A Possibility of the Long Range Nuclear Potential by the NN π Three Body Approach and the Pion-Deuteron Scattering Length

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Abstract:

Difference between the above and below the three-body break up threshold is emphasized. Below the NNπ three-body break up threshold, the Born term of the three-body equation makes the one pion exchange NN potential which is an energy dependent two-body quasi (E2Q) potential. Such an E2Q potential could be Fourier transformed, and gives the r-space potential with the energy dependent. After an energy average, we obtain the usual r-space potential which is characterized by the well known Yukawa-type for the short range, but for the long range, the 1/r^2-type, or the 1/r^6-type, or the 1/r^7-type potentials corresponding to the different parameters in the energy average treatment. The 1/r^2 potential leads the Efimov-like states, and the other two are the London-type and the Cassimere-type Van der Waals potentials. The E2Q potential method will be applied to the NN' and the pion-deuteron scattering length calculations.

Motivation

Nuclear reaction:

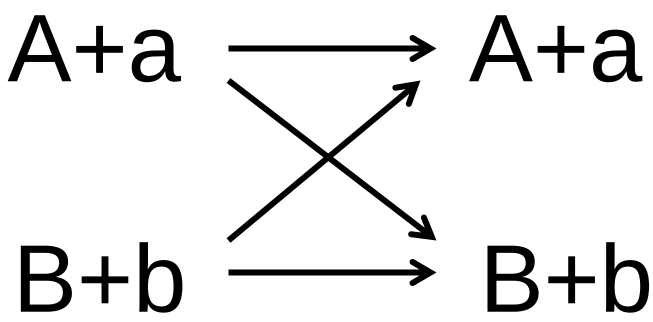
Multi-channel Lippmann-Schwinger equation

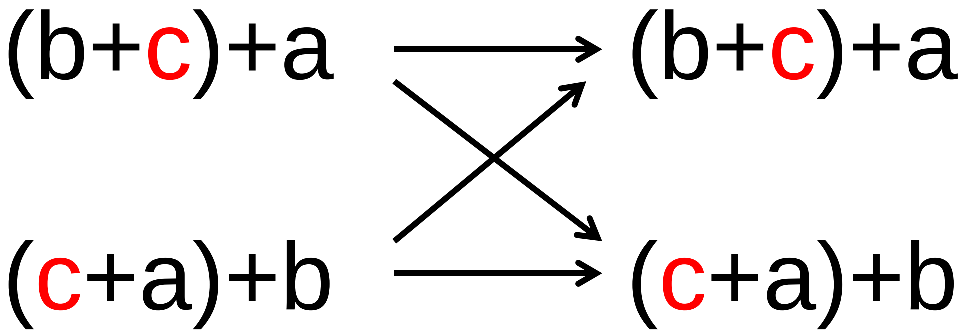
$$\begin{tikzcd}
A+a \arrow[r] \arrow[dr] & A+a \\
B+b \arrow[r] \arrow[ur] & B+b
\end{tikzcd}$$

Motivation

Nuclear reaction:

Multi-channel Lippmann-Schwinger equation





Motivation

Nuclear reaction:

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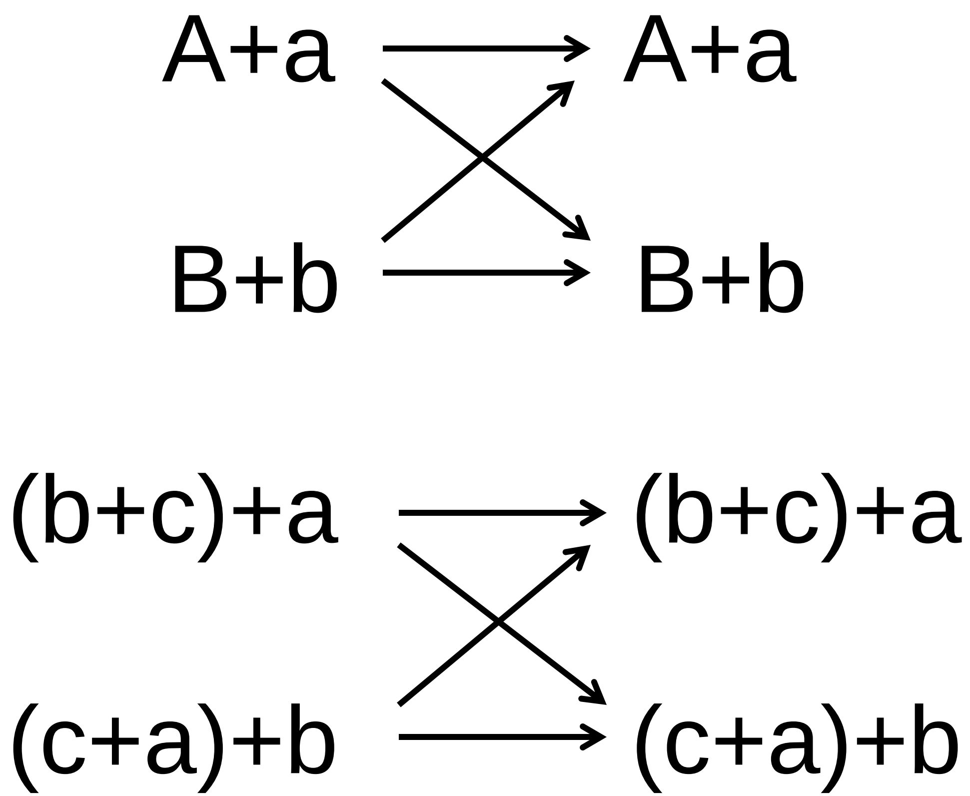


three-Body Faddeev equation

Motivation

Nuclear reaction:

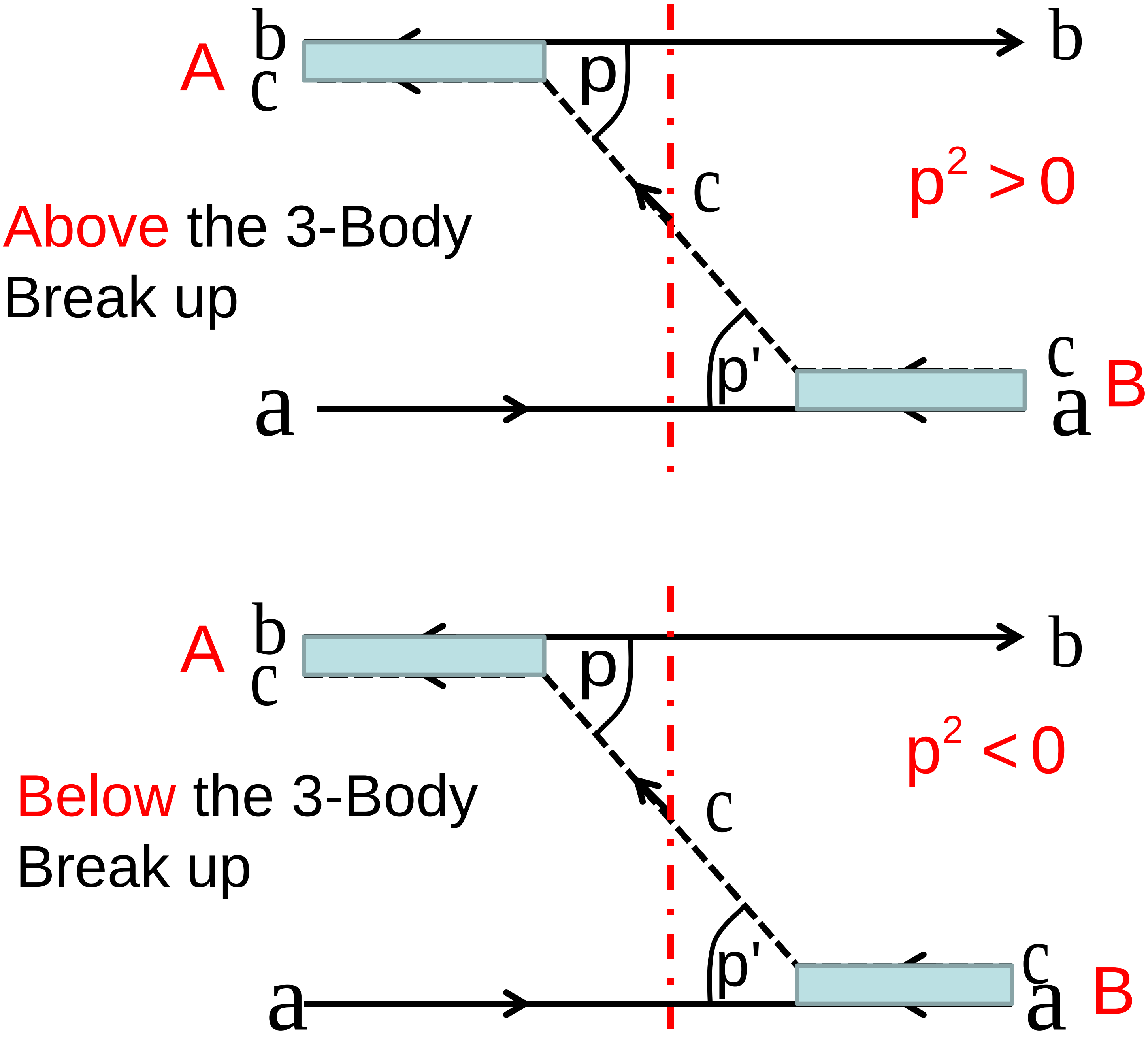
Multi-channel Lippmann-Schwinger equation

$$A+a \rightarrow A+a$$
$$A+a \rightarrow B+b$$
$$B+b \rightarrow A+a$$
$$B+b \rightarrow B+b$$

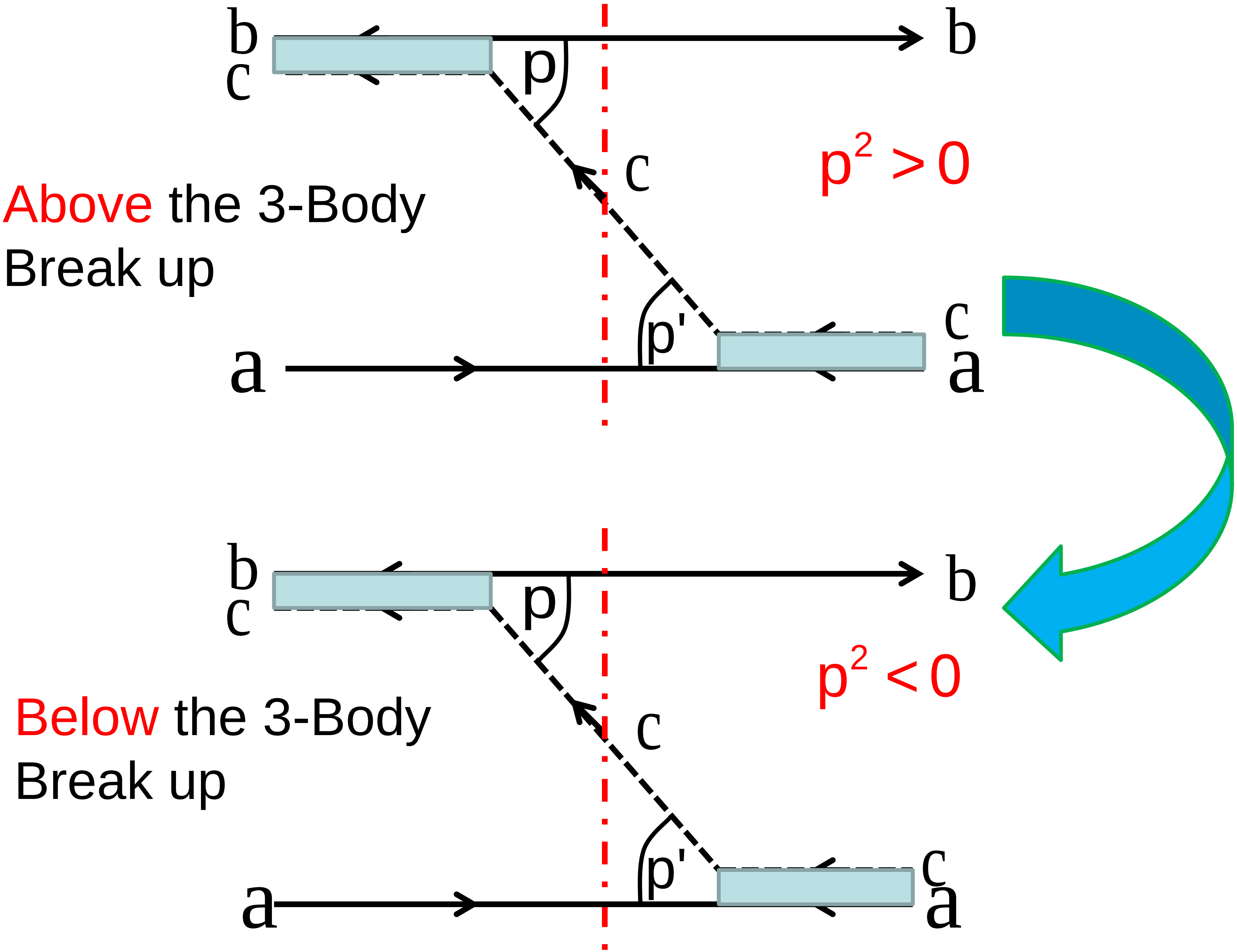
should be smoothly contineud to

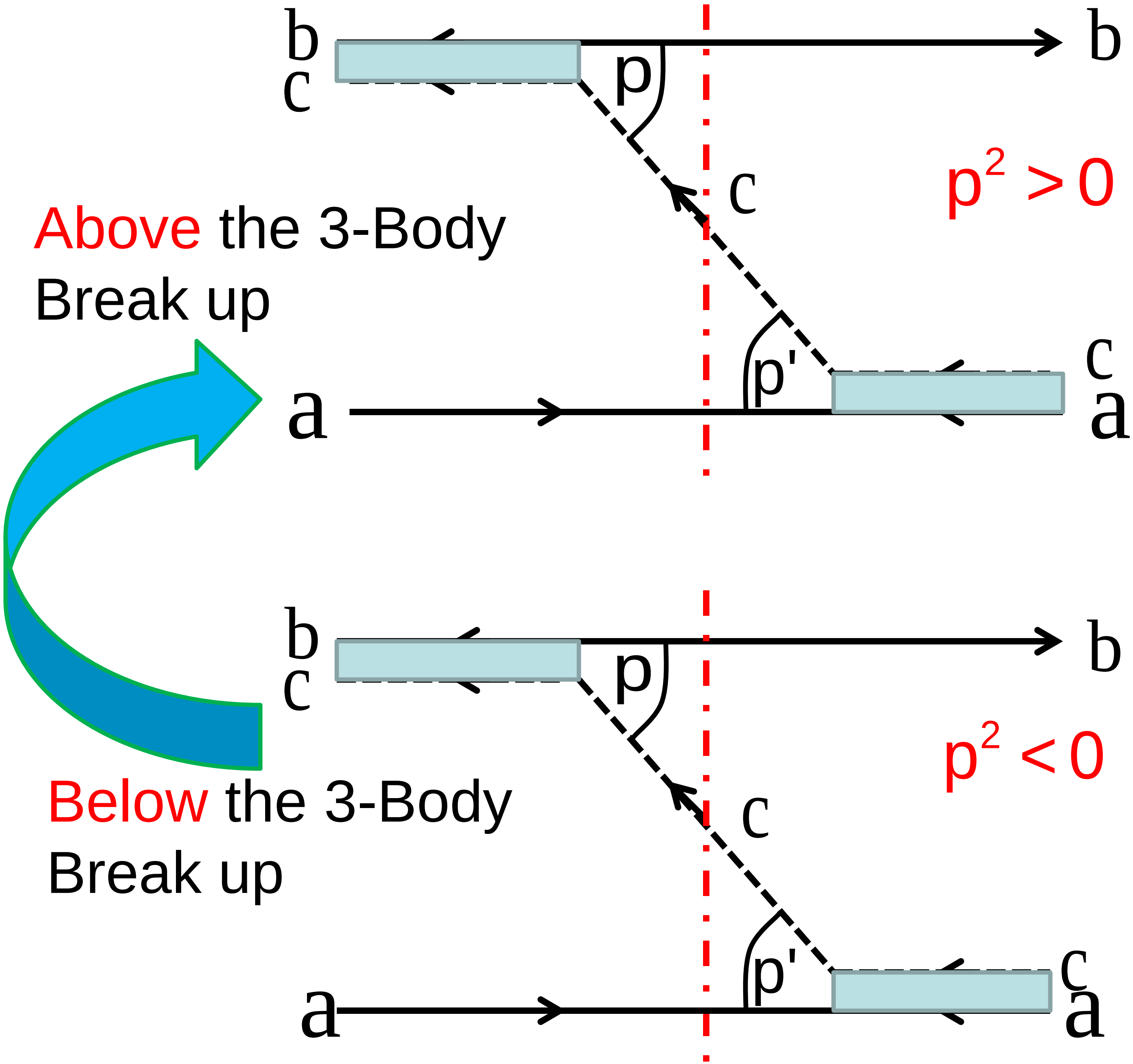
$$(b+c)+a \rightarrow (b+c)+a$$
$$(b+c)+a \rightarrow (c+a)+b$$
$$(c+a)+b \rightarrow (b+c)+a$$
$$(c+a)+b \rightarrow (c+a)+b$$

three-Body Faddeev equation

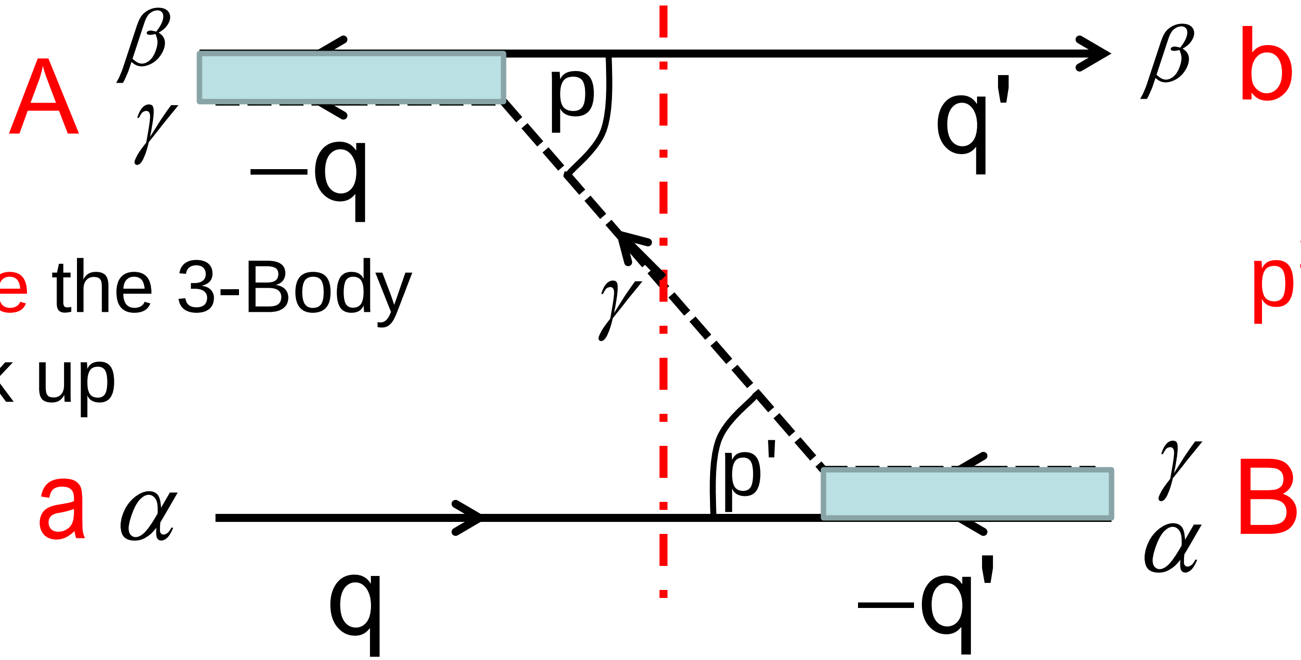


A
b
c
b
p
c
$p^2 > 0$
Above the 3-Body Break up
p'
c
a
a
B
A
b
c
b
p
$p^2 < 0$
c
Below the 3-Body Break up
p'
c
a
a
B


b
c
p
b
c
$p^2 > 0$
Above the 3-Body Break up
c
p'
a
a
b
c
p
b
$p^2 < 0$
Below the 3-Body Break up
c
p'
c
a
a



b
c
b
p
c
$p^2 > 0$
Above the 3-Body Break up
p'
c
a
a
b
c
b
p
$p^2 < 0$
c
Below the 3-Body Break up
p'
c
a
a



Above the 3-Body Break up

$p^2 > 0$

Faddeev Born

$$Z_{\alpha n,\beta m}(-\mathrm{q},\mathrm{q}';E)=\frac{-g_{\alpha n}(\mathrm{p})\,m_\gamma\,g_{\beta m}(\mathrm{p}')\,\bar{\delta}_{\alpha,\beta}}{E-\left(\dfrac{\mathrm{q}^2}{2m_\alpha}+\dfrac{\mathrm{q}'^2}{2m_\beta}+\dfrac{(\mathrm{q}-\mathrm{q}')^2}{2m_\gamma}\right)}$$

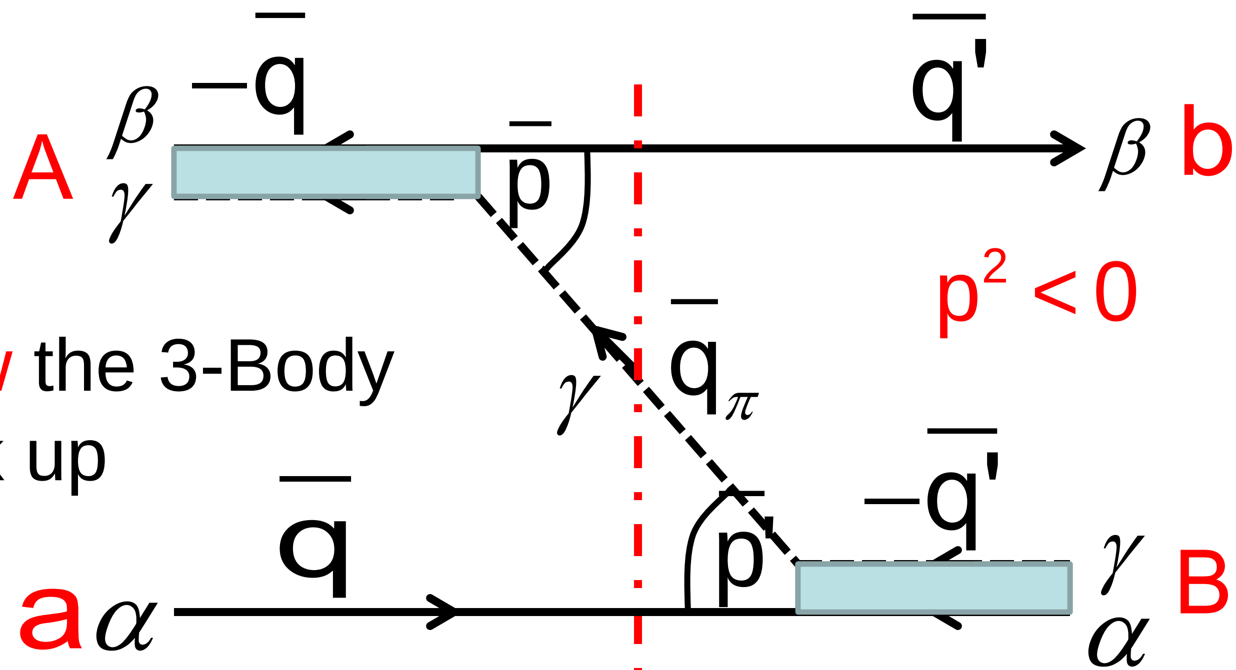
$$Z_{\alpha n,\beta m}(-\mathrm{q},\mathrm{q}';E)=\frac{-g_{\alpha n}(\mathrm{p})m_\gamma g_{\beta m}(\mathrm{p}')\overline{\delta}_{\alpha,\beta}}{(E+\varepsilon_B)-\left(\frac{\mathrm{q}^2}{2m_\alpha}+\frac{\mathrm{q}'^2}{2m_\beta}+\frac{(\mathrm{q}-\mathrm{q}')^2}{2m_\gamma}+\varepsilon_B\right)}$$

$$=\frac{-g_{\alpha n}(\mathrm{p})m_\gamma g_{\beta m}(\mathrm{p}')\overline{\delta}_{\alpha,\beta}}{E_{\mathrm{cm}}-\left(\frac{\overline{\mathrm{q}}^2}{2m_\alpha}+\frac{\overline{\mathrm{q}}'^2}{2m_\beta}+\frac{(\overline{\mathrm{q}}-\overline{\mathrm{q}}')^2}{2m_\gamma}\right)}$$

$E<0,$

$0\le E_{\mathrm{cm}}$ or $E_{\mathrm{cm}}<0$

E2Q Potential



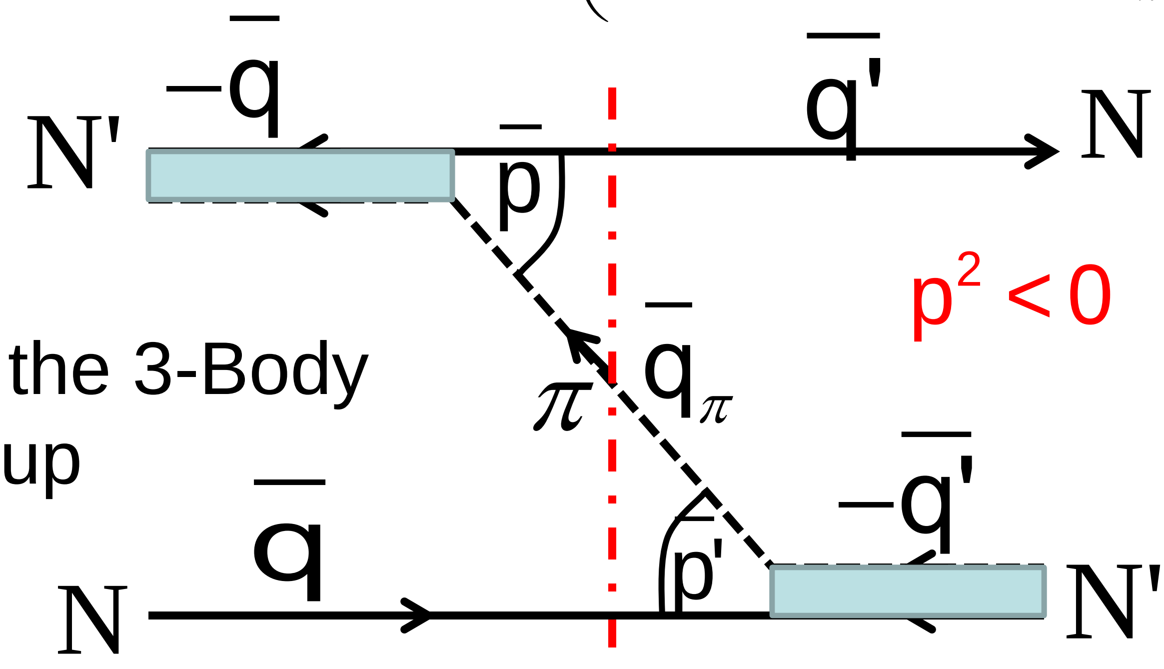
Below the 3-Body Break up

$NN\pi - system$

$$Z_{n,m}(-\mathrm{q},\mathrm{q}';E) = \frac{-g_n(\mathrm{p})m_\pi g_m(\mathrm{p}')}{(E+m_\pi) - \left(\frac{\mathrm{q}^2}{2M} + \frac{\mathrm{q}'^2}{2M} + \frac{(\mathrm{q}-\mathrm{q}')^2}{2m_\pi} + m_\pi\right)}$$

E2Q Potential

$$= \frac{-g_n(\mathrm{p})m_\pi g_m(\mathrm{p}')}{E_{\mathrm{cm}} - \left(\frac{\bar{\mathrm{q}}^2}{2M} + \frac{\bar{\mathrm{q}}'^2}{2M} + \frac{(\bar{\mathrm{q}}-\bar{\mathrm{q}}')^2}{2m_\pi}\right)}$$



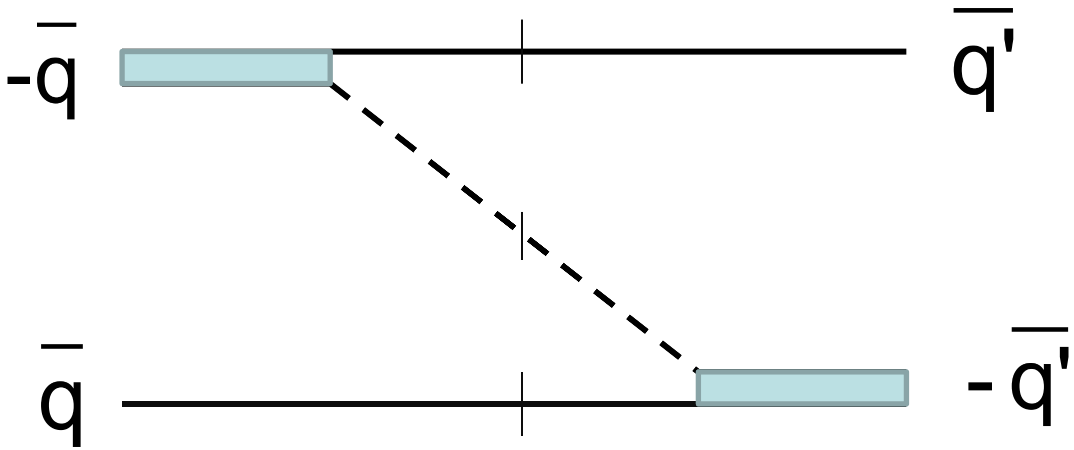
Below the 3-Body Break up

Hereafter Let us use momentum without bar for simplicity

$\bar{\mathrm{q}} \to \mathrm{q}$

$\bar{\mathrm{q}}' \to \mathrm{q}'$

1) It is clear that the difference between above and below the three-body break up threshold in the 3-body treatment, and the smooth continuation from the Faddeev to the Lippmann-Schwinger equation is completed.



2) 3-body Faddeev equation is reduced to the coupled channel 2-body Lippmann-Schwinger (LS) equations with energy dependent 2-body quasi (E2Q) potentials.

3) 3-body Faddeev-Born is smoothly continued to

E2Q (Energy Dependent 2-body Quasi) Potential

$$Z_{\alpha n,\beta m}(-\mathrm{q},\mathrm{q}';E)=\frac{-g_{\alpha n}(\mathrm{p})m_\pi g_{\beta m}(\mathrm{p}')\overline{\delta}_{\alpha,\beta}}{qq'(\chi'-x)}$$

$\boxed{\text{E2Q Potential}}$

with $\Lambda=1+{m_\pi}/{M}=1+\Delta=1+0.147$

$$\chi'=\frac{\boxed{-2m_\pi E_{\mathrm{cm}}}+\Lambda q^2+\Lambda q'^2}{2qq'}\rightarrow\frac{\boxed{-S}+q^2+q'^2}{2qq'}\Lambda=\chi\Lambda$$

$$x={\mathbf{qq'}}/{qq'};\quad \chi=\frac{\boxed{-S}+q^2+q'^2}{2qq'};\quad \boxed{-S=\frac{-2m_\pi E_{\mathrm{cm}}}{\Lambda}\equiv\sigma^2}$$

Below the Break up threshold: 3-body free energy: $E<0$,

a) NN'-bound state: $E_{\mathrm{cm}}=-|E|+m_\pi\leq 0$ (or $0<\sigma^2=-S$)

b) NN'-elastic scattering & scat. length cal.: $0\leq E_{\mathrm{cm}}=-|E|+m_\pi$

c) π + d scattering length calculation: $0\leq E_{\mathrm{cm}}=-|E|+\varepsilon_d$

Part 1
Bound State Problem
by E2Q

a) NN'-bound state : $E_{\mathrm{cm}} = -|E| + m_\pi \leq 0$ (or $0 < \sigma^2 = -S$)

$$\sigma^2 = \frac{-2m_\pi(E + \varepsilon_{\mathrm{B}})}{\Lambda} \equiv \frac{-2m_\pi E_{\mathrm{cm}}}{\Lambda} \geq 0$$

Refered from APFB Conf. Seoul 2011.

a) NN'-binding energy near the $E_{\rm cm}$ threshold: $E_{\rm cm} \le 0$

$$Z_{\alpha n,\beta m}(-\mathrm{q},\mathrm{q}';E) = \frac{-g_{\alpha n}(\mathrm{p}) m_\pi g_{\beta m}(\mathrm{p}')\bar{\delta}_{\alpha,\beta}}{qq'(\chi' - x)} :$$

$\boxed{\text{E2Q Potential}}$

with $\Lambda = 1 + {m_\pi}/{M} = 1 + \Delta = 1 + 0.147$

$$\chi' = \frac{-2m_\pi E + \Lambda q^2 + \Lambda q'^2}{2qq'} \to \frac{\sigma^2 + q^2 + q'^2}{2qq'}\Lambda = \chi\Lambda$$

$$x = {\mathbf{qq'}}/{qq'} ; \qquad \chi = \frac{\sigma^2 + q^2 + q'^2}{2qq'} ;$$

$$\sigma^2 = \frac{-2m_\pi(E + \varepsilon_{\rm B})}{\Lambda} \equiv \frac{-2m_\pi E_{\rm cm}}{\Lambda} \ge 0$$

E: 3-body free energy,

$\varepsilon_{\rm B} = \varepsilon_{N\pi} = m_\pi$: binding energy of $(N\pi)$ sub-system.

Since the Green's function with $\Lambda = 1 + \Delta$

$(\Delta \equiv m_\pi / M = 0.147)$;

$$(\chi' - x)^{-1} = (\Lambda\chi - x)^{-1} = (\chi + \Delta\chi - x)^{-1}$$

then Δ expansion of Green's function leads,

$$Z_{\alpha n,\beta m}(\text{-q, q'}; E) = 2C_{\alpha n,\beta m} \sum_{j=0}^{\infty} \frac{(-\Delta)^j (\sigma^2 + q^2 + q'^2)^j}{[\sigma^2 + (\mathrm{q} - \mathrm{q'})^2]^{j+1}}$$

This is a two-body like potential with energy dependence.

Introduce Fourier tranceform;

$$\mathcal{F}\left[Z_{\alpha n,\beta m}(\text{-q, q'}; E) \right] = \frac{\delta(\mathrm{R}) C_{\alpha n,\beta m}}{4\pi(2+\Delta)} U(\Delta, \sigma; r)$$

$$U(\Delta,\sigma;r)=\frac{1}{r}e^{-\sigma r/2}+\left(\frac{\Delta}{2+\Delta}\right)\frac{(-1)}{1!2^2}\sigma e^{-\sigma r/2}$$

$$+\left(\frac{\Delta}{2+\Delta}\right)^2\frac{(-1)^2(\sigma r/2+1)}{2!2^3}\sigma e^{-\sigma r/2}$$

$$+\left(\frac{\Delta}{2+\Delta}\right)^3\frac{(-1)^3(\sigma^2 r^2/2+3\sigma r+6)}{3!2^5}\sigma e^{-\sigma r/2}$$

$$+\ \ldots..$$

$$\equiv U^{(0)}(\Delta,\sigma;r)+U^{(1)}(\Delta,\sigma;r)+U^{(2)}(\Delta,\sigma;r)+\ldots.$$

The two-body potential reduction has the energy dependence.

For the standerdization, we adopt a statistical average with a weight :

$$P = \frac{\sigma^{2\gamma+1} e^{-a\sigma}}{\rho}$$

$$\rho = \int_0^\infty \sigma^{2\gamma+1} e^{-a\sigma} d\sigma = \frac{\Gamma(2\gamma+2)}{a^{2\gamma+2}}$$

$$\mathcal{L}\left\{U^{(0)}(\Delta,\sigma;r)\right\} \equiv \frac{1}{\rho}\int_0^\infty \sigma^{2\gamma+1} e^{-a\sigma} \frac{e^{-\sigma r/2}}{r} d\sigma$$

$$= \frac{a^{2\gamma+2}}{r(r/2+a)^{2\gamma+2}}$$

Weight function $\sigma^{2\gamma+1}e^{-a\sigma}/\rho$ denotes

nucleon structure (or form factor) effects.

1) Van der Waals type: $\sigma^{2\gamma+1}e^{-a\sigma} \to \sigma^{4}e^{-a\sigma}$

by $\gamma = 3/2$ and for $2a \equiv a_0$

$$\mathcal{L}\left\{U^{(0)}(\Delta,\sigma;r)\right\} = \frac{a_0^5}{r(r+a_0)^5}$$

$$\to \frac{e^{-5r/a_0}}{r} \quad \text{for } r \ll a_0$$

$$\to \frac{a_0^5}{r^6} \quad \text{for } r \gg a_0$$

2) Monotonic: $\sigma^{2\gamma+1}e^{-a\sigma} \rightarrow 1e^{-a\sigma}$ by $\gamma = -1/2$

$$\mathcal{L}\left\{U^{(0)}(\Delta,\sigma;r)\right\} = \frac{a_0}{r(r+a_0)} \qquad (\text{with } 2a = a_0)$$

$$\rightarrow \frac{e^{-\mu_0 r}}{r} \qquad (\text{for } a_0 >> r \quad \text{with } \mu_0 = 1/a_0)$$

$$\rightarrow \frac{a_0}{r^2} \qquad (\text{for } a_0 << r \quad)$$

3) Yukawa potential: $\sigma^{2\gamma+1}e^{-a\sigma} \rightarrow \delta(\sigma - 2\mu_0)$

$$\mathcal{L}\left\{U^{(0)}(\Delta,\sigma;r)\right\} = \frac{e^{-\mu_0 r}}{r}$$

I. Simple Fourier-Laplace transformation:

Therefore the energy independent r-space potential by Faddeev–Born term is obtained by Fourier-Laplace transform;

$$\mathcal{L}\mathcal{F}\left\{Z_{\alpha n,\beta m}(\text{-q},\text{q}';E)\right\} \equiv V_0 \mathcal{L}\left\{U^{(0)}(\Delta,\sigma;r)\right\}$$

$$= V(\Delta;r) \equiv \sum_{k=0}^{\infty} V^{(k)}(\Delta;r)$$

where the lowest order potential is

$$V^{(0)}(\Delta;r) = \frac{V_0 a_0}{r(r+a_0)}$$

II. Solution of the long range potential at large distance :

The Schrödinger equation is

$$\left\{\frac{d^2}{dr^2}+\left(\beta^2-\frac{\nu^2-1/4}{r^2}\right)\right\}\chi_l(r)=0$$

with $\beta^2=-\kappa^2=2mE/\hbar^2$

$$-\nu^2=\mu^2=\frac{2m\left|V_0\right|a_0}{\hbar^2}-(l+\frac{1}{2})^2$$

The solution is the modified Bessel function;

$$\chi(r)=\sqrt{\kappa r}Z_\nu(i\kappa r)\equiv\sqrt{\kappa r}K_{i\mu}(\kappa r)$$

Boundary condition leads

$$\kappa = \frac{2}{a} e^{(C - n\pi)/\mu} \equiv \kappa_n$$

$$E_n = -\frac{\kappa_n^2}{2m} = \left(\frac{2}{ma^2} e^{2C/\mu} \right) e^{-2\pi n/\mu}$$

then

$$E_{n+1} = E_n \, e^{-2\pi/\mu}$$

Energy sequence

rms radius is

$$r_{n+1} = r_n \, e^{\pi/\mu}$$

sequence reversed

where

$$\mu^2 = \frac{2m\left|V_0\right|a_0}{\hbar^2} - (l + \frac{1}{2})^2$$

$$= 1.7435 - (l + \frac{1}{2})^2 > 0$$

only $l = 0$ satisfies, and we obtain

$$\mu = 1.2221$$

$(\textit{while} \;\; \mu = 1.0 \;\;\; \textit{is the Efimov's condition})$

Then

$$e^{2\pi/\mu} = 170.98 \qquad\qquad e^{\pi/\mu} = 13.076$$

$$\frac{E_n}{E_{n+1}} = e^{2\pi/\mu} = 170.98$$

$$E_{n+1} = \frac{E_n}{170.98} = 5.85 \times 10^{-3} E_n$$

$$E_1 = 2.226\,\text{MeV}$$

$$E_2 = 5.85 \times 10^{-3} \times 2.226\,\text{MeV} = 13.0\,\text{keV}$$

$$E_3 = 5.85 \times 10^{-3} \times 13.0\,\text{keV} = 76.14\,\text{eV}$$

$$E_4 = 5.85 \times 10^{-3} \times 76.14\,\text{eV} = 0.44533\,\text{eV}$$

$$\boxed{E_5 = 5.85 \times 10^{-3} \times 0.44533\,\text{eV} = 0.002605\,\text{eV}}$$

$$E_6 = 5.85 \times 10^{-3} \times 0.002605\,\text{eV} = 1.5236 \times 10^{-5}\,\text{eV}$$

$$E_7 = 5.85 \times 10^{-3} \times 1.5236 \times 10^{-5}\,\text{eV} = 8.9108 \times 10^{-11}\,\text{eV}$$

$$\frac{\langle r\rangle_n}{\langle r\rangle_{n+1}} = e^{-\pi/\mu} = \frac{1}{13.076} \quad \langle r\rangle_{n+1} = 13.076\langle r\rangle_n$$

$$\langle r\rangle_1 = 1.97\times10^{-15}\text{m}$$

$$\langle r\rangle_2 = 13.076\times1.97\times10^{-15}\text{m} = 2.576\times10^{-14}\text{m}$$

$$\langle r\rangle_3 = 13.076\times25.76\times10^{-15}\text{m} = 3.37\times10^{-13}\text{m}$$

$$\langle r\rangle_4 = 13.076\times3.37\times10^{-13}\text{m} = 4.40\times10^{-12}\text{m}$$

Bohr radius

$$\boxed{\langle r\rangle_5 = 13.076\times4.40\times10^{-12}\text{m} = 5.76\times10^{-11}\text{m}}$$

$$\langle r\rangle_6 = 13.076\times5.76\times10^{-11}\text{m} = 7.53\times10^{-10}\text{m}$$

$$\langle r\rangle_7 = 13.076\times7.53\times10^{-10}\text{m} = 9.85\times10^{-9}\text{m}$$

Numerical calculation by Schroedinger equation:

	MeV		fm	
n	E_n	E_n/E_{n-1}	$\langle r_n^2 \rangle^{1/2}$	$\langle r_n^2 \rangle^{1/2} / \langle r_{n-1}^2 \rangle^{1/2}$
1	-2.222		2.516	
2	-1.271×10^{-2}	174.8	3.652×10^{1}	14.52
3	-7.433×10^{-5}	171.0	4.812×10^{2}	13.18
4	-4.347×10^{-7}	171.0	6.296×10^{3}	13.08
5	-2.543×10^{-9}	171.0	8.233×10^{4}	13.08
6	-1.487×10^{-11}	171.0	1.077×10^{6}	13.08
7	-8.697×10^{-14}	171.0	1.408×10^{7}	13.08
8	-5.087×10^{-16}	171.0	1.841×10^{8}	13.08
9	-2.975×10^{-18}	171.0	2.407×10^{9}	13.08
10	-1.740×10^{-20}	171.0	3.147×10^{10}	13.08

Our analytic prediction fits to the numerical solution.

Part 2
NN & π-d
Scattering Length

b) NN' - elastic scattering & scat. length cal. : $0 \le E_{\mathrm{cm}} = -|E| + m_{\pi}$

c) π + d scattering length calculation : $0 \le E_{\mathrm{cm}} = -|E| + \varepsilon_{d}$

c') π + d elastic scattering : $0 < E$

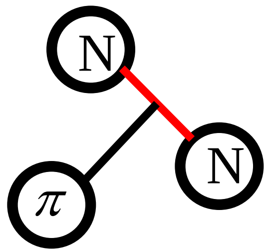
c') is not belong to E2Q, but original Faddeev which is chosen to confirm the precision of our Faddeev program.

Refered from APFB Conf. Adelaide 2014.

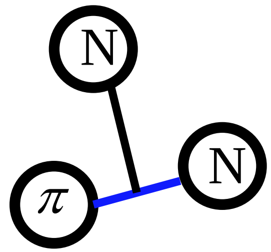
π^+D elastic scattering

This is not the E2Q example, but adopted to check the original Faddeev program

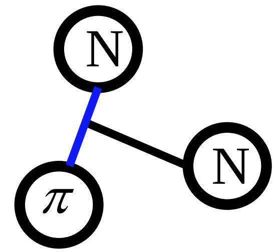
potential of π^+D elastic scattering



α channel



β channel



γ channel

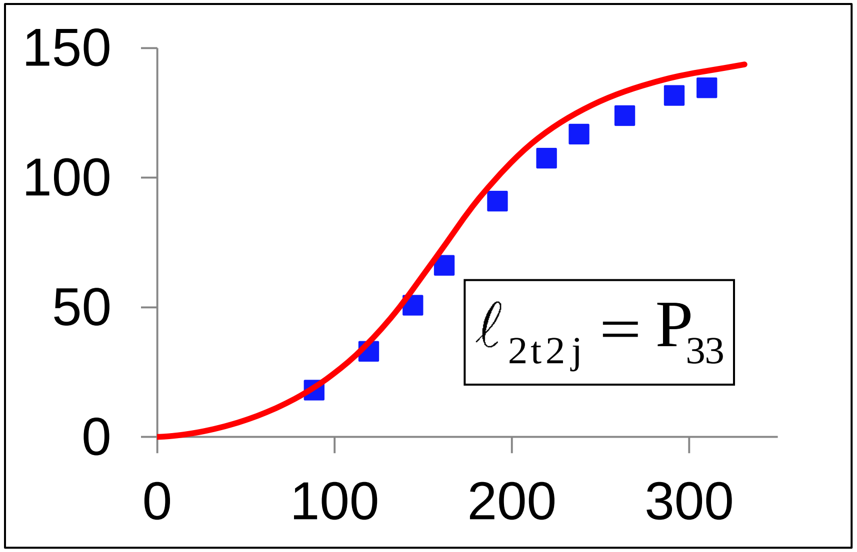
①Nuclear potential → Argonne υ18
R. B. Wiringa et al., PRC 51, 38 (1995)

②pion-N potential → $S_{11}, S_{31}, P_{11}, P_{13}, P_{31}, P_{33} = \ell_{2t2j}$ $\begin{cases} \text{type A} \\ \text{type B} \end{cases}$

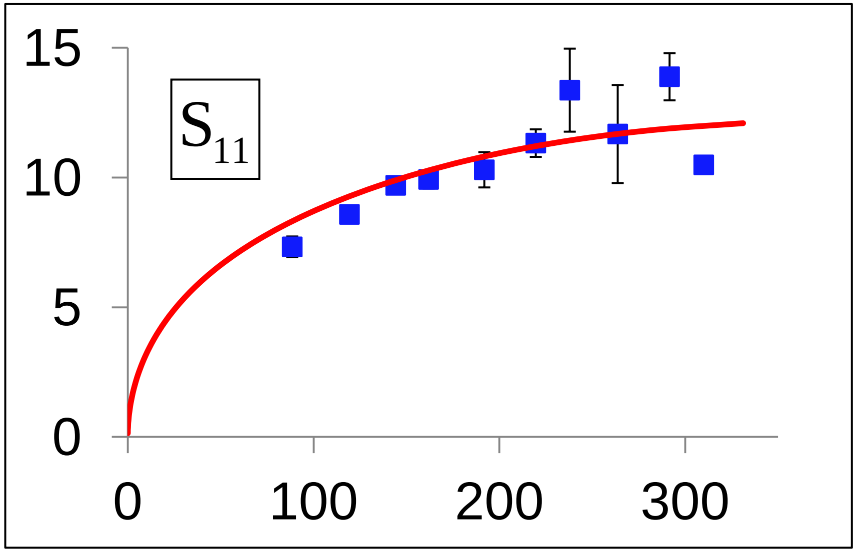
ℓ	: angular momentum
t	: pair isospin
j	: total angular momentum

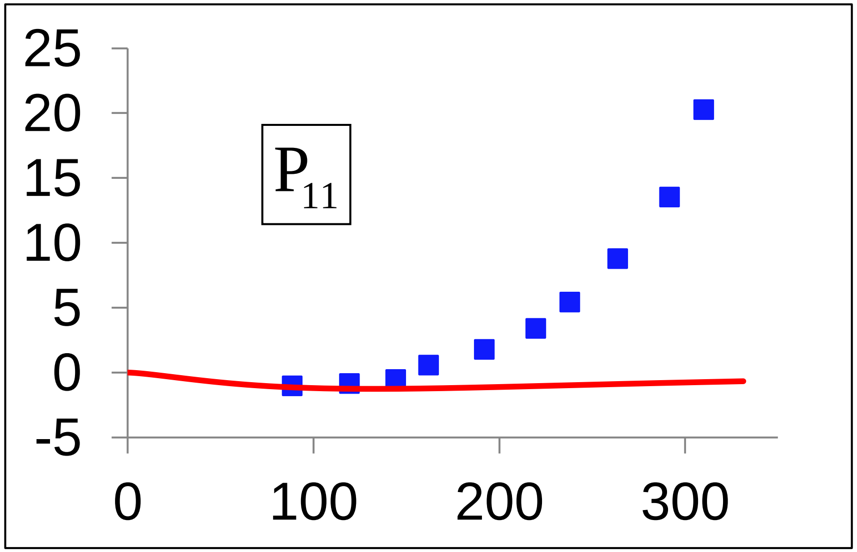
pion-Nucleon phase shift (type A)
pion Lab kinetic energy (MeV) vs phase shift (deg)

ℓ : angular momentum
$\mathbf{t}$: pair isospin
$\mathbf{j}$: total angular momentum



①type A
A. W. Thomas, NPA258, 417 (1976)
②EXP
J. R. Carter et.al., NP B58, 378 (1973)





pion-Nucleon phase shift (type A)
pion Lab kinetic energy (MeV) vs phase shift (deg)

ℓ : angularmomentum
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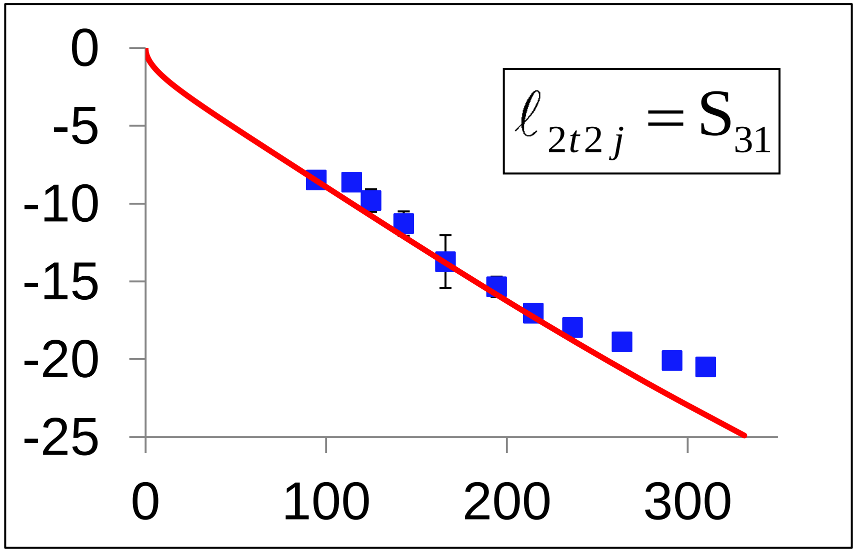

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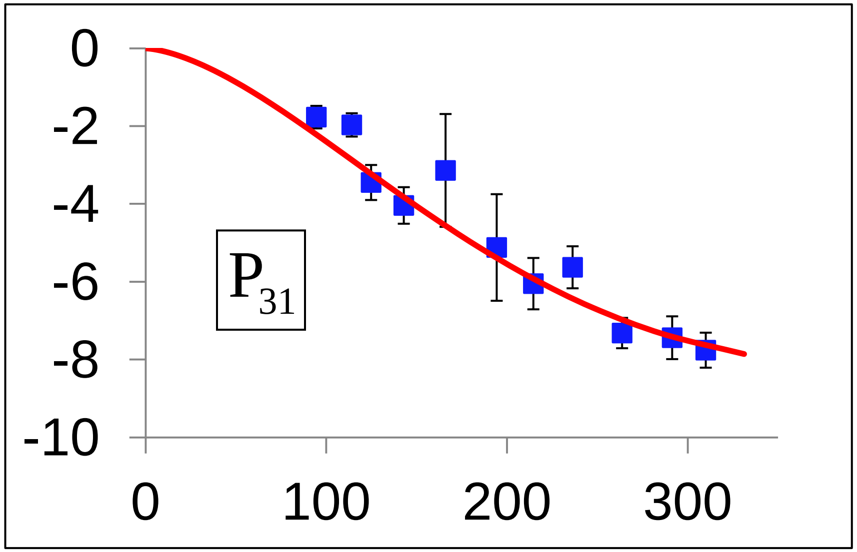
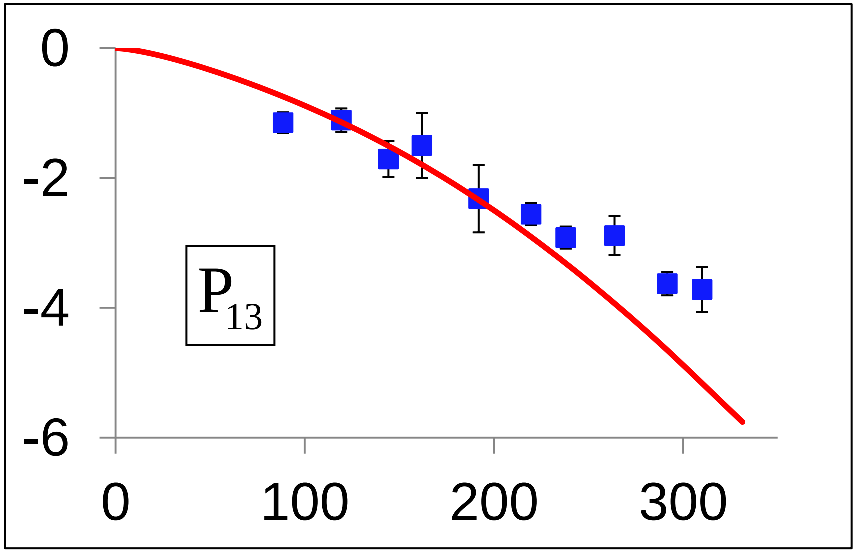
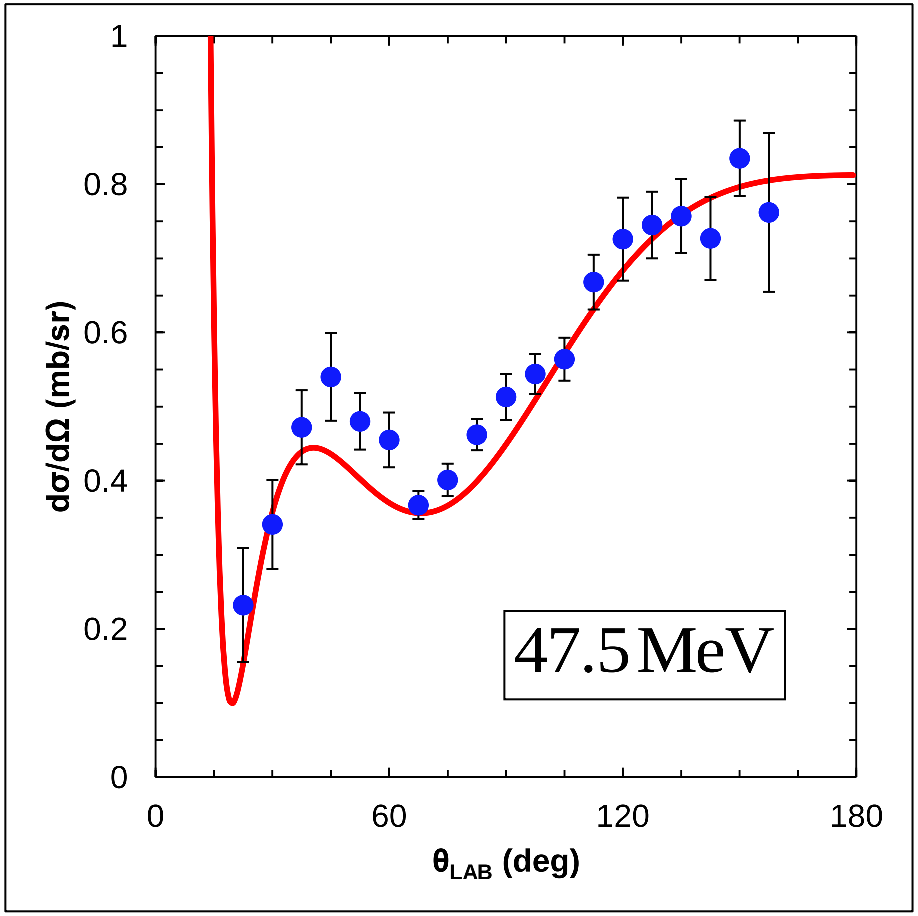
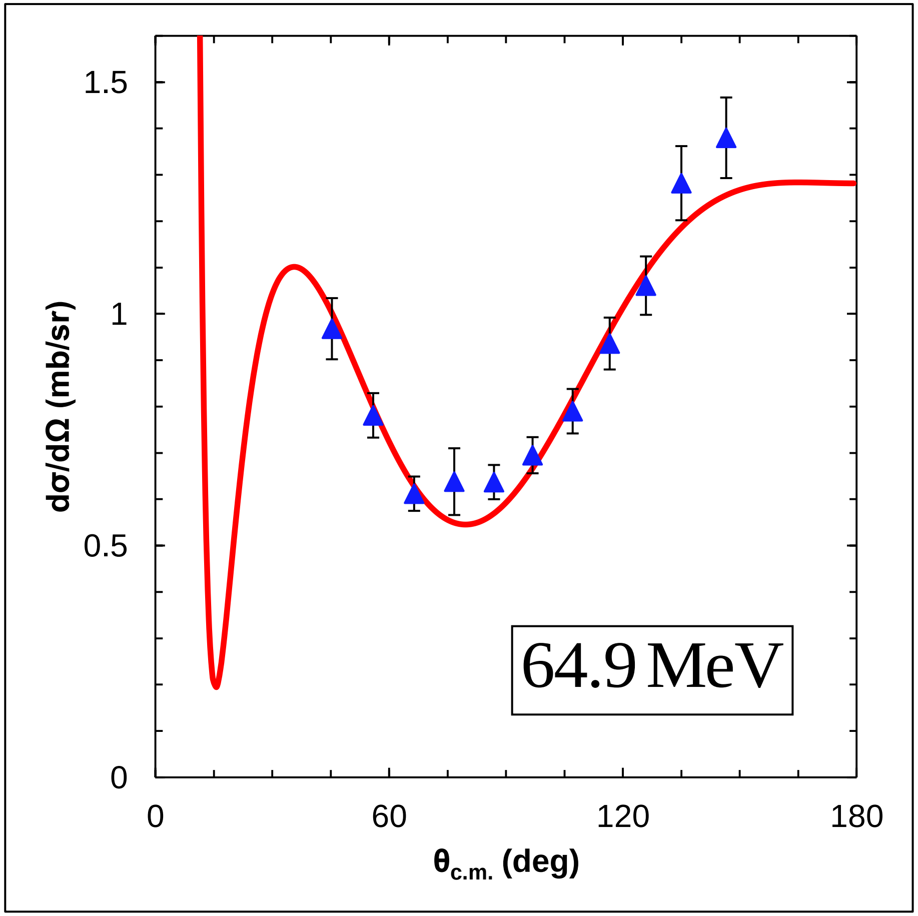




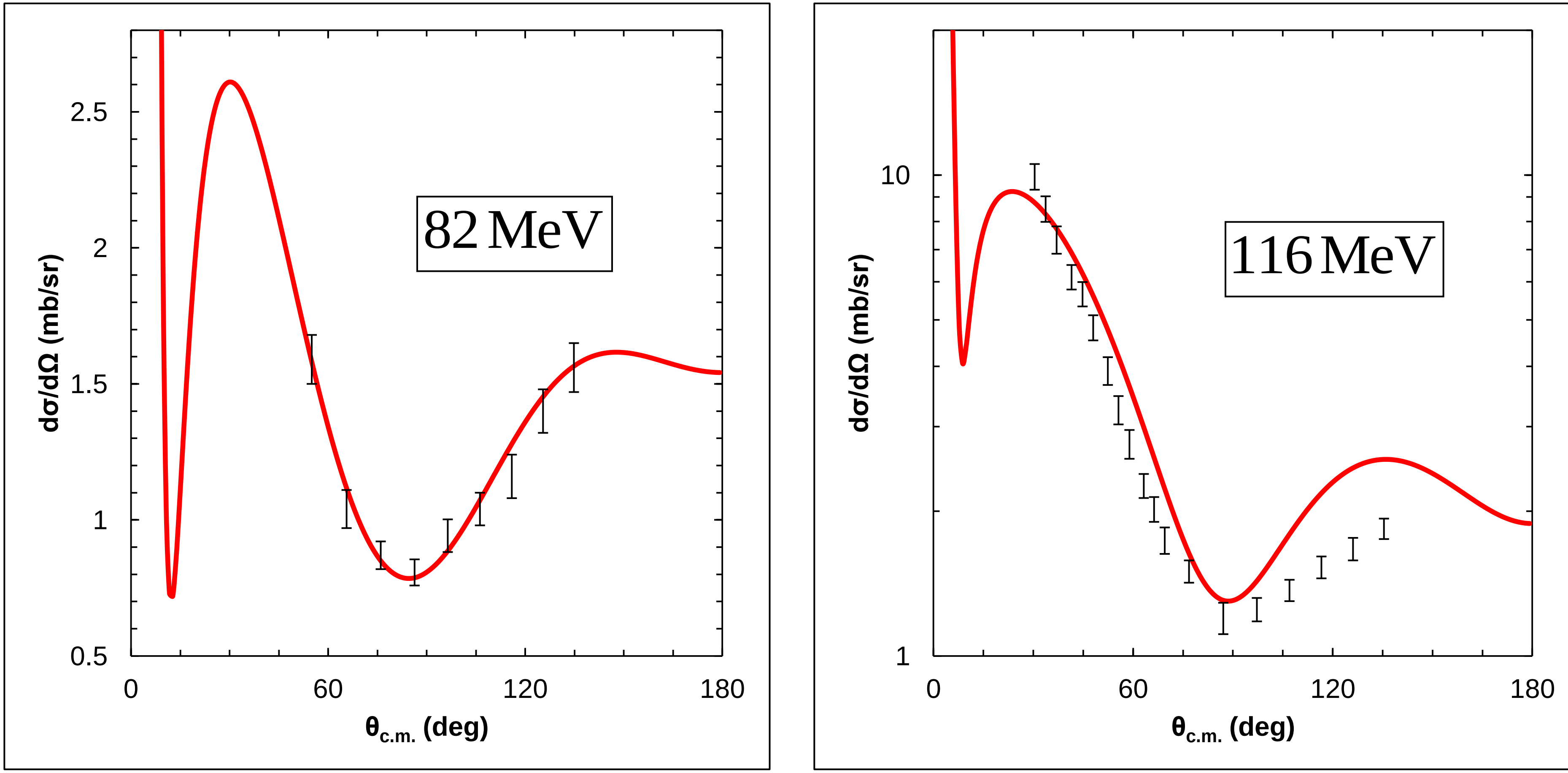
$\pi^+ D$ elastic scattering (type A; P_{33} resonance)





D. Axen et al., Nucl. Phys. A256, 387-413 (1976);
B. Balestri et al., Nucl. Phys. A392, 217-321 (1976).

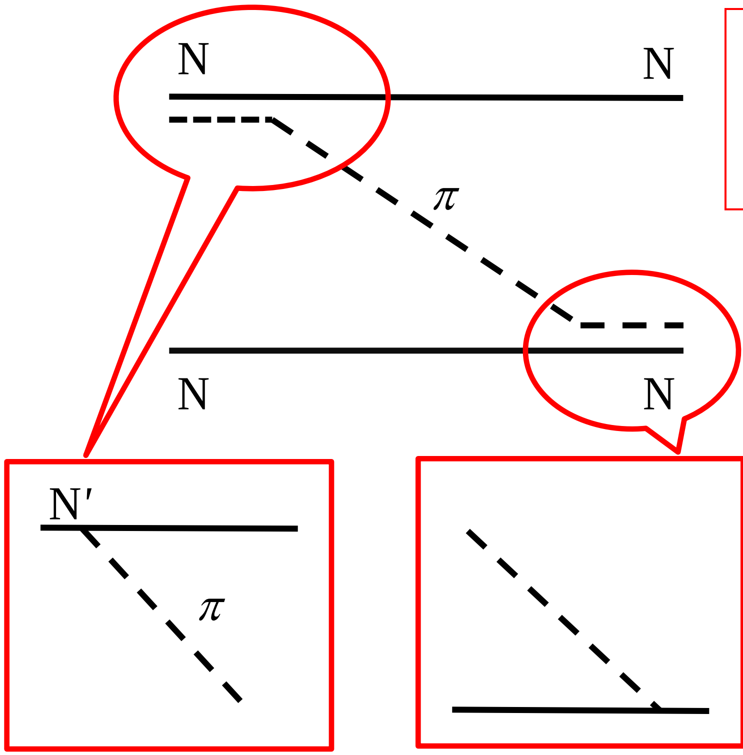
$\pi^+ D$ elastic scattering (type A; P_{33} resonance)



K. Gabathuler et al., Nucl. Phys. A350, 253-264 (1980).

back to introduction

One of our aims is to investigate the low energy NN interaction by 3-body NNpion equation



P_{11}

pion creation & annihilation/absorption

type B: M. G. Fuda, PRC52, 2875(1995)

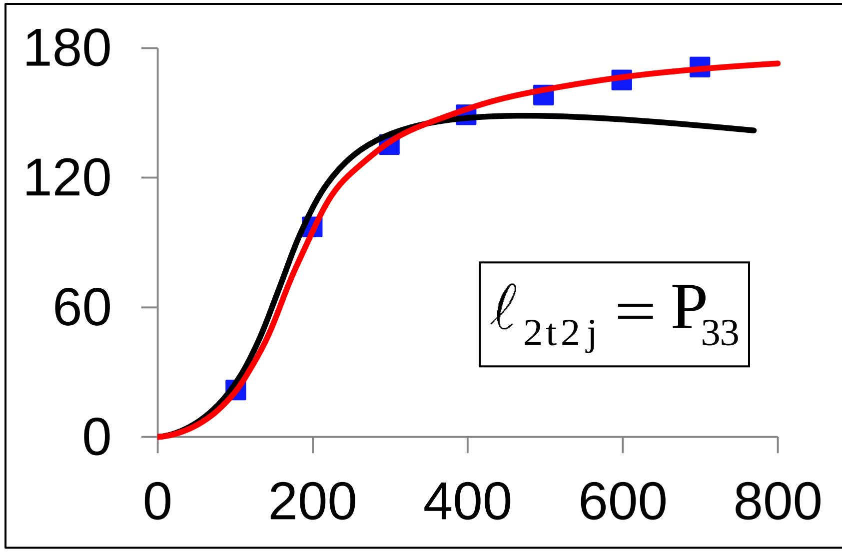
① P_{11} bound state

② $S_{11}(1535)$, $P_{11}(1440)$, $P_{33}(1232)$ resonances

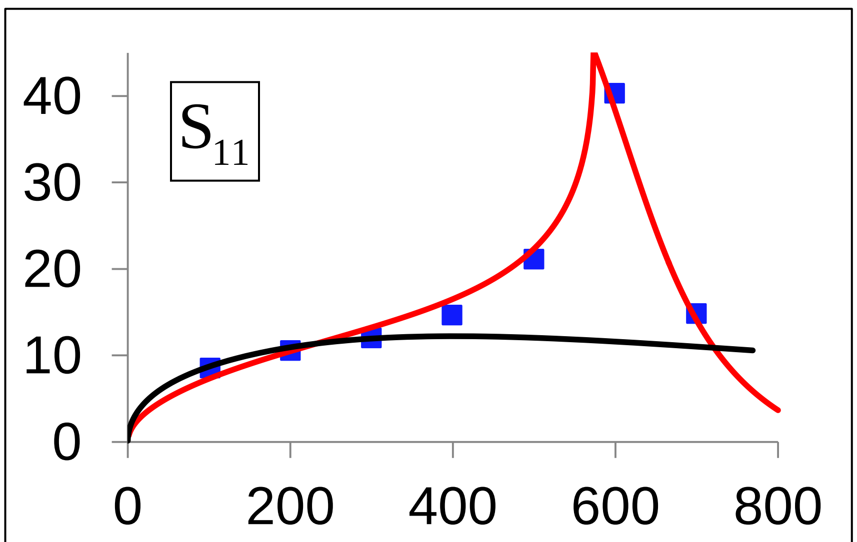
③ high energy experimental data are reproduced

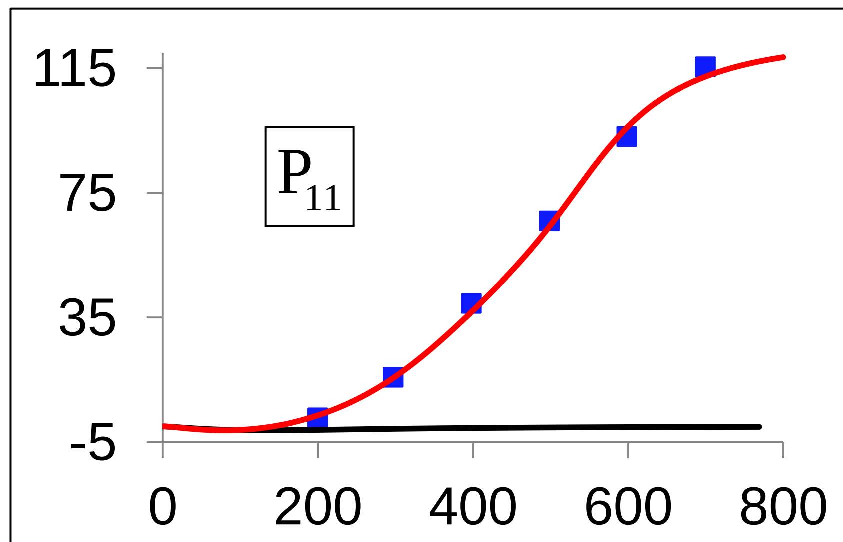
pion-Nucleon phase shift (type B)
pion Lab kinetic energy (MeV) vs phase shift (deg)

ℓ : angular momentum
t : pair isospin
j : total angular momentum



①type A
A. W. Thomas
②type B
Michael G. Fuda,
PRC, 52, 2875(1995)
③EXP
R. A. Arndt et al.,
SAID program (1995)





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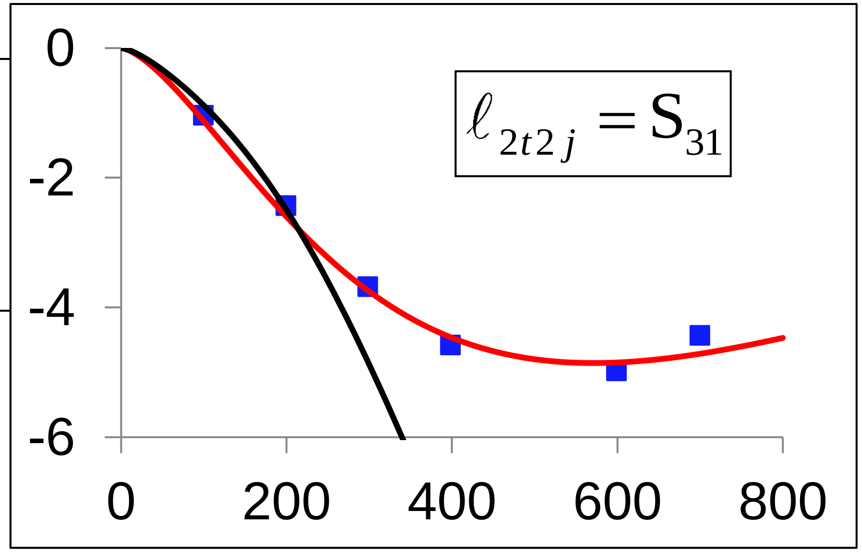

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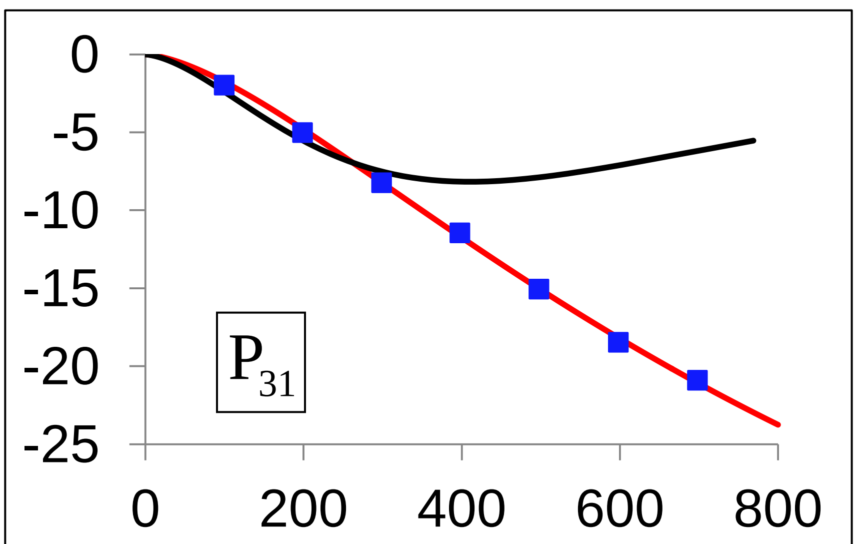
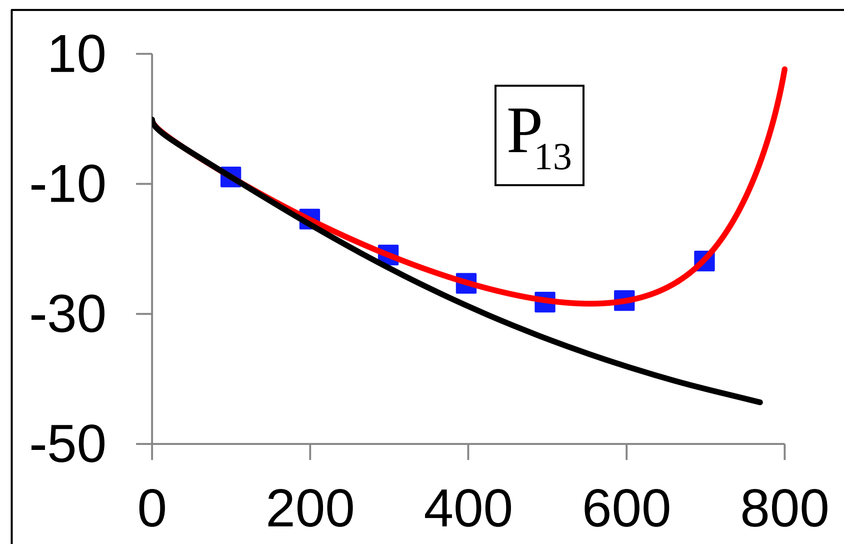




πD scattering length

E2Q Example

πD scattering length by 3-body Faddeev (without E2Q)

	Scattering length [fm]
type A P_{33} resonance	0.033
type B S_{11}, P_{11}, P_{33} resonance P_{11} bound state	$-0.019 \quad +0.019i$
EXP	$-0.038 \quad +0.009i$ (1) $-0.038 \quad +0.008i$ (2)

Faddeev

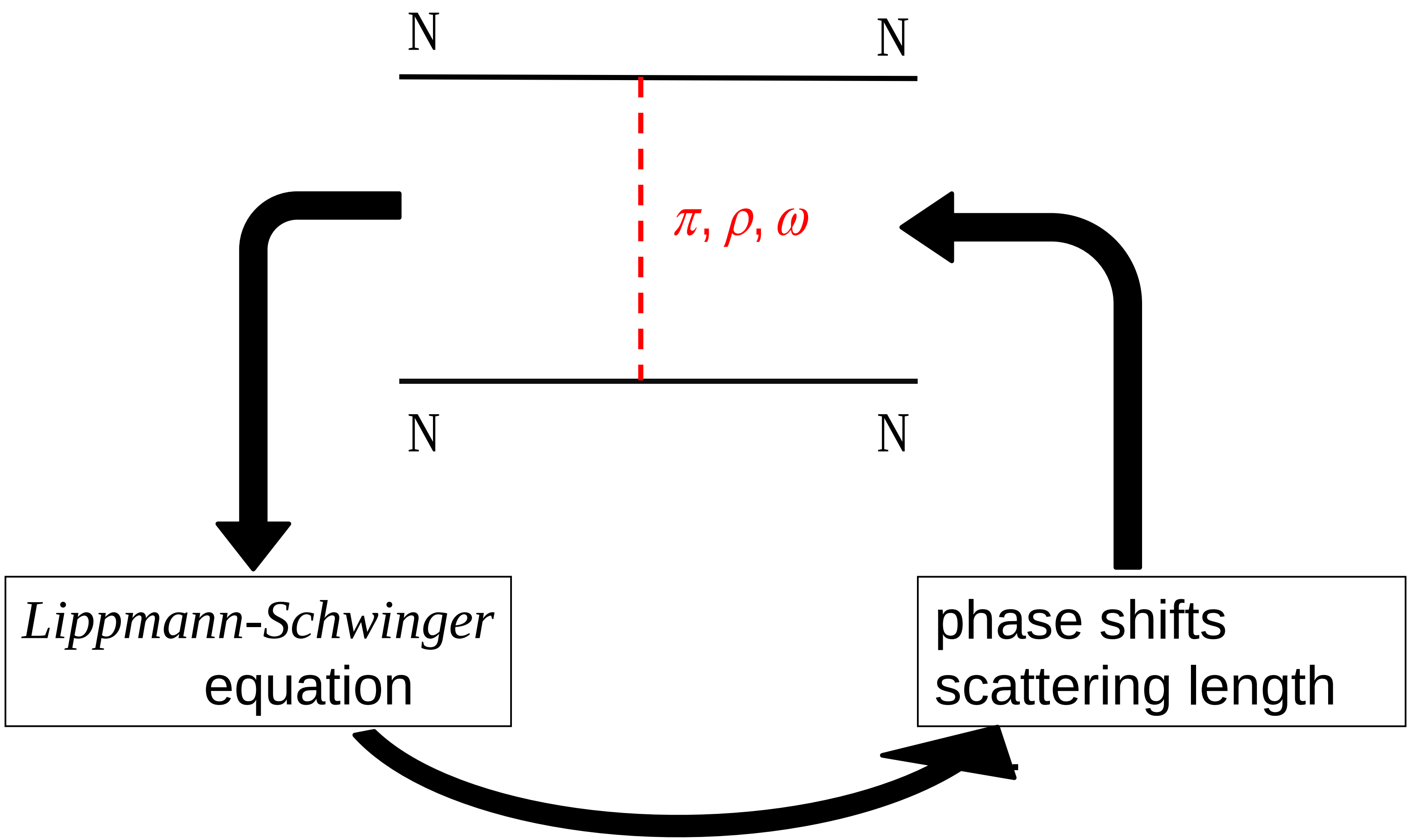
Faddeev

(1) P. Hauser et al., Phys. Rev. C58, R1869 (1998);
(2) D. Chatellard et al., Nucl. Phys. A625, 855 (1997).

neutron-proton scattering length
&
deuteron

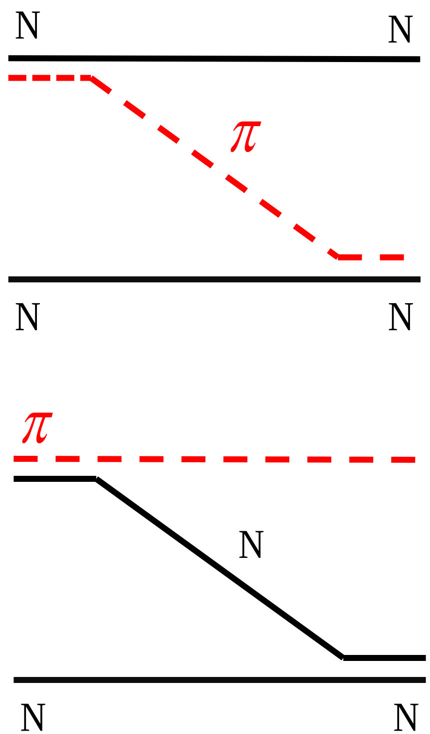
E2Q Example

NN interaction in 2-body effective potential



NN' interaction in 3-body

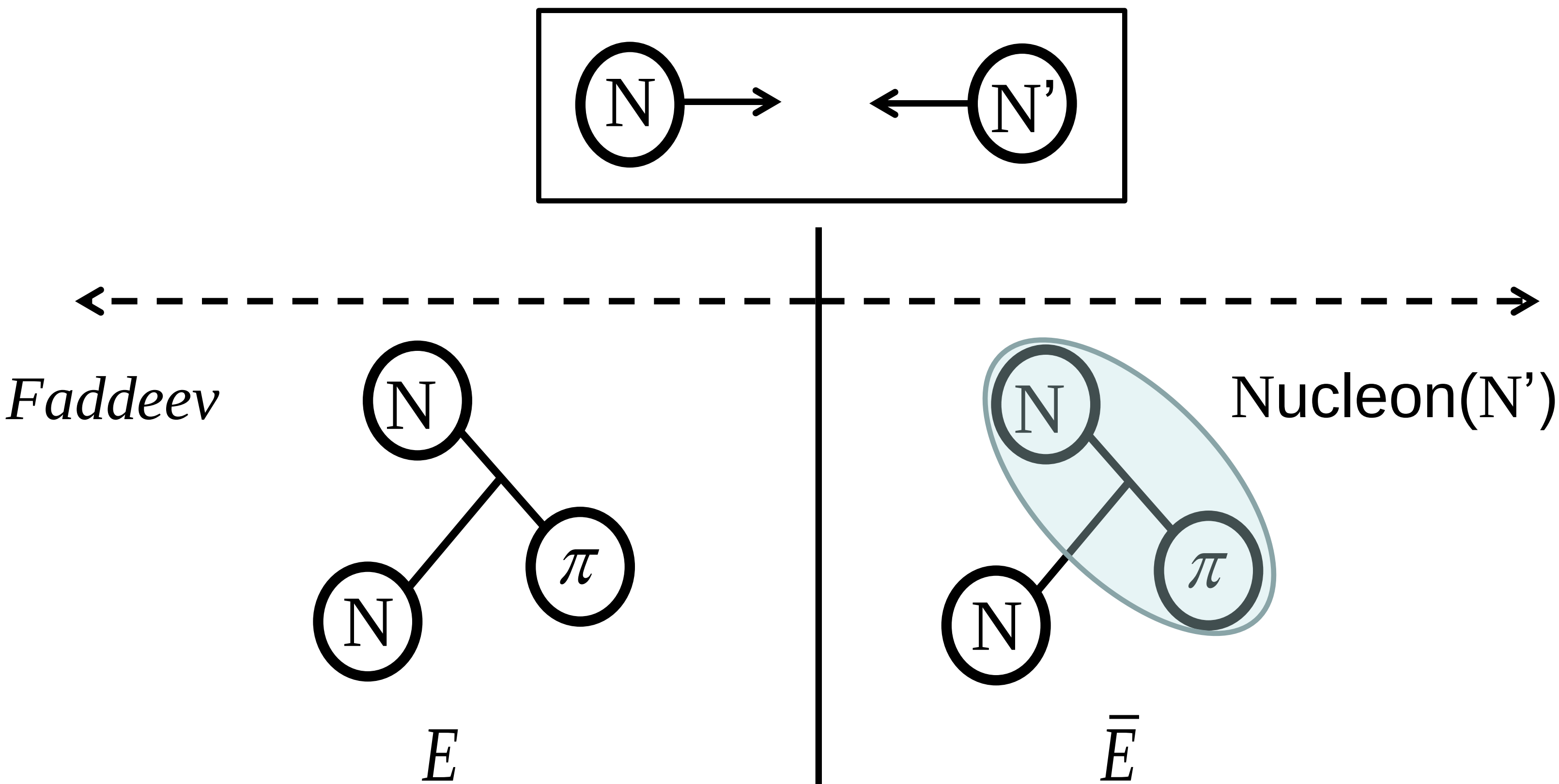
ρ, ω
→Multi Channel
Faddeev equation
S. Oryu et al., (1997)



Faddeev equation

phase shifts
scattering length

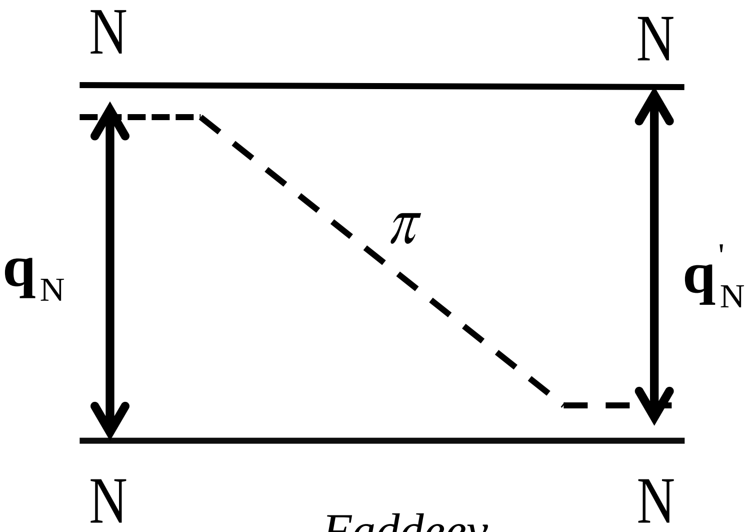
3-body energy of NNπ in NN' scattering



above 3-body break up threshold

below 3-body break up threshold

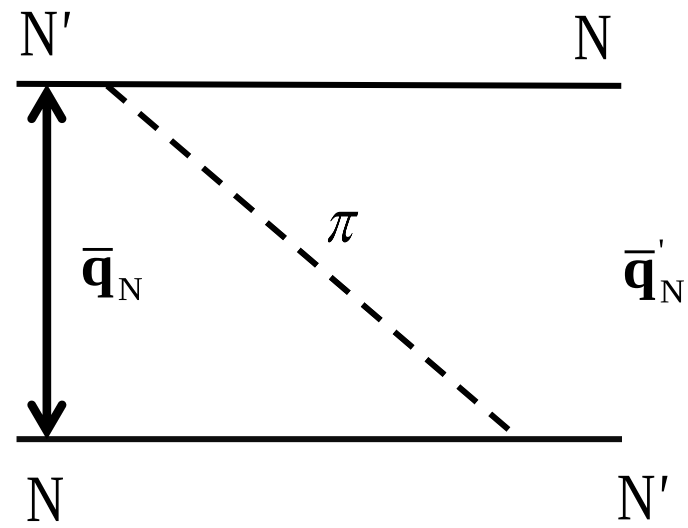
$$E = \bar{E} - m_\pi \equiv E_{\mathrm{cm}} - m_\pi$$



Faddeev

above 3-body breakup threshold

$$\frac{g\left(\mathbf{p}_{\pi \mathrm{N}}\right) g\left(\mathbf{p}_{\pi \mathrm{N}}\right)}{E-\frac{\mathbf{q}_{\mathrm{N}}^{2}}{2 m_{\mathrm{N}}}-\frac{\mathbf{q}_{\mathrm{N}}^{\prime 2}}{2 m_{\mathrm{N}}}-\frac{\left(\mathbf{q}_{\mathrm{N}}+\mathbf{q}_{\mathrm{N}}^{\prime}\right)^{2}}{2 m_{\pi}}}$$



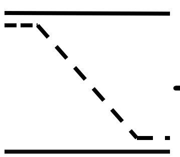
below 3-body breakup threshold

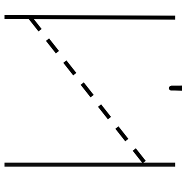
$$\frac{g\left(\overline{\mathbf{p}}_{\pi \mathrm{N}}\right) g\left(\overline{\mathbf{p}}_{\pi \mathrm{N}}\right)}{E_{\mathrm{cm}}-\frac{\overline{\mathbf{q}}_{\mathrm{N}}^{2}}{2 m_{\mathrm{N}}}-\frac{\overline{\mathbf{q}}_{\mathrm{N}}^{\prime 2}}{2 m_{\mathrm{N}}}-\frac{\left(\overline{\mathbf{q}}_{\mathrm{N}}+\overline{\mathbf{q}}_{\mathrm{N}}^{\prime}\right)^{2}}{2 m_{\pi}}}$$

Energy dependent
2-body Quasi potential (E2Q)

S. Oryu, PRC86, 044001(2012)

neutron-proton triplet scattering length by Faddeev & E2Q





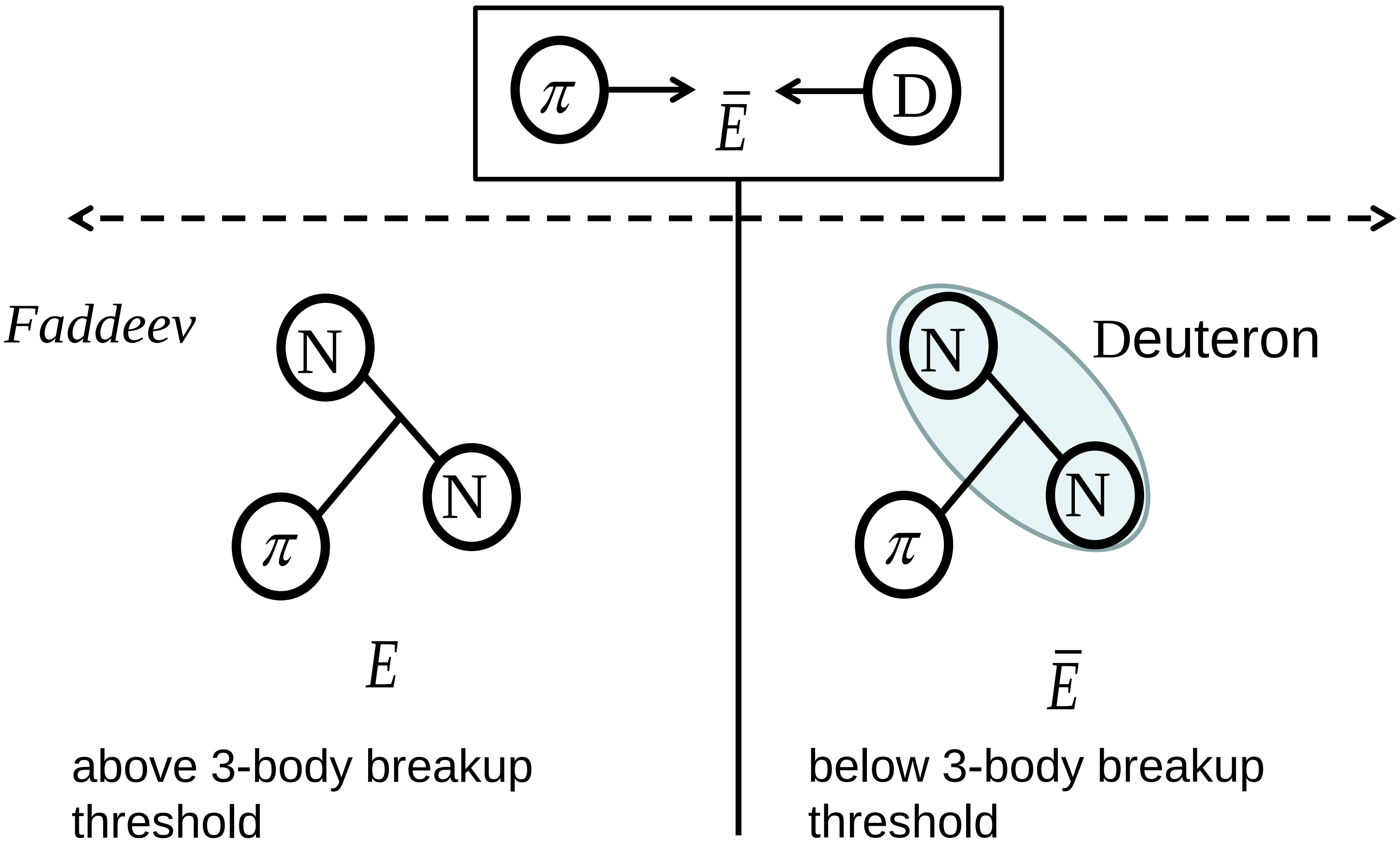
	Scattering length [fm]	
Faddeev (type A)	0.280	*Faddeev*
Faddeev (type B; S_{11}, P_{11} resonance P_{11} bound state)	2.85	*Faddeev*
E2Q (type B; S_{11}, P_{11} resonance P_{11} bound state)	4.66	*E2Q*
EXP	5.419±0.007	

T. L. Houk, PRC3, 1886 (1971); W. Dilg, PRC11,103 (1975);
S. Klarsfeld et al., JPG10, 165 (1984)

Back to πD scattering

E2Q Example

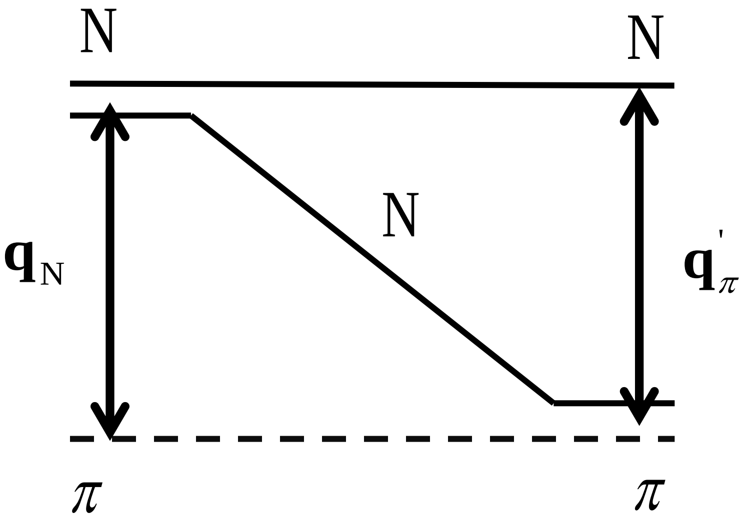
3-body energy of NNπ in πD scattering



$$E = \bar{E} - \varepsilon_{\mathrm{D}} \equiv E_{\mathrm{cm}} - \varepsilon_{\mathrm{D}}$$

Deuteron binding energy

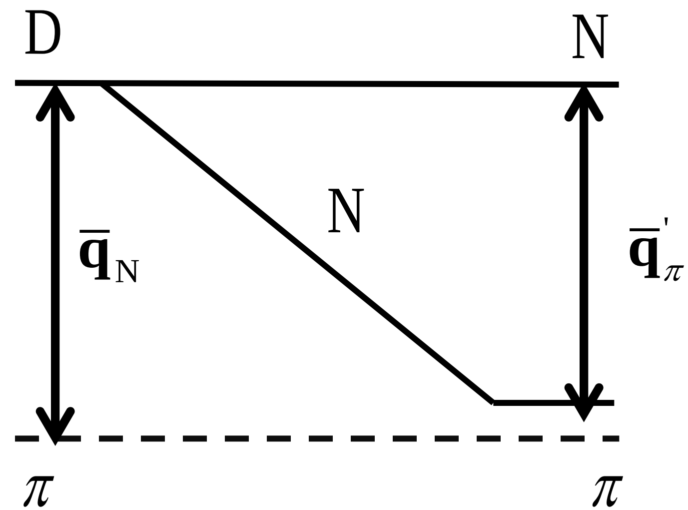
πD elastic scattering



Faddeev

above 3-body breakup threshold

$$\frac{g\left(\mathbf{p}_{\mathrm{NN}}\right)g\left(\mathbf{p}_{\mathrm{N}\pi}\right)}{E-\frac{\mathbf{q}_{\mathrm{N}}^{2}}{2m_{\mathrm{N}}}-\frac{\mathbf{q}_{\pi}^{'2}}{2m_{\pi}}-\frac{\left(\mathbf{q}_{\mathrm{N}}+\mathbf{q}_{\pi}^{'}\right)^{2}}{2m_{\mathrm{N}}}}$$



E2Q

below 3-body breakup threshold

$$\frac{g\left(\bar{\mathbf{p}}_{\mathrm{NN}}\right)g\left(\bar{\mathbf{p}}_{\mathrm{N}\pi}\right)}{E_{\mathrm{cm}}-\frac{\bar{\mathbf{q}}_{\mathrm{N}}^{2}}{2m_{\mathrm{N}}}-\frac{\bar{\mathbf{q}}_{\pi}^{'2}}{2m_{\pi}}-\frac{\left(\bar{\mathbf{q}}_{\mathrm{N}}+\bar{\mathbf{q}}_{\pi}^{'}\right)^{2}}{2m_{\mathrm{N}}}}$$

πD scattering length by 3-body

	Scattering length [fm]
Faddeev (type A; P_{33} resonance)	0.033
Faddeev (type B; S_{11}, P_{11}, P_{33} resonance P_{11} bound state)	–0.019 $+0.019i$
E2Q (type B; S_{11}, P_{11}, P_{33} resonance P_{11} bound state)	–0.023 $+0.019i$
EXP	–0.038 $+0.009i$ –0.038 $+0.008i$

Faddeev

Faddeev

E2Q

P. Hauser et al., Phys. Rev. C58, R1869 (1998);
D. Chatellard et al., Nucl. Phys. A625, 855 (1997).

summary

1) In 2012, we insisted that the original 3-body Faddeev treatment should be changed by the E2Q method below the three-body break up threshold.
2) In 1976, A. W. Thomas accomplished 47.5MeV $\pi^+ \mathrm{D}$ elastic scattering. We could reproduce not only the 47.5 MeV result but also some other energy data very well. It means that the accuracy of our Faddeev program is confirmed. However, this is not the case of E2Q because of the above the threshold.
3) In 1995, M. G. Fuda proposed a new type $\pi \mathrm{N}$ potential which is including P_{11} bound state etc. . The Fuda's potential brings about rather good results for NN, and πD scattering length data. Since the Fuda's potential is represented only by the rank 1, then our results by E2Q method may be improved by increasing ranks or off-shell effects.

4) For the scattering lengths of NN' and $\pi^{+}\mathrm{D}$, E2Q leads better results than the original Faddeev's method.
5) Our deuteron calculation by E2Q is preliminary, however, the result seems to reproduce the experimental value very well, although the deuteron binding energy fitting in the N-N-pion system is generally very hard.
6) E2Q potential is transformed to the r-space potential with the energy dependent.

   We adopted an energy average method below the NN' threshold by using the dumping function. The potential indicates the Yukawa-type potential for the shorter range, however, $1/r^2$-type potential, or the Van der Waals potential for the longer range depending on the parameter γ . Even if the Efimov-like states by the $1/r^2$-type potential could not be found experimentally, it seems that the existence of the long range $1/r^2$ or Van der Waals-types are not theoretically denied in the NN' interaction.

Thank you very much!

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Discussion

1) The divergence of the E2Q potential on the NN' threshold is similar to the Coulomb potential:

$$\lim_{\sigma\to 0} U(\Delta,\sigma;r) = \frac{1}{r}e^{-\sigma r/2} + \ ..... = \frac{1}{r}$$

On the other hand, at the slightly small energy below NN' threshold, the potential should be a screened Coulomb potential:

$$\lim_{\sigma\to +0} U(\Delta,\sigma;r) = \frac{1}{r}e^{-\sigma(E)r/2} + \ .....$$

Since, the screening range contains the eigen energy which is presumably very difficult to fix, because the shallower energy levels are close one another under near the NN' threshold in the numerical calculation.

Therefore, the energy average of the potential could be one of the most useful methods to estimate the energy levels under near the NN' threshold.

2) These shallower binding energy levels could be understood as a reflection of the long range potentials.

3) Our prediction of the shallow binding energies is lesser than several keV for NN’ system, however, those energy region (0~-50keV :preliminary) has not being measured yet. Such a very low energy measurement seems to be rather hard.
So far as the proton-proton phase shift is concerned, data lesser than -50keV are not shown (J. Phys. G:Nucl. Part. Phys.39 (2012) 045101 12pp).

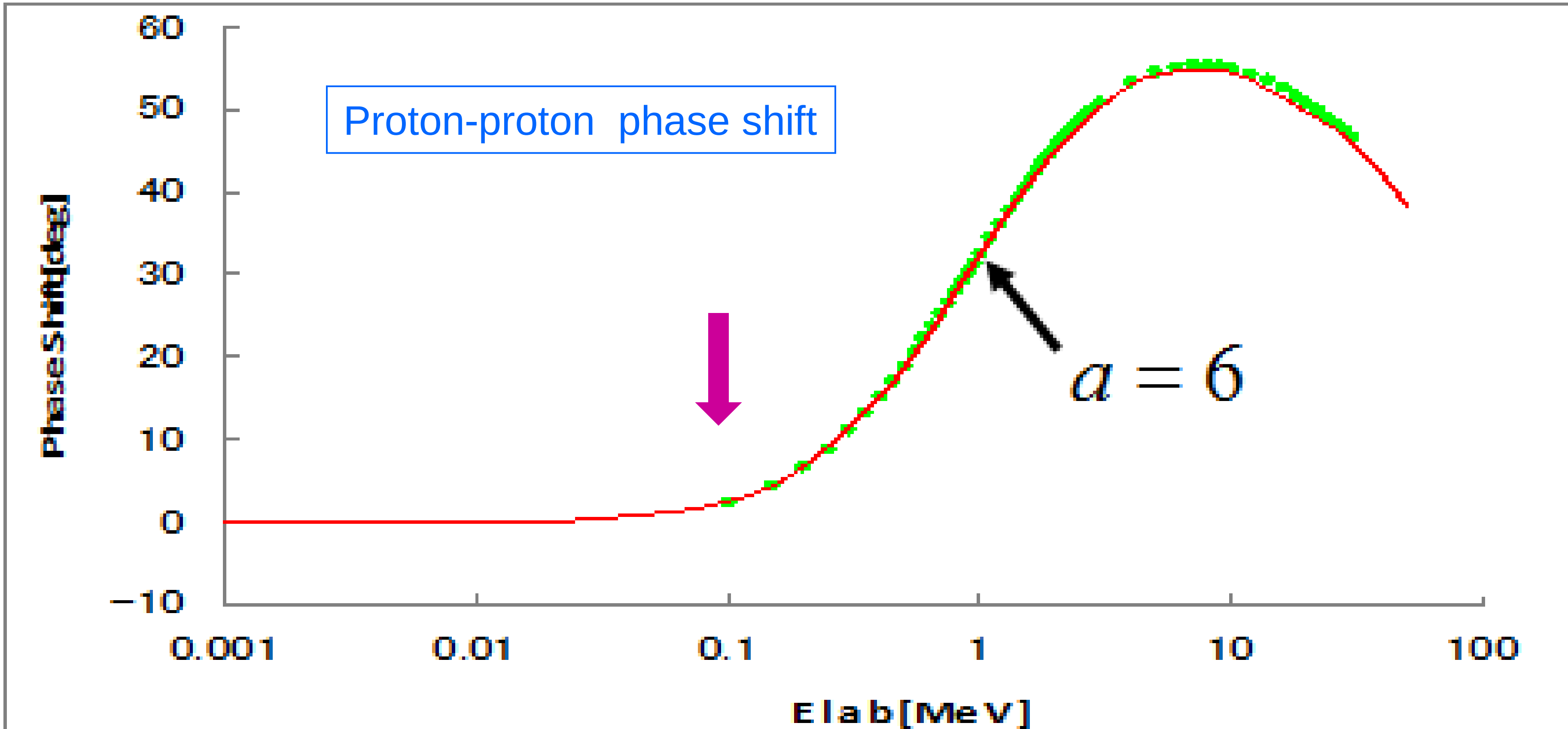
Determine the screened Coulomb range parameter;

$$R_{c\ell} = \exp(a\gamma)/2k$$

$$T^{(R)} = (V^S + V^R) + (V^S + V^R)G_0T^{(R)},$$

$$T^R = V^R + V^R G_0 T^R,$$

$$\delta = \tan^{-1}\left(\frac{\mathrm{Im}(T^{(R)} - T^R)}{\mathrm{Re}(T^{(R)} - T^R)}\right)$$



J. R. Bergervoet, P. C. van Campen, W. A. van der Sanden, and J. J. de Swart, Phys. Rev. C38, 15 (1988)