Correlated-basis approach to nuclear five- and six-body problems

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Variational calculation for many-body quantum system

- Many-body wave function Ψ has all information of the nucleon dynamics
- Solve many-body Schoedinger equation
  ⇔ Eigenvalue problem with Hamiltonian matrix
  $$H\Psi = E\Psi$$
- Variational principle $\langle\Psi|H|\Psi\rangle = E \geqq E_0$ (“Exact” energy)
  （Equal holds if Ψ is the “exact” solution）

- Contents of my talk
  - Correlated-basis approach to few-body quantum physics
    - Example: *Ab initio* calculation for ^{4}He
  - ^{12}C+4N five-body calculations: Cluster and shell competition
    - Semi-microscopic
      Valence nucleon can occupy any orbit except the occupied orbits in the core nucleus
  - Six-body calculation for ^{6}He: Halo and clustering
    - Fully microscopic

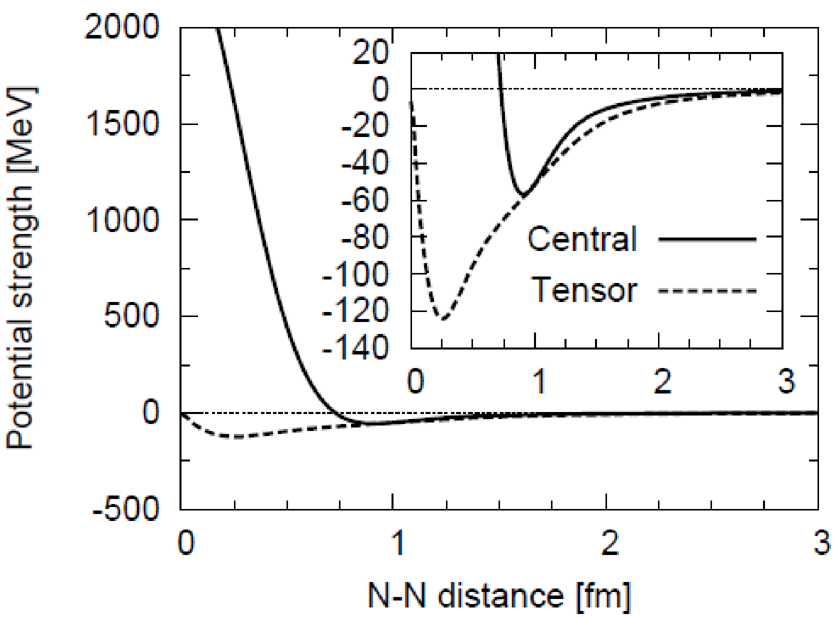
Hamiltonian and nuclear forces

Hamiltonian

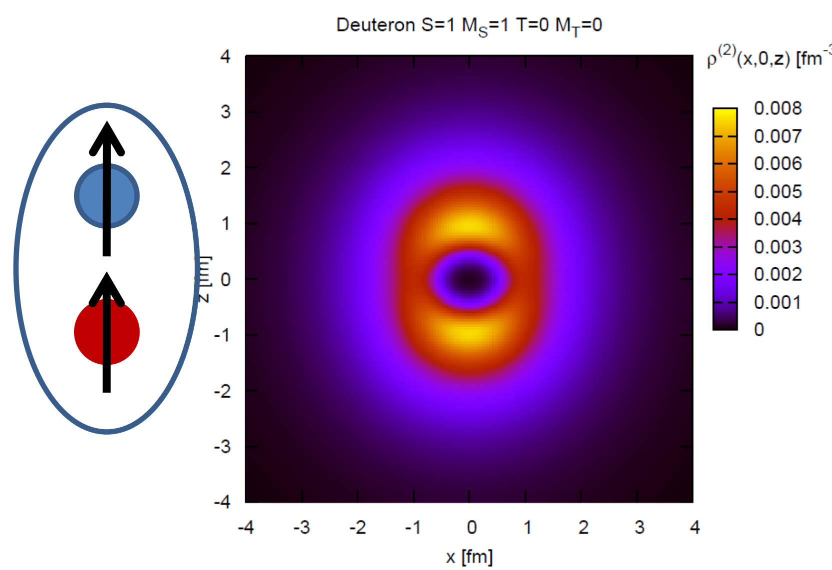
$$H = \sum_{i=1}^{A} T_i - T_{\rm cm} + \sum_{i<j}^{A} v_{ij} + \sum_{i<j<k}^{A} v_{ijk}$$

$$v_{12} = V_c(r) + V_{\rm Coul.}(r) P_{1\pi} P_{2\pi} + V_t(r) S_{12} + V_b(r) \boldsymbol{L} \cdot \boldsymbol{S}$$

- Argonne v8 type interactions (AV8’ , G3RS); “bare” interaction
  central, tensor, spin-orbit
- Three-nucleon force (3NF) E. Hiyama et al. PRC70, 031001(R) (2002)
  → reproduce binding energies of ^{3}H, ^{4}He



Deuteron (pn) density



Hamiltonian and nuclear forces

Hamiltonian

$$H=\sum_{i=1}^{A}T_i-T_{\rm cm}+\sum_{i<j}^{A}v_{ij}+\sum_{i<j<k}^{A}v_{ijk}$$

$$v_{12}=V_c(r)+V_{\rm Coul.}(r)P_{1\pi}P_{2\pi}+V_t(r)S_{12}+V_b(r)\boldsymbol{L}\cdot\boldsymbol{S}$$

- Argonne v8 type interactions (AV8' , G3RS); "bare" interaction
  central, tensor, spin-orbit
- Three-nucleon force (3NF) E. Hiyama et al. PRC70, 031001(R) (2002)
  → reproduce binding energies of ^{3}H, ^{4}He, inelastic form factor of ^{4}He

Basis function

$$\Psi_{(LS)JM_JTM_T}=\mathcal{A}\left\{\left[\psi_L^{(\rm space)}\psi_S^{(\rm spin)}\right]_{JM_J}\psi_{TM_T}^{(\rm isospin)}\right\}$$

$$\psi_{SM_S}^{(\rm spin)}=\left|[\cdots[[[\tfrac{1}{2}\tfrac{1}{2}]_{S_{12}}\tfrac{1}{2}]_{S_{123}}]\cdots]_{SM_S}\right\rangle$$

$\psi_{LM}^{(\rm space)}$: correlated Gaussian combined with two global vectors

Y. Suzuki, W.H., M. Orabi, K. Arai, FBS42, 33-72 (2008)

$$F_{(L_1L_2)LM}(u_1,u_2,A,\boldsymbol{x})=\exp\left(-\frac{1}{2}\widetilde{\boldsymbol{x}}A\boldsymbol{x}\right)[\mathcal{Y}_{L_1}(\widetilde{u_1}\boldsymbol{x})\mathcal{Y}_{L_2}(\widetilde{u_2}\boldsymbol{x})]_{LM}$$

Explicitly correlated basis approach

Correlated Gaussian with two global vectors

Y. Suzuki, W.H., M. Orabi, K. Arai, FBS42, 33-72 (2008)

$$\phi^{\pi}_{(L_1 L_2)LM_L}(A, u_1, u_2) = \exp(-\tilde{x}Ax)[\mathcal{Y}_{L_1}(\tilde{u}_1 x)\mathcal{Y}_{L_2}(\tilde{u}_2 x)]_{LM_L}$$

x: any relative coordinates (cf. Jacobi) $\quad \mathcal{Y}_\ell(\boldsymbol{r}) = r^\ell Y_\ell(\hat{\boldsymbol{r}})$

$$\tilde{x}Ax = \sum_{i,j=1}^{N-1} A_{ij} x_i \cdot x_j$$

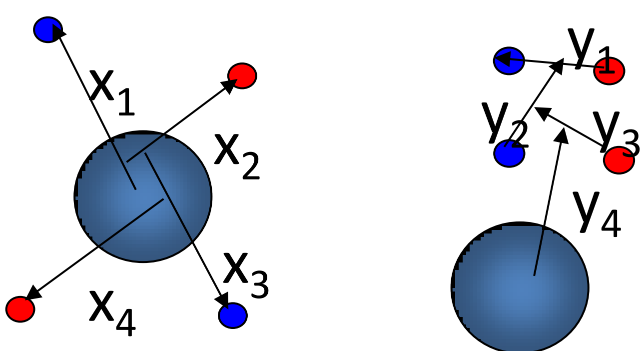
$$\tilde{u}_i x = \sum_{k=1}^{N-1} (u_i)_k x_k$$

Formulation for N-particle system
Analytical expression for matrix elements

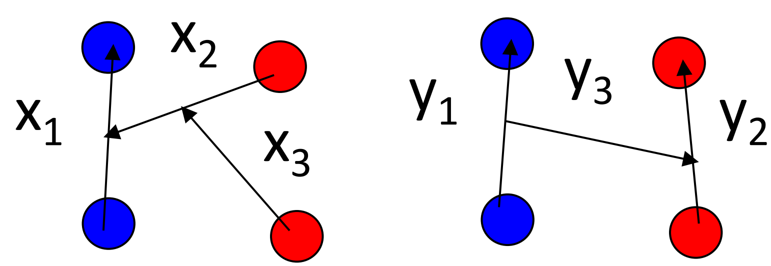
Functional form does not change under any coordinate transformation

$$y = Tx \quad \Longrightarrow \quad \tilde{y}By = \tilde{x}\tilde{T}BTx \qquad \tilde{v}y = \tilde{\tilde{T}v}x$$

Shell and cluster structure



Rearrangement channels



See Recent Review: J. Mitroy et al., Rev. Mod. Phys. 85, 693 (2013)

Basis optimization:
Stochastic Variational Method

Possibility of the stochastic optimization

1. increase the basis dimension one by one
2. set up an optimal basis by trial and error procedures
3. fine tune the chosen parameters until convergence

1. **Generate** $(A_k^1, A_k^2, \cdots\cdots, A_k^m)$ **randomly**

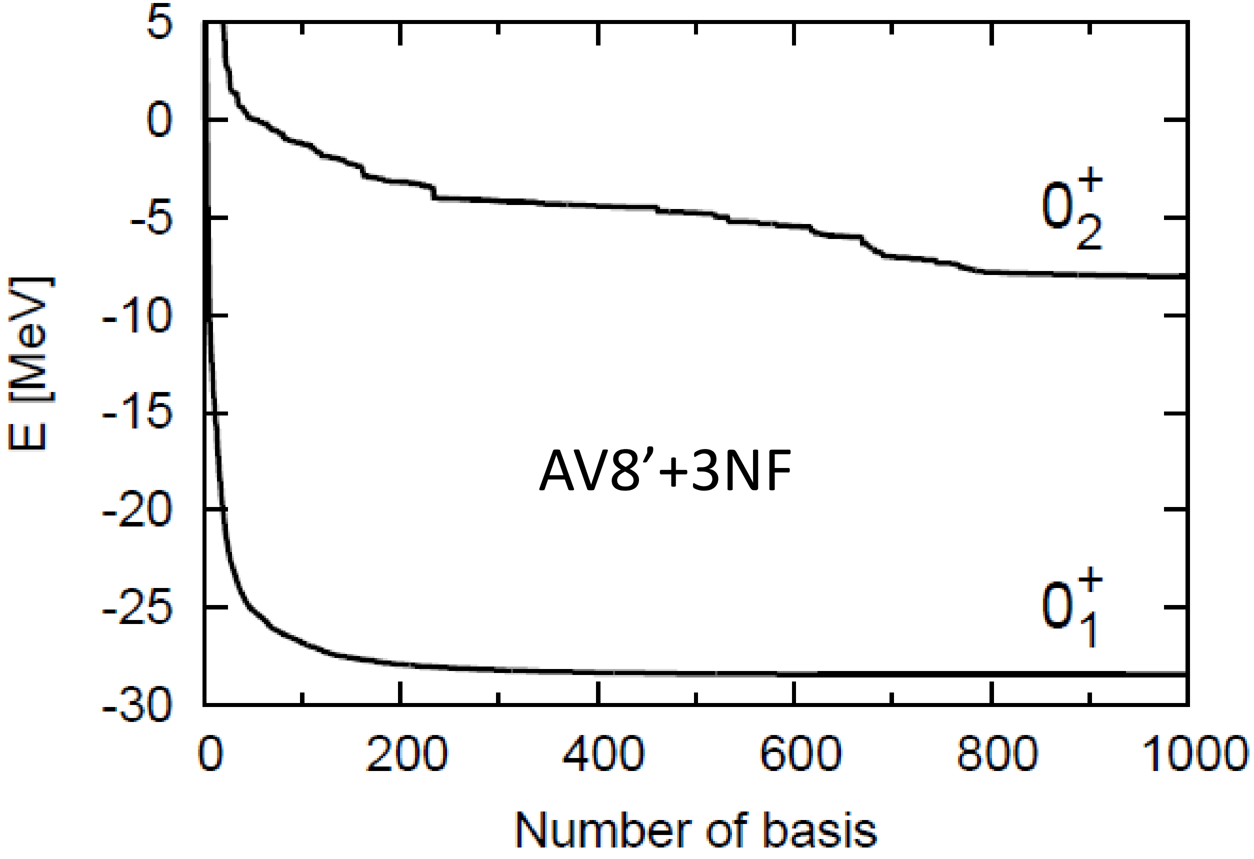
2. **Get the eigenvalues** $(E_k^1, E_k^2, \cdots\cdots, E_k^m)$

3. **Select** A_k^n corresponding to the lowest E_k^n
and **Include** it in a basis set

4. **k → k+1**

Y. Suzuki and K. Varga, Stochastic variational approach to quantum-mechanical few-body problems, LNP 54 (Springer, 1998).
K. Varga and Y. Suzuki, Phys. Rev. C52, 2885 (1995).

Energy convergence of ^{4}He



Ground state energy agrees with the other accurate methods (FY, GFMC, NCSM,…) within 60 keV

H. Kamada et al., PRC64, 044001 (2001)

Application to photonuclear reaction

WH, Y. Suzuki, PRC85, 054002 (2012)

Photoabsorption cross section

$$\sigma_\gamma(E_\gamma) = \frac{4\pi^2}{\hbar c} E_\gamma \frac{1}{3} S(E_\gamma)$$

S(E$_\gamma$): E1 strength function

The continuum $J^\pi T=1^-1$ state is expanded in several thousand of basis states including explicit decay to two- and three-body channels.

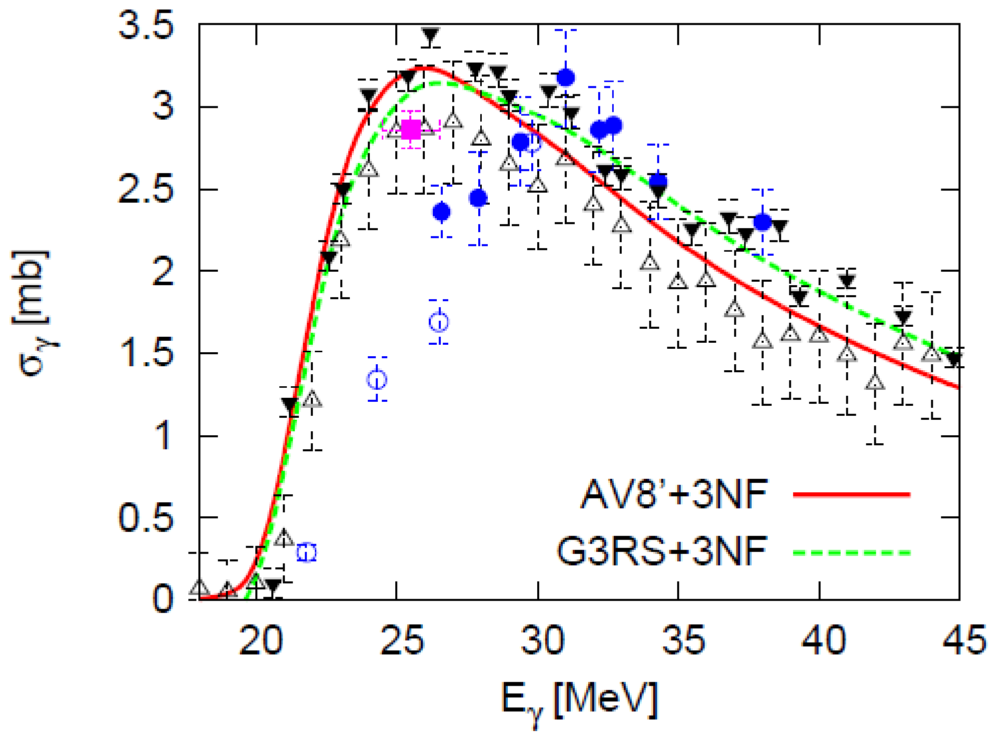
Complex scaling method:
Rotation in complex plane

$$\boldsymbol{r}_j \to \boldsymbol{r}_j e^{i\theta}, \quad \boldsymbol{p}_j \to \boldsymbol{p}_j e^{-i\theta}$$

→ outgoing-wave B.C.

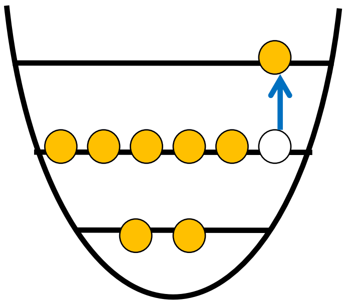
Unifying bound and continuum states

Comparison with the measurements
→ good agreement above 30 MeV

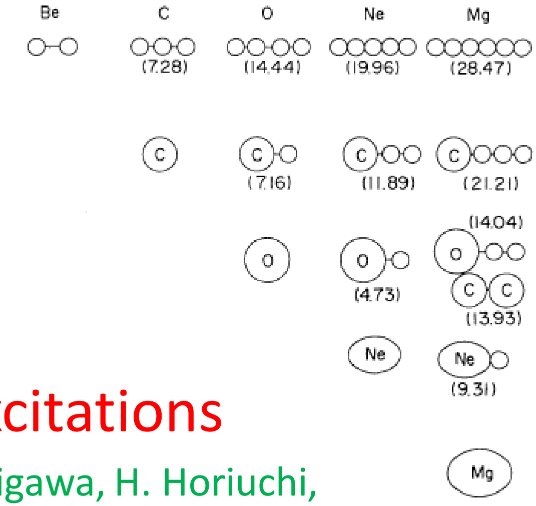


△ S. Nakayama et al., (2007)
■ D.P. Wells et al. (1992)
▼ Y. M. Arkatov et al.,(1974).
○ T. Shima et al., (2005).
● T. Shima et al., new measurement

Motivation: Coexistence of two aspects



Particle-hole excitations



Cluster excitations

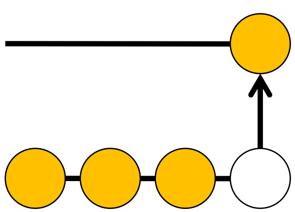
K. Ikeda, N. Takigawa, H. Horiuchi, PTPS52 (1972)

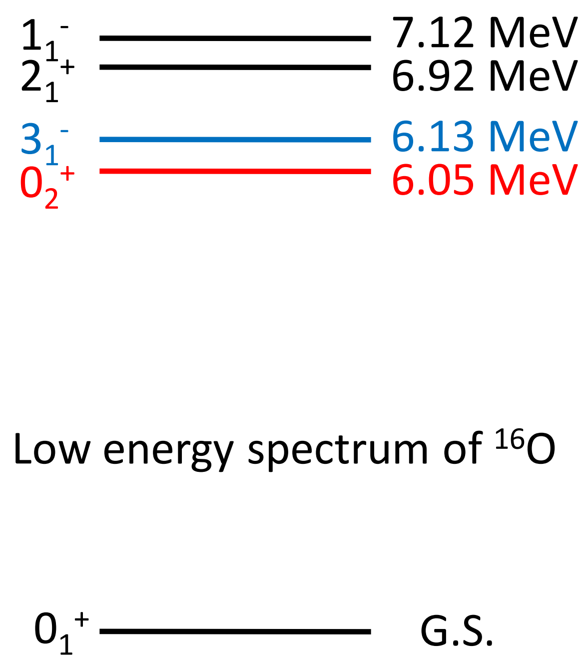
"Shape coexistence"

Mysterious 0^+ state in ^{16}O

– Simple 1p-1h negative parity?

– First excited state -> 0^+





Low energy spectrum of ^{16}O

Difficult to reproduce them even in modern large scale calculations

Theoretical works for the 0_2^+ state

- Status of the state-of-art theoretical models
  - No core shell-model
    P. Maris, J.P. Vary, and A.M. Shirokov, Phys. Rev. C79, 014308 (2009).
  - Coupled-cluster theory
    M. Wloch et al., Phys. Rev. Lett. 94, 212501 (2005)
    - The energy of the state is too high
    - Not converged even more than 4p-4h conf.

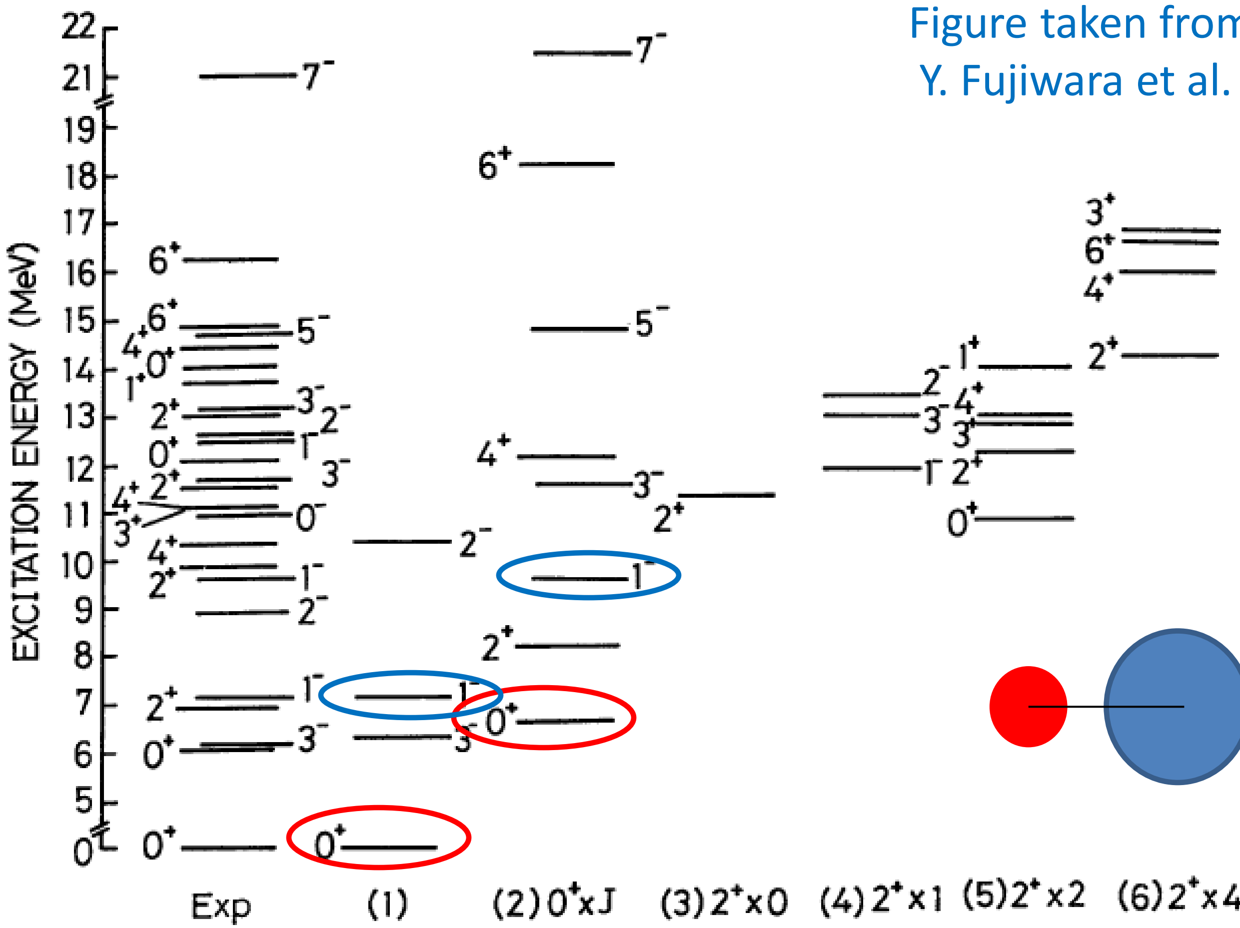
- Beyond the mean field approach
  - Configuration mixing of Slater determinants
    - M. Bender, P.-H Heenen, Nucl. Phys. A713, 390 (2003)
      - The energy is well reproduced, 4p-4h conf. is small
    - S. Shinohara et al., Phys. Rev. C74, 054315 (2006)
      - The energy is too high, 15-17 MeV

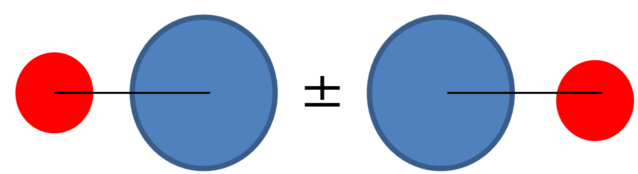
Low-lying spectra of ^{16}O:^{12}C+α picture

^{12}C+α model: coupled channel OCM Y. Suzuki, PTP55 (1976) 1751.

→ Levels are reproduced very well including 0_2^+

Figure taken from
Y. Fujiwara et al. PTPS 68(1980)





Towards resolving the ^{16}O problem

^{12}C+p+p+n+n five body model

Hamiltonian $$H = \sum_{i=1}^{N} T_i - T_{\rm cm} + \sum_{i=1}^{N-1} U_i + \sum_{i<j} v_{ij} + \sum_{i=1}^{N-1} \Gamma_i$$

N-N potential $$v_{ij} = v_c + v_{\rm Coul.} P_{i\pi} P_{j\pi}$$ Minnesota force

N-^{12}C potential $$U_i = U_c + U_{\rm Coul.} P_{i\pi} + U_b \boldsymbol{\ell}_i \cdot \boldsymbol{s}_i$$

Woods-Saxon form, reproduce the levels 1/2-, 1/2+, 5/2+ of ^{13}C

Pauli constraint $$\Gamma_i = \lambda \sum_{jm} |f_{jm}(i)\rangle \langle f_{jm}(i)|, \quad \lambda \to \infty$$

HO 0s1/2 and 0p3/2

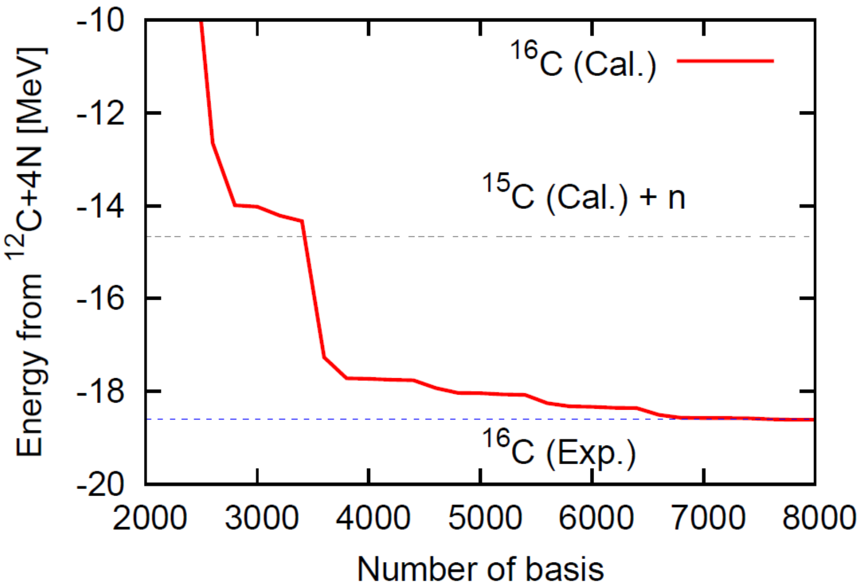
Kukulin and Pomerantsev (1978)

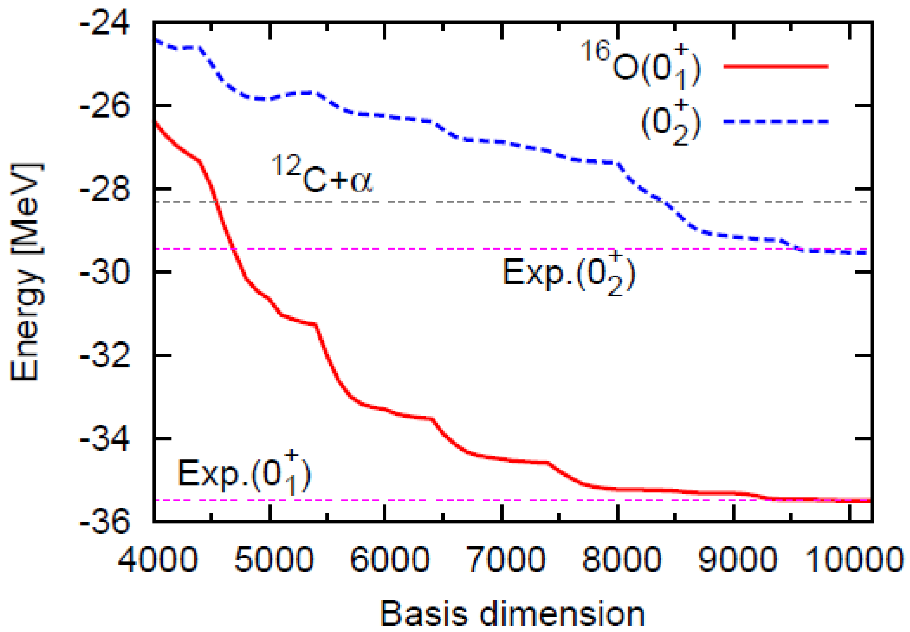
- Four particles can occupy any states except for the occupied orbits in the core
- No assumption of alpha cluster nor shell model conf.

Basis function $$\Phi = \mathcal{A}\left\{\left[\left[\psi_L^{(\rm space)} \psi_S^{(\rm spin)}\right]_{\mathcal{J}} \phi_I(^{12}{\rm C})\right]_{JM} \psi_{TM_T}^{(\rm isospin)}\right\}$$

No core excitation (I=0) $$\psi_{SM_S}^{(\rm spin)} = \left|\left[\cdots\left[\left[\left[\tfrac{1}{2}\tfrac{1}{2}\right]S_{12}\tfrac{1}{2}\right]S_{123}\right]\cdots\right]SM_S\right\rangle$$

Energy curves of ^{16}C and ^{16}O





- ^{16}C: converged (7000 basis states)
  - Good agreement with experiment
- ^{16}O: converged at around 10000 basis states

Discretizing on grids → Variational parameters $D \sim N^{10+4+4} \times 3 \times 2$

$N=4$, $D \sim 4 \times 10^{11}$

- Energies of the ground and first excited states are reproduced very well

Expectation values of Hamiltonian terms

	$^{16}\mathrm{C}\ (0_1^+)$	$^{16}\mathrm{O}\ (0_1^+)$	$^{16}\mathrm{O}\ (0_2^+)$	α
E	-18.47	-35.47	-29.52	-28.30
$E_{\mathrm{exp.}}$	-18.59	-35.46	-29.41	-28.30
$\langle T_{cv}\rangle$	17.81	11.55	7.16	–
$\langle V_{cv}\rangle$	-82.49	-79.55	-29.22	–
$\langle T_{v}\rangle$	53.53	72.93	67.46	56.92
$\langle V_{v}\rangle$	-7.32	-40.41	-74.92	-85.22

N-N potential

$$v_{ij} = v_c + v_{\mathrm{Coul.}} P_{i\pi} P_{j\pi}$$

Core-N potential

$$U_i = U_c + U_{\mathrm{Coul.}} P_{i\pi} + U_b \boldsymbol{\ell}_i \cdot \boldsymbol{s}_i$$

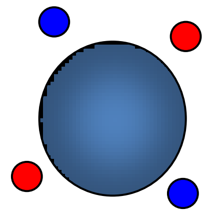
- ^{16}C: shell model like state
- ^{16}O (0_1^+): coexistence of shell and cluster states
  - α cluster is strongly distorted → $\langle V_v \rangle$ ∼1/2 of free α
- ^{16}O (0_2^+): well developed α+^{12}C cluster

  $\langle V_{cv}\rangle$ ^{16}O (0_2^+) ∼ $\langle V_v\rangle$ ^{4}He

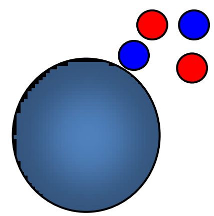
  ∼ $\langle V_{cv}\rangle$ ^{16}O(0_1^+)
  - Balance of Core-N and N-N interactions

Density distributions of ^{12}C-4N and 4N

- ^{16}C : weak NN interaction
  - Shell model like states
    - Delocalized 4N density



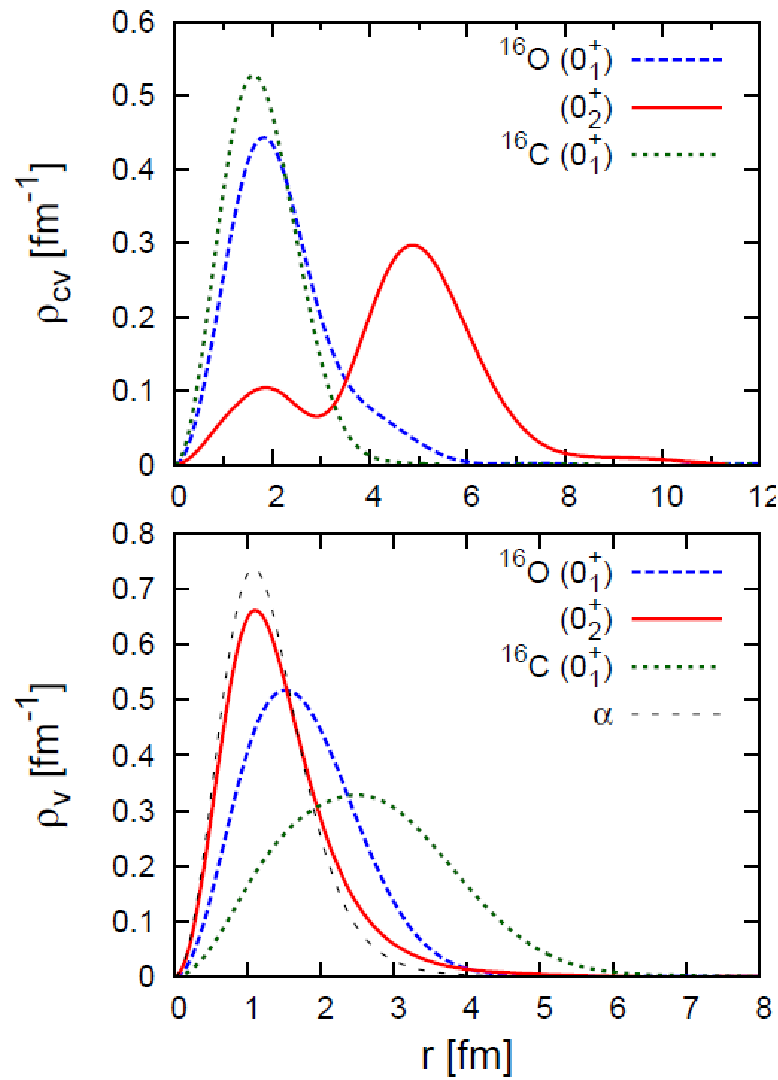
- ^{16}O: strong NN interaction
  - 0_1^+: Shell and 4N correlation
  - 0_2^+: Strong 4N correlation
    - Second peak far from the core
      Peak at ^{12}C+α touching distance ～4.9 fm
    - Similar 4N density distribution to that of ^{4}He



Rms distance between ^{12}C and 4N and radius of 4N system

	^{16}C (0_1^+)	^{16}O (0_1^+)	^{16}O (0_2^+)	α
$\sqrt{\langle r_{cv}^2 \rangle}$	1.94	2.54	4.86	–
$\sqrt{\langle r_v^2 \rangle}$	2.88	1.90	1.62	1.43

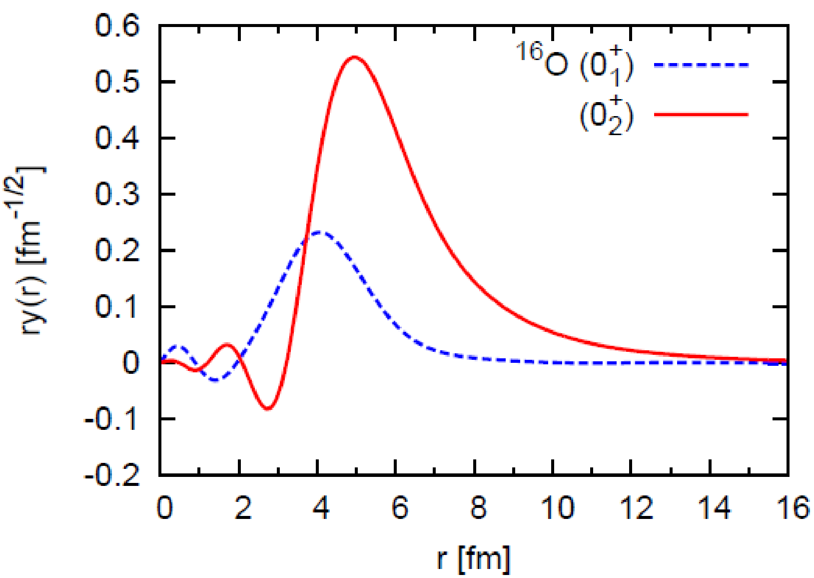
Density distributions between ^{12}C and 4N system



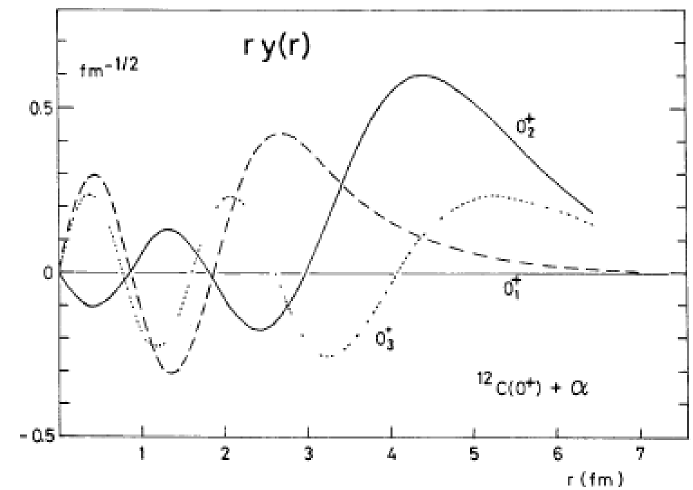
Density distributions of 4N system from their CM

^{12}C+α spectroscopic amplitude

$y(r)=\langle {}^{12}C+\alpha \,|\, {}^{16}O \rangle$



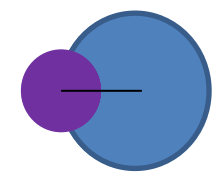
^{12}C+α OCM: Y. Suzuki, PTP56 (1976) 111

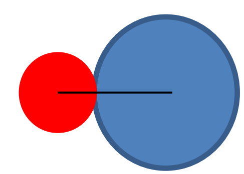


- 0_1^+: Alpha cluster is strongly distorted by the core nucleus
- 0_2^+: Large amplitude beyond the touching distance of ^{12}C+α (∼4.9 fm)
  - Very long tail that characterizes the cluster structure
- Spectroscopic factor

  0_1^+: 0.105 (OCM: 0.300)

  0_2^+: 0.680 (OCM: 0.679)





Distribution of harmonic oscillator quanta

Components of the harmonic oscillator quanta Q in the A=16 wave functions

Oscillator frequency is set to be the same as the occupied (forbidden) states in ^{12}C

^{16}C: Q≧6

Shell model state: four neutrons in *p* and *sd* shells

Average: M_Q=7.0

Standard deviation: σ_Q=2.1

^{16}O: Q≧4

0_1^+: Shell model or intermediate state between shell and cluster.

M_Q=5.5

σ_Q=2.9

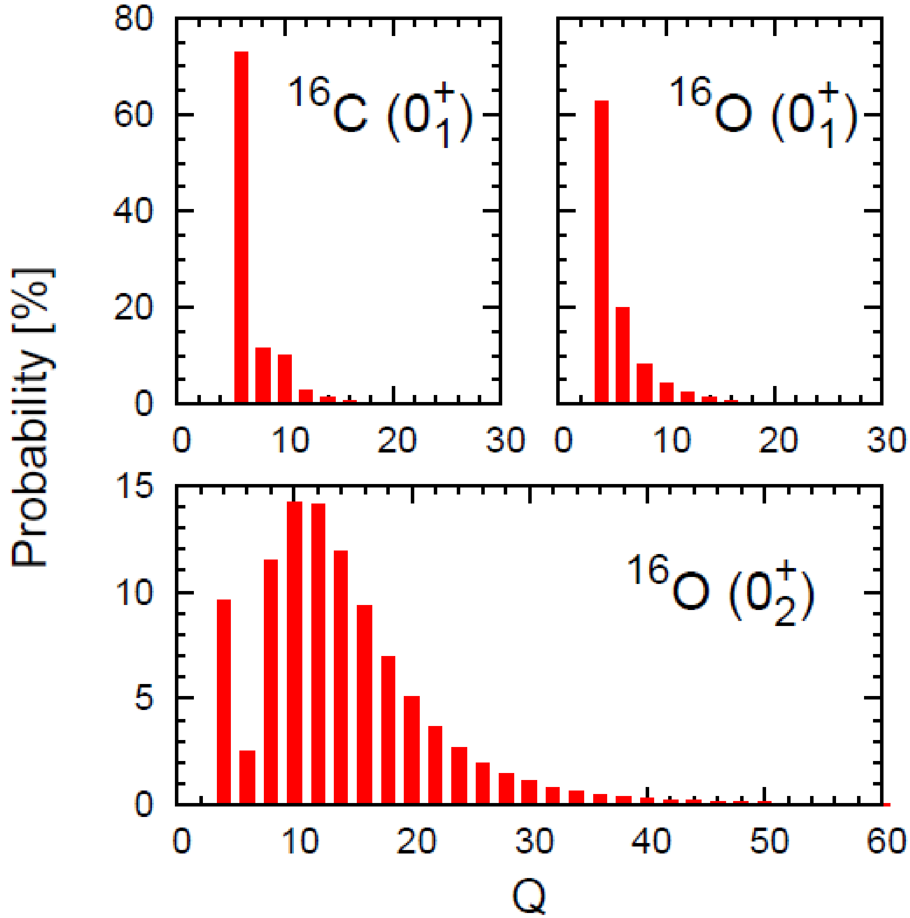
0_2^+: **Cluster state**

Peak : 10-12 hw

2-4 hw larger than 4p-4h conf.

M_Q=14.3

σ_Q=8.3



Difficult to describe it with the conventional shell model truncation

Monte Carlo, symmetry adaption, importance truncation, etc.

Summary I

- ^{16}O: a ^{12}C+n+n+p+p five body model
  - No assumption of alpha cluster
  - Energies of 0_1^+, 0_2^+ states in ^{16}O are reproduced well
  - Shell model (delocalized) and cluster structure in A=16 systems
    - 0_1^+, 0_2^+: balance of N-^{12}C and N-N potentials
    - Two different types of structure coexist in the spectrum of ^{16}O
    - Well developed ^{12}C+α cluster in the first excited states of ^{16}O
      - Density distributions
      - Spectroscopic amplitude (long tail, large α-width)
    - Large major shell mixing in the cluster state

WH and Y. Suzuki, PRC89, 011304(R) (2014)

- Correlated Gaussian with global vectors: Both the shell model like and cluster states can be described in a single scheme

Electric dipole response of halo nuclei

- Exploring new phenomena in unstable nuclei

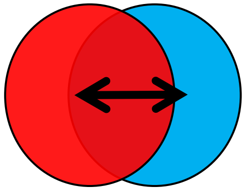
  Halo nucleus: typical examples ^{11}Li, ^{6}He, ^{14}Be, ^{22}C,..

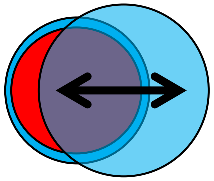
  I. Tanihata et al. PRL55, 2676; PLB160, 380; PLB206, 592 (1985)
  K. Tanaka et al. PRL104, 062701 (2010)

  Small binding energy ⇔ Large matter radius

- Soft dipole resonance (SDR) or Pigmy dipole resonance (PDR)
  - Possible new mode in neutron rich unstable nuclei
  - Vibration of valence nucleons against the core

  A variant of the macroscopic picture of giant dipole resonance (GDR)
  - Goldhaber-Teller, Steinwedel-Jensen models

GDR 

SDR 

P.G. Hansen and B. Jonson, Europhys. Lett. 4, 409 (1987)
K. Ikeda, INS Report No. (1988) (in Japanese)
Y. Suzuki, K. Ikeda, and H. Sato PTP83, 180 (1990).

Purpose of this work

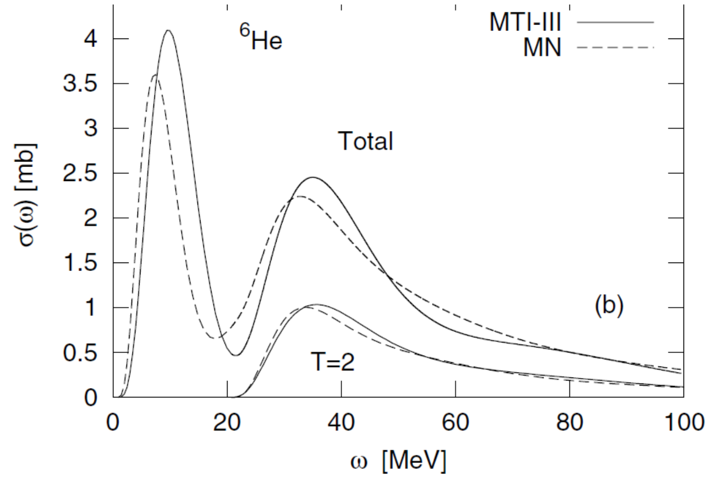
- Study electric dipole (E1) response of ^{6}He
  Energy region: from SDR to GDR
  – Possible excitation modes?
  – Core excitation?
    ⇔ α+n+n three-body model

Two phase equivalent potentials gives the different results

T. Myo et al,. PRC63, 054313 (2001), D. Baye et al., PRC79, 024607 (2009)
E.C. Pinilla, NPA865, 43 (2011)

→Fully microscopic six-body calculation

– Theoretical indication of SDR
  - S. Bacca et al. PRL89, 052502 (2001)
    EIHH+LIT
    Two peak structure is found

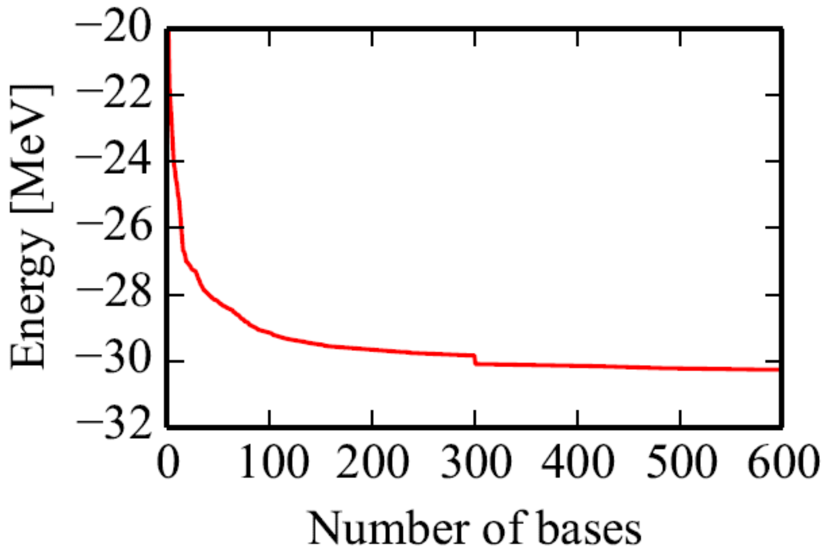


Ground state properties of ^{6}He

$$H = \sum_{i=1}^{N} T_i - T_{\rm cm} + \sum_{i<j} v_{ij}$$

Free of spurious c.m. motion

v_{ij}: Minnesota potential (u=1.05)



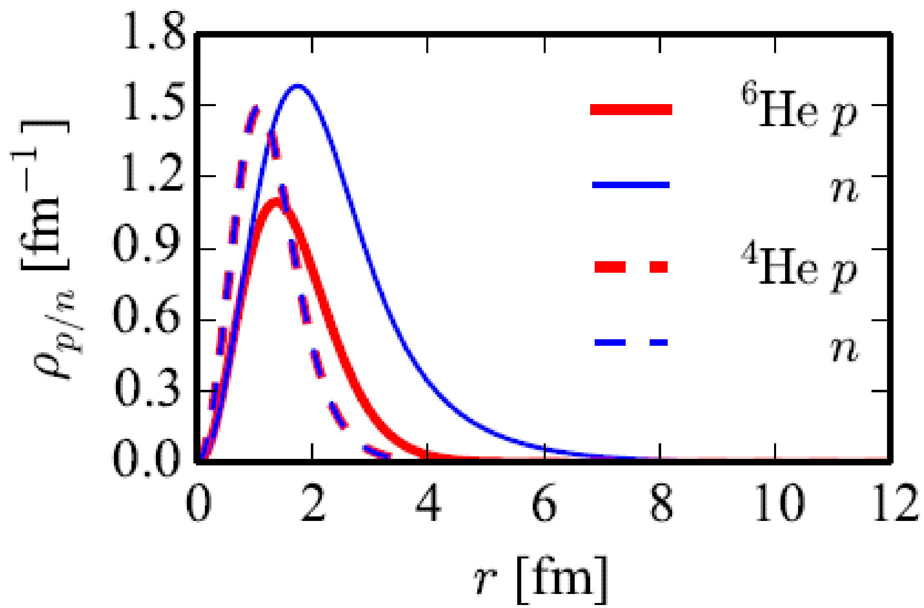
Converged only with 600 basis states

Note: 15 parameters on each basis

E= -30.98 MeV (Expt. -29.27 MeV)

S_{2n}= 1.01 MeV (Expt. 0.974 MeV)

Density distributions → “Neutron halo”

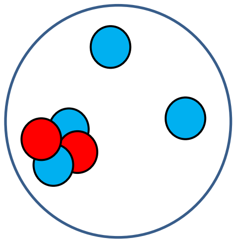


r_m: 2.41(Expt. 2.37 (10)) fm

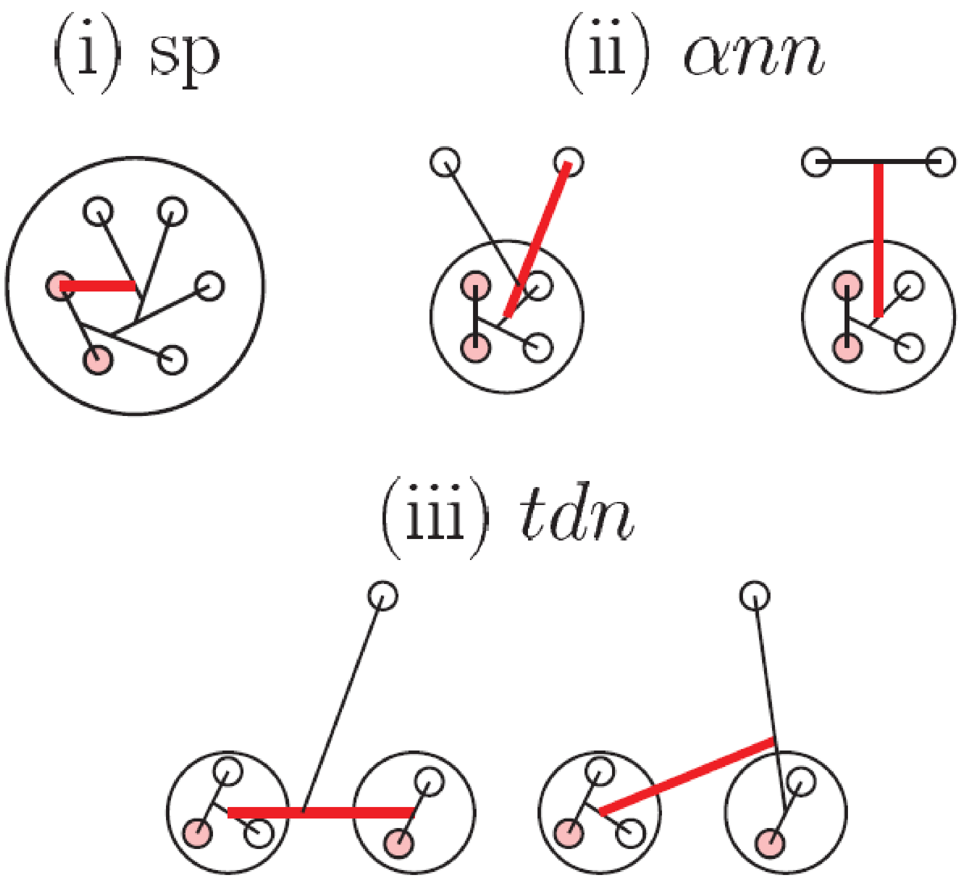
r_p : 1.83 (Expt. 1.87(1)) fm

r_{pp}: 2.49 fm > 2.37 (^{4}He) fm

5% of core swelling



Configurations for the final state



W.H, Y. Suzuki, K. Arai, PRC85, 054002 (2012)

E1 operator

$$\mathcal{M}_{1\mu} = e \sum_{i \in p} (\boldsymbol{r}_i - \boldsymbol{x}_6)_\mu = \sqrt{\frac{4\pi}{3}} e \sum_{i \in p} \mathcal{Y}_{1\mu}(\boldsymbol{r}_i - \boldsymbol{x}_6)$$

(i)Single particle excitations

$M_{1\mu}(E1)\Psi_i(^6\mathrm{He})$

"Coherent E1 state"

Well account for the E1 sumrule

600 basis states

(ii)α+n+n disintegration

$\Psi_i(^4\mathrm{He})\chi(R, r)$

SDR configuration

8100 basis states

(iii)t+d+n three-body disintegration

$\Psi_i(^3\mathrm{H})\Psi_j(^2\mathrm{H})\chi(R, r)$

GDR configuration

5250 basis states

Correlated Gaussian + global vector

$$F_{LM_L}(v, A, \boldsymbol{x}) = \exp\left(-\tfrac{1}{2}\tilde{\boldsymbol{x}} A \boldsymbol{x}\right) \mathcal{Y}_{LM_L}(\tilde{v}\boldsymbol{x})$$

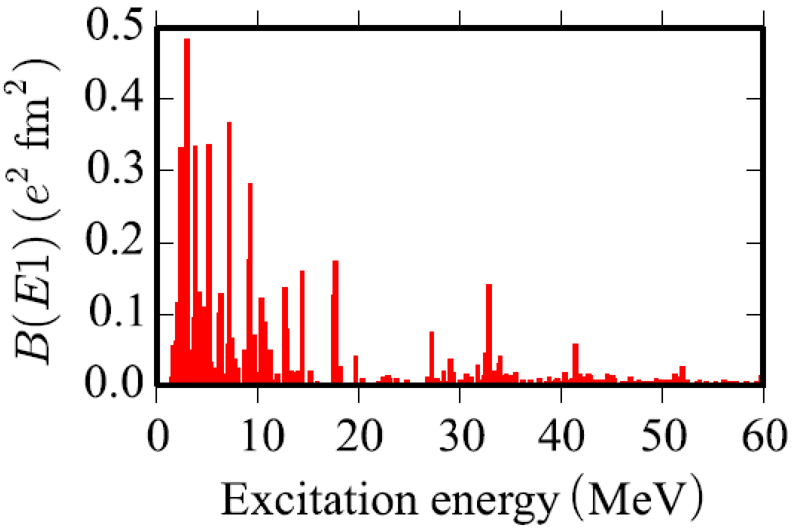
$$\tilde{\boldsymbol{x}} A \boldsymbol{x} = \sum_{i,j=1}^{N-1} A_{ij} \boldsymbol{x}_i \cdot \boldsymbol{x}_j \qquad \tilde{v}\boldsymbol{x} \left(= \sum_{i=1}^{N-1} v_i \boldsymbol{x}_i\right).$$

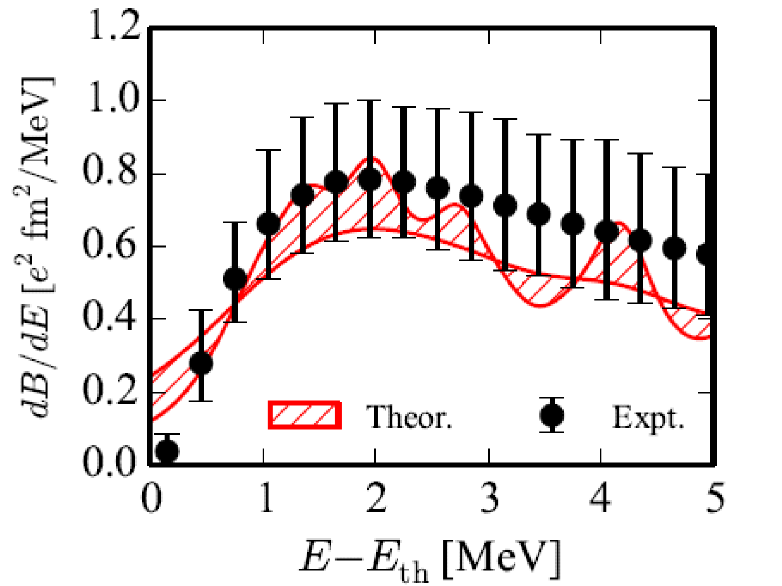
Electric dipole strength function

Reduced E1 strength $B(E1,\nu) = \sum_{M\mu} |\langle \Psi_{1M}(E_\nu)|\mathcal{M}_{1\mu}|\Psi_0 \rangle|^2$

Expt. T. Aumann et al.,PRC59, 1252 (1999)

Diagonalization of 13950 basis states





$$\frac{dB(E1,E)}{dE} = \sum N(E_\nu,\Gamma)L(E,E_\nu,\Gamma)B(E1,\nu)$$

$$L(E,E_\nu,\Gamma) = \frac{\Gamma}{2\pi}\frac{1}{(E-E_\nu)^2+(\Gamma/2)^2},$$

$$N(E_\nu,\Gamma) = \frac{1}{1-\int_{-\infty}^{E_{\rm th}} L(E',E_\nu,\Gamma)dE'}$$

Γ = 0.75 to 2.0 MeV

Sum rule 99.9 %

$$\sum_\nu B(E1,\nu) = e^2\left(Z^2\langle r_p^2\rangle - \frac{Z(Z-1)}{2}\langle r_{pp}^2\rangle\right)$$

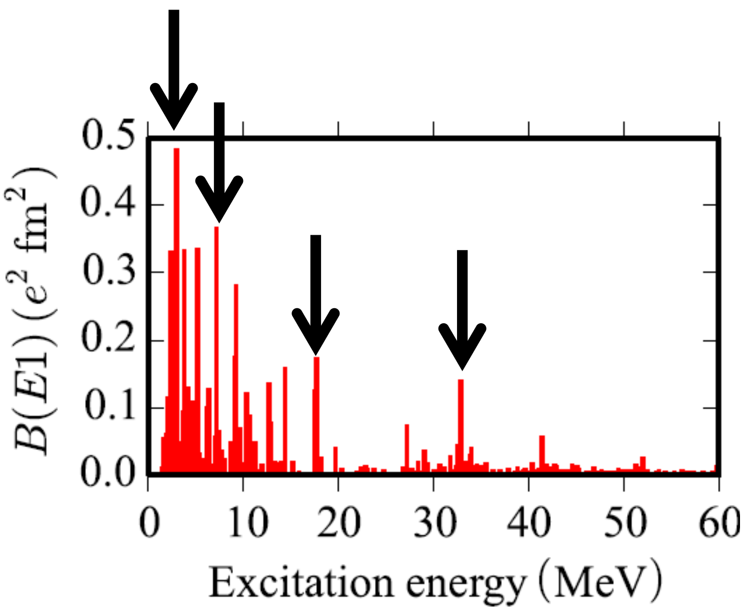
New excitation modes in ^{6}He?

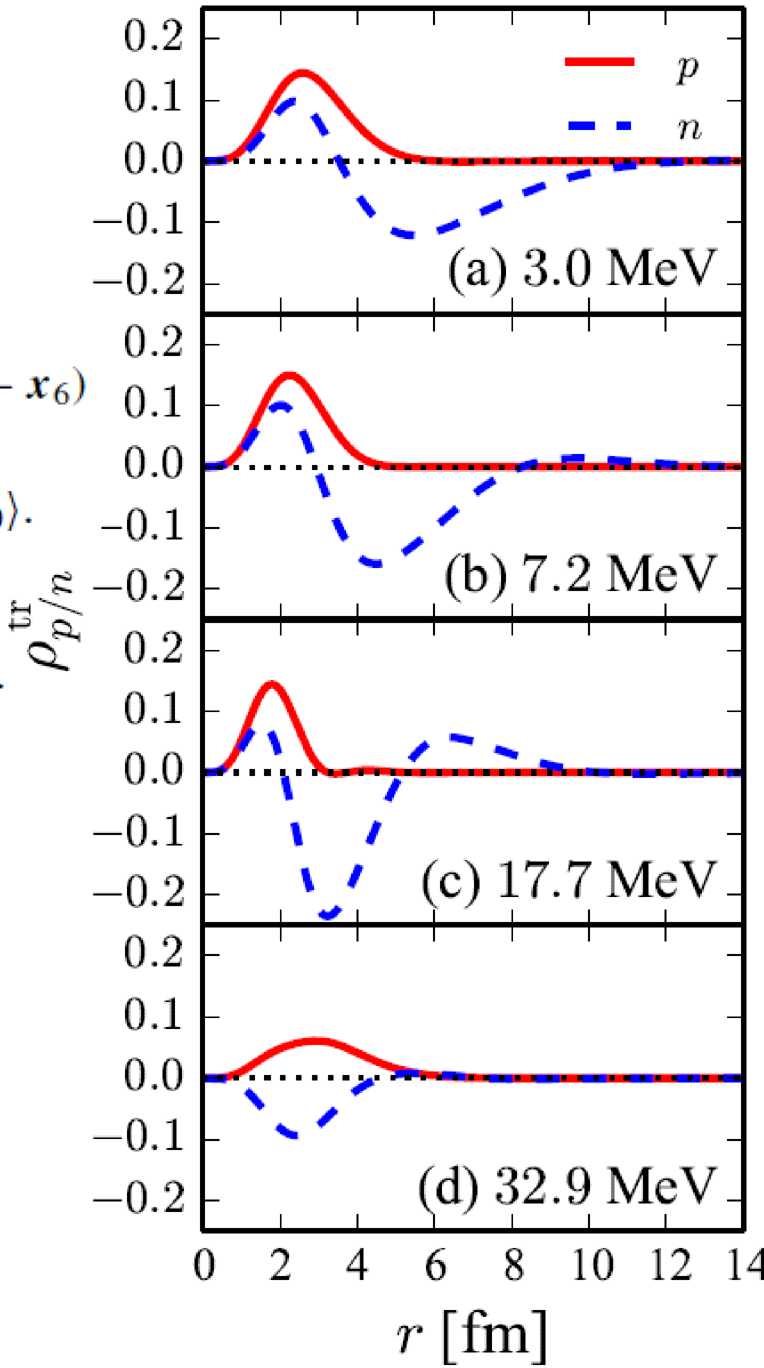
Transition density (x_6: c.m of ^{6}He)

$$\rho^{\mathrm{tr}}_{p/n}(E_\nu, r) = \langle \Psi_1(E_\nu) \| \sum_{i\in p/n} \mathcal{Y}_1(\boldsymbol{r}_i - \boldsymbol{x}_6) \times \delta(|\boldsymbol{r}_i - \boldsymbol{x}_6| - r) \| \Psi_0\rangle.$$

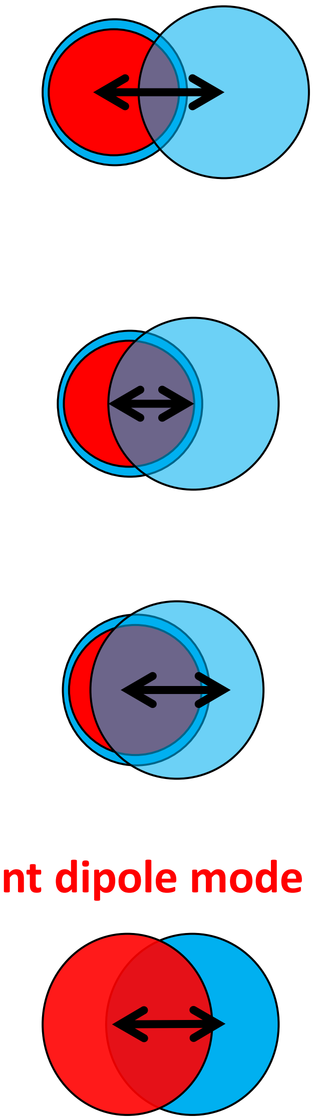
E1 matrix element

$$\langle \Psi_1(E_\nu)\|\mathcal{M}_1\|\Psi_0\rangle = e\sqrt{\frac{4\pi}{3}}\int_0^\infty \rho_p^{\mathrm{tr}}(E_\nu, r)dr.$$





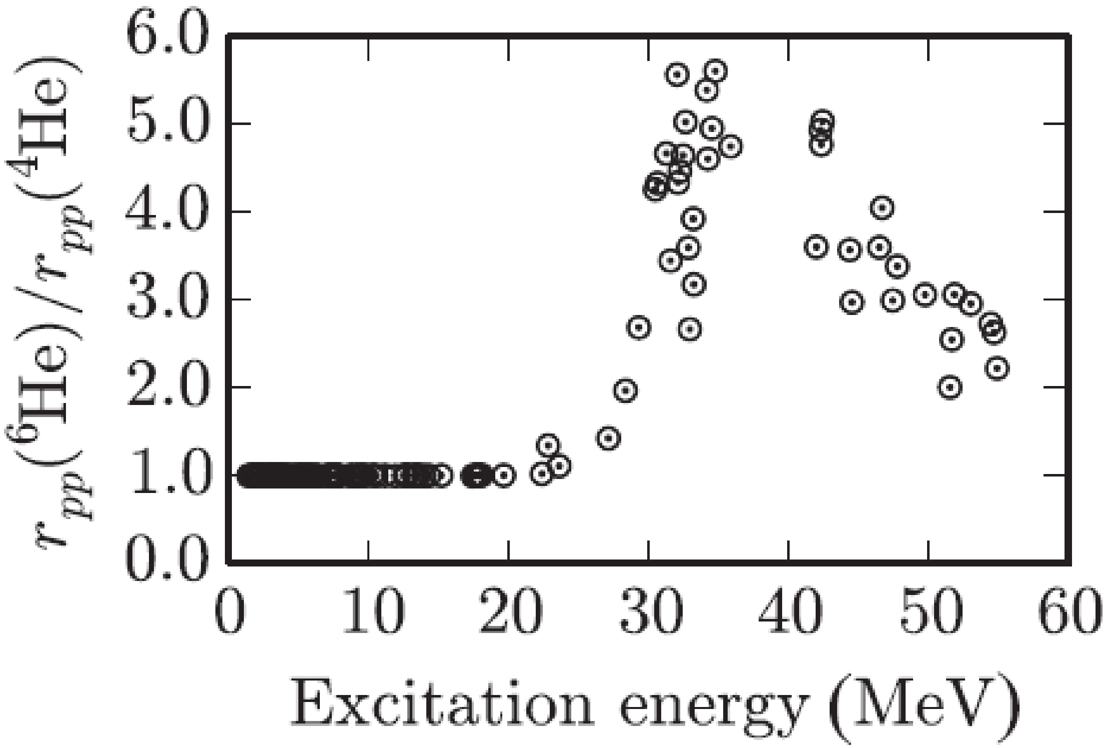
“Soft” dipole mode



Giant dipole mode

Core excitations in ^{6}He

Ratio of r_{pp} of ^{6}He and ^{4}He → Validity of three-body model



- The ratio is unity up to 20 MeV

  No core swelling in the excited state of ^{6}He (Note: 5% in the g.s)

  → Three-body model for the excited state is valid up to 20 MeV
- Large core distortion in the GDR region

Compressional dipole modes

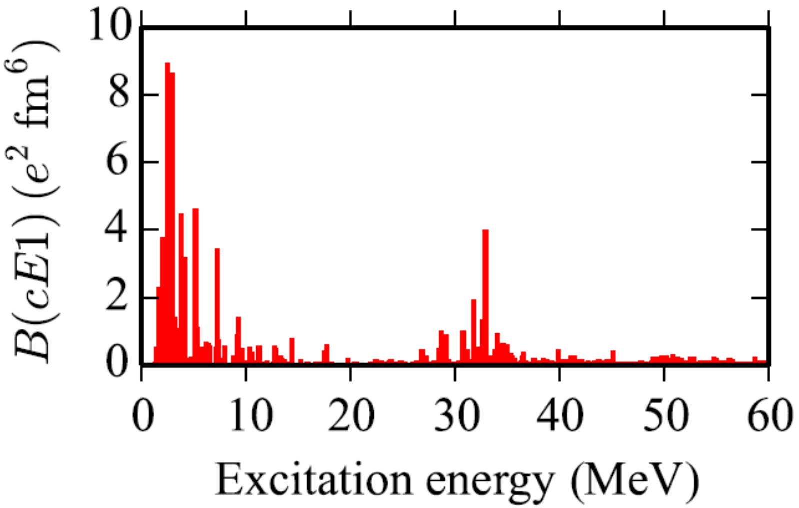
Operator $$\mathcal{M}^{\text{comp.}}_{1\mu} = e \sum_{i\in p} |\boldsymbol{r}_i - \boldsymbol{x}_6|^3 Y_{1\mu}(\widehat{\boldsymbol{r}_i - \boldsymbol{x}_6})$$

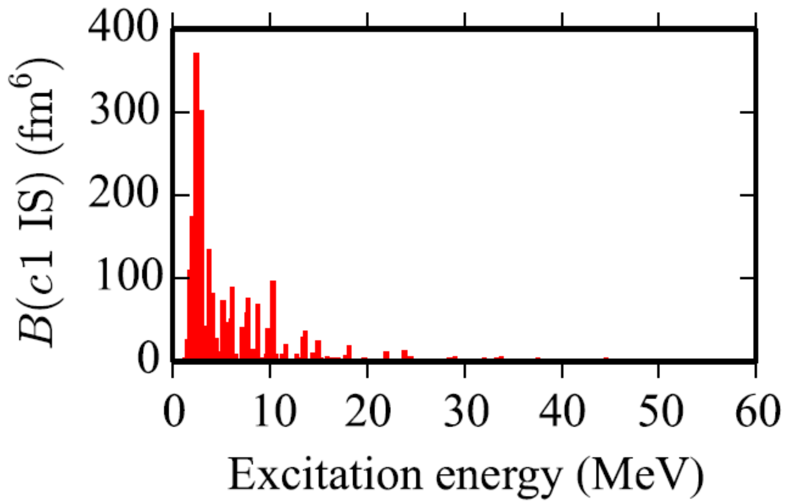
Transition matrix element of compressional E1 (cE1) mode

$$\langle \Psi_1(E_\nu) \| \mathcal{M}^{\text{comp.}}_1 \| \Psi_0 \rangle = e \int_0^\infty r^2 \rho_p^{\text{tr}}(E_\nu, r) dr$$

Isoscalar compressional dipole (IS c1) mode

$$\int_0^\infty r^2 \left(\rho_p^{\text{tr}}(E_\nu, r) + \rho_n^{\text{tr}}(E_\nu, r)\right) dr.$$





Clearer two peak structure
Inelastic proton scattering, etc.

Only low-lying peak
Inelastic α scattering

Summary II

- A fully microscopic six-body calculation for the electric dipole response of ^{6}He D. Mikami, WH, Y. Suzuki, Phys. Rev. C 89, 046303 (2014)
  - Explicitly correlated Gaussians
  - Final state interactions
    - Single-particle excitations
    - Three-body disintegrations

- Two-peak structure is found in the E1 strength
  - Low-energy peak: Soft dipole resonance (SDR)
  - High-energy peak: Giant dipole mode

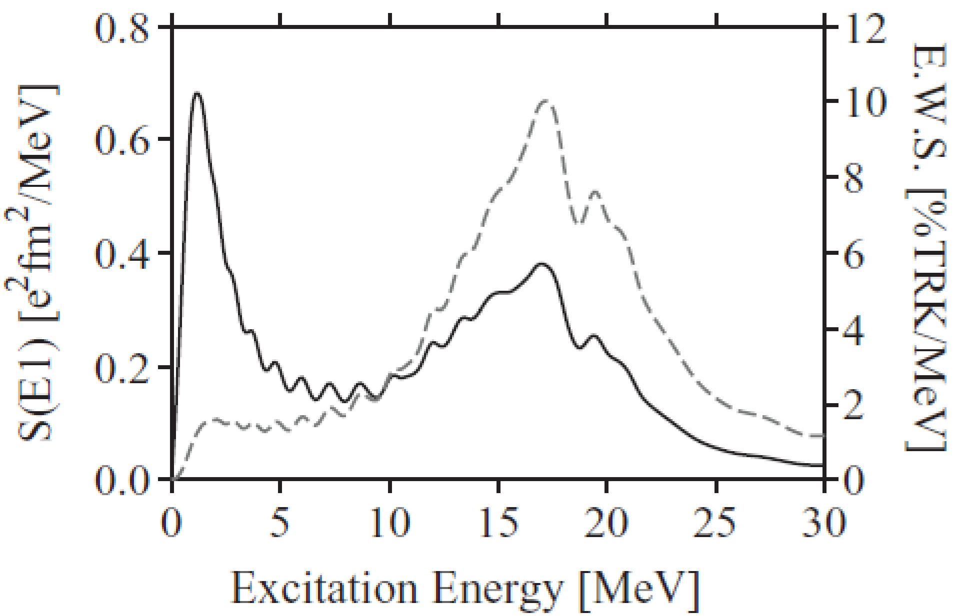
    New mode: Vibrational excitation of SDR
  - SDR is more disclosed by the isoscalar compressional dipole mode

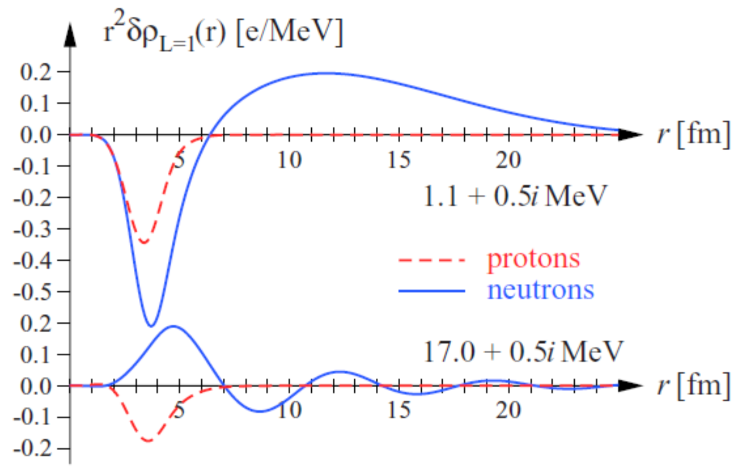
    α inelastic scattering measurement

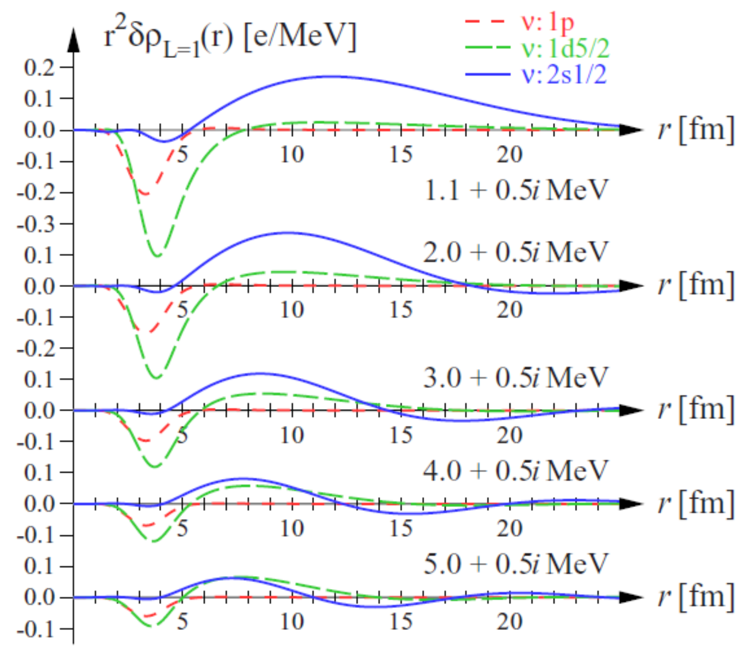
^{22}C case

T. Inakura, W.H., Y. Suzuki, T. Nakatsukasa, PRC89, 064316 (2014)

- Skyrme-Hartree-Fock + RPA on 3D coordinate space
- Fermi level -0.5 MeV (modified SIII)
- "Pigmy is taller than Giant"







Importance of core excitation

Summary

- Explicitly correlated Gaussian with global vectors is powerful method to describe complex nuclear many body problems
  - Formulation of N particle system
  - No change of functional form by any coordinate transformation

- Outlook: Possible applications
  - More particle systems (A=6, 7,…)
  - Heavier core plus N-nucleon systems
  - Exotic nucleus (Hyper nucleus, …)
  - Atomic and molecule systems