Formation of Clusters in stable and unstable nuclei explored by Antisymmetrized Molecular Dynamics

M. Kimura (Hokkaido University)

Collaborators: Y. Chiba, T. Baba (Hokkaido Univeristy)

Plan of this talk

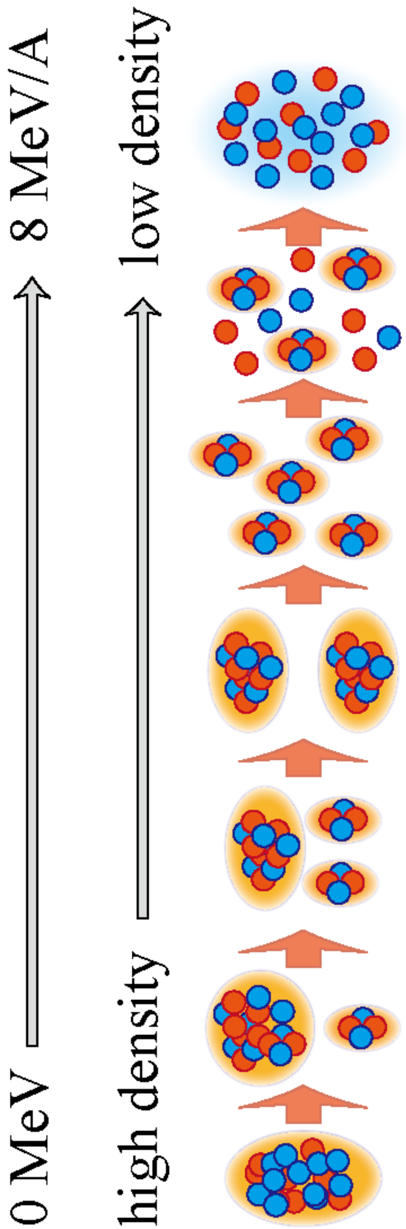
◎ **Introduction**
- Clustering phenomena and typical examples
- Theoretical framework of
  antisymmetrized molecular dynamics (AMD)

◎ **Clusters in stable nuclei (^{24}Mg)**
- Clusters in the highly excited states of ^{24}Mg
- IS monopole transitions to probe them

◎ **Clusters in neutron-rich nuclei**
- Nuclear molecule with molecular-orbital bonding
- From dimers to trimers and tetramers

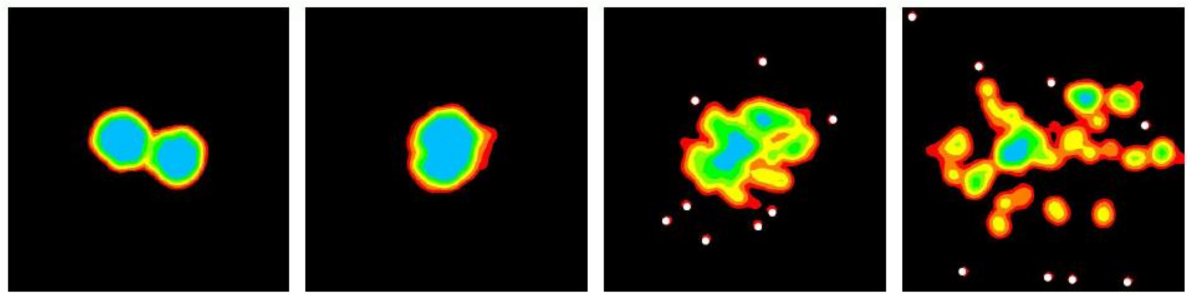
◎ **Summary**

Introduction: Evolution of clusters



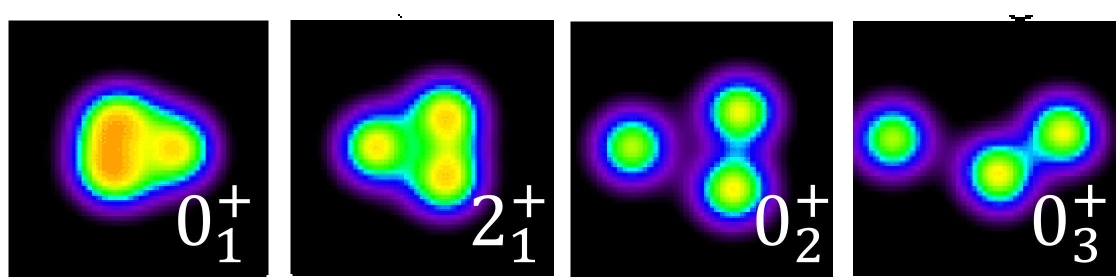
Multi fragmentation in Heavy Ion Collision

^{129}Xe+Sn E/A 50 MeV, A. Ono, PRC66 (2002).

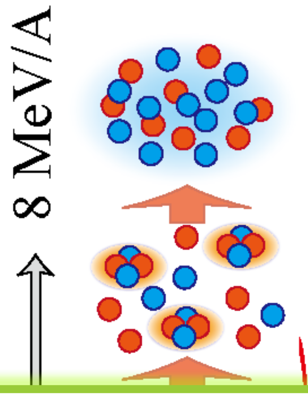


Clustering in the excited states of nuclei

^{12}C excited states, Y. Kanada-En'yo, PRL81 (1998)



Introduction: Clustering phenomena



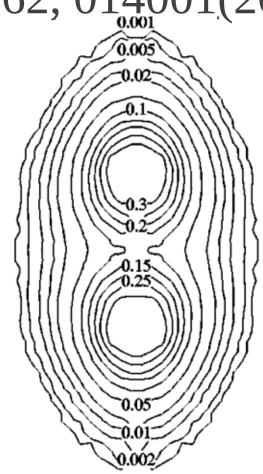
Degrees-of-freedom of nuclear excitation

◎ Single particle excitation
◎ Collective excitation
◎ **Cluster excitation**

Famous cluster states in light stable nuclei

^{8}Be: $\alpha+\alpha$

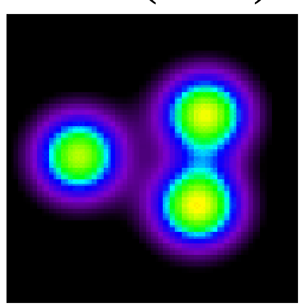
GF Monte Carlo
R.B.Wiringa, et al.,
PRC62, 014001(2000).



^{12}C*(0_2^+): 3α

Hoyle state
3α BEC state

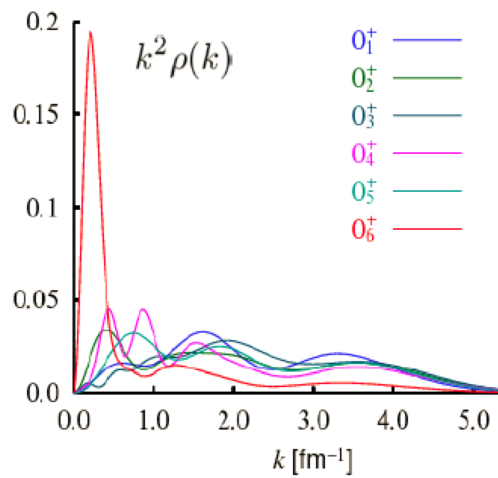
A. Tohsaki et al,
PRL87 (2001).

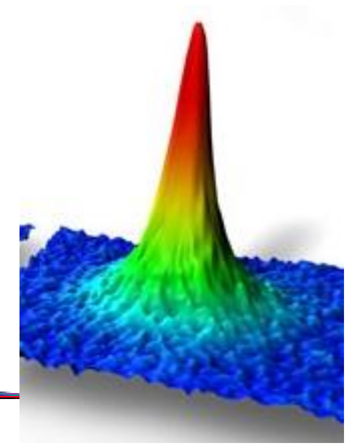


^{16}O*(0_6^+):

4α BEC state

Y. Funaki, PRL101, (2008).

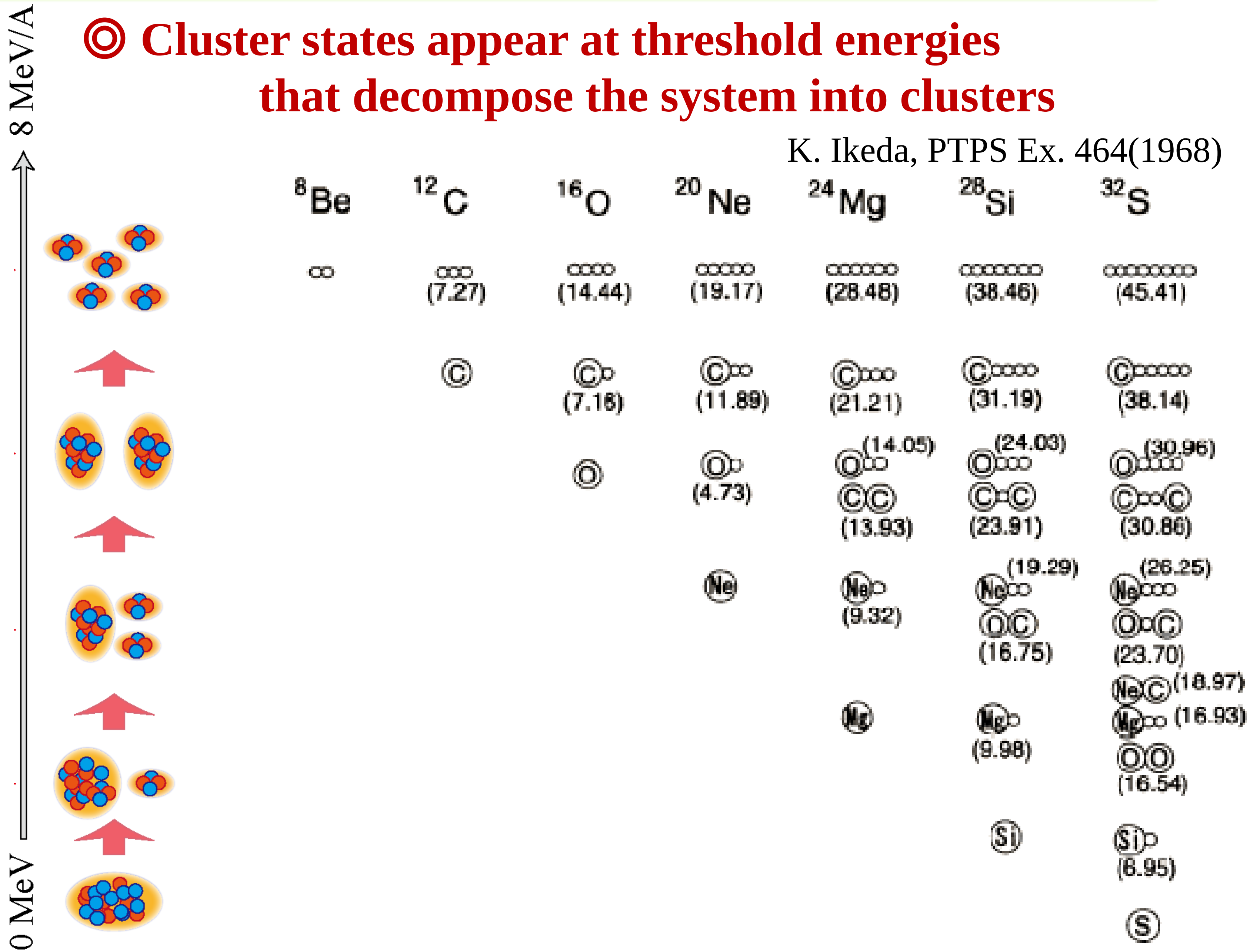




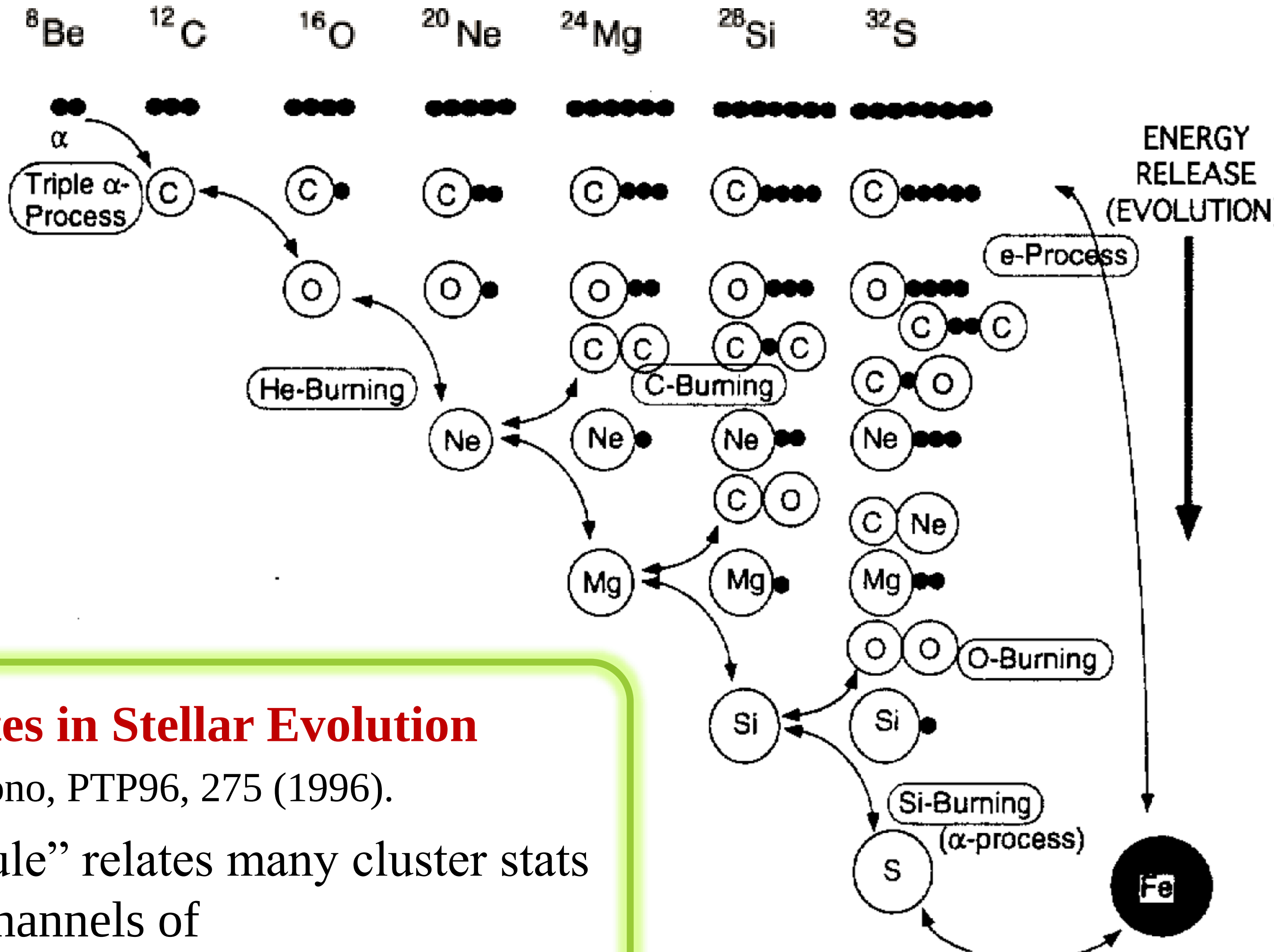
Threshold Energy Rule of Clustering

◎ Cluster states appear at threshold energies that decompose the system into clusters

K. Ikeda, PTPS Ex. 464(1968)



Cluster Nucleosyntheis



Cluster states in Stellar Evolution

S. Kubono, PTP96, 275 (1996).

“Threshold rule” relates many cluster stats to reaction channels of He-, C-, O-Burning processes

Theoretical Framework of Antisymmetrized Molecular Dynamics

◎ Wave function (Spatially Localized Gaussians)

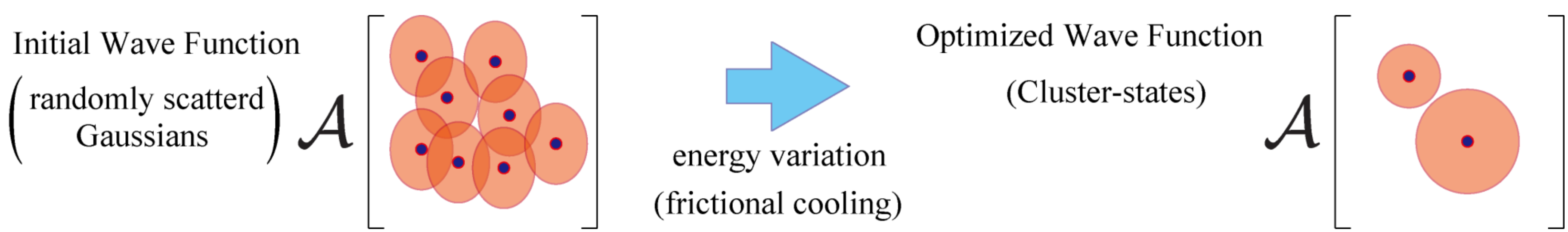
$$\Psi^{\pi} = \frac{1+\pi\hat{P}_r}{2}\Psi_{int} = \frac{1+\pi\hat{P}_r}{2}\mathcal{A}\{\varphi_1, \varphi_2, ..., \varphi_A\}$$

$$\varphi_i(\boldsymbol{r}) \propto \exp\left\{-\boldsymbol{\nu_x}\left(x - \frac{\boldsymbol{Z_{ix}}}{\sqrt{\boldsymbol{\nu_x}}}\right)^2 - \boldsymbol{\nu_y}\left(y - \frac{\boldsymbol{Z_{iy}}}{\sqrt{\boldsymbol{\nu_y}}}\right)^2 - \boldsymbol{\nu_z}\left(z - \frac{\boldsymbol{Z_{iz}}}{\sqrt{\boldsymbol{\nu_z}}}\right)^2\right\} \otimes \{\boldsymbol{a_i}|\uparrow\rangle + \boldsymbol{b_i}|\uparrow\rangle\} \otimes (|n\rangle \text{ or } |p\rangle)$$

$$\text{variational parameters}: X_i = \underbrace{\boldsymbol{Z}_1, ..., \boldsymbol{Z}_A}_{\text{centroids of wave packets}}, \underbrace{a_1, ..., a_A, b_1, ..., b_A}_{\text{spin directions}}, \underbrace{\nu_x, \nu_y, \nu_z}_{\text{width of wave packets}}$$

◎ No a-priori assumption on cluster structure

◎ Both of the single-particle and clustering nature are described within a single framework

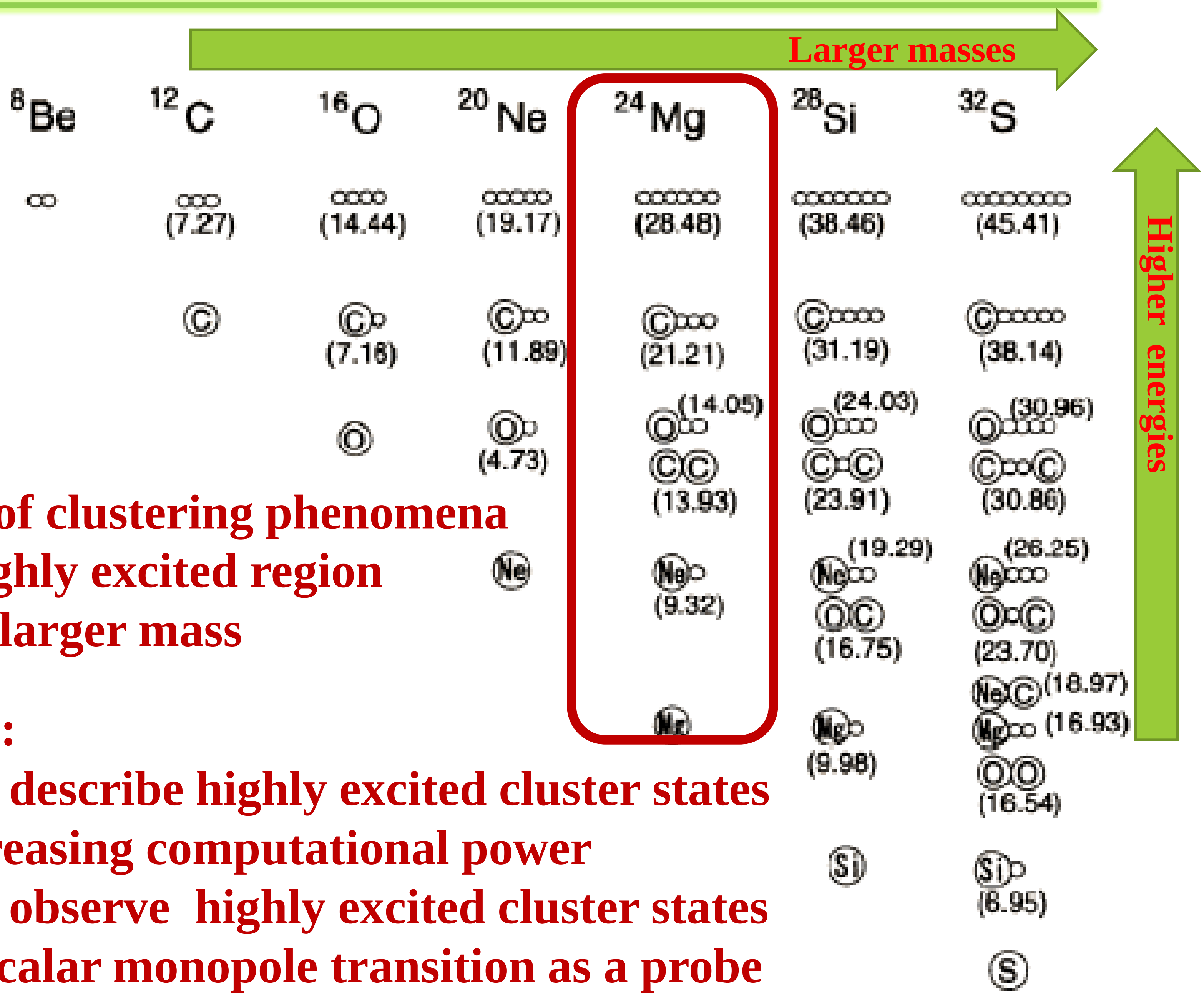


Y. Kanada-En'yo, M. K and A. Ono, *Prog. Theor. Exp. Phys.* (2012) 01A202.
"Antisymmetrized molecular dynamics and its applications to cluster phenomena"

Clusters in stable nuclei

- Clusters in the highly excited states of ^{24}Mg
- IS monopole transitions to probe them

A Frontier of Nuclear Cluster Physics



◎ **Survey of clustering phenomena in highly excited region with larger mass**

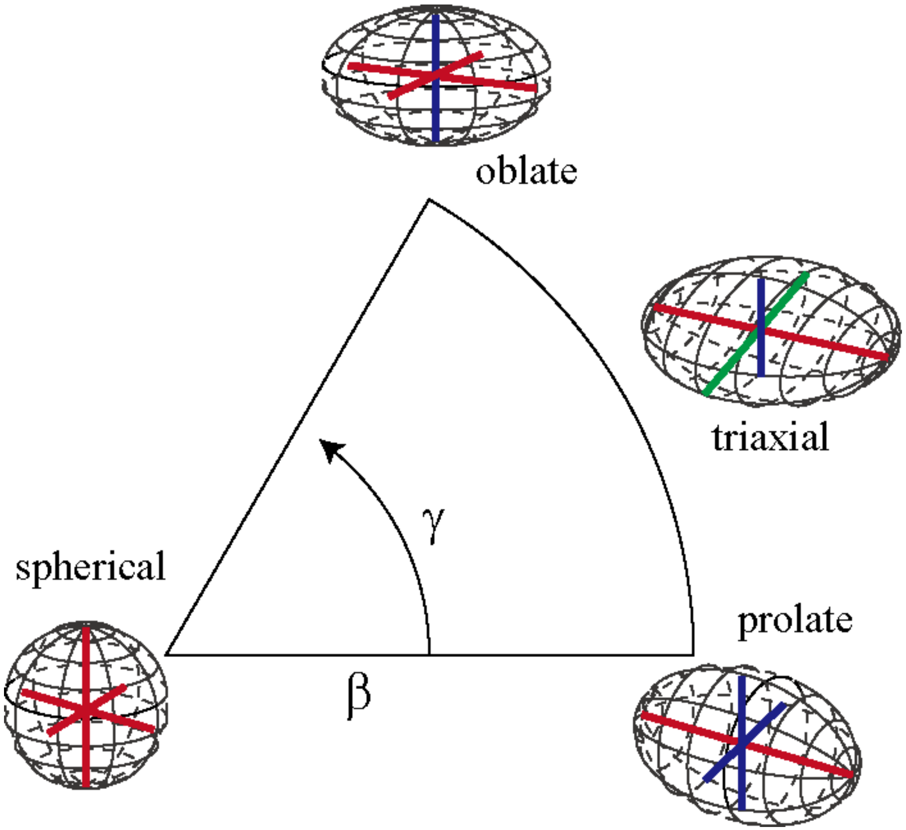
Challenges:

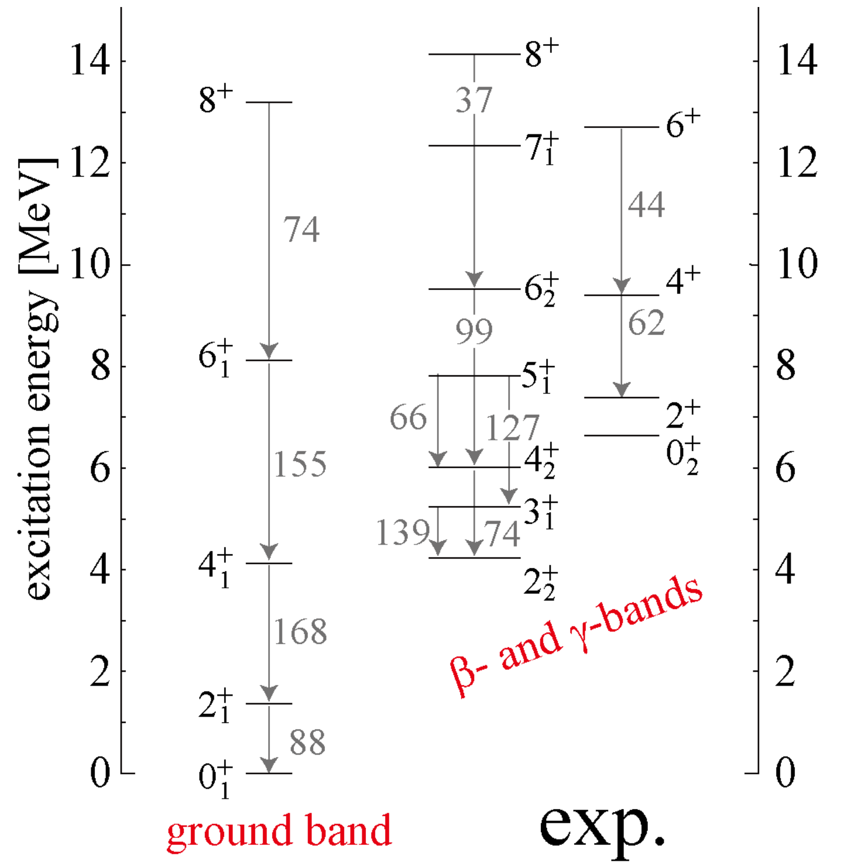
◎ **How to describe highly excited cluster states**
⇒ **increasing computational power**

◎ **How to observe highly excited cluster states**
⇒ **isoscalar monopole transition as a probe**

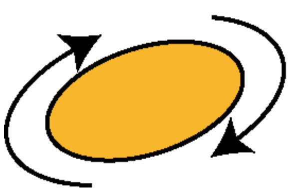
^{24}Mg low-lying quadrupole collectivity

◎ **^{24}Mg: well-known low-lying collectivity, triaxial deformation**

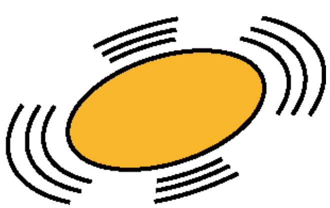




rotation



vibration



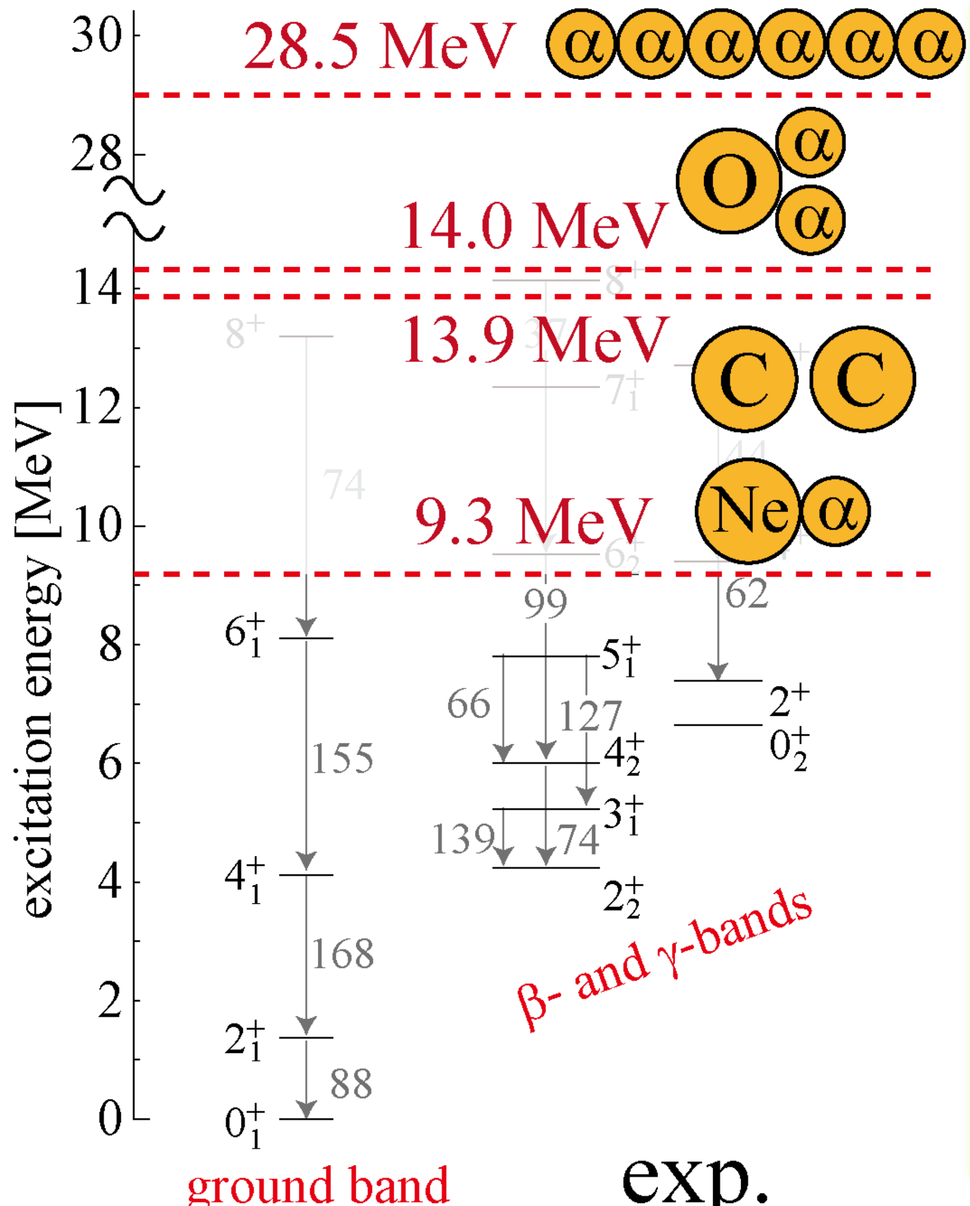
Many many works…

HFB calc. : M.Girod et al., PRC27 (1983).

⋮

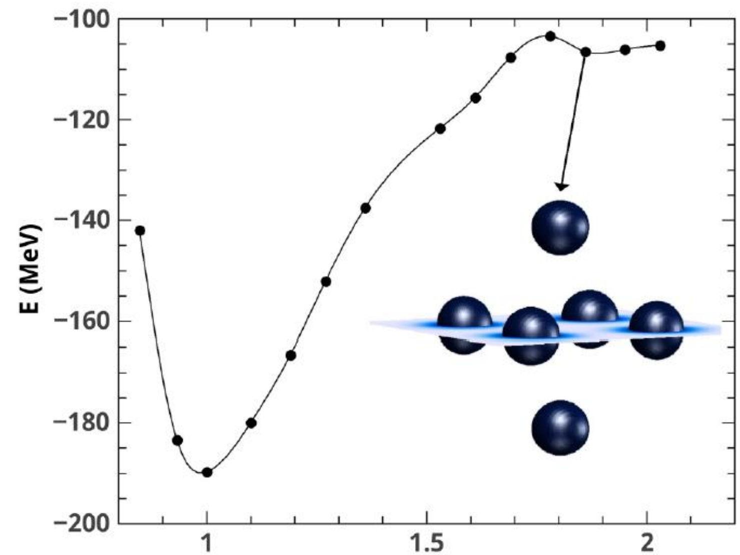
Beyond MF calc.: M.Bender et al., PRC 78 (2008).

^{24}Mg highly excited cluster states



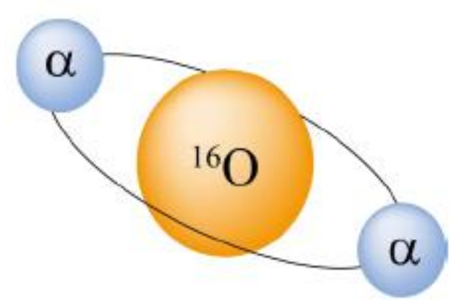
◎ 6α cluster candidate by HFB

M. Girod et al, PRL111 (2013).



◎ 2α condensate around ^{16}O

N. Itagaki, et al., PRC75 (2007).



Theoretical Framework of Antisymmetrized Molecular Dynamics

◎ Microscopic Hamiltonian (A-nucleons)

Gogny D1S interaction,   No spurious center-of-mass energy

$$\hat{H} = \sum_{i}^{A} \hat{t}_i - \hat{t}_{c.m.} + \sum_{i<j}^{A} \hat{v}_{\mathrm{GognyD1S}}(r_{ij}) + \sum_{i<j}^{Z} \hat{v}_{\mathrm{Coulomb}}(r_{ij})$$

◎ Wave function (Spatially Localized Gaussians)

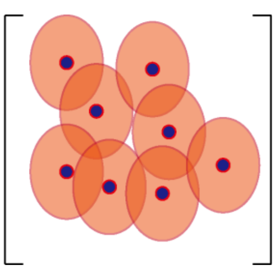
No a-priori assumption on cluster structure

$$\Psi^{\pi} = \frac{1+\pi\hat{P}_r}{2}\Psi_{int} = \frac{1+\pi\hat{P}_r}{2}\mathcal{A}\{\varphi_1, \varphi_2, ..., \varphi_A\}$$

$$\varphi_i(\boldsymbol{r}) \propto \exp\left\{-\boldsymbol{\nu_x}\left(x - \frac{\boldsymbol{Z_{ix}}}{\sqrt{\boldsymbol{\nu_x}}}\right)^2 - \boldsymbol{\nu_y}\left(y - \frac{\boldsymbol{Z_{iy}}}{\sqrt{\boldsymbol{\nu_y}}}\right)^2 - \boldsymbol{\nu_z}\left(z - \frac{\boldsymbol{Z_{iz}}}{\sqrt{\boldsymbol{\nu_z}}}\right)^2\right\} \otimes \{\boldsymbol{a_i}|\uparrow\rangle + \boldsymbol{b_i}|\uparrow\rangle\} \otimes (|n\rangle \text{ or } |p\rangle)$$

$$\text{variational parameters}: X_i = \underbrace{\boldsymbol{Z}_1, ..., \boldsymbol{Z}_A}_{\text{centroids of wave packets}}, \underbrace{a_1, ..., a_A, b_1, ..., b_A}_{\text{spin directions}}, \underbrace{\nu_x, \nu_y, \nu_z}_{\text{width of wave packets}}$$

Initial Wave Function

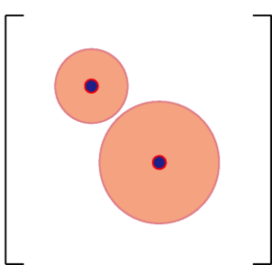
$\left(\begin{array}{c}\text{randomly scatterd}\\ \text{Gaussians}\end{array}\right) \mathcal{A}$ 

energy variation

(frictional cooling)

Optimized Wave Function

(Cluster-states)

$\mathcal{A}$ 

^{24}Mg low-lying quadrupole collectivity

Step 1: Energy variation with constraint

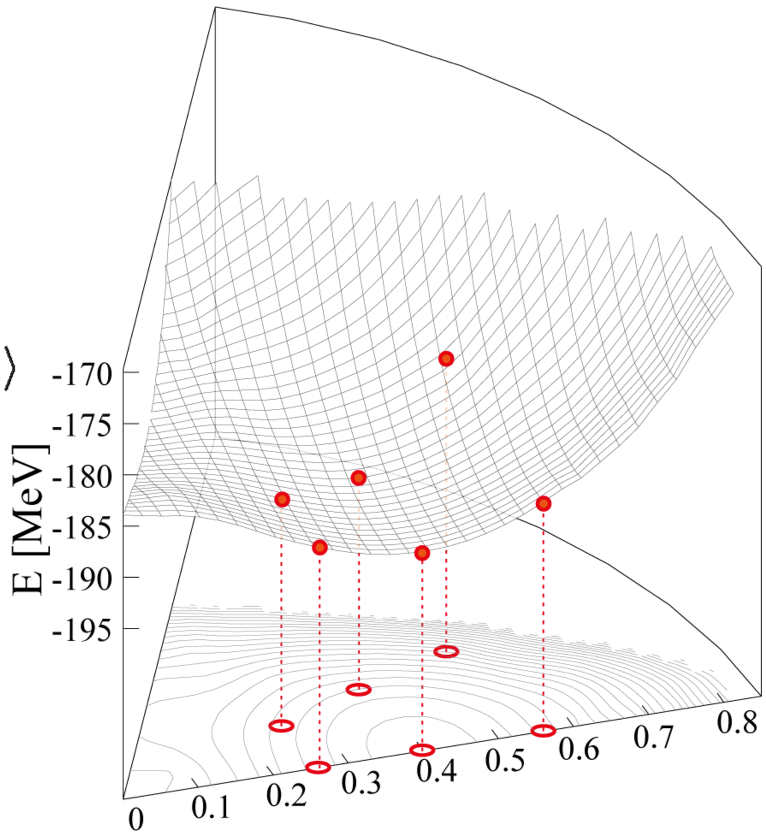
$$\text{variational parameters}: X_i = \underbrace{\boldsymbol{Z}_1,...,\boldsymbol{Z}_A}_{\text{centroids of wave packets}}, \underbrace{a_1,...,a_A,b_1,...,b_A}_{\text{spin directions}}, \underbrace{\nu_x,\nu_y,\nu_z}_{\text{width of wave packets}}$$

Survey low-lying quadrupole collectivity

- Energy variation with the constraint on the quadrupole deformation parameters (β,γ)

Equations for "frictional cooling method"

$$\frac{d}{d\tau}X_i = -\lambda\frac{\partial}{\partial X_i^*}\langle\Psi^\pi(\boldsymbol{Z},\boldsymbol{\nu},a,b)|\hat{H}|\Psi^\pi(\boldsymbol{Z},\boldsymbol{\nu},a,b)\rangle$$



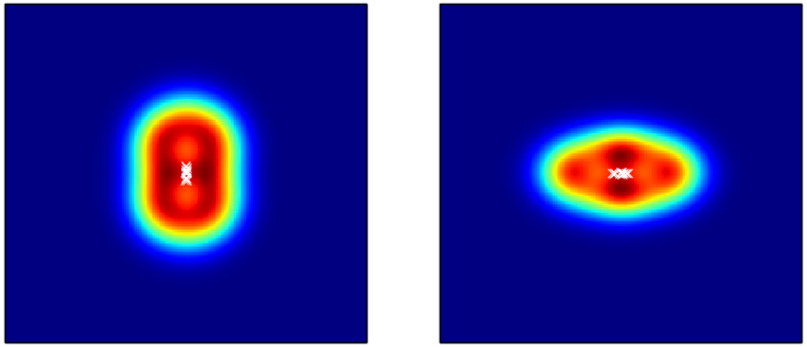
^{24}Mg low-lying quadrupole collectivity

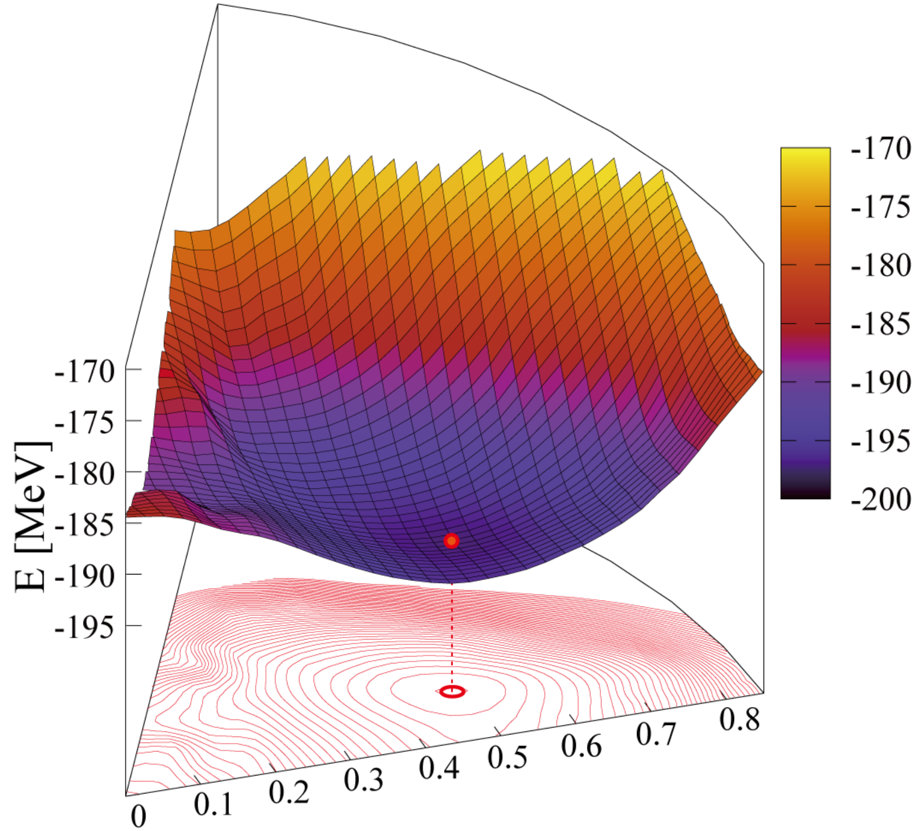
Step 2: Angular momentum projection

Optimized wave functions are projected to the eigenstates of $\hat{J}$

$$\hat{P}^J_{MK}\Psi^\pi(\beta,\gamma) = \frac{2J+1}{8\pi^2}\int d\Omega D^{J*}_{MK}\hat{R}(\Omega)\Psi^\pi(\beta,\gamma)$$

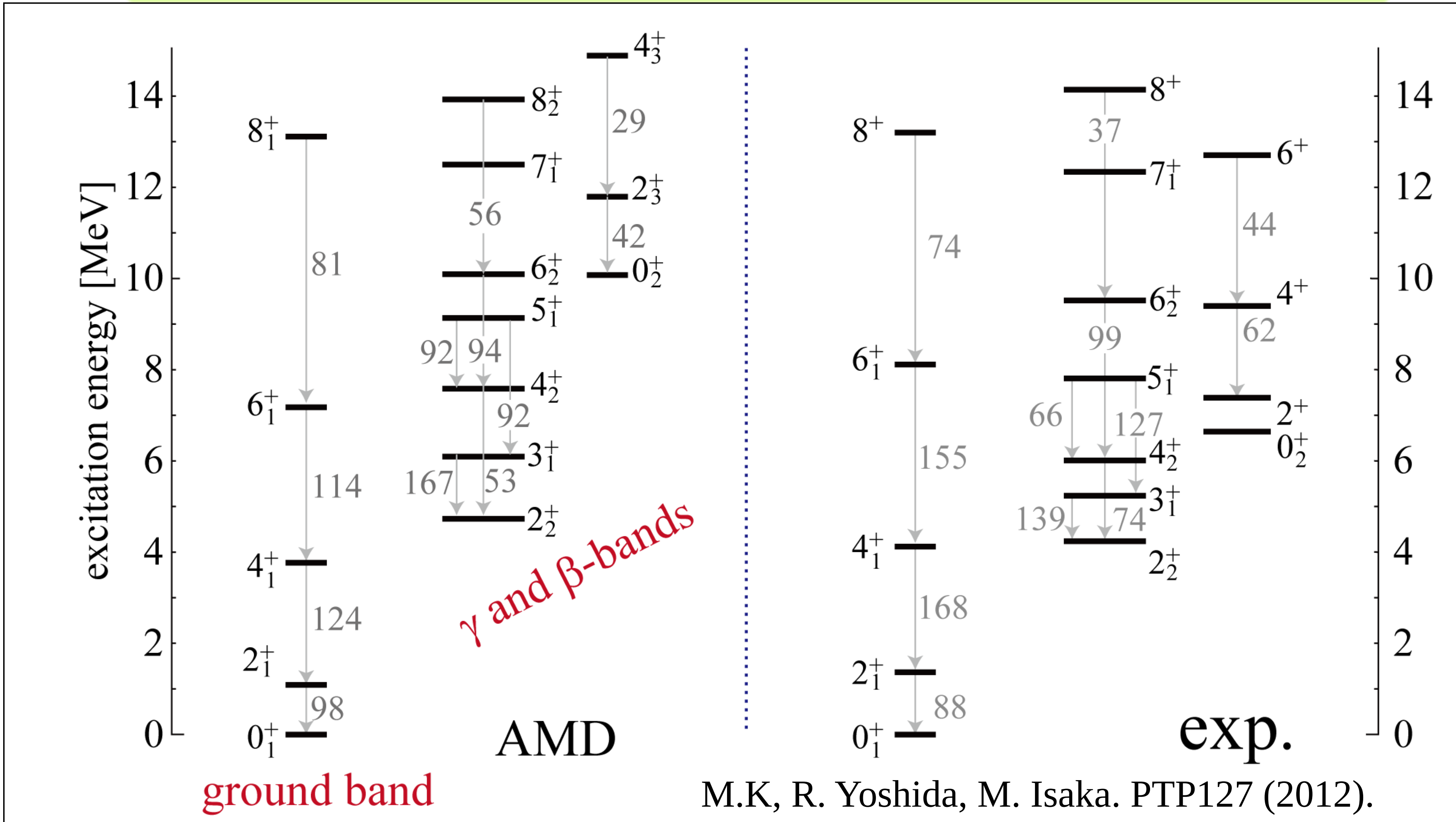
- Triaxially deformed energy minimum





⇒ Low-lying quadrupole collectivity

^{24}Mg low-lying quadrupole collectivity



M.K, R. Yoshida, M. Isaka. PTP127 (2012).

- Good descri

No Clusters until now

Description of highly excited cluster states

- The cluster thresholds are higher than quadrupole collective states

$\Rightarrow$ Need to explore higher energy region with many-particle and many-hole configurations

- An alternative method

Constraint on H.O. quantum numbers

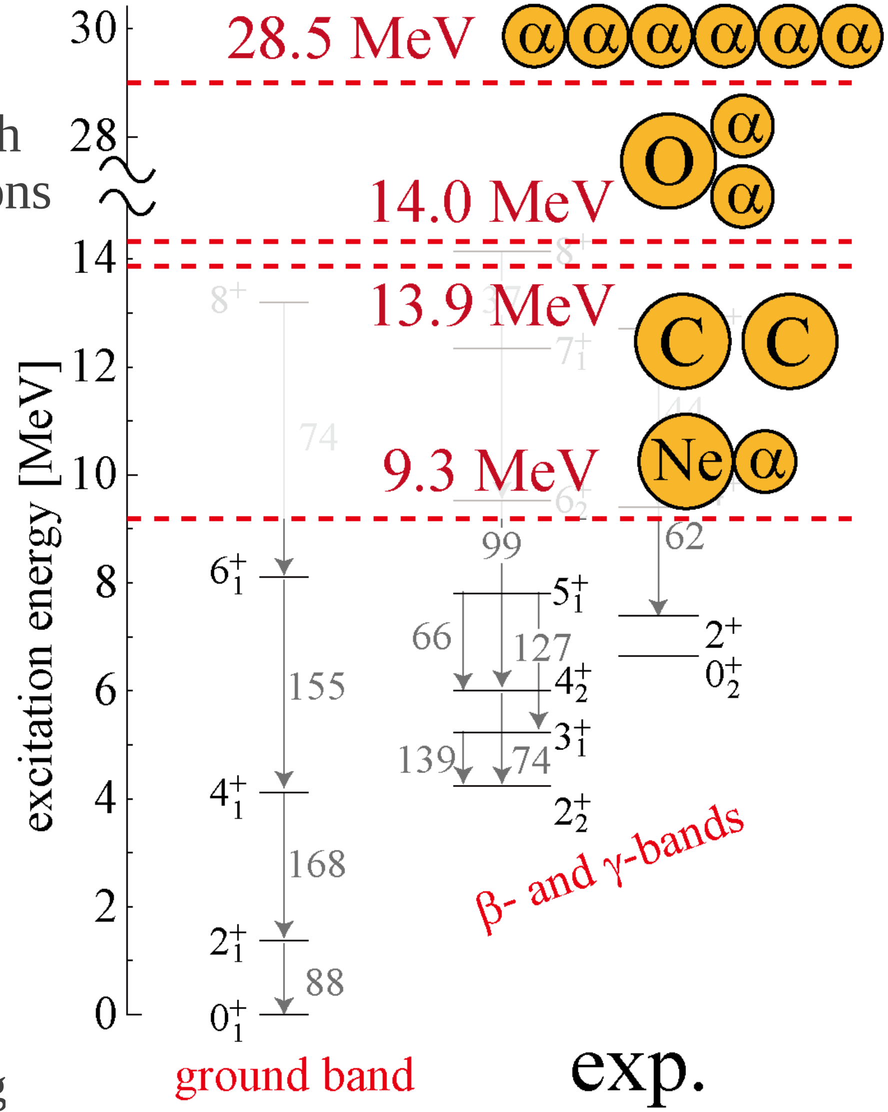
$$N_x = \langle \Psi^\pi | \hat{a}_x^\dagger \hat{a}_x | \Psi^\pi \rangle, \quad N_y = \langle \Psi^\pi | \hat{a}_y^\dagger \hat{a}_y | \Psi^\pi \rangle, \ \dots$$

$$\hat{a}_x = \sqrt{\frac{m\omega}{2\hbar}}\hat{x} + i\sqrt{\frac{1}{2m\hbar\omega}}\hat{p}_x.$$

$$\hbar\omega = \frac{2\hbar(\nu_x\nu_y\nu_z)^{1/3}}{m}$$

size of H.O. is determined from the g.s. wave function

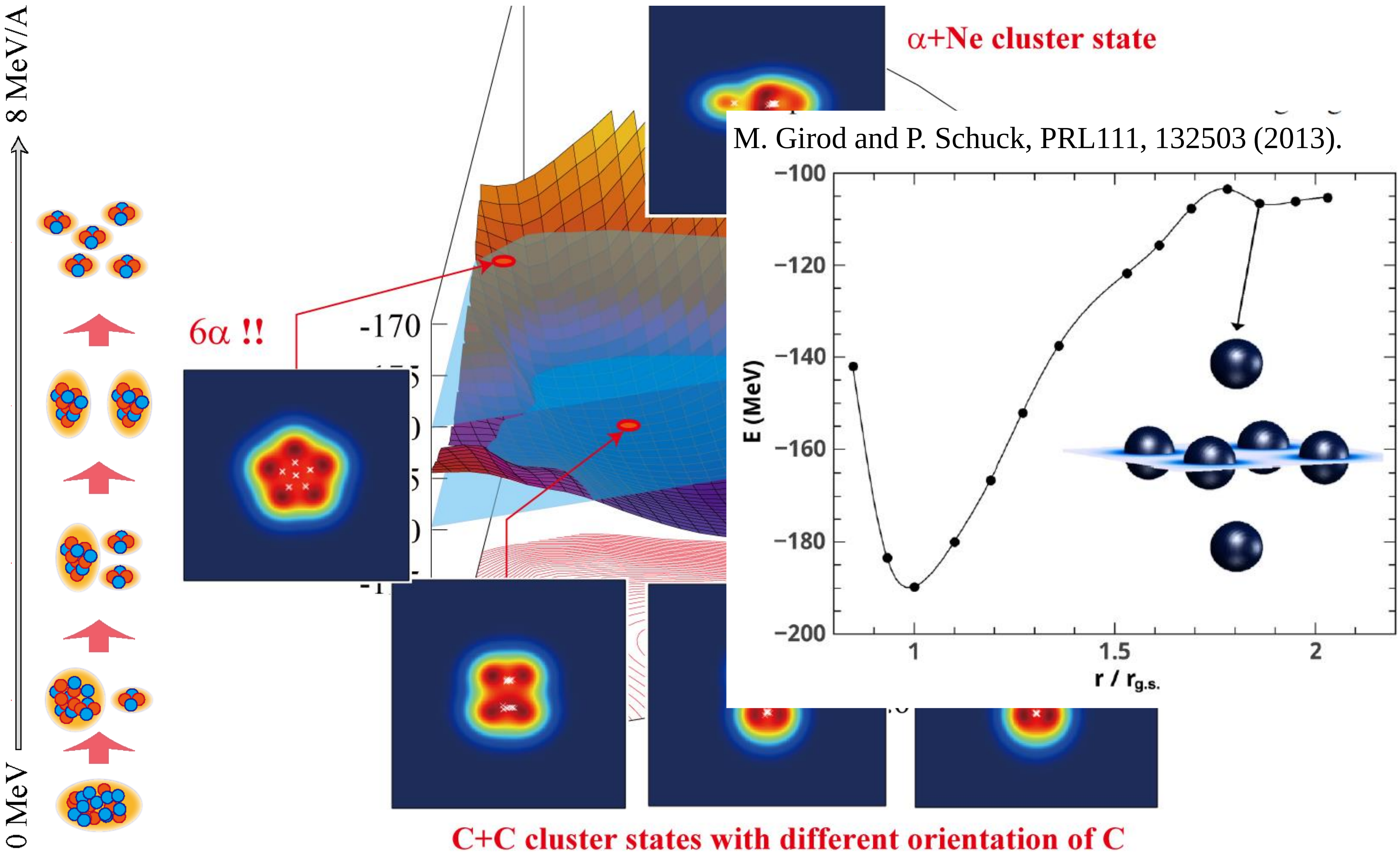
$\Rightarrow$ Many states with different particle-hole config with large N_x, N_y, N_z are generated



Description of highly excited cluster states

Various structure that are never seen on the β-γ plane

- **A Rich Variety of Clusters !** : C+C, α+Ne, 2α+O, 6α
- **Clustering is a fundamental degrees-of-freedom of nuclear excitation**

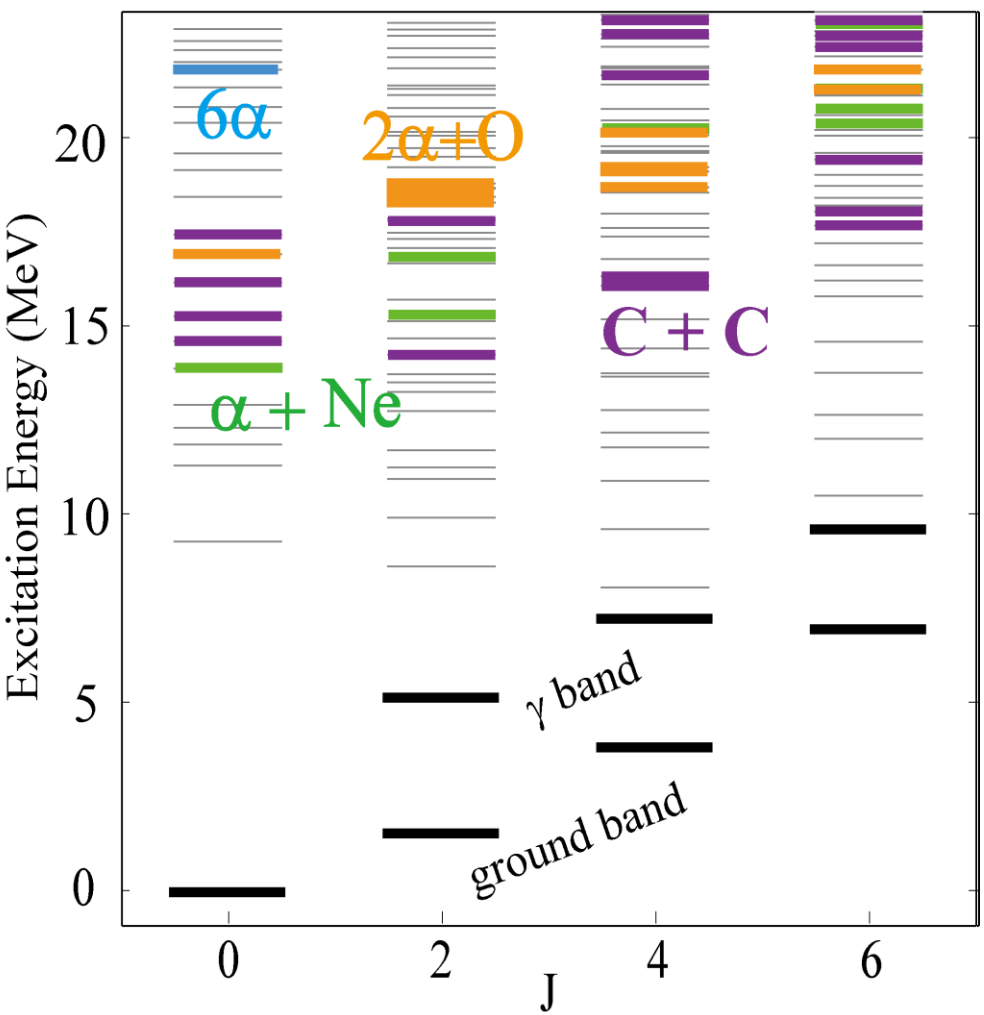


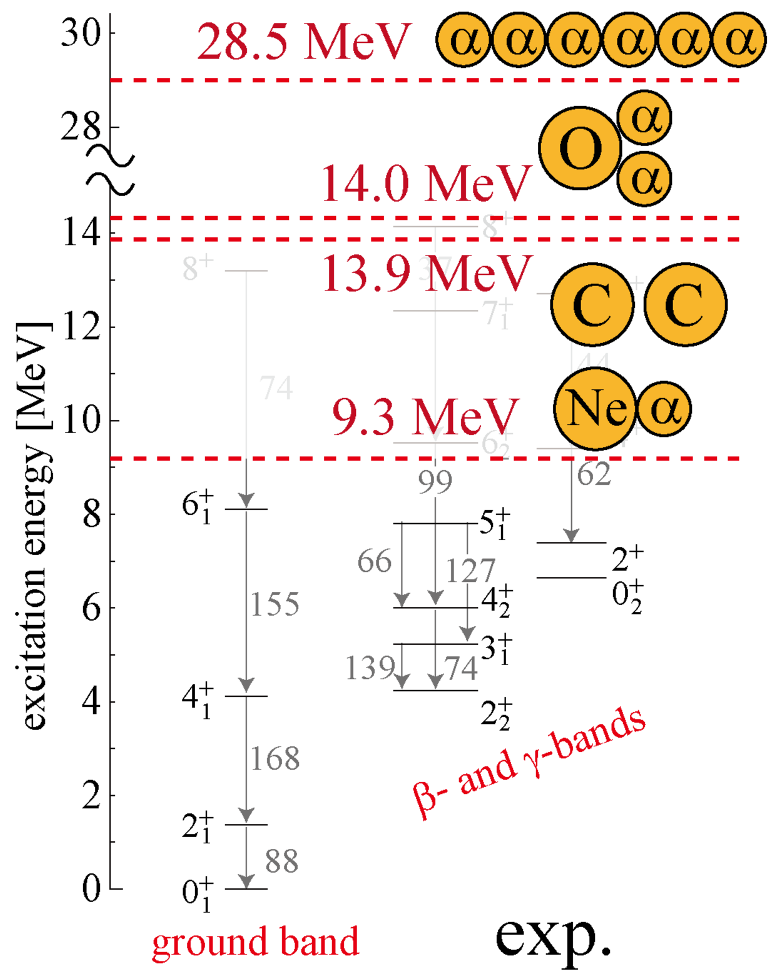
Description of highly excited cluster states

All the basis wave functions (collective states and clusters) are superposed

⇒ **Most CPU demanding part in AMD calculation (～ 500 basis wave functions)**

Cluster states in the vicinity of the thresholds





How to observe them? A key observable: isoscalar monopole transition

Isoscalar monopole transition strength is a good prove for Clustering

T. Yamada,et al., PTP120, 1139 (2008).

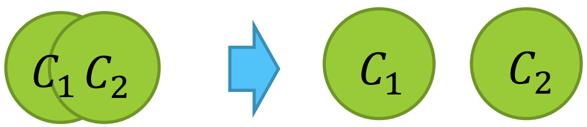
$$\hat{O} = \Sigma_{i=1}^{A} \underline{(r_i - r_{c.m.})^2}$$

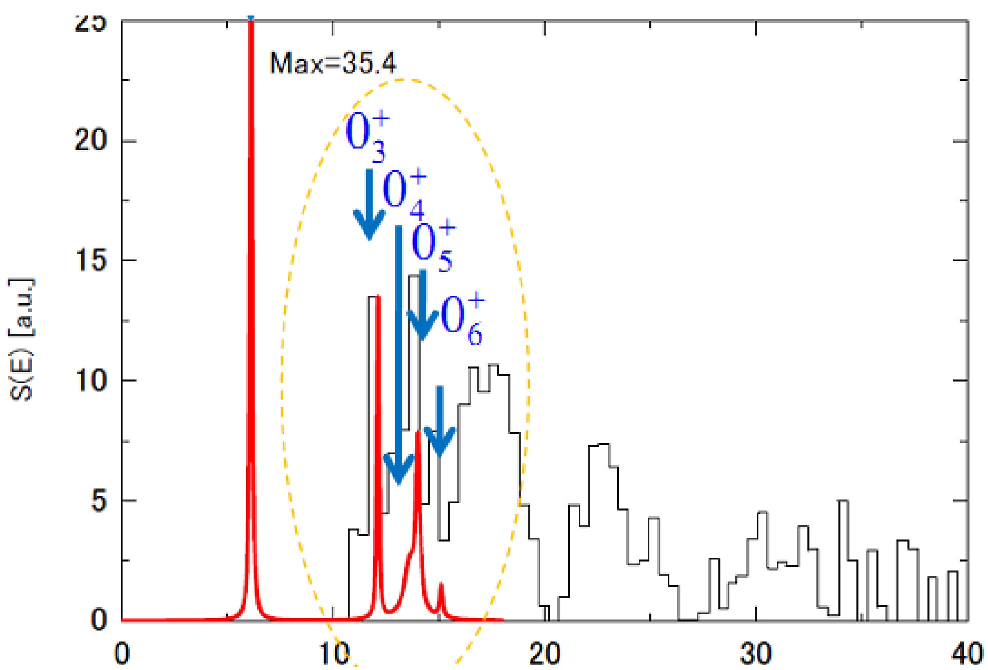
Single particle $2\hbar\omega$ excitation

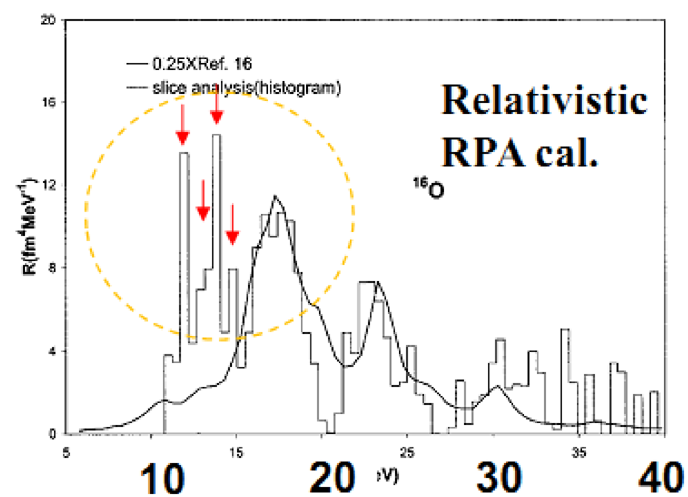
$$= \underline{\Sigma_{i\in C_1}(r_i - r_{C_1})^2 + \Sigma_{i\in C_2}(r_i - r_{C2})^2} + \frac{C_1 C_2}{C_1 + C_2}\underline{(r_{C_1} - r_{C_2})}^2$$

Single particle $2\hbar\omega$ excitations within clusters C_1 and C_2

Cluster excitation mode



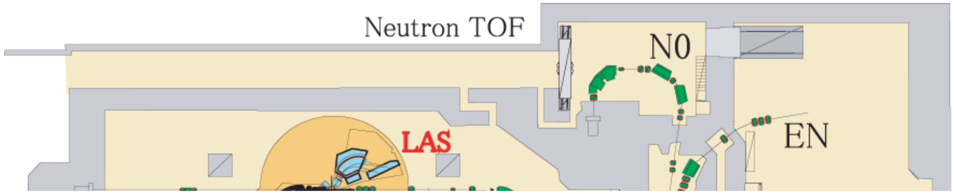




Exp. condition: $E_x > 10$ MeV

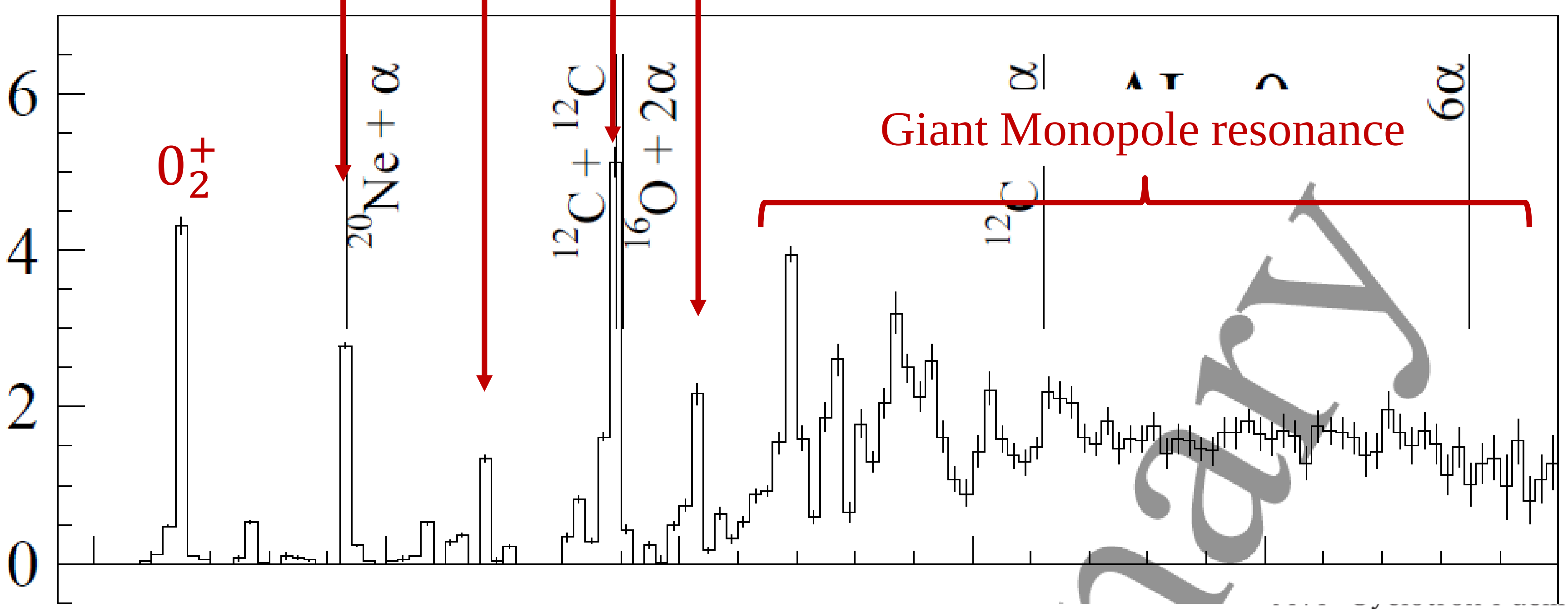
A Key Observable: Isoscalar Monopole Transition Strength

Its already measured @ RCNP
$^{24}Mg(\alpha, \alpha')^{24}Mg^*$ at 0 degree



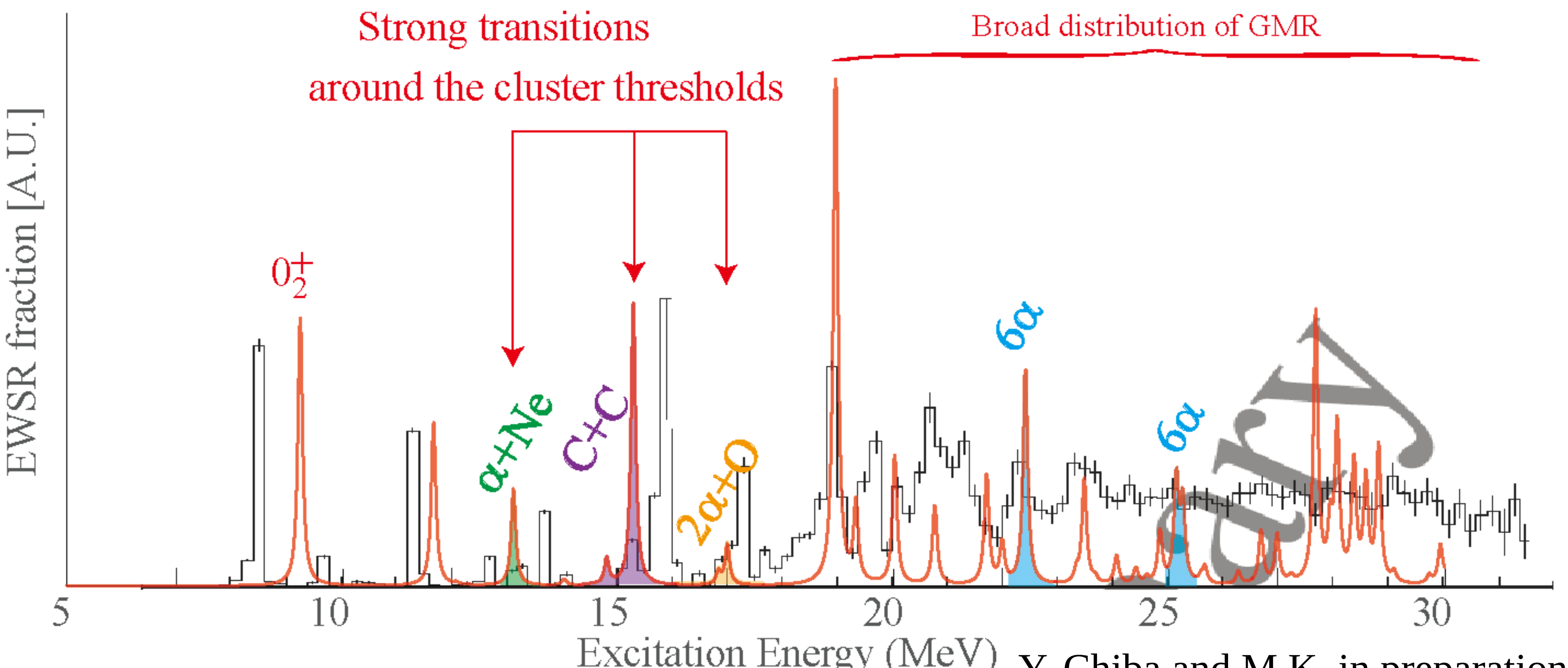
T. Kawabata, Proceedings of Cluster12 Conf.

Strong transitions observed around the cluster thresholds



A Key Observable: Isoscalar Monopole Transition Strength

- The full AMD calculation shows several states with strong isoscalar monopole transitions
- Those states are associated with various cluster states
- 6α states is embedded in the giant resonace
- A good accordance with the observed data (Promising !)

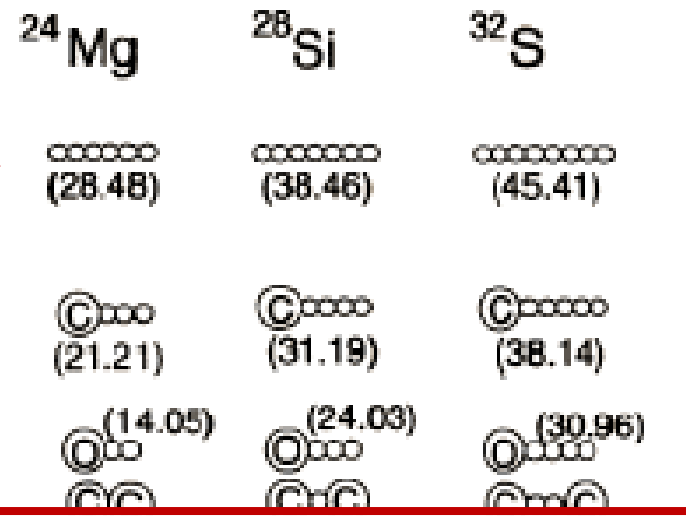


Y. Chiba and M.K. in preparation.

Prospects in study of highly excited cluster states

◎ Highly excited cluster states are now theoretically and experimentally accessible !

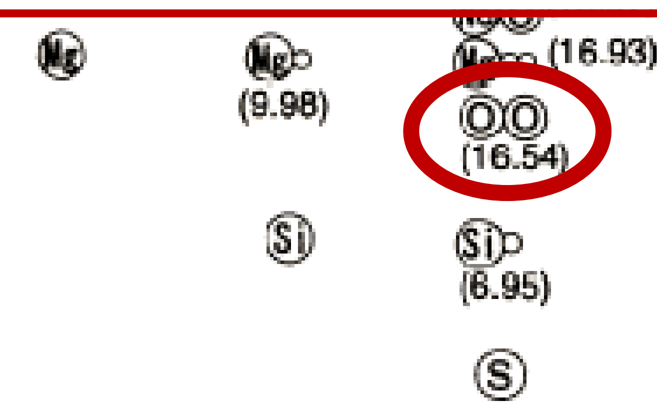
Many fascinating topics



An example

PHYSICAL REVIEW C **88**, 064313 (2013)

Isoscalar giant resonance strengths in ^{32}S and possible excitation of superdeformed and ^{28}Si + α cluster bandheads



Clusters in neutron-rich nuclei

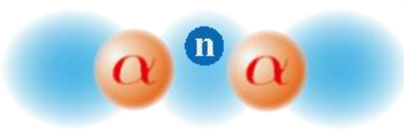
- Nuclear molecule with molecular-orbital bonding
- From dimers to trimers and tetramers

Frontiers of Nuclear Cluster Physics

◎ Clusters in neutron-rich nuclei ⇒ "nuclear molecules"

stable → n drip-line

^{8}Be | ^{9}Be | ^{10}Be | ^{11}Be | ^{12}Be | ^{14}Be



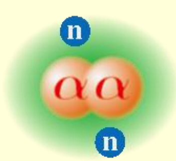
molecular-orbital bonding rules clusters instead of threshold energies

^{10}Be

0^+_2

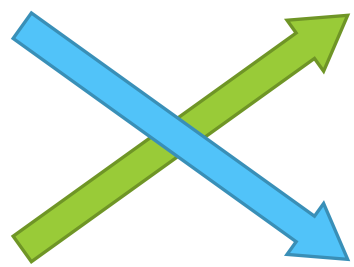
molecular-orbital bonding

0^+_1



Compact shell state

inversion,
vanishing N=8 shell gap

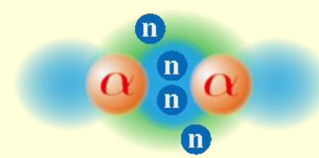


^{12}Be

0^+_2

Compact shell state

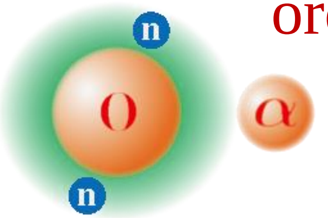
0^+_1



molecular-orbital bonding

Nuclear molecules with heavier masses

◎ Extension of “molecular-orbital bonding” to heavier clusters

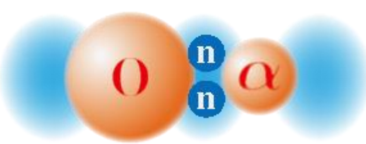


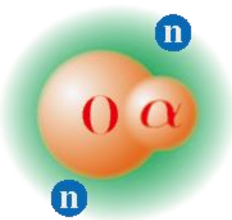
ordinary α-clustering

$\alpha + {}^{18}\mathrm{O}$

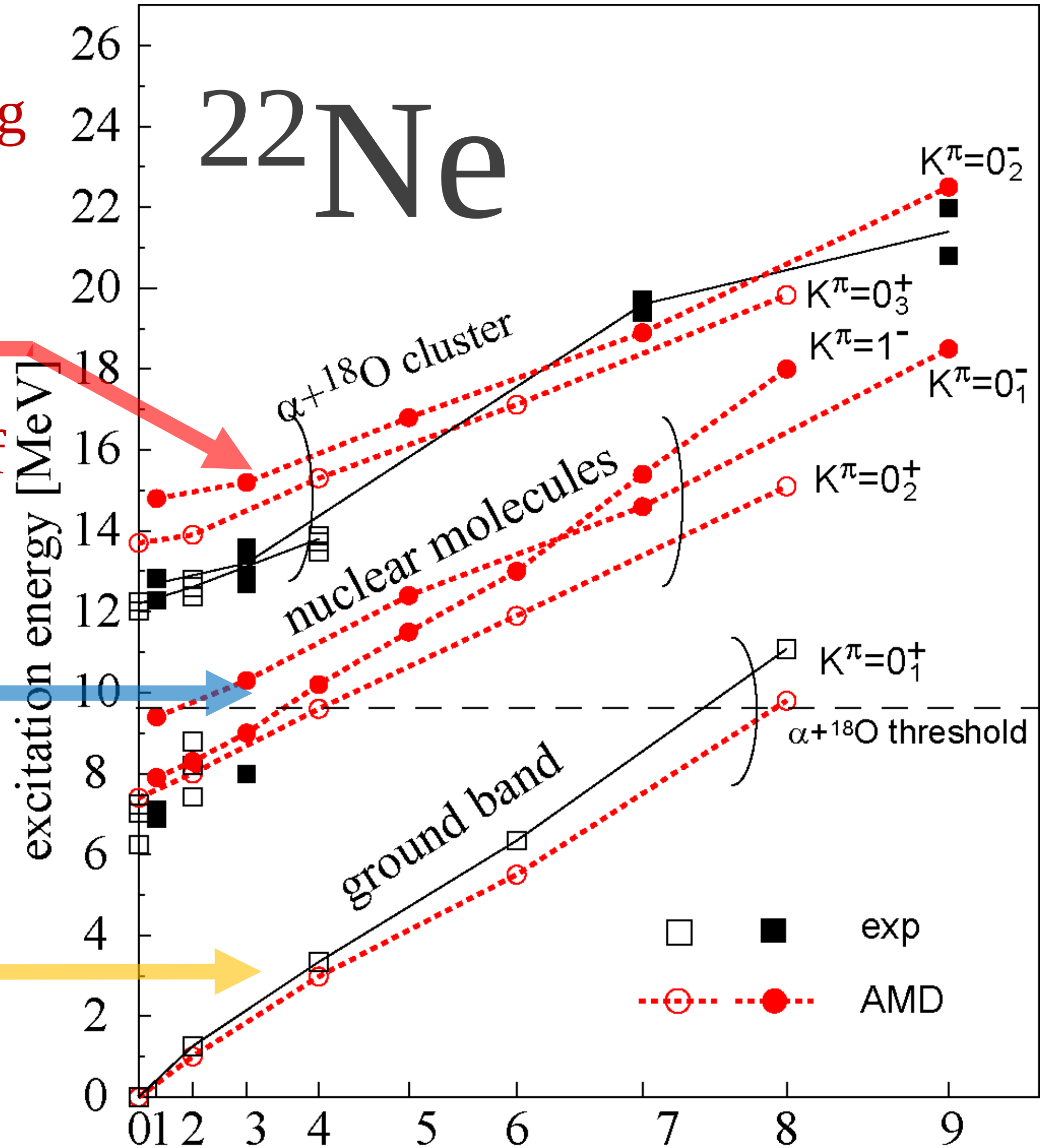
molecular-orbital bonding of

α and ${}^{16}\mathrm{O}$





Compact shell state

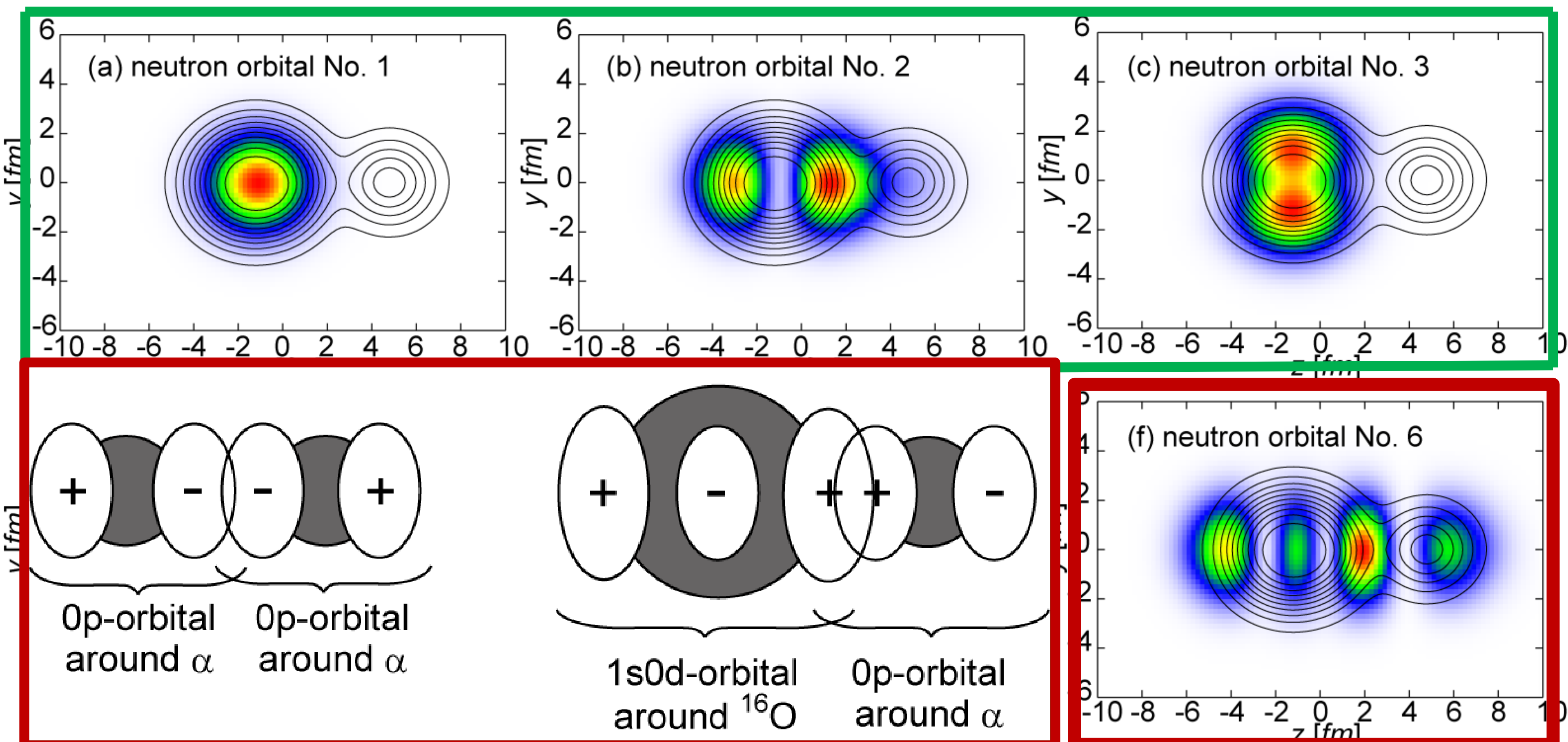


M.K., PRC75 (2007)

Nuclear molecules with heavier masses

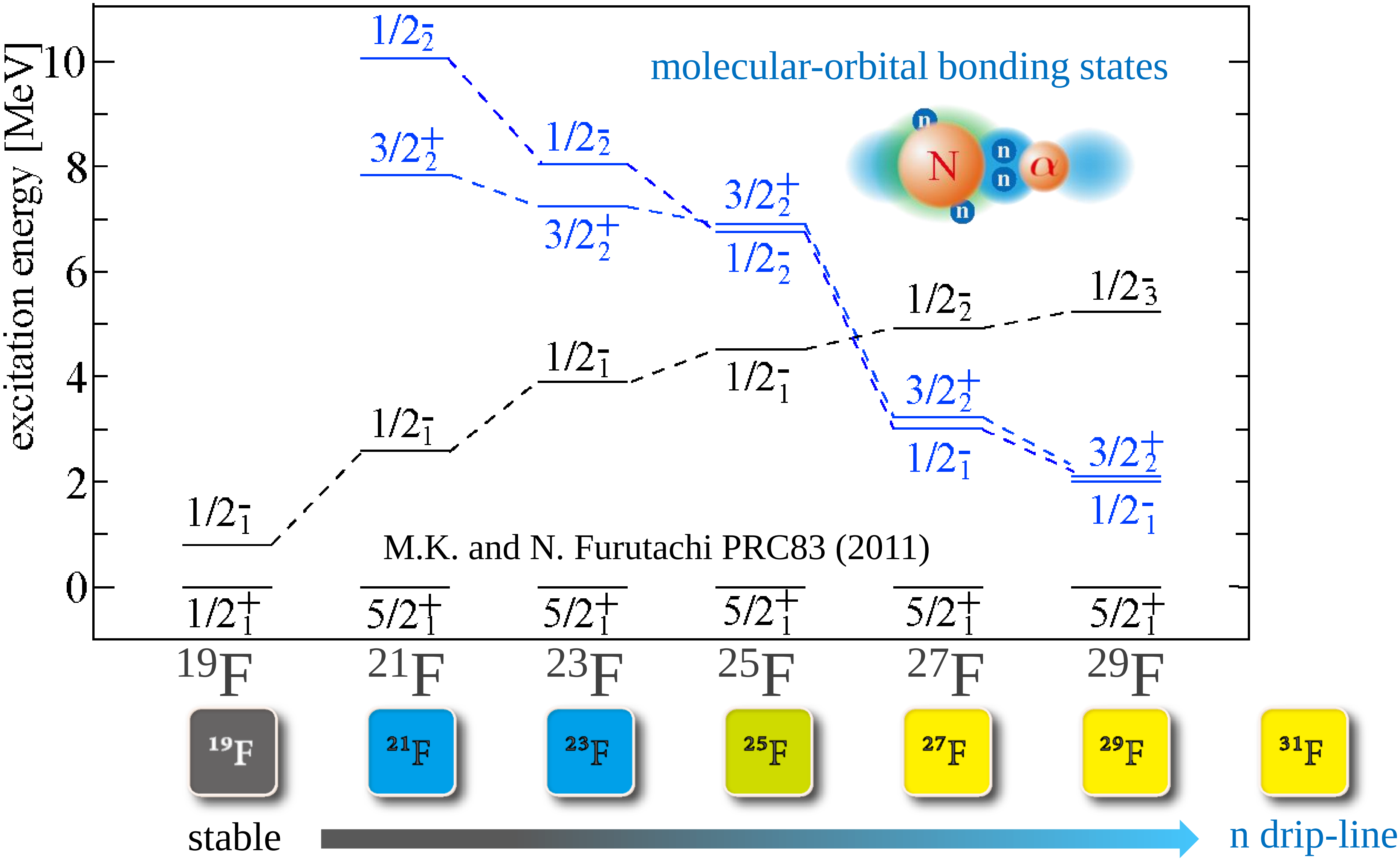
- deeply bound 10 neutrons are well confined within α and ^{16}O clusters
- 2 valence neutrons distribute entire system (molecular-orbital bonding)

Analogous to σ-molecular orbit in Be isotopes



Nuclear molecules with heavier masses

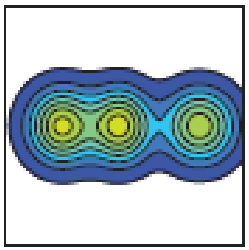
Reduction of excitation energies of molecular-orbital bonding states



Dimers, trimers and tetramers

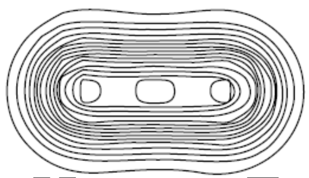
◎ Extension of "molecular-orbital bonding" to many clusters
⇒ trimers and tetramers

dimer (^{10}Be) | ^{6}He | trimer (^{16}C)



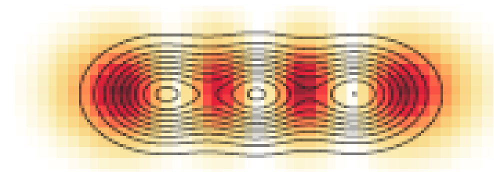
^{14}C : AMD

T. Suhara et al. PRC82(2010)



^{16}C : Hartree-Fock

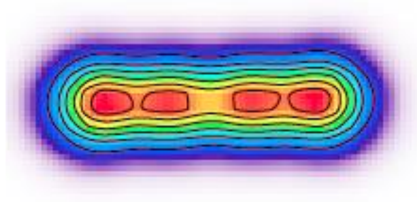
J. A. Maruhn et al. NPA833(2010)



^{16}C : AMD

T. Baba et al. submitted to PRC

trimer (^{16}C) | ^{6}He | tetramer (^{22}O)



^{16}O (4α): Cranked Hartree-Fock

T. Ichikawa, et al., PRL 107 (2011).

Summary

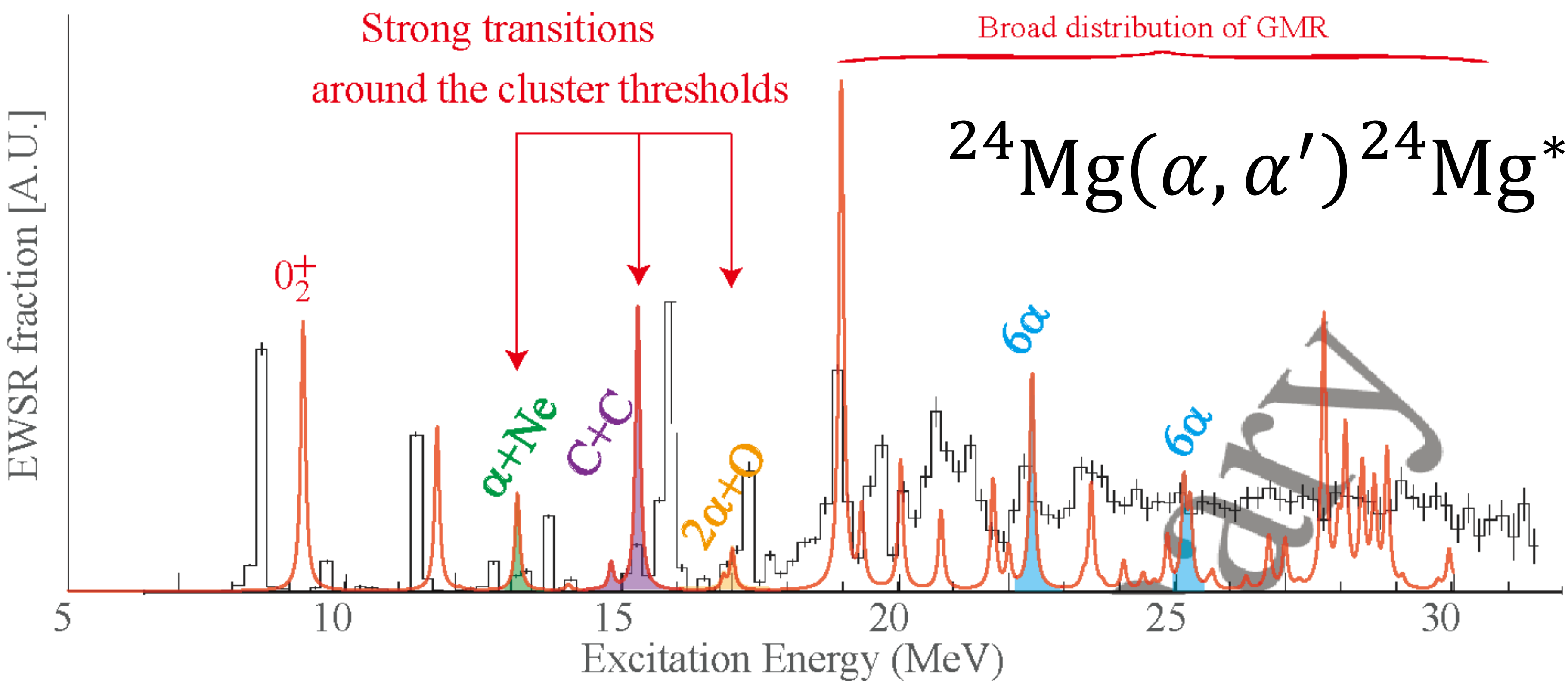
◎ Introduction
- Clustering phenomena and typical examples
- Antisymmetrized molecular dynamics (AMD)

◎ Clusters in stable nuclei
- Evolution of clusters as function of excitation energy
- Clusters in ^{24}Mg
- Monopole transition strengths as a probe of clusters

◎ Clusters in neutron-rich nuclei
- Nuclear molecule with molecular-orbital bonding
- From dimers to trimers and tetramers

◎ Summary

Summary & Perspective



◎ Highly excited cluster states are now theoretically and experimentally accessible !

backups

IS strength Calculation

IS monopole strength Calc.

1. Basis wave function

$\Psi^{\pi}(\beta_0,\gamma_0),\ \Psi^{\pi}(\beta_1,\gamma_1),\ \dots,$

on β–γ energy surface

2. Solve GCM equation

$$\sum_{jK'} H_{iKjK'} f_{jK'\alpha} = E_{\alpha} \sum_{jK'} N_{iKjK'} f_{jK'\alpha}$$

energyies and wave functions for 0^+ states

$$\Psi^{J\pi\alpha}_{\mathrm{GCM}} = \sum_{iK} f_{iK\alpha} \hat{P}^{J}_{MK} \Psi^{\pi}(\beta_i,\gamma_i)$$

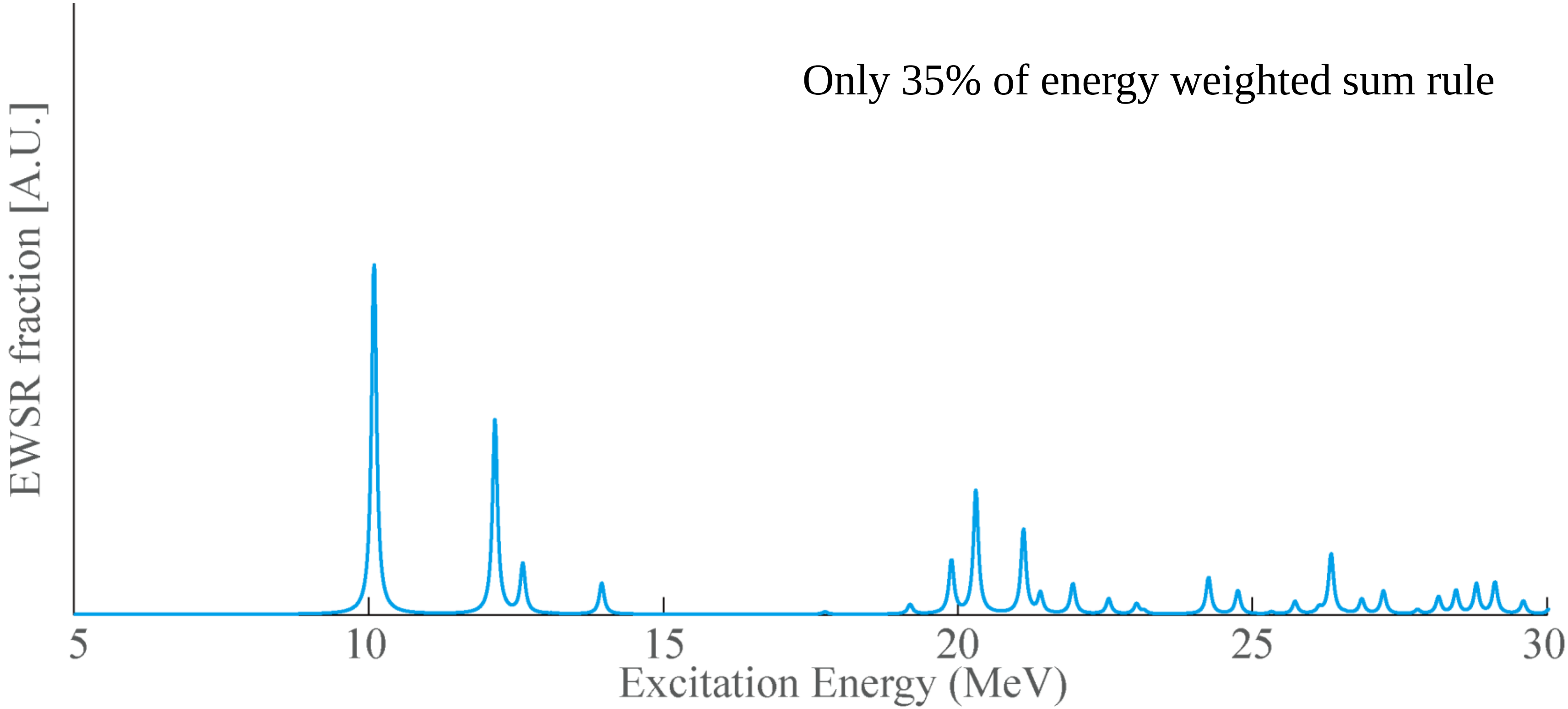
3. IS0 strengh

calculate B(IS0;$0^+_{\mathrm{g.s.}} \rightarrow 0^+_{\mathrm{n}}$)

for every 0+ states

strength of each peak is smeared

by Lorenzian with Γ=0.1 MeV



IS monopole strength Calc.

1. Basis wave function

$\Psi^\pi(\beta_0,\gamma_0), \Psi^\pi(\beta_1,\gamma_1), \ldots$

on β–γ energy surface

$\Psi^\pi(N_{x0}, N_{y0}, N_{z0}), \Psi^\pi(N_{x1}, N_{y1}, N_{z1}), \ldots,$

obtained by H.O. constraint
clusters, mp-mh configurations

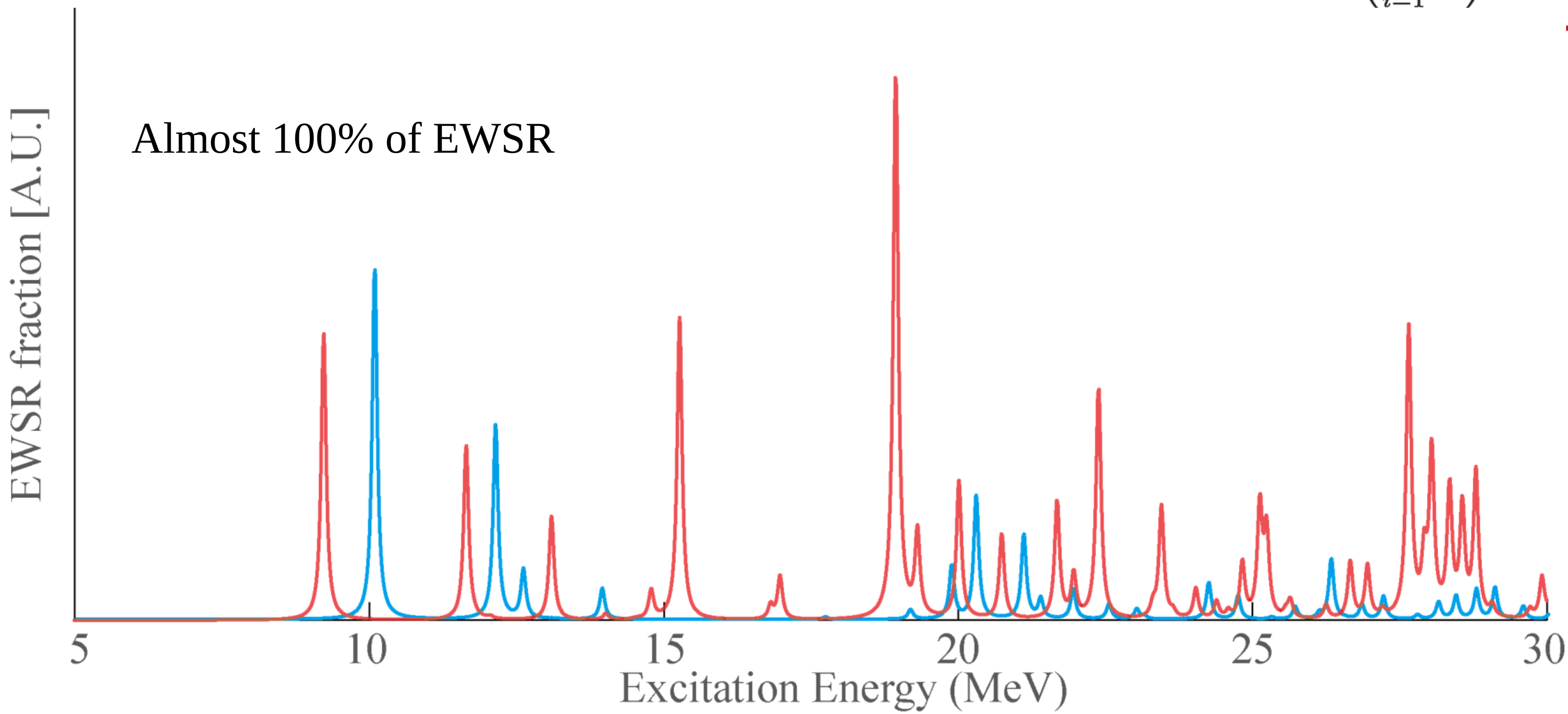
$\exp(\alpha\hat{O})\Psi^\pi(\beta_0,\gamma_0), \exp(\alpha\hat{O})\Psi^\pi(\beta_1,\gamma_1), \ldots$

IS monopole operator × β–γ basis

$$\hat{O} = \sum_{i=1}^{A} \hat{r}_i^2,$$

$$\exp(\alpha\hat{O})\Psi^\pi(\beta_0,\gamma_0) \simeq \Psi^\pi(\beta_0,\gamma_0) + \alpha\left(\sum_{i=1}^{A}\hat{r}_i^2\right)\Psi^\pi(\beta_0,\gamma_0),$$

basis



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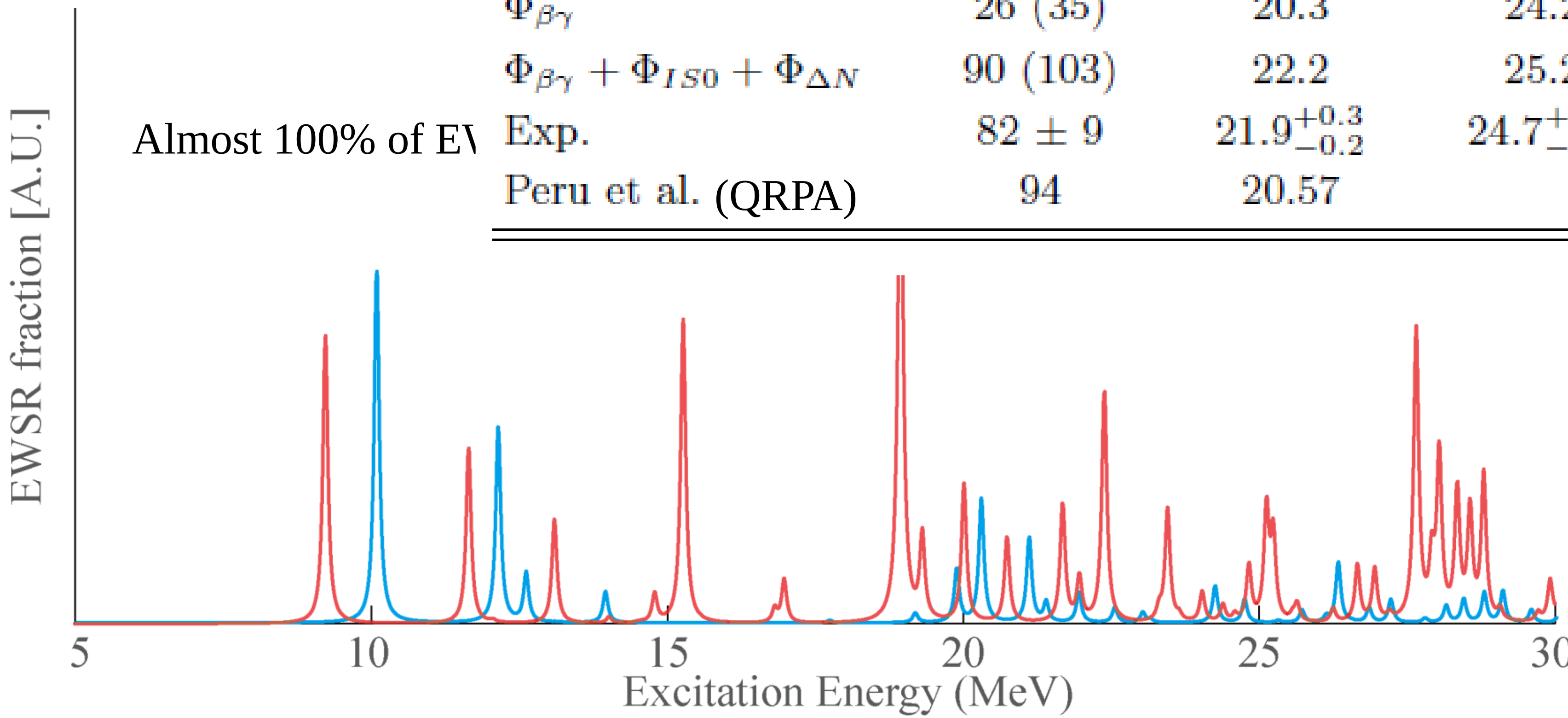
$$\hat{O} = \sum_{i=1}^{A} \hat{r}_i^2,$$

Basis	m_1	m_1/m_0	$\sqrt{m_3/m_1}$
$\Phi_{\beta\gamma}$	26 (35)	20.3	24.2
$\Phi_{\beta\gamma} + \Phi_{IS0} + \Phi_{\Delta N}$	90 (103)	22.2	25.2
Exp.	82 ± 9	$21.9^{+0.3}_{-0.2}$	$24.7^{+0.5}_{-0.3}$
Peru et al. (QRPA)	94	20.57	

Almost 100% of EW



IS monopole strength

- New peaks are Clusters !
  α + Ne: around 13 MeV
  C + C : around 15 MeV
  2α + O: around 17 MeV
  6α: 22 and 25 MeV