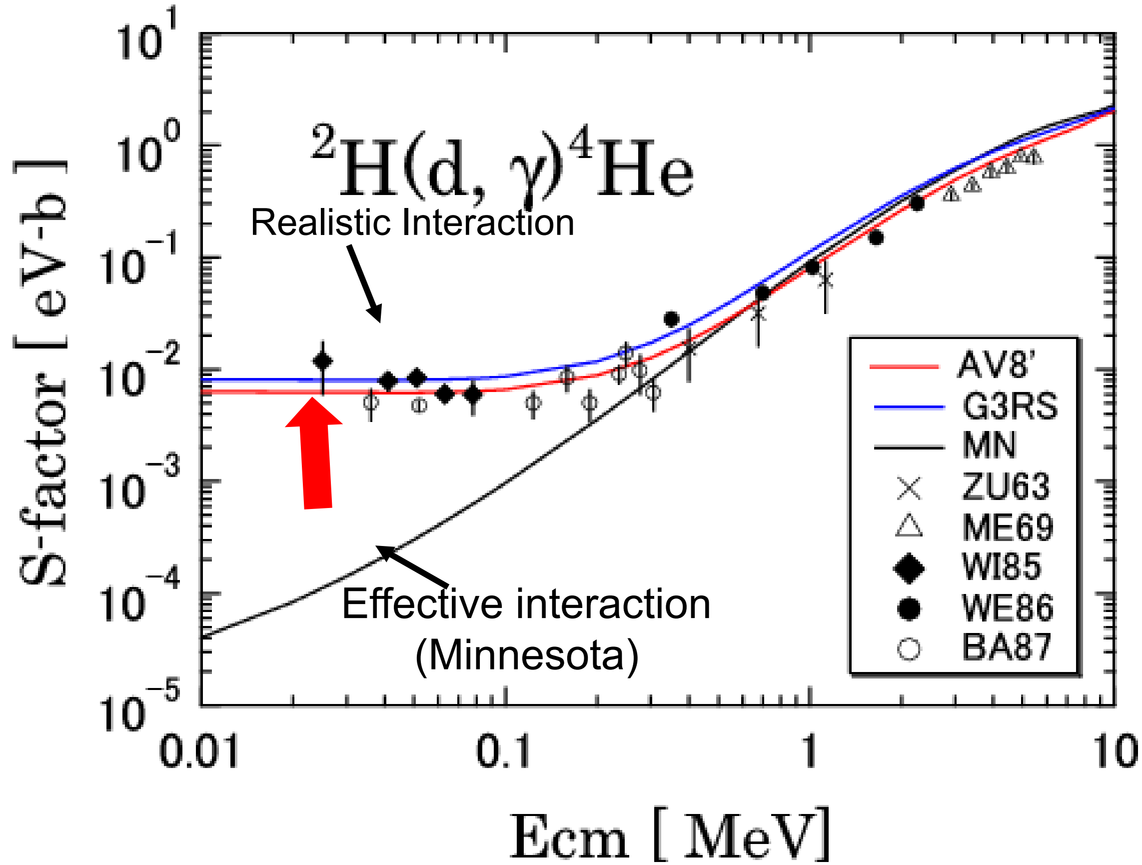
$$[[I_1\ I_2]_I\ l\ ]_{JM}$$

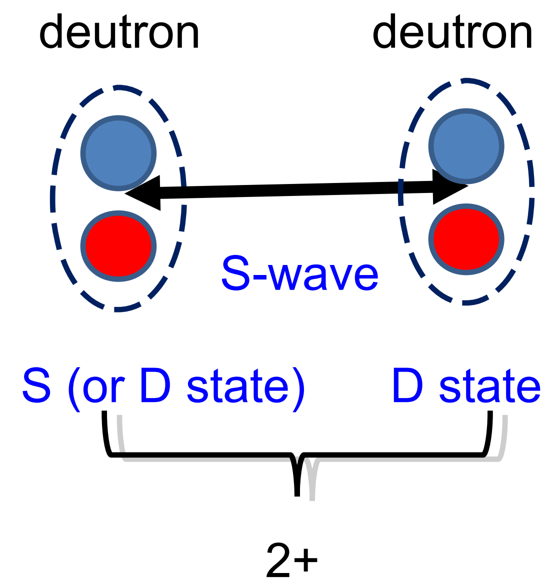
- a : channel raidus (13-15fm)
- $x_3 < a$ --- Gaussian expansion
- $x_3 > a$ --- $I_1(ka)\ \delta_{\alpha\alpha'} - S_{\alpha\alpha'}\ O_1(ka)$ *or* $W_{l+1/2,\eta}(2ka)$

e.g. D. Baye, P. -H.Heenen, M. Libert-Heinemann, NPA291(1977).

Radiative capture

K. Arai, S. Aoyama, Y. Suzuki, P. Descouvemont and D. Baye, PRL107 (2011) 132502.

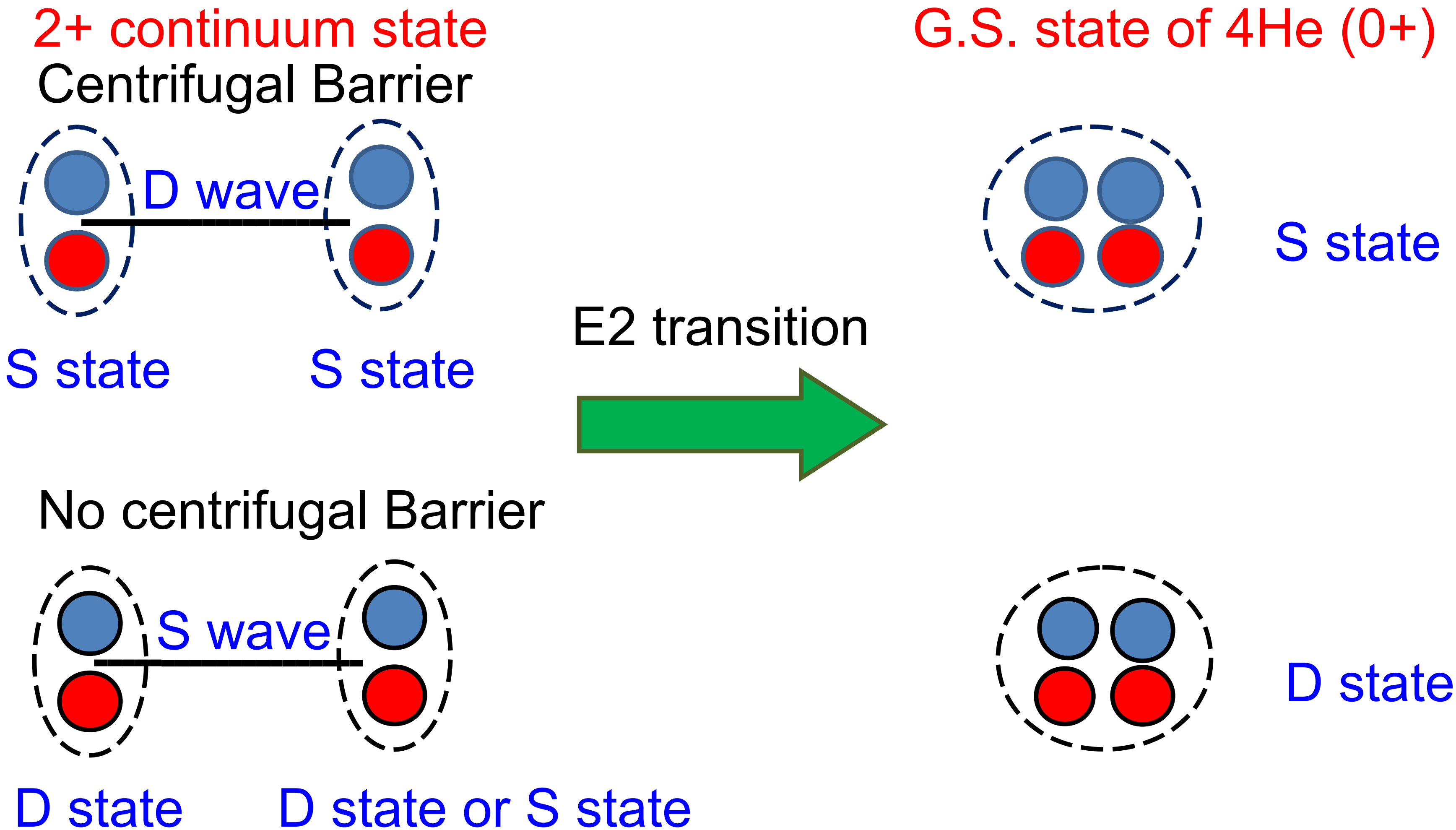




E2 transition is not reduced so much because of d-wave component.

d(1+;T=0) + d(1+; T=0) => ^{4}He (T=0) + γ

We can add a new evidence of D-wave components (tensor) of deutron and ^{4}He.



Initial dd state in S-wave

$$|^5S_2 : J = 2 > \; \sim \; (1 - P_D(d))|L = 0, S = 2 > \; + \; \sqrt{2P_D(d)}\left\{\sqrt{\frac{1}{5}}|L = 2, S = 0 > + \dots\right\}$$

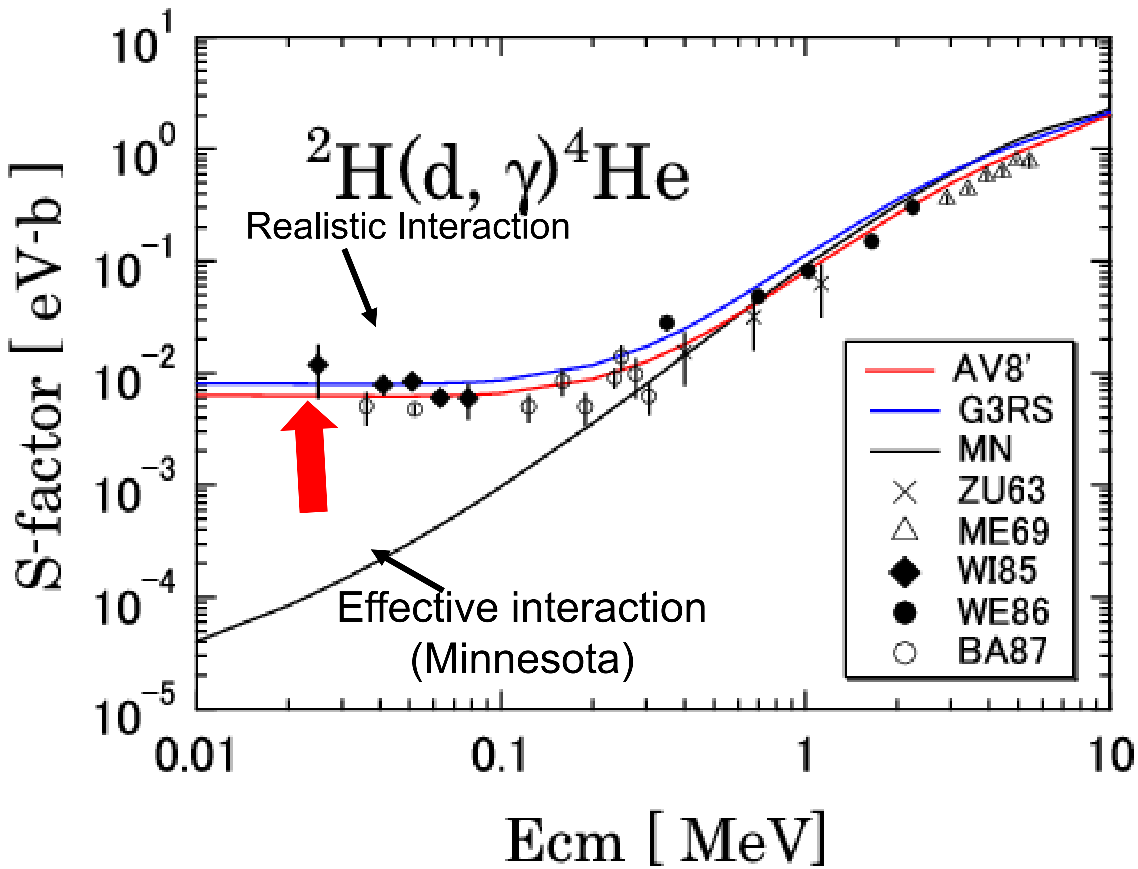
E2

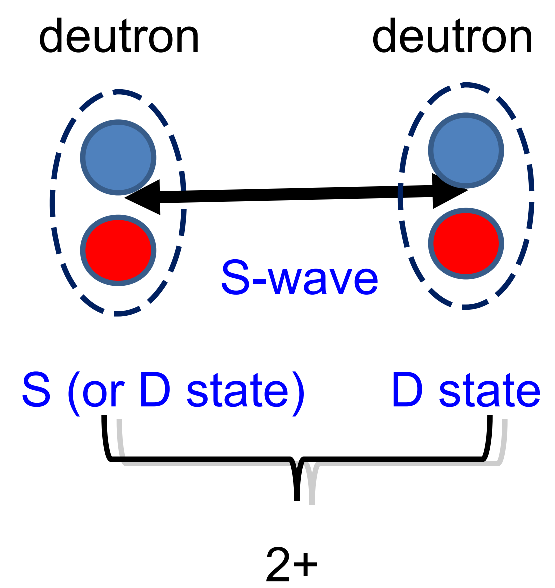
Final ^{4}He state

$$|^4\mathrm{He} : J = 0 > \; \sim \; \sqrt{1 - P_D(\alpha)}|L = 0, S = 0 > \; + \; \sqrt{P_D(\alpha)}|L = 2, S = 2 >$$

Radiative capture

K. Arai, S. Aoyama, Y. Suzuki, P. Descouvemont and D. Baye, PRL107 (2011) 132502.





E2 transition is not reduced so much because of d-wave component.

d(1+;T=0) + d(1+; T=0) => ^{4}He (T=0) + γ

E1 transition

First term of E1 is an iso-vector operator

$$\begin{aligned} \mathcal{M}^{E}_{1\mu} &= e\sum_{i_1}^{4} g_l^{(i)} r_i Y_{1\mu}(\hat{\boldsymbol{r}}_i) \\ &\approx e\sum_{i=1}^{4} t_{i3}\boldsymbol{r}_i \\ &\propto \boldsymbol{R}^{n}_{c.m.} - \boldsymbol{R}^{p}_{c.m} \end{aligned}$$

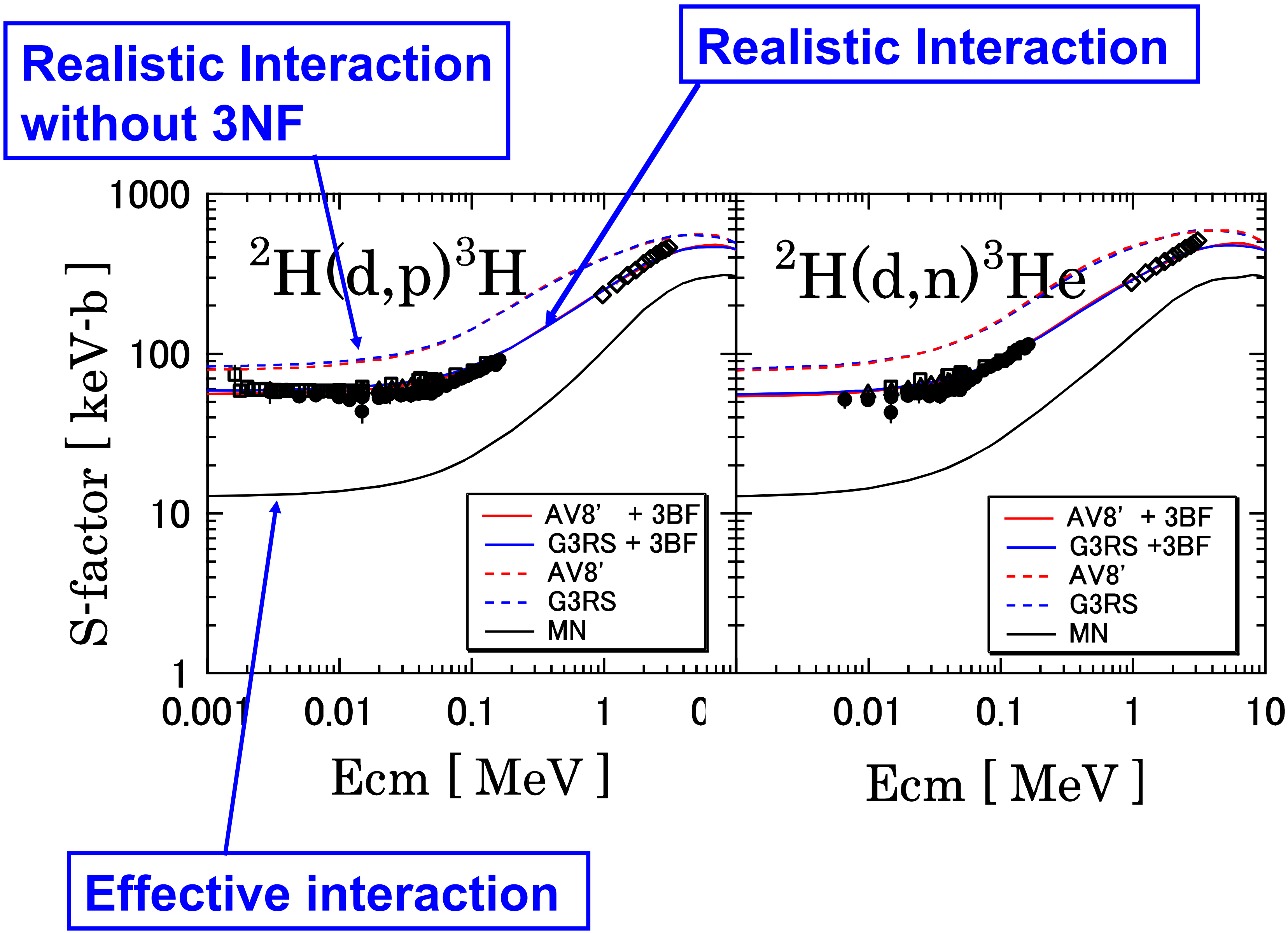
Second term of E1 is an iso-scalar

$$\mathcal{M}^{E}_{1\mu} \approx -e\sum_{i}^{A} t_{i3} r_i Y_{1\mu}(\hat{\boldsymbol{r}}_i) - e\frac{k^2}{60}\sum_{i}^{A} r_i^3 Y_{1\mu}(\hat{\boldsymbol{r}}_i)$$

D.Baye, PRC86(2012)039306

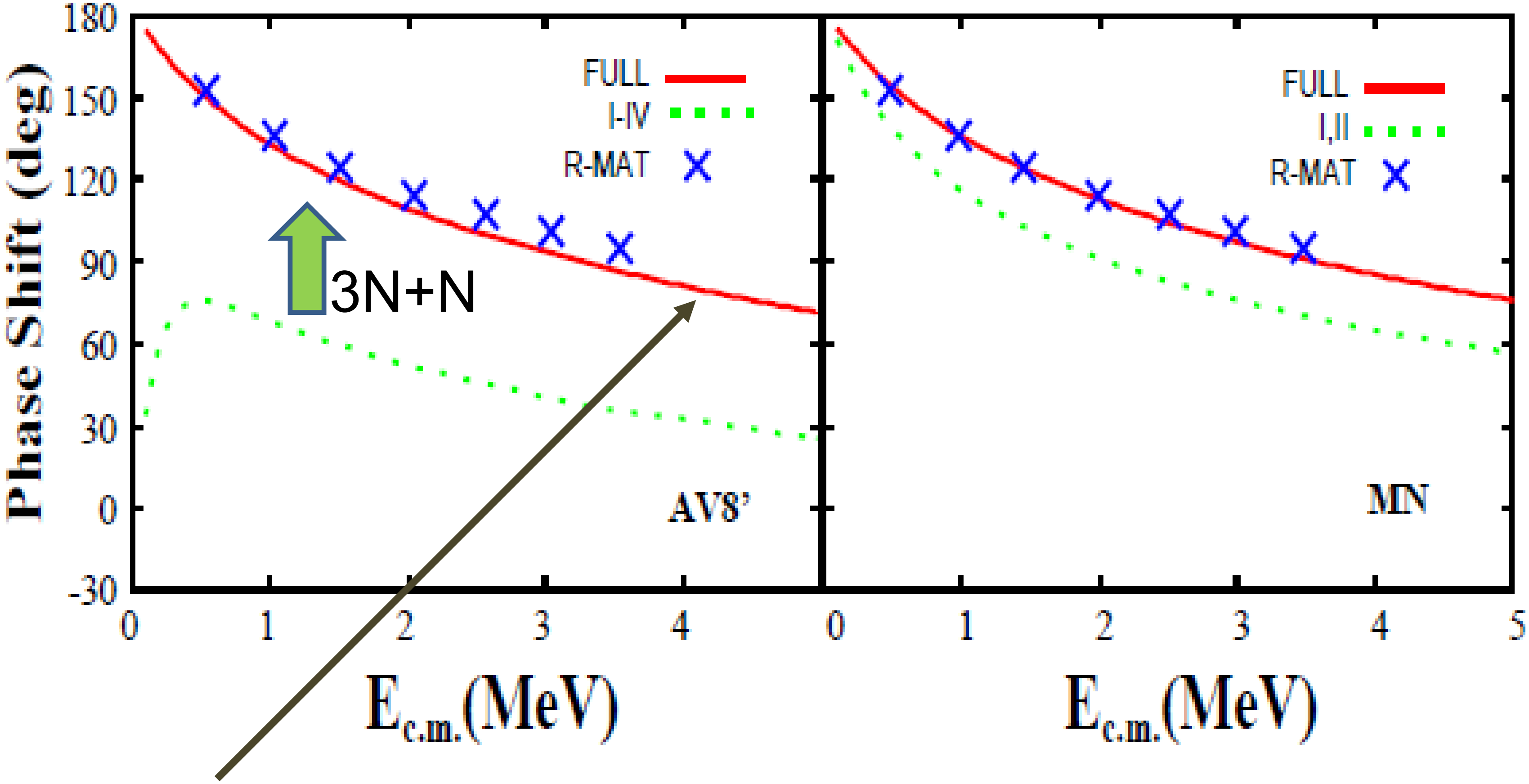
order is 1/30 x $(kr)^2$ x E1

Transfer reaction



S. Aoyama, K. Arai, Y. Suzuki, P. Descouvemont, D. Baye, FBS52, (2012)97.

$^{1}S_{0}$ d+d elastic phase shift (0+)

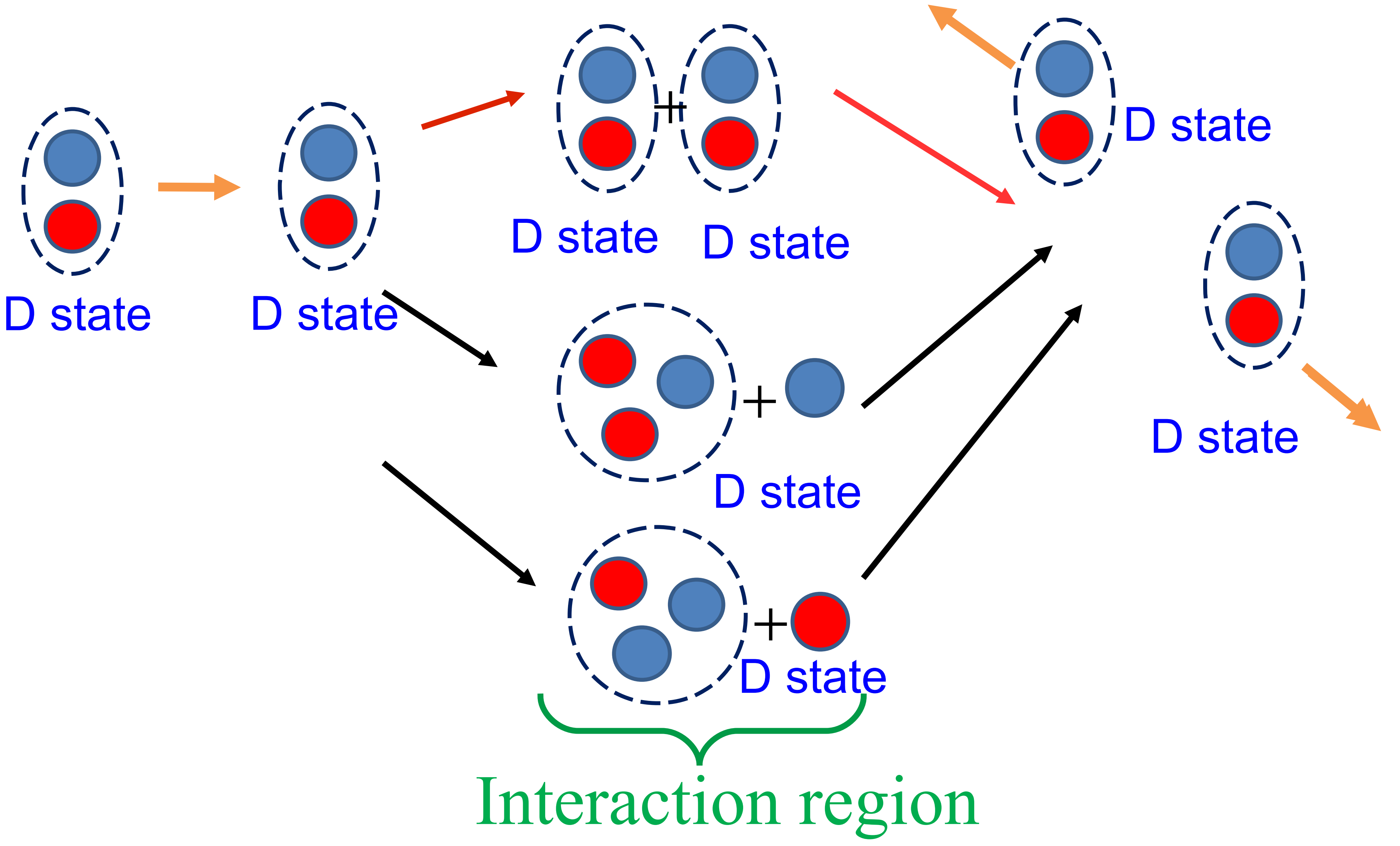


For effective interaction, d+d scattering picture is good!

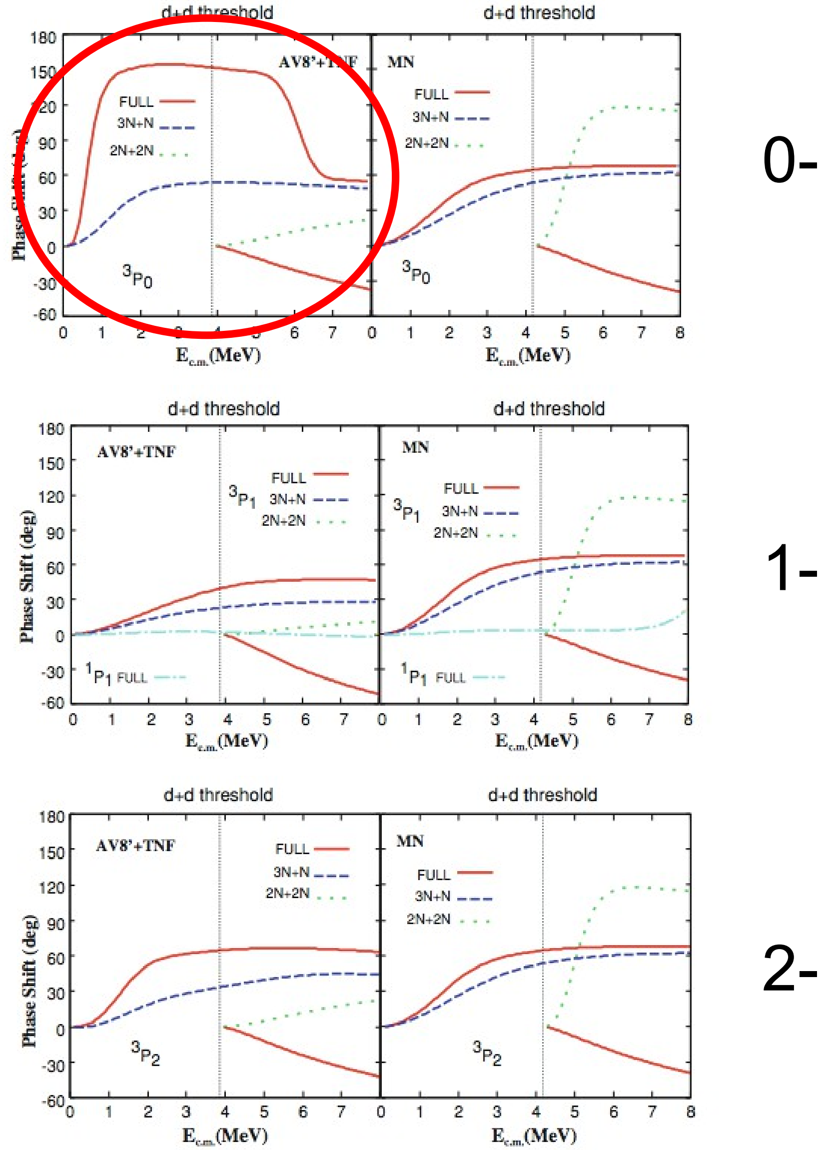
R.-Matrix analyses : Hofmann, Hale, PRC77(2008)044002

Coupling between d+d channel and 3N+N channels

Tensor force makes the coupling in the scattering strong



Realistic int. Effective int.



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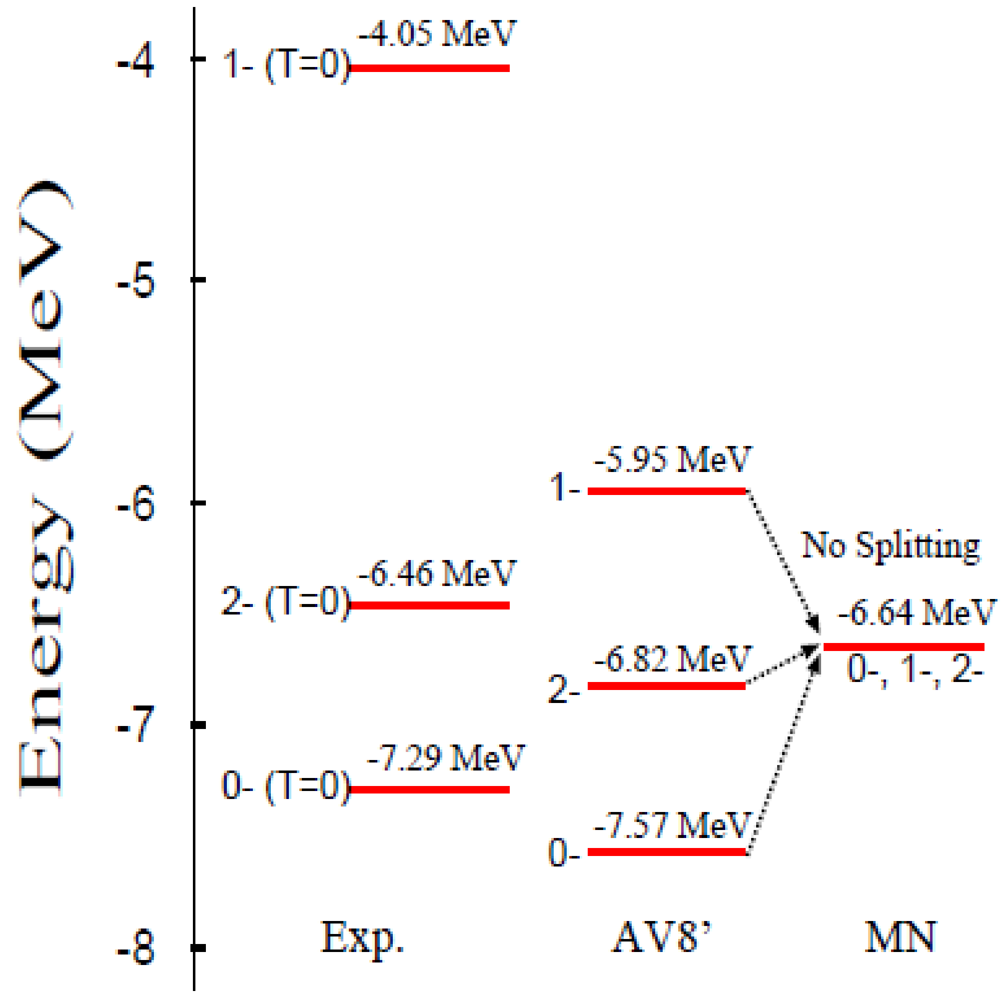
Energy levels for negative parity states



Effective interaction (MN) gives same phase shift for 0-.1-.2- !

Included channels in the present calculation

model			channel
FULL	$2N+2N$	I	$d(1^+)+d(1^+)$
			$d(1^+)+d^*(1^+)$
			$d^*(1^+)+d^*(1^+)$
		II	$\bar{d}(0^+)+\bar{d}(0^+)$
			$\bar{d}(0^+)+d^*(0^+)$
			$d^*(0^+)+d^*(0^+)$
		III	$d^*(2^+)+d^*(1^+)$
			$d^*(2^+)+d^*(2^+)$
		IV	$d^*(3^+)+d^*(1^+)$
			$d^*(3^+)+d^*(2^+)$
			$d^*(3^+)+d^*(3^+)$
		V	$2n(0^+)+2p(0^+)$
			$2n(0^+)+2p^*(0^+)$
			$2n^*(0^+)+2p(0^+)$
			$2n^*(0^+)+2p^*(0^+)$
	$3N+N$	1	$t(\frac{1}{2}^+)+p(\frac{1}{2}^+)$
			$t^*(\frac{1}{2}^+)+p(\frac{1}{2}^+)$
		2	$h(\frac{1}{2}^+)+n(\frac{1}{2}^+)$
			$h^*(\frac{1}{2}^+)+n(\frac{1}{2}^+)$

Thanks to the reduction of basis function by SVM for the sub-system. We can reduce the dimension of matrix elements very much!

Dimensions of matrix elements for FULL in the LS-coupled case

	N
0+	6660
1+	16680
2+	22230
0-	4200
1-	11670
2-	12480

The number of M.E. is N(N+1)/2.

For 2+, it takes about 200 days with 1CPU(1Core). And we need about 20Gbyte memory for the MRM calculation(half day).

All pseudo states (discretized continuum state) are employed in the MRM calculation.

Summary

1. For astrophysical S-factor in d(d, γ)^{4}He, d(d, p)^{3}H, d(d,n)^{3}He reactions, tensor interaction is important to reproduce experiment.

2. In the d+d elastic reaction, the breaking of deuteron due to tensor interaction is large.

Next

5-nucleon systems