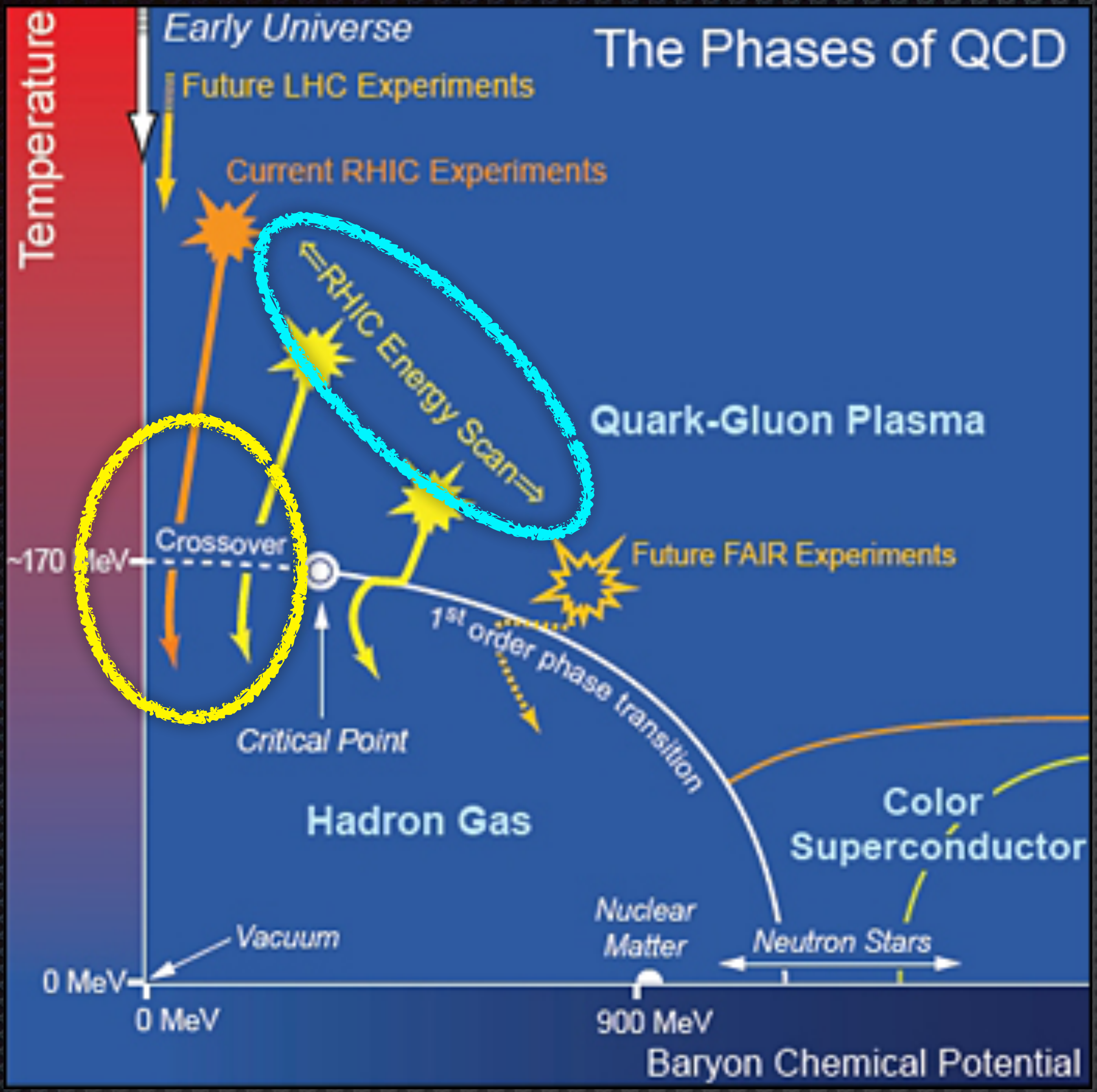
Baryon number distribution in Lattice QCD

Keitaro Nagata KEK

KN, S. Motoki, Y. Nakagawa, A. Nakamura, T. Saito [PTEP01A103(2012)]
A. Nakamura, KN [arXiv:1305.0760]
KN, K. Kashiwa, A. Nakamura, S. M. Nishigaki arXiv:1410.0783

Our scope



The Phases of QCD
Temperature
Early Universe
Future LHC Experiments
Current RHIC Experiments
⇐RHIC Energy Scan⇒
Quark-Gluon Plasma
~170 MeV
Crossover
Future FAIR Experiments
1st order phase transition
Critical Point
Hadron Gas
Color Superconductor
Nuclear Matter
Vacuum
Neutron Stars
0 MeV
0 MeV
900 MeV
Baryon Chemical Potential

Number distribution and Canonical approach

- A grand canonical ensemble ~ superposition of canonical ensembles

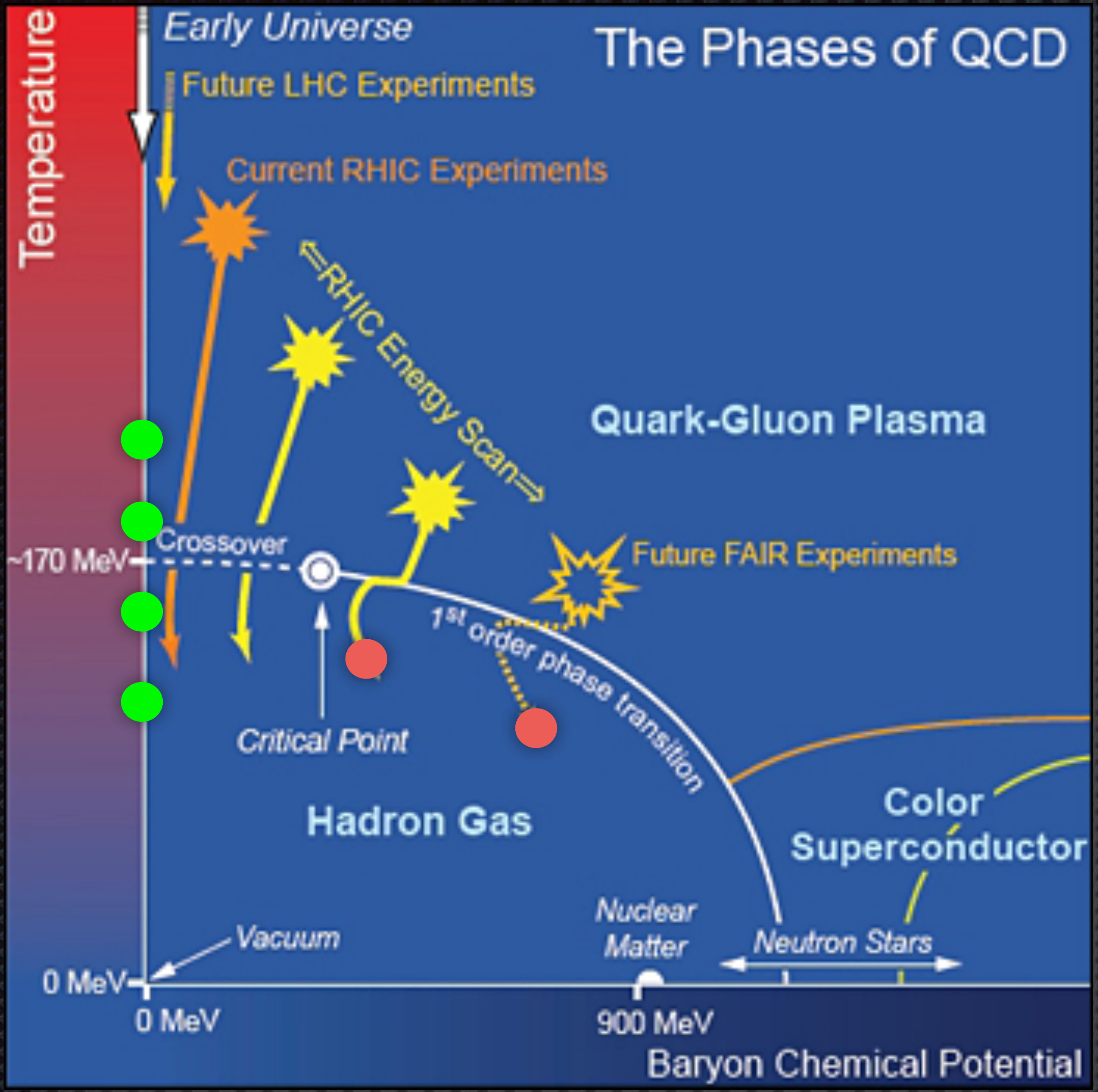
$$Z(\mu) = \mathrm{tr} e^{-\beta(\hat{H}-\mu\hat{N})}$$

$$= \sum_{n=-N}^{N} Z_n e^{n\mu/T}$$

(net quark/baryon) number distribution
= probability for an n-particle state

- μ-dependence of Z(μ) is determined.
- Once having Zn, Z(μ) can be obtained for any μ

What does it provide us with?



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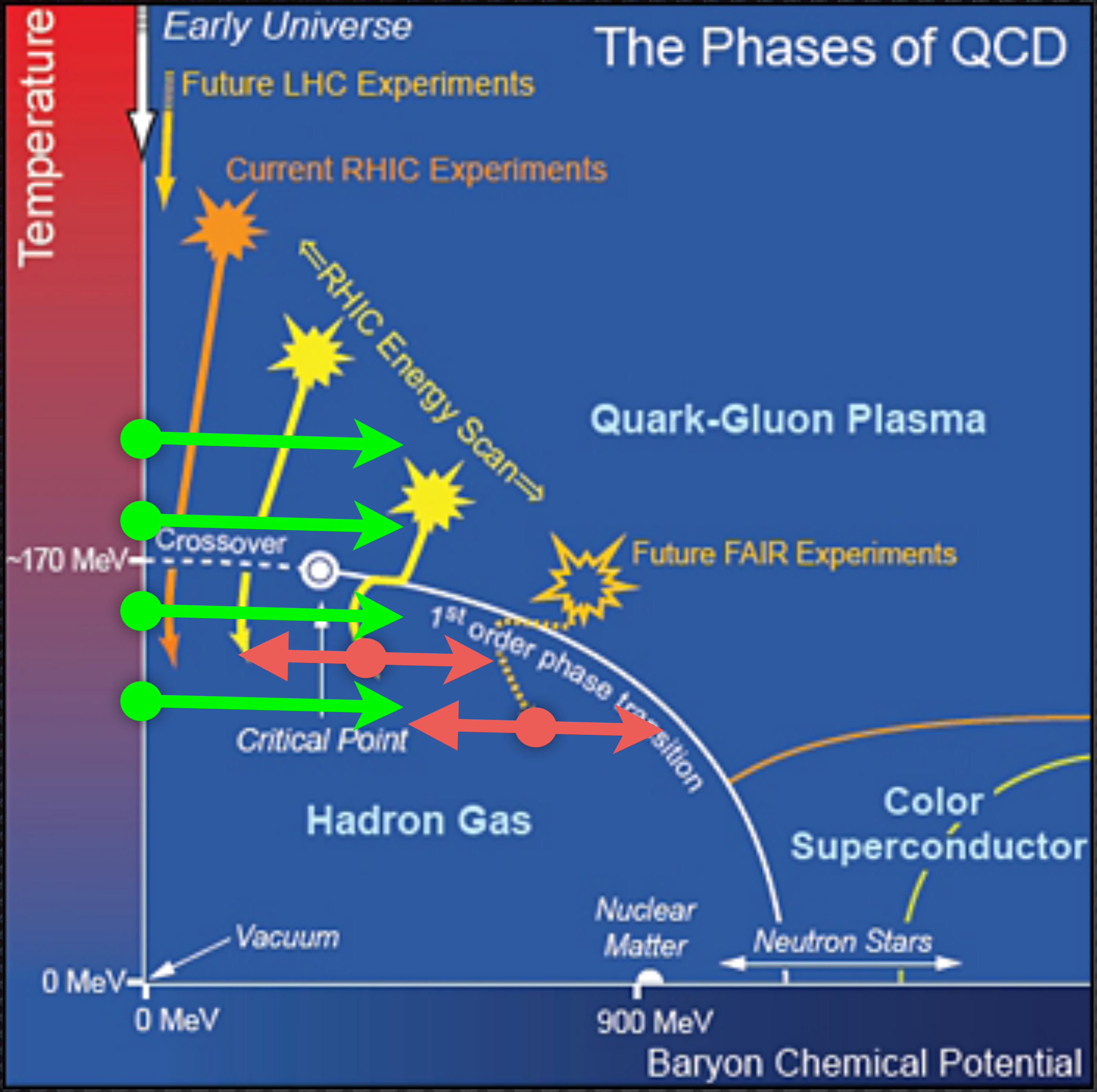


$$Z(\mu) = \mathrm{tr} e^{-\beta(\hat{H} - \mu \hat{N})}$$

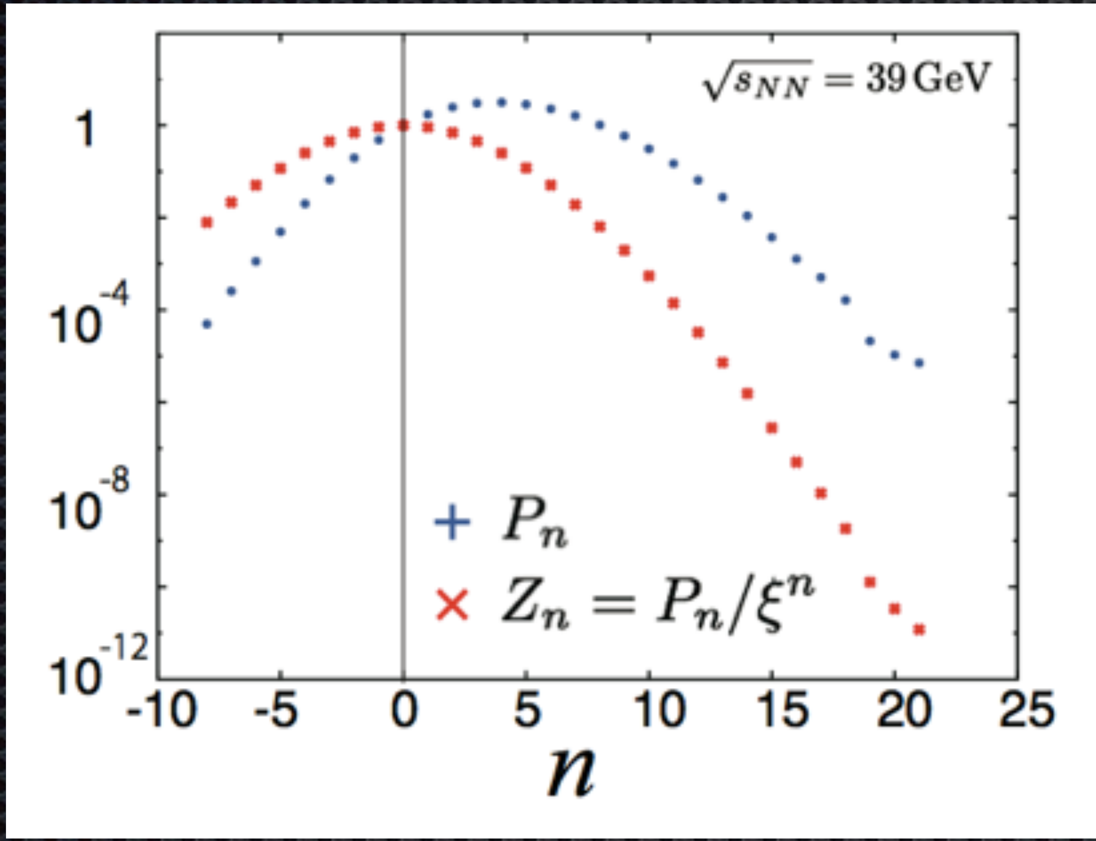
$$= \sum_{n=-N}^{N} Z_n e^{n\mu/T}$$

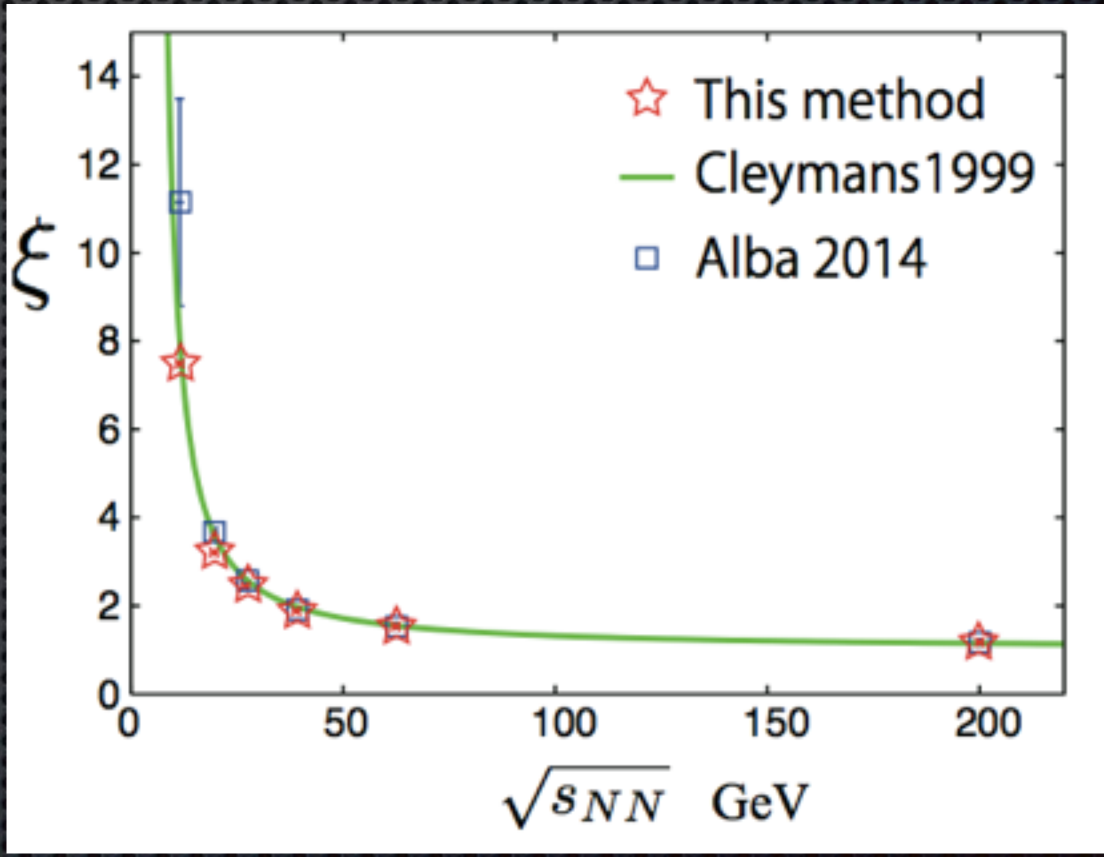
Canonical Partition Functions in experimental data

Extraction of Zn from Pn

using CP invariance : Z(n)= Z(-n)

A. Nakamura(on 10/5 morning)

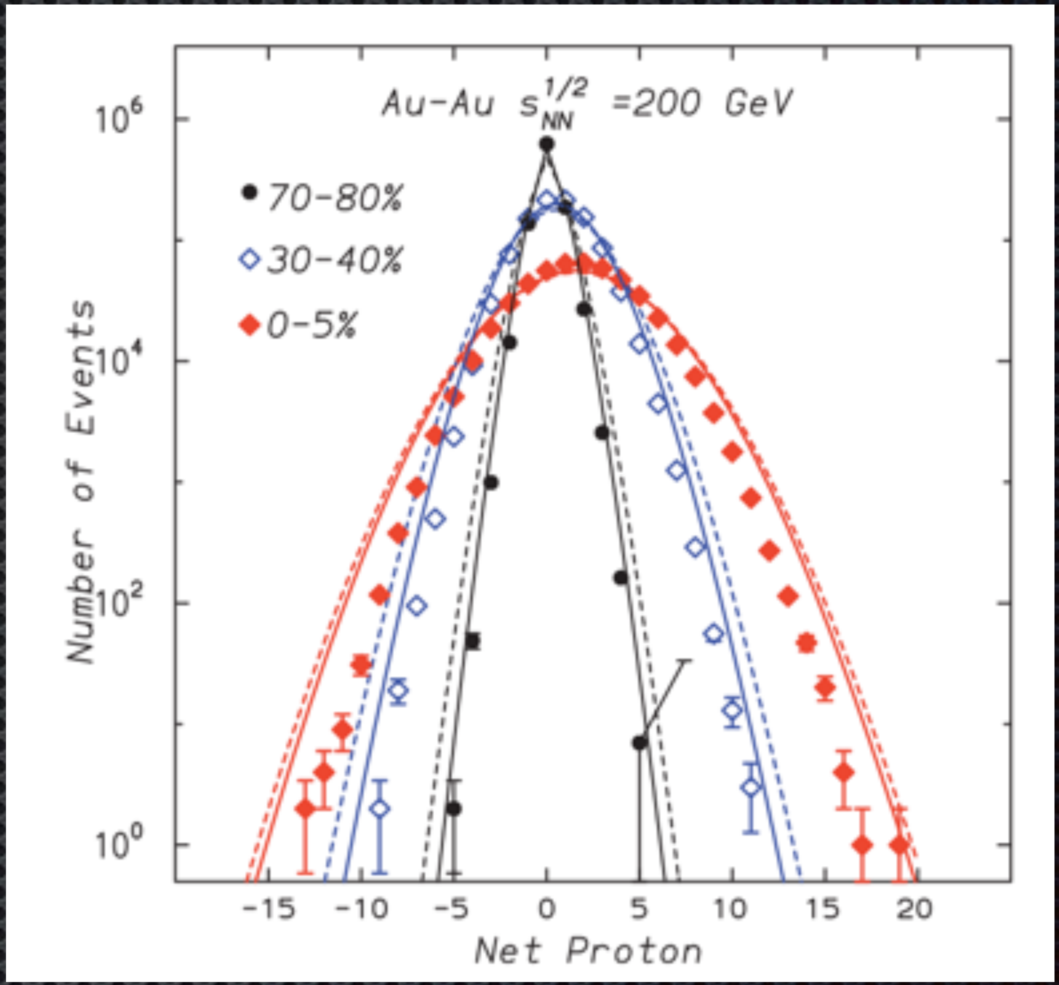




What's the situation ?

For some cases, the distribution is approximately a Skallam type (two-Poisson)

- **Proton (not baryon) multiplicity at RHIC v.s. HRG's Distribution (Skellam with parameters determined from freeze-out ones)**
- **HRG consistent with BES data at 200 GeV peripheral collision**



Braun-Munzinger, et al.

PRC84, 064911(2011)

Canonical partition function in Lattice QCD simulations

Computational Nuclear Physics 10/05-07, Hilton Waikoloa Village, Hawaii

Calculation of Zn in lattice QCD simulation

- Canonical approach ~ valuable tool to circumvent sign problem

$$Z(\mu) = \mathrm{tr} e^{-\beta(\hat{H}-\mu\hat{N})}$$

$$= \sum_{n=-N}^{N} Z_n e^{n\mu/T}$$

Calculable at μ=0

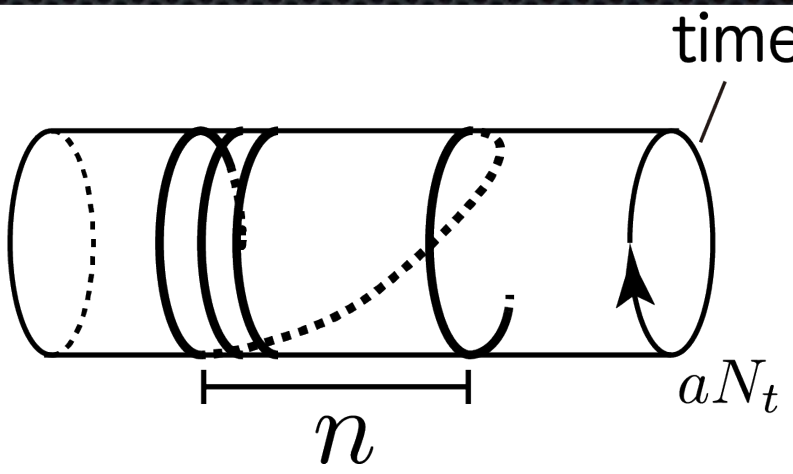
Canonical approach in lattice QCD

- Barbour et. al. / Hasenfratz & Touissaint / de Forcrand & Kratochvila / Alexandru, Li, Liu (Kentucky)/ Ejiri/KN, Nakamura

Calculation of Zn in lattice QCD simulation

$$\det \Delta = \xi^{-N_{\rm red}/2} C_0 \prod (\xi + \lambda_n)$$

$$= C_0 \sum_{n=-N_{\rm red}/2}^{N_{\rm red}/2} c_n \xi^n$$



eigenvalues of a transfer matrix

[Gibbs ('86). Hasenfratz, Toussaint('92). Adams('03, '04), Borici('04). KN&AN('10), Alexandru &Wenger('10)]

we use gauge configurations at μ=0 (reweighting)

$$Z_n = \left\langle \frac{C_0^{N_f} d_n}{(\det \Delta(0))^{N_f}} \right\rangle$$

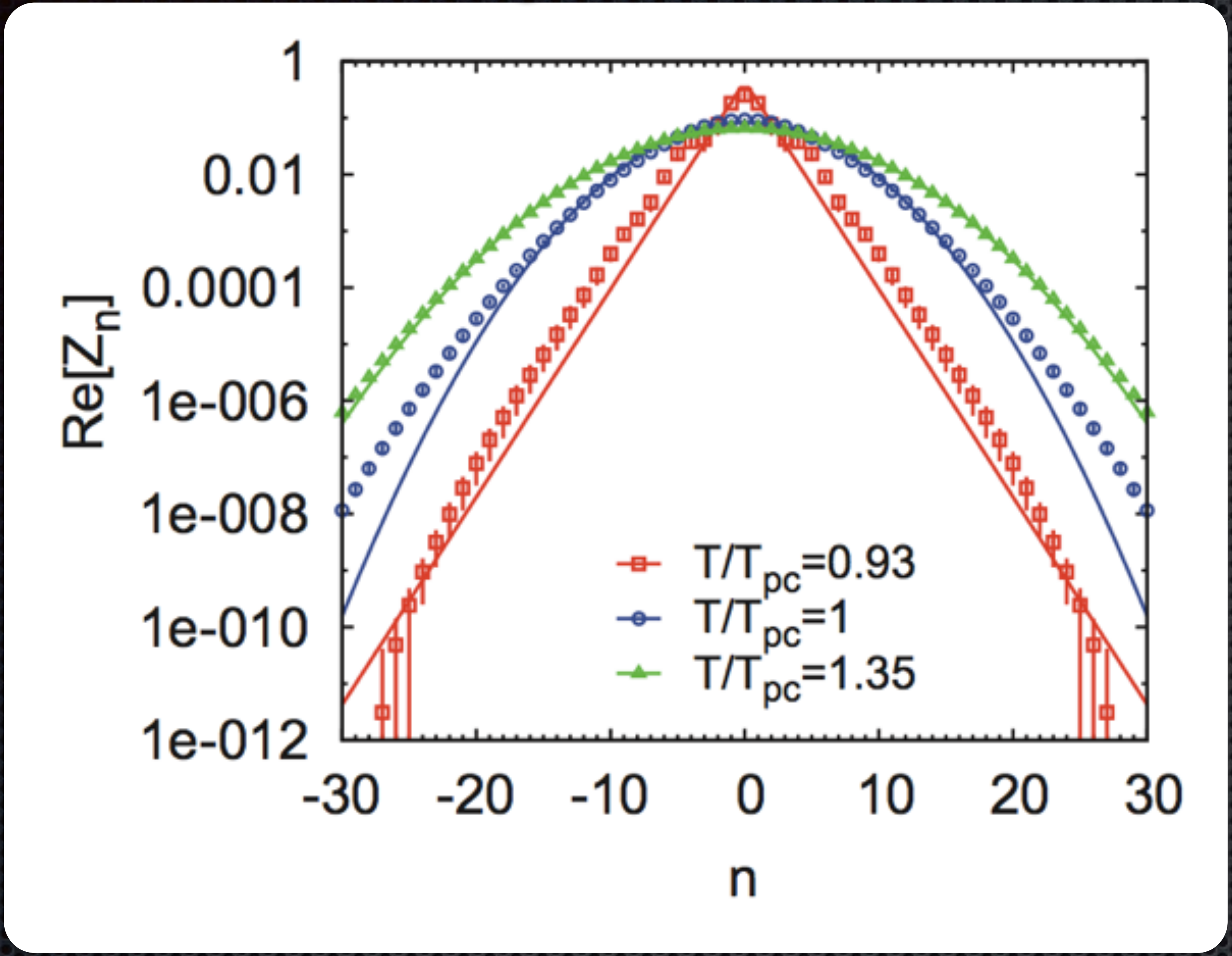
KN, S. Motoki, Y. Nakagawa, A. Nakamura, T. Saito [PTEP01A103(2012)], references therein.

Lattice simulations

- reweghting in μ
- volume : 8^3x4, 10^3x4
- mass : mps/mv~0.8
- action : clover-improved Wilson fermion +renormalization improved gauge
- # of statistics : 400 (20 trajectoriy-intervals, 3000 therm.)

Canonical partition functions in lattice QCD

- KN, S. Motoki, Y. Nakagawa, A. Nakamura, T. saito, PTEP(2012).

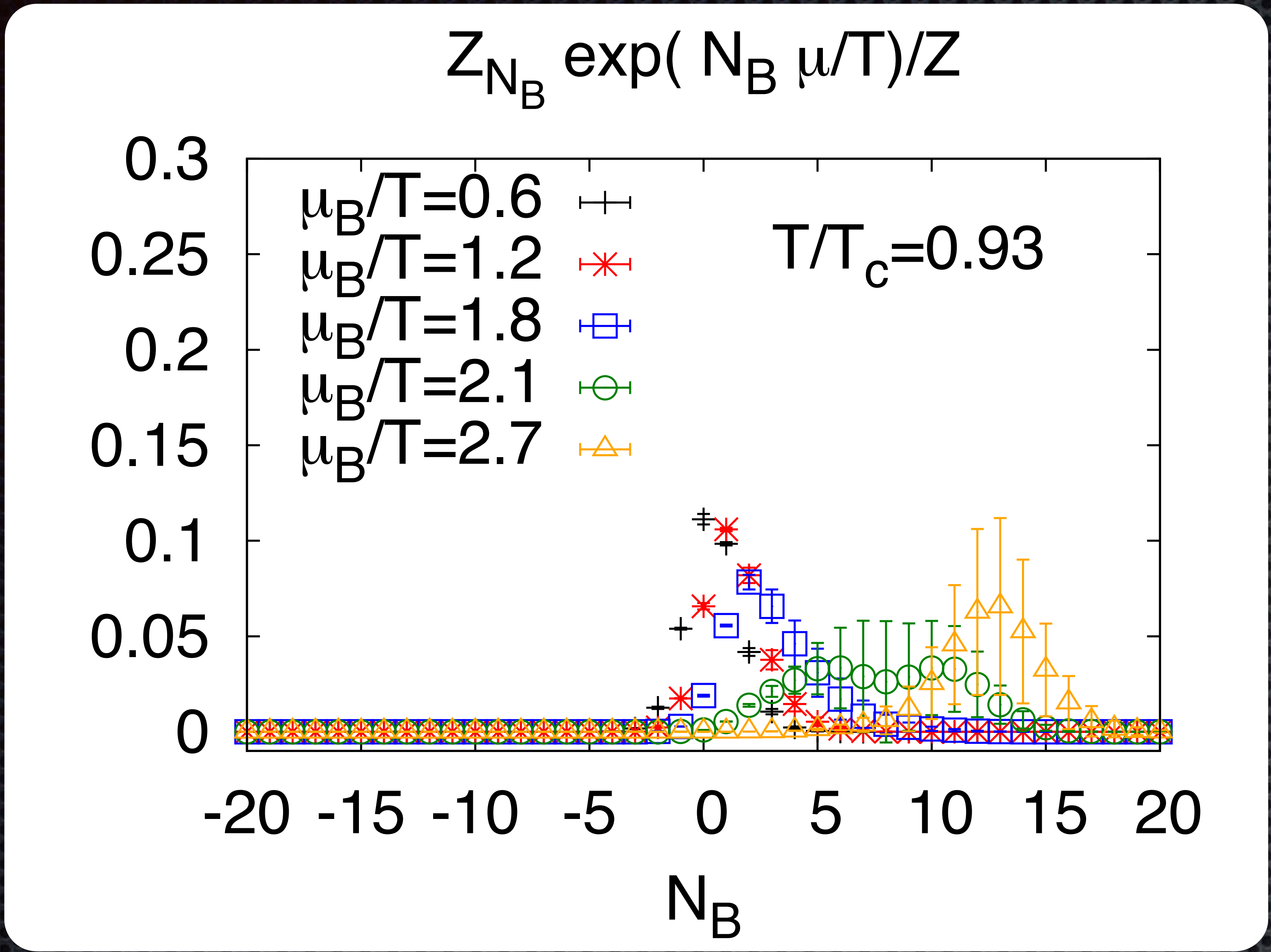


high T : Gaussian

low T : linear like

Effective d.o.f increases as T increases

Baryon number distribution & fluctuations



T/Tc = 0.93
μ_B/T = 1.8 : right tail
μ_B/T = 2.1 : flat
μ_B/T = 2.7 : left tail

(At high T, BND is Gaussian)

Shape of the canonical partition function

μ-dependence of the baryon number distribution

Lee-Yang zeros : from CPF to Phase transition

- **Lee-Yang zeros [Lee & Yang 1952]**

$$Z(\mu) = \sum Z_n e^{n\mu/T}$$
$$\propto \prod (1 - \xi/\xi_i)$$

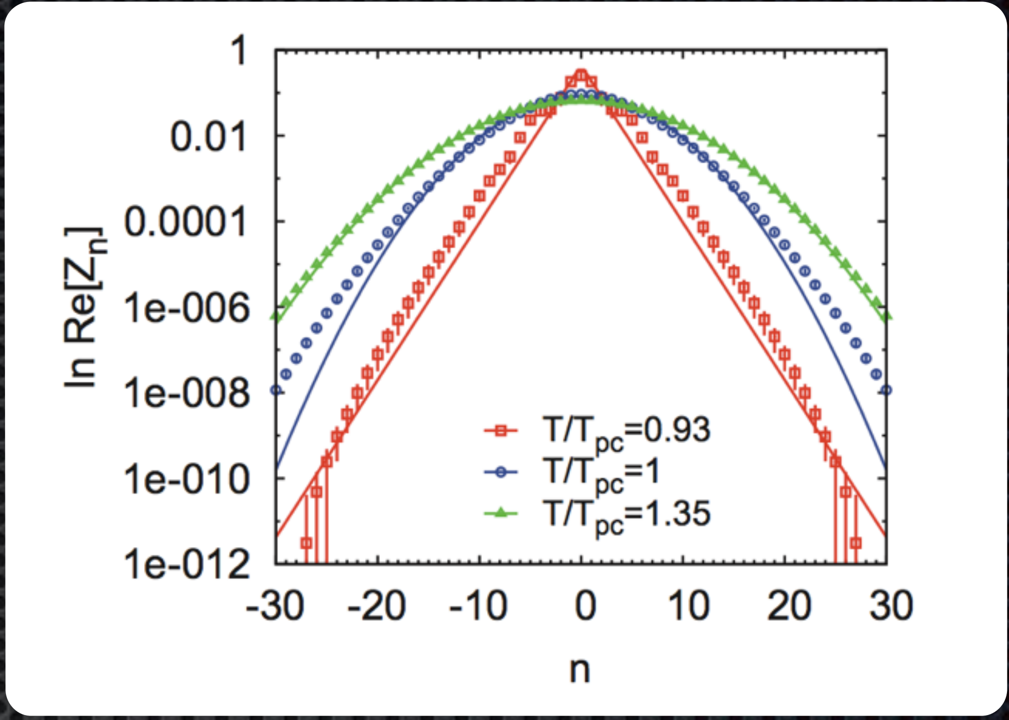
- **Z(μ) = 0 is an origin of a thermodynamic non-analyticity**

How to achieve LY zeros ?

- **Calculation of Zn : truncation is inevitable**

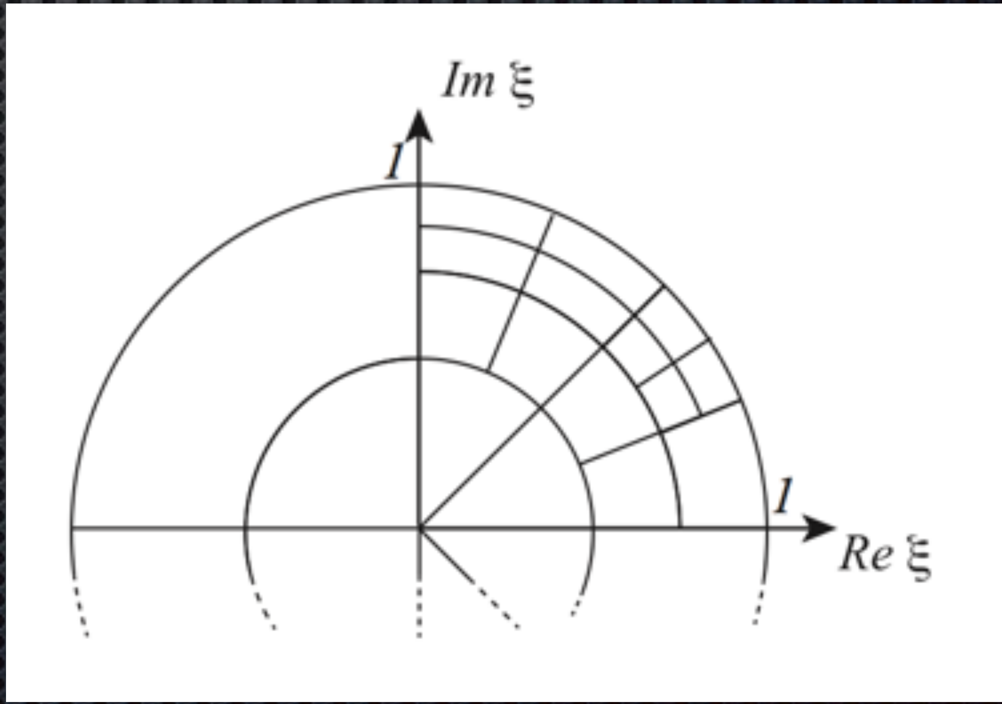
$$Z(\mu) = \sum_{n=-\infty}^{\infty} Z_n e^{n\mu/T}$$

$$\rightarrow \sum_{n=-n_0}^{n_0} Z_n e^{n\mu/T}$$



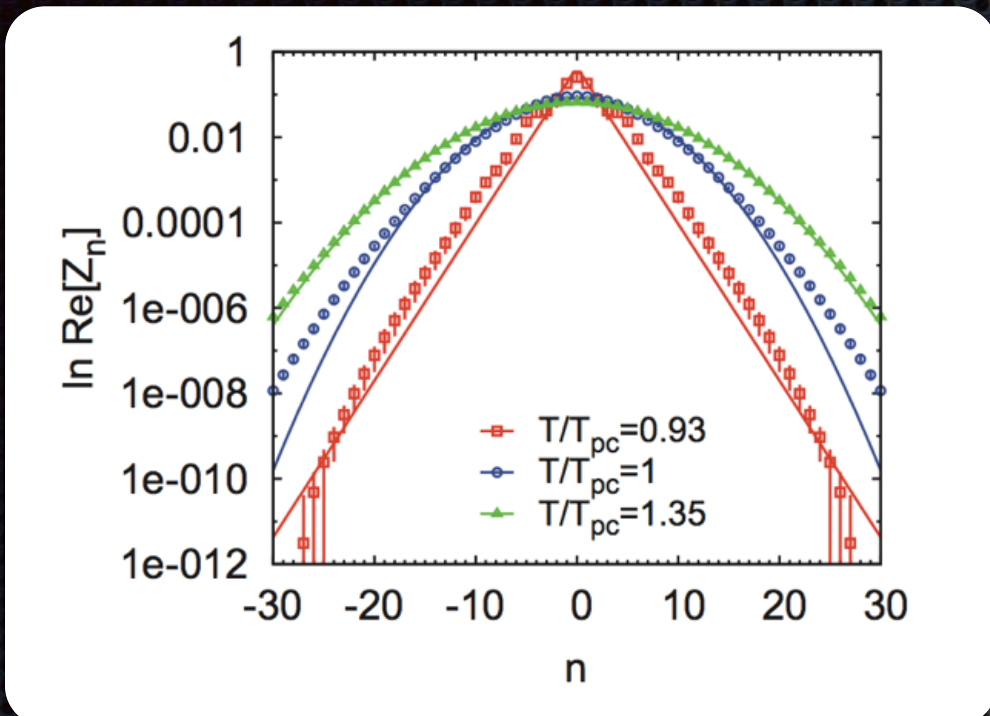
- **Cauchy integral + recursive division + multi-precision arithmetic**

$$Z(\mu) = \sum Z_n e^{n\mu/T} \rightarrow \prod (1 - \xi/\xi_i)$$



Canonical partition functions in lattice QCD

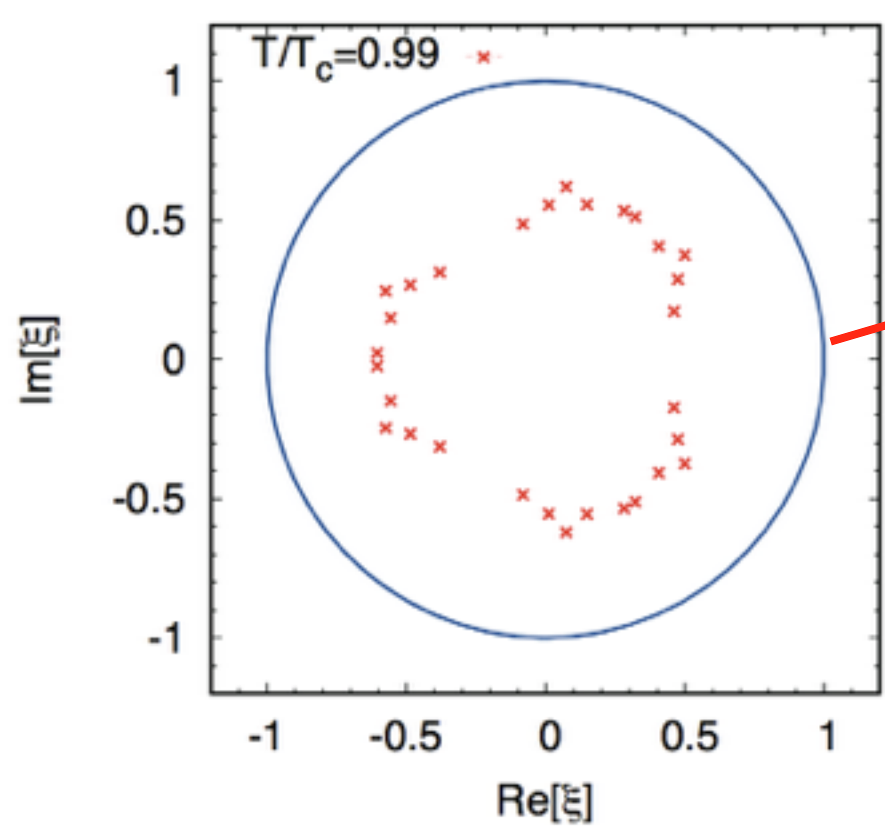
- [Nakamura, Nagata(2013)]



CPF , LYZ

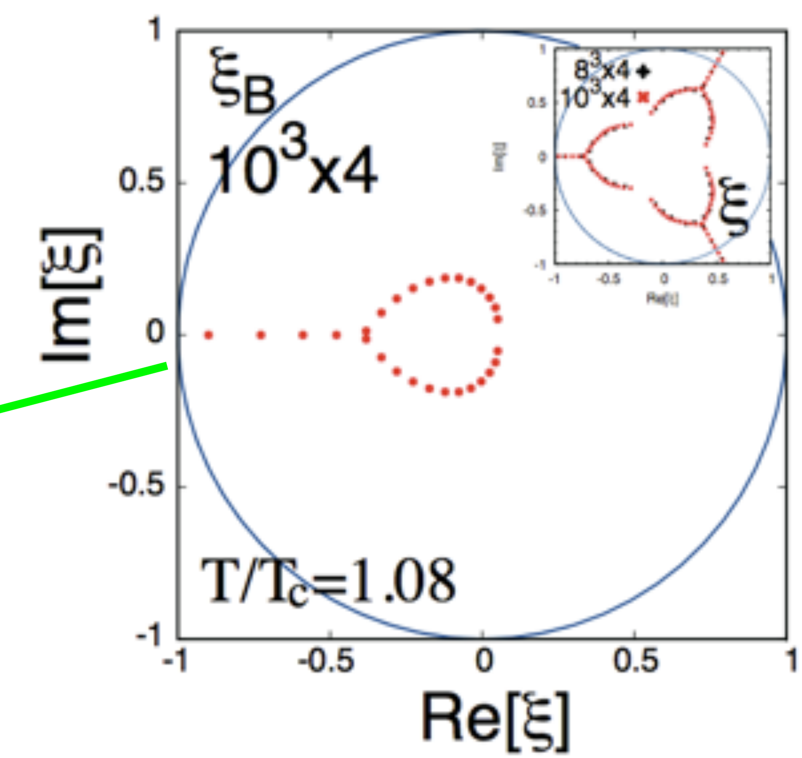
low T : linear like, circle-like

high T : Gaussian, radial line(RW)



crossover

1st order (RW phase transition)



Questions

- We truncated the fugacity polynomial

$$Z(\mu) = \sum_{n=-\infty}^{\infty} Z_n e^{n\mu/T} \to \sum_{n=-n_0}^{n_0} Z_n e^{n\mu/T} + (|n| > n_0)$$

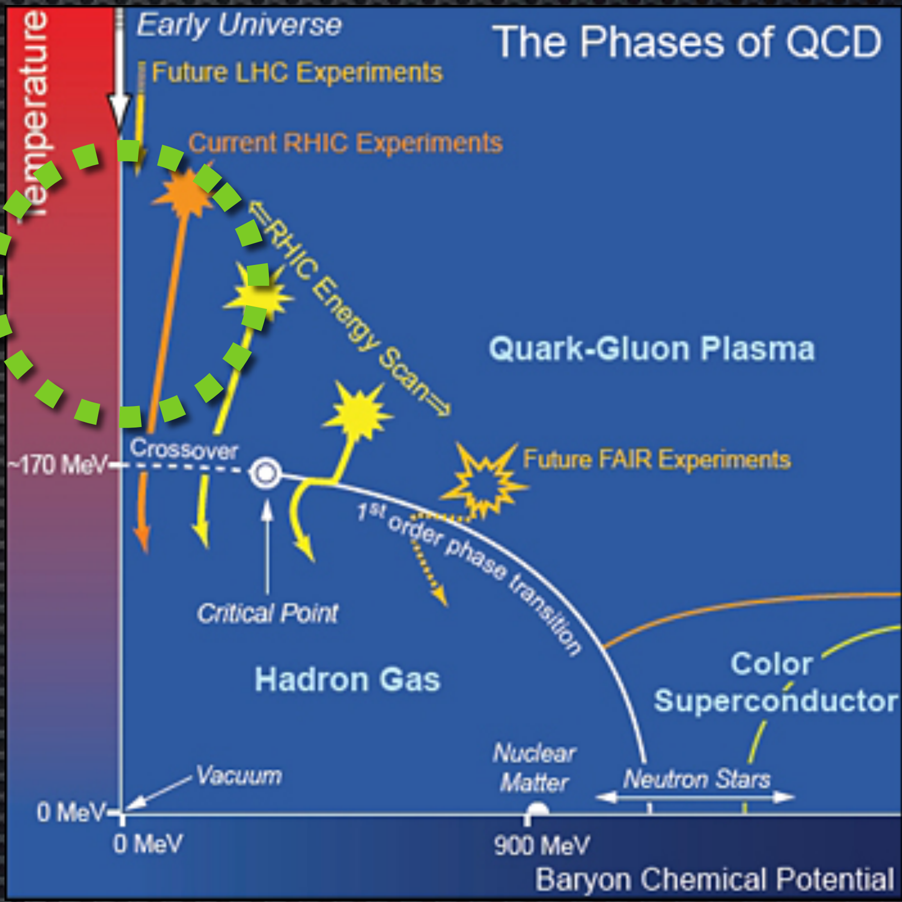
  - convergence/statistical errors have to be examined to obtain quantitative results

- Volume scaling is necessary to understand the order of phase transition
  - (e.g. 1/V for 1st)

LY zero distribution of high temperature QCD

To answer the question, [arXiv:1410.0783]

We focus on high T region, and



1. solve Lee-Yang zeros analytically

2. examine the convergence and numerical error of lattice data

Derivation of CPF at high T

We derive an analytic expression of CPF from the Stefan-Boltzmann type of free energy at high T.

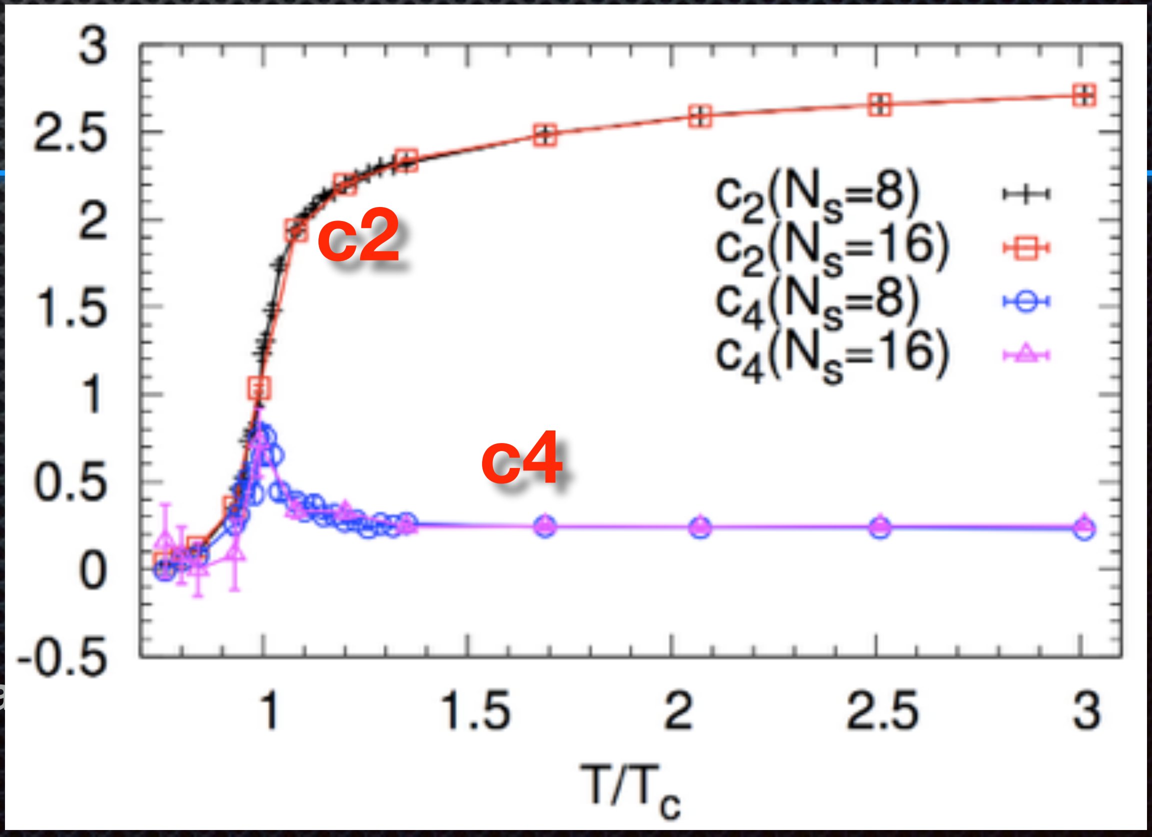
$$Z(\mu) = \sum_n Z_n e^{n\mu/T} \qquad \mu/T = i\theta$$

$$Z_n = \int d\theta e^{-Vf(\theta)/T} e^{in\theta}$$

$$-f(\mu) = c_0 + c_2(\mu/T)^2 + c_4(\mu/T)^4$$
$$= c_0 - c_2\theta^2 + c_4\theta^4$$

$$Z_n = \int d\theta e^{-VT^3\theta^2} e^{in\theta}$$

We can use a saddle point approximation



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$$Z_n = \int d\theta e^{-VT^3\theta^2} e^{in\theta}$$

$$Z_n = Ce^{-n^2/(4T^3Vc_2)}, \quad (n \equiv 0 \mod 3)$$

RWperiodicity(Z(3))

Derivation of Lee-Yang zeros

Gaussian type of an infinite series ~ Jacobi-theta function

$$\vartheta(z,\tau) = \sum_{n=-\infty}^{\infty} e^{-\pi i n^2 \tau + 2\pi i n z}$$

$$\vartheta(z,\tau) = 0 \leftrightarrow z = m + n\tau + \frac{1}{2} + \frac{\tau}{2}$$

$$Z(\mu) = C \sum e^{-9n^2/(4VT^3 c_2) + 3n\mu/T}$$

$$= C\vartheta(z,\tau)$$

$$\frac{\mu}{T} = \frac{(2k+1)\pi i}{3} - \frac{3(2l+1)}{4VT^3 c_2}$$

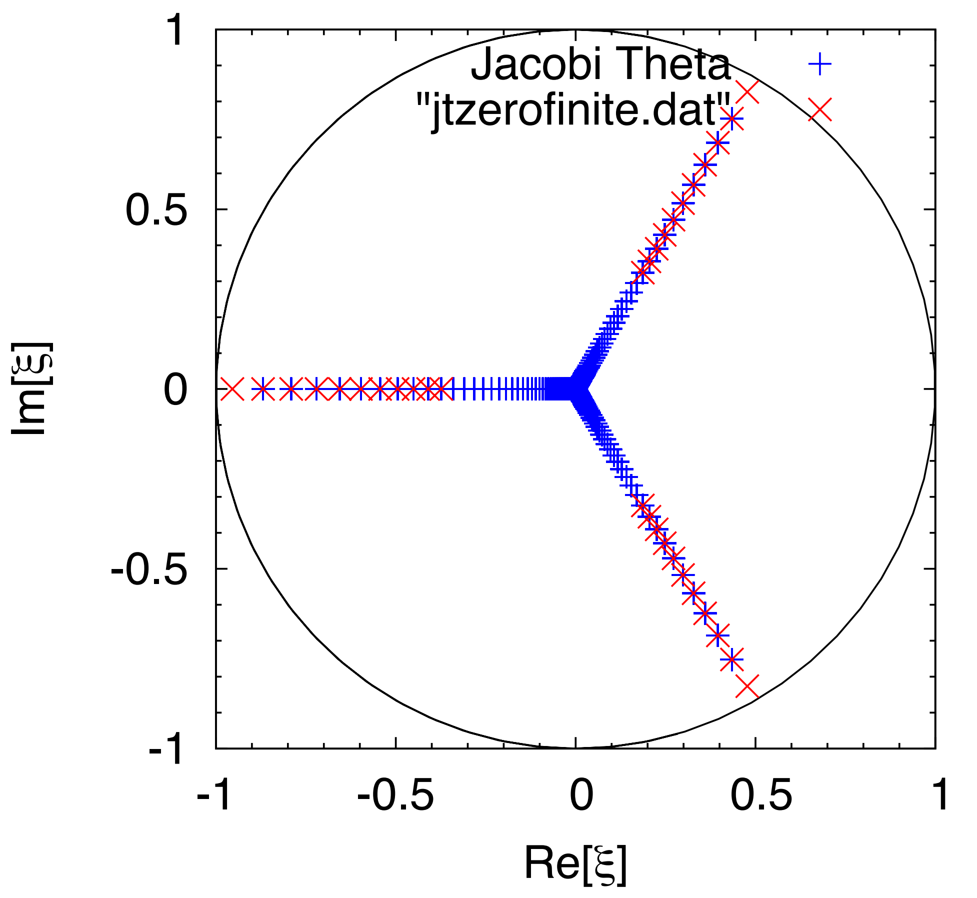
Lee-Yang zero distribution of theta function

We find an analytic expression of Lee-Yang zeros

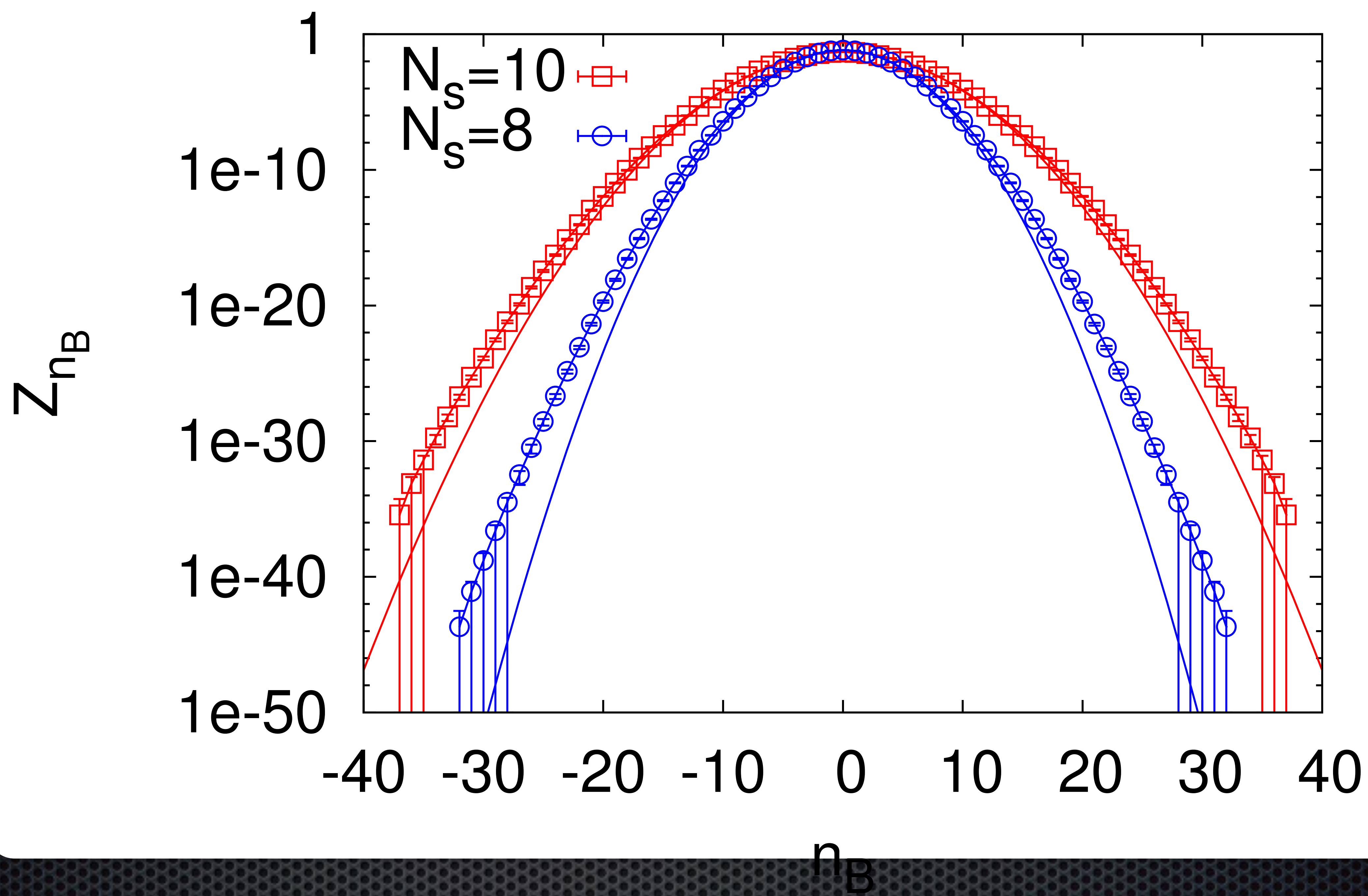
$$\frac{\mu}{T} = \frac{(2k+1)\pi i}{3} - \frac{3(2l+1)}{4VT^3 c_2}$$

$$\xi = \exp(-\mu/T)$$

phase **radial**



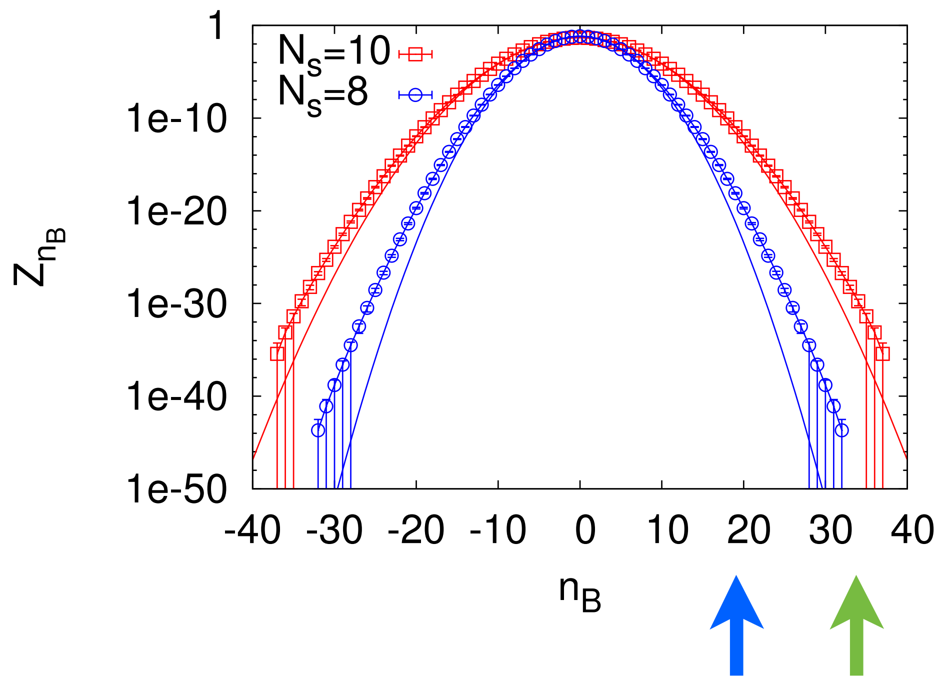
Saddle point approximation vs data (T/Tc=1.2)

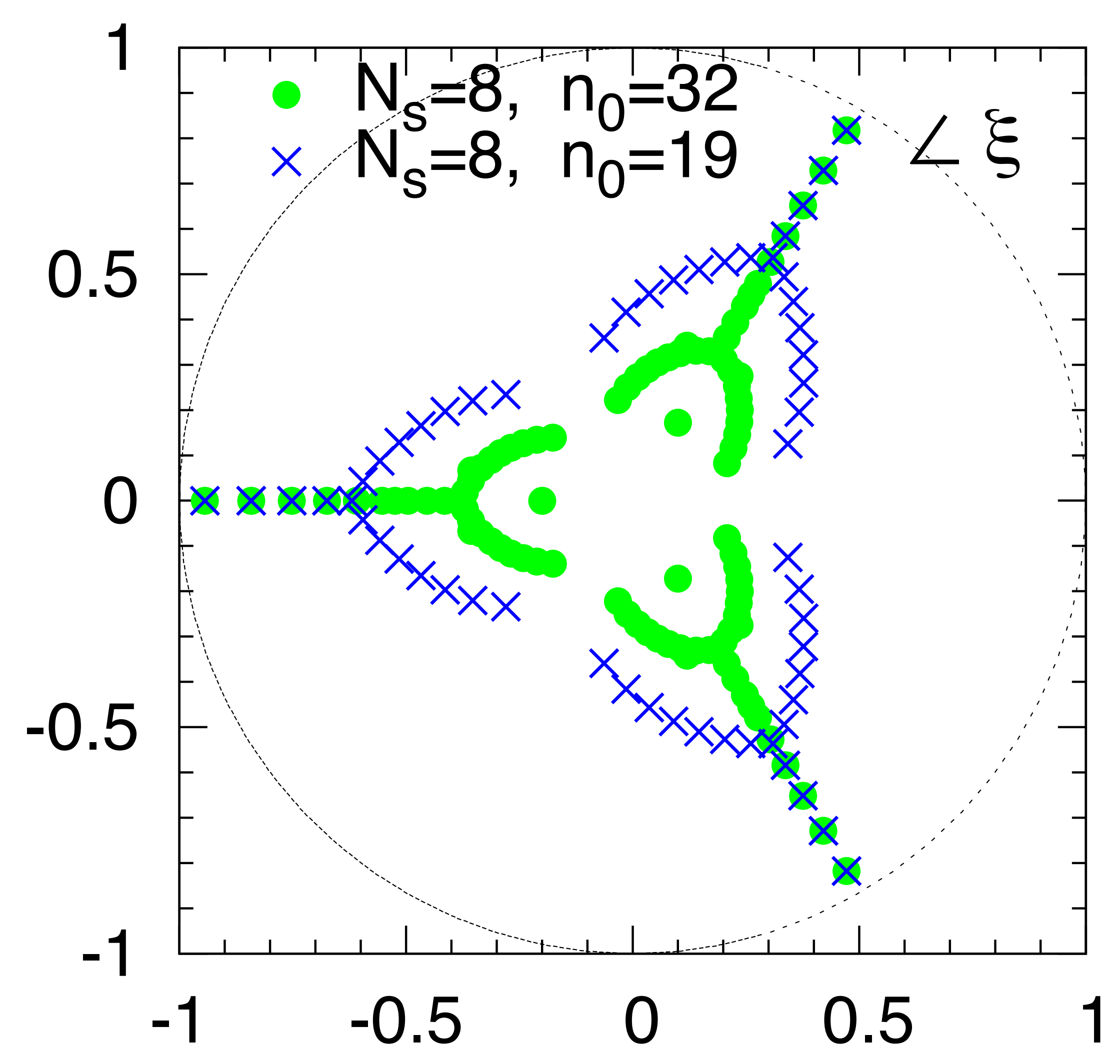


Solid lines Gaussian function, where c2 are obtained from lattice simulation

To confirm convergence

$$Z(\mu) = \sum_{|n|\leq n_0} Z_n \xi^n$$

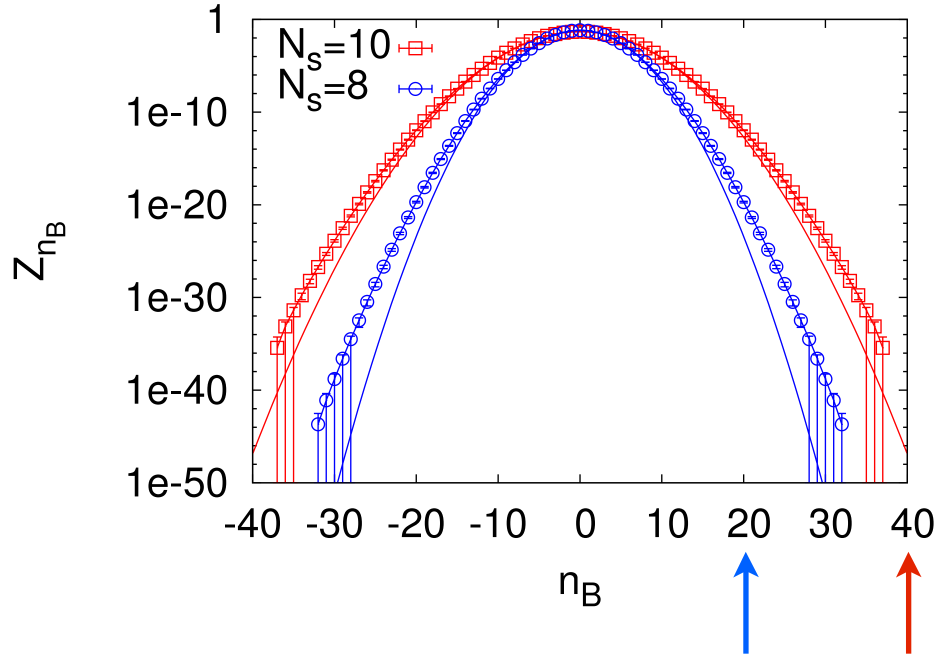




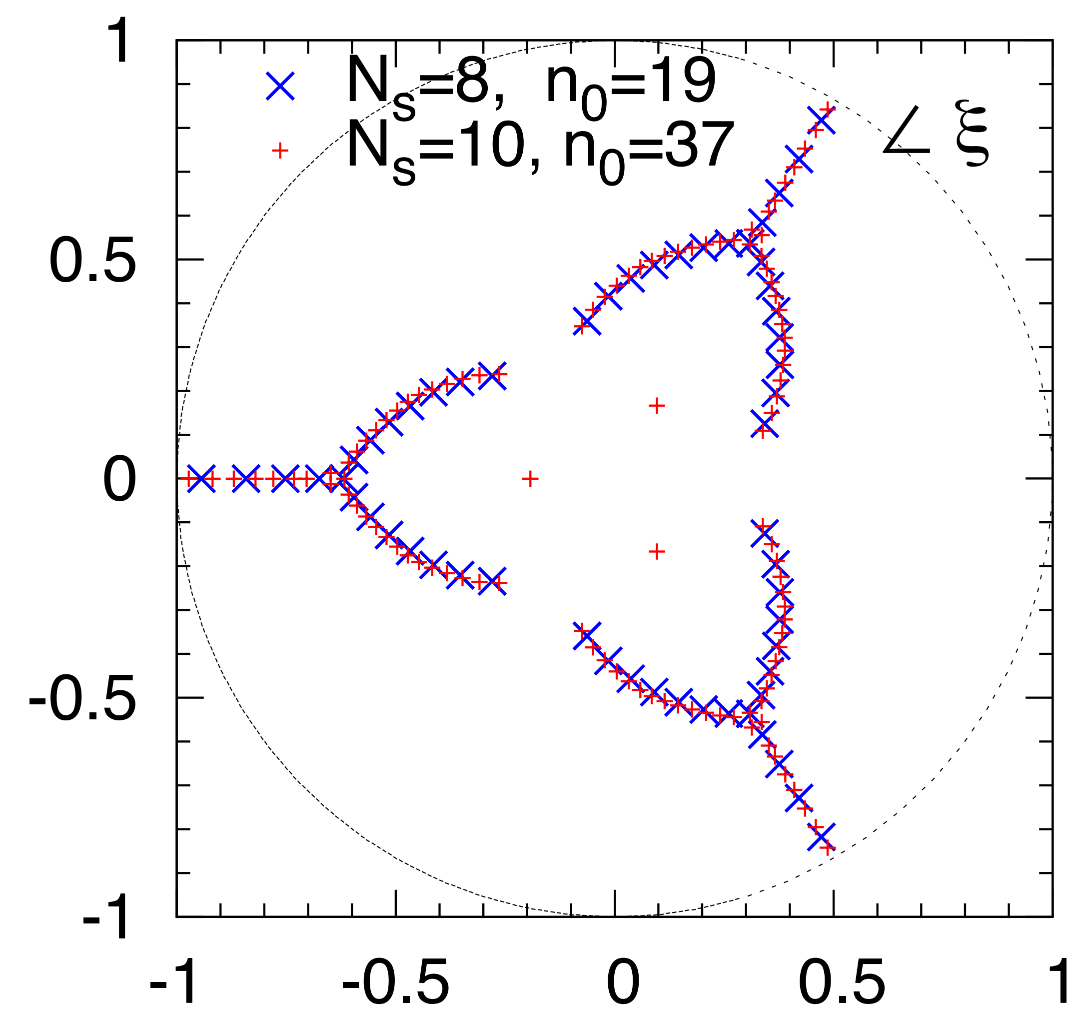
We compare n0=19 and 32 for Ns=8

To see volume scaling

$$Z(\mu) = \sum_{|n| \le n_0} Z_n \xi^n$$



$n_0 \propto V$



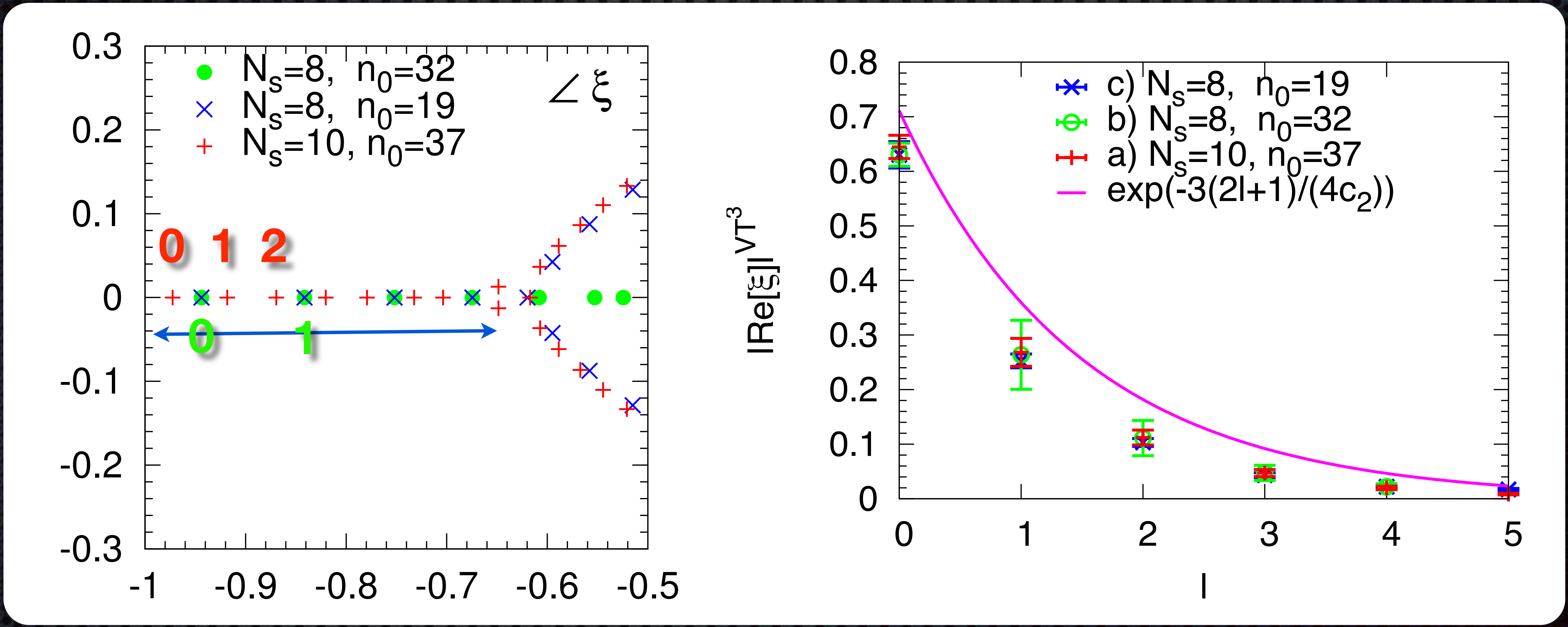
We compare (n0,Ns)=(19, 8) and (37, 10)

Error analysis and Lattice vs Analytic

A scaling predicted by theta function

$$|\mathrm{Re}[\xi]|^{VT^3} = \exp(-(3l+1)/(4c_2))$$

Statistical error is estimated by a bootstrap method(1000 samples)

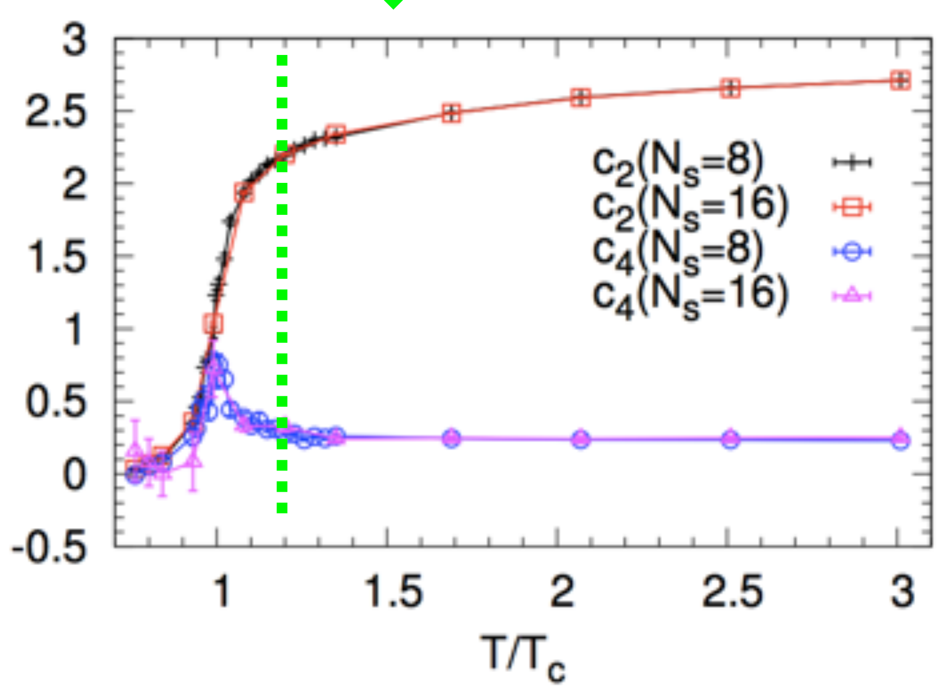


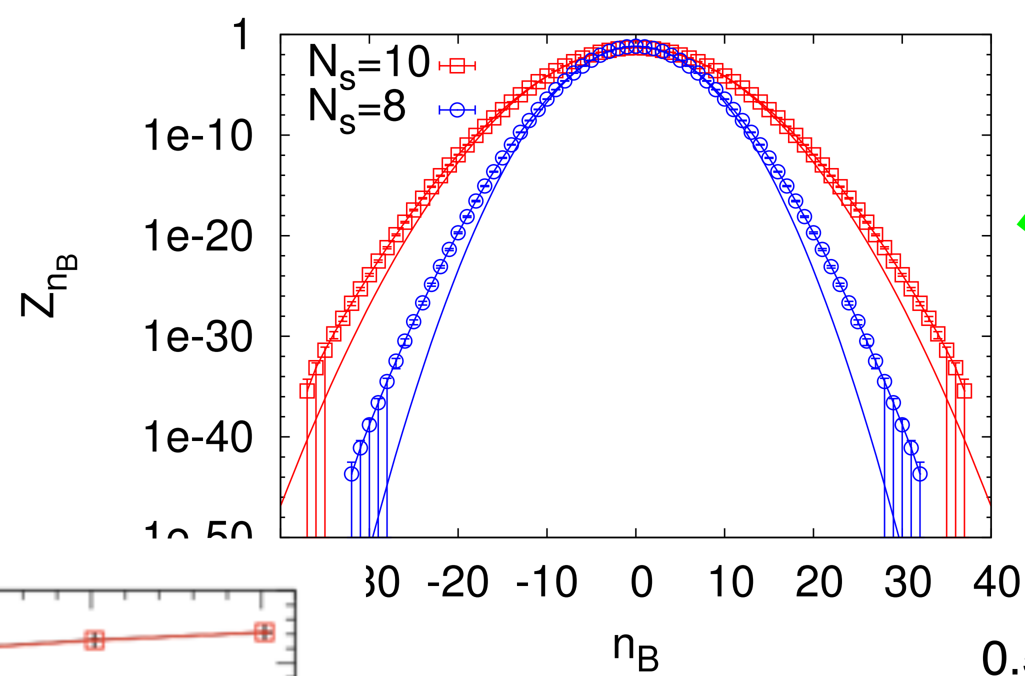
What does the result tell?

Indication

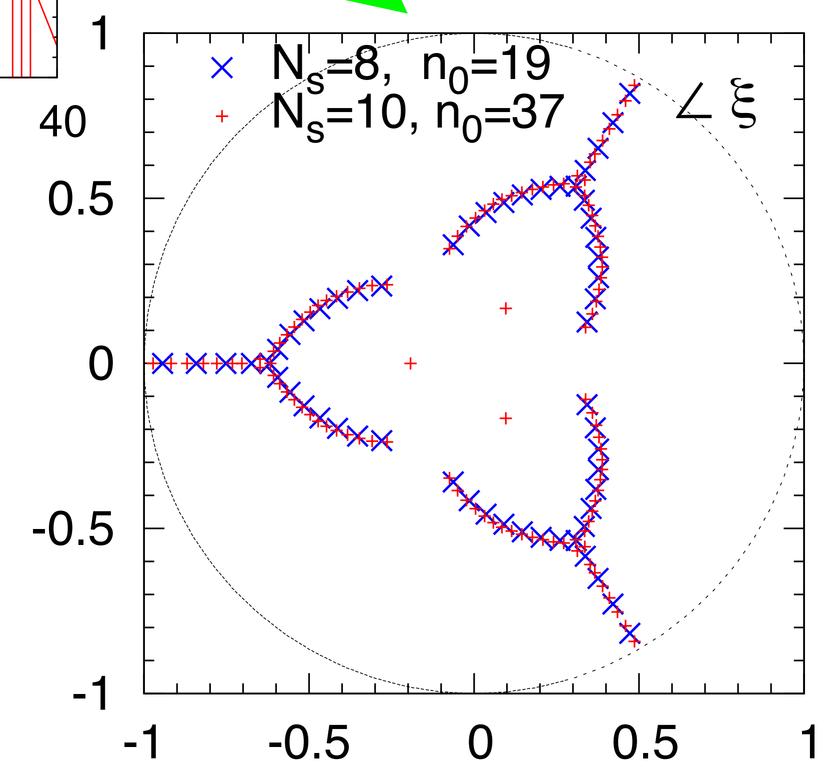
- Gaussian type of canonical partition function is an indication of completion of deconfinement transition

SPA





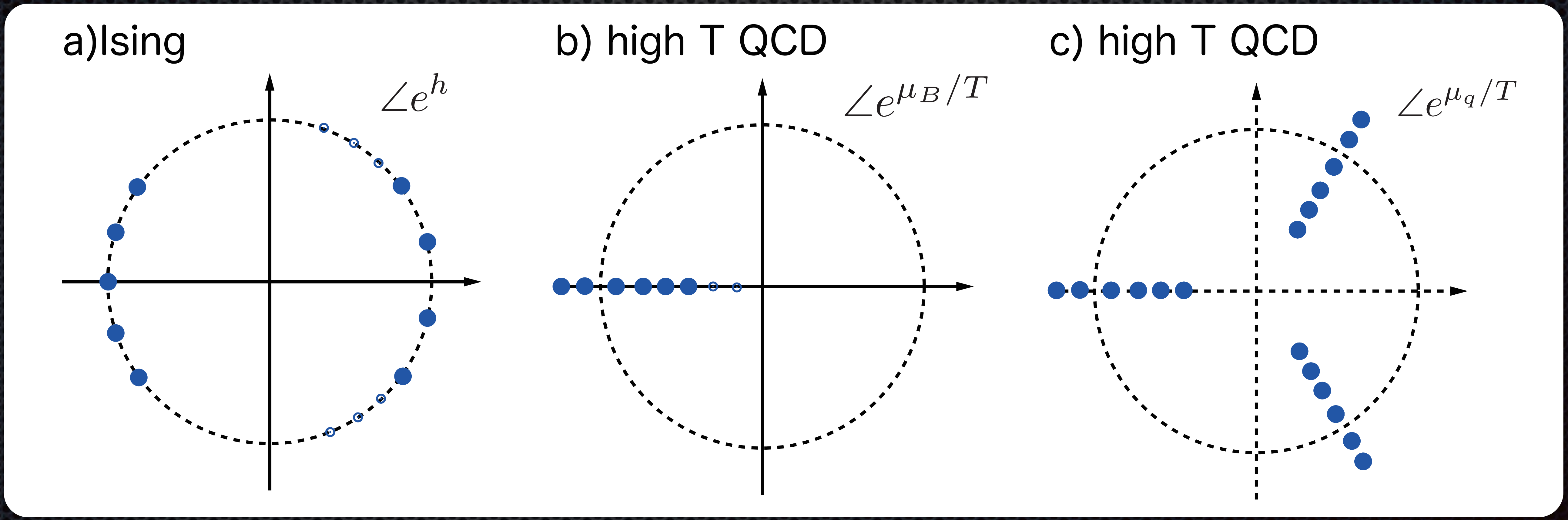
Theta function



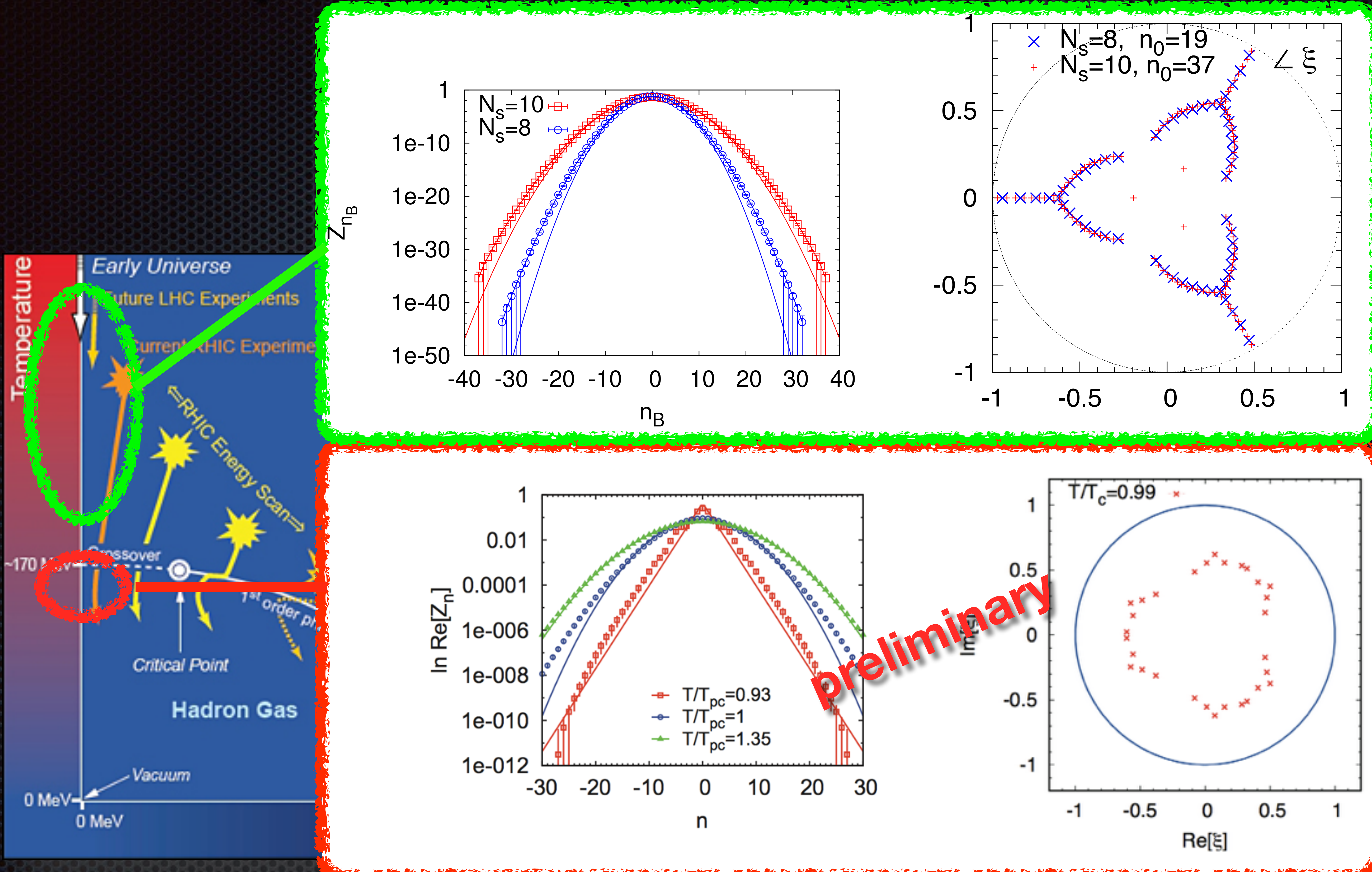
From the view point of Lee-Yang zeros

Gaussian type of canonical partition functions are exceptional cases of Lee-Yang zero circle theorem.

e.g. free fermions at small density



Summary



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N_s=10
N_s=8
Z_{n_B}
n_B
N_s=8, n_0=19
N_s=10, n_0=37
∠ ξ
T/T_{pc}=0.93
T/T_{pc}=1
T/T_{pc}=1.35
ln Re[Z_n]
n
preliminary
T/T_c=0.99
Im[ξ]
Re[ξ]

Summary

- Canonical approach :
  - useful as a tool to investigate wide range of chemical potentials.
  - applications : fluctuations, Lee-Yang zeros

- Our findings :
- Gaussian distribution and RW phase transitions at high temperatures : indication of completion of deconfinement
- Non-trivial distribution, circle—like zeros at low temperatures (further confirmation is necessary)
- A concrete example of an analytic calculation of Lee-Yang zeros