# Measurements of the hyperfine structure constant for laser-cooled $^{11}Be^+$ ions

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## Hyperfine Structure Constant of the Neutron Halo Nucleus $^{11}Be^+$

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The hyperfine splittings of ground state $^{11}Be^+$ have been measured precisely by laser-microwave double resonance spectroscopy for trapped and laser cooled beryllium ions. The ions were produced at relativistic energies and subsequently slowed down and trapped at mK temperatures. The magnetic hyperfine structure constant of $^{11}Be^+$ was determined to be $A_{11} = -2677.302\,988(72)$ MHz from the measurements of the $m_F - m_F{}' = 0$–$0$ field independent transition. This measurement provides essential data for the study of the distribution of the halo neutron in the single neutron halo nucleus $^{11}$Be through the Bohr-Weisskopf effect.

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**Motivation & Background**

Electromagnetically probe the magnetisation of $^{11}$Be

**Experimental Procedure**

Online spectroscopy at prototype of SLOWRI

**Results**

# Electromagnetic Interaction between nucleus and atomic electrons

$$H_{em} = \int \rho(\vec{r})\phi(\vec{r})d^3\vec{r} - \int \vec{j}(\vec{r}) \cdot \vec{A}(\vec{r})d^3\vec{r}$$

$$= H_{em}^{\mathrm{E0}} + H_{em}^{\mathrm{M1}} + H_{em}^{\mathrm{E2}} + \cdots$$

$\rho$ : nuclear charge density

$\vec{j}$ : current charge density

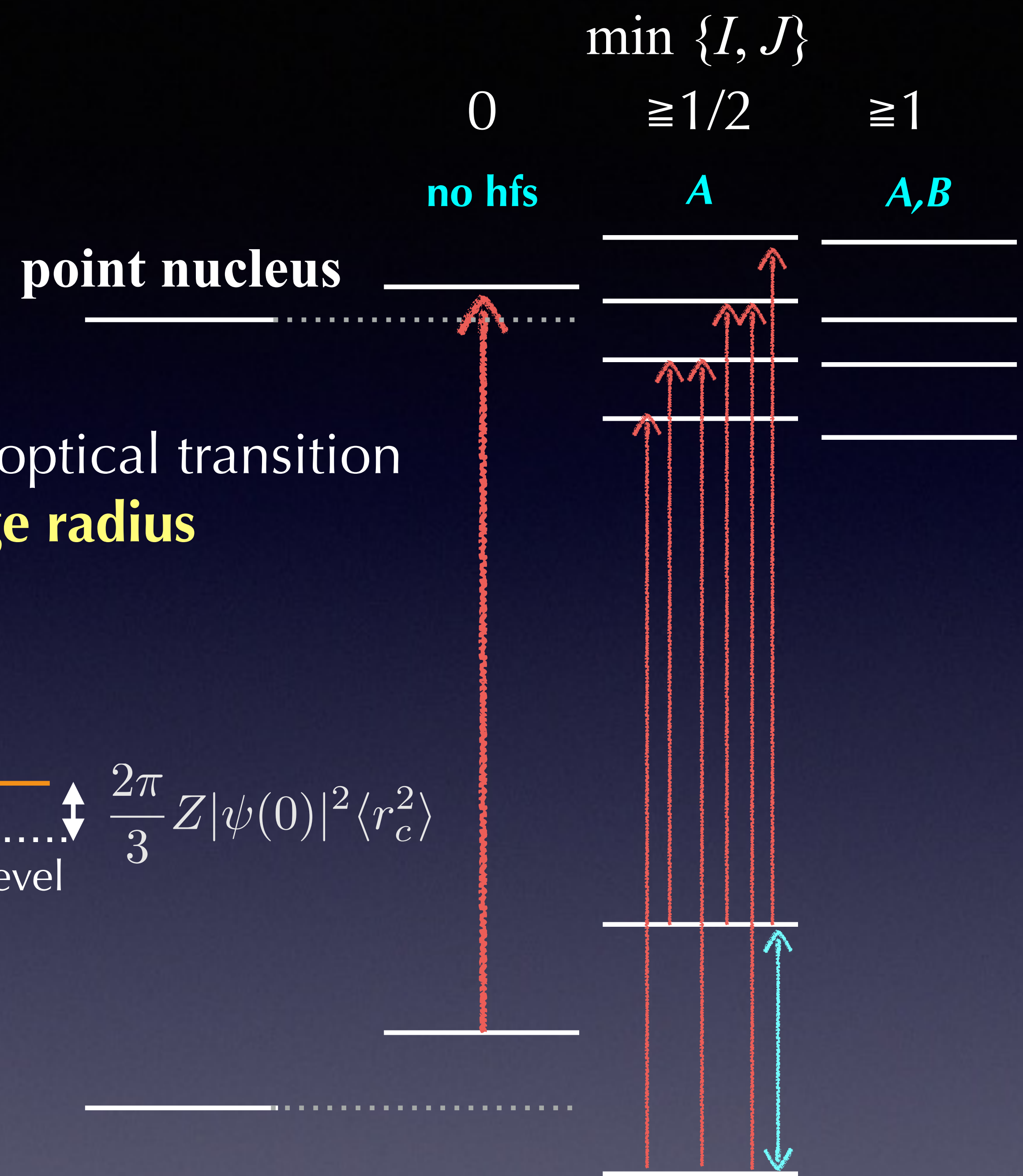
$\left.\begin{array}{ll}\phi & \text{: electric potential} \\ \vec{A} & \text{: vector potential}\end{array}\right\}$ by electron

$H_{em}^{\mathrm{E0}} = Ze\phi(0)$ **Field Shift** $\langle r_c^2 \rangle$

$H_{em}^{\mathrm{M1}} = -\vec{B}(0) \cdot \vec{\mu}$ **Magnetic Dipole Hyperfine Structure** $A$

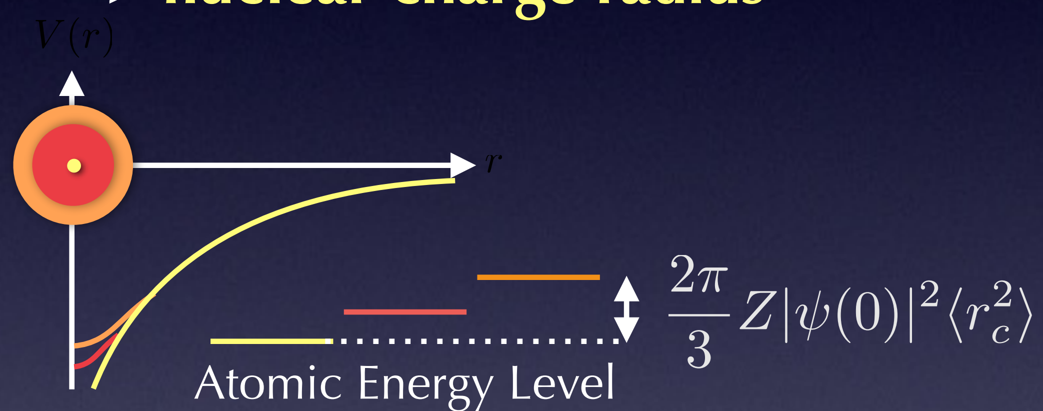
$H_{em}^{\mathrm{E2}} = \frac{1}{6}\sum_{ij} V_{ij}Q_{ij}$ **Electric Quadrupole Hyperfine Structure** $B$

# Nuclear Effects in Atomic Spectra




Number of hyperfine splitting states
—> **nuclear spin**

Isotope shift of optical transition
—> **nuclear charge radius**




Hyperfine constant $A$,$B$,$C$,…
—> **nuclear moment (with sign)**

$$A = \frac{\mu_I B(0)}{IJ}$$

$$B = eQ_s\langle\frac{\partial^2 V_e}{\partial z^2}\rangle$$

Isotope shift of hfs
—> **nuclear magnetization radius**

**Electromagnetic probe for ground(/isomeric) state properties of nuclei**

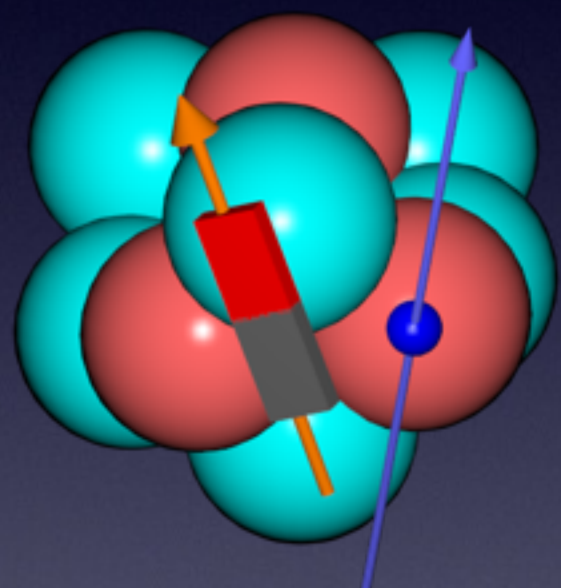
# Magnetic Hyperfine Interaction

| hfs A | = | nuclear *g*-factor | × | hyperfine field by valence (s-)electron | / | Atomic spin |
|---|---|---|---|---|---|---|
| $A$ | | $\mu_I/I$ | | $B(0)$ | | $J$ |

If B(0)=uniform or nucleus is a point dipole,

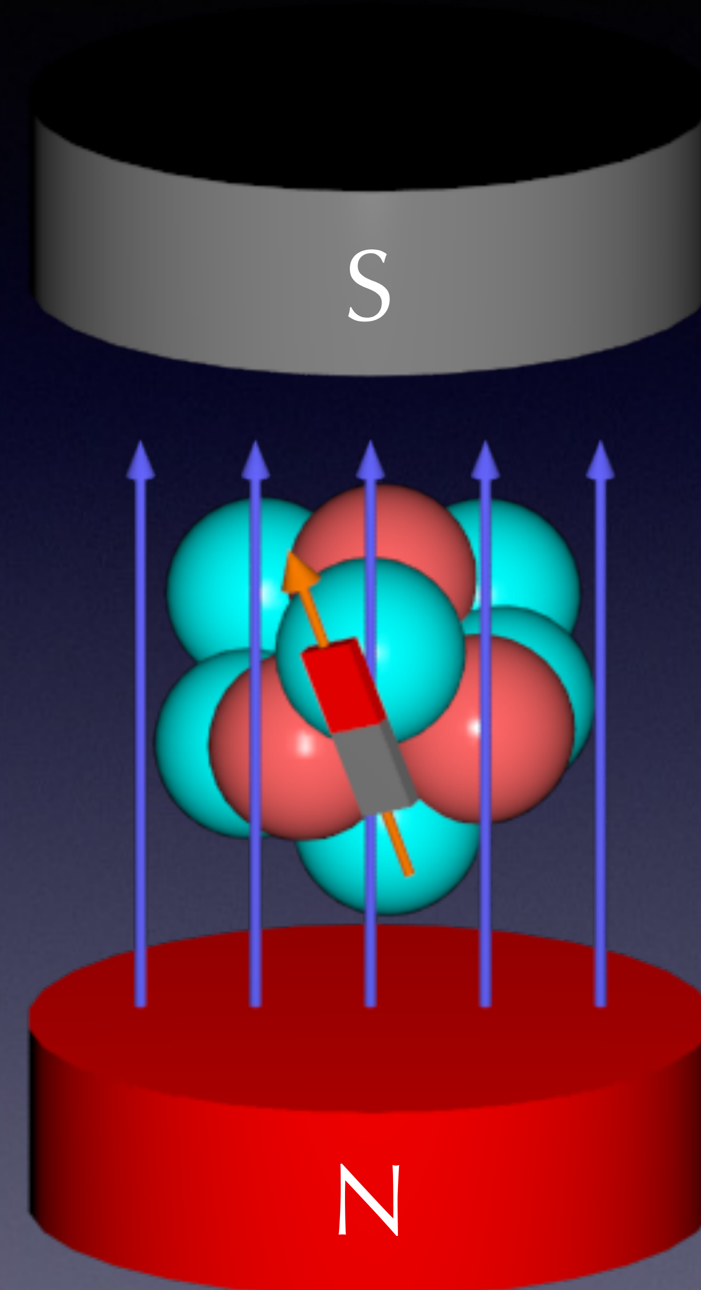
$$\frac{A}{g_I/I} = \text{const.}$$ among isotopes.

## Hyperfine Constant



probed by **inhomogeneous** magnetic field due to (s-)electron

## Nuclear *g*-factor




probed by **homogeneous** magnetic field externally applied

# Magnetic Hyperfine Interaction

| hfs A | = | nuclear g-factor | × | hyperfine field by valence (s-)electron | / | Atomic spin |
|---|---|---|---|---|---|---|
| $A$ | | $\mu_I/I$ | | $B(0)$ | | $J$ |

If B(0)=uniform or nucleus is a point dipole,

$$\frac{A}{g_I/I} = \text{const.}$$ among isotopes.

**Not in real nuclei : Anomaly**

$$A = A_p(1+\epsilon)$$

point dipole

"Bohr-Weisskopf effect"

**Hyperfine Anomaly**

$$^{1}\Delta^{2} = \epsilon_1 - \epsilon_2 \approx \frac{A_1/(\mu_{I_1}/I_1)}{A_2/(\mu_{I_2}/I_2)} - 1$$

e.g. for Rb

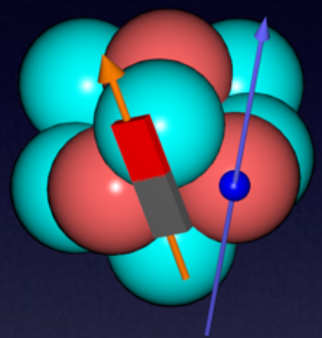
$$^{85}\Delta^{87} = 0.3514(3)\ \%$$

# Nuclear Volume Effect on HFS *A*

## Bohr-Weisskopf effect

" $\epsilon$ depends on the nuclear magnetization distribution."

A. Bohr and V. W. Weisskopf, Phys. Rev. **77**, 94 (1950)



**inhomogeneous** magnetic field due to a valence electron
⇒ detection of magnetisation distribution

## Breit-Rosenthal(-Crawford-Schawlow) effect

Nuclear charge distribution changes the wave functions of atomic electrons.
⇒ Shift on HFS

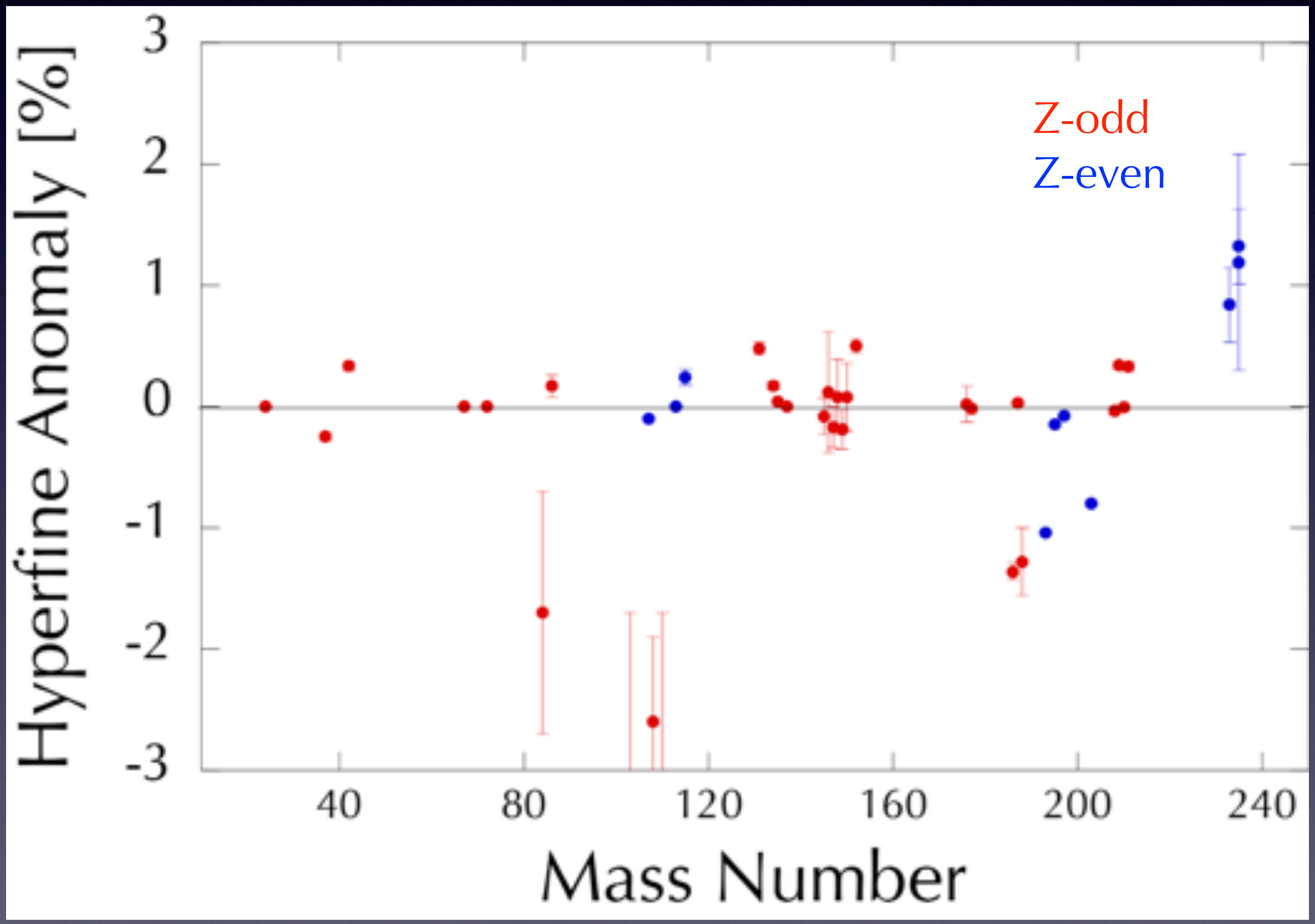
$$A = A_p(1 + \epsilon_{\mathrm{BW}})\cancel{(1 + \epsilon_{\mathrm{BR}})}$$

$$\epsilon_{\mathrm{BW}} >> \epsilon_{\mathrm{BR}}$$

H. J. Rosenberg and H. H. Stroke, PRA **5**, 1992 (1972)

# Hyperfine Anomaly

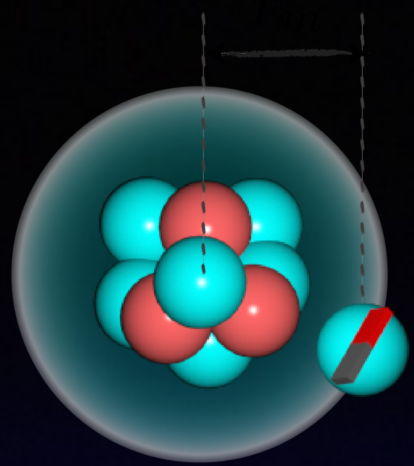
$$ {}^{1}\Delta^{2} = \epsilon_1 - \epsilon_2 \approx \frac{A_1/(\mu_{I_1}/I_1)}{A_2/(\mu_{I_2}/I_2)} - 1 $$




HFA: 0.1~1%

Independent measurements of $A, \mu_I$
w/ an accuracy higher than $10^{-4}$

# Hyperfine Anomaly of $^{11}$Be

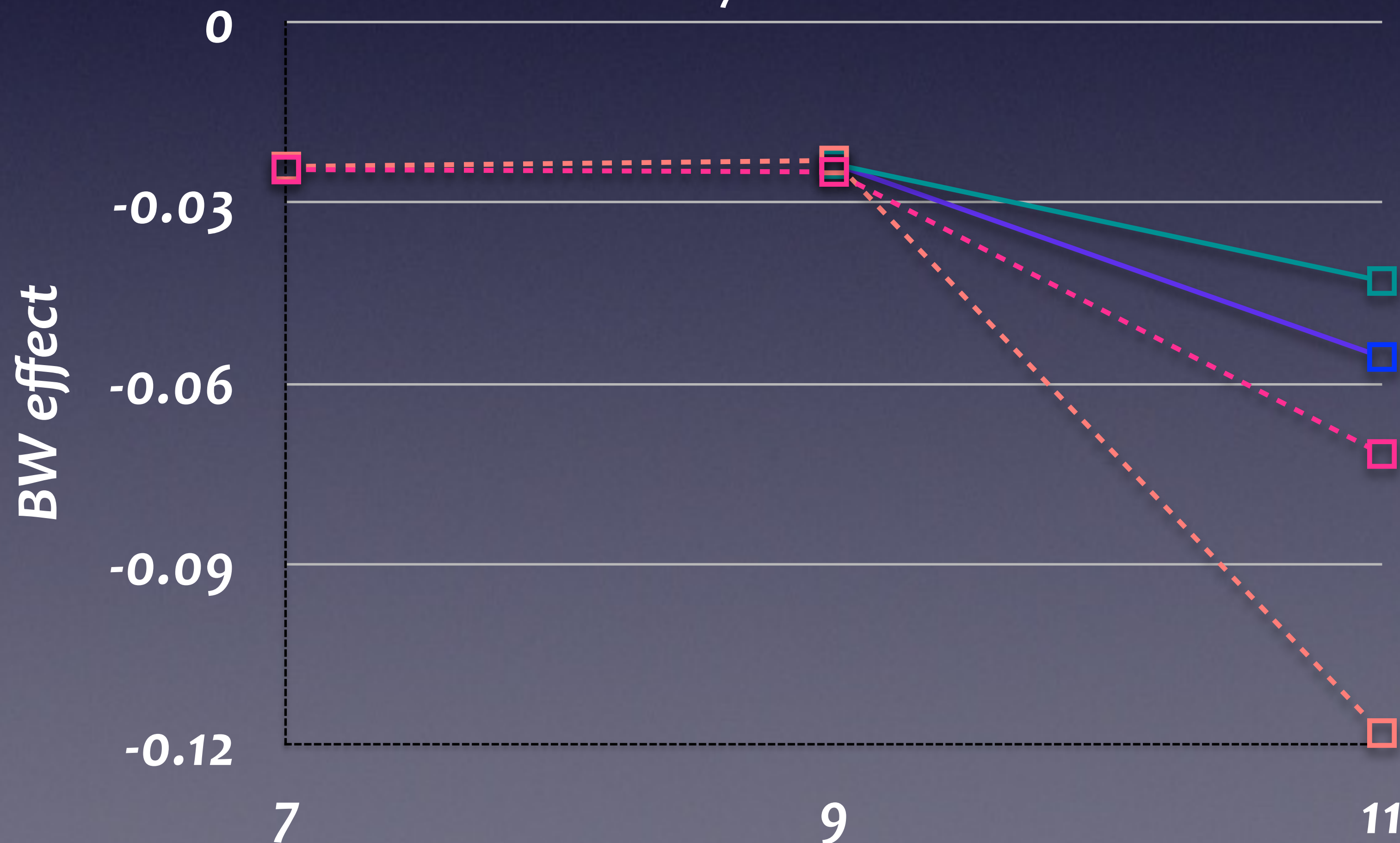


Z=4, N=7    1 neutron halo

Unique Probe for Neutron Halo!

M. Wada *et al.*, Nucl. Phys. A **626**, 365 (1997)

T. Fujita, K. Ito, T. Suzuki (a) — core + neutron type wave function
T. Fujita, K. Ito, T. Suzuki (b) — single-particle model with core polarization — PRC **59,** 210 (1999)
Y. Parfenova et al. ws=0.5
Y. Parfenova et al. ws=0.72 — cluster model with a halo effect — PRC **72,** 024312 (2005)

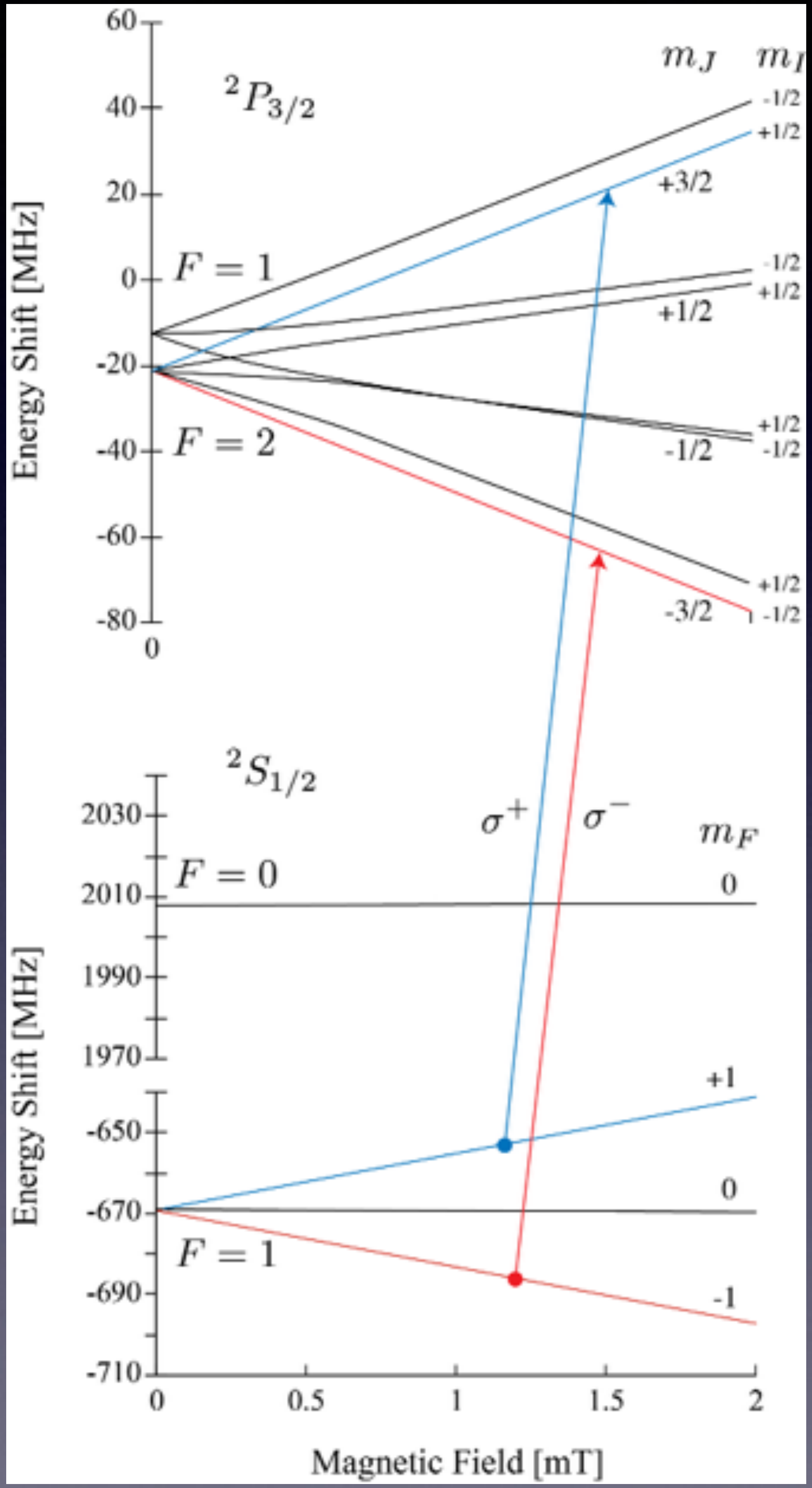



HFA ~ 0.01-0.1 %

$A, \mu_I$

w/ accuracies higher than **$10^{-5}$**

# HFS of $^{11}Be^{+}$




$A_{11} \sim 2.6$ GHz

$\nu(2s_{1/2} - 2p3/2) \sim 957$ THz

Doppler Width for Be ions @300 K

$$\Delta\nu_D = \frac{\nu}{c}\sqrt{\frac{8k_B T \ln 2}{m}} \sim 3.7 \text{ THz} >> \text{HFS}$$

Collinear Laser Spectroscopy $\Delta\nu \sim$ 1MHz

**Direct measurement of HFS via microwave spectroscopy for laser-cooled ions**

# Magnetization Distribution

VOLUME 74, NUMBER 12 PHYSICAL REVIEW LETTERS 20 MARCH 1995

## Magnetic Moment Distributions in Tl Nuclei

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(Received 10 June 1994)

$$h_{\mathrm{BW}}^{\mathrm{hfs}} = \int \left\{ \eta(R - r) \left[ g_s \left( \frac{\mathbf{S}_n}{r^2} - \sqrt{10}\,(\mathbf{S}_n \mathbf{C}_n^2)^1 \frac{r}{R^3} \right) + g_L \mathbf{L}_n \left( \frac{1}{r^2} - \frac{r}{R^3} \right) \right] \times \boldsymbol{\alpha} \right\} \rho_m(R) R^2 \, dR$$

$$\Delta_{\mathrm{BW}} = b_2 \delta \langle r_m^2 \rangle + b_4 \delta \langle r_m^4 \rangle + \cdots .$$

$$\Delta_{\mathrm{BW}} = b_{2s} d_2 \delta \langle r^2 \rangle + b_{4s} d_4 \delta \langle r^4 \rangle + \cdots = b_{2s} \lambda_m$$

$$\lambda_m = \delta \langle r^2 \rangle \left( d_2 + \frac{b_{4s} d_4}{b_{2s}} \frac{\delta \langle r^4 \rangle}{\delta \langle r^2 \rangle} + \cdots \right).$$

$$\lambda_{c,m} = \lambda_m + 1.91(1) \lambda_c .$$

$$^{203:205}\delta \langle r_m^2 \rangle = 0.26(2)\ \mathrm{fm}^2$$

TABLE II. Hyperfine anomalies: The theoretical values give the relative effect of the magnetic moment distributions on the various terms shown in Table I and are given in terms of $b_{2s}$ factors, i.e., as $\Delta/\lambda_{c,m}$. The experimental hyperfine anomalies were obtained using the magnetic moments [8] $^{203}\mu = 1.622\,257\,87 \mu_N$ and $^{205}\mu = 1.638\,314\,61 \mu_N$. The $\lambda_{c,m}$ values shown in the last line were extracted by combining theoretical and experimental values, but the error bars do not reflect the theoretical uncertainty.

| | $6p_{1/2}$ | $6p_{3/2}$ | $7s$ |
|---|---|---|---|
| $\Delta/\lambda_{c,m} (10^{-4}/\mathrm{fm}^2)$ | | | |
| DF(BO) | −2.26 | 0 | −7.95 |
| RPA | −4.89 | −5.02 | −5.86 |
| Corr | −4.12 | −4.24 | −1.63 |
| Total | −2.48 | 43.0 | −7.62 |
| Experiment | | | |
| $\Delta(10^{-4})$ | −1.04[a] | 16.26[b] | −3.4(18)[c] |
| $\lambda_{c,m}$ ($\mathrm{fm}^2$) | 0.42 | 0.38 | 0.45(24) |

[a]Lurio and Prodell, Ref. [3].
[b]Gould, Ref. [4].
[c]Hermann *et al.*, Z. Phys. D **28**, 127 (1993).

# Magnetization Distribution

VOLUME 83, NUMBER 5 PHYSICAL REVIEW LETTERS 2 AUGUST 1999

## Hyperfine Anomaly Measurements in Francium Isotopes and the Radial Distribution of Neutrons

J. S. Grossman, L. A. Orozco, M. R. Pearson, J. E. Simsarian,* G. D. Sprouse, and W. Z. Zhao†
*Department of Physics and Astronomy, State University of New York, Stony Brook, New York 11794-3800*
(Received 5 April 1999)

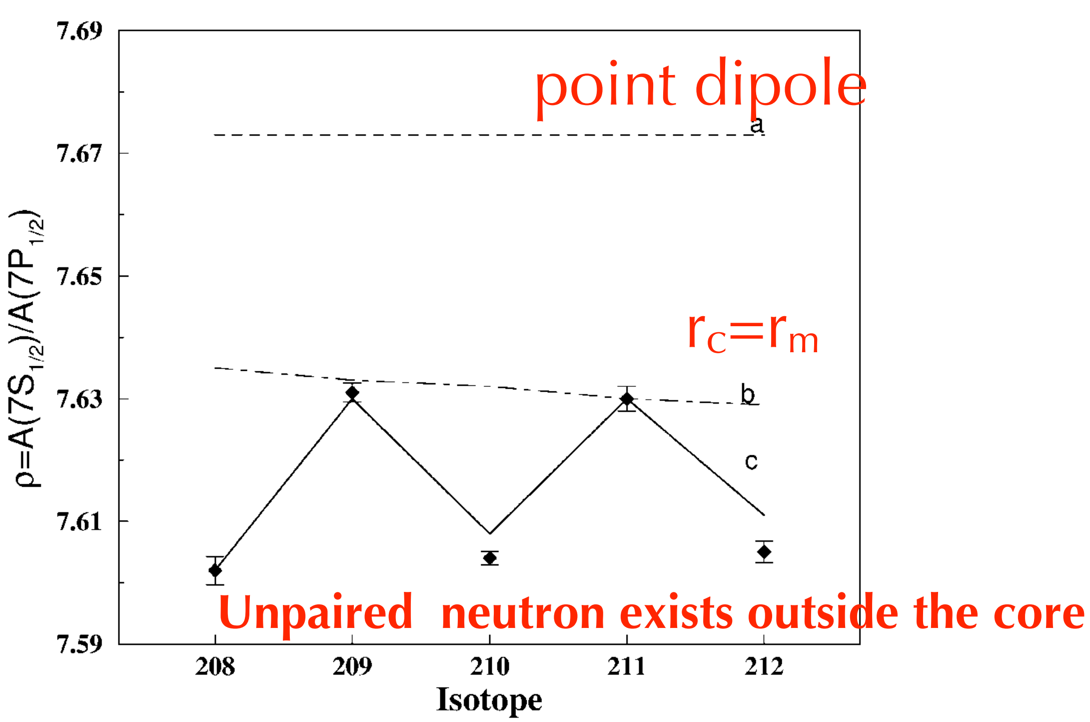



FIG. 3. Ratio of hyperfine $A$ magnetic dipole constants of $7S_{1/2}$ and $7P_{1/2}$ states and differential changes observed for five different Fr isotopes. *a*: point nucleus; *b*: charge radius equal to magnetic radius; *c*: Stroke calculation method.

$A/\mu_I$ :unknown

compare $A_s/A_p$

$$\rho_A = \frac{W^S_{\text{extended}}}{W^P_{\text{extended}}} = \frac{W^S_{\text{point}}[1 + \epsilon(A,S)]}{W^P_{\text{point}}[1 + \epsilon(A,P)]},$$

$$\rho_A \approx \rho_0[1 + \epsilon(A,S) - \epsilon(A,P)],$$

$$\rho = \rho_0(1 - \lambda_m \langle r^2 \rangle_m^{1/2})$$

for Fr $\lambda_m = +0.0046\ \text{fm}^{-1}$

# Magnetization Distribution

In the non relativistic QED approach, relativistic and QED corrections are expressed in terms of an effective Hamiltonian, so the expansion in the fine-structure constant $\alpha$ is of the form

$$E_{\mathrm{hfs}} = \langle H^{(4)}_{\mathrm{hfs}} \rangle + \langle H^{(5)}_{\mathrm{hfs}} \rangle + 2 \left\langle H^{(4)} \frac{1}{(E-H)'} H^{(4)}_{\mathrm{hfs}} \right\rangle + \langle H^{(6)}_{\mathrm{hfs}} \rangle + \langle H^{(6)}_{\mathrm{rad}} \rangle + \langle H^{(7)}_{\mathrm{hfs}} \rangle + \cdots$$

If the nucleus is described by the electric $\rho_E(r)$ and the magnetic $\rho_M(r)$ form factors,

$$H^{(5)}_{\mathrm{hfs}} = -H^{A}_{\mathrm{hfs}} 2Z\alpha m r_Z$$

$$\text{Zeemach radius: } r_Z = \int d^3r\, d^3r'\, \rho_E(r)\rho_M(r')|\vec{r}-\vec{r}'|$$

The more accurate formula goes beyond the elastic form factor treatment.

$$H^{(5)}_{\mathrm{hfs}} = \frac{\pi\alpha^2}{2} \sum_a \delta^3(r_a) \int d^3r\, d^3r' \langle \rho(\vec{r}), \vec{\sigma}_a \cdot (\vec{r}-\vec{r}') \times \vec{j}(\vec{r}')|\vec{r}-\vec{r}'|\rangle$$

$$= -H^{A}_{\mathrm{hfs}} 2Z\alpha m \tilde{r}_z$$

Both formulas include the same feature: linear dependence on the average distance of the magnetic moment density from the charge density.

TABLE III. Contributions in MHz to the hyperfine splitting constant $A$ in $^{6,7}$Li. Used constants are $g = 2.002\,319\,304\,361\,53(53)$, $\alpha^{-1} = 137.035\,999\,074(44)$, the next-to-last row is a Zemach radius inferred from comparison of experiment (expt) [4] with theoretical (theor) value for the point nucleus.

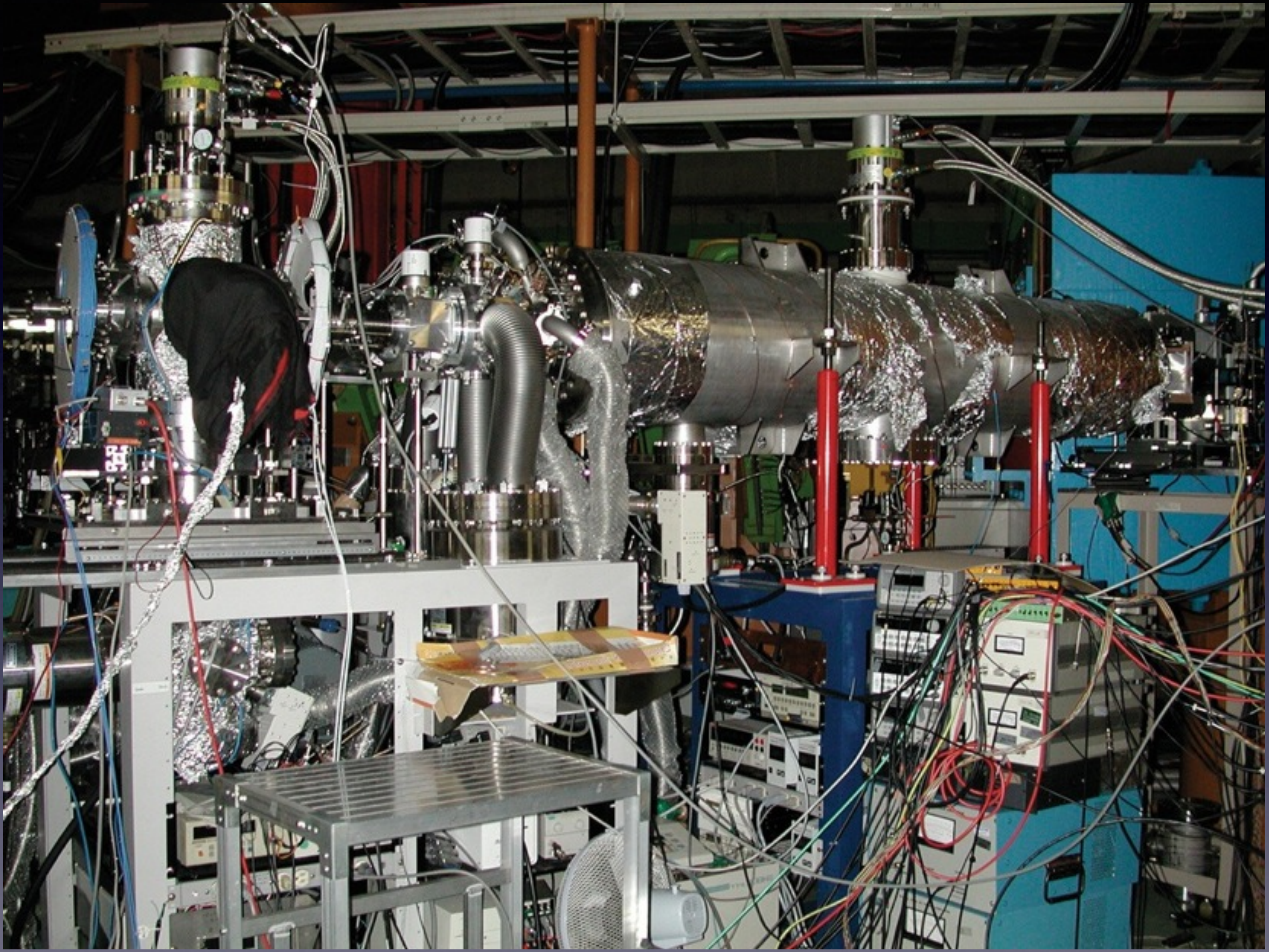
| | $^7$Li | $^6$Li |
|---|---|---|
| $\varepsilon \times 10^{-9}$ | 24.348 067(13) | 9.219 580(7) |
| $\varepsilon\alpha^4 g A^{(4)}/2$ | 401.654 08(21) | 152.083 69(11) |
| $\varepsilon\alpha^5 A^{(5)}_{\mathrm{rec}}$ | −0.004 14 | −0.001 80 |
| $\varepsilon\alpha^6 A^{(6)}$ | 0.260 08(2) | 0.098 48(1) |
| $\varepsilon\alpha^7 A^{(7)}$ | −0.010 2(13) | −0.003 9(5) |
| $A_{\mathrm{theor}}$ (point nucleus) | 401.899 8(13) | 152.176 5(5) |
| Reference [6] | 401.903(11) | 152.177 8(42) |
| $A_{\mathrm{expt}}$ | 401.752 043 3(5) | 152.136 839(2) |
| $(A_{\mathrm{expt}} - A_{\mathrm{theor}})/A_{\mathrm{expt}}$ | −368(3) ppm | −261(3) ppm |
| Reference [6] (nuclear calculations) | −369(23) ppm | −368(60) ppm |
| $\tilde{r}_Z$ | 3.25(3) fm | 2.30(3) fm |
| $r_E$ | 2.390(30) fm | 2.540(28) fm |

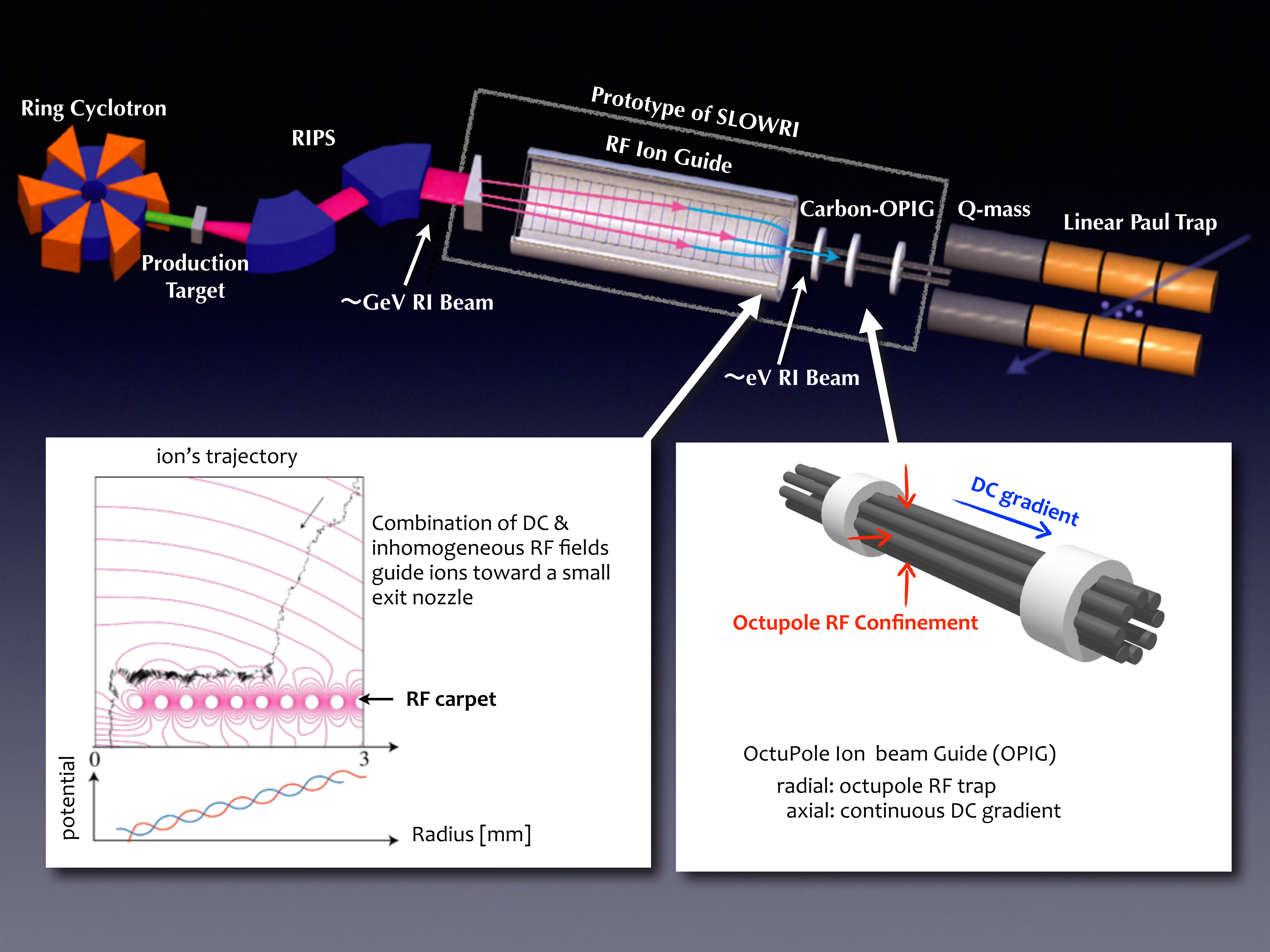
TABLE IV. Contributions in MHz to the hyperfine splitting constant $A$ in $^9$Be$^+$; physical constants are $g = 2.002\,319\,304\,361\,53(53)$ and $\alpha^{-1} = 137.035\,999\,074(44)$. The second uncertainty of $A_{\mathrm{theor}}$ comes from the nuclear magnetic moment.

| | $^9$Be$^+$ |
|---|---|
| $\mu[\mu_N]$ (Ref. [25]) | − 1.177 432(3) |
| Atomic mass $[u]$ (Ref. [26]) | 9.012 182 20(43) |
| $g_N$ | − 1.755 335 5(25) |
| $\varepsilon \times 10^{-9}$ | − 6.602 679(17) |
| $\varepsilon\alpha^4 g/2 A^{(4)}$ | − 624.600 44 |
| $\varepsilon\alpha^5 A^{(5)}_{\mathrm{rec}}$ | 0.006 85 |
| $\varepsilon\alpha^6 A^{(6)}$ | − 0.820 96 |
| $\varepsilon\alpha^7 A^{(7)}$ | 0.021 8(36) |
| $A_{\mathrm{theor}}$ (point nucleus) | − 625.392 7(36)(16) |
| Ref. [23] | − 625.401(22) |
| $A_{\mathrm{expt}}$ (Ref. [27]) | − 625.008 837 048(10) |
| $(A_{\mathrm{expt}} - A_{\mathrm{theor}})/A_{\mathrm{expt}}$ | −614(6)(3) ppm |
| Ref. [23] (theory) | −514(16) ppm |
| $\tilde{r}_Z$ | 4.07(5)(2) fm |
| $r_E$ (Ref. [28]) | 2.519(12) fm |

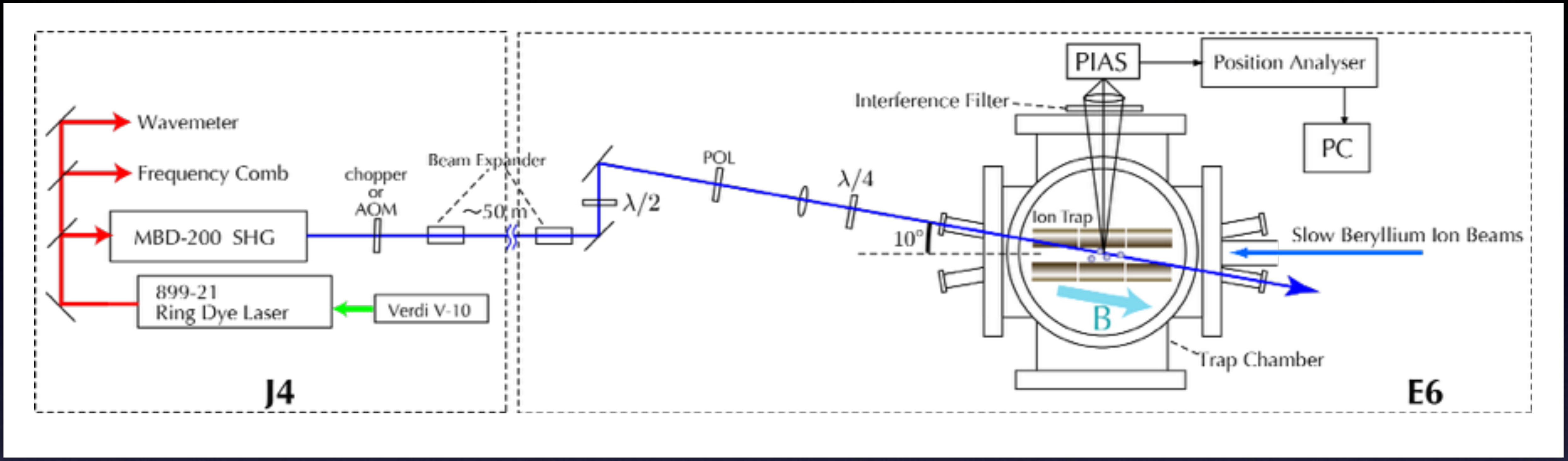
by M. Puchalski and K. Pachucki
PRL **111**, 243001 (2013) and PRA **89**, 032510 (2014)

# Experimental Procedure

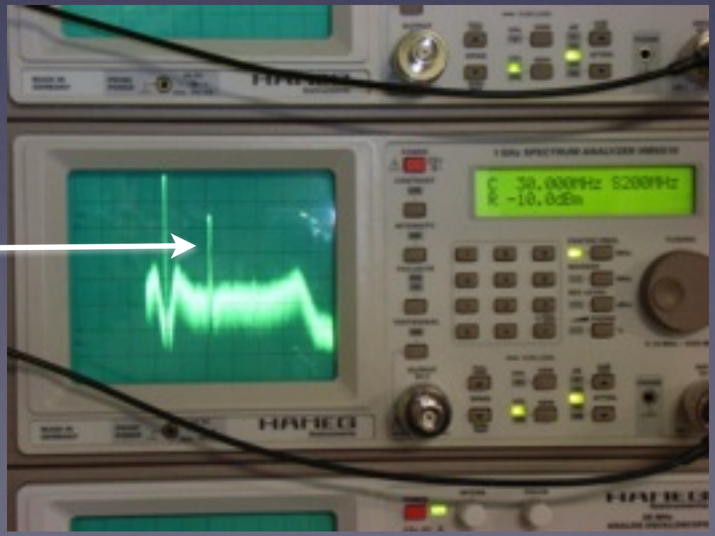


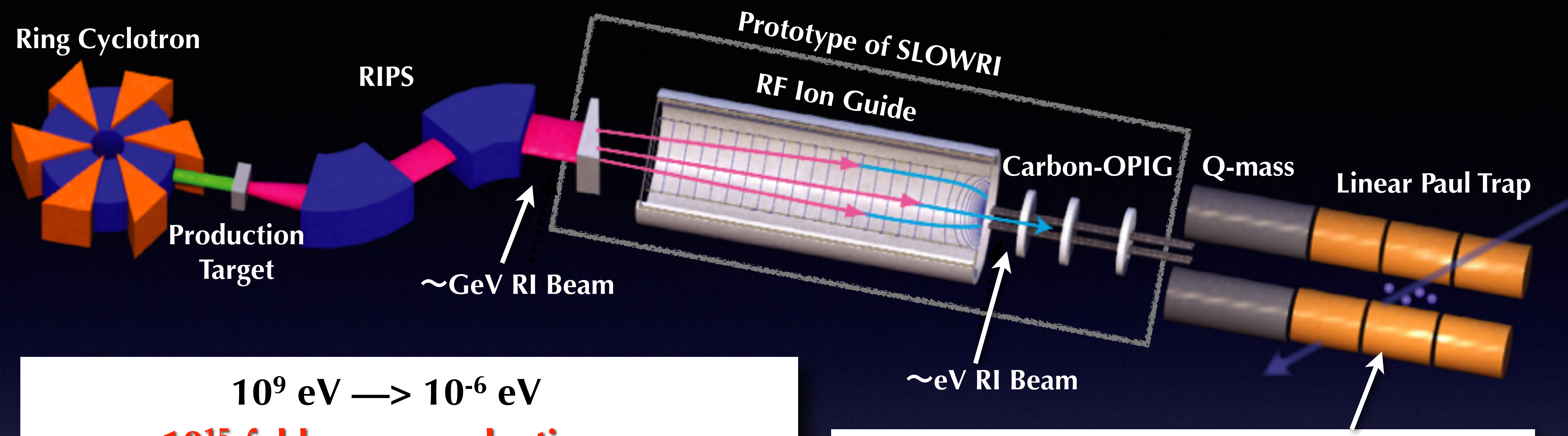

Prototype of SLOWRI
Ring Cyclotron
RIPS
RF Ion Guide
Carbon-OPIG
Q-mass
Linear Paul Trap
Production Target
~GeV RI Beam
~eV RI Beam
ion's trajectory
Combination of DC & inhomogeneous RF fields guide ions toward a small exit nozzle
RF carpet
0
3
potential
Radius [mm]
DC gradient
Octupole RF Confinement
OctuPole Ion beam Guide (OPIG)
radial: octupole RF trap
axial: continuous DC gradient

# Optical System



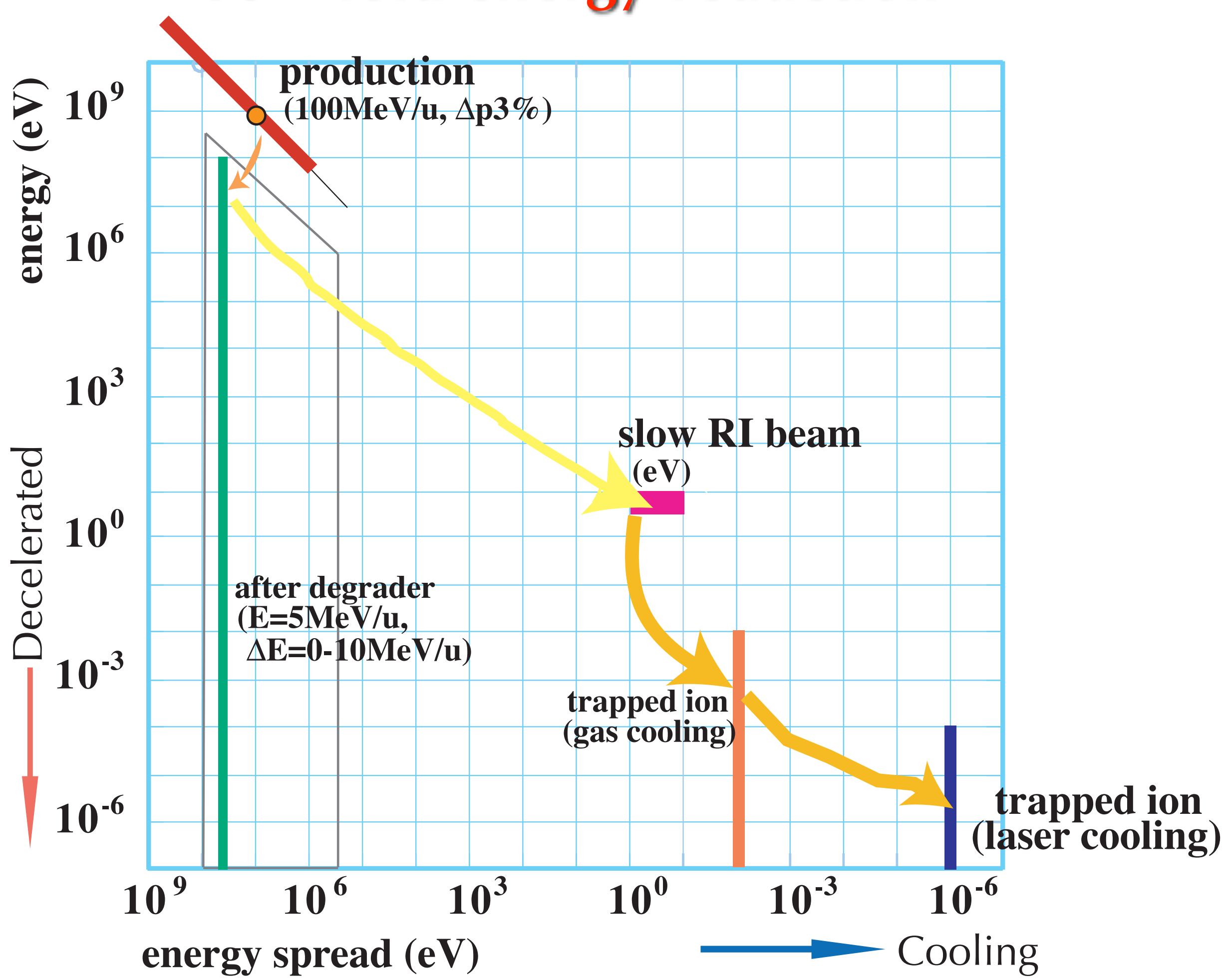

beat signal between
frequency comb & laser

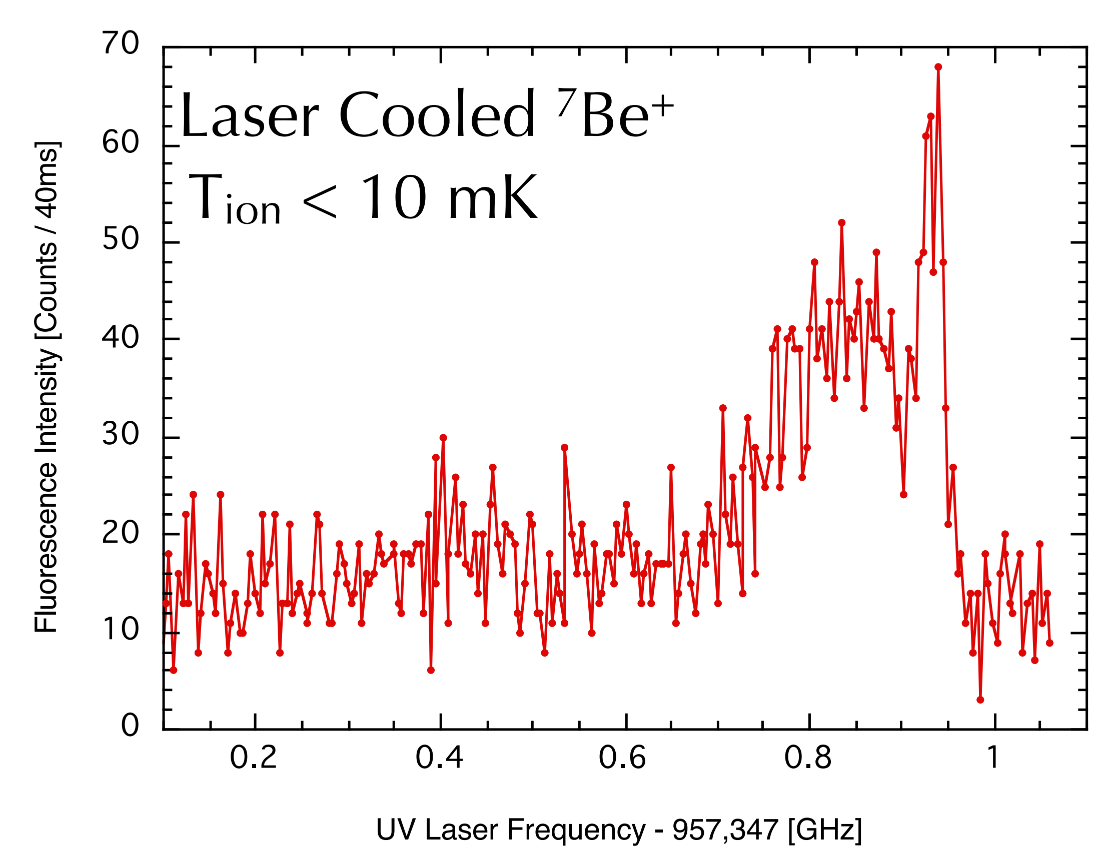





$10^9$ eV —> $10^{-6}$ eV

**$10^{15}$-fold energy reduction**

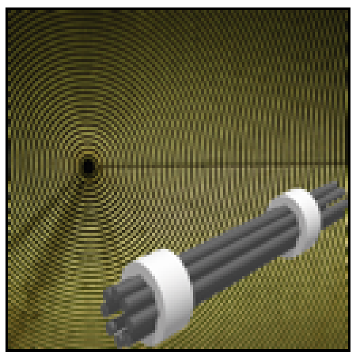



T. Nakamura *et al.*, PRA **74**, 052503 (2004)

K. Okada *et al.*, PRL **101**, 212502 (2008)

# How do you trap a very high-energy ion?



**Precision Measurement of the Hyperfine Structure of Laser-Cooled Radioactive $^{7}Be^{+}$ Ions Produced by Projectile Fragmentation**

K. Okada, M. Wada, T. Nakamura, A. Takamine, V. Lioubimov, P. Schury, Y. Ishida, T. Sonoda, M. Ogawa, Y. Yamazaki, Y. Kanai, T. M. Kojima, A. Yoshida, T. Kubo, I. Katayama, S. Ohtani, H. Wollnik, and H. A. Schuessler

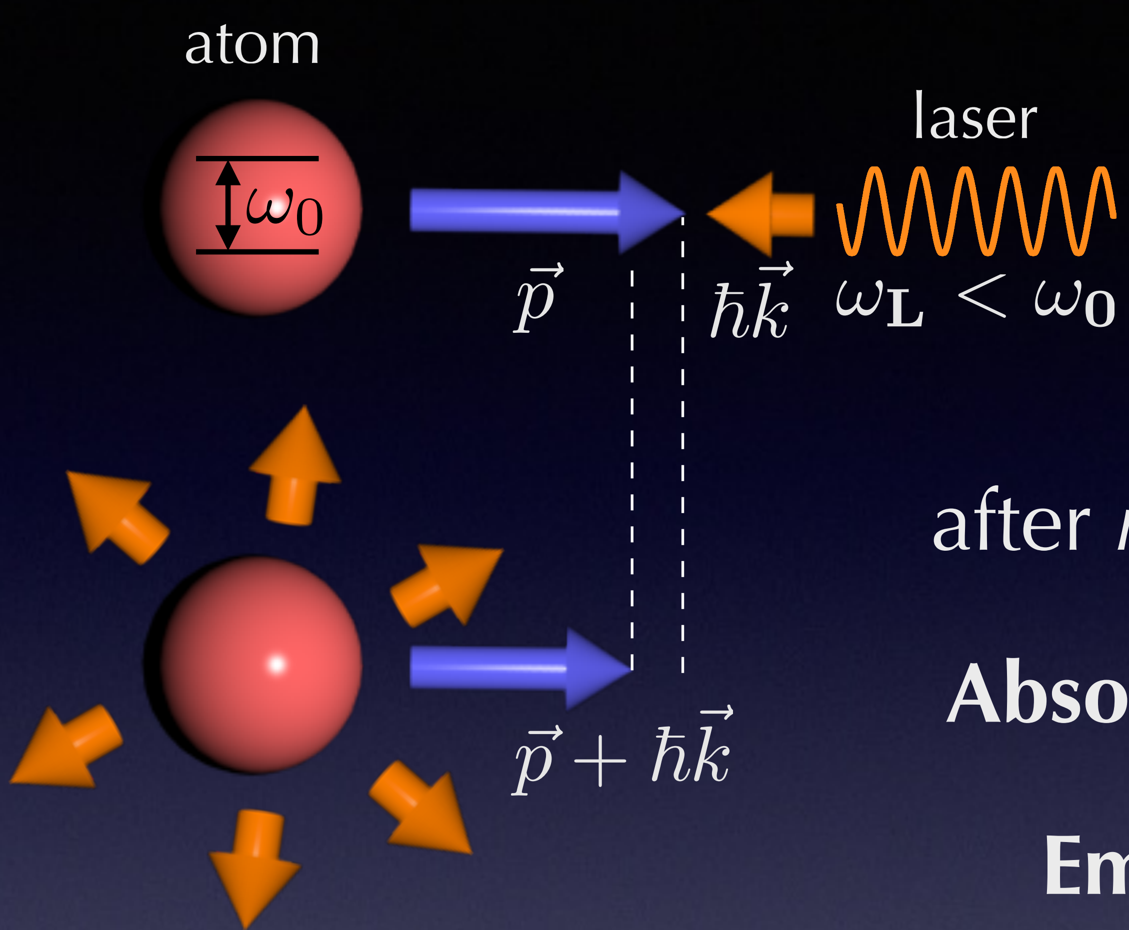
Phys. Rev. Lett. 101, 212502 (Published November 18, 2008)

ShareThis • Nuclear Physics

Measurements of nuclear moments give details about nuclear structure that cannot be obtained in any other way. However, traditional methods like nuclear magnetic resonance (NMR) and nuclear quadrupole resonance (NQR) require large numbers of stable nuclei to make a measurement and cannot be applied to unstable radioactive nuclei, which are usually produced in very small numbers. Instead, these unstable nuclei are best measured in traps, where atoms can be held for a long enough time to make sensitive measurements. The challenge is to take nuclei that were created in a high-energy collision and slow, trap, and cool them to make a precision measurement.

Writing in *Physical Review Letters*, a group at the newly commissioned Slow Radioactive Ion (SLOWRI) facility at RIKEN in Japan reports they have trapped and measured the magnetic moment of unstable $^{7}\mathrm{Be}$ ions. The group starts with $^{7}\mathrm{Be}$ ions from a high-energy fragmentation reaction and cools away 15 orders of magnitude in their kinetic energy, leaving trapped ions with temperatures less than 10 $\mathrm{mK}$. The RIKEN team then used a laser method to measure the atomic hyperfine structure of the ions to deduce the nuclear magnetic moment of $^{7}\mathrm{Be}$.

# How to cool an atom by laser?




**Laser Frequency in the atom's rest frame**

$$\omega_A = \omega_L\,(1+\beta)$$

**Doppler Effect**

after *n* cycles…

**Absorption:** $\sum \Delta\vec{p} = n \cdot \hbar\vec{k}$ 1 direction

**Emission:** $\sum \Delta\vec{p} = \vec{0}$ isotropically

**Cooled in total**

# How cool can the atom be?

**Doppler Cooling Limit**

$$T_{\mathrm{D}} = \frac{\hbar\Gamma}{2k_{\mathrm{B}}}$$

for $Be^{+}$ 2s→2p
$( \ \Gamma = 18 \ \mathrm{MHz} \ )$

$T_{\mathrm{D}} = 438 \ \mu K$ ⇔ 39 neV
in theory

$\Gamma$ : natural width

balance with heating effects
practically

# Laser Cooling of $Be^+$

## Requirement for laser cooling

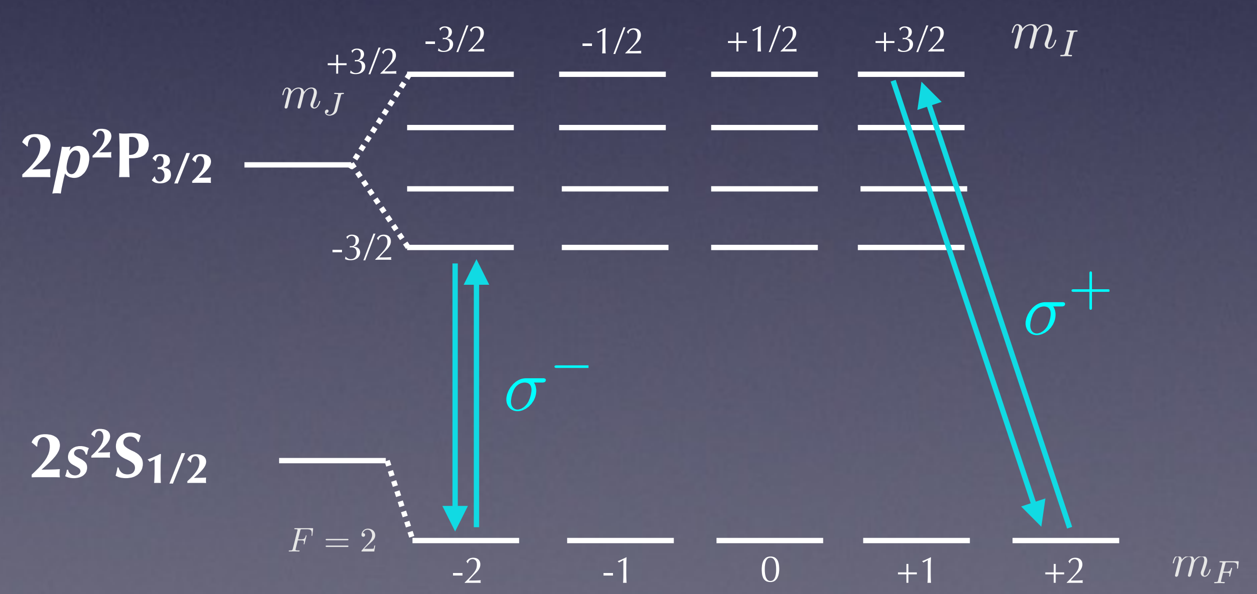
- **quick cycle**

  Absorption: 41 ns @ 100 mW/cm$^2$
  Emission: 8.8 ns
  } ⇒1 cycle 50 ns

  $|\Delta\vec{p}| \sim 128\ \mathrm{mm/s}$ in one cycle

  To cool $Be^+$ ~500 m/s @ 300 K ⇒ cooling time > 0.2 ms

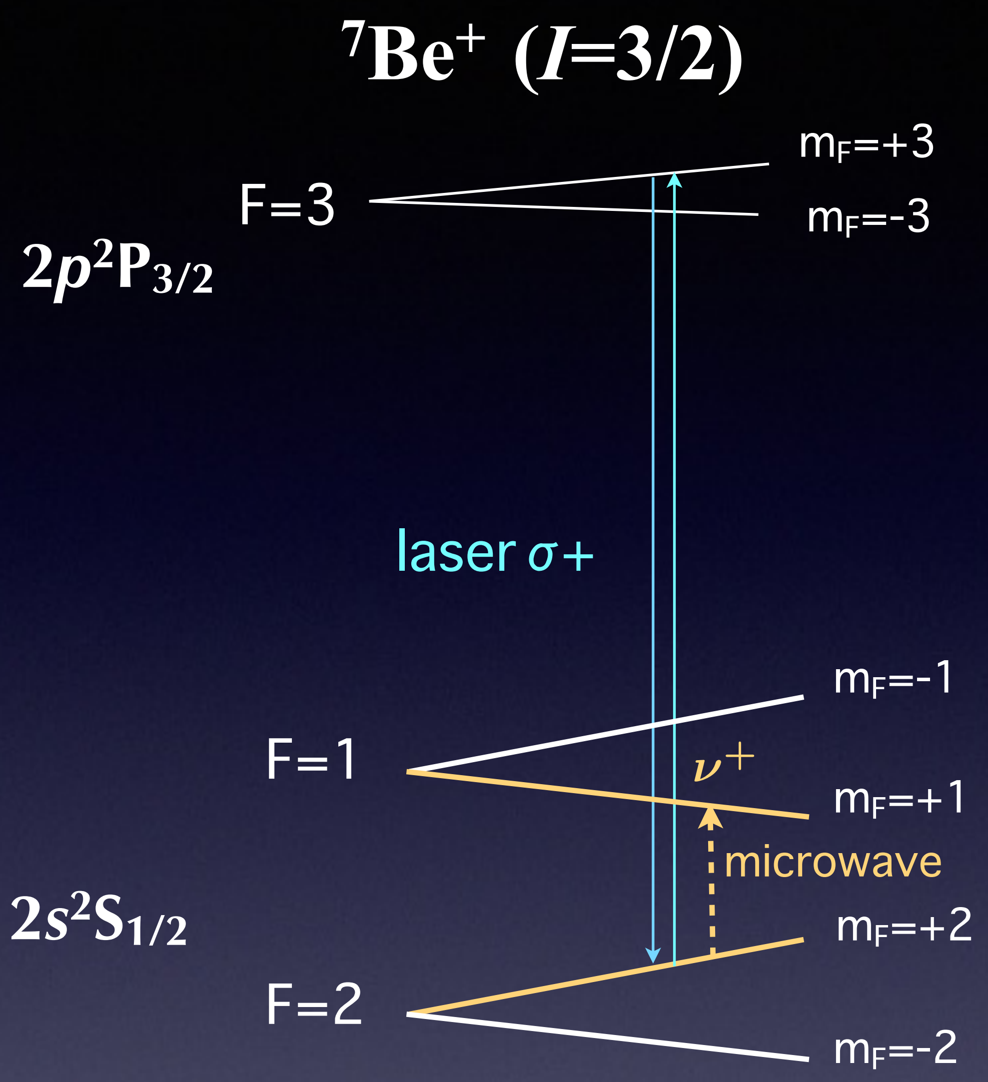
- **closed cycle (w/ one or a few lasers)**

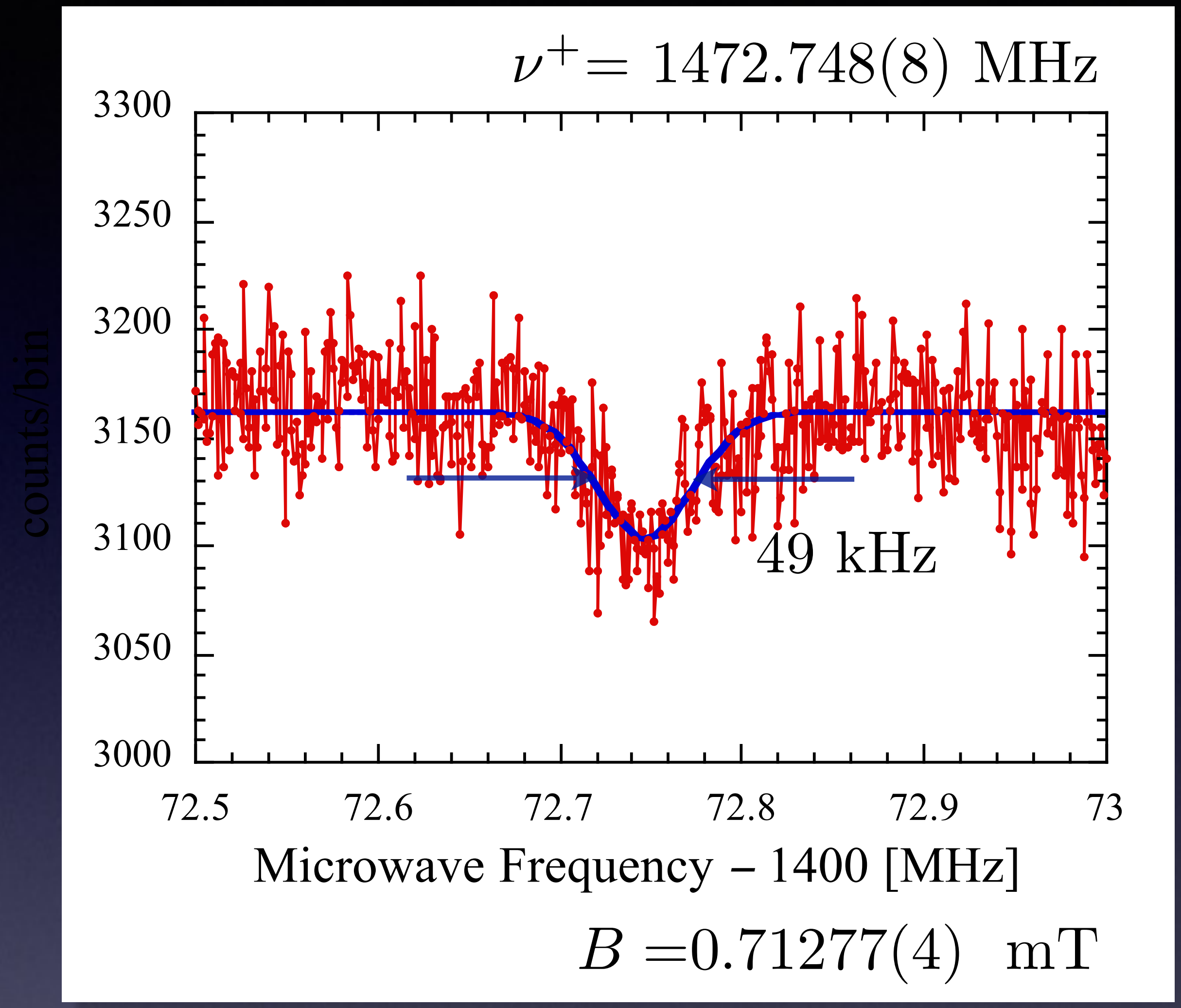



- **available wavelength** dye laser 626 nm $\xrightarrow{\text{SHG}}$ 313 nm

# HFS Spectroscopy of $^7Be^+$

K. Okada *et al.*, PRL **101**, 212502 (2008)



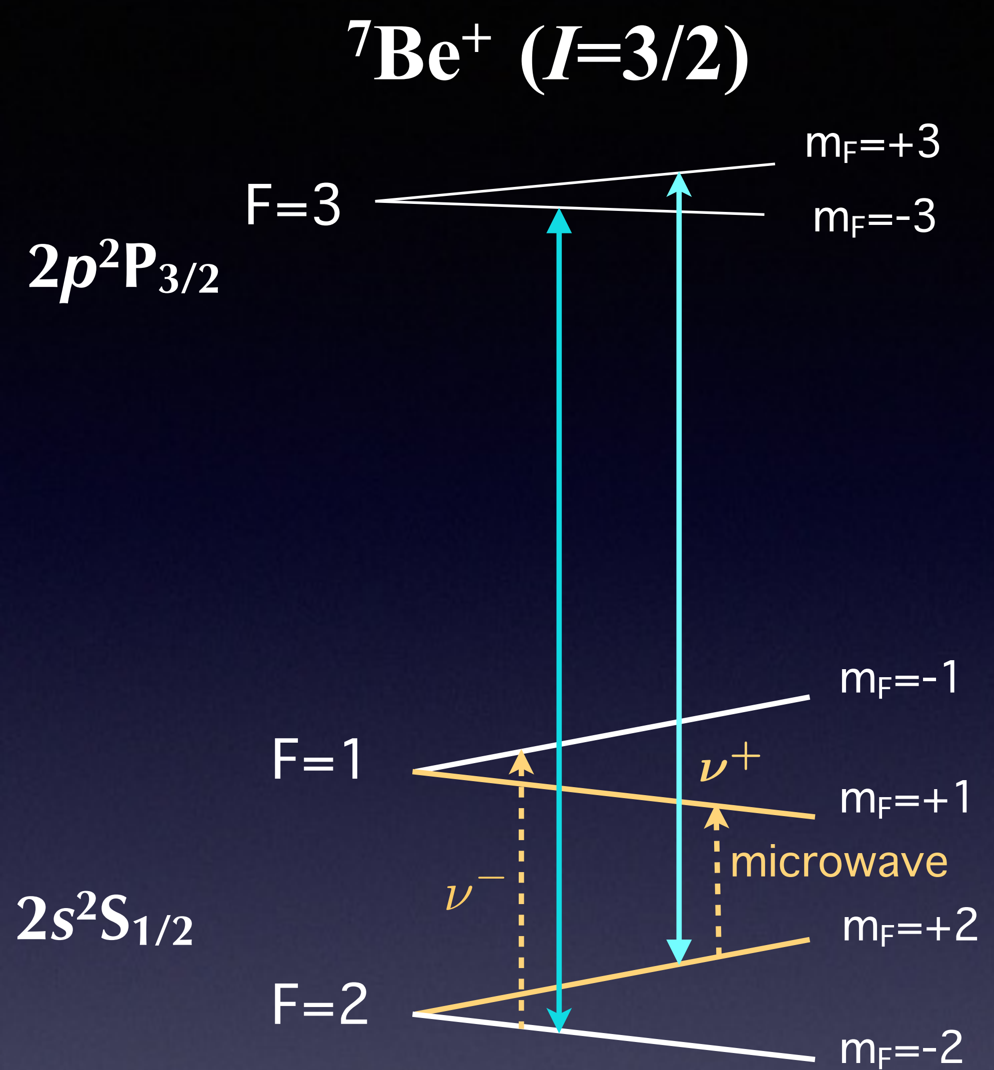




**Laser-Microwave Double Resonance**

1. Optical Pumping to Recyclable State by σ+ or σ- Laser
2. Laser Cooling
3. Microwave induces hf transition
4. Fluorescence detects population

# HFS Spectroscopy of $^7Be^+$

K. Okada *et al.*, PRL **101**, 212502 (2008)




| Quantity | $I_{\rm coil} = 12$ A | $I_{\rm coil} = 14$ A |
|---|---|---|
| $B$ (mT) | 0.6106 1(13) | 0.7127 7(4) |
| $\nu^+$ (MHz) | 1472.745 4(32) | 1470.613 1(13) |
| $\nu^-$ (MHz) | 1498.413 8(46) | 1500.577 1(13) |

Breit-Rabi formulae

$$\nu^+ = -A + \tfrac{1}{2}\sqrt{4A^2 - 2Ab(-1+\gamma) + b^2(-1+\gamma)^2} + \tfrac{1}{2}b(1-5\gamma)$$

$$\nu^- = -A + \tfrac{1}{2}\sqrt{4A^2 - 2Ab(-1+\gamma) + b^2(-1+\gamma)^2} - \tfrac{1}{2}b(1+\gamma)$$

$$b = g_J \mu_B B$$

$$\Rightarrow A = -742.772\,28(43)\ \mathrm{MHz}(5\cdot 10^{-7})$$

$$\frac{\mathrm{d}\nu}{\mathrm{d}B} = \mu_B \frac{4I}{2I+1} = 21\ \mathrm{MHz/mT} \Rightarrow I = 3/2$$

$$A \Rightarrow \mu_I = -1.39928(1) \quad \text{if } |{}^7\Delta^9| < 10^{-5}$$

# HFS Spectroscopy of $^7Be^+$

K. Okada *et al.*, PRL **101**, 212502 (2008)

## Comparison with Theory

TABLE II: Nuclear magnetic moment of $^7$Be [$\mu_N$]

| | | |
|---|---|---|
| Present experimental value | −1.39928(1) | |
| *Shell model* | | |
| Cohen-Kurath (8-16)POT [19] | −1.3787 | 1.5% |
| Large-basis shell model [20] | −1.132 | 20% |
| *Ab-initio* non-core shell model [21] | −1.138 | |
| *Quantum Monte Carlo* | | |
| Variational quantum Monte Carlo [22] | −1.110(2) | 20% |
| *Empirical* | | |
| Linear relations $\gamma_p = \alpha\gamma_n + \beta$ [24] [a] | −1.462 | 5% |
| Isospin doublet ($\langle\Sigma\sigma_z\rangle = 1$) | −1.377 | |

[a] $\gamma_n = \mu(^7\mathrm{Li})/I = +3.2564625(4)/(3/2)$ [18] is used.


[19] S. Cohen and D. Kurath, Nucl. Phys. 73, 1 (1965).

[20] P. Navŕatil and B.R. Barrett, Phys. Rev. C **57**, 3119 (1998).

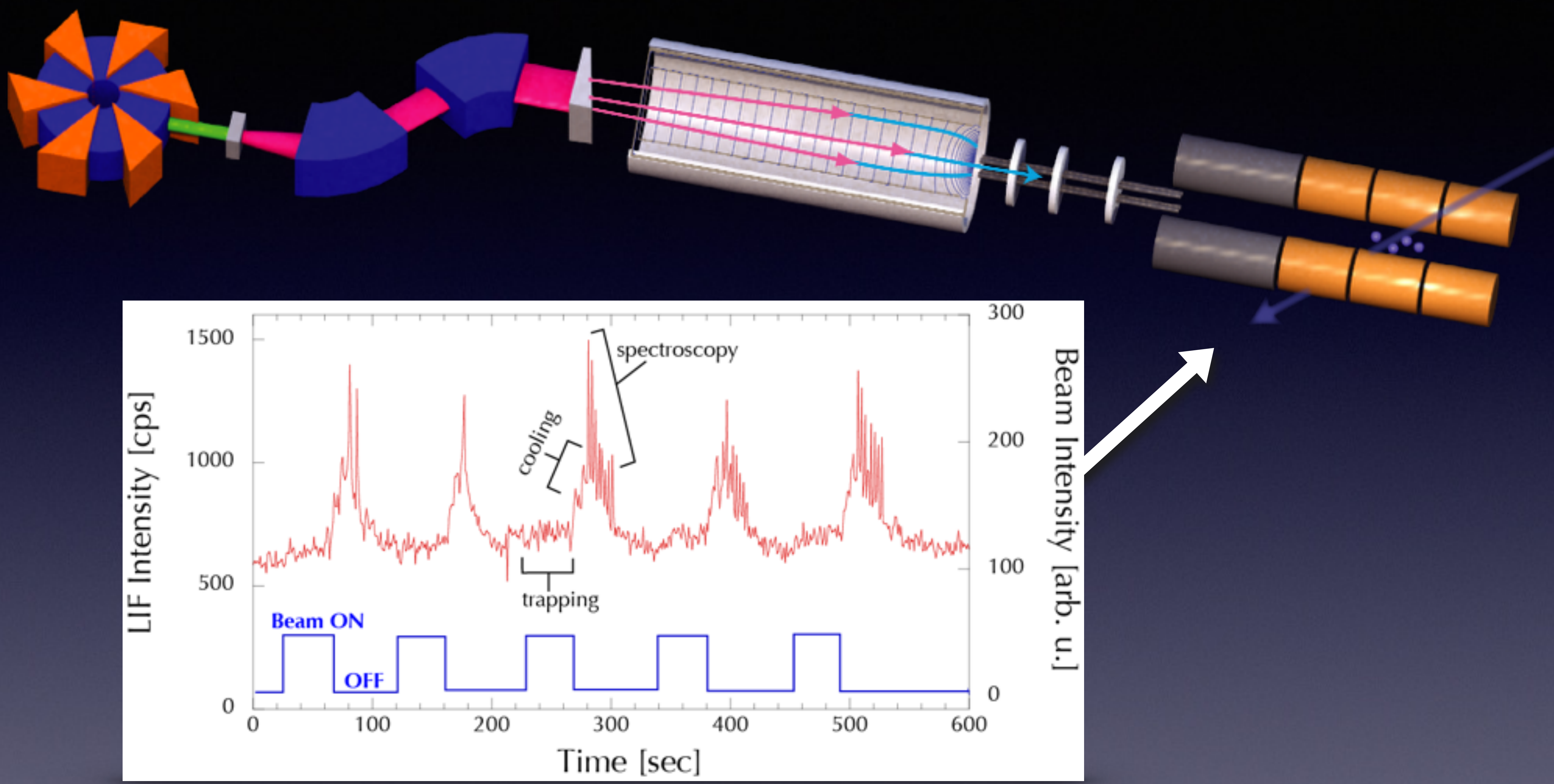
[21] P. Navŕatil and W.E. Ormand, Phys. Rev. C **68**, 034305 (2003).

[22] B. S. Pudliner, V.R. Pandharipande, J. Carlson, S.C. Pieper, and R.B. Wiringa, Phys. Rev. C **56**, 1720 (1997).

[23] R. Schiavilla, V.R. Pandharipande, and D.O. Riska, Phys. Rev. C**40**, 2294 (1989).

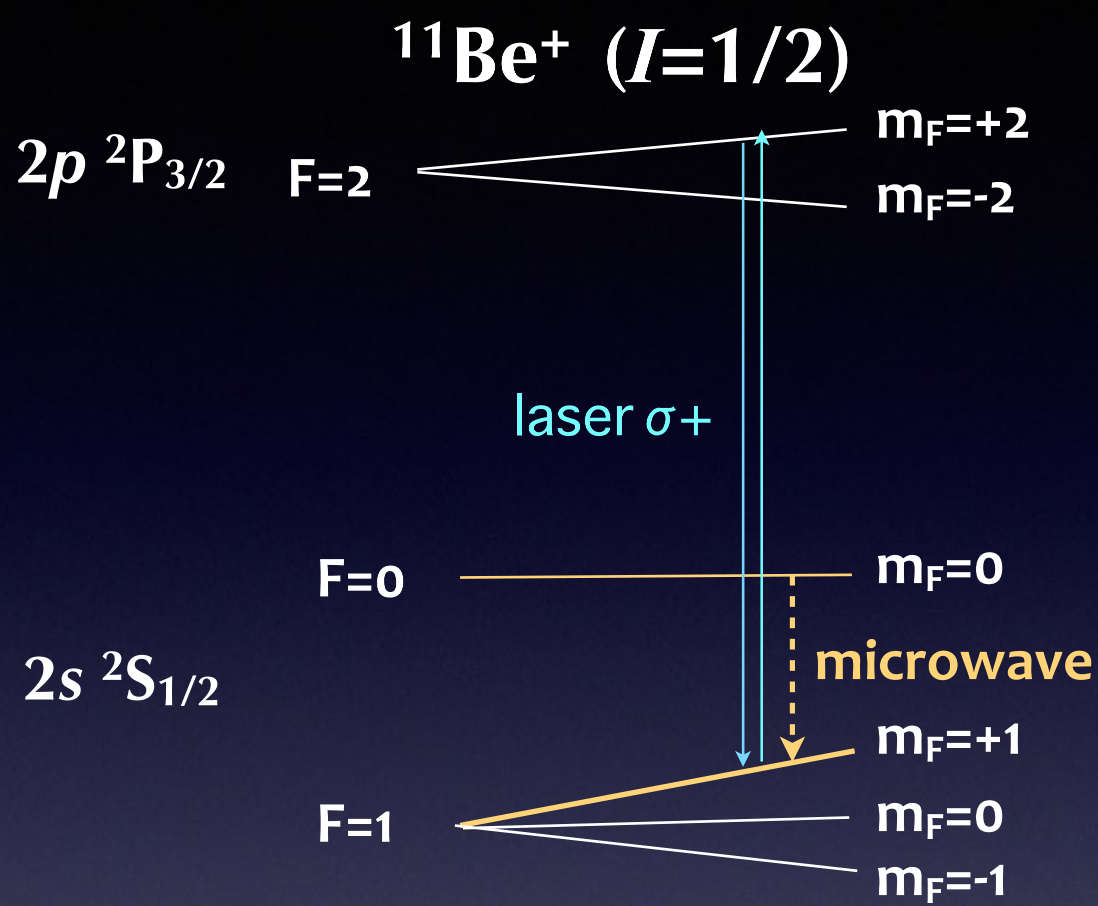
[24] B. Buck, A. C. Merchant, and S. M. Perez, Phys. Rev. C **63**, 037301 (2001).

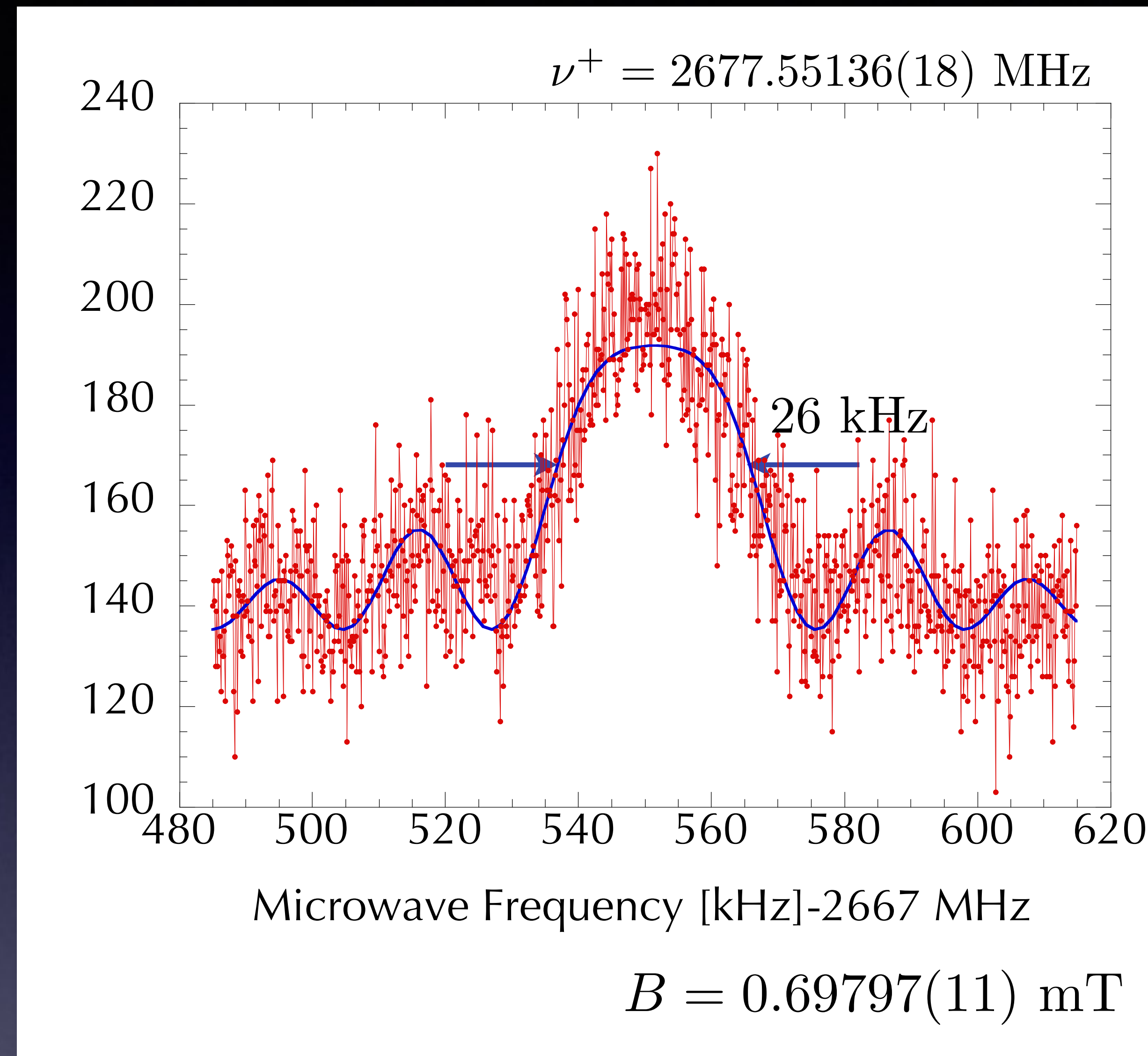
# Time Sequence of $^{11}$Be Experiment




1. Trapping for 40 s
2. Laser cooling (frequency chirping) in a few seconds
3. Spectroscopy: 2s scan x ~20

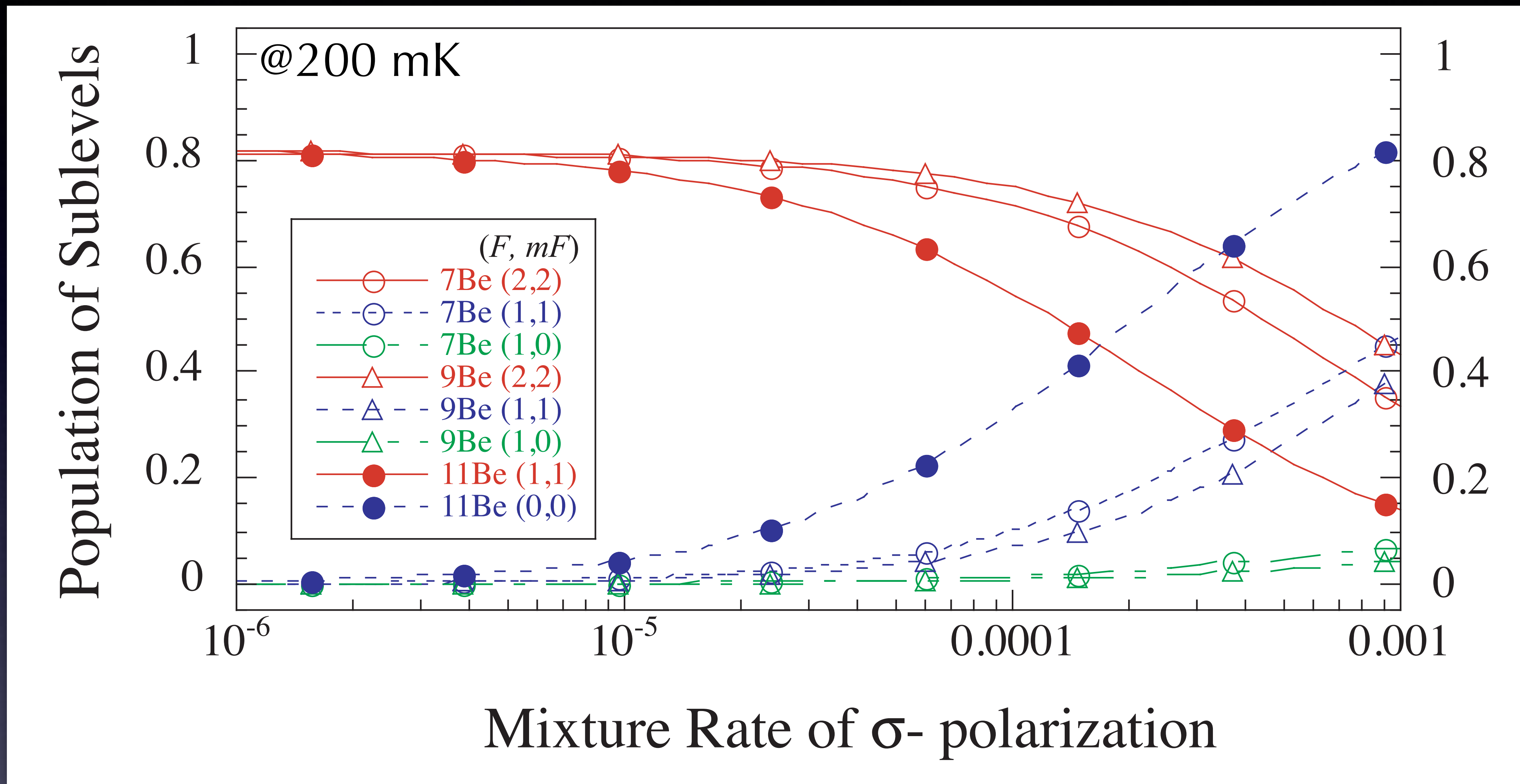
# HFS Spectroscopy of $^{11}Be^+$




**Why not a dip but a peak?**

# Optical Pumping of Be isotopes



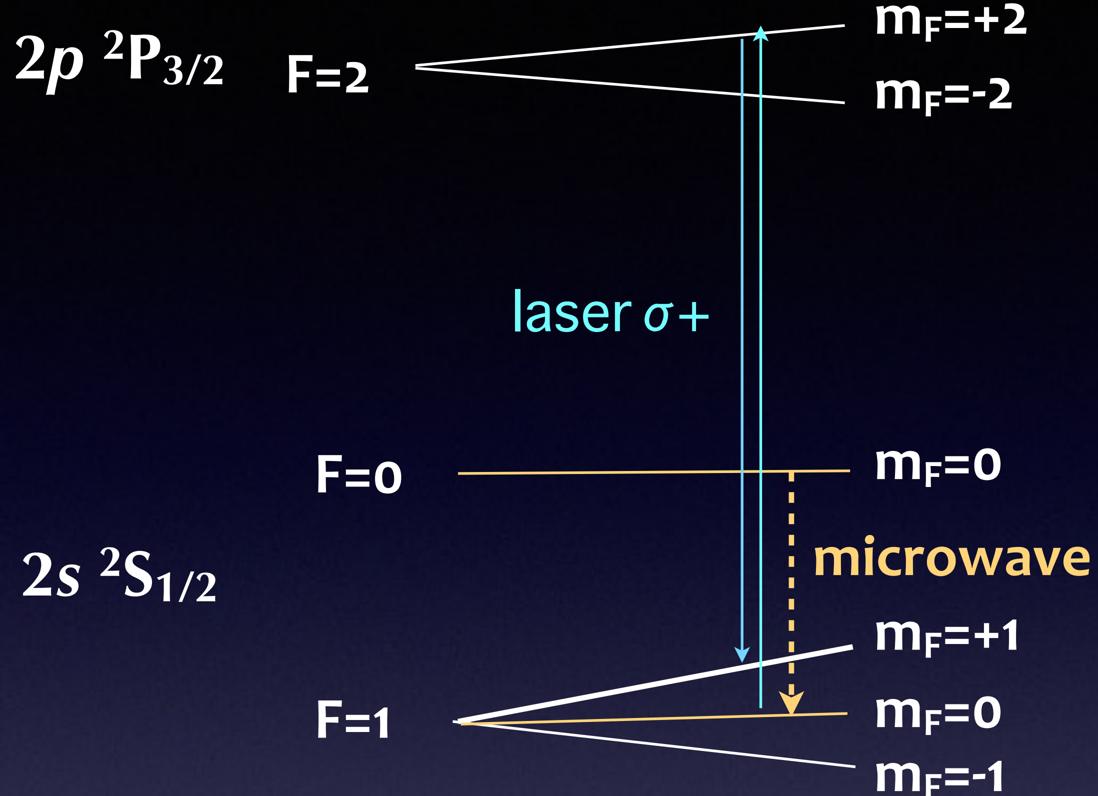

$^{11}Be^{+}$ ions tend to remain in smaller *F* state compared to $^{7,9}Be^{+}$

Laser cooling by only one laser

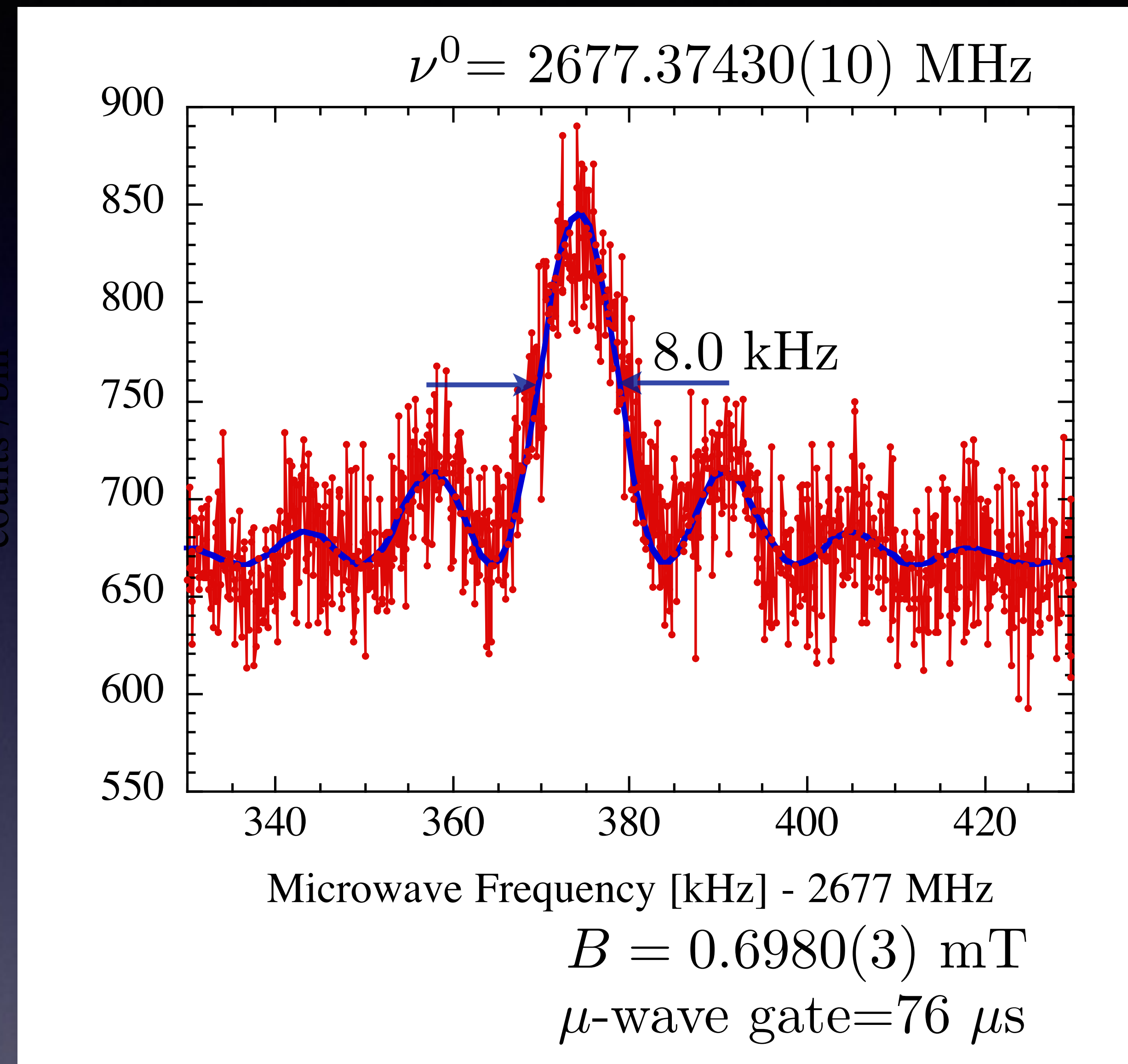
⇒ $^{11}$Be is difficult to optically pump because its hfs is larger than others?

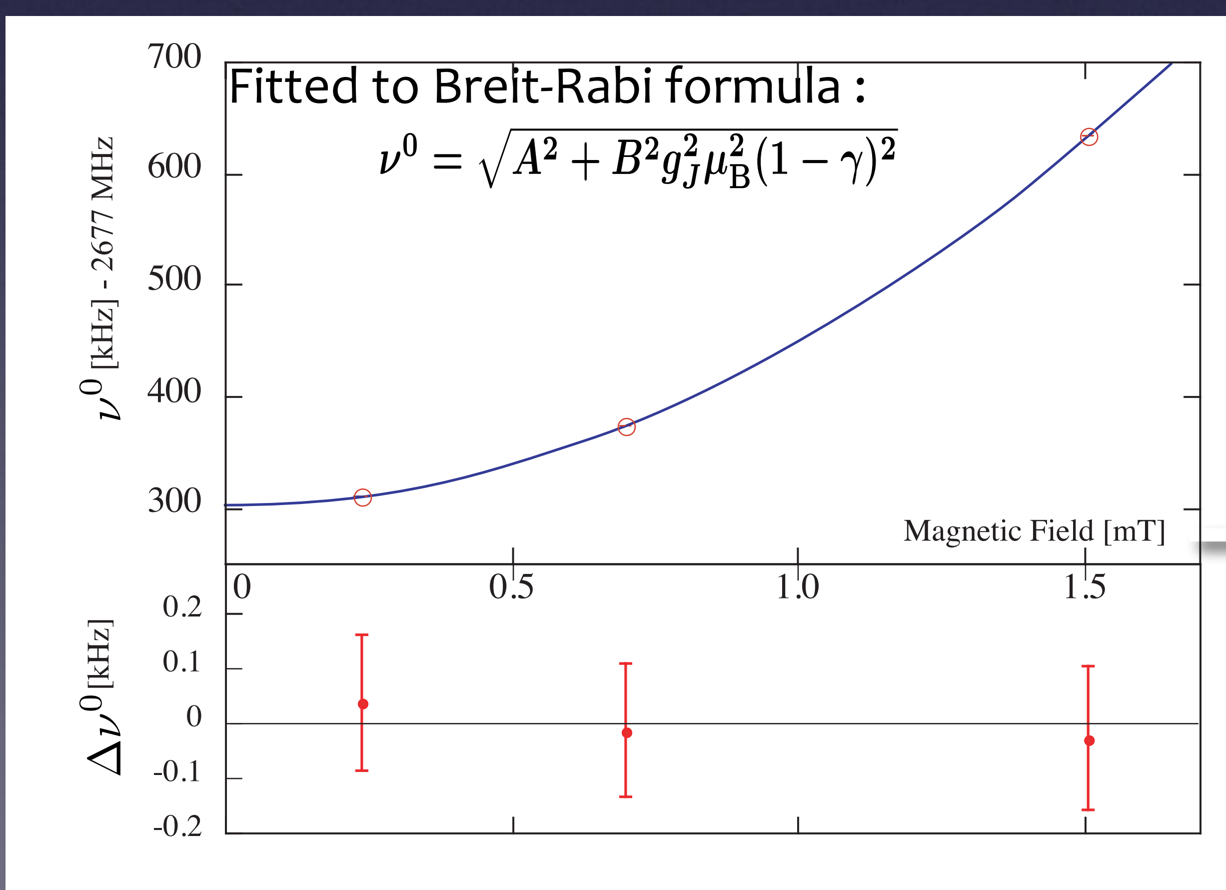
# HFS Spectroscopy of $^{11}Be^+$

## $^{11}Be^+$ ($I$=1/2)




A. Takamine *et al.*, PRL **112**, 162502 (2014)




$$A_{11} = -2677.302988(72)\mathrm{MHz}\ \ (3\cdot 10^{-8})$$

$$A \Rightarrow \mu_I = -1.68166(11)\ \text{n.m.}\quad \text{if } |^9\Delta^{11}| < 10^{-4}$$

$$\frac{d\nu}{dB} = \mu_B \frac{4I}{2I+1} = 14\ \mathrm{MHz/mT} \Rightarrow I = 1/2$$

# Systematic Errors for $^{11}Be^+$ HFS $A$

- Light shift? No. Laser is chopped off.
- Magnetic field? Measured from $^9Be^+$ HFS
  Feedback control to <10 ppm
- Microwave synthesizer? Locked to GPS
- AC Stark shift due to rf voltage?

second-order Doppler shift

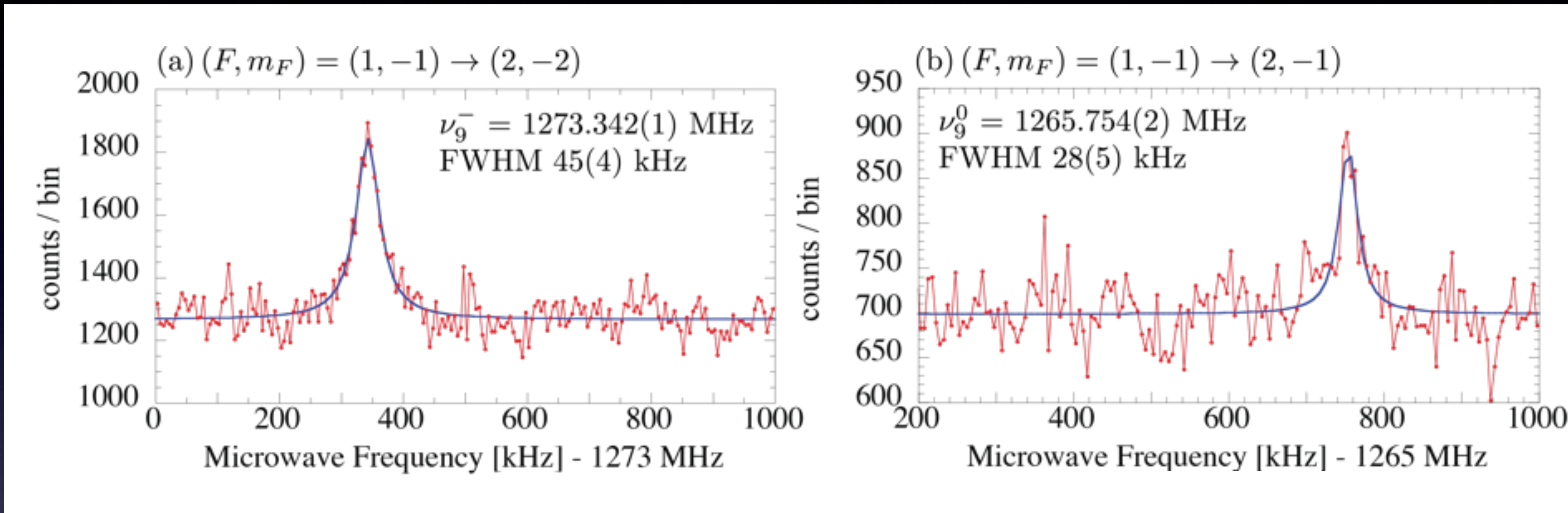
$$\Delta\nu_D = -\frac{3k_BT}{2mc^2}\left(1+\frac{2}{3}\left(\frac{\omega_s}{\omega_m}\right)^2\right)\cdot\nu = -5.6\times10^{-13}\ \mathrm{MHz}$$

quadratic Stark shift

$$\Delta\nu_S = -4k_s\left(\frac{\omega_s}{\omega_m}\right)^2\frac{m\Omega^2}{e^2}k_BT\cdot\nu = -5.0\times10^{-23}\ \mathrm{MHz}$$

# Magnetic Field Determination

## HFS spectroscopy of $^{9}Be^{+}$




$$b = -\frac{1}{2}\left\{(\nu_9^- + A_9)\left(\frac{1}{\gamma_9}+1\right) + \frac{A}{2}\left(-\frac{1}{\gamma_9}+1\right)\right\} + \sqrt{\frac{1}{4}\left\{(\nu_9^- + A_9)\left(\frac{1}{\gamma_9}+1\right) + \frac{A}{2}\left(-\frac{1}{\gamma_9}+1\right)\right\}^2 - \frac{1}{\gamma_9}(\nu_9^- + A_9)^2 + \frac{1}{\gamma_9}A_9^2}$$

$$\Delta b = \left| -\frac{1}{2}\left(\frac{1}{\gamma_9} + \frac{1}{2}\frac{\frac{1}{2}\left\{(\nu_9^- + A_9)\left(\frac{1}{\gamma_9}+1\right) + \frac{A_9}{2}\left(-\frac{1}{\gamma_9}+1\right)\right\}\left(\frac{1}{\gamma_9}+1\right) - \frac{2}{\gamma_9}(\nu_9^- + A_9)}{\sqrt{\frac{1}{4}\left\{(\nu_9^- + A_9)\left(\frac{1}{\gamma_9}+1\right) + \frac{A}{2}\left(-\frac{1}{\gamma_9}+1\right)\right\}^2 - \frac{1}{\gamma_9}(\nu_9^- + A_9)^2 + \frac{1}{\gamma_9}A_9^2}}\right)\right| \Delta\nu_9^-$$

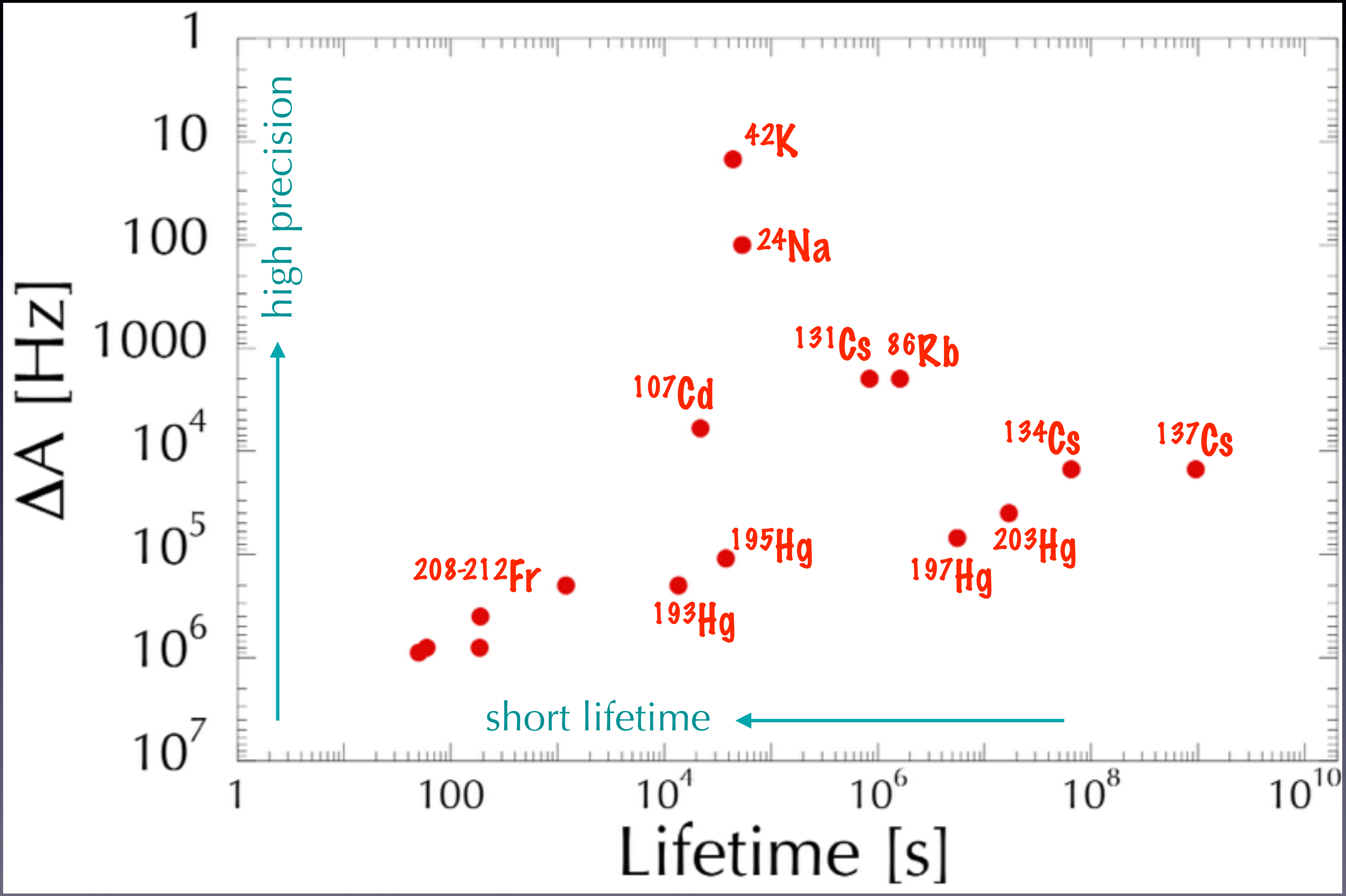
$$B = 1.102938(59)\ \mathrm{mT}$$

$$b = \frac{-A_9 - \sqrt{(\nu_9^{0-})^2 - 3A_9^2}}{1-\gamma_9}$$

$$\Delta b = \left| -\frac{\nu_9^{0-}}{(1-\gamma_9)\sqrt{(\nu_9^{0-})^2 - 3A_9^2}} \right| \Delta\nu_9^{0-}$$

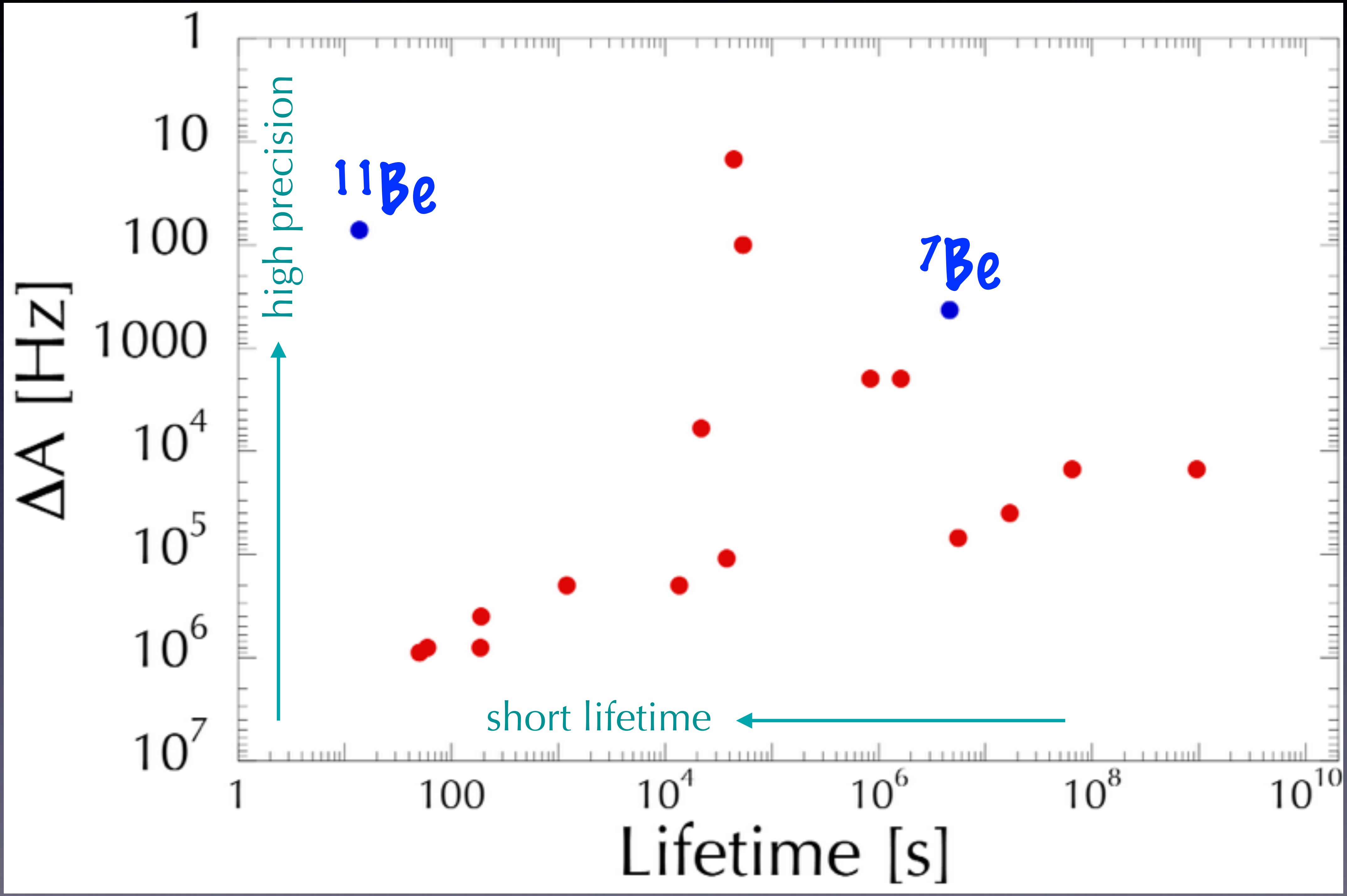
$$B = 1.10310(12)\ \mathrm{mT}$$

# Precision of hfs constant *A* for RI (hfa-measured nuclei)




J. R. Persson, Atomic Data and Nucelar Data Tables 99, 62 (2013)

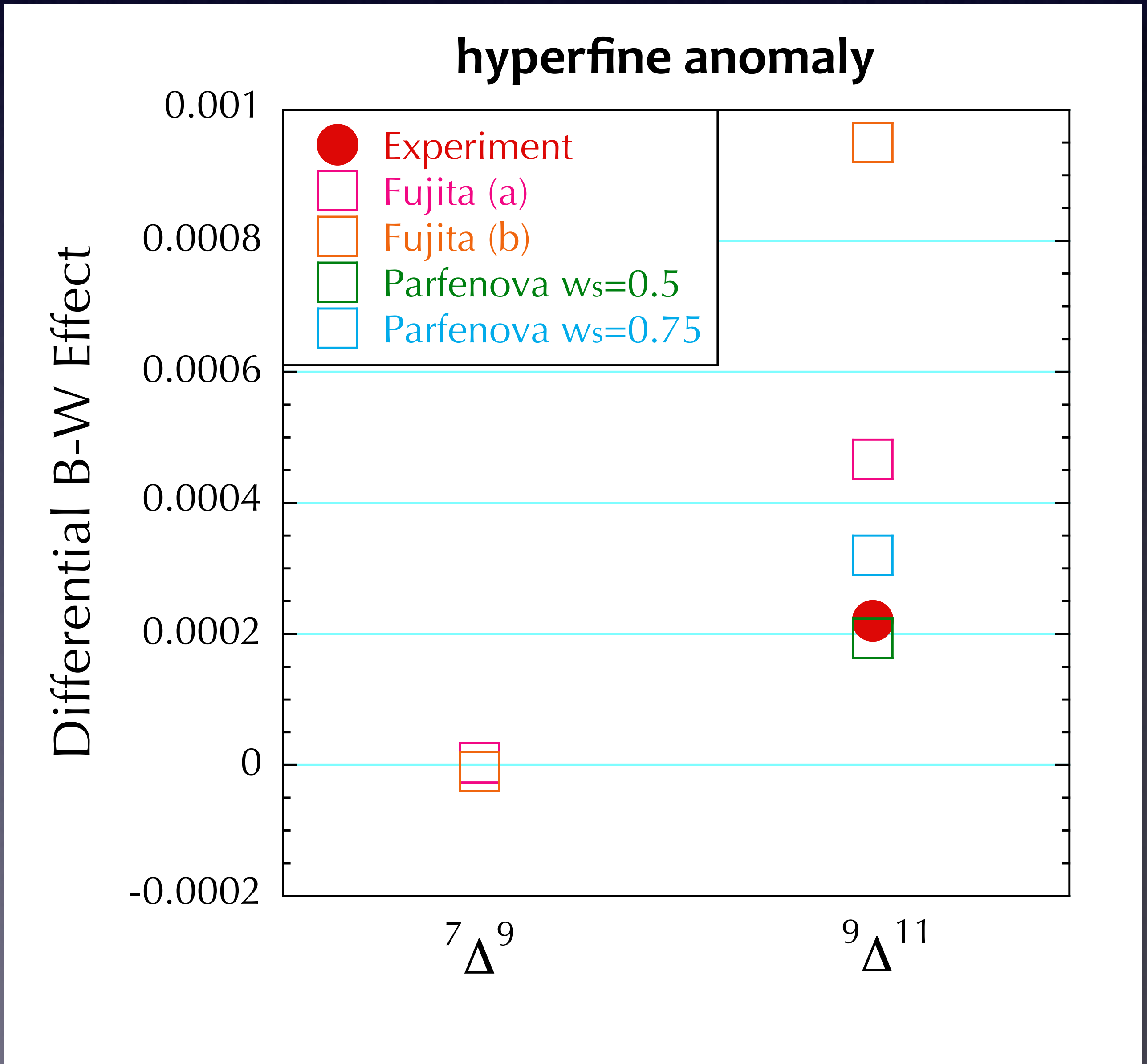
# Precision of hfs constant *A* for RI



# Results of Be HFS Spectroscopy

Our Works

| | Be-7 | Be-9 | Be-11 |
|---|---|---|---|
| HFS constant | -742.77228(43) | -625.008837048(10) | -2677.302988(72) |
| Nuclear Mag. Moment [n.m] | | -1.177432(3) | (-)1.6816(8) |
| {deduced from | { -1.39928(1) } | | { -1.68166(11) } |

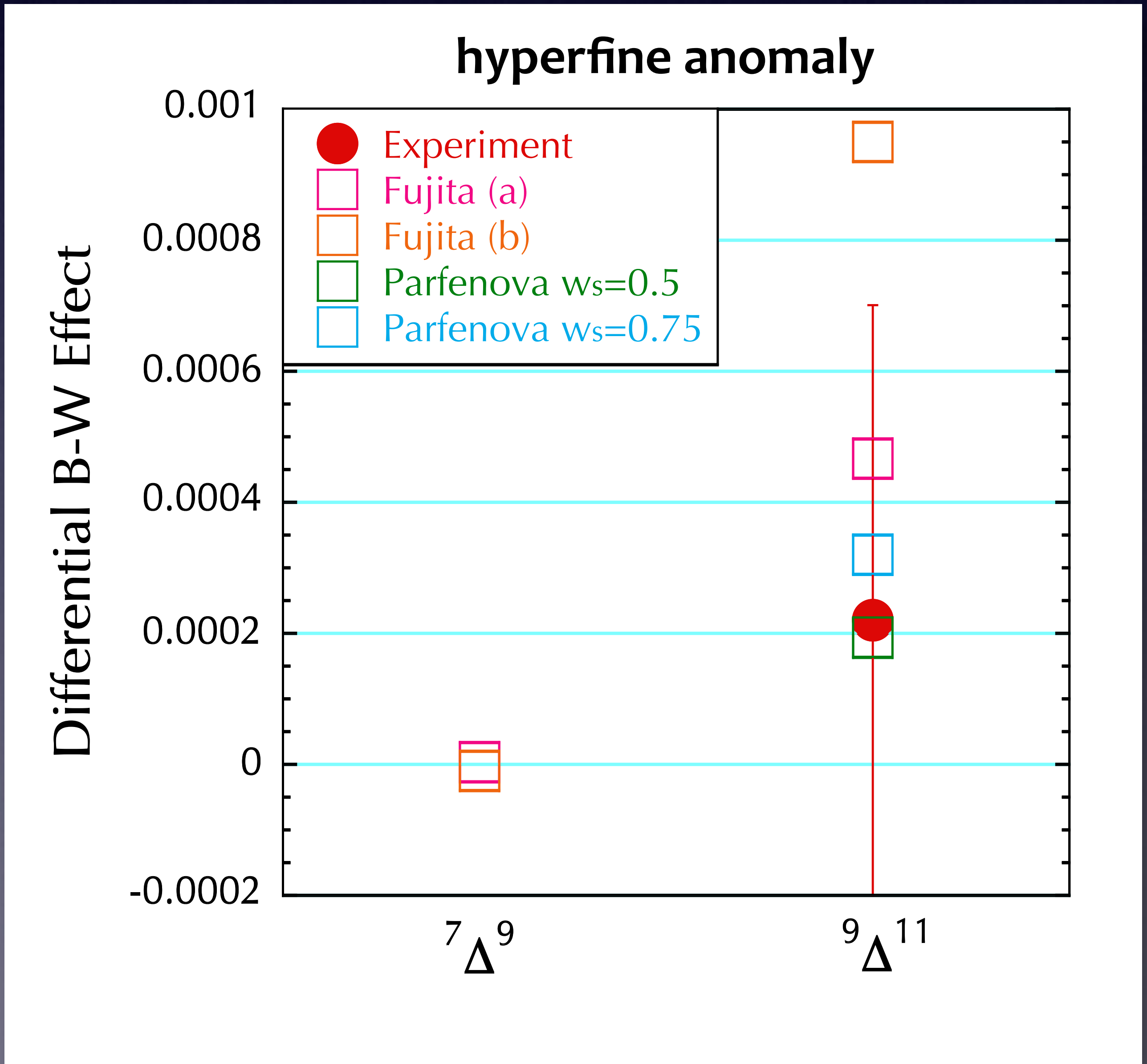



$$^9\Delta^{11} = \frac{A_9/(\mu_9/I_9)}{A_{11}/(\mu_{11}/I_{11})} - 1 = 2.2 \quad \times 10^{-4}$$

# Results of Be HFS Spectroscopy

Our Works

| | Be-7 | Be-9 | Be-11 |
|---|---|---|---|
| HFS constant | -742.77228(43) | -625.0088370529(11) | -2677.302988(72) |
| Nuclear Mag. Moment [n.m] | | -1.177432(3) | (-)1.6816(8) |
| {deduced from | { -1.39928(1) } | | { -1.68166(11) } |



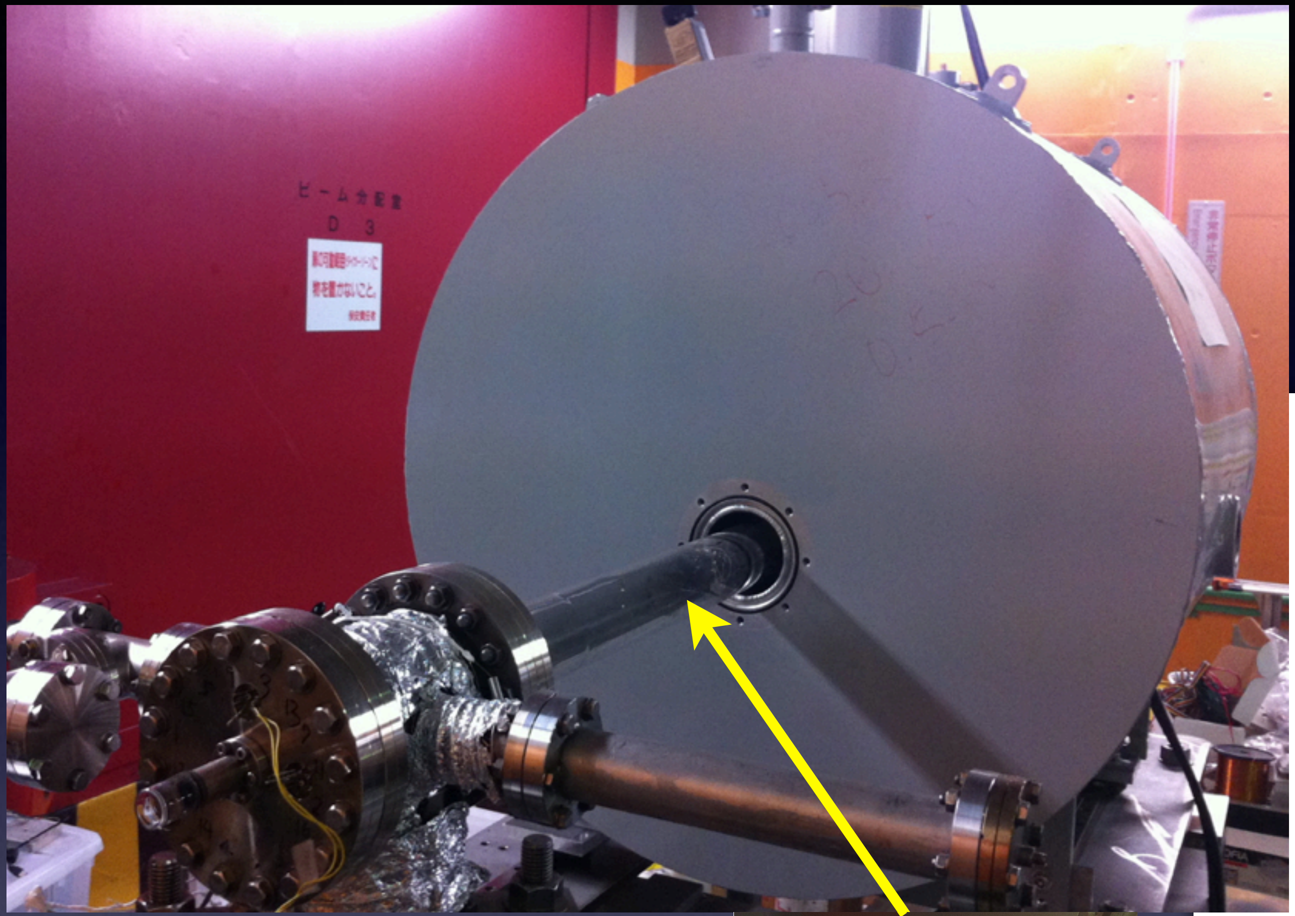

$$^9\Delta^{11} = \frac{A_9/(\mu_9/I_9)}{A_{11}/(\mu_{11}/I_{11})} - 1 = 2.2(47) \times 10^{-4}$$

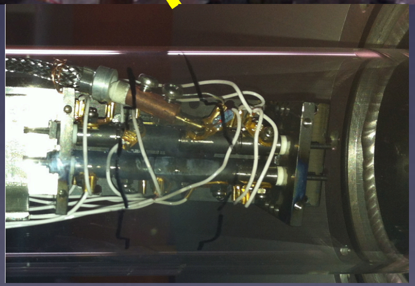
- More than one order of magnitude better accuracy for $\mu_I$ is required.
- More seriously, β-NMR method cannot be applied to Be-7.

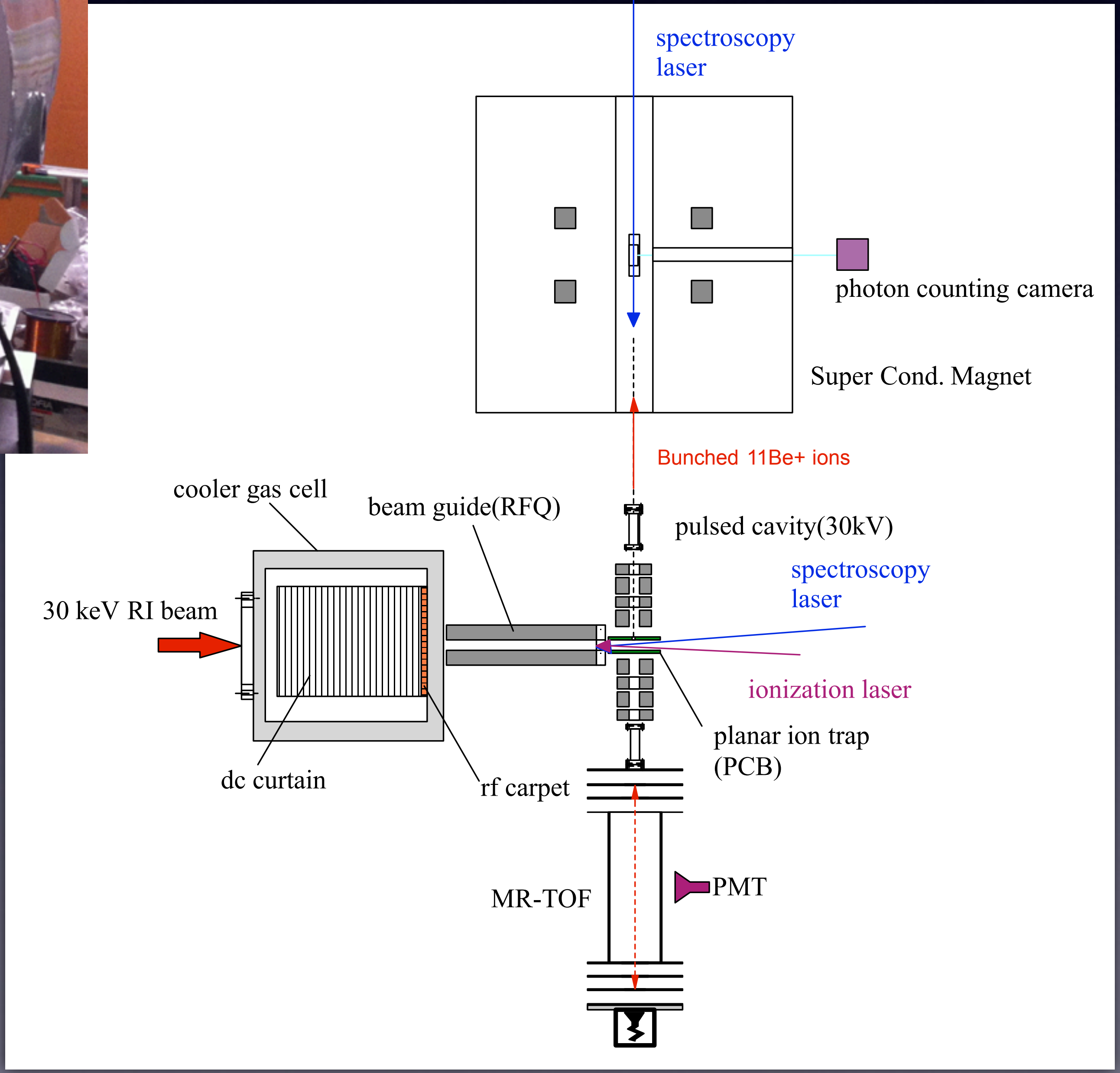
→ **How to measure it ?**

# HFS spectroscopy under a higher magnetic field

Superconducting Helmholtz Magnet ~1 T

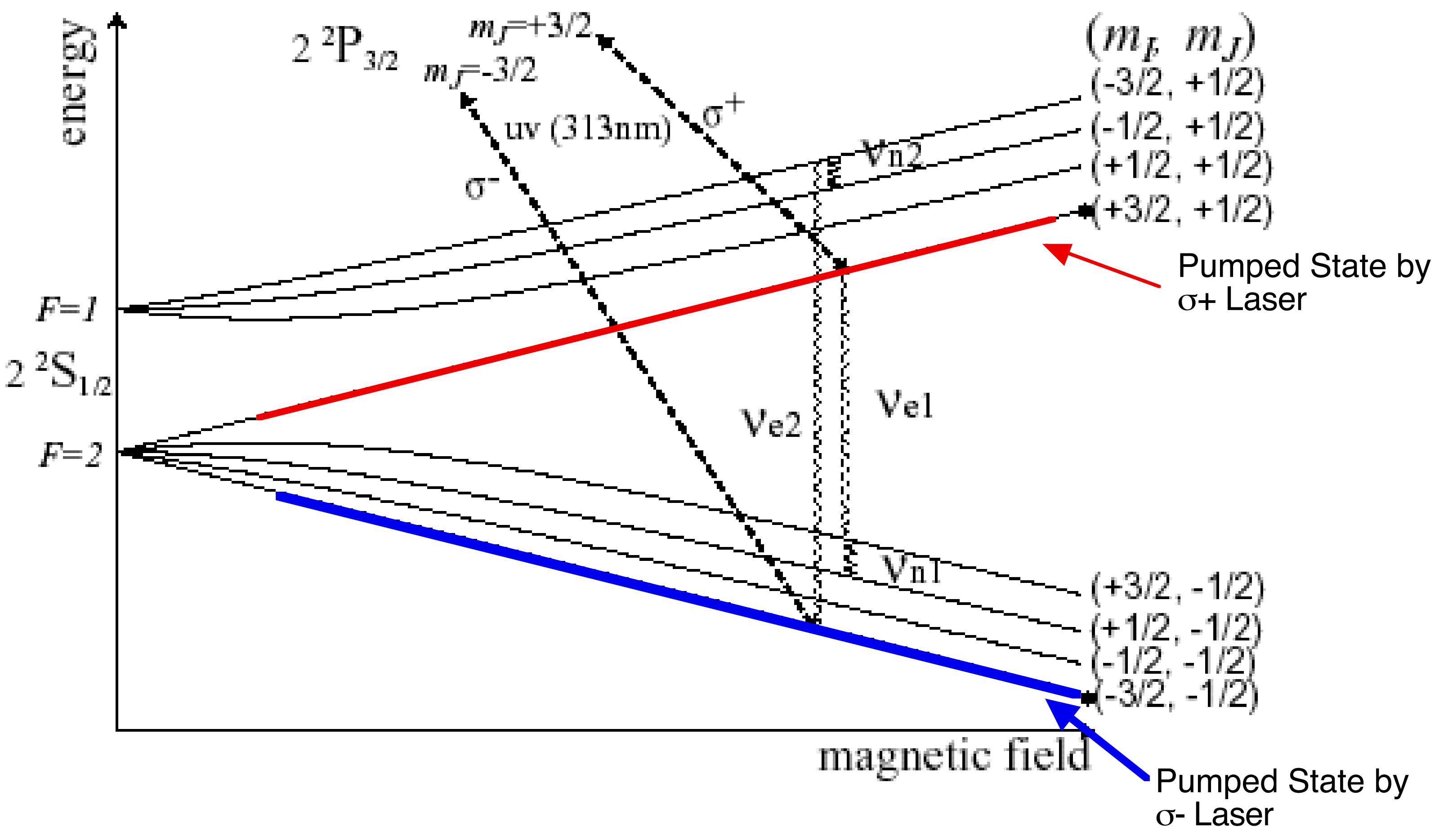


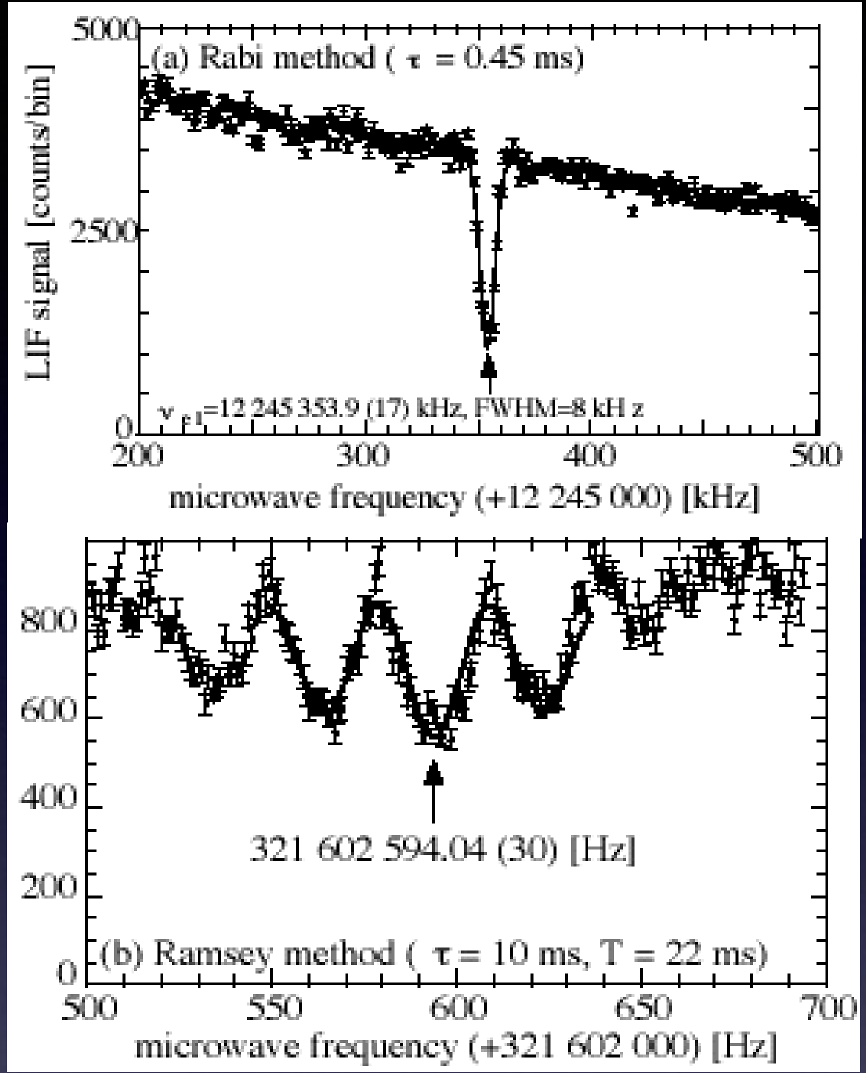




# Accurate and Independent Measurement of $\mu_I$ and $A$

Zeeman Splittings of the Ground-State Hyperfine Structure of $^9$Be




Breit-Rabi's Formula:

$$W_F(m_J, m_I, b) = -\frac{A}{4} - (m_J + m_I)\gamma b$$
$$+ m_J\sqrt{A^2(\tfrac{1}{2}+I)^2 + 2A(m_J+m_I)(\gamma-1)b + (\gamma-1)^2 b^2}$$
$$b = g_J \mu_B B_0 / h, \quad \gamma = g_I{}' / g_J$$

$$A = -625\,008\,835.23\ (75)\ \mathrm{Hz}$$

$$g_I' / g_J = 2.134\,780\,33\ (28) \times 10^{-4}$$

**T. Nakamura *et al.*, Opt. Comm 205,329 (2002)**

→ $^7Be^+$, $^{11}Be^+$
→ B-W effect

TO DO

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