hadron spectroscopy from lattice QCD

Jozef Dudek

Old Dominion University

Jefferson Lab

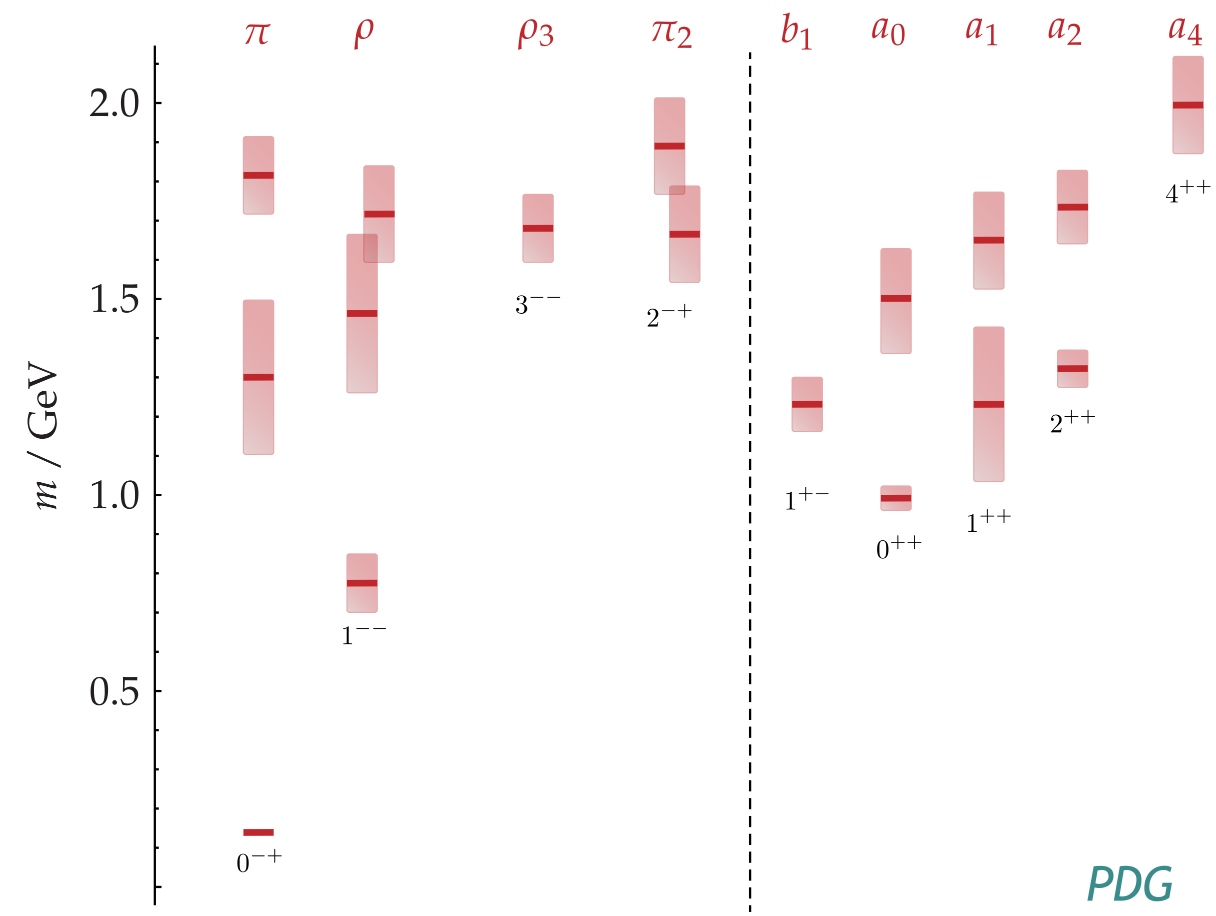
understanding the hadron spectrum

- how are mesons and baryons assembled from quarks and gluons ?
  - e.g. why only isospin ≤ 1 ?
  - do excited gluonic fields play a role?

QUANTUM CHROMODYNAMICS

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi$$
$$-g\,\bar{\psi}\gamma^\mu\psi\, A_\mu$$
$$-\tfrac{1}{2}\mathrm{tr}\, F_{\mu\nu}F^{\mu\nu}$$

ISOSPIN=1 MESON SPECTRUM



hadrons from lattice QCD

- discretize fields on a finite Euclidean hypercubic grid
- integrate out the quark fields
- Monte Carlo sample 100s of configurations of gluon fields

- spectrum from two-point correlation functions

$$C(t) = \langle 0 | \mathcal{O}(t)\, \mathcal{O}^\dagger(0) | 0 \rangle$$

$$C(t) = \langle 0 | \mathcal{O}(0) e^{-Ht} \mathcal{O}^\dagger(0) | 0 \rangle$$

$$C(t) = \langle 0 | \mathcal{O}(0) e^{-Ht} \sum_{\mathfrak{n}} |\mathfrak{n}\rangle\langle\mathfrak{n}| \mathcal{O}^\dagger(0) | 0 \rangle$$

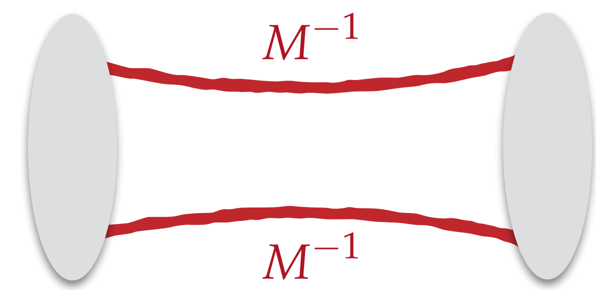
$$C(t) = \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t} \langle 0 | \mathcal{O}(0) | \mathfrak{n}\rangle\langle\mathfrak{n}| \mathcal{O}^\dagger(0) | 0 \rangle$$

hadrons from lattice QCD

- spectrum from two-point correlation functions

$$C(t) = \langle 0 | \mathcal{O}(t)\, \mathcal{O}^\dagger(0) | 0 \rangle$$

$$\mathcal{O} \sim \bar{\psi} \boldsymbol{\Gamma} \psi$$



$$\int d^4x\, \bar{\psi}(x) i\gamma^\mu (\partial_\mu + igA_\mu(x)) \psi(x) \xrightarrow{\text{discretize}} \sum_{xy} \bar{\psi}_x M_{xy} \psi_y$$

integrate out quark fields

$$\psi_x \bar{\psi}_y \rightsquigarrow M^{-1}_{xy}$$

$$\langle \bar{\psi}_t \boldsymbol{\Gamma}_t \psi_t\, \bar{\psi}_0 \boldsymbol{\Gamma}_0 \psi_0 \rangle \rightsquigarrow M^{-1}_{0t} \boldsymbol{\Gamma}_t M^{-1}_{t0} \boldsymbol{\Gamma}_0$$

“Wick contract”

matrix multiply and trace

excited states from correlators

- how to get at excited QCD eigenstates ?
  - optimal operator for state $|\mathfrak{n}\rangle$: $\Omega_{\mathfrak{n}}^\dagger \sim \sum_i v_i^{(\mathfrak{n})} \mathcal{O}_i^\dagger$

    for a basis of meson operators $\{\mathcal{O}_i\}$

  - can be obtained (in a variational sense) from the matrix of correlators

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t)\, \mathcal{O}_j^\dagger(0) | 0 \rangle$$

  - by solving a generalized eigenvalue problem

$$C(t) v^{(\mathfrak{n})} = C(t_0) v^{(\mathfrak{n})} \lambda_{\mathfrak{n}}(t)$$

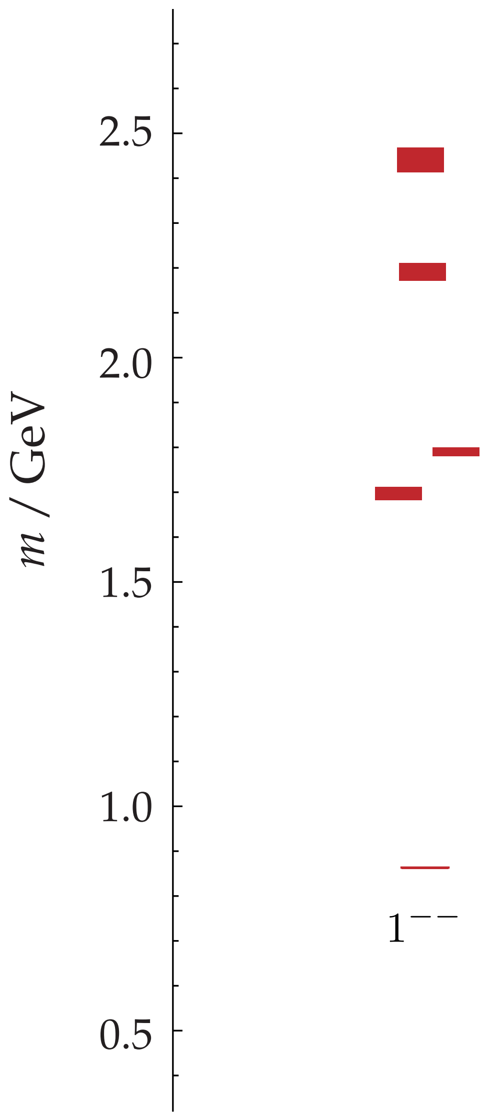
eigenvalues $\lambda_{\mathfrak{n}}(t) \sim e^{-E_{\mathfrak{n}}(t-t_0)}$

  - a large basis can be constructed using covariant derivatives :

$$\mathcal{O} \sim \bar{\psi} \Gamma \overleftrightarrow{D} \dots \overleftrightarrow{D} \psi$$

excited state spectra

- for example vector meson sector, using more than 20 operators



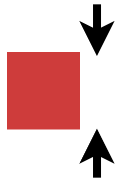
$m_\pi \sim 391\,\mathrm{MeV}$

$a \sim 0.12\,\mathrm{fm}$

$24^3 \times 128$

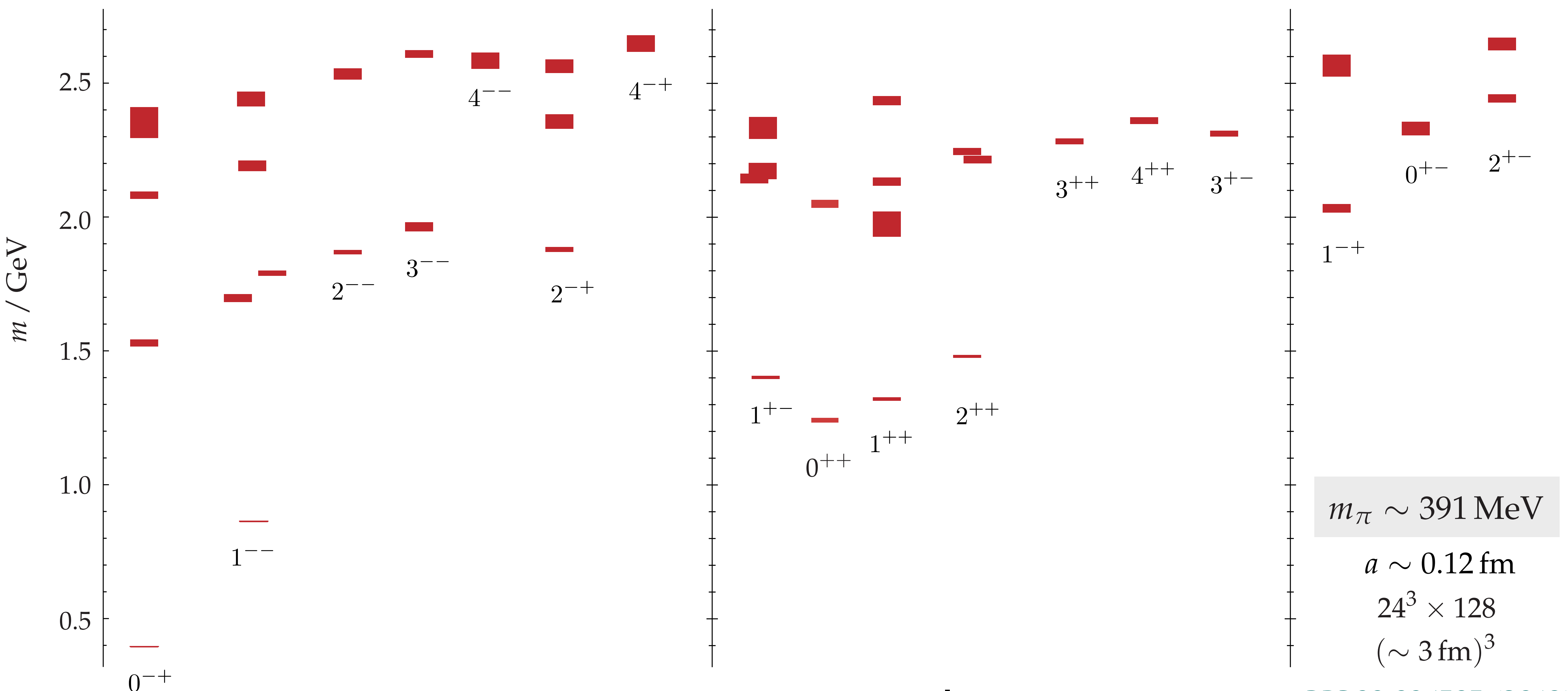
$(\sim 3\,\mathrm{fm})^3$

PRD88 094505 (2013)

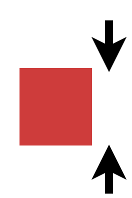


excited state spectra

- meson spectrum for a range of J^{PC}



PRD88 094505 (2013)



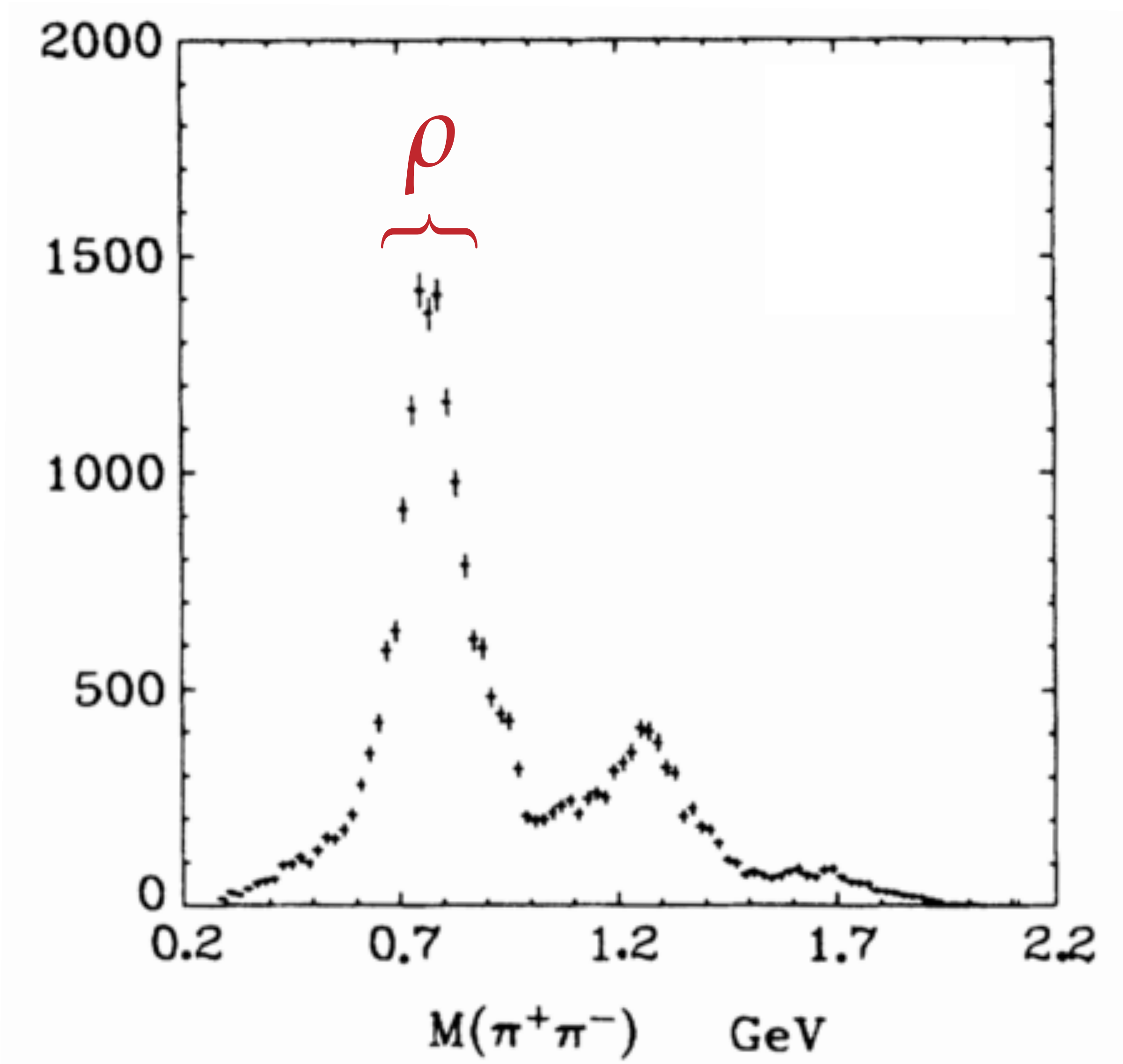
Monte Carlo
stat. uncertainty

excited hadrons are resonances

PHYSICAL REVIEW D VOLUME 7, NUMBER 5 1 MARCH 1973

$\pi\pi$ Partial-Wave Analysis from Reactions $\pi^+p \to \pi^+\pi^-\Delta^{++}$ and $\pi^+p \to K^+K^-\Delta^{++}$ at 7.1 GeV/c†

S. D. Protopopescu,* M. Alston-Garnjost, A. Barbaro-Galtieri, S. M. Flatté,‡
J. H. Friedman,§ T. A. Lasinski, G. R. Lynch, M. S. Rabin,‖ and F. T. Solmitz
Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720
(Received 25 September 1972)



expand angular dependence
in *partial waves*

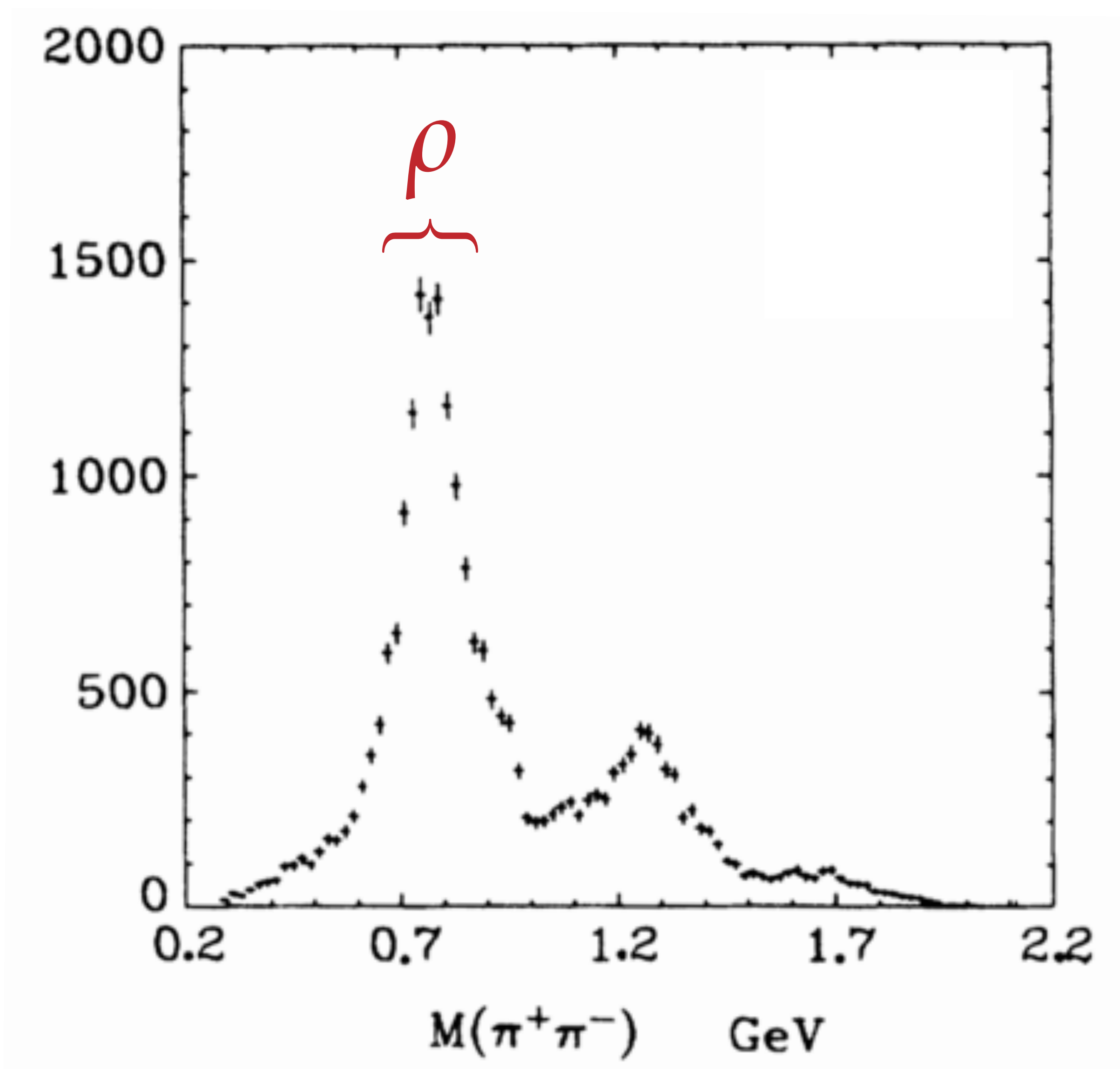
PARTIAL WAVE AMPLITUDE

$$f_\ell = \frac{1}{2i}\left(\eta_\ell\, e^{2i\delta_\ell} - 1\right)$$

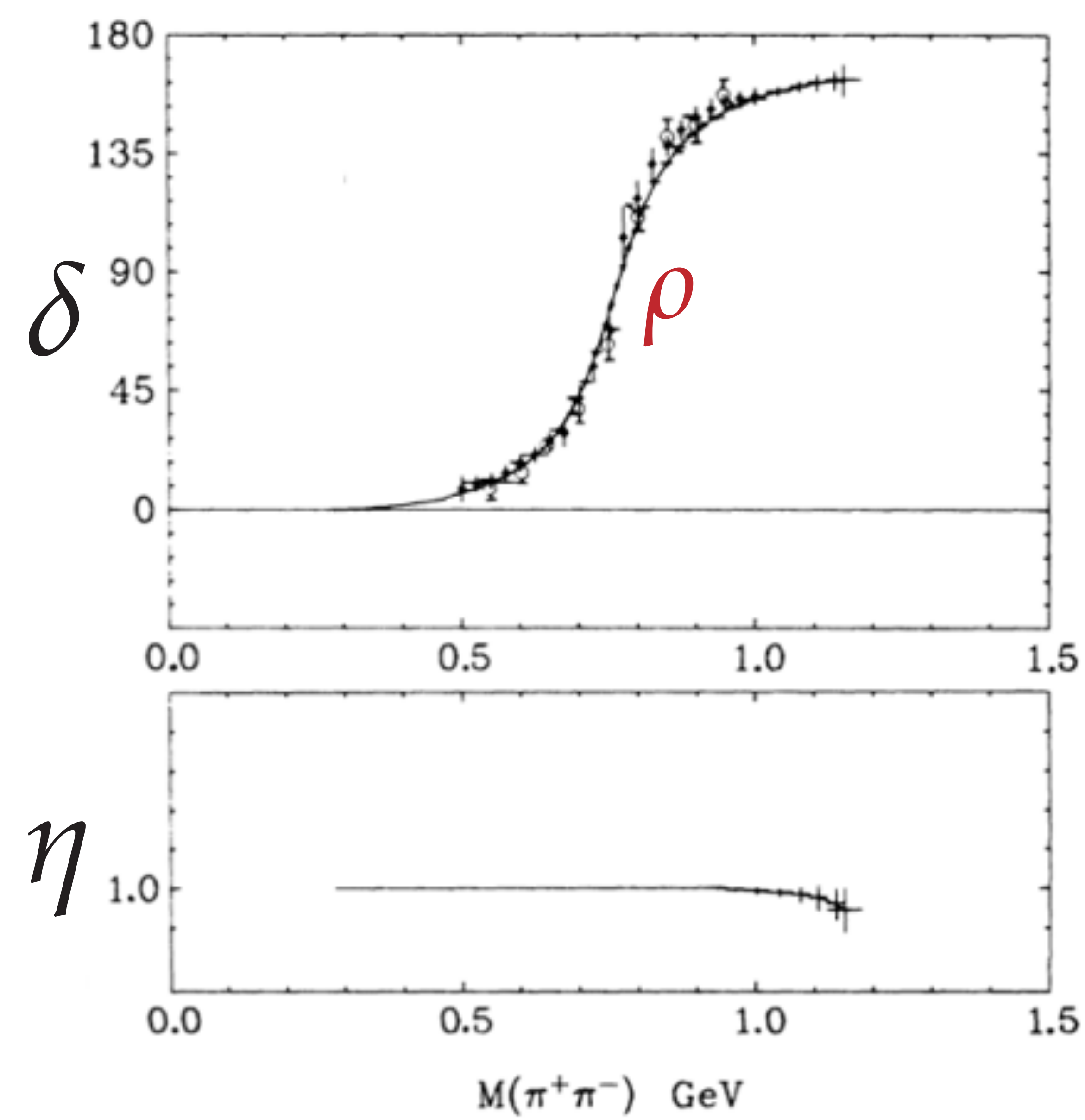
$\eta = 1$ elastic

$\eta \leq 1$ inelastic

excited hadrons are resonances



RESONANT PHASE SHIFT



scattering in a finite cubic volume

- expect a discrete spectrum in a finite periodic volume $\psi(x+L) = \psi(x)$

e.g. free particle $e^{ip(x+L)} = e^{ipx}$

quantized momentum $p = \frac{2\pi}{L} n$

- for an interacting theory

$$\cot \delta_\ell(E) = \mathcal{M}_\ell(E, L)$$

LÜSCHER ...

elastic scattering phase-shift

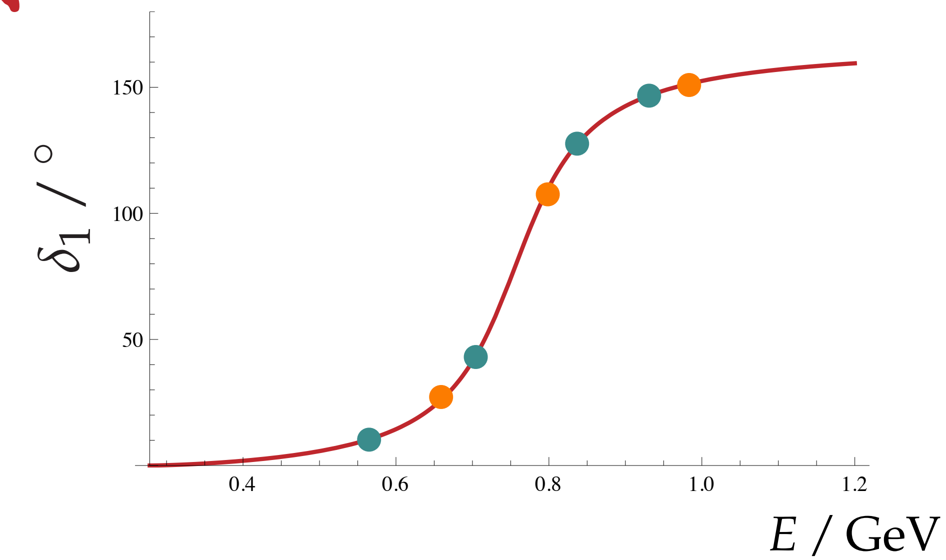
known function

discrete energies in a finite-volume ⟷ discrete values of the phase-shift

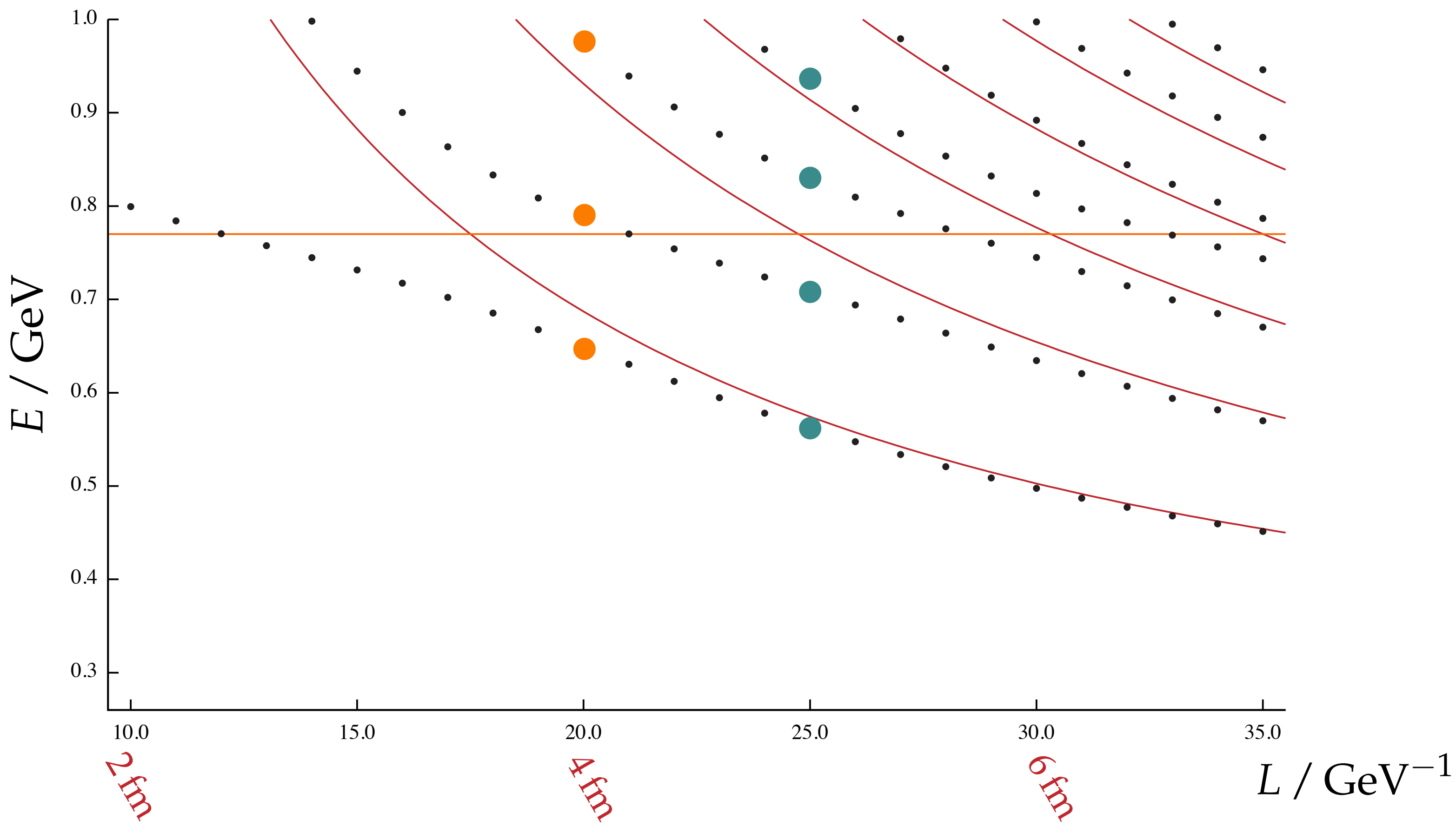
scattering in a finite cubic volume

- e.g. experimental $\pi\pi$ I=1 P-wave scattering amplitude

P-WAVE PHASE SHIFT



CUBIC BOX SPECTRUM



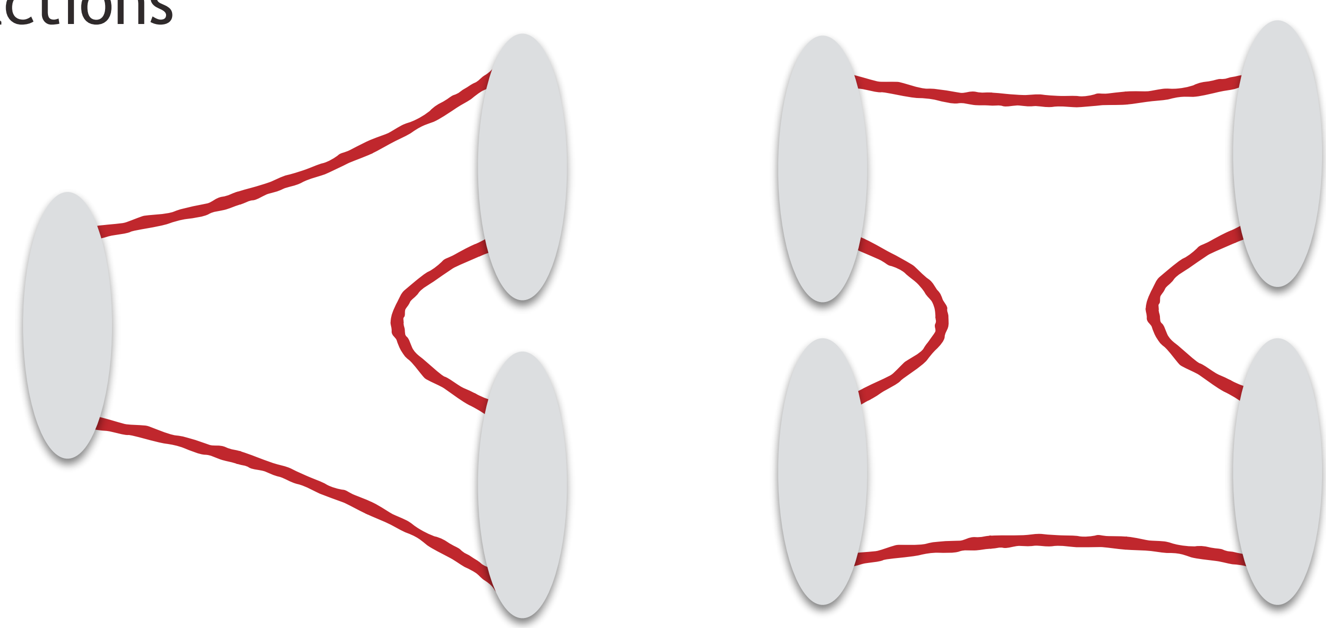
determining the finite-volume spectrum

- include operators which resemble a pair of pions $\sum_{\hat{k}_1,\hat{k}_2} C(\Lambda,\vec{P};\vec{k}_1,\vec{k}_2)\,\pi^\dagger(\vec{k}_1)\,\pi^\dagger(\vec{k}_2)$

  $\pi^\dagger \sim \bar{\psi}\boldsymbol{\Gamma}\psi$

- form correlator matrix with both $\bar{\psi}\boldsymbol{\Gamma}\psi$ and $\pi\pi$-like
  - more complicated Wick contractions

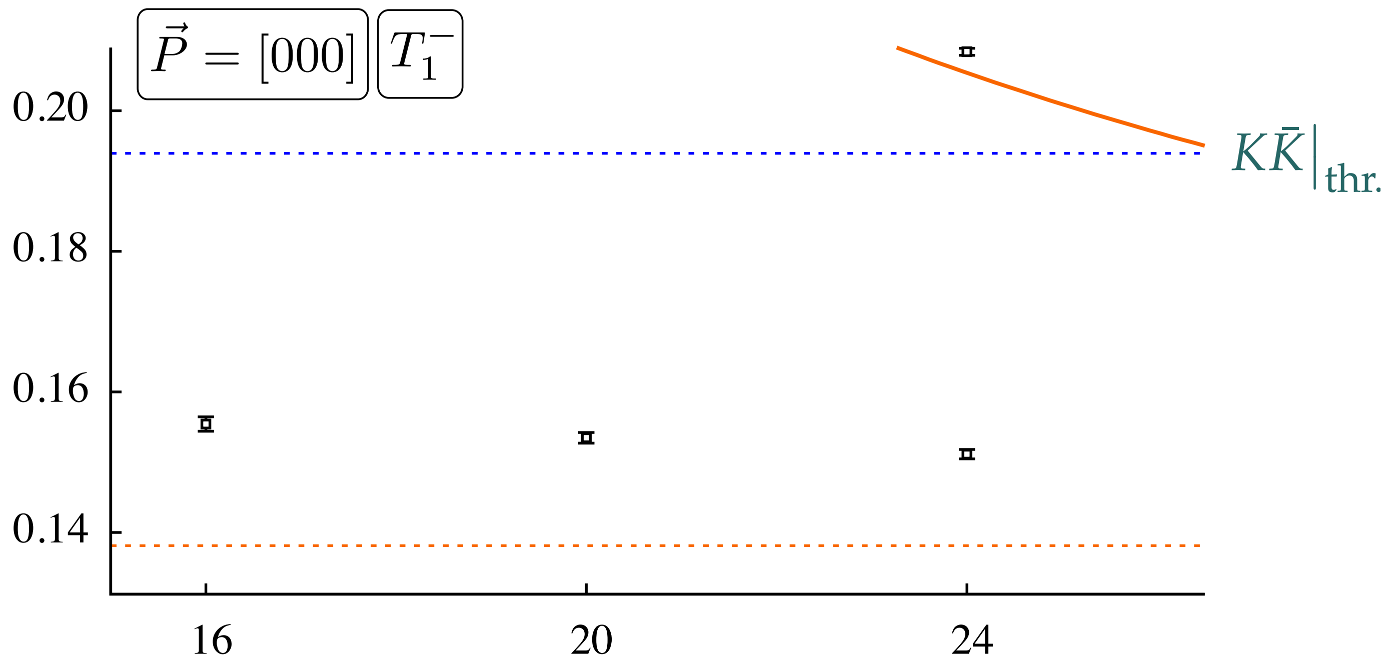
e.g.

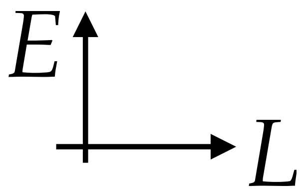


  - quark 'annihilation' lines have in the past been a computational challenge M^{-1}_{tt}

finite-volume spectrum

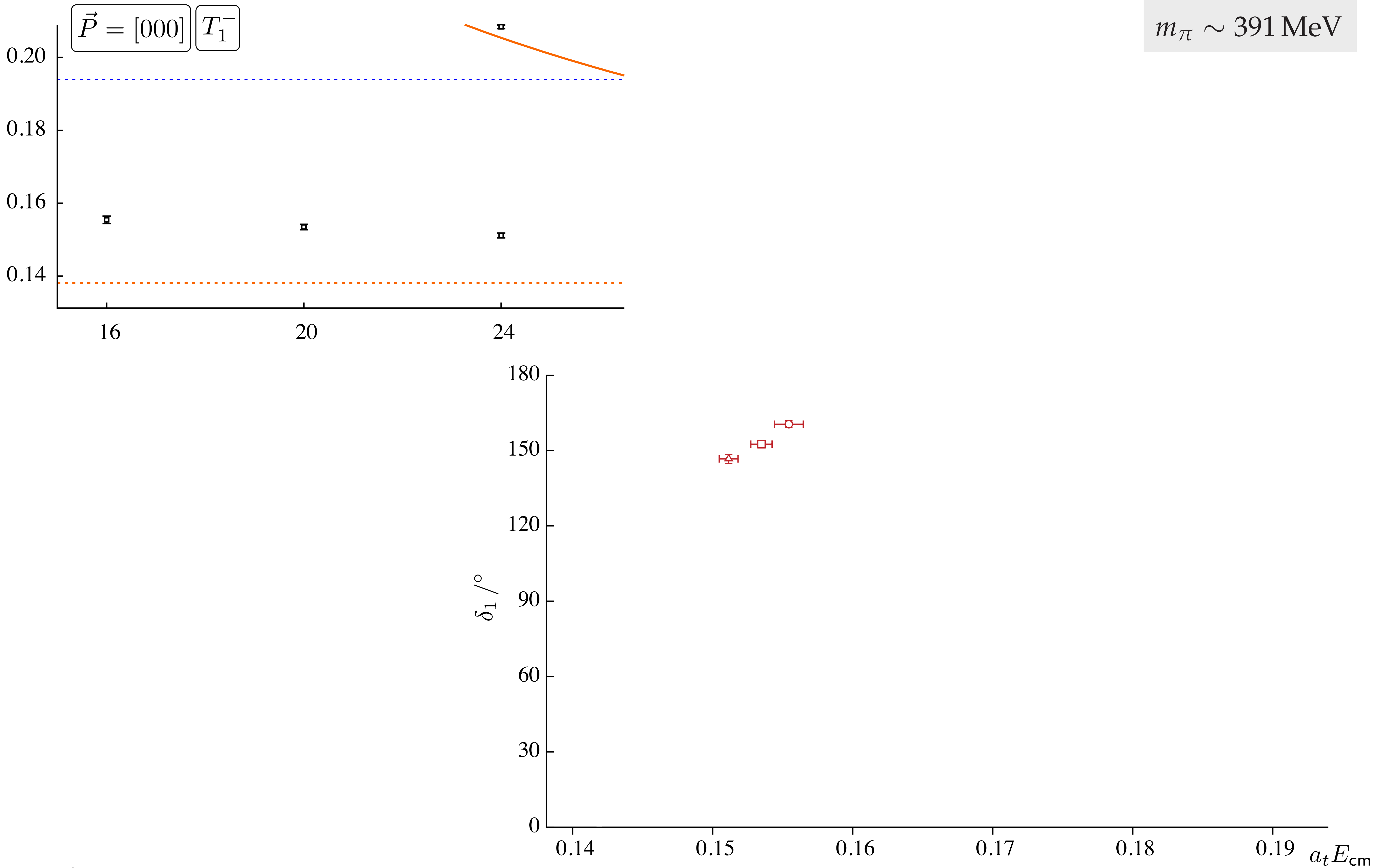
$m_\pi \sim 391\,\mathrm{MeV}$





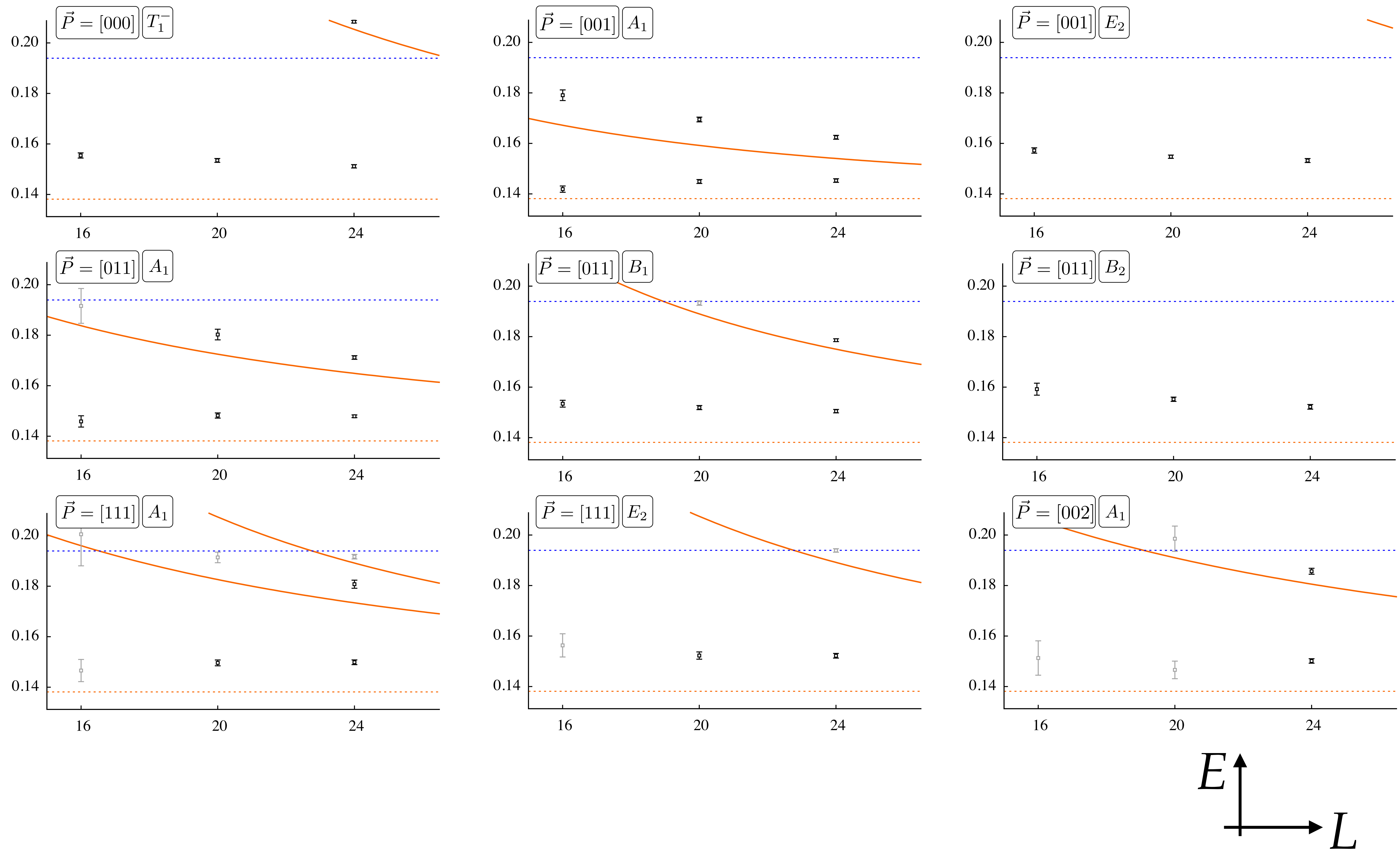
finite-volume spectrum

$m_\pi \sim 391\,\mathrm{MeV}$

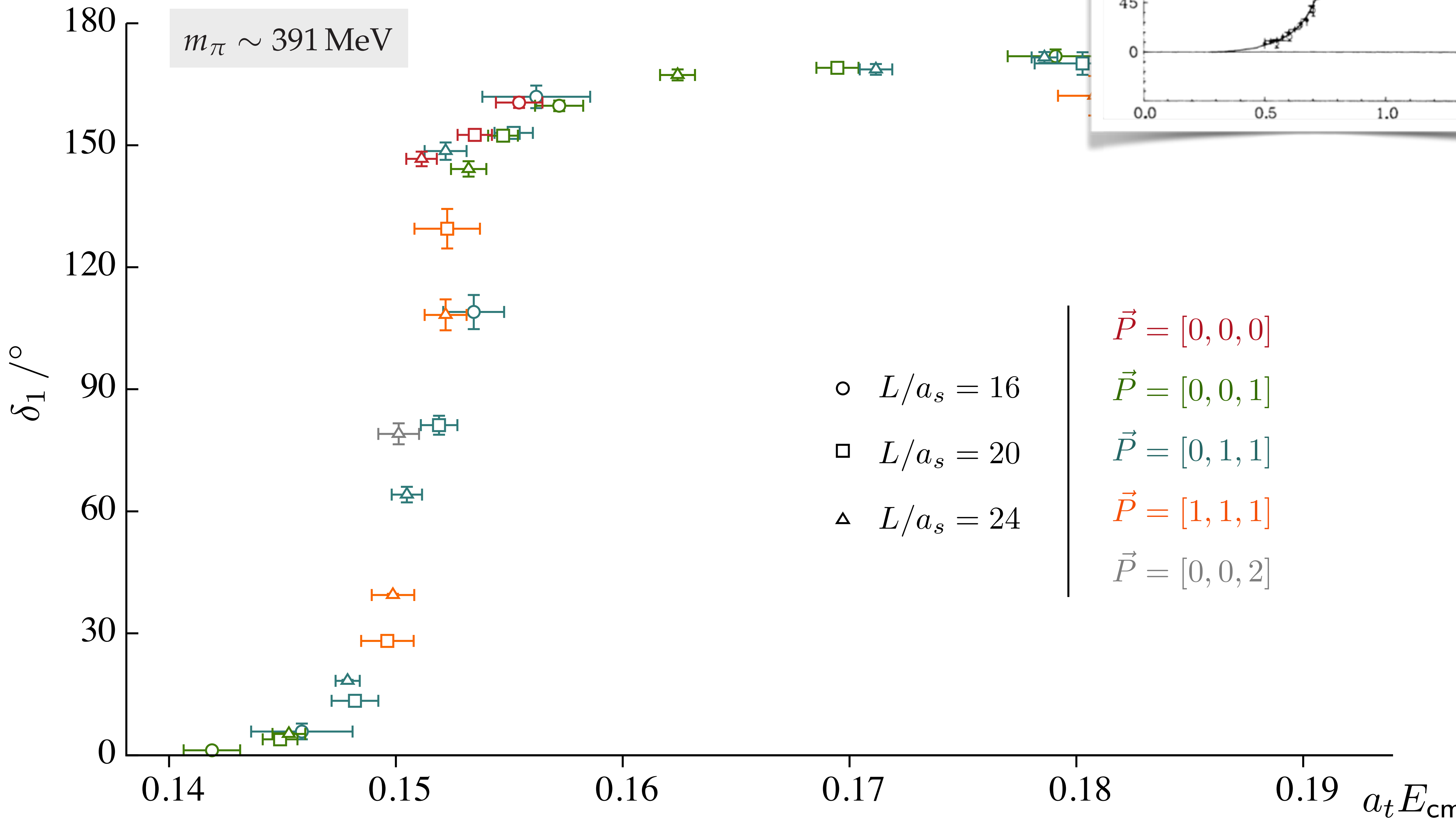


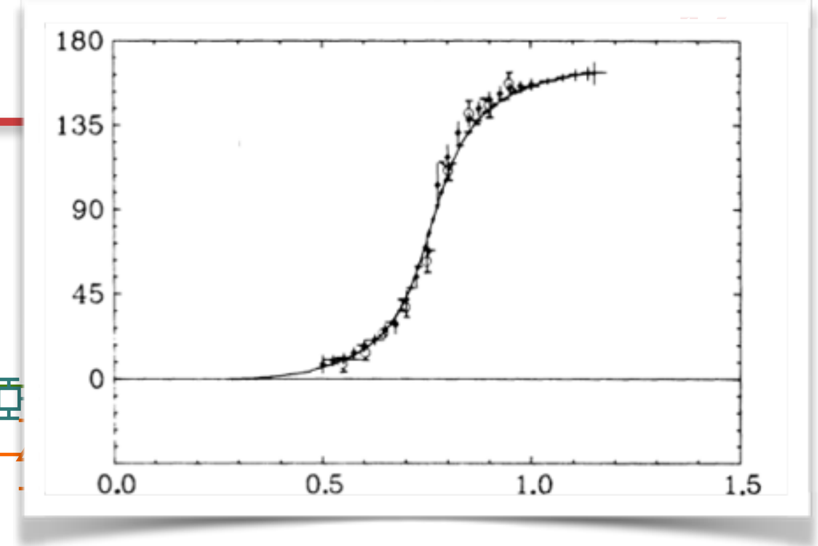
finite-volume spectrum - moving frames

$m_\pi \sim 391\,\text{MeV}$



ππ *P*-wave phase-shift

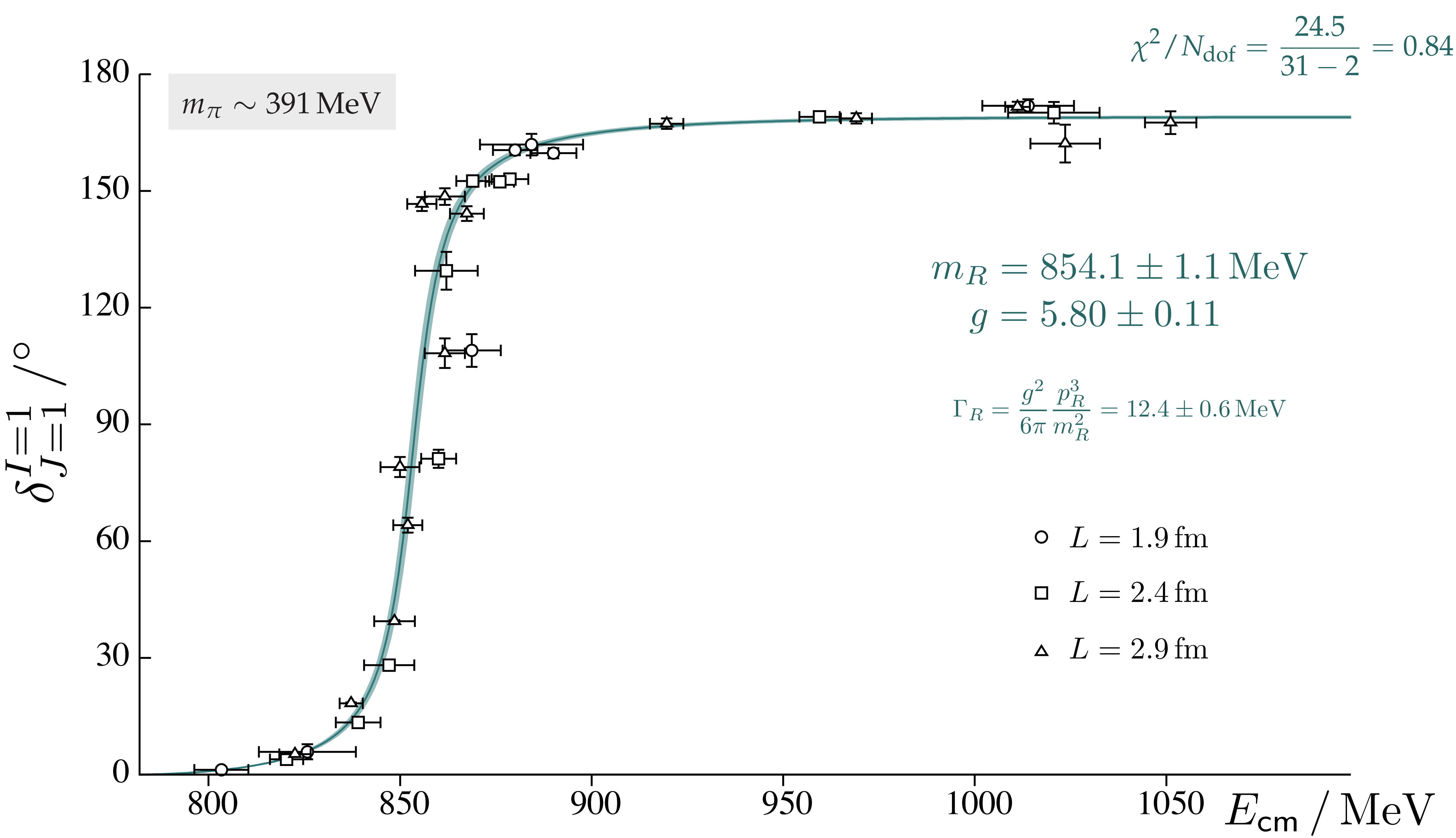




(other lattice groups have computed this in less detail)

PRD87 034505 (2013)
& erratum to appear

ρ resonance



$g_{\mathrm{phys.}} = 5.95(2)$ *PDG*

coupled-channel scattering

He, JHEP 0507 011
Hansen, PRD86 016007
Briceno, PRD88 094507
Guo, PRD88 014051

- finite-volume formalism recently derived (multiple methods)

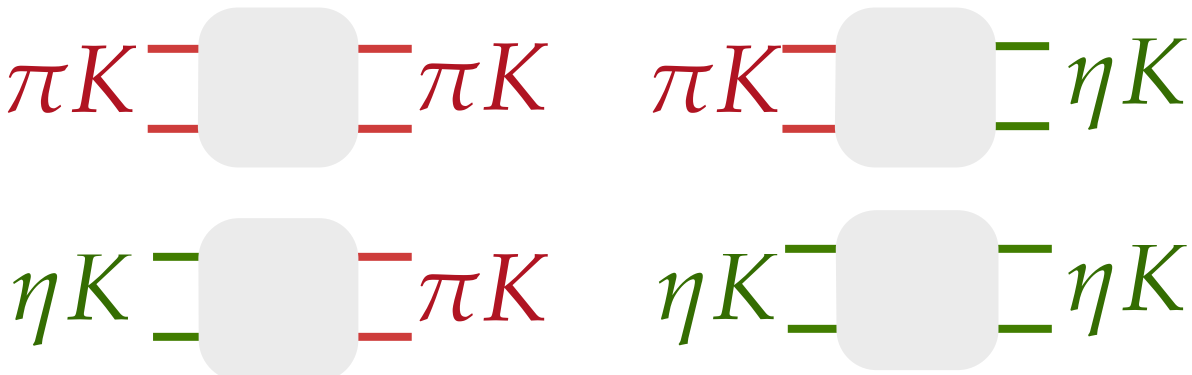
$$\det\left[\left(\left[t^{(\ell)}(E)\right]^{-1}_{ij} + i\rho_i(E)\,\delta_{ij}\right) - \delta_{ij}\,\mathcal{M}_\ell\big(p_i(E)L\big)\right] = 0$$

scattering matrix — phase space — known functions

- but this is one equation for multiple unknowns (per energy level)
  - parameterize the energy dependence of *t*
  - try to describe a spectrum globally

$\pi K/\eta K$ scattering & kaon resonances

- an example of coupled-channel scattering

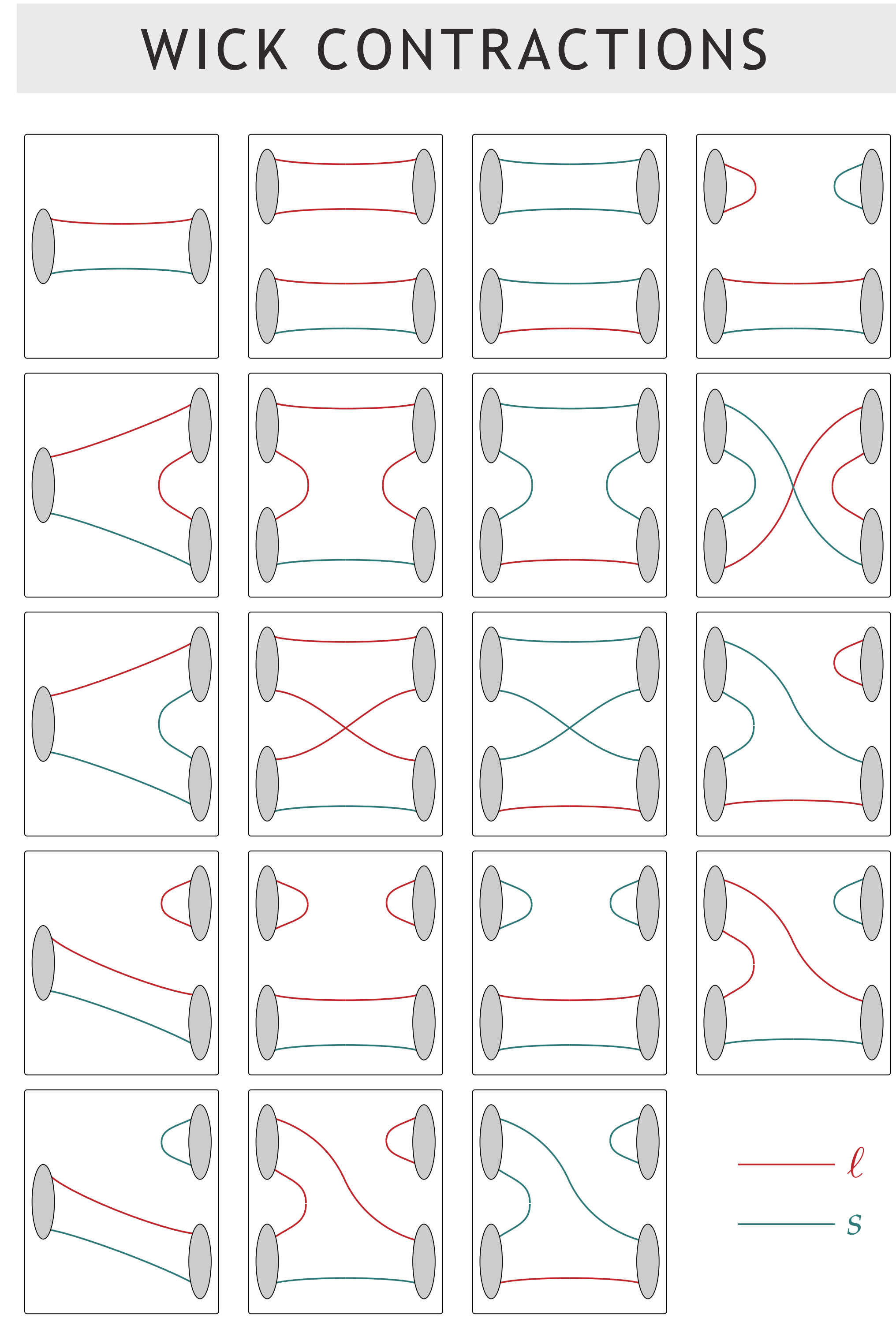


- compute finite-volume spectrum

$$\bar{u}\boldsymbol{\Gamma}s$$

$$\sum_{\hat{k}_1,\hat{k}_2} C(\Lambda,\vec{P};\vec{k}_1,\vec{k}_2)\;\pi^\dagger(\vec{k}_1)\;K^\dagger(\vec{k}_2)$$

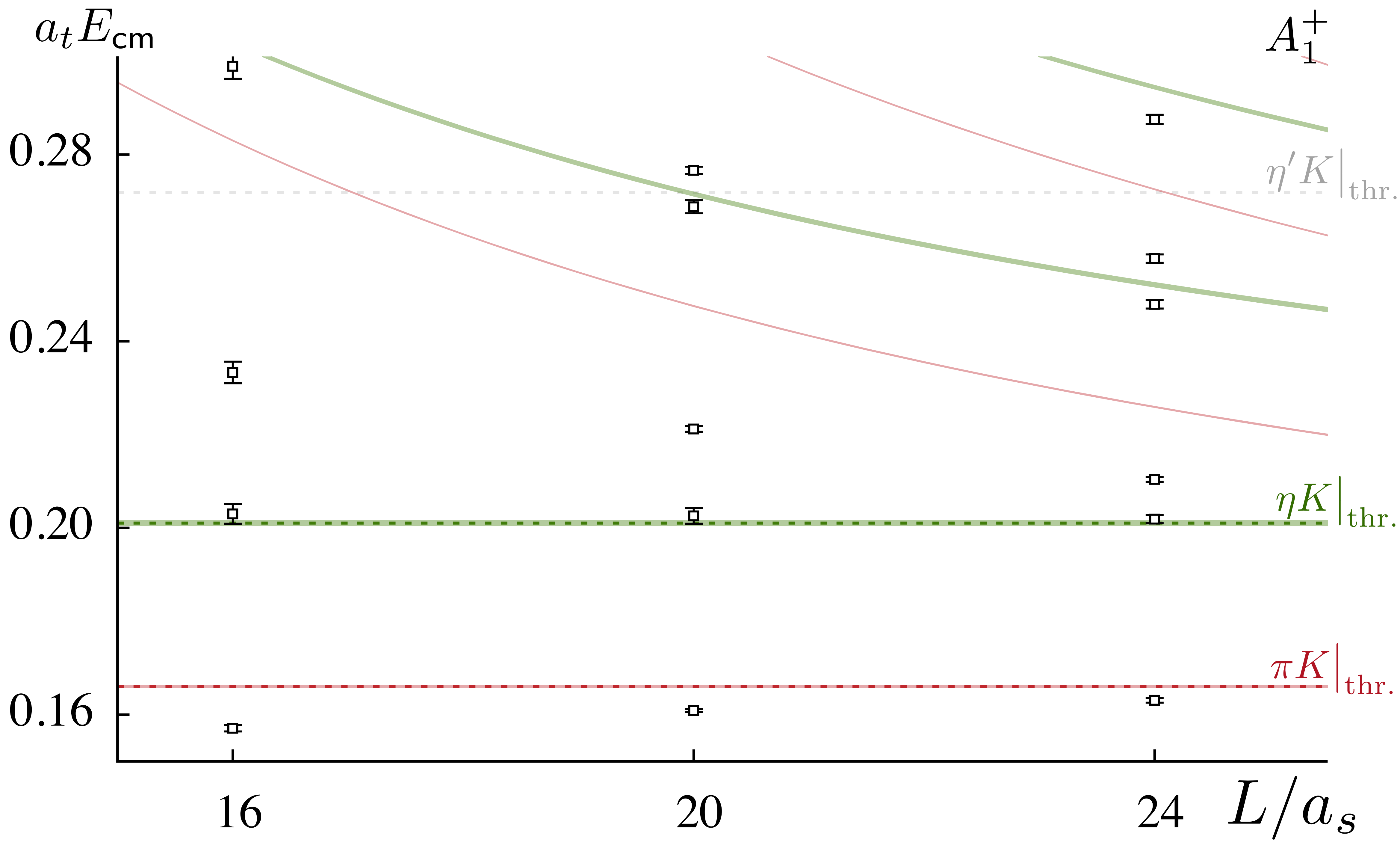
$$\sum_{\hat{k}_1,\hat{k}_2} C(\Lambda,\vec{P};\vec{k}_1,\vec{k}_2)\;\eta^\dagger(\vec{k}_1)\;K^\dagger(\vec{k}_2)$$



$\pi K/\eta K$ scattering & kaon resonances

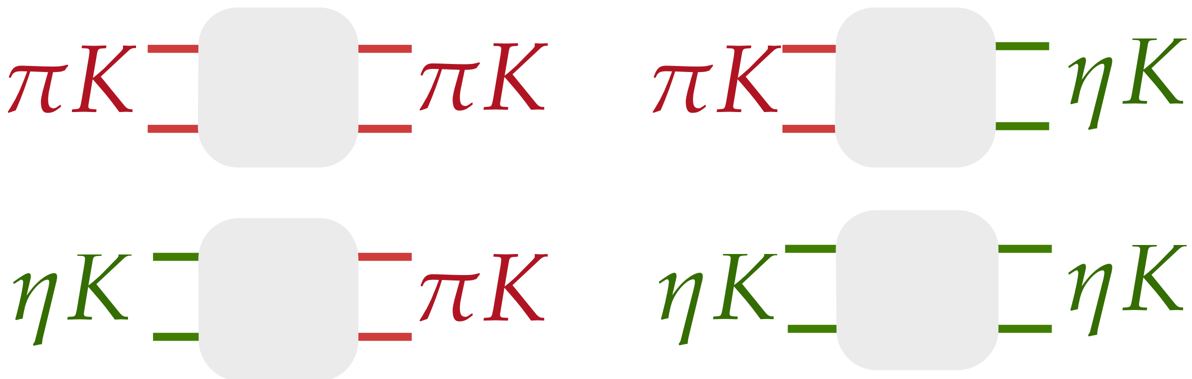
- rest frame "S-wave" spectrum

$m_\pi \sim 391\,\mathrm{MeV}$



$\pi K/\eta K$ scattering & kaon resonances

- parameterize the t-matrix in a unitarity conserving way



$$t_{ij}^{-1}(E) = K_{ij}^{-1}(E) + \delta_{ij}\, I_i(E)$$

$$K_{ij}(E) = \frac{g_i\, g_j}{m^2 - E^2} + \gamma_{ij}$$

  - vary the parameters, solving

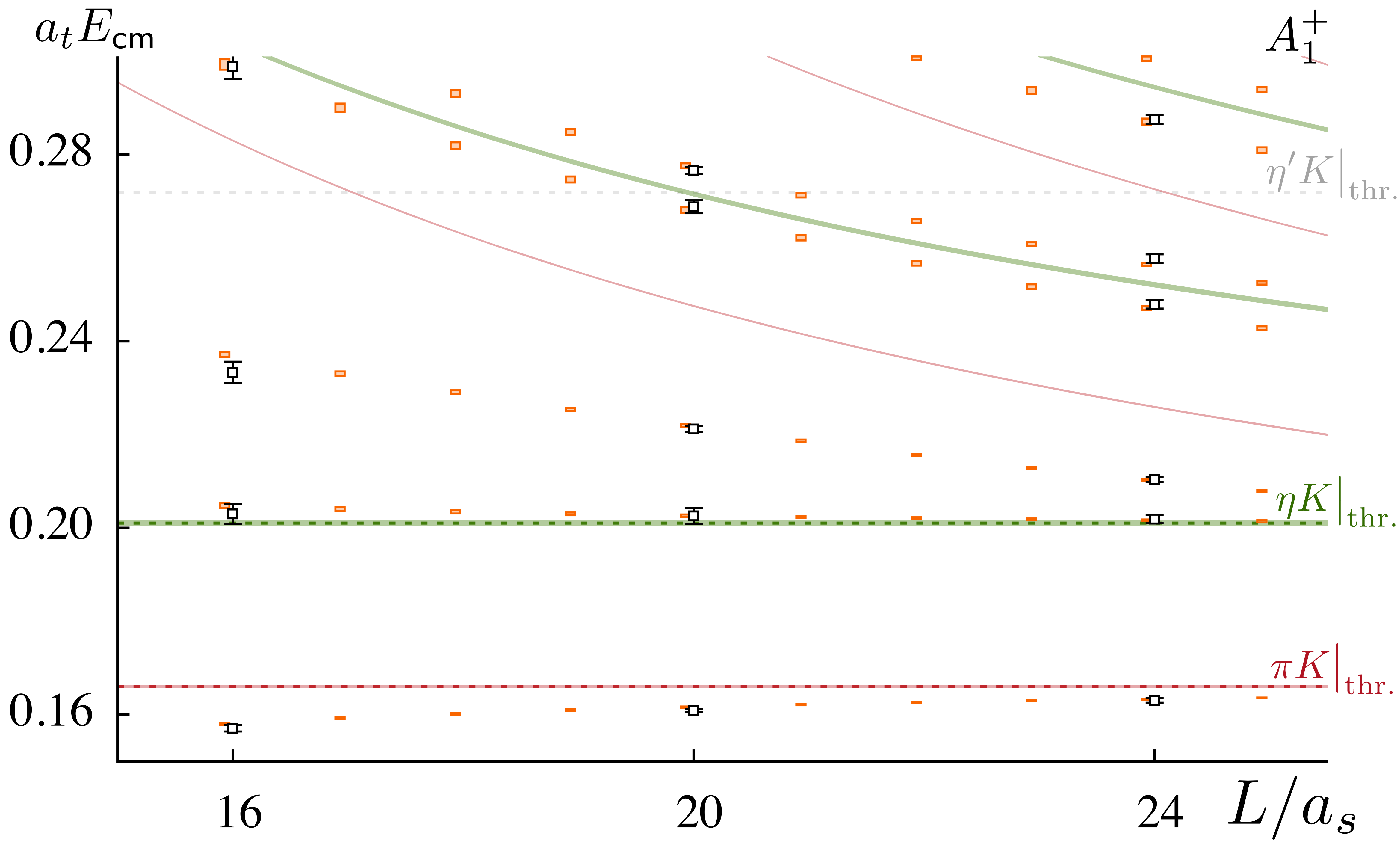
$$\det\left[\left(\left[t^{(\ell)}(E)\right]_{ij}^{-1} + i\rho_i(E)\,\delta_{ij}\right) - \delta_{ij}\,\mathcal{M}_\ell(E,L)\right] = 0$$

for the spectrum each time

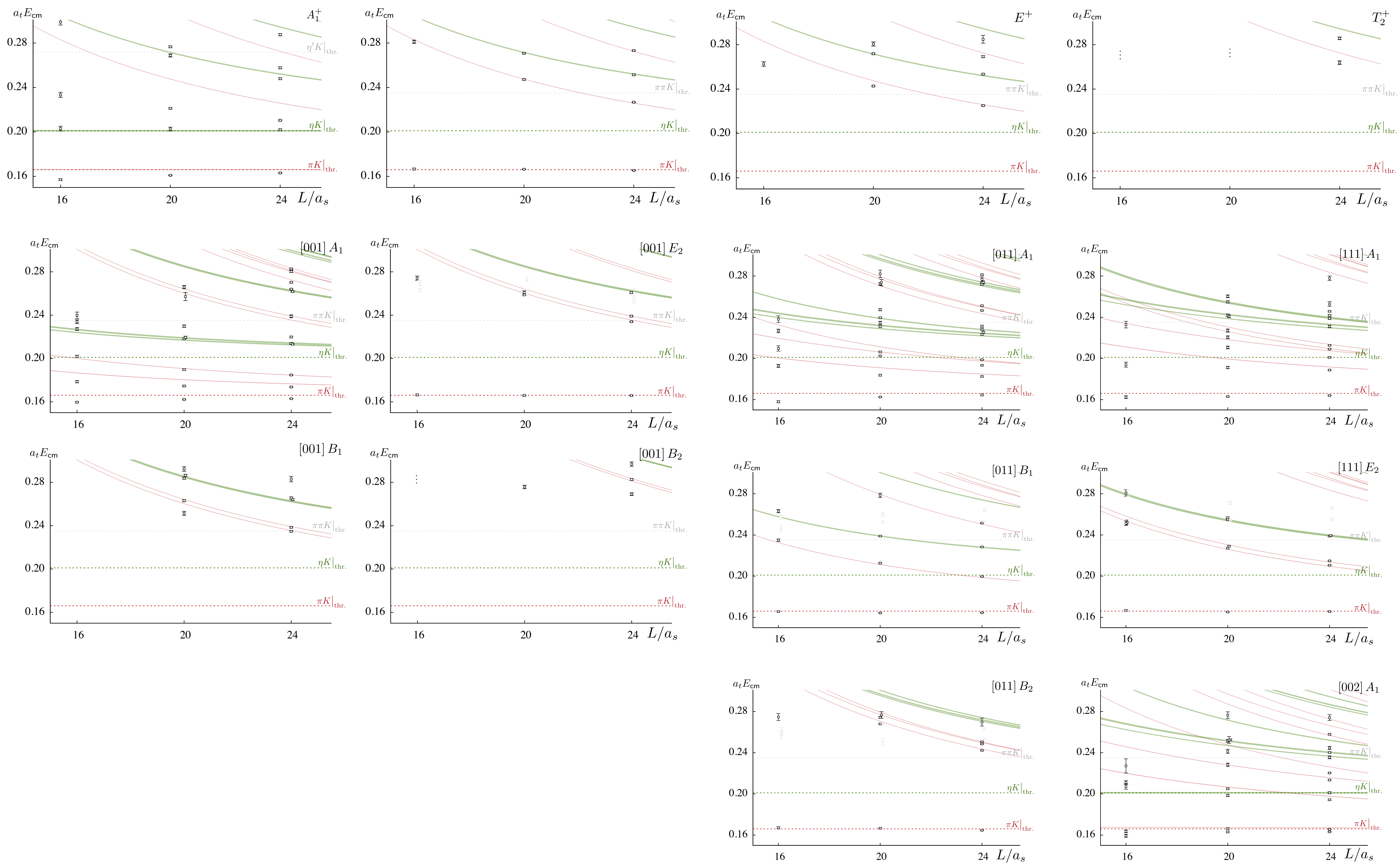
$\pi K/\eta K$ scattering

model description $\chi^2/N_{\rm dof} = \frac{6.40}{15-6} = 0.71$

$m_\pi \sim 391\,\text{MeV}$



spectra in moving frames



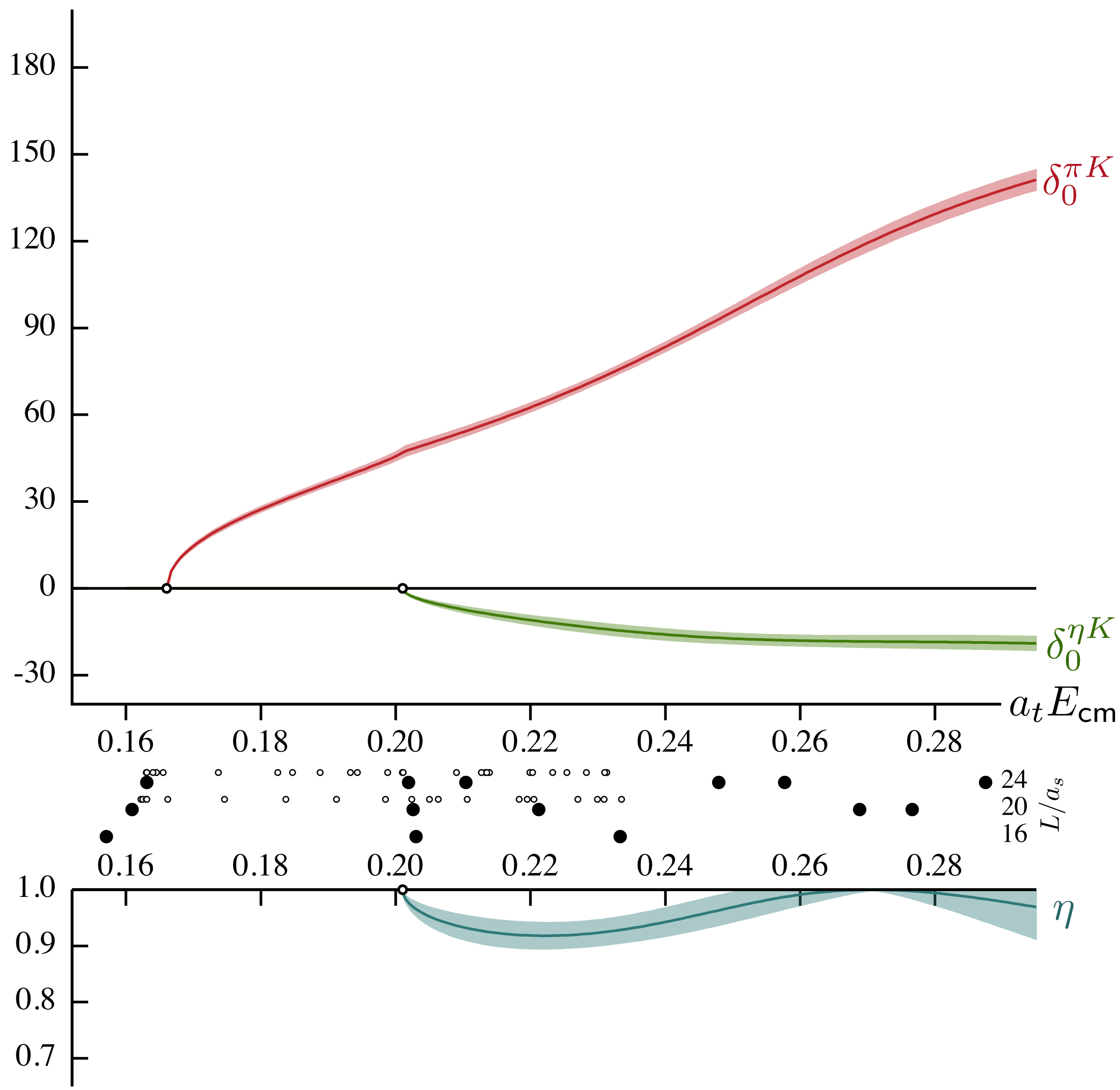
$\pi K/\eta K$ scattering

- describe all the finite-volume spectra

$$\chi^2/N_{\mathrm{dof}} = \frac{49.1}{61-6} = 0.89$$

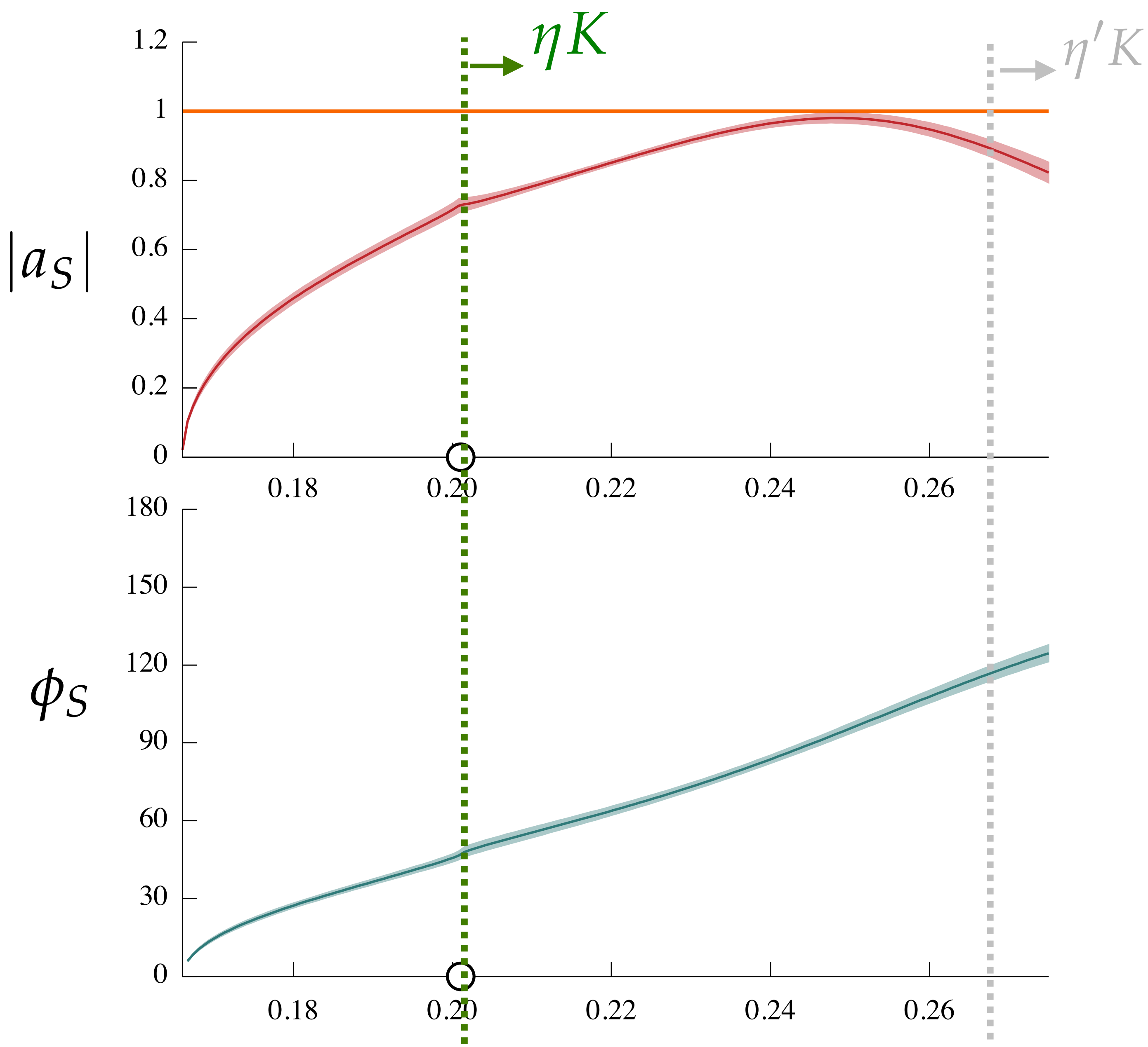
$$S_{\pi K,\pi K} = \eta\, e^{2i\delta^{\pi K}}$$
$$S_{\eta K,\eta K} = \eta\, e^{2i\delta^{\eta K}}$$

S-WAVE πK/ηK SCATTERING

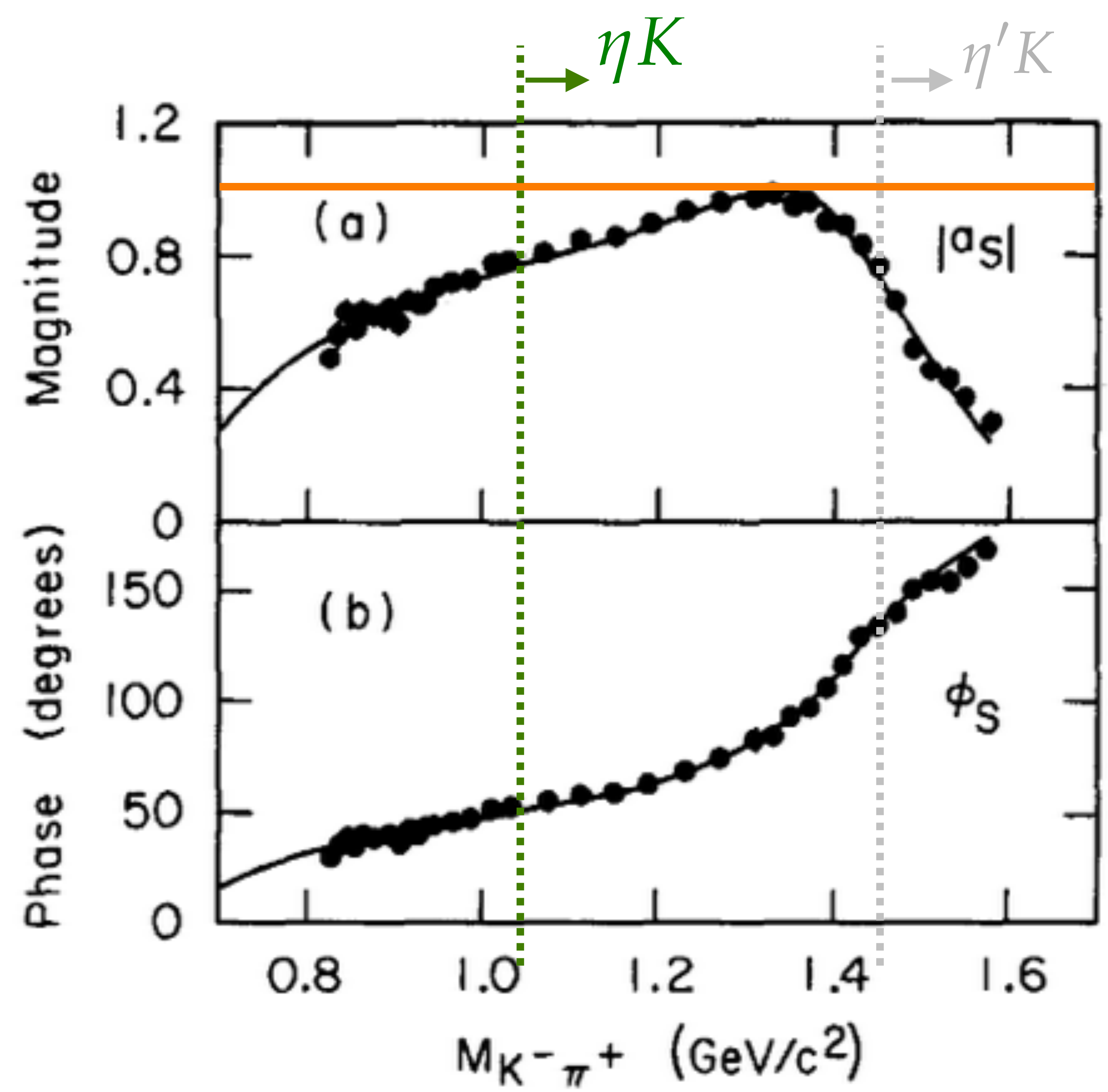


versus experimental scattering

S-WAVE πK→πK AMPLITUDE



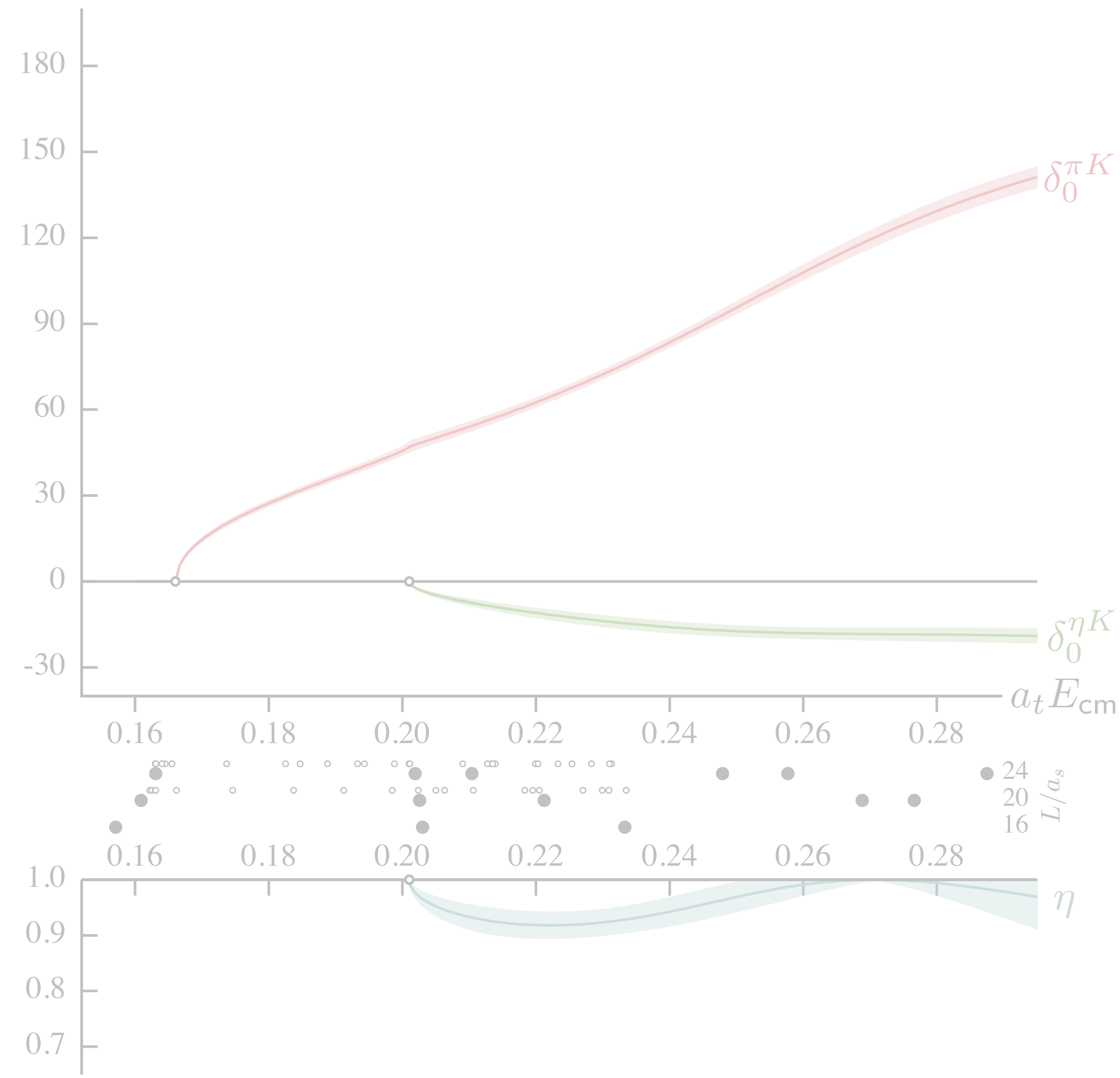
$m_\pi \sim 391$ MeV



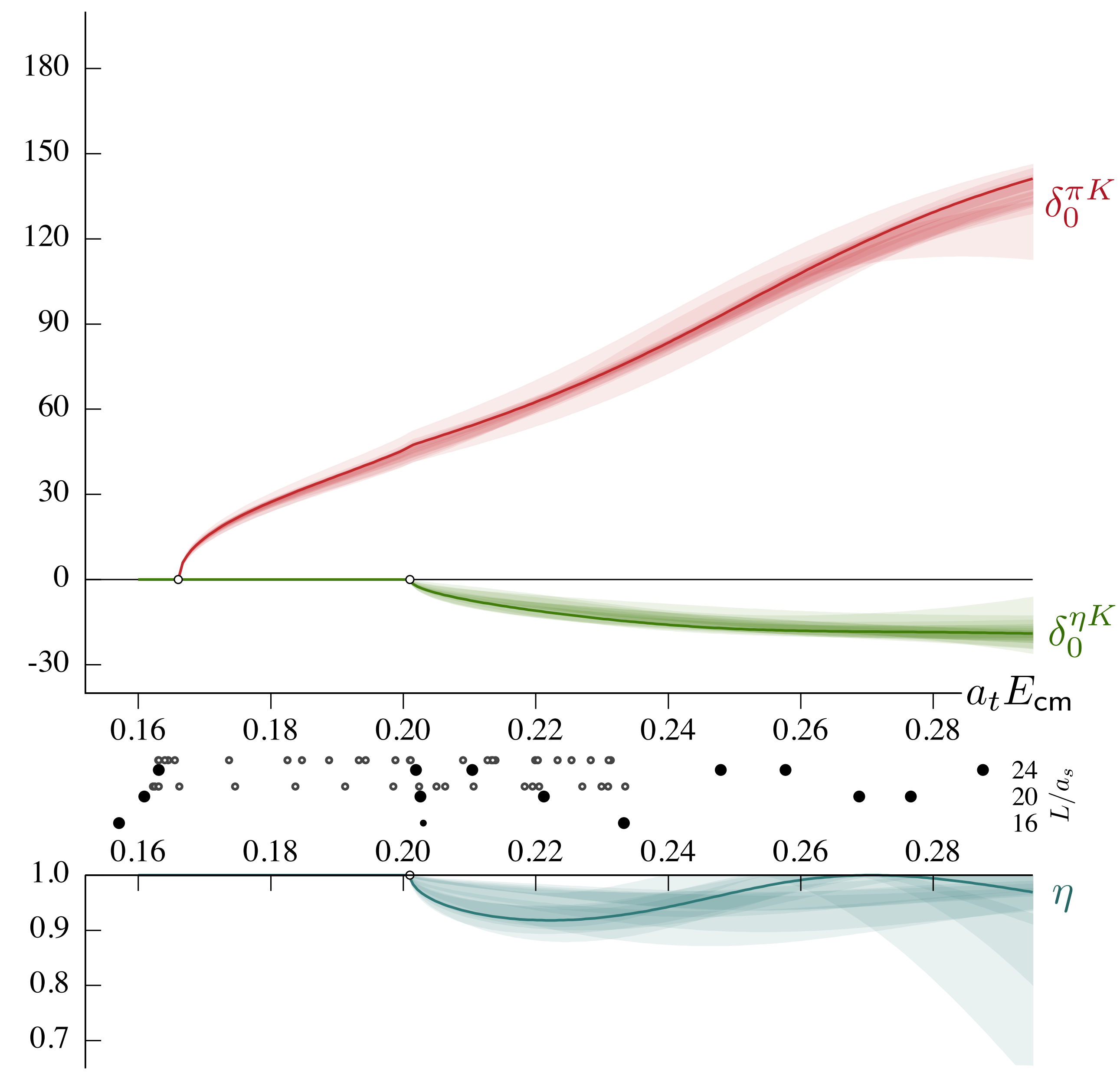
LASS, NPB296 493

πK/ηK scattering

- are the result parameterization dependent ?
  - try a range of parameterizations …



S-WAVE πK/ηK SCATTERING

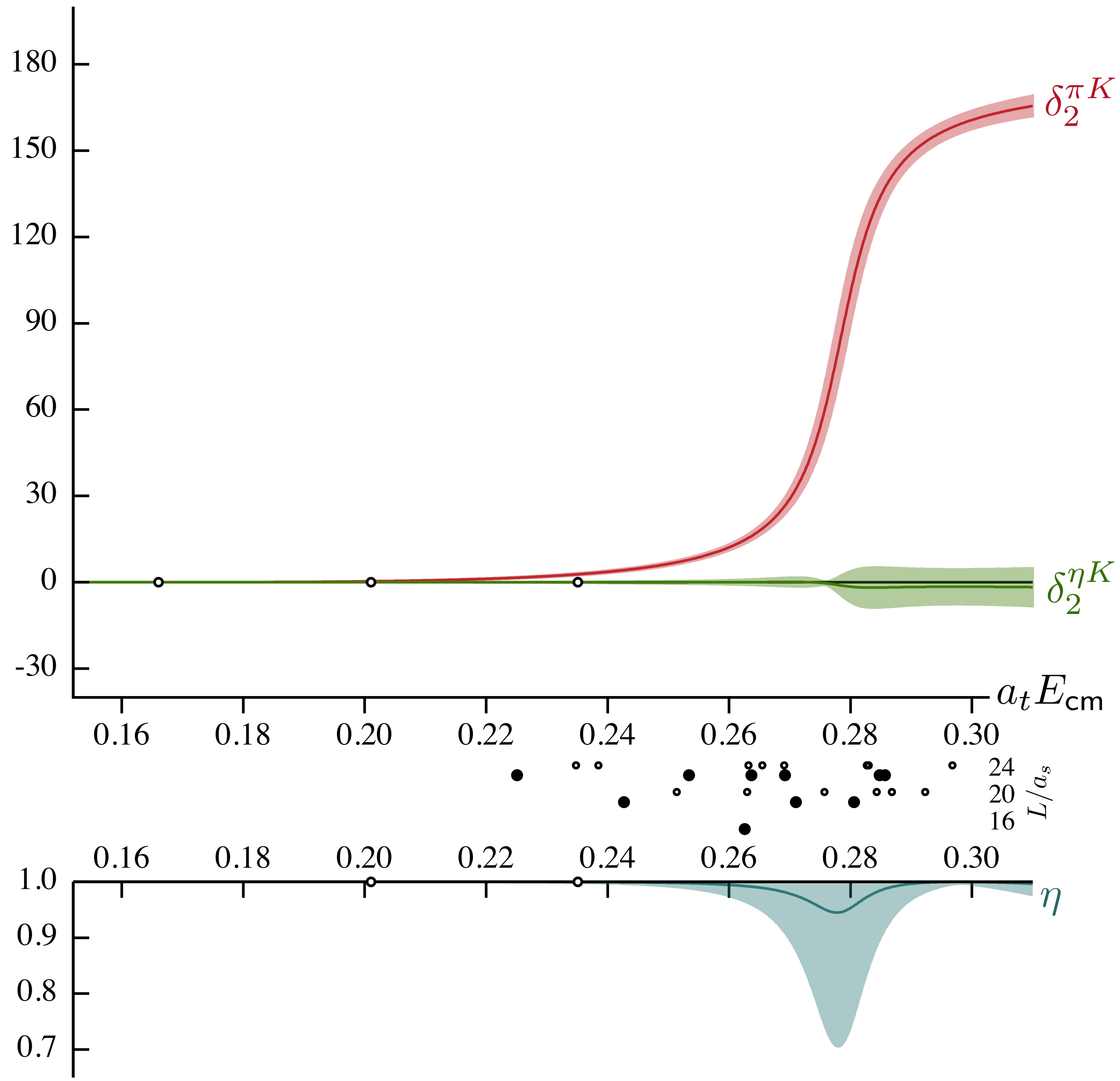


- gross features are robust

πK/ηK scattering

- clear narrow resonance in *D*-wave scattering

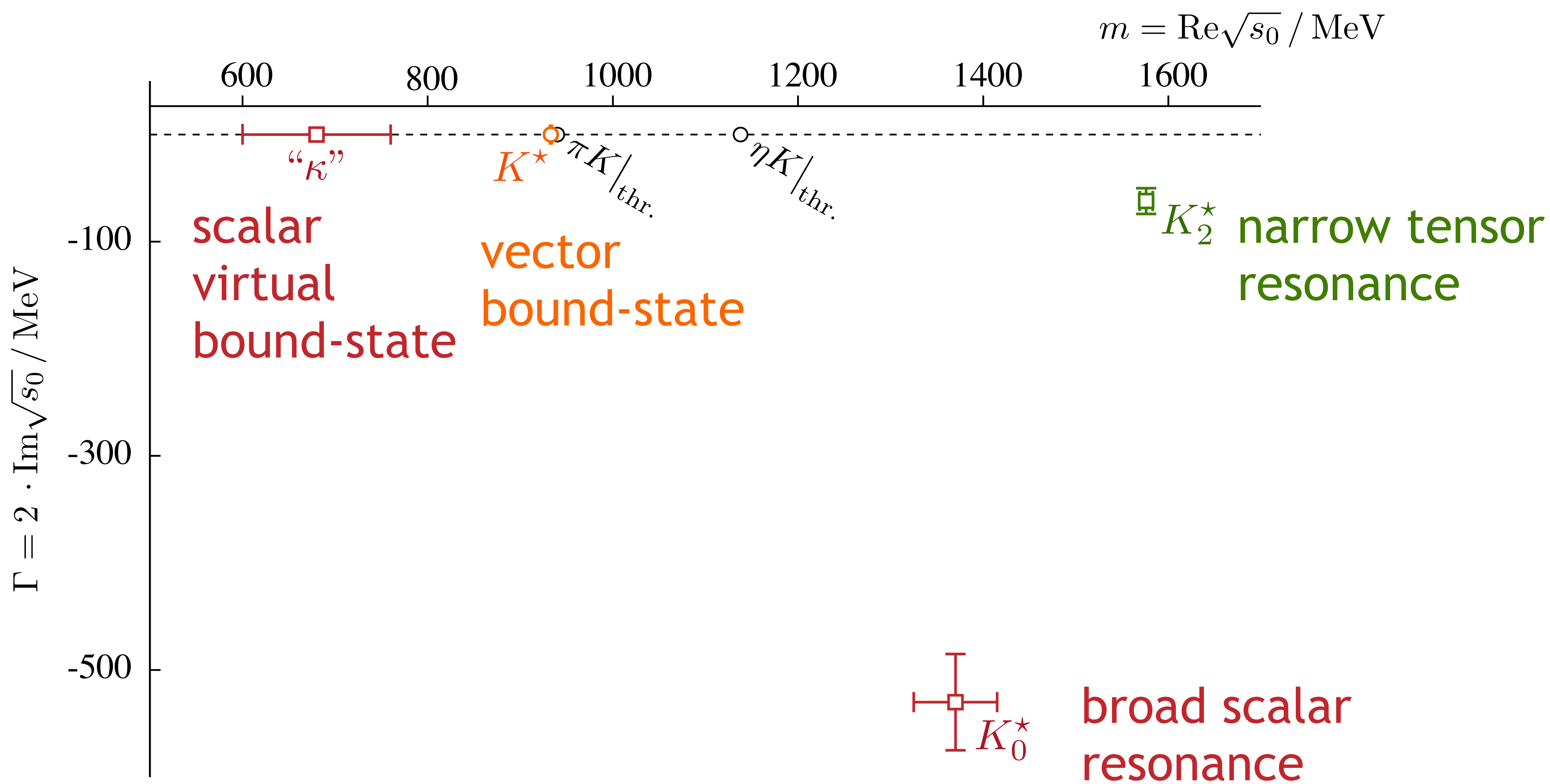
D-WAVE πK/ηK SCATTERING



$m_\pi \sim 391\,\mathrm{MeV}$

singularity content

- t-matrix poles as least model-dependent characterization of resonances



$m_\pi \sim 391\,\mathrm{MeV}$

hadron spectroscopy from lattice QCD

- excited hadron spectra from lattice QCD
  - qualitative features of the hadron spectrum
  - hybrid hadrons seem to play a role

- hadrons as resonances in scattering amplitudes from lattice QCD
  - elastic & coupled-channel
  - but at large quark masses - challenges to go lower

computational - big volumes, expensive

physics - multi-hadron final states