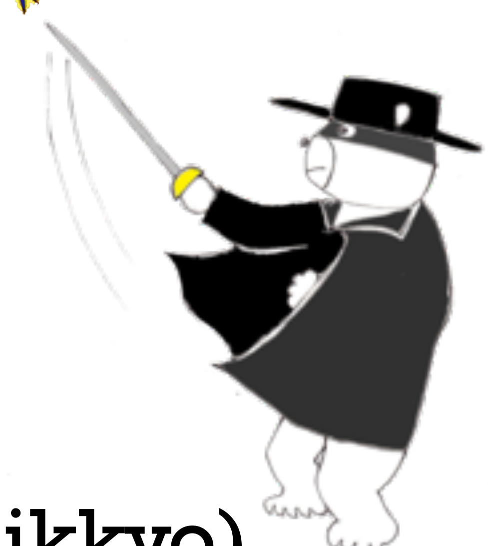
Numerical Study of QCD Phase Diagram with High Multi-precision Arithmetic

Advances and perspectives
in computational nuclear physics
Oct. 5-7, 2014, Hawaii
Kohala 3



Atsushi Nakamura
in collaboration with
Zn Collaboration
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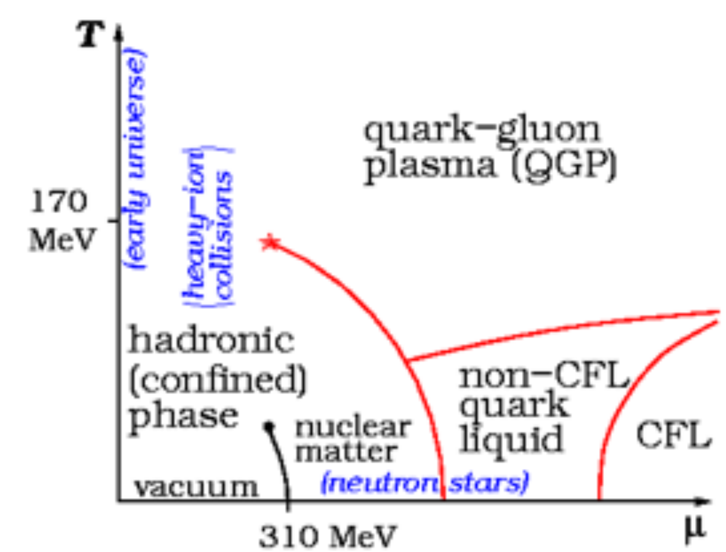
S.Sakai (Kyoto), Y. Taniguchi (Tsukuba)
K. Nagata(KEK), and K. Morita(Yukawa)



What is Z_N ?

Canonical Partition Function.

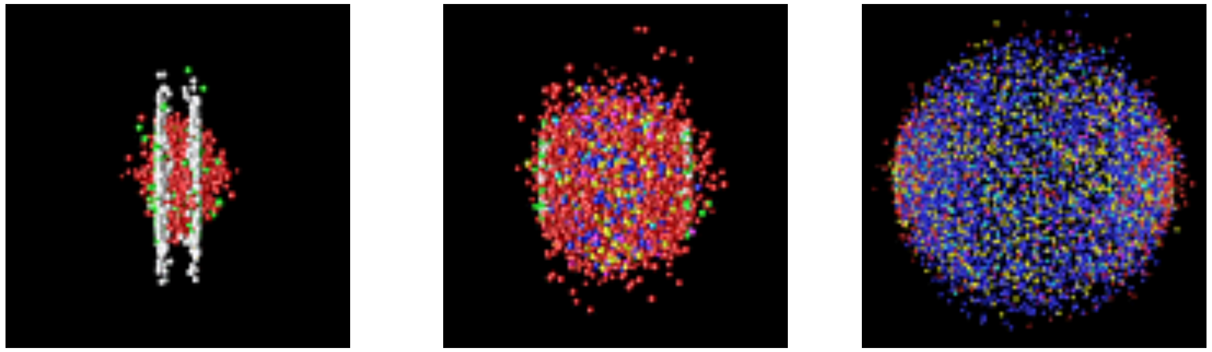
We will see it later.

Fireballs created in High Energy Nuclear Collisons are described as a Statistical System.

with Two Parameters:

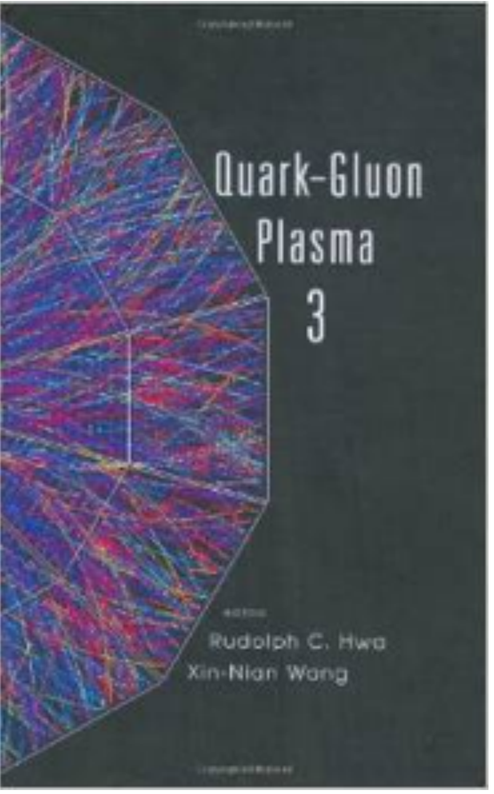
Chemical Potential, μ

and Temperature, T



$$Z(\mu, T)$$

Grand Canonical Partition Function



P. Braun-Munzinger , K. Redlich and J. Stachel
Quark Gluon Plasma 3, 491
arXiv:nucl-th/0304013

$$\ln Z(T,V,\vec{\mu}) = \sum_i \ln Z_i(T,V,\vec{\mu}),$$

$$\ln Z_i(T,V,\vec{\mu}) = \frac{V g_i}{2\pi^2} \int_0^\infty \pm p^2 dp \ln[1 \pm \lambda_i \exp(-\beta\epsilon_i)],$$

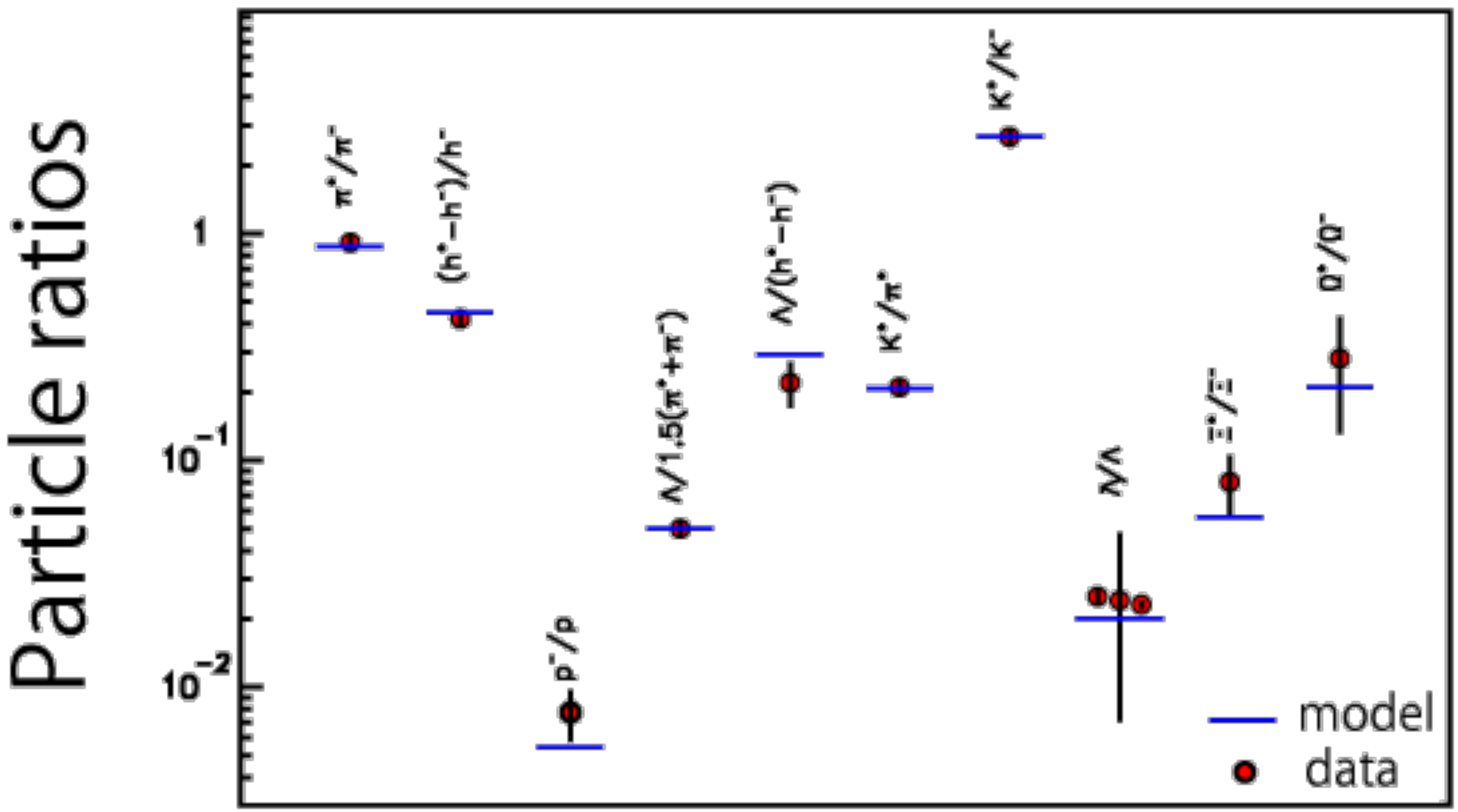
g_i spin--isospin degeneracy factor

(+) for fermions, (-) for bosons

$$\epsilon_i = \sqrt{p^2 + m_i^2}$$

$$\lambda_i(T,\vec{\mu}) = \exp\left(\frac{B_i\mu_B + S_i\mu_S + Q_i\mu_Q}{T}\right)$$

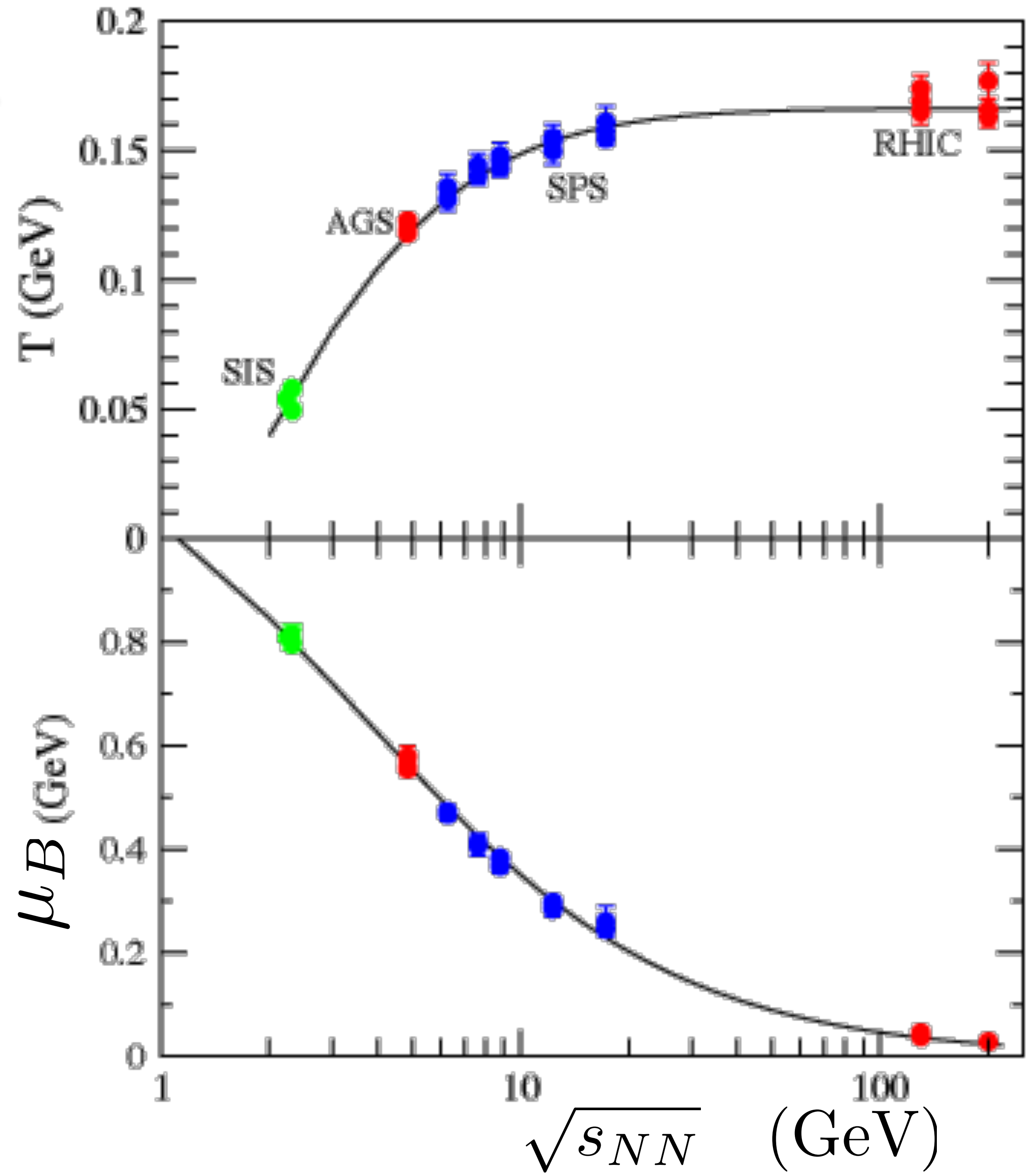
Parameters: T and μ

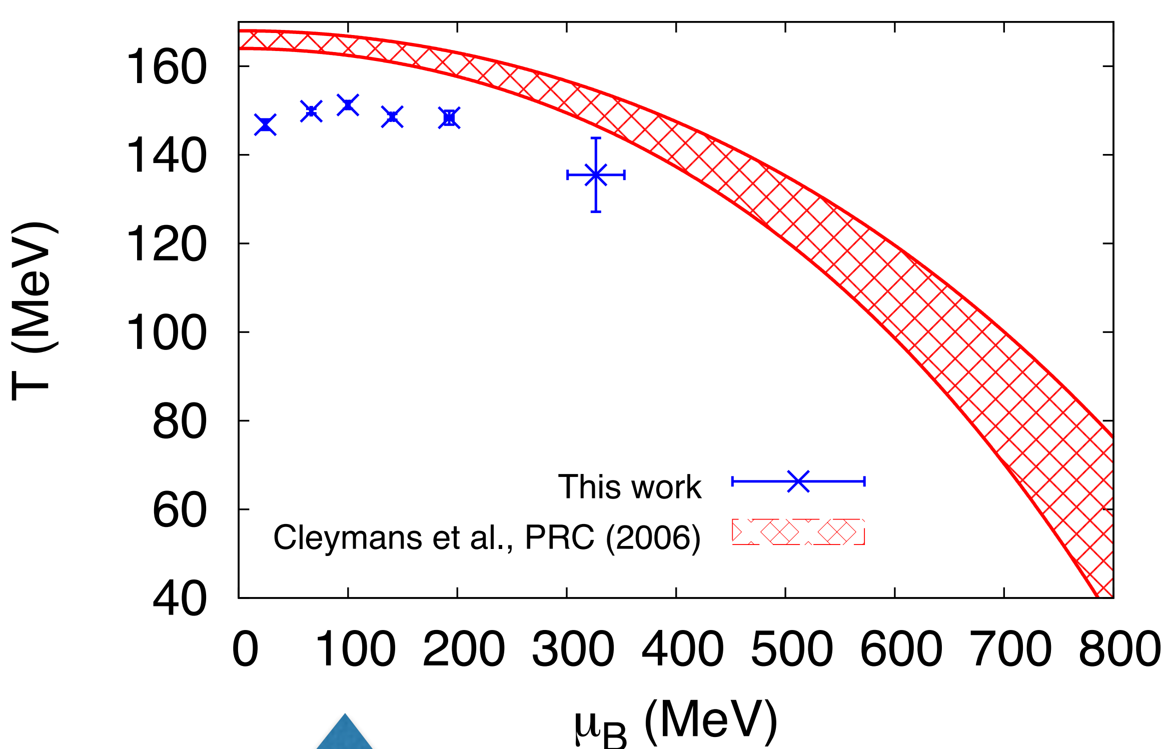


Pb–Pb collisions at 40 GeV/nucleon.
The thermal model calculations are obtained with T = 148 MeV and μB = 400 MeV

Freeze-out Analysis

J.Cleymans et al., Phys. Rev. C73, (2006) 034905.





Alba et al., arXiv:1403.4903

including also higher moments of multiplicities

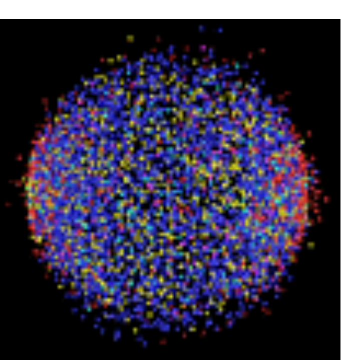
Statistical Description is good at least as a first approximation

with Two Parameters Chemical Potential, μ
and Temperature, T

$Z_{GC}(\mu, T)$ Grand Canonical Partition Function

Alternative: Number, n and Temperature, T

$Z_C(n, T)$ Canonical Partition Function



or Z_N

They are equivalent and related as

$$Z(\xi, T) = \sum_n Z_n(T)\, \xi^n$$

$$\xi \equiv e^{\mu/T}$$ Fugacity

(Probably) Well-known and easy to prove

This is very useful relation.

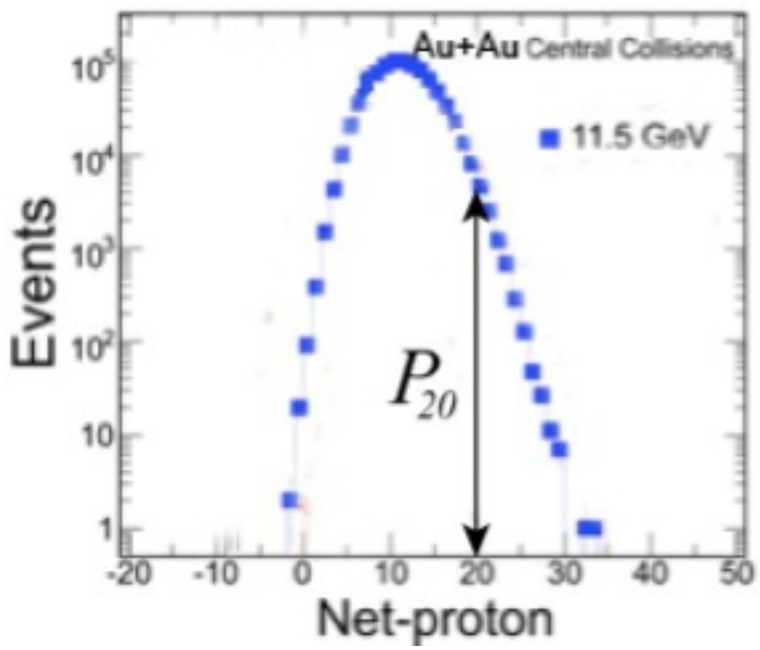
The partition function stands for the Probability

$$Z_{GC}(\mu, T) = \sum_n Z_n(T)\xi^n$$

System with μ and T

Probability to find (net-)baryon number= n

We extract Z_n from experimental multiplicity at RHIC



$$P_n = Z_n \xi^n$$

$$\left(\xi \equiv e^{\mu/T}\right)$$

ξ unknown

$$Z_n = P_n / \xi^n$$

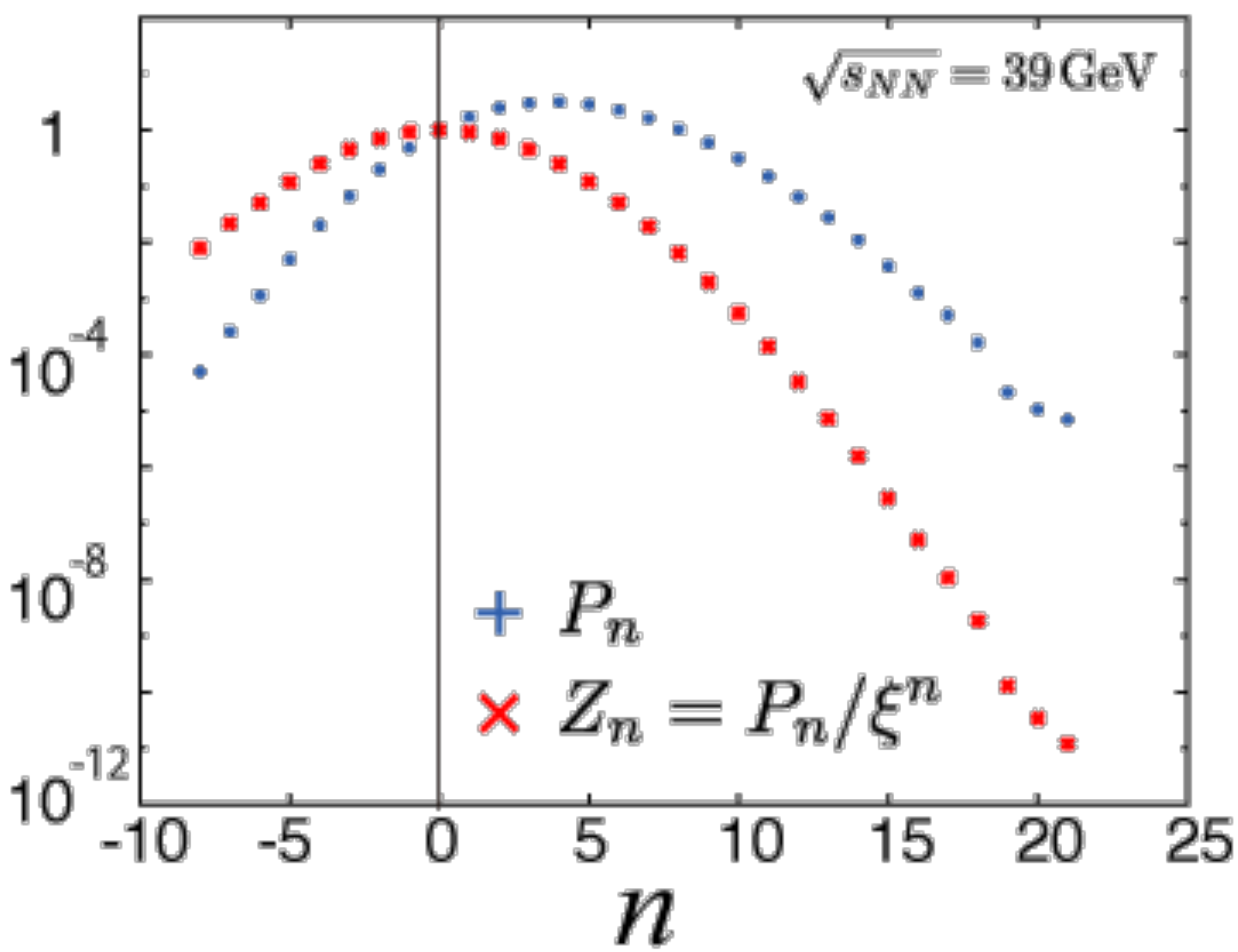
RHIC provides us Z_n

We require

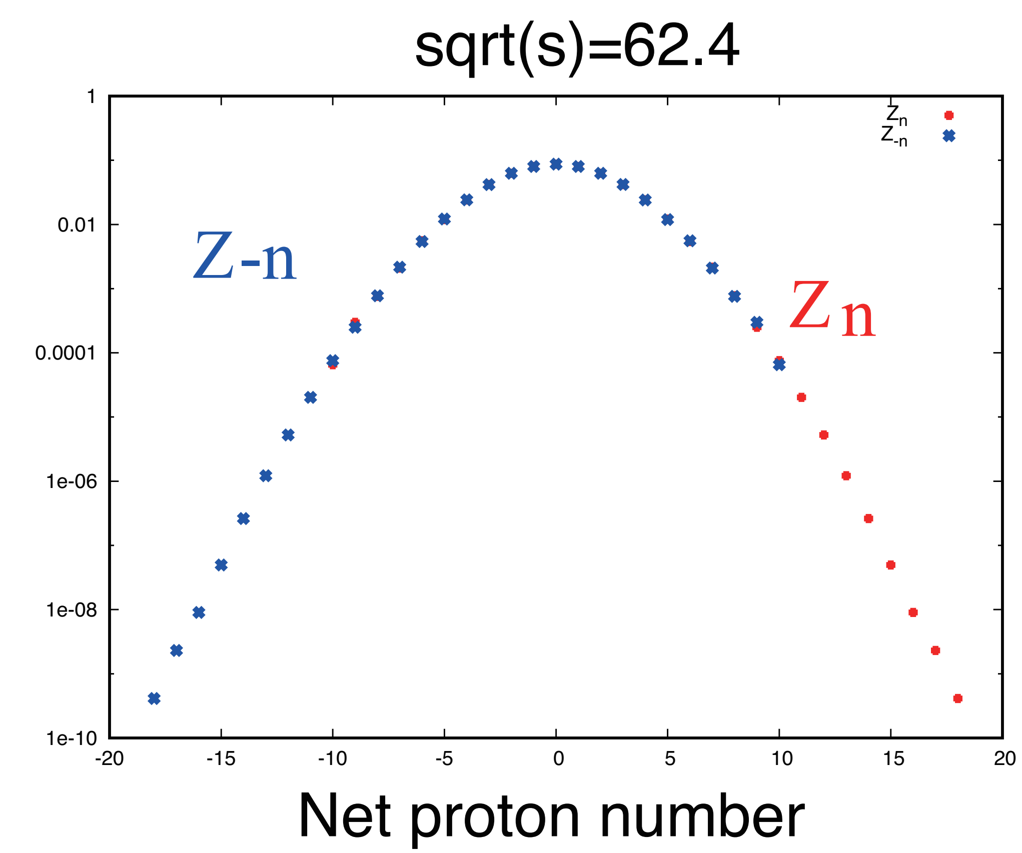
$$Z_{+n} = Z_{-n}$$

Particle-AntiParticle Symmetry)

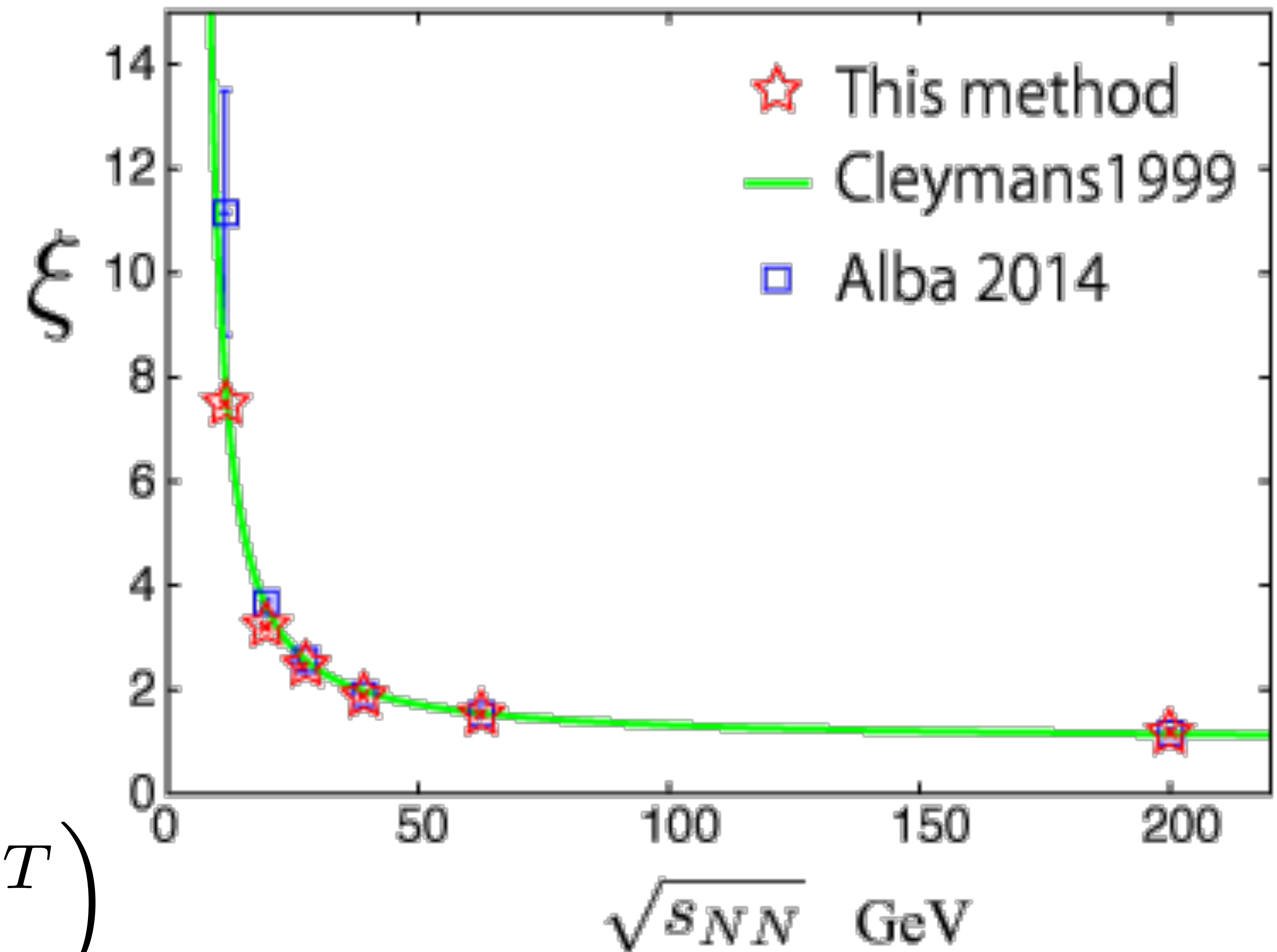
Demand $Z_{+n} = Z_{-n}$



$\xi = 1.88336$



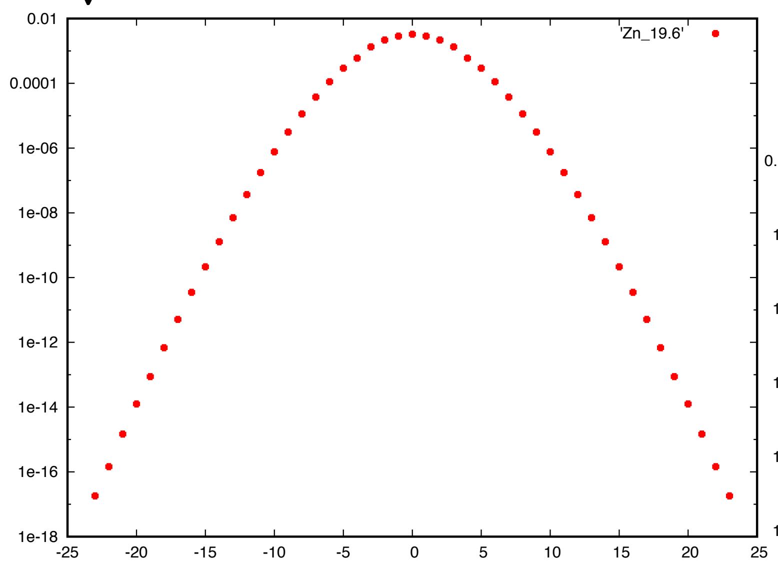
Fitted ξ are very consistent with those by Freeze-out Analysis.



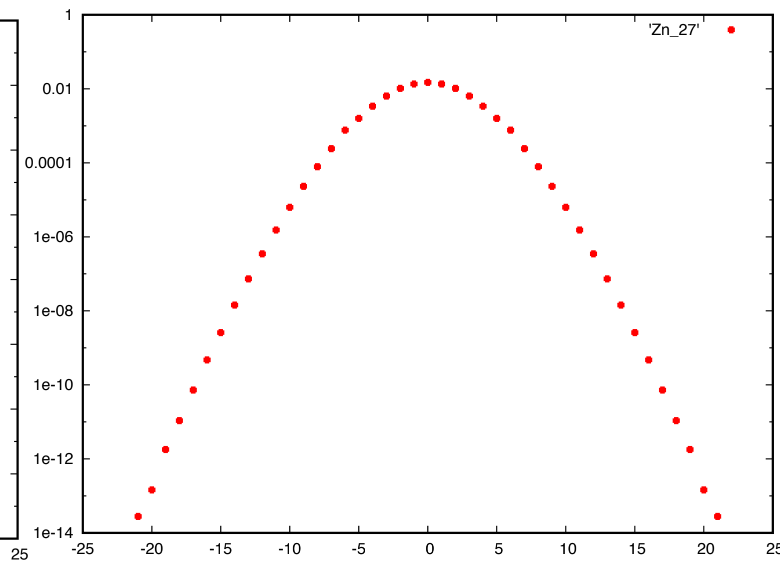
$$\left(\xi \equiv e^{\mu/T}\right)$$

Z_n from RHIC data

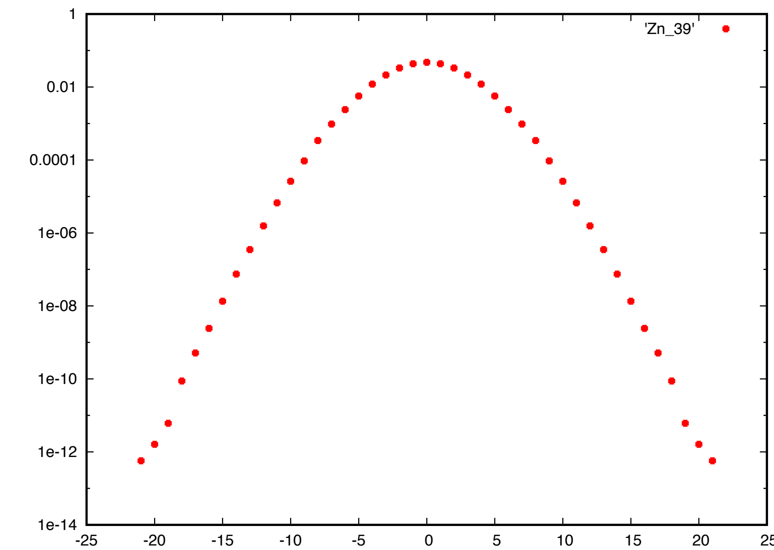
$\sqrt{s} = 19.6\mathrm{GeV}$



$\sqrt{s} = 27\mathrm{GeV}$

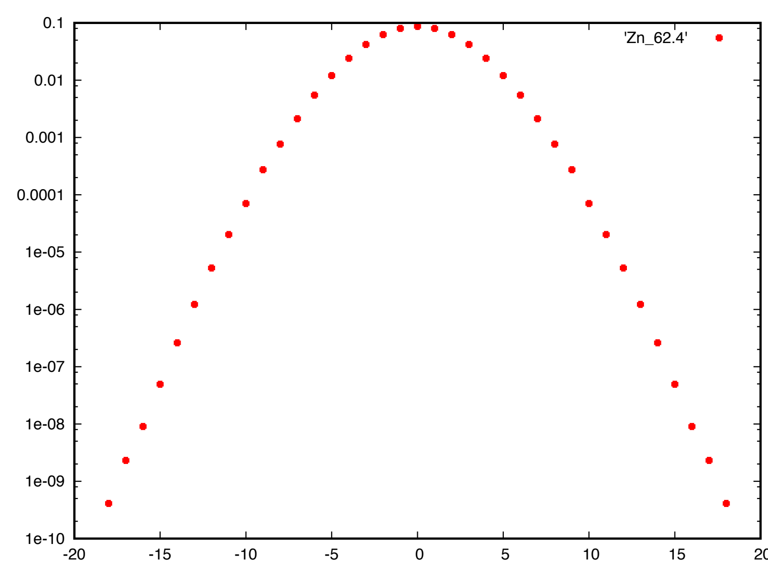


$\sqrt{s} = 39\mathrm{GeV}$

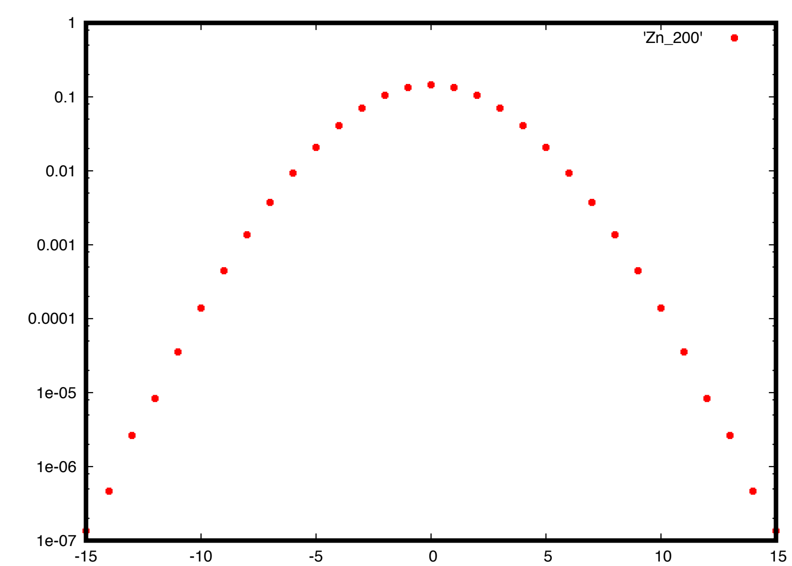


Can I see Difference?

$\sqrt{s} = 62.4\mathrm{GeV}$



$\sqrt{s} = 200\mathrm{GeV}$



Yes, You Can !
We will see it.

Yes, very useful, because

$$Z(\xi, T) = \sum_n Z_n(T)\,\xi^n$$

($\xi \equiv e^{\mu/T}$: Fugacity)

$Z_n(T) \leftarrow Z(\xi, T)$ at some ξ and T

$Z_n(T) \rightarrow Z(\xi, T)$ at ANY ξ

for both Experiments and Lattice

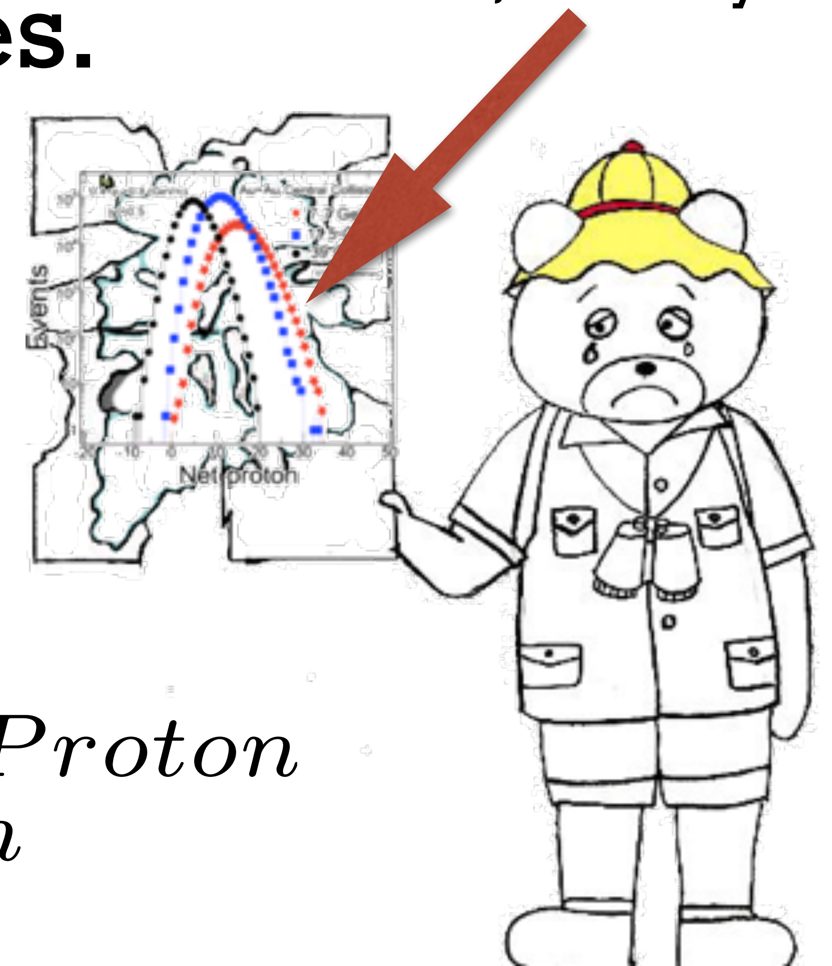
(Current) Weak Points

1) Experimental Multiplicity Data

Net-Proton and Not net-Baryon

One can prove $Z(\xi, T) = \sum_n Z_n(T)\, \xi^n$ only for Conserved Quantities.

Proton, not Baryon



Possible approaches:

i) Wait for Net-Baryon data, or Net-Charge data.

ii) Study and analyze data assuming $Z_n^{Baryon} \sim Z_n^{Proton}$

2) N_{max} is not very large.

$$Z(\xi, T) = \sum_{n=-N_{max}}^{+N_{max}} Z_n(T)\,\xi^n$$

Lower estimation of larger density contribution.

We can calculate also by Lattice QCD Z_n

But Sign Problem on Lattice ?

$$Z_{GC}(\mu, T) = \int \mathcal{D}(\text{Gluon Fields}) \times \boxed{\det D(\mu)}\; e^{-(\text{Gluon Action})}$$

Complex if μ is real.

For Pure Imaginary μ → $\det D$ **real**

A.Hasenfratz and Toussant, 1992

$$Z_C(n,T) = \int \frac{d\theta}{2\pi} e^{i\theta n} Z_{GC}(\theta \equiv \frac{\mathrm{Im}\mu}{T}, T)$$

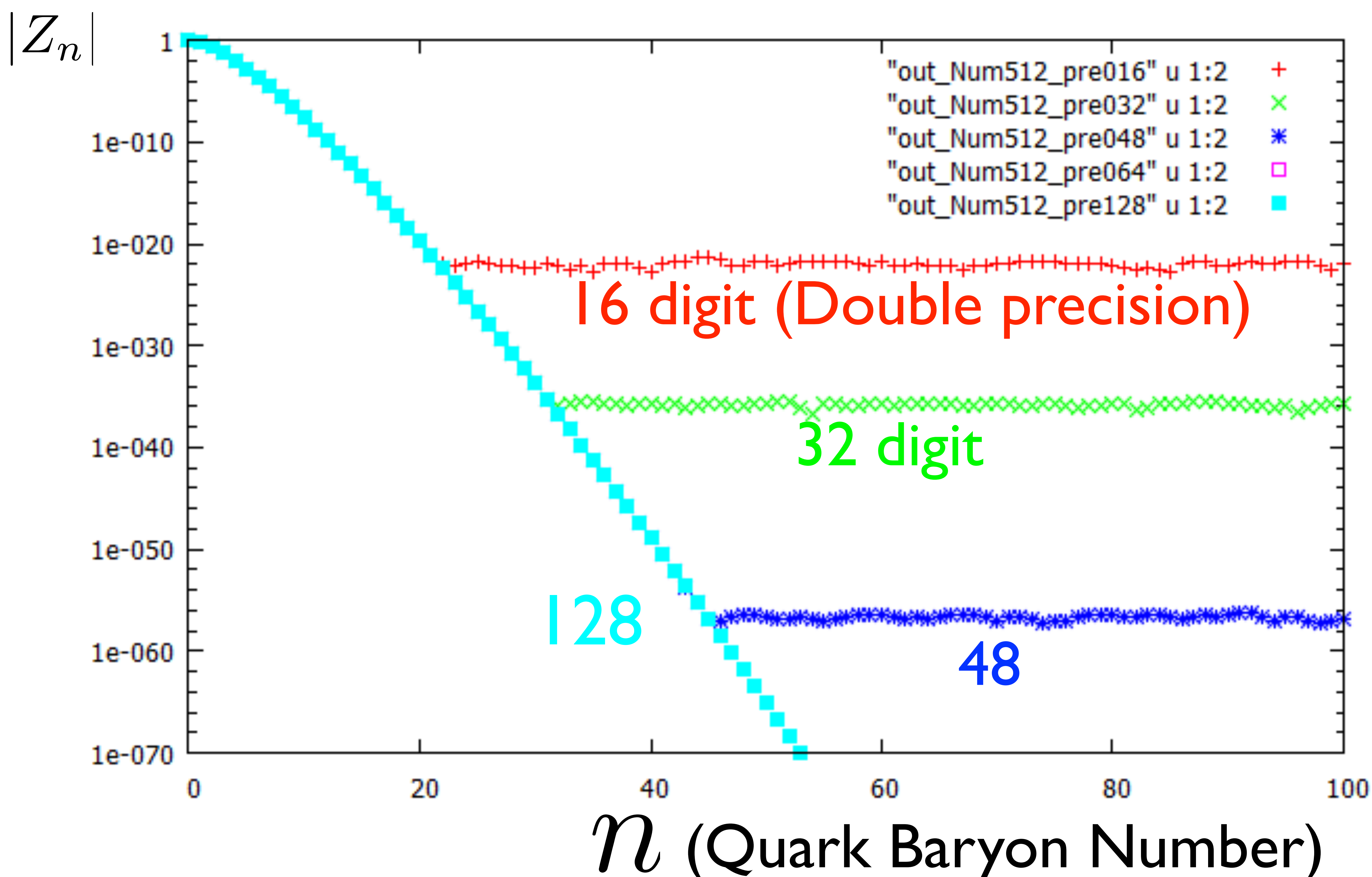
Great Idea ! But practically it did not work.

Zn Collaboration Method:

$$Z_C(n,T) = \int \frac{d\theta}{2\pi} \int \frac{\det(\theta)}{\det(\theta_0)} \det(\theta_0) e^{-(\mathrm{Gluon\ Action})}$$

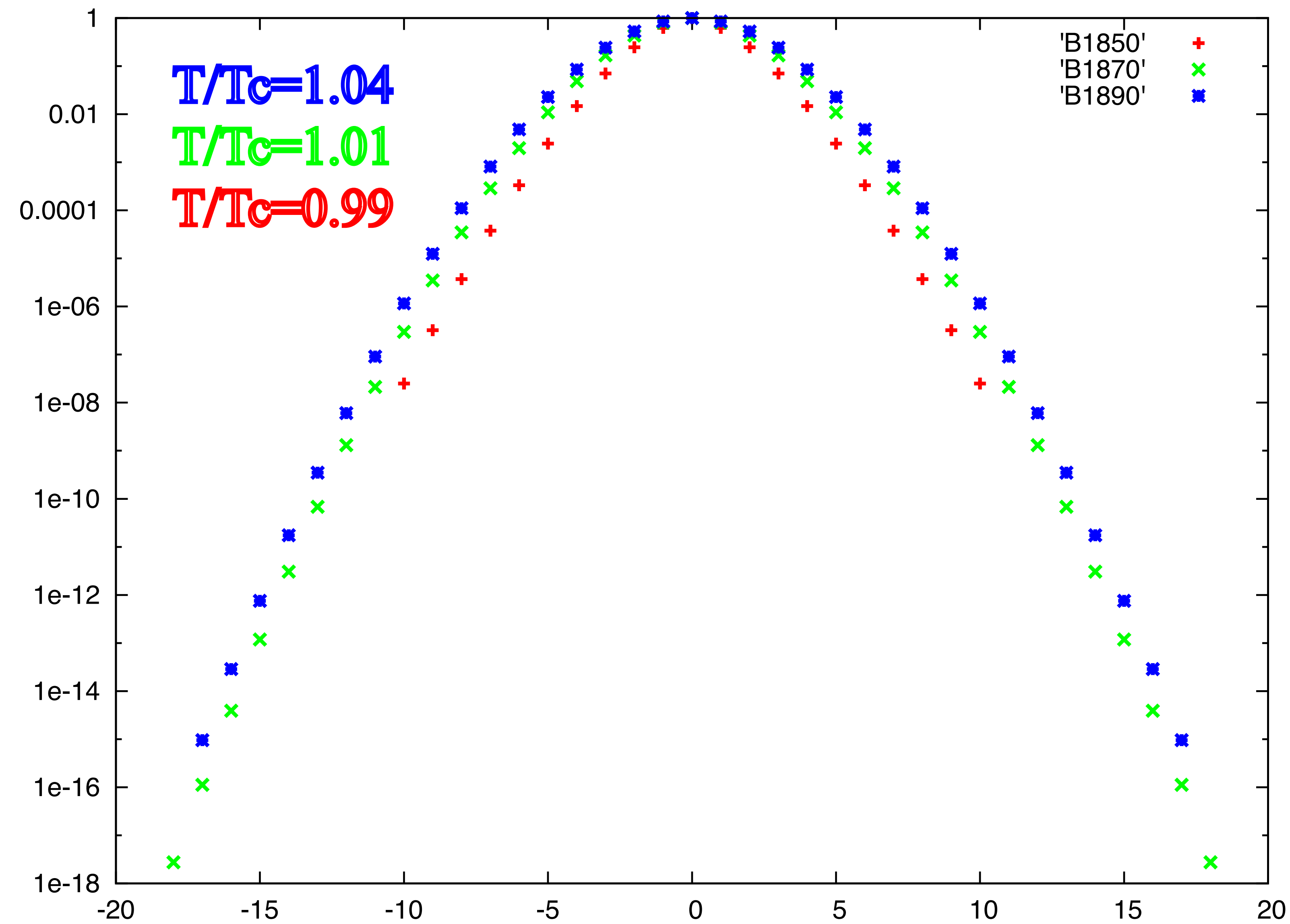
θ integration → Multi-Precision (50 - 100)

Fourier Transformation with multi-precision



Lattice Data

Zn



Yes, You Can !
Wait a moment.

$$Z(\xi, T) = \sum_n Z_n(T)\, \xi^n$$

$$\xi \equiv e^{\mu/T}$$

Is this useful ?

Yes, because

1) We can calculate Z at any ξ (i.e., μ)

2) We can calculate Z even at complex ξ

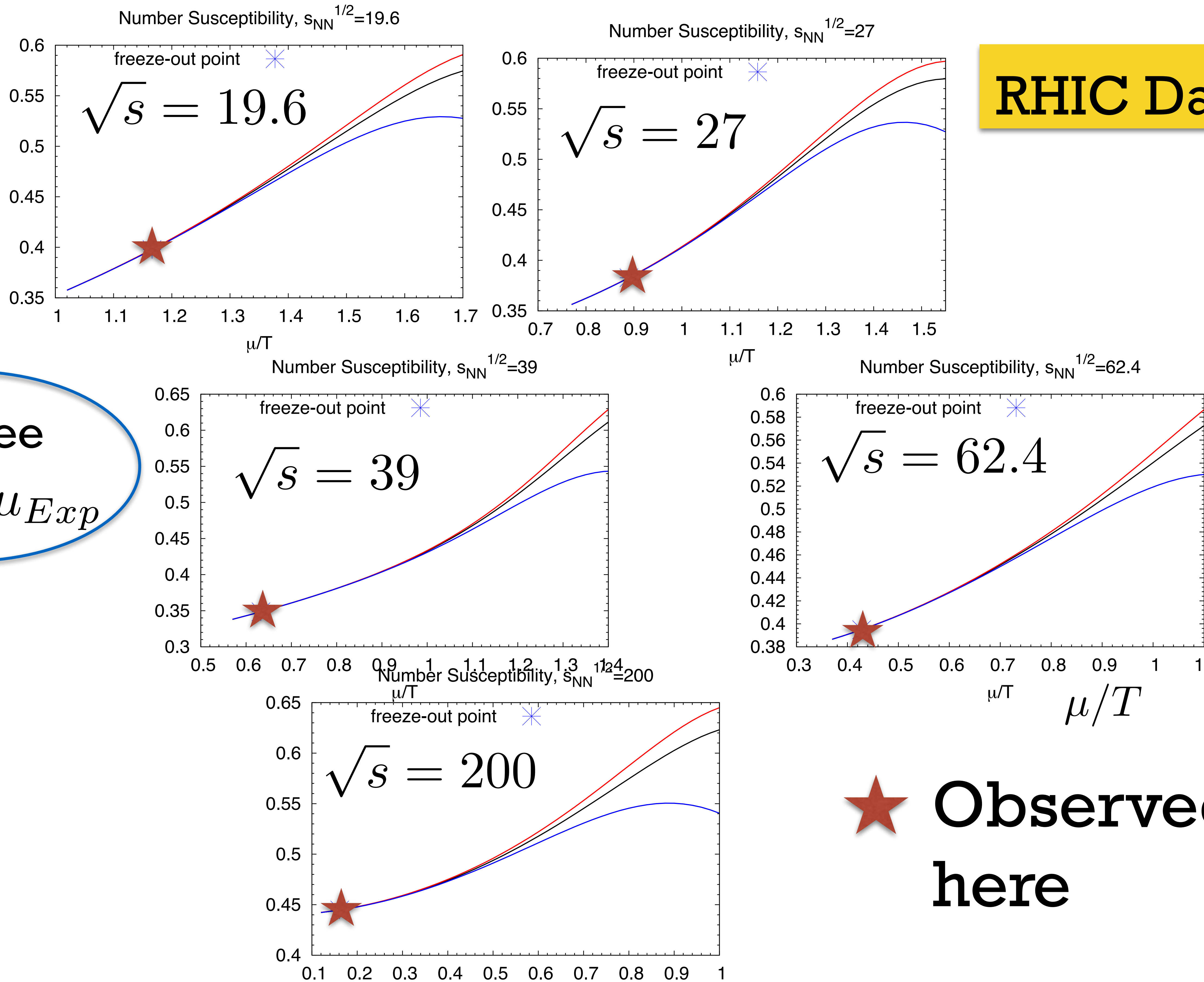
Moments λ_k

$$Z(\xi, T) = \sum_{n=-N_{max}}^{+N_{max}} Z_n(T)\,\xi^n$$

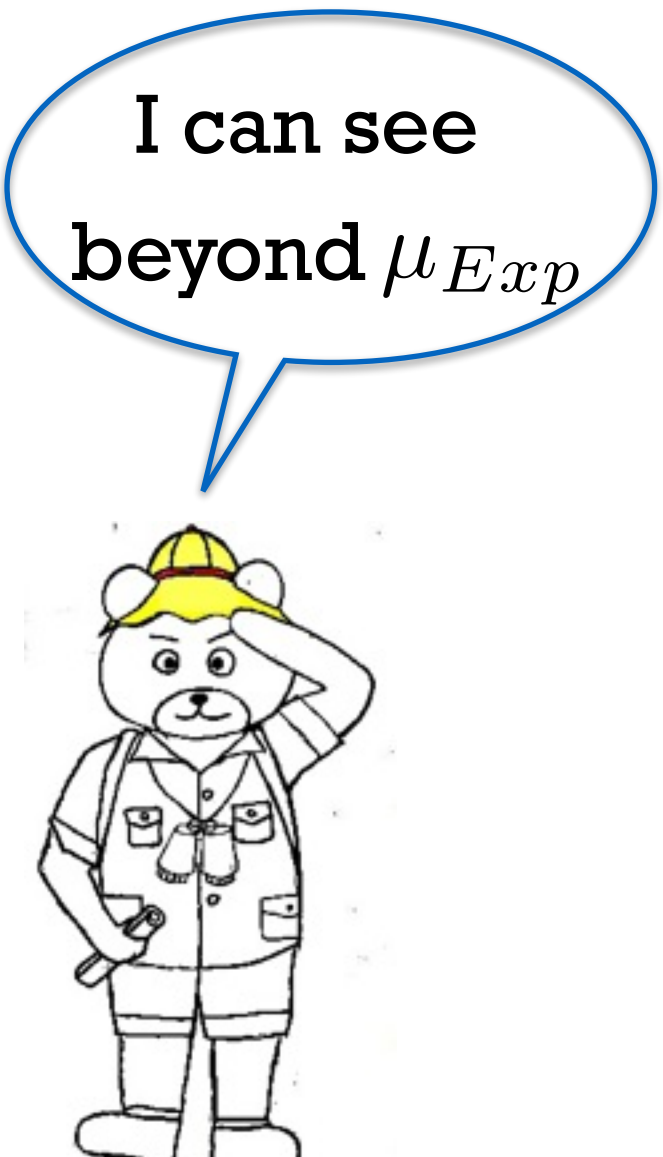
$$\lambda_k \equiv \left(T\frac{\partial}{\partial \mu}\right)^k \log Z$$

$$= \left(\xi\frac{\partial}{\partial \xi}\right)^k \log Z$$

Susceptivility as a function of μ/T



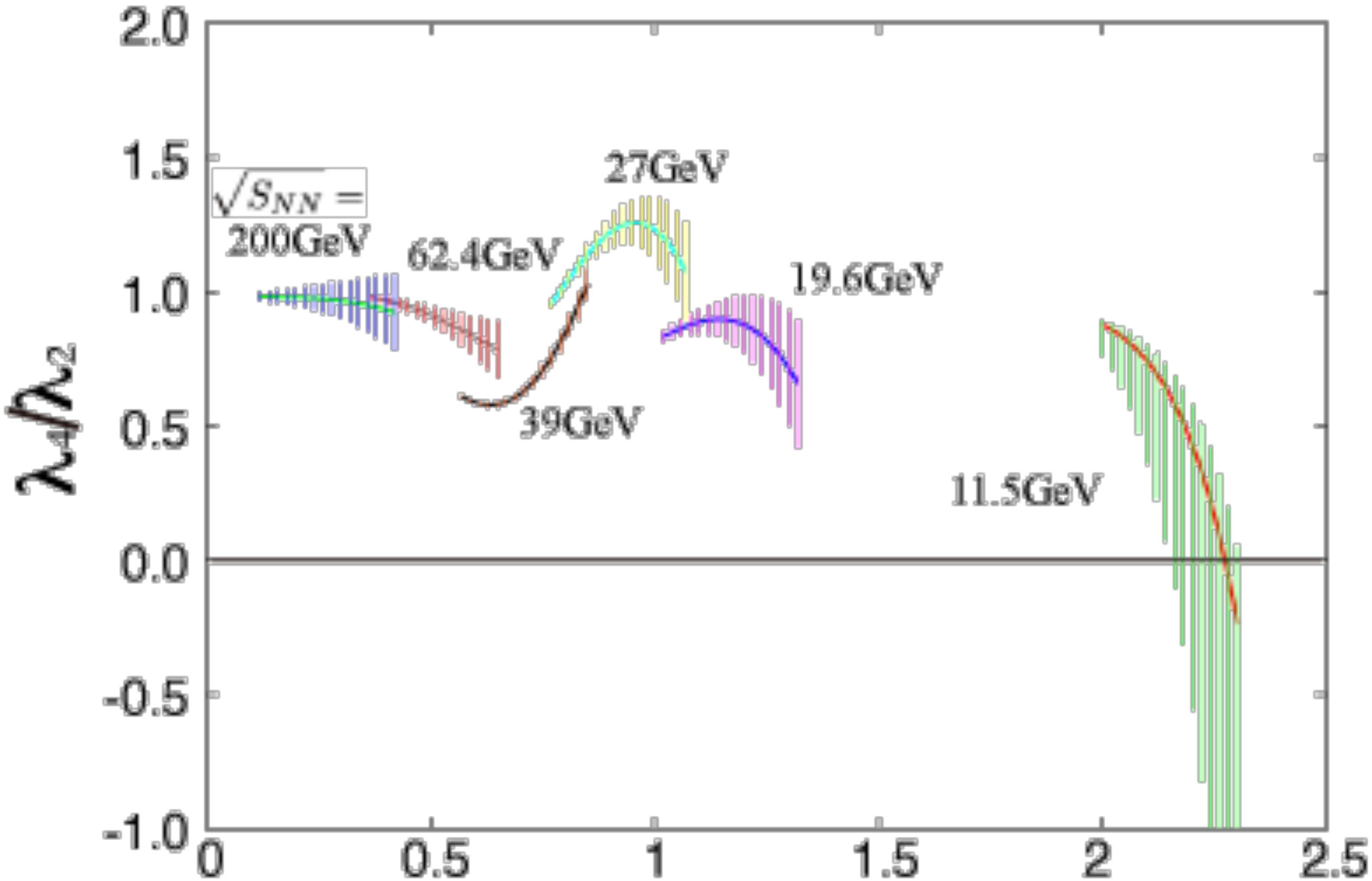
RHIC Data



★ **Observed here**

RHIC Data

Kurtosis $\frac{\lambda_4}{\lambda_2}$ as a function of $\frac{\mu}{T}$



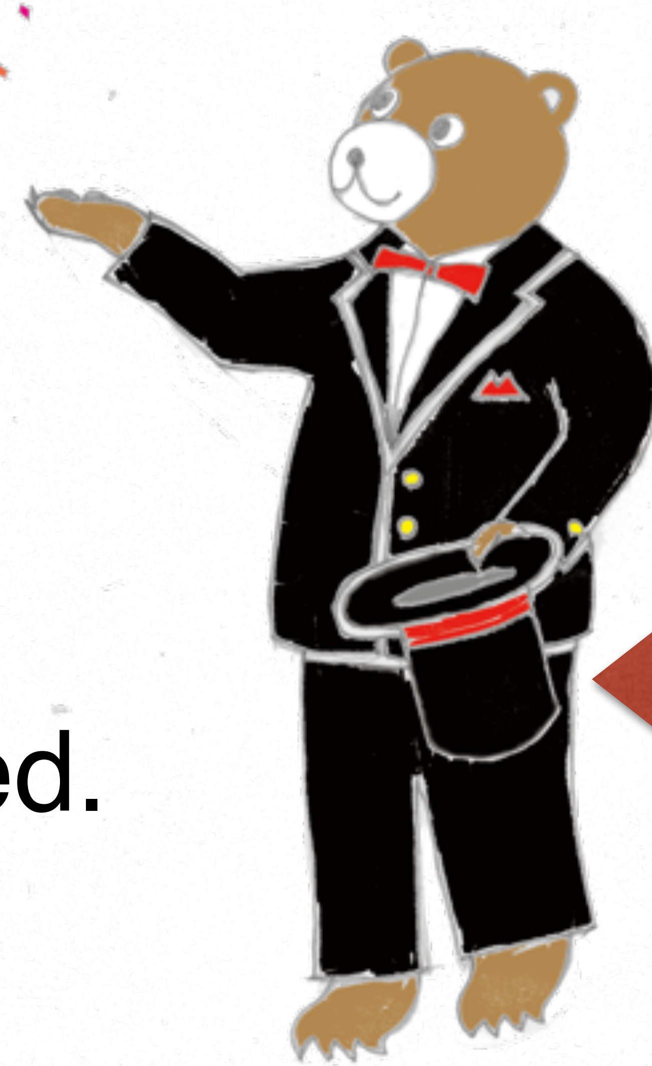
$$\frac{\mu}{T}$$

Data are taken at μ_0
You calculate the moments at $\mu > \mu_0$

Magic ?
or Cheating ?

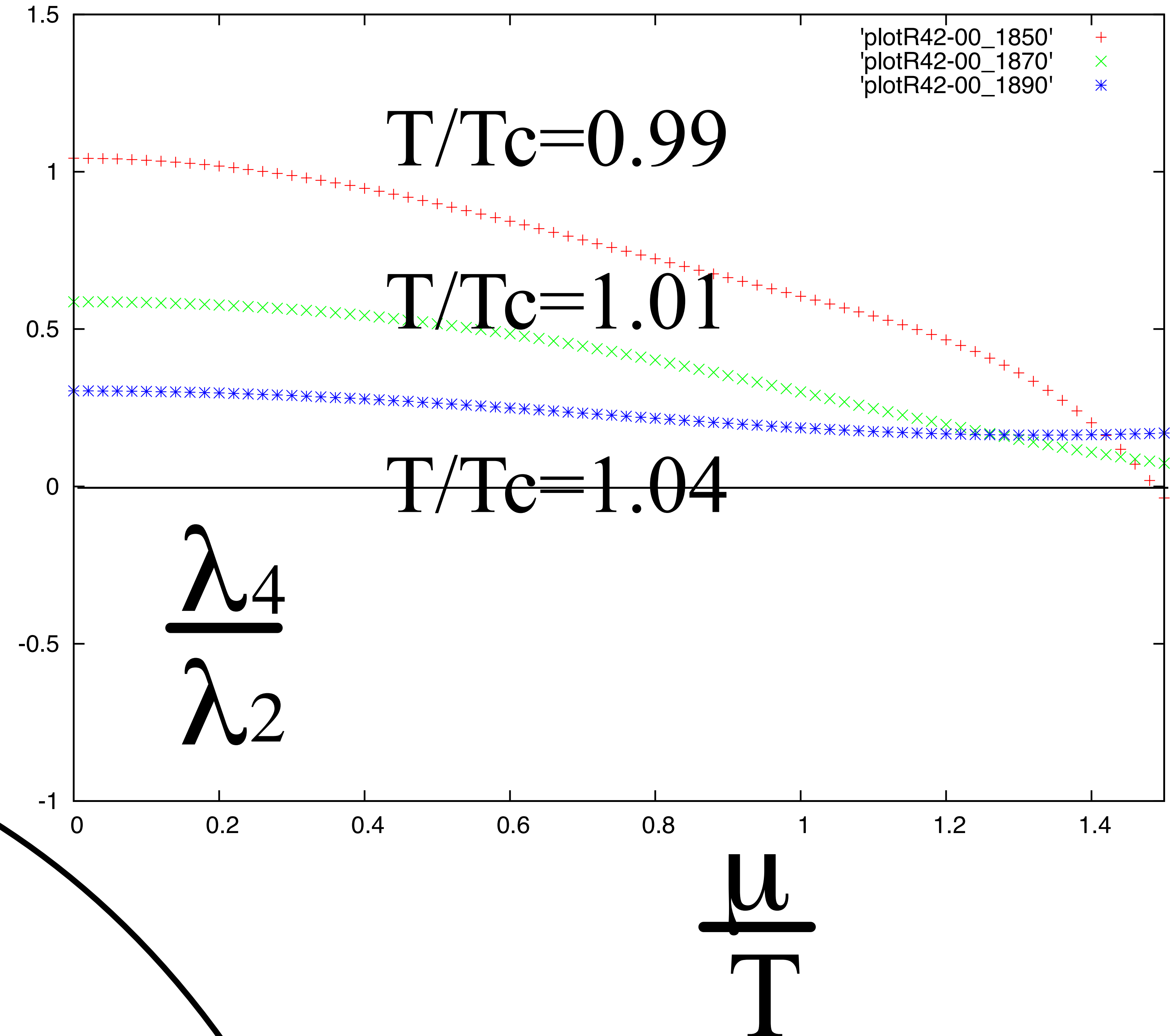
Moments at $\mu > \mu_0$

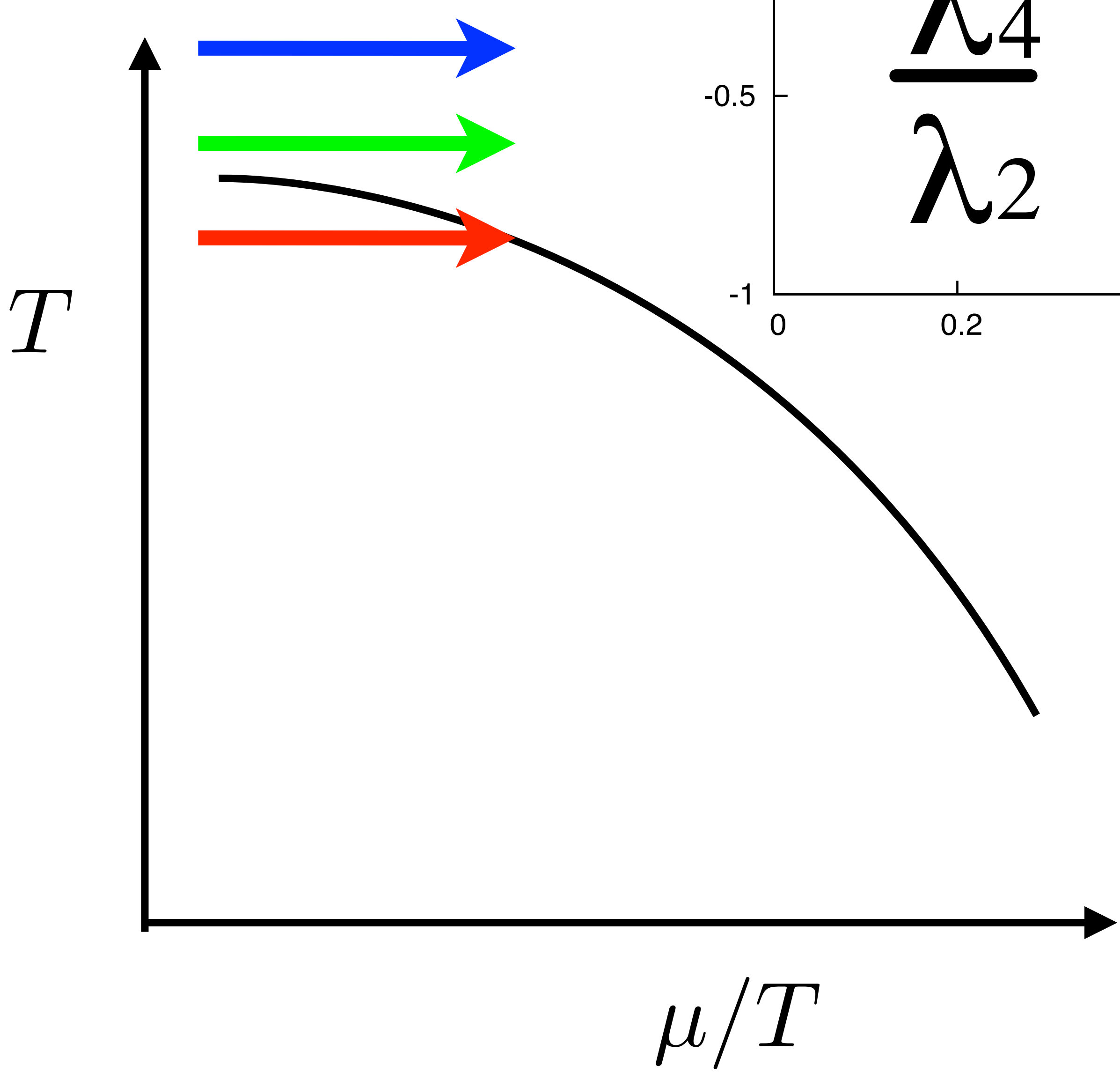
No Magic !
We use all Z_n data,
$(-N_{max} \le n \le +N_{max})$
that are usually not employed.



Data at μ_0

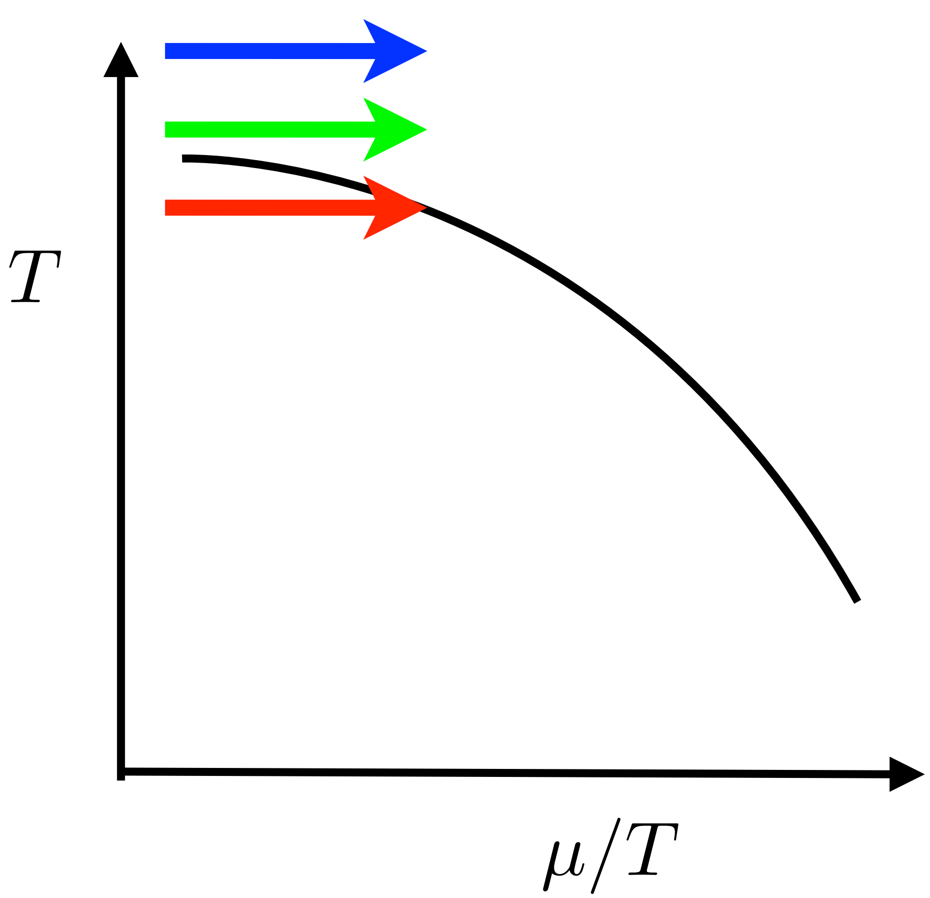
Lattice

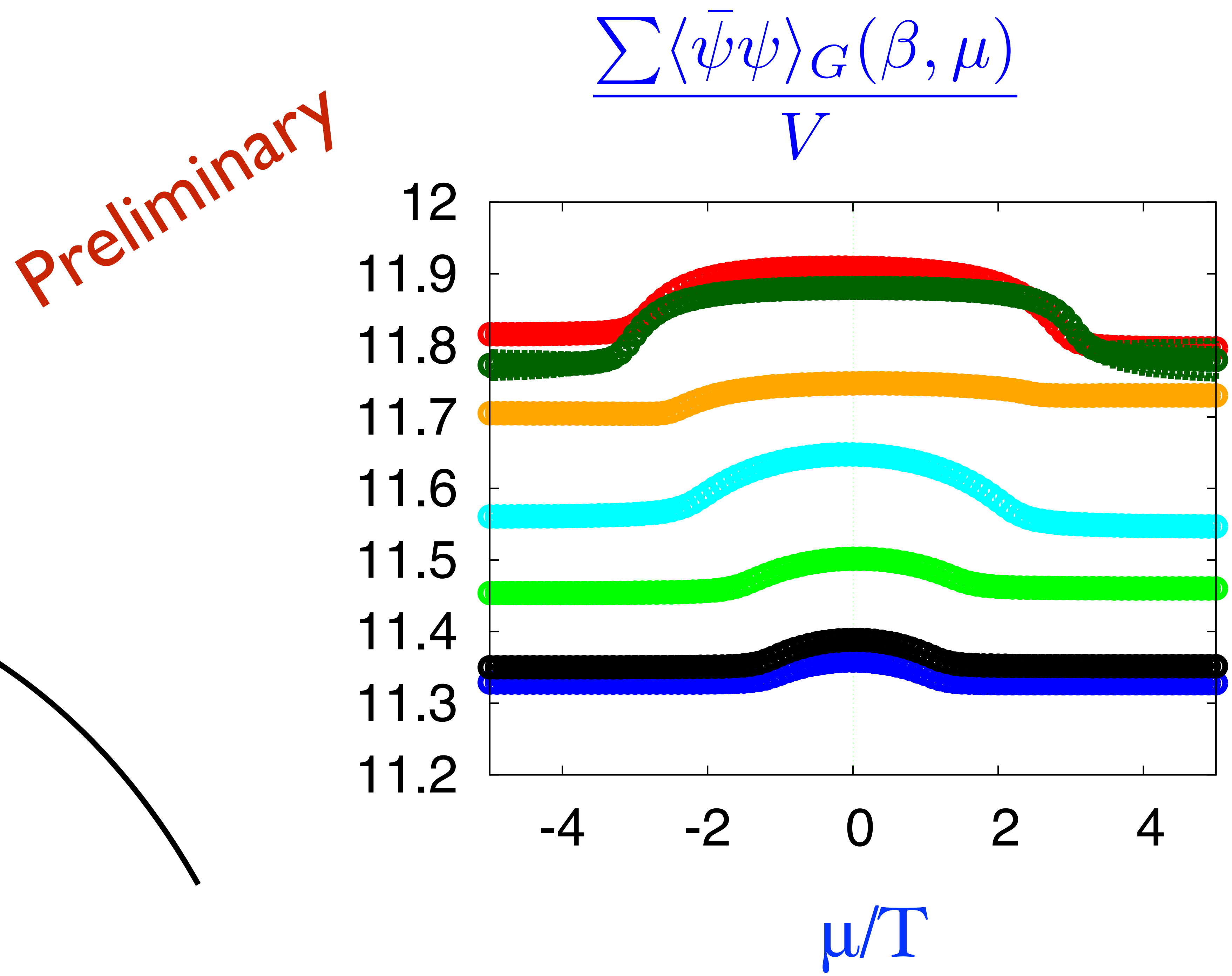

1.5
1
0.5
0
-0.5
-1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
'plotR42-00_1850'
'plotR42-00_1870'
'plotR42-00_1890'
T/Tc=0.99
T/Tc=1.01
T/Tc=1.04
λ4 / λ2
μ / T


T
μ/T

Chiral Condensate

Lattice





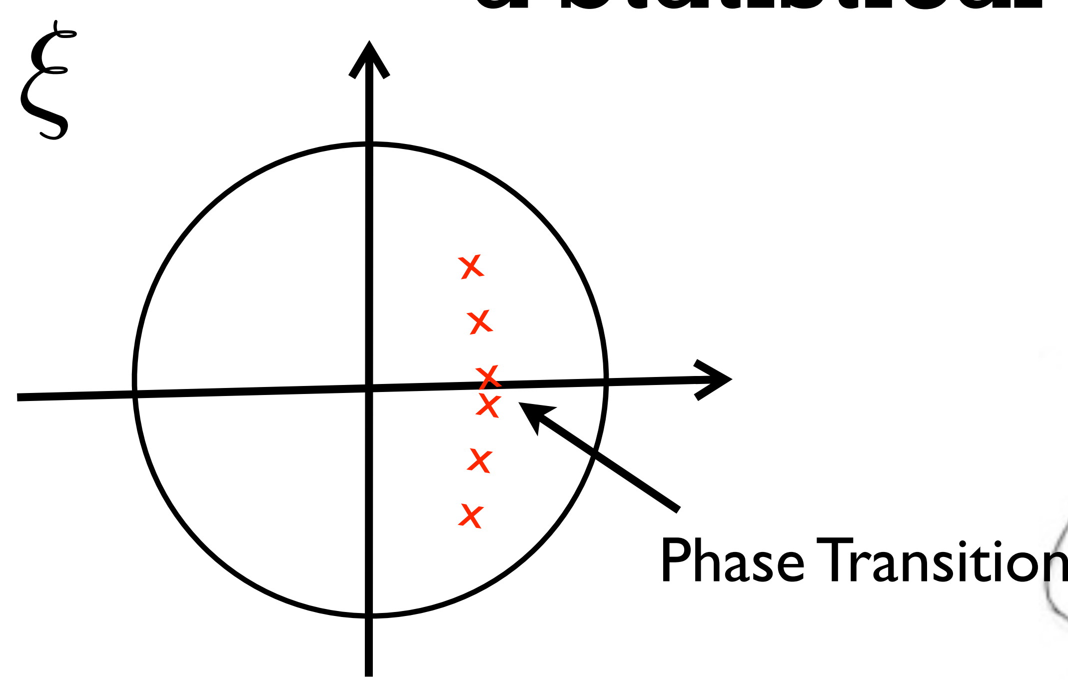
$$Z(\xi,T) = \sum_{n=-N_{max}}^{+N_{max}} Z_n(T)\,\xi^n$$

Lee-Yang Zeros (1952)

Zeros of $Z(\xi)$ in Complex Fugacity Plane.

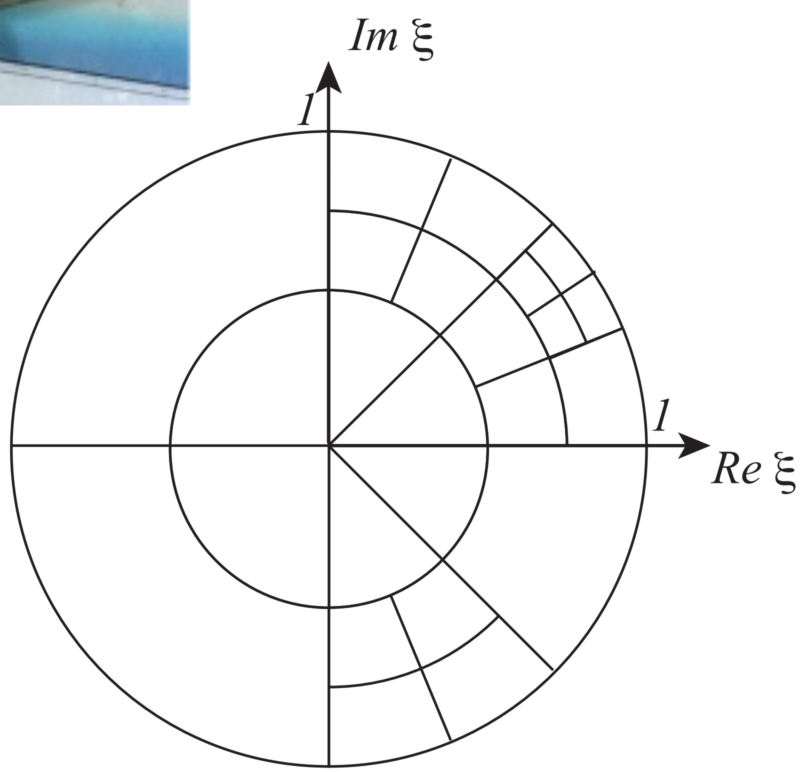
$$Z(\alpha_k) = 0$$

Great Idea to investigate a Statistical System



and

cut Baum-Kuchen (cBK) Algorithm



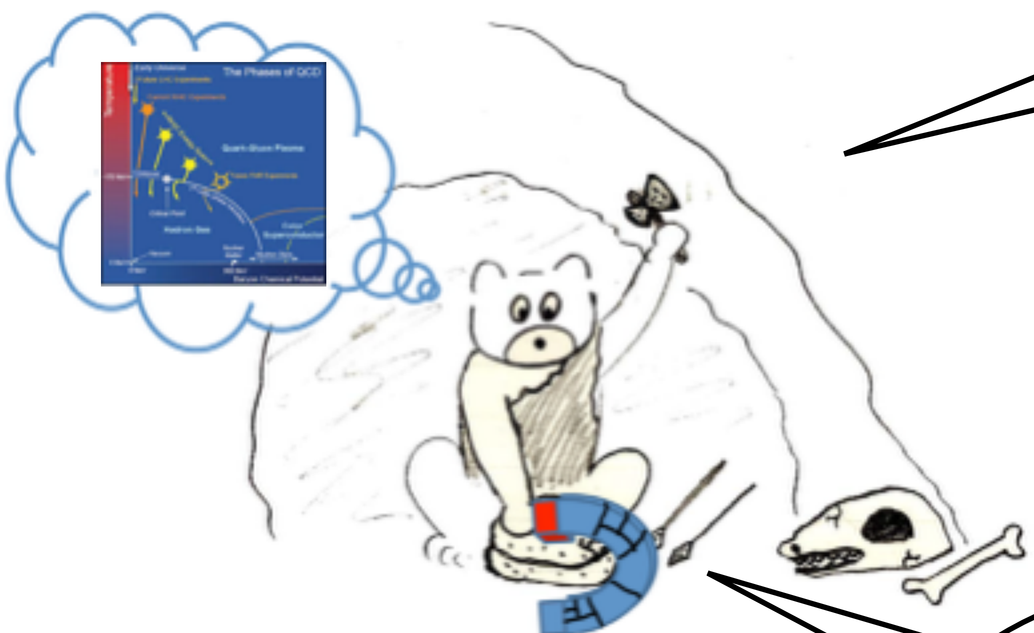
$$f(\xi) = \prod_k (\xi - \alpha_k)$$

$$\frac{f'}{f} = \sum_k \frac{1}{\xi - \alpha_k}$$

$$\frac{1}{2\pi i}\oint_C \frac{f'}{f} d\xi = \left(\text{Number of Zeros in Contour } C \right)$$

50 - 100 number of significant digits

A Coutour is cut into four pieces if there are zeros inside.



Is this my Original ?

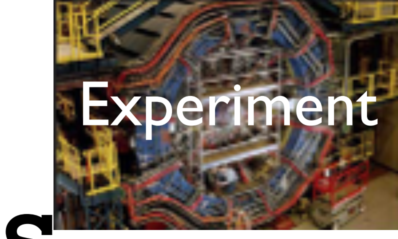
Let us wait until someone claims.

I donot think so.

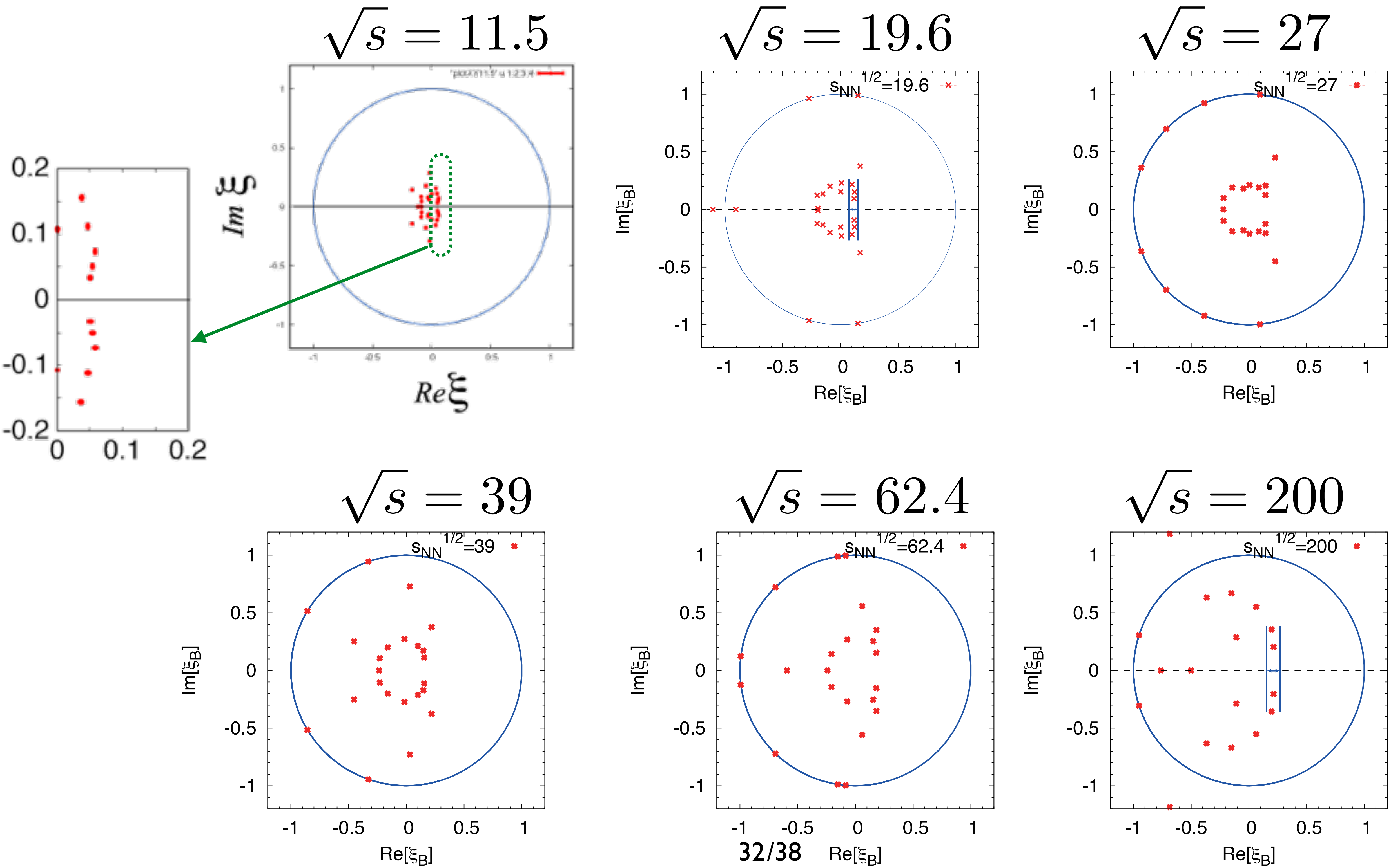
It’s ME

Riemann

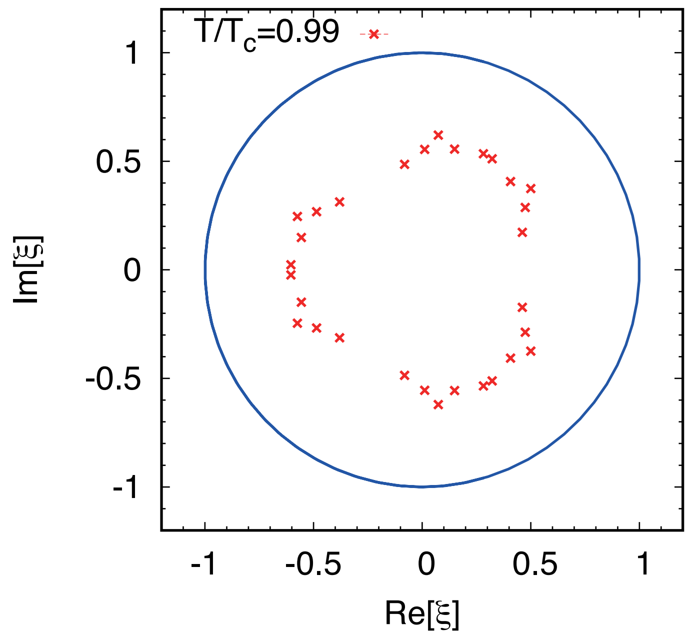
Lee-Yang Zeros Experimental Data (RHIC)



Lee-Yang Zeros: RHIC Experiments

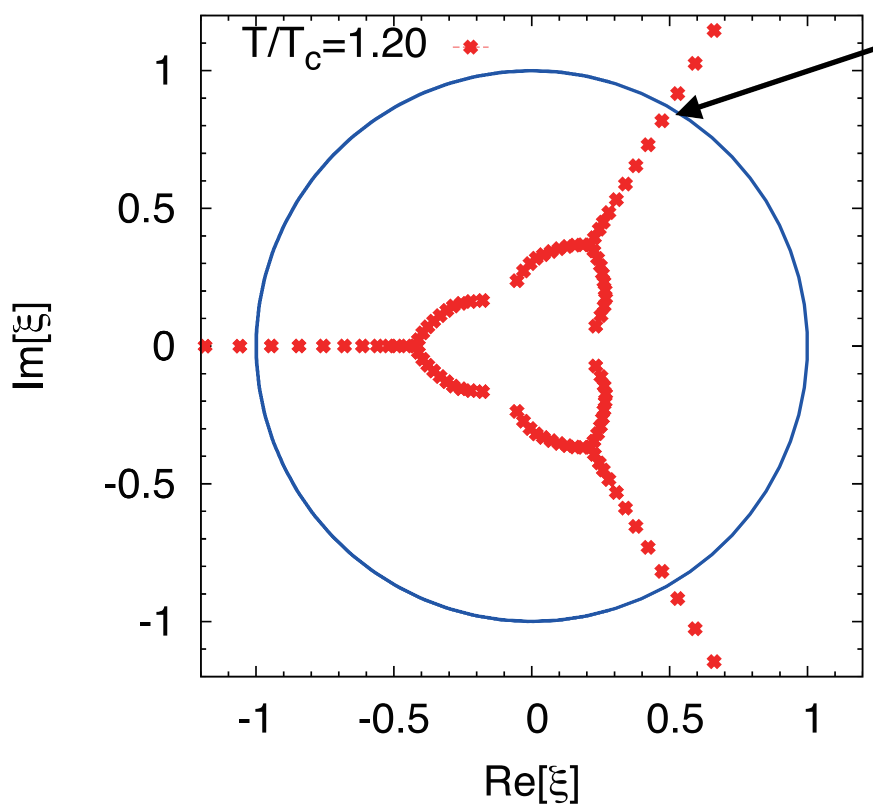


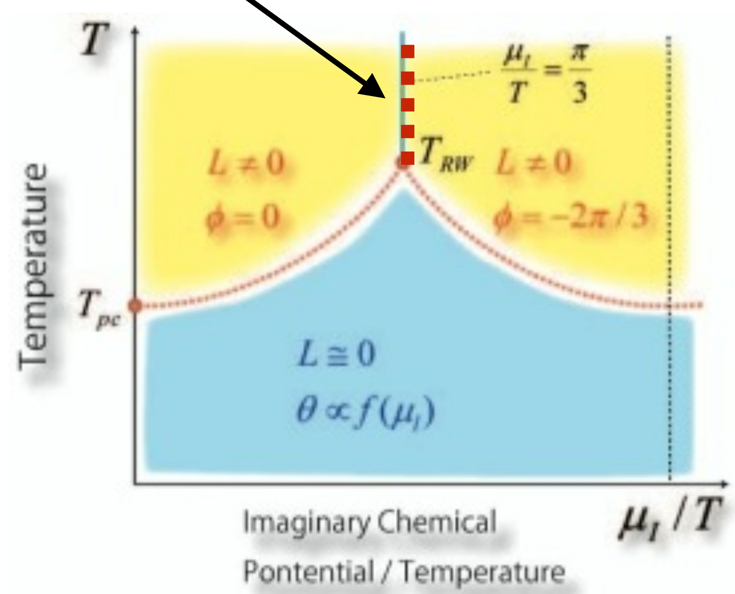
Lattice Data



$\beta = 1.85$ $\qquad T/T_c \sim 0.99$

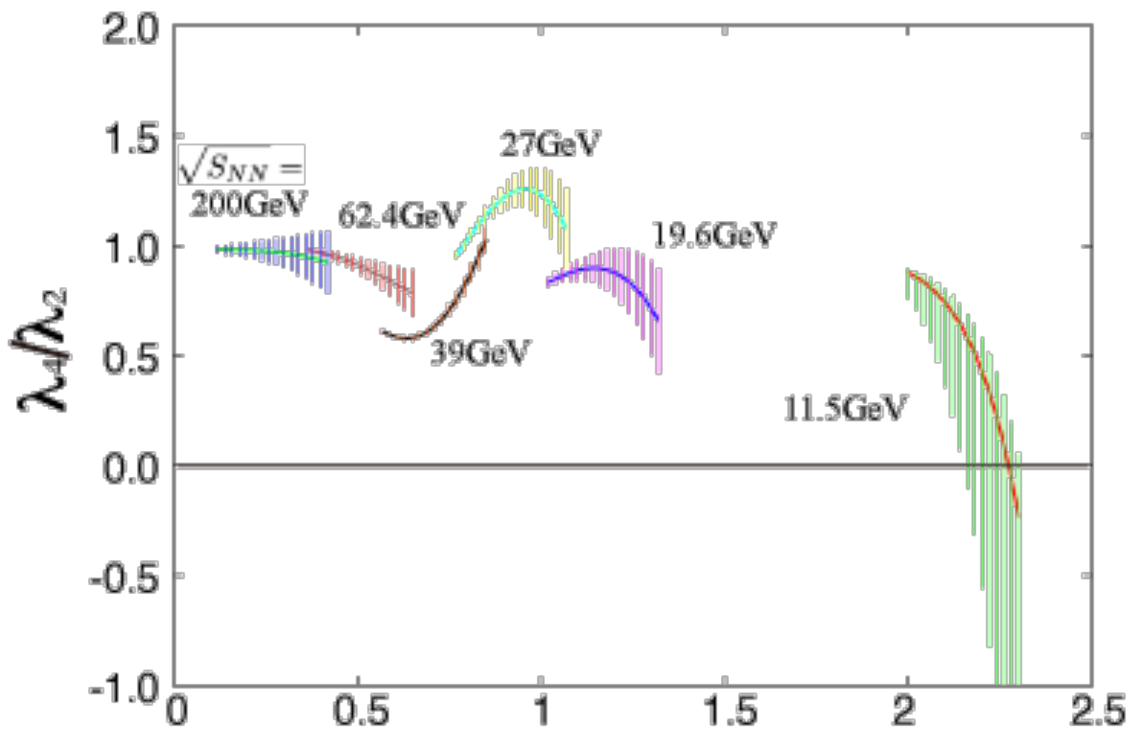
Roberge-Weise Transition !

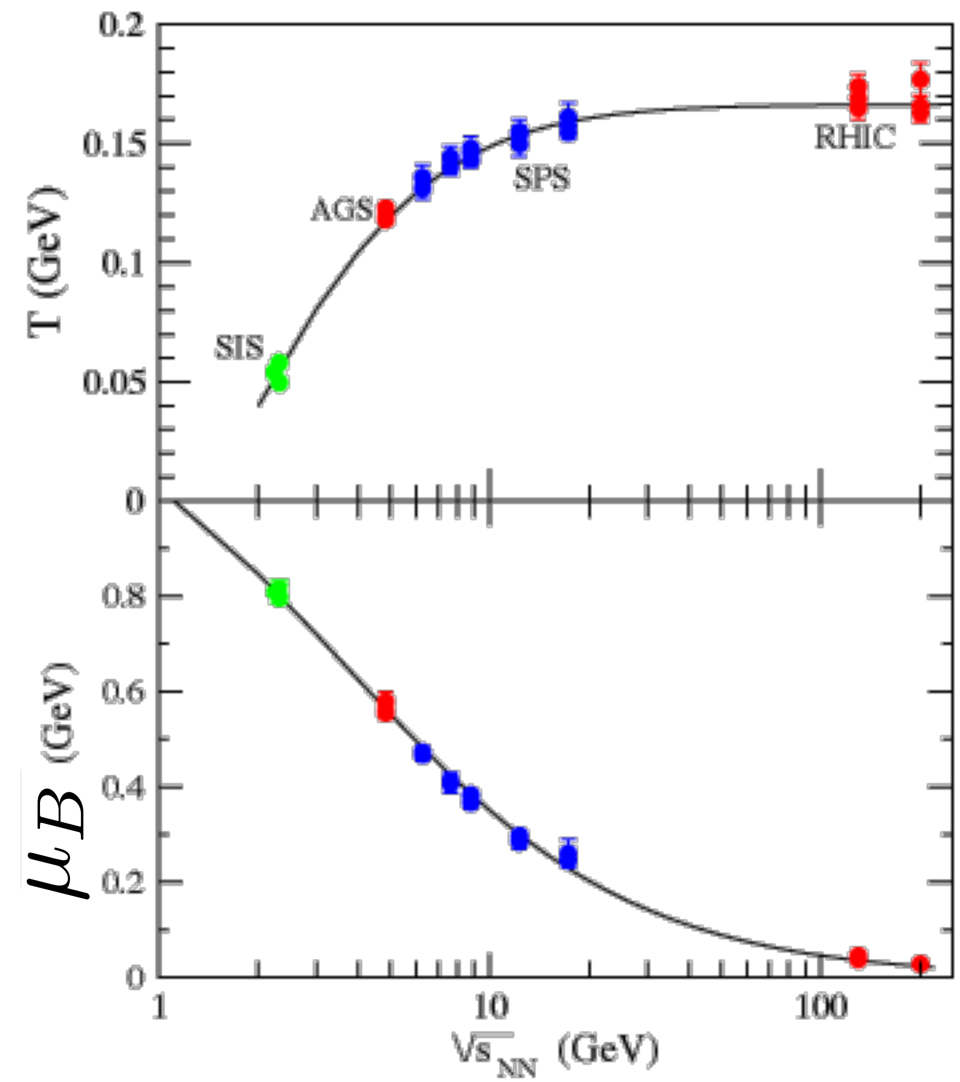




$T/T_c \sim 1.20$

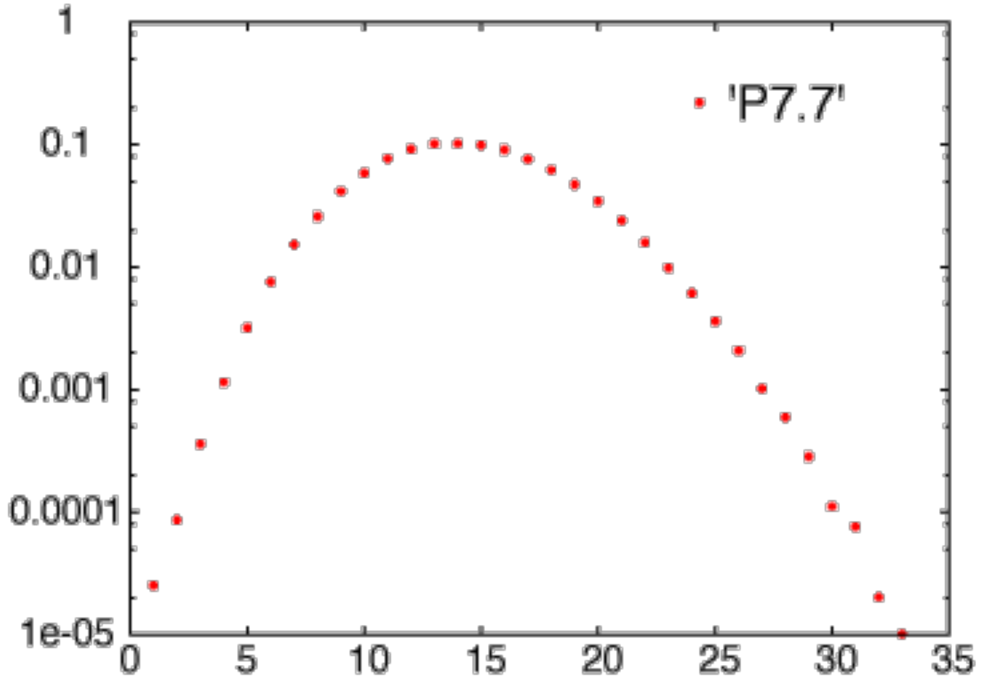
Lower Energy looks interesting.



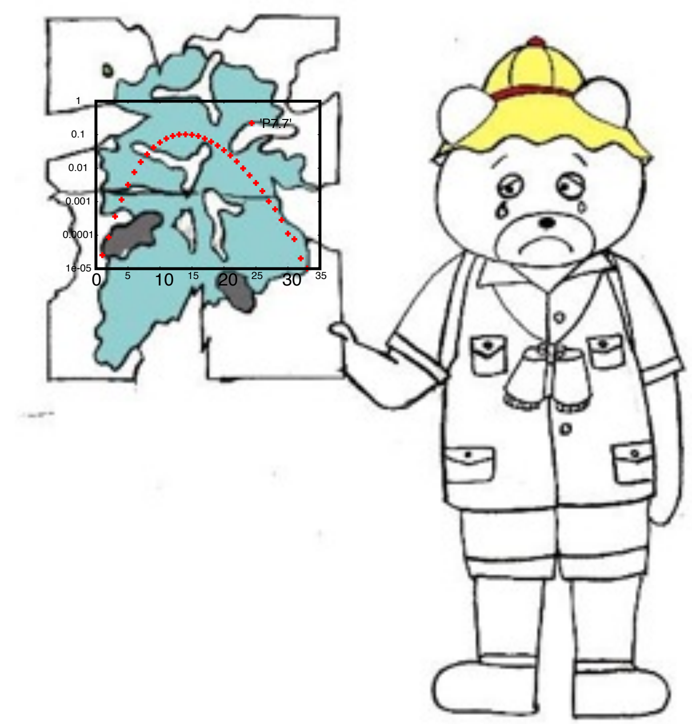


Why you do not investigate $\sqrt{S_{NN}} = 7.7$?

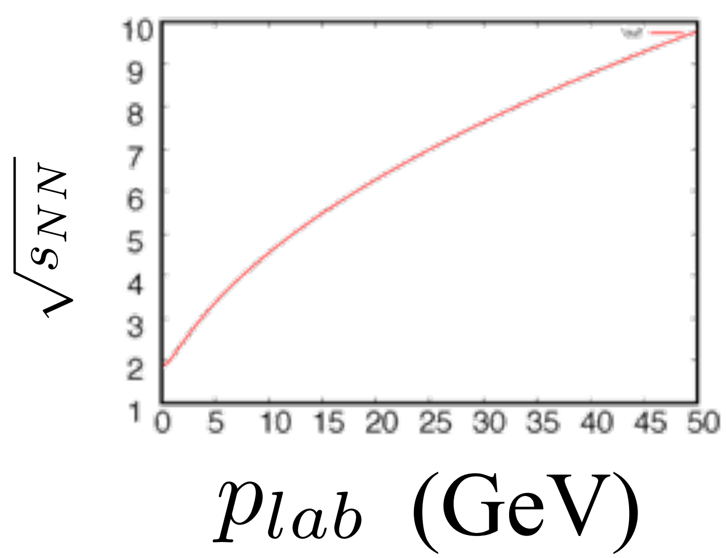
J.Cleymans et al.,
Phys. Rev. C73, (2006) 034905.



No Data for $n < 0$



J-PARC search regions ?



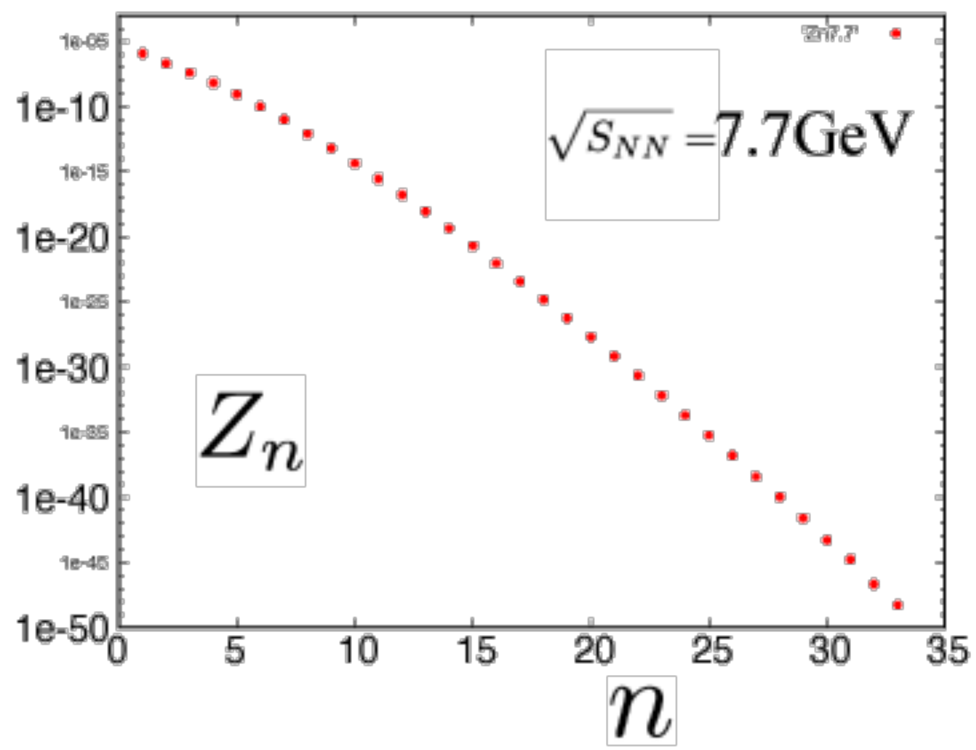
I cannot determine ξ

I visited J-Parc
on Sept.18, 2014

Why don't you borrow ξ from Freeze-out Analysis

$$Z_n = P_n/\xi^n$$

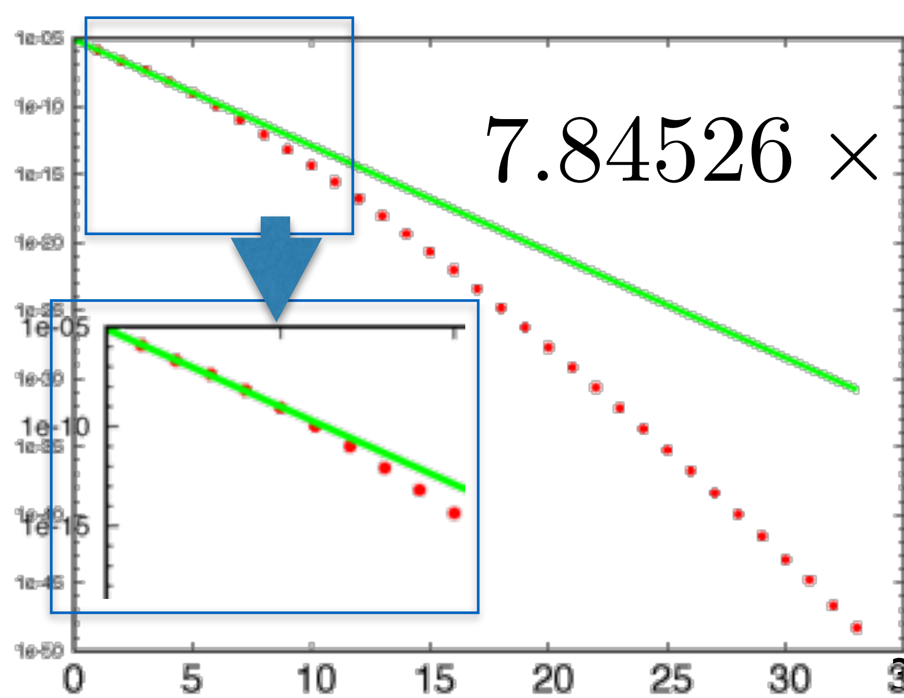
ξ =20.4944 (Cleymans et al.)



No Data for $n = 0$

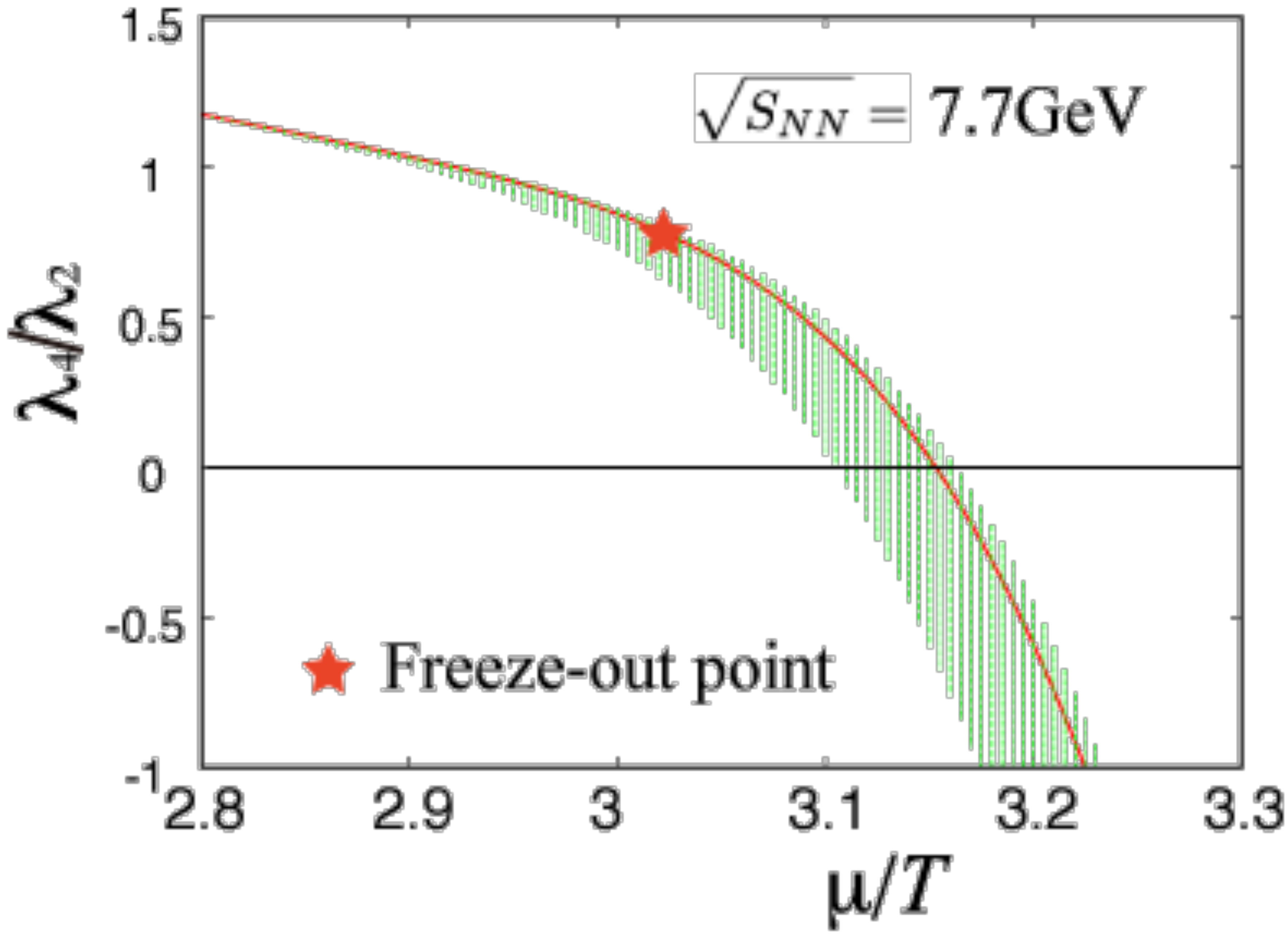
Around $n = 0$
Zn can be approximated by

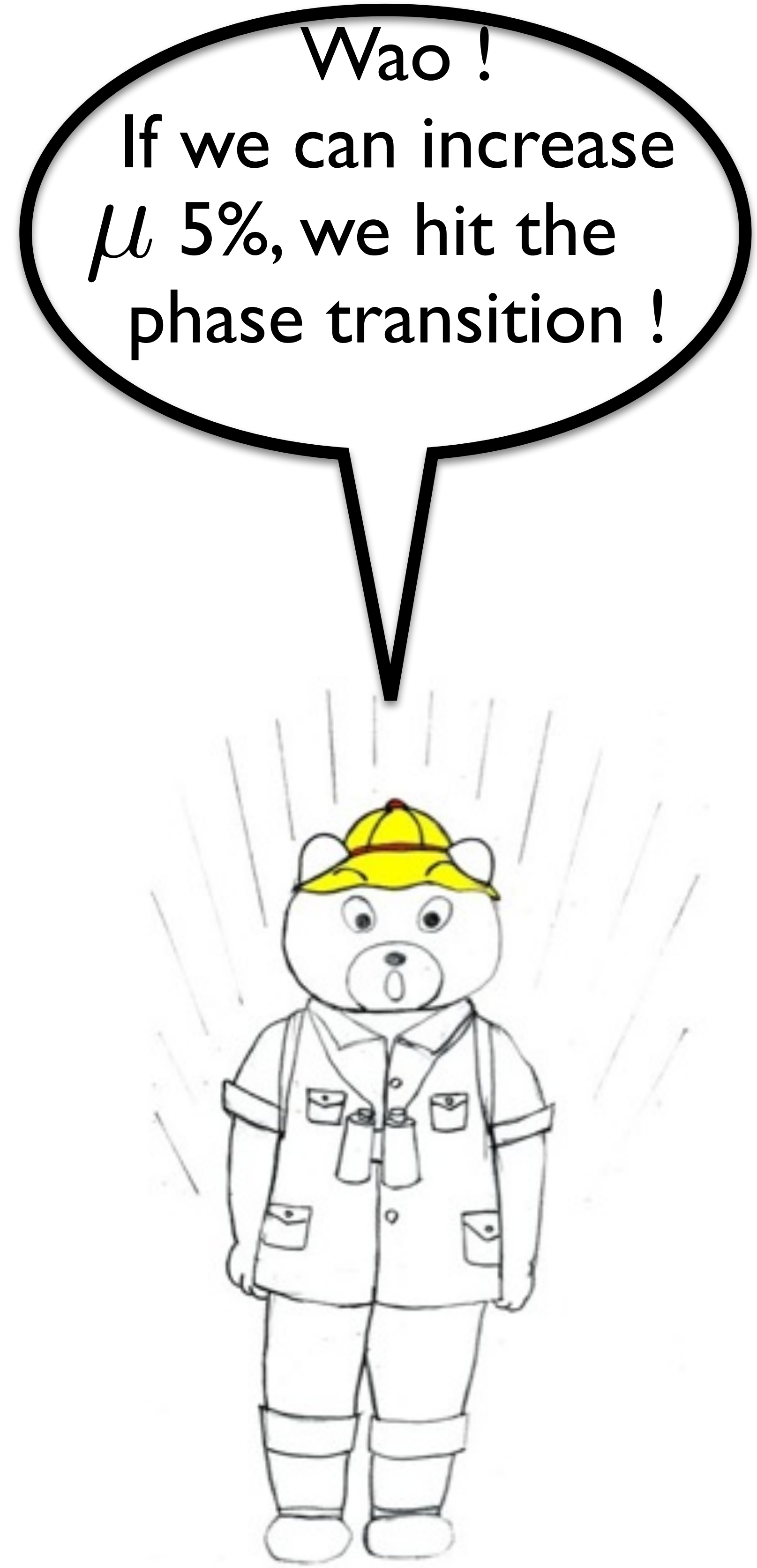
$$7.84526 \times 10^{-6} \times \exp(-1.79351n)$$



Kurtosis at $\sqrt{s_{NN}} = 7.7$ GeV

Very Preliminary
(calculated between Tokyo and Hawaii)





Summary

- A+A collision data at RHIC around 10 GeV indicate we are near the QCD phase transition line.
  If J-PARC may join this challenge, it will contribute a lot.
- Since Zn decrease rapidly, high multi-precision is essential.
- Zn analysis give us a power to predict higher density.
  - ☆ Large statistic at large n is important
- Lattice QCD has now power to calculate high density, and helpful to understand experiments.

Backup Slide

$$Z(\mu, T) \Leftrightarrow Z_n(T)$$

Grand Canonical — Canonical

$$Z(\mu,T) = \mathrm{Tr}\, e^{-(H-\mu\hat{N})/T}$$

$$= \sum_n \langle n| e^{-(H-\mu\hat{N})/T} |n\rangle$$

If $[H, \hat{N}] = 0$

$$= \sum_n \langle n| e^{-H/T} |n\rangle \, e^{\mu n/T}$$

$$= \sum_n Z_n(T) \xi^n \qquad \left(\xi \equiv e^{\mu/T}\right)$$

Fugacity

Comparison of obtained ξ

$$\xi \equiv e^{\mu/T}$$

$\sqrt{s_{NN}}$ GeV	Cleymans(06)	Aba(14)	Our
11.5	8.04	11.1	7.48
19.6	3.62	3.65	3.21
27	2.62	2.58	2.43
39	1.98	1.93	1.88
62.4	1.55	1.53	1.53
200	1.18	1.18	1.18

Sign Problem
One Slide Review

$$Z_{GC}(\mu, T) = \int \mathcal{D}(\text{Gluon Fields})\ \det D\ e^{-(\text{Gluon Action})}$$

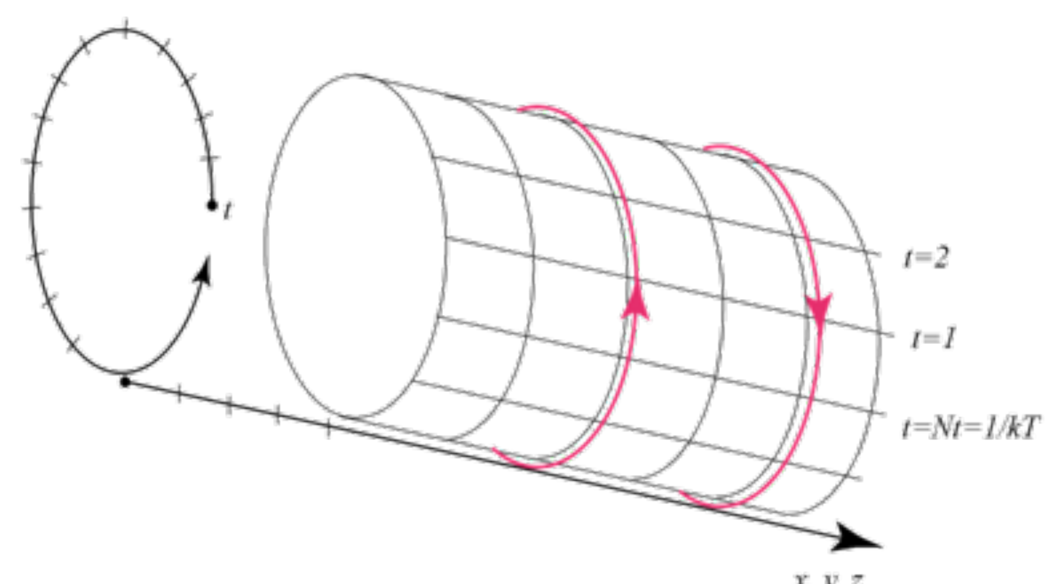
$$\begin{aligned}\det D &= \exp(\mathrm{Tr}\log D) \\ &= \exp\left(e^{+\mu/T}Q^{+} + e^{-\mu/T}Q^{-} + \cdots\right)\end{aligned}$$

Q^{+} ⟷ Q^{-}

Complex Conjugate

If $\mu = 0$ → $\det D$ real

$\mu \neq 0$ → $\det D$ complex



$$\det D = \exp\left(e^{+\mu/T}Q^{+} + e^{-\mu/T}Q^{-} + \cdots\right)$$

Q^{+} ⟷ Q^{-} Complex Conjugate

If μ Pure Imaginary → $\det D$ real

A.Hasenfratz and Toussant, 1992

$$Z_C(n,T) = \int \frac{d\theta}{2\pi} e^{i\theta n} Z_{GC}(\theta \equiv \frac{\mathrm{Im}\mu}{T}, T)$$

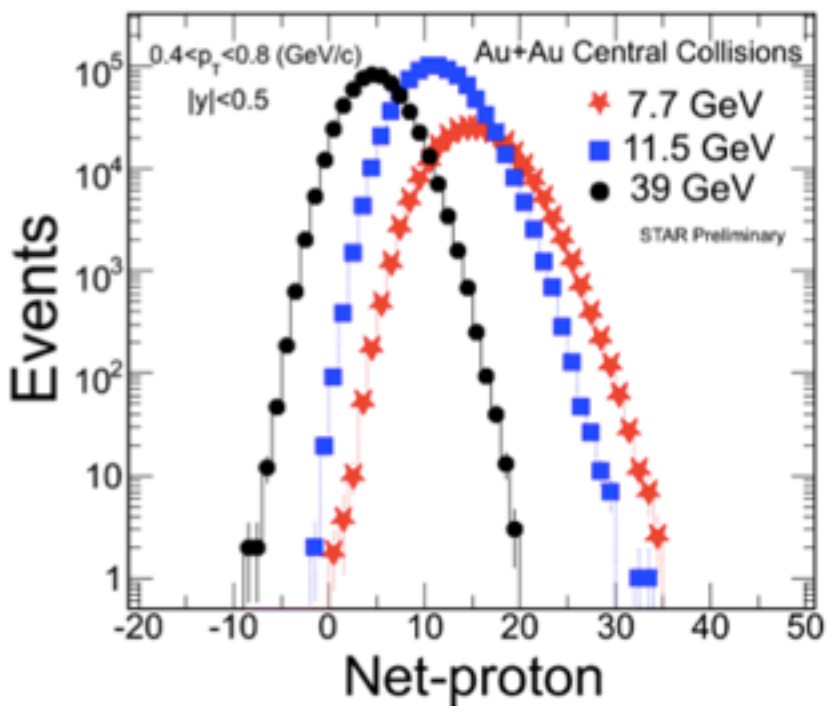
Great Idea ! But practically it did not work.

Zn Collaboartion Method:

$$Z_C(n,T) = \int \frac{d\theta}{2\pi} \int \frac{\det(\theta)}{\det(\theta_0)} \det(\theta_0) e^{-(\text{Gluon Action})}$$

θ integration → Multi-Precision (50 - 100)

What (and How) are Multiplicity Distributions telling us on QCD Phase Diagram ?



Experimental Data

↓

Extract $Z_n(T)$

↓

Construct

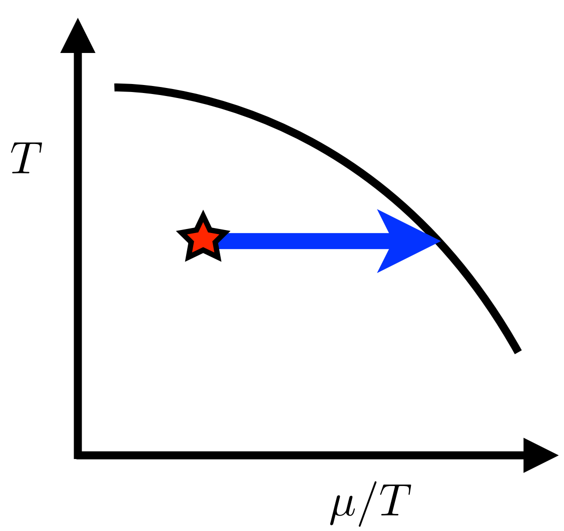
$$Z(\xi, T) = \sum_n Z_n(T)\, \xi^n$$

Construct

$$Z(\xi, T) = \sum_n Z_n(T)\, \xi^n$$

Calculate Moments at $\mu \geq \mu_{\text{Experiment}}$

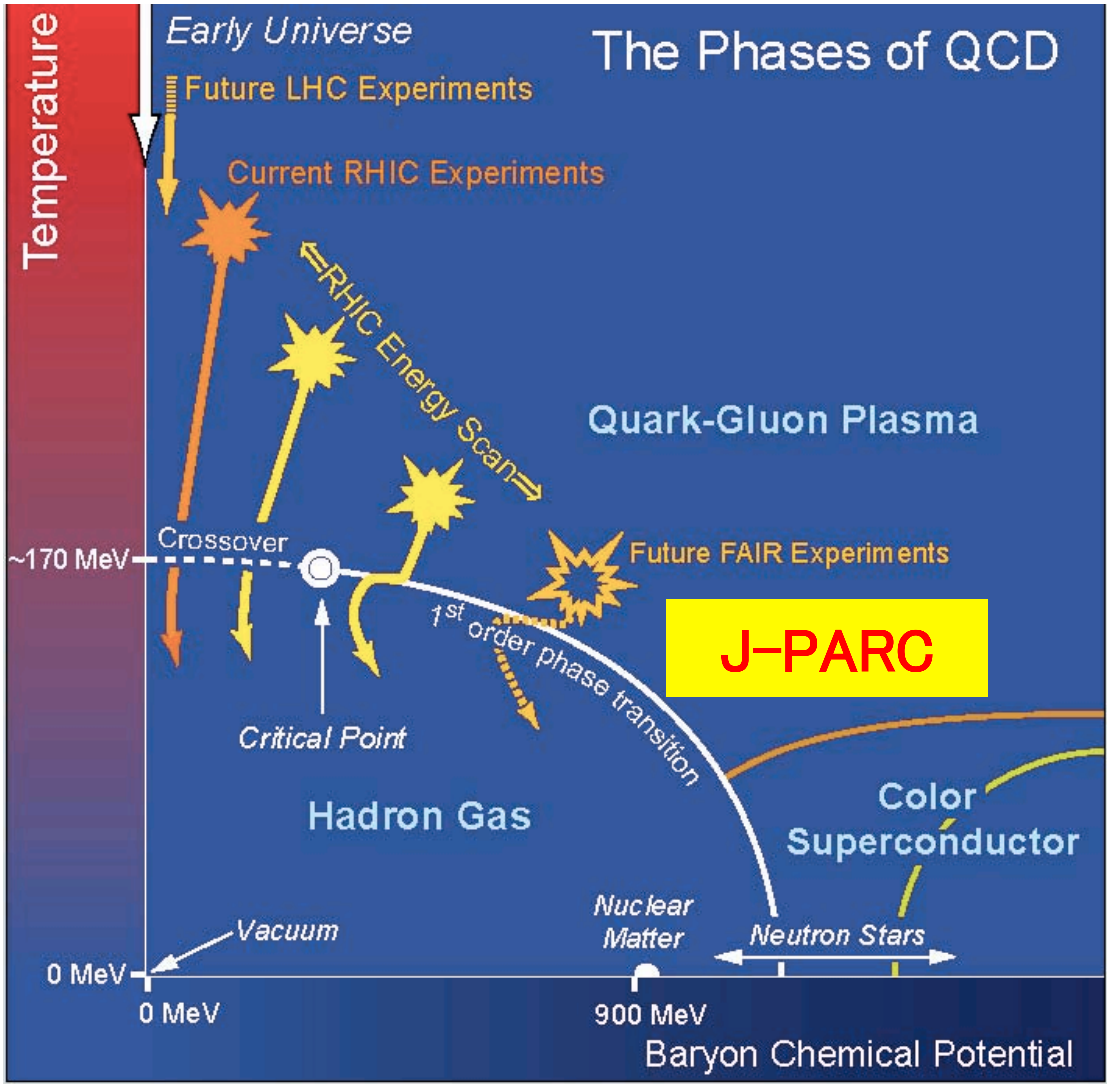
Construct Lee-Yang Zeros



The current Net-Proton data is a Test-Bed.
But even they suggest the phase boundary.

Sako@QM2014

“Towards the Heavy-Ion Program at J-PARC”



Hadron Seminar @J-Parc Takao Sakaguchi

“High Enegy” Program (50 GeV MR)

- Ion species
  - p, Si, Cu, Au, U
  Au→U
  - Baryon density
    - $7.5\rho_0 \rightarrow 8.6\rho_0$ (JAM)
  - Duration at $\rho>5\rho_0$
    - 4 → 7 fm/c
- Beam energy
  - 1 - 11.6 AGeV (U) ($\sqrt{s}\downarrow NN = 4.9GeV$
  - Possibly 19 AGeV($\sqrt{s}\downarrow NN = 6.2GeV$)
- Rate
  - 10^{10}-10^{11} ions per cycle (~a few sec)

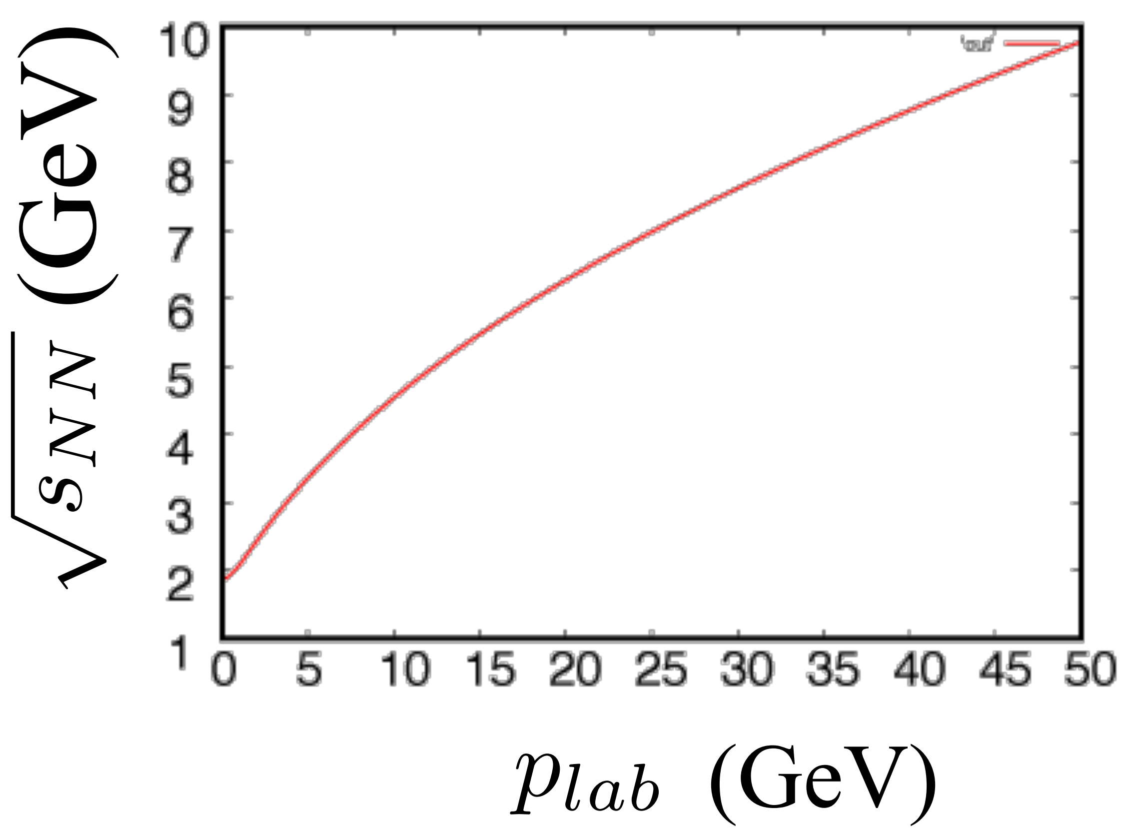
$A+B$ in a lab. frame (fixed target)

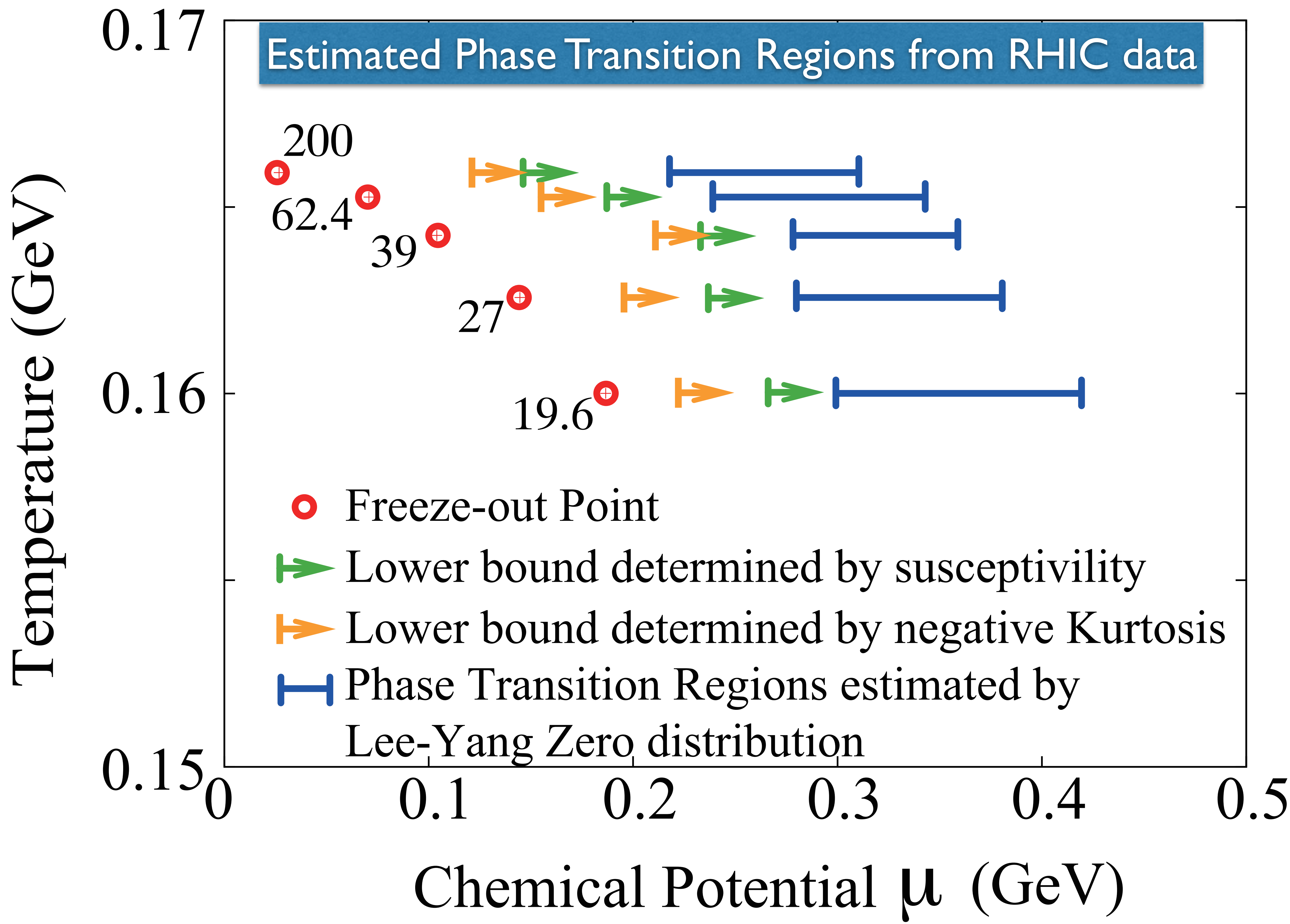
$$s = (p_a + p_b)^2 = M_A^2 + M_B^2 + 2E_a M_b$$

For simplicity, $A = B$

$$M_A = M_B = A m_N \quad E_a = A\sqrt{\vec{p}_{lab}^{\,2} + m_N^2}$$

$$\sqrt{s_{NN}} = \frac{\sqrt{s}}{A} = \sqrt{2\left(m_N + \sqrt{\vec{p}_{lab}^{\,2} + m_N^2}\right) m_N}$$



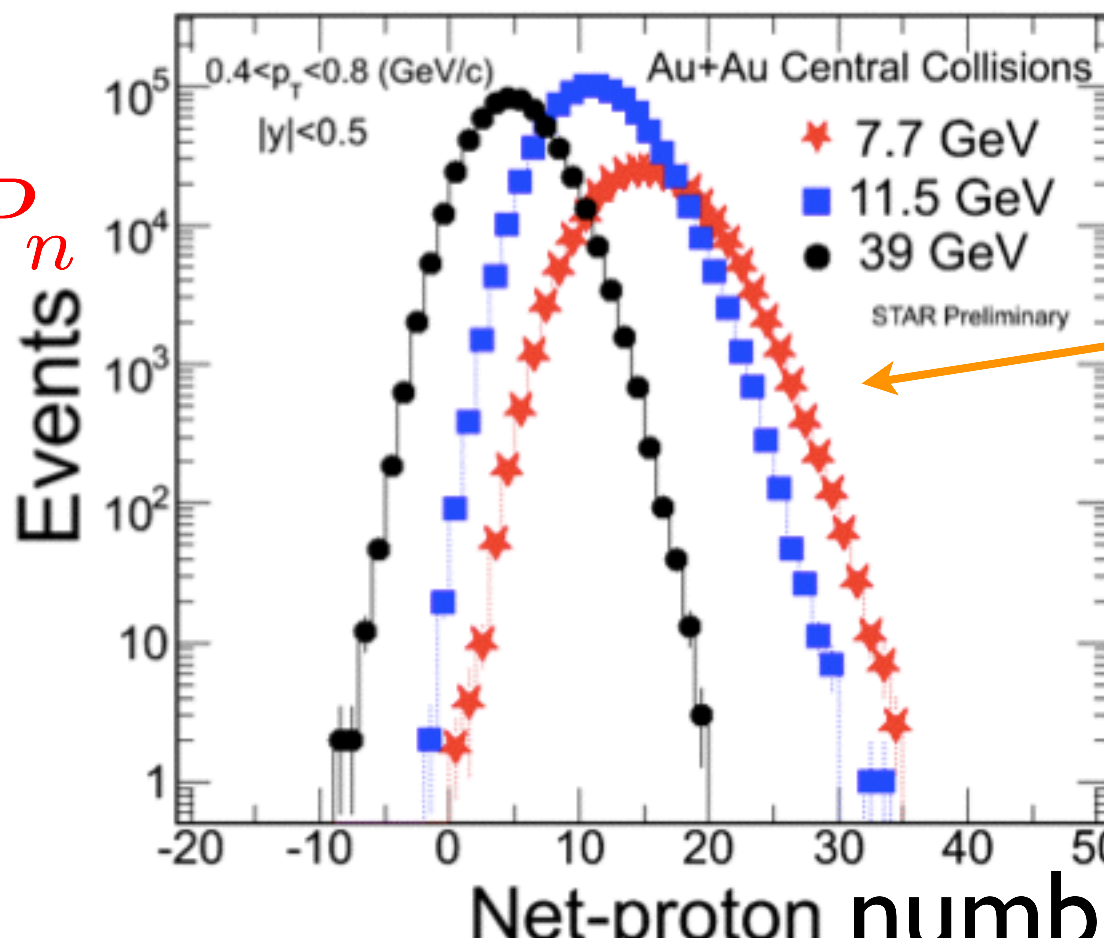


Estimated Phase Transition Regions from RHIC data
0.17
0.16
0.15
0
0.1
0.2
0.3
0.4
0.5
Temperature (GeV)
Chemical Potential μ (GeV)
200
62.4
39
27
19.6
Freeze-out Point
Lower bound determined by susceptivility
Lower bound determined by negative Kurtosis
Phase Transition Regions estimated by
Lee-Yang Zero distribution

$$Z(\xi, T) = \sum_n Z_n(T)\,\xi^n$$

Partition Function is
Sum of the Probabilities
P_n with n ...
If I know ξ, then I have Z_n.

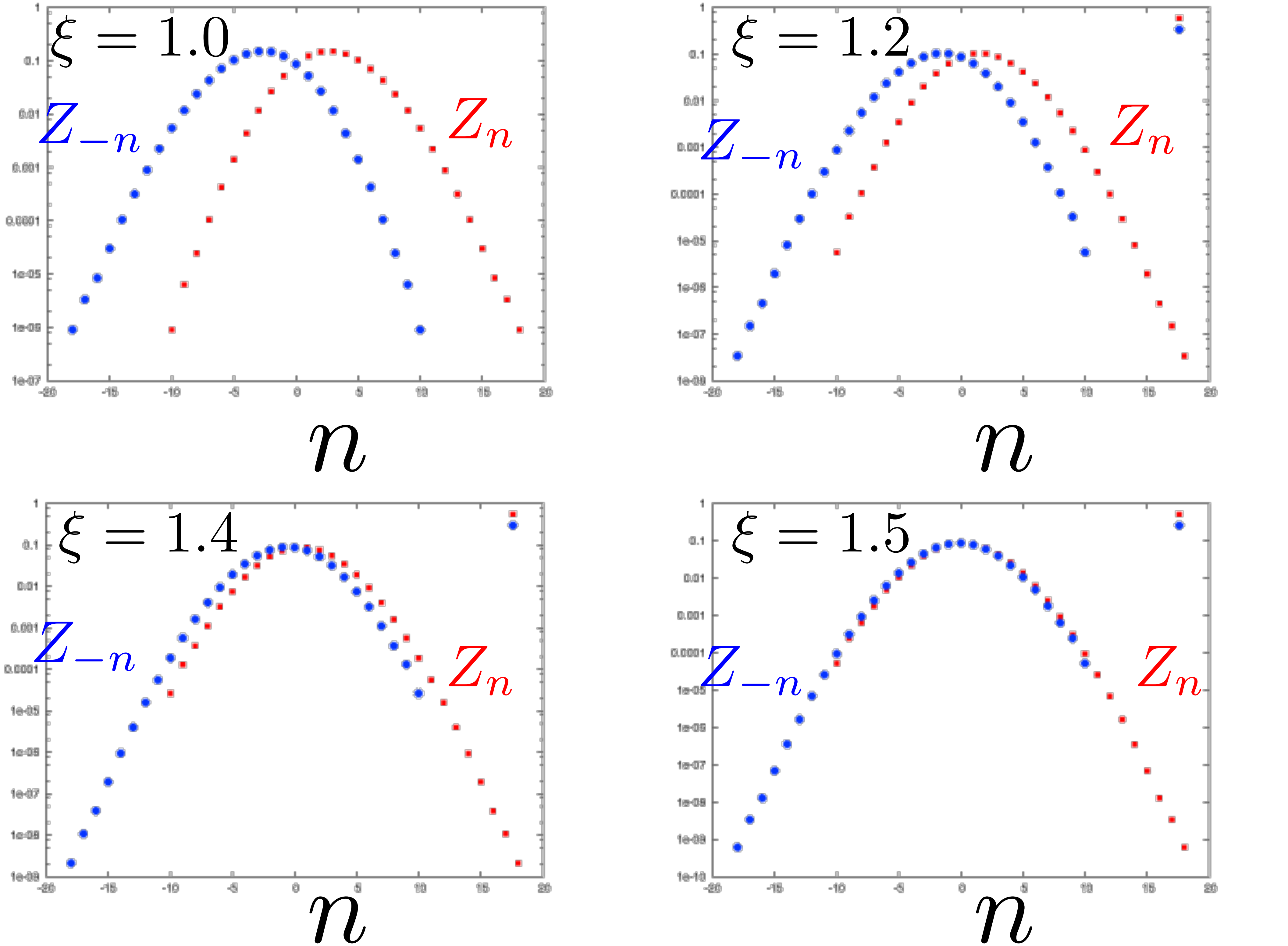
P_n



number

From Experiment

$$Z_n = P_n / \xi^n$$

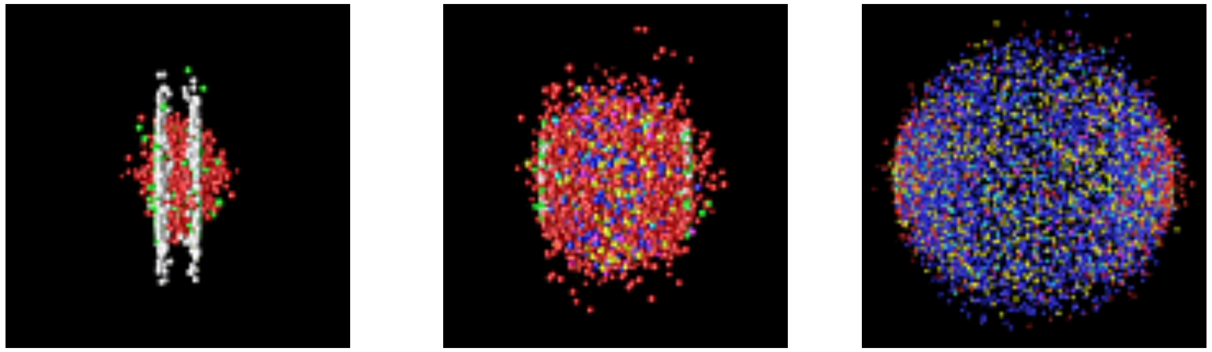


Final Value $\xi = 1.534$

We assume

the Fireballs created in High Energy Nuclear Collisons are described as a Statistical System.

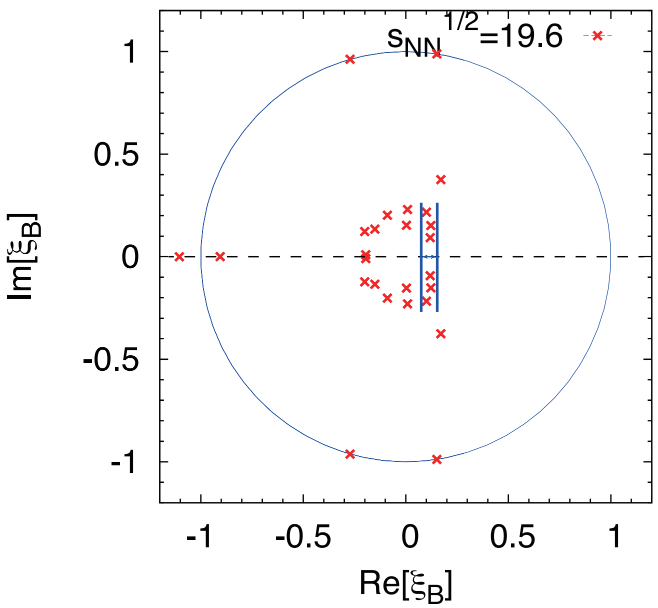
with μ (chemical Potential)
and T (Temperature)

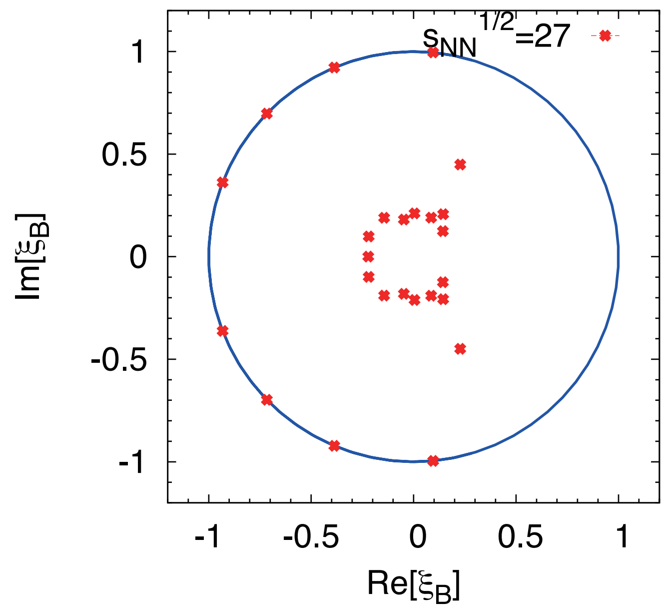


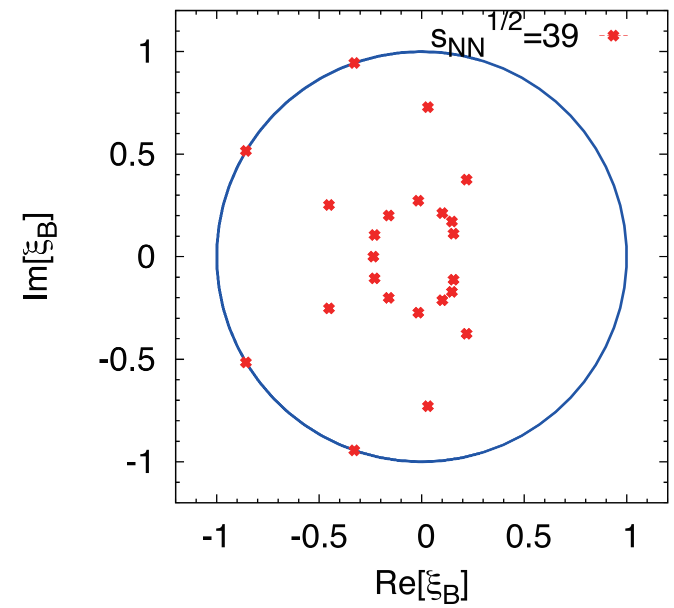
$$Z(\mu, T)$$

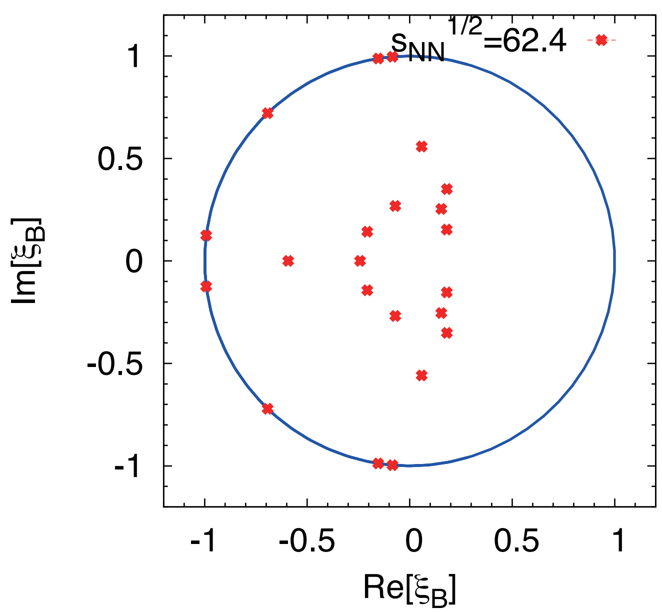
Grand Canonical Partition Function

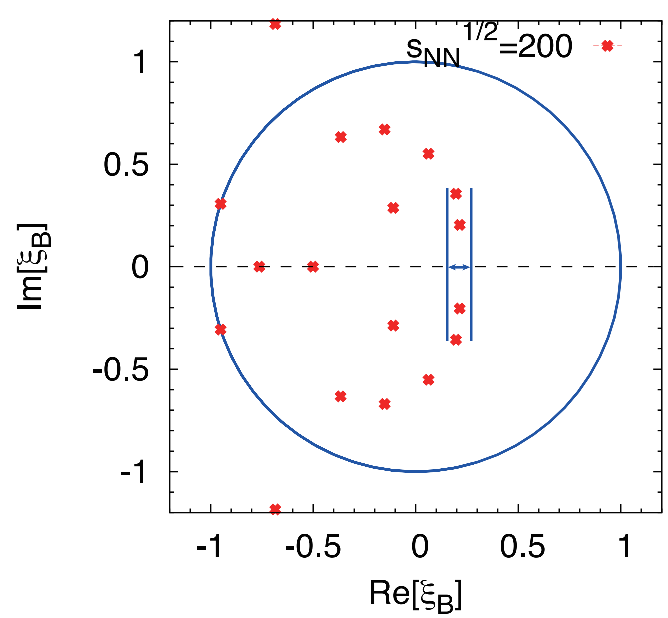
Lee-Yang Zeros: RHIC Experiments











Hunting the QCD Phase Transition Regions

Find Rooms where No Criminal.

→ The Target is in other Room.