Di-neutron correlation and two-neutron decay of nuclei beyond the neutron drip line

Kouichi Hagino

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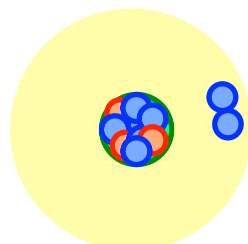
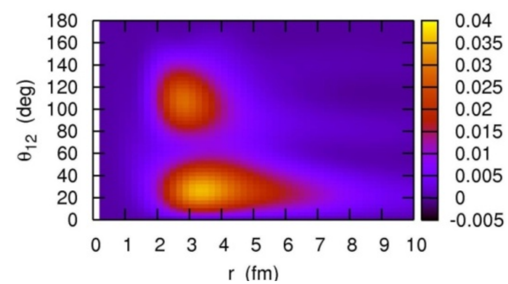




1. Introduction: physics of neutron-rich nuclei

2. Di-neutron correlation: what is it?

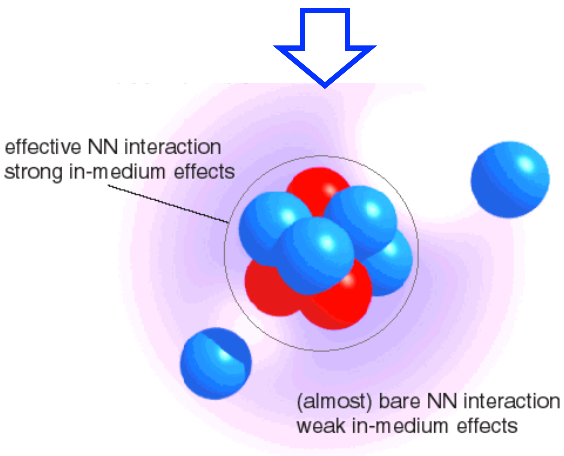
3. Coulomb breakup

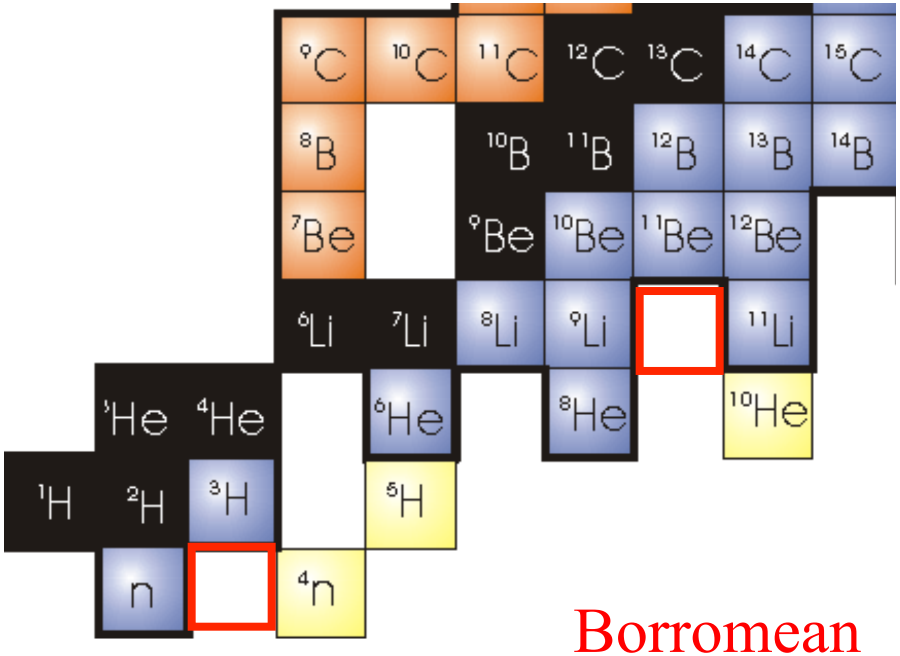
4. Two-neutron decay of unbound nucleus ^{26}O

5. Summary

Introduction: neutron-rich nuclei

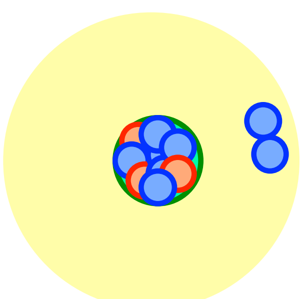
- *What is the spatial structure of the valence neutrons?*
- *To what extent is this picture correct?*





Borromean nuclei

di-neutron correlations



What is Di-neutron correlation?

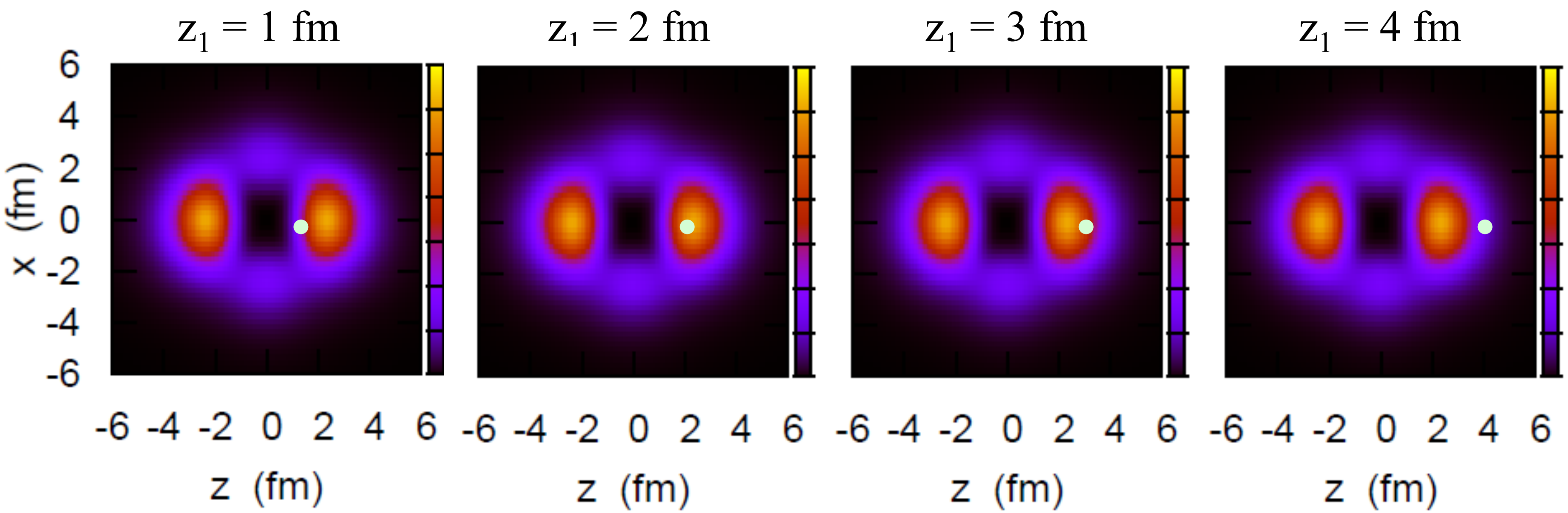
Correlation : $\langle AB \rangle \neq \langle A \rangle \langle B \rangle$

Example: ^{18}O = ^{16}O + n + n

cf. ^{16}O + n : 3 bound states ($1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$)

i) Without nn interaction: $|nn\rangle = |(1d_{5/2})^2\rangle$

Distribution of the 2nd neutron when the 1st neutron is at z_1 :



✓Two neutrons move independently

✓No influence of the 2nd neutron from the 1st neutron

$$\langle AB \rangle = \langle A \rangle \langle B \rangle$$

What is Di-neutron correlation?

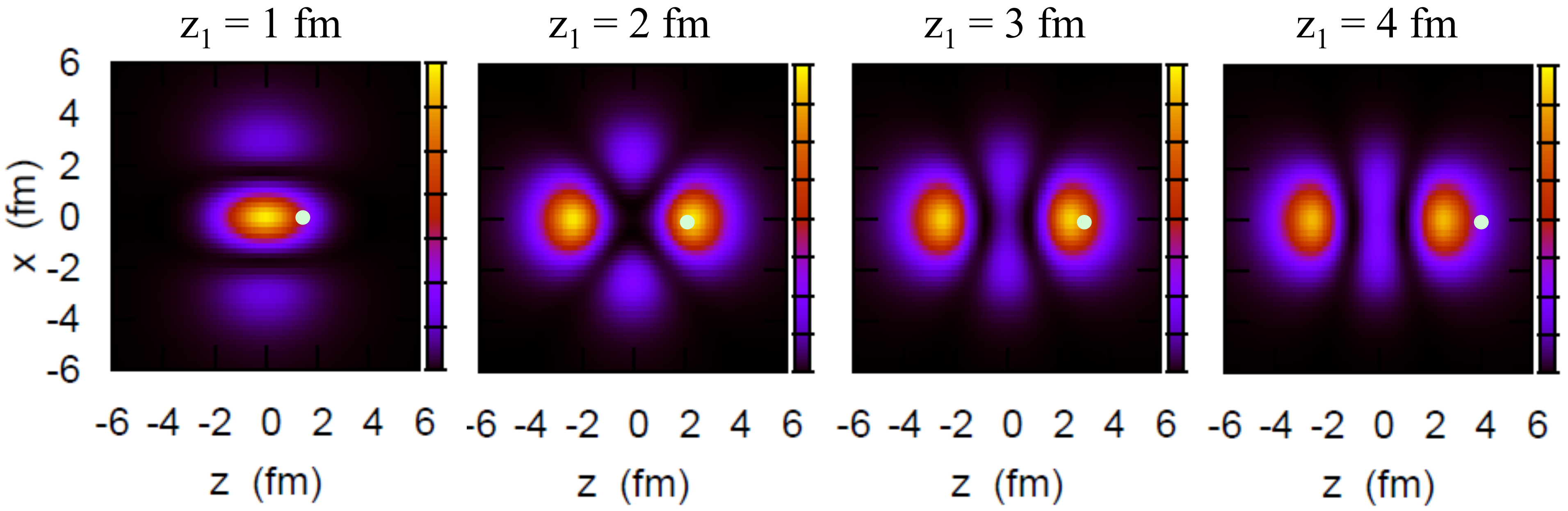
Correlation: $\langle AB \rangle \neq \langle A \rangle \langle B \rangle$

Example: $^{18}O = ^{16}O + n + n$

cf. $^{16}O + n$: 3 bound states ($1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$)

ii) nn interaction: works only on the positive parity (bound) states

$$|nn\rangle = \alpha |(1d_{5/2})^2\rangle + \beta |(2s_{1/2})^2\rangle + \gamma |(1d_{3/2})^2\rangle$$



✓distribution changes according to the 1st neutron (nn correlation)

✓but, the distribution of the 2nd neutron has peaks both at z_1 and $-z_1$

⟶ this is NOT called the di-neutron correlation

What is Di-neutron correlation?

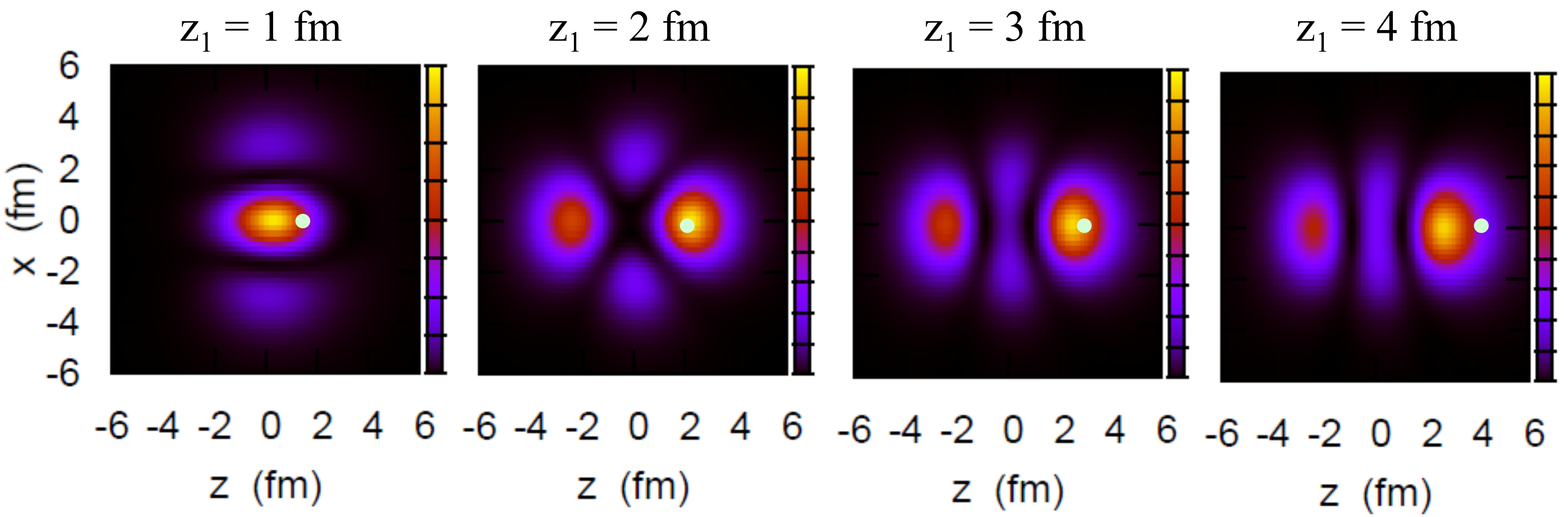
Correlation: $\langle AB \rangle \neq \langle A \rangle \langle B \rangle$

Example: $^{18}O = {}^{16}O + n + n$

cf. $^{16}O + n$: 3 bound states ($1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$)

iii) nn interaction: works also on the continuum states

$$|nn\rangle = \sum_{n,n',j,l} C_{nn'jl} |(nn'jl)^2\rangle$$

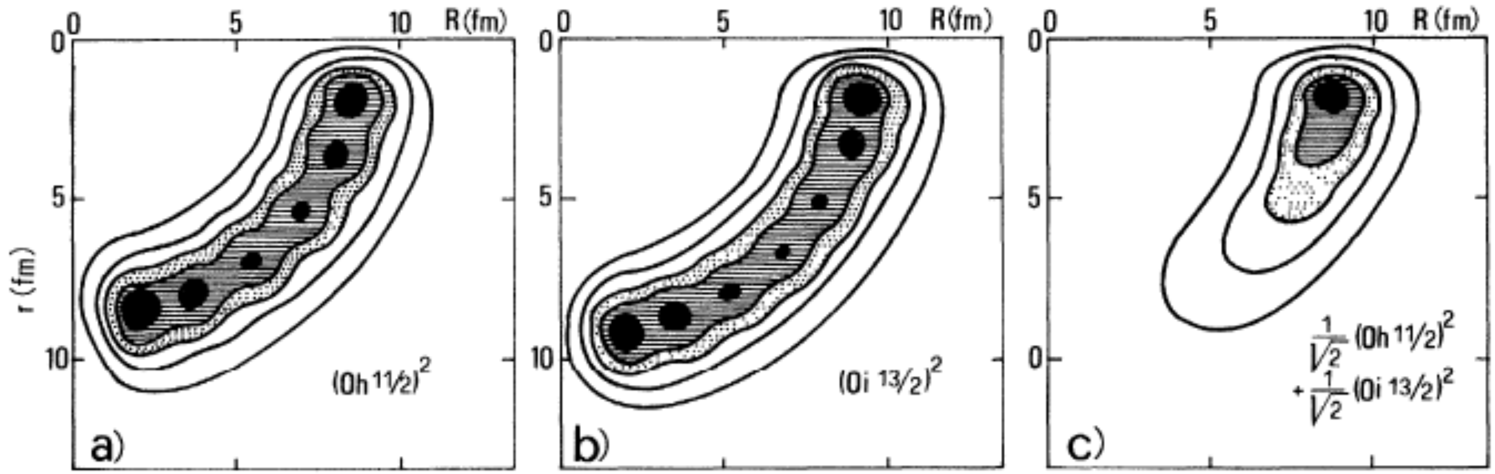


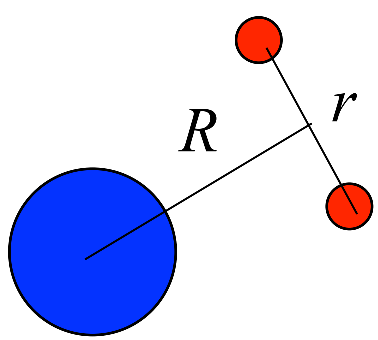
✓spatial correlation: the density of the 2nd neutron localized close to the 1st neutron (dineutron correlation)

✓parity mixing: essential role

cf. F. Catara et al., PRC29('84)1091

dineutron correlation: caused by the admixture of different parity states





F. Catara, A. Insolia, E. Maglione, and A. Vitturi, PRC29('84)1091

One dimensional 3-body model

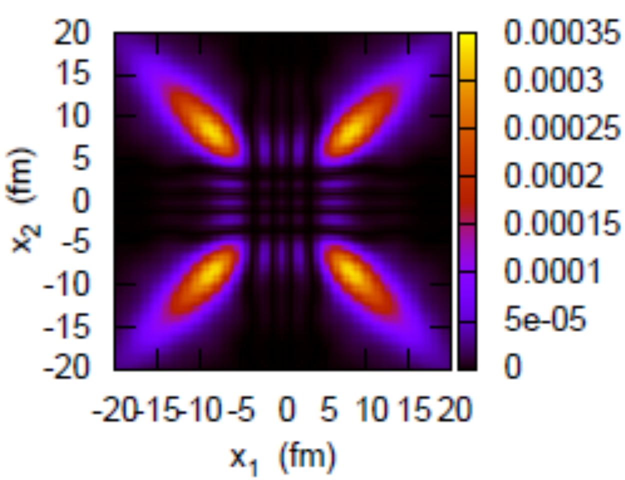
K.H., A. Vitturi, F. Perez-Bernal, and H. Sagawa, J. of Phys. G38 ('11) 015015

$$\Psi_{\rm gs}(x_1,x_2) = \Psi_{\rm ee}(x_1,x_2) + \Psi_{\rm oo}(x_1,x_2)$$

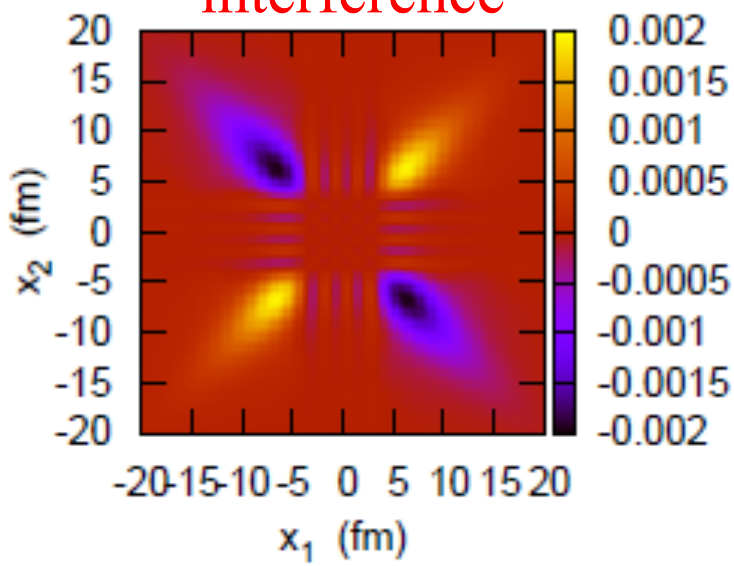
$$\longrightarrow \quad \rho_2(x_1,x_2) = |\Psi_{\rm ee}(x_1,x_2)|^2 + |\Psi_{\rm oo}(x_1,x_2)|^2 + 2\Psi_{\rm ee}(x_1,x_2)\Psi_{\rm oo}(x_1,x_2)$$

$|\Psi_{\rm ee}(x_1,x_2)|^2$

$|\Psi_{\rm oo}(x_1,x_2)|^2$



interference



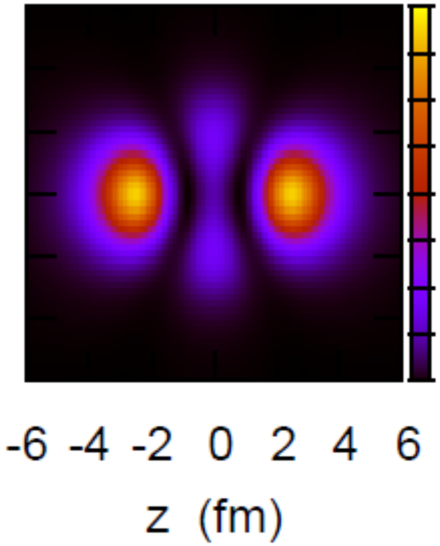
$$\Psi_{\rm ee}(x_1,x_2)\Psi_{\rm oo}(x_1,x_2) = -\Psi_{\rm ee}(x_1,-x_2)\Psi_{\rm oo}(x_1,-x_2)$$

spatial localization of two neutrons
(dineutron correlation)

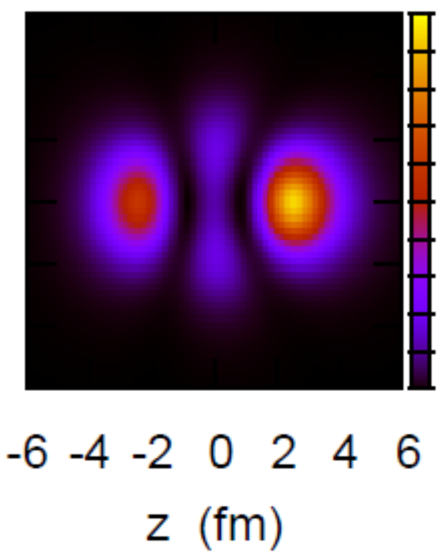
cf. Migdal, Soviet J. of Nucl. Phys. 16 ('73) 238
Bertsch, Broglia, Riedel, NPA91('67)123

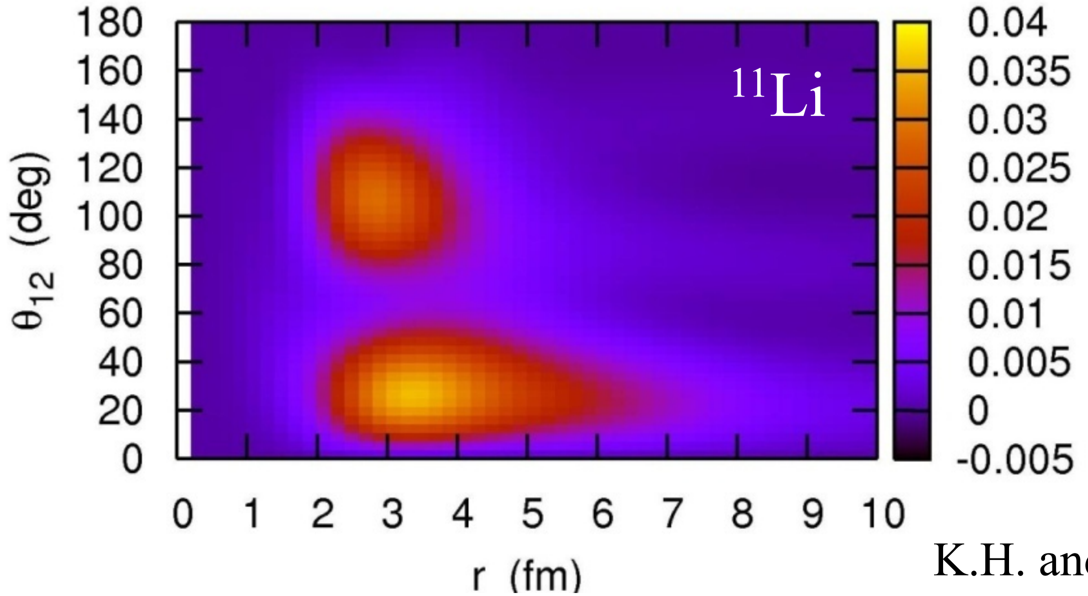
weakly bound systems

→easy to mix different parity states due to the continuum couplings
+ enhancement of pairing on the surface



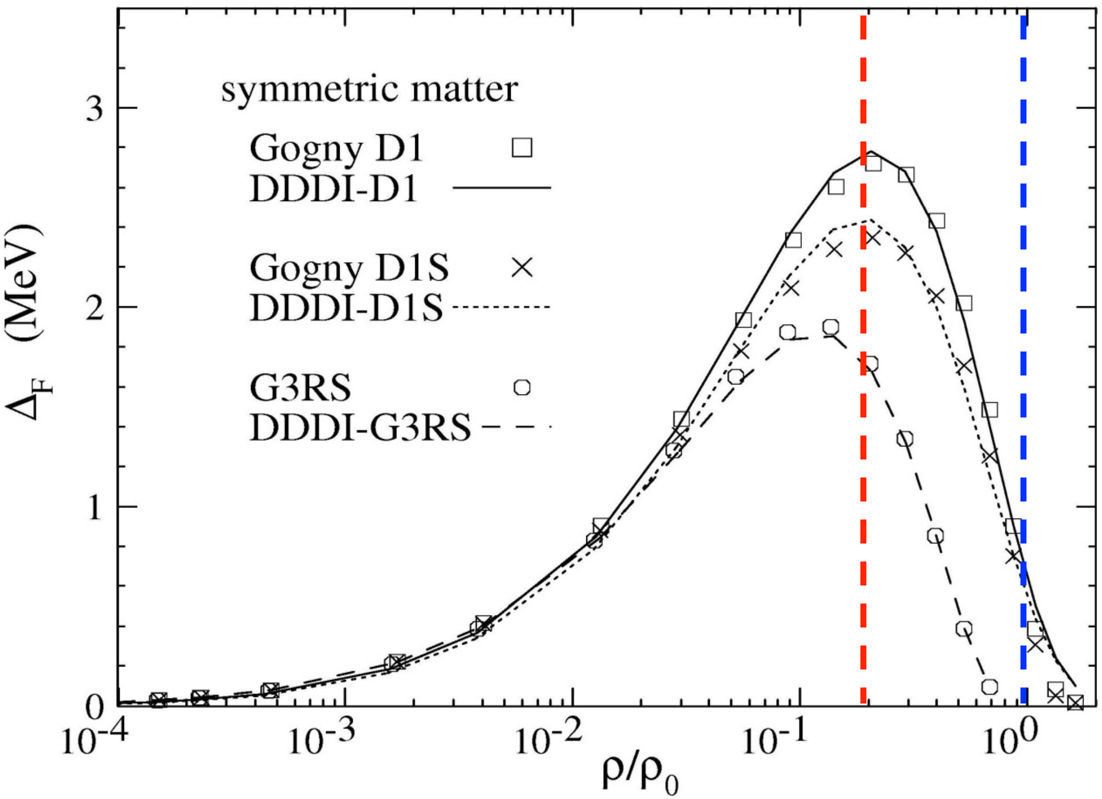
parity mixing





K.H. and H. Sagawa, PRC72('05)044321

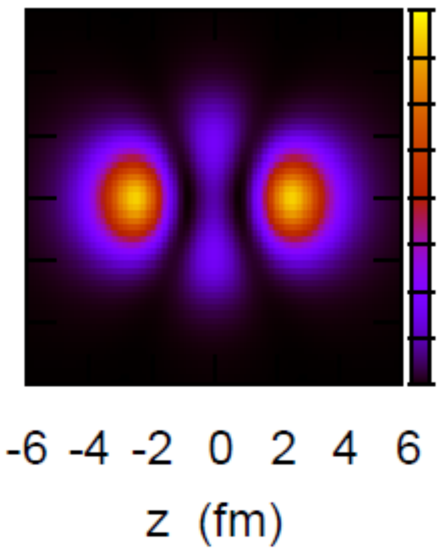
pairing gap in infinite nuclear matter



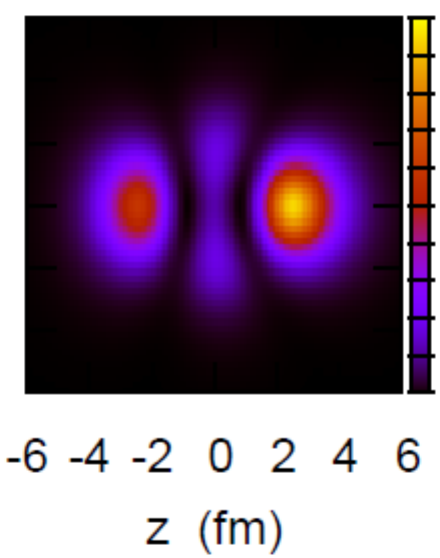
M. Matsuo, PRC73('06)044309

spatial localization of two neutrons
(dineutron correlation)

cf. Migdal, Soviet J. of Nucl. Phys. 16 (‘73) 238
Bertsch, Broglia, Riedel, NPA91(‘67)123



parity mixing



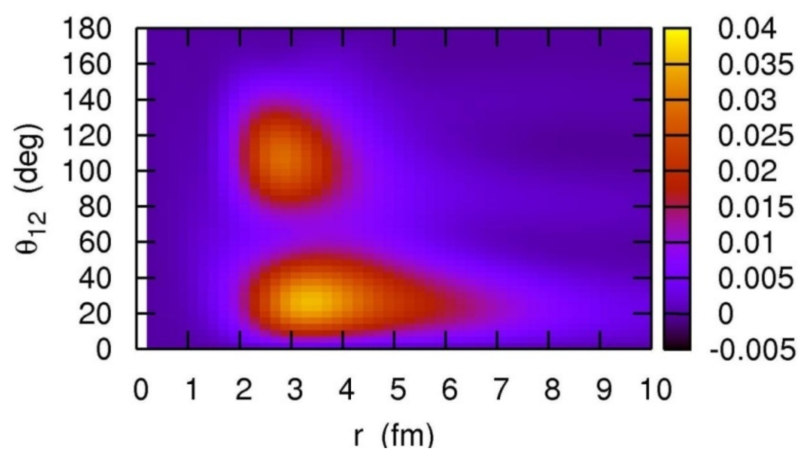
weakly bound systems

→easy to mix different parity states due to the continuum couplings

\+ enhancement of pairing on the surface

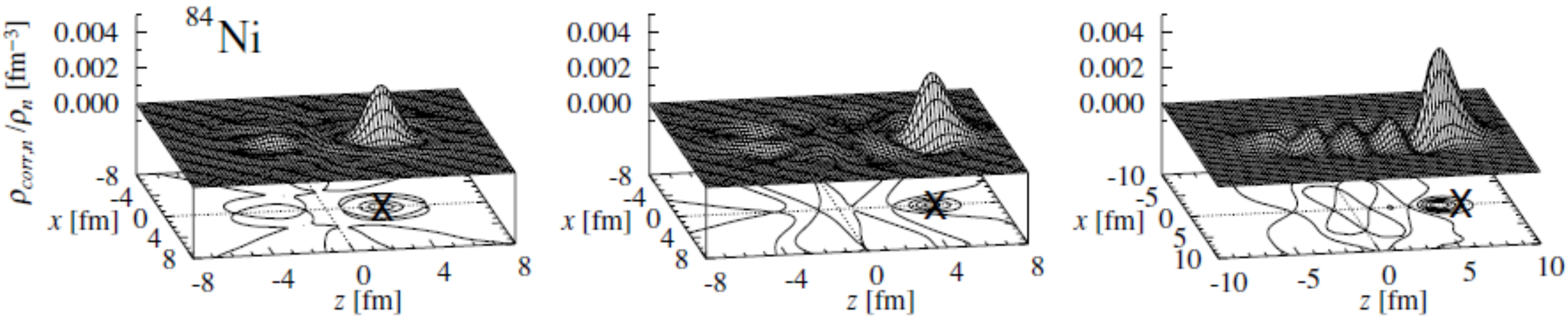
→ dineutron correlation: enhanced

cf. - Bertsch, Esbensen, Ann. of Phys. 209(’91)327
- M. Matsuo, K. Mizuyama, Y. Serizawa, PRC71(‘05)064326

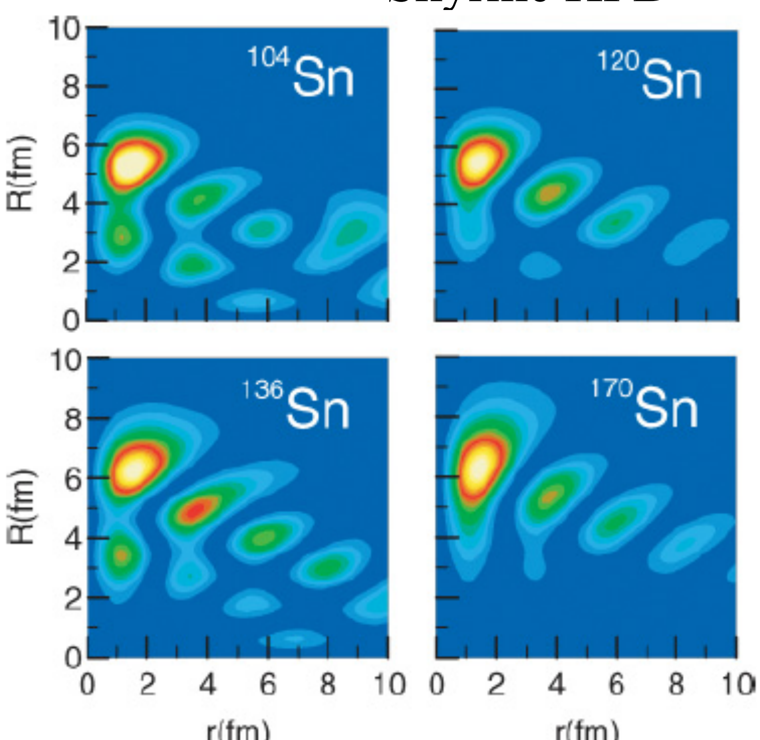


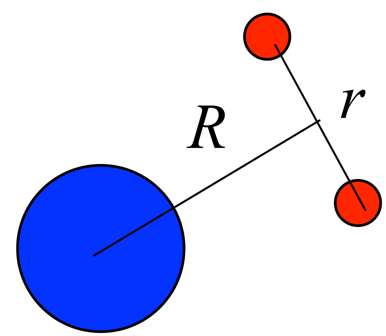
K.H. and H. Sagawa, PRC72(’05)044321

dineutron correlation in heavy neutron-rich nuclei



M. Matsuo, K. Mizuyama, and Y. Serizawa, PRC71(‘05)064326
Skyrme HFB

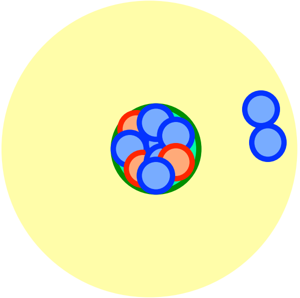




N. Pillet, N. Sandulescu, and P. Schuck,
PRC76(‘07)024310
Gogny HFB

Dineutron correlation in the momentum space

$$\boxed{\Psi(\boldsymbol{r},\boldsymbol{r}') = \alpha\,\Psi_{s^2}(\boldsymbol{r},\boldsymbol{r}') + \beta\,\Psi_{p^2}(\boldsymbol{r},\boldsymbol{r}')} \longrightarrow \theta_r = 0\text{: enhanced}$$

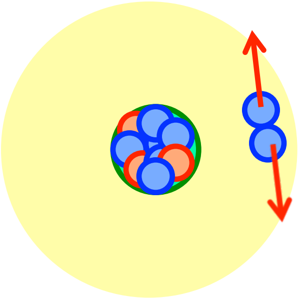


$\longrightarrow$ Fourier transform

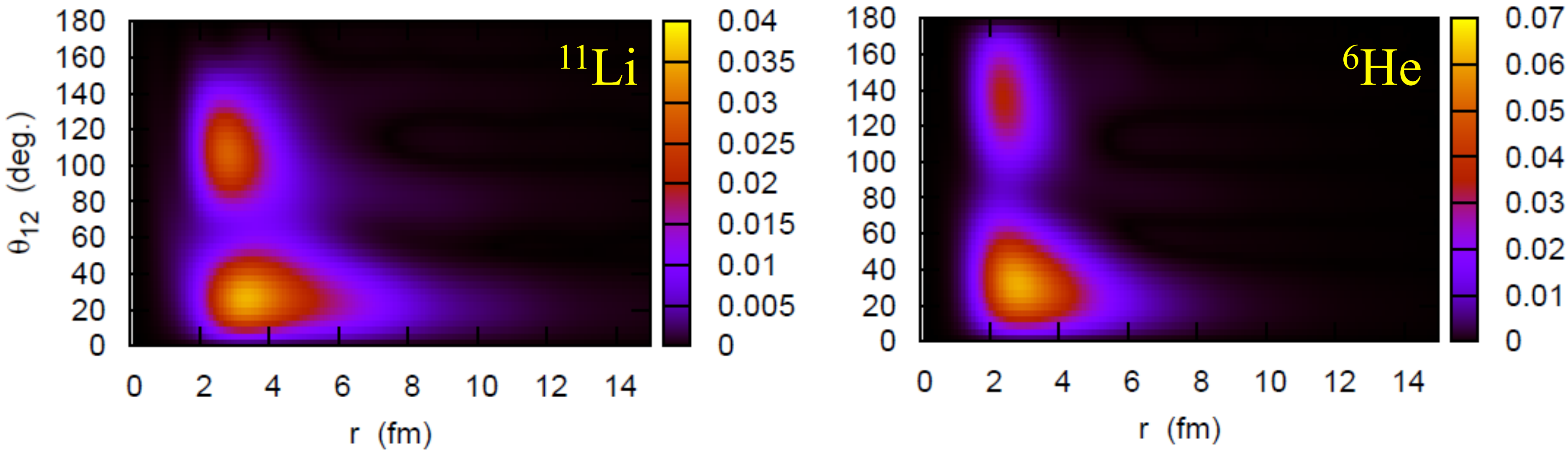
$$\tilde{\Psi}(\boldsymbol{k},\boldsymbol{k}') = \int e^{i\boldsymbol{k}\cdot\boldsymbol{r}} e^{i\boldsymbol{k}'\cdot\boldsymbol{r}'} \Psi(\boldsymbol{r},\boldsymbol{r}') d\boldsymbol{r} d\boldsymbol{r}'$$

$$e^{i\boldsymbol{k}\cdot\boldsymbol{r}} = \sum_l (2l+1) i^l \ldots\ldots \longrightarrow \underset{r}{i^l} \cdot \underset{r'}{i^l} = i^{2l} = (-)^l$$

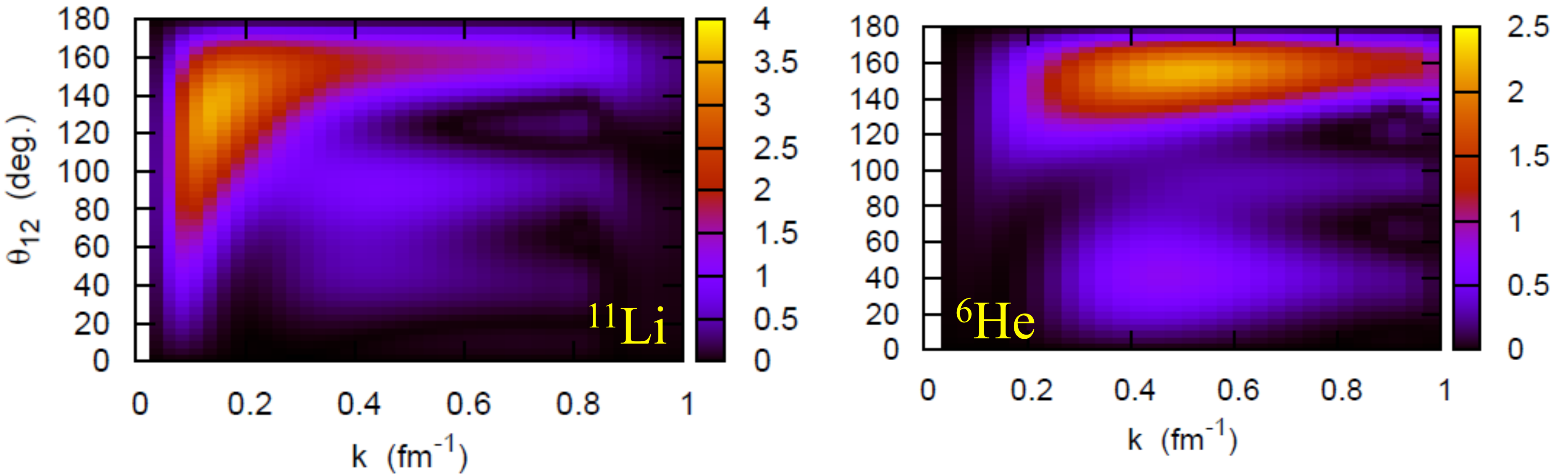
$$\boxed{\tilde{\Psi}(\boldsymbol{k},\boldsymbol{k}') = \alpha\,\tilde{\Psi}_{s^2}(\boldsymbol{k},\boldsymbol{k}') - \beta\,\tilde{\Psi}_{p^2}(\boldsymbol{k},\boldsymbol{k}')} \longrightarrow \theta_k = \pi\text{: enhanced}$$



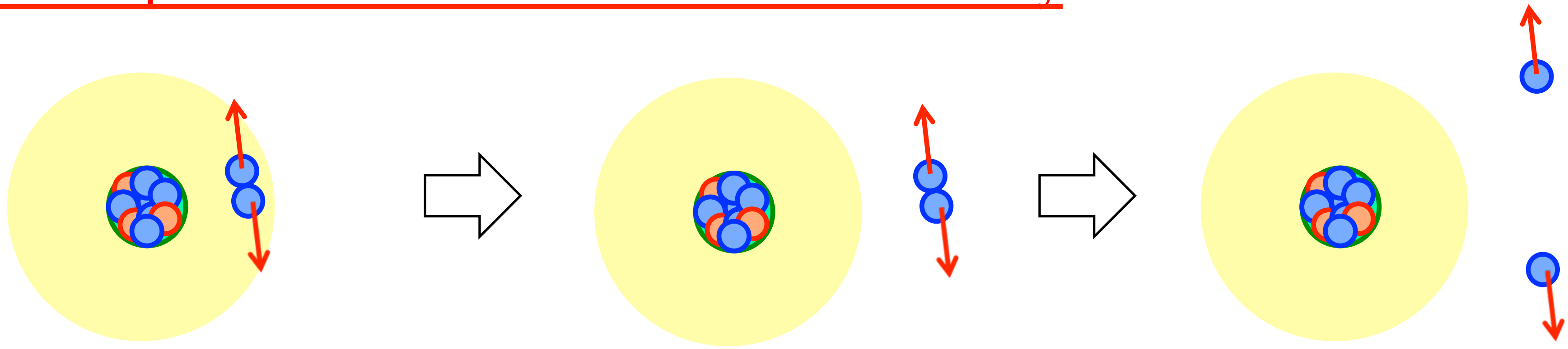
Two-particle density in the *r* space: $8\pi^2 r^4 \sin\theta \cdot \rho(r, r, \theta)$



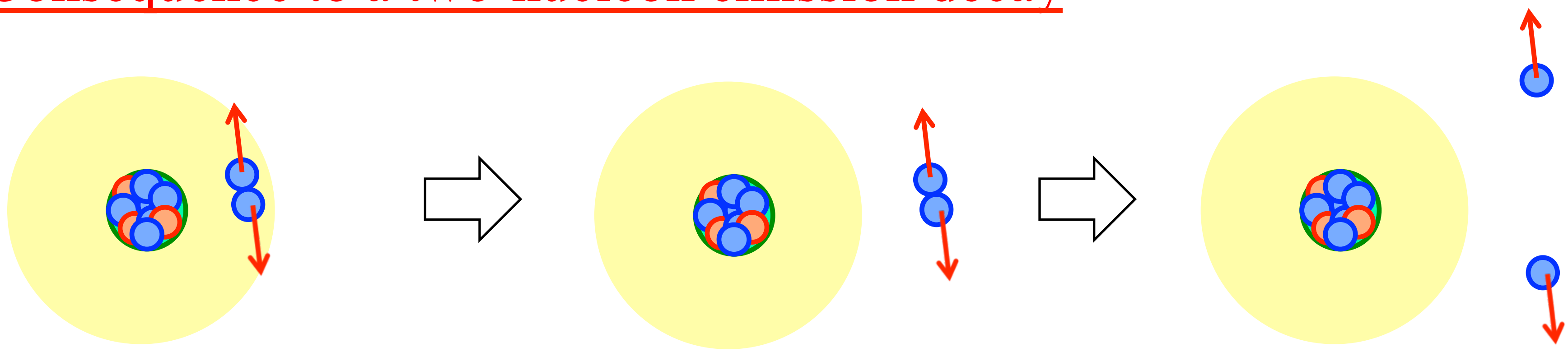
Two-particle density in the *p* space: $8\pi^2 k^4 \sin\theta \cdot \rho(k, k, \theta)$



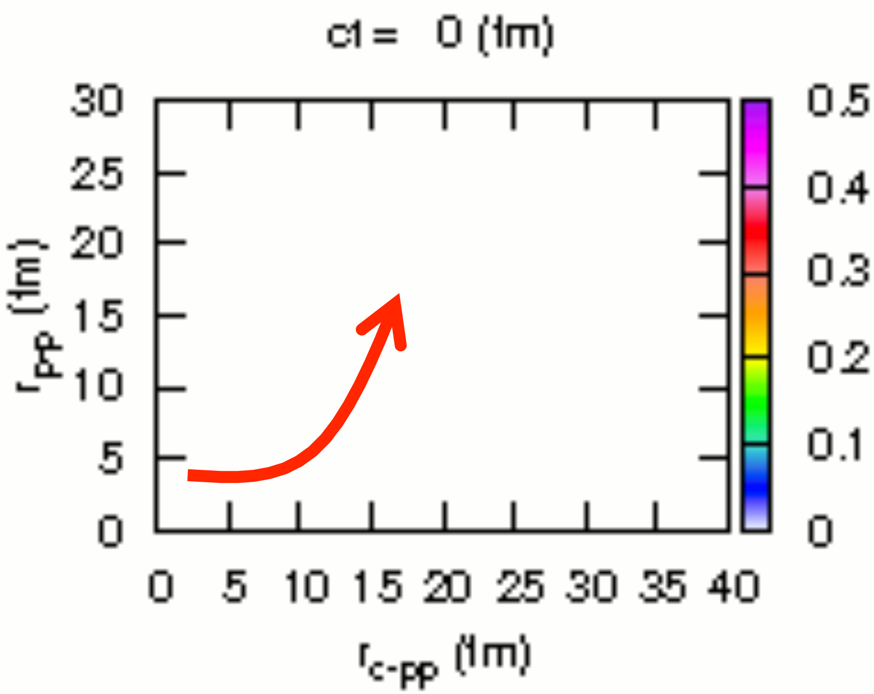
Consequence to a two-nucleon emission decay



Consequence to a two-nucleon emission decay



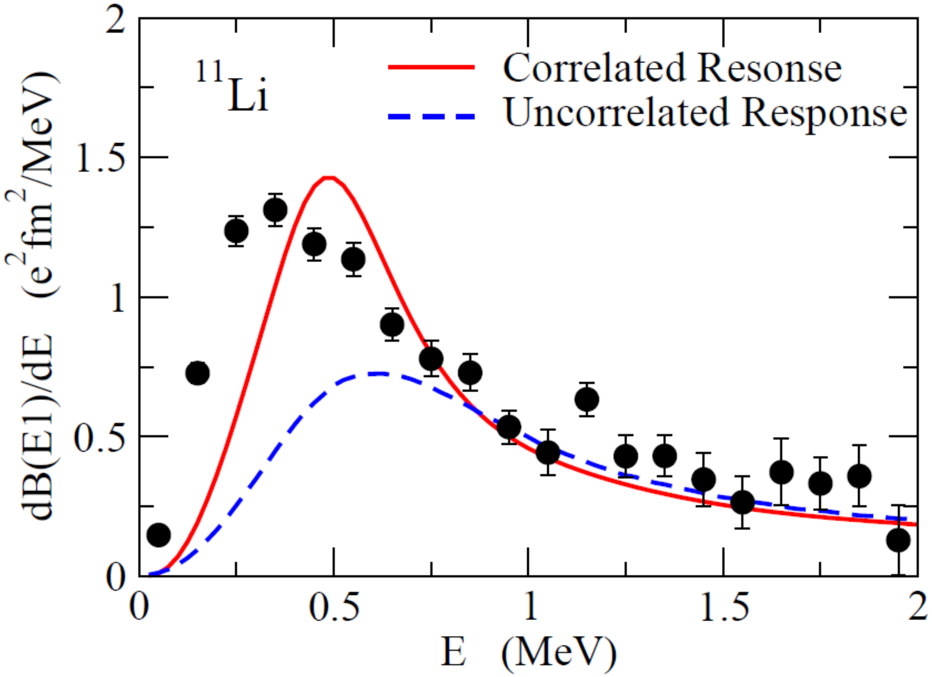
2p decay of ^{6}Be
: time-dependent calculations

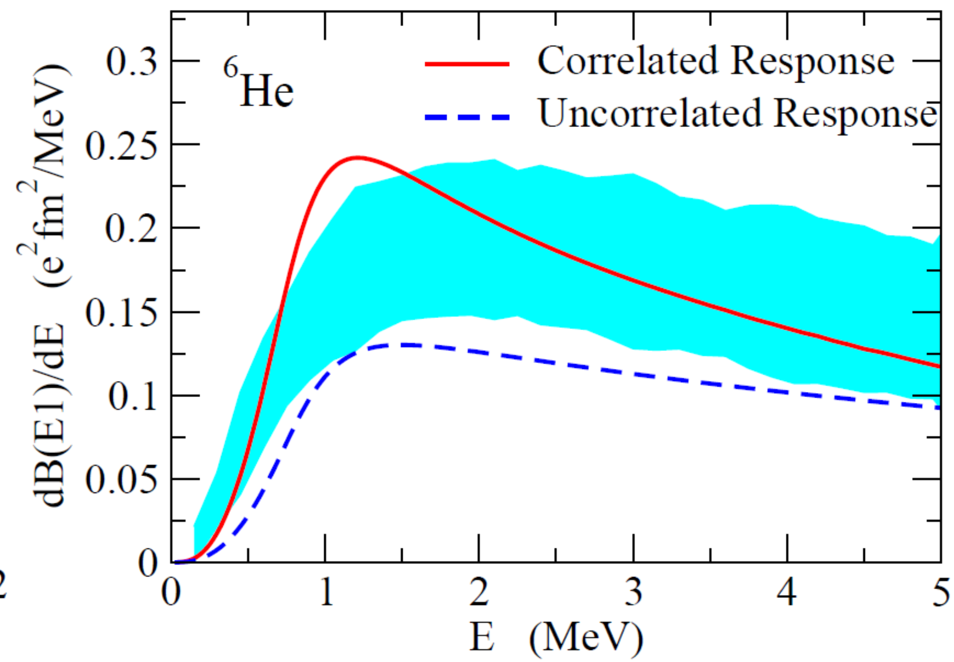


T. Oishi (Tohoku → Jyvaskyla),
K.H., H. Sagawa,
arXiv:1403.3019

Coulomb breakup of 2-neutron halo nuclei

How to probe the dineutron correlation? ⟶ Coulomb breakup





Experiments:

T. Nakamura et al., PRL96('06)252502

T. Aumann et al., PRC59('99)1252

3-body model calculations:

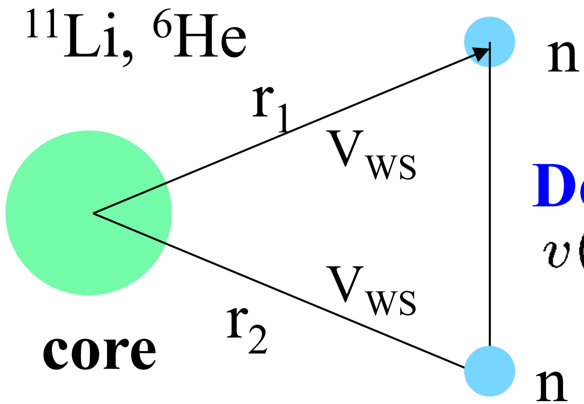
K.H., H. Sagawa, T. Nakamura, S. Shimoura, PRC80('09)031301(R)

cf. Y. Kikuchi et al., PRC87('13)034606 ⟵ structure of the core nucleus (^{9}Li)

also for ^{22}C, ^{14}Be, ^{19}B etc. (T. Nakamura et al.)

3-body model calculation for Borromean nuclei

G.F. Bertsch and H. Esbensen,
Ann. of Phys. 209('91)327; PRC56('99)3054



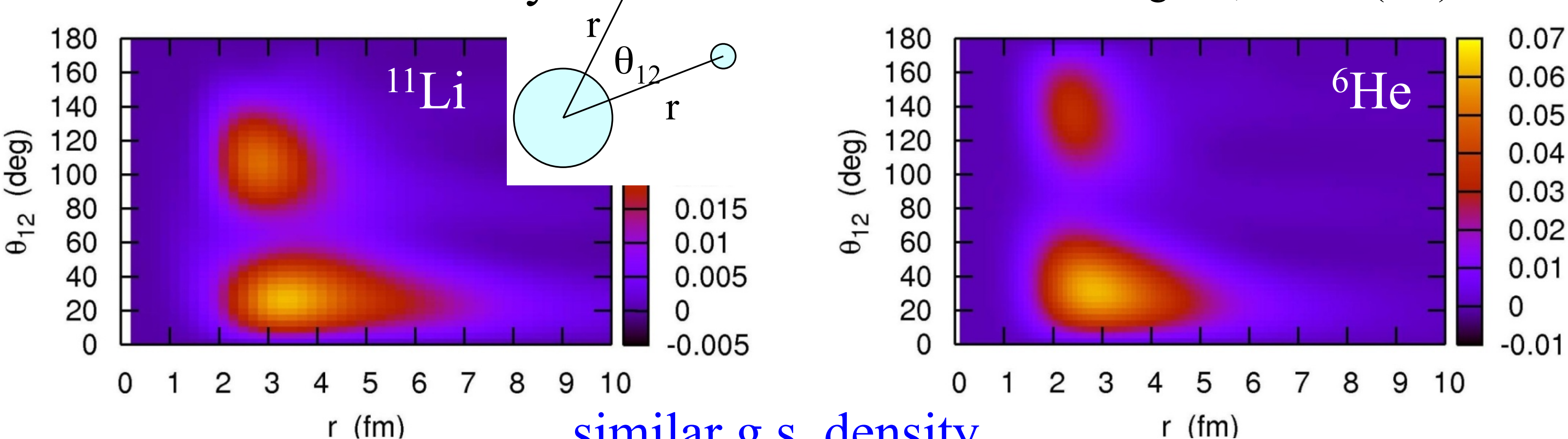
Density-dependent delta-force

$$v(\boldsymbol{r}_1, \boldsymbol{r}_2) = v_0(1 + \alpha\rho(r)) \times \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)$$

$$\boxed{H = \frac{\boldsymbol{p}_1^2}{2m} + \frac{\boldsymbol{p}_2^2}{2m} + V_{nC}(r_1) + V_{nC}(r_2) + V_{nn} + \frac{(\boldsymbol{p}_1 + \boldsymbol{p}_2)^2}{2A_c m}}$$

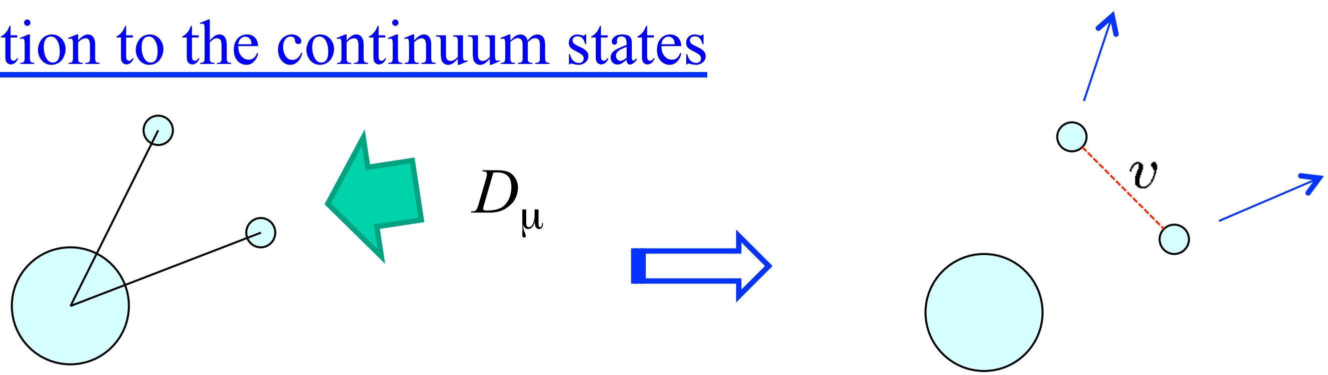
Ground state density

K.H. and H. Sagawa, PRC72('05)044321



similar g.s. density

E1 excitation to the continuum states



$$M(E1) = \langle (j_1 j_2)^1_\mu | (1 - vG_0 + vG_0 vG_0 - \cdots) D_\mu | \Psi_{gs} \rangle$$

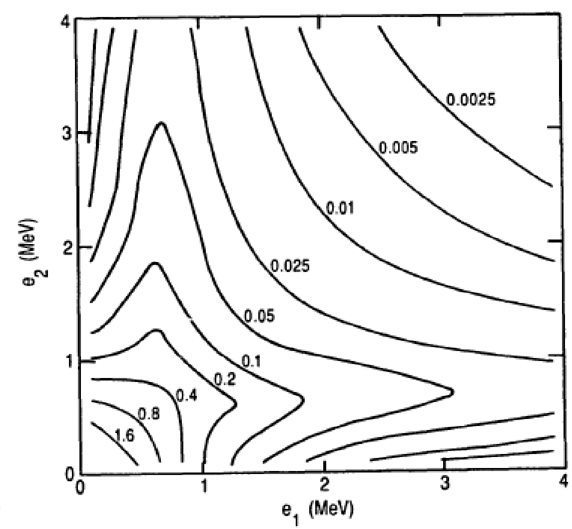
$$= \langle (j_1 j_2)^1_\mu | \underbrace{(1 + vG_0)^{-1}}_{\text{FSI}} D_\mu | \Psi_{gs} \rangle$$

dipole operator

unperturbed continuum wf

$$G_0(E) = \sum_{\mu, f.st.} \frac{\left| (j_1 j_2)^1_\mu \right\rangle \left\langle (j_1 j_2)^1_\mu \right|}{e_1 + e_2 - E - i\eta}$$

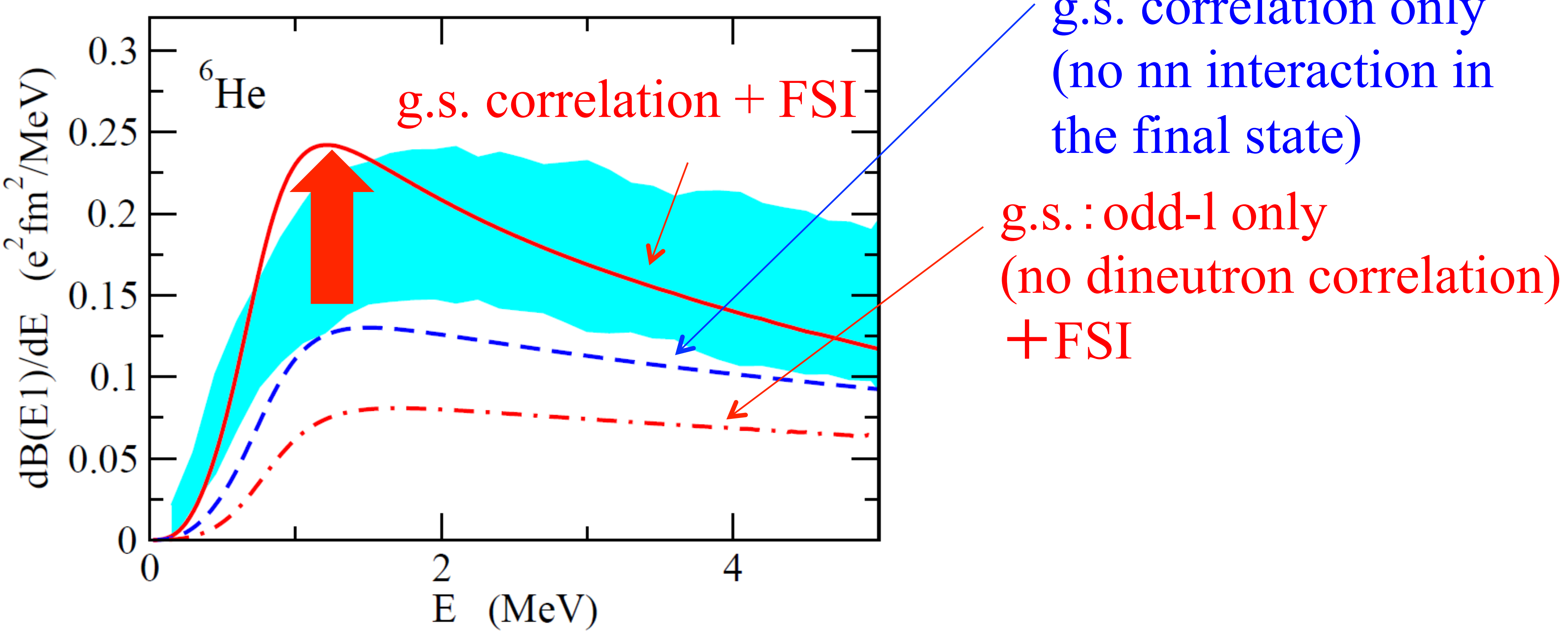
$$\boxed{\frac{d^2 B(E1)}{de_1 de_2} = 3 \sum_{l_1 j_2 l_2 j_2} |M(E1)|^2 \frac{dk_1}{de_1} \frac{dk_2}{de_2}}$$



H. Esbensen and G.F. Bertsch, NPA542('92)310

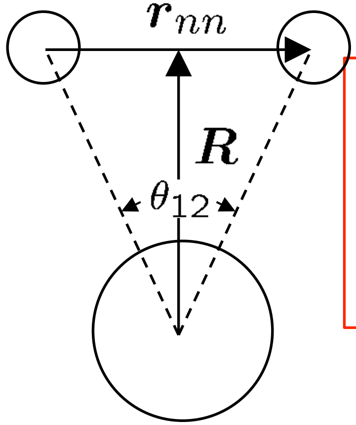
g.s. correlation? or correlation in excited states?

$^6He(0^+) \rightarrow {}^6He(1^-) \rightarrow \alpha + n + n$



✓Both FSI and dineutron correlations: important role in E1 strength

Geometry of Borromean nuclei

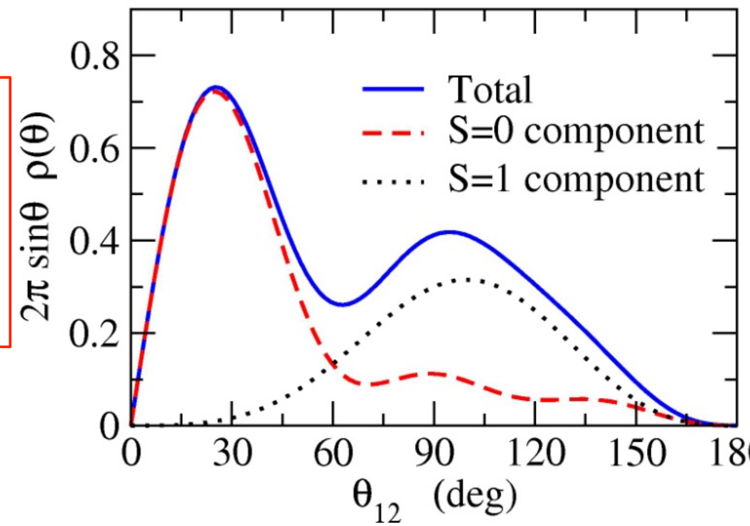


Cluster sum rule

$$B_{\rm tot}(E1) = \sum_f |\langle \Psi_f | \hat{T}_{\rm E1} | \Psi_0 \rangle|^2 \sim \frac{3}{\pi}\left(\frac{Z_c e}{A_c + 2}\right)^2 \langle R^2 \rangle$$

reflects the g.s. correlation

3-body model calculations



$$\langle \theta_{12} \rangle = 65.29 \quad \text{deg.}$$

"experimental data" for opening angle

$\sqrt{\langle R^2 \rangle}$ ← $B_{tot}(E1)$

$\sqrt{\langle r_{\rm nn}^2 \rangle}$ ← matter radius or HBT

$$\boxed{\begin{aligned} \langle \theta_{12} \rangle &= 65.2 \pm 12.2 \quad (^{11}\text{Li}) \\ &= 74.5 \pm 12.1 \quad (^{6}\text{He}) \end{aligned}}$$

K.H. and H. Sagawa,PRC76('07)047302

$\langle \theta_{12} \rangle$: significantly smaller than 90 deg.

↓

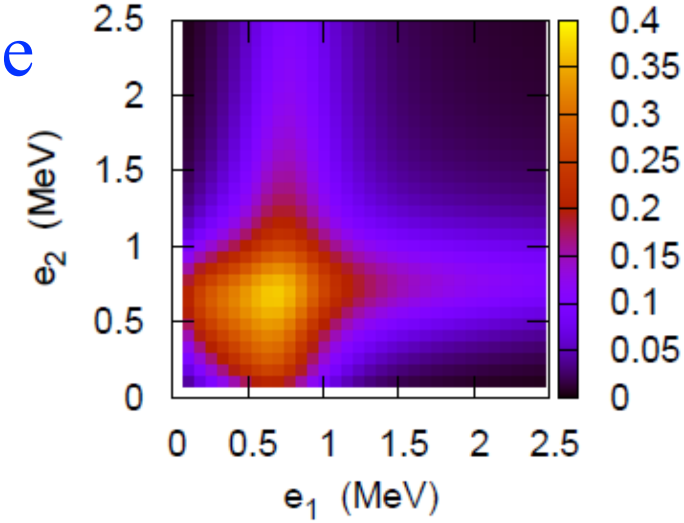
suggests dineutron corr.

(but, an average of small and large angles)

cf. T. Nakamura et al., PRL96('06)252502
C.A. Bertulani and M.S. Hussein, PRC76('07)051602

Energy distribution of emitted neutrons

^{6}He

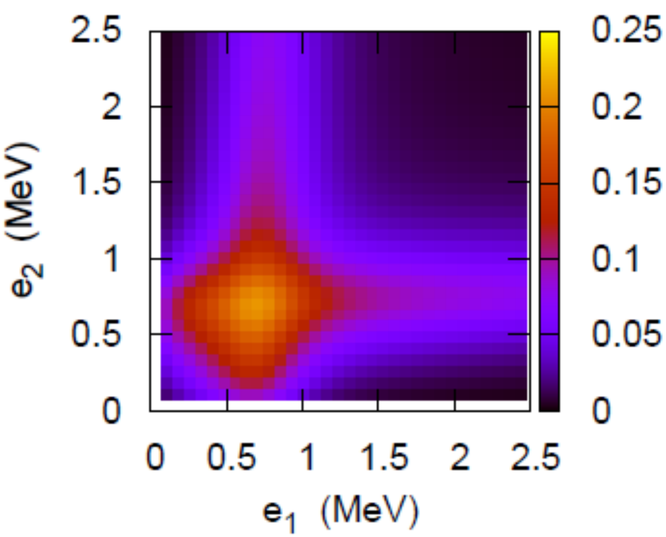


✓shape of distribution: insensitive to the nn-interaction (except for the absolute value)

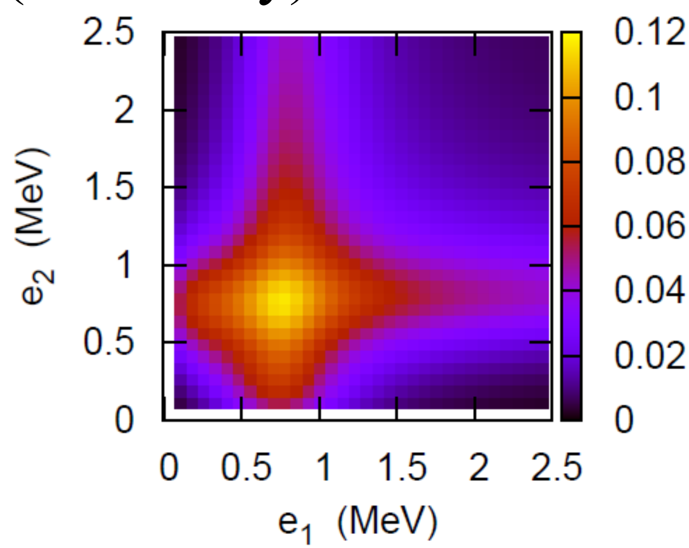
✓strong sensitivity to V_{nC}

✓similar situation in between ^{11}Li and ^{6}He

$v_{nn} = 0$

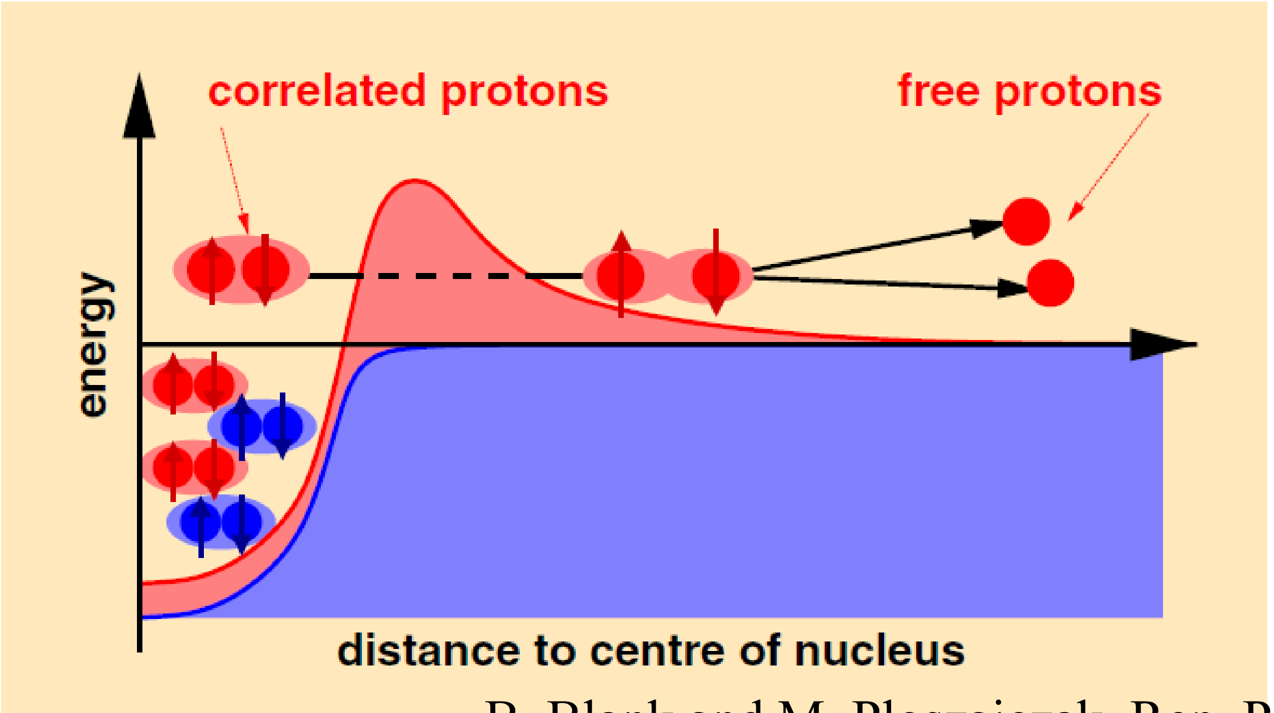


no di-neutron corr. in the g.s. (odd-l only)



K.H., H. Sagawa, T. Nakamura, S. Shimoura, PRC80(‘09)031301(R)

2-proton radio activity

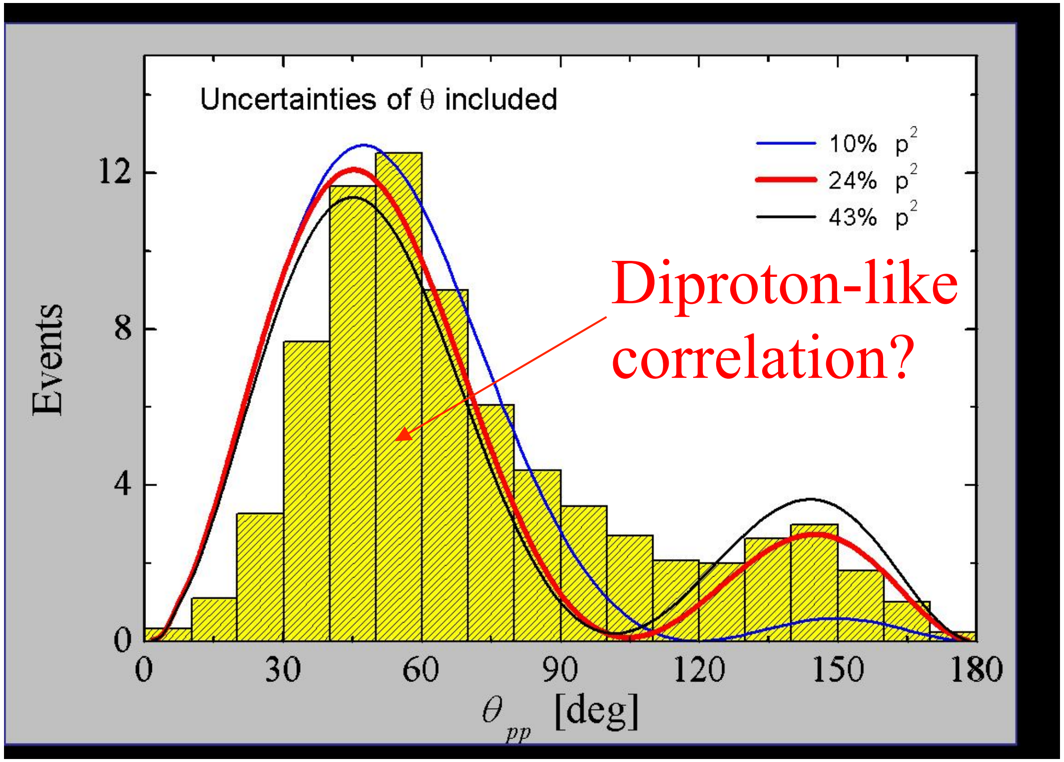


B. Blank and M. Ploszajczak, Rep. Prog. Phys. 71('08)046301

✓ probing correlations from energy and angle distributions of two emitted protons?

✓ Coulomb 3-body system
  - Theoretical treatment: difficult
  - how does FSI disturb the g.s. correlation?

2-proton decay of ^{45}Fe

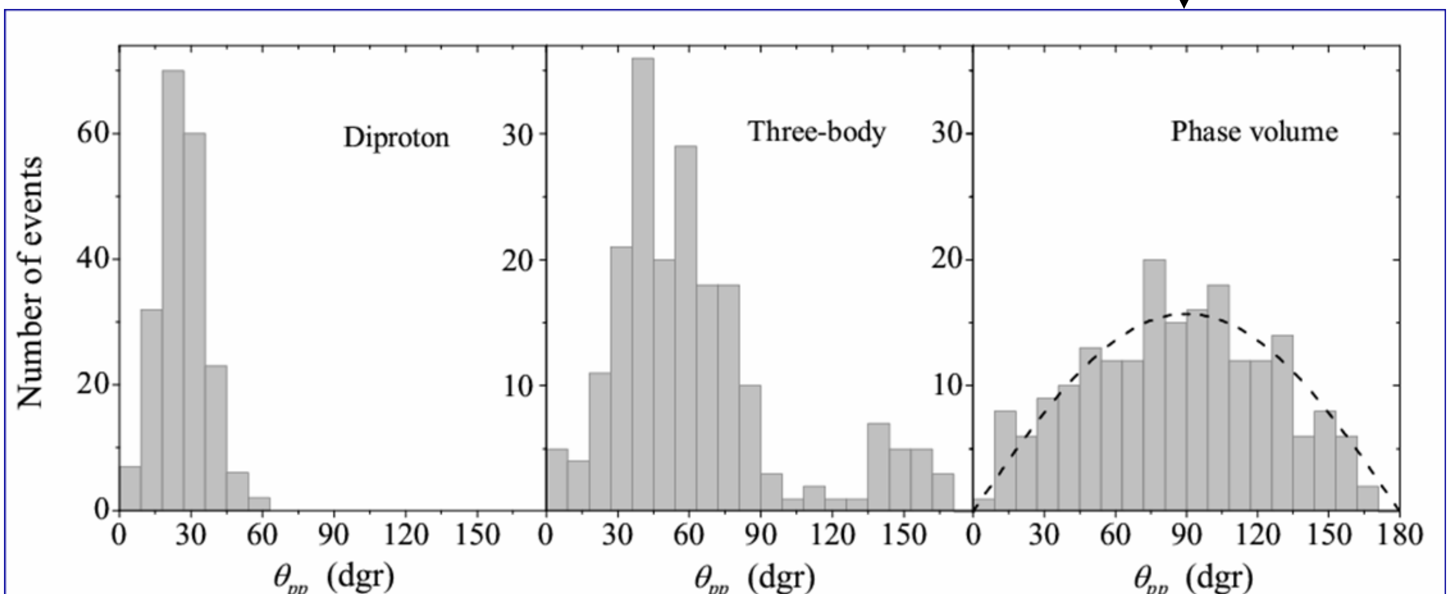


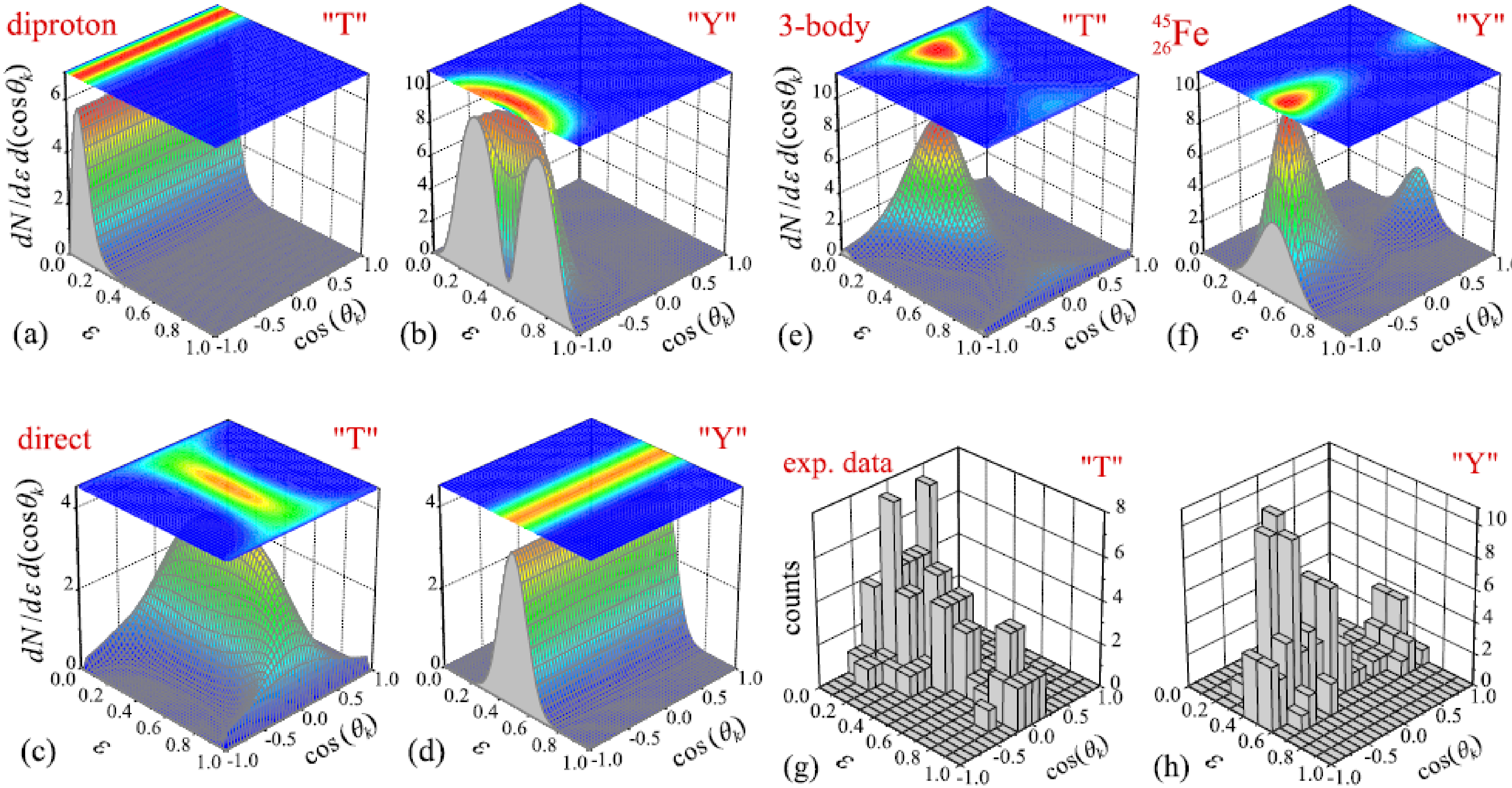
K. Miernik et al., PRL99 ('07) 192501

experimental data

calculations (Grigorenko)

RMP 84 ('12) 567



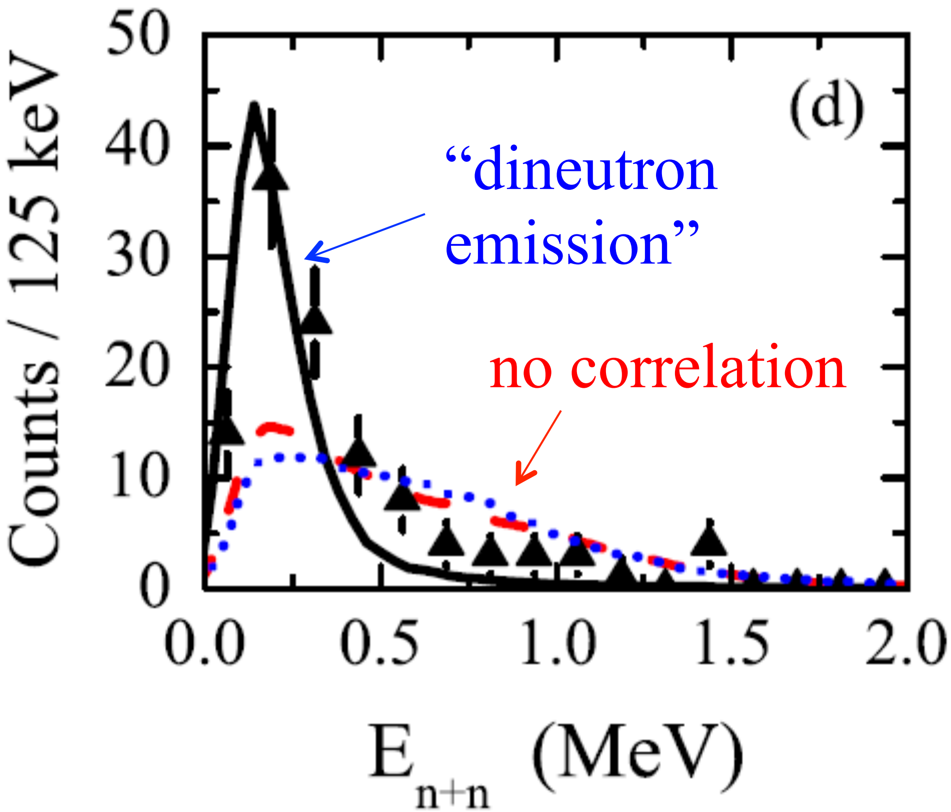


M. Pfutzner, M. Karny, L.V. Grigorenko, K. Riisager, Rev. Mod. Phys. 84 ('12) 567

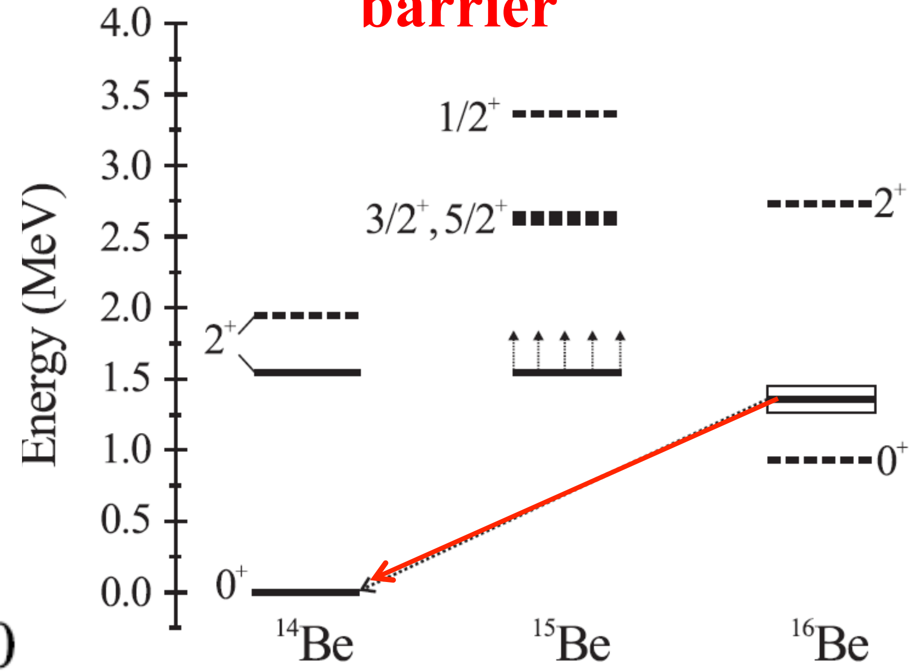
→ diproton correlation: unclear in many other systems (theoretical calculations: not many)

2-neutron decay（MoNA@MSU）

$^{17}B \rightarrow {}^{16}Be \rightarrow {}^{14}Be + n + n$



3-body resonance due to the **centrifugal barrier**



A. Spyrou et al., PRL108(‘12) 102501

Other data:

^{13}Li (Z. Kohley et al., PRC87(‘13)011304(R)) $^{14}Be \rightarrow {}^{13}Li \rightarrow {}^{11}Li + 2n$

^{26}O (E. Lunderbert et al., PRL108(‘12)142503) $^{27}F \rightarrow {}^{26}O \rightarrow {}^{24}O + 2n$

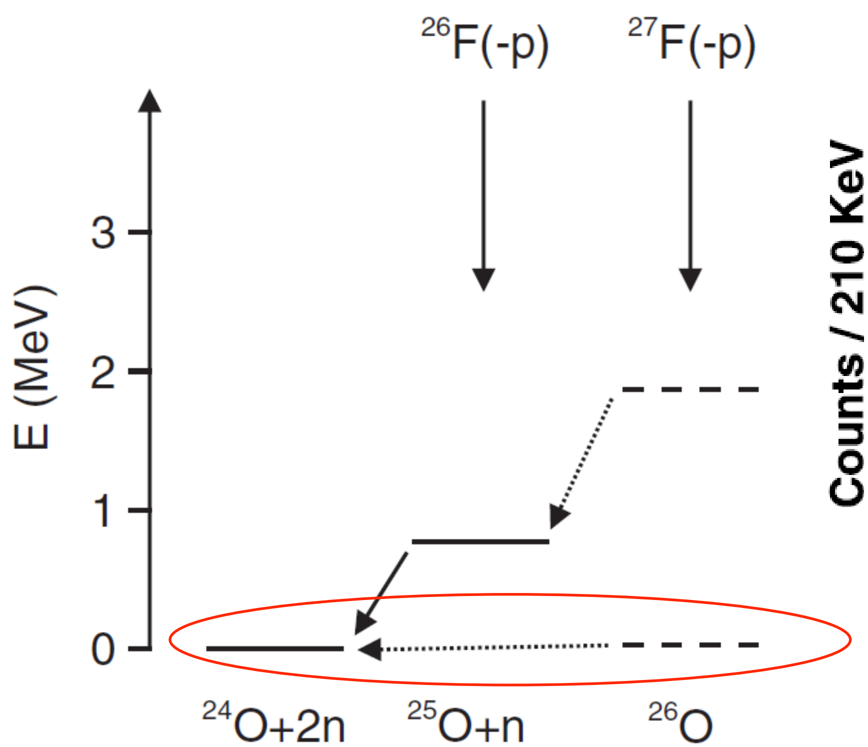
3-body model calculation with nn correlation: required

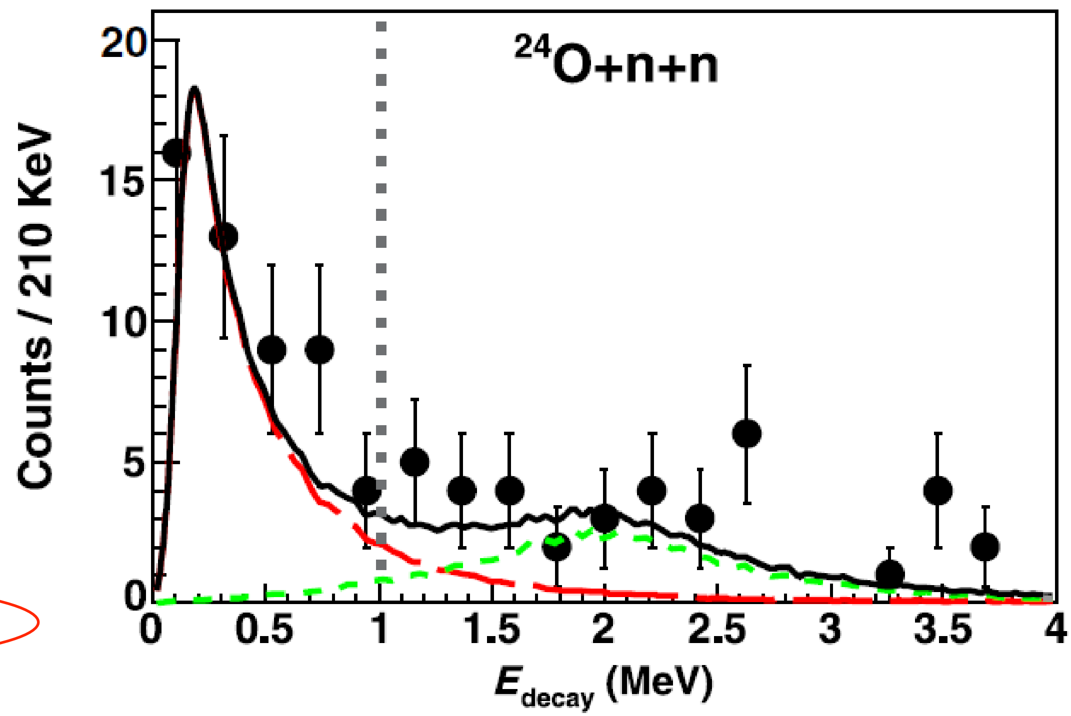
Two-neutron decay of ^{26}O

- the simplest among ^{16}Be, ^{13}Li, ^{26}O (MSU)
  ^{16}Be: deformation, ^{13}Li: treatment of ^{11}Li core

E. Lunderberg et al., PRL108 ('12) 142503
Z. Kohley et al., PRL 110 ('13)152501

^{27}F (82 MeV/u) + ^{9}Be → ^{26}O → ^{24}O + n + n

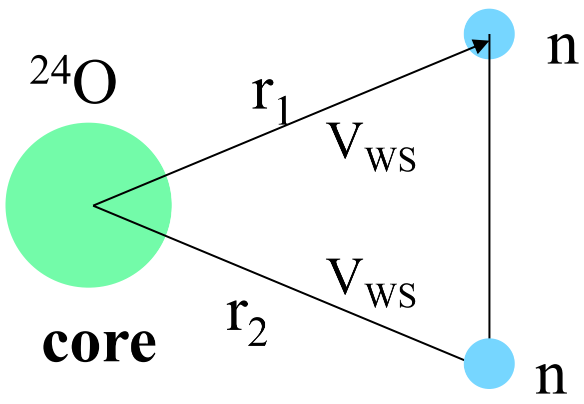




cf. C. Caesar et al., PRC88 ('13) 034313 (GSI exp.)
Y. Kondo et al., (SAMURAI)

$E_{decay} = 150\ ^{+50}_{-150}$ keV

i) Decay energy spectrum



➤ $^{24}O + n$ potential

Woods-Saxon potential to reproduce

$e_{2s1/2} = -4.09\ (13)$ MeV,

$e_{1d3/2} = +\ 770^{+20}_{-10}$ keV,

$\Gamma_{1d3/2} = 172(30)$ keV

➤ nn interaction

density-dep. contact interaction

E (^{27}F) = -2.69 MeV

$$\boxed{\frac{dP_I}{dE} = \sum_k |\langle\Psi_k^{(I)}|\Phi_{\rm ref}^{(I)}\rangle|^2\,\delta(E-E_k)}$$

$$= -\frac{1}{\pi}\Im\langle\Phi_{\rm ref}^{(I)}|G^{(I)}(E)|\Phi_{\rm ref}^{(I)}\rangle,$$

overlap with a ref. state $\longleftarrow$ 2n config. with $^{25}F + n + n$

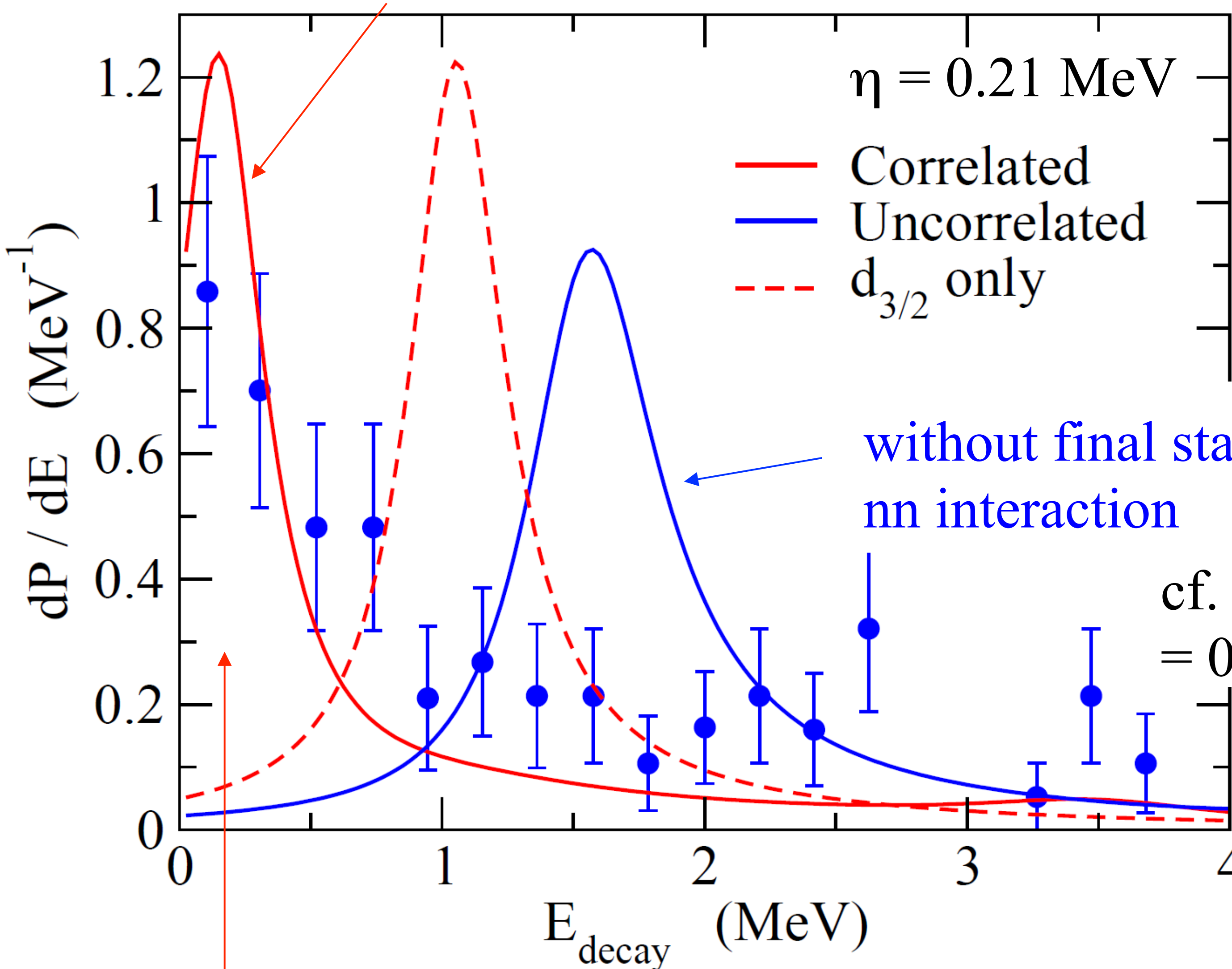
$$\boxed{G^{(I)}(E) = G_0^{(I)}(E) - G_0^{(I)}(E)v(1+G_0^{(I)}(E)v)^{-1}G_0^{(I)}(E)}$$

$\longleftarrow$ continuum effects

i) Decay energy spectrum

K.H. and H. Sagawa, PRC89 ('14) 014331

with final state nn interaction



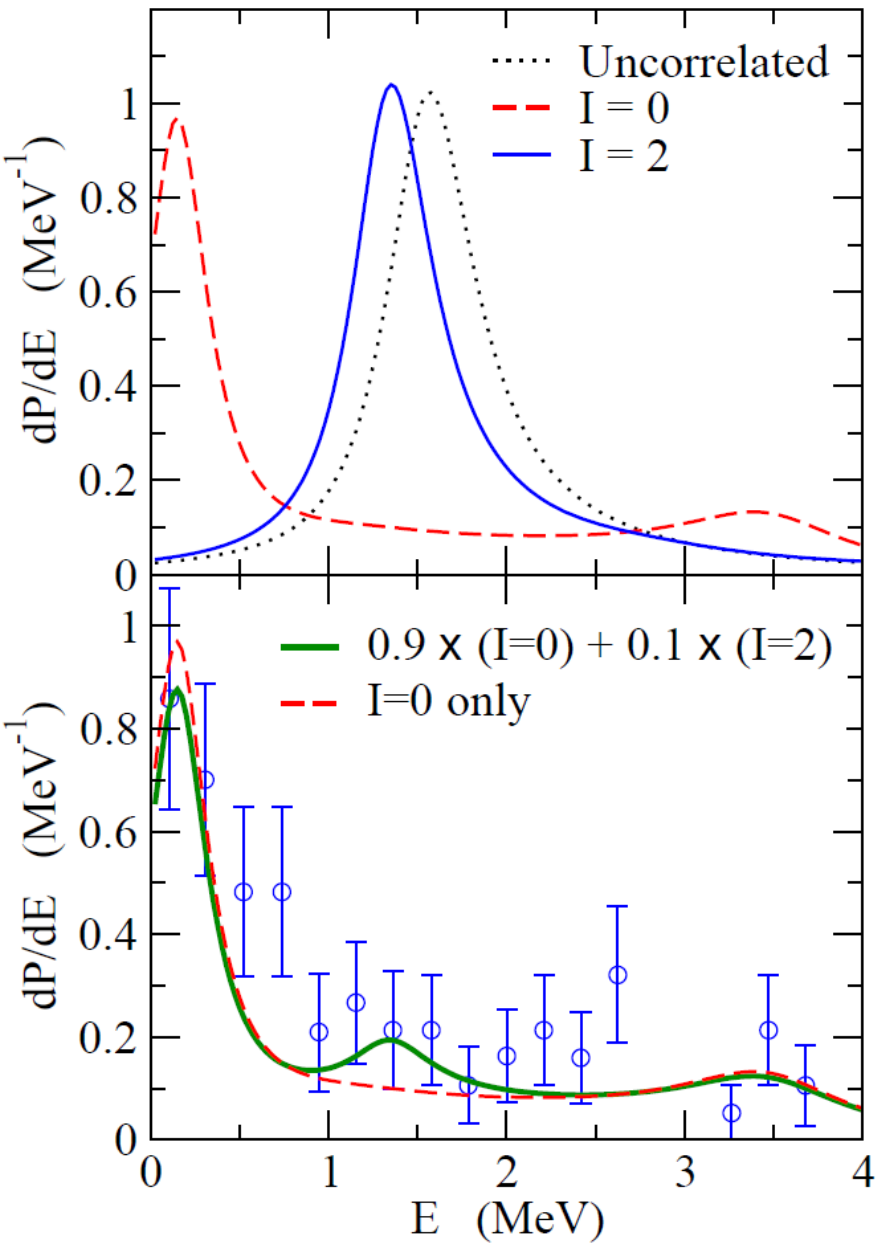
without final state nn interaction

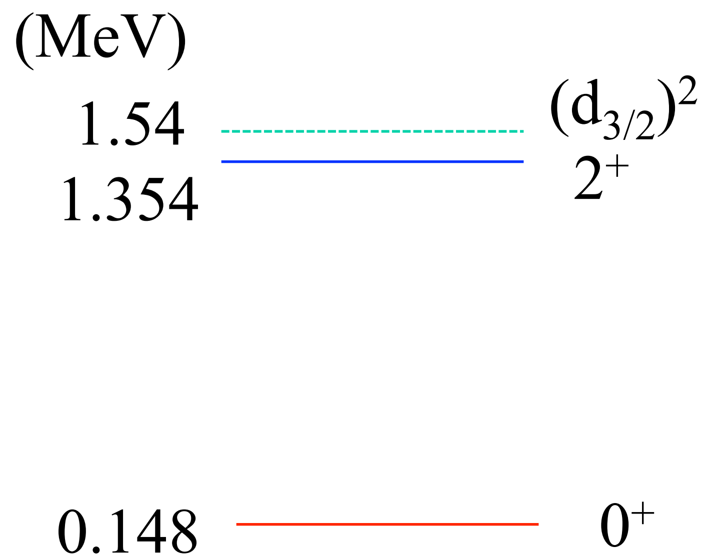
cf. $e_{1d3/2}(^{25}O)$ = 0.770 MeV

very narrow three-body resonance state ($\Gamma_{exp} \sim 10^{-10}$ MeV)

E_{peak} = 0.14 MeV with this setup for the Hamiltonian

2^+ state of ^{26}O   Kondo et al. : a prominent second peak at $E \sim 1.3$ MeV

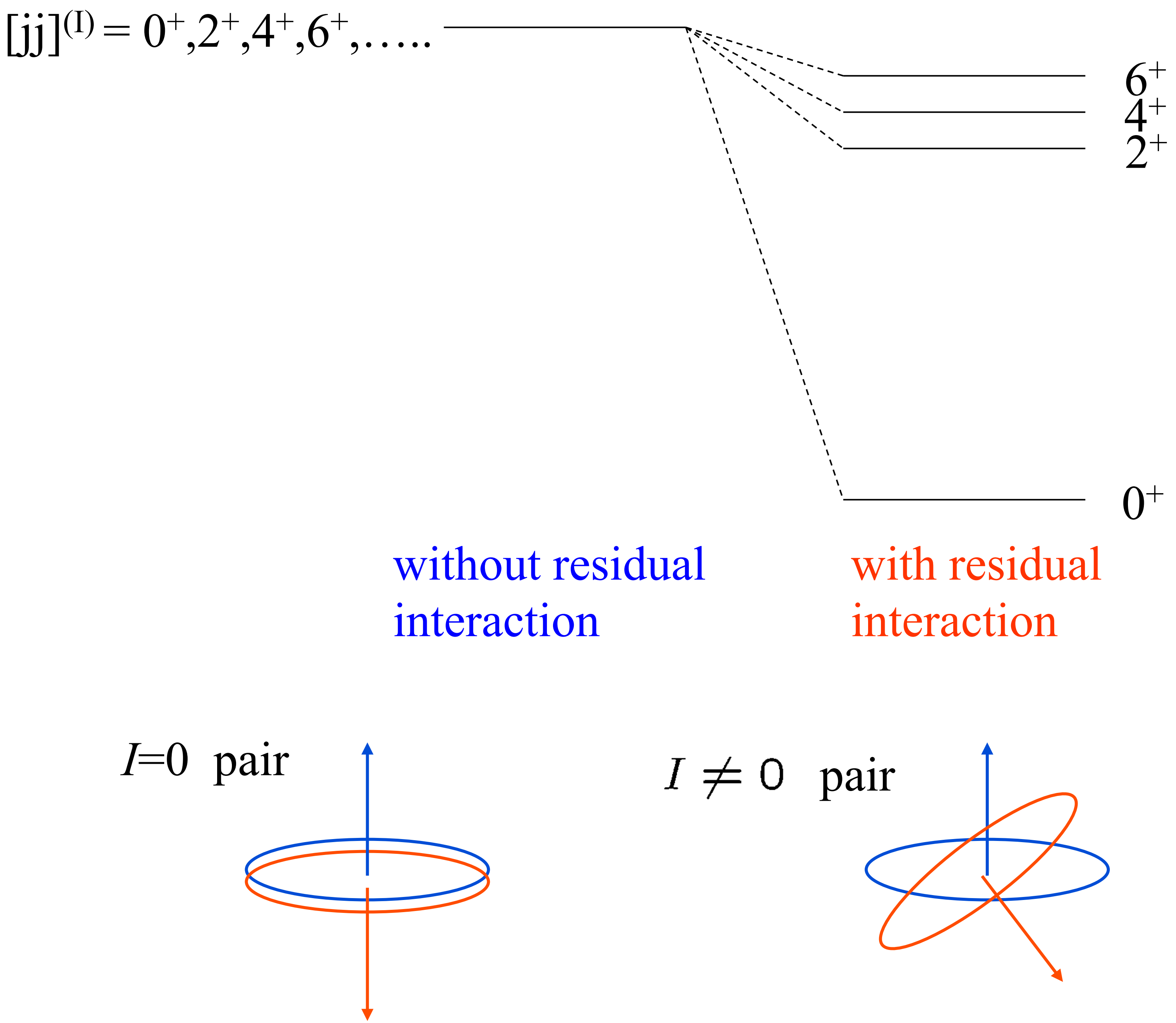




a textbook example
of pairing interaction!

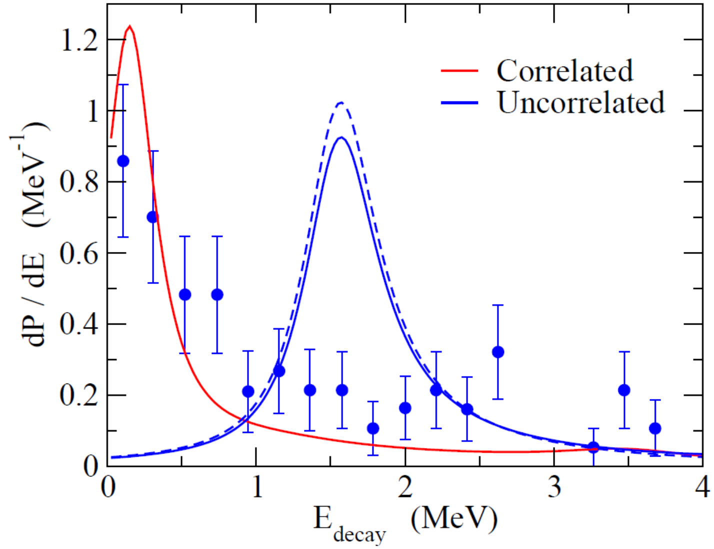
cf. another set of parameters:
$E(0^+) = 5$ keV
$E(2^+) = 1.338$ MeV

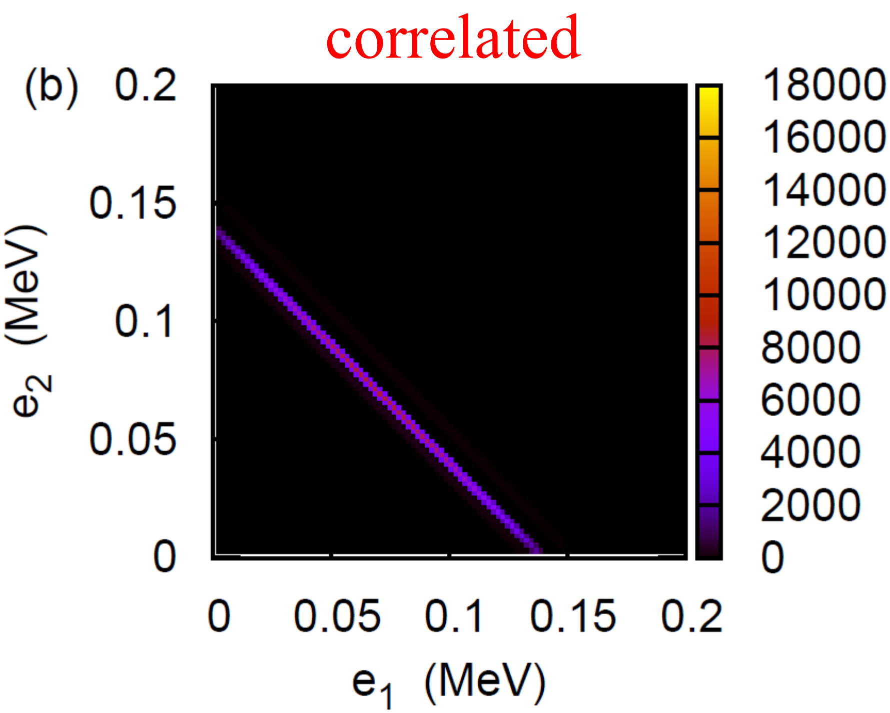
K.H. and H. Sagawa, in preparation

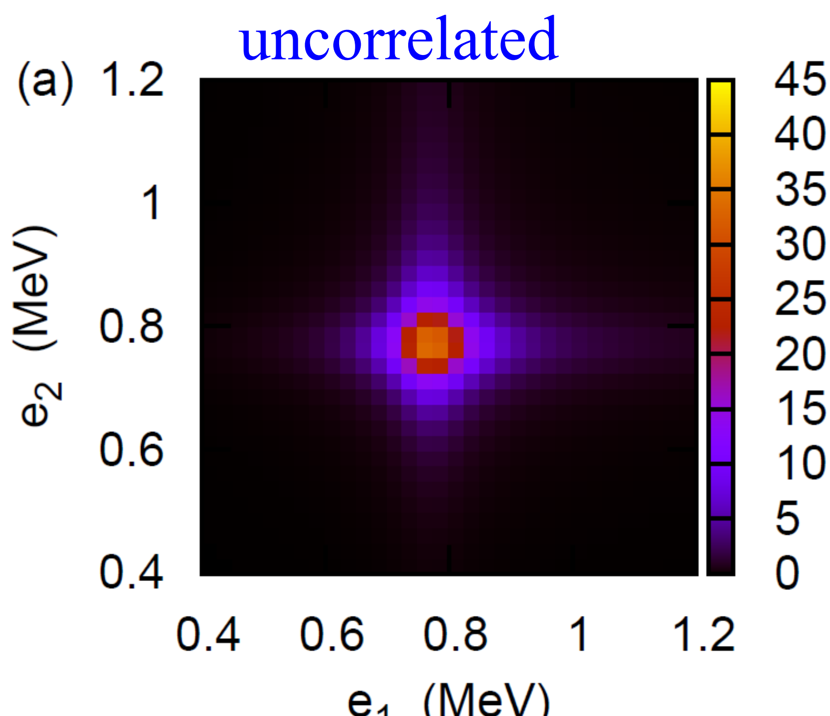

$[jj]^{(I)} = 0^+, 2^+, 4^+, 6^+, \ldots\ldots$
6^+
4^+
2^+
0^+
without residual interaction
with residual interaction
$I=0$ pair
$I \neq 0$ pair

ii) Energy spectrum of the emitted neutrons

K.H. and H. Sagawa,
PRC89 ('14) 014331

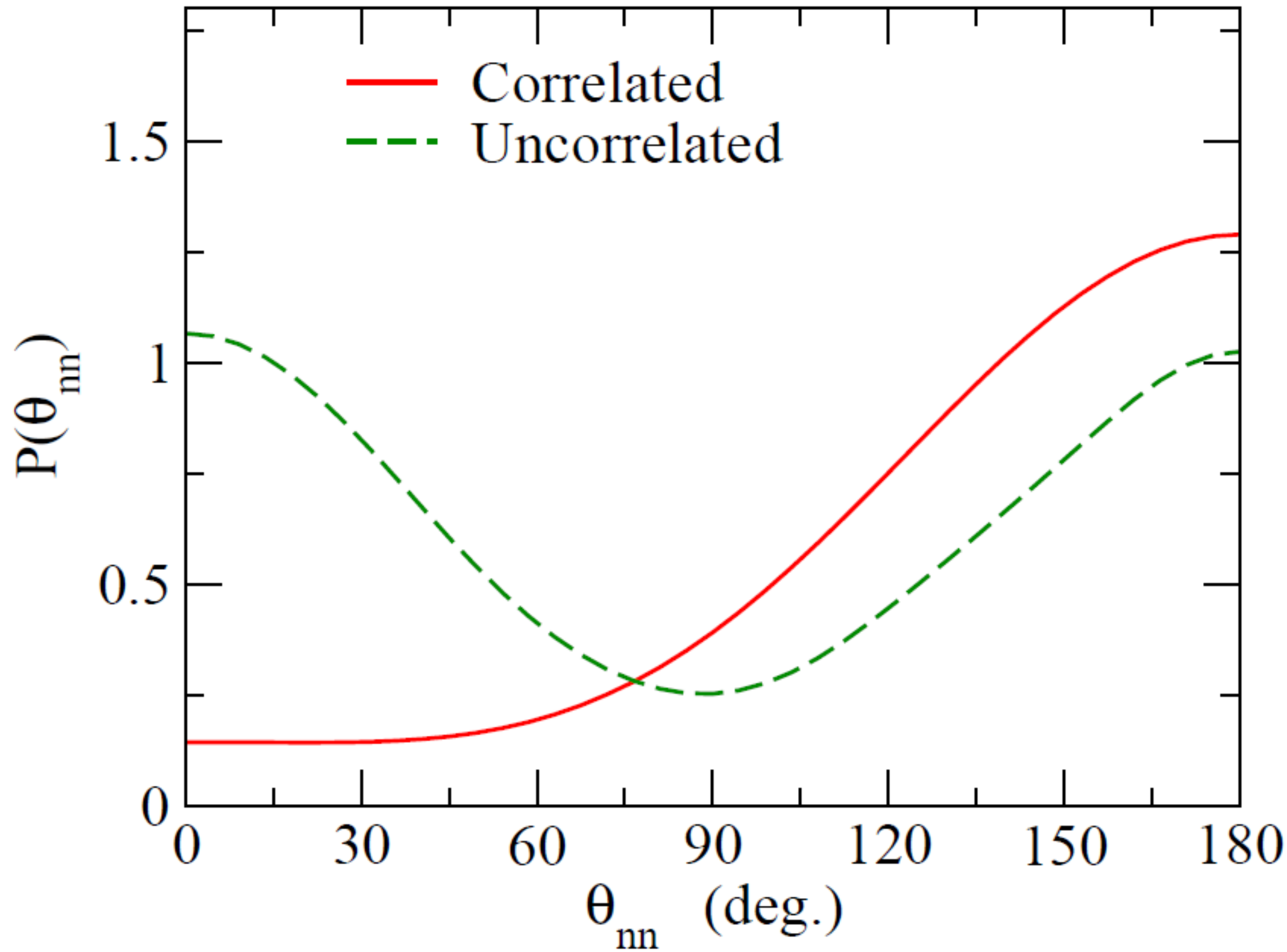






iii) angular correlations of the emitted neutrons

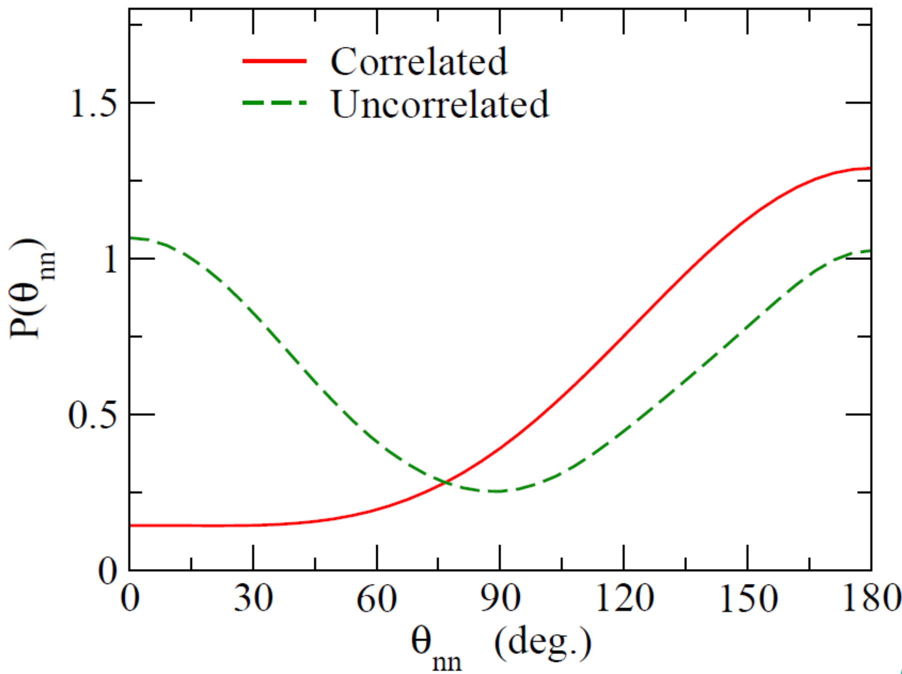
K.H. and H. Sagawa, PRC89 ('14) 014331



correlation → enhancement of back-to-back emissions

$$\langle \theta_{nn} \rangle = 115.3^\circ$$

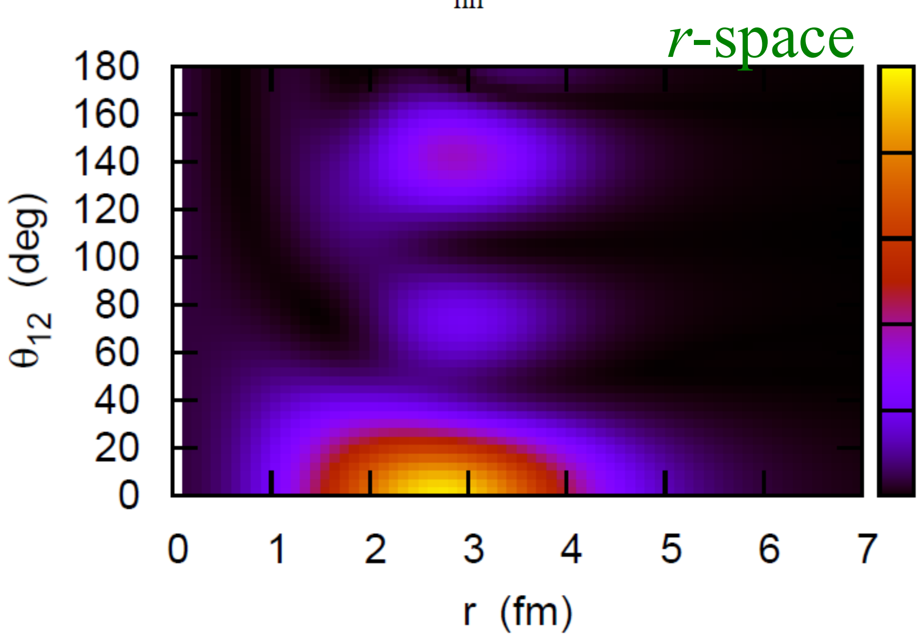
iii) distribution of opening angle for two-emitted neutrons

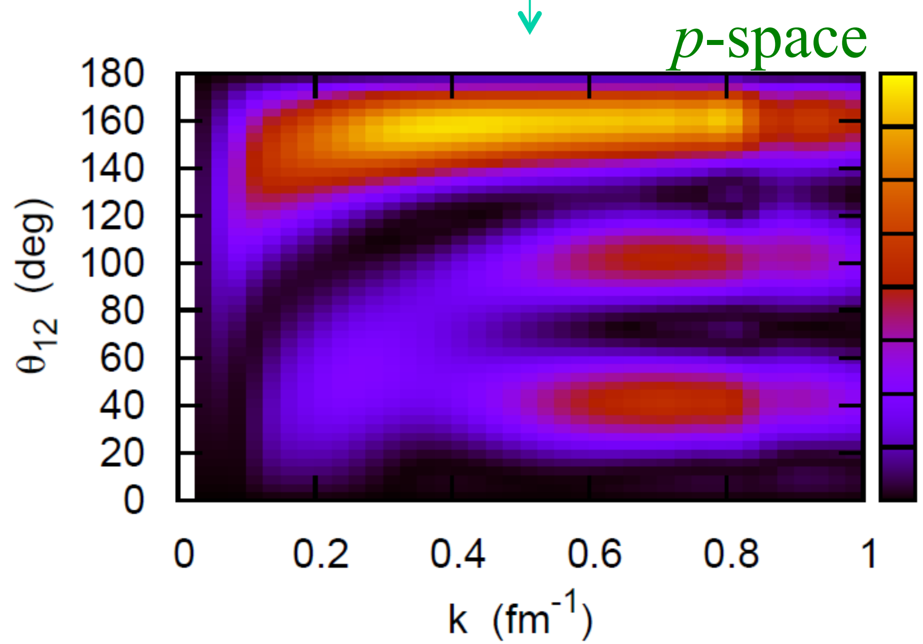


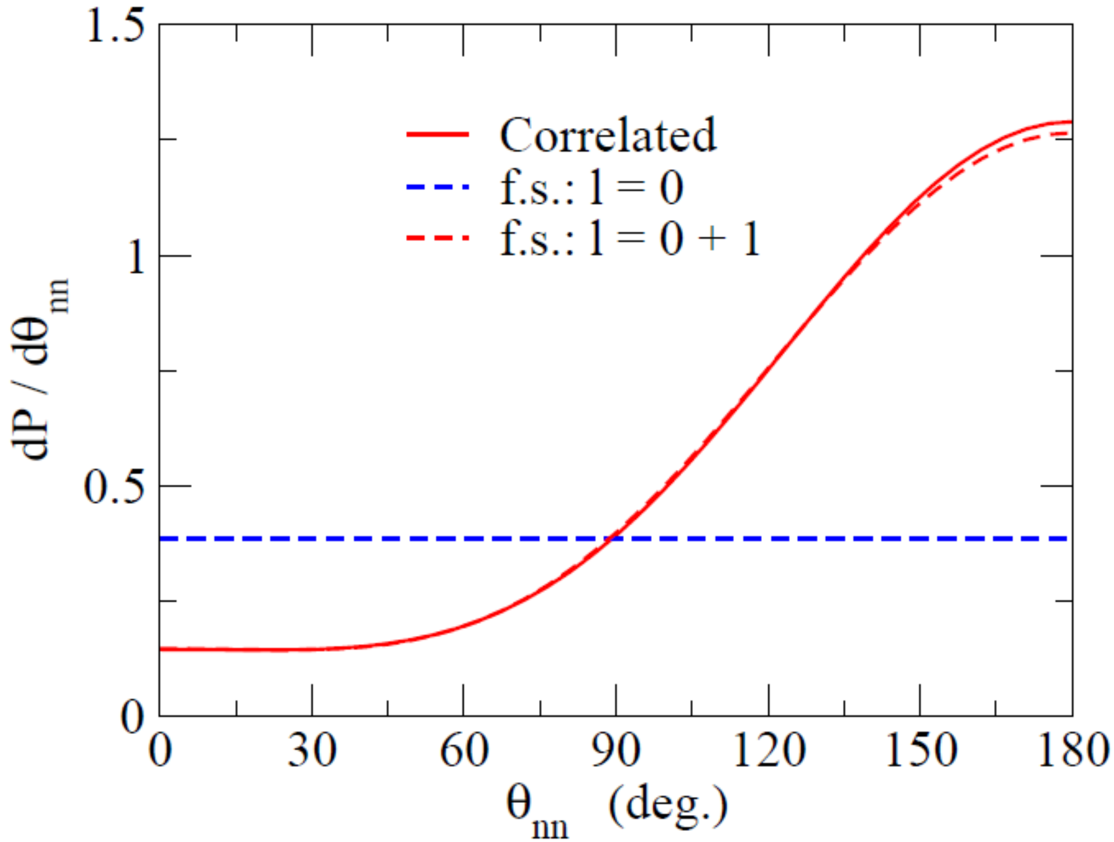
density of the resonance state (with the box b.c.)

$\rho(r, r, \theta)$

$8\pi^2 k^4 \sin\theta \cdot \rho(k, k, \theta)$





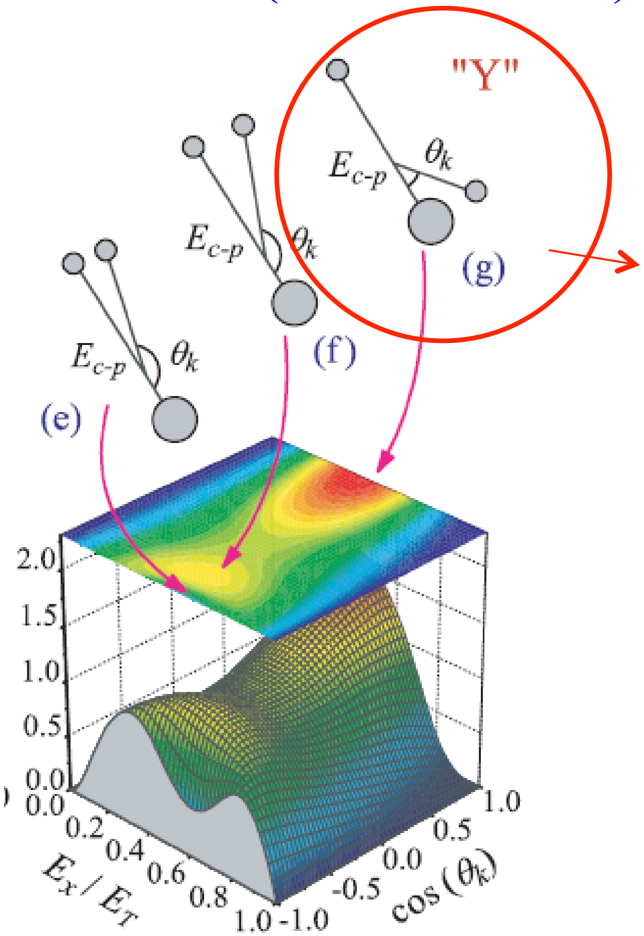


main contributions: *s*- and *p*-waves in three-body wave function
(no or low centrifugal barrier)

*higher *l* components: largely suppressed due to the centrifugal pot.
(E_{decay} ~ 0.14 MeV, e_1 ~ e_2 ~ 0.07 MeV)

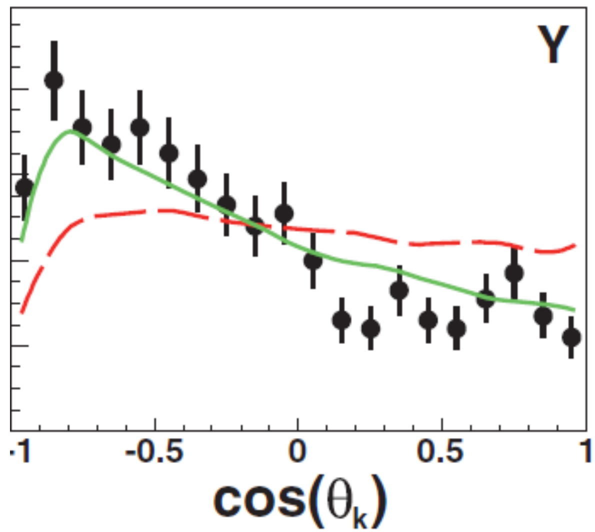
➤Discussions: back-to-back? or forward angles?

two-proton decay from ^{6}Be (back-to-back)



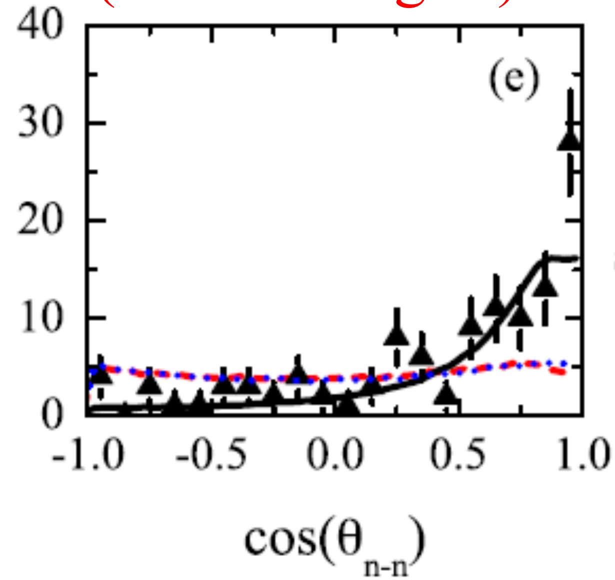
L.V. Grigorenko et al., PRC80 (‘09) 034602

2n decay of ^{13}Li (forward angles)



Z. Kohley et al., PRC87(‘13)011304(R)

2n decay of ^{16}Be (forward angles)



A. Spyrou et al., PRL108(‘12) 102501

✓Q-value effect? (cf. nuclear phase shifts)

✓core excitations?

↑

open problem

Summary

di-neutron correlation : spatial localization of two neutrons

✓parity mixing
✓neutron-rich nuclei: scattering to the continuum states
enhancement of pairing on the surface

how to probe it?

- Coulomb breakup

✓enhancement of B(E1) due to the correlation
✓Cluster sum rule (only with the g.s. correlation)
✓opening angle of two neutrons

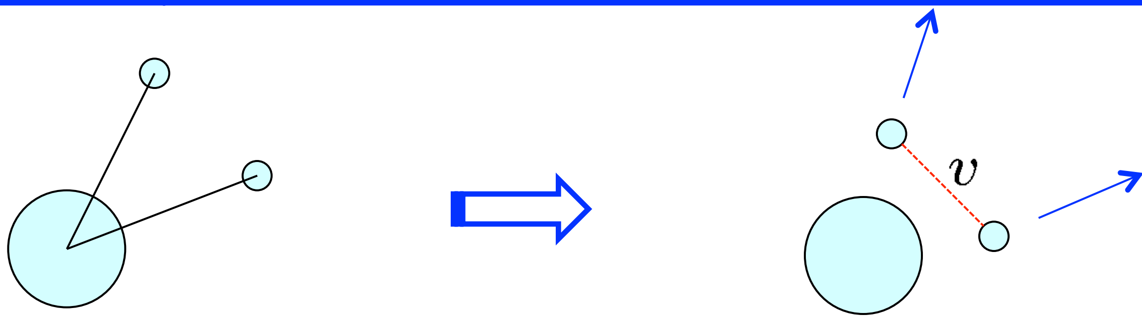
- 2-neutron emission decay

✓ decay energy spectrum
✓ energy spectrum of two emitted neutrons
✓ opening angle of two emitted neutrons (back-to-back)

$\longleftrightarrow$ dineutron correlation

Three-body model calculations: extension of continuum E1 for ^{11}Li

K.H. and H. Sagawa, PRC89 ('14) 014331



initial
meta-stable state

$$
\begin{aligned}
M_{fi} &= \langle (j_1 j_2)^{J=0} | (1 - vG_0 + vG_0 vG_0 - \cdots) | \Psi_i \rangle \\
&= \langle (j_1 j_2)^{J=0} | \underbrace{(1 + vG_0)^{-1}}_{\text{FSI}} | \Psi_i \rangle = \langle \Psi_{j_1 j_2}^{(J=0)} | \Psi_i \rangle
\end{aligned}
$$

unperturbed continuum wf

$$G_0(E) = \sum_{f.st.} \frac{\left|(j_1 j_2)^{J=0}\right\rangle \left\langle (j_1 j_2)^{J=0}\right|}{e_1 + e_2 - E - i\eta}$$

$$\boxed{\frac{d^2P}{de_1 de_2} = \sum_{l_1 j_2 l_2 j_2} |M_{fi}|^2 \frac{dk_1}{de_1} \frac{dk_2}{de_2}}$$

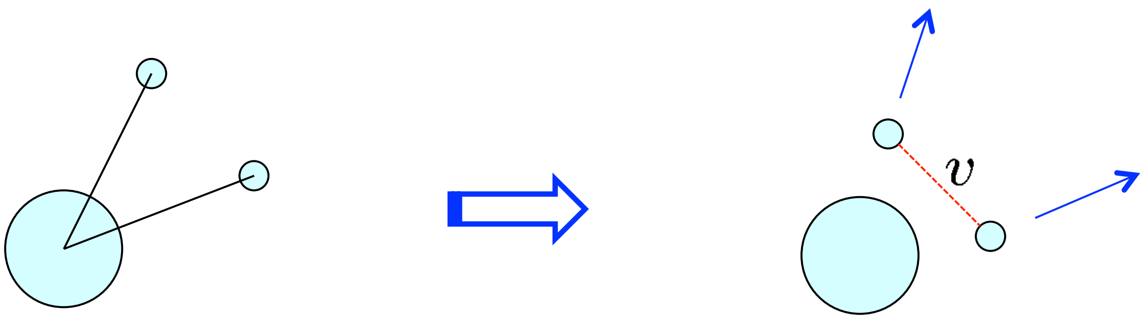
$$\boxed{\frac{dP}{dE} = \int de_1 de_2 \frac{d^2P}{de_1 de_2} \delta(E - e_1 - e_2)}$$

*Green function method: also for angular correlation

* approximate treatment for the recoil term

Three-body model calculations: extension of continuum E1 for ^{11}Li

K.H. and H. Sagawa,
PRC89 ('14) 014331



initial
meta-stable state

$$M_{fi} = \langle (j_1 j_2)^{J=0} | (1 - vG_0 + vG_0 vG_0 - \cdots) | \Psi_i \rangle$$

$$= \langle \underbrace{(j_1 j_2)^{J=0}}_{\text{unperturbed continuum wf}} | \underbrace{(1 + vG_0)^{-1}}_{\text{FSI}} | \Psi_i \rangle = \langle \Psi_{j_1 j_2}^{(J=0)} | \Psi_i \rangle$$

$$G_0(E) = \sum_{f.st.} \frac{\left| (j_1 j_2)^{J=0} \right\rangle \left\langle (j_1 j_2)^{J=0} \right|}{e_1 + e_2 - E - i\eta}$$

cf. Continuum E1 response:

E1 operator

$$M(E1) = \langle (j_1 j_2)^1_\mu | (1 - vG_0 + vG_0 vG_0 - \cdots) D_\mu | \Psi_{gs} \rangle$$

Initial state : the bound ground state for a 3-body model (^{25}F + n + n)

cf. Expt. : ^{27}F (82 MeV/u) + ^{9}Be → ^{26}O → ^{24}O + n + n

- ^{25}F + n potential

(^{24}O + n) potential + δV_{ls}

pn tensor interaction

T. Otsuka et al., PRL95('05)232502

$e_{1d3/2}$ (^{26}F) = - 0.811 MeV

cf. $e_{1d3/2}$ (^{25}O) = + 770^{+20}_{-10} keV

- pairing strength

⟶ E (^{27}F) = -2.69 MeV

cf. E_{exp} (^{27}F) = -2.80(18) MeV

➡ sudden proton removal

(keep the nn configuration for ^{25}F+n+n, and suddenly change the core from ^{25}F to ^{24}O)

$$M_{fi} = \langle (j_1 j_2)^{J=0} | (1 + vG_0)^{-1} | \Psi_i \rangle$$

Initial state : 3-body model ($^{25}F + n + n$)

$\longrightarrow$ sudden proton removal : $^{27}F \rightarrow {}^{26}O$

$\hookrightarrow$ spontaneous decay

cf. $\Psi_{nn}(^{27}F)$: is not an eigenstate of $H_{nn}(^{26}O)$

Propagation & final uncorrelated state : 3-body model ($^{24}O + n + n$)

- $^{24}O + n$ potential

Woods-Saxon potential to reproduce

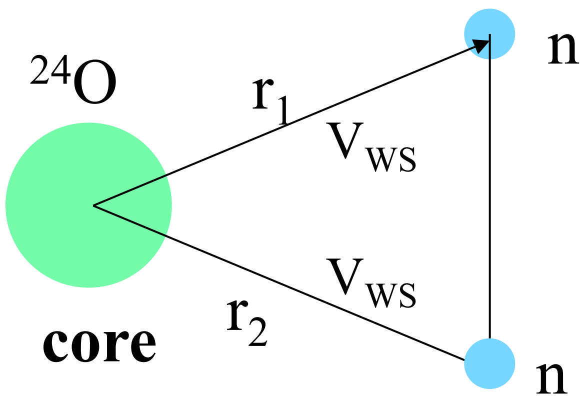
C.R. Hoffman et al., PRL100('08)152502

$e_{2s1/2} = -4.09\ (13)$ MeV,

$e_{1d3/2} = +\ 770^{+20}_{-10}$ keV, $\Gamma_{1d3/2} = 172(30)$ keV

$a = 0.95$ fm $\rightarrow \Gamma_{1d3/2} = 141.7$ keV

i) Decay energy spectrum



➢ ^{24}O + n potential

Woods-Saxon potential to reproduce

$e_{2s1/2}$ = -4.09 (13) MeV,

$e_{1d3/2}$ = + 770^{+20}_{-10} keV,

$\Gamma_{1d3/2}$ = 172(30) keV

➢ nn interaction

density-dep. contact interaction

E (^{27}F) = -2.69 MeV

$$\begin{aligned}
\frac{dP_I}{dE} &= \sum_k |\langle \Psi_k^{(I)} | \Phi_{\rm ref}^{(I)} \rangle|^2 \, \delta(E - E_k) \\
&= -\frac{1}{\pi} \Im \sum_k \langle \Phi_{\rm ref}^{(I)} | \Psi_k^{(I)} \rangle \, \frac{1}{E_k - E - i\eta} \, \langle \Psi_k^{(I)} | \Phi_{\rm ref}^{(I)} \rangle, \\
&= -\frac{1}{\pi} \Im \langle \Phi_{\rm ref}^{(I)} | G^{(I)}(E) | \Phi_{\rm ref}^{(I)} \rangle,
\end{aligned}$$

$$f_{h_1h_2}(\boldsymbol{k}_1,\boldsymbol{k}_2) = \sum_{j,l} e^{-il\pi} e^{i(\delta_1+\delta_2)} \frac{1}{k_1k_2} \langle \Psi^{(00)}_{k_1k_2jl}|\Psi_i\rangle \times \langle [\mathcal{Y}_{jl}(\hat{\boldsymbol{k}}_1)\mathcal{Y}_{jl}(\hat{\boldsymbol{k}}_2)]^{(00)}|\chi_{h_1}\chi_{h_2}\rangle$$

$$\frac{d^2P}{d\hat{\boldsymbol{k}}_1 d\hat{\boldsymbol{k}}_2} = \sum_{h_1,h_2} \int k_1^2 dk_1 k_2^2 dk_2 \, \left| f_{h_1h_2}(\boldsymbol{k}_1,\boldsymbol{k}_2)\right|^2$$