The 33rd International Symposium on Lattice Field Theory

14-18.07.2015, Kobe, Japan

Light flavour physics

Andreas Jüttner

UNIVERSITY OF Southampton

erc

Light flavours

$$u, d, s$$

Standard:

- *standard* MEs
- well controlled
- *precision!*

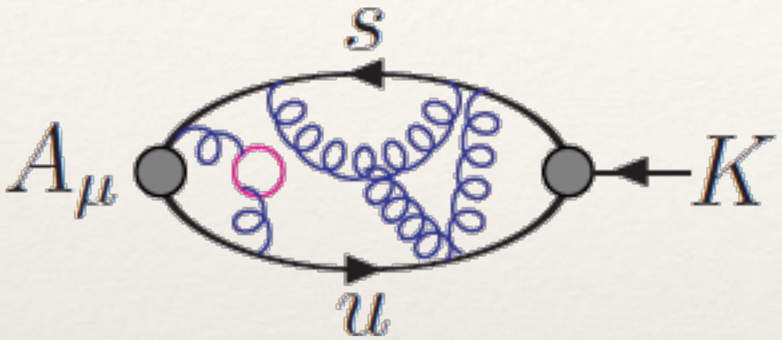
Beyond Standard:

- next level of sophistication
- hopefully well controlled
- just doable

tree K and π decays

“tree” kaon/pion decays

Leptonic kaon decay:



$$\langle 0|\bar{s}/\bar{d}\gamma_\mu\gamma_5 u|K^+/\pi^+(p)\rangle = i f_{K^+/\pi^+} p_\mu$$

ratio of decay constants → ratio of CKM MEs:

$$\frac{\Gamma(K^+\to l^+\nu_l(\gamma))}{\Gamma(\pi^+\to l^+\nu_l(\gamma))} = \left(\frac{|V_{us}|}{|V_{ud}|}\frac{f_{K^+}}{f_{\pi^+}}\right)^2 \frac{m_K\left(1-m_l^2/m_K^2\right)^2}{m_\pi\left(1-m_l^2/m_\pi^2\right)^2} \underbrace{\left(1+\delta_{\rm EM}^{\rm ChPT}\right)}_{0.9930(35)}$$

Marciano, PRL. 93 (2004) 231803
hep-ph/0402299

experimental status:

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_{K^+}}{f_{\pi^+}} = 0.2758(5)$$

FLAVIA Kaon WG EPJ C 69, 399-424 (2010)
arXiv:1005.2323 KTeV, Istra, KLOE

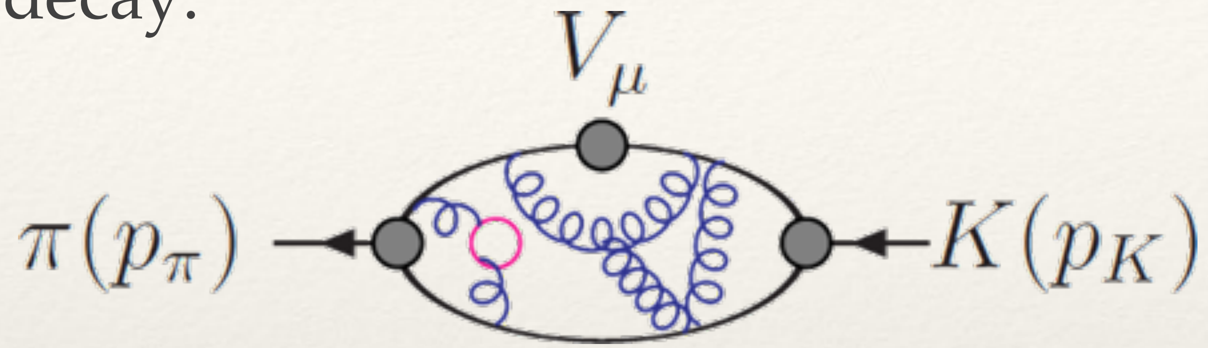
update e.g. Moulson

$$\left|\frac{V_{\rm us}}{V_{\rm ud}}\right|\frac{f_{K^+}}{f_{\pi^+}} = 0.2760(4)$$

arXiv:1411.5252

“tree” kaon/pion decays

semi-leptonic kaon decay:



matrix element and form factors:

$$\langle\pi(p_\pi)|V_\mu(0)|K(p_K)\rangle = f_+^{K\pi}(q^2)(p_K+p_\pi)_\mu + f_-^{K\pi}(q^2)(p_K-p_\pi)_\mu$$

$$\Gamma(K\to\pi l\nu) = C_K^2\frac{G_F^2 m_K^5}{192\pi^2}S_{\mathrm{EW}}(1+\delta_{SU(2)}^{\mathrm{ChPT}}+\delta_{\mathrm{EM}}^{\mathrm{ChPT}})^2\, I\,\left(f_+^{K^0\pi^-}(0)|V_{us}|\right)^2$$

charged and neutral
kaon decays

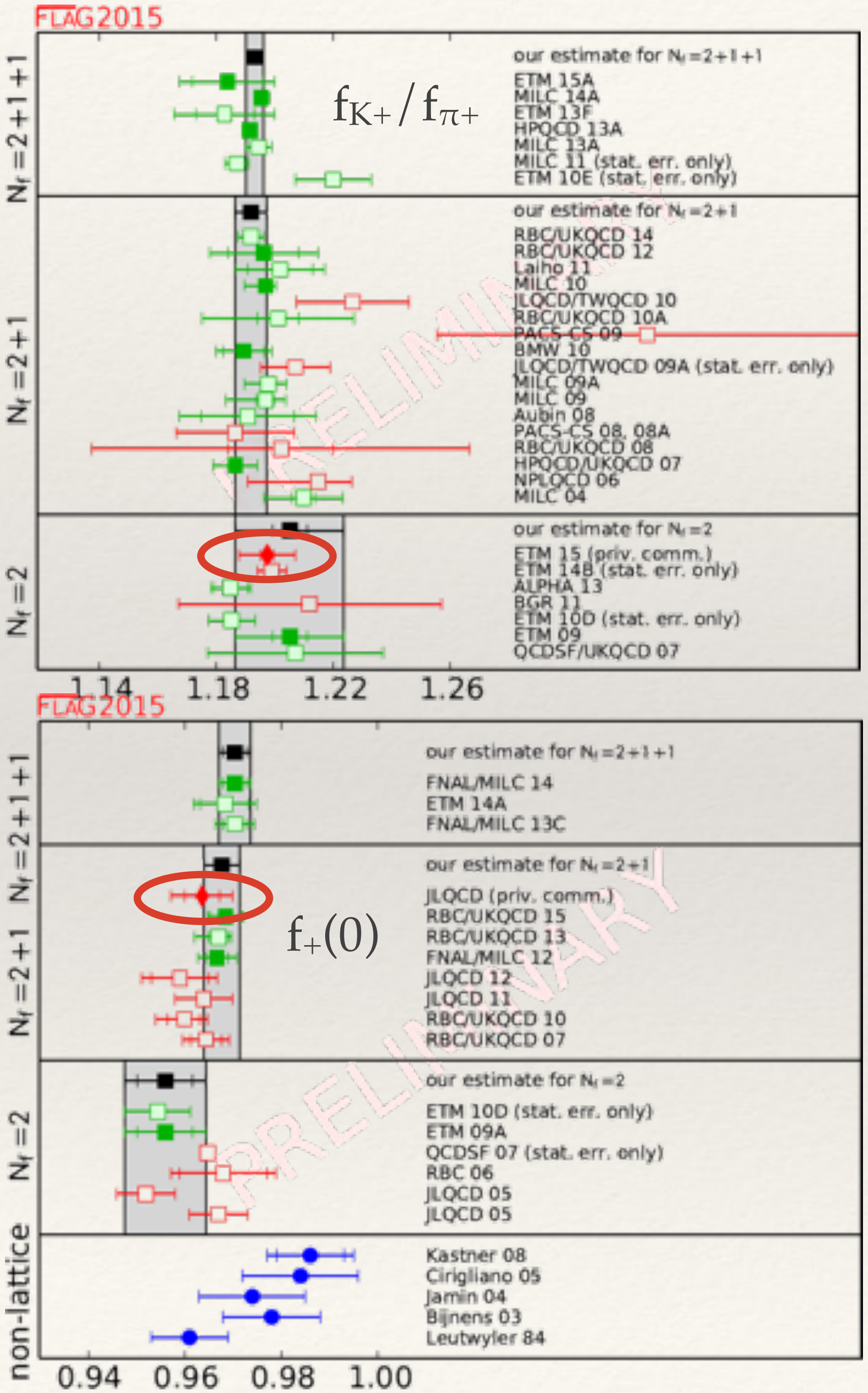
experimental status:

$$|V_{us}|f_+^{K^0\pi^-}(0) = 0.2163(5)$$

FLAVIA Kaon WG EPJ C 69, 399-424 (2010) **arXiv:1005.2323**
KTeV, Istra, KLOE

update e.g. Moulson
$|V_{us}|f_+^{K^0\pi^-}(0) = 0.2165(4)$
arXiv:1411.5252

“tree” kaon/pion decays



red diamonds added for this talk (not FLAG)

- thanks to FLAG V_{us} Working Group for prelim. plots and numbers (Boyle, Kaneko, Simula)

	f_{K^+}/f_{π^+}	$f_+(0)$
$N_f=2+1+1$	1.1933(29)	0.9704(32)
$N_f=2+1$	1.1919(54)	0.9677(37)
$N_f=2$	1.205(18)	0.9560(84)

- precision reaches down to below 0.3%!

- most of recent results from analysis including physical m_l
  dominant error then stat./a^2/FV

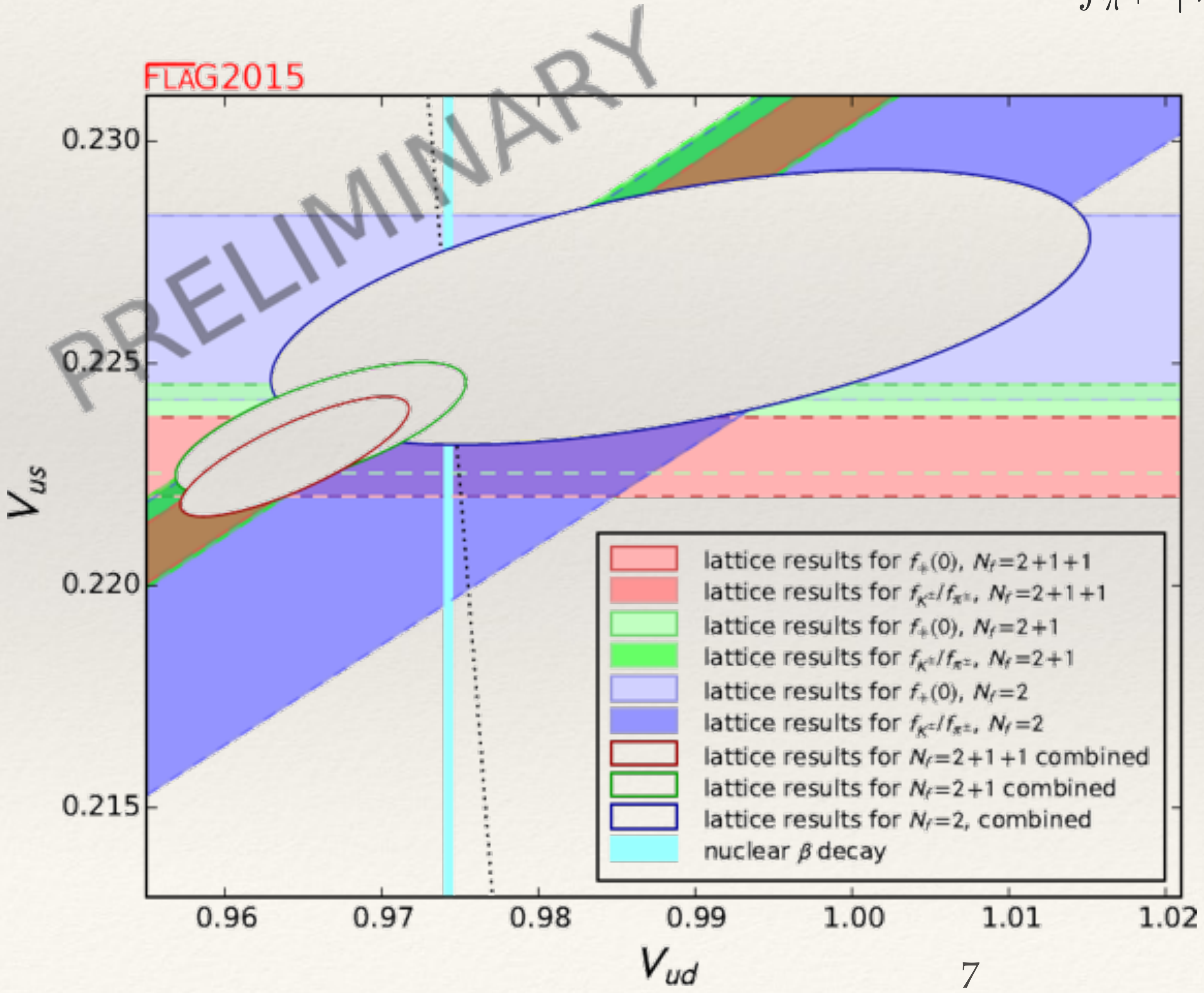
- completing the $N_f = 2$ program with physical m_l results and hence smaller errors would be very nice → ETM is now producing $N_f=2$ results with physical m_l

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \approx |V_{ud}|^2 + |V_{us}|^2 \overset{?}{=} 1$$

FLAG V$_{us}$ Working Group (Boyle, Kaneko, Simula)

$$|V_{us}| f_+^{K^0\pi^-}(0) = 0.2163(5) \qquad \frac{f_{K^+}}{f_{\pi^+}} \frac{|V_{us}|}{|V_{ud}|} = 0.2758(5)$$

FLAVIANet Kaon WG
EPJ C 69, 399-424 (2010)
arXiv:1005.2323



high precision test of SM unitarity - no worrisome tension at sub-percent-level precision

Analysis paradigm change?

Results with physical mass pions are quickly becoming standard

This is having an impact on the analysis strategies in use:

A. do a global analysis over physical AND unphysical m_l QCD simulations and make predictions in terms of global fit

B. take physical m_l result as what it is and use unphysical point results merely to correct (i.e. interpolate) for mistuning in quark masses

A. seems preferable if model/EFT is 100% trustworthy → reduce stat. error

B. preferable in cases where reliability of fit models/EFT is questionable
→ exclude unknown systematics?

Example (I) - f_K/f_π

RBC/UKQCD	arXiv:1411.7017	2+1	dwf	**“overweighting”**
HPQCD	PRD88, 074504 (2013) arXiv:1303.1670	2+1+1	stagg.	**“global analysis”**
FNAL/MILC	PRD90 (2014) 7, 074509 arXiv:1407.3772	2+1+1	stagg.	**“physical-mass”**

RBC/UKQCD “overweighting”

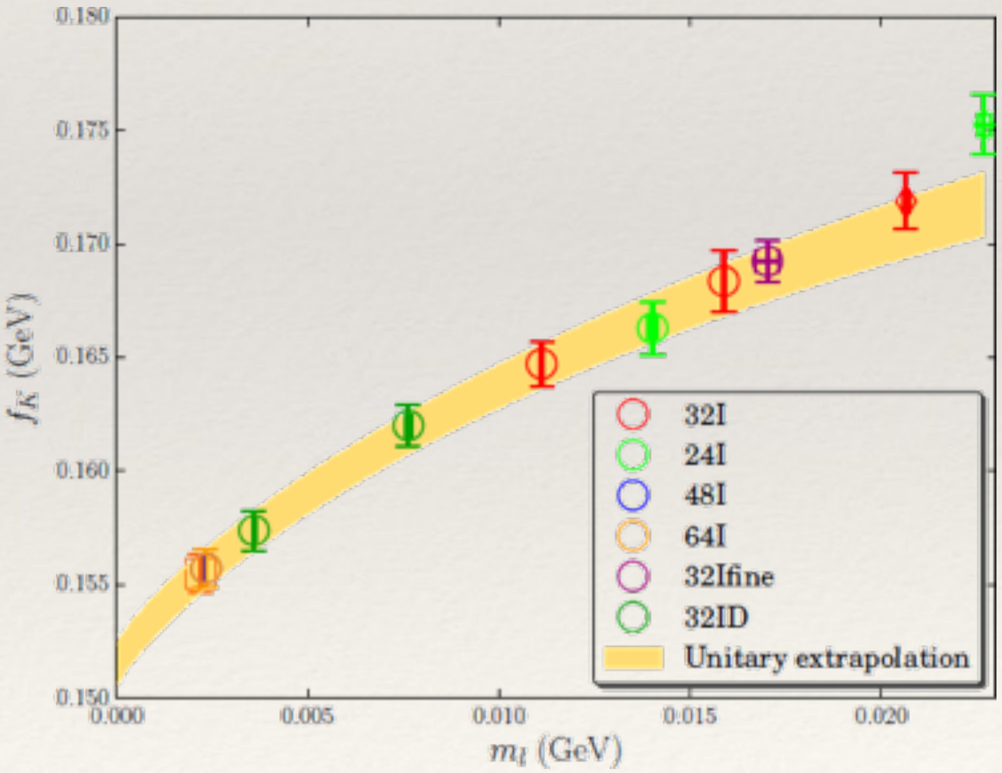
- heavy meson ChPT or polynomials
- global analysis but *overweight* physical quark mass ensembles

$$\chi^2 = \sum_i \frac{\omega_i \left[y_i - f(x_i, \vec{a})\right]^2}{\sigma_i^2}$$

$\omega_i =$ large for physical m_l ensembles

$\omega_i = 1$ otherwise

→ fit will go exactly through physical m_l ensemble and resampling ensures that stat. error after extrapolation not artificially deflated



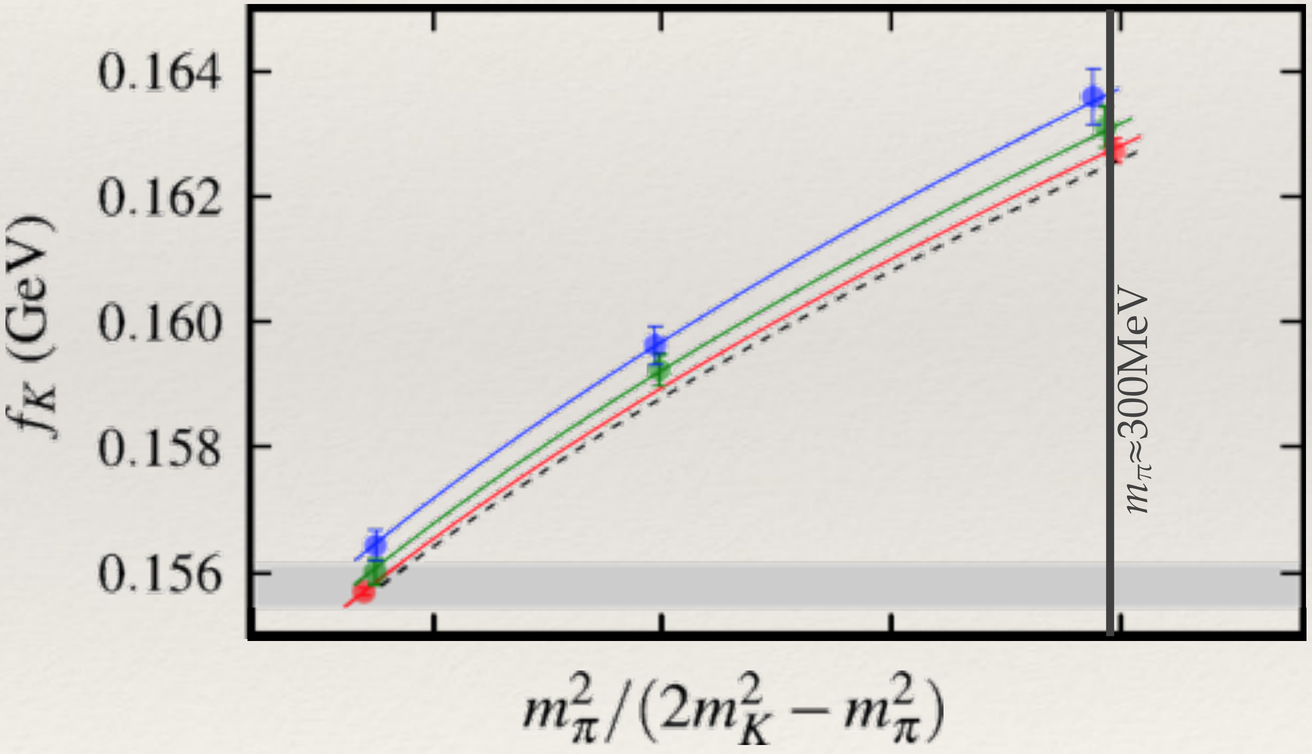
→ final error budget completely dominated by stat. error

Example (I) - f_K/f_π

RBC/UKQCD	arXiv:1411.7017	2+1	dwf	**“overweighting”**
HPQCD	PRD88, 074504 (2013) arXiv:1303.1670	2+1+1	stagg.	**“global analysis”**
FNAL/MILC	PRD90 (2014) 7, 074509 arXiv:1407.3772	2+1+1	stagg.	**“physical-mass”**

HPQCD “global analysis”:

- global fit including large dataset and many observables
- NLO PQChPT + higher order models
- large set of model parameters

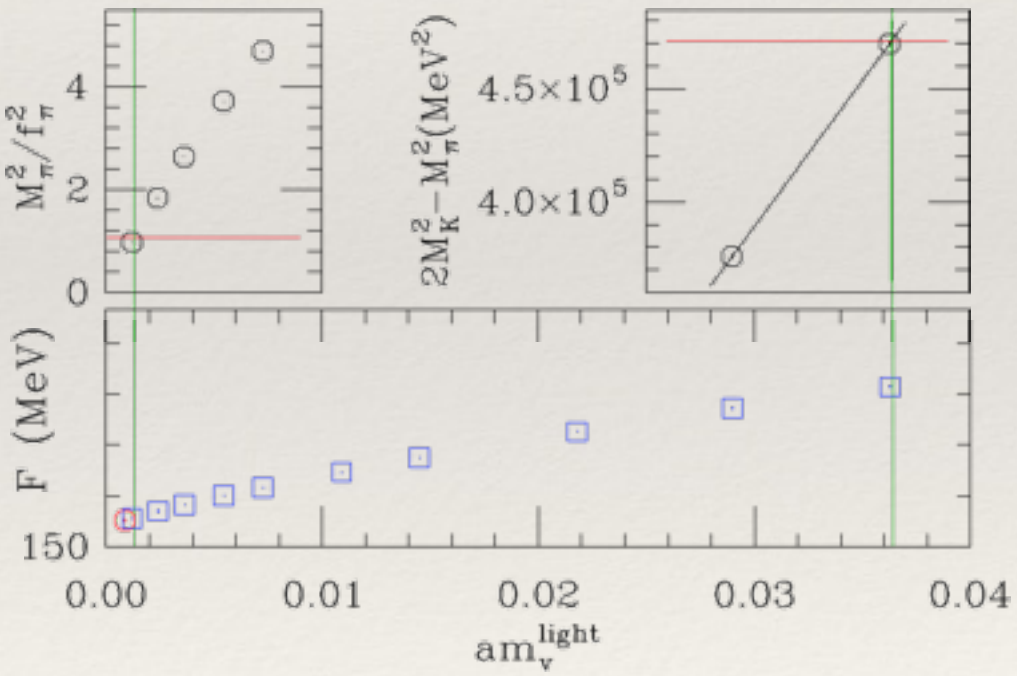


Example (I) - f_K/f_π

RBC/UKQCD	arXiv:1411.7017	2+1	dwf	**“overweighting”**
HPQCD	PRD88, 074504 (2013) **arXiv:1303.1670**	2+1+1	stagg.	**“global analysis”**
FNAL/MILC	PRD90 (2014) 7, 074509 **arXiv:1407.3772**	2+1+1	stagg.	**“physical-mass”**

FNAL/MILC “physical-mass” analysis:

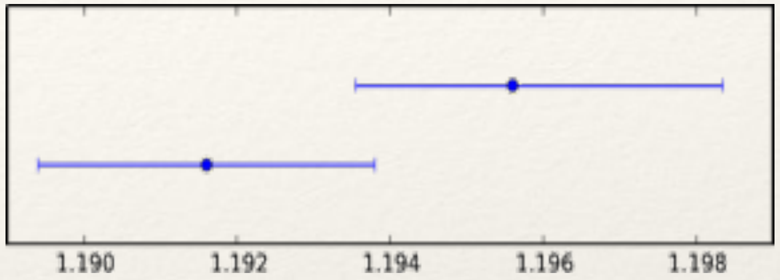
- PQ analysis on physical sea m_l
- no global fit:
  m_l, m_s and a^{-1} obtained for individual ensembles based on continuum NLO PQChPT
  subsequent continuum limit
- **reduced impact of *heavy* m_l data**
  reduced model dependence



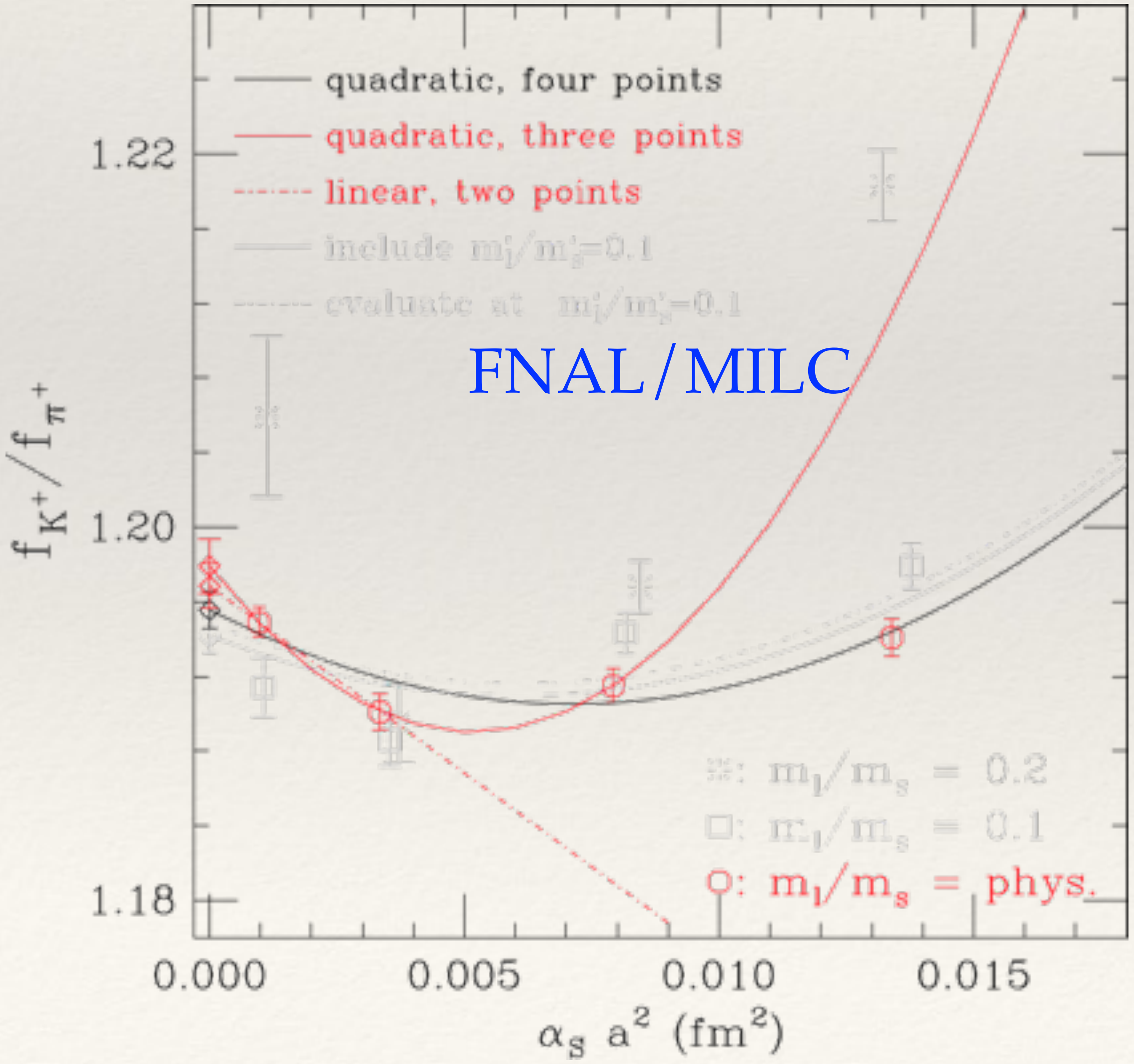
Example (I) - f_K/f_π

$$\text{FNAL/MILC}\ f_{K^+}/f_{\pi^+} = 1.1956(10)_{\rm stat}\ \left({}^{+23}_{-14}\right)_{a^2-\rm extrapol}\ (10)_{\rm FV}\ \ (5)_{\rm EM}$$

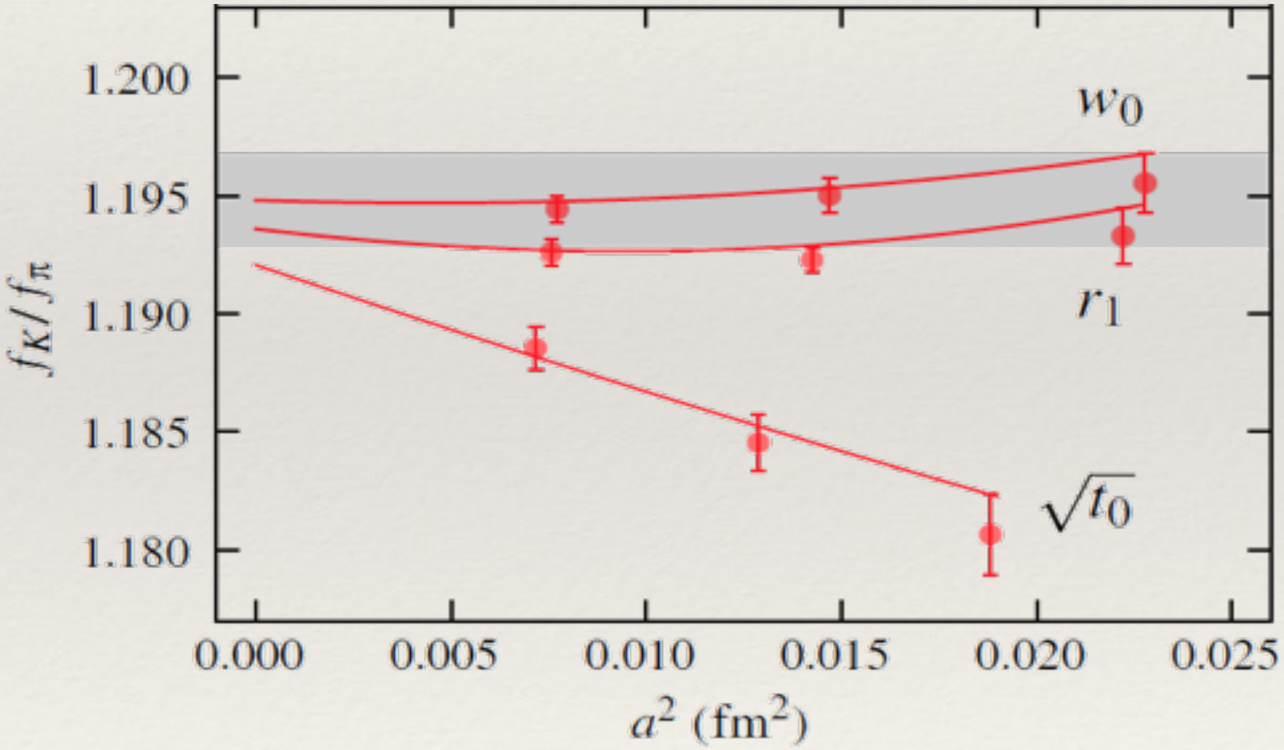
$$\text{HPQCD}\ f_{K^+}/f_{\pi^+} = 1.1916(15)_{\rm stat}\ (12)_{a^2-\rm extrapol}\ (1)_{\rm FV}\ (10)_{\rm other}$$



PRD90 (2014) 7, 074509
arXiv:1407.3772



PRD88, 074504 (2013)
arXiv:1303.1670



tension due to cutoff effect?

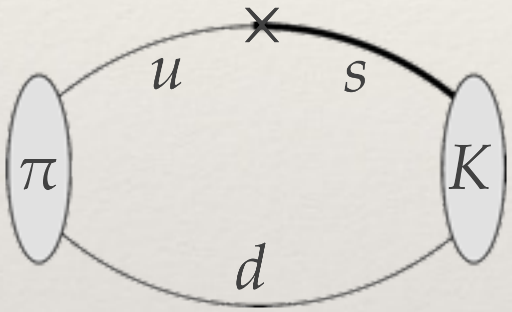
Example (II) - $f_+(0)$

RBC/UKQCD JHEP06(2015)164
arXiv:1504.01692

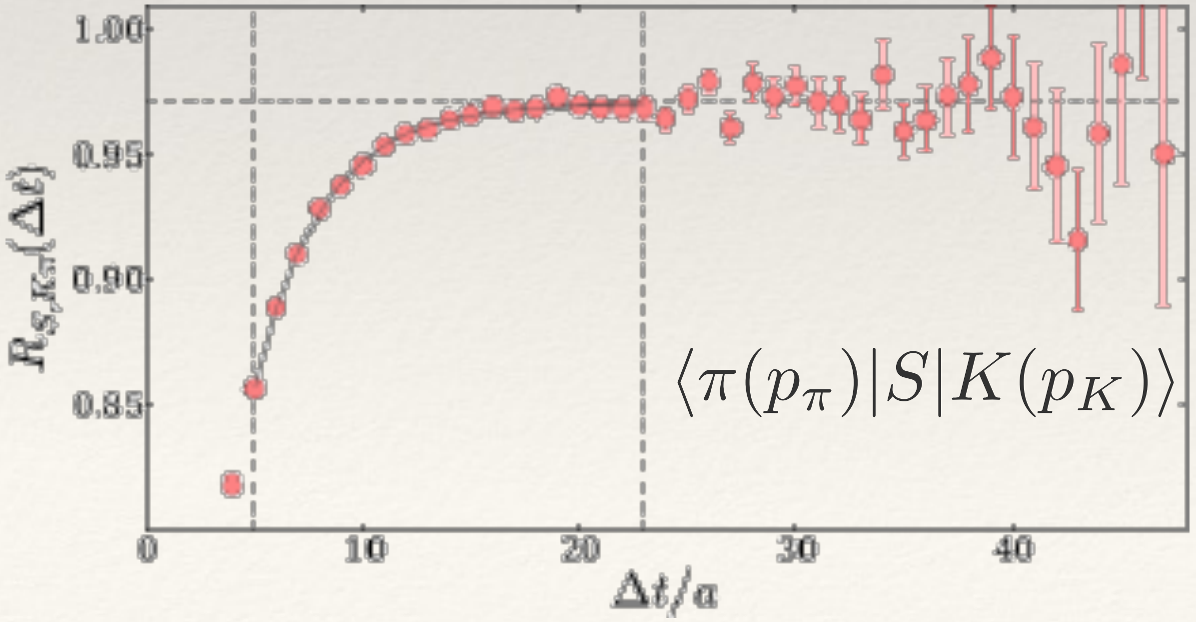
- $N_f = 2+1$ DWF
- $m_\pi \approx m_\pi^{phys}$
- $a \approx 0.08, 0.11 fm$

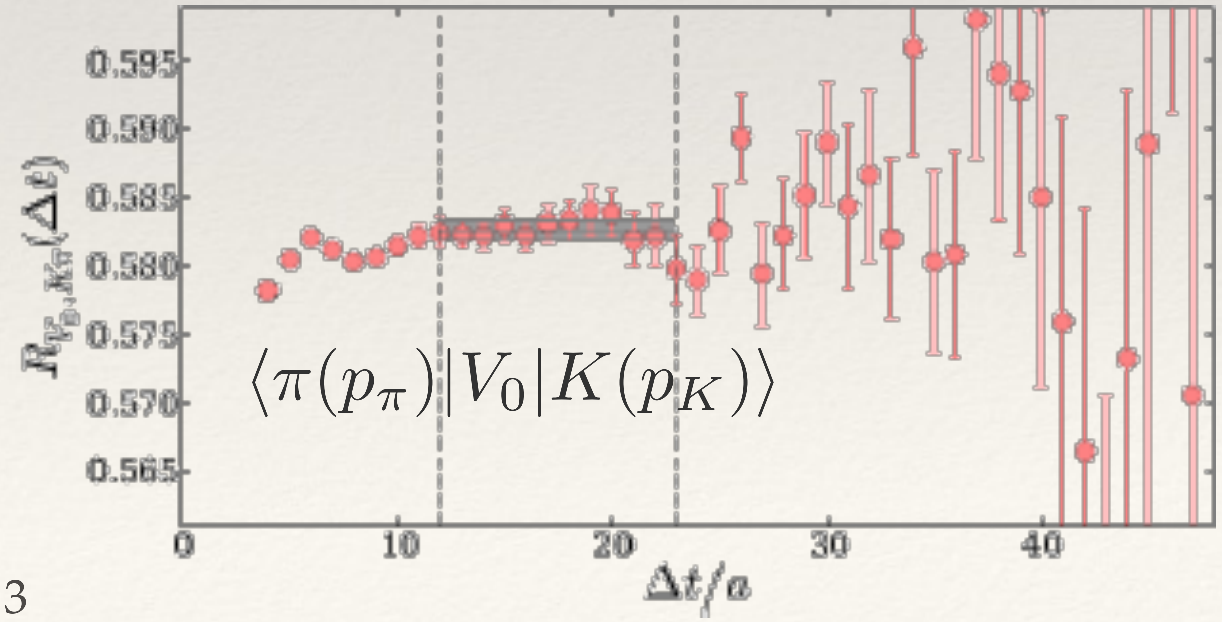
$$\langle \pi(p_\pi)|V_\mu|K(p_K)\rangle = f_+^{K\pi}(q^2)(p_K+p_\pi)_\mu + f_-^{K\pi}(q^2)(p_K-p_\pi)_\mu$$

$$\langle \pi(p_\pi)|S|K(p_K)\rangle|_{q^2=0} = f_0^{K\pi}(0)\frac{m_K^2-m_\pi^2}{m_s-m_u}$$



- $K \to \pi$ ME extracted from ratios of suitable meson 3pt functions
- generate 3pts for all all possible src-sink separations
  → **allows to study excited states contamination**





Example (II) - $f_+(0)$

RBC/UKQCD JHEP06(2015)164
arXiv:1504.01692

- expectation in NLO ChPT

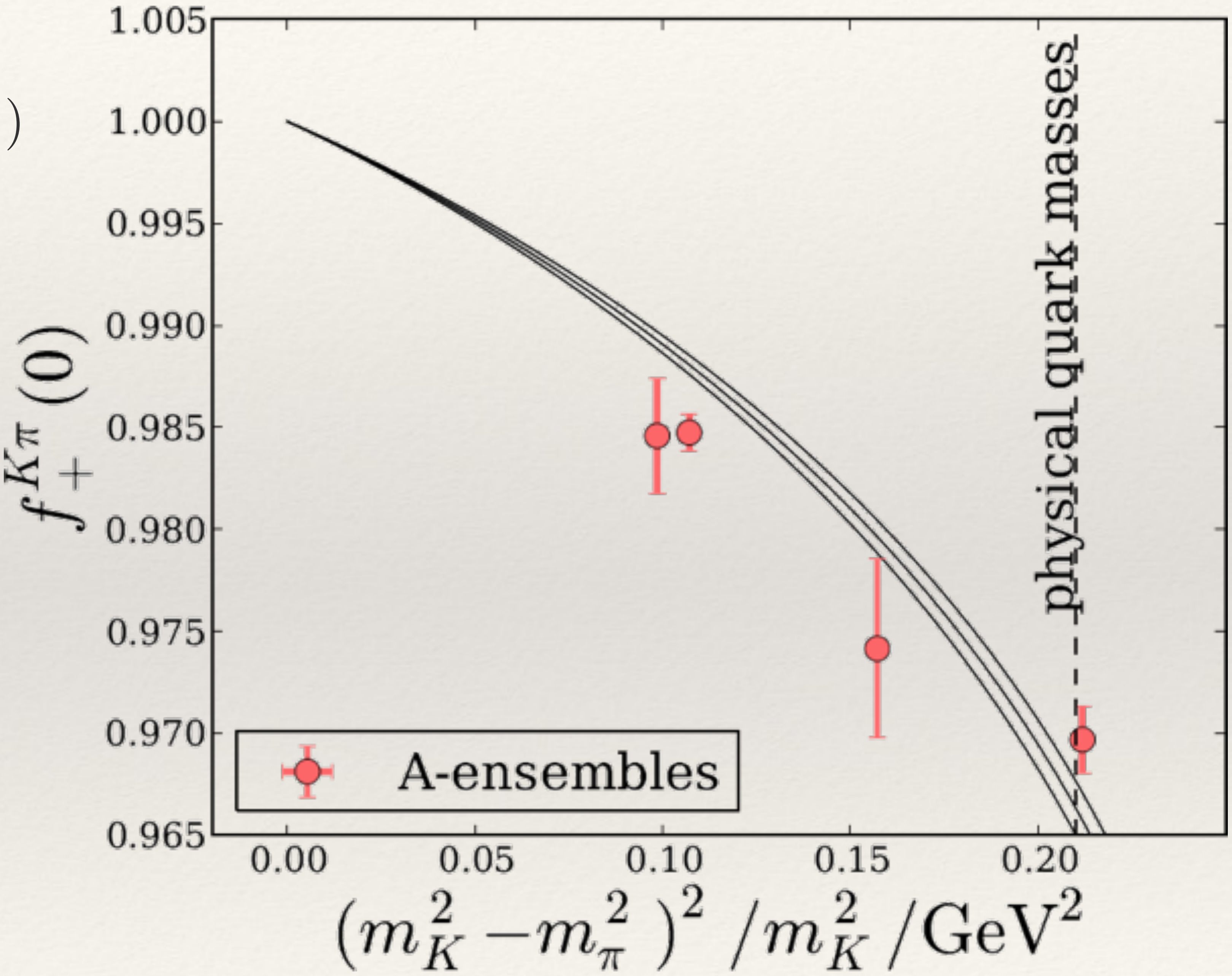
  $$f_+^{K\pi}(0) = 1 + f_2(f^2, m_\pi^2, m_K^2)$$

  Gasser & Leutwyler Nucl.Phys. B250 (1985) 517-538

  found to be incompatible with data

- need to add NNLO terms

- or: data suggests much simpler, linear ansatz to interpolate to the phys. value of the meson masses

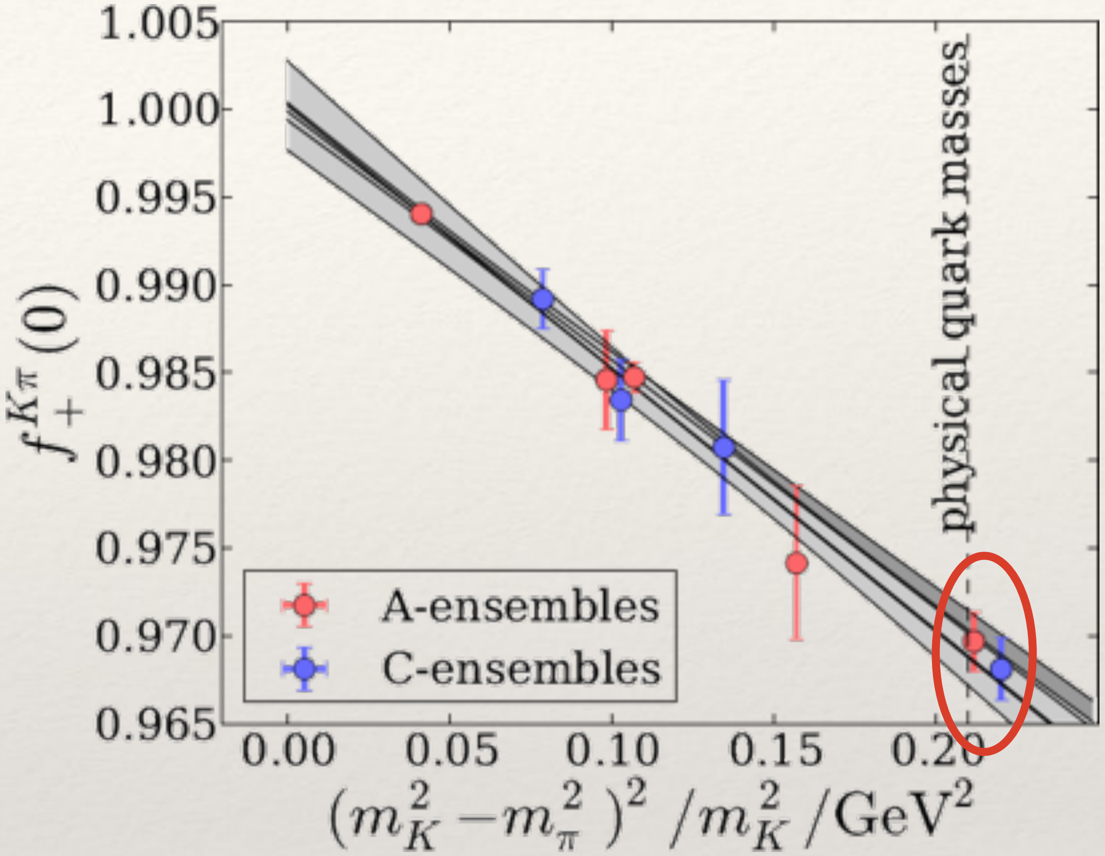


Example (II) - $f_+(0)$

RBC/UKQCD JHEP06(2015)164
arXiv:1504.01692

mass-interpolation

- just linear in SU(3)-breaking
- only tiny correction



- dependence on largest included m_π:

m_π^{min}/MeV	355	450	600
$f_+^{K\pi}(0)$	0.9690(33)	0.9689(25)	0.9691(22)

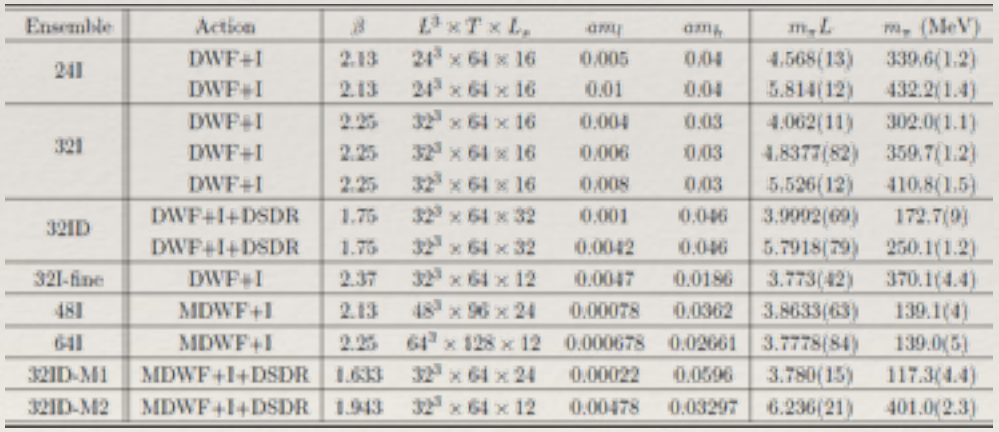
high-m_π cut: small stat., potentially syst. interpolation error
low-m_π cut : larger stat., basically no syst. interpolation error

we shouldn't forget LECs …

D. Murphy, Thu 10:40 Chiral
B. Mawhinney, Thu 11:00 Chiral

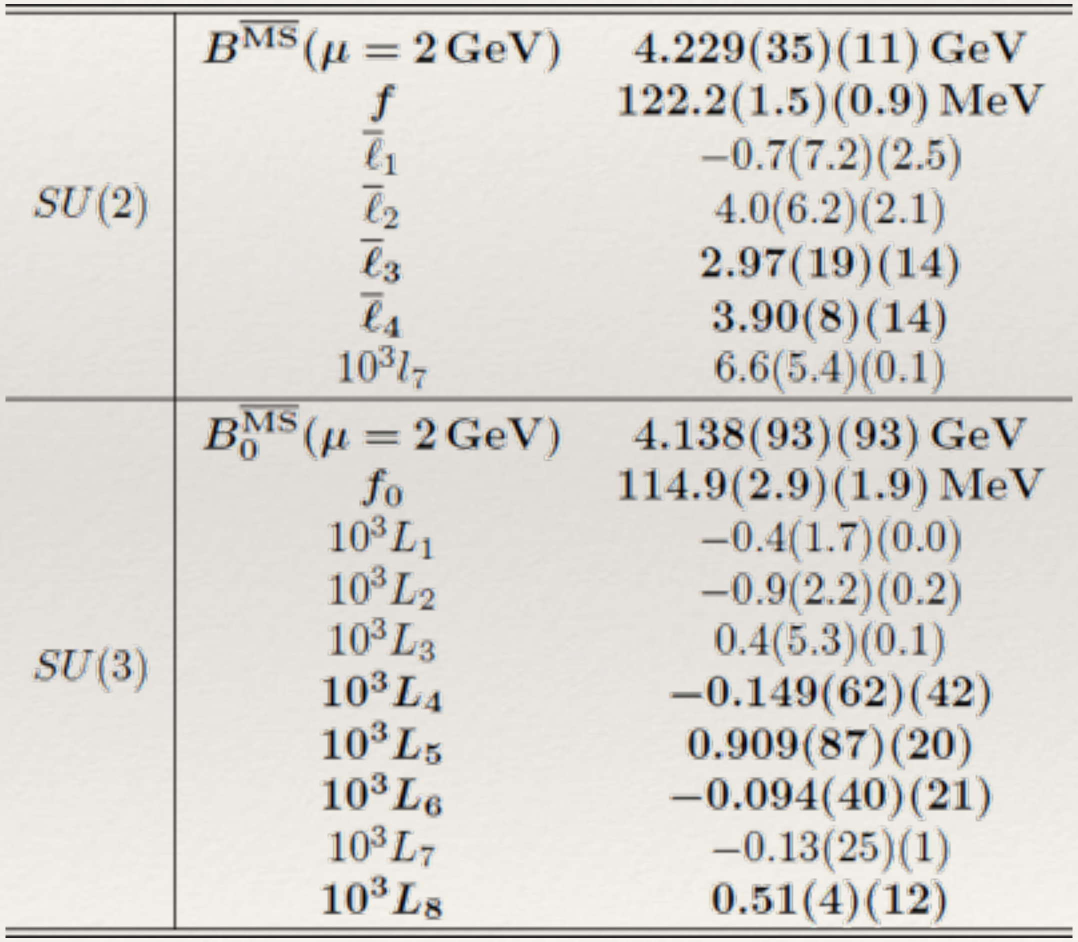
"Please do not be content with reaching physical quark masses. Extract the dependence on them, determine the LECs !" — Heiri Leutwyler at Chiral Dynamics 2015, Pisa

- comprehensive set of data for masses and decay constants based on RBC/UKQCD DWF data **arXiv:1411.7017**



Ensemble	Action	β	$L^3 \times T \times L_s$	am_l	am_h	$m_\pi L$	m_π (MeV)
24I	DWF+I	2.13	$24^3 \times 64 \times 16$	0.005	0.04	4.568(13)	339.6(1.2)
	DWF+I	2.13	$24^3 \times 64 \times 16$	0.01	0.04	5.814(12)	432.2(1.4)
32I	DWF+I	2.25	$32^3 \times 64 \times 16$	0.004	0.03	4.062(11)	302.0(1.1)
	DWF+I	2.25	$32^3 \times 64 \times 16$	0.006	0.03	4.8377(82)	359.7(1.2)
	DWF+I	2.25	$32^3 \times 64 \times 16$	0.008	0.03	5.526(12)	410.8(1.5)
32ID	DWF+I+DSDR	1.75	$32^3 \times 64 \times 32$	0.001	0.046	3.9992(69)	172.7(9)
	DWF+I+DSDR	1.75	$32^3 \times 64 \times 32$	0.0042	0.046	5.7918(79)	250.1(1.2)
32I-fine	DWF+I	2.37	$32^3 \times 64 \times 12$	0.0047	0.0186	3.773(42)	370.1(4.4)
48I	MDWF+I	2.13	$48^3 \times 96 \times 24$	0.00078	0.0362	3.8633(63)	139.1(4)
64I	MDWF+I	2.25	$64^3 \times 128 \times 12$	0.000678	0.02661	3.7778(84)	139.0(5)
32ID-M1	MDWF+I+DSDR	1.633	$32^3 \times 64 \times 24$	0.00022	0.0596	3.780(15)	117.3(4.4)
32ID-M2	MDWF+I+DSDR	1.943	$32^3 \times 64 \times 12$	0.00478	0.03297	6.236(21)	401.0(2.3)

- SU(2) and SU(3) PQ ChPT NLO, NNLO



$SU(2)$	$B^{\overline{\mathrm{MS}}}(\mu = 2\,\mathrm{GeV})$	$4.229(35)(11)\,\mathrm{GeV}$
	f	$122.2(1.5)(0.9)\,\mathrm{MeV}$
	$\bar{\ell}_1$	$-0.7(7.2)(2.5)$
	$\bar{\ell}_2$	$4.0(6.2)(2.1)$
	$\bar{\ell}_3$	$2.97(19)(14)$
	$\bar{\ell}_4$	$3.90(8)(14)$
	$10^3 l_7$	$6.6(5.4)(0.1)$
$SU(3)$	$B_0^{\overline{\mathrm{MS}}}(\mu = 2\,\mathrm{GeV})$	$4.138(93)(93)\,\mathrm{GeV}$
	f_0	$114.9(2.9)(1.9)\,\mathrm{MeV}$
	$10^3 L_1$	$-0.4(1.7)(0.0)$
	$10^3 L_2$	$-0.9(2.2)(0.2)$
	$10^3 L_3$	$0.4(5.3)(0.1)$
	$10^3 L_4$	$-0.149(62)(42)$
	$10^3 L_5$	$0.909(87)(20)$
	$10^3 L_6$	$-0.094(40)(21)$
	$10^3 L_7$	$-0.13(25)(1)$
	$10^3 L_8$	$0.51(4)(12)$

Nice consitency check: LECs used to predict Kπ scattering length, agrees with recent result with physical m_l

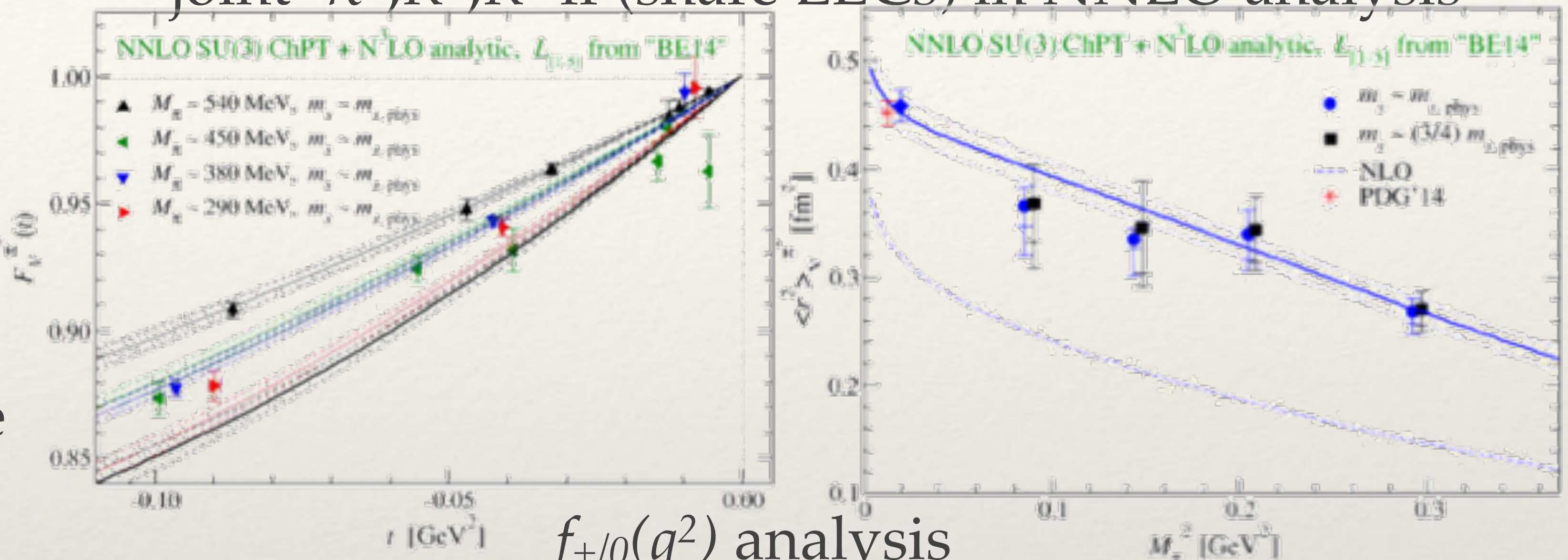
T. Janowski, Tue 14:40 HadSpec

we shouldn’t forget LECs …

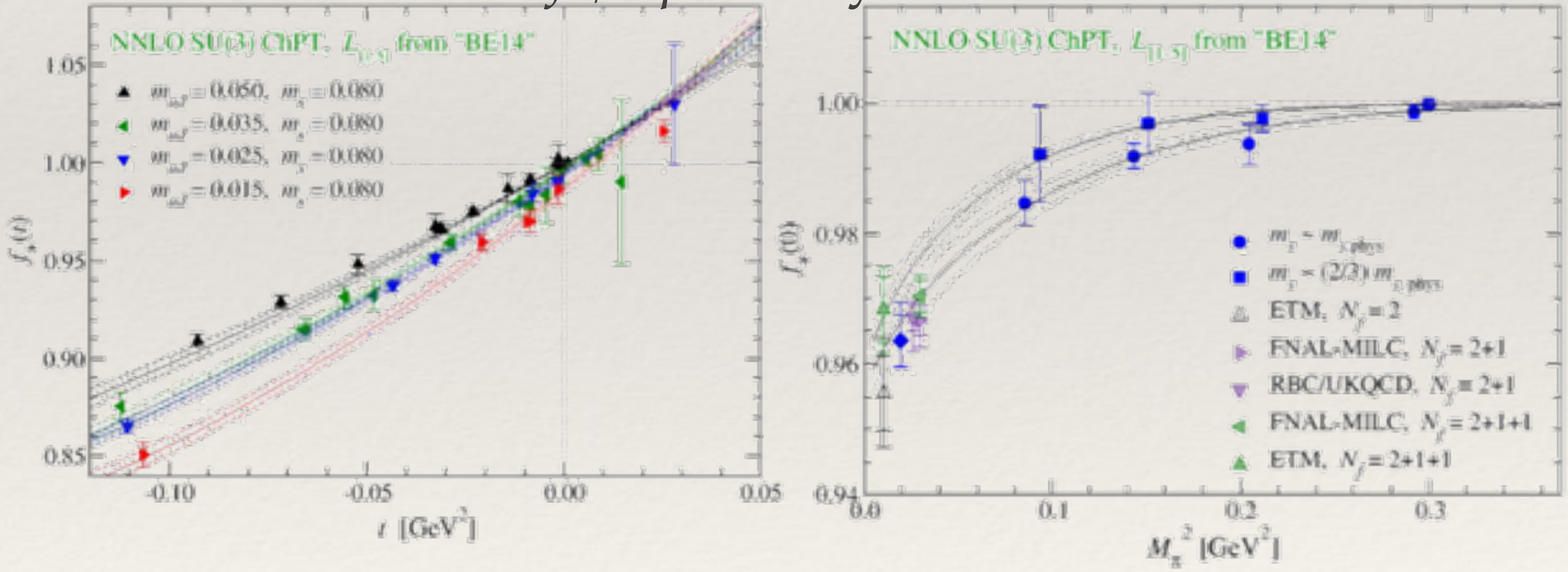
T. Kaneko, Tue 16:30, Weak

- JLQCD
- N_f=2+1 overlap fixed Q
- a=0.11fm
- m_π=290-540MeV
- $m_\pi L$≳4
- except for a^2 cont. ChPT applicable here: NNLO (+N³LO terms) with $L_{1\text{-}8}$ from Bijnens, Ecker Ann.Rev.Nucl.Part.Sci. 64 (2014) 149-174 [arXiv:1405.6488]
- EM ff and $K\to\pi$ ff share LECs do combined fit
- besides results for charge radii and $K\to\pi$ ff also predictions for ff shape and LECs

joint π^+,K^+,K^0 ff (share LECs) in NNLO analysis



$f_{+/0}(q^2)$ analysis



$$f_+(0) = 0.9636(36)_{\rm stat} \left(^{+0}_{-45}\right)_{\rm N^3NLO} \left(^{+41}_{-3}\right)_{L_i} (29)_{a\neq 0} = 0.9636 \left(^{+62}_{-65}\right)$$

ETM 2+1+1 $f_+(0)$ L. Riggio, Fri 14:00, Weak

Including QED in meson decay MEs

- Precision on *standard* MEs such that EM and strong isospin effects important
- we should go beyond ChPT treatment
- need to understand conceptually how this can be done (beyond spectral quantities)

- leptonic decay at $O(\alpha^0)$:

$$\Gamma(\pi^+ \to l^+ \nu_l) = \frac{G_F^2 |V_{ud}|^2 f_\pi^2}{8\pi} m_\pi m_l^2 \left(1 - \frac{m_l^2}{m_\pi^2}\right)^2$$

- including elm. effects @ $O(\alpha)$:

$$\begin{aligned} \Gamma(\pi^+ \to l^+ \nu_l(\gamma)) &= \Gamma(\pi^+ \to l^+ \nu_l) + \Gamma(\pi^+ \to l^+ \nu_l \gamma) \\ &\equiv \quad\quad \Gamma_0 \quad\quad + \quad\quad \Gamma_1 \end{aligned}$$

IR div. cancel between terms on r.h.s. between virtual and real photons (Bloch Nordsieck)

Including QED in meson decay MEs

Carrasco et al. PRD 91 074506 (2015) **arXiv:1502.00257**

- cut on small photon momentum < $\Delta E \rightarrow \gamma$ *sees* point-like π
  $\Delta E \approx 20$MeV experimentally accessible and π point like

point approximation

$$\Gamma(\Delta E) = \lim_{V\to\infty} \boxed{\left(\Gamma_0 - \Gamma_0^{\mathrm{pt}}\right)} + \lim_{V\to\infty} \boxed{\left(\Gamma_0^{\mathrm{pt}} + \Gamma_1^{\mathrm{pt}}(\Delta E)\right)}$$

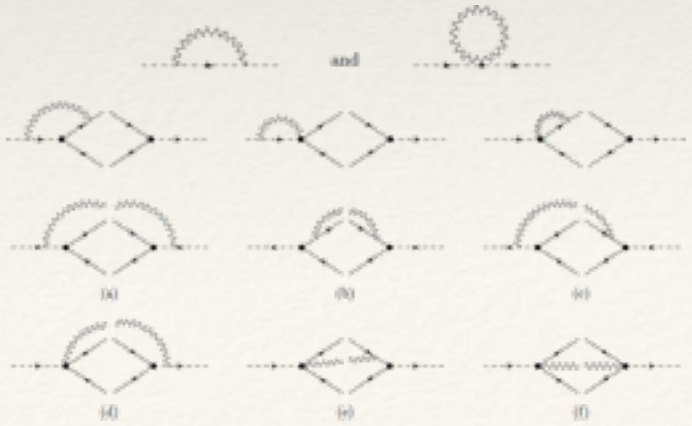
$\Gamma(\pi^+ \rightarrow l^+ \nu_l)$
lattice and analytical
finite V

$\Gamma(\pi^+ \rightarrow l^+ \nu_l \gamma(\Delta E))$
analytically in $V \rightarrow \infty$

both terms separately IR finite, gauge invariant on its own

- analytical calculation for pt. approximation is done:

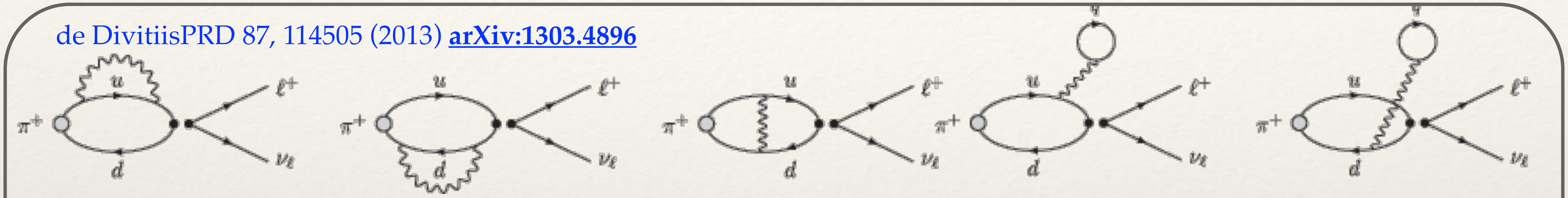
$$\mathcal{L}_{\pi\text{-}\ell\text{-}\nu_\ell} = i\, G_F f_\pi V_{ud}^* \left\{(\partial_\mu - ieA_\mu)\pi\right\} \left\{\bar{\psi}_{\nu_\ell} \frac{1+\gamma_5}{2} \gamma^\mu \psi_\ell\right\} + \mathrm{H.C.}\,.$$



Including QED in meson decay MEs

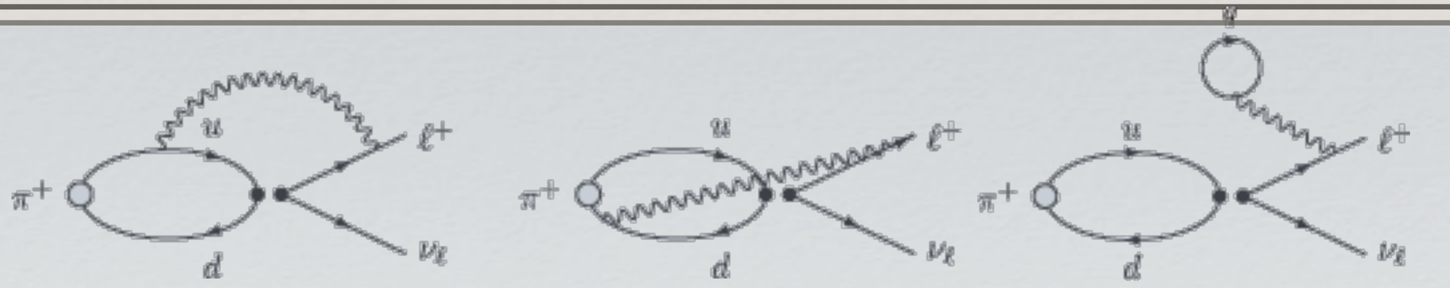
C. Sachrajda, Tue 16:50, Weak

de DivitiisPRD 87, 114505 (2013) **arXiv:1303.4896**



$$C_1(t) = -\frac{1}{2}\int d^3\vec{x}d^4x_1d^4x_2\sum_{\vec{x}}\langle 0|T\left\{J_W^\nu(0)j_\mu(x_1)j_\mu(x_2)\phi^\dagger(\vec{x},-t)\right\}|0\rangle\Delta(x_1,x_2)$$

$$C_0(t)+C_1(t) \simeq \frac{Z^\phi}{2m_\pi}e^{-m_\pi t}\mathcal{A} \simeq \frac{Z_0^\phi+\delta Z^\phi}{2(m_\pi^0+\delta m_\pi)}e^{-m_\pi^0 t}\left(1-\delta m_\pi t\right)\left(\mathcal{A}_0+\delta\mathcal{A}\right)$$



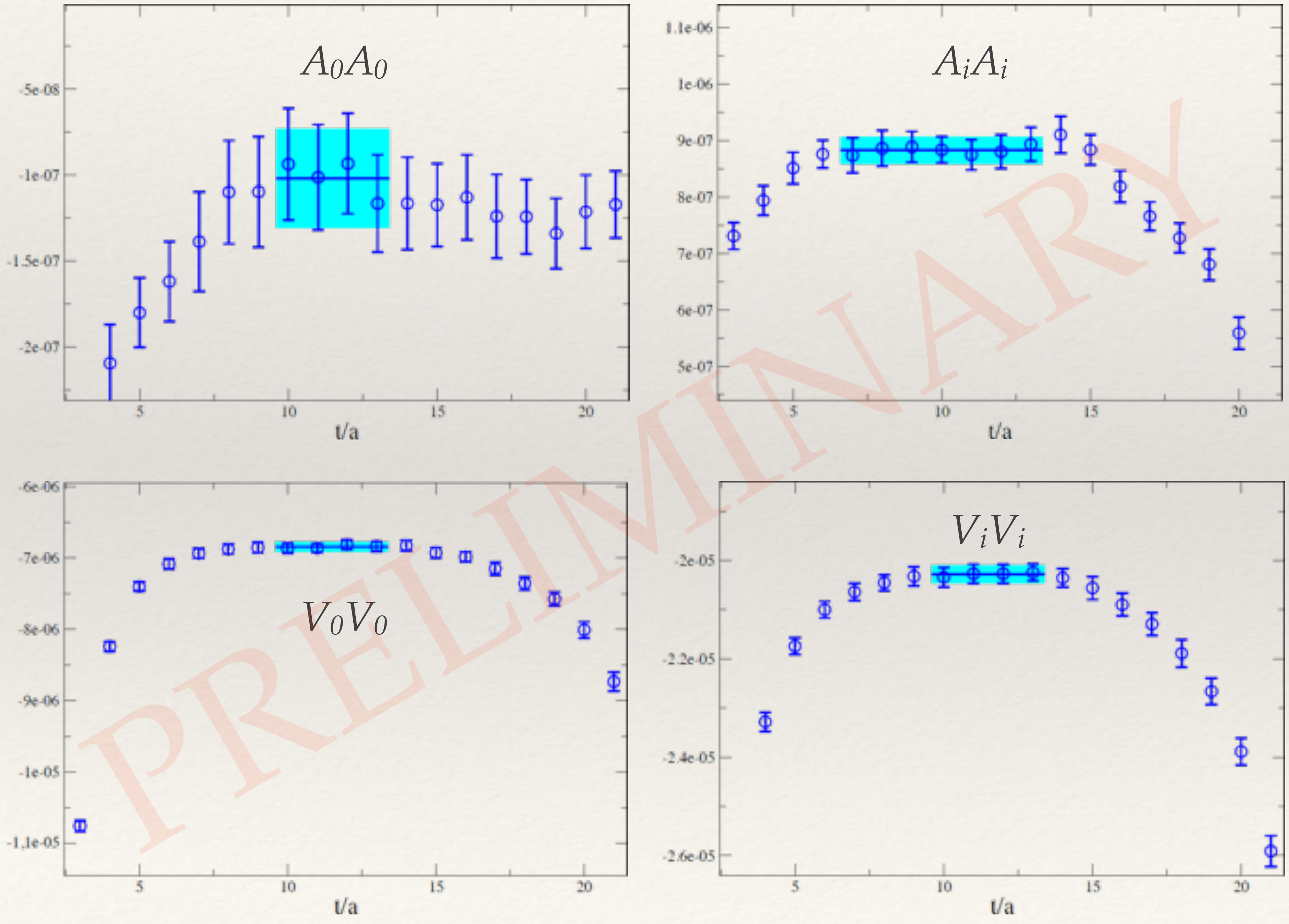
$$C_1(t)_{\alpha\beta} = -\int d^3\vec{x}d^4x_1d^4x_2\langle 0|T\left\{J_W^\nu(0)j_\mu(x_1)\phi^\dagger(\vec{x},-t)\right\}|0\rangle$$
$$\times\Delta(x_1,x_2)\left(\gamma_n u(1-\gamma^5)S(0,x_2)\gamma_\mu\right)_{\alpha\beta}e^{E_l t_2-i\vec{p}_l\cdot\vec{x}_2}$$

convergent by
energy / momentum conservation

Including QED in meson decay MEs

24^3; $m_\pi \approx 500$MeV

C. Sachrajda, Tue 16:50, Weak



- first time ever conceptually clean attempt of calculation of leptonic decay at $O(\alpha)$
- disconnected pieces need to be included
- Γ_0 works, now needs to be combined wt. analytical results for Γ_0^{pt}, $\Gamma_1^{\mathrm{pt}}(\Delta E)$
- ~20% stat. error would be sufficient for use in phenomenology

neutral kaon system

Neutral meson mixing - Kaon

Weak eigenstate:

$$K_L = \frac{1}{\sqrt{1+|\bar{\epsilon}|^2}}\left(K_2 + \bar{\epsilon} K_1\right)$$

CP-odd CP-even

$$\begin{aligned} K_1 &= \tfrac{1}{\sqrt{2}}\left(K^0 + \bar{K}^0\right) \\ K_2 &= \tfrac{1}{\sqrt{2}}\left(K^0 - \bar{K}^0\right) \end{aligned}$$

CP-even $\pi\pi$

direct → ε′

KTeV, NA48 1999

C. Kelly, Wed 12:00 plenary

CP-even $\pi\pi$

indirect → $\epsilon_K = \bar{\epsilon} + i\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}$

Cronin Fitch 1964

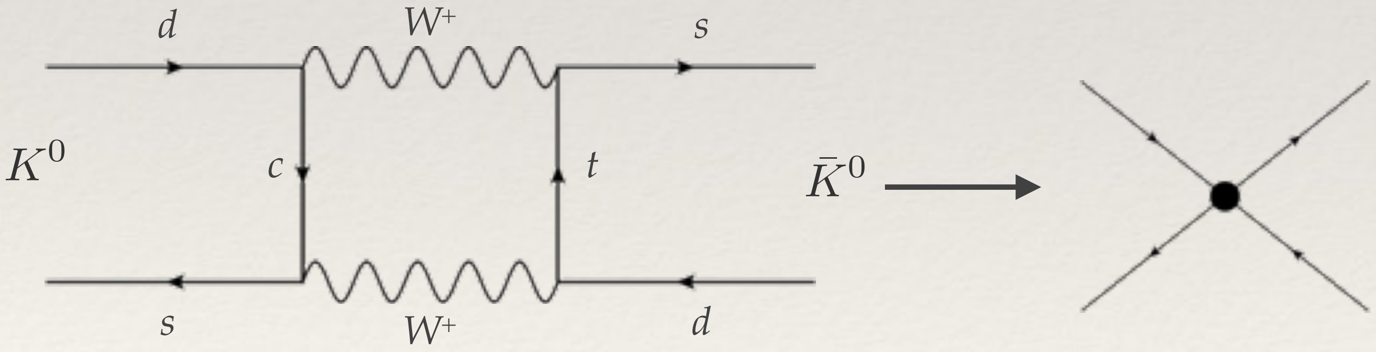
experimentally: $|\epsilon_K| = 2.228(11)\cdot 10^{-3}$ PDG

Neutal kaon mixing - time evolution

$$i\frac{d}{dt}\begin{pmatrix} K^0 \\ \bar{K}^0 \end{pmatrix} = \left[\begin{pmatrix} M_{00} & M_{0\bar{0}} \\ M_{\bar{0}0} & M_{\bar{0}\bar{0}} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{00} & \Gamma_{0\bar{0}} \\ \Gamma_{\bar{0}0} & \Gamma_{\bar{0}\bar{0}} \end{pmatrix}\right]\begin{pmatrix} K^0 \\ \bar{K}^0 \end{pmatrix}$$

to second order in H_W:

$$\begin{aligned} M_{\alpha',\alpha} &= \delta_{\alpha,\alpha'} m_K + \langle\alpha|H_W|\alpha'\rangle + \mathcal{P}\sum_\lambda \frac{\langle\alpha'|H_W|\lambda\rangle\langle\lambda|H_W|\alpha\rangle}{m_K - E_\lambda} \\ \Gamma_{\alpha',\alpha} &= 2\pi\sum_\alpha \langle\alpha'|H_W|\lambda\rangle\langle\lambda|H_W|\alpha\rangle\delta(E_\lambda - m_K) \end{aligned}$$



$$\epsilon_K = e^{i\phi}\sin\phi\left(\frac{\mathrm{Im}\langle\bar{K}^0|H_W^{\Delta S=2}|K^0\rangle}{2\mathrm{Re}M_{0\bar{0}}} + \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)$$

Conventional analysis:

- ϕ, ΔM_K, ξ=Im A_0/Re A_0 from experiment and $\mathrm{Im}\langle\bar{K}^0|H_W^{\Delta S=2}|K^0\rangle$ from the lattice

Neutral meson mixing - kaon

In SM W-boson exchange implies V – A structure, beyond SM other operators possible e.g. RBC/UKQCD PRD 86 (2012), ETM JHEP 1303 (2013) 08, SWME PRD 88 (2013) 07

$$\begin{pmatrix} \boxed{1_{(27,1)}} & 0 & 0 \\ 0 & \boxed{(2\times 2)_{(6,\bar{6})}} & 0 \\ 0 & 0 & \boxed{(2\times 2)_{(8,8)}} \end{pmatrix}$$

$$\begin{aligned} O_1^{\Delta S=2} &= (\bar{s}_\alpha \gamma_\mu (1-\gamma_5) d_\alpha)(\bar{s}_\beta \gamma_\mu (1-\gamma_5) d_\beta) \\ O_2^{\Delta S=2} &= (\bar{s}_\alpha (1-\gamma_5) d_\alpha)(\bar{s}_\beta (1-\gamma_5) d_\beta) \\ O_3^{\Delta S=2} &= (\bar{s}_\alpha (1-\gamma_5) d_\beta)(\bar{s}_\beta (1-\gamma_5) d_\alpha) \\ O_4^{\Delta S=2} &= (\bar{s}_\alpha (1-\gamma_5) d_\alpha)(\bar{s}_\beta (1+\gamma_5) d_\beta) \\ O_5^{\Delta S=2} &= (\bar{s}_\alpha (1-\gamma_5) d_\beta)(\bar{s}_\beta (1+\gamma_5) d_\alpha) \end{aligned}$$

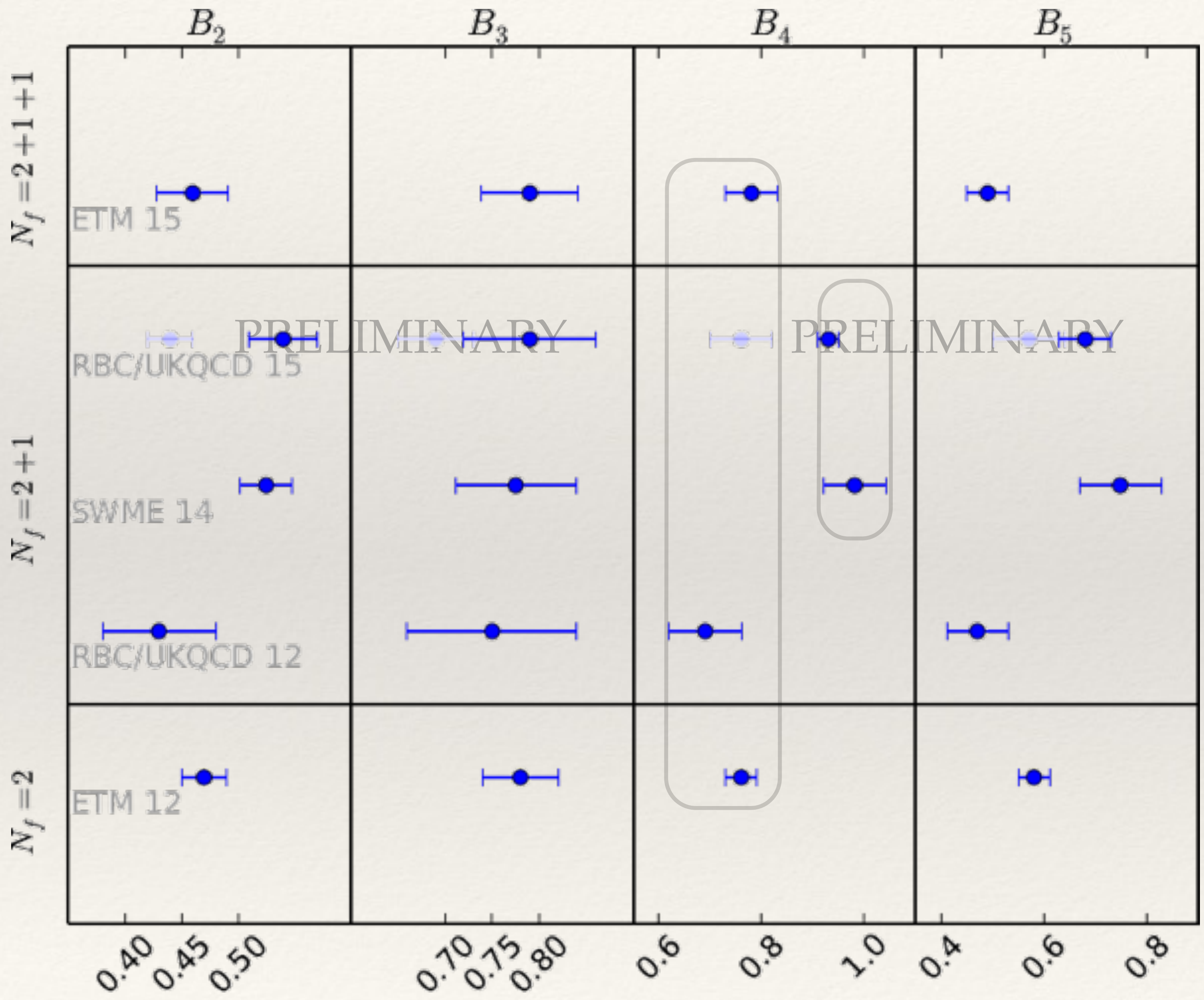
$$B_1(\mu) = \frac{\langle \bar{K}^0 | O_1^{\Delta S=2} | K^0 \rangle}{\frac{8}{3} m_K^2 f_K^2} \qquad B_i(\mu) = \frac{\langle \bar{K}^0 | O_i^{\Delta S=2} | K^0 \rangle}{N_i \frac{m_K^4 f_K^2}{(m_s+m_d)^2}}$$

$$C_i(\Lambda) \propto \frac{1}{\Lambda}$$

very stringent bounds on NP scales ~TeV!

(e.g. ETM JHEP03(2013)089)

BSM-B_K



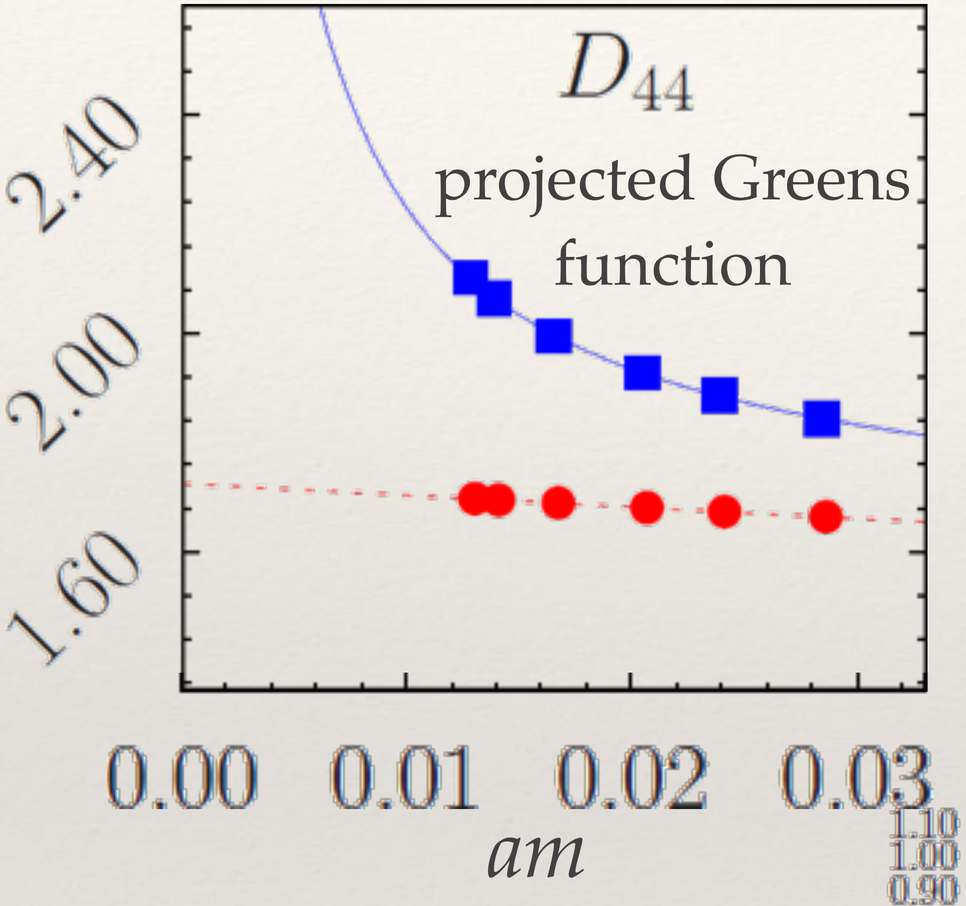
- tension between groups for e.g. B_4
- ETM consistent(ish) between 2- and 2+1+1 flavour
- quite some movement in RBC/UKQCD

some systematic effect? candidates:

- **NPR?**
- **matching?**
- **continuum limit?**

NPR - RI/MOM

ETM arXiv:1505.06639

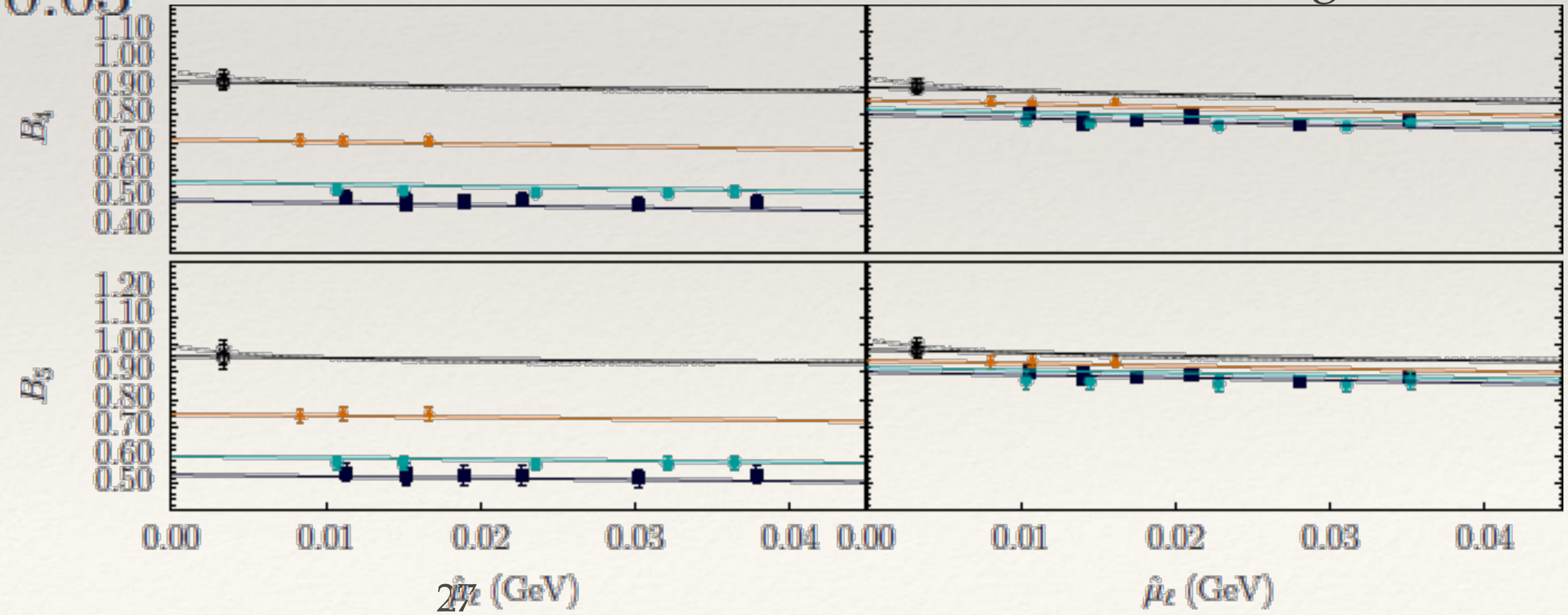


projected Greens function

ETM sticks to RI/MOM and needs to subtract Goldstone pole with fit but this seems to be under reasonable control

$$D_{ij}(a^2\tilde{p}^2; m_{\rm PS}^2) = \mathcal{A}(a^2\tilde{p}^2) + \mathcal{B}(a^2\tilde{p}^2)m_{\rm PS}^2$$
$$+\mathcal{C}(a^2\tilde{p}^2)/m_{\rm PS}^2 + \mathcal{D}(a^2\tilde{p}^2)/m_{\rm PS}^4$$

2 definitions of the continuum limits agree



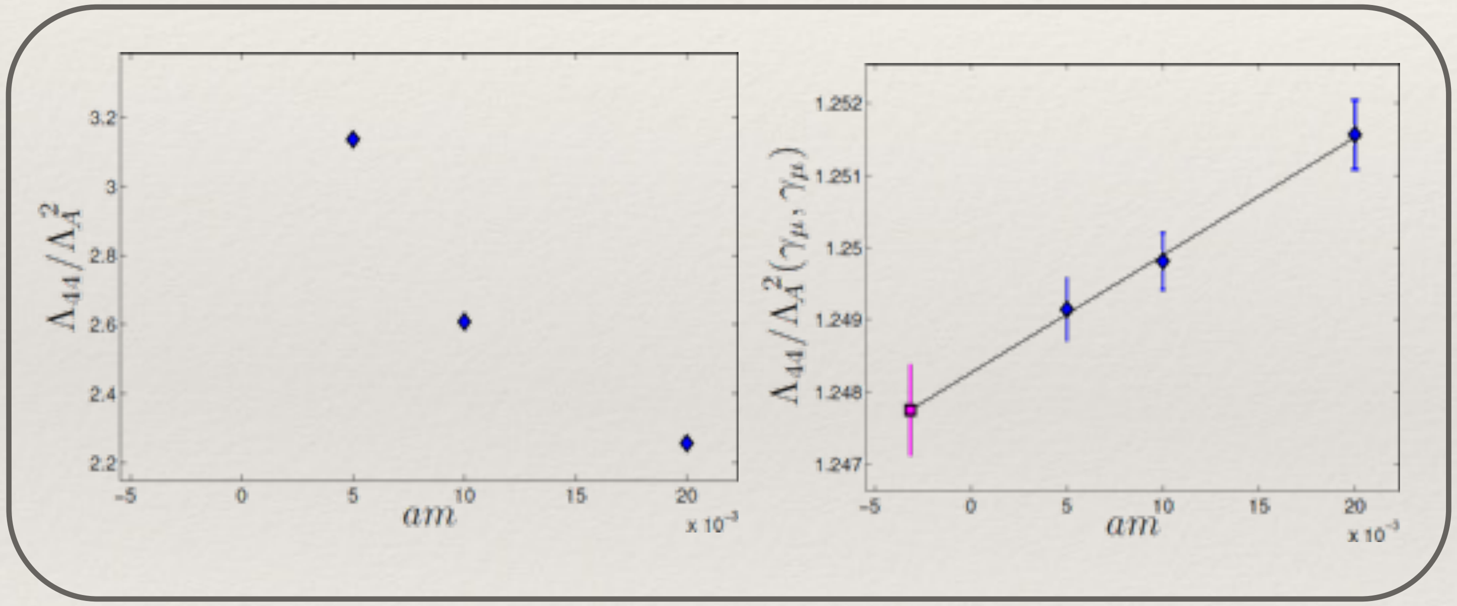
RBC/UKQCD

R. Hudspith, Tue 17:10 Weak
J. Frison, Wed 14:40 SM Params

RBC/UKQCD

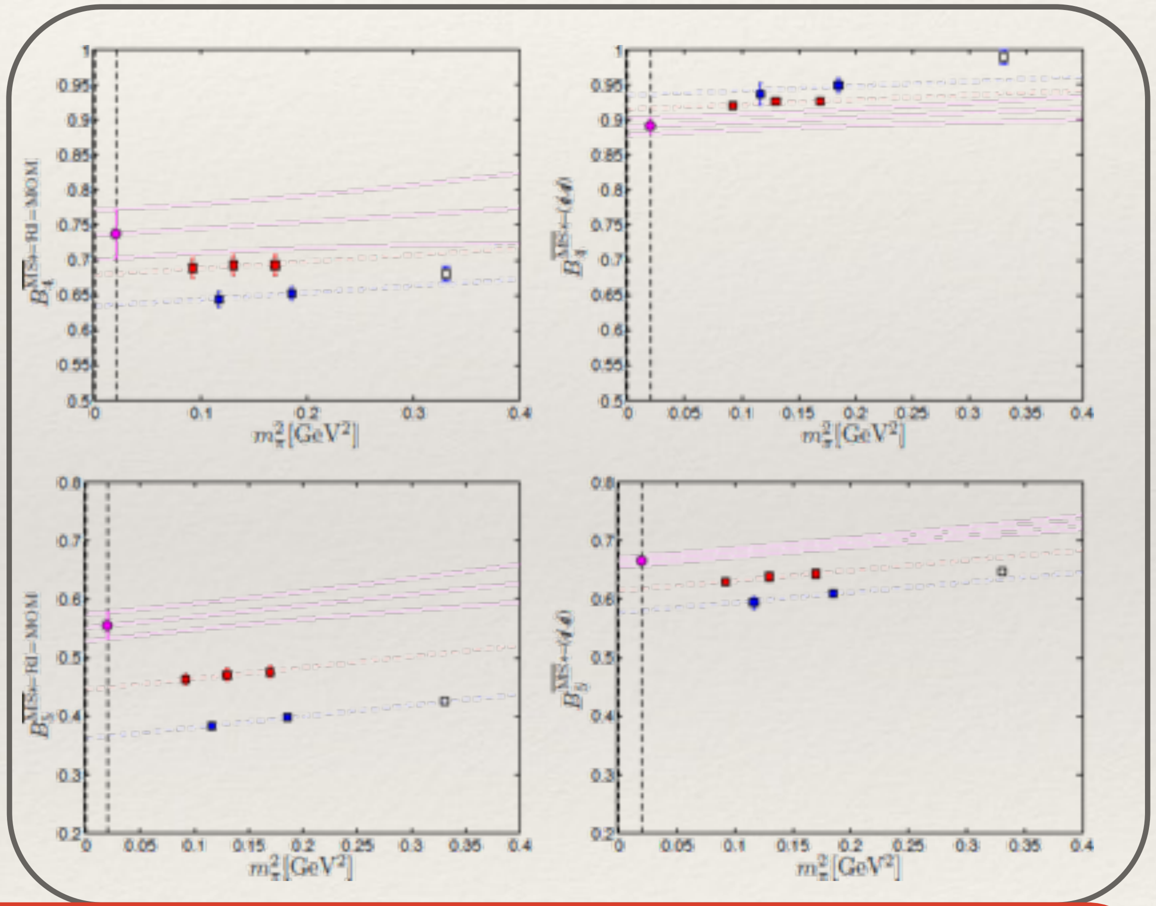
- wt. respect to 2012 changed renormalisation scheme:
  RI/MOM → RI/SMOM PRD 78, 054510 (2008) arXiv:0712.1061
  PRD 80, 014501 (2009) arXiv:0901.2599
- continuum limit

absence of Goldstone Pole



Thanks to RBC/UKQCD Nicolas Garron

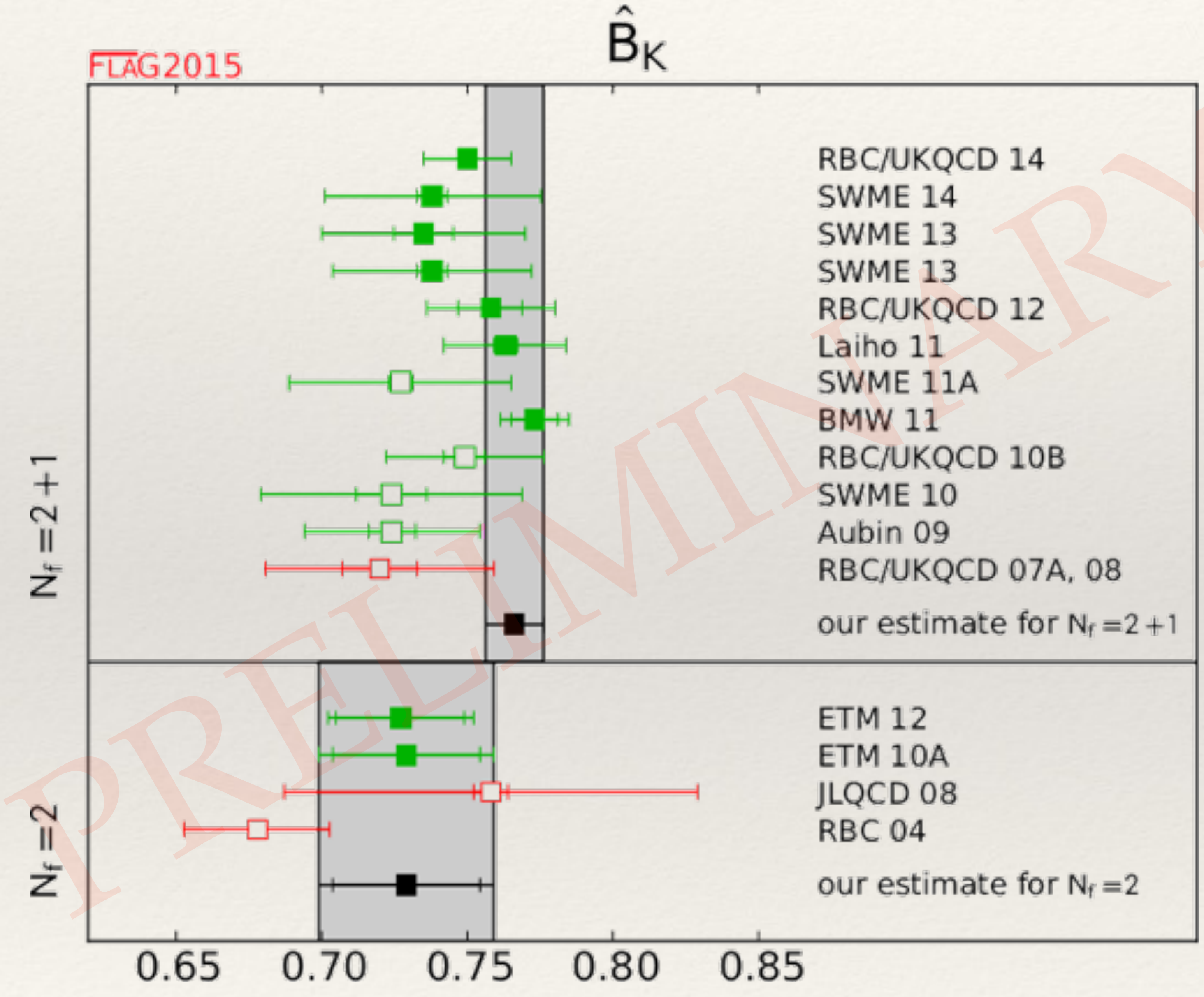
continuum limit



- discrepancy between RI/MOM and RI/SMOM only in some channels
- would ETM's results also move if they used RI/SMOM?
- RBC/UKQCD's SMOM results agree with SWME's pert. ren. results - indicative?

Neutral meson mixing - kaon

FLAG B_K Working Group Dimopoulos, Mawhinney, Wittig



grey band still old average (FLAG2013)

contrary to BSM bags here excellent agreement at the **1.3%-level**

SM prediction for ε_K

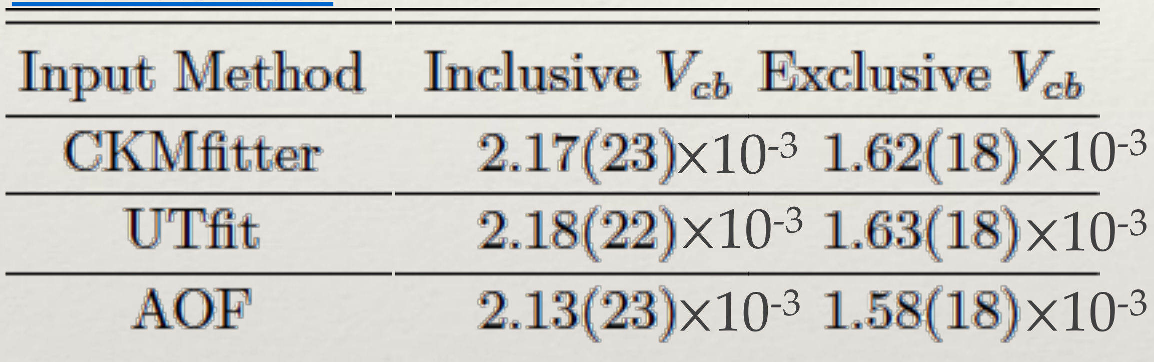
W. Lee, Poster #5

$$\epsilon_K = e^{i\phi}\sin\phi\left(\frac{\mathrm{Im}\langle\bar{K}^0|O^{\Delta S=2}|K^0\rangle}{2\mathrm{Re}M_{0\bar{0}}} + \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)$$

~ΔM_K, $|V_{cb}|$

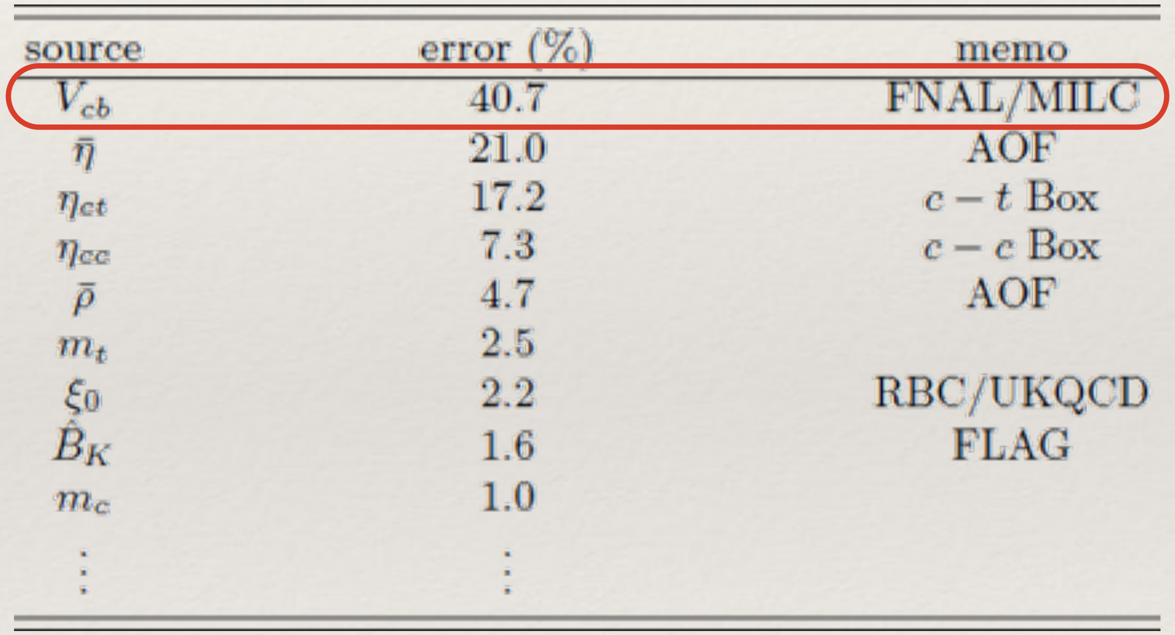
from lattice
$K \to \pi\pi\,(\Delta I = 3/2)$
and experiment

arXiv:1503.05388



Input Method	Inclusive V_{cb}	Exclusive V_{cb}
CKMfitter	$2.17(23)\times10^{-3}$	$1.62(18)\times10^{-3}$
UTfit	$2.18(22)\times10^{-3}$	$1.63(18)\times10^{-3}$
AOF	$2.13(23)\times10^{-3}$	$1.58(18)\times10^{-3}$

$|\varepsilon_K| = 2.228(11)\times10^{-3}$ (PDG)



source	error (%)	memo
V_{cb}	40.7	FNAL/MILC
$\bar{\eta}$	21.0	AOF
η_{ct}	17.2	$c-t$ Box
η_{cc}	7.3	$c-c$ Box
$\bar{\rho}$	4.7	AOF
m_t	2.5	
ξ_0	2.2	RBC/UKQCD
$\hat{B}_K$	1.6	FLAG
m_c	1.0	
⋮	⋮	

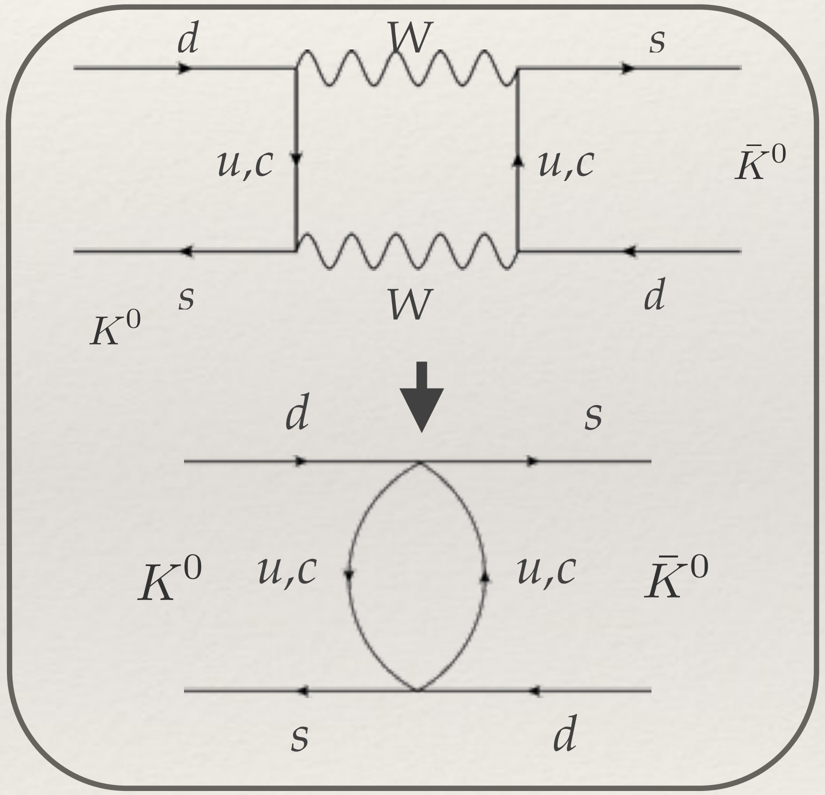
there is a **3.6σ** tension in the lattice analysis(exclusive V_{cb});
tension disappears if inclusive V_{cb} used

Neutral kaon mixing on the lattice beyond the bag parameter?

Beyond short distance ΔM_K, ε_K

$$\boxed{\Delta M_K = m_{K_S} - m_{K_L} = \boxed{2\mathrm{Re}M_{0\bar{0}}}} \qquad M_{\bar{0}0} = \mathcal{P}\sum_{\lambda} \frac{\langle \overline{K^0}|H_W|\lambda\rangle\langle\lambda|H_W|K^0\rangle}{m_K - E_\lambda}$$

- experimentally $\Delta M_K = 3.483(6) \times 10^{-12}$ MeV (PDG)
- suppressed by 14 orders of magnitude with respect to QCD $\rightarrow$ poses strong BSM constraints (e.g. $(1/\Lambda)^2\ \bar{s}d\bar{s}d$ BSM contribution) knowing ΔM_K at 10%-level $\rightarrow$ **$\Lambda \geq 10^4$ TeV**
- SD about 70% of experimental value - rest LD?
- PT large contributions at $\mu \sim m_c$ where PT turns out to converge badly (NLO->NNLO constitutes 36% correction) Brod, Gorbahn PRL 108 121801 (2012) arXiv:1108.2036

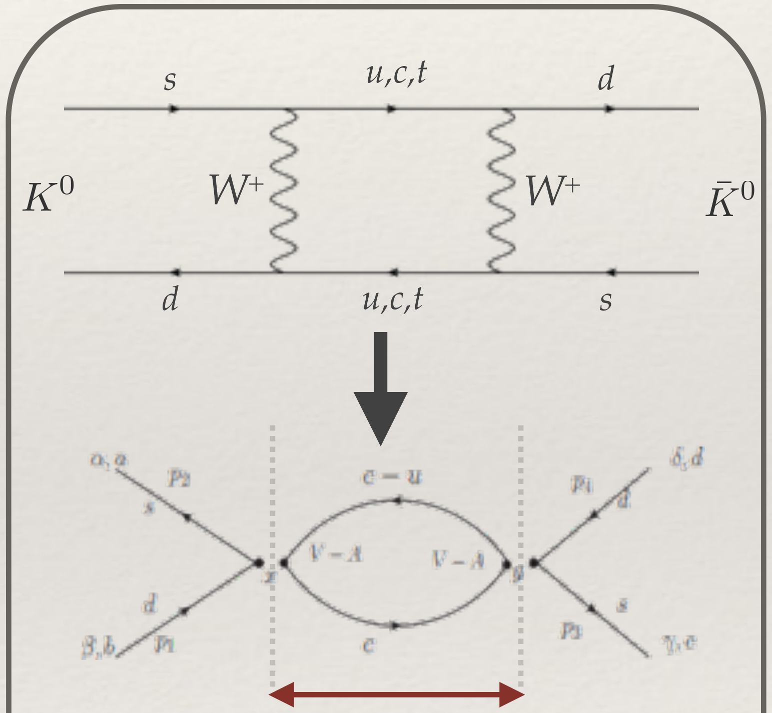


Beyond short distance ΔM_K, ε_K

$$\boxed{\epsilon_K = e^{i\phi}\sin\phi\left(\frac{\boxed{\mathrm{Im}M_{0\bar{0}}}}{2\mathrm{Re}M_{0\bar{0}}} + \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)}$$

$$M_{\bar{0}0} = \mathcal{P}\sum_{\lambda}\frac{\langle\overline{K^0}|H_W|\lambda\rangle\langle\lambda|H_W|K^0\rangle}{m_K - E_\lambda}$$

- precise prediction would constitute important SM-test
- experiment: $|\varepsilon_K| = 2.228(11)\times10^{-3}$ and there is the mentioned tension with the SM prediction
- long distance contribution estimated to be only some percent but lattice computation seems still an interesting challenge



~$1/\Lambda_{\mathrm{QCD}}$

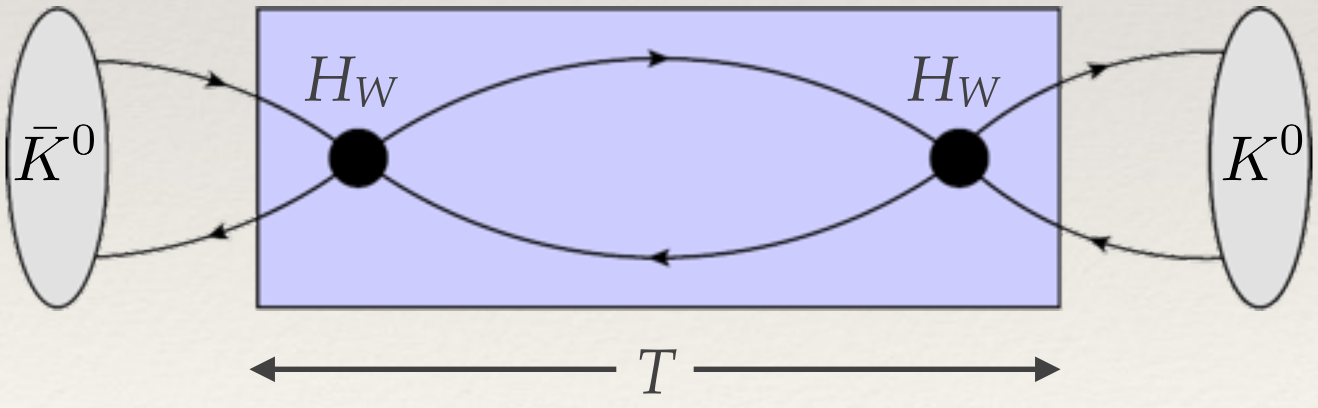
long distance effect

Beyond short distance ΔM_K, ε_K

$$M_{\bar{0}0} \quad = \quad \mathcal{P}\sum_{\lambda} \frac{\langle \overline{K^0}|H_W|\lambda\rangle\langle\lambda|H_W|K^0\rangle}{m_K - E_\lambda}$$

$$\mathcal{A} = \langle 0|T\left\{K^0(t_f)\frac{1}{2}\int_{t_A}^{t_B} dt_2 \int_{t_A}^{t_B} dt_1 H_W(t_2)H_W(t_1){K^0}^\dagger(t_i)\right\}|0\rangle$$

Integrate operators (here H_W) over time interval where initial and final kaon dominate



Beyond short distance ΔM_K, ε_K

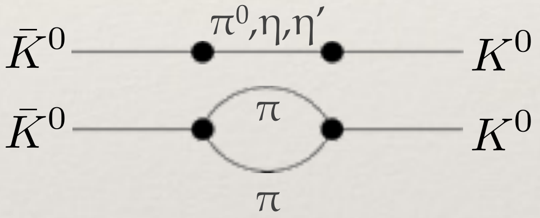
N. Christ et al. PRD 88 (2013) 014508 **arXiv:1212.5931**
Bai et al. PRL 113 (2014) 112003 **arXiv:1406.0916**

$$\mathcal{A} = N_K^2 e^{-M_K(t_f-t_i)} \sum_n \frac{\langle \bar{K}^0|H_W|n\rangle\langle n|H_W|K^0\rangle}{M_K-E_n}\left(-T - \frac{1}{M_K-E_n} + \frac{e^{(M_K-E_n)T}}{M_K-E_n}\right)$$

amplitude Δm_K^{FV}

irrelevant constant

exponential term needs to be subtracted

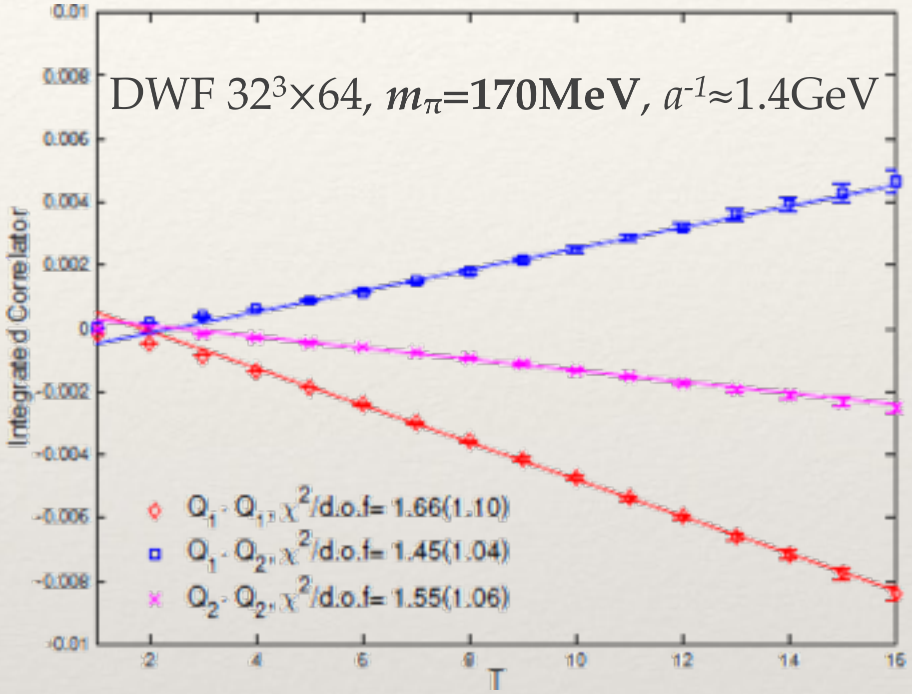


- n runs over all sort of intermediate states: vacuum, π^0, η, $\pi\pi$, …
- in order to get amplitude, need to subtract them
- Subtraction methods:
  a) compute $<n\,|\,H_W\,|\,K^0>$ from correlation functions
  b) use V or A WI and subtract $\bar{s}d$ and $\bar{s}\gamma_5 d$ with suitably tuned prefactor
- finite volume corrections from two-particle intermediate state can be sizeable
  N. Christ et al. PRD91 (2015) 114510 **arXiv:1504.01170** also: Briceno, Hansen **arXiv:1502.04314**
  extension of Lellouch-Lüscher correction to 2nd order weak MEs
  $$\Delta^{\mathrm{FV}}\left(\Delta M_K\right) = -\cot\left(\phi(M_K)+\delta_0(M_K)\right)\frac{d(\phi(E)+\delta_0(E))}{dE}\Big|_{E=M_K}|\langle\bar{K}^0|H_W|\pi\pi, M_K\rangle^{\mathrm{V}'}|^2$$
- what happens when the two H_W approach each other (GIM in action)?

K_L-K_S mass difference

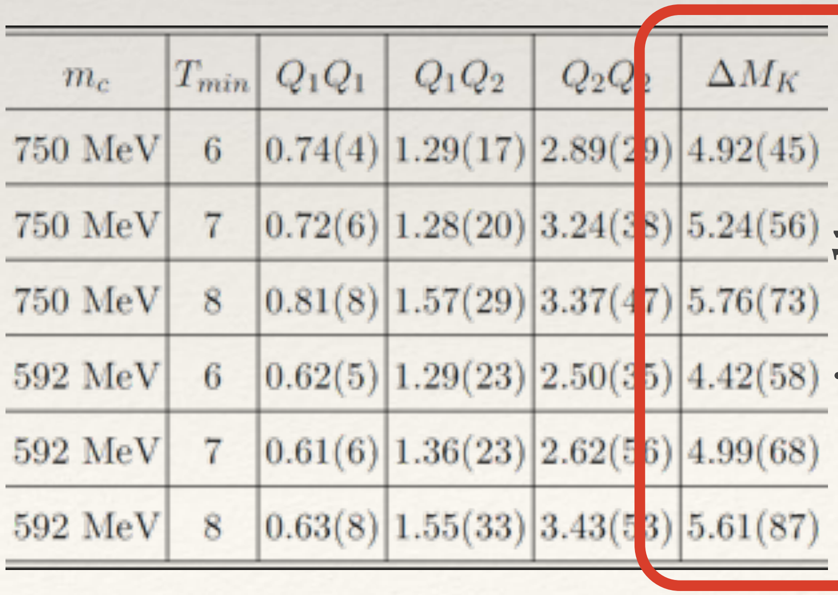
ongoing RBC/UKQCD project

- subtractions of intermediate states works:
  - vacuum and η: $$\begin{aligned}\langle 0|H_W + c_P\bar{s}\gamma_5 d|K\rangle &= 0\\ \langle\eta|H_W + c_S\bar{s}d|K\rangle &= 0\end{aligned}$$
  - ππ: compute and subtract $\langle\pi\pi_{I=0,2}|H'_W|K\rangle$
  - π: compute and subtract $\langle\pi|H'_W|K\rangle$


DWF $32^3\times64$, m_π=**170MeV**, $a^{-1}\approx$1.4GeV

- GIM subtraction + *V*-*A* → no divergence wt. chiral fermions

- doable calculation and preliminary results are in the right ballpark
- need to go to physical charm and physical m_l



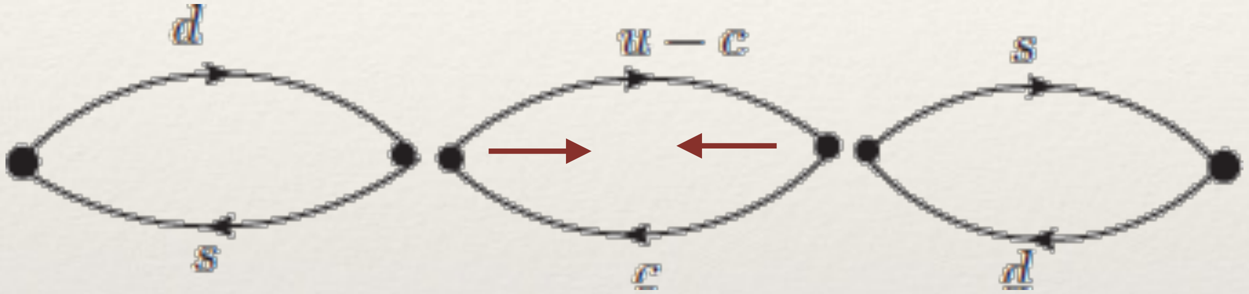
m_c	T_{min}	Q_1Q_1	Q_1Q_2	Q_2Q_2	ΔM_K
750 MeV	6	0.74(4)	1.29(17)	2.89(29)	4.92(45)
750 MeV	7	0.72(6)	1.28(20)	3.24(38)	5.24(56)
750 MeV	8	0.81(8)	1.57(29)	3.37(47)	5.76(73)
592 MeV	6	0.62(5)	1.29(23)	2.50(35)	4.42(58)
592 MeV	7	0.61(6)	1.36(23)	2.62(56)	4.99(68)
592 MeV	8	0.63(8)	1.55(33)	3.43(53)	5.61(87)

signal!

A direct computation of ε_K

N. Christ, Fri 15:00 Weak

- log divergence arises when two weak operators approach each other



- short-distance behaviour when the two H_W approach each other well described by local 4-quark operator O_{LL} - allows for subtraction procedure at large energy scale μ^2:

$$\mathcal{A} \to \mathcal{A} - \left(E^{\text{lat}}(\mu^2) - E^{\text{cont}}(\mu^2)\right) \sum_x \langle \bar{K}^0 | O_{LL}(x) | K^0 \rangle$$

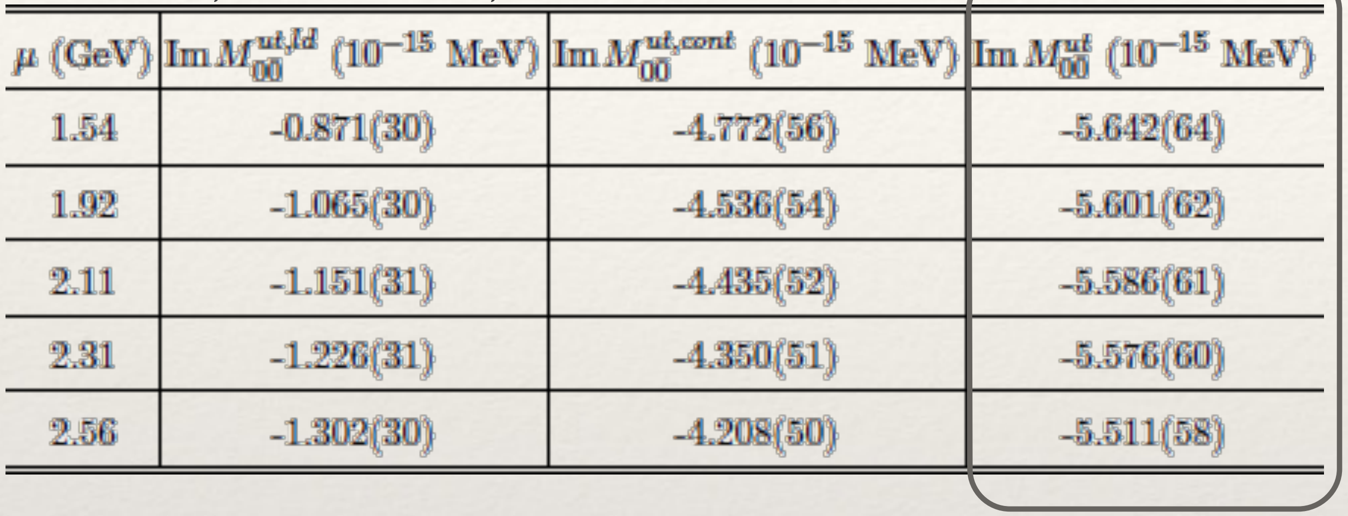
where E^{lat} and E^{cont} are computed in lattice NPR and the continuum perturbation theory, respectively

A direct computation of ε_K

First results of study on reduced set of Wick contractions:

N. Christ, Fri 15:00 Weak

DWF 24^3, $a^{-1}\approx 1.7$GeV, $m_\pi\approx 330$MeV



μ (GeV)	Im $M_{00}^{ut,ld}$ (10^{-15} MeV)	Im $M_{00}^{ut,cont}$ (10^{-15} MeV)	Im M_{00}^{ut} (10^{-15} MeV)
1.54	-0.871(30)	-4.772(56)	-5.642(64)
1.92	-1.065(30)	-4.536(54)	-5.601(62)
2.11	-1.151(31)	-4.435(52)	-5.586(61)
2.31	-1.226(31)	-4.350(51)	-5.576(60)
2.56	-1.302(30)	-4.208(50)	-5.511(58)

Result after subtraction

Note insensitivity of the final result to the value of the subtraction point μ

Result for ε_K on connected Wick contractions for QCD with $m_\pi\approx 329$MeV and $m_K\approx 575$MeV

$$|\epsilon_K| = (\overset{tt}{1.806} + \overset{ut_{sd}}{0.892} + \overset{ut_{ld}}{0.209} + \overset{\text{Im}\,A_0}{0.111}) \times 10^{-3} = 3.019 \times 10^{-3}$$

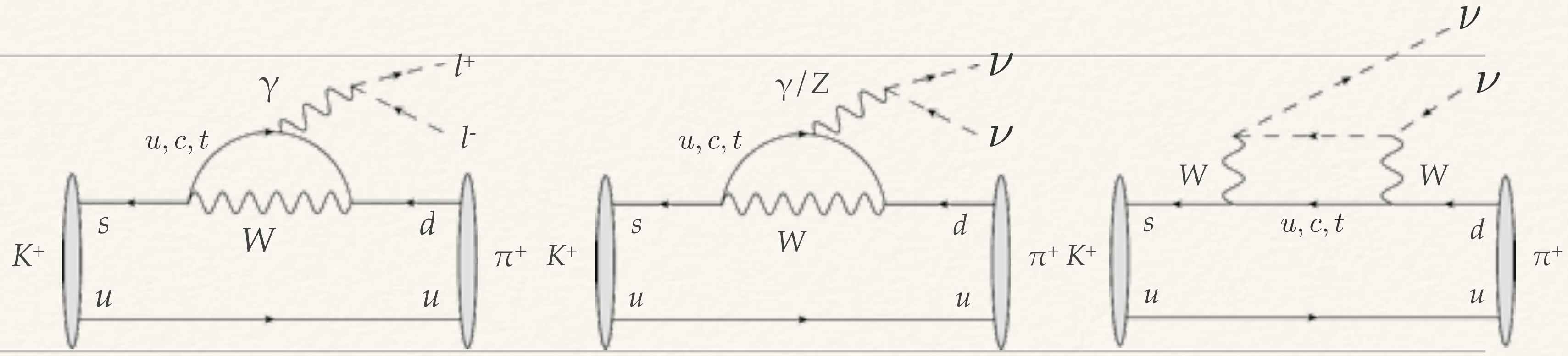
50% larger than the 2.228(11) × 10^{-3} experimental result because only LO terms for SD part are included so far and simulation away from physical point

Plan:

- include all diagrams and do subtraction for other diagrams
- go towards physical situation (lighter pions, dynamical charm, larger volume…)

rare kaon decays

Rare kaon decays



Two new experiments dedicated to rare kaon decays NA62 (CERN) and KOTO (J-PARC) are running

- FCNC (W-W or γ/Z-exchange diagrams)
- deep probe into flavour mixing and SM/BSM due to suppression in the SM
- can determine V_{td}, V_{ts} and test SM

$K_L \to \pi^0 \nu\bar{\nu}$

- KOTO (J-PARC)
- direct CP violation
- exp. BR $\le 2.6 \times 10^{-8}$
  theory BR $3.0(3) \times 10^{-11}$
- GIM → top dominated and charm suppressed, purely SD

$K^+ \to \pi^+ \nu\bar{\nu}$

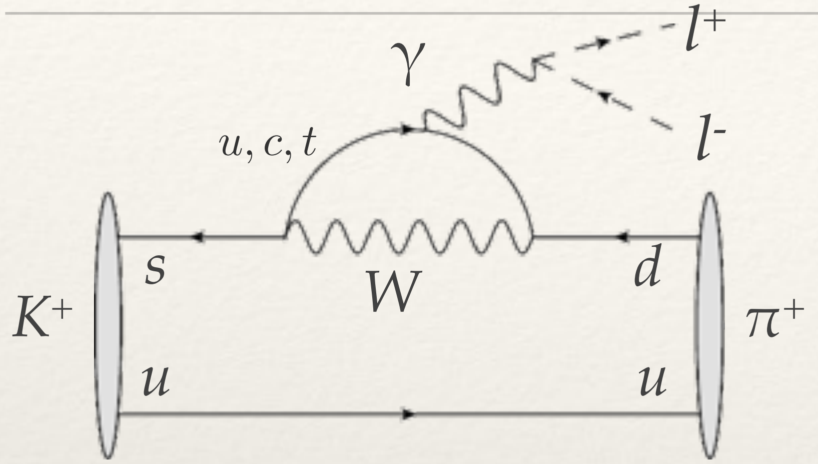
- NA62 (CERN)
- CP conserving
- exp. BR $1.73(^{+1.15}_{-1.05}) \times 10^{-10}$
  theory BR $0.911(72) \times 10^{-10}$
- small LD contribution, candidate for lattice

compute in lattice QCD

$K^+ \to \pi^+ l^+ l^-$ $K_S \to \pi^0 l^+ l^-$

- 1-photon exchange LD dominated
- indirect contribution to CP-violating rare K_L decay
- SM prediction mainly ChPT
- lattice can predict ME and LECs
- experimenters will be able to look at these channels as well

Rare kaon decays $K^+ \to \pi^+ l^+ l^-$



A. Lawson, Fri 14:20 Weak

N. Christ et al. arXiv:1507.03094

LD contribution given through $K \to \gamma^*$ contributionwhich is computed as

$$\mathcal{A}_\mu = (q^2) \int d^4x \langle \pi(p) | T\left[J_\mu(0) H_W(x)\right] | K(k) \rangle$$

$$Q_1^q = \left(\bar{s}_i \gamma_\mu^L d_i\right)\left(\bar{q}_j \gamma_\mu^L q_j\right), \qquad Q_2^q = \left(\bar{s}_i \gamma_\mu^L q_i\right)\left(\bar{q}_j \gamma_\mu^L d_j\right)$$

Decay amplitude in terms of elm. transition form factor

$$A_i = -\frac{G_F \alpha}{4\pi} V_i(z)(k+p)^\mu \bar{u}_l(p_-)\gamma_\mu \nu_l(p_+) \qquad (i = +, S)$$

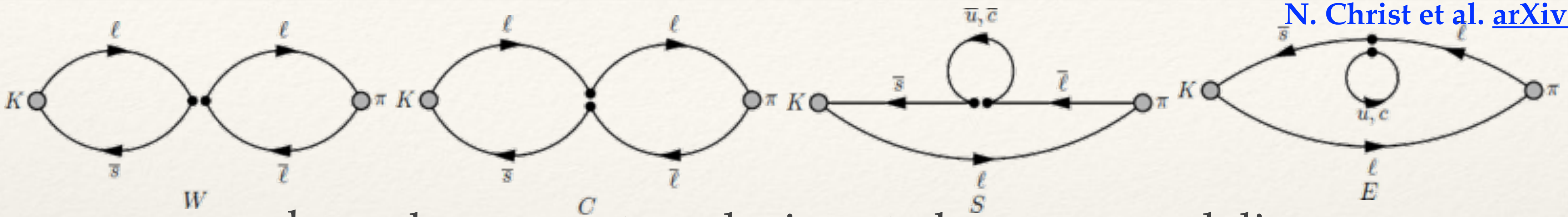
$$V_i(z) = a_i + b_i z + V_i^{\pi\pi}(z)$$

- the a_S and a_+ can be extracted from experiment or lattice
- a_S parameterises also the CP-violating contribution to the K_L decay

Rare kaon decays $K^+ \to \pi^+ l^+ l^-$

A. Lawson, Fri 14:20 Weak

N. Christ et al. arXiv:1507.03094



where elm. current can be inserted on any quark line

- Integral similar to ΔM_K and ε_K:

$$
\begin{aligned}
I^j_\mu(q^2) \;=\; & -\int_0^\infty dE\,\frac{\rho(E)}{2E}\,\frac{\langle \pi^j(\mathbf{p})|J_\mu(0)|E,\mathbf{k}\rangle\langle E,\mathbf{k}|H_W(0)|K^j(\mathbf{k})\rangle}{E_K(\mathbf{k})-E}\left(1-e^{(E_K(\mathbf{k})-E)T_a}\right)\\
& + \int_0^\infty dE\,\frac{\rho_S(E)}{2E}\,\frac{\langle \pi^j(\mathbf{p})|H_W(0)|E,\mathbf{p}\rangle\langle E,\mathbf{p}|J_\mu(0)|K^j(\mathbf{k})\rangle}{E-E_\pi(\mathbf{p})}\left(1-e^{(E-E_\pi(\mathbf{p}))T_b}\right)
\end{aligned}
$$

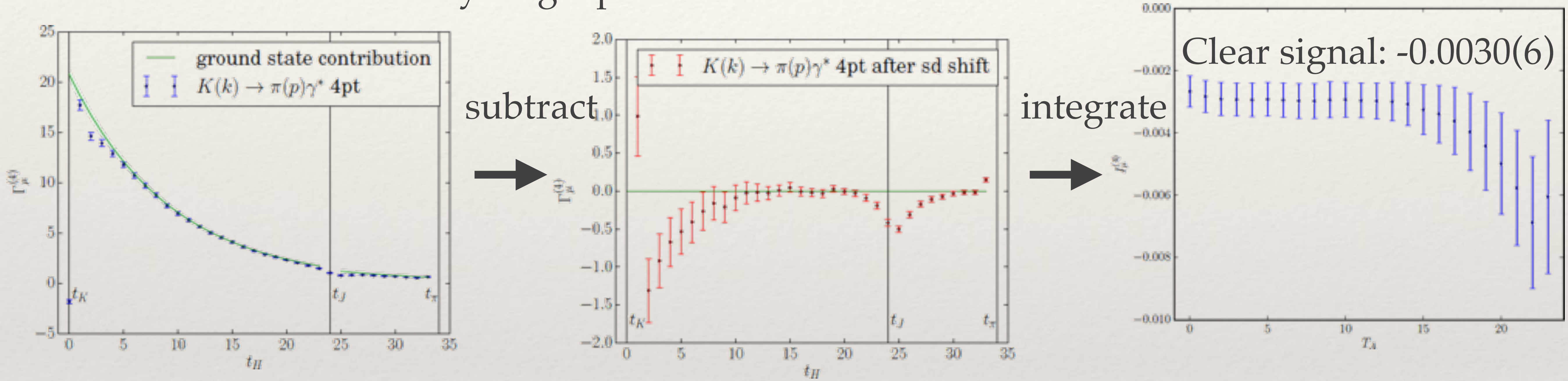
- elm. gauge invariance and GIM ensure no logarithmic divergence
- exponentially growing terms for $E<E_K(\boldsymbol{k})$
  - 1 pion: subtract like for ΔM_K
  - 2 pion: $O(4)$ invariance tells us these can't contribute
  - 3 pion, …: expected to be small but needs to be studied in detail

Rare kaon decays $K^+ \to \pi^+ l^+ l^-$

A. Lawson, Fri 14:20 Weak

N. Christ et al. arXiv:1507.03094

- Pilot: $24^3 \times 64$ DWF, $a^{-1} \approx 1.7$GeV, $m_\pi \approx 421$MeV, $m_K \approx 600$MeV, only C and W diagrams
- in this situation only single pion contribution to subtract



subtract

integrate

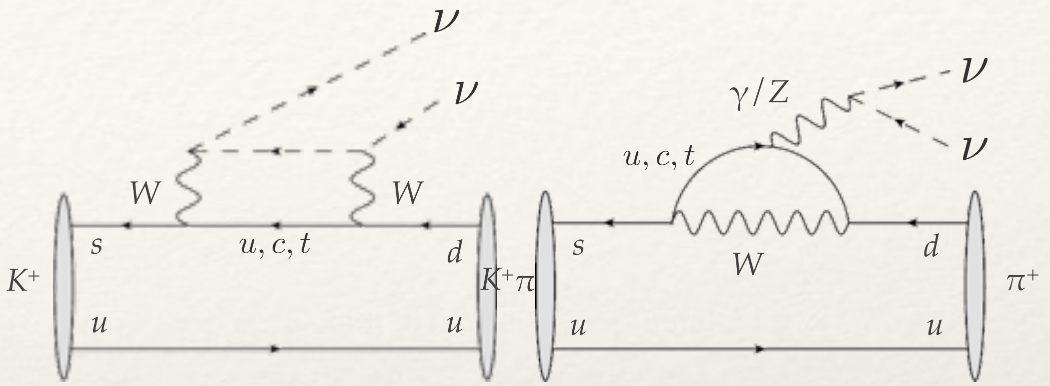
Clear signal: -0.0030(6)

- Pilot shows that calculation is possible
- all diagrams need to be included now
- eventually needs to move to calculations with lighter pions

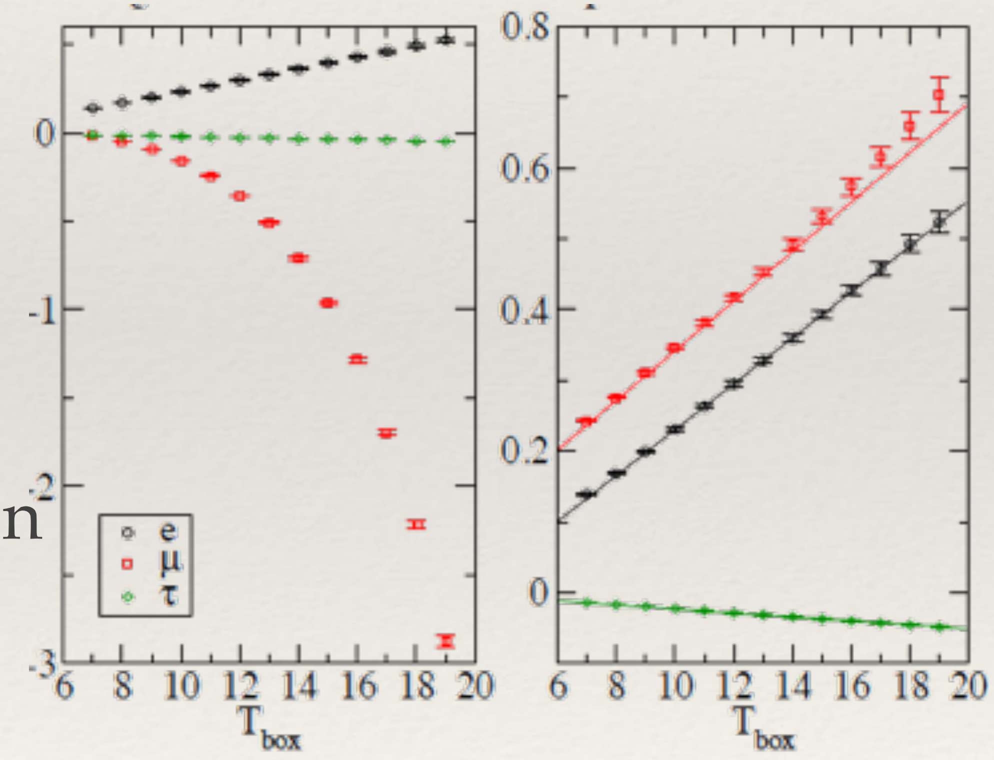
Rare kaon decays $K^+ \to \pi^+ \nu\bar{\nu}$

X. Feng, Fri 14:40 Weak

- More diagrams - *Z*-exchange and *WW*-diagrams
- GIM will reduce divergence when H_W get close, to logarithmic → treatment similar to ε_K



Example *WW*-diagram: before and after subtraction of intermediate states for different neutrino final states. r.h.s. plot nicely shows linear dependence on fiducial integration volume as expected; its slope give she form factor



Developed techniques work, now go to lighter pion masses and dynamical charm!

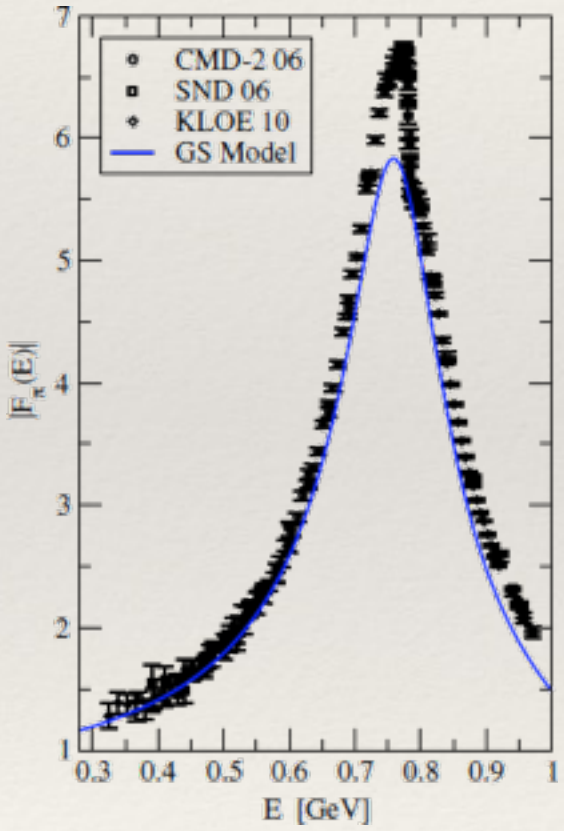
pion form factors

Timelike pion vector form factor

Feng et al. PRD 91, 054504 **arXiv:1412.6319**

- $\pi\pi$ elastic scattering phase shift δ_1 (via Lüscher from E_n) is complex phase of $F_\pi(s)$
- Harvey Meyer derived a relation between $|\langle 0|j|\pi\pi, n\rangle_V|^2$ and $|F_\pi(s)|^2$ PRL 107, 072002 (2011) similar to Lellouch-Lüscher, extended here to the moving frame with momentum **P**

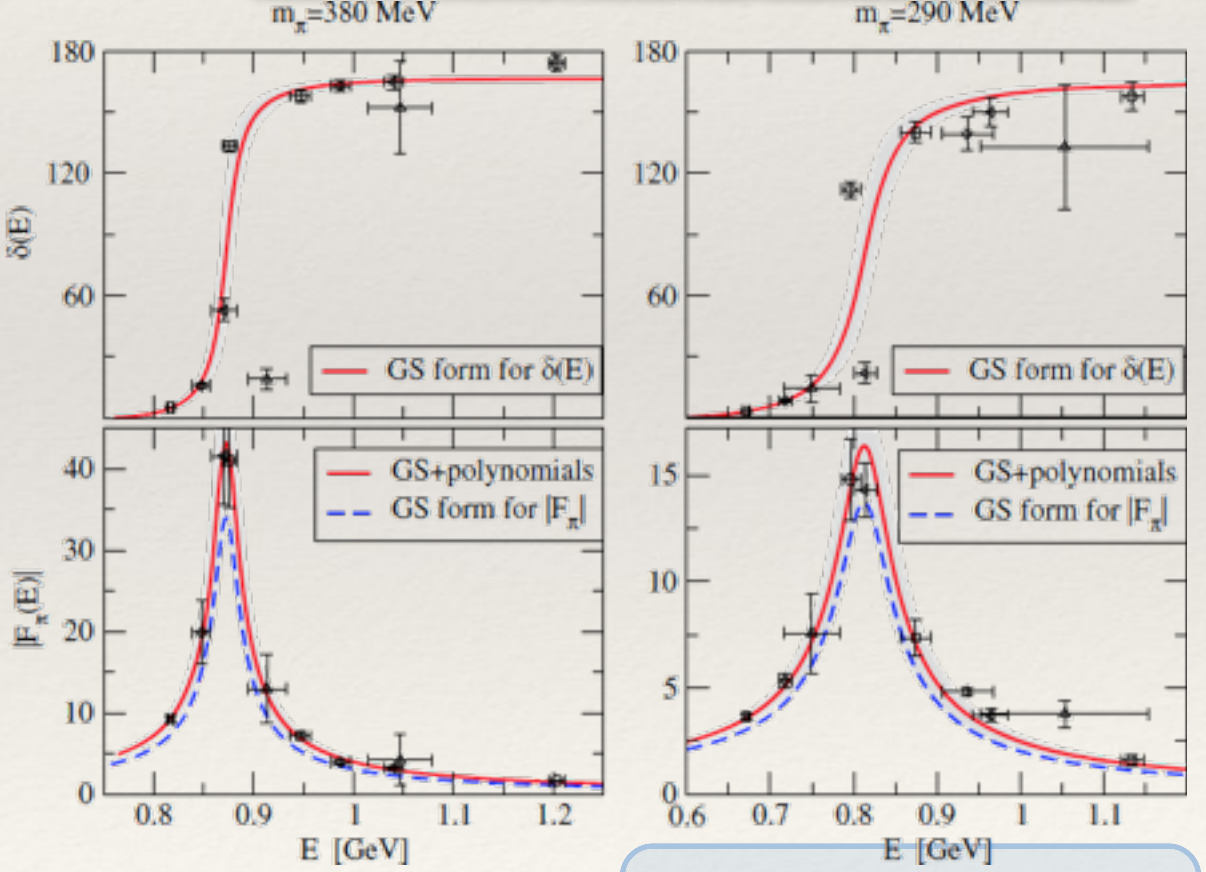
Iwasaki + N_f=2+1 overlap (fixed topology)
a=0.11fm, m_π=380,290MeV, $m_\pi L \gtrsim 4$
ρ decays in both cases and



- variational basis of vector and $\pi\pi$ opts
- ground- and excited states + 5 momenta → 10 energy values

Results:

- $g_{\rho\pi\pi}$ and m_ρ from δ(E)
- good signal for ff with clear ρ-peak



related:
M. Werner, poster #7
Y. Chen, Tue 15:20 HadSpec

Summary/Conclusions

- lattice data with physical light quark masses allows for a new quality of data analysis
- in particular analysis strategies which reduce the impact of data from un-physically heavy light quarks seem very attractive
- for simulations with physical quark masses statistical/finite size/continuum extrapolation errors are now the dominant source of uncertainty in pure QCD
- QCD simulations so precise that it's worth studying isospin breaking effects in matrix elements in more detail in a conceptually clean way
- new challenges have been identified: long distance effects in
    - neutral kaon system
    - rare kaon decays (new experimental facilities!!!)

  with loads of new questions and theoretical problems and potential impact on SM and BSM phenomenology
- time-like region accessed for pion form factor via Lellouch-Lüscher method; should be looked at in other channels as well

34th International Symposium on
Lattice Field Theory

University of Southampton
24–30 July 2016

LATTICE
2016

Looking forward to see you in Southampton!

http://www.southampton.ac.uk/lattice2016/

What's next – f_K/f_π ?

tension in treatment of $\delta_{\mathrm{SU(2)}}$?

- SU(3) ChPT Phys.Lett. B700 (2011) 7-10 **arXiv:1102.0563**

$$\delta_{\mathrm{SU}(2)} \approx \frac{3}{4} R \left[-\frac{4}{3}\left(f_K/f_\pi - 1\right) + \frac{2}{3(4\pi)^2 f_0^2}\left(M_K^2 - M_\pi^2 - M_\pi^2 \ln\frac{M_K^2}{M_\pi^2} \right) \right] \approx -0.004(1)$$

- ETM PRD87 (2013) 11, 114505 **arXiv:1303.4896** $\delta_{\mathrm{SU}(2)} = -0.0080(4)$
  expand action in m_u-m_d and compute 2pt function with insertions
- ETM PRD91 (2015) 5, 054507 **arXiv:1411.7908** correct with slope of f_K in m_l determined in two ways $\delta_{\mathrm{SU}(2)} = -0.0080(38)$
- HPQCD PRD 88, 074504 (2013) **arXiv:1303.1670** $\delta_{\mathrm{SU}(2)} = -0.0054(14)$

EM corrections taken into account in ChPT: $\frac{f_{K^+}}{f_{\pi^+}} \frac{|V_{us}|}{|V_{ud}|} = 0.27598(35)_{\mathrm{Br}} \boxed{(25)_{\mathrm{EM}}}$
Cirigliano, Neufeld PLB700 (2011) 7-10 **arXiv:1102.0563**

What's next – K→π ff. ?

$$\Gamma_{K\to\pi l\nu} = C_K^2 \frac{G_F^2 m_K^5}{192\pi^2} S_{\rm EW}(1+\Delta_{SU(2)}+\Delta_{\rm EM})^2\, I\, |f_+^{K\pi}(0)|^2 |V_{us}|^2$$

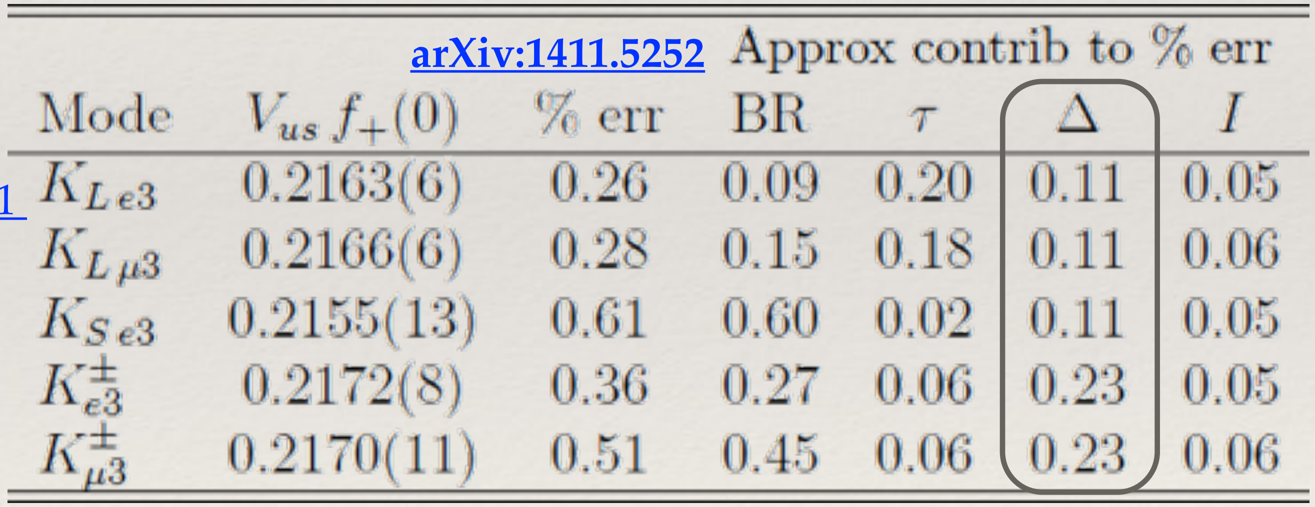
Take N_f=2+1+1 FLAG average $f_+(0)$=0.970(3) and $|V_{us}|f_+^{K^0\pi^-}(0) = 0.2163(5)$

FLAVIA Kaon WG EPJ C 69, 399-424 (2010)
KTeV, Istra, KLOE

$\boxed{\delta_{\rm exp}\approx 0.2\%\quad \delta_{\rm th}\approx 0.3\%}$

What's to be expected from the experimental side?

- Moulson@CKM 2014 **arXiv:1411.5252**
  Cirigliano et al. Rev.Mod.Phys. 84 (2012) 399 arXiv:1107.6001
  update: $|V_{us}|f_+^{K^0\pi^-}(0) = 0.2165(4)$
- BR, τ and $\delta_{\rm EM}$ and $\delta_{SU(2)}$ are dominant src. of uncertainty
- there is old data (e.g KLOE) and there are new experiments (NA62, KOTO, OKA) who could potentially improve the experimental number but seems that it is not on the top of todos/limited manpower



arXiv:1411.5252

			Approx contrib to % err			
Mode	$V_{us}\, f_+(0)$	% err	BR	τ	Δ	I
K_{Le3}	0.2163(6)	0.26	0.09	0.20	0.11	0.05
$K_{L\mu3}$	0.2166(6)	0.28	0.15	0.18	0.11	0.06
K_{Se3}	0.2155(13)	0.61	0.60	0.02	0.11	0.05
$K_{e3}^{\pm}$	0.2172(8)	0.36	0.27	0.06	0.23	0.05
$K_{\mu3}^{\pm}$	0.2170(11)	0.51	0.45	0.06	0.23	0.06