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Topology and glueballs observables in $SU(7)$ Yang-Mills with open boundary conditions

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Lattice 2015 - Kobe

Outline

Introduction

- Boundary conditions are required in lattice simulation, but should not affect the final result.
- Every choice of BC carries a number of effects that must be addressed appropriately.
- Often such effects can be positively used to mitigate technical issues, e.g. Schrodinger Functional.
- Periodic Boundary Conditions (PBC) are widely used in LQCD: translational invariance.
- Problem: frozen topology in the continuum limit.

Open Boundary Conditions – so far

- First studies showed a reduction of τ_{int} for the slowly varying modes of the theory. [Lüscher, Schaefer '11,'13]
- Influence of OBC on χ_{TOP}, scalar/pseudoscalar glueball masses for $SU(3)$ [Chowdhury et al. '14,'15]
- Meson/baryon spectroscopy with $N_f = 2 + 1$ improved Wilson fermions and OBC [Bruno et al '15]

Open Boundary Conditions – This Work

- Motivation: Large-N ’t-Hooft limit of QCD approached from lattice simulation;
- As N is increased, slow down of numerical simulation is proven to be even worse;
- We focus on $SU(7)$ Yang-Mills theory with Open/Periodic BC and study:
  - Instanton distribution;
  - Autocorrelation of Topological charge;
  - Glueball correlators for 0^{++} and 0^{-+} states.

Simulations Details

Wilson Action

$$S = \beta \sum_{i,\mu<\nu} \left(1 - \mathcal{R}\mathrm{e}\,\mathrm{Tr}\,U_{\mu\nu}(i)\right) \qquad \beta \equiv 2N/g^2$$

- 1 HB + 4 OR ($N_{\mathrm{skip}} = 200$)
- $\beta = 34.8343$
- $a \simeq 0.94$ fm
  with $a\sqrt{\sigma} = 0.2093(1)$
  [Bali et al., '13]
- Code: `QDP-JIT/PTX`
  (on Tesla M2090)
  [Winter et al., '14]

Volume	BC	$N_{\mathtt{CFG}}$
$16^3 \times 32$	PBC	383
	OBC	787
$16^3 \times 48$	PBC	438
	OBC	592
$16^3 \times 64$	PBC	2973
	OBC	2981

Observables

Topological Charge

$$Q = \frac{1}{32\pi^2} \sum_{i,\mu<\nu} \epsilon_{\mu\nu\rho\sigma} U_{\mu\nu}(i) U_{\rho\sigma}(i) = \sum_{t=0}^{N_t-1} q(t)$$

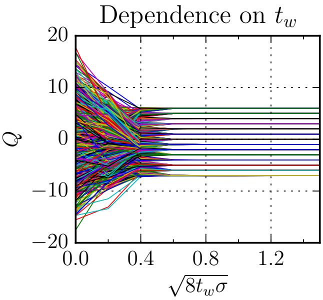
Smoothing provided by:

- Cooling

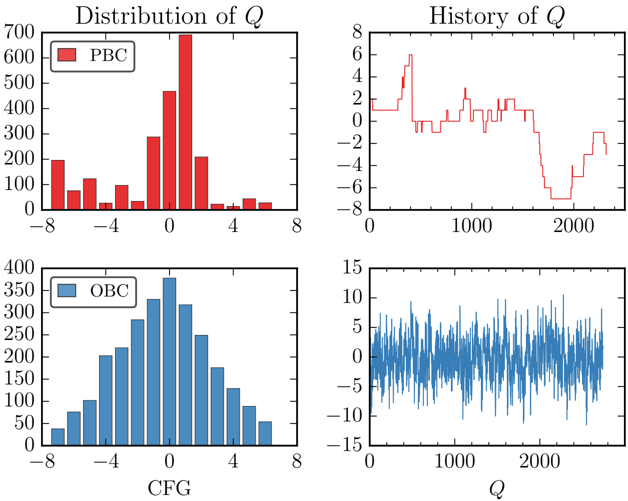
  [Teper, '85]

- Gradient Flow

  [Lüscher, '10]



Topological Charge: History



Observables

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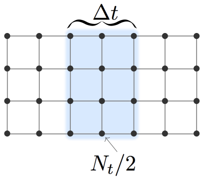
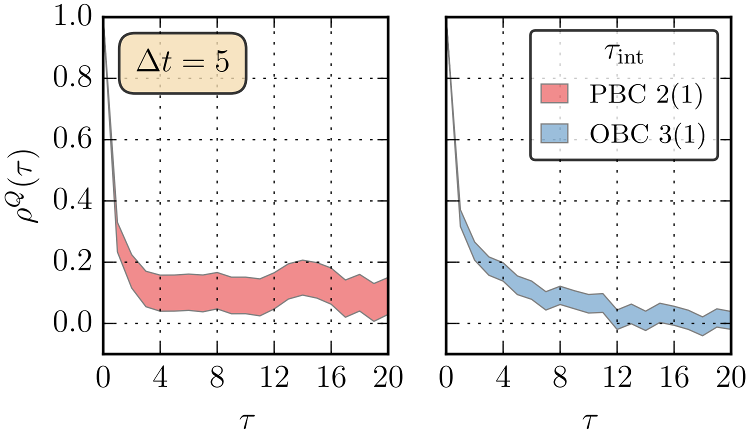
- Autocorrelation [Wolff, '04]:

$$\Gamma(\tau) = \frac{1}{N_{\mathtt{CFG}} - \tau} \sum_{i=0}^{N_{\mathtt{CFG}}-\tau} (O^i - \bar{O})(O^{i+\tau} - \bar{O}) \quad \rho(\tau) = \frac{\Gamma(\tau)}{\Gamma(0)}$$

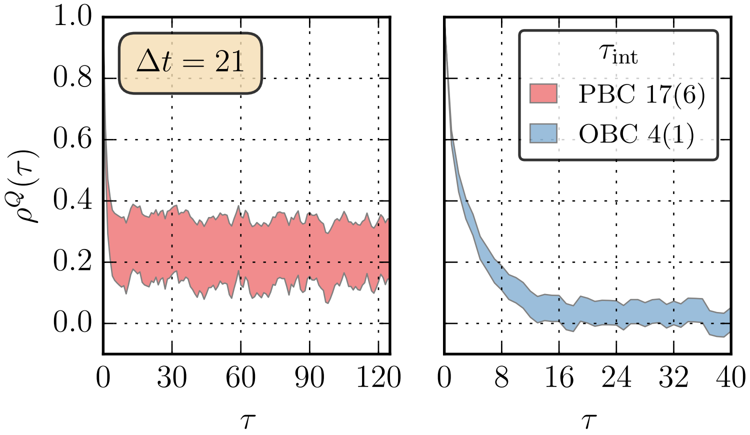
$$O \equiv \sum_{t=(N_t-\Delta t)/2}^{(N_t+\Delta t)/2} q(t)$$



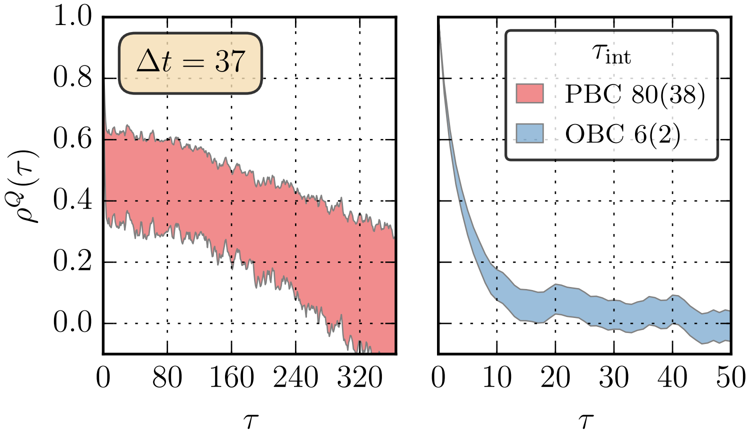
Topological Charge: Autocorrelation



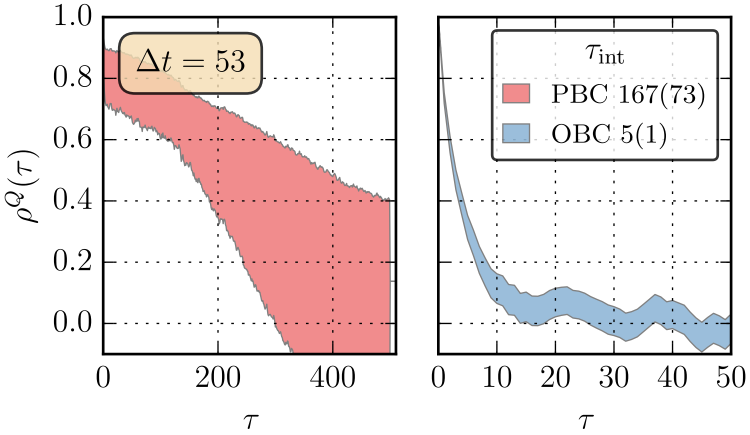
Topological Charge: Autocorrelation



Topological Charge: Autocorrelation



Topological Charge: Autocorrelation



Instanton Analysis

- In terms of the instantons, the topological charge:

$$Q = N^{+} - N^{-}$$

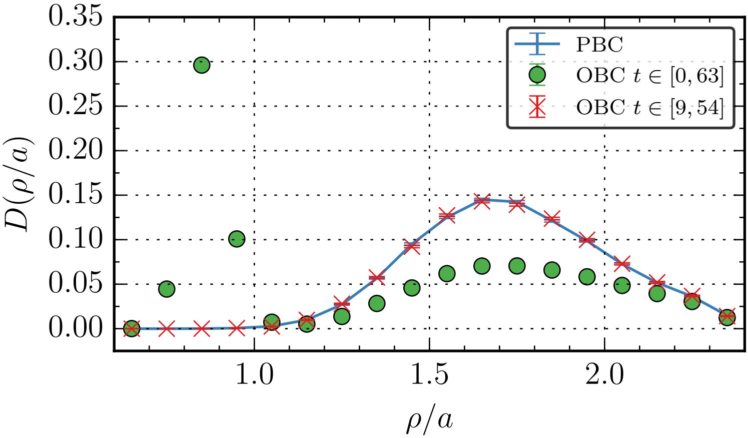
- Small Instantons suppressed at large N

$$D(\rho) \propto \rho^{\frac{11}{3}N-5}$$

- Instantons size is determined from local peaks of the topological charge [Smith, Teper, '98]

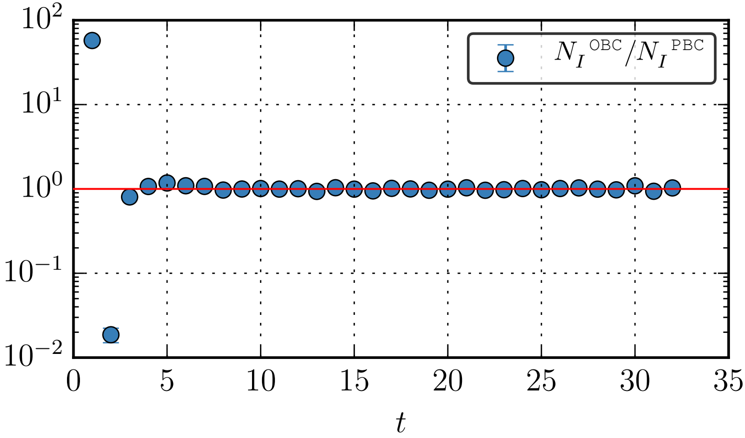
$$\frac{\rho}{a} = \left(\frac{6}{\pi^2 Q_{\text{peak}}}\right)^{1/4}$$

Instanton Size Distribution



Instantons Position in time

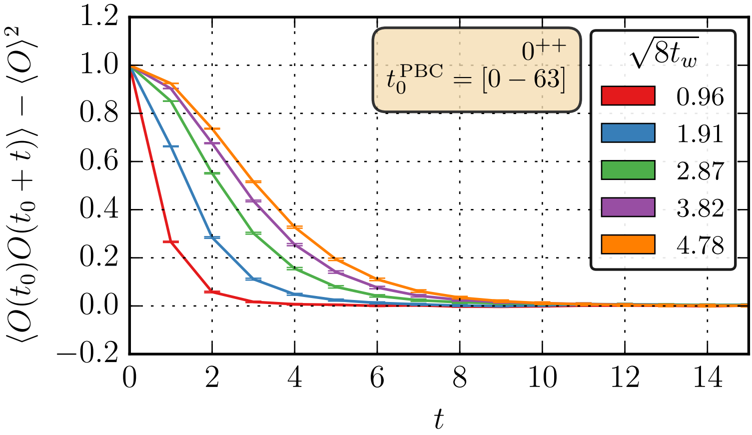
$$N_I = (n_{I^+} + n_{I^-})$$



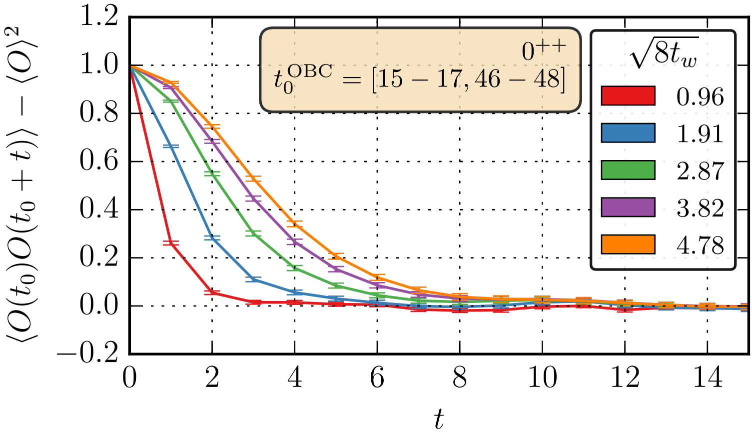
Glueballs Correlators

- Motivation: slow dynamic of Q might affect some spectral quantities
- Channels studied:
  - 0^{++}: plaquette-plaquette correlation function.
  - 0^{-+}: correlation of topological charge per timeslice.
- Smoothing
  - Gradient flow

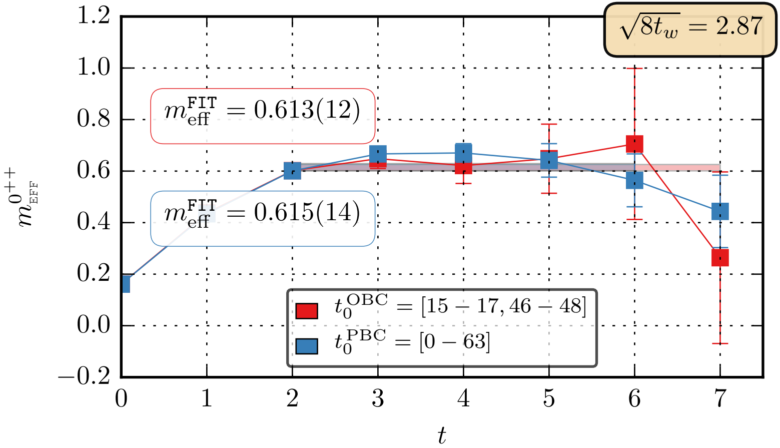
0^{++} – Periodic BC



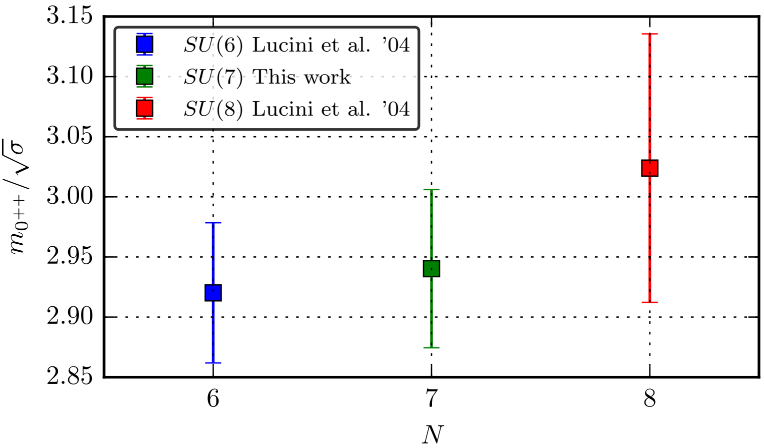
0^{++} – Open BC


0^{++}
$t_0^{\mathrm{OBC}} = [15-17, 46-48]$
$\sqrt{8t_w}$
0.96
1.91
2.87
3.82
4.78
$\langle O(t_0)O(t_0+t)\rangle - \langle O\rangle^2$
t
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0
2
4
6
8
10
12
14

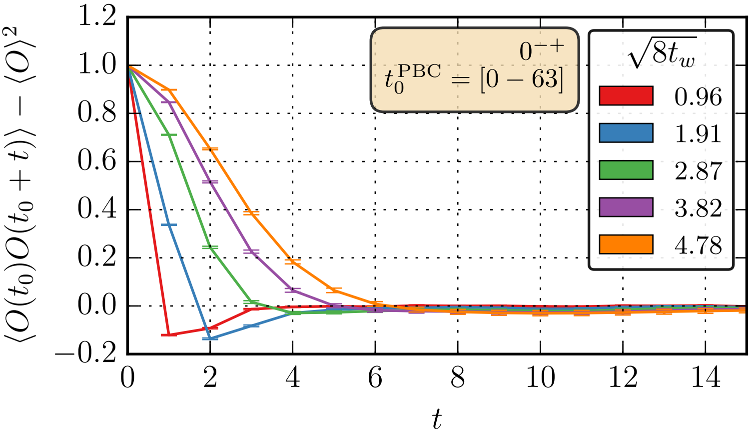
0^{++} – Effective mass



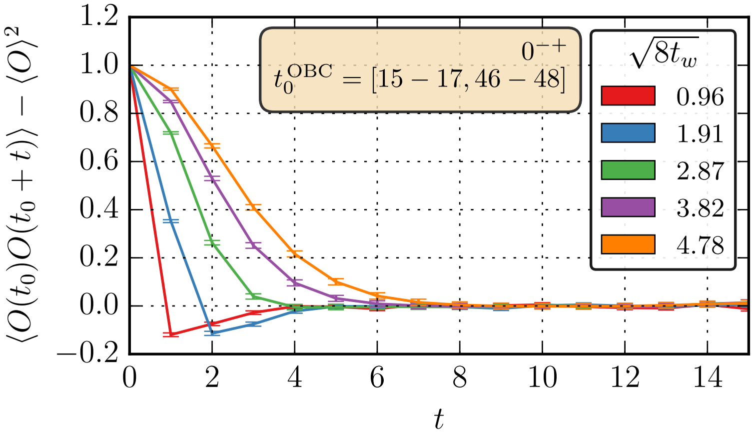
0^{++} – Effective mass



0^{-+} – Periodic BC



0^{-+} – Open BC


0^{-+}
$t_0^{\mathrm{OBC}} = [15-17, 46-48]$
$\sqrt{8t_w}$
0.96
1.91
2.87
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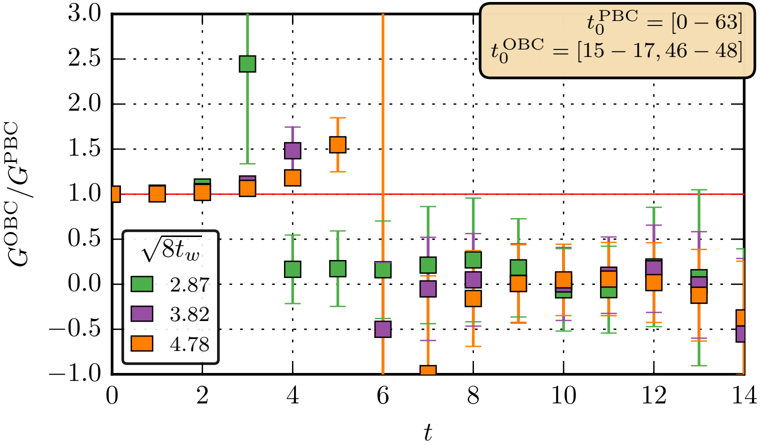
4.78
$\langle O(t_0)O(t_0+t)\rangle - \langle O\rangle^2$
t
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0
2
4
6
8
10
12
14

0^{-+} – Ratio of Correlators

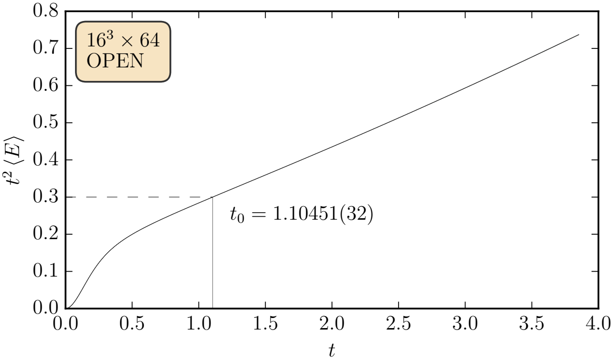


Conclusions

- OBC effectively reduces correlation of Q even in the $SU(7)$ case where creation of small instantons is highly suppressed
- OBC agrees very well with PBC in the bulk
- A great number of small dislocations are concentrated on the boundaries for OBC which then grow to physical size in the bulk
- Wilson flow competitive for extraction of glueball 0^{++} mass

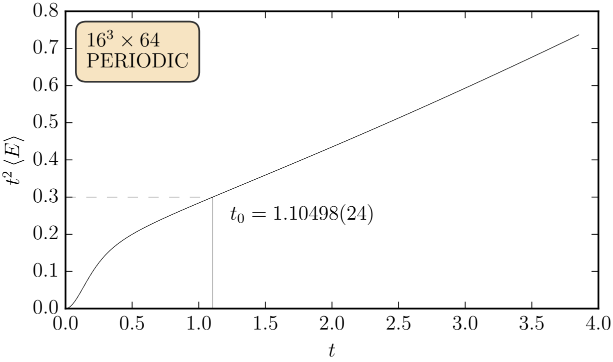
Thanks!

t_0 scale



$16^3 \times 64$
OPEN
$t_0 = 1.10451(32)$
$t^2 \langle E \rangle$
t
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0

t_0 scale



$16^3 \times 64$
PERIODIC
$t_0 = 1.10498(24)$
$t^2 \langle E \rangle$
t
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0

Q as a function of t_w