Reconstructing the EOS with gravitational-wave observations

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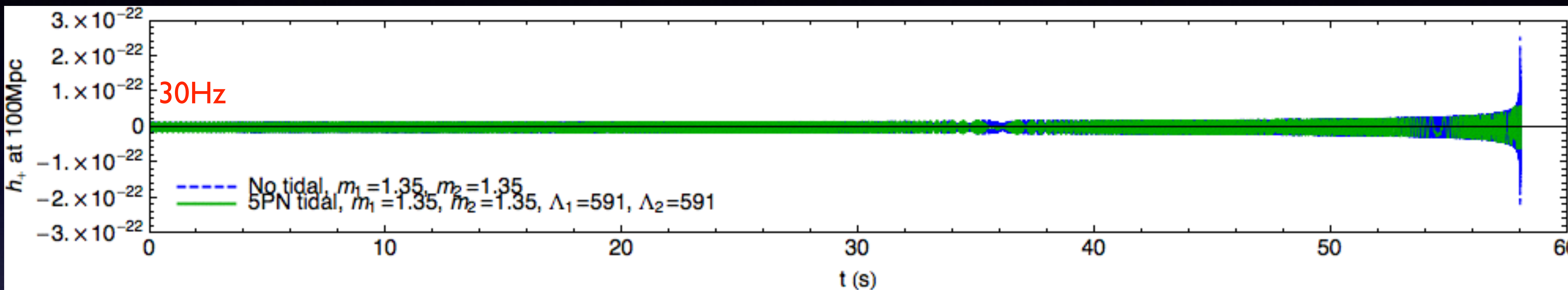
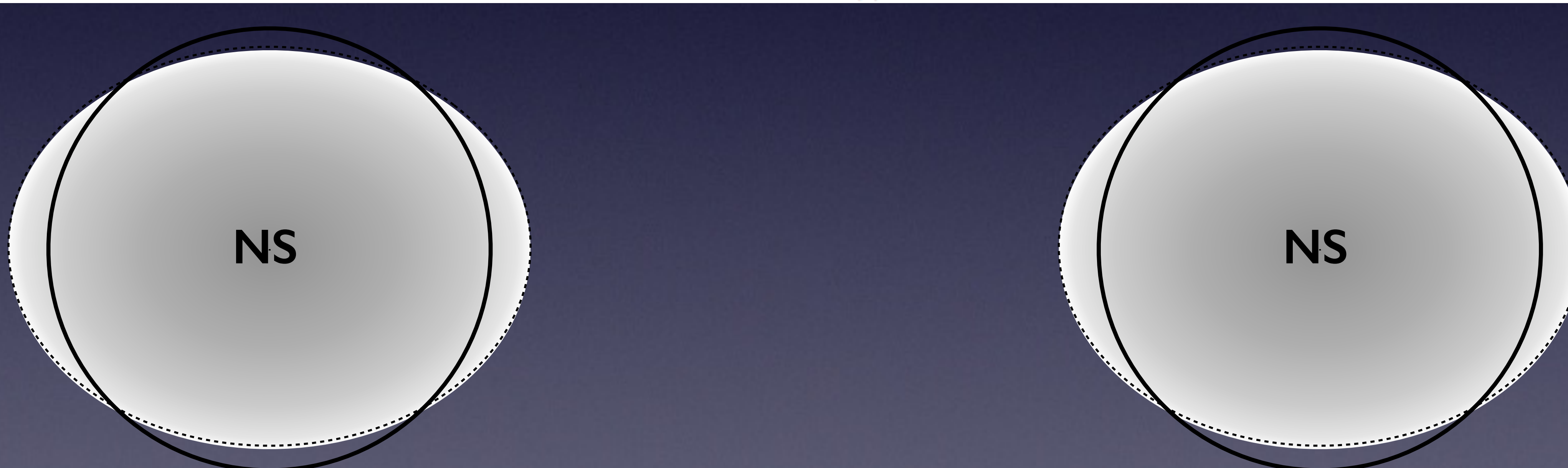
Second generation gravitational-wave detectors

- Will reach design sensitivity around end of decade
- Sensitive between 10s Hz and a few kHz

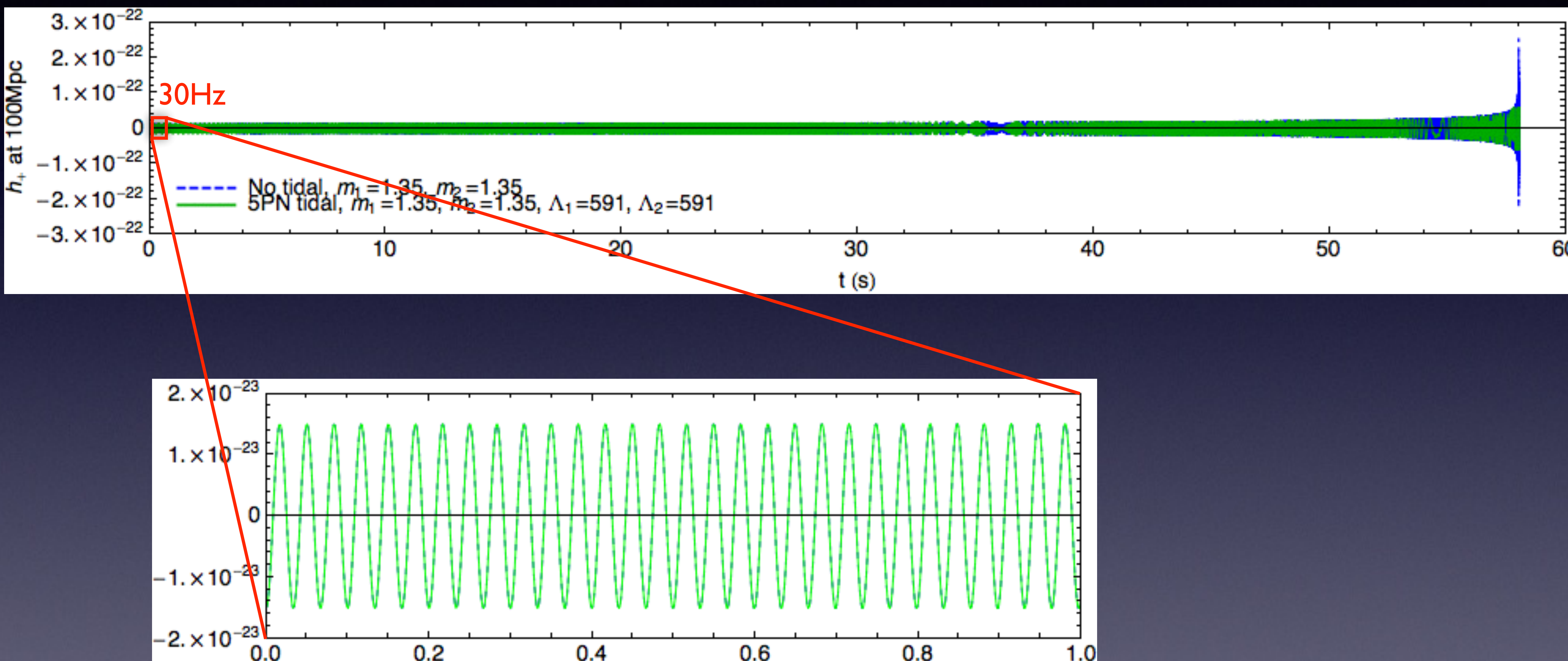
Stages of BNS coalescence





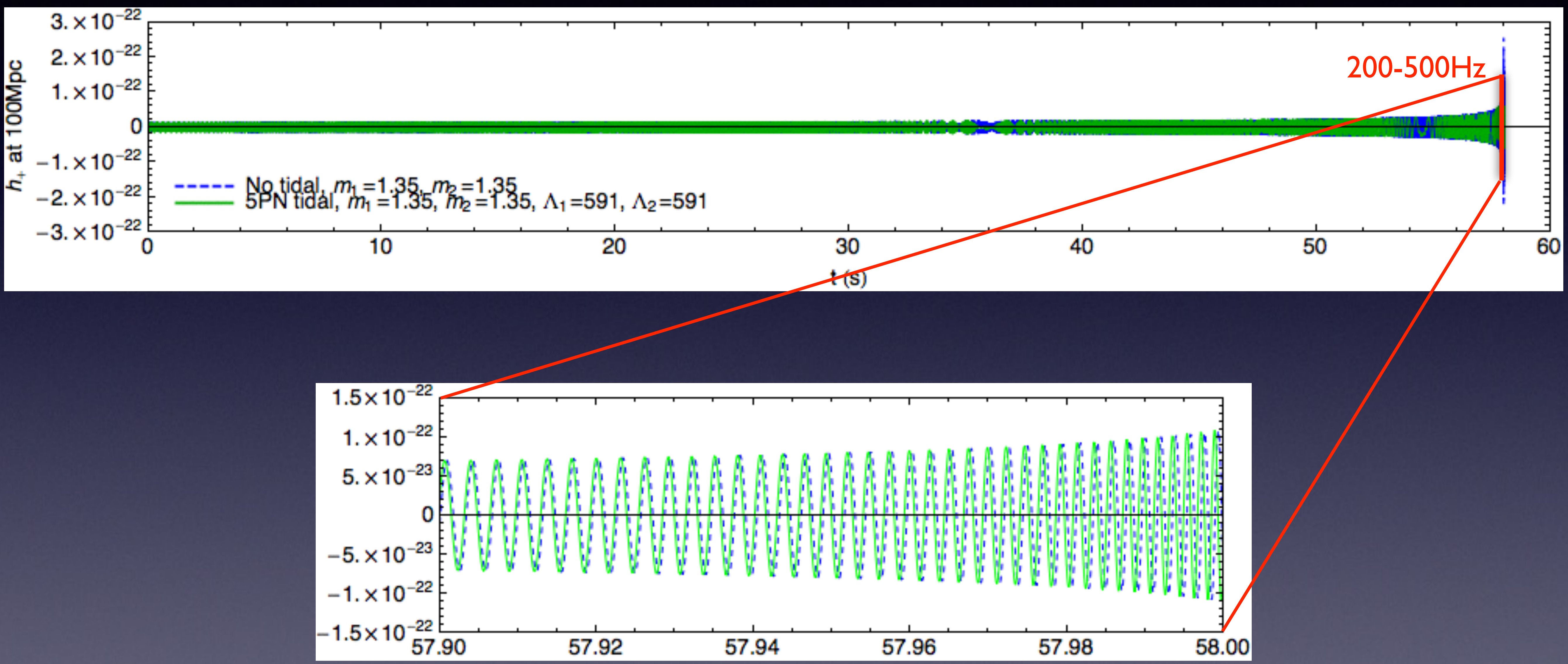
- Advanced LIGO sensitive to last minute of inspiral

Stages of BNS coalescence



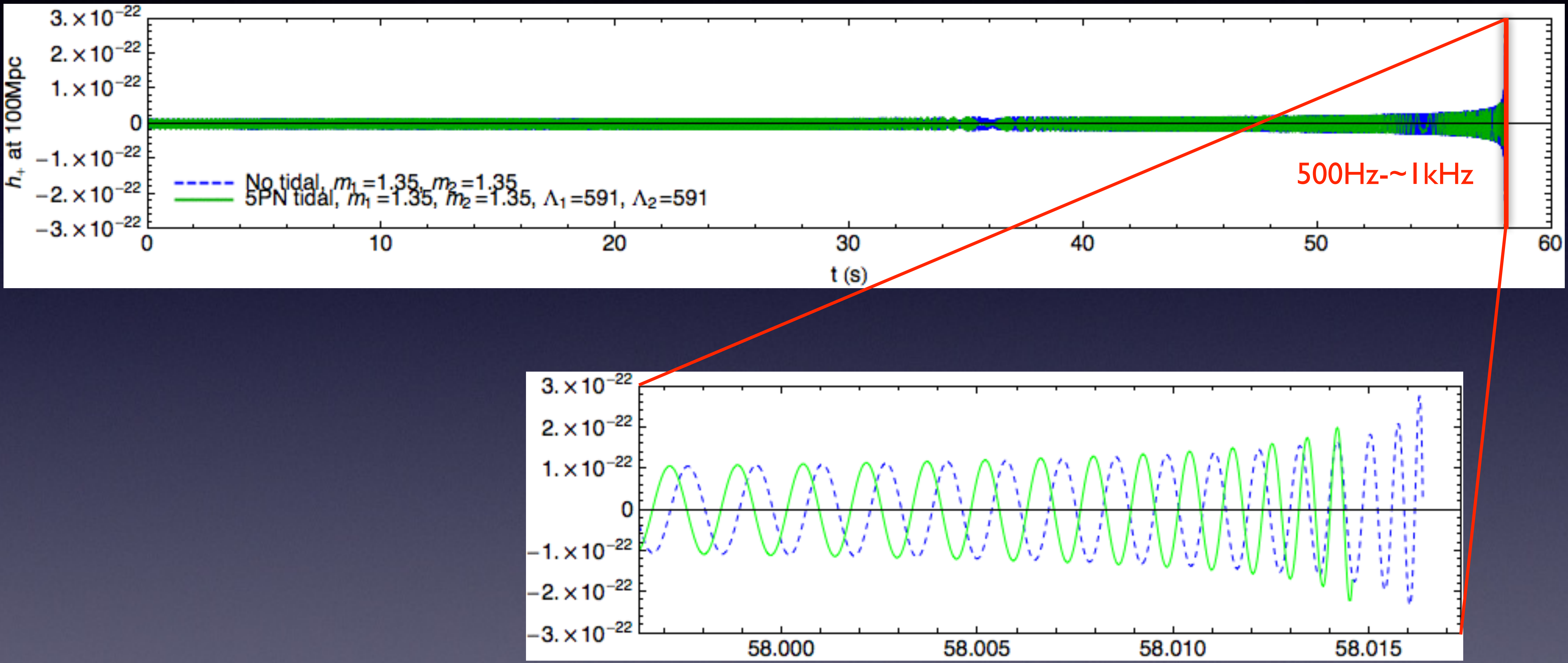
- Early inspiral: Evolution depends on chirp mass $\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$ and symmetric mass ratio $\eta = \frac{m_1 m_2}{(m_1 + m_2)^2}$

Stages of BNS coalescence



- Late inspiral: EOS-dependent tidal interactions lead to phase shift of ~1 radian

Stages of BNS coalescence



- Last 20 cycles: Tidal interactions lead to phase shift of ~1 GW cycle

Tidal interactions during inspiral

- Tidal field $\mathcal{E}_{ij}$ of each star induces quadrupole moment Q_{ij} in other star
- Amount of deformation depends on the stiffness of the EOS via the tidal deformability λ

$$Q_{ij} = -\lambda(\mathrm{EOS}, M)\mathcal{E}_{ij}$$

- Interaction makes binary more tightly bound
- Additional quadrupole moments increase gravitational radiation $\dot{E} = -(1/5)\langle \dddot{Q}^{\mathrm{T}}_{ij} \dddot{Q}^{\mathrm{T}}_{ij} \rangle$

Tidal interactions during inspiral

- Intrinsic parameters encoded in phase evolution of waveform

Amplitude

Phase

$$\tilde{h}(f) = \frac{A(\alpha, \delta, \iota, \psi)}{d_L} \mathcal{M}^{5/6} f^{-7/6} e^{i\psi(f)}$$

Tidal interactions during inspiral

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Amplitude

Phase

$$\tilde{h}(f) = \frac{A(\alpha,\delta,\iota,\psi)}{d_L}\mathcal{M}^{5/6} f^{-7/6} e^{i\psi(f)}$$

Newtonian 5PN 6PN

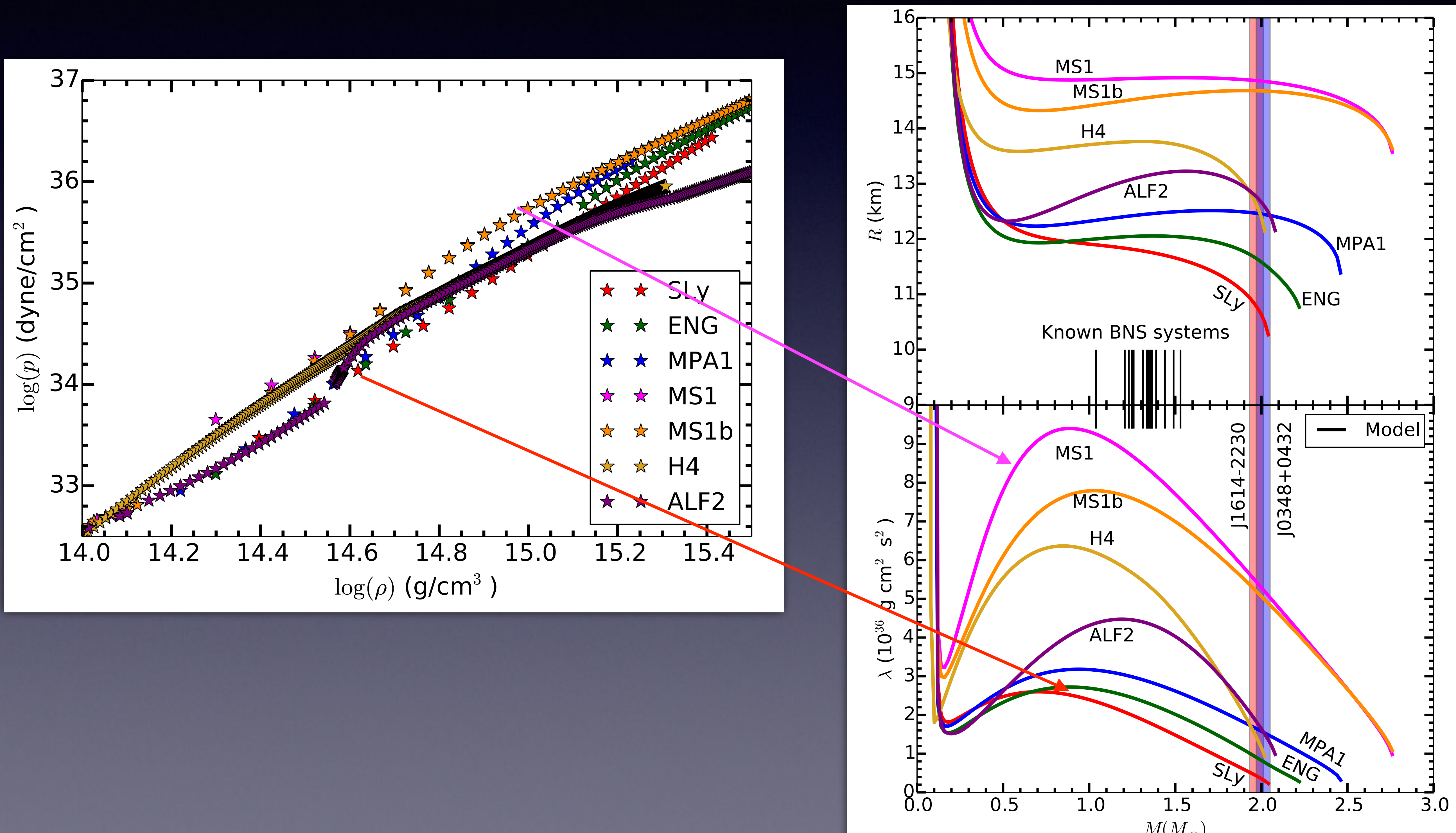
$$\psi(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128\eta x^{5/2}}\left[1 + (\text{PP-PN}) + \frac{39}{2}\tilde{\Lambda} x^5 + \left(\frac{3115}{64}\tilde{\Lambda} - \frac{659}{364}\delta\tilde{\Lambda}\right) x^6\right]$$

$$x = (\pi M f)^{2/3} \sim \left(\frac{v}{c}\right)^2$$

$$\tilde{\Lambda} = \frac{8}{13}\left[(1+7\eta-31\eta^2)(\Lambda_1+\Lambda_2) + \sqrt{1-4\eta}(1+9\eta-11\eta^2)(\Lambda_1-\Lambda_2)\right]$$

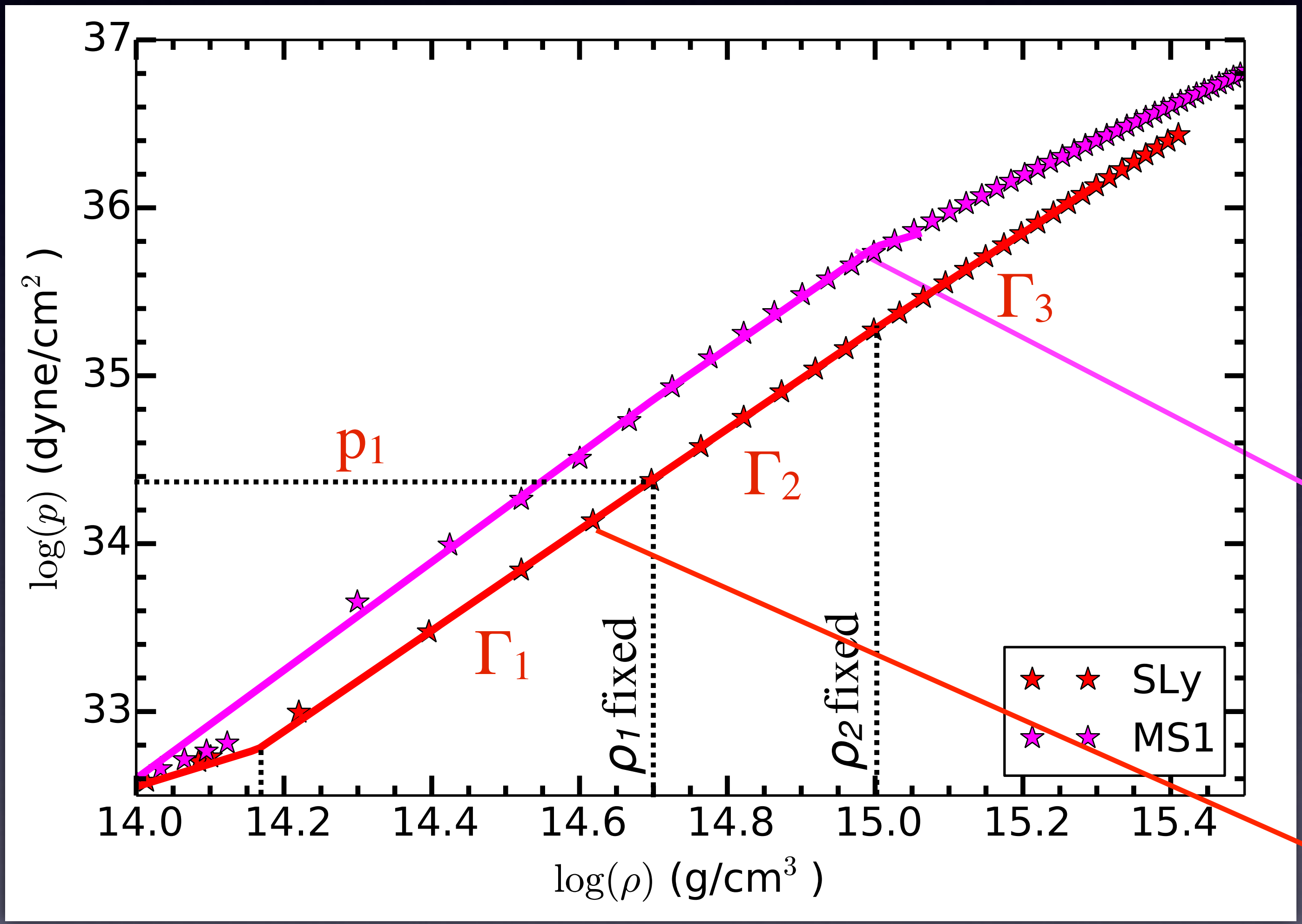
EOS fit

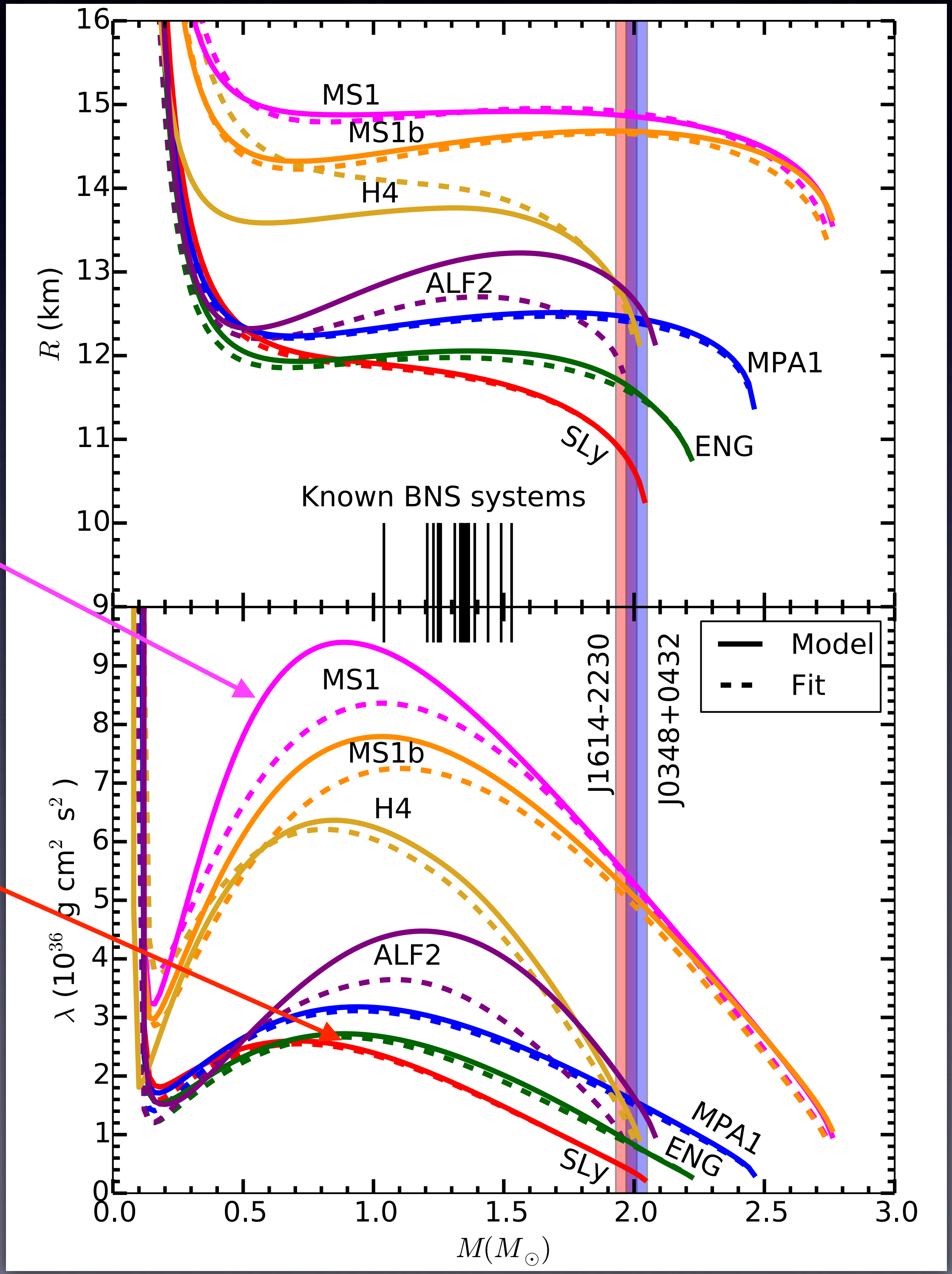
- One-to-one relation between EOS and radius-mass curves
- As well as between EOS and tidal deformability-mass curves



EOS fit

- Purely phenomenological EOS with 4 free parameters
- Methods apply to any EOS with free parameters





$$p(\rho) = \begin{cases} K_1 \rho^{\Gamma_1}, & \rho_0 < \rho < \rho_1 \\ K_2 \rho^{\Gamma_2}, & \rho_1 < \rho < \rho_2 \\ K_3 \rho^{\Gamma_3}, & \rho > \rho_2 \end{cases}$$

Step 1: Estimate masses and tidal deformability

- Can estimate parameters of each BNS inspiral from Bayes' Theorem:

Posterior, Prior, Likelihood

$$p(\vec{\theta}|d_n) = \frac{p(\vec{\theta})p(d_n|\vec{\theta})}{p(d_n)}$$

Evidence

- $\vec{\theta} = \{d_L, \alpha, \delta, \psi, \iota, t_c, \phi_c, \mathcal{M}, \eta, \tilde{\Lambda}, \delta\tilde{\Lambda}\}$
- d_n: data from nth BNS event

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$$p(\vec{\theta}|d_n) = \frac{p(\vec{\theta})p(d_n|\vec{\theta})}{p(d_n)}$$

- Time series of stationary, Gaussian noise has the distribution

$$p_n[n(t)] \propto e^{-(n,n)/2} \qquad (a,b) = 4\mathrm{Re}\int_0^\infty \frac{\tilde{a}(f)\tilde{b}(f)}{S_n(f)}\,df$$

- Likelihood of observing data d for gravitational wave model $m(t;\vec{\theta})$ with parameters $\vec{\theta}$

$$p(d|\vec{\theta}) \propto e^{-(d-m,d-m)/2}$$

  - where (data) = (noise) + (GW signal)

Step 1: Estimate masses and tidal deformability

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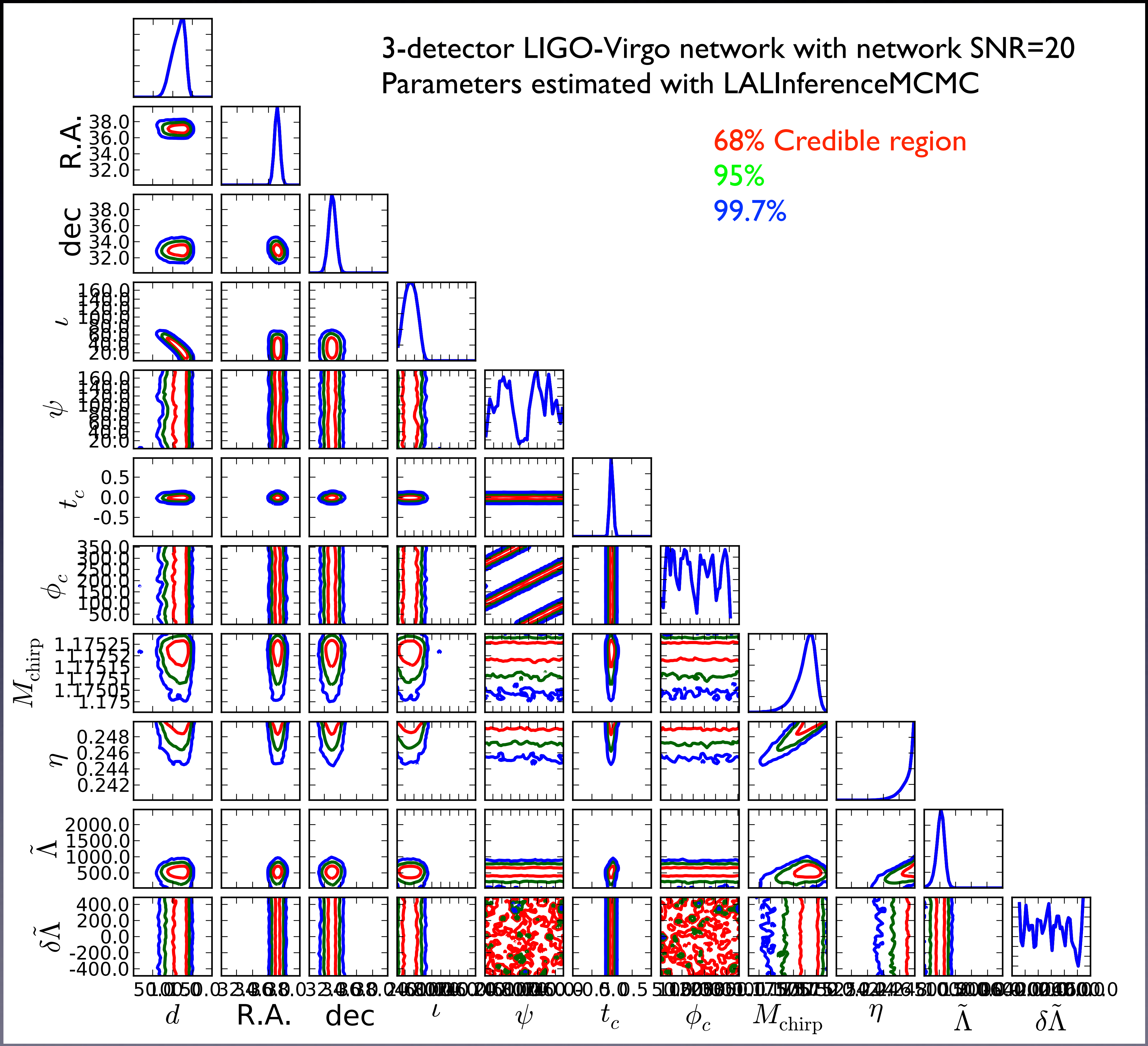
Posterior, Prior, Likelihood, Evidence

$$p(\vec{\theta}|d_n) = \frac{p(\vec{\theta})p(d_n|\vec{\theta})}{p(d_n)}$$

- Use Markov Chain Monte Carlo (MCMC) to sample posterior and marginalize over nuisance parameters

$$p(\mathcal{M}, \eta, \tilde{\Lambda}|d_n) = \int p(\vec{\theta}|d_n) d\vec{\theta}_{\text{nuisance}}$$

Step 1: Estimate masses and tidal deformability



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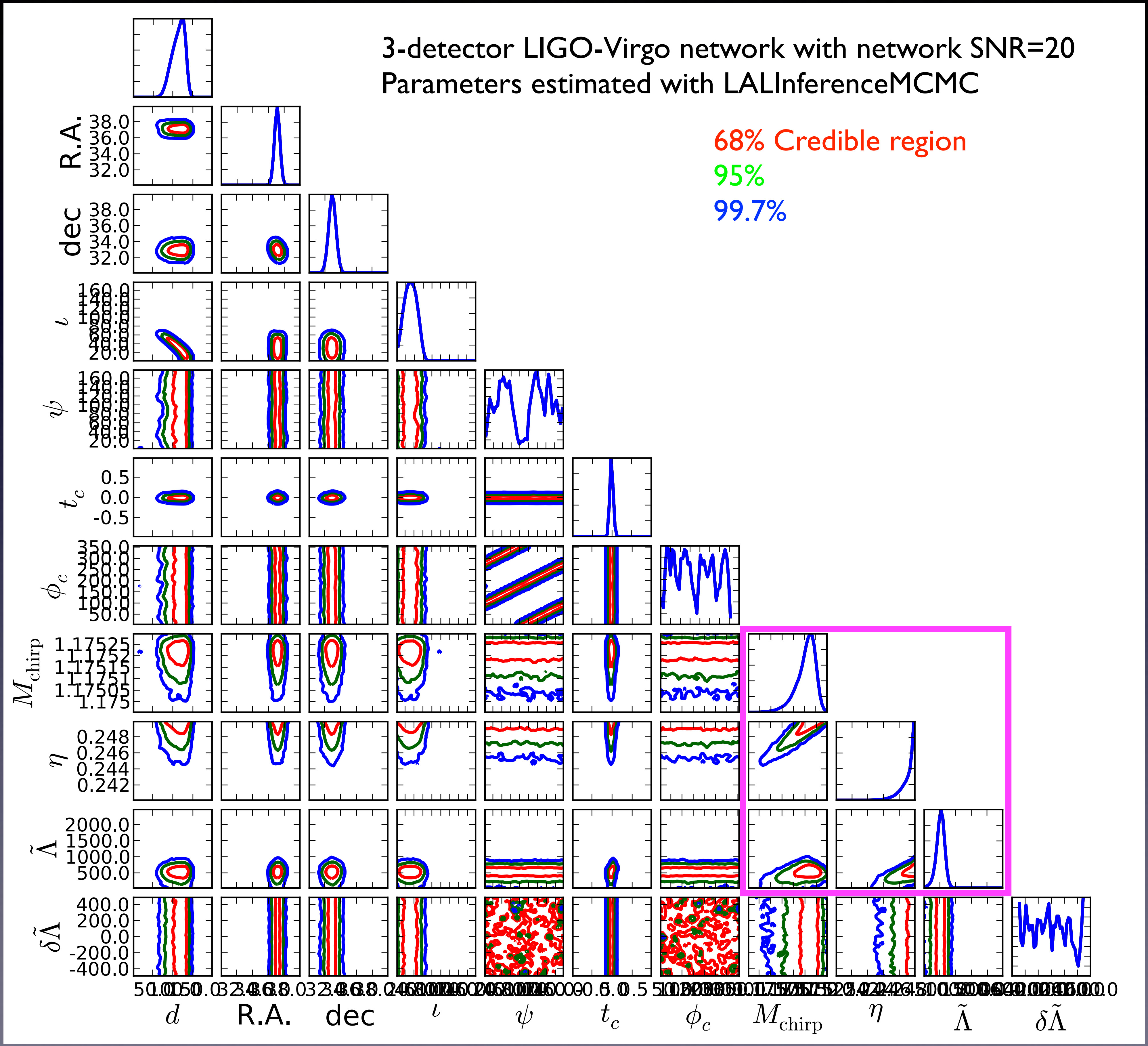

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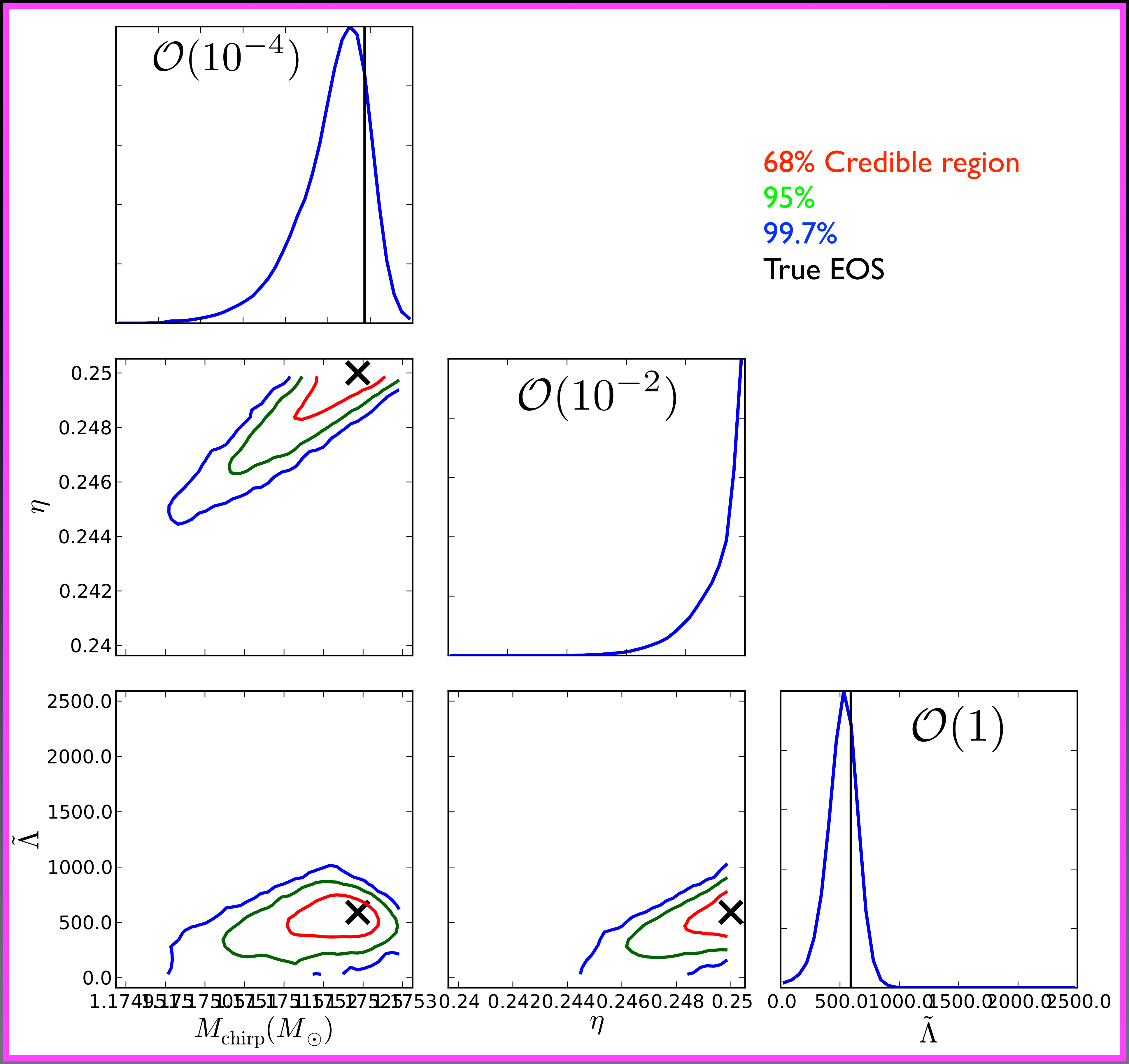

Step 2: Estimate EOS parameters

- Use Bayes' theorem again to estimate masses and EOS parameters:

$$\underbrace{p(\vec{x}|d_1 \ldots d_N)}_{\text{Posterior}} = \frac{\overbrace{p(\vec{x})}^{\text{Prior}}\overbrace{p(d_1 \ldots d_N|\vec{x})}^{\text{Likelihood}}}{\underbrace{p(d_1 \ldots d_N)}_{\text{Evidence}}}$$

$$\vec{x} = \{\log(p_1), \Gamma_1, \Gamma_2, \Gamma_3, \mathcal{M}_1, \eta_1, \ldots, \mathcal{M}_N, \eta_N\}$$

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Posterior — Prior — Likelihood

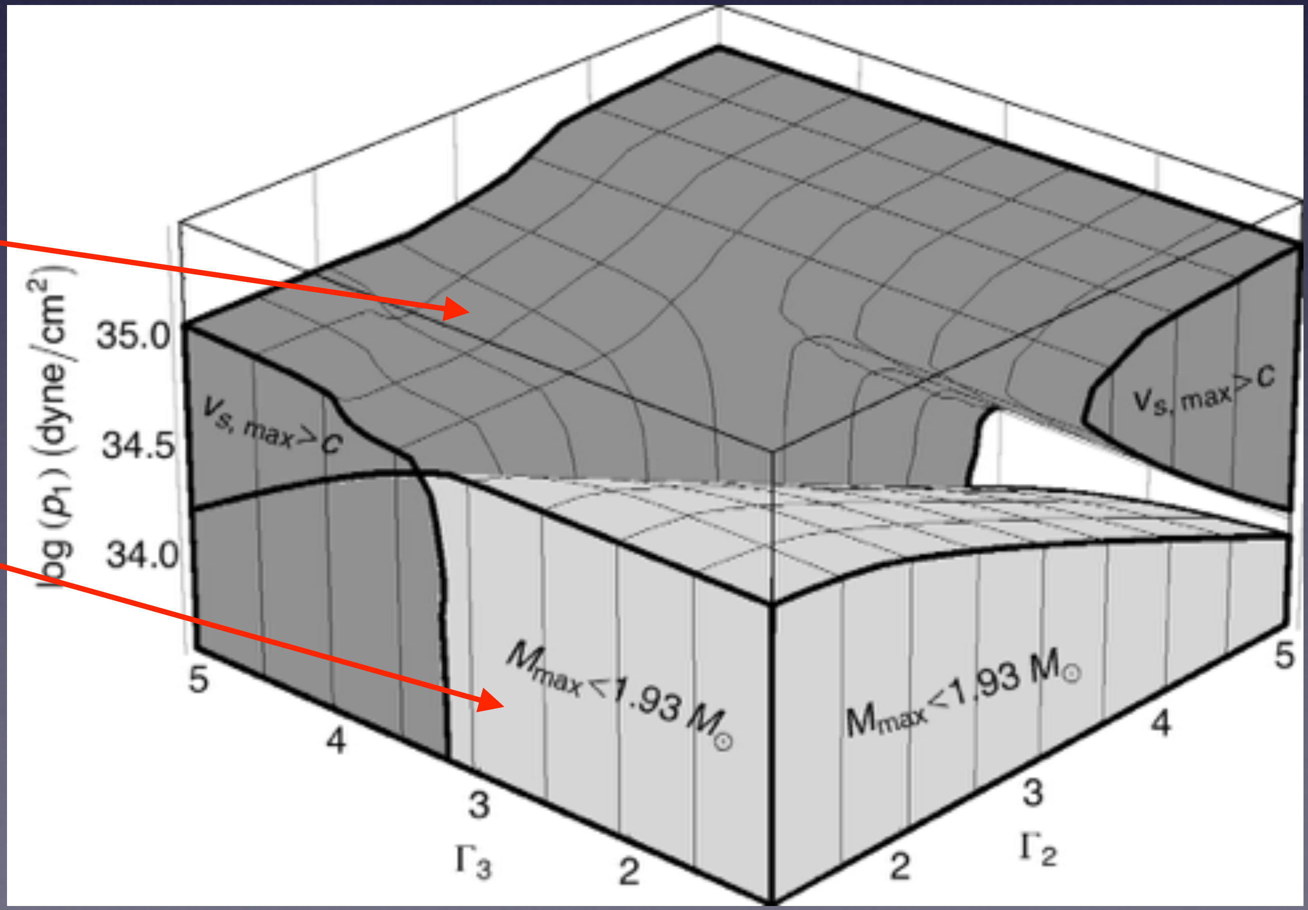
$$p(\vec{x}|d_1 \dots d_N) = \frac{p(\vec{x})p(d_1 \dots d_N|\vec{x})}{p(d_1 \dots d_N)}$$

Evidence

- Causality: Speed of sound must be less than the speed of light

$$v_s = \sqrt{dp/d\epsilon} < c$$

- Maximum mass: EOS must support observed stars with masses greater than $1.93 M_{\odot}$



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Posterior — Prior — Likelihood — Evidence

$$p(\vec{x}|d_1 \ldots d_N) = \frac{p(\vec{x})p(d_1 \ldots d_N|\vec{x})}{p(d_1 \ldots d_N)}$$

- Total likelihood is product of likelihoods for each independent event
- Rewritten in terms of the EOS parameters instead of tidal deformability

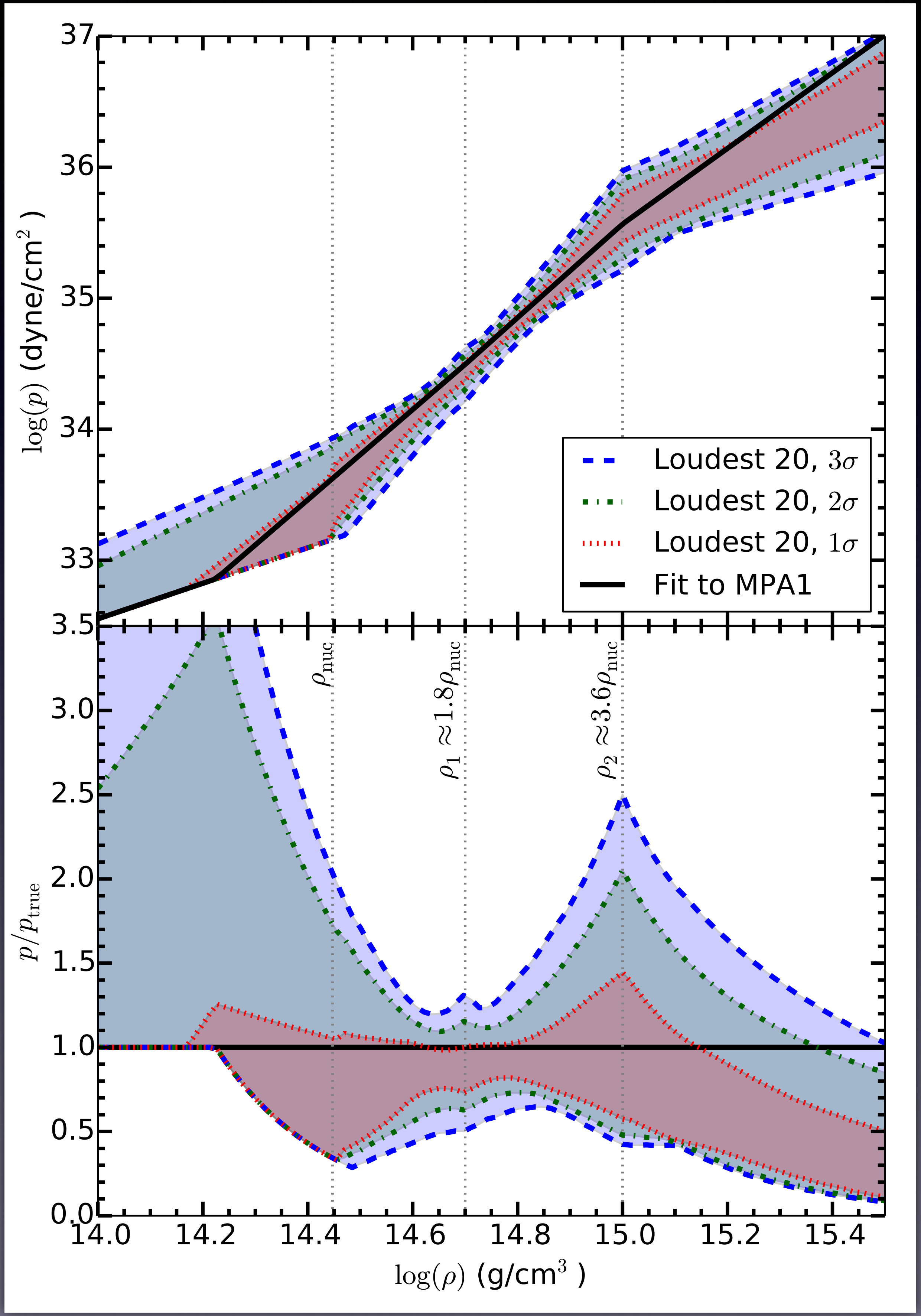
Marginalized posterior for single event

$$p(d_1, \ldots, d_N|\vec{x}) = \prod_{n=1}^{N} p(\mathcal{M}_n, \eta_n, \tilde{\Lambda}_n|d_n)|_{\tilde{\Lambda}_n = \tilde{\Lambda}(\mathcal{M}_n, \eta_n, \mathrm{EOS})}$$

Simulating a population of BNS events

- Sampled a year of data using the standard “realistic” event rate
  - ~40 BNS events/year for single detector with SNR>8
- Masses sampled uniformly in $[1.2M_\odot, 1.6M_\odot]$
- Chose MPA1 to be “true” EOS when calculating tidal parameters for these events
- Injected waveforms into simulated noise for the 3-detector LIGO-Virgo network

Results for 1 year of data



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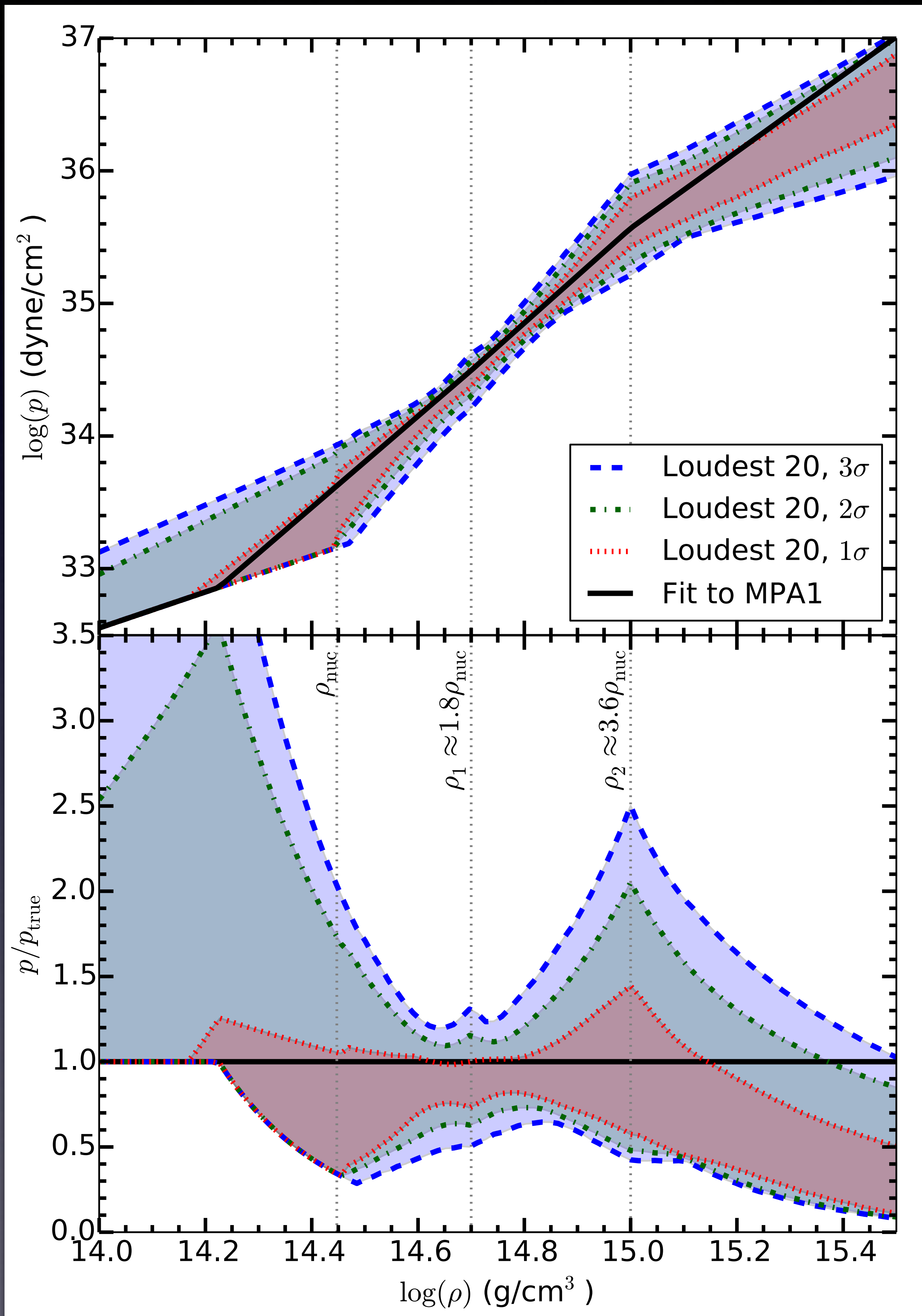
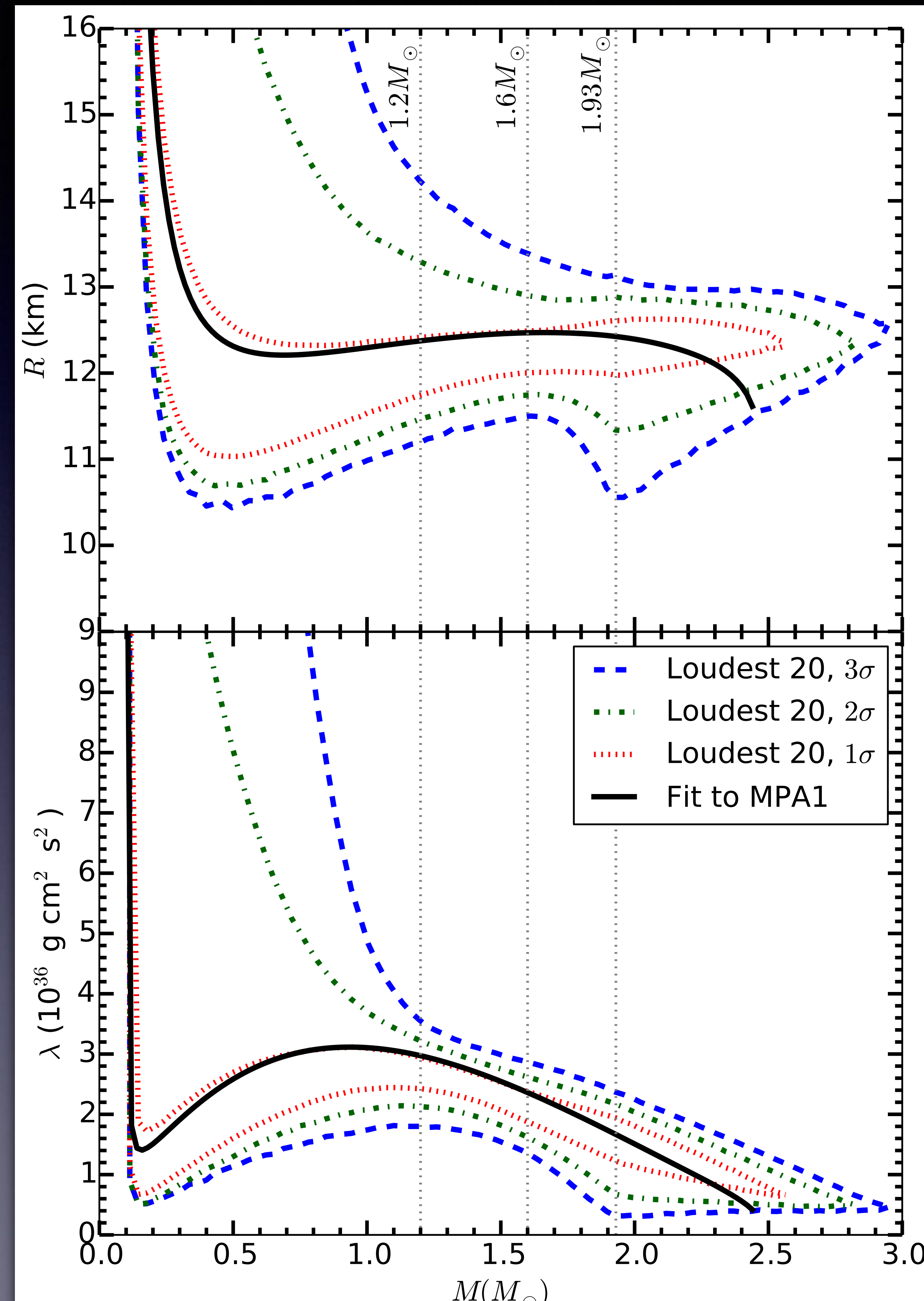
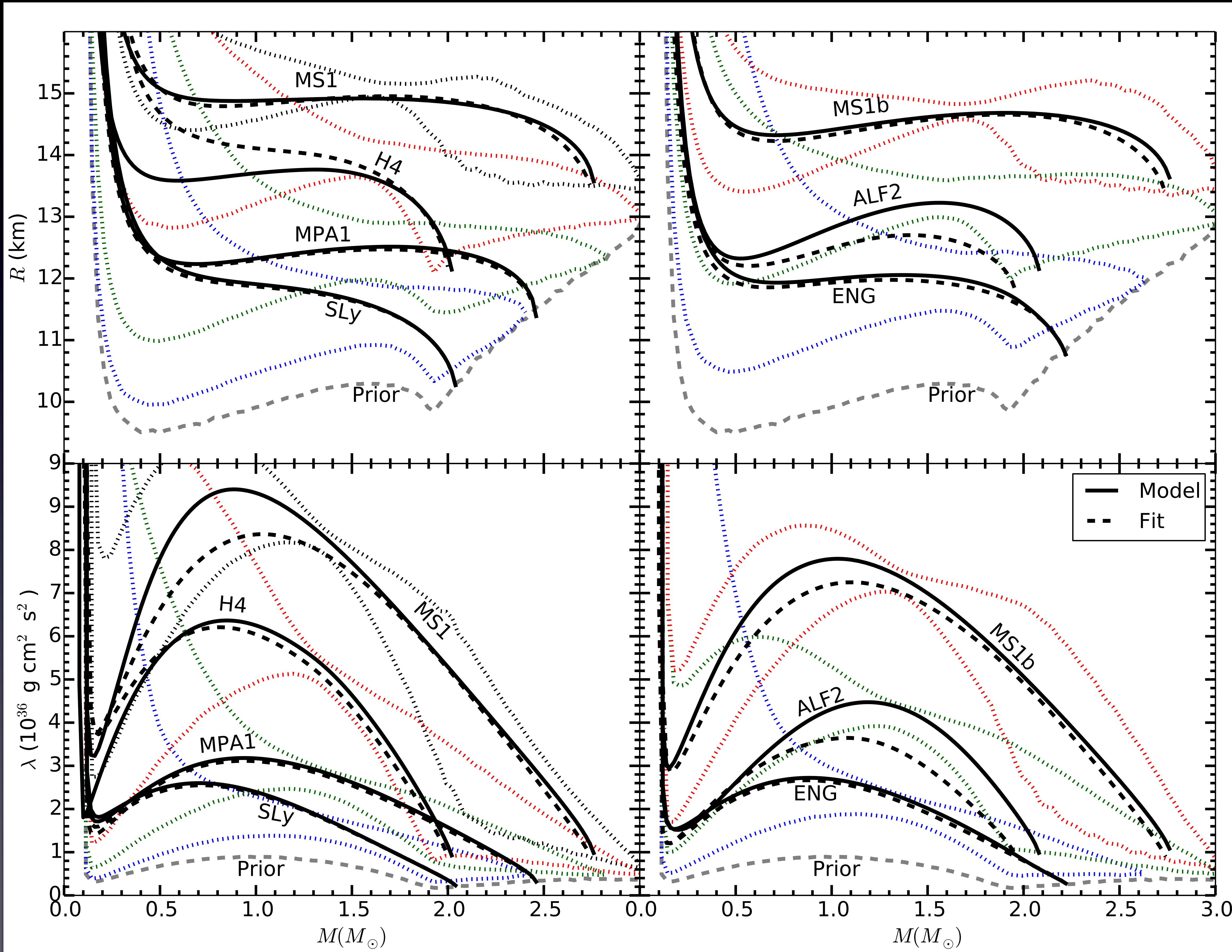




Other EOS models



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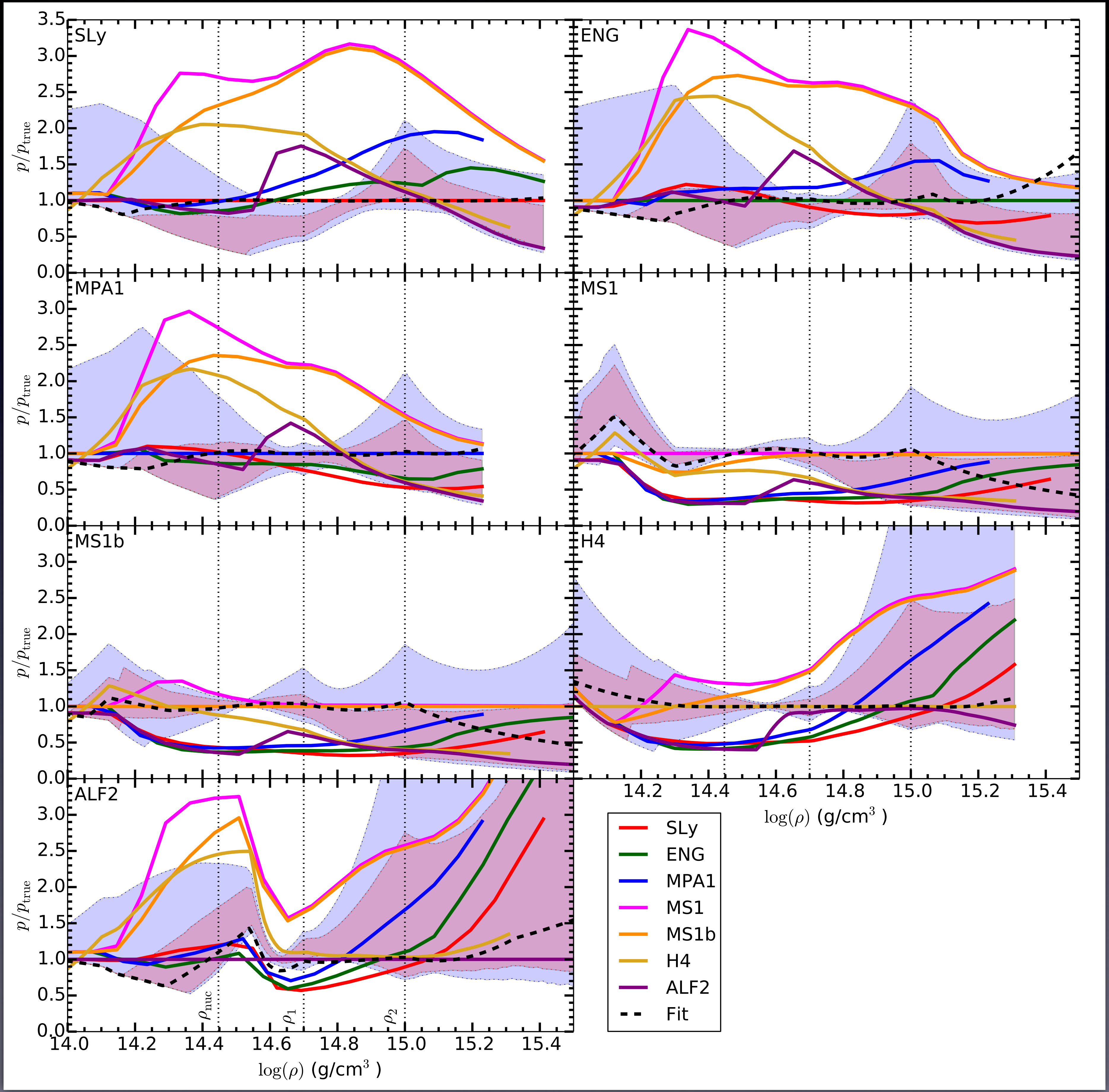
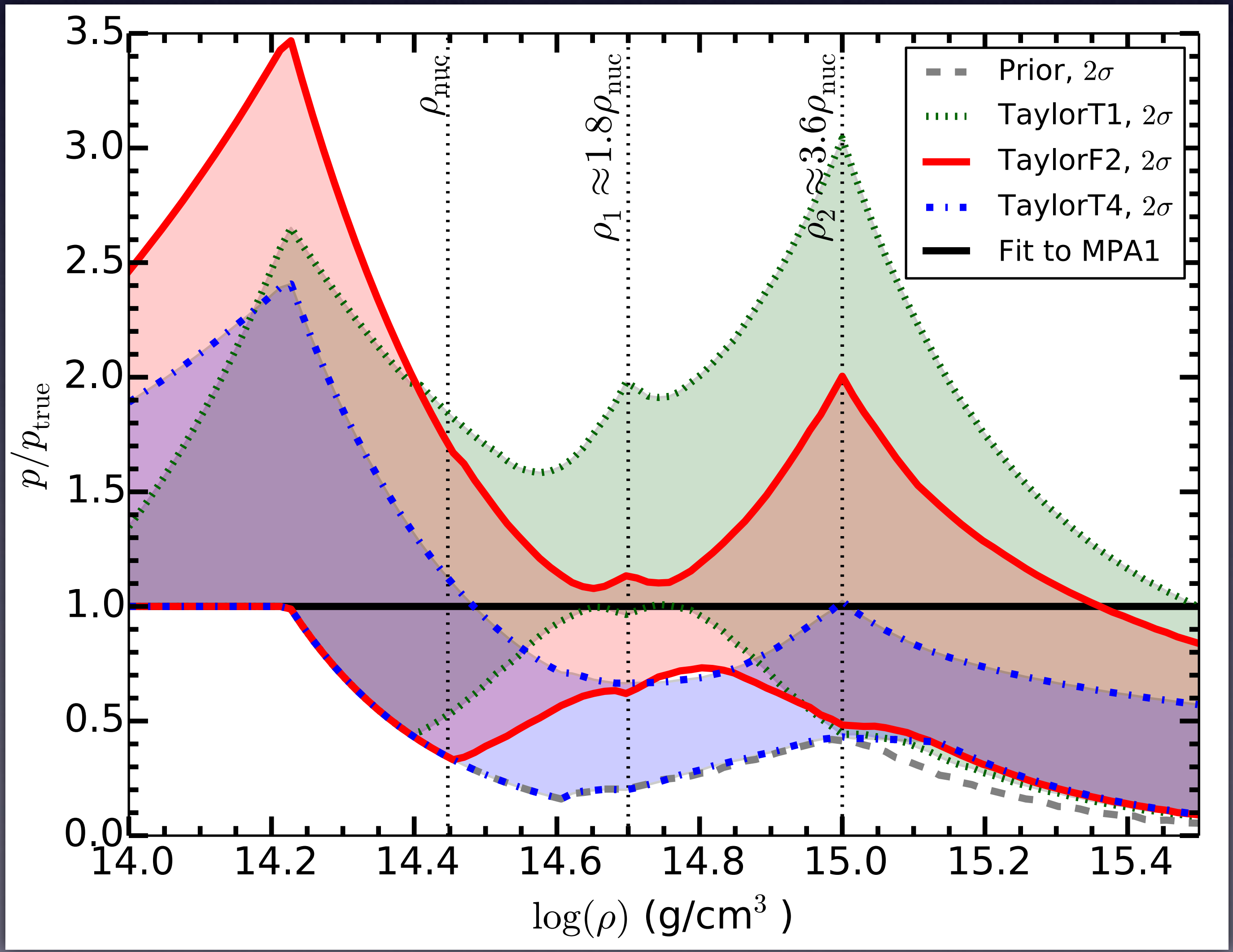
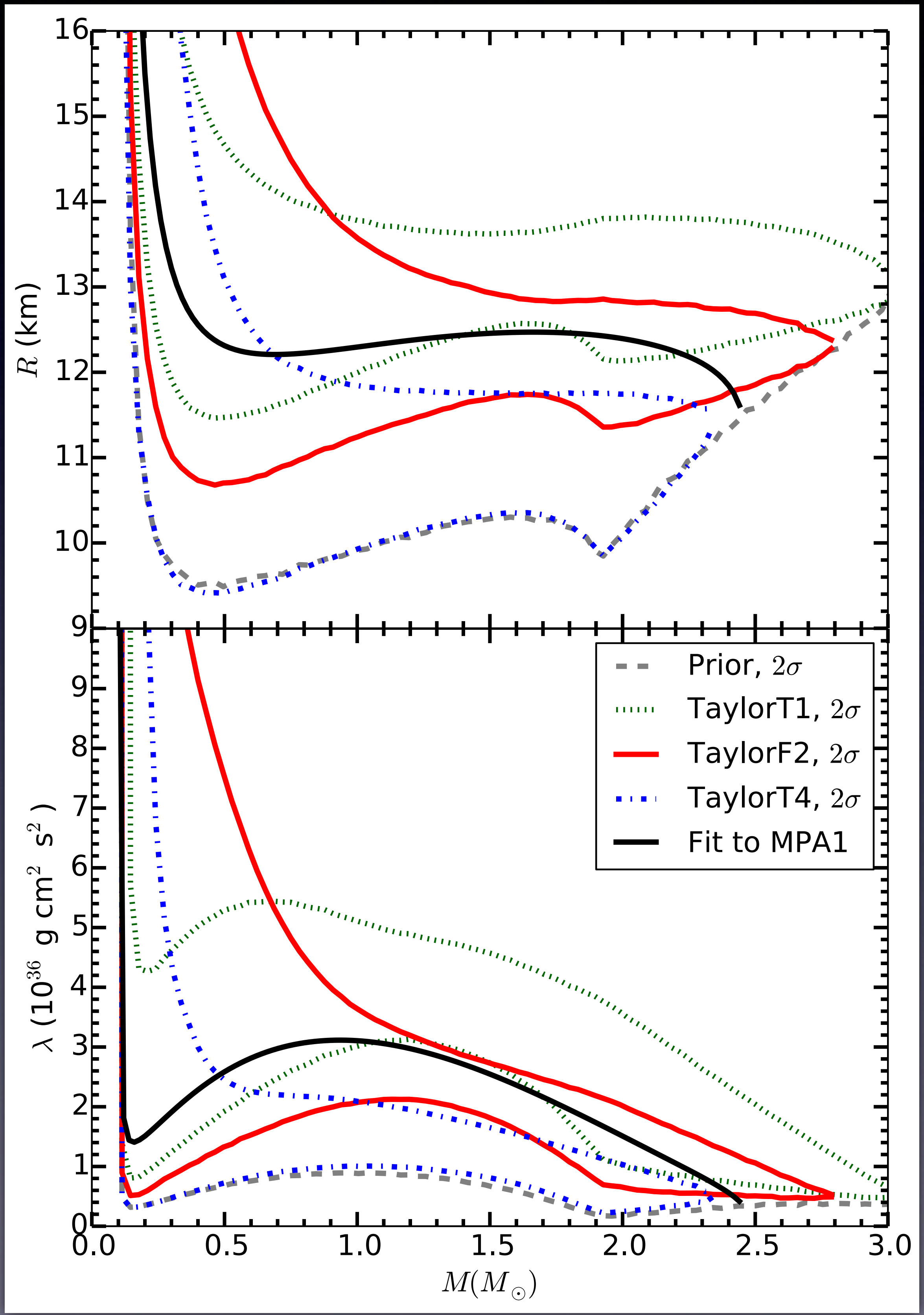


Systematic errors

- Several ways to calculate waveform phase from energy and luminosity expressions
- Injected TaylorF2, TaylorT1, TaylorT4 waveform models
- Used TaylorF2 as template





Conclusions

- The BNS inspiral waveform contains detailed EOS information
- 1 year of data will be sufficient to measure (statistical error):
  - Pressure to less than a factor of 2
  - Radius to +/- 1 km
- Systematic errors from inexact waveform templates will be primary difficulty in measuring the EOS
  - Will be reduced in the near future with improved waveform models

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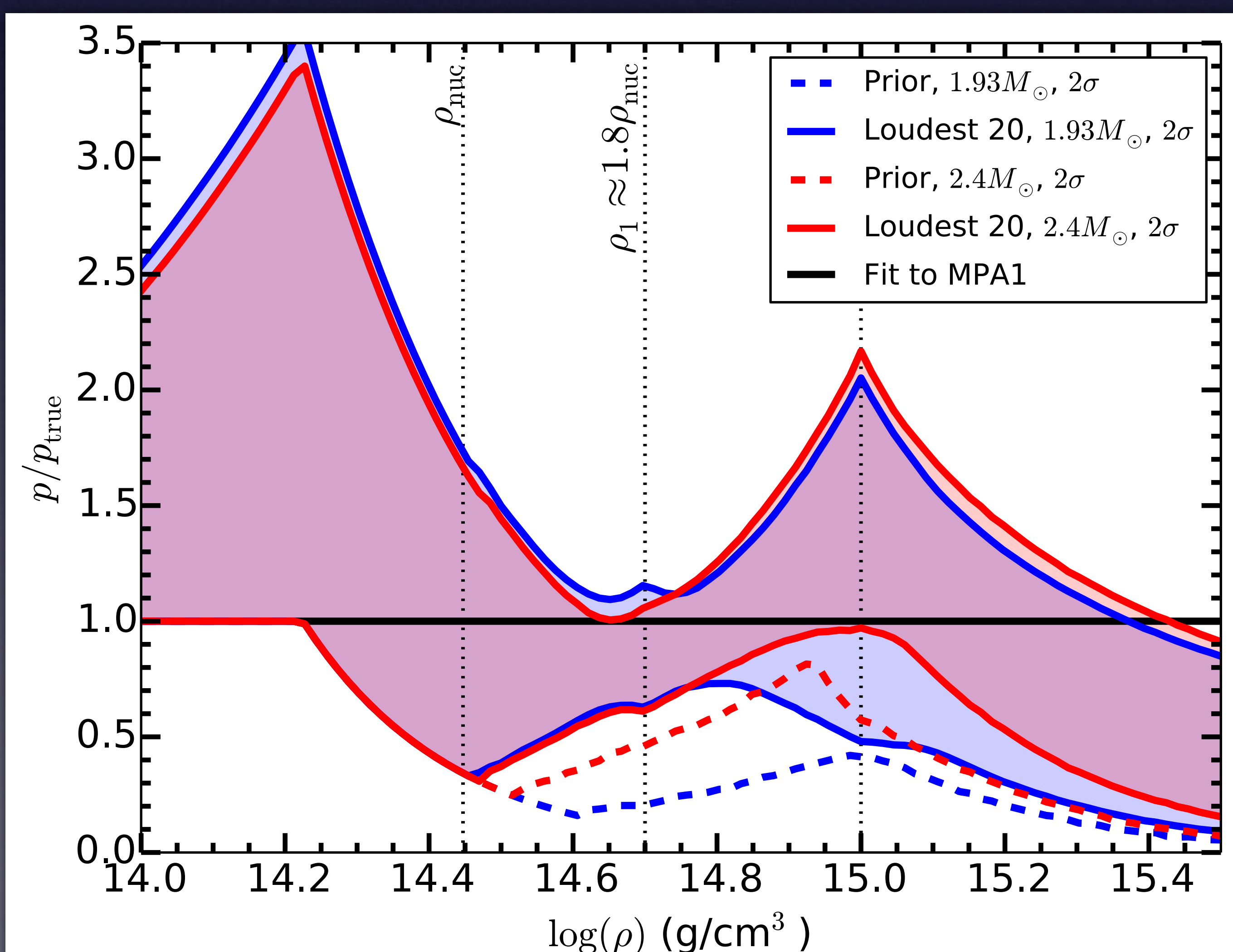
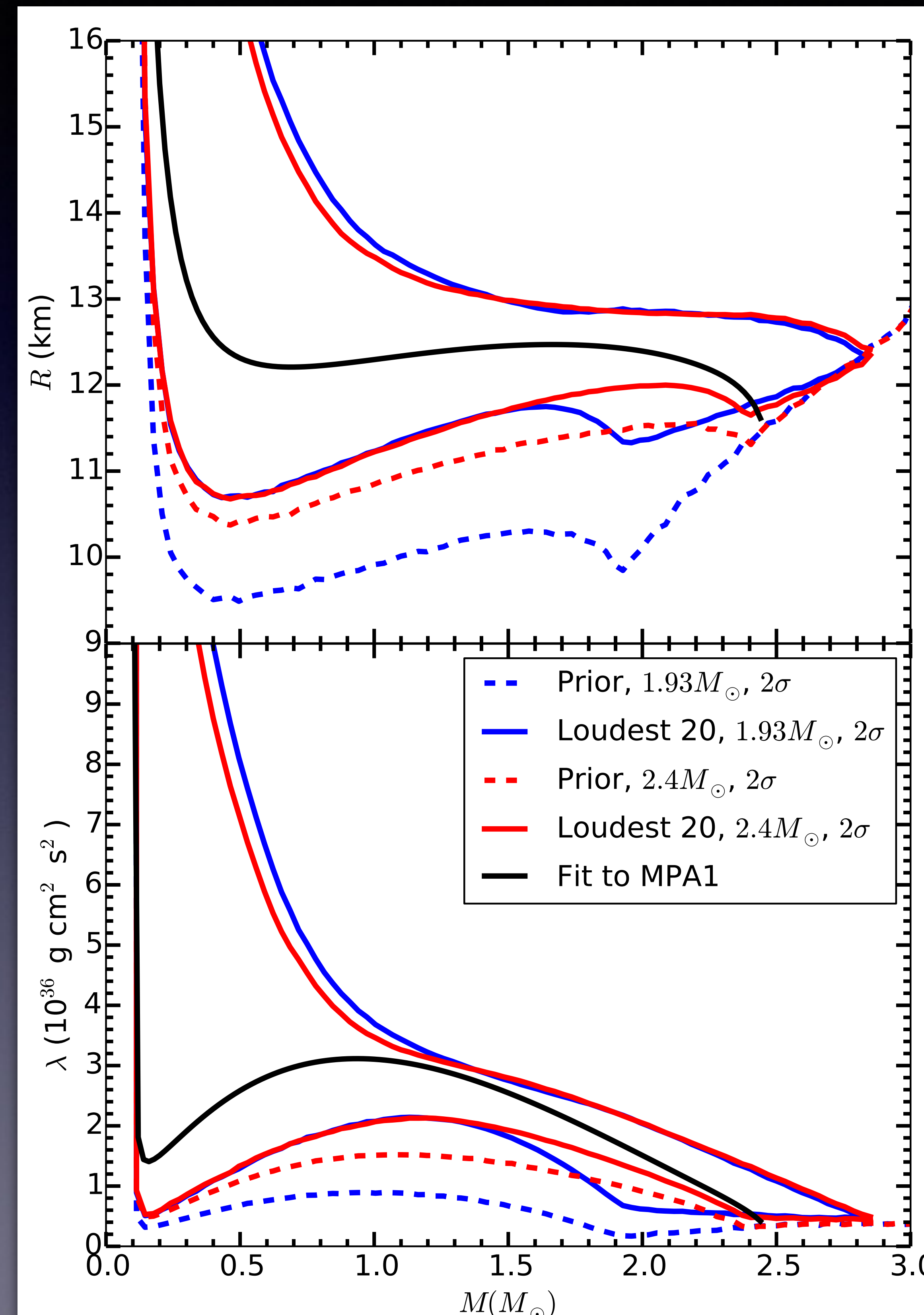
Thank you

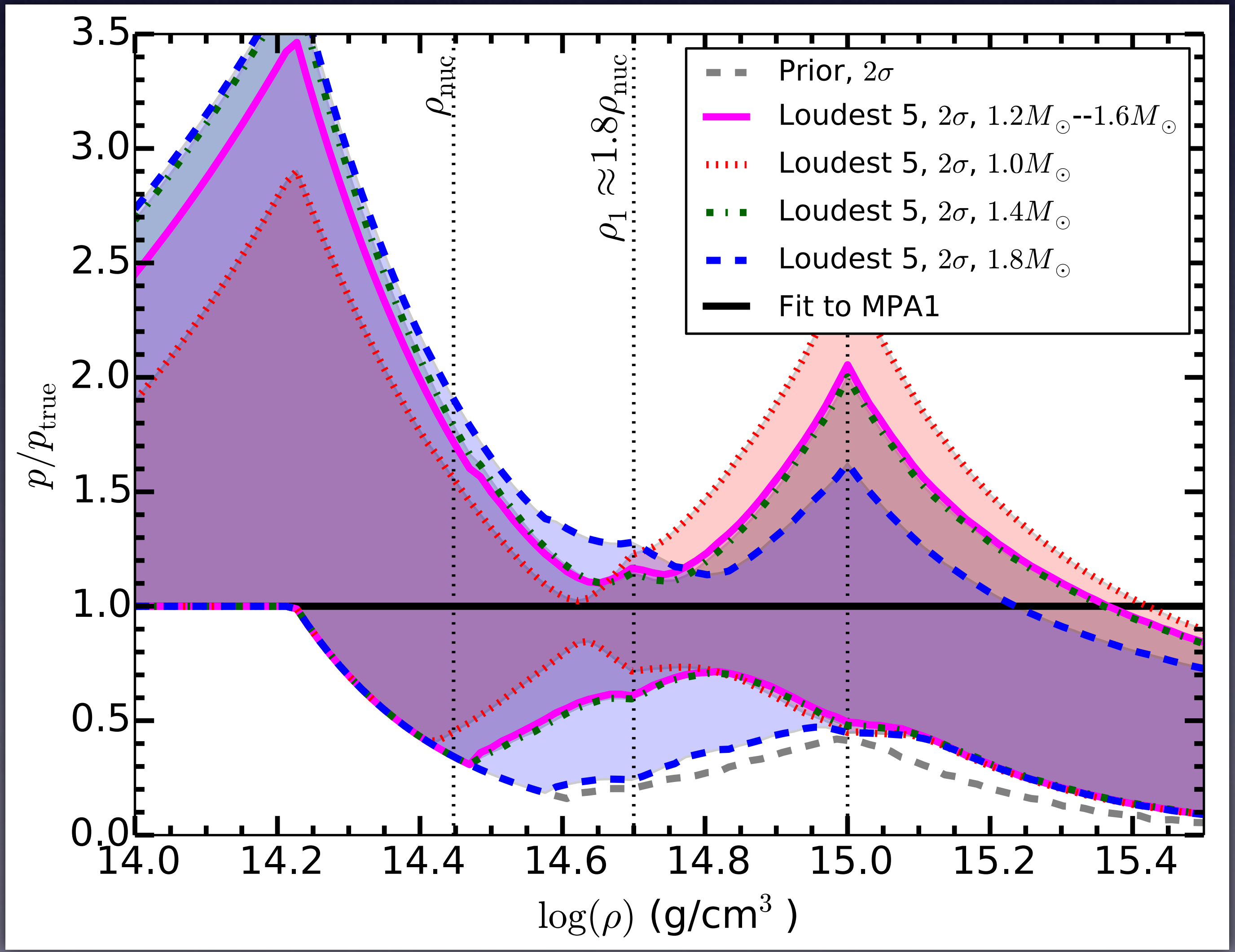
Extra Slides

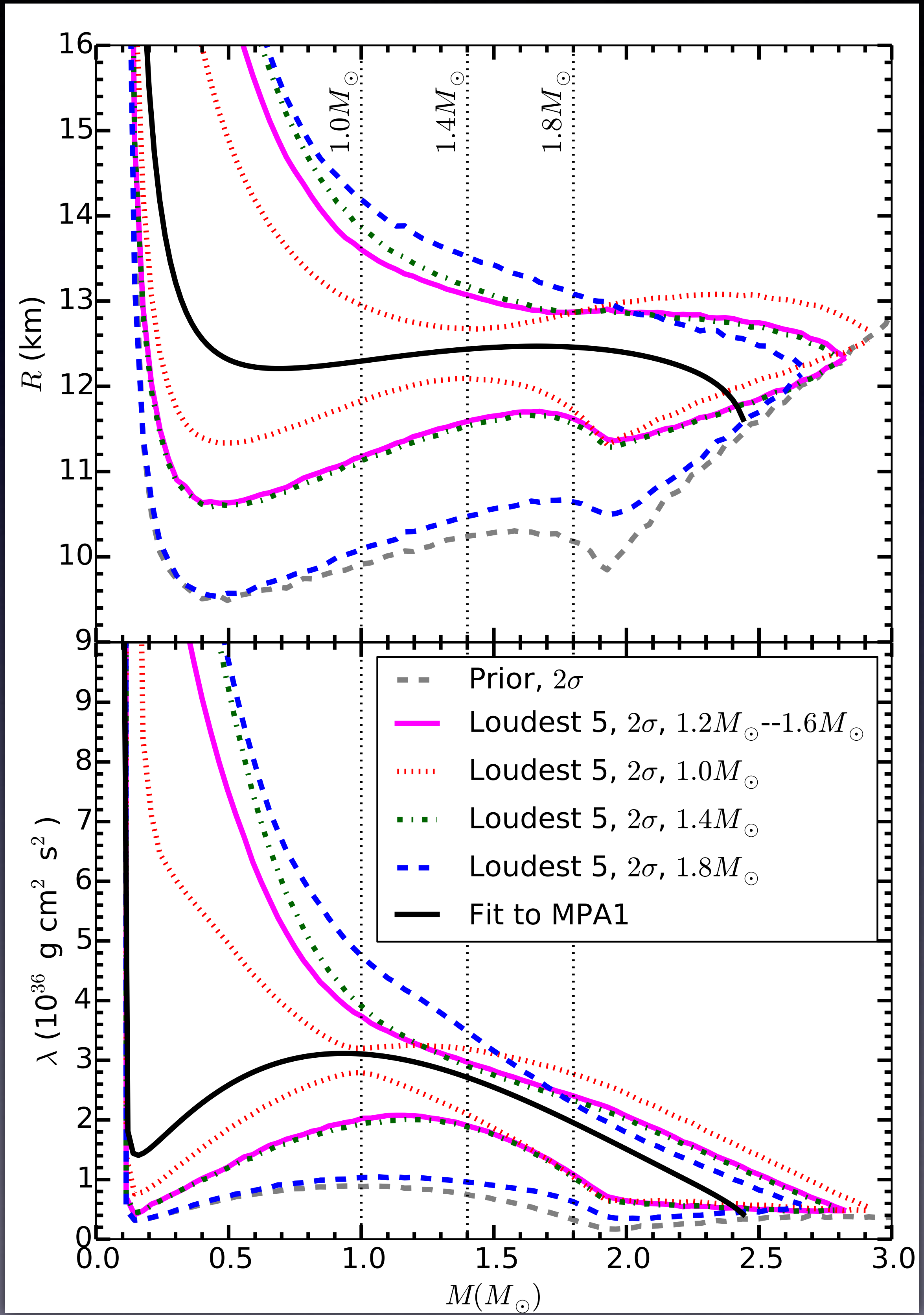
Higher mass NS observations





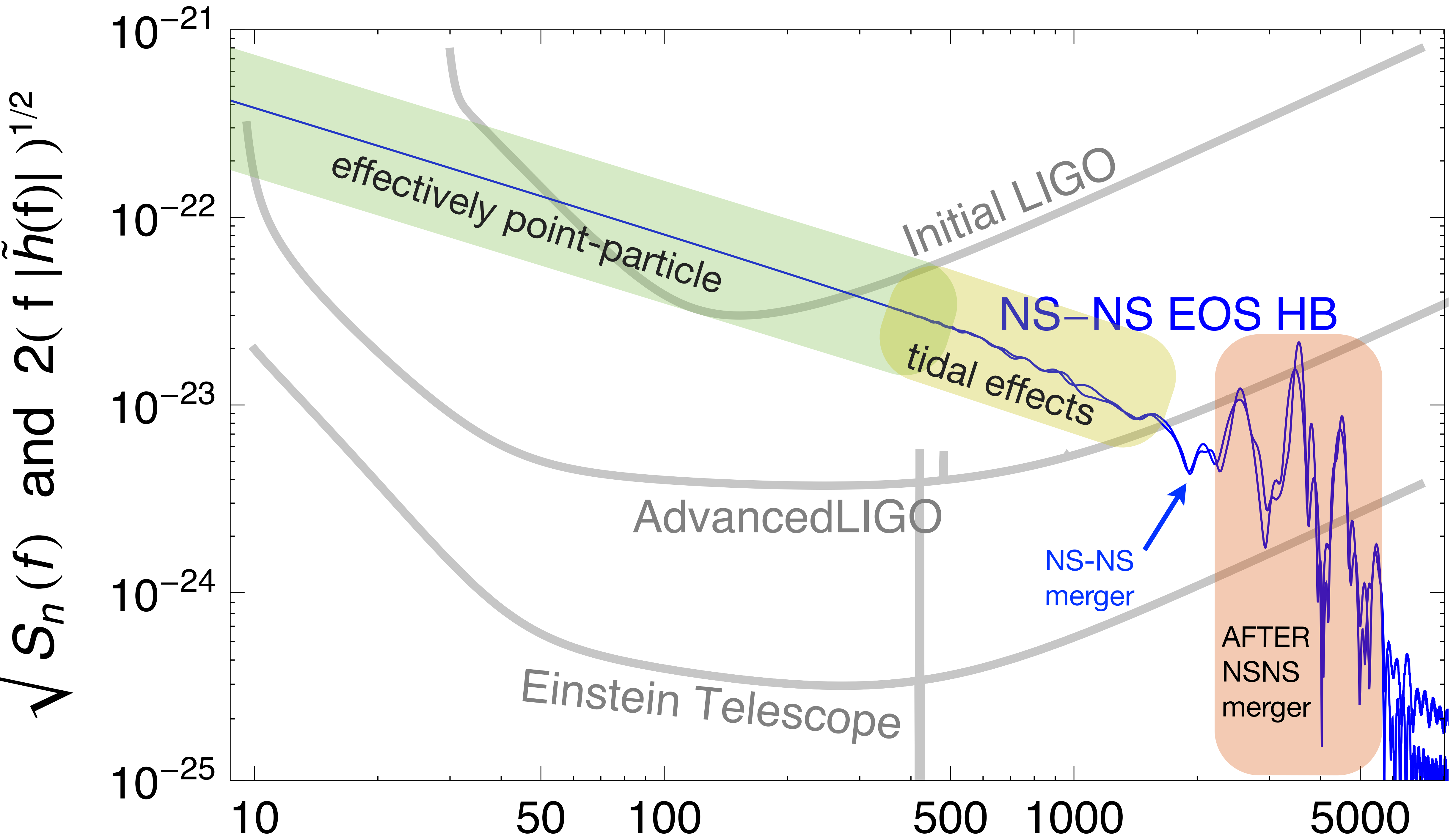
Range of sampled of BNS masses





Stages of BNS coalescence

Fourier transform of waveform:



Waveforms from SACRA and WHISKY codes
(Credit: Jocelyn Read, arXiv:1306.4065)