陽子スピンの分解

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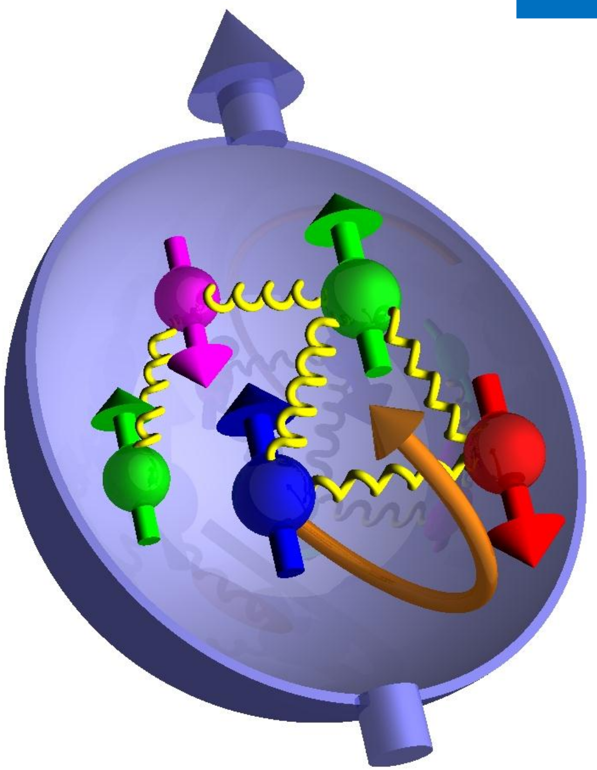
Outline

- QCD spin physics
- Proton spin decomposition: Problems and resolution
- Orbital angular momentum
- Twist analysis
- Transverse polarization
- Method to compute ΔG on a lattice

1101.5989 (PRD)
1111.3547 (PLB)
1207.5332 (JHEP) with Shinsuke Yoshida（吉田信介）
1211.2918 (JHEP) with Kazuhiro Tanaka（田中和廣）and S. Yoshida
1310.4263 (PRD)   with Xiangdong Ji（季向東）and Yong Zhao（趙勇）

The proton spin problem

The proton has spin ½.
The proton is not an elementary particle.



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_z$$

Quarks' helicity

Gluons' helicity

Oribital angular Momentum (OAM)

Quark model prediction: $\Delta\Sigma = 1$

$\Delta\Sigma \approx 0.6$ with relativistic effects

`Spin crisis’

In 1987, EMC (European Muon Collaboration) announced a very small value of the quark helicity contribution

$$\Delta\Sigma = 0.12 \pm 0.09 \pm 0.14 \quad !?$$

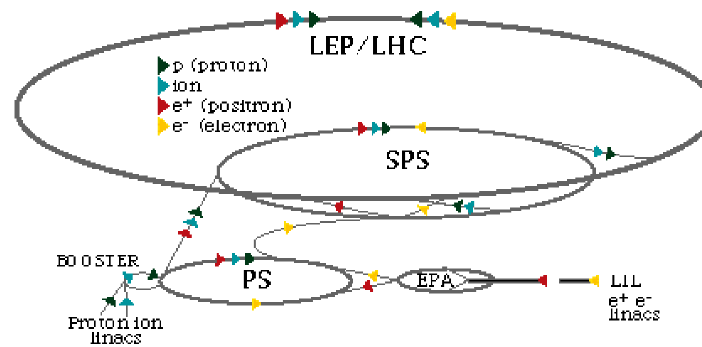
Recent results from NLO QCD global analysis $\Delta\Sigma \approx 0.3$

World’s facilities for high energy spin physics

BNL-RHIC : PHENIX, STAR, BRAHMS pp

Single (double) spin asymmetry

CERN-SPS : EMC, COMPASS μp DIS



DESY-HERA : HERMES ep DIS, DVCS

J-Lab : CLAS, Hall-A ep DVCS, SIDIS

J-PARC : pp Drell-Yan, SSA

How to measure $\Delta\Sigma$

Longitudinal spin asymmetry in polarized DIS

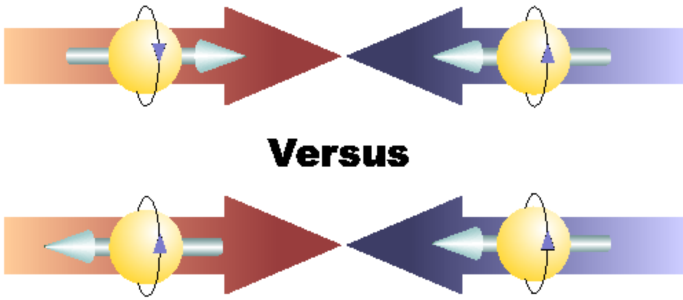
$$A_{LL} \equiv \frac{\mu^{\uparrow}p^{\uparrow} - \mu^{\uparrow}p^{\downarrow}}{\mu^{\uparrow}p^{\uparrow} + \mu^{\uparrow}p^{\downarrow}} = \left(1 + \frac{\sigma_L}{\sigma_T}\right)\frac{g_1}{F_1}$$

$$\begin{aligned}\int_0^1 dx g_1(x, Q^2) &= \frac{1}{4S^+}\sum_f e_f^2 \langle PS|\bar{q}_f \gamma_5 \gamma^+ q_f|PS\rangle \\ &= \frac{1}{9}\underline{\Delta\Sigma} + \frac{1}{6}F + \frac{1}{18}D + \mathcal{O}(\alpha_s)\end{aligned}$$

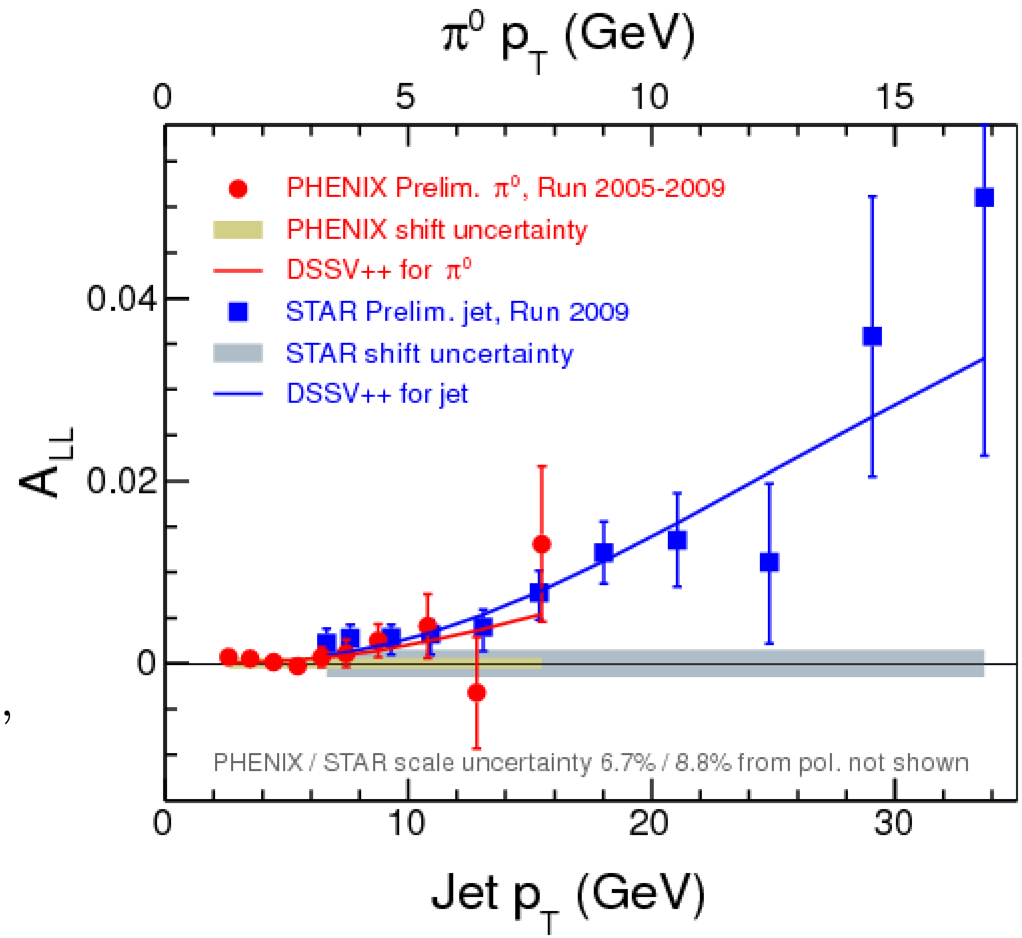
Determination of ΔG

PHENIX, STAR, COMPASS, JLab…

Longitudinal spin asymmetry in pp

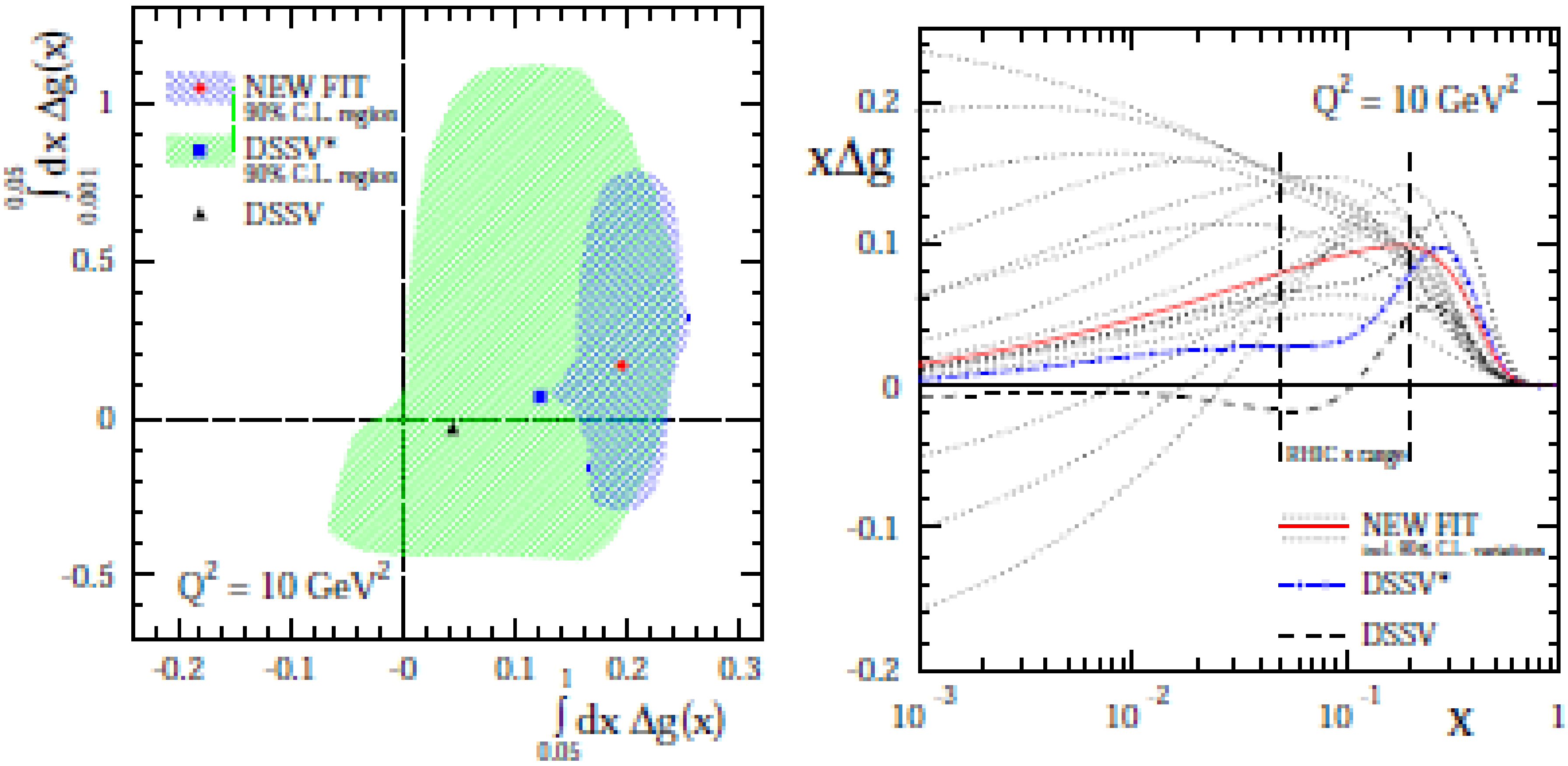


$$A_{LL}^{\pi} = \frac{d\sigma^{++} - d\sigma^{+-}}{d\sigma^{++} + d\sigma^{+-}}$$
$$\sim \sum_{a,b} \Delta f_a \quad \Delta f_b \quad \Delta\sigma_a,$$



Latest global QCD analysis

DeFlorian, Sassot, Stratmann, Vogelsang, 1404.4293



QCD angular momentum tensor

QCD Lagrangian → Lorentz invariant $x^\mu \longrightarrow x^\mu + \omega^{\mu\nu} x_\nu$

→ Noether current $\partial_\mu M^{\mu\nu\lambda}_{can} = 0$

QCD angular momentum tensor

$$M^{\mu\nu\lambda}_{can} = x^\nu T^{\mu\lambda}_{can} - x^\lambda T^{\mu\nu}_{can} - \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}\bar{\psi}\gamma_5\gamma_\rho\psi + F^{\mu\lambda}A^\nu - F^{\mu\nu}A^\lambda$$

quark spin gluon spin

canonical energy momentum tensor

$$T^{\mu\nu}_{can} = \bar{\psi} i \gamma^\mu \overleftrightarrow{\partial}^\nu \psi - F^{\mu\alpha}\partial^\nu A^\alpha - g^{\mu\nu}\mathcal{L}$$

→ Quark OAM → Gluon OAM

Jaffe-Manohar decomposition (1990)

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^{q}_{can} + L^{g}_{can}$$

Based on the canonical energy momentum tensor

Operators NOT gauge invariant.

Partonic interpretation in the light-cone gauge $A^{+} = 0$

Ji decomposition (1997)

Improved (Belinfante) energy momentum tensor

$$\widetilde{T}^{\mu\nu} = T^{\mu\nu}_{can} + \partial_\rho G^{\rho\mu\nu}$$

← One can add a total derivative.

$$= \bar{\psi} i \gamma^{(\mu} \overleftrightarrow{D}^{\nu)} \psi - F^{\mu\rho} F^{\nu}{}_{\rho} - g^{\mu\nu} \mathcal{L}$$

quark part gluon part

$$\frac{1}{2} = J_q + J_g$$

Further decomposition in the quark part

$$\bar{\psi} i \gamma^{(\mu} \overleftrightarrow{D}^{\nu)} \psi = \bar{\psi} i \gamma^{\mu} \overleftrightarrow{D}^{\nu} \psi - \frac{1}{4} \epsilon^{\mu\nu\rho\sigma} \partial_\rho (\bar{\psi} \gamma_5 \gamma_\sigma \psi)$$

$$J_q = \frac{1}{2} \Delta\Sigma + L_q$$

Generalized parton distributions (GPD)

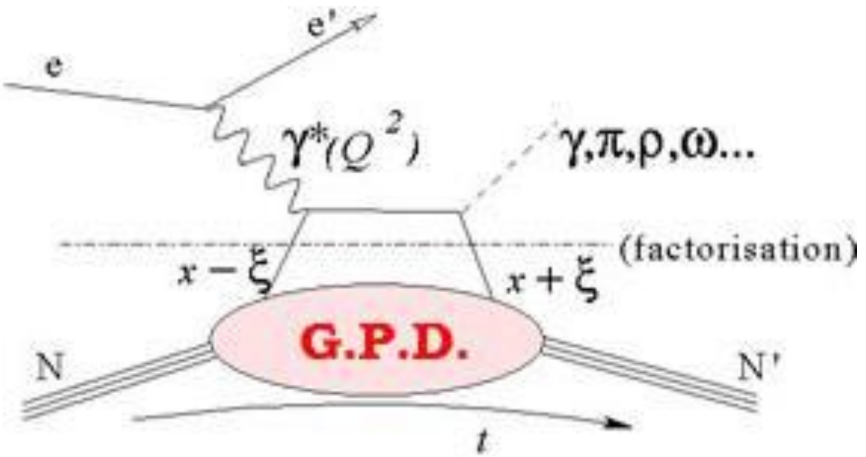
Non-forward proton matrix element

$$\int d\lambda e^{i\lambda x}\frac{d\lambda}{2\pi}\langle P'S'|\bar{\psi}(0)\gamma^{\mu}\psi(\lambda)|PS\rangle$$

$$= H_q(x)\bar{u}(P'S')\gamma^{\mu}u(PS) + E_q(x)\bar{u}(P'S')\frac{i\sigma^{\mu\nu}\Delta_{\nu}}{2m}u(PS)$$

$$J^q = \frac{1}{2}\int dx x(H_q(x) + E_q(x)) \qquad J^g = \frac{1}{4}\int dx(H_g(x) + E_g(x))$$

Measurable in
Deeply Virtual Compton Scattering



Two spin communities divided

measured by PHENIX, STAR, COMPASS, HERMES

Jaffe-Manohar

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q_{can} + L^g_{can}$$

common and well-known

not measured yet
not even well-defined?

Define rigorously.
Must be related to GPDs!

Ji

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$$

accessible from GPD at COMPASS, HERMES, JLab, J-PARC...
also calculated in lattice QCD

Complete decomposition

Chen, Lu, Sun, Wang, Goldman (2008)
Wakamatsu (2010)
Y.H. (2011)

$$M^{\mu\nu\lambda}_{\text{quark-spin}} = -\frac{1}{2}\epsilon^{\mu\nu\lambda\sigma}\bar{\psi}\gamma_5\gamma_\sigma\psi\,,$$

$$M^{\mu\nu\lambda}_{\text{quark-orbit}} = \bar{\psi}\gamma^\mu(x^\nu iD^\lambda_{\text{pure}} - x^\lambda iD^\nu_{\text{pure}})\psi\,,$$

$$M^{\mu\nu\lambda}_{\text{gluon-spin}} = F^{\mu\lambda}_a A^{\nu a}_{\text{phys}} - F^{\mu\nu}_a A^{\lambda a}_{\text{phys}}\,,$$

$$M^{\mu\nu\lambda}_{\text{gluon-orbit}} = F^{\mu\alpha}_a\left(x^\nu(D^\lambda_{\text{pure}}A^{\text{phys}}_\alpha)_a - x^\lambda(D^\nu_{\text{pure}}A^{\text{phys}}_\alpha)_a\right)$$

My choice $A^\mu_{phys} = \frac{1}{D^+}F^{+\mu}$ $D^\mu_{pure} = D^\mu - iA^\mu_{phys}$

Gauge invariant completion of Jaffe-Manohar

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q_{can} + L^g_{can}$$

OAM from the Wigner distribution

Wigner distribution in QCD   Belitsky, Ji, Yuan (2003)

$$W(x,q) = \int \frac{d^4 z}{(2\pi)^4} e^{iqz} \bar{\psi}\left(x - \frac{z}{2}\right) \gamma^\mu \psi \left(x + \frac{z}{2}\right)$$

position   momentum

Need a Wilson line !

Define

$$\vec{L} = \int dq \, \vec{x} \times \vec{q} \, \langle W(x,q) \rangle$$

Lorce, Pasquini (2011)

Which OAM is this??

Canonical OAM from the light-cone Wilson line   YH (2011)

$$\int dq\,\vec{x}\times\vec{q}\,\langle W_{light-cone}(x,q)\rangle = \langle\bar{\psi}\gamma^{\mu}\vec{x}\times i\overleftrightarrow{D}_{pure}\psi\rangle$$

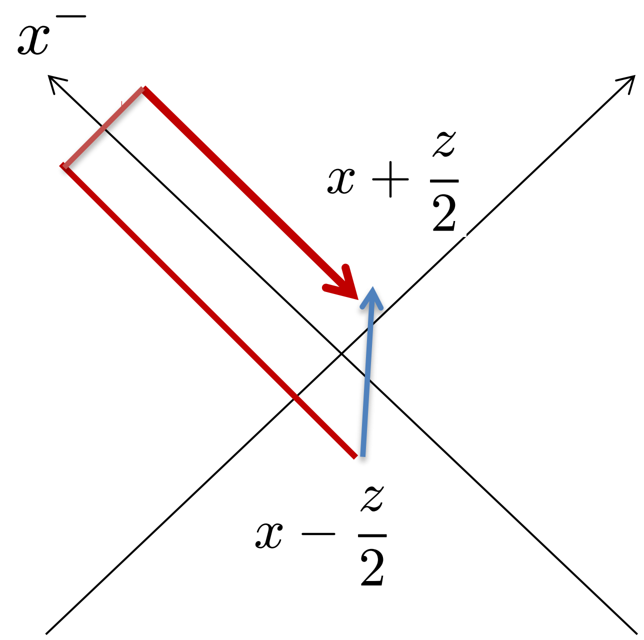
Naturally defined in infinite momentum frame.
Parton interpretation possible.

Kinetic OAM from the straight Wilson line

Ji, Xiong, Yuan (2012)

$$\int dq\,\vec{x}\times\vec{q}\,\langle W_{straight}(x,q)\rangle$$

$$= \langle\bar{\psi}\gamma^{\mu}\vec{x}\times i\overleftrightarrow{D}\psi\rangle$$



Twist analysis

YH, Yoshida (2012)

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q_{can} + L^g_{can}$$

Understand these relations at the density level

$$\Delta\Sigma = \sum_f \int dx \Delta q_f(x) \qquad \Delta G = \int dx \Delta G(x)$$

$$L^q_{can} = \int dx L^q_{can}(x)$$??

c.f. $\Delta q(x) = \frac{1}{4\pi S^+}\int dz e^{ixPz} \langle PS|\bar{\psi}(z)\gamma^+\gamma_5\psi(0)|PS\rangle$

Density of OAM

Ji's OAM canonical OAM `potential OAM'

$$\langle \bar{\psi} x \times D\psi \rangle = \langle \bar{\psi} x \times D_{pure}\psi \rangle + ig\langle \bar{\psi} x \times A_{phys}\psi \rangle$$

$$A^{\mu}_{phys} = \frac{1}{D^+}F^{+\mu}$$

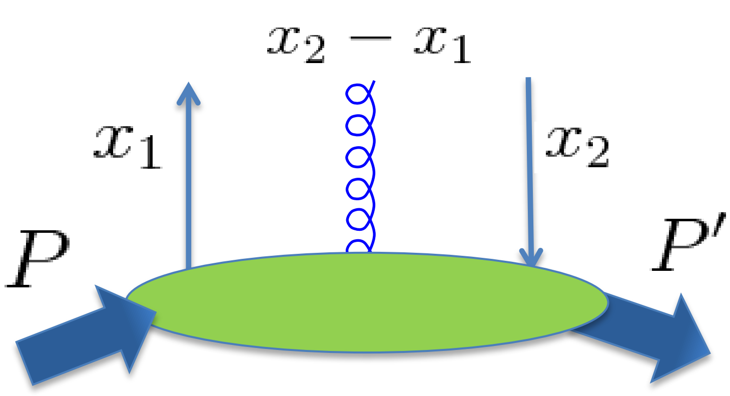
``F-type"

For a 3-body operator, it is natural to define the double density.

$$\int d\lambda d\mu e^{i\frac{\lambda}{2}(x_1+x_2)+i\mu(x_1-x_2)}\langle P'S'|\bar{\psi}(-\lambda/2)D^i(\mu)\psi(\lambda/2)|PS\rangle$$

$$\sim \epsilon^{ij}\Delta_j S^+ \Phi_D(x_1, x_2)$$

``D-type"



The D-type and F-type correlators are related.

Eguchi, Koike, Tanaka (2006)

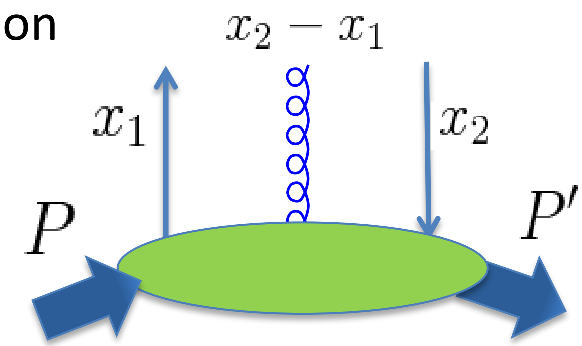
$$\langle \bar{\psi} x \times D\psi \rangle = \langle \bar{\psi} x \times D_{pure}\psi \rangle + ig\langle \bar{\psi} x \times A_{phys}\psi \rangle$$

doubly-unintegrate

$$\Phi_D(x_1, x_2) = \delta(x_1 - x_2) L^q_{can}(x_1) + \mathcal{P}\frac{1}{x_1 - x_2}\Phi_F(x_1, x_2)$$

Canonical OAM density

The gluon has zero energy
→ density interpretation



No unique density for Ji's OAM.

Relation to twist-3 GPD

$$\int d\lambda e^{i\lambda x} \frac{d\lambda}{2\pi} \langle P'S'|\bar{\psi}(0)\gamma^\mu \psi(\lambda)|PS\rangle$$

$$= H_q(x)\bar{u}(P'S')\gamma^\mu u(PS) + E_q(x)\bar{u}(P'S')\frac{i\sigma^{\mu\nu}\Delta_\nu}{2m}u(PS)$$ twist-2

$$+\underline{G_3(x)}\bar{u}(P'S')\gamma^\mu_\perp u(PS) + \cdots$$ ← twist-3

From the equation of motion,

$$x(H_q(x) + E_q(x) + \underline{G_3(x)}) =$$

$$\Delta q(x) + \underline{L^q_{can}(x)} + \int dx' \mathcal{P}\frac{1}{x - x'}\left(\Phi_F(x, x') + \tilde{\Phi}_F(x, x')\right)$$

Quark canonical OAM density

Wandzura-Wilczek part

$$L^q_{can}(x) = x\int_x^{\epsilon(x)} \frac{dx'}{x'}(H_q(x') + E_q(x')) - x\int_x^{\epsilon(x)} \frac{dx'}{x'^2}\Delta q(x')$$

$$-x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \Phi_F(x_1, x_2)\mathcal{P}\frac{3x_1 - x_2}{x_1^2(x_1 - x_2)^2}$$

$$-x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \tilde{\Phi}_F(x_1, x_2)\mathcal{P}\frac{1}{x_1^2(x_1 - x_2)} .$$

genuine twist-three

First moment: $J^q = \frac{1}{2}\Delta\Sigma + L^q_{can} + L_{pot}$

The bridge between JM and Ji

Gluon canonical OAM density

Relation between F- and D-type three-gluon correlators

$$\frac{M_D(x_1,x_2)}{x_1} = \frac{M_F(x_1,x_2)}{x_1(x_1-x_2)} - \delta(x_1-x_2)\underline{L^g_{can}(x_1)}$$

Related to a twist-3 gluon GPD

$$L^g_{can}(x) = \frac{x}{2}\int_x^{\epsilon(x)} \frac{dx'}{x'^2}(H_g(x')+E_g(x')) - x\int_x^{\epsilon(x)} \frac{dx'}{x'^2}\Delta G(x')$$
$$+2x\int_x^{\epsilon(x)} \frac{dx'}{x'^3}\int dX\,\Phi_F(X,x') + 2x\int_x^{\epsilon(x)} dx_1\int_{-1}^{1} dx_2 \tilde{M}_F(x_1,x_2)\mathcal{P}\frac{1}{x_1^3(x_1-x_2)}$$
$$+2x\int_x^{\epsilon(x)} dx_1\int_{-1}^{1} dx_2 M_F(x_1,x_2)\mathcal{P}\frac{2x_1-x_2}{x_1^3(x_1-x_2)^2}$$

First moment: $J^g + L_{pot} = \Delta G + L^g_{can}$

Transverse spin decomposition

It's important to use the Pauli-Lubanski vector

$$W^\mu = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma} P_\nu \int d^3x M^+{}_{\rho\sigma}$$

instead of the angular momentum tensor $M^{+\mu\nu}$ itself.

$$J^{q,g} = \frac{1}{2}(A_{q,g} + B_{q,g}) + \frac{P^3}{2(P^0 + M)}\bar{C}_{q,g}$$

Ji (1996)
Ji, Xiong, Yuan (2012)

YH, Tanaka, Yoshida (2012)
Leader (2012)

Frame dependent!

Complete transverse spin decomposition?

Longitudinal

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q_{can} + L^g_{can}$$

same!

Transverse

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^{q+g}_{can}$$

cannot be separated in a frame-independent way

New development: Measuring PDF on a lattice?

Consider the usual quark distribution function.

$$q_f(x,Q^2) = \frac{1}{4\pi}\int dy^- e^{ixP^+y^-}\langle P|\bar{q}_f(0)W[0,y^-]\gamma^+ q_f(y^-)|P\rangle_{Q^2}$$

Nonlocal operator along the light-cone → Real-time problem

Local operator after taking the moment

$$\begin{aligned} q_f(j,Q^2) &= \int_0^1 dx x^{j-1} q_f(x,Q^2) \\ &= \frac{1}{2(P^+)^j}\langle P|\bar{q}_f(0)\gamma^+(iD^+)^{j-1}q_f(0)|P\rangle \end{aligned}$$

Computing ΔG on a lattice

Ji, Zhang, Zhao (2013)
YH, Ji, Zhao (2013)

$$\Delta G = \int dx \Delta G(x)$$
$$= \int dx \frac{i}{2xP^+} \int \frac{dy^-}{2\pi} e^{-ixP^+y^-} \langle PS|F^{+\mu}(y^-)W[y^-,0]\tilde{F}_\mu{}^+(0)|PS\rangle$$

Nonlocal even though it is a x-moment!

??

$$\Delta\widetilde{G}(P_z) = \frac{1}{2P^0}\langle PS|\epsilon^{ij}F^{i0}A^j|PS\rangle$$

Local, but gauge variant.
Calculable on a lattice.

Calculate $\Delta\widetilde{G}(P_z)$ in a universality class of gauges and do the matching.

$$\Delta\tilde{G}(P^z,\mu) = Z_{gg}(P^z/\mu)\Delta G(\mu) + Z_{gq}(P^z/\mu)\Delta\Sigma(\mu)$$

x-dependence may also be calculable Ji (2013)

Summary

- Complete gauge invariant decomposition of nucleon spin now available in QCD, even at the density level.

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q_{can} + L^g_{can}$$

- Relation between the two decomposition schemes (JM vs Ji) fully revealed. The connection to twist-3 GPDs clarified.