# Influence of clusters and symmetry energy on pion production at 300 MeV/nucleon

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Workshop on Science with SpiRIT TPC

June 5, 2015, RIKEN

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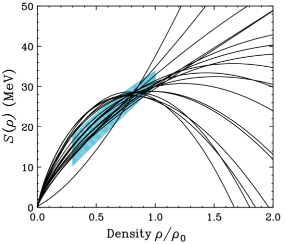
# Symmetry energy at various densities

## Nuclear EOS (at $T = 0$)

$$(E/A)(\rho_p, \rho_n) = (E/A)_0(\rho) + S(\rho)\delta^2 + \cdots$$

$$\rho = \rho_p + \rho_n, \quad \delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

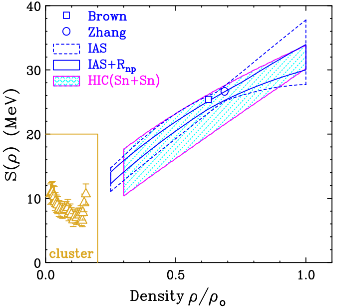
- $S_0 = S(\rho_0)$
- $L = 3\rho_0 (dS/d\rho)_{\rho=\rho_0}$




$S(\rho)$ for Skyrme interactions

Constrains on $S(\rho)$

Horowitz et al., J. Phys. G: Nucl. Part. Phys. 41 (2014) 093001.




$\sim \frac{1}{10}\rho_0$ $\rho > \rho_0$

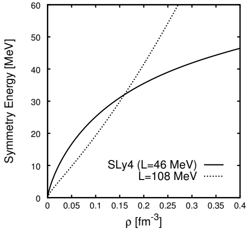
- Uncertainties at high densities.
- Clusters at low densities, at least.

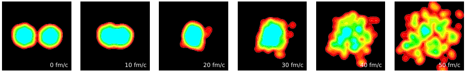
## General purpose

Explore the symmetry energy at $\rho \sim 2\rho_0$.

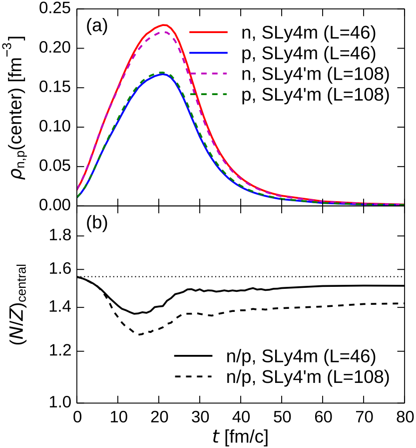
$S(\rho) \Rightarrow$ Compression stage $\Rightarrow$ Obs.(?)

- Nucleon observables
- Pion observables



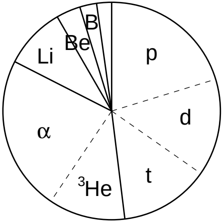




$^{132}$Sn + $^{124}$Sn, $E/A = 300$ MeV, $b \sim 0$

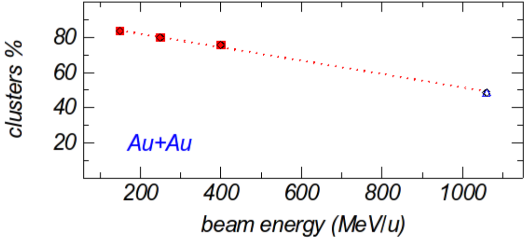


# Clusters at relatively high energies

Reisdorf.et al., NPA612(1997)493.

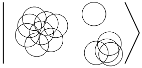



Au + Au at 250 MeV/u




- Clusters are important at least in the late stage of collisions.
- What about the cluster correlations at earlier times?

# Antisymmetrized Molecular Dynamics (with NN collisions)



## AMD wave function

$$|\Phi(Z)\rangle = {\color{red}\det_{ij}}\Big[\exp\Big\{-\nu\Big(\mathbf{r}_j - \frac{\mathbf{Z}_i}{\sqrt{\nu}}\Big)^2\Big\}\chi_{\alpha_i}(j)\Big]$$

$$\mathbf{Z}_i = \sqrt{\nu}\mathbf{D}_i + \frac{i}{2\hbar\sqrt{\nu}}\mathbf{K}_i$$

$\nu$ : Width parameter $= (2.5\ \text{fm})^{-2}$

$\chi_{\alpha_i}$ : Spin-isospin states $= p\uparrow, p\downarrow, n\uparrow, n\downarrow$

Time-dependent variational principle

$$\delta\int_{t_1}^{t_2}\frac{\langle\Phi(Z)|(i\hbar\frac{d}{dt} - H)|\Phi(Z)\rangle}{\langle\Phi(Z)|\Phi(Z)\rangle}dt = 0, \qquad \delta Z(t_1) = \delta Z(t_2) = 0$$

## Equation of motion for the wave packet centroids $Z$

$$\frac{d}{dt}\mathbf{Z}_i = \{\mathbf{Z}_i, \mathcal{H}\}_{\text{PB}} \quad + \quad {\color{red}\text{(NN collisions)}}$$

$\{\mathbf{Z}_i, \mathcal{H}\}_{\text{PB}}$: Motion of wave packets in the mean field

$$\mathcal{H} = \frac{\langle\Phi(Z)|H|\Phi(Z)\rangle}{\langle\Phi(Z)|\Phi(Z)\rangle} + \text{(c.m. correction)},$$

$H$: Effective interaction (e.g. Skyrme force)

# Zero-range effective interaction

## Skyrme force

$$v_{ij} = t_0(1 + x_0 P_\sigma)\delta(\mathbf{r}) + \tfrac{1}{2} t_1(1 + x_1 P_\sigma)[\delta(\mathbf{r})\mathbf{k}^2 + \mathbf{k}^2\delta(\mathbf{r})] \qquad \mathbf{r} = \mathbf{r}_i - \mathbf{r}_j$$

$$+ t_2(1 + x_2 P_\sigma)\mathbf{k}\cdot\delta(\mathbf{r})\mathbf{k} + t_3(1 + x_3 P_\sigma)[\rho(\mathbf{r}_i)]^\alpha \delta(\mathbf{r}) \qquad \mathbf{k} = \tfrac{1}{2\hbar}(\mathbf{p}_i - \mathbf{p}_j)$$

Expectation value of interaction energy can be written by using several kinds of densities.

$$\langle V\rangle = \int \mathcal{V}\Big(\rho(\mathbf{r}), \tau(\mathbf{r}), \triangle\rho(\mathbf{r}), \mathbf{j}(\mathbf{r})\Big) d\mathbf{r}$$

$$\rho_\alpha(\mathbf{r}) = \int f_\alpha(\mathbf{r},\mathbf{p}) \frac{d\mathbf{p}}{(2\pi\hbar)^3} = \left(\tfrac{2\nu}{\pi}\right)^{3/2} \sum_{i\in\alpha}\sum_{j\in\alpha} e^{-2\nu(\mathbf{r}-\mathbf{R}_{ij})^2} B_{ij} B^{-1}_{ji}, \qquad \mathbf{R}_{ij} = \tfrac{1}{2\sqrt{\nu}}(\mathbf{Z}_i^* + \mathbf{Z}_j)$$

$$\mathbf{j}_\alpha(\mathbf{r}) = \int \frac{\mathbf{p}}{M} f_\alpha(\mathbf{r},\mathbf{p}) \frac{d\mathbf{p}}{(2\pi\hbar)^3} = \left(\tfrac{2\nu}{\pi}\right)^{3/2} \sum_{i\in\alpha}\sum_{j\in\alpha} \frac{\mathbf{P}_{ij}}{M} e^{-2\nu(\mathbf{r}-\mathbf{R}_{ij})^2} B_{ij} B^{-1}_{ji}, \qquad \mathbf{P}_{ij} = i\hbar\sqrt{\nu}(\mathbf{Z}_i^* - \mathbf{Z}_j)$$

$$\tau_\alpha(\mathbf{r}) = \int \frac{\mathbf{p}^2}{M^2} f_\alpha(\mathbf{r},\mathbf{p}) \frac{d\mathbf{p}}{(2\pi\hbar)^3} = \left(\tfrac{2\nu}{\pi}\right)^{3/2} \sum_{i\in\alpha}\sum_{j\in\alpha} \frac{\mathbf{P}_{ij}^2 + 3\hbar^2\nu}{M^2} e^{-2\nu(\mathbf{r}-\mathbf{R}_{ij})^2} B_{ij} B^{-1}_{ji}$$

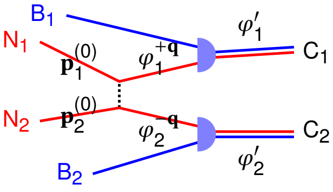
Momentum dependence has been corrected for high energy collisions, in a similar way to Gale, Bertsch, Das Gupta, PRC 35 (1987) 1666.

# NN collisions with cluster correlations

$$\mathrm{N}_1 + \mathrm{B}_1 \; + \; \mathrm{N}_2 + \mathrm{B}_2 \; \to \mathrm{C}_1 + \mathrm{C}_2$$

- $\mathrm{N}_1$, $\mathrm{N}_2$ : Colliding nucleons
- $\mathrm{B}_1$, $\mathrm{B}_2$ : Spectator nucleons/clusters
- $\mathrm{C}_1$, $\mathrm{C}_2$ : $N$, $(2N)$, $(3N)$, $(4N)$ (up to $\alpha$ cluster)

Similar to Danielewicz et al., NPA533 (1991) 712.




## Transition probability

$$W(\mathrm{NBNB} \to \mathrm{CC}) = \frac{2\pi}{\hbar} |\langle \mathrm{CC}|V|\mathrm{NBNB}\rangle|^2 \delta(E_f - E_i)$$

$$v d\sigma \propto |\langle \varphi'_1 | \varphi_1^{+\mathbf{q}} \rangle|^2 |\langle \varphi'_2 | \varphi_2^{-\mathbf{q}} \rangle|^2 |M|^2 \delta(E_f - E_i) p_{\mathrm{rel}}^2 \, dp_{\mathrm{rel}} \, d\Omega$$

$|M|^2 = |\langle \mathrm{NN}|V|\mathrm{NN}\rangle|^2$: Matrix elements of NN scattering

$\Leftarrow (d\sigma/d\Omega)_{\mathrm{NN}}$ in medium

$$\mathbf{p}_{\mathrm{rel}} = \tfrac{1}{2}(\mathbf{p}_1 - \mathbf{p}_2) = p_{\mathrm{rel}} \hat{\boldsymbol{\Omega}}$$

$$\mathbf{q} = \mathbf{p}_1 - \mathbf{p}_1^{(0)} = \mathbf{p}_2^{(0)} - \mathbf{p}_2$$

$$\varphi_1^{+\mathbf{q}} = \exp(+i\mathbf{q}\cdot\mathbf{r}_{\mathrm{N}_1}) \varphi_1^{(0)}$$

$$\varphi_2^{-\mathbf{q}} = \exp(-i\mathbf{q}\cdot\mathbf{r}_{\mathrm{N}_2}) \varphi_2^{(0)}$$

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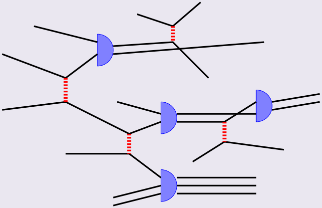
In this study, free NN cross sections are adopted.

# NN collisions with cluster correlations (more explanations)



For each NN collision, cluster formation is considered.

$$\mathrm{N}_1 + \mathrm{B}_1 + \mathrm{N}_2 + \mathrm{B}_2 \to \mathrm{C}_1 + \mathrm{C}_2$$

$$W_{i \to f} = \frac{2\pi}{\hbar} |\langle \mathrm{CC} | V_{NN} | \mathrm{NBNB} \rangle|^2 \delta(E_f - E_i)$$

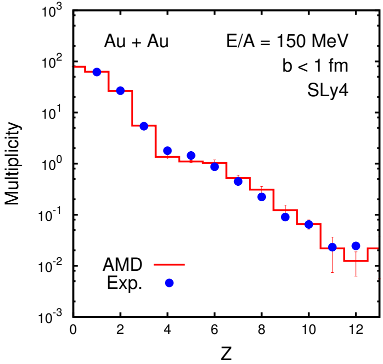
AO, J. Phys. Conf. Ser. 420 (2013) 012103

- Clusters in the final states are represented by placing Gaussian wave packets at the same phase space point.
- Consequently the processes such as the elastic and inelastic scatterings of “Cluster + Nucleon” and “Cluster + Cluster” are automatically taken into account.
- There are many possibilities to from clusters in the final states. Non-orthogonality of the final states should be carefully handled.
- It is also important to consider the process that several clusters form bound light nuclei.

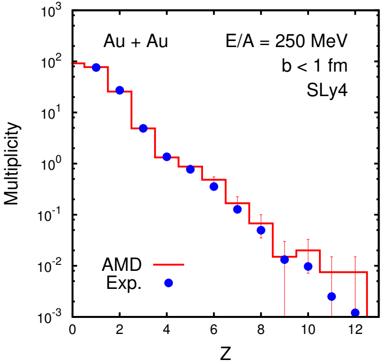
$$\text{e.g.} \quad \alpha + t \to {}^7\mathrm{Li} \qquad (\text{B.E.} = -2.5\ \text{MeV})$$

# AMD results: Au + Au central collisions at 150 and 250 MeV/nucleon

$E/A = 150$ MeV




$E/A = 250$ MeV

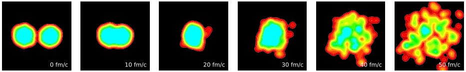



| | with C & C-C | FOPI |
|---|---|---|
| $M(p)$ | 32.8 | 26.1 |
| $M(\alpha)$ | 20.1 | 21.0 |
| $Z_{\text{gas}}/Z_{\text{tot}}$ | 71% | 73% |

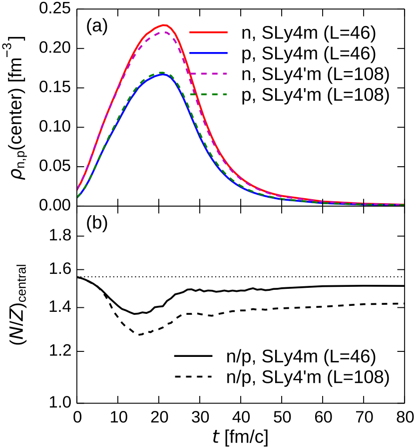
| | with C & C-C | FOPI |
|---|---|---|
| $M(p)$ | 42.0 | 31.9 |
| $M(\alpha)$ | 19.4 | 18.2 |
| $Z_{\text{gas}}/Z_{\text{tot}}$ | 80% | 83% |

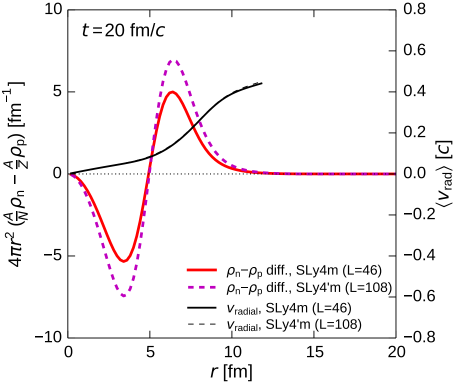
FOPI data: Reisdorf et al., NPA 612 (1997) 493.

# Compression and expansion in collisions at 300 MeV/nucleon




$^{132}\mathrm{Sn} + {}^{124}\mathrm{Sn}$, $E/A = 300$ MeV, $b \sim 0$




- Neutron-proton density diff. (fn of $r$)

$$4\pi r^2 \left[ \frac{A}{N}\rho_n(r) - \frac{A}{Z}\rho_p(r) \right]$$

- Radial expansion velocity $v_{\rm rad}(r)$

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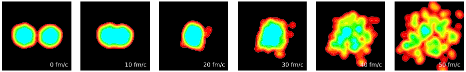


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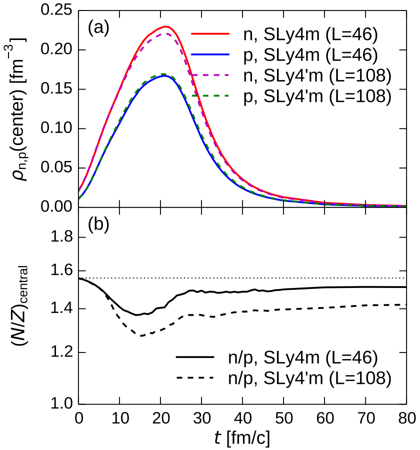


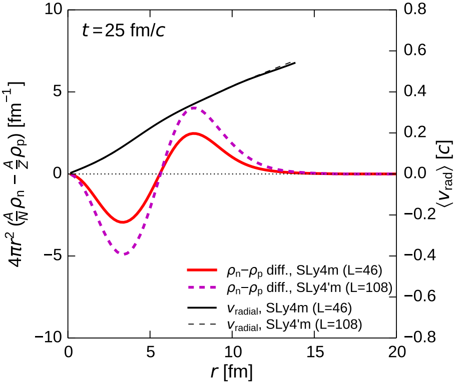


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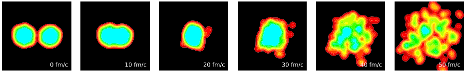


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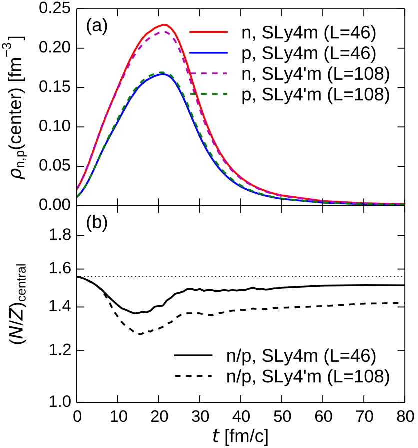


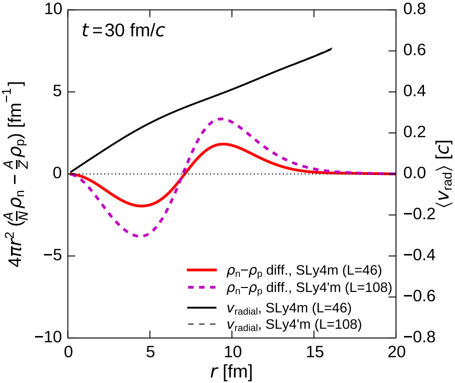


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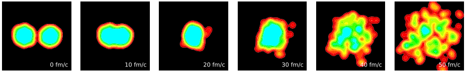


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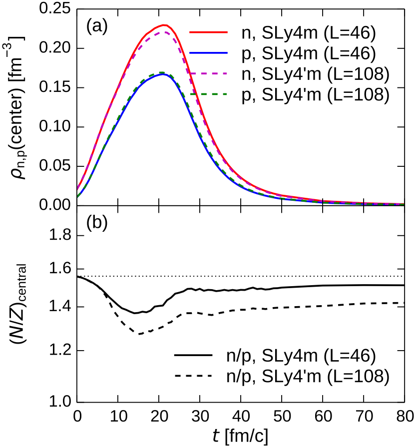


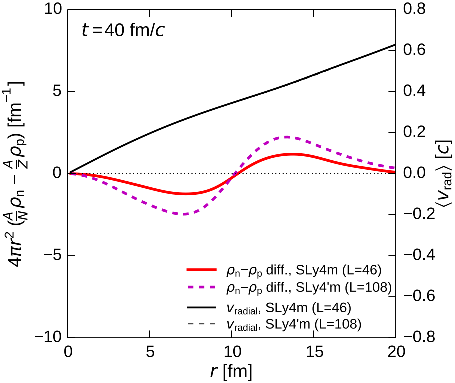


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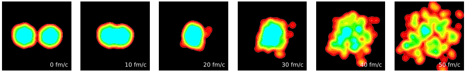


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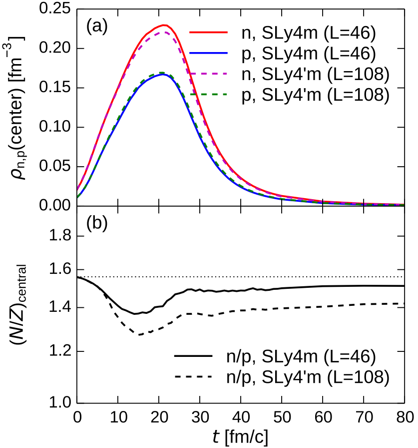


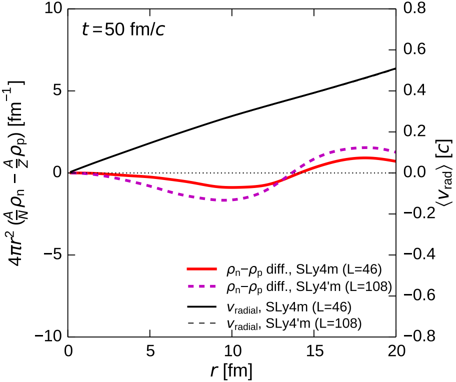


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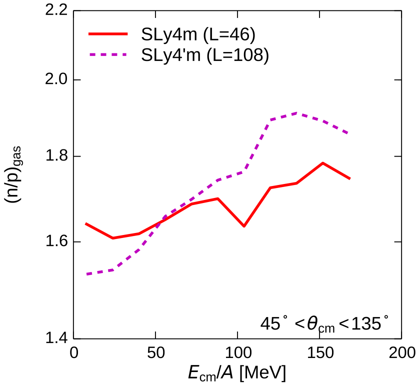
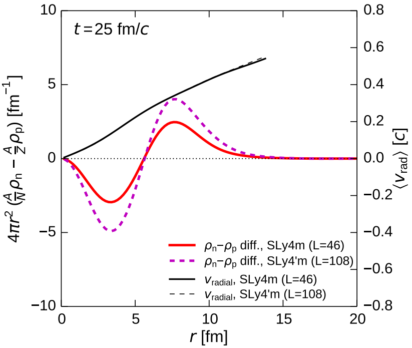
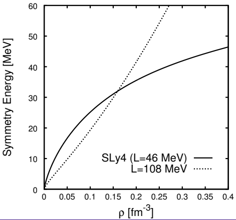
Effect remains until late times ⇒ Observable?

# $N/Z$ Spectrum Ratio — an observable




$$\left(\frac{N}{Z}\right)_{\rm gas} = \frac{Y_n(v)+Y_d(v)+2Y_t(v)+Y_h(v)+2Y_\alpha(v)}{Y_p(v)+Y_d(v)+Y_t(v)+2Y_h(v)+2Y_\alpha(v)}$$




$N/Z$ of spectrum of emitted particles is similar to the neutron-proton density difference at the compression stage.

# Nucleon distributions in **r**- and **p**-spaces, With/without clusters

## With Clusters

- Stiff symmetry energy

  ⇔ High density part: $N/Z \Downarrow$

  ⇔ High momentum part: $N/Z \Uparrow$

- Expansion is simple:

  **r**-space ⇔ **p**-space ⇒ Obs.

## Without Clusters

- Stiff symmetry energy

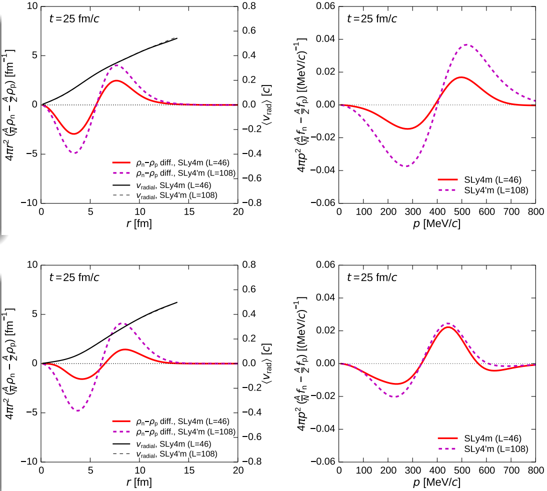
  ⇔ High density part: $N/Z \Downarrow$

  ⇔ High momentum part: ???

- Expansion is not so simple.

  **r**-space ⇏ n/p spectrum

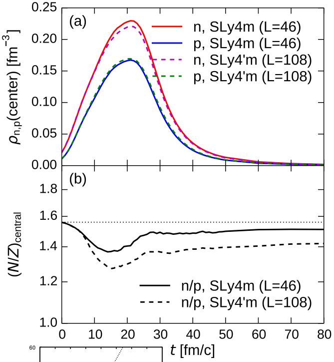
$n-p$ in **r**-space    $n-p$ in **p**-space

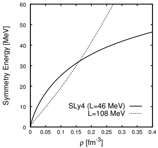



c.f. Comparison of AMD and SMF at 50 MeV/u: Colonna, Ono, Rizzo, PRC82 (2010) 054613.

$^{132}Sn + ^{124}Sn$ collisions

$E/A = 300$ MeV, $b < 1$ fm




Bao-An Li, PRL88 (2002) 192701.

Pion ratio $\pi^-/\pi^+$ has been proposed as a good probe of

- symmetry energy at high densities
- $\rho_n/\rho_p$ ratio in the high density region

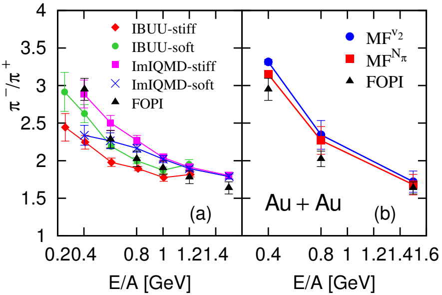
Production/absorption of $\Delta$ and $\pi$:

$$N + N \to N + \Delta$$
$$N + N \leftarrow N + \Delta$$
$$\Delta \to N + \pi$$
$$\Delta \leftarrow N + \pi$$

Simple expectation: $\pi^-/\pi^+ \approx (N/Z)^2$

- First-chance $NN \to N\Delta \to NN\pi$
- Chemical equilibrium

# Symmetry energy and pions studied by other models




- Bao-An Li, PRL 88 (2002) 192701. IBUU
- Ferini et al., PRL 97 (2006) 202301.
- Reisdorf et al., NPA 781 (2007) 459. Data & IQMD
- Z. Xiao et al., PRL 102 (2009) 062502. IBUU04
- Z.Q. Feng and G.M. Jin, PLB 683 (2010) 140. ImIQMD
- Hong and Danielewicz, PRC 90 (2014) 024605. pBUU
- ...

| | asy-stiff | | asy-soft |
|---|---|---|---|
| IBUU | $\pi^-/\pi^+$ | $<$ | $\pi^-/\pi^+$ |
| ImIQMD | $\pi^-/\pi^+$ | $>$ | $\pi^-/\pi^+$ |
| pBUU | $\pi^-/\pi^+$ | $\approx$ | $\pi^-/\pi^+$ |

Model predictions do not agree.

$(N/Z)^2_{\mathrm{sys}} = 2.231$ for the Au + Au system.

- In the data and in some calculations, $\pi^-/\pi^+ > (N/Z)^2_{\mathrm{sys}}$ at low energies.
- The high density region is less n-rich $(N/Z)^2 < (N/Z)^2_{\mathrm{sys}}$.

How can this happen?

Relations should be clarified: $S(\rho) \iff \rho_n/\rho_p \iff \Delta^-/\Delta^0/\Delta^+/\Delta^{++} \iff \pi^-/\pi^+$

# Transport equations for $N$, $\Delta$ and $\pi$

Collaboration with Natsumi Ikeno, Yasushi Nara, Akira Ohnishi

## Coupled equations for $f_N(\mathbf{r},\mathbf{p},t)$, $f_\Delta(\mathbf{r},\mathbf{p},t)$, $f_\pi(\mathbf{r},\mathbf{p},t)$

$$\frac{\partial f_N}{\partial t} + \frac{\partial h_N}{\partial \mathbf{p}}\cdot\frac{\partial f_N}{\partial \mathbf{r}} - \frac{\partial h_N[f_N,f_\Delta,f_\pi]}{\partial \mathbf{r}}\cdot\frac{\partial f_N}{\partial \mathbf{p}} = I_N[f_N,f_\Delta,f_\pi]$$

$$\frac{\partial f_\Delta}{\partial t} + \frac{\partial h_\Delta}{\partial \mathbf{p}}\cdot\frac{\partial f_\Delta}{\partial \mathbf{r}} - \frac{\partial h_\Delta[f_N,f_\Delta,f_\pi]}{\partial \mathbf{r}}\cdot\frac{\partial f_\Delta}{\partial \mathbf{p}} = I_\Delta[f_N,f_\Delta,f_\pi]$$

$$\frac{\partial f_\pi}{\partial t} + \frac{\partial h_\pi}{\partial \mathbf{p}}\cdot\frac{\partial f_\pi}{\partial \mathbf{r}} - \frac{\partial h_\pi[f_N,f_\Delta,f_\pi]}{\partial \mathbf{r}}\cdot\frac{\partial f_\pi}{\partial \mathbf{p}} = I_\pi[f_N,f_\Delta,f_\pi]$$

$NN \to NN$

$NN \leftrightarrow N\Delta$

$\Delta \leftrightarrow N\pi$

Assumption: $\Delta$ and pion productions are rare (in low-energy collisions).

$$I_N[f_N,f_\Delta,f_\pi] = I_N^{\text{el}}[f_N,0,0] + \lambda I'_N[f_N,f_\Delta,f_\pi]$$

$$f_N = f_N^{(0)} + \lambda f_N^{(1)} + \cdots, \qquad f_\Delta = O(\lambda), \quad f_\pi = O(\lambda)$$

# Perturbative treatment of pion production

Assumption: $\Delta$ and pion productions are rare (in low-energy collisions).

$$I_N[f_N, f_\Delta, f_\pi] = I_N^{\text{el}}[f_N, 0, 0] + \lambda I'_N[f_N, f_\Delta, f_\pi]$$

$$f_N = f_N^{(0)} + \lambda f_N^{(1)} + \cdots, \qquad f_\Delta = O(\lambda), \quad f_\pi = O(\lambda)$$

## Zeroth order equation for $f_N$ — solved by AMD

$$\frac{\partial f_N^{(0)}}{\partial t} + \frac{\partial h_N}{\partial \mathbf{p}} \cdot \frac{\partial f_N^{(0)}}{\partial \mathbf{r}} - \frac{\partial h_N[f_N^{(0)}, 0, 0]}{\partial \mathbf{r}} \cdot \frac{\partial f_N^{(0)}}{\partial \mathbf{p}} = I_N^{\text{el}}[f_N^{(0)}, 0, 0]$$

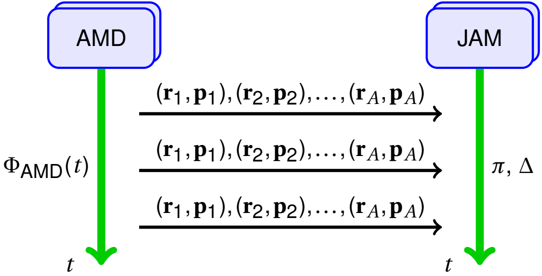
## First order equations for $f_\Delta$ and $f_\pi$ — solved by JAM for given $f_N^{(0)}$

$$\frac{\partial f_\Delta}{\partial t} + \frac{\partial h_\Delta}{\partial \mathbf{p}} \cdot \frac{\partial f_\Delta}{\partial \mathbf{r}} - \frac{\partial h_\Delta[f_N^{(0)}, f_\Delta, f_\pi]}{\partial \mathbf{r}} \cdot \frac{\partial f_\Delta}{\partial \mathbf{p}} = I_\Delta[f_N^{(0)}, f_\Delta, f_\pi]$$

$$\frac{\partial f_\pi}{\partial t} + \frac{\partial h_\pi}{\partial \mathbf{p}} \cdot \frac{\partial f_\pi}{\partial \mathbf{r}} - \frac{\partial h_\pi[f_N^{(0)}, f_\Delta, f_\pi]}{\partial \mathbf{r}} \cdot \frac{\partial f_\pi}{\partial \mathbf{p}} = I_\pi[f_N^{(0)}, f_\Delta, f_\pi]$$

# AMD + JAM

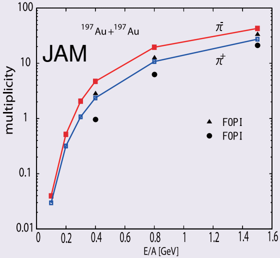
Send nucleon test particles from AMD to JAM at every 2 fm/$c$, with corrections for the conservation of baryon number and charge.



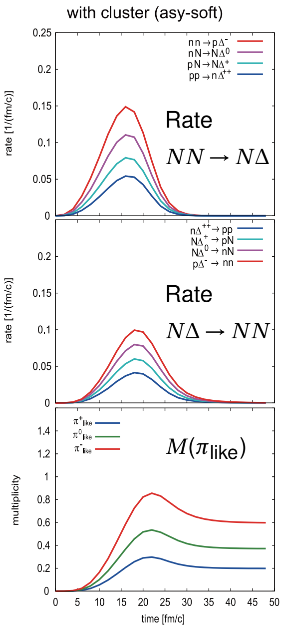

## JAM: Jet AA Microscopic transport model

Nara, Otuka, Ohnishi, Niita, Chiba, PRC 61 (2000) 024901.

- Has been applied to high-energy collisions (1 ~ 158 A GeV).
- Hadron-Hadron reactions are based on experimental data and the detailed balance.
- No mean field (default)
- $s$-wave pion production is turned off.

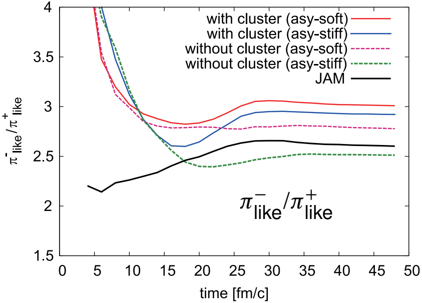


# AMD+JAM calculations for $^{132}\mathrm{Sn} + {}^{124}\mathrm{Sn}$ at 300 MeV/nucleon




Calculations for $^{132}\mathrm{Sn} + {}^{124}\mathrm{Sn}$, $E/A = 300$ MeV, $b < 1$ fm

1. AMD+JAM with cluster (asy-soft) $L = 46$ (SLy4)
2. AMD+JAM with cluster (asy-stiff) $L = 108$
3. AMD+JAM without cluster (asy-soft) $L = 46$ (SLy4)
4. AMD+JAM without cluster (asy-stiff) $L = 108$
5. JAM (no mean field)

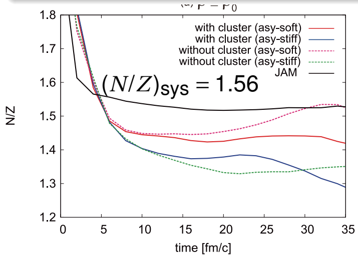


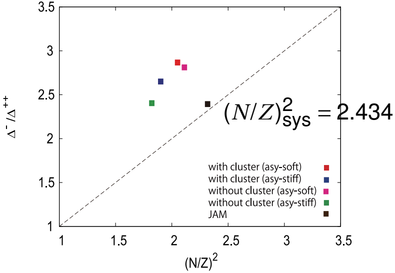

$$\left(\frac{N}{Z}\right)^2_{\mathrm{sys}} = 2.434$$

$$\pi^-_{\mathrm{like}} = \pi^- + \Delta^- + \tfrac{1}{3}\Delta^0, \quad \pi^+_{\mathrm{like}} = \pi^+ + \Delta^{++} + \tfrac{1}{3}\Delta^+$$

# Relation between n/p and $\Delta^-/\Delta^{++}$

Nucleons in the sphere $\rho(r) \geq \rho_0$ centered at the c.m.




$$(N/Z)^2 \equiv \frac{\int_0^\infty N(t)^2 dt}{\int_0^\infty Z(t)^2 dt}$$

$N(t)$, $Z(t)$: Number of nucleons that satisfy the condition.

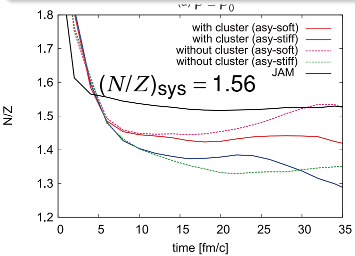
$$\Delta^-/\Delta^{++} \equiv \frac{\int_0^\infty \mathrm{Rate}(nn \to p\Delta^-)dt}{\int_0^\infty \mathrm{Rate}(pp \to n\Delta^{++})dt}$$

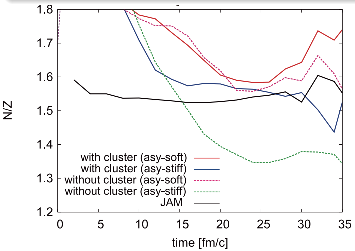
Simply expect $\Delta^-/\Delta^{++} \approx (N/Z)^2$.

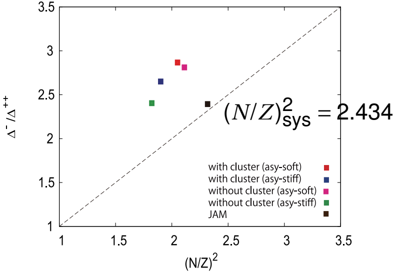
$\Delta^-/\Delta^{++} \neq (N/Z)^2 \Rightarrow$ We are not looking at the correct region of nucleons for $\Delta$ production.

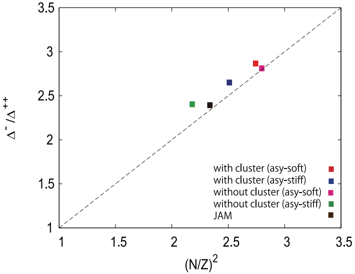
Nucleons in the sphere $\rho(r) \geq \rho_0$ centered at the c.o.m.

Nucleons in the sphere $\rho(r) \geq \rho_0$ with high momentum $p \geq 500$ MeV/$c$



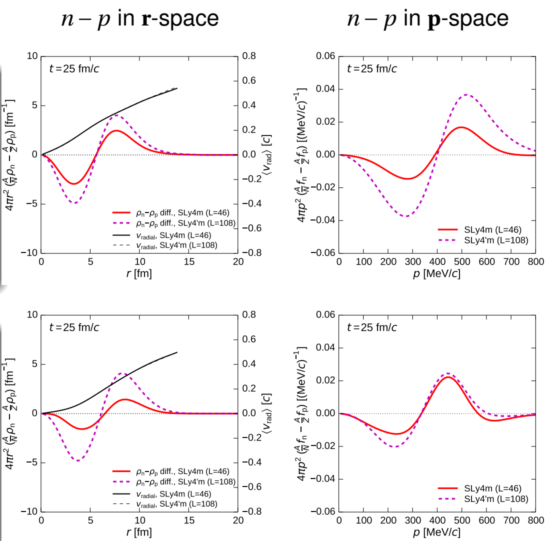
# Nucleon distributions in $\mathbf{r}$- and $\mathbf{p}$-spaces, With/without clusters

## With Clusters

- Stiff symmetry energy
  - $\Leftrightarrow$ High density part: $N/Z \Downarrow$
  - $\Leftrightarrow$ High momentum part: $N/Z \Uparrow$
- Expansion is simple:
  $\mathbf{r}$-space $\Leftrightarrow$ $\mathbf{p}$-space $\Rightarrow$ Obs.

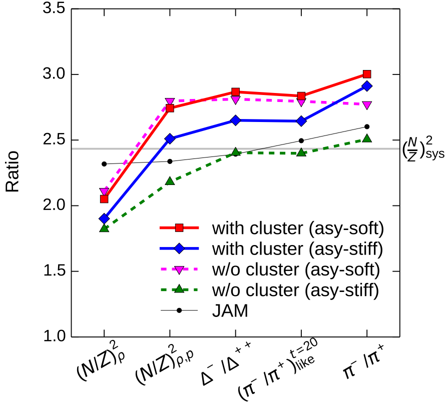
## Without Clusters

- Stiff symmetry energy
  - $\Leftrightarrow$ High density part: $N/Z \Downarrow$
  - $\Leftrightarrow$ High momentum part: ???
- Expansion is not so simple.
  $\mathbf{r}$-space $\not\Leftrightarrow$ n/p spectrum




c.f. Comparison of AMD and SMF at 50 MeV/u: Colonna, Ono, Rizzo, PRC82 (2010) 054613.

# Final $\pi^-/\pi^+$

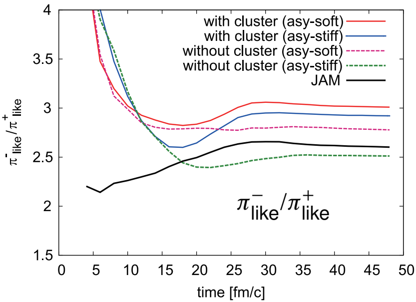



Strong correlation of $\Delta^-/\Delta^{++}$ to $(N/Z)^2$ almost remains in $\pi^-/\pi^+$, but it is weakened a little.

- Cluster correlation $\Rightarrow (\pi^-/\pi^+) \uparrow$
- Stiff $S(\rho) \Rightarrow (\pi^-/\pi^+) \uparrow$

$$(N/Z)^2_{\text{cond}} \equiv \frac{\int_0^\infty N(t)^2_{\text{cond}}\,dt}{\int_0^\infty Z(t)^2_{\text{cond}}\,dt}$$

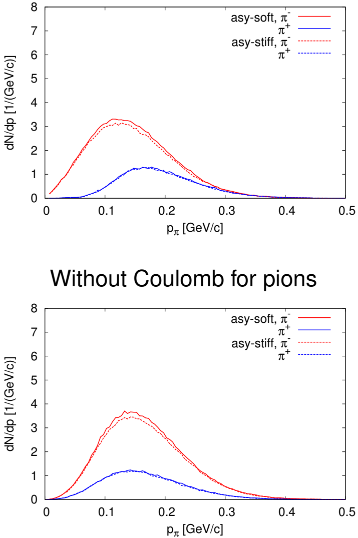
$$\Delta^-/\Delta^{++} \equiv \frac{\int_0^\infty \text{Rate}(nn \to p\Delta^-)\,dt}{\int_0^\infty \text{Rate}(pp \to n\Delta^{++})\,dt}$$



# Pion spectrum, Coulomb effect

With Coulomb for pions

Without Coulomb for pions




Coulomb acceleration/deceleration of produced pions

- affects the charged pion spectra, but
- has almost no effect on the pion multiplicities and the pion ratio.

| | $\pi^-$ | $\pi^+$ | $\pi^-/\pi^+$ |
|---|---|---|---|
| with Coulomb | 0.60 | 0.20 | 3.00(2) |
| without Coulomb | 0.61 | 0.20 | 3.04(2) |

AMD+JAM with cluster (asy-soft)

# About potentials for $\Delta$ and $\pi$

| $N_{\tau_1} + N_{\tau_2}$ | $\longleftrightarrow$ | $N_{\tau_3} + \Delta_{\tau_4}$ |
|---|---|---|
| $U^{(N)}_{\tau_1} + U^{(N)}_{\tau_2}$ | | $U^{(N)}_{\tau_3} + U^{(\Delta)}_{\tau_4}$ |
| $+q_1 U_C + q_2 U_C$ | | $+q_3 U_C + q_4 U_C$ |

| $\Delta_{\tau_1}$ | $\longleftrightarrow$ | $N_{\tau_3} + \pi_{\tau_4}$ |
|---|---|---|
| $U^{(\Delta)}_{\tau_1}$ | | $U^{(N)}_{\tau_3} + U^{(\pi)}_{\tau_4}$ |
| $+q_1 U_C$ | | $+q_3 U_C + q_4 U_C$ |

- $U^{(*)}_{\tau}$: Isospin($\tau$)-dependent potential due to the strong interaction
- $U_C$: Coulomb potential

In JAM, reaction thresholds are the same as in free space. Therefore AMD+JAM assumes

$$U^{(N)}_{\tau_1} + U^{(N)}_{\tau_2} = U^{(N)}_{\tau_3} + U^{(\Delta)}_{\tau_4}, \qquad U^{(\Delta)}_{\tau_1} = U^{(N)}_{\tau_3} + U^{(\pi)}_{\tau_4} \qquad \text{for } \tau_1(+\tau_2) = \tau_3 + \tau_4$$

This is satisfied in case

$$U^{(N,\Delta)}_{\tau} = U_0(\mathbf{r}) + \tau U_{\text{sym}}(\mathbf{r}), \qquad U^{(\pi)}_{\tau} = \tau U_{\text{sym}}(\mathbf{r})$$

c.f. pBUU : Hong and Danielewicz, PRC 90 (2014) 024605.

# Summary

Observables that reflect the symmetry-energy effects at high densities were studied by AMD combined with JAM for central $^{132}\mathrm{Sn} + {}^{124}\mathrm{Sn}$ collisions at 300 MeV/nucleon.

- If cluster correlation is strong, the high-density effect is reflected almost directly in the n/p spectrum ratio.
- The $\pi^-/\pi^+$ and $\Delta^-/\Delta^{++}$ ratios are related to the $(n/p)^2$ ratio in high-density and high-momentum region.

  Cluster, $E_{\mathrm{sym}}(\rho) \Leftrightarrow (N/Z)^2 \Leftrightarrow \Delta^-/\Delta^{++} \Leftrightarrow \pi^-/\pi^+$

  - Selection of high-momentum nucleons: $(\pi^-/\pi^+)\uparrow$
  - Stiff $E_{\mathrm{sym}}$: $(\pi^-/\pi^+)\Downarrow$, but $(\pi^-/\pi^+)\uparrow$ in late stage
  - Cluster correlation: $(\pi^-/\pi^+)\uparrow$
  - $E_{\mathrm{sym}}$ effect were stronger without cluster correlations.

Todo:

Other systems.

Improve $NN \to NN\pi$ cross section near threshold.