Workshop on Science with SπRIT TPC
RIKEN, 5-6 June 2015

Fission Dynamics of Exotic Nuclei

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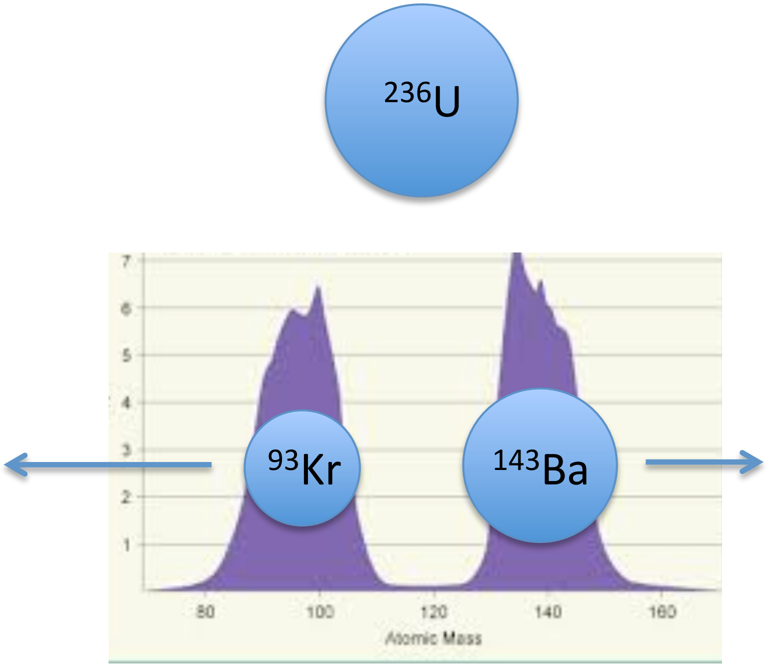
Nuclear Fission

Nuclear fission was discovered on December 17, 1938,
in Berlin by *Otto Hahn* and his assistant *Fritz Strassmann*,
and it was interpreted theoretically in January 1939
by *Lise Meitner* and her nephew *Otto Robert Frisch*



$Q \approx 200$ MeV

Lise Meitner Otto Hahn

109
Mt
Meitnerium
278

Fission Dynamics of Exotic Nuclei

Fission dynamics

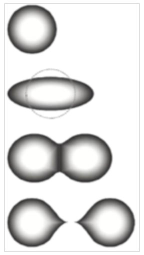
Fission of neutron-deficient nuclei

Nuclear fission is a result of shape dynamics

Lise Meitner & O.R. Frisch, Nature 143 (1939) 239:

Disintegration of Uranium by Neutrons:
a New Type of Nuclear Reaction

``.. An essentially classical picture .. suggests itself. .. The particles in a heavy nucleus would be expected to move In a collective way which has some resemblance to the movement of a liquid drop. If the movement is made sufficiently violent by adding energy, such a drop may divide itself into two smaller drops ..''



N. Bohr & J.A. Wheeler, Phys Rev 56 (1939) 426:

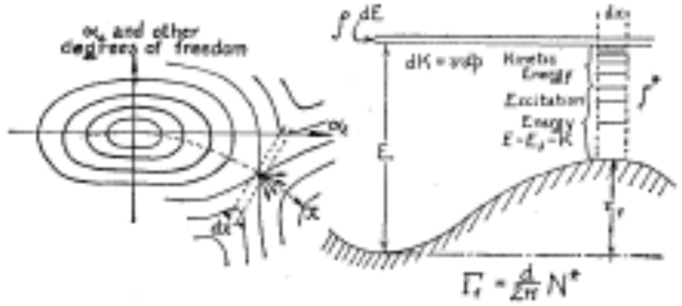
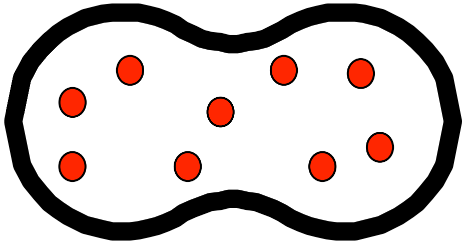


FIG. 3. The potential energy associated with any arbitrary deformation of the nuclear form may be plotted as a function of the parameters which specify the deformation, thus giving a contour surface which is represented schematically in the left-hand portion of the figure. The pass or saddle point corresponds to the critical deformation of unstable equilibrium. To the extent to which we may use classical terms, the course of the fission process may be symbolized by a ball lying in the hollow at the origin of coordinates (spherical form) which receives an impulse (neutron capture) which sets it to executing a complicated Lissajous figure of oscillation about equilibrium. If its energy is sufficient, it will in the course of time happen to move in the proper direction to pass over the saddle point (after which fission will occur), unless it loses its energy (radiation or neutron re-emission). At the right is a cross section taken through the fission barrier, illustrating the calculation in the text of the probability per unit time of fission occurring.

A nucleus is characterized by its *shape* and its *thermal energy*



$$E = M + U(\text{shape}) + E^*$$

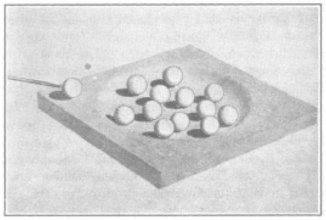
Total energy

Ground-state energy: Mass

Potential energy of deformation

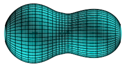
Statistical excitation

Niels Bohr, Nature 137 (1936) 344:
Compound nucleus



Nuclear shape dynamics

1) Parametrized family of nuclear shapes: $\boldsymbol{q} = (q_1, q_2, \ldots\ )$



N

2) Potential energy of deformation: $U(\boldsymbol{q}) = U(q_1, q_2, \ldots)$

1

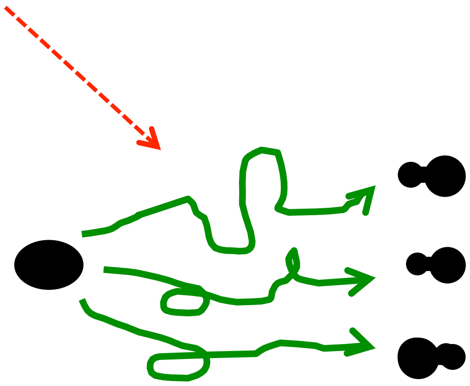
3) Inertial mass tensor: $\boldsymbol{M}(\boldsymbol{q}) = \{M_{ij}(q_1, q_2, \ldots)\}$

N x N

4) Dissipation tensor: $\gamma(\boldsymbol{q}) = \{\gamma_{ij}(q_1, q_2, \ldots)\}$

N x N

Langevin equation of motion: $d\boldsymbol{p}/dt = \boldsymbol{F}_{cons} + \boldsymbol{F}_{diss}$



Examples (incomplete list) [all *macroscopic and <5D*]:

Kramers, Physica 7 (1940) 284, …

Fröbrich & Gontchar, Phys Rep 292 (1998) 131

Chadhuri & Pal, Phys Rev C63 (2001) 064603

Karpov, Nadtouchy, Vanin, Adeev, Phys Rev C63 (2001) 054610

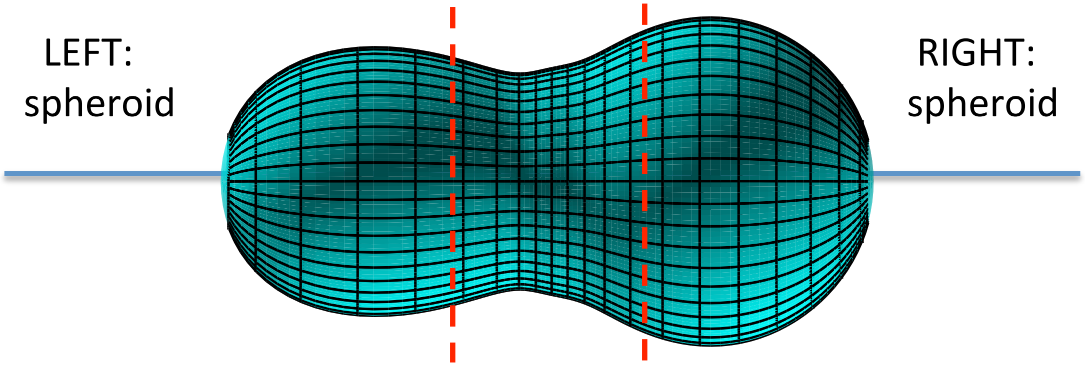
Nadtouchy, Adeev, Karpov, Phys Rev C65 (2002) 064615

Nadtouchy, Kelic, Schmidt, Phys Rev C75 (2007) 064614

Nuclear shape families

Three quadratic surfaces [Nix 1968]

5 independent shape parameters

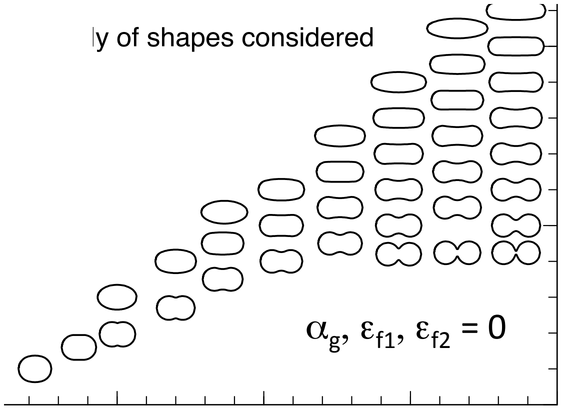


MIDDLE:
spheroid/hyperboloid

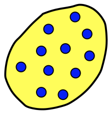
LEFT:
spheroid

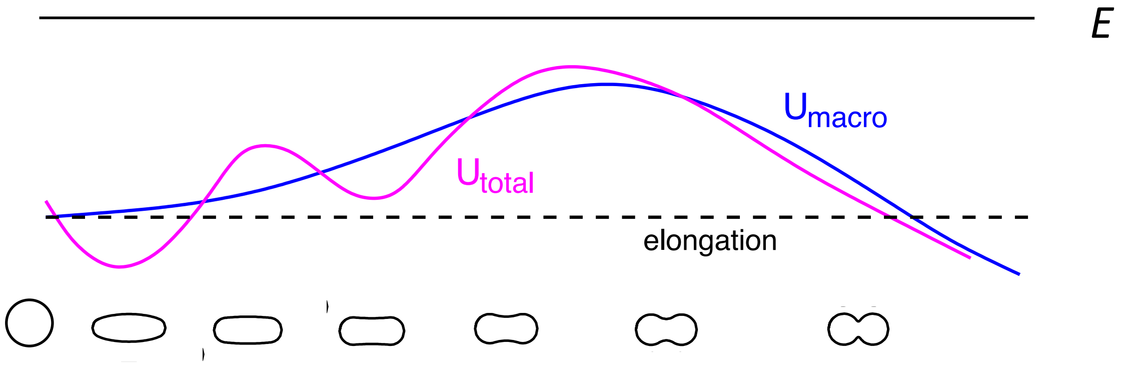
RIGHT:
spheroid

Overall elongation
Constriction (neck)
Reflection asymmetry
Fragment deformations



Potential energy: Macroscopic-microscopic method





$$U(Z,N,\text{shape}) = U_{\text{macro}}(Z,N,\text{shape}) + U_{\text{micro}}(Z,N,\text{shape})$$

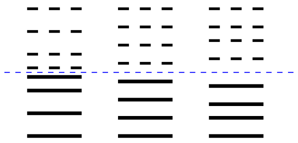
Swiatecki 1963
Strutinsky 1966

Finite-range liquid drop:

$$U_{\text{macro}} = E_{\text{vol}} + E_{\text{surf}} + E_{\text{coul}} + \ldots$$

Shell & pairing:

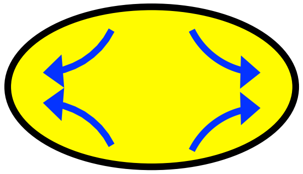
$$U_{\text{micro}} = E_{\text{shell}} + E_{\text{pair}}$$



Single-particle levels in the effective field

Inertial mass tensor for nuclear shape motion

Kinetic energy associated with shape changes:
$$K(\dot{\boldsymbol{q}},\boldsymbol{q}) \;=\; \tfrac{1}{2}\dot{\boldsymbol{q}}\cdot\boldsymbol{M}(\boldsymbol{q})\cdot\dot{\boldsymbol{q}} \;=\; \tfrac{1}{2}\sum_{ij}\dot{q}_i\,M_{ij}(\boldsymbol{q})\,\dot{q}_j$$



Macroscopic fluid:
$$K[\rho(\boldsymbol{r}),\boldsymbol{v}(\boldsymbol{r})] \;=\; \tfrac{1}{2}m\int\rho(\boldsymbol{r})\,\boldsymbol{v}(\boldsymbol{r})^2 d^3\boldsymbol{r}$$

The nuclear fluid is incompressible => density is constant: $\rho(\boldsymbol{r}) = \rho_0$

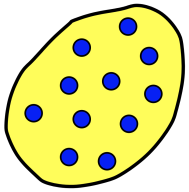
Assume irrotational flow => ***v***(***r***) depends only on shape change

$$\boxed{\Rightarrow\; \boldsymbol{M}(\boldsymbol{q})}$$

The inertial-mass tensor is still poorly understood
and the above is only a very rough approximation
- but it may be less important for the dynamics

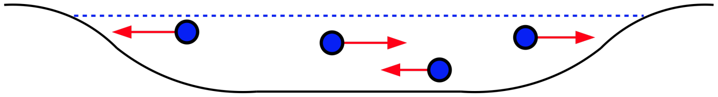
Dissipation for nuclear shape motion: one-body

Individual nucleons move in common one-body mean field
(while occasionally experiencing Pauli-suppressed collisions)



$$h[f](\boldsymbol{r},\boldsymbol{p}) = \frac{p^2}{2m^*} + U[\rho](\boldsymbol{r})$$

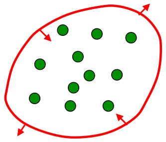
Single-particle Hamiltonian



Nucleons in mean field

One-body dissipation:

Density $\rho(\boldsymbol{r})$ changes => mean field changes => nucleons adjust quickly

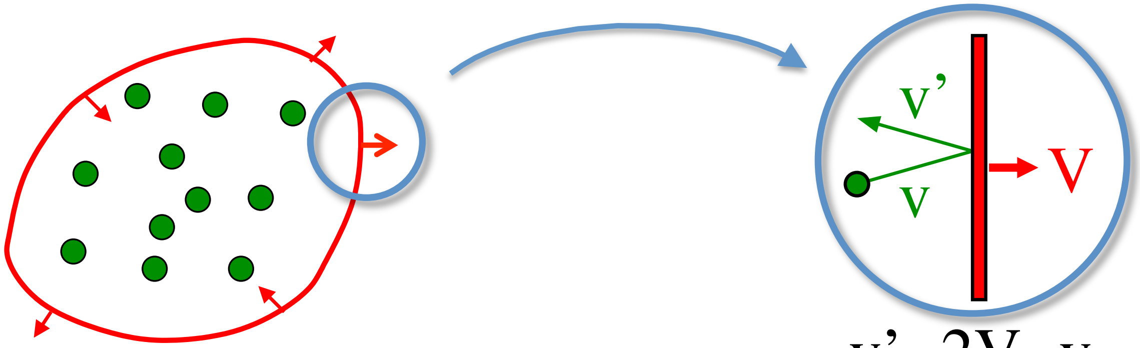


The interaction between the individual nucleon and the residual system is concentrated in the *surface* region: *Fermi gas in a deforming container*

One-body dissipation in a mononucleus:

Wall formula for the dissipation rate

Slowly deforming nucleus:



v'=2V−v

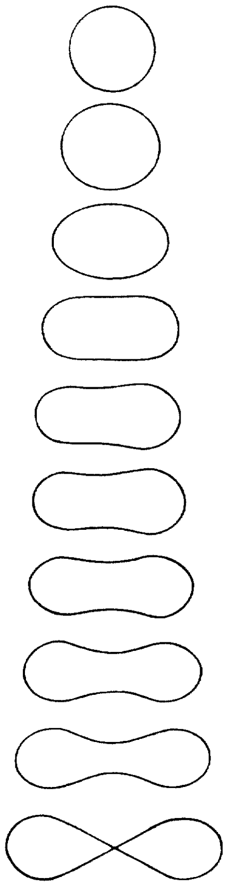
Dissipation rate: $$\dot{Q}^{\mathrm{wall}}(\boldsymbol{q},\dot{\boldsymbol{q}}) = \boxed{m\rho_0\bar{v}}\oint V^2 d^2\sigma = \sum_{ij}\dot{q}_i\gamma_{ij}^{\mathrm{wall}}(\boldsymbol{q})\dot{q}_j \ ,$$ *STRONG!*

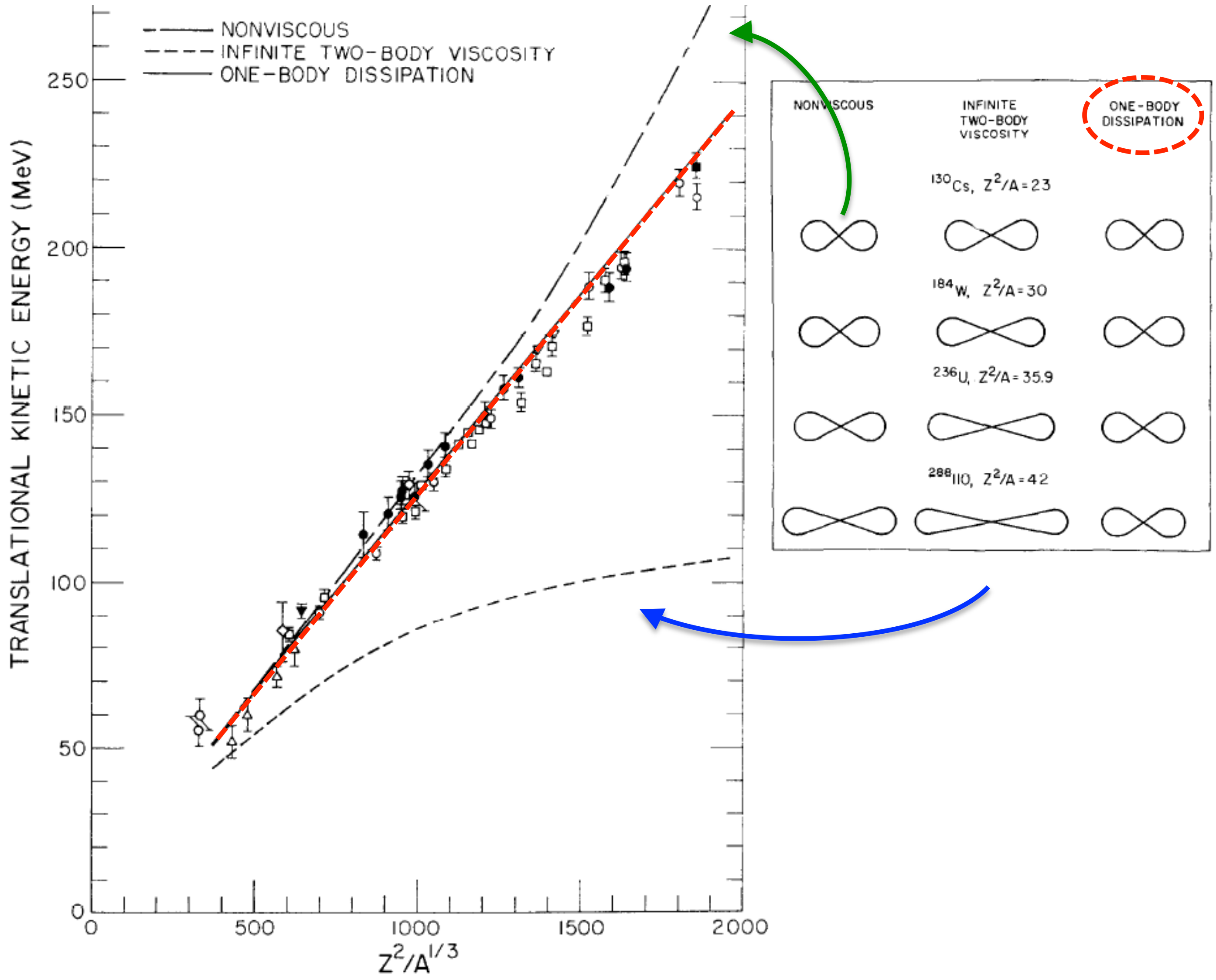
Dissipation tensor: $$\gamma_{ij}^{\mathrm{wall}}(\boldsymbol{q}) = 2\pi m\rho_0\bar{v}\int_{z_{\mathrm{min}}}^{z_{\mathrm{max}}}(\rho\frac{\partial\rho}{\partial q_i})(\rho\frac{\partial\rho}{\partial q_j})\left[\rho^2+(\rho\frac{\partial\rho}{\partial z})^2\right]^{-\frac{1}{2}}dz$$

ρ(*z*):

J. Blocki, Y. Boneh, J. R. Nix, J. Randrup, M. Robel, A. J. Sierk, and W. J. Swiatecki,
Ann Phys **113** (1978) 330: *One-Body Dissipation and the Super-Viscidity of Nuclei*

Fission fragment kinetic energy





J. Blocki, Y. Boneh, J. R. Nix, J. Randrup, M. Robel, A. J. Sierk, and W. J. Swiatecki,
Ann Phys **113** (1978) 330: *One-Body Dissipation and the Super-Viscidity of Nuclei*

Strongly damped nuclear shape dynamics => Metropolis walk

★ Dissipation is *strong* => Creeping evolution => Acceleration and (velocity)^2 are small => The inertial mass ***M*** is unimportant

If the inertial mass is unimportant then the Langevin equation simplifies to the Smoluchowski equation:

$$\underset{\substack{\uparrow \\ \text{driving} \\ \text{force}}}{\boldsymbol{F}^{\rm pot}} + \underbrace{\boldsymbol{F}^{\rm fric} + \boldsymbol{F}^{\rm ran}}_{\substack{\text{dissipative} \\ \text{force}}} \doteq \mathbf{0}$$

Brownian motion

$$\left\{\begin{array}{l} \boldsymbol{F}^{\rm pot} = -\partial U/\partial\boldsymbol{\chi} \\ \boldsymbol{F}^{\rm fric} = -\boldsymbol{\gamma}\cdot\dot{\boldsymbol{\chi}} \\ \langle \boldsymbol{F}^{\rm ran}(t)\rangle = \mathbf{0} \\ \langle F_i^{\rm ran}(t)F_j^{\rm ran}(t')\rangle \;=\; 2T\gamma_{ij}\delta(t-t') \end{array}\right.$$

$$\dot{\boldsymbol{\chi}} = \boldsymbol{\mu}(\boldsymbol{\chi})\cdot[\boldsymbol{F}^{\rm pot}(\boldsymbol{\chi}) + \boldsymbol{F}^{\rm ran}(\boldsymbol{\chi})]$$

mobility tensor: $\boldsymbol{\mu}(\boldsymbol{\chi}) \equiv \boldsymbol{\gamma}(\boldsymbol{\chi})^{-1}$

Easy to simulate:

$$\delta\boldsymbol{\chi} = \int_t^{t+\Delta t} \dot{\boldsymbol{\chi}}\, dt \;:\quad \delta\chi_i \;=\; \sum_n \tilde{\chi}_i^{(n)} \left[\Delta t\, \tilde{\boldsymbol{\chi}}^{(n)}\cdot \boldsymbol{F}^{\rm pot} + \sqrt{2T\Delta t}\,\xi_n\right]$$

mobility eigenvector

random number $\langle\xi_n\rangle = 0$ $\langle\xi_n^2\rangle = 1$

Strongly damped nuclear shape dynamics => Metropolis walk

- Dissipation is *strong* => Creeping evolution => Acceleration and (velocity)^2 are small => The inertial mass ***M***(χ) is unimportant

If the inertial mass is unimportant then the Langevin equation simplifies to the Smoluchowski equation:

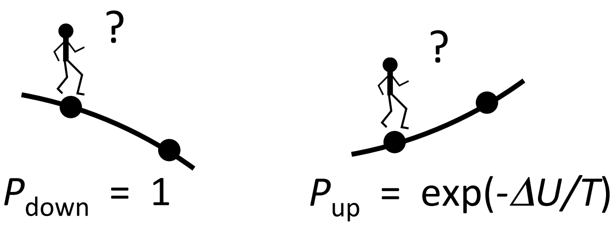
$$\boldsymbol{F}^{\rm pot} + \boldsymbol{F}^{\rm fric} + \boldsymbol{F}^{\rm ran} \doteq \mathbf{0}$$

Brownian motion

$$\dot{\chi} = \boldsymbol{\mu}(\chi) \cdot [\boldsymbol{F}^{\rm pot}(\chi) + \boldsymbol{F}^{\rm ran}(\chi)]$$

- Dissipation is *strong* => Large degree of shape equilibration => Little sensitivity to the structure of the dissipation tensor $\gamma(\chi)$ (?)

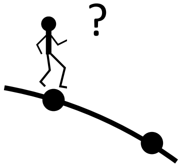
If the dissipation tensor is isotropic then a Metropolis random walk on the potential-energy lattice provides an exact simulation of the Smoluchowski shape evolution:



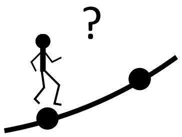
$P_{\rm down}$ = 1 $\quad$ $P_{\rm up}$ = exp($-\Delta U/T$)

Metropolis *et al.* (1953)

Metropolis walk …



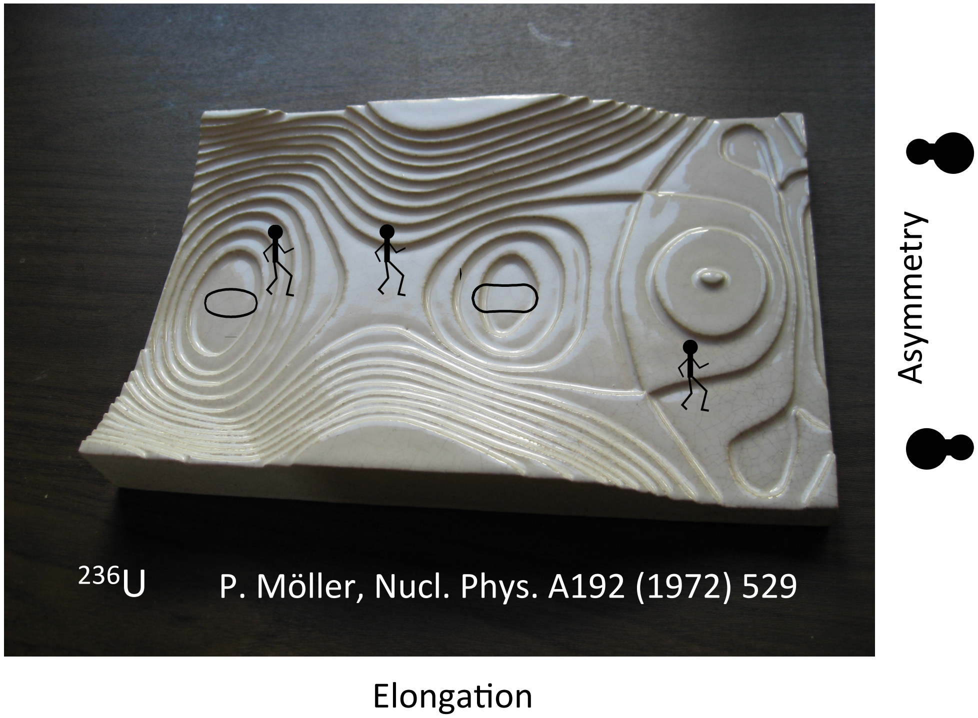
$P_{\text{down}} = 1$



$P_{\text{up}} = \exp(-\Delta U/T)$

… on the potential-energy surface:

Start at ground-state (or isomeric) minimum

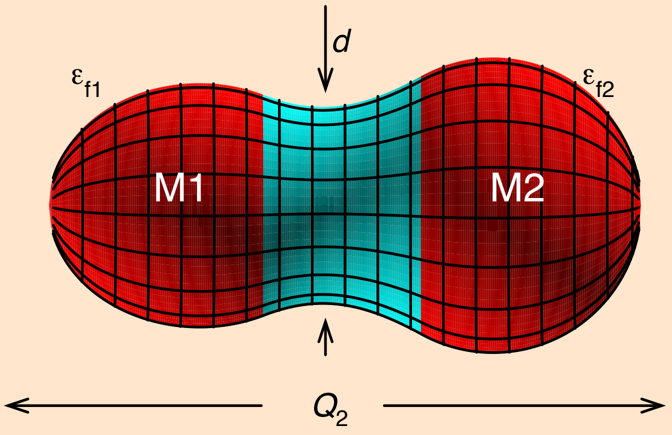


P. Möller, Nucl. Phys. A192 (1972) 529

Walk until the neck has become thin

Three-quadratic-surfaces shape family [Nix 1968]

Five Essential Fission Shape Coordinates

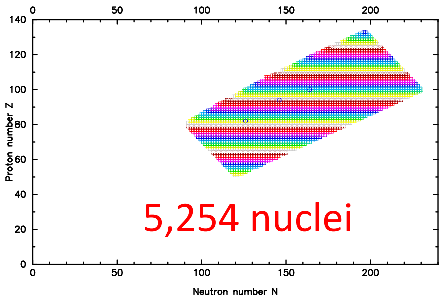


5D Cartesian
shape lattice
by P. Möller:

45	Q_2 ~	Elongation (fission direction)
⊗		
35	α_g ~	(M1-M2)/(M1+M2) Mass asymmetry
⊗		
15	ε_{f1} ~	Left fragment deformation
⊗		
15	ε_{f2} ~	Right fragment deformation
⊗		
15	d ~	Neck

⇒ 5 315 625 grid points – 306 300 unphysical points

⇒ 5 009 325 physical grid points



5,254 nuclei

$U = U_{\text{macro}} + U_{\text{micro}}$

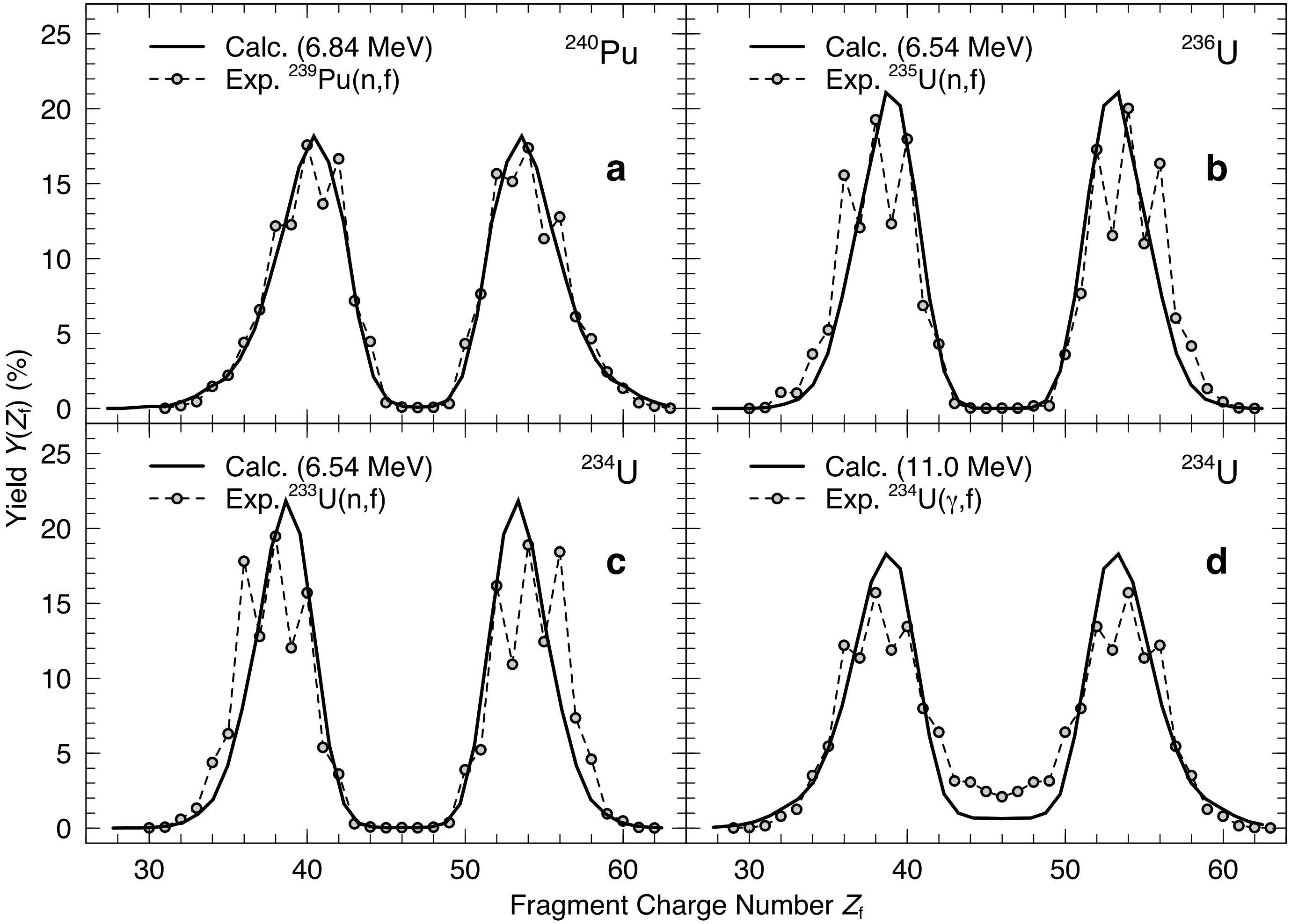
=> 5D table of the potential energy:

$\{U_{IJKLM}\}$

[Möller 2009]

$P(A_f)$ from $^{240}Pu^*$ and $^{236,234}U^*$

5D Metropolis walks

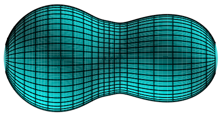


Odd-even staggering:
PRC90 (2014) 014601

J. Randrup & P. Möller, PRL 106 (2011) 132503

Energy dependence:
PRC88 (2013) 064606

Nuclear potential energy: fission barrier landscape



5D potential-energy surfaces reduced to two dimensions:

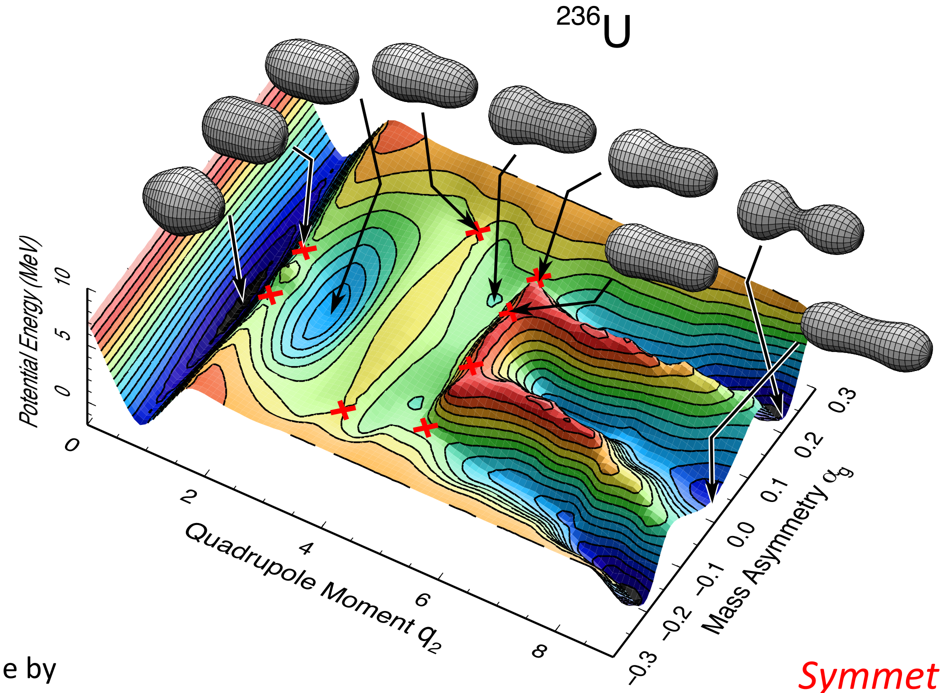
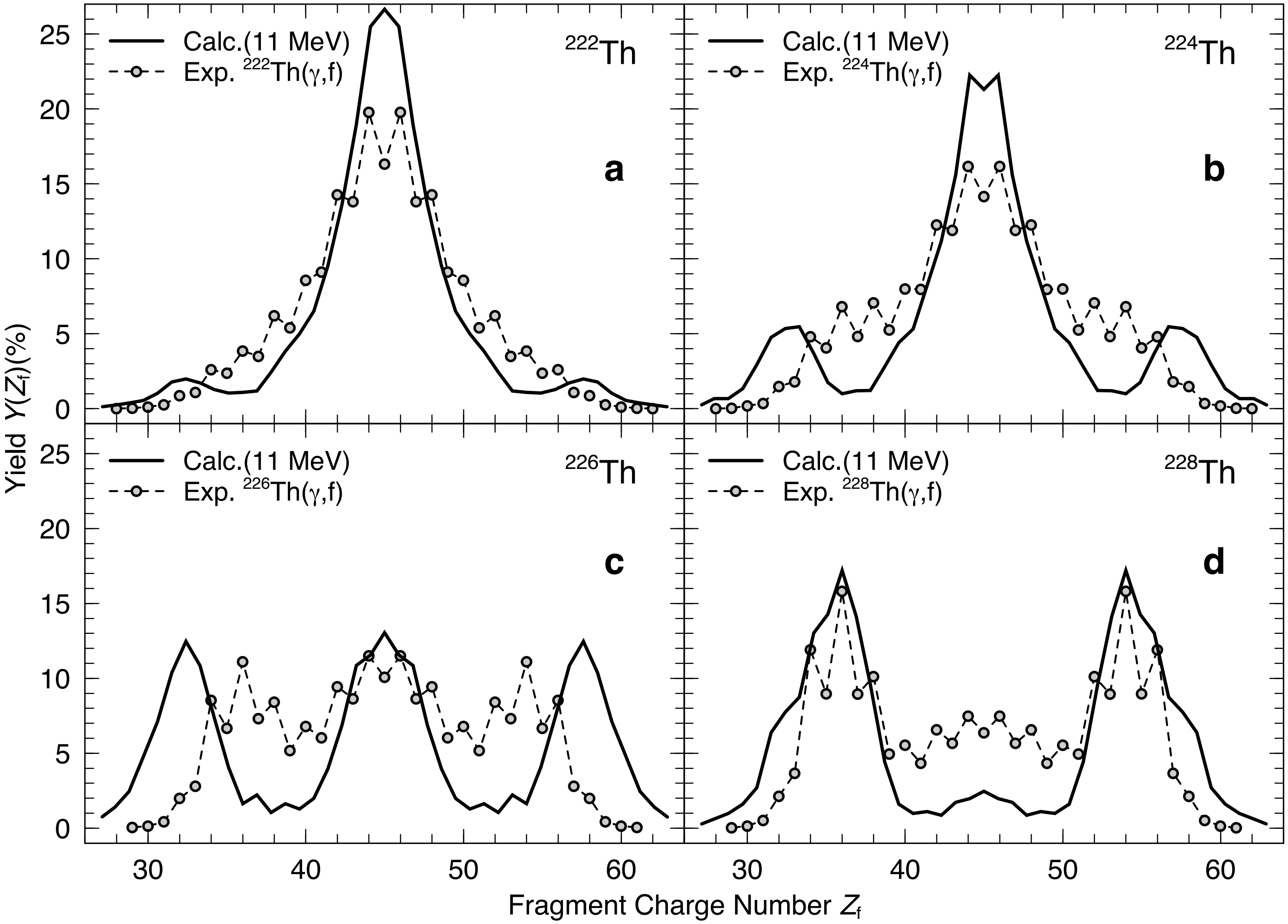


Figure made by
Takatoshi Ichikawa

Symmetric split blocked;
deep asymmetric valley

$P(A_f)$ for $^{222,224,226,228}Th(\gamma,f)$

5D Metropolis walks

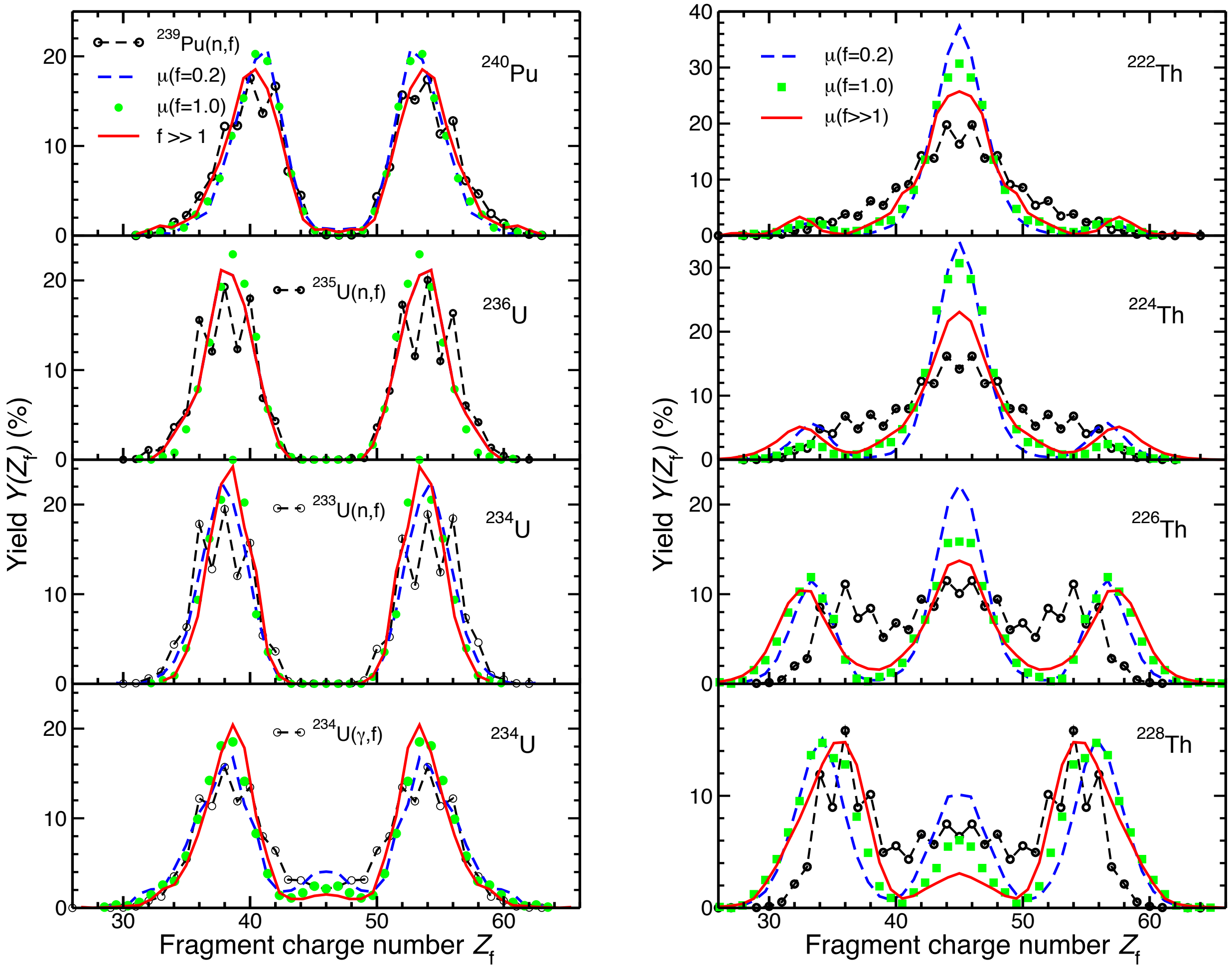


J. Randrup & P. Möller, PRL 106 (2011) 132503

Dependence of $P(A_f)$ on the structure of the dissipation tensor?

Solve the Smoluchowski equation for various forms of $\gamma(\boldsymbol{q})$

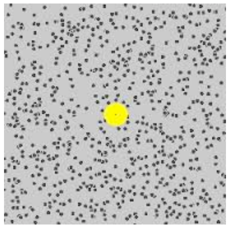
To get $\gamma(\boldsymbol{q})$, interpolate between $\gamma_{ij} \sim \delta_{ij}$ and $\gamma_{ij} \sim \gamma_{ij}^{\text{wall}}(\boldsymbol{q})$



Main points (so far):

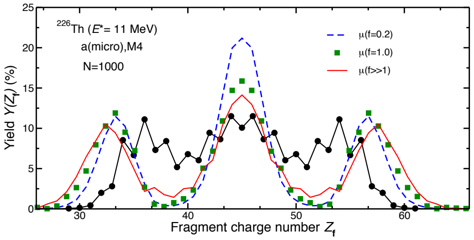
If shape motion is strongly damped => ignore inertial masses
=> Smoluchowski equation (Brownian motion)

$$\boldsymbol{F}_{\mathrm{pot}} + \boldsymbol{F}_{\mathrm{fric}} + \boldsymbol{F}_{\mathrm{ran}} = \mathbf{0}$$



If $P(A_f)$ depends only weakly on the dissipation tensor
=> Metropolis walk is a reasonable starting point

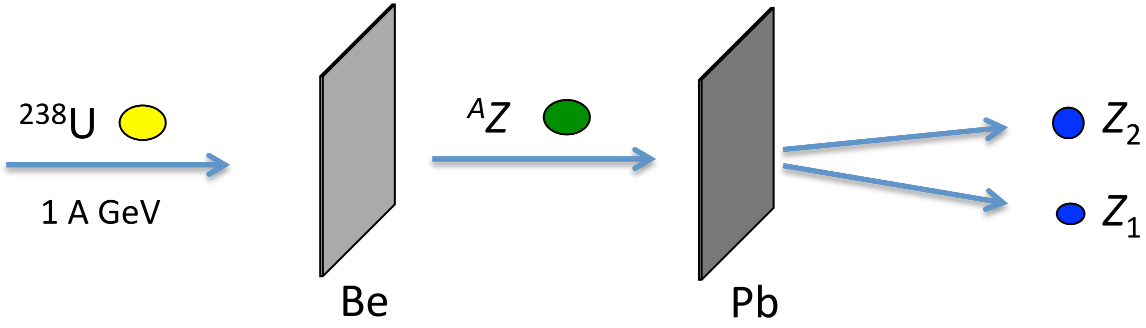
The dependence on the dissipation anisotropy provides an estimate of the uncertainty on the calculated $P(A_f)$



Comparison with experimental data …

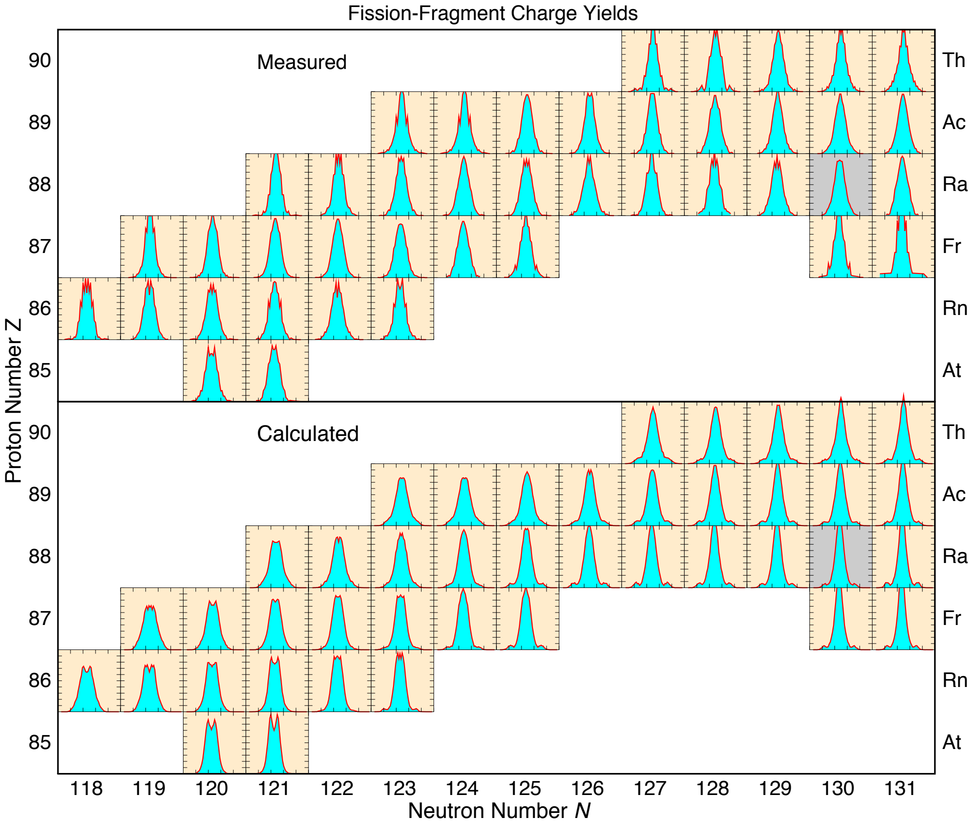
K.-H. Schmidt *et al.*, Nucl. Phys. A 665 (2000) 221:

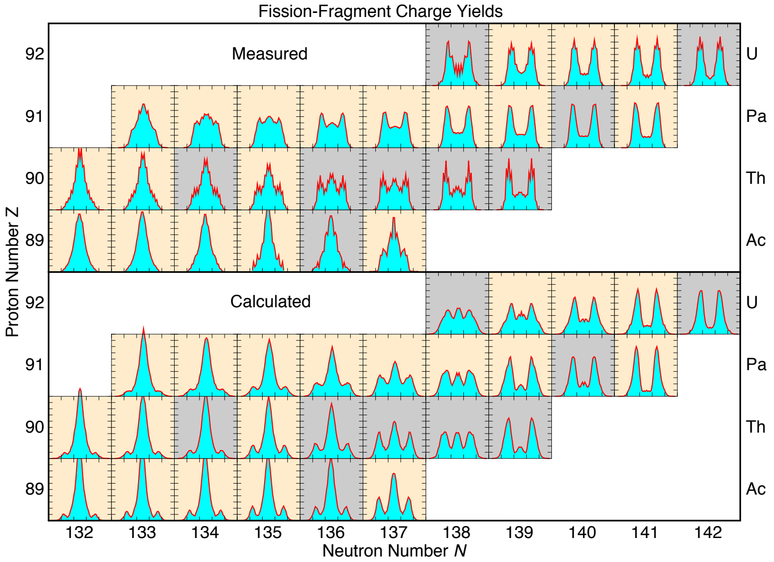
Fission fragment charge distributions
for *seventy* radioactive nuclei,
using the secondary-beam facility at GSI



Comparison with experimental data

J. Randrup & P. Möller, Phys. Rev. C **88**, 064606 (2013)

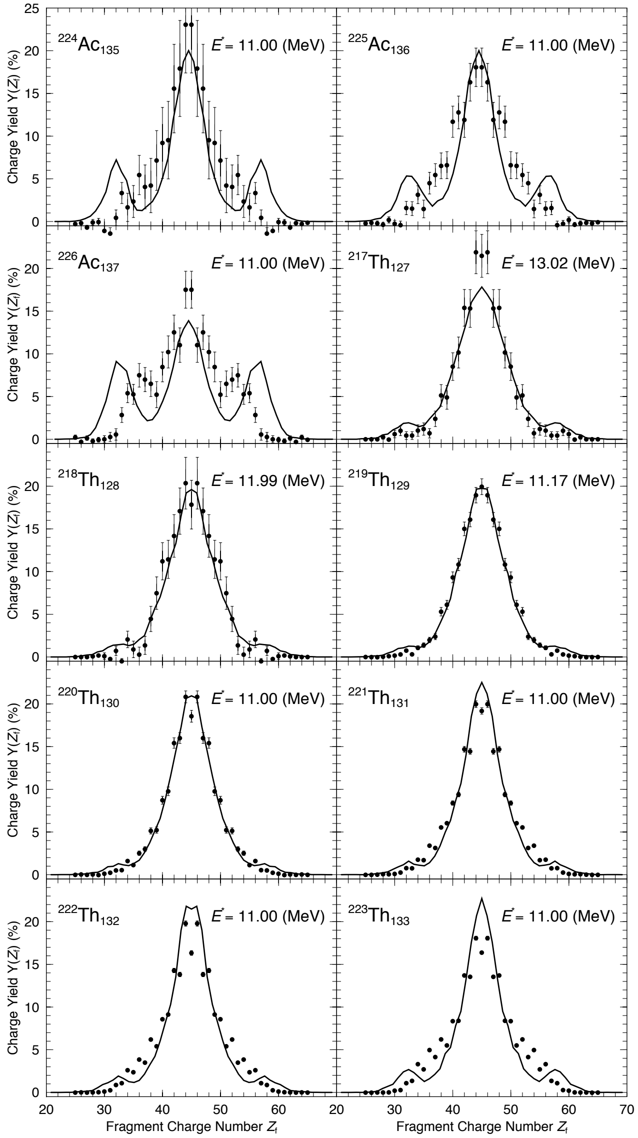




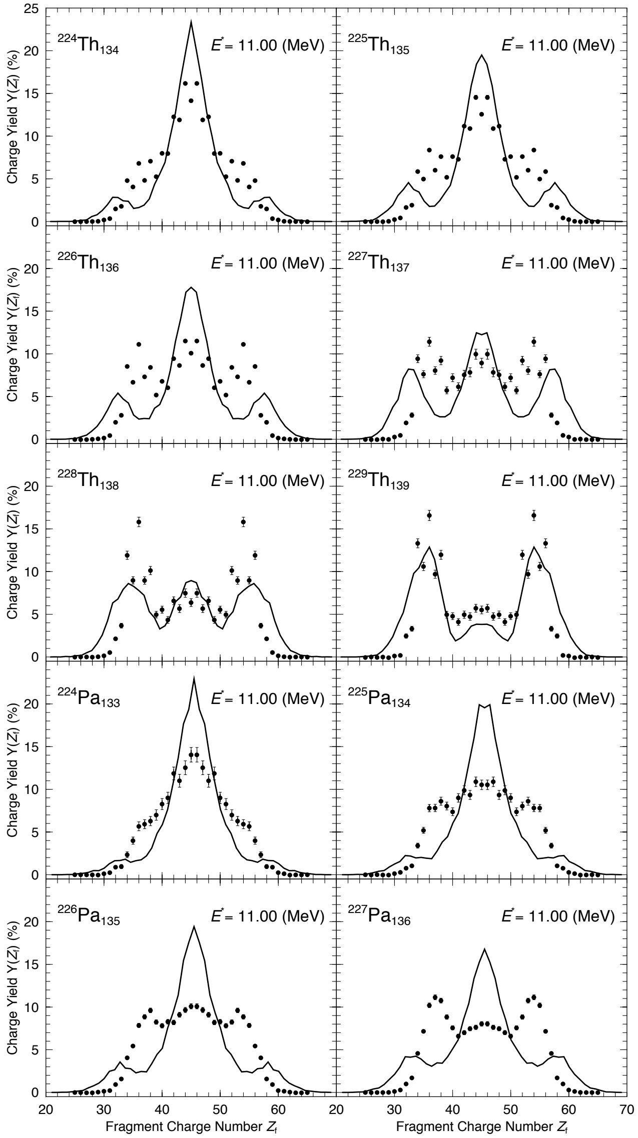
Data from K.-H. Schmidt *et al.*,
Nucl. Phys. **A 665** (2000) 221

Comparison with data from K.H. Schmidt *et al.*, NPA 665 (2000) 221

$P(Z_f)$ for isotopes at E^*=11 MeV



$P(Z_f)$ for isotopes at E^*=11 MeV



J. Randrup & P. Möller, Phys. Rev. C **88**, 064606 (2013)

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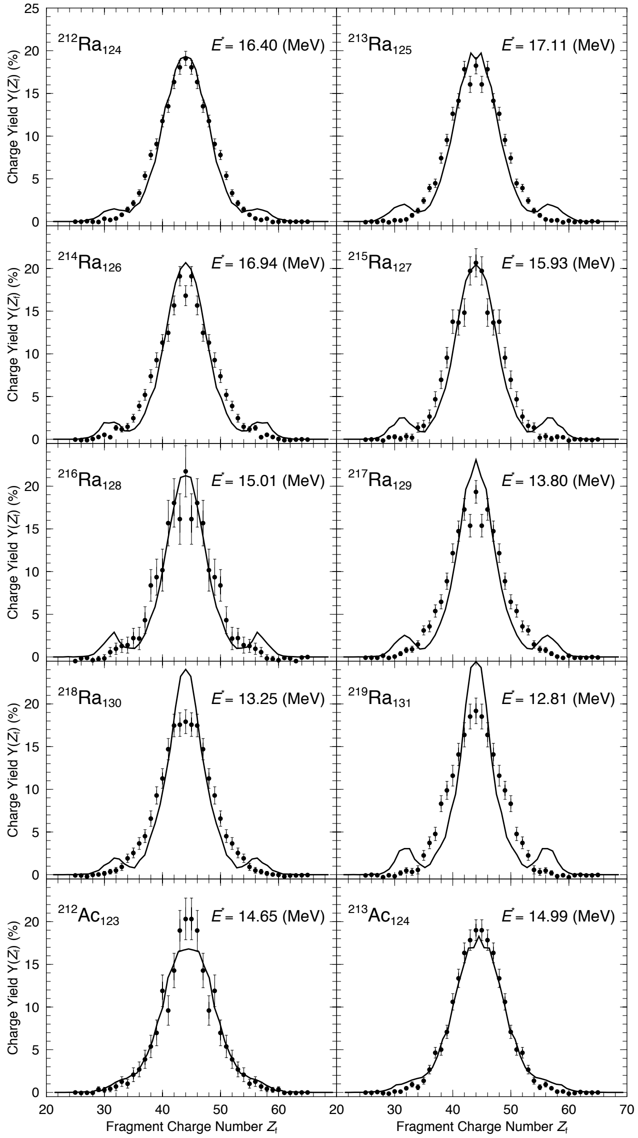
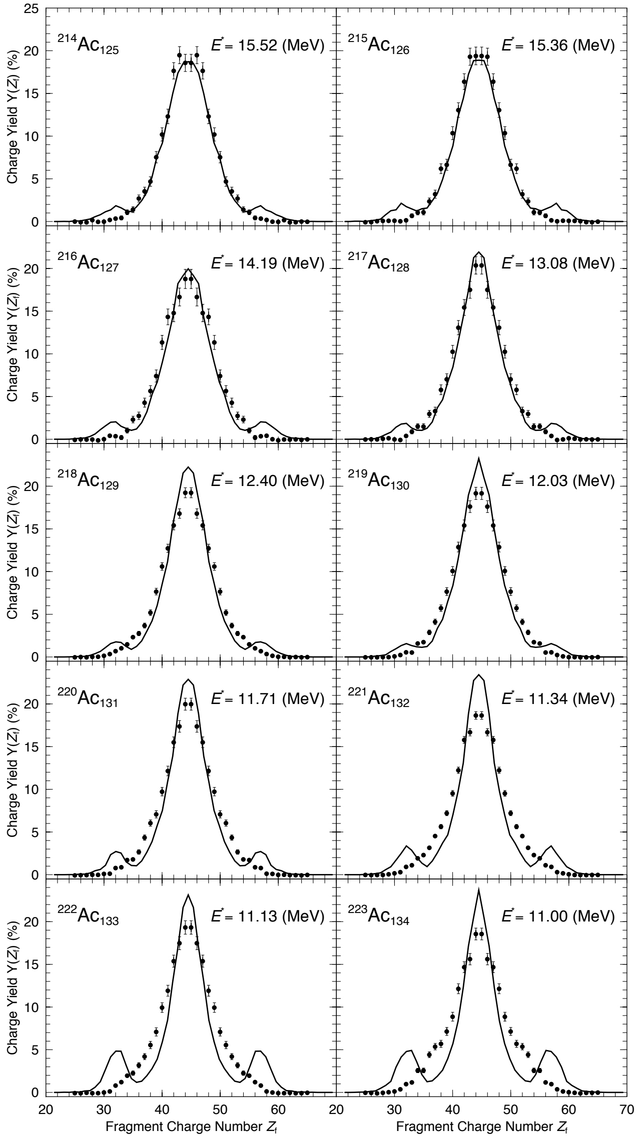


$P(Z_f)$ for isotopes at E^*=11 MeV



J. Randrup & P. Möller, Phys. Rev. C **88**, 064606 (2013)

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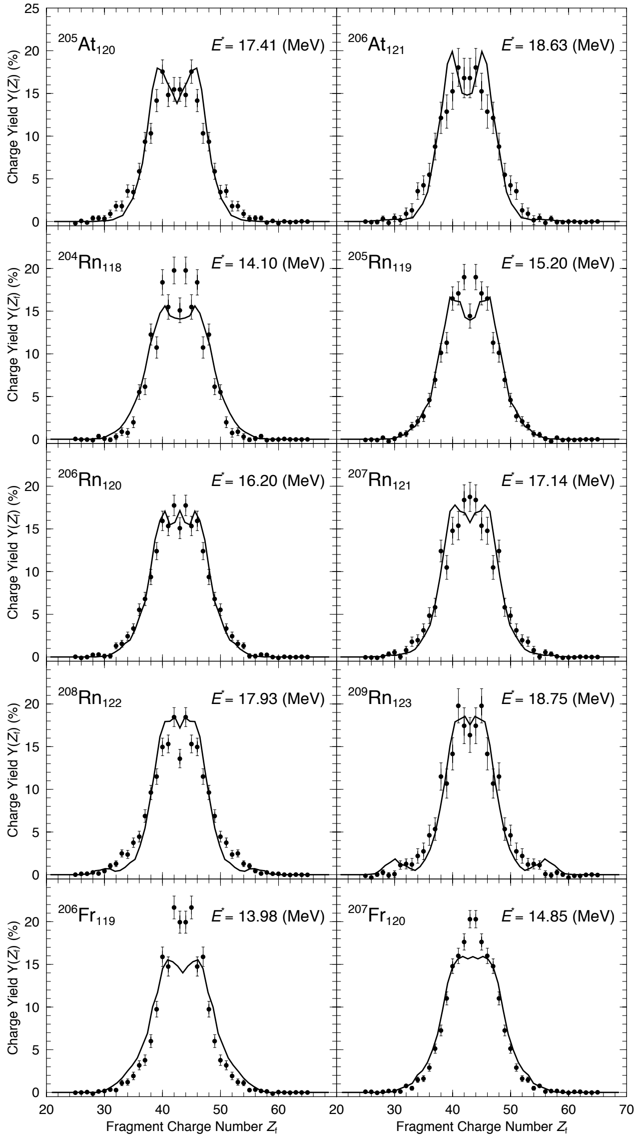
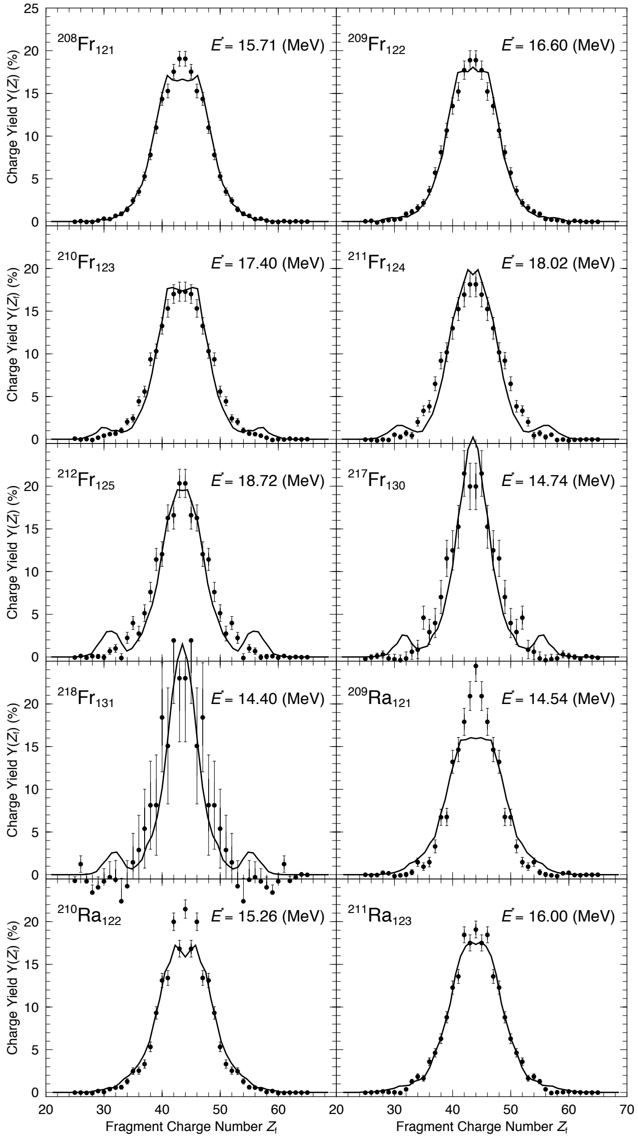


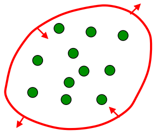
$P(Z_f)$ for isotopes at E^*=11 MeV



J. Randrup & P. Möller, Phys. Rev. C **88**, 064606 (2013)

Summary and perspectives

Accumulating evidence suggest that the nuclear shape dynamics at moderate excitations is *highly dissipative*



If so, the evolution resembles (generalized) *Brownian motion* *(N* dims, non-uniform & anisotropic medium, external force)

$\boldsymbol{F}_{\text{pot}}+\boldsymbol{F}_{\text{fric}}+\boldsymbol{F}_{\text{ran}}=\mathbf{0}$

Because the nuclear shape evolves so slowly,
its evolution can be approximated by a *Metropolis walk*
on the *N*-dimensional potential-energy surface

Novel method for calculating fission fragment mass distributions

- … can immediately be applied to >5,000 nuclei
- … requires only modest computer power
- … has unprecedented predictive power

5D tables of *U* exist
min-hrs on a laptop
no free parameters

While the Metropolis method presents a very useful tool,
it is merely a rough, preliminary, and idealized treatment.
But its success provides hope that it is possible to develop
a quantitative theory of nuclear shape dynamics

Langevin:
dissipation &
mass tensors

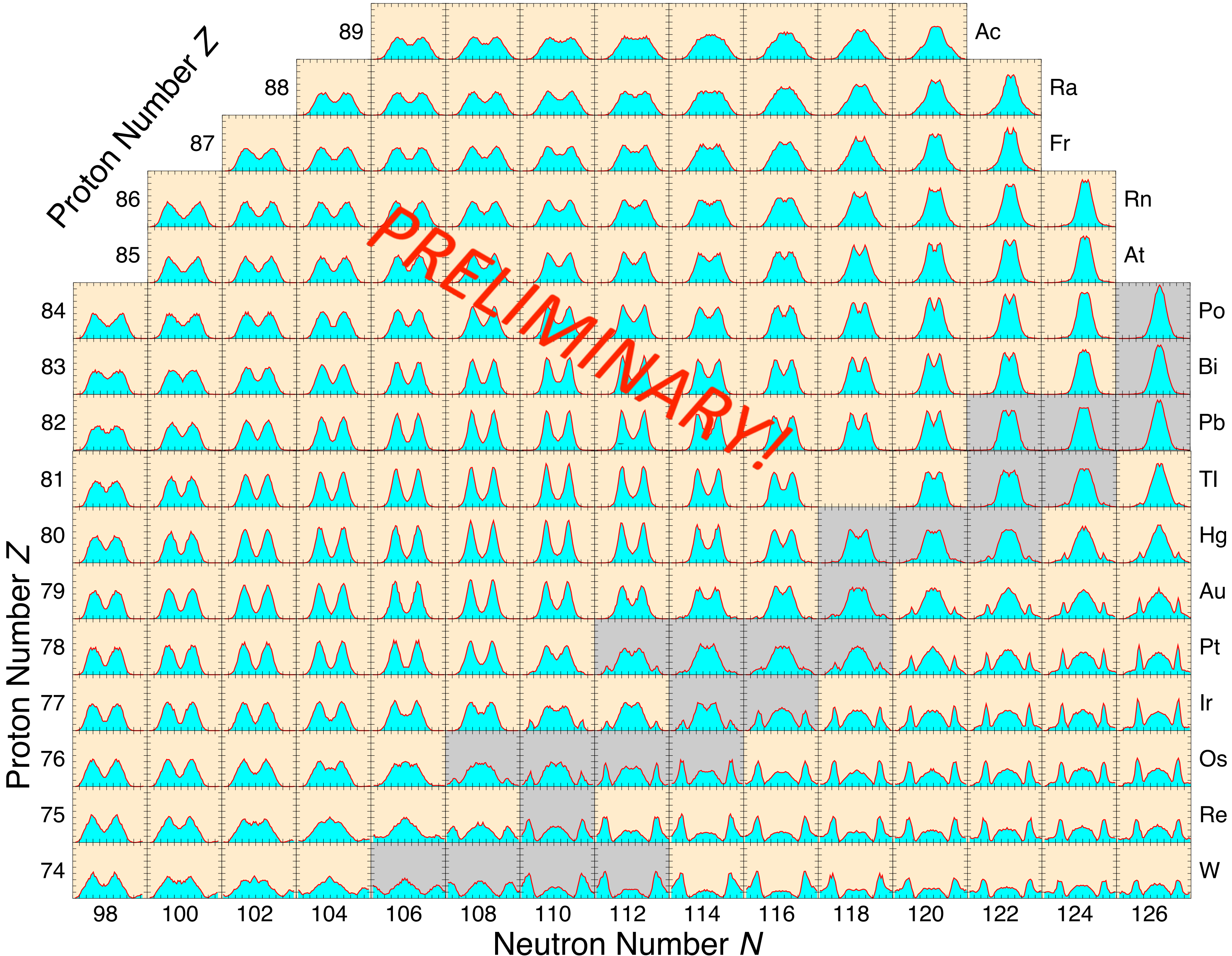
Predictions …

✔ Fission-fragment mass distributions in the *neutron-deficient* region of the nuclear chart?

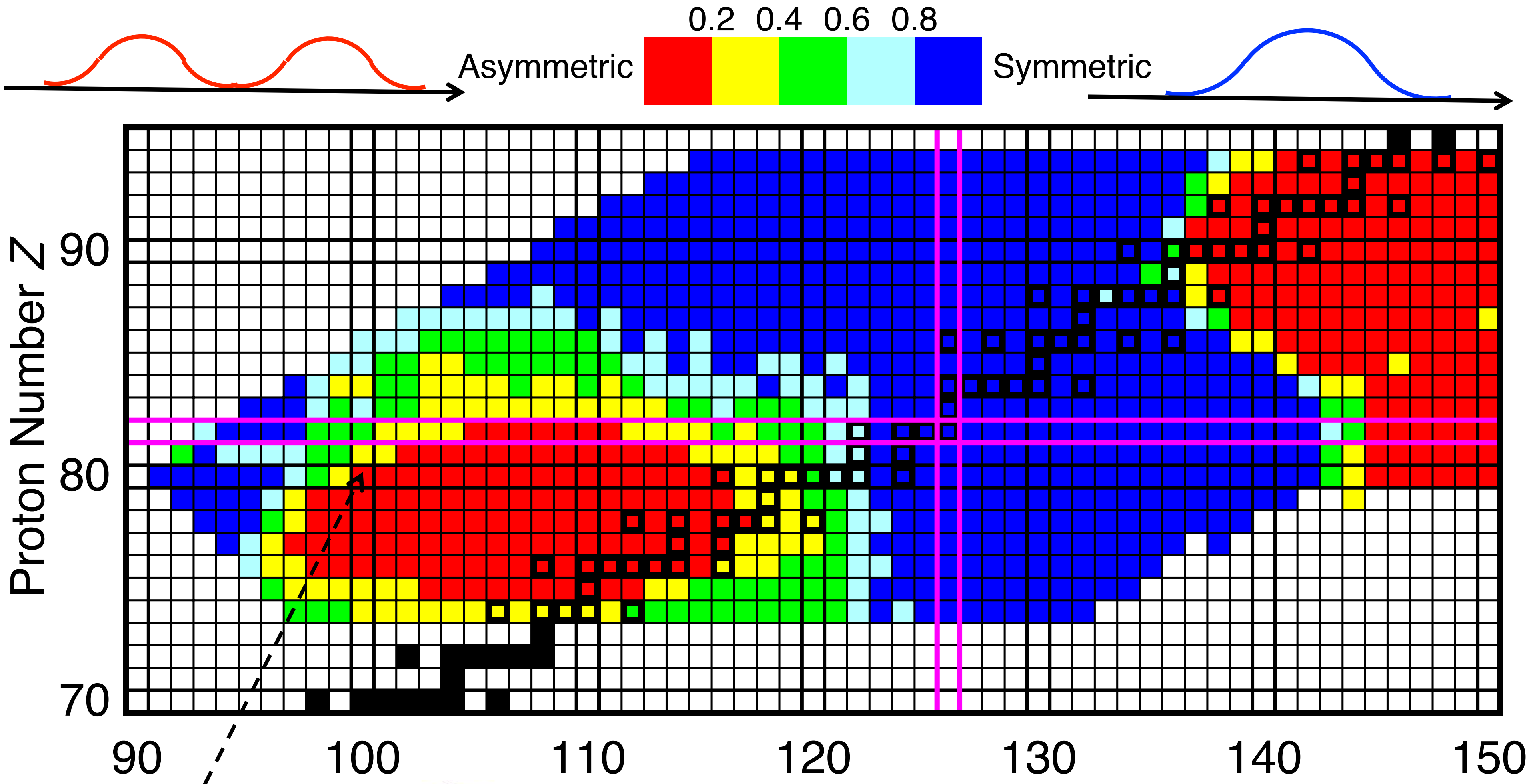
Fission-fragment mass distributions in the *r-process* region of the nuclear chart?

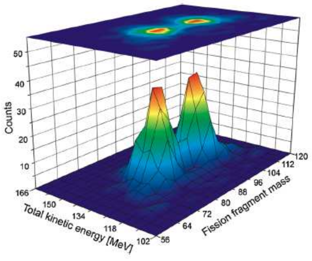
. . .

Calculated Fission-Fragment Mass Distributions



Fission-Fragment Symmetric-Yield to Peak-Yield Ratio

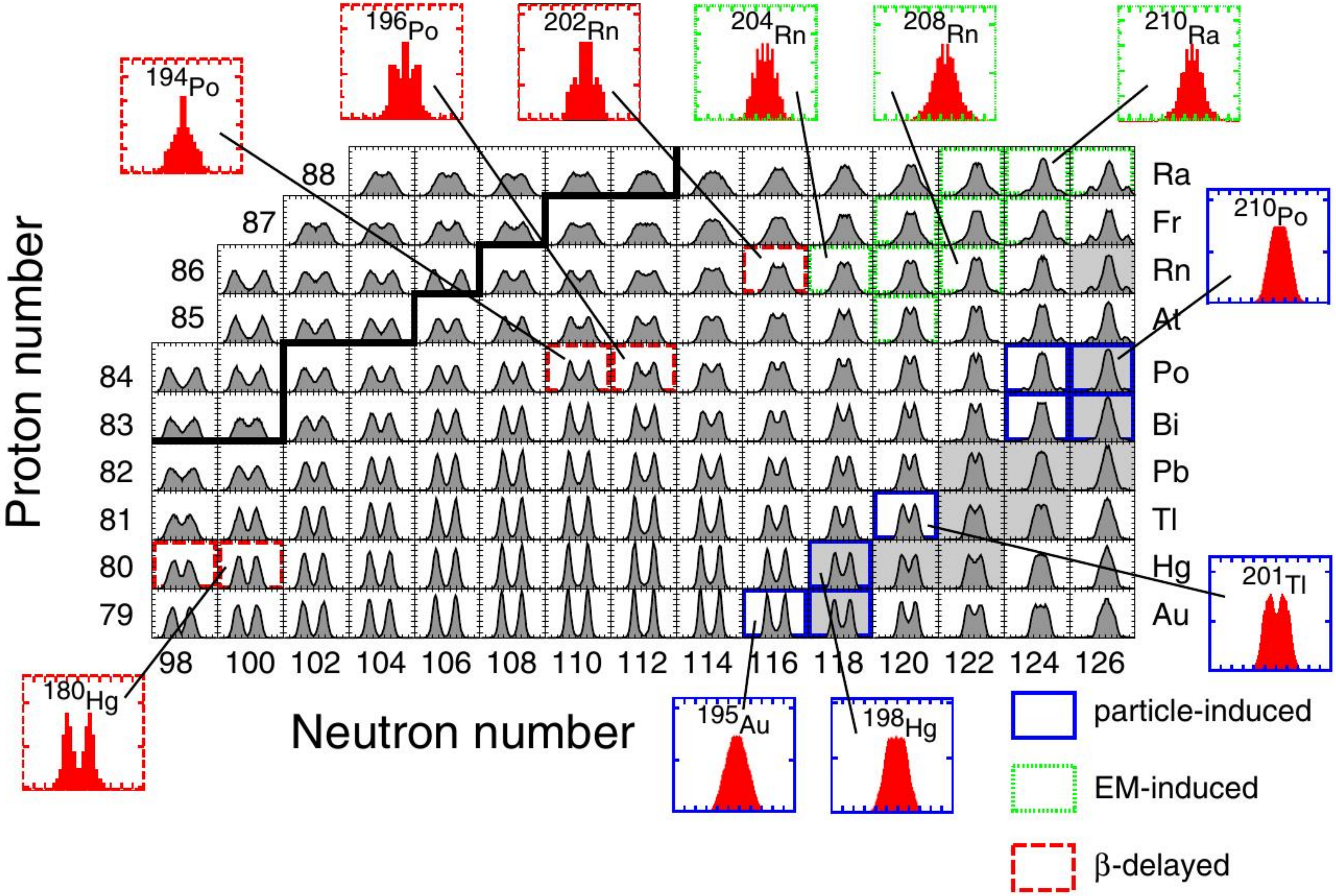




^{180}Hg

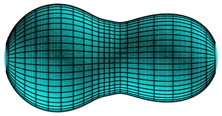
A.N. Andreyev *et al.*,
PRL **105**, 252502 (2010)

P. Möller & J. Randrup,
PRC **91**, 044316 (2015)

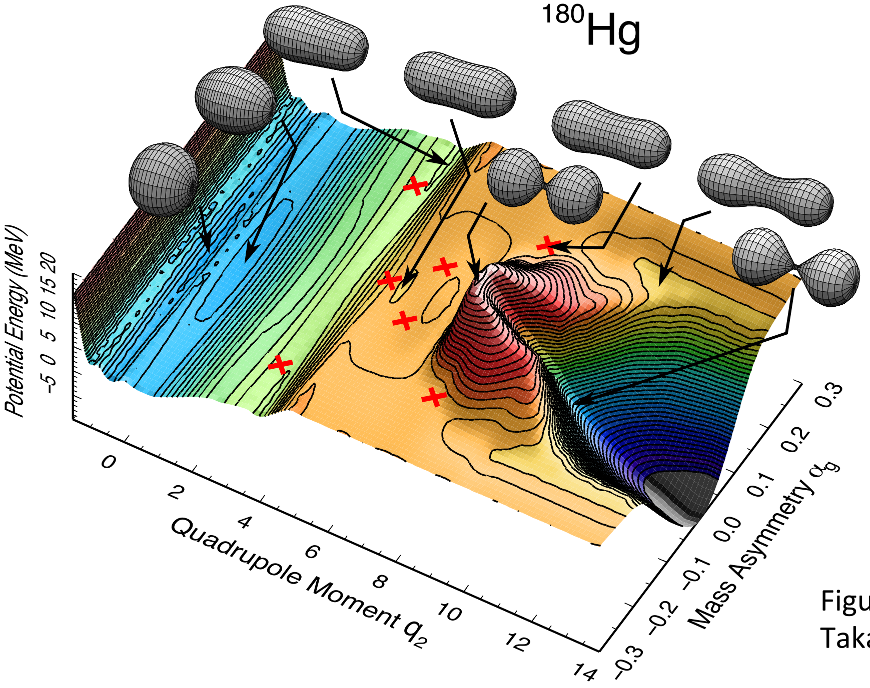


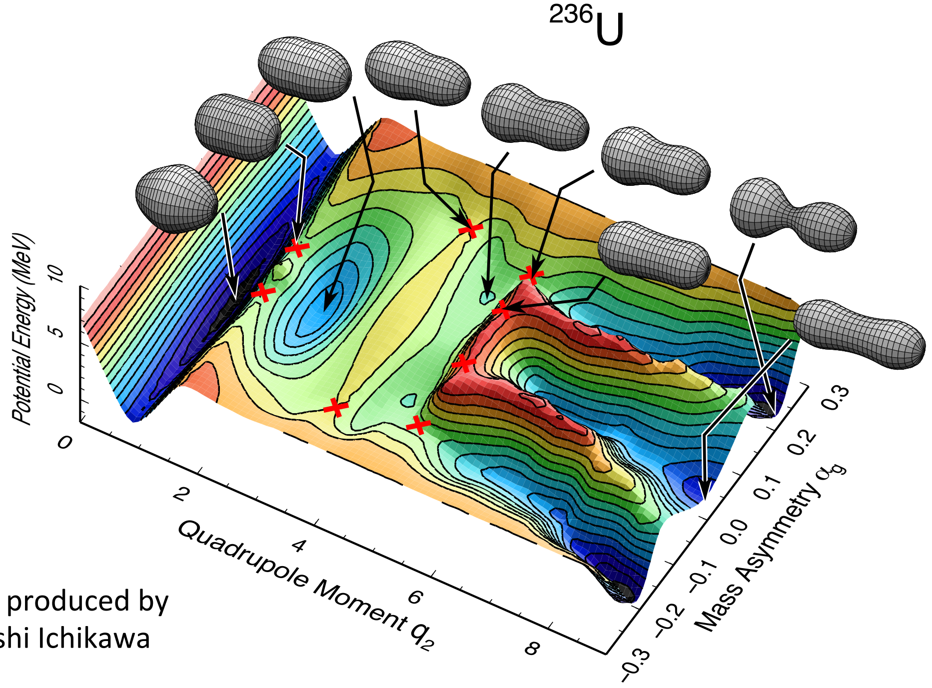
L. Ghys et al, Phys. Rev. C 90, 041301(R) (2014)

Nuclear potential energy: fission barrier landscape



5D potential-energy surfaces reduced to two dimensions





Figures produced by
Takatoshi Ichikawa

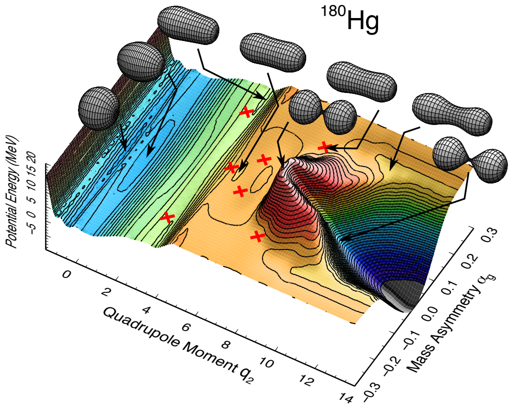
Symmetric split is blocked;
asymmetric landscape is flat

Symmetric split is blocked;
deep asymmetric valley

Why is it interesting to study fission in the neutron-deficient region?

The character of the fragment mass distribution exhibits a significant sensitivity to model specifics

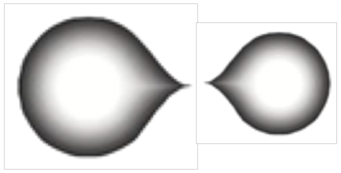
⇒ Good prospects for testing & improving our understanding of nuclear dynamics



Workshop on Science with SπRIT TPC
RIKEN, 5-6 June 2015

Fission Dynamics of Exotic Nuclei

Jørgen Randrup
LBNL, Berkeley, California



Why is it interesting to study fission with SπRIT TPC?

It is possible to make event-by-event measurements of the fission fragment mass, charge, and kinetic energy, providing valuable information for constraining models*

Additional measurements of neutrons and/or photons, e.g. mean neutron multiplicity and total photon energy for each mass and charge partition, will help further

ADVERTISEMENT:

* The Monte-Carlo simulation code FREYA provides large samples of *complete* fission events, quickly (both product nuclei & all neutrons and photons):

Jerome M. Verbeke, Jørgen Randrup, Ramona Vogt,
Computer Physics Communications 191 (2015) 178