



# Hadron Structure from Lattice QCD

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The University of Adelaide

Lattice 2015, July 14-18, 2015, Kobe, Japan

# Acknowledgements

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- **Thanks to everyone who sent data, plots, references**
  - **Special thanks to .....**
    - **Alex Chambers**
    - **Jack Dragos**
    - **Tim Harris**
    - **Huey-Wen Lin**
    - **Rudolf Rödl**
    - **Shoichi Sasaki**
- **Sorry if I don't cover your work, there are way too many topics to cover in 40mins**

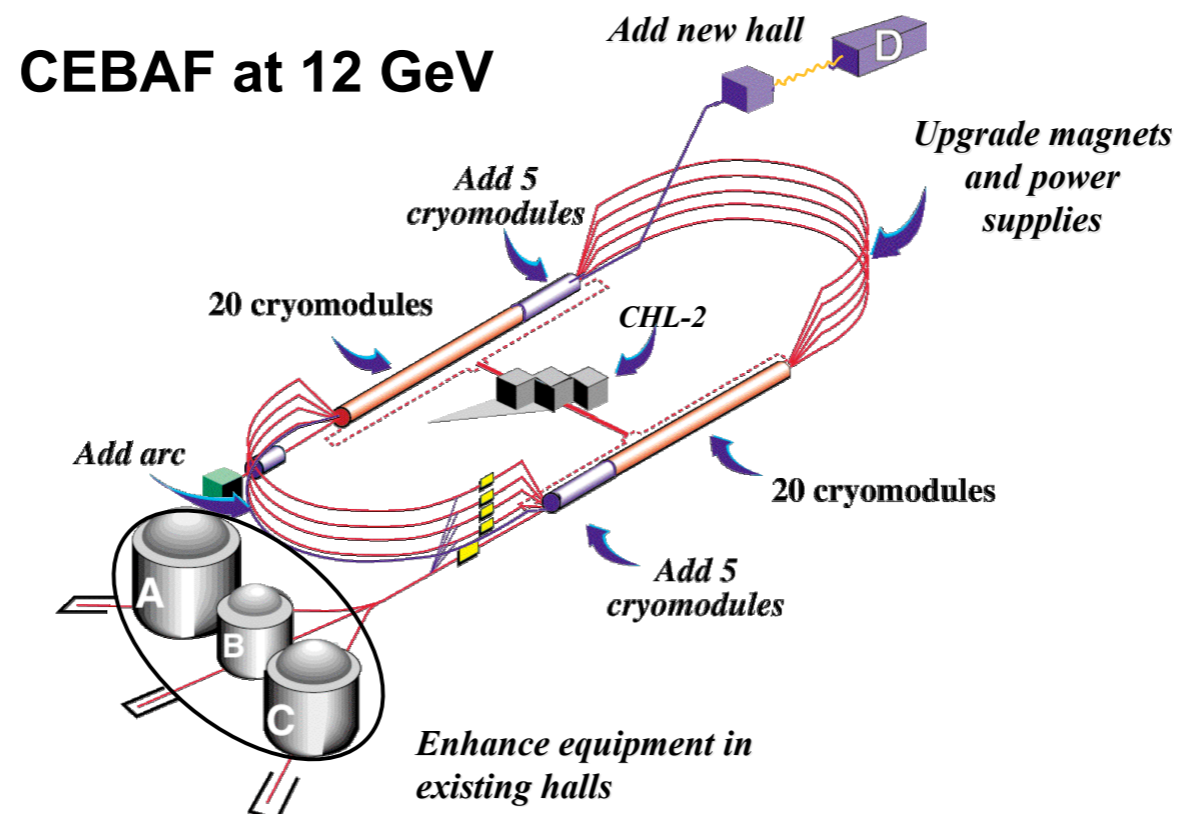
# Outline

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- **Motivation** - Experimental challenges & theoretical issues
- Nucleon charges (Axial, tensor, scalar)
- **Momentum fraction**
- Spin content, including  $\Delta_S$   $\Delta_G$
- **EM form factors** (including strangeness, light nuclei, charge symmetry)
- Pion,  $\rho \rightarrow \pi\gamma$  form factors
- **Transverse Momentum Distributions & “quasi”-PDFs**
- **Summary & outlook**

# Motivation

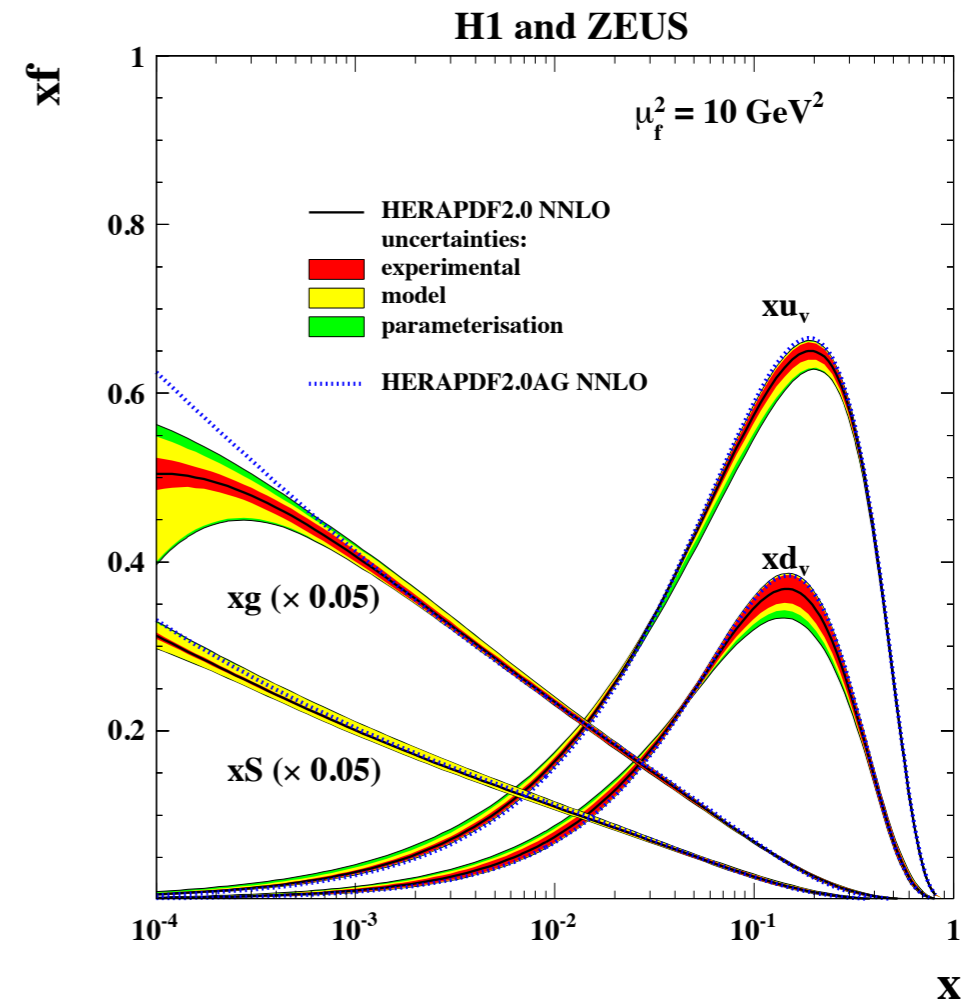
- **Major goal of nuclear physics community**
  - **understand the structure and behaviour of strongly interacting matter in terms of its basic constituents - quarks and gluons**
- **An important step towards this goal is the characterisation of the internal structure of the nucleon**
- **Driving force behind several experimental programs, e.g. JLab 12 GeV upgrade**



# Deep inelastic scattering (DIS) experiments

→ **Parton distribution functions**

[HERA, 1506.06042]



→ reveal the breakdown of the momentum of a (fast-moving) nucleon terms of its **quark** and **gluon** (parton) constituents

- The same cannot be said of our understanding of the nucleon spin, mass, magnetic moment, ....

# Spin of the Proton

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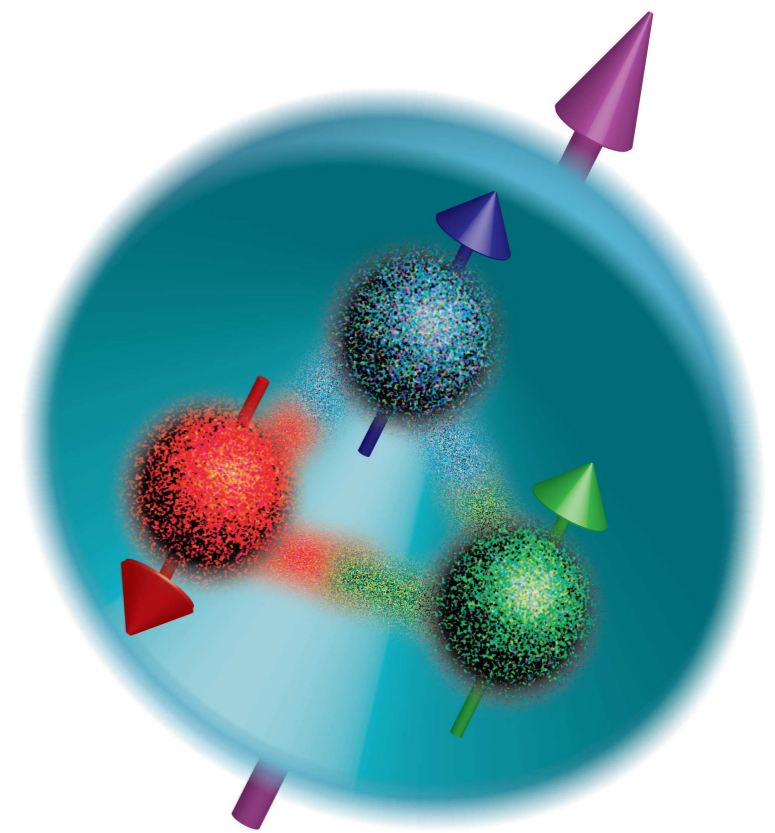
**Proton has spin  $1/2$**

# Spin of the Proton

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**Quark Model predicts: only due to the spin of 3 valence quarks (also spin- $1/2$ )**



# Spin of the Proton

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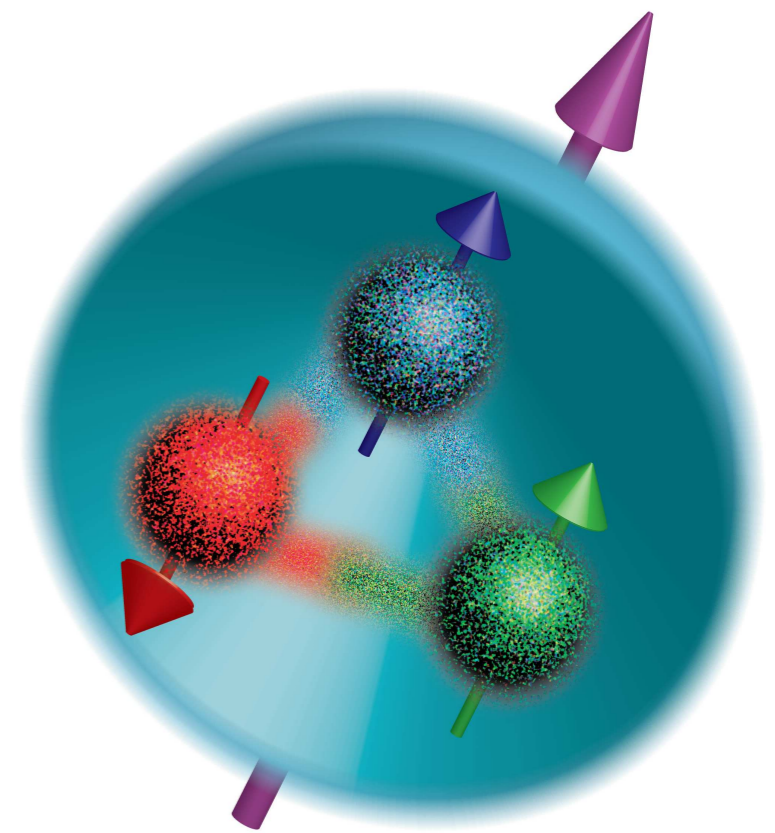
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**EMC (1988) found this only accounted for  $1 \pm 12 \pm 24\%$**

**COMPASS (2007) improved this to  $33 \pm 3 \pm 5\%$**

 **“Spin crisis”?**





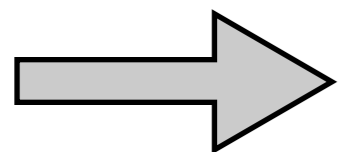
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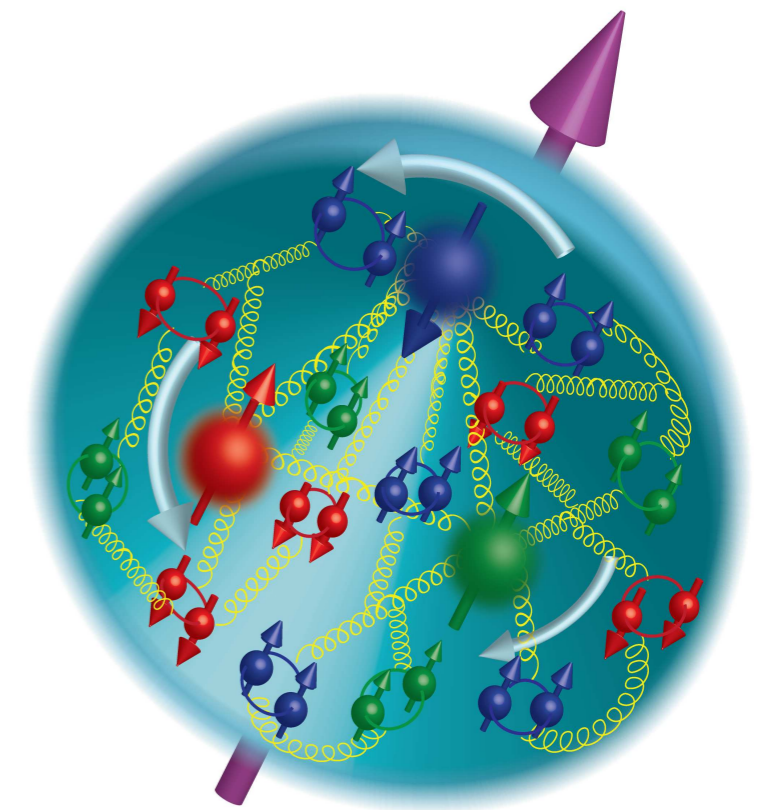
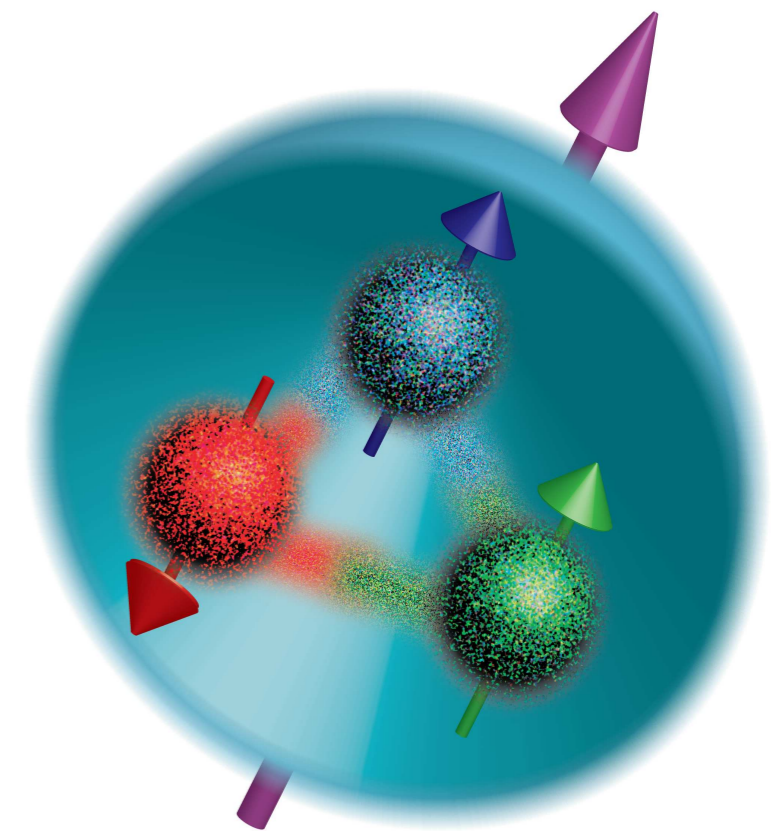
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**“Spin crisis”?**

**No, only a “spin puzzle”**



(EIC white paper [1212.1701])

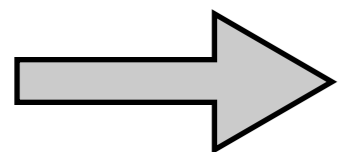
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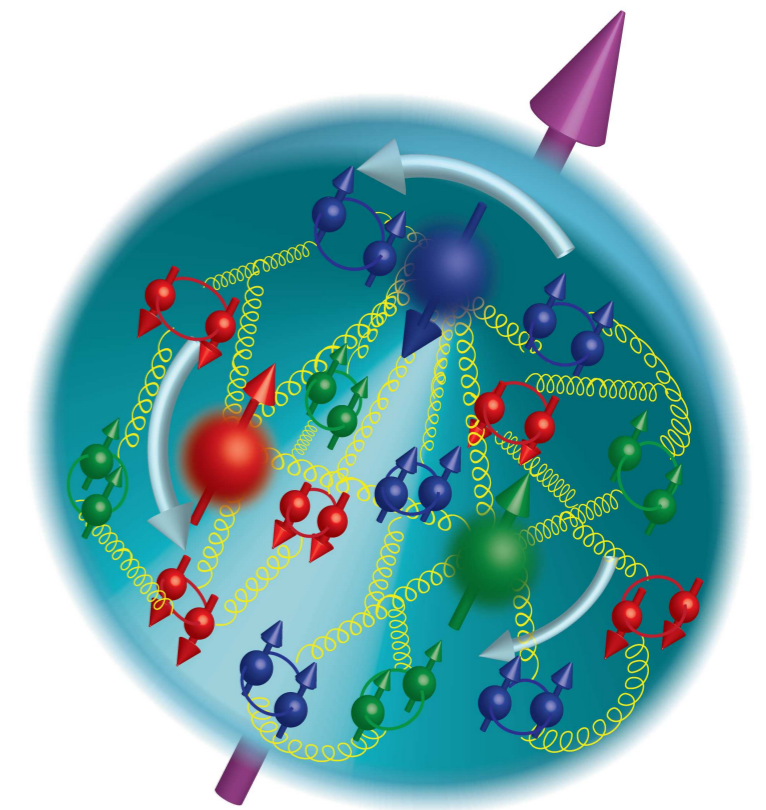
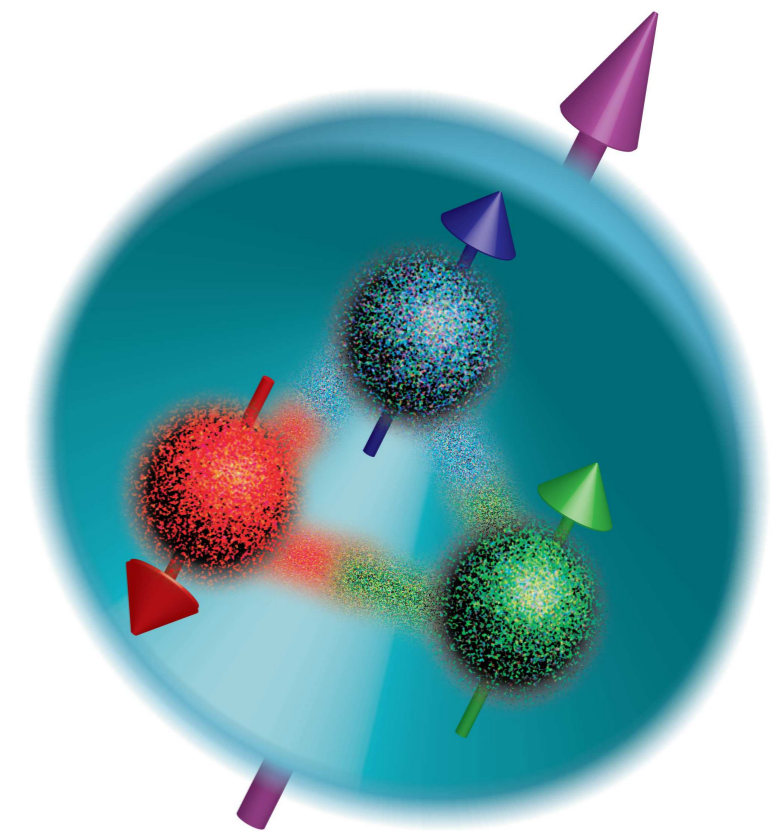
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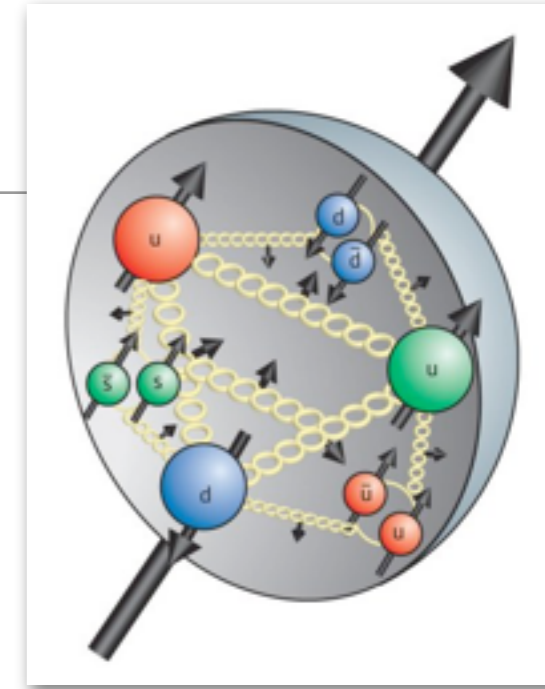
**No, only a “spin puzzle”**



(EIC white paper [1212.1701])

**But how is the other 70% of the proton spin distributed between these components?**

# Spin of the Proton



**How is the spin of the proton distributed between its constituents?**

**Jaffe & Manohar (1990):**  $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \mathcal{L}_q + \Delta G + \mathcal{L}_g$

**X. Ji (1997):**  $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + J_g$

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s = 0.33(3)(5)$$

**Why so small?**

**Due to large negative  $\Delta s$  ?**

**Much effort to determine  $\Delta s$  experimentally**  $\int_0^1 g_1^p(x) dx = \frac{1}{36}(4a_0 + 3a_3 + a_8)$

**e.g. COMPASS, HERMES**

$$x \geq 0.004 \quad x \geq 0.02$$

Also  $g_A$  and semileptonic hyperon decays assuming SU(3) symmetry

**→**  $-0.15 \leq \Delta s \leq -0.03$

# Spin of the Proton

How is the spin of the proton distributed between its constituents?

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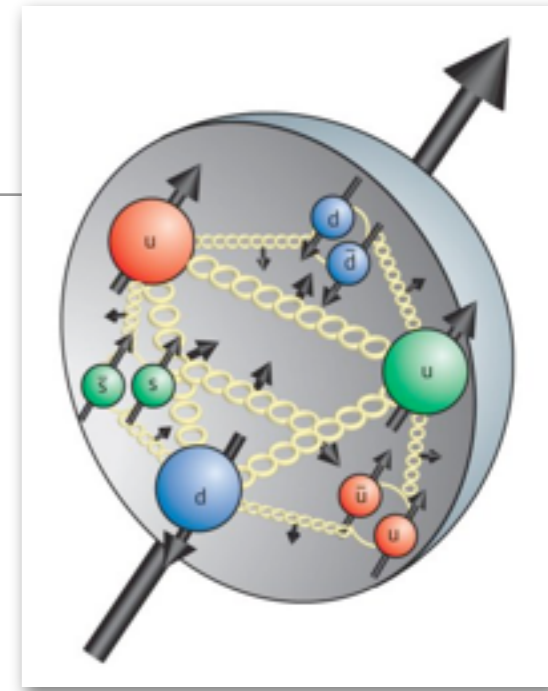
Large  $\Delta G$  ?

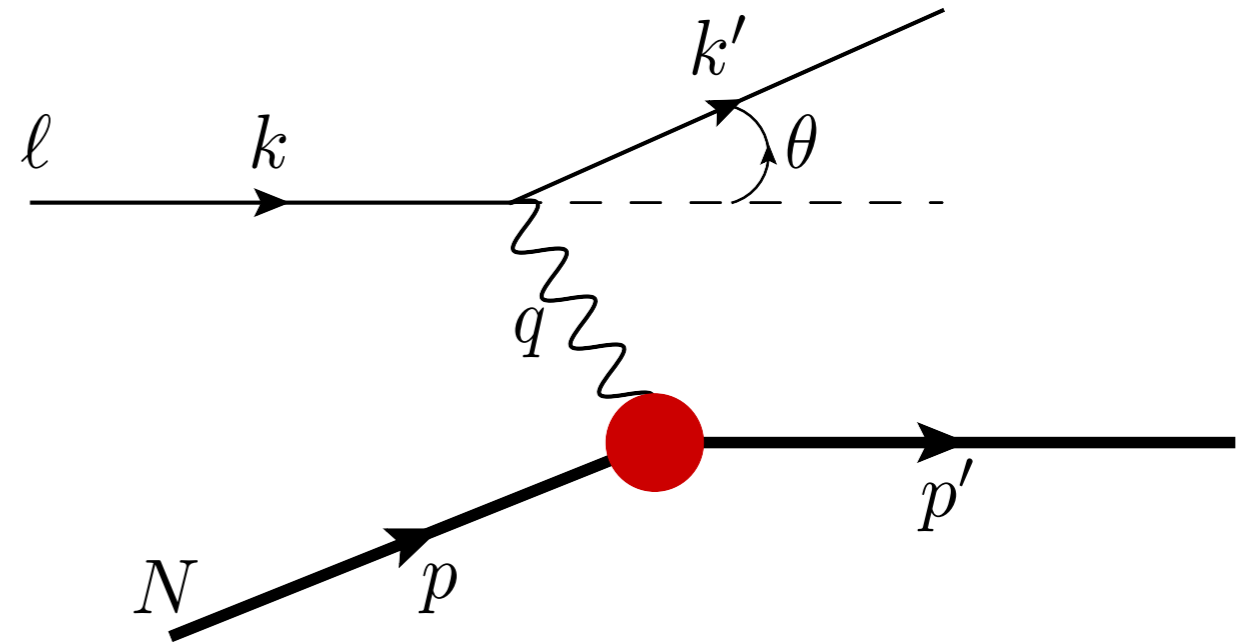
**Recent global analysis of RHIC (PHENIX & STAR) and COMPASS data**

[de Florian et al., PRL113 012001((2014)]

$$\Delta G \approx \int_{0.005}^{0.2} \Delta g(x) dx = 0.1^{+0.06}_{-0.07}$$

(negligible contribution  $x > 0.2$ , poorly constrained  $0.001 < x < 0.05$ )





## Electromagnetic Form Factors

$$\langle p', s' | J^\mu(\vec{q}) | p, s \rangle = \bar{u}(p', s') \left[ \gamma^\mu F_1(q^2) + i\sigma^{\mu\nu} \frac{q_\nu}{2m} F_2(q^2) \right] u(p, s)$$

$$G_E(Q^2) = F_1(Q^2) - \tau F_2(Q^2)$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

# Elastic Scattering - Polarisation Transfer

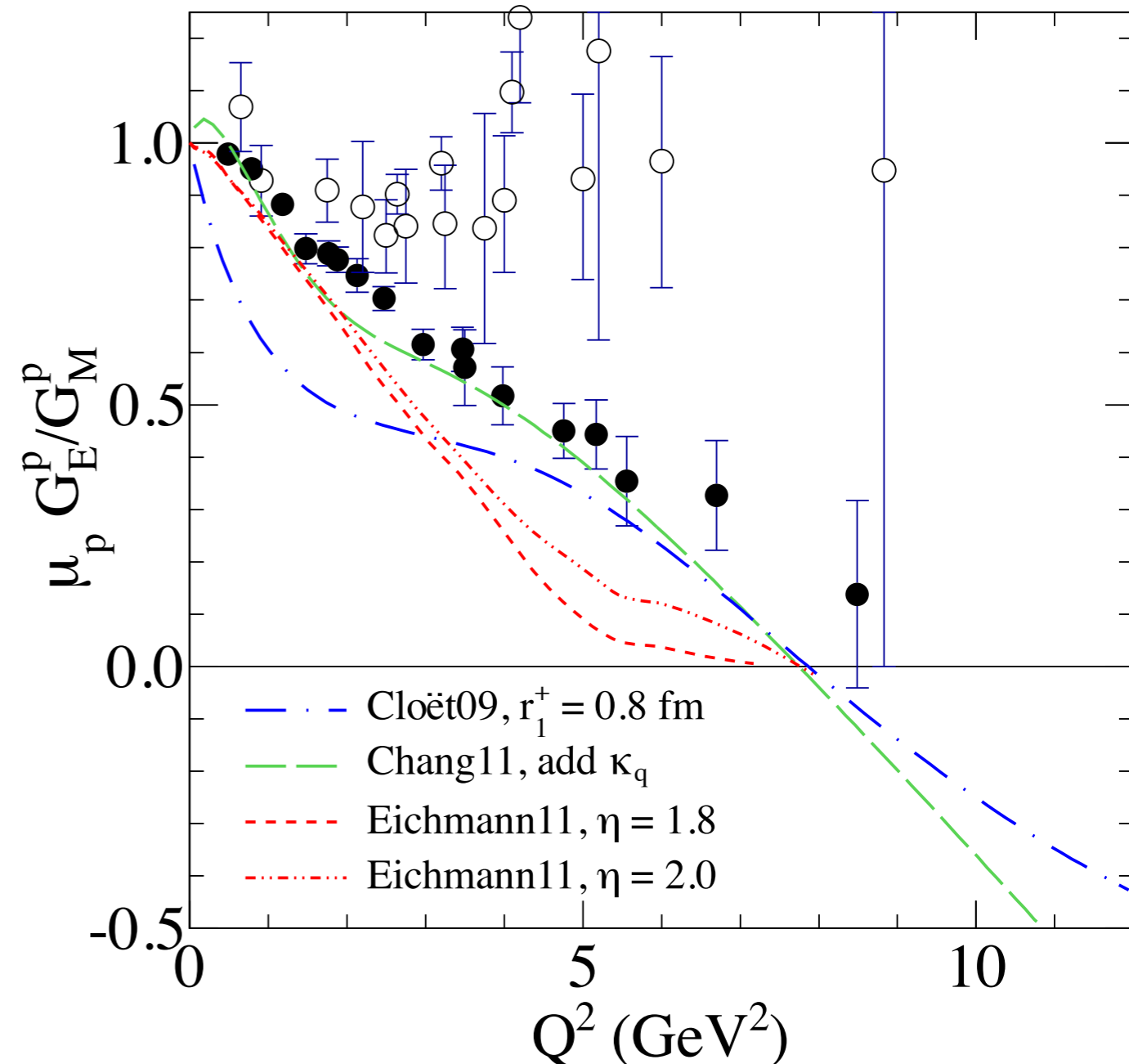
Polarisation transfer experiments at JLab revealed a surprising behaviour for  $G_E/G_M$

Precise results now available up to 8-9  $\text{GeV}^2$

[JLab, Hall A, PRC85 (2012) 045203]

What is the origin of the linear fall-off?

Does  $G_E^p$  change sign?

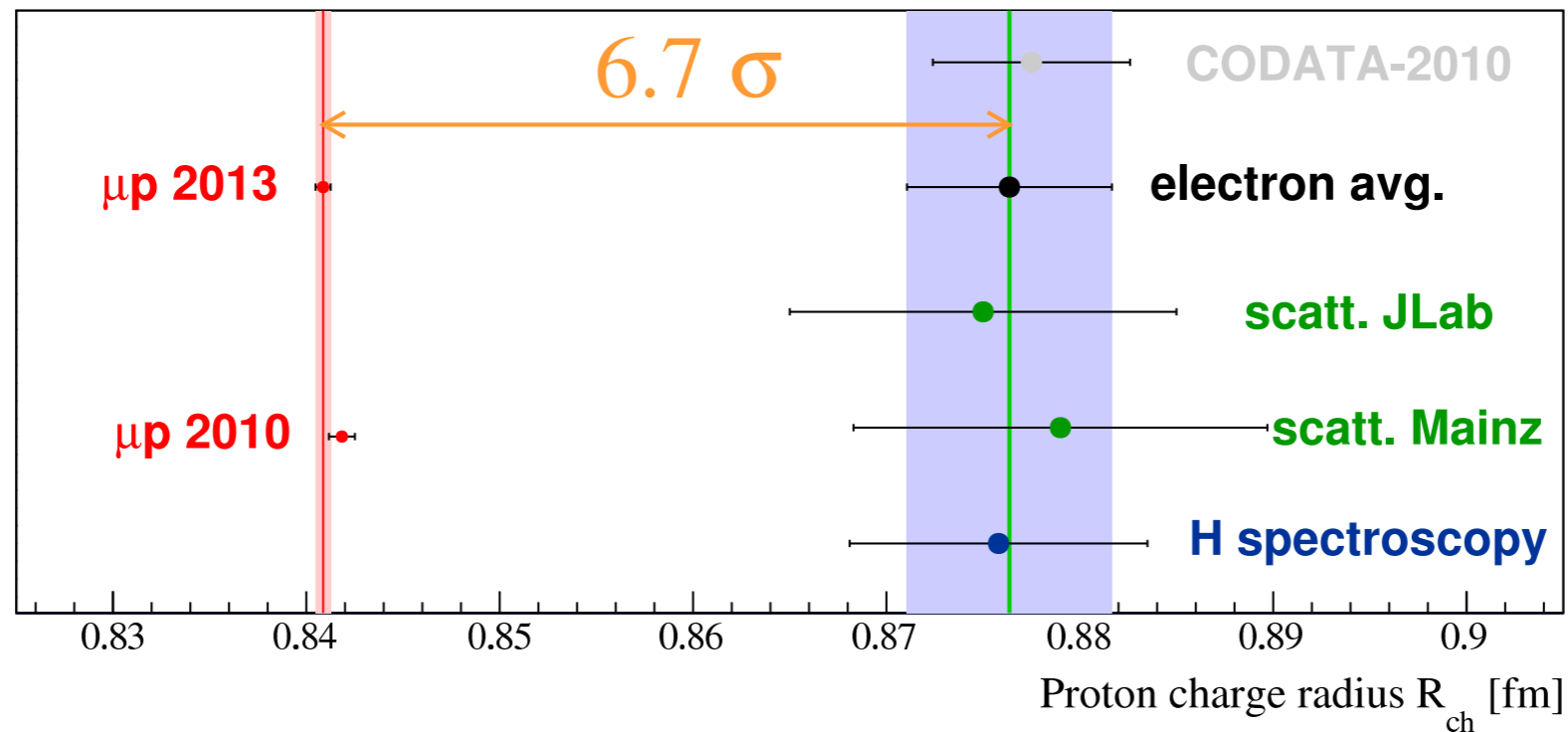


# Size of the Proton

- Charge radius of the nucleon

$$G_E^p(Q^2) = 1 - \frac{Q^2}{6} \langle r^2 \rangle_E + p + \dots$$

- $\sim 7\sigma$  discrepancy between  $\mu\text{H}$  and  $\text{H} / \text{e-p}$  scattering



[Arrington, arXiv:1506.00873]

# Hadron Structure - new experiments

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- **An understanding of nucleon spin is a major goal of many current and future experimental programs**
- **12 GeV JLab upgrade**
- **Origins of quark confinement**
- **Spin and flavour structure of the proton and neutron (PDF's, GPD's, TMD's...)**
- **Quark structure of nuclei**
- **Probe potential new physics through high precision tests of the Standard Model**
- **Flavour separation of EM form factors**



# Hadron Structure - new experiments

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- **An understanding of nucleon spin is a major goal of many current and future experimental programs**
- 12 GeV JLab upgrade
- **COMPASS (II)**
- **Proton spin (GPDs, Gluon)**
- **TMDs**
- **Pi and K polarisabilities**

# Hadron Structure - new experiments

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- **An understanding of nucleon spin is a major goal of many current and future experimental programs**
- 12 GeV JLab upgrade
- COMPASS (II)
- **RHIC**
- **(PHENIX, STAR)**
- **spin and orbital angular momentum of quarks and gluons**
- **transverse spin structure of the proton**

# Hadron Structure - new experiments

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- **An understanding of nucleon spin is a major goal of many current and future experimental programs**
- 12 GeV JLab upgrade
- COMPASS (II)
- RHIC
- **MAMI (Mainz):**
  - **EM form factors, polarisabilities**
  - **Structure of nuclei**
  - **Parity violating - strange EM, axial form factor**

# Hadron Structure - new experiments

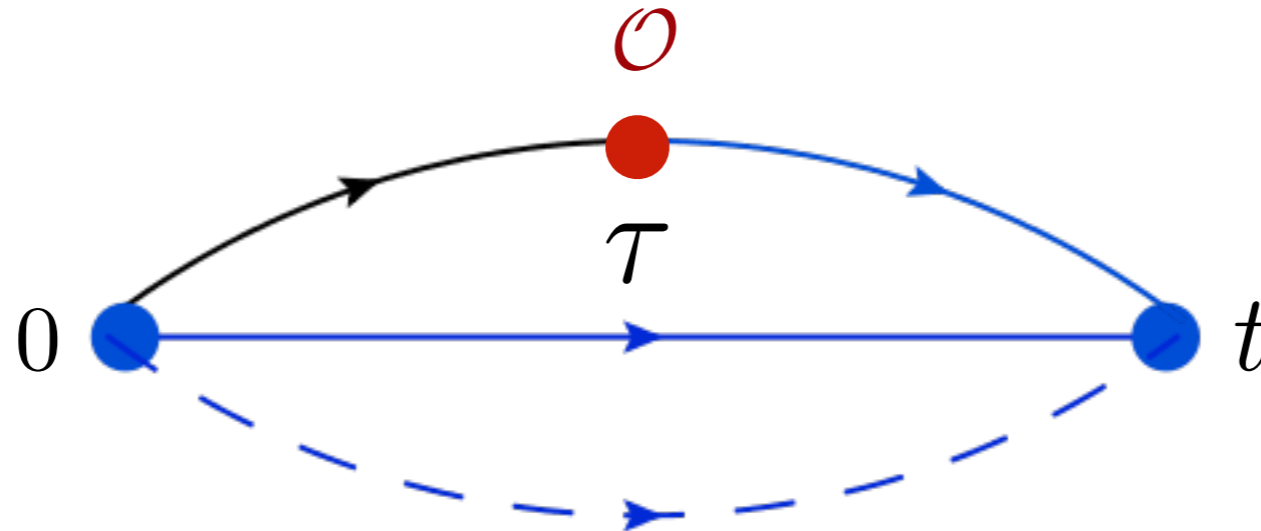
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- **An understanding of nucleon spin is a major goal of many current and future experimental programs**
- 12 GeV JLab upgrade
- COMPASS (II)
- RHIC
- MAMI (Mainz)
- **Electron-Ion Collider (JLab or BNL)**
- **Dramatic improvement in understanding role of sea quarks and glue**
- **“Missing spin” provided by gluons**
- **High-energy probes of partons’ transverse momenta - contribution from orbital motion**

# Lattice Hadron Structure

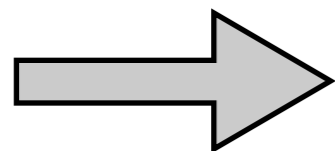
# Lattice 3pt function

**Most common method for determining matrix elements relevant for hadron structure calculations - 3pt function**

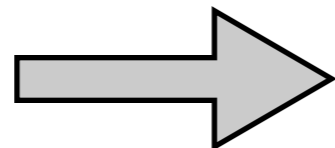


$$G(t, \tau, \vec{p}, \vec{p}') = \sum_{s, s'} e^{-E_{\vec{p}'}(t-\tau)} e^{-E_{\vec{p}}\tau} \Gamma_{\beta\alpha} \langle \Omega | \chi_{\alpha}(0) | N(p', s') \rangle \langle N(p', s') | \mathcal{O}(\vec{q}) | N(p, s) \rangle \langle N(p, s) | \bar{\chi}_{\beta}(0) | \Omega \rangle$$

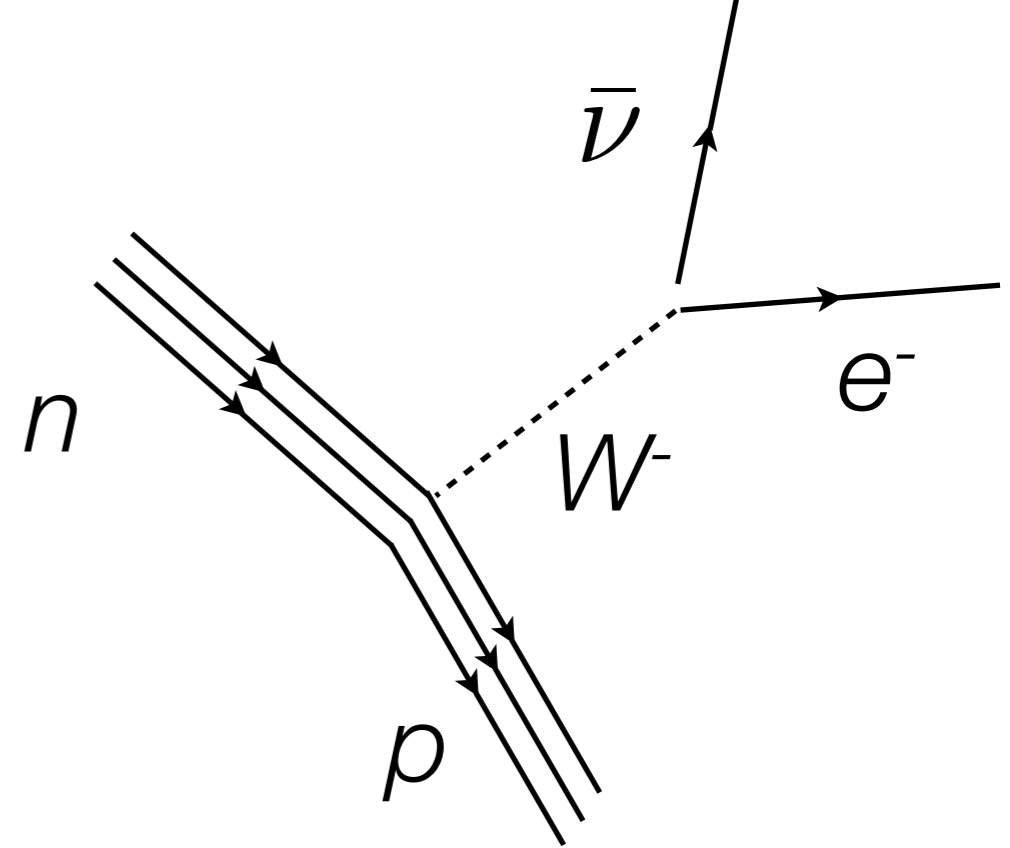
**For large times**  $1 \ll \tau$   $1 \ll t - \tau$



**Extract matrix element**



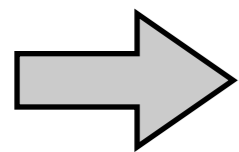
**Determine form factors, charges, moments, ...**



## Nucleon axial charge

$$g_A^{\text{exp}} = 1.2701(25)$$

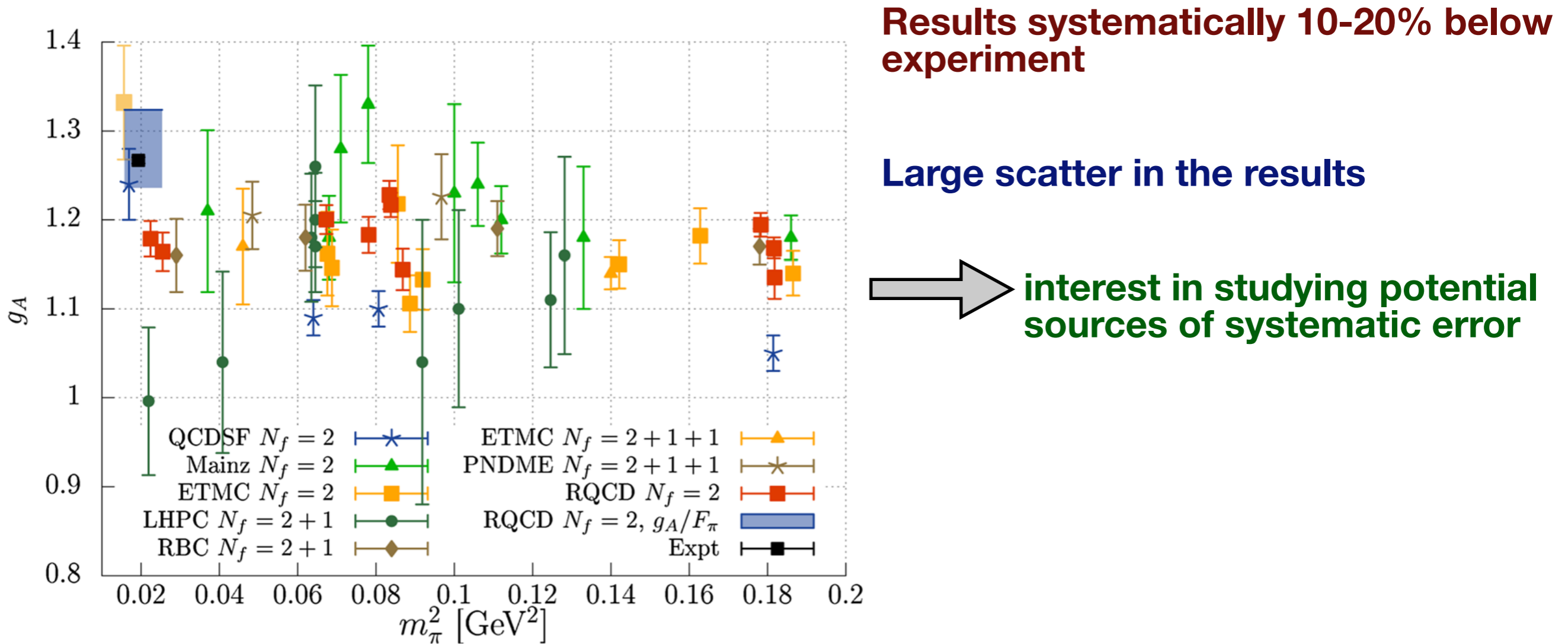
**Relatively simple to compute on the lattice (p=q=0, isovector)**



**Good benchmark for hadron structure (understanding systematic errors)**

# Determination of $g_A$ on the Lattice

## Summary plot: RQCD PRD91 (2015) 054501





# Determination of $g_A$ on the Lattice

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Thorough investigations of

**Lattice spacing**

**Quark mass**

**Finite volume**

**Contamination from excited states**

**Improved axial current**

[Dragos, Tue 17:30]

[Gupta, Thu 10:40]

[Harris, Sat 10am]

[Ohta, Wed 18:10]

[von Hippel, Sat 10:40]

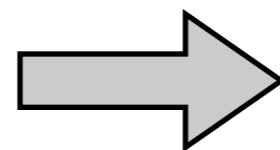
[Yamanaka, Wed 17:10]

[Yamazaki, Wed 14:20 (Had Spec & Int)]

[Schiller, Sat 10:20]

$g_A$  appears to be very sensitive to Lattice systematics

Lots of effort in reducing systematic errors



flow on for other quantities

# Determination of $g_A$ on the Lattice

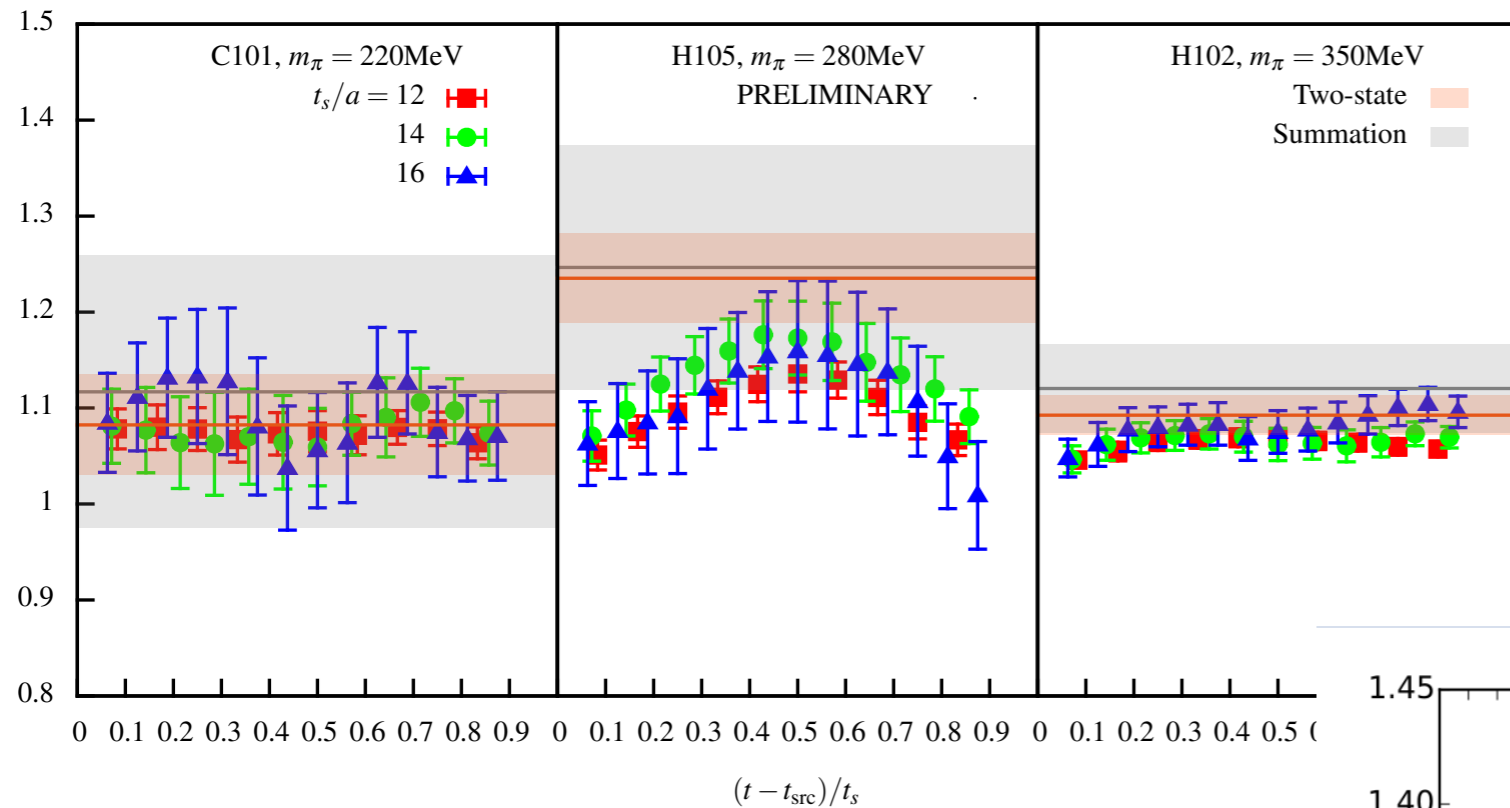
## Excited state contamination

$g_A, N_f = 2 + 1$  O( $a$ )-improved Wilson,  $a \approx 0.086\text{fm}$

Mainz (Harris, Sat 10:00)

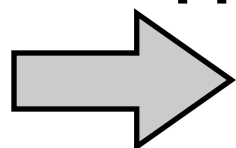
Multiple tsink

Summation and 2-exponential fits

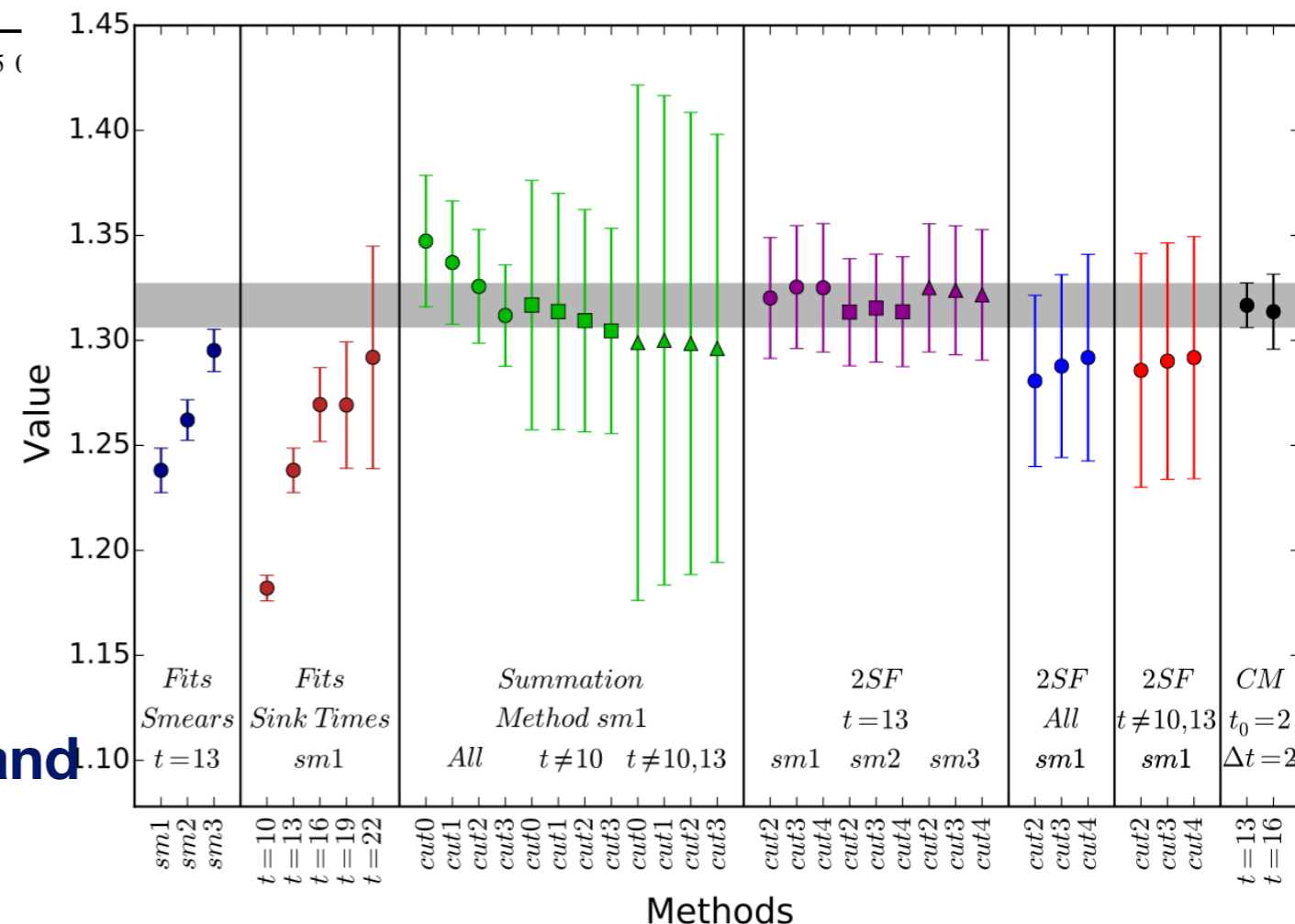


CSSM/QCDSF (Dragos, Tue 17:30)

Comparison of various methods for suppressing excited states



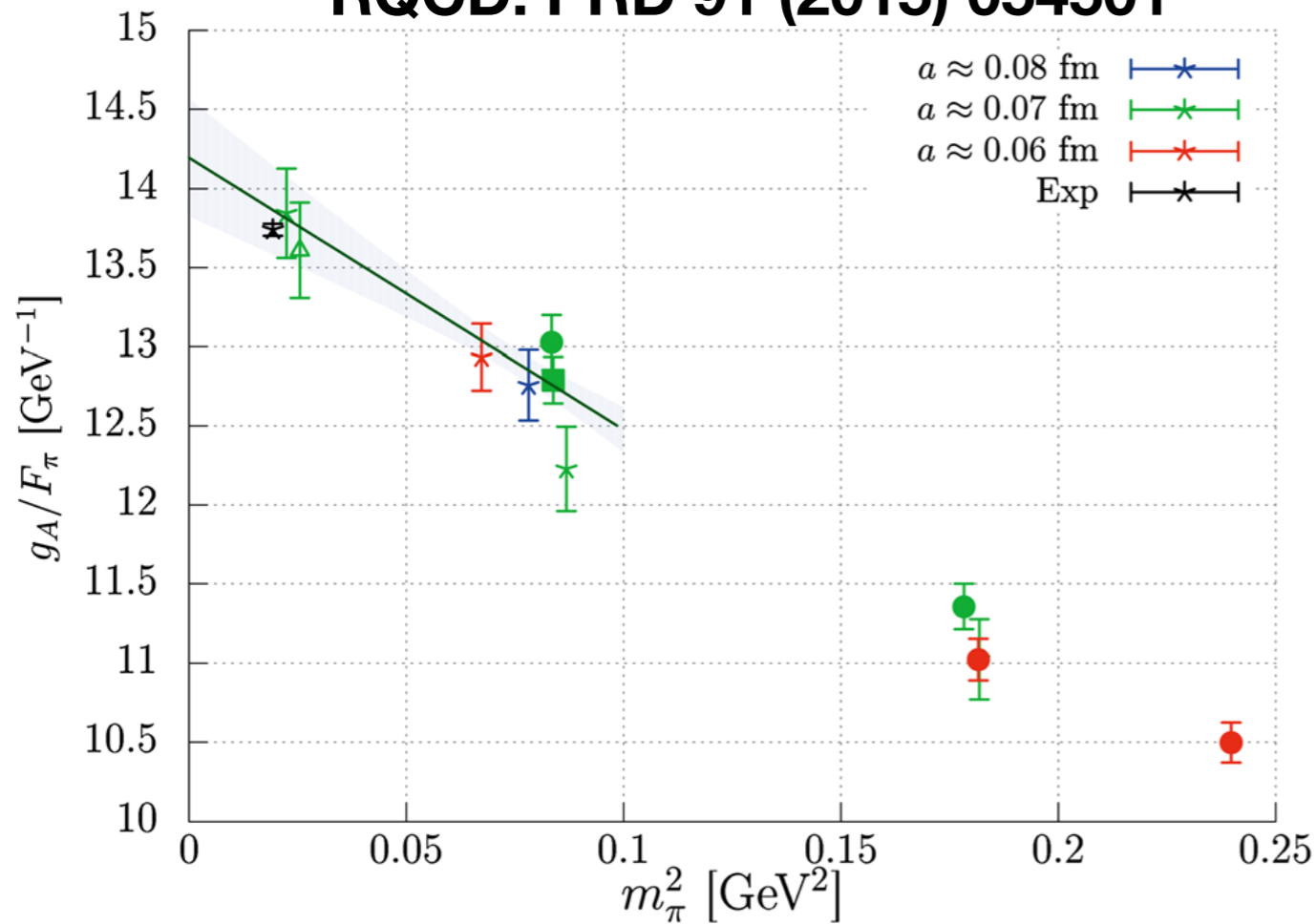
Small suppression of  $g_A$  for small times and poor operator choice



# Determination of $g_A$ on the Lattice

Renormalisation free ratio

RQCD: PRD 91 (2015) 054501



QCDSF: PLB 732 (2014) 41

$g_A/f_\pi$  Ratio extrapolation agrees with exp

FV effects cancel in HBChPT

# Spin of the Proton

**How is the spin of the proton distributed between its constituents?**

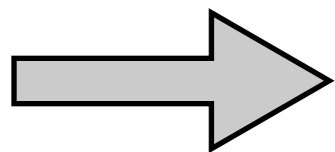
**Recall Ji's sum rule:**  $\frac{1}{2} = \sum_q J_q(\mu^2) + J_g(\mu^2)$

**Express in terms of moments of Generalised Parton Distributions**

$$J_{q/g} = \frac{1}{2} \left[ A_{20}^{q/g}(q^2 = 0) + B_{20}^{q/g}(q^2 = 0) \right]$$

**which are obtained from the matrix elements of the energy momentum tensor**

$$\langle P' | T^{\mu\nu} | P \rangle = \bar{U}(P') \left\{ \gamma^\mu \bar{P}^\nu A_{20}(q^2) + \frac{i\sigma^{\mu\rho} q_\rho \bar{P}^\nu}{2m_N} B_{20}(q^2) + \frac{q^\mu q^\nu}{m_N} C_{20}(q^2) \right\} U(P)$$

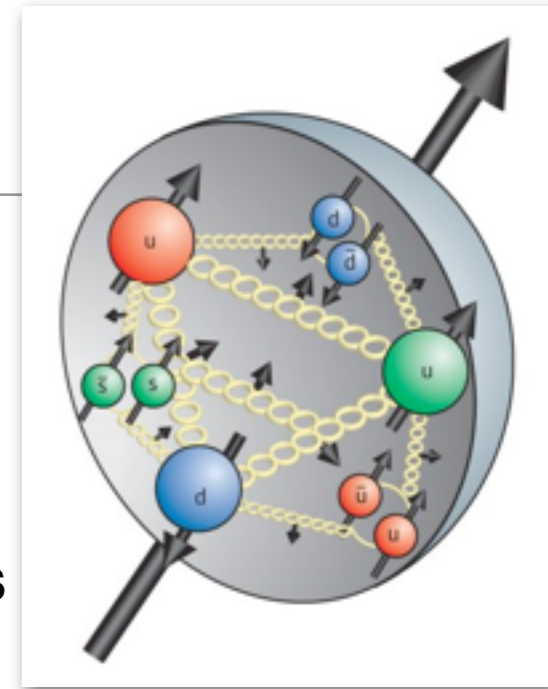


**Determine the quark orbital angular momentum**

$$L_q = \frac{1}{2} \left[ A_{20}^q(q^2 = 0) + B_{20}^q(q^2 = 0) - \Delta q \right]$$

**But**

$$\langle x \rangle^{q/g} = A_{20}^{q/g}(q^2 = 0)$$

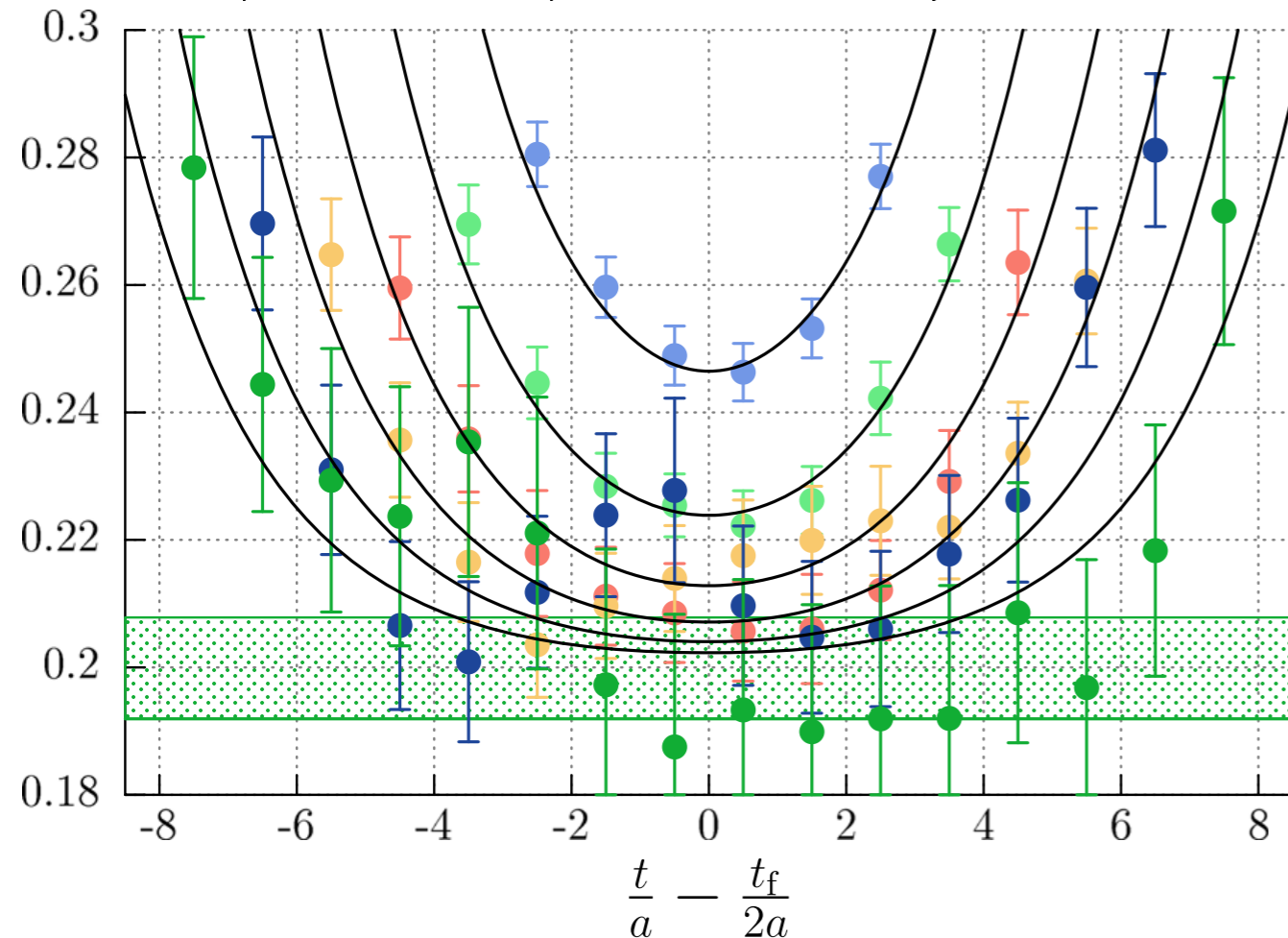


# Quark Momentum Fraction

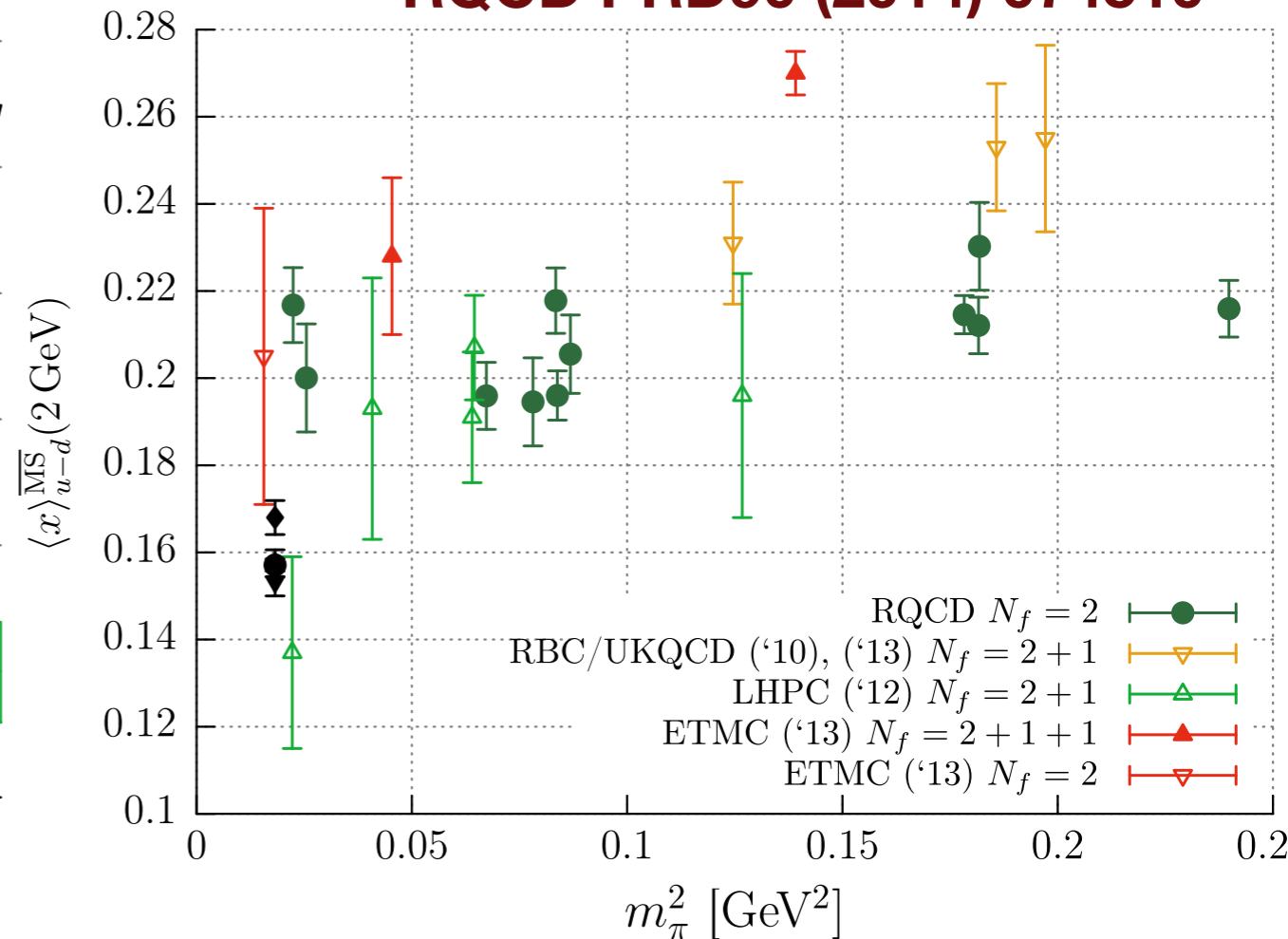
$$\langle x \rangle_q = \int_0^1 dx x (q(x) + \bar{q}(x))$$

- **First moment of the (isovector) nucleon parton distribution function**
- **Notorious for producing lattice results  $\approx 2x$  too large for isovector nucleon**
- **Known to be sensitive to excited state contamination**
- **Results near physical mass inconclusive**

IV,  $a \sim 0.07$  fm,  $m_\pi \sim 295$  MeV,  $Lm_\pi = 3.42$



**RQCD PRD90 (2014) 074510**



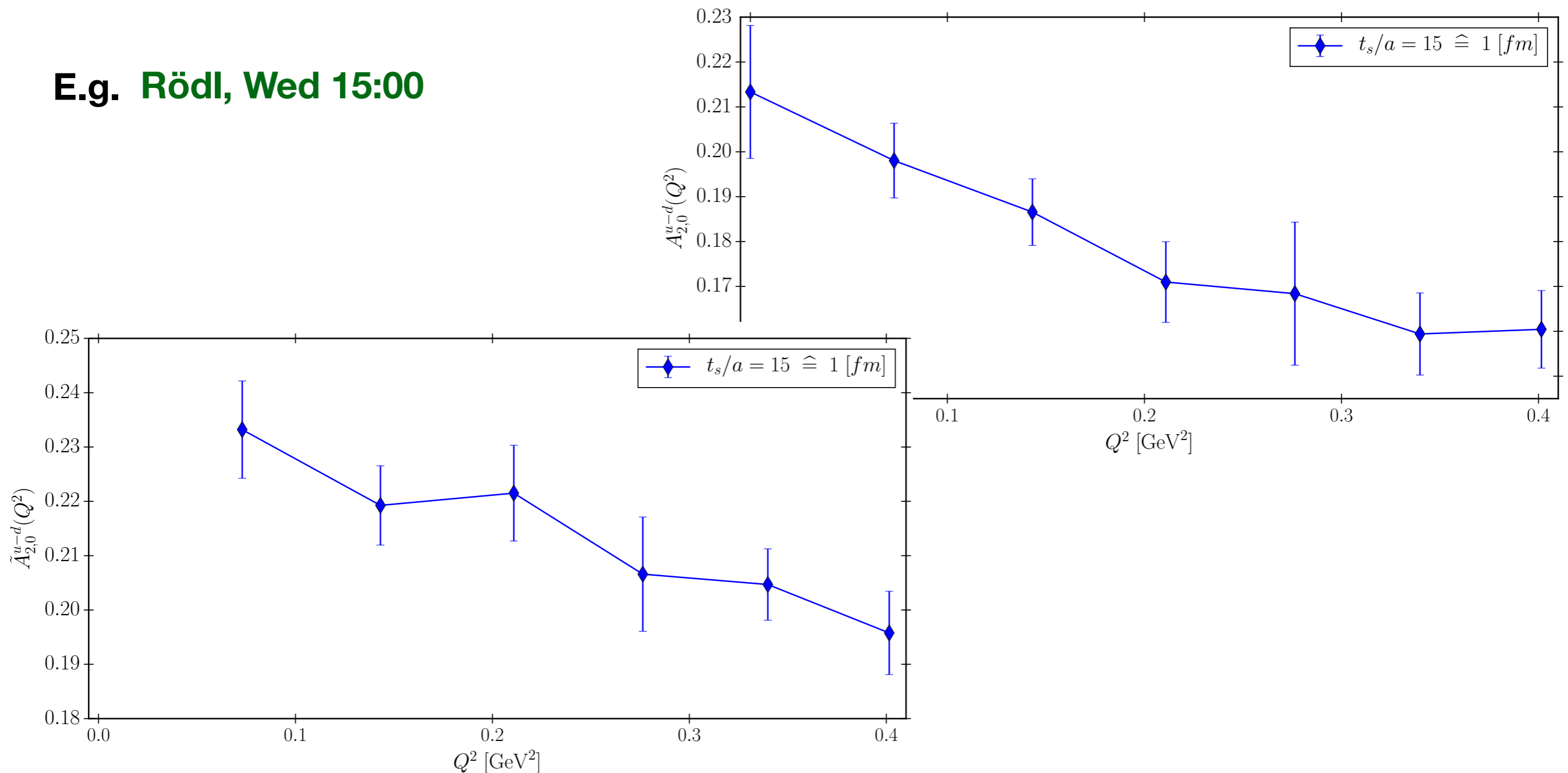
# Spin of the Proton

## Ji's sum rule

Recall Ji's sum rule requires moments of Generalised Parton Distributions

$$J_{q/g} = \frac{1}{2} \left[ \langle x \rangle^{q/g} + B_{20}^{q/g}(q^2 = 0) \right] \quad \langle x \rangle^{q/g} = A_{20}^{q/g}(q^2 = 0)$$

E.g. Rödl, Wed 15:00

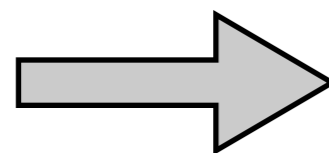


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$$L_q = \frac{1}{2} \left[ \langle x \rangle^q + B_{20}^q(q^2 = 0) - \Delta q \right]$$

For reliable results for  $L_q$



need to control systematic errors for  $g_A$  and  $\langle x \rangle^{u-d}$



disconnected contributions for:

$$\Delta q$$

$$[g_A = \Delta u - \Delta d]$$

$$\langle x \rangle^q$$

# Spin of the Proton

$$\Delta s = \int_0^1 dx [\Delta s(x) + \Delta \bar{s}(x)]$$

$\Delta s$  is a purely quark-line disconnected contribution

A challenge on the lattice

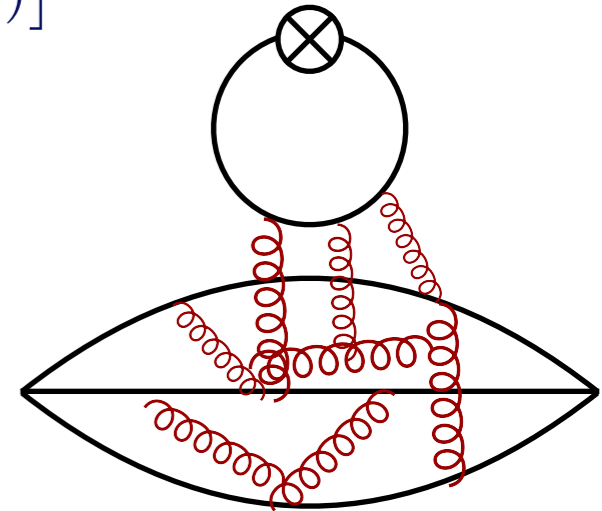
Standard procedure: Use stochastic (random noise) sources

e.g.

[QCDSF/RQCD PRL108 (2012) 222001]

$$m_\pi = 285 \text{ MeV}$$
$$\overline{\text{MS}} \quad \mu = \sqrt{7.4} \text{ GeV}$$

$$\Delta s = -0.020(10)(4)$$





# Spin of the Proton

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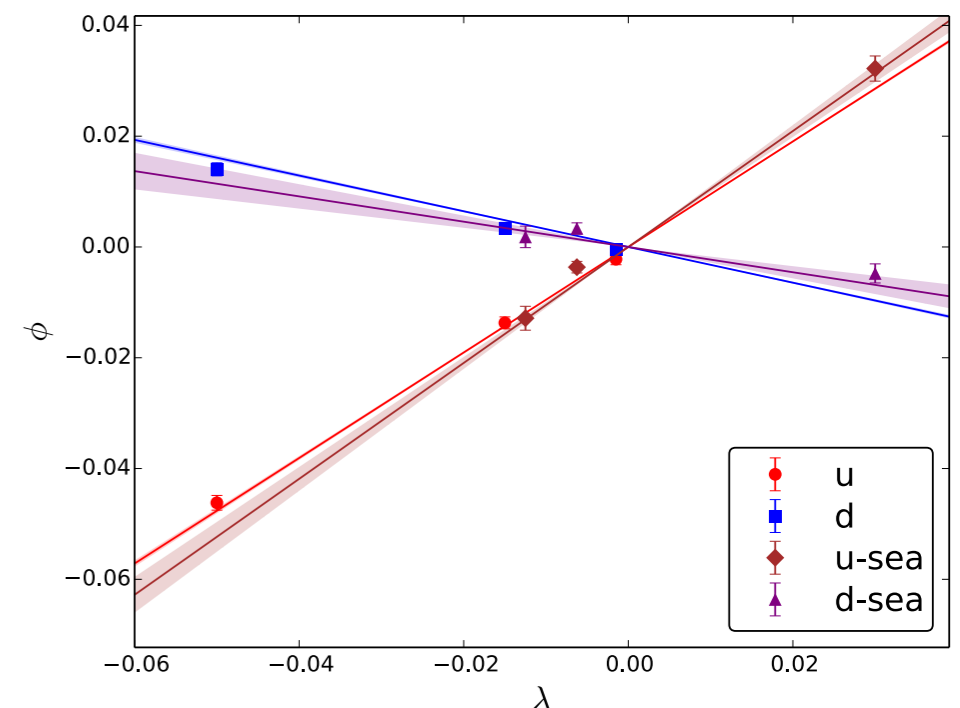
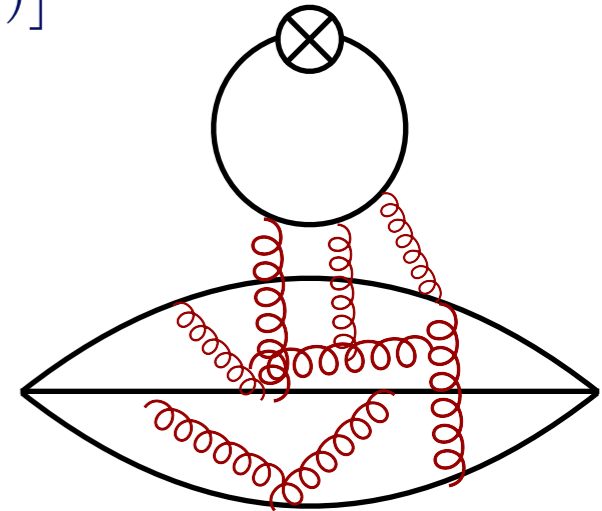
$$m_\pi = 285 \text{ MeV} \quad \Delta s = -0.020(10)$$

$$\overline{\text{MS}} \quad \mu = \sqrt{7.4} \text{ GeV}$$

An alternative: Feynman-Hellmann method

[CSSM/QCDSF PRD90 (2014) 014510]

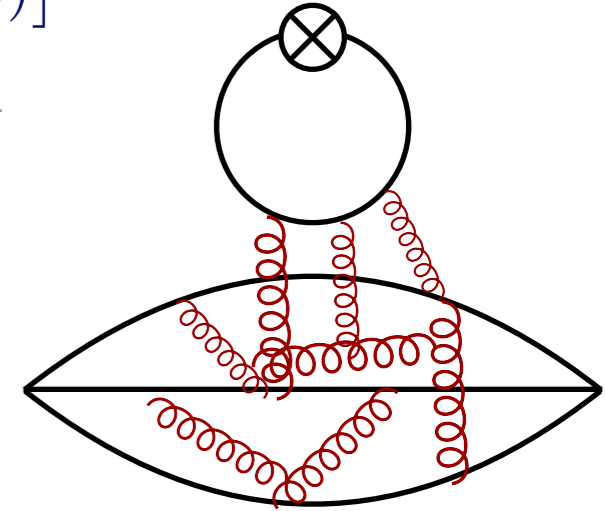
$$\frac{\partial E_H(\lambda)}{\partial \lambda} = \frac{1}{2E_H(\lambda)} \left\langle H \left| \frac{\partial S(\lambda)}{\partial \lambda} \right| H \right\rangle$$



# Spin of the Proton

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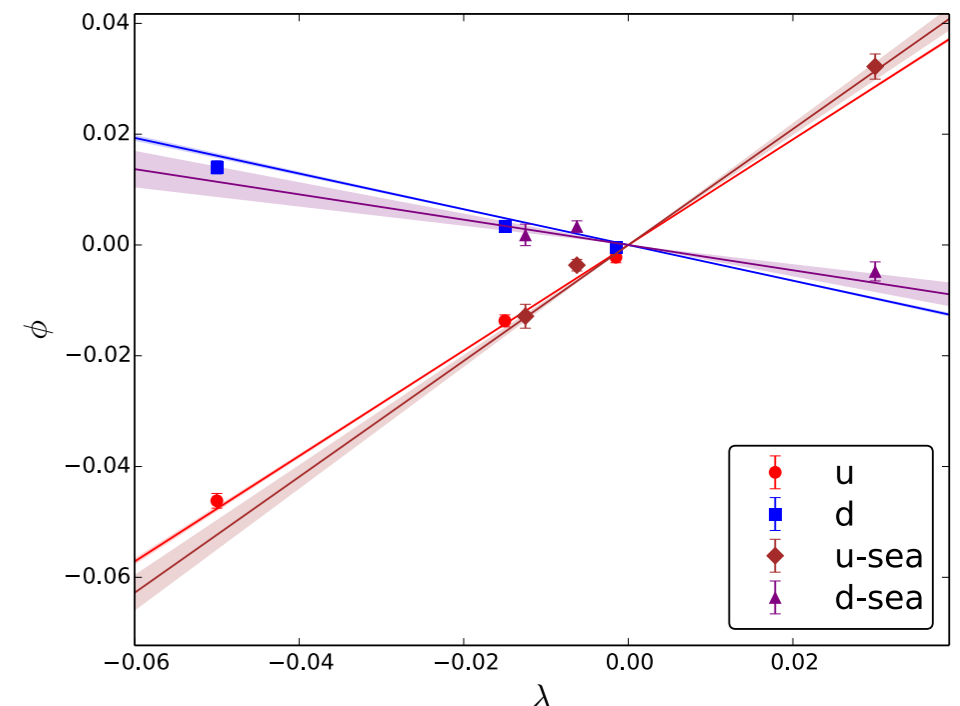
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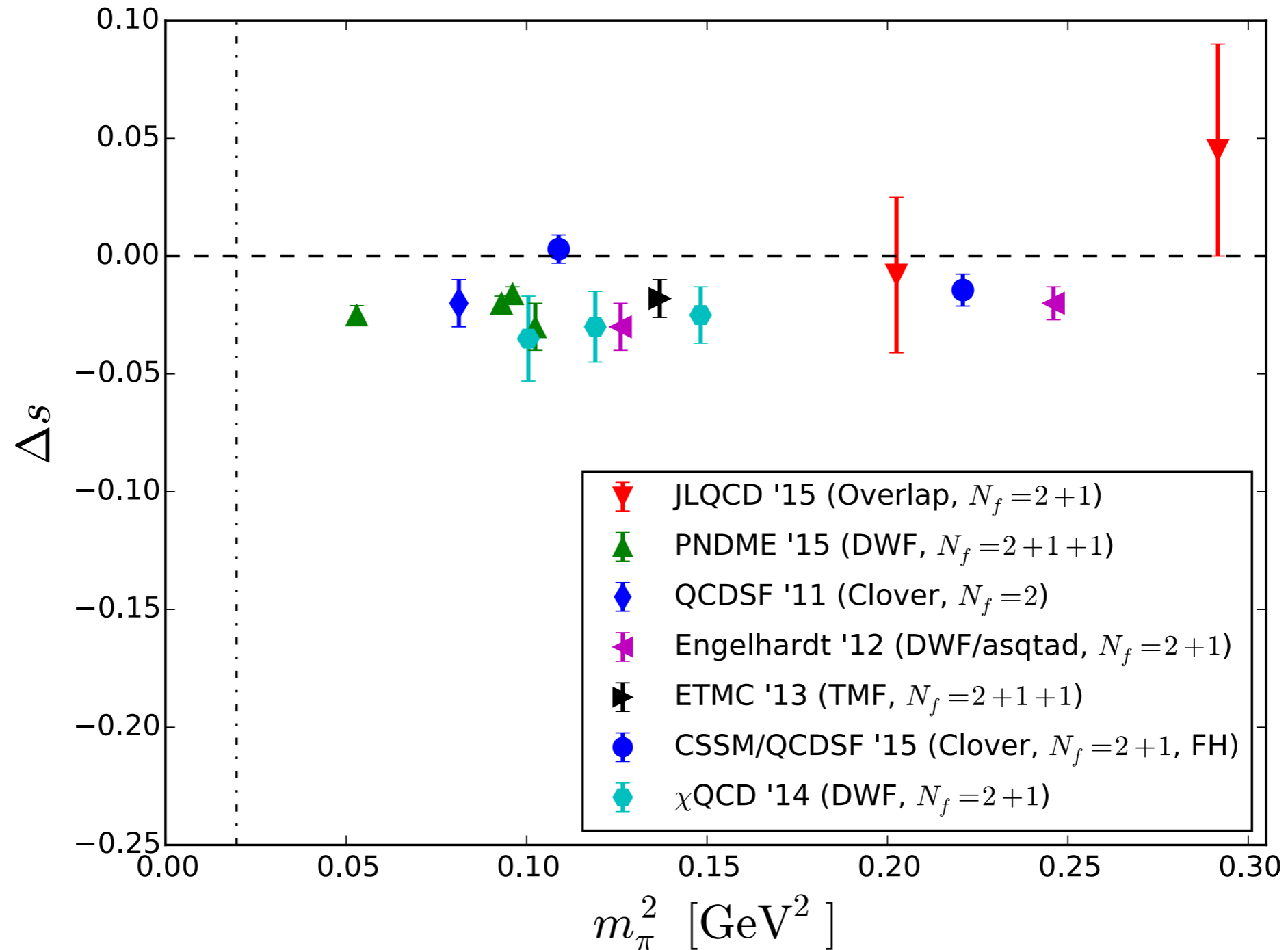
Also allows for full nonperturbative determination of singlet renormalisation constants

$$g_A = Z_A^{\text{NS}} g_A^{\text{latt}}$$

$$\Delta \Sigma = Z_A^{\text{S}} \Delta \Sigma^{\text{latt}}$$

[CSSM/QCDSF, PLB740 (2015) 30]

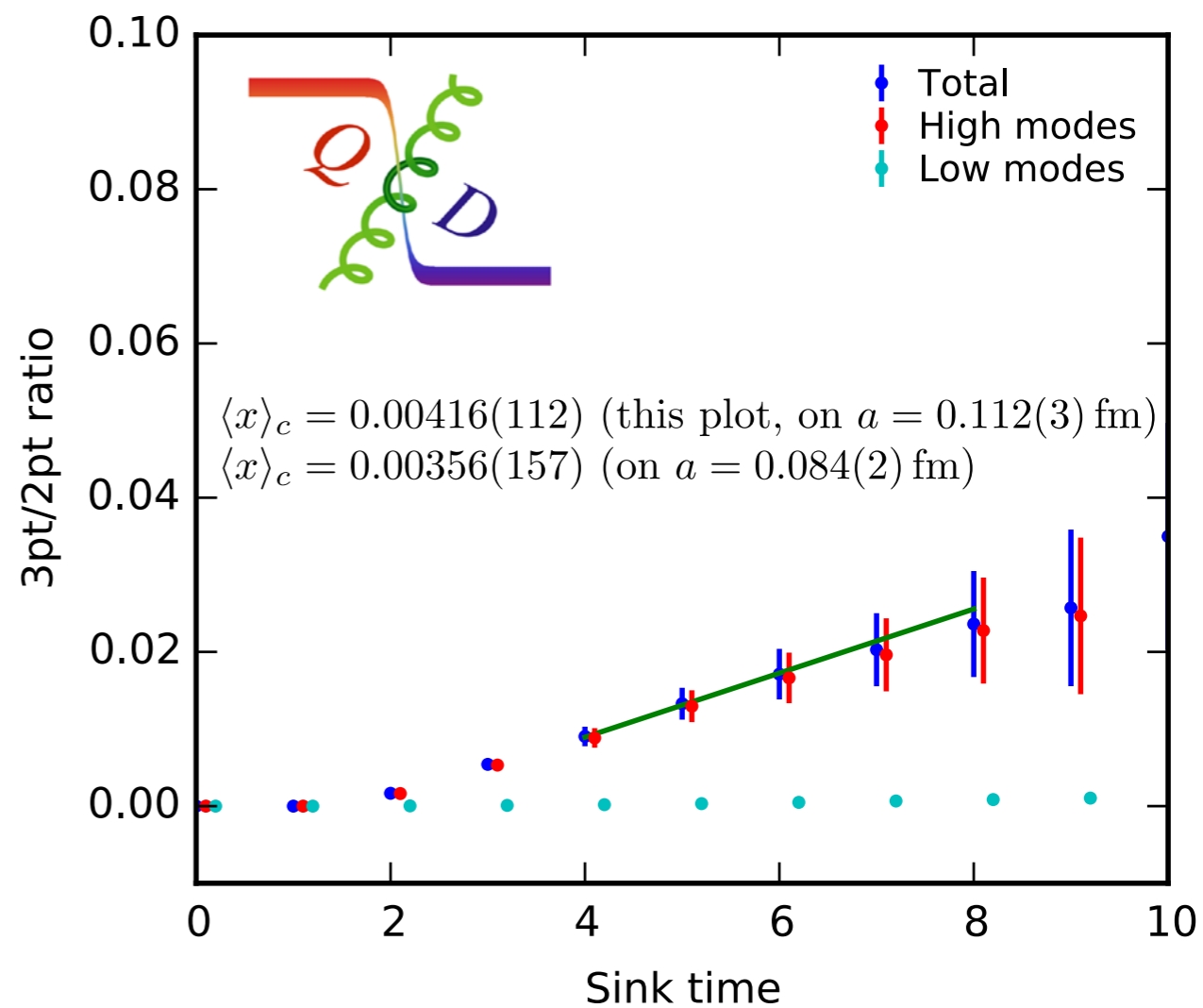
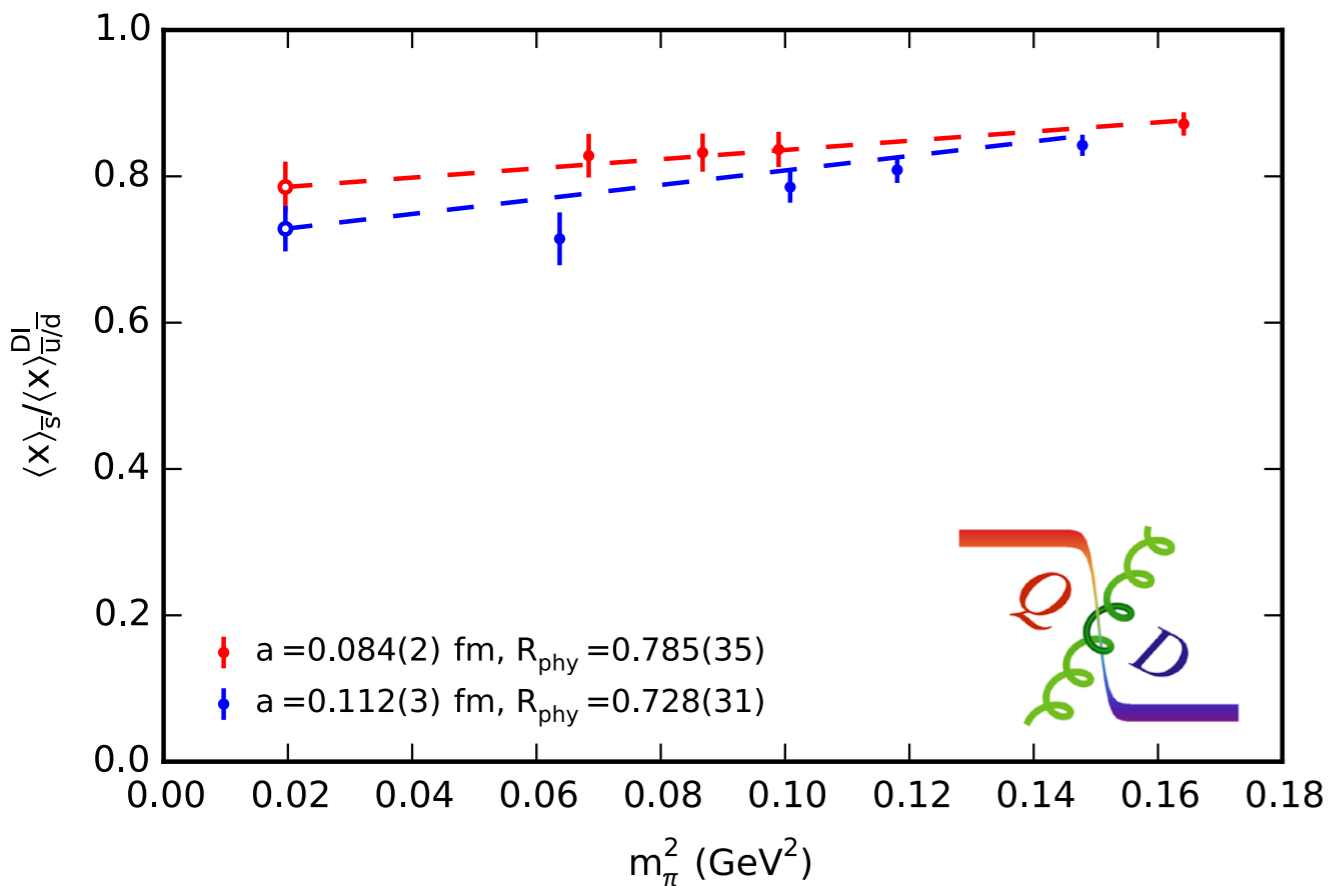
# Disconnected Spin Contributions

 $\Delta s$ 

**favours a small and negative  $\Delta s$**

# Disconnected Contributions

$$\langle x \rangle^q$$



**non-zero charm contribution**

# Glue contribution to proton spin

Yang Thu 9:50am

Ji et al. [PRL 111 (2013) 112002] proposed a glue spin density operator

$$\vec{S}_G = \int d^3x \text{Tr} [\vec{E}_C \times \vec{A}_C] \quad (\text{gauge-invariant})$$

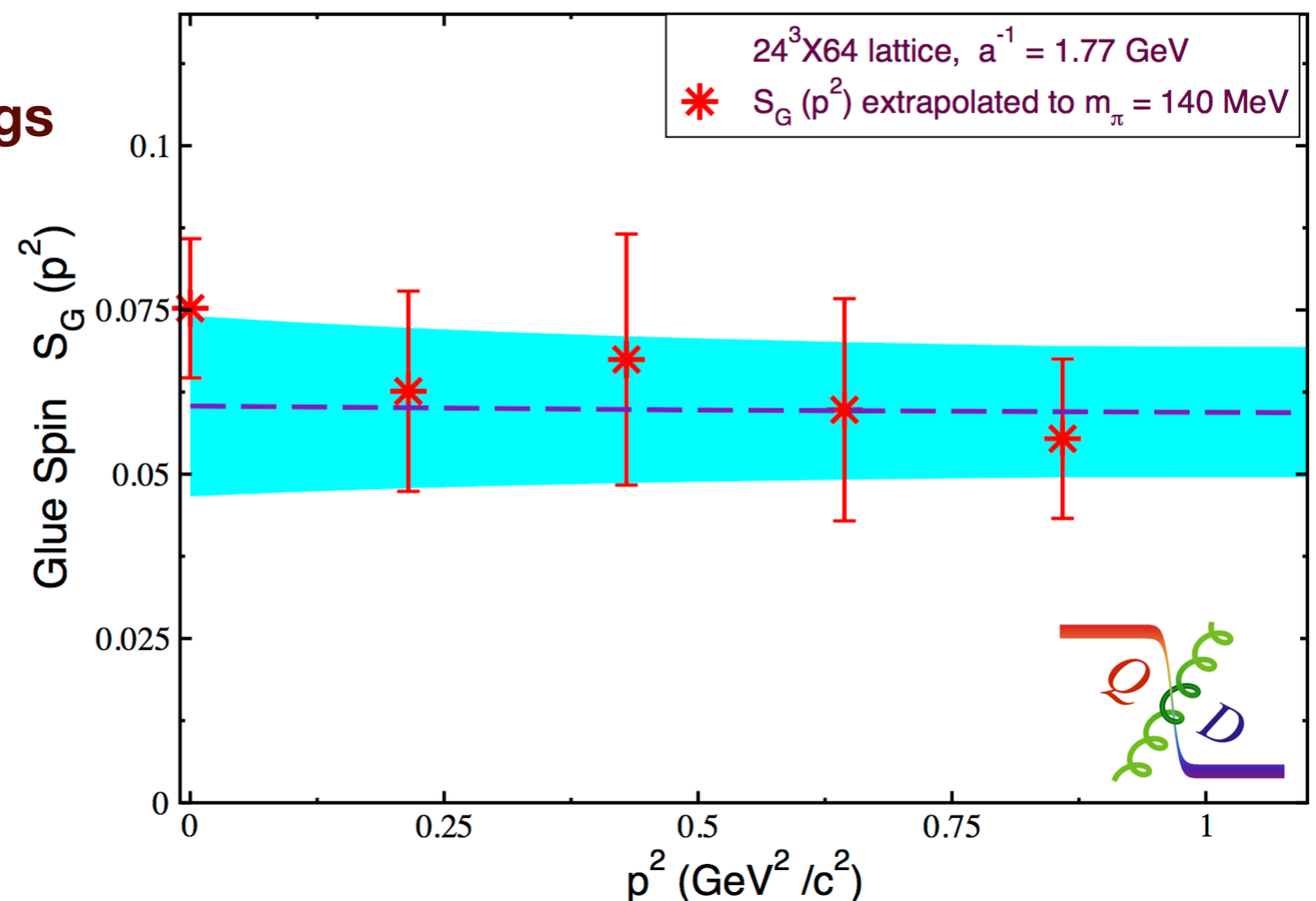


RBC/UKQCD 2+1 24<sup>3</sup>x48 DWF configs

Overlap definition of  $F_{\mu\nu}$

Extrapolated to  $m_\pi^{\text{phys}}$

Renormalisation soon!



# Tensor Charge, $g_T = \int dx [\delta u(x) - \delta d(x)]$

---

Recent interest driven by

Phenomenological determinations now available using **SIDIS data** (HERMES, COMPASS)

Implications for new physics

Precision neutron  $\beta$  decay studies sensitive to possible BSM scalar and tensor interactions  
**[PNDME PRD85 (2012) 054512]**

 **Require neutron matrix elements of appropriate low-energy operators**

i.e.  $g_T^{u-d}$ ,  $g_S^{u-d}$  (to  $\sim 10\%$ )

Quantify the contribution of the quark EDM to the nEDM and set bounds on new sources of CP violation  
**[PNDME 1506.04196, 1506.06411]**

$$d_n = d_u g_T^u + d_d g_T^d + d_s g_T^s$$

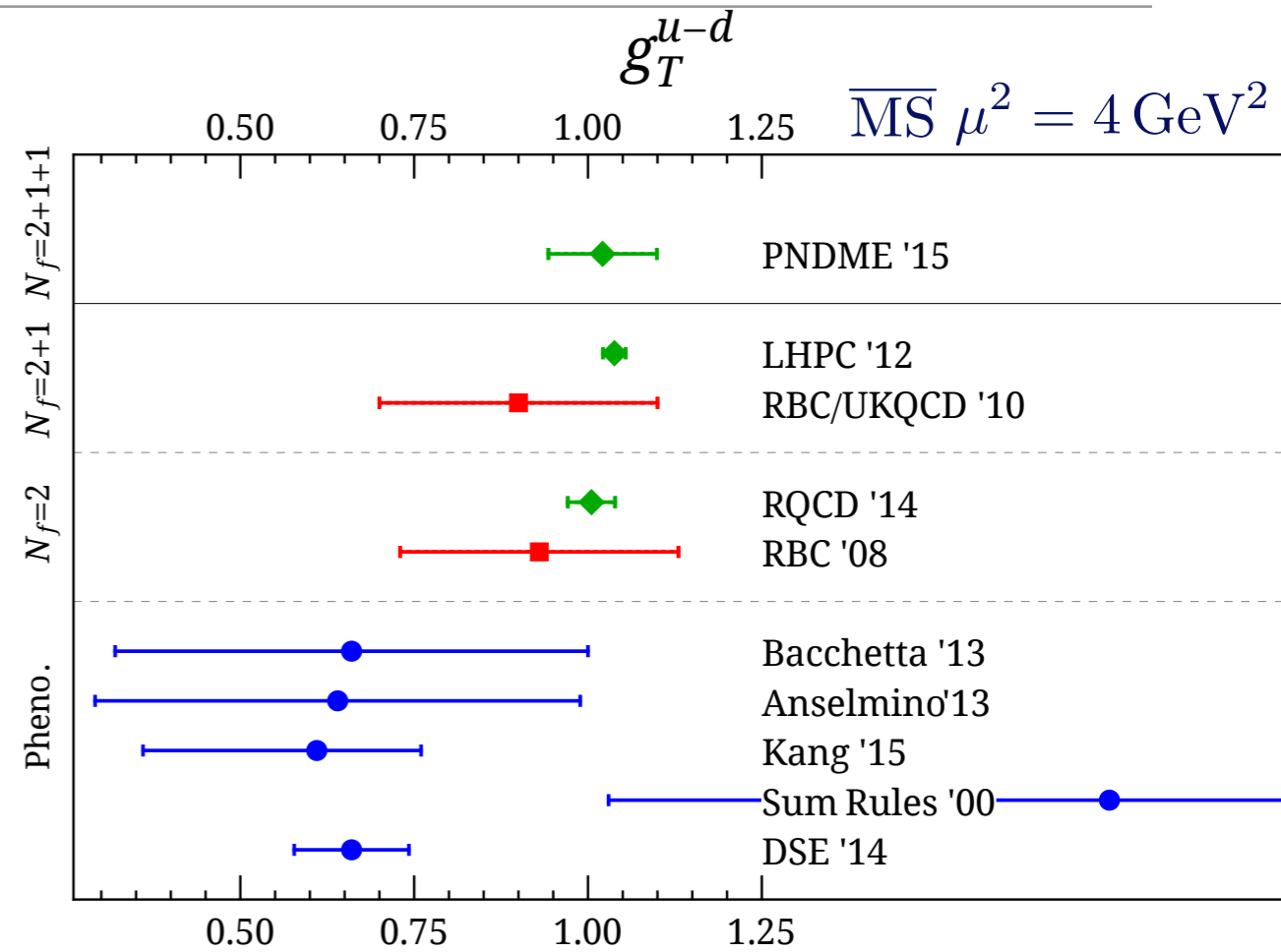
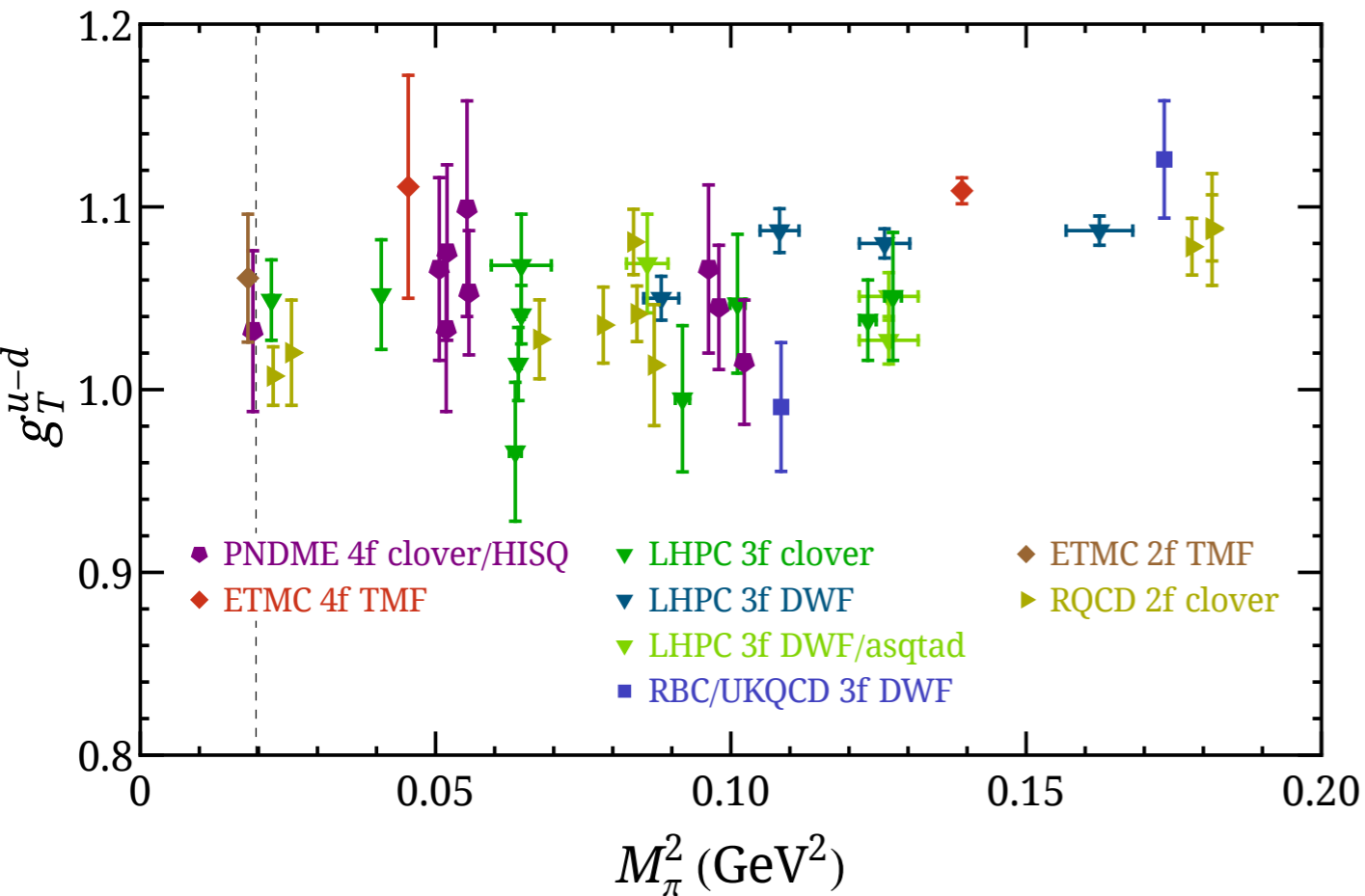
(including disconnected)

 **See plenary talk by T. Izubuchi (Tue 11:45)**

# Tensor Charge, $g_T = \int dx [\delta u(x) - \delta d(x)]$

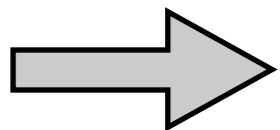
[Gupta, Thu 10:40]

## PNDME 1506.06411



**Comprehensive study of systematics by PNDME**

**Introduced 'FLAG-like' colour-coding system for each systematic error**



**10% uncertainty target achieved**

# Scalar Charge, $g_S$

$$\langle N | \bar{q}q | N \rangle$$

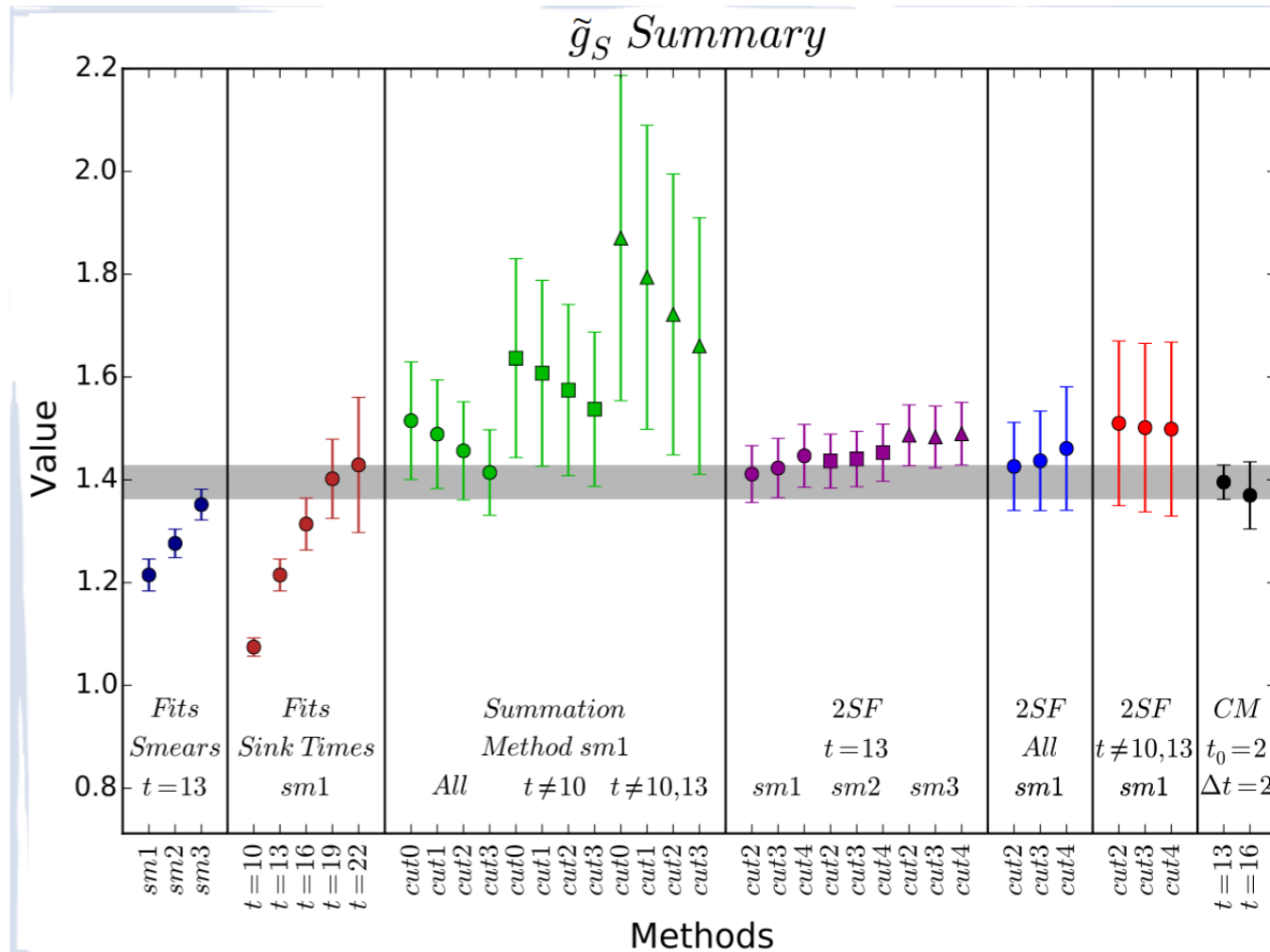
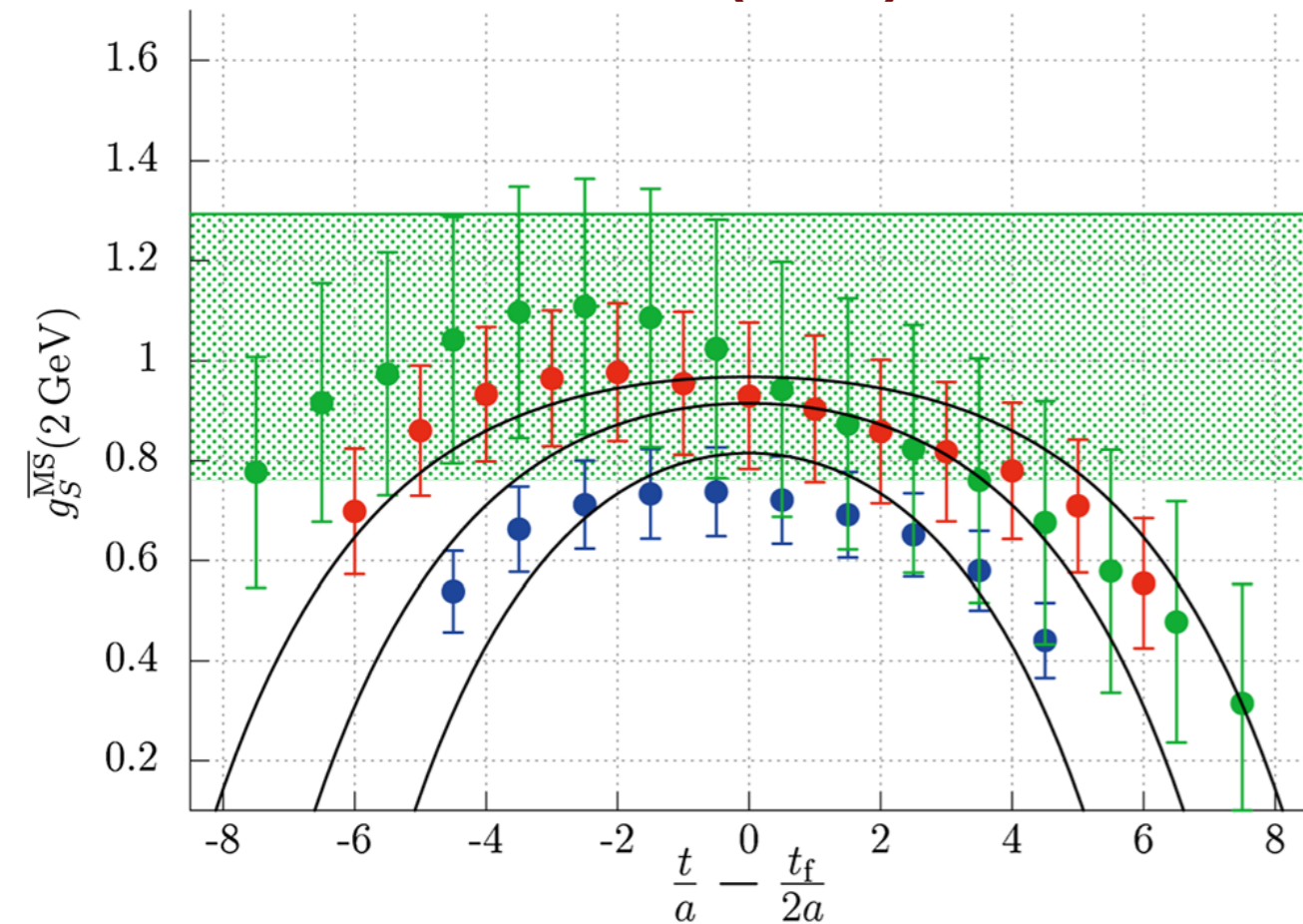
**Commonly used for  $\sigma$  terms**  $\sigma_l^H = m_l \langle H | (\bar{u}u + \bar{d}d) | H \rangle$        $\sigma_s^H = m_s \langle H | \bar{s}s | H \rangle$   
 [R.Young, Latt'12 review]

**Recent interest: new physics contributions to  $\beta$  decay:  $g_S^{u-d}$  to 10%**  
 [PNDME PRD85 (2012) 054512]

**Severe excited state contamination - take care!**

**CSSM/QCDSF (Dragos, Tue 17:30)**

**RQCD PRD91 (2015) 054501**





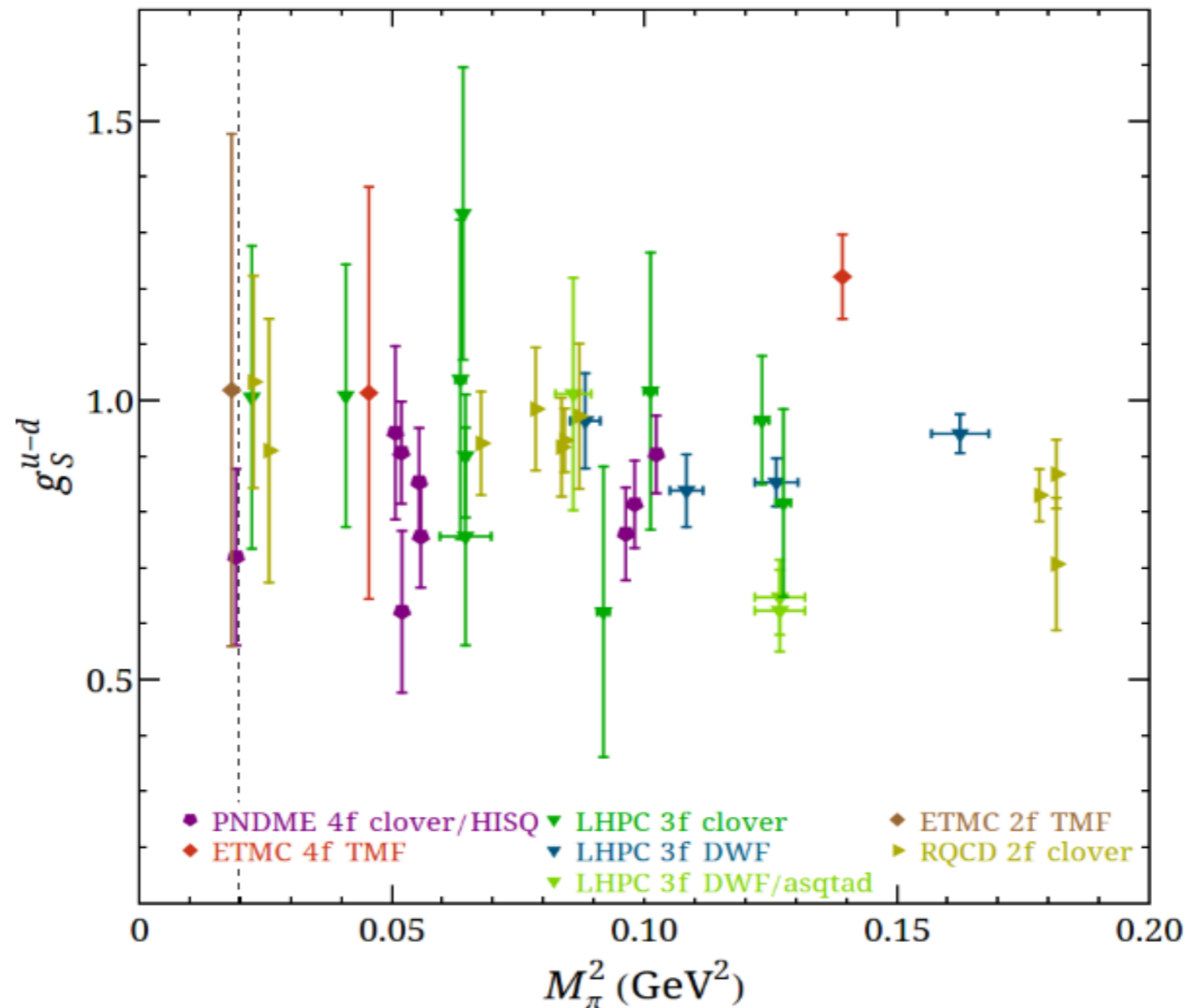
# Scalar Charge, $g_S$

$$\langle N | \bar{q}q | N \rangle$$

Large scatter in lattice determinations

Only determined to ~30%

(Plot from H-W Lin)



# Electromagnetic Form Factors

---

**Several new simulations with near-physical quark masses**

**LHPC [PRD90 (2014) 074507]**  $m_\pi \approx 149 \text{ MeV}$

**Mainz [arXiv:1504.04628]**  $m_\pi \approx 193 \text{ MeV}$

**Shintani, Wed 17:30**

**PACS-CS**  $m_\pi \approx 145 \text{ MeV}$

**Yamazaki, Wed 14:20  
(Had Spec & Int)**

**PNDME**  $m_\pi \approx 130 \text{ MeV}$

**Gupta, Thu 10:40**

**Progress in accounting for excited state contamination**

# Electromagnetic Form Factors

Shintani, Wed 17:30

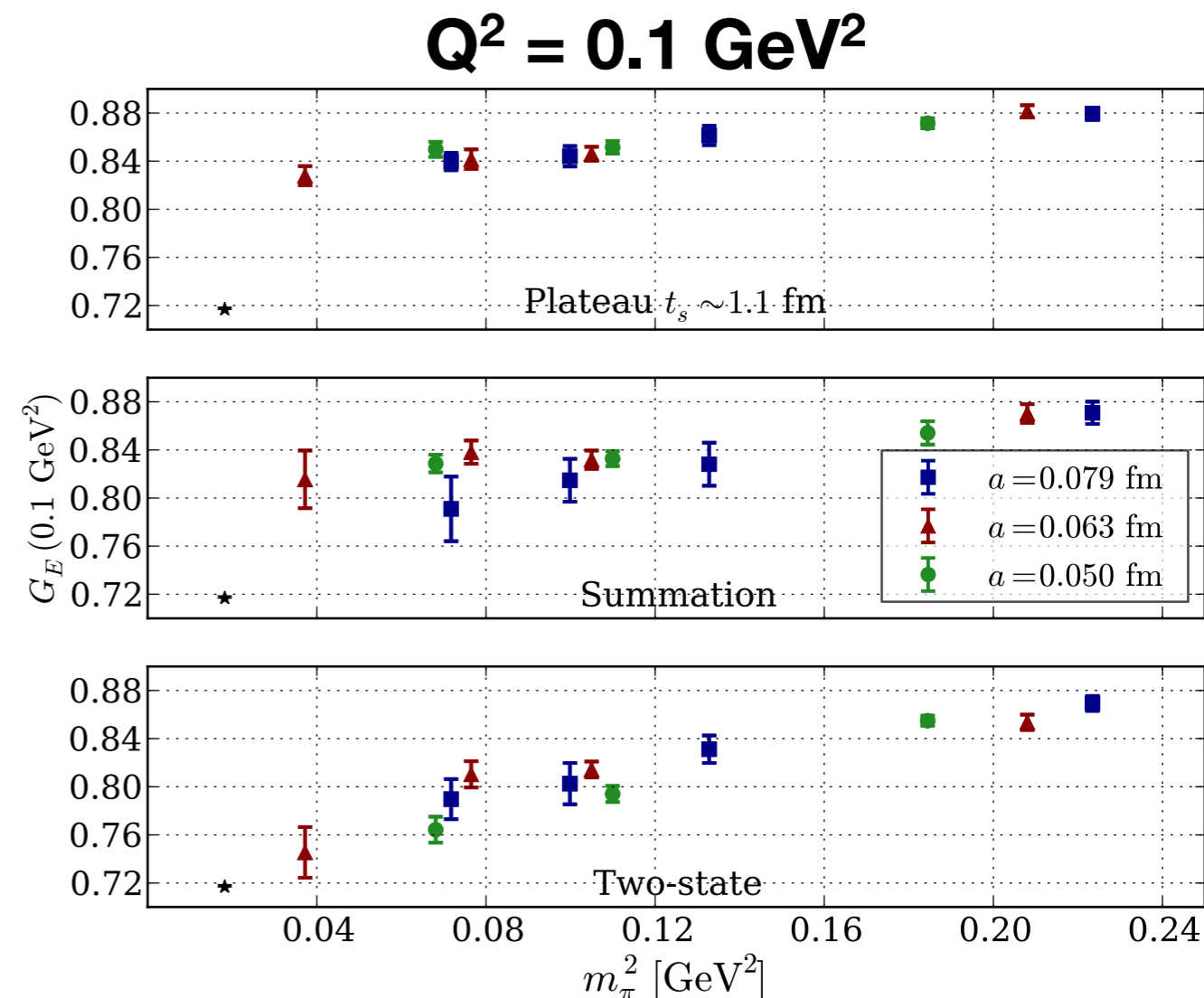
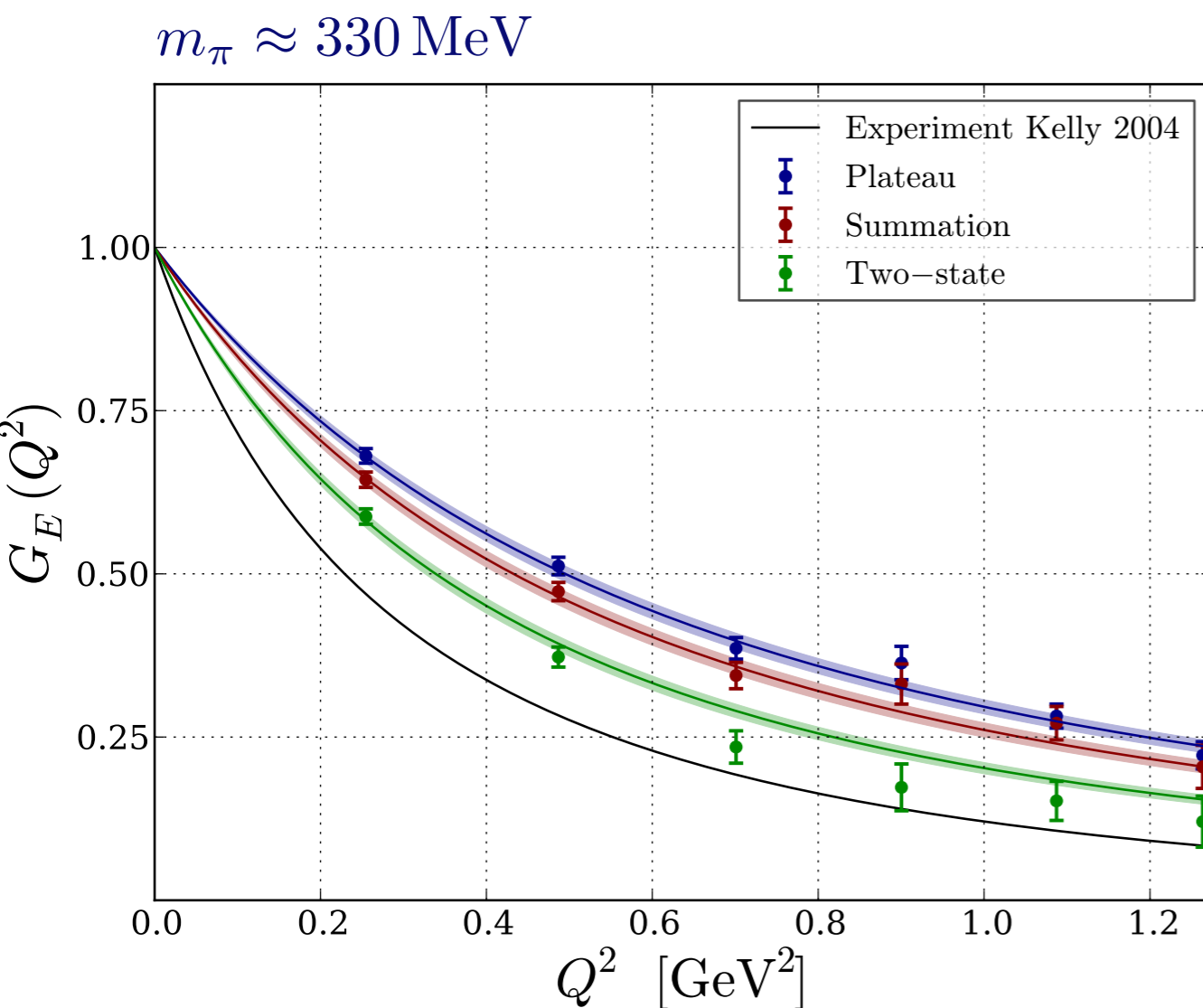
Mainz [arXiv:1504.04628]

Evidence of excited state contamination in EM FFs

Effect becomes more problematic at light quark masses

Effect increases  $G_E$  at each  $Q^2$

Improved approach to physical point



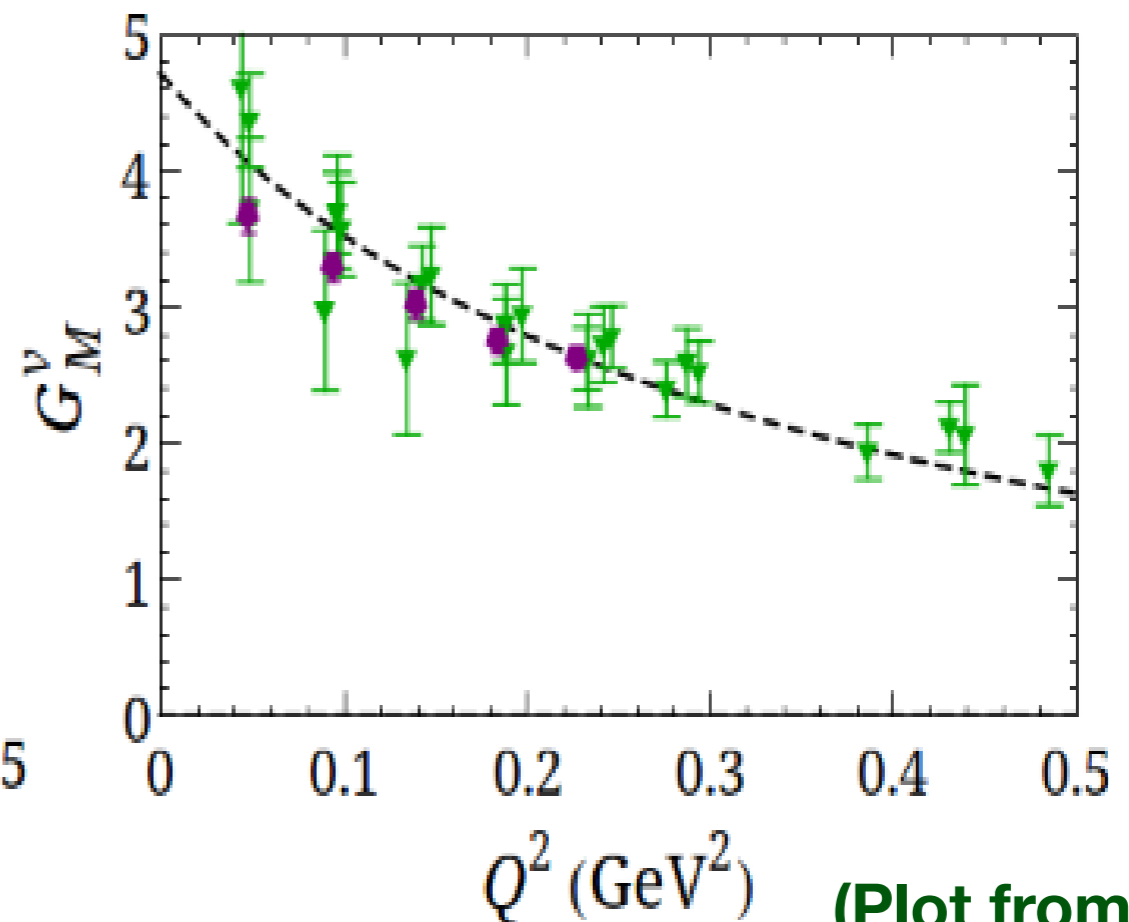
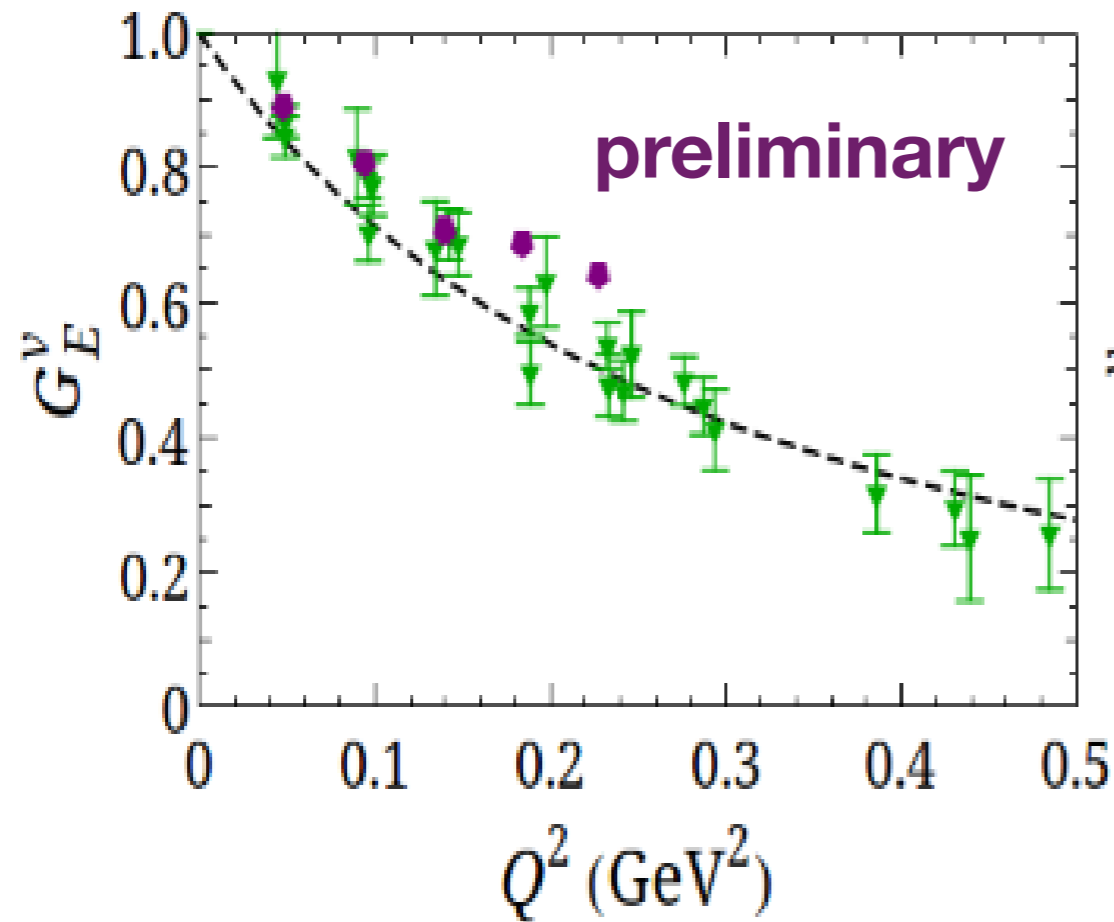
# Electromagnetic Form Factors

Comparison of **LHPC** ( $m_\pi \approx 149$  MeV)

**PRD90 (2014) 074507**

**PNDME** ( $m_\pi \approx 130$  MeV)

**Gupta, Thu 10:40**



(Plot from H-W Lin)

Good agreement with parameterisation of experimental data

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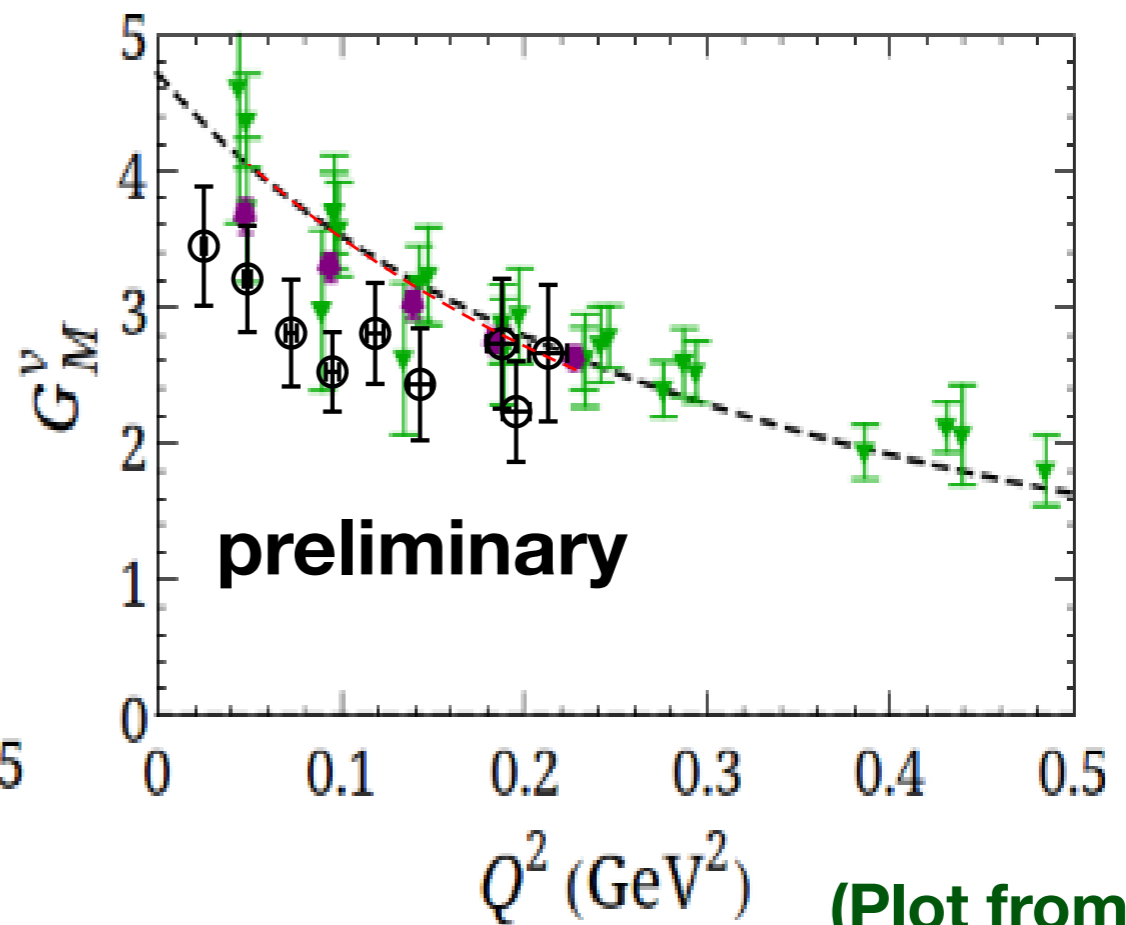
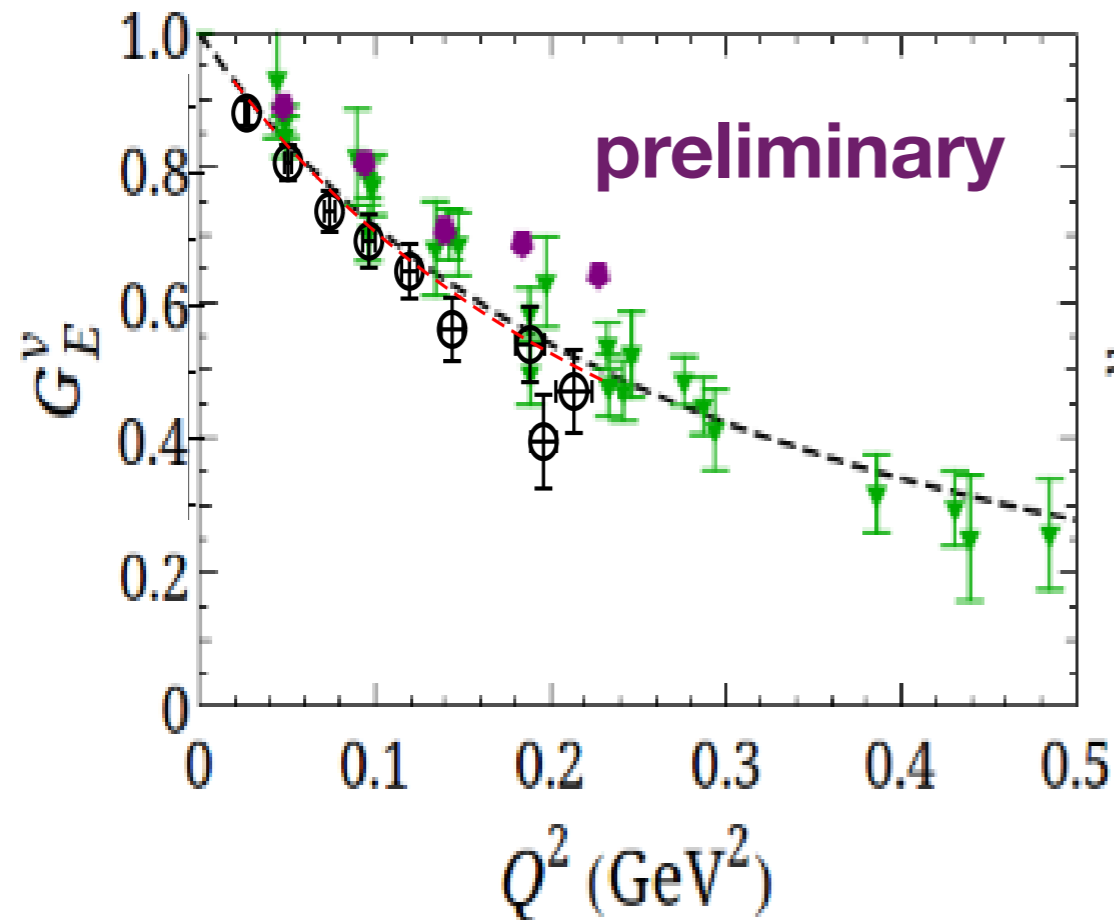
**PNDME** ( $m_\pi \approx 130$  MeV)

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**PRD90 (2014) 074507**

**Gupta, Thu 10:40**

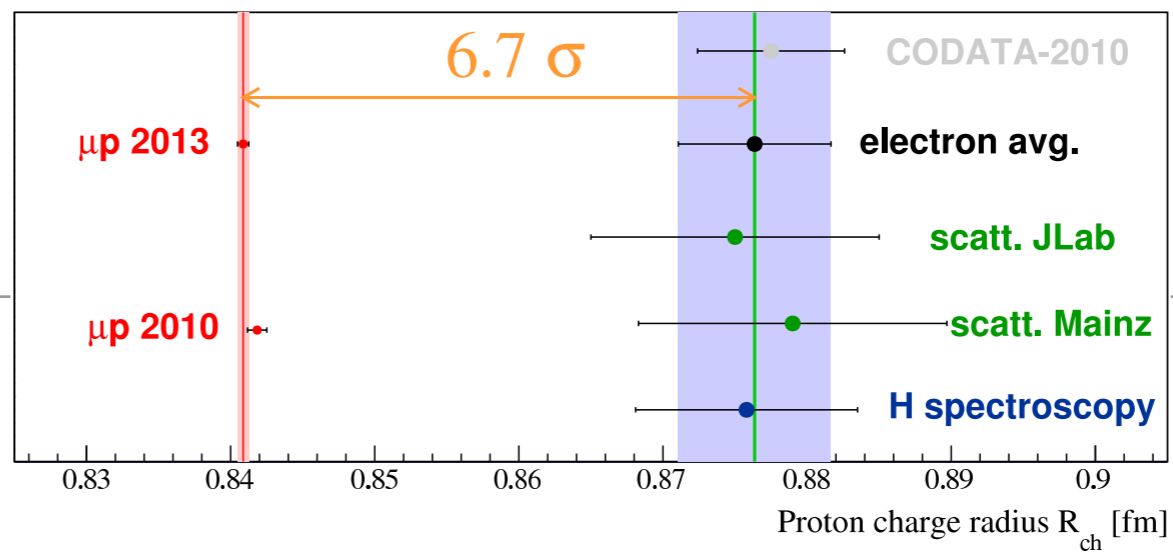
**Yamazaki, Wed 14:20 (Had Spec & Int)**



(Plot from H-W Lin)

**Good agreement with parameterisation of experimental data**

# Charge Radius



**Lattice form factors now available with physical masses**

**(Direct or small chiral extrapolation)**

**Determine accurate radii @  $m_\pi \approx 140$  MeV :**

**Use finite-volume and excited-state corrected results**

**Need small  $Q^2 < 4m_\pi^2 \sim 0.08$  GeV<sup>2</sup> ( $2\pi$  threshold)**

**dipole fit to large  $Q^2$  data introduces unwanted model dependence**

**“z-fit” [Hill & Paz, PRD82 (2010) 113005 ] more reliable**

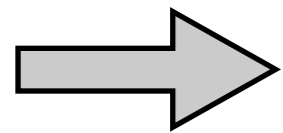
# Large $Q^2$

---

Lattice form factors difficult to determine at large  $Q^2 > 2-3 \text{ GeV}^2$  (even at large  $m_\pi$ )

Standard method:

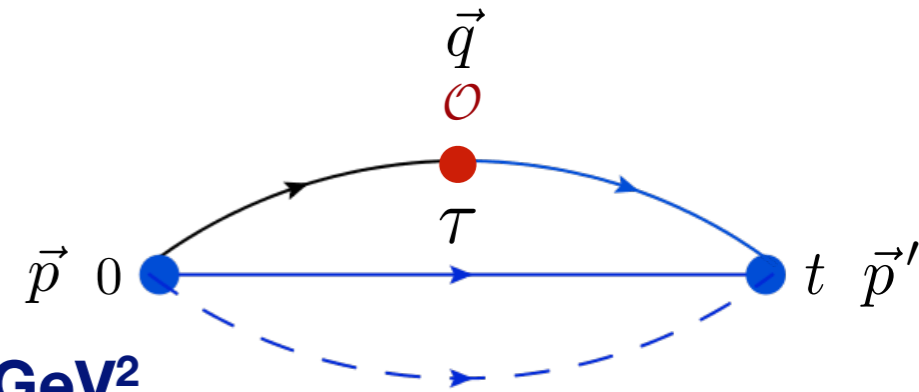
$$\text{fix } \vec{p}' = 0 \quad \text{vary } \vec{q} = -\vec{p}$$



Large nucleon momenta required for  $Q^2 > 2-3 \text{ GeV}^2$

Breit frame  $\vec{p}' = -\vec{p}$  to maximise  $Q^2$  at fixed  $\vec{p}'$ ?

New methods?



# Large $Q^2$

[R. Young, Thu 8:30am]

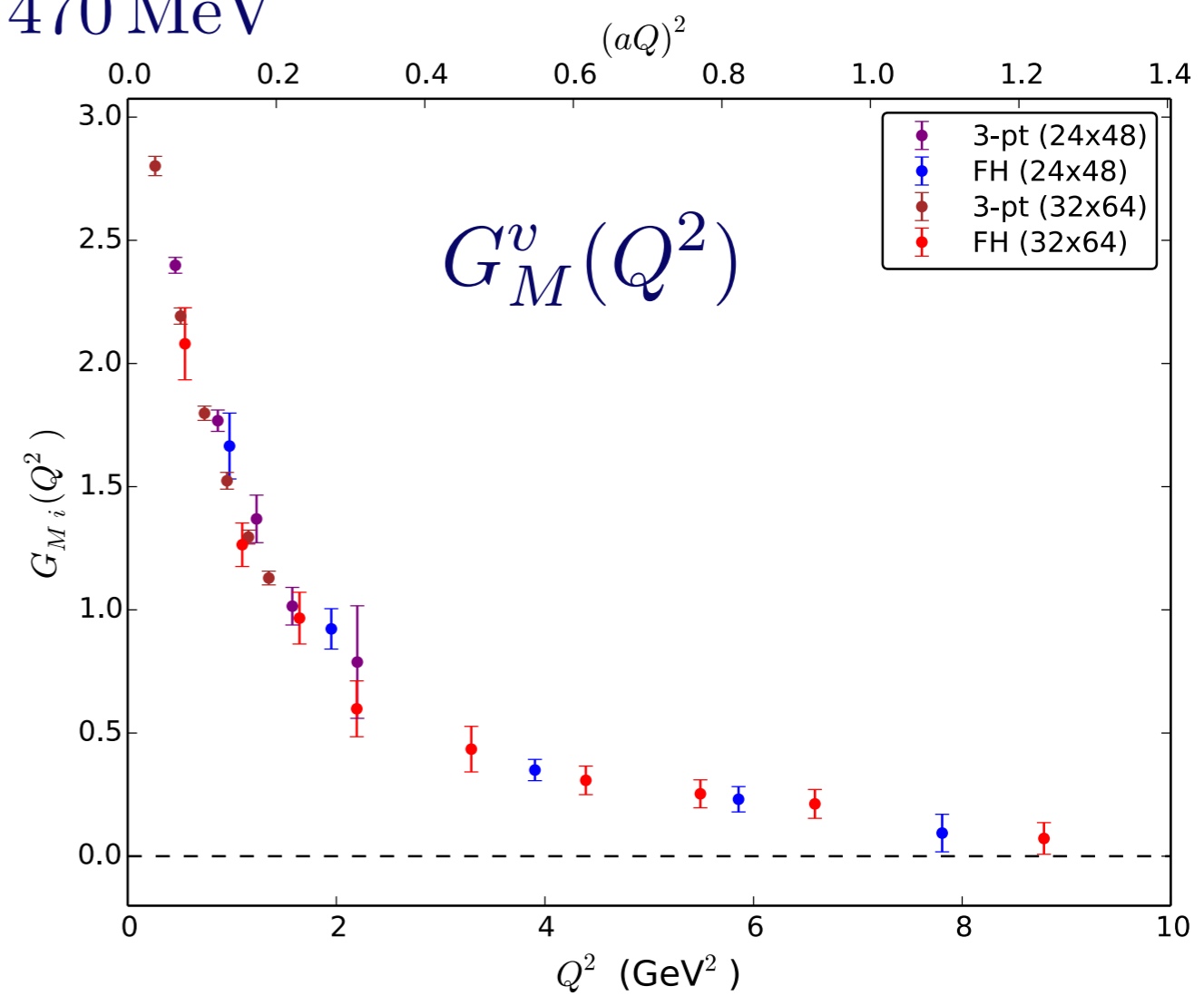
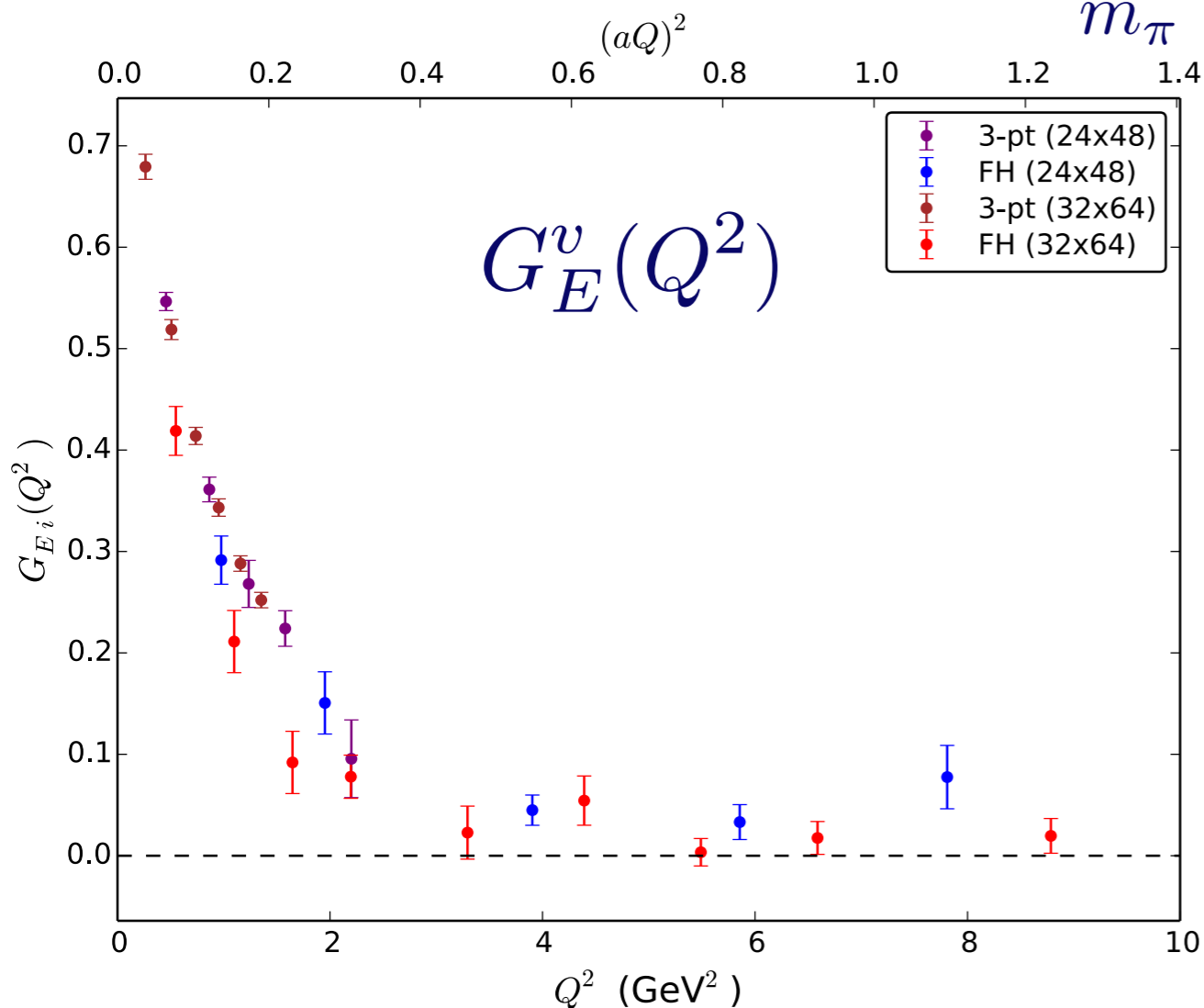
## Application of Feynman-Hellmann to form factors

Only require 2-point functions  $\longrightarrow$  simple excited state removal

Choose Breit frame kinematics  $|\vec{q}| = 2|\vec{p}| = 2|\vec{p}'|$

$\longrightarrow$  single exponential  $\longrightarrow$  look for energy shifts

$$m_\pi \approx 470 \text{ MeV}$$





# Large $Q^2$

[R. Young, Thu 8:30am]

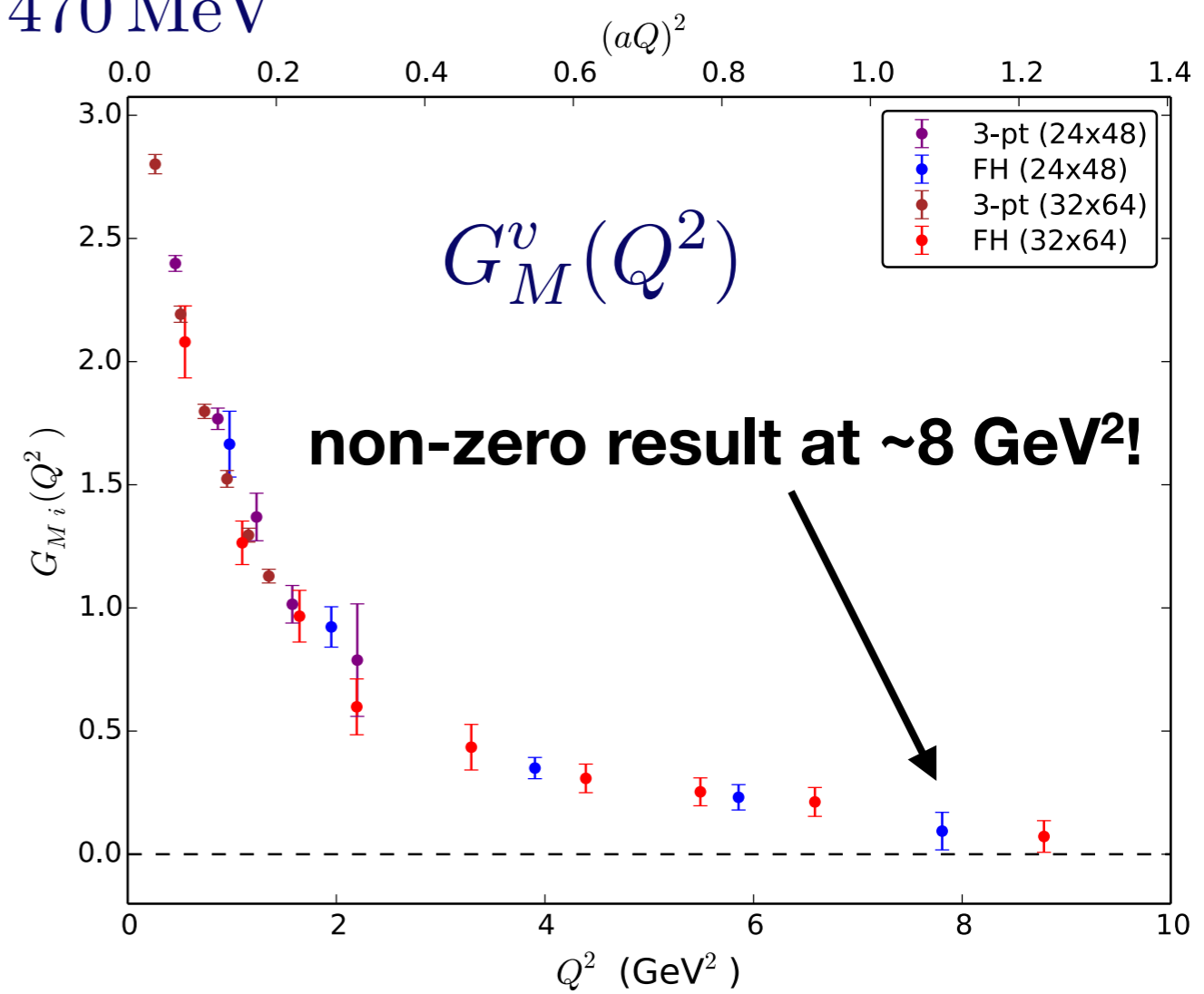
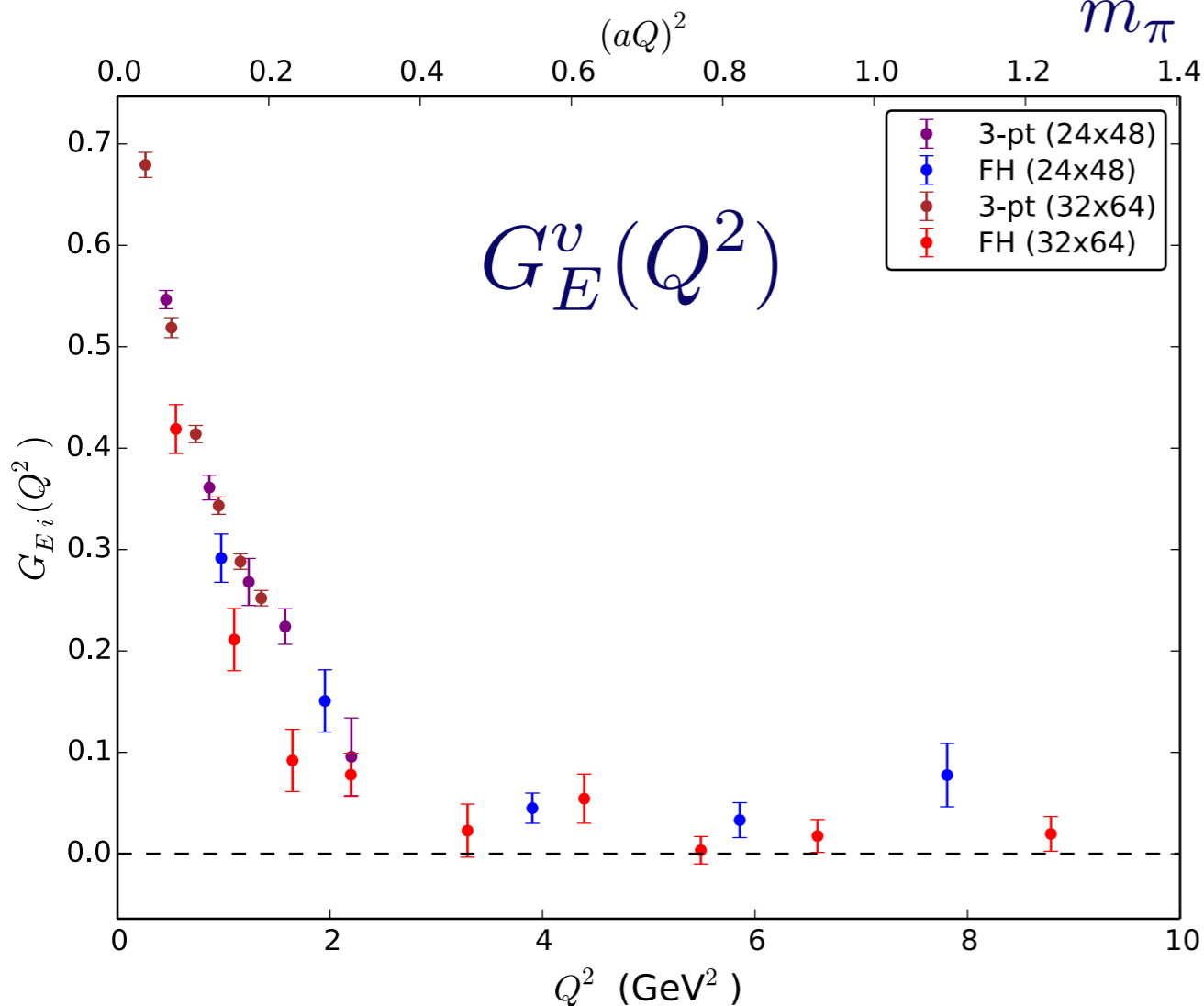
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$\longrightarrow$  single exponential  $\longrightarrow$  look for energy shifts

$$m_\pi \approx 470 \text{ MeV}$$



# Neutron Form Factors

**Experimental determination of neutron EM form factors difficult since**

**Neutron has no charge**  $\longrightarrow$   $G_E^n$  naturally small

**Lack of free neutron target**  $\longrightarrow$  use  $^2\text{H}$ ,  $^3\text{H}$ ,  $^3\text{He}$

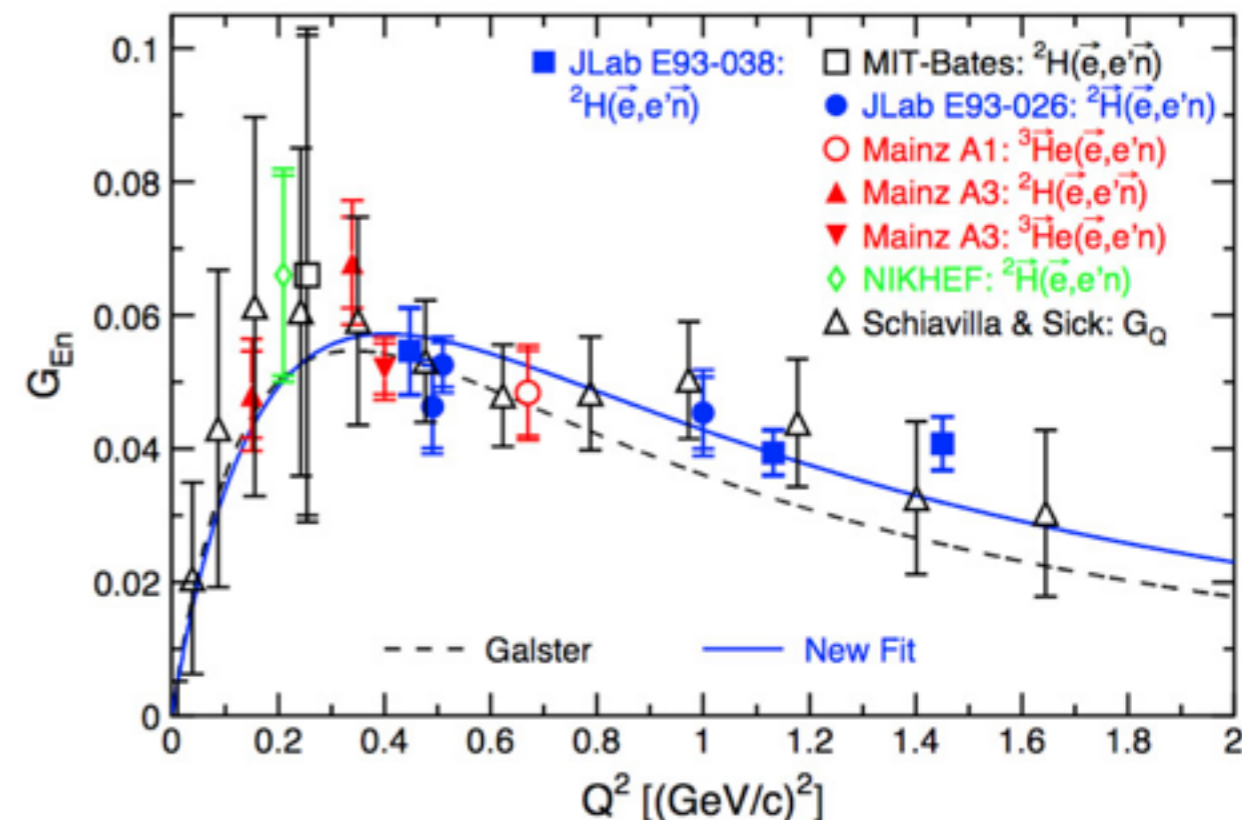
$\longrightarrow$  **requires corrections for nuclear effects (Fermi motion, Final State Interactions, Meson Exchange Currents) and proton contributions**

$\longrightarrow$  **Model dependence**

**Opportunity for Lattice:**

**Direct  $G_{E,M}^n(Q^2)$  determination**

**Determine  $^2\text{H}$ ,  $^3\text{H}$ ,  $^3\text{He}$  form factors**



# Magnetic Structure of Light Nuclei

[NPLQCD PRL 113 (2014) 252001 & 1506.05518]

**Magnetic moments and polarizabilities of nucleons and light nuclei with  $A \leq 4$**

**Employ background magnetic fields**

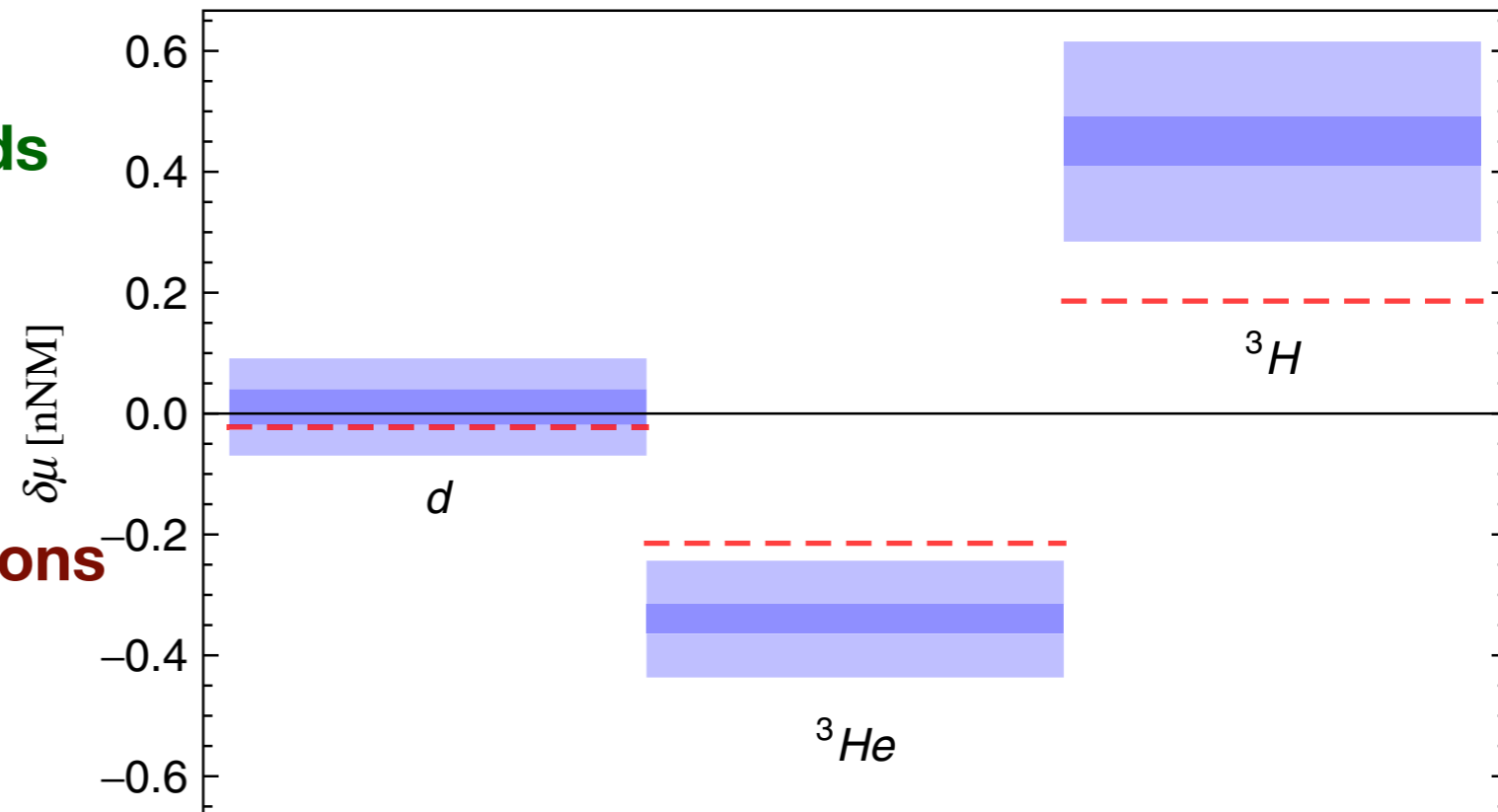
$$m_\pi \approx 800 \text{ MeV}$$

**Compare with shell model predictions**

$$\mu_d^{\text{SM}} = \mu_p + \mu_n$$

$$\mu_{^3\text{H}}^{\text{SM}} = \mu_p$$

$$\mu_{^3\text{He}}^{\text{SM}} = \mu_n$$



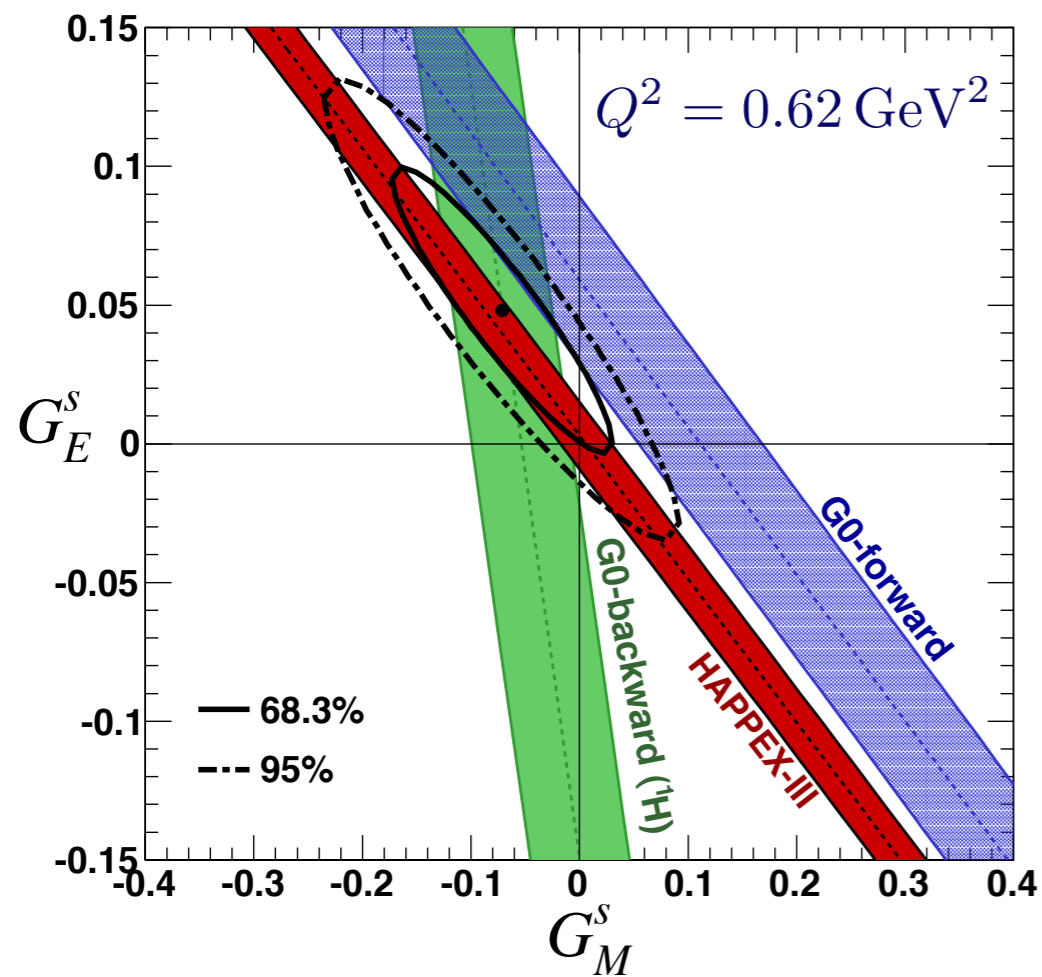
→ **shell-model configuration captures their dominant structures**

**Deviations similar to experiment**

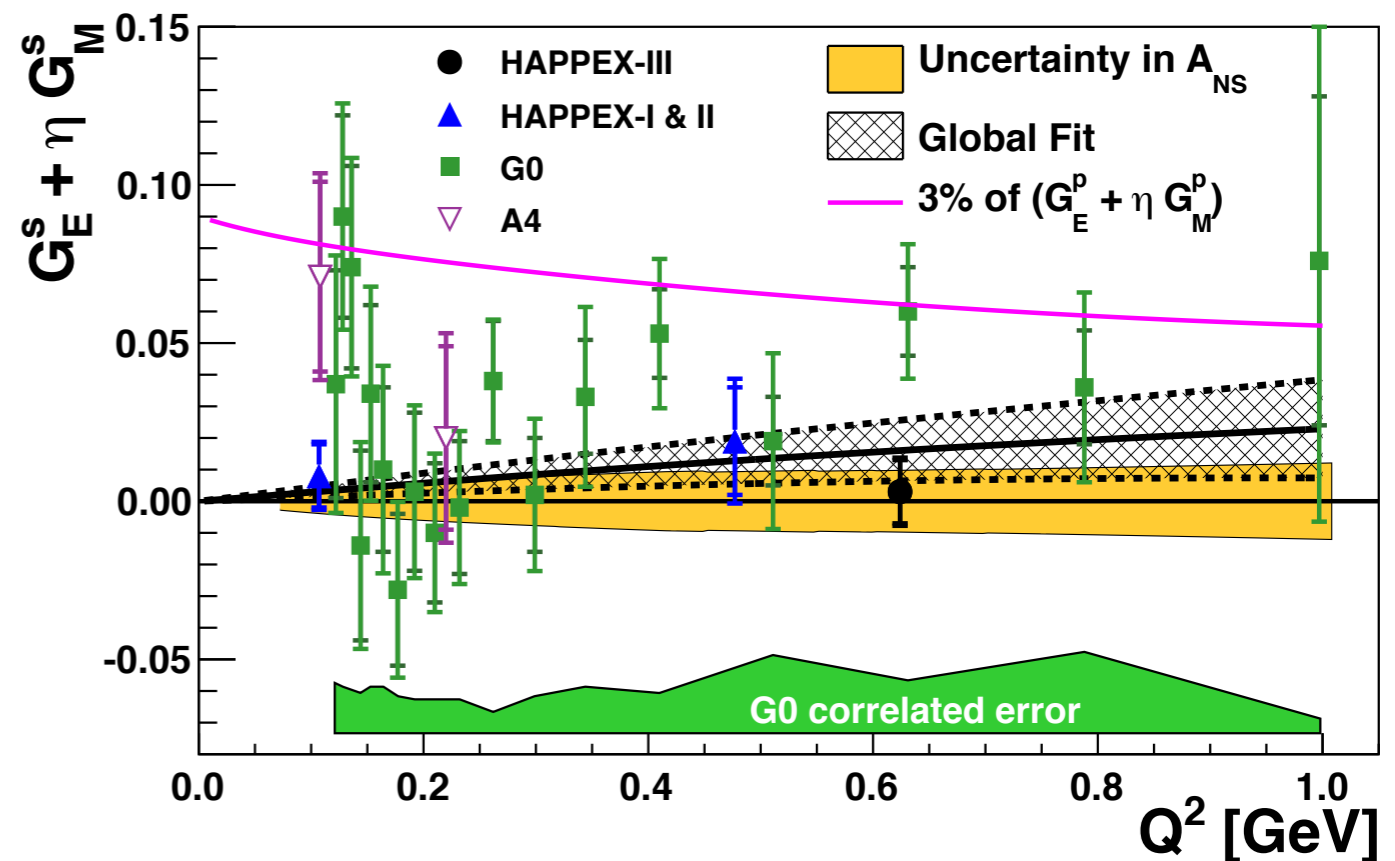
**Implications for experiments using  $^2\text{H}$ ,  $^3\text{H}$ ,  $^3\text{He}$  for neutron properties**

# Strangeness Form Factors

- **Understanding hidden flavour** - A fundamental challenge of hadronic physics
- Contributions arise entirely through interactions with QCD vacuum
- **Extensive experimental searches**
  - JLAB (G0, HAPPEX), MIT-Bates (SAMPLE), Mainz (A4)



[HAPPEX-III PRL108 (2012) 102001]



# Strangeness Form Factors

**Qweak experiment (JLab)**

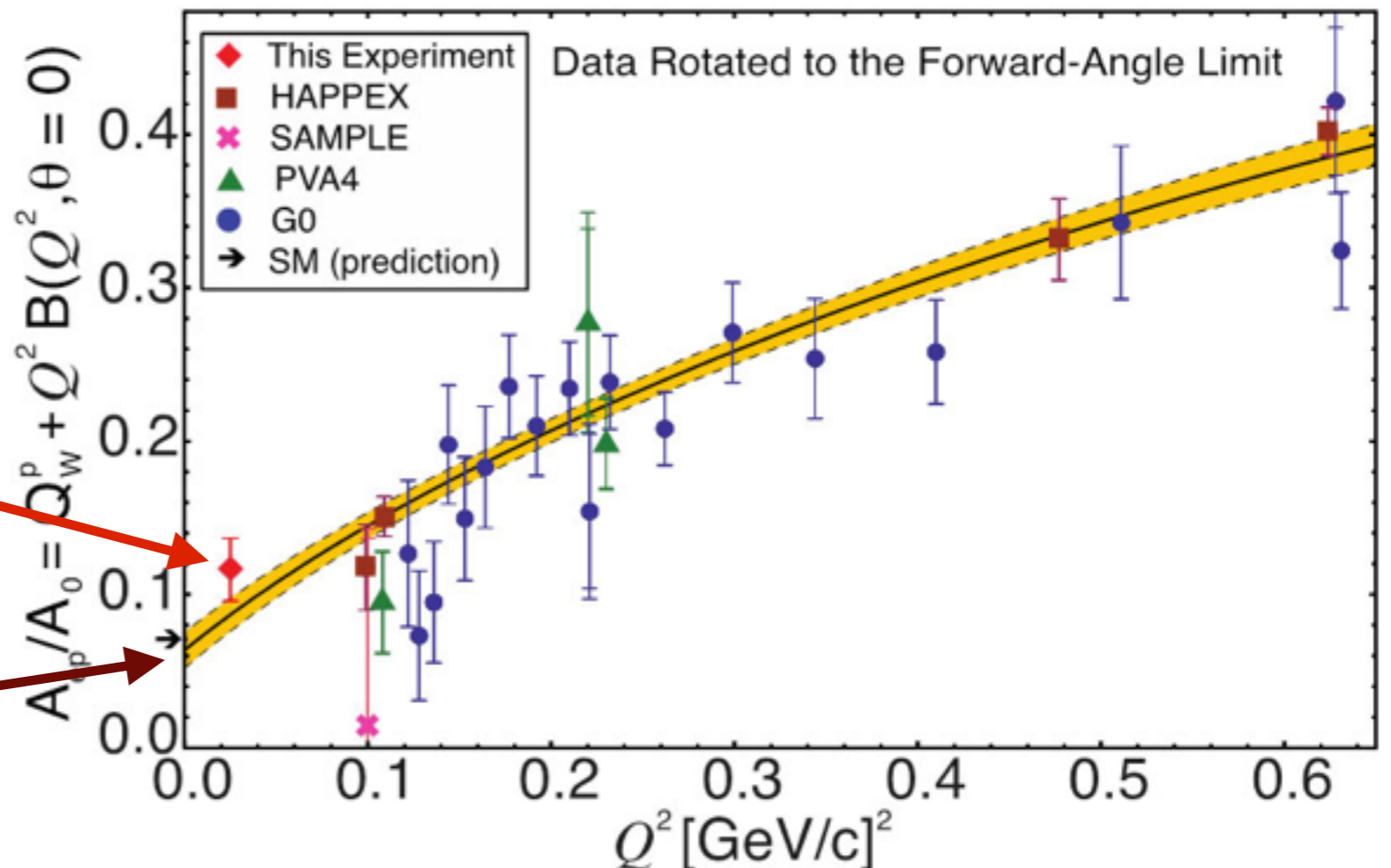
- Implications for experimental determination of the proton's weak charge  $Q_W^p$
- $Q_W^p$  : neutral current analog to the proton's electric charge
- Precisely predicted to be small in the SM

➔ **Constrain new parity-violating (PV) physics between electrons and light quarks**

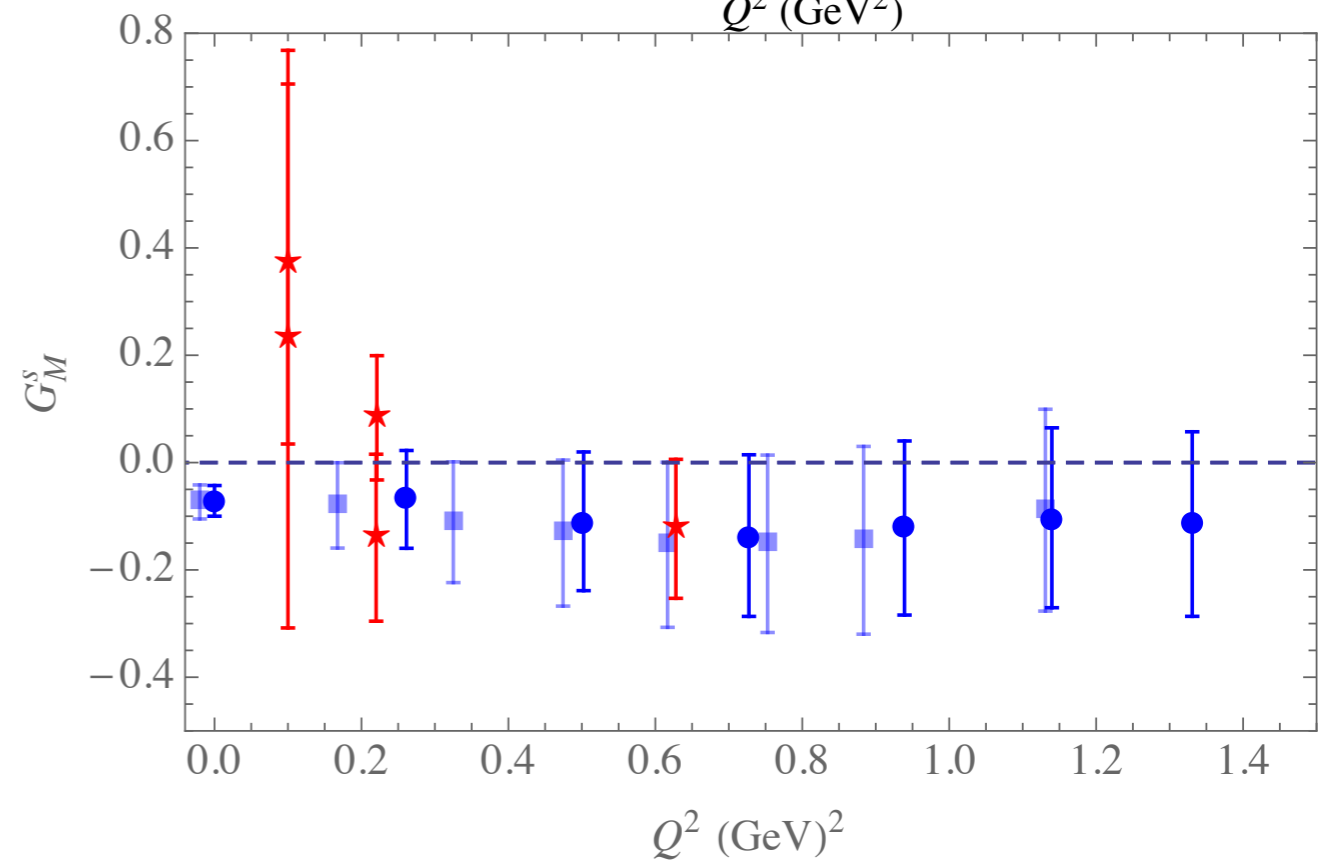
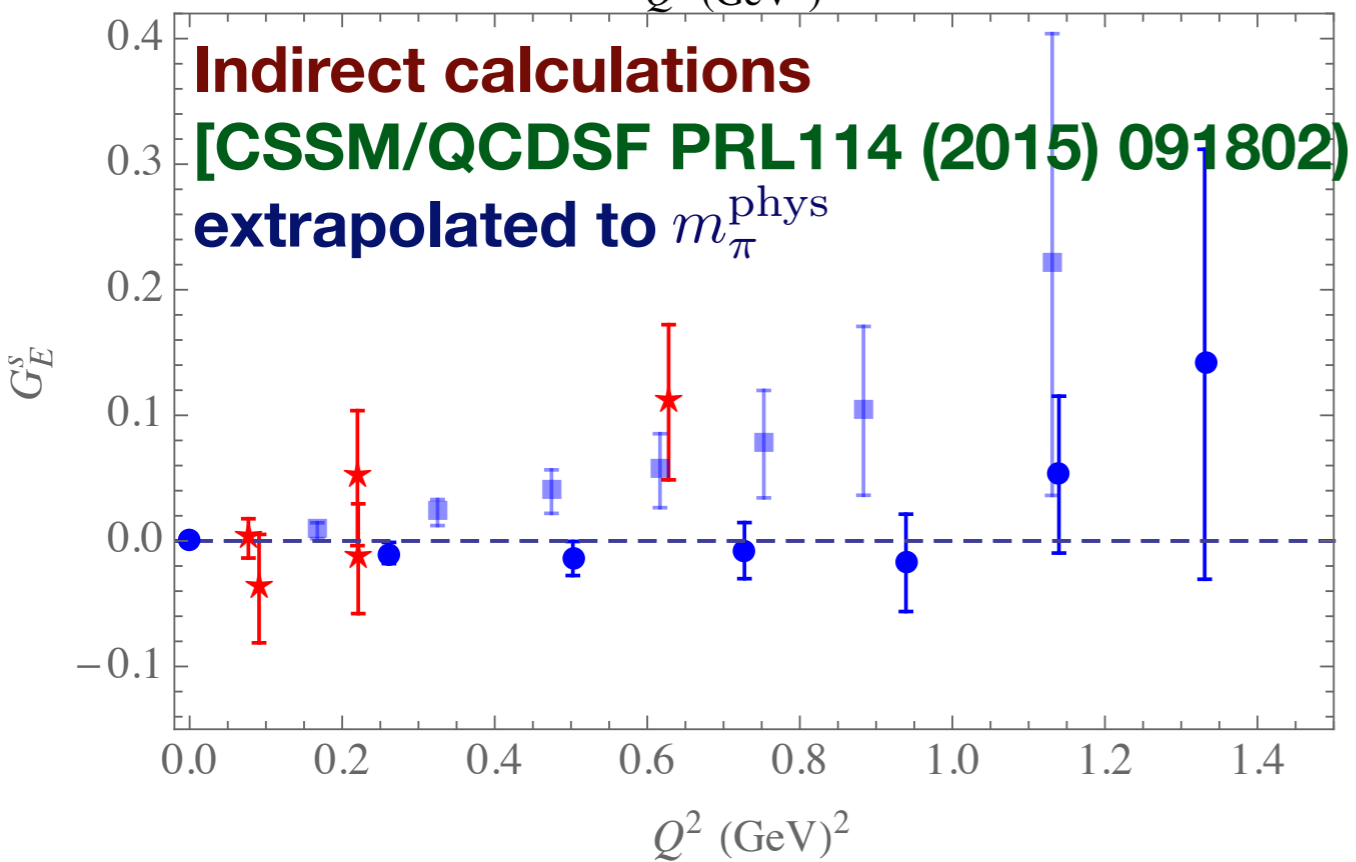
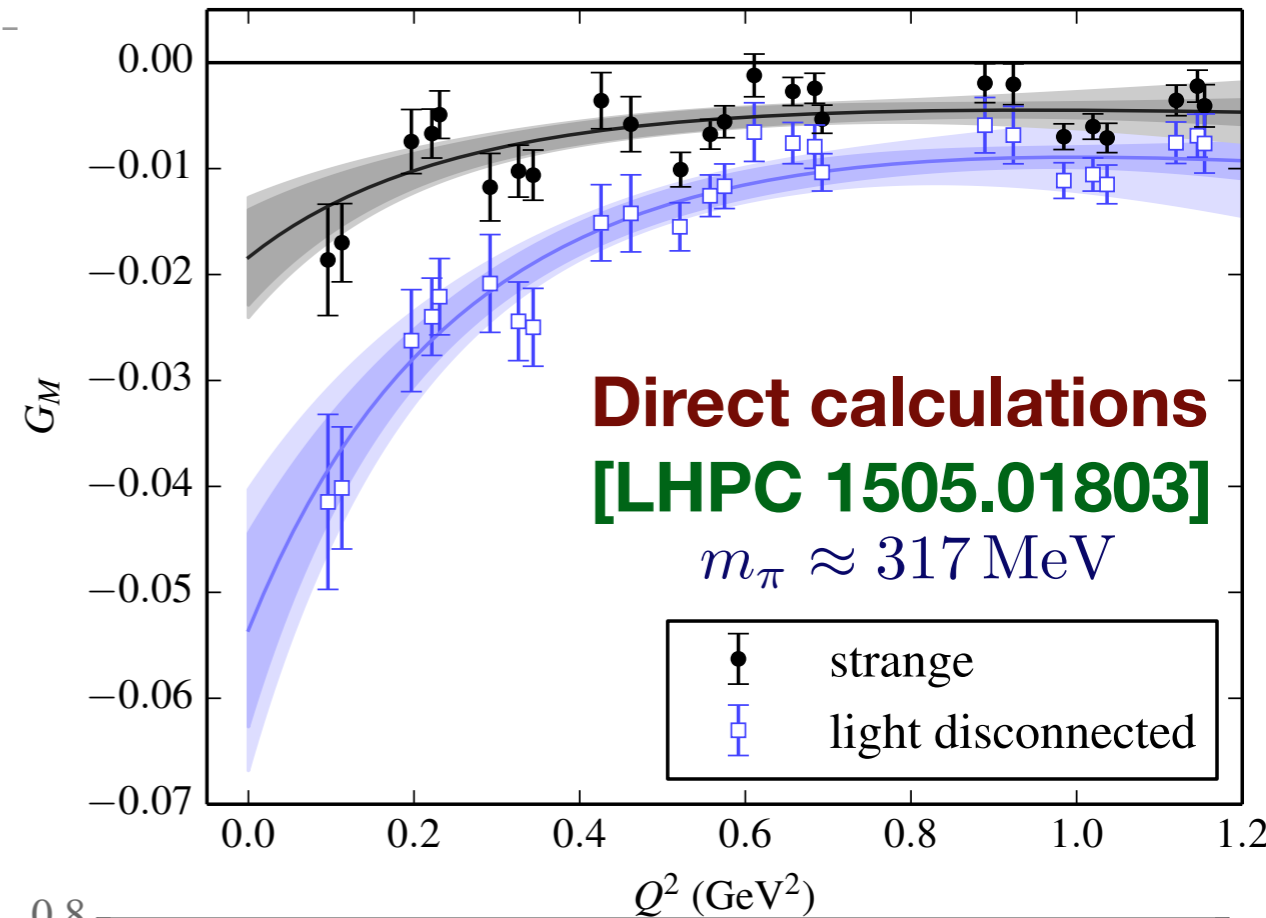
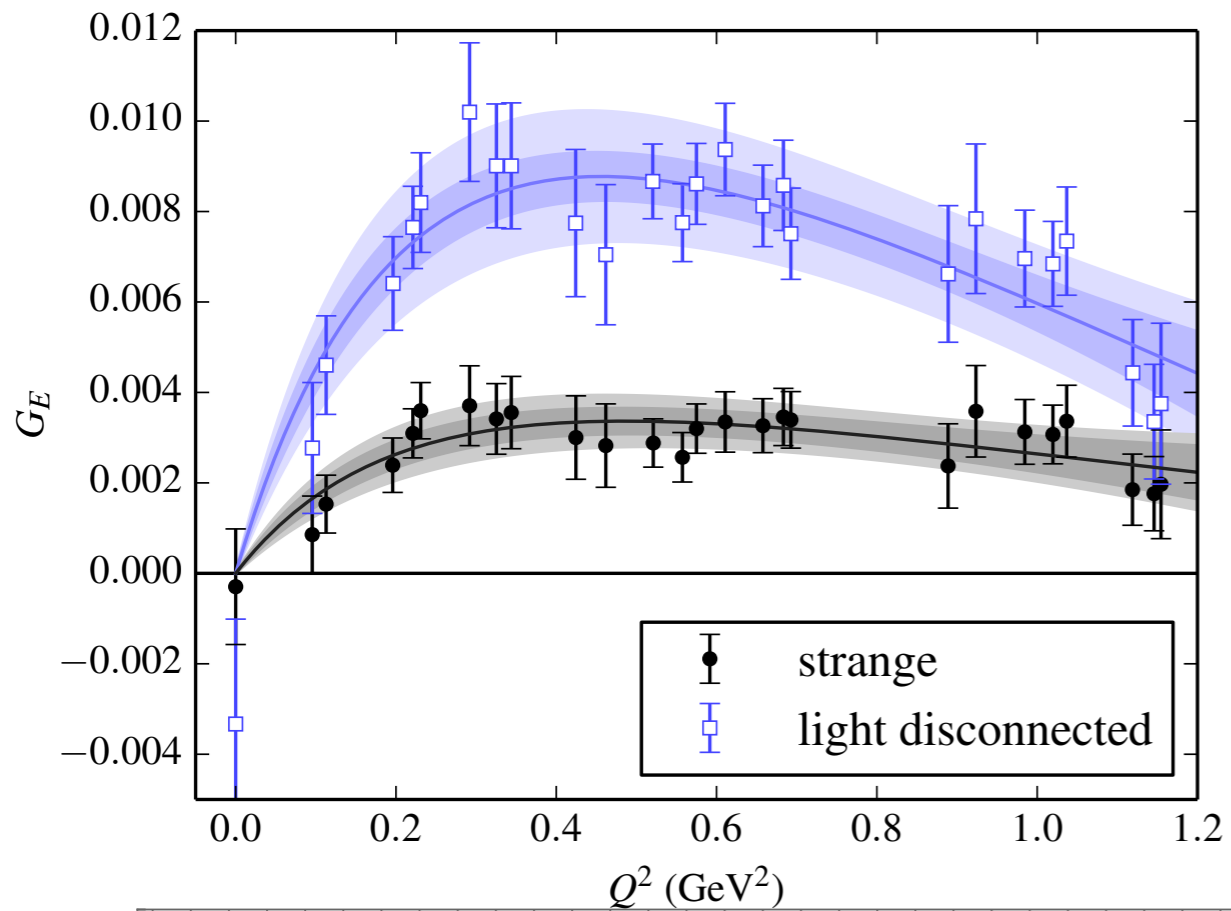
[Qweak PRL111 (2013) 141803]

4% data analysed

$Q_W^p$



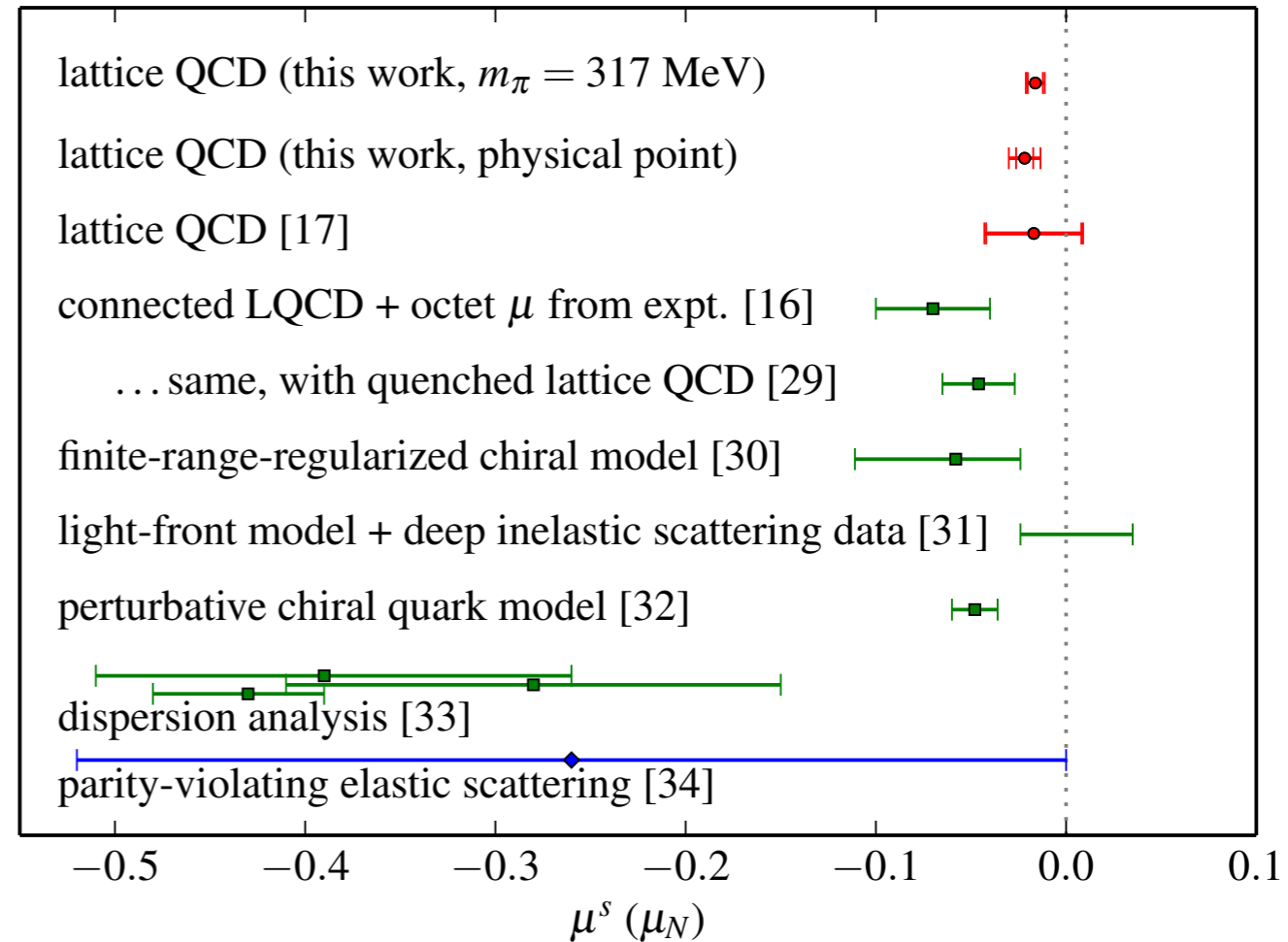
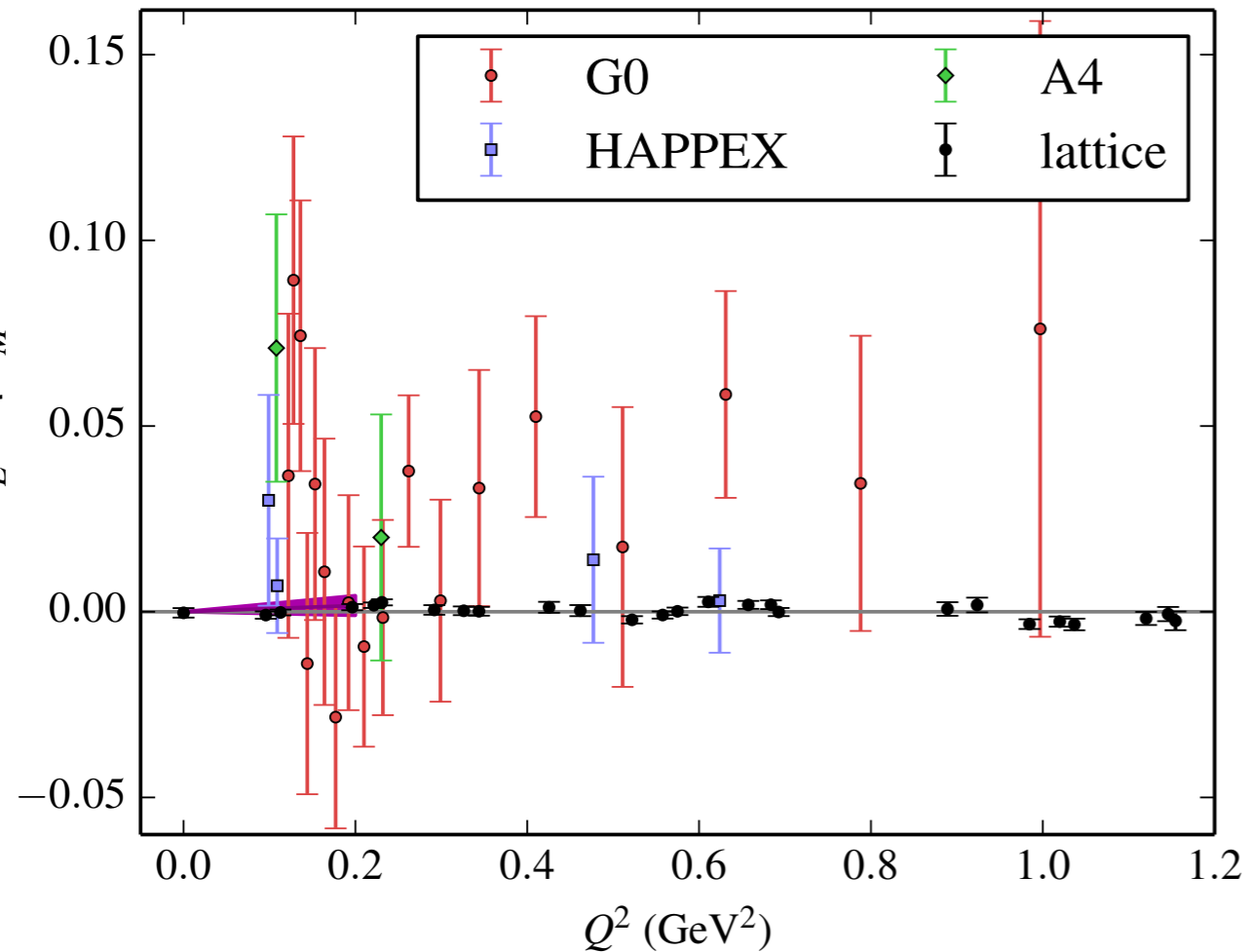
# Strangeness Form Factor



# Strangeness Form Factor

[LHPC 1505.01803]

## Comparison with experimental constraints



Presents a challenge for the next generation of experiments

Reached a precision where violations of charge symmetry will become important

# Charge Symmetry

---

$u$  quarks in the proton  $\equiv$   $d$  quarks in the neutron

**Experimental determinations of  $G_{E,M}^s(Q^2)$  rely on charge symmetry**

**(As does the inclusion of nuclear data for Qweak)**

**EM and weak interactions give access to different combinations of  $G^{p,(u/d/s)}$**

$$G^{p,\gamma} = \frac{2}{3}G^{p,u} - \frac{1}{3}(G^{p,d} + G^{p,s})$$

$$G^{p,Z} = \left(1 - \frac{8}{3}\sin^2\theta_W\right)G^{p,u} - \left(1 - \frac{4}{3}\sin^2\theta_W\right)(G^{p,d} + G^{p,s})$$

**Assume charge symmetry** ( $G^{p,u} = G^{n,d}$ ,  $G^{p,d} = G^{n,u}$ ,  $G^{p,s} = G^{n,s}$ )

  $G_{E/M}^{p,s} = (1 - 4\sin^2\theta_W)G_{E/M}^{p,\gamma} - G_{E/M}^{n,\gamma} - G_{E/M}^{p,Z}$



# Charge Symmetry Violation

[CSSM/QCDSF PRD91 (2015) 113006]

---

**Determine the degree to which charge symmetry is violated in EM form factors by**

**Combining chiral perturbation theory fits to isospin-averaged hyperon FFs**

**Input  $m_u/m_d$  from experiment (or FLAG)**

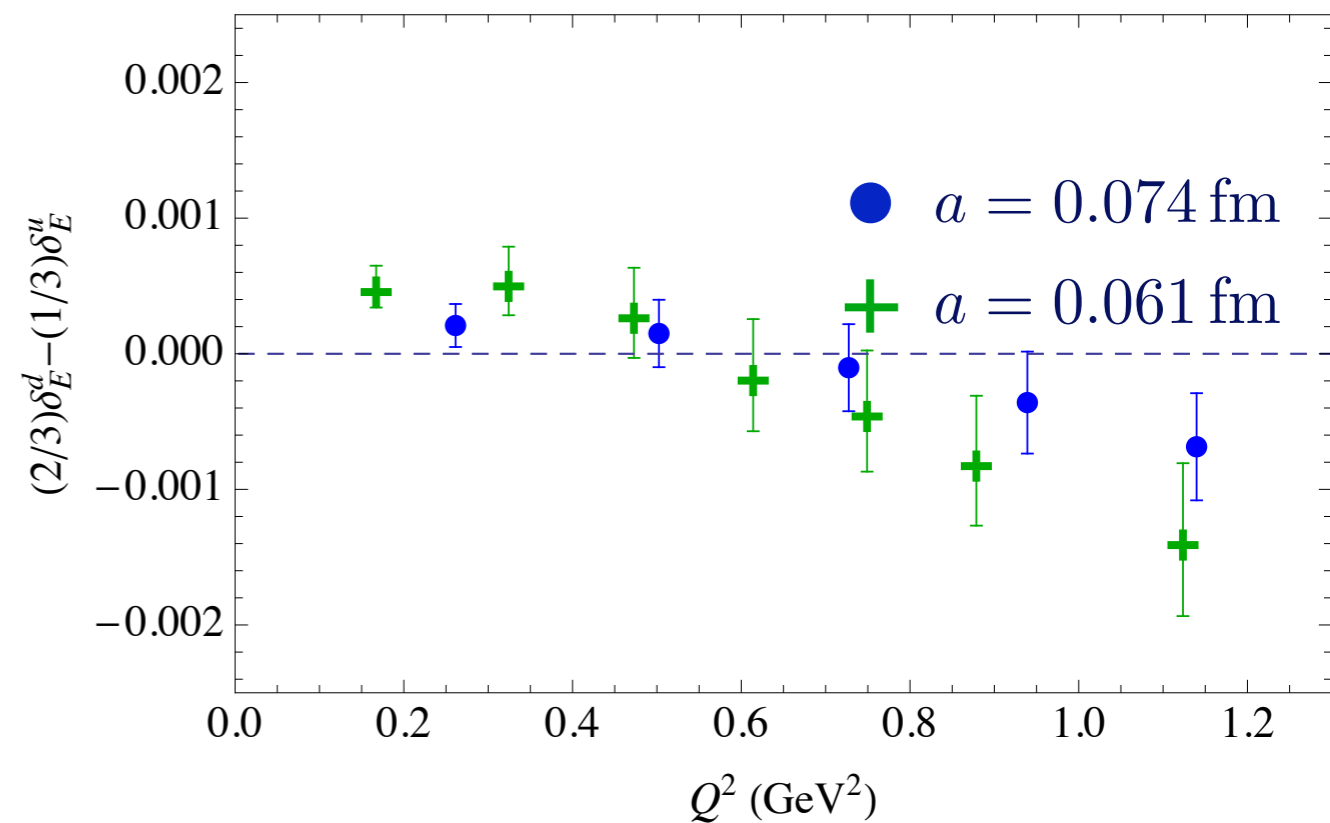
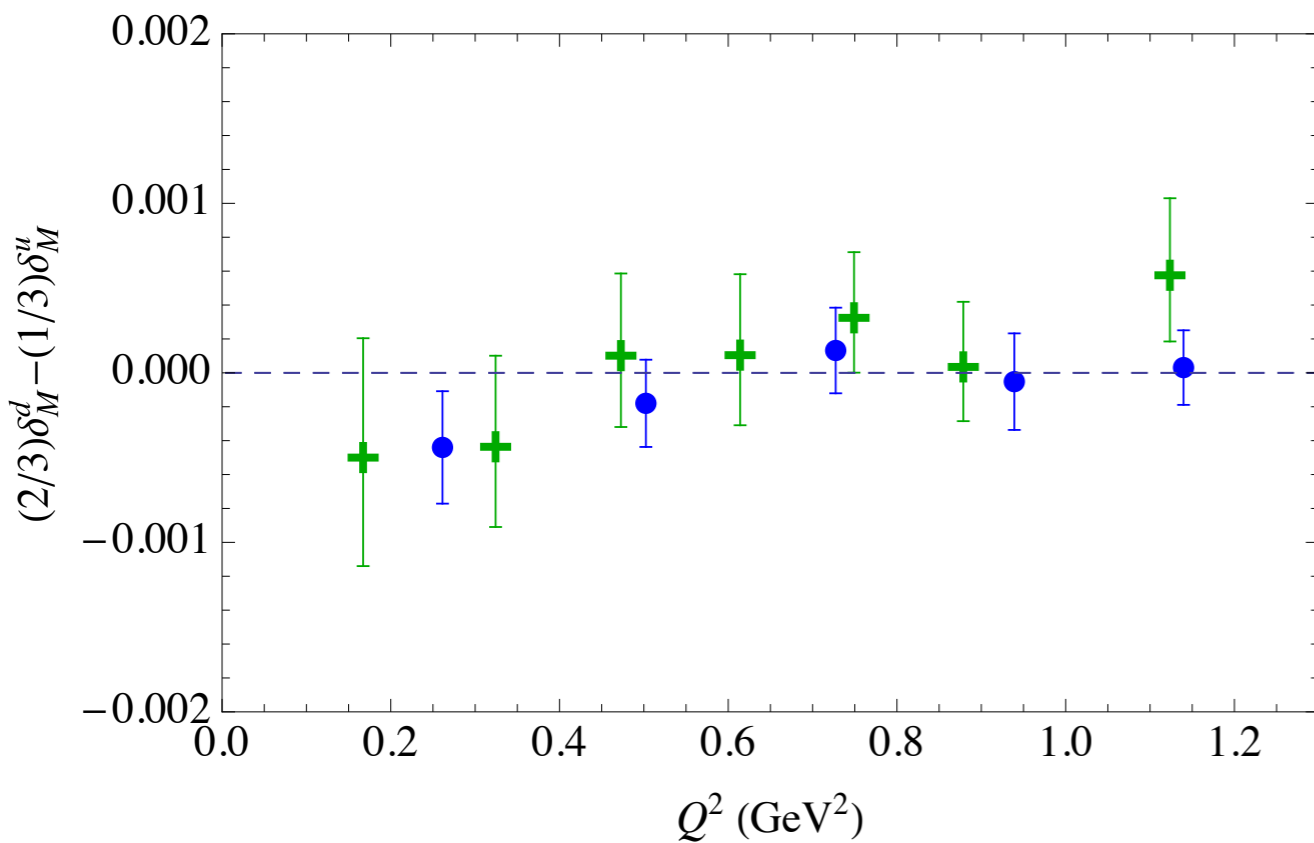
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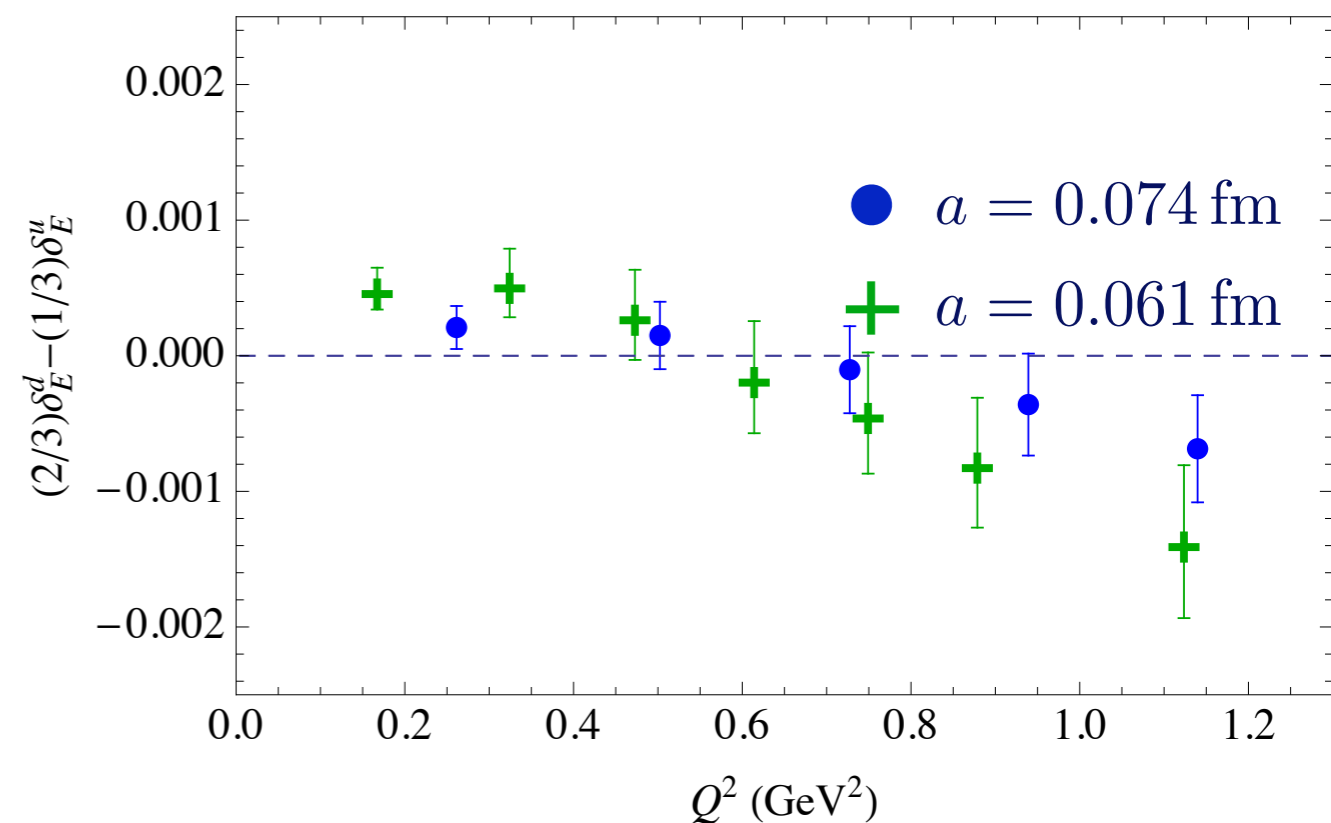
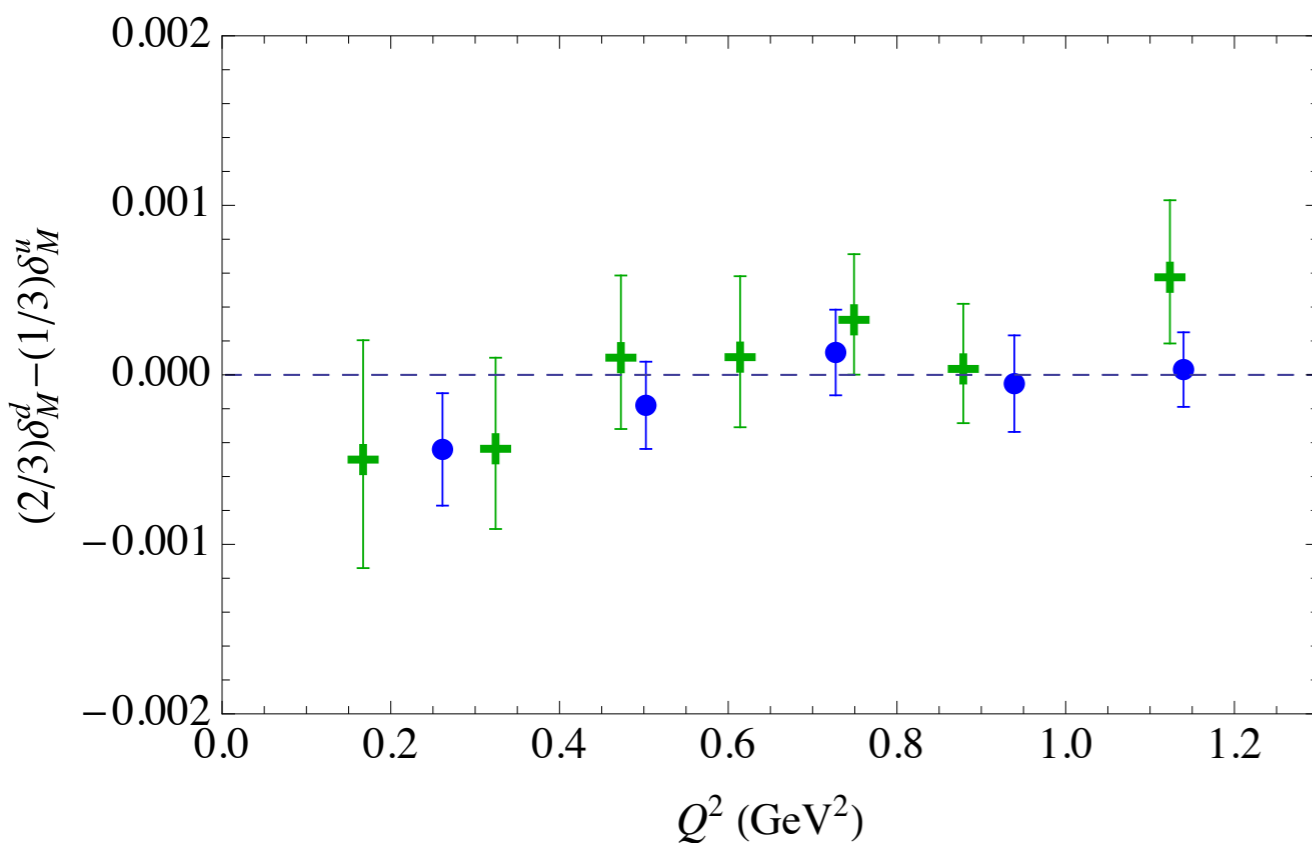
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Input  $m_u/m_d$  from experiment (or FLAG)



Charge symmetry satisfied to better than **0.2% (QCD)**

Violations due to **QED**? Challenge for future QCD+QED simulations

# Pion Form Factor

- **Asymptotic normalisation known from  $\pi \rightarrow \mu + \nu$  decay**

$$F_\pi(Q^2 \rightarrow \infty) = \frac{16\pi\alpha_s(Q^2)f_\pi^2}{Q^2}$$

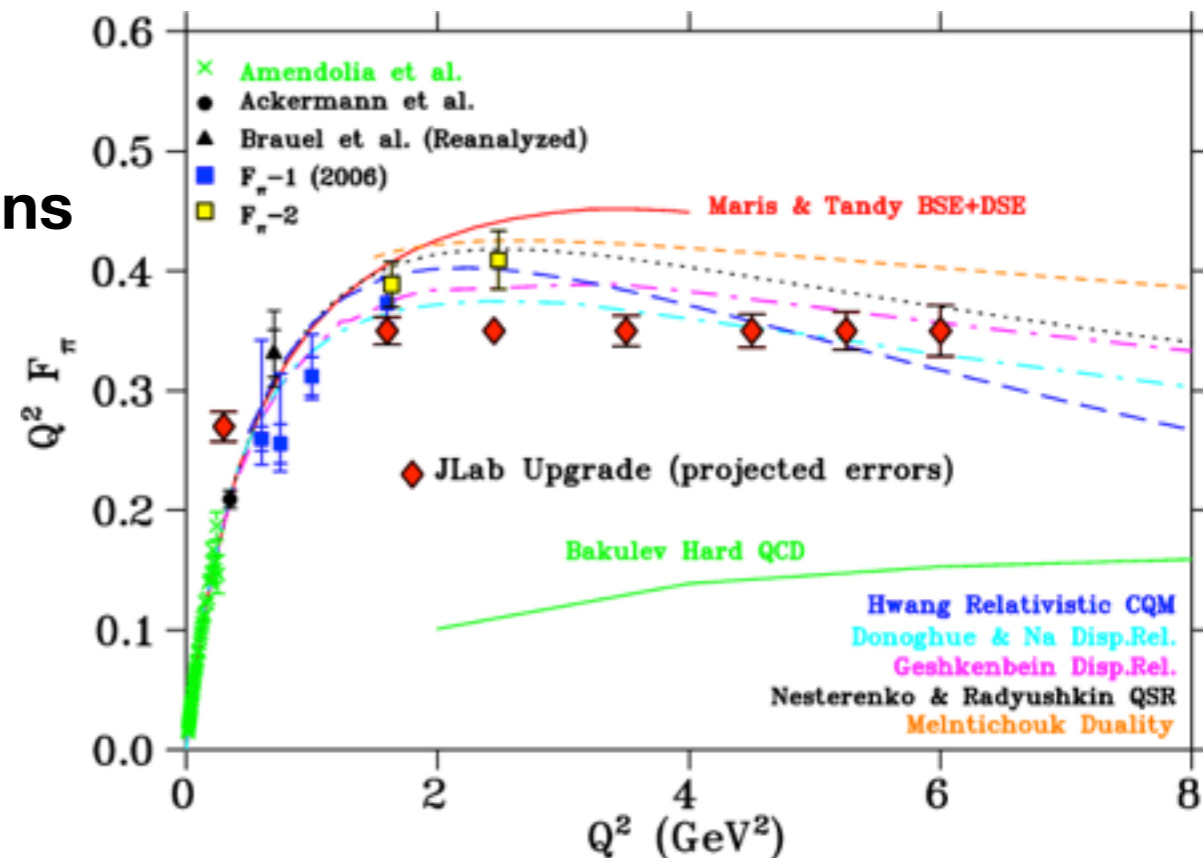
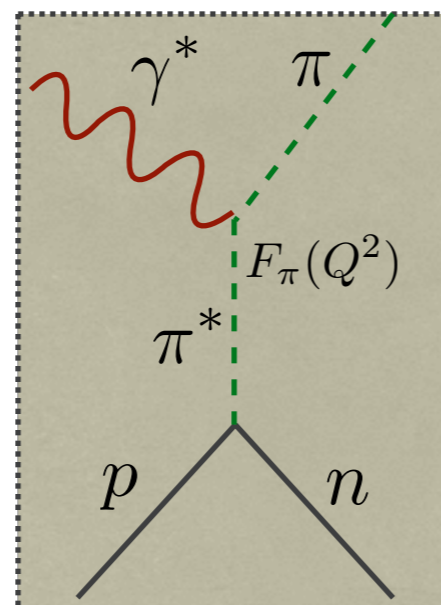
- **Allows to study the transition from the soft to hard regimes**

- **Low  $Q^2$ : measured directly by scattering high energy pions from atomic electrons**

➔ **precise determination of  $\langle r_\pi^2 \rangle$**

- **High  $Q^2$ : quasi-elastic scattering off virtual pions**

➔ **Model dependence**



# Pion Form Factor

[Bijnens-Ecker, 2014]

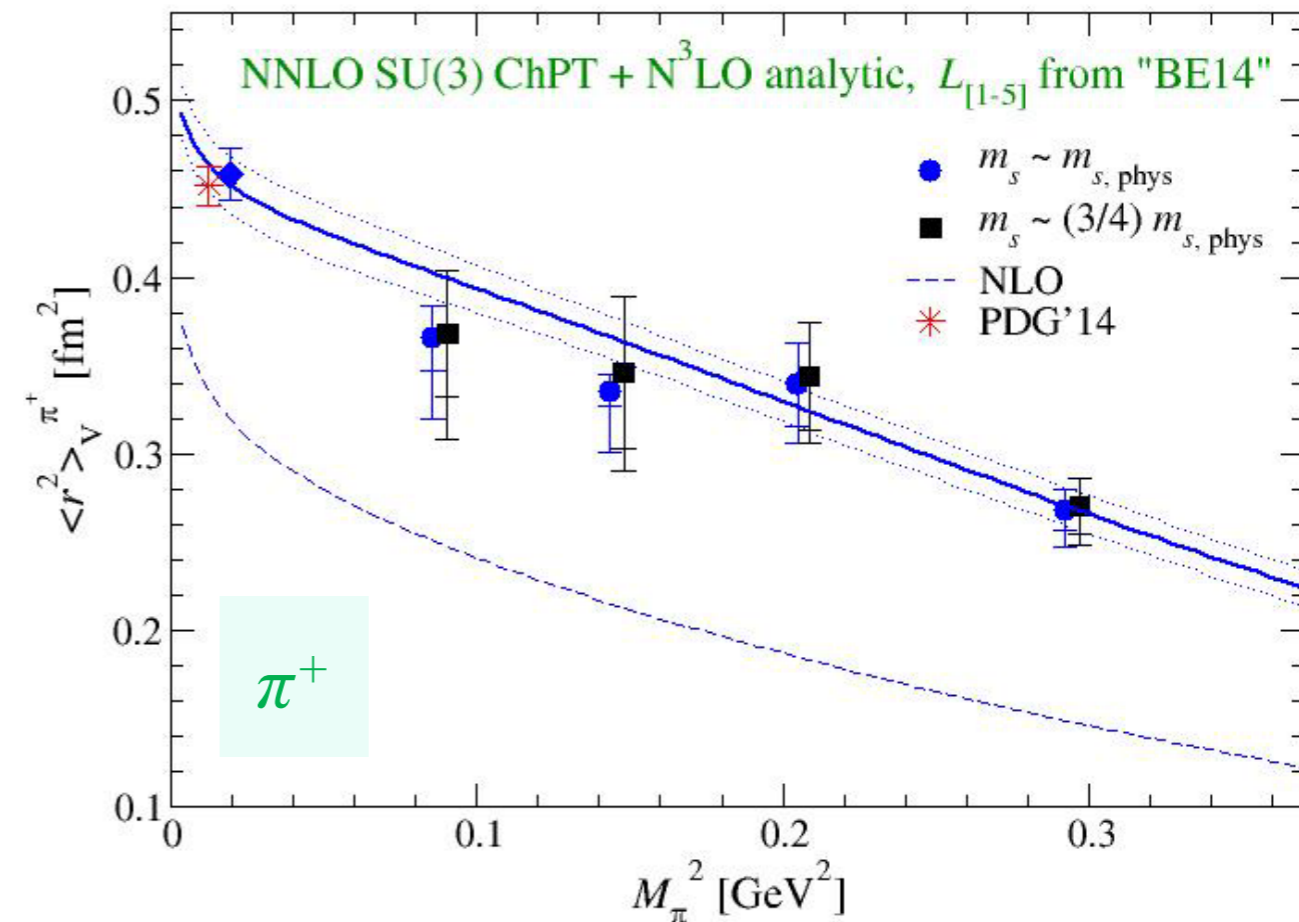
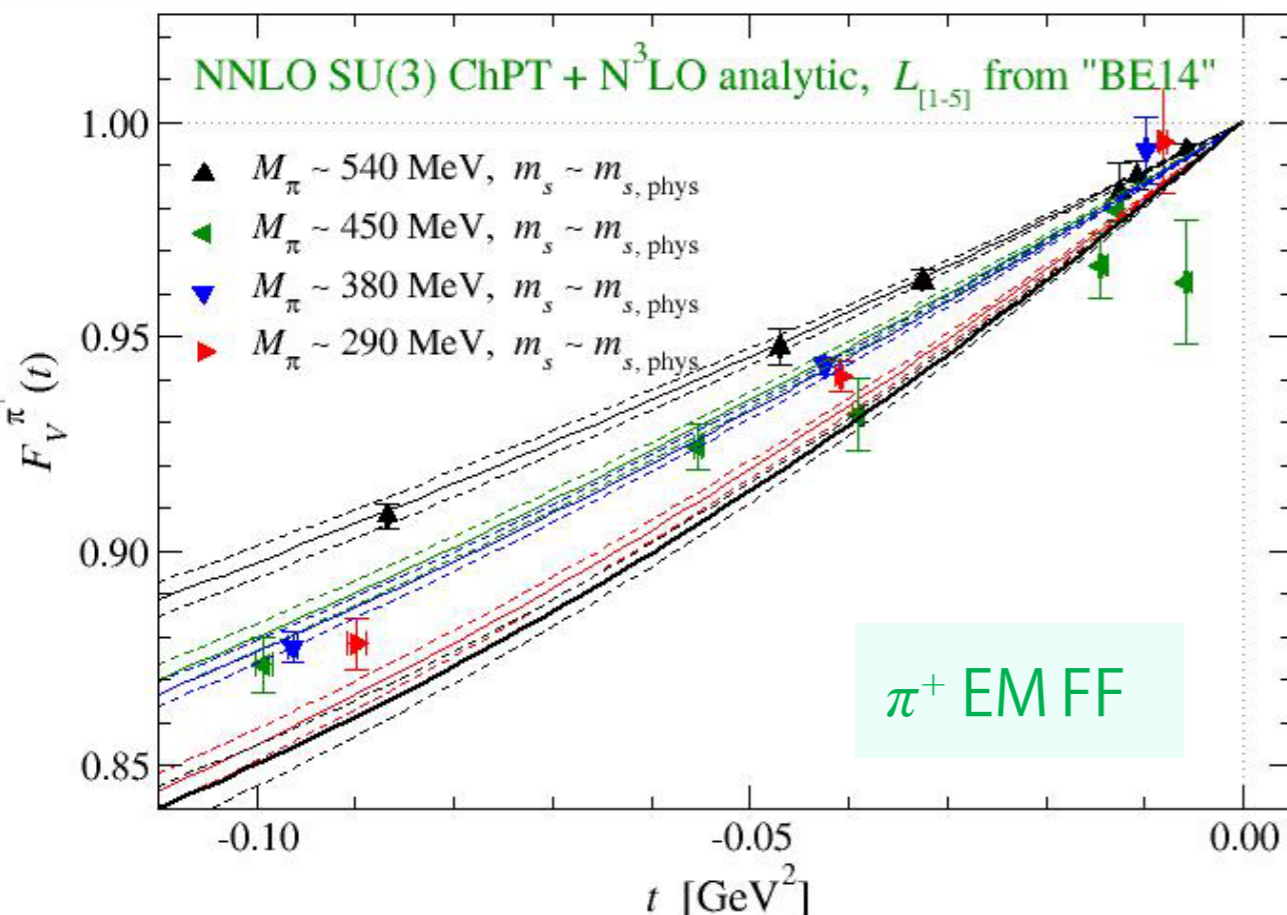
[T. Kaneko, Tue 16:30 (WD & ME)]

- **JLQCD: overlap quarks,  $290 \leq m_\pi \leq 540$  MeV (all-2-all propagators)**
- **NNLO SU(3) ChPT + N<sup>3</sup>LO analytic [Bijnens-Ecker, 2014] combined fit to**

$$F_{\pi^+}(q^2), F_{K^+}(q^2), F_{K^0}(q^2)$$

$$\langle r_{\pi^+}^2 \rangle = 0.458(15)(38) \text{ fm}^2, \langle r_{K^+}^2 \rangle = 0.380(12)(32) \text{ fm}^2, \langle r_{K^0}^2 \rangle = -0.055(11)(45) \text{ fm}^2$$

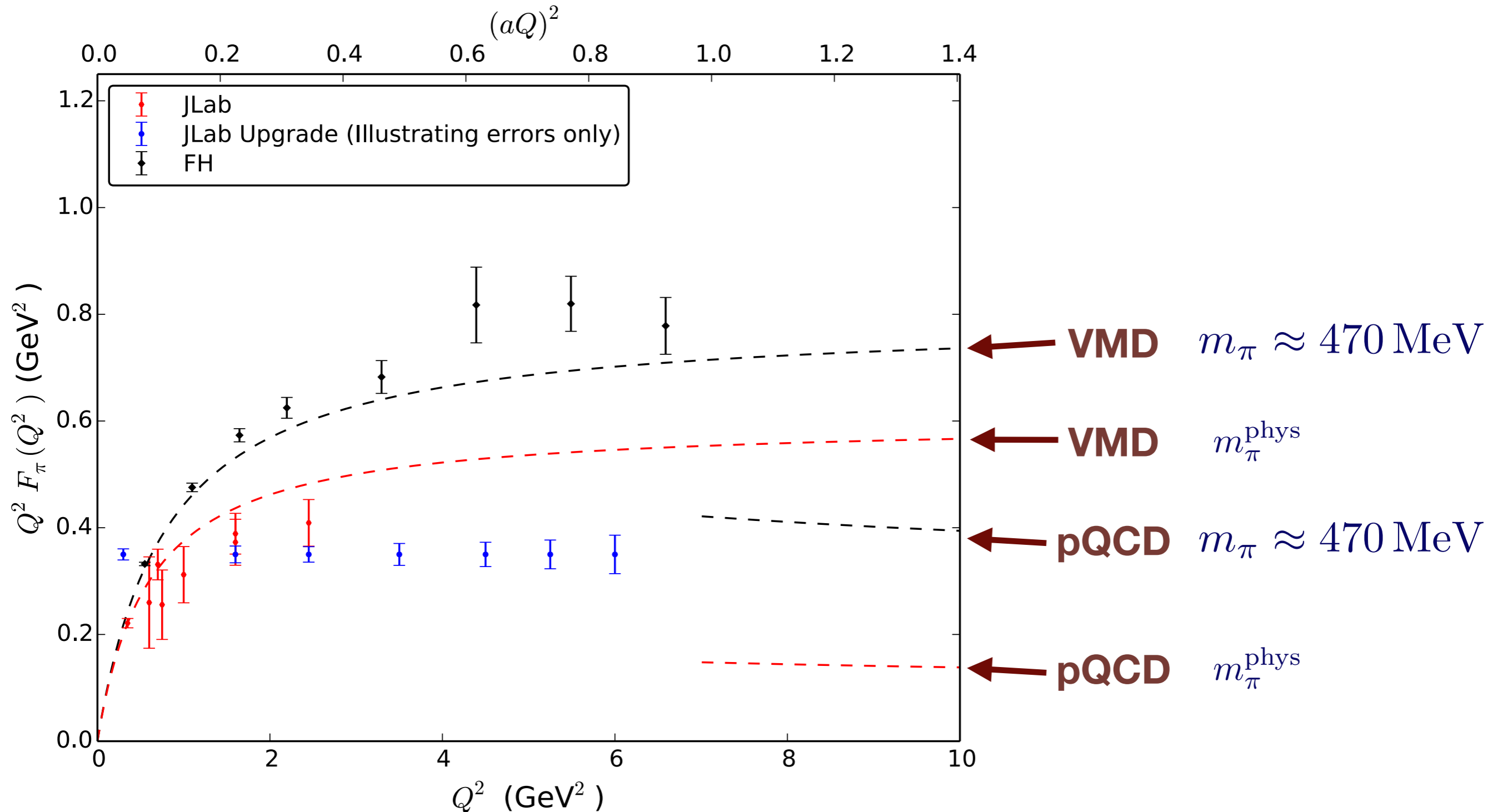
**PDG'14:**  $\langle r_{\pi^+}^2 \rangle = 0.452(11) \text{ fm}^2, \langle r_{K^+}^2 \rangle = 0.314(35) \text{ fm}^2, \langle r_{K^0}^2 \rangle = -0.077(10) \text{ fm}^2$

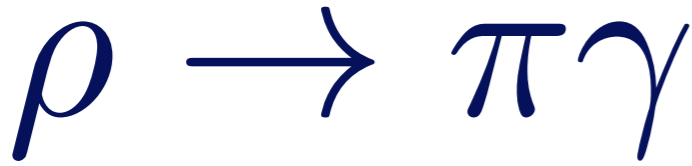


# Pion Form Factor

[R. Young, Thu 8:30am]

Access large  $Q^2$  via Feynman-Hellmann

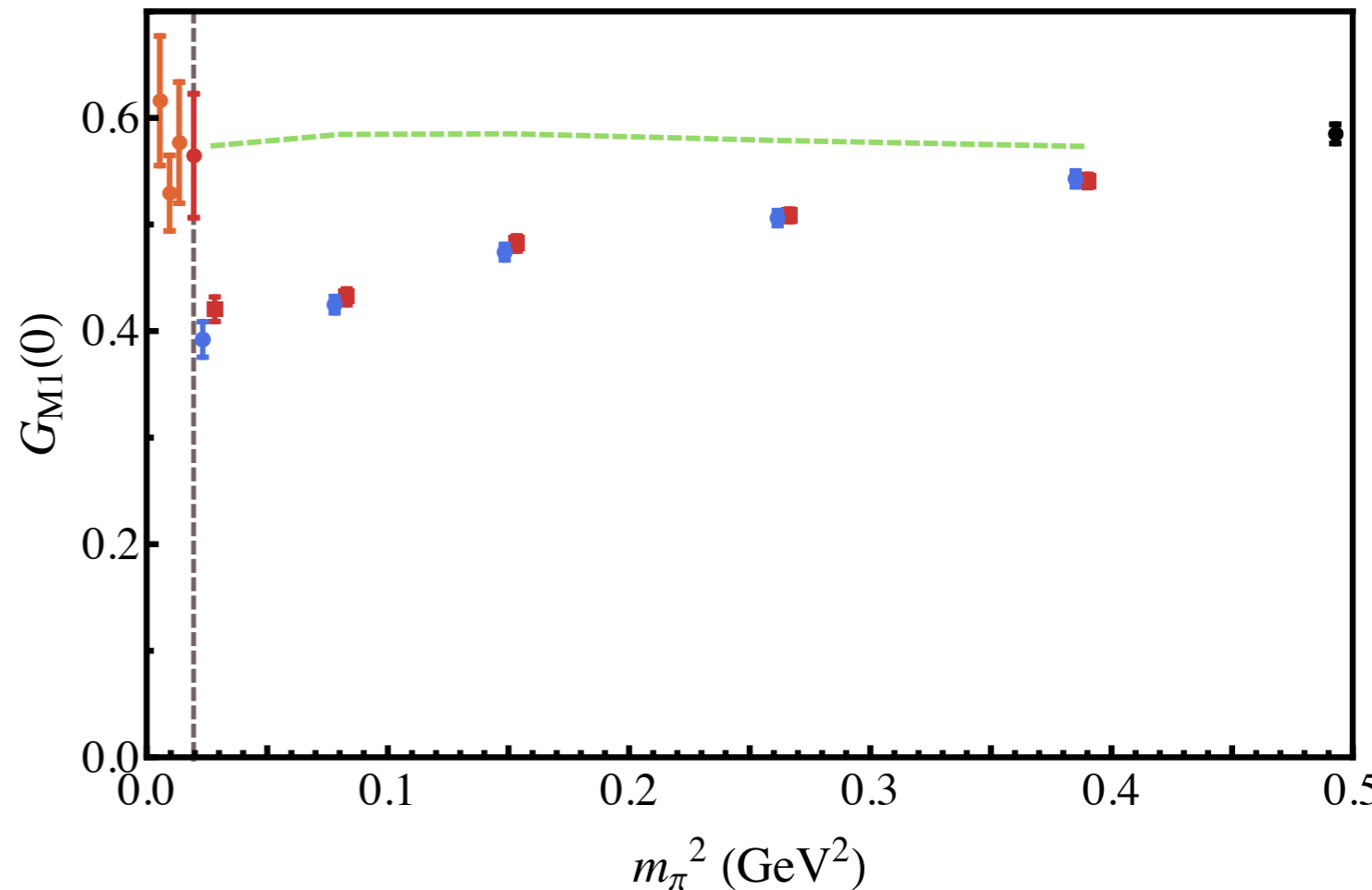
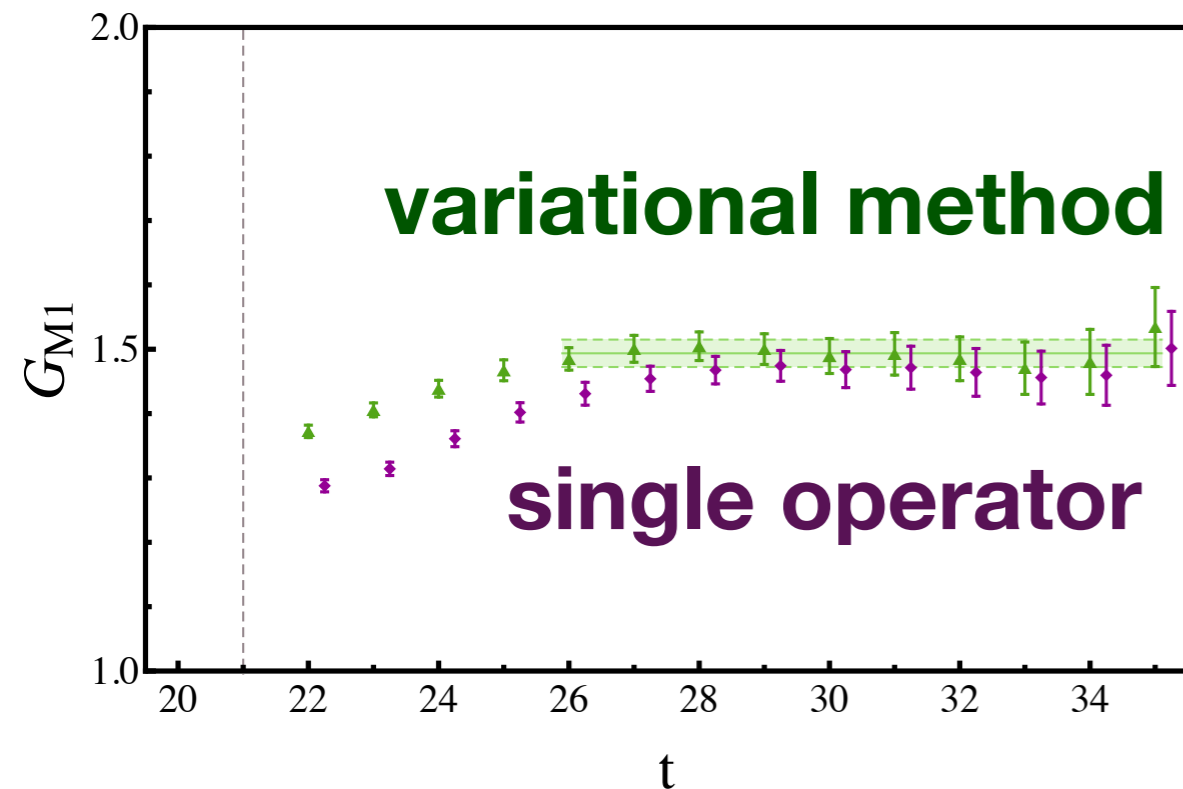




[CSSM 1505.02876]

Relevant for experiments at JLab and Mainz studying hadronic excitations

Develop framework for other radiative processes, e.g.  $N^* \rightarrow N \gamma$



PACS-CS 2+1 Clover

$$156 \leq m_\pi \leq 700 \text{ MeV}$$

Variational method to remove excited states

Discrepancy due to  
 disconnected diagrams?  
 multiparticle operators?  
 finite volume?

# Quark Distribution Functions

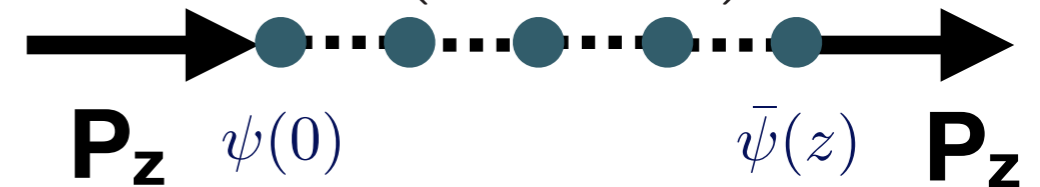
Recent proposal by Ji:

“Quasi”-PDFs

[PRL 110 (2013) 262002]

$$\tilde{q}_{\text{lat}}(x, \Lambda, P_z) = \int \frac{dz}{4\pi} e^{-izk} h(z, \Lambda, P_z),$$

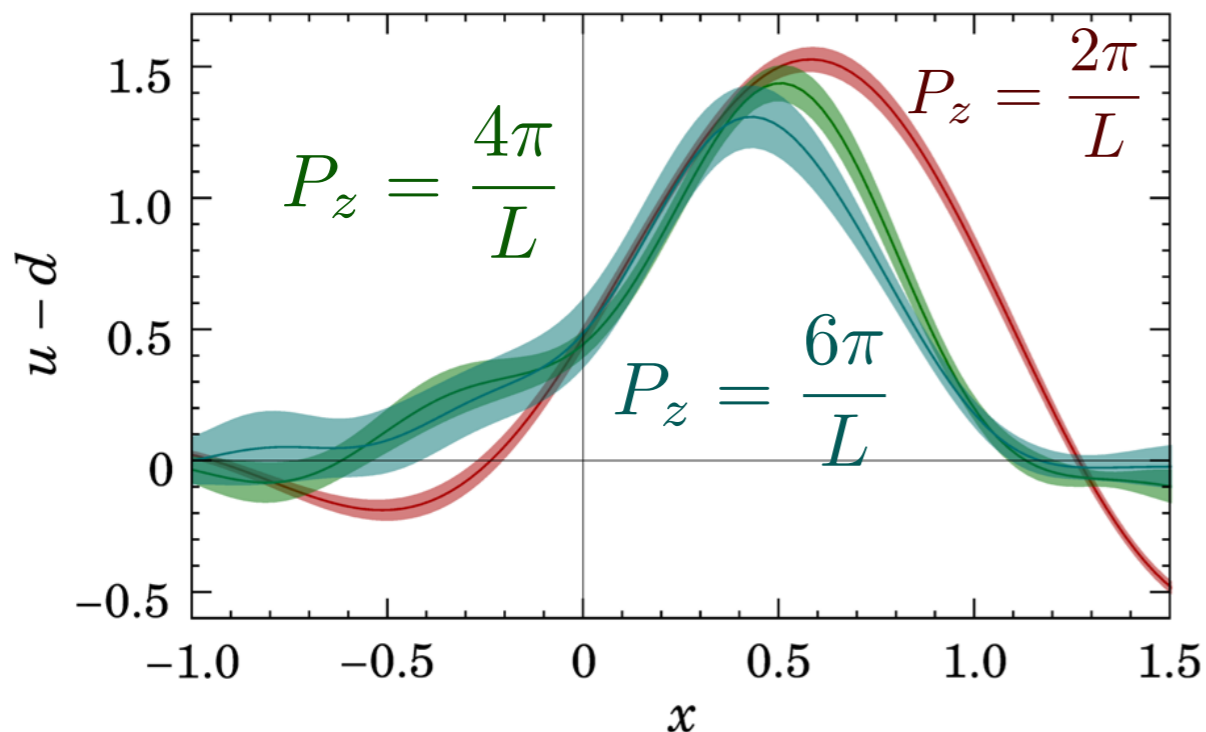
$$h(z, \Lambda, P_z) = \langle \vec{P} | \bar{\psi}(z) \gamma_z \left( \prod_n U_z(n\hat{z}) \right) \psi(0) | \vec{P} \rangle,$$



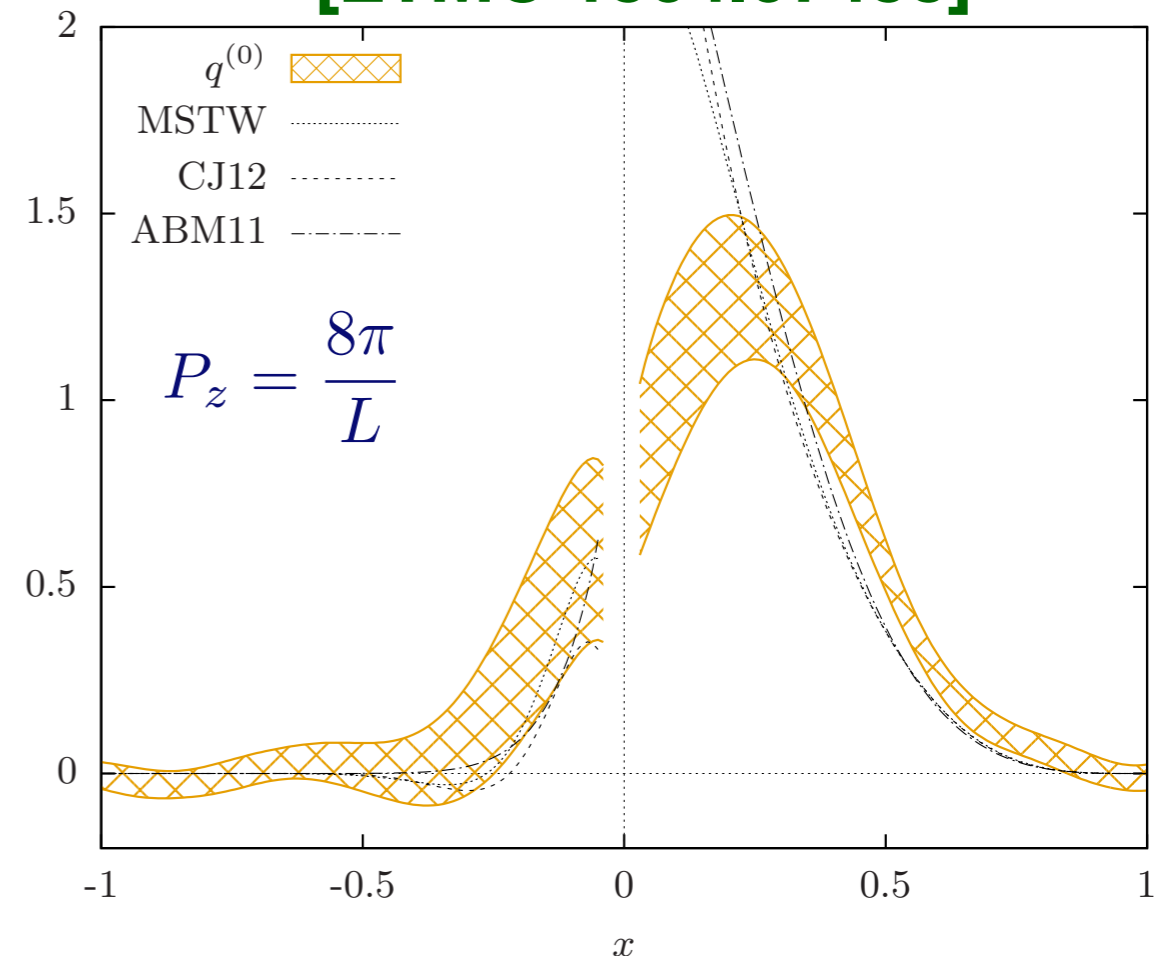
Connection to continuum through perturbative matching

2 lattice calculations

[H-W.Lin et al. PRD91 (2015) 054510]



[ETMC 1504.07455]





# Transverse Momentum Distributions

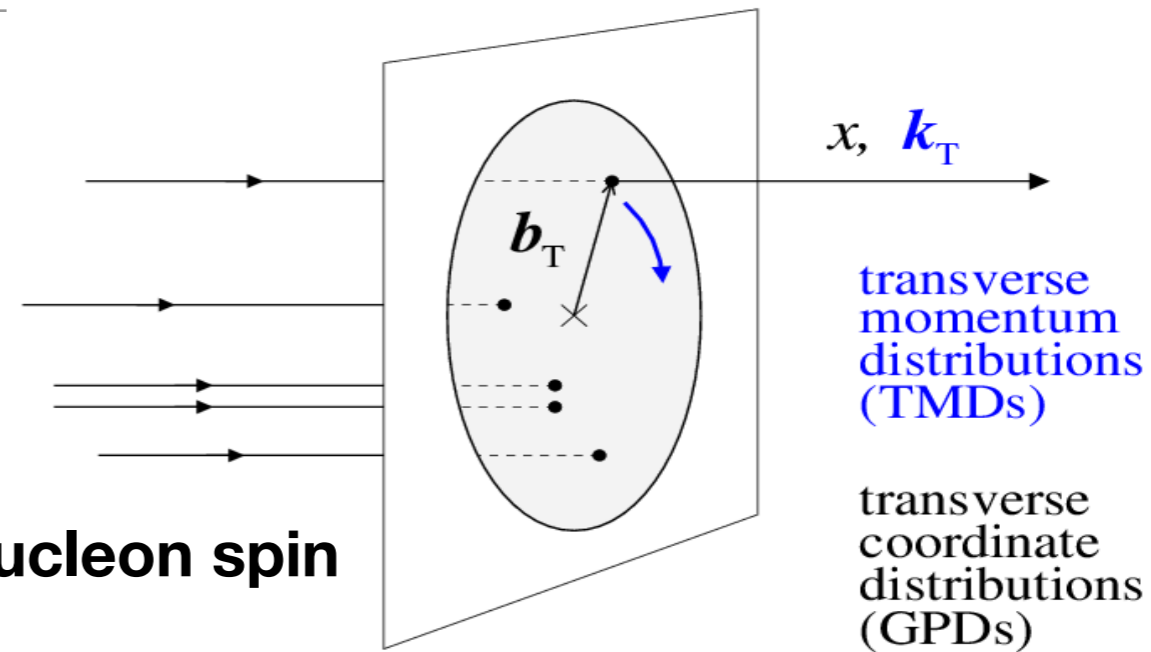
[Engelhardt, Wed 14:40]

## 3D imaging in momentum space

### Relevant for:

orbital angular momentum contributions to nucleon spin

spin-orbit correlations in hadrons



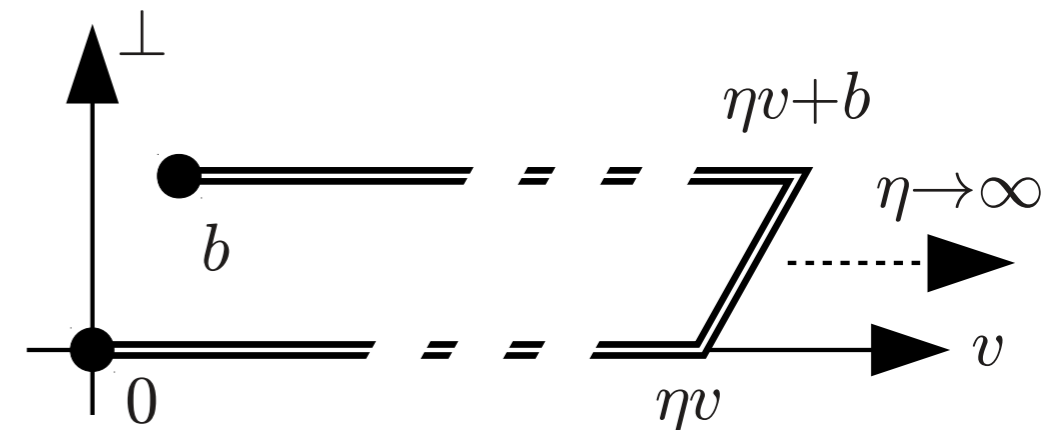
Accessed via Semi-Inclusive Deep Inelastic Scattering or Drell-Yan  
[JLab, HERMES, COMPASS, RHIC]

### Relevant matrix elements:

$$\langle P | \bar{q}(0) \Gamma \mathcal{U}[0, \eta v, \eta v + b, b] q(b) | P \rangle$$

Gauge link “along the light cone”

[LHPC: PRD85 (2012) 094510]

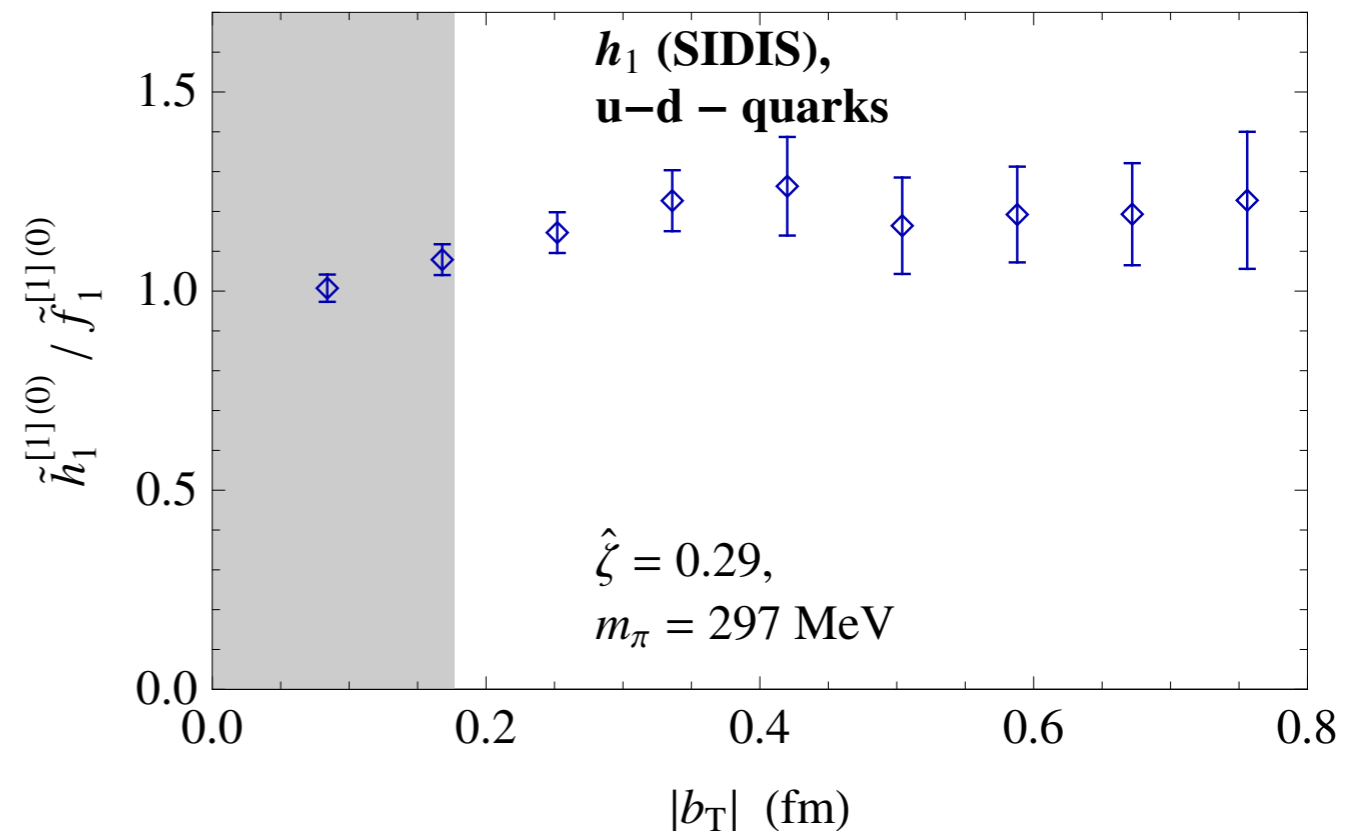
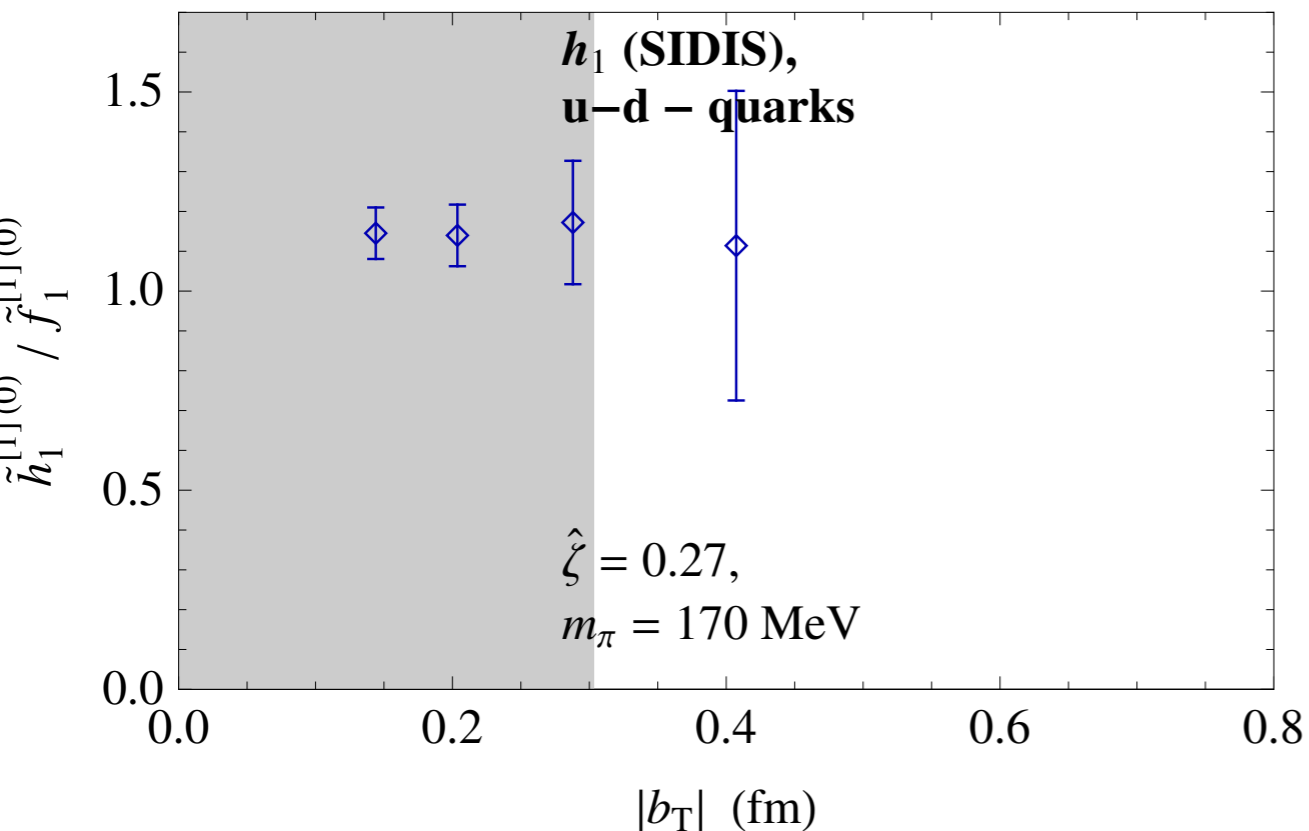


# Transverse Momentum Distributions

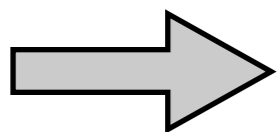
[Engelhardt, Fri 15:40 (WD & ME)]

**Nucleon update with  $m_\pi \approx 170$  MeV (2+1 DWF)**

Dependence of SIDIS/DY limit on  $|b_T|$



**Recent investigated pion TMD [LHPC 1506.07826]**

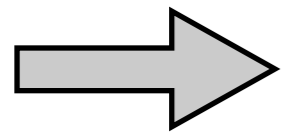


understand approach to “light cone”

# Summary & Outlook

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**Excellent progress in understanding and controlling systematic errors**



**Precise results at the physical point now achievable for**

**isovector charges**

**electromagnetic form factors**

**With additional progress in computing disconnected contributions**

**Improved understanding of long term questions, e.g.**

**decomposition of proton spin**

**role of hidden flavour**

**Right time to tackle more “exotic” quantities relevant for upcoming experiments**

**quasi-PDFs, TMDs, nuclear effects, ...**

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with dedicated lattice sessions on nuclear physics, hadron structure, finite density, ...

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