Convergent Perturbation Theory for the lattice ϕ^4-model

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Motivation

We study convergent series for lattice ϕ^4-model

- To check the method of the convergent series on the simple example, allowing one a direct comparison with the Monte Carlo simulations. The method was developed for continuum scalar field theories **[A. Ushveridze, Phys. Let. B, 1984]** and recently reformulated for QCD **[V. Sazonov, arXiv:1503.00739]**.
- To design new methods for lattice computations, which may help to avoid the Sign problem.
- To compare results with the Borel resummation

Lattice ϕ^4-model

Continuous theory in the Euclidean space-time

$$S = \int dx \Big(\frac{1}{2} (\partial \phi)^2 + \frac{1}{2} M^2 \phi^2 + \frac{1}{4!} \lambda \phi^4 \Big)$$

Theory on the lattice

$$S = \sum_{n=0}^{V} \Big[-\frac{1}{2} \sum_{\mu} \Big(\phi_n \phi_{n+\mu} + \phi_n \phi_{n-\mu} - 2\phi_n^2 \Big) + \frac{1}{2} M^2 \phi_n^2 + \frac{\lambda}{4!} \phi_n^4 \Big]$$

In the following we write the quadratic part of the action as

$$\sum_{n=0}^{V} \Big[-\frac{1}{2} \sum_{\mu} \Big(\phi_n \phi_{n+\mu} + \phi_n \phi_{n-\mu} - 2\phi_n^2 \Big) + \frac{1}{2} M^2 \phi_n^2 \Big] \equiv \|\phi\|^2 .$$

Calculations

We calculate the observable

$$\langle \phi_n^2 \rangle \,,$$

using the

- Monte Carlo method **[M. Creutz, B. Freedman, Annals Phys. 1981]**
- Borel resummation of the standard perturbation theory **[Jean Zinn-Justin arXiv:1001.0675v1 2010]**
- Convergent series **[A. Ushveridze, Phys.Let.B, 1984]**

Another ways to obtain convergent series

- **V. Belokurov, V. Kamchatny, E. Shavgulidze, Y. Solovyov, Mod.Phys.Let. A, 1997**
- **Y. Meurice, arxiv.org/abs/hep-th/0103134v3, 2002**

Ushveridze method. Main ideas

- New non-perturbed part of the action
- Positive determined series for the perturbation
- Interconnection between new and standard perturbation theory

Ushveridze method

Let's split the action as

$$S[\phi_n] = N[\phi_n] + P[\phi_n] = N[\phi_n] + (S[\phi_n] - N[\phi_n])\,.$$

Then the partition function can be calculated in the following way

$$Z = \prod_n^V \int [d\phi_n]\, e^{-S[\phi_n]} = \prod_n^V \int [d\phi_n]\, e^{-N[\phi_n]+(N[\phi_n]-S[\phi_n])} =$$

$$= \prod_n^V \int [d\phi_n]\, e^{-N[\phi_n]} \sum_{l=0}^{\infty} \frac{(N[\phi_n] - S[\phi_n])^l}{l!}$$

$$\mathbf{N}[\phi_\mathbf{n}] \geq \mathbf{S}[\phi_\mathbf{n}]$$

Ushveridze method

The partition function after interchanging of integration and summation is

$$Z = \sum_{l=0}^{\infty} \prod_{n}^{V} \int [d\phi_n]\, e^{-N[\phi_n]} \frac{(N[\phi_n] - S[\phi_n])^l}{l!} \,.$$

Let us choose the non-perturbed part of the action as

$$N[\phi_n] = \|\phi\|^2 + \sigma \|\phi\|^4$$

How to find σ

The action and its non-perturbed part are

$$S = \|\phi\|^2 + \sum_{n=0}^{V} \frac{\lambda}{4!}\phi_n^4,$$

$$N = \|\phi\|^2 + \sigma\|\phi\|^4$$

So, for σ we have

$$N[\phi_n] \geq S[\phi_n] \Longleftrightarrow \sigma\|\phi\|^4 \geq \sum_{n=0}^{V} \frac{\lambda}{4!}\phi_n^4 \Longrightarrow$$

$$\Longrightarrow \sigma \geq \frac{\lambda}{6M^4}$$

How to solve new initial approximation

The observable $\langle \phi_n^2 \rangle$ is the sum of terms of the following type

$$\prod_n^V \int [d\phi_n]\ \phi_n^2\ e^{-N[\phi_n]} \frac{(N[\phi_n] - S[\phi_n])^l}{l!} =$$

using the δ-function we change $\|\phi\|$ to a new variable t

$$= \int_0^\infty dt\ \exp\Big(- t^2 - \sigma t^4 \Big) \prod_n^V \int [d\phi_n]\ \phi_n^2\ \delta(t - \|\phi\|) \times$$

$$\times \frac{(\sigma t^4 - \sum_{n=0}^V \frac{\lambda}{4!} \phi_n^4)^l}{l!}$$

How to solve new initial approximation

$$\int_0^\infty dt \, \exp\Big(-t^2 - \sigma t^4\Big) \prod_n^V \int [d\phi_n] \; \phi_n^2 \; \delta(t - \|\phi\|) \times$$

$$\times \frac{(\sigma t^4 - \sum_{n=0}^V \frac{\lambda}{4!}\phi_n^4)^l}{l!}$$

We rescale field ϕ as $t\phi$, expand brakets $(....)^l$ and end up with the sum of the integrals like

$$\Big(t - \mathit{depending\ integral}\Big) \cdot \prod_n^V \int [d\phi_n] \; \phi_{n_1} ... \phi_{n_k} \delta(1 - \|\phi\|)$$

1-dimensional results, 5 loops vs Monte Carlo

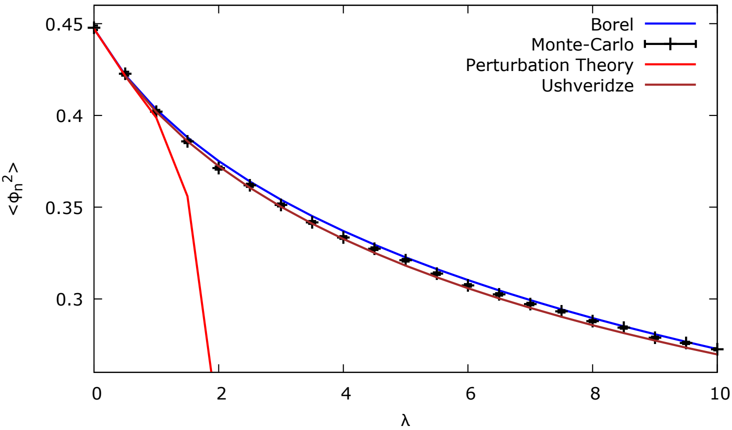

Figure: Comparison of the results for the 1d case on the $V = 100$ lattice

Behavior of results in dependence on order

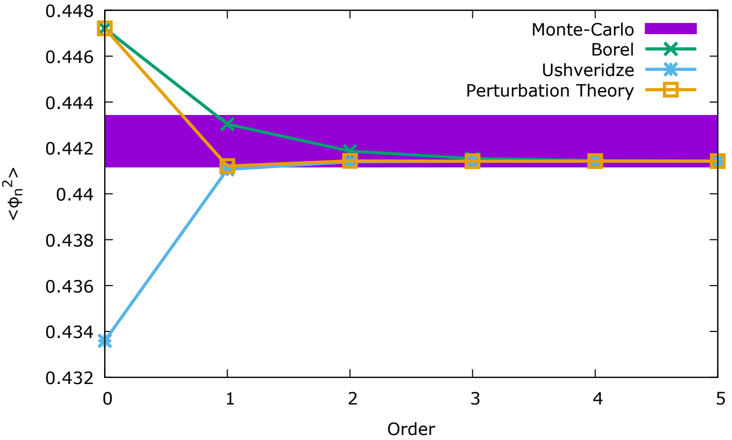

Figure: 1d case for $\lambda = 0.1$

Behavior of results in dependence on order

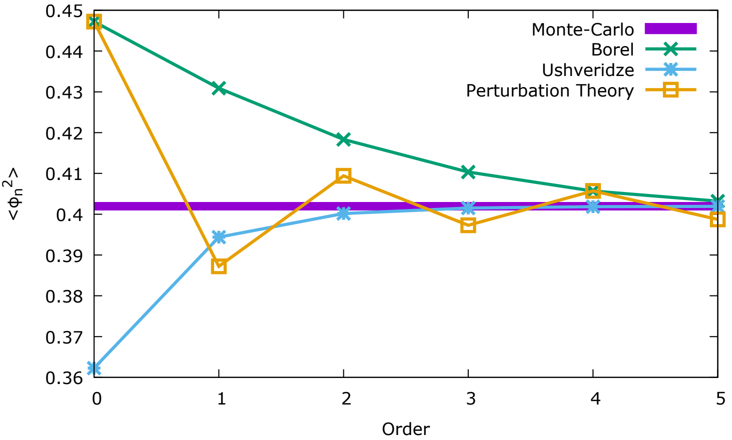

Figure: 1d case for $\lambda = 1$

Behavior of results in dependence on order

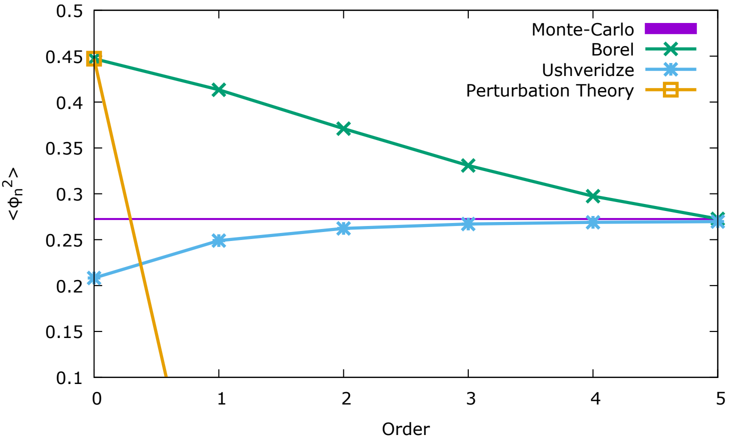

Figure: 1d case for $\lambda = 10$

2-dimensional results, 5 loops vs Monte Carlo

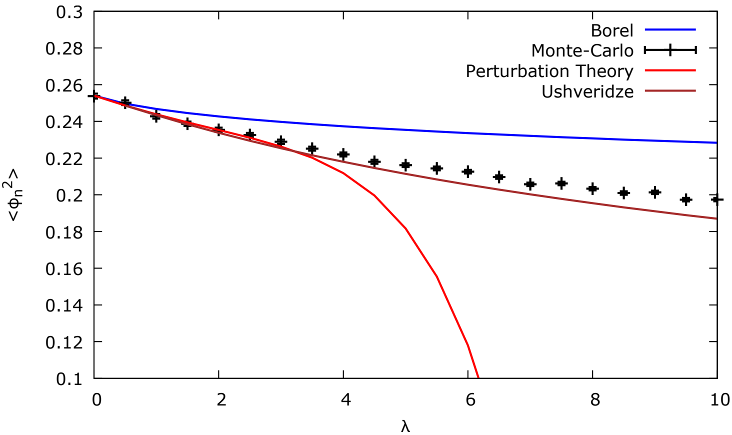

Figure: Comparison of the results for the 2d case on the $V = 10 \times 10$ lattice

Behavior of results in dependence on order

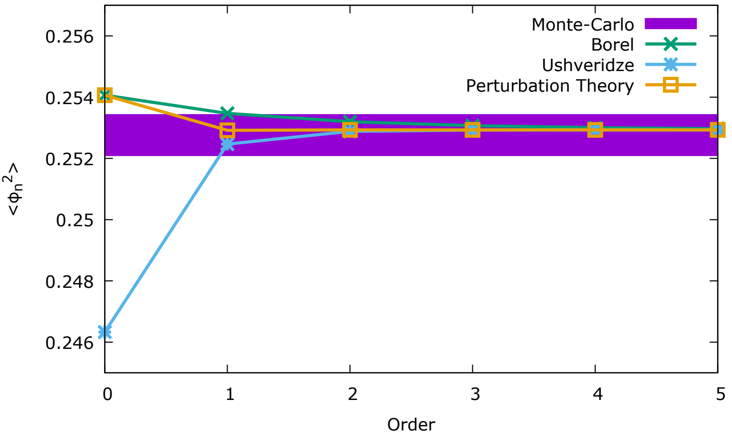

Figure: 2d case for $\lambda = 0.1$

Behavior of results in dependence on order

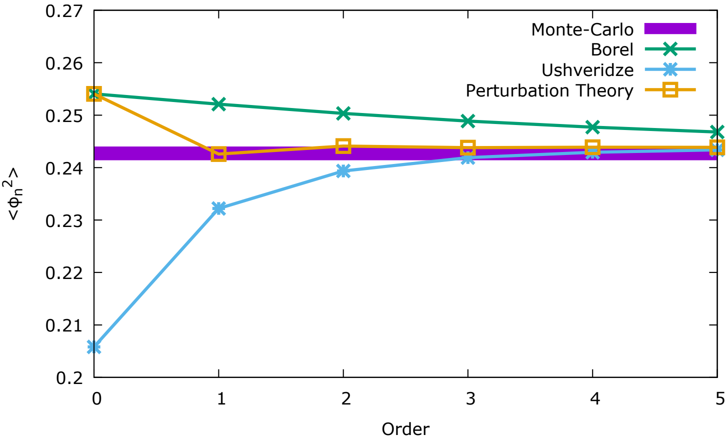

Figure: 2d case for $\lambda = 1$

Behavior of results in dependence on order

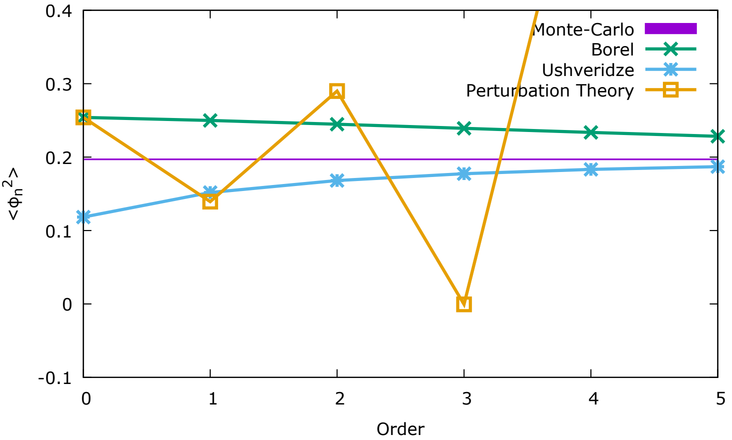

Figure: 2d case for $\lambda = 10$

Conclusions

- We have checked the convergent series method in the application to the lattice ϕ^4-model.
- The results of 5-loop calculations of $\langle \phi_n^2 \rangle$ are in the good agreement with Monte Carlo data in the wide range of the coupling constants.
- This supports the further utilization of this method for continuum QFT (including Yang-Mills, QCD...)
- and opens new ways for the computations on the lattice, which probably can help to avoid Sign problem.