A stochastic approach to the reconstruction of spectral functions in lattice QCD

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Lattice 2015

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Outline

- Introduction & Motivation
- Stochastic Optimization Method (SOM)
  - ✧ General description
  - ✧ Mock data test
  - ✧ Results from Lattice data and comparisons
- Summary & Conclusion

Spectral functions

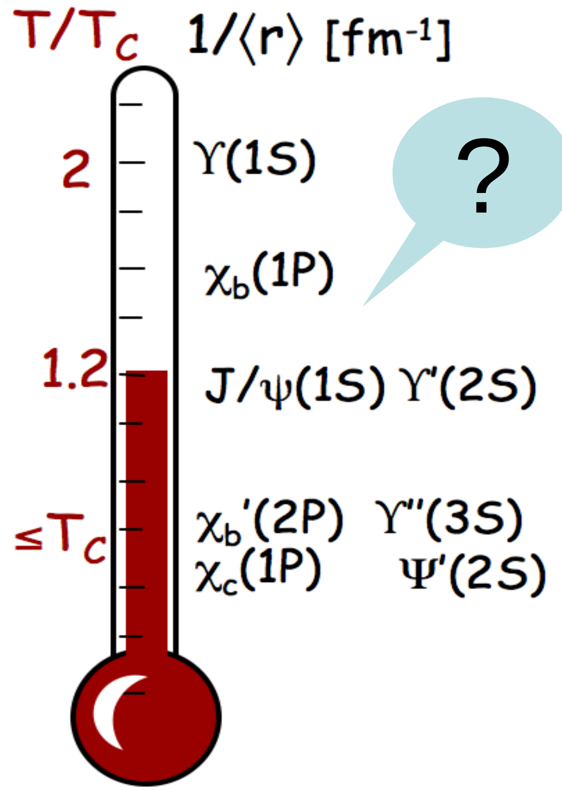
- Thermal di-lepton/photon rates (QGP probe)
- Transport properties of strongly-interacting matter
- Dissociation patterns of hadrons
- …

Thermal di-lepton production :

$$\frac{dW}{d\omega d^3p}=\frac{5\alpha}{54\pi^3}\frac{1}{\omega^2(e^{\omega/T}-1)}\rho_V(\omega,\vec{p},T)$$

Heavy quark diffusion coefficient :

$$D=\frac{\pi}{6\chi_{00}}\lim_{\omega\to 0}\frac{\rho_{ii}(\omega,\vec{p}=0,T)}{\omega}$$



A. Mocsy, Eur.Phys.J.C61:705-710,2009

How to extract spectral functions

Lattice observables:

$$J_H(\tau,\vec{x}) = \vec{q}(\tau,\vec{x})\Gamma_H q(\tau,\vec{x})$$

$$G(\tau,\vec{p},T)=\sum_{\vec{x}} \exp(-i\cdot\vec{p}\cdot\vec{x})\langle J_H(0,0)J_H{}^{+}(\tau,\vec{x})\rangle$$

$q(\tau,\vec{x})$ Γ_H $\bar{q}(\tau,\vec{x})$

$\bar{q}(0,0)$ Γ_H $q(0,0)$

$$K(\tau,\omega,T) = \frac{\cosh(\omega(\tau-1/2T))}{\sinh(\omega/2T)}$$

$$G(\tau,\vec{p},T)=\int_0^{\infty}\frac{d\omega}{2\pi}\rho(\omega,\vec{p},T)\,K(\tau,\omega,T)$$

Discretized ~O(10)

Continuous >=O(10^3)

χ^2 fitting inconclusive

Methods to solve this ill-posed problem:

- ✓ **Maximum Entropy Method** [M. Jarrell and J. E. Gubernatis, Phy. Rep.269,133(1996)]
  [M. Asakawa et al., Prog.Part.Nucl.Phys. 46(2001) 459-508]
- ✓ **Stochastic Analytical Inference** [S. Fuchs et al., 10.1103/PhysRevE.81.056701]
  See H. Ohno's talk today 14:20-40
- ✓ **Backus Gilbert Method** [A. Francis, Hard Probes 2015]
- ✓ ……
- ✓ **Stochastic Optimization Method (SOM)** [A. S. Mishchenko et al., PRB 62(2000)6317]

Comparison of SOM with MEM

Maximum Entropy Method

① Based on Bayesian Inference.

② Default model is needed.

③ Energy space is discretized.

④ The most probable solution is given.

⑤ This solution can be unique.

⑥ Integrate over α:

$$A_{out} = \int [dA] \int d\alpha\, A(\omega) P[A|DH\alpha m] P[\alpha|DHm]$$

Stochastic Optimization Method

① Based on Central Limit Theorem.

② No prior information is needed.

③ Energy space is continuous.

④ All possible solutions are given.

⑤ These solutions are independent.

⑥ Average over many solutions with equal weight:

$$A_{out} = \frac{1}{L} \sum_{j=1}^{L} \tilde{A}_j(\omega)$$

How to obtain a possible solution by SOM

Define a target function: $Q = \sum_{i,j}^{N_{conf}} (\bar{G}_i - G_i)\,[C^{-1}]_{i,j}(\bar{G}_j - G_j)$

C: covariance matrix

↓

Initialize spectral function; Calculate Q_0;
Initialize "Temperature" T_0

↓

Reshape spectral function stochastically by elementary updates; Calculate Q_i

↓

$\delta Q = Q_i - Q_{i-1}$

↓

Accept Q_i with possibility: P=min{1, exp($-\delta Q / T_i$)}

↓

Decrease "Temperature": $T_{i+1} = T_i \cdot$ decayscale

↓

$T_i < T_{stop}$

NO → back to "Reshape spectral function stochastically by elementary updates"

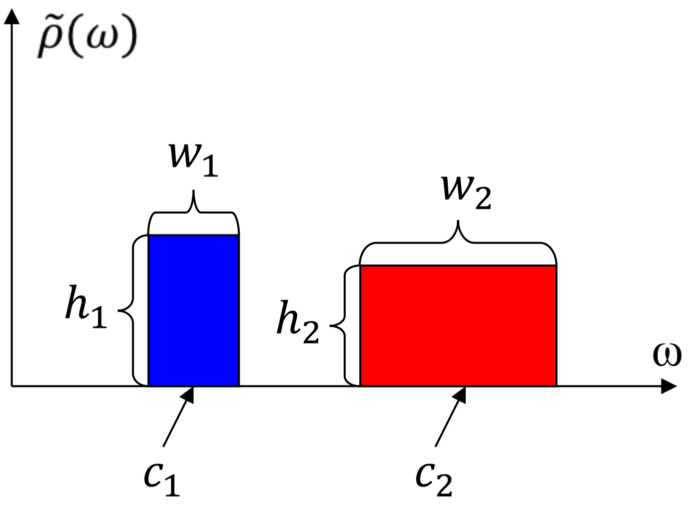
YES ↓

One possible solution is obtained

Finally average over all possible solutions

Stochastic Optimization Method

- Generate randomly-distributed boxes $\{P_t\} = \{c_t, w_t, h_t\}$



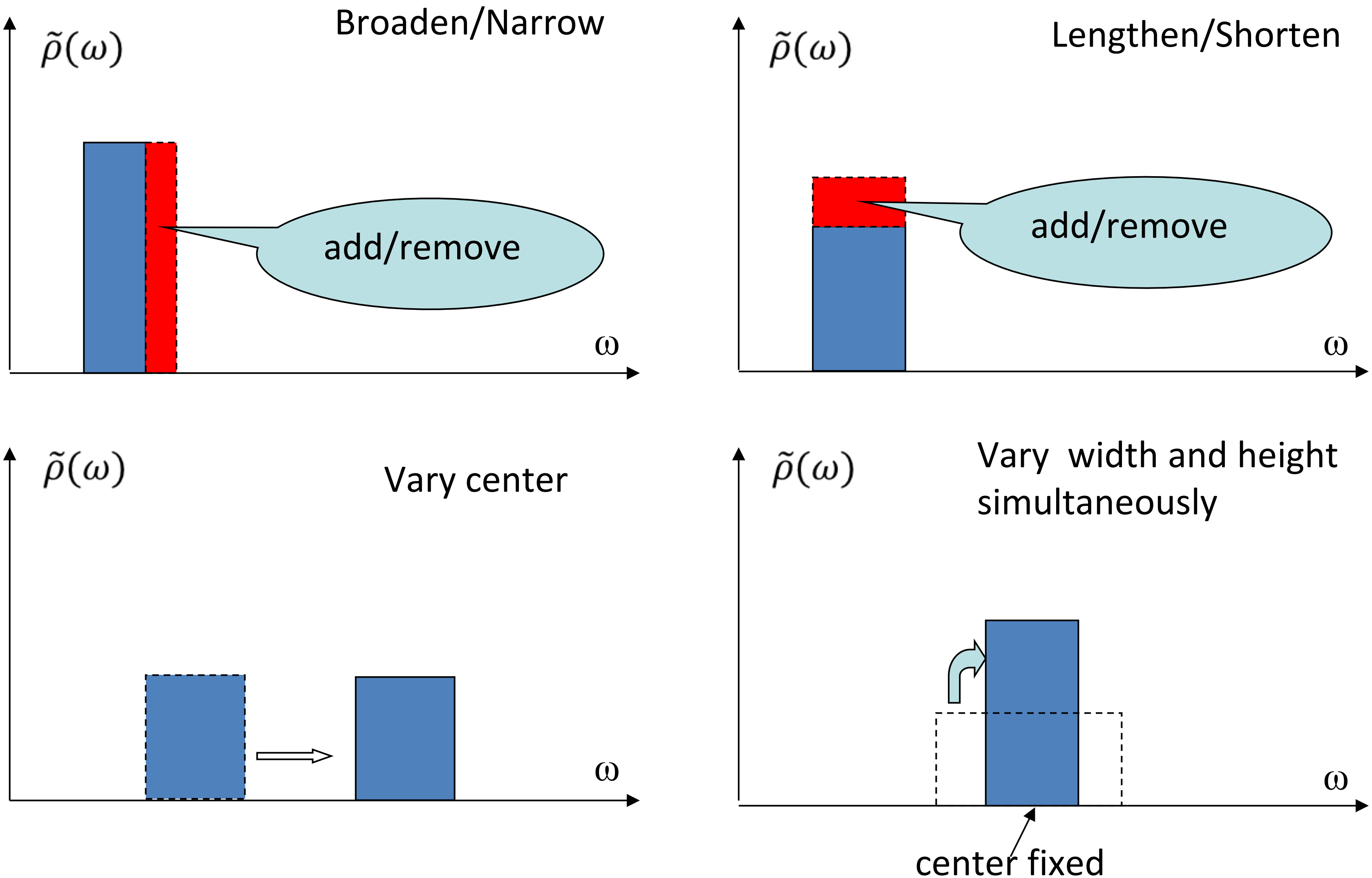
c_t: *center*
w_t: *width*
h_t: *height*

- Parameterize the spectral function $\tilde{\rho}(\omega)$ as a sum of boxes

$$\tilde{\rho}(\omega) = \sum_t^K \eta\,(P_t)(\omega), \quad \eta(P_t)(\omega) = \begin{cases} h_t, & \omega \in [c_t - w_t/2, c_t + w_t/2,] \\ 0, & otherwise \end{cases}$$

Stochastic Optimization Method

✧ **Perform elementary updates to minimize target function:**



Stochastic Optimization Method

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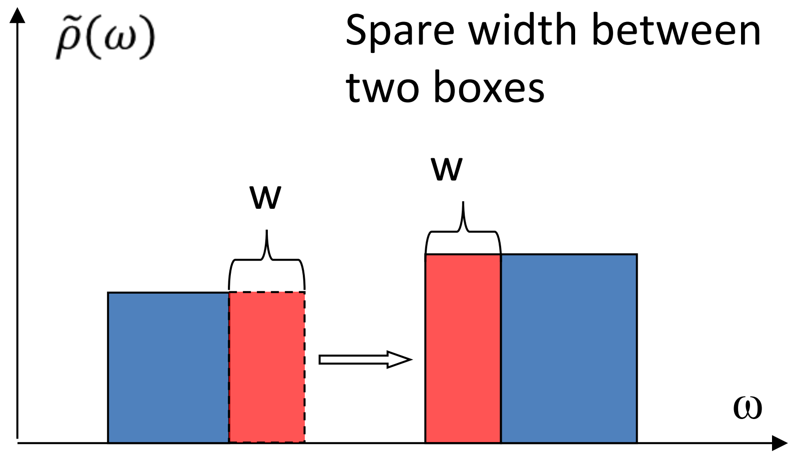
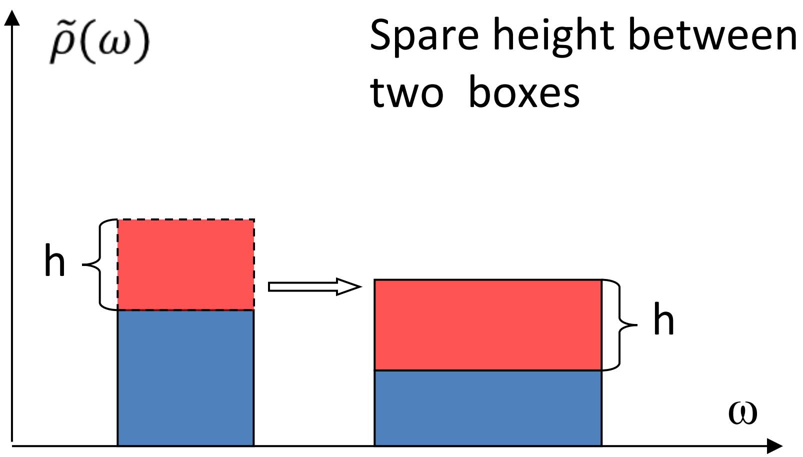
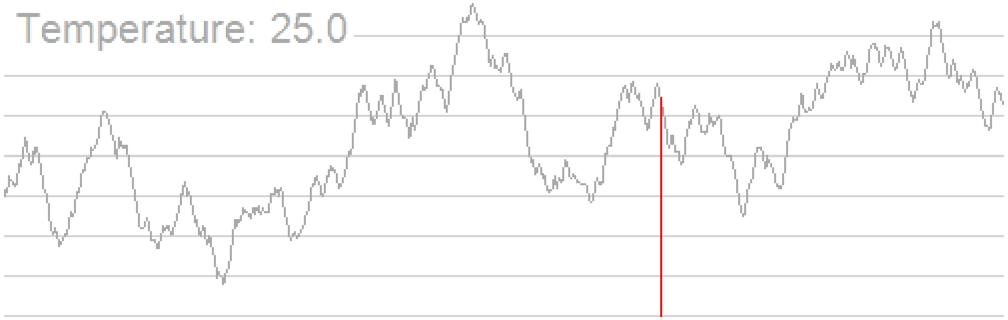


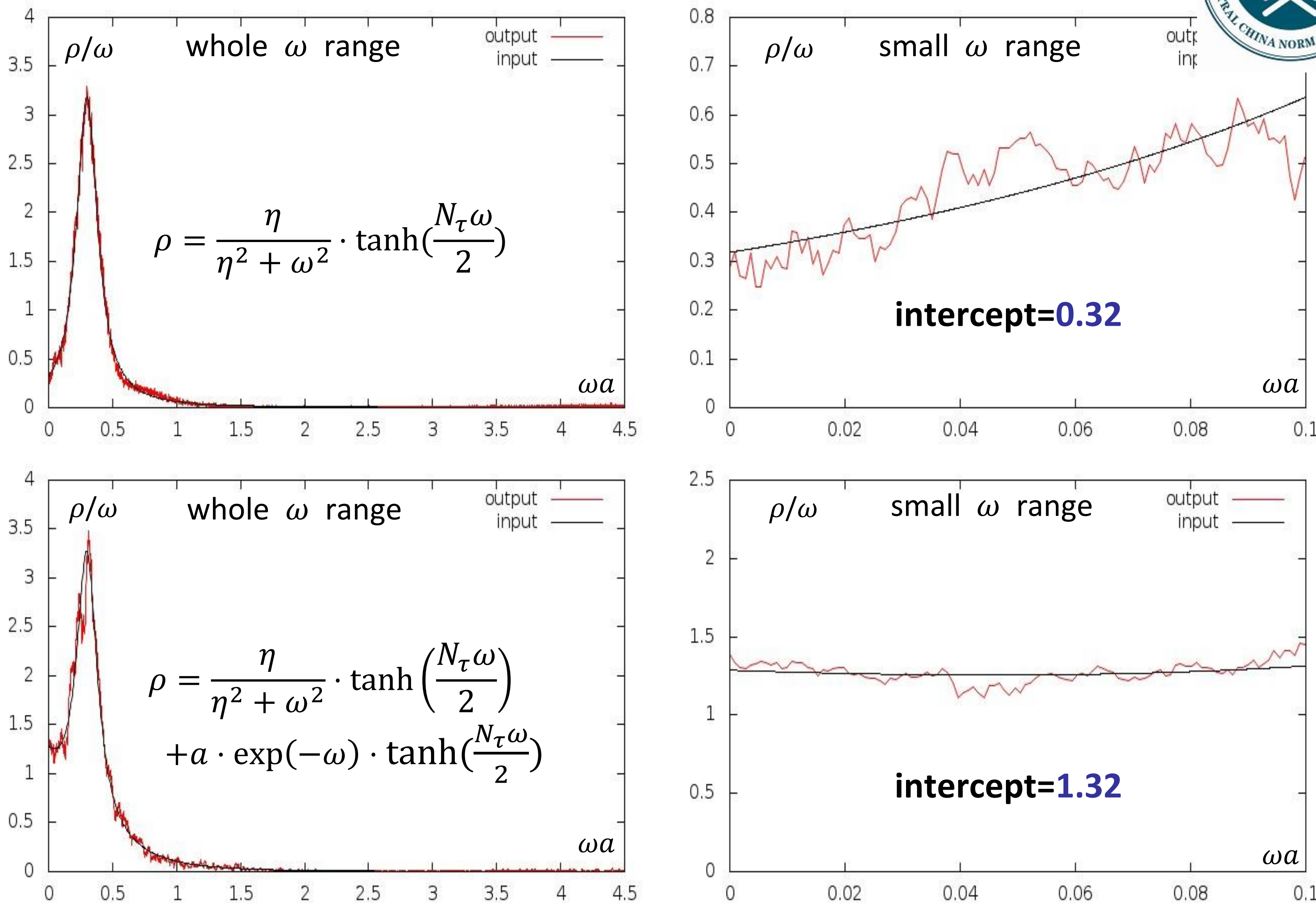


✧ **Avoid local minima by Simulated Annealing Algorithm**



MOCK DATA TEST on transport peaks with N_τ=64

1 possible solution



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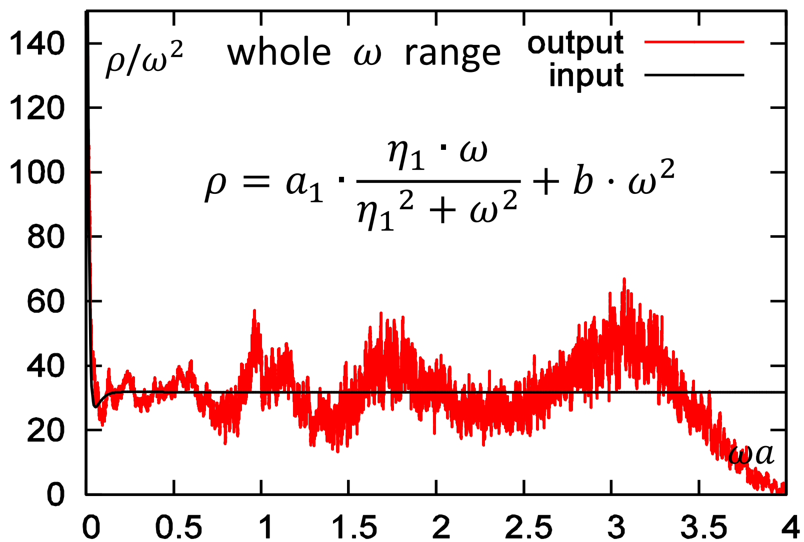
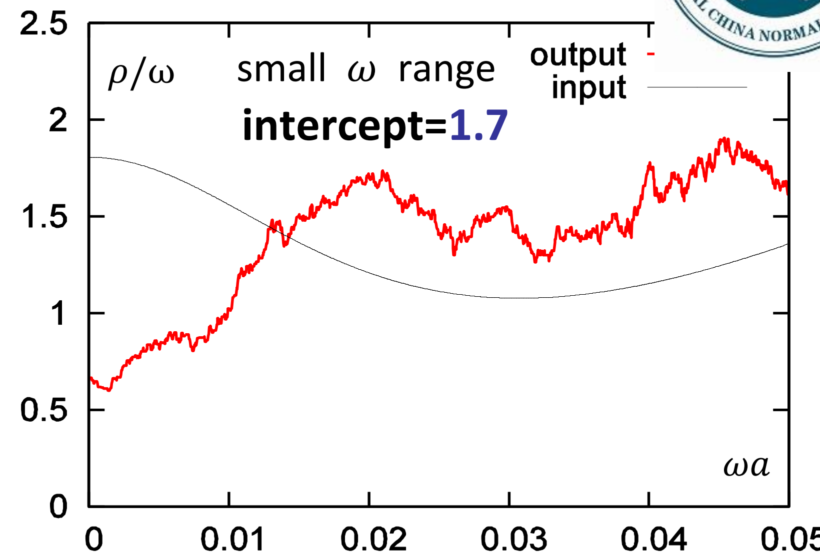
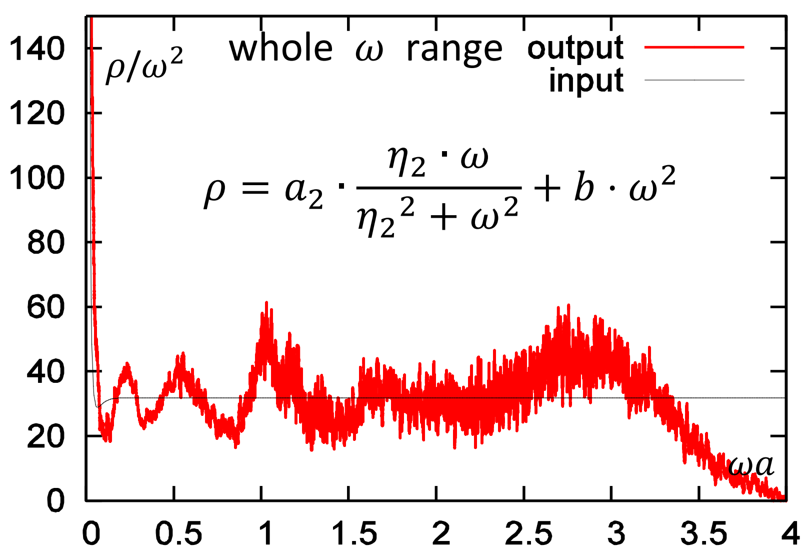
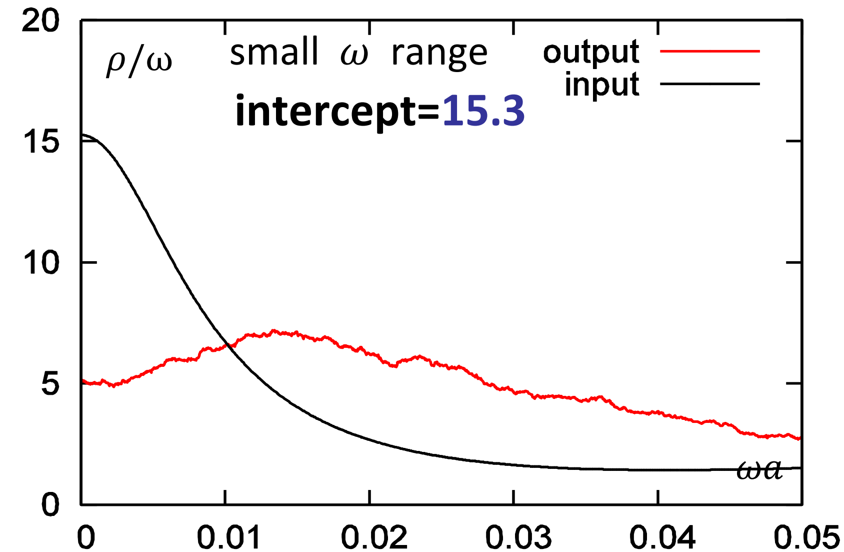


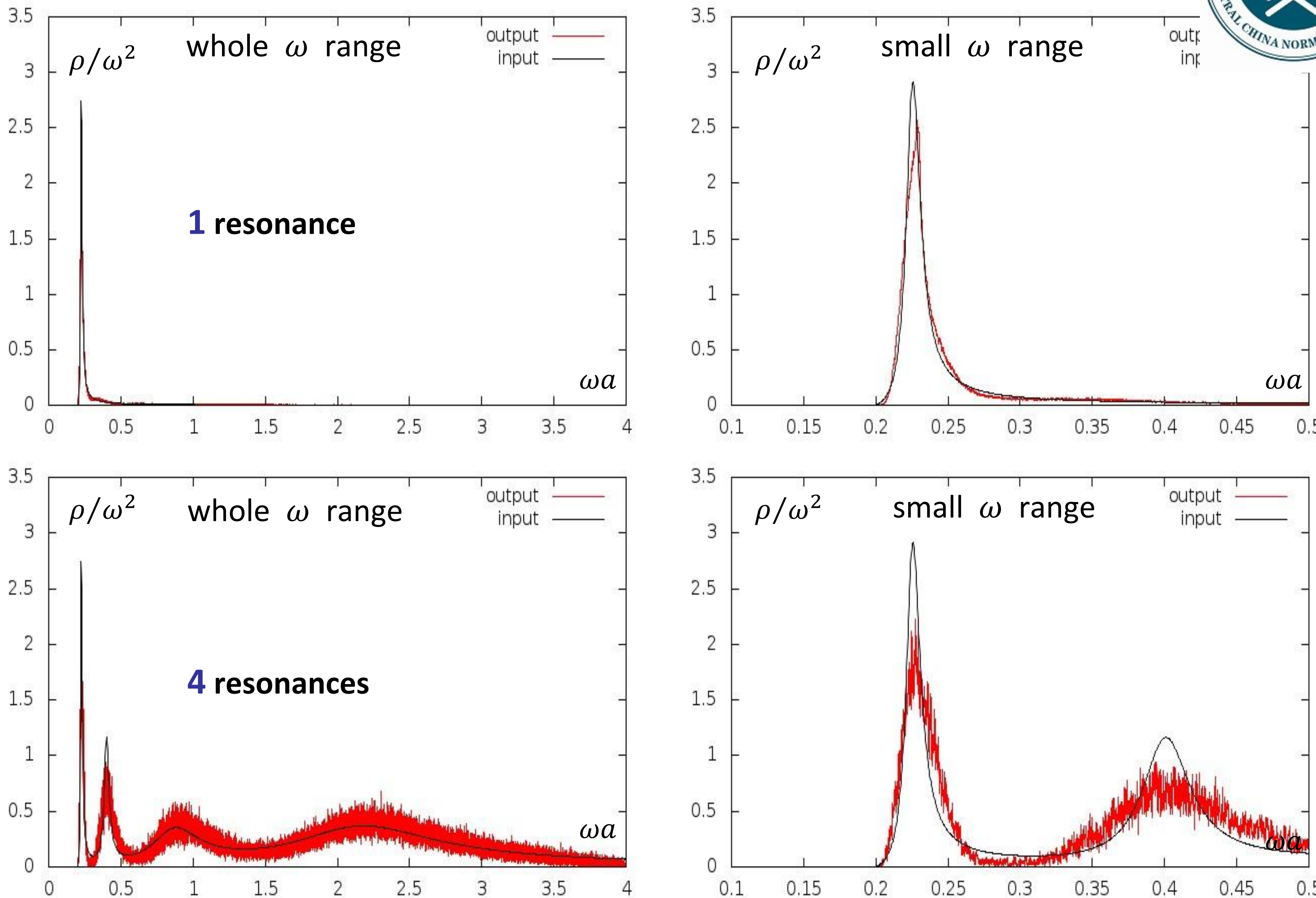




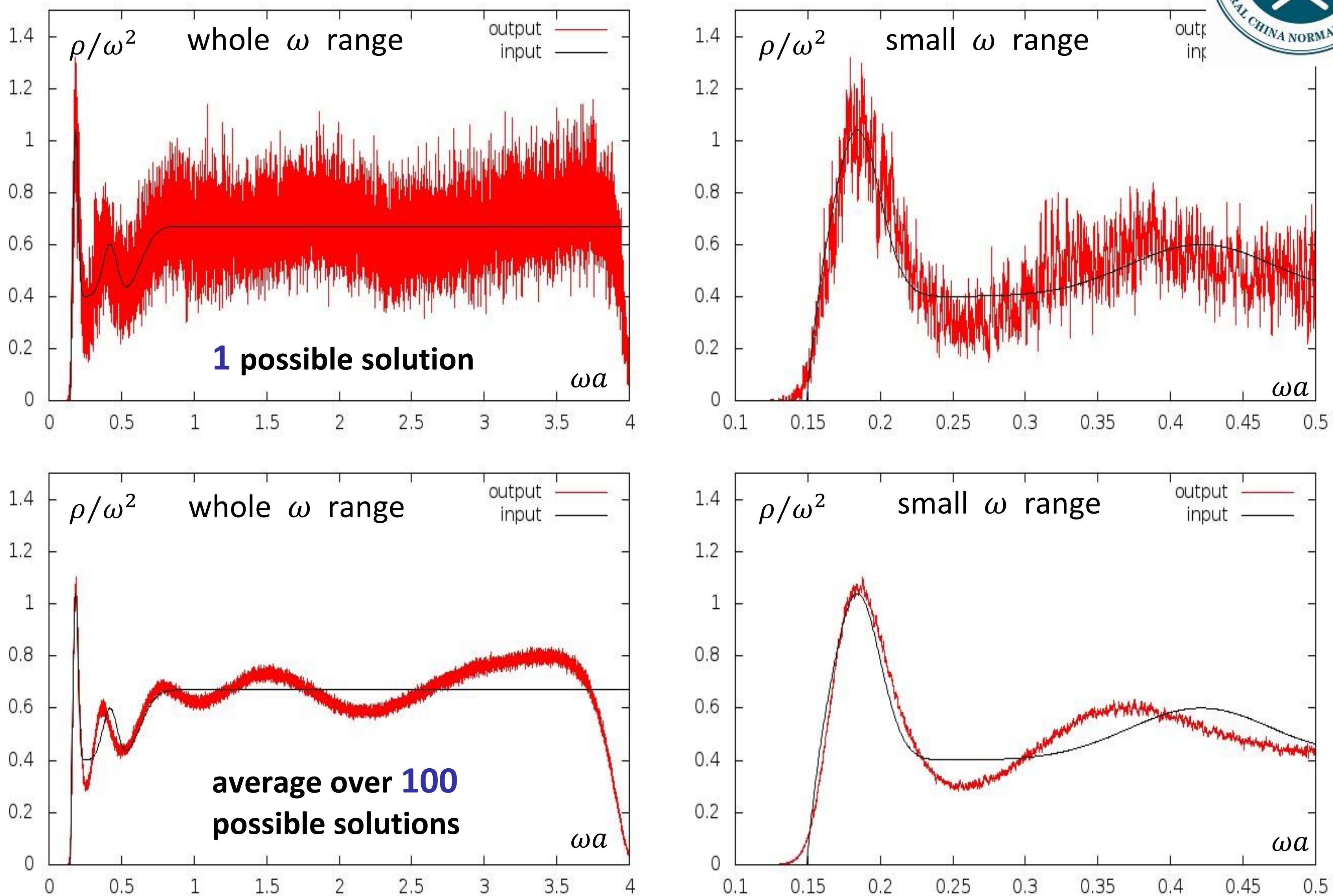


MOCK DATA TEST on sharp resonance(s) with N_τ=96

1 possible solution

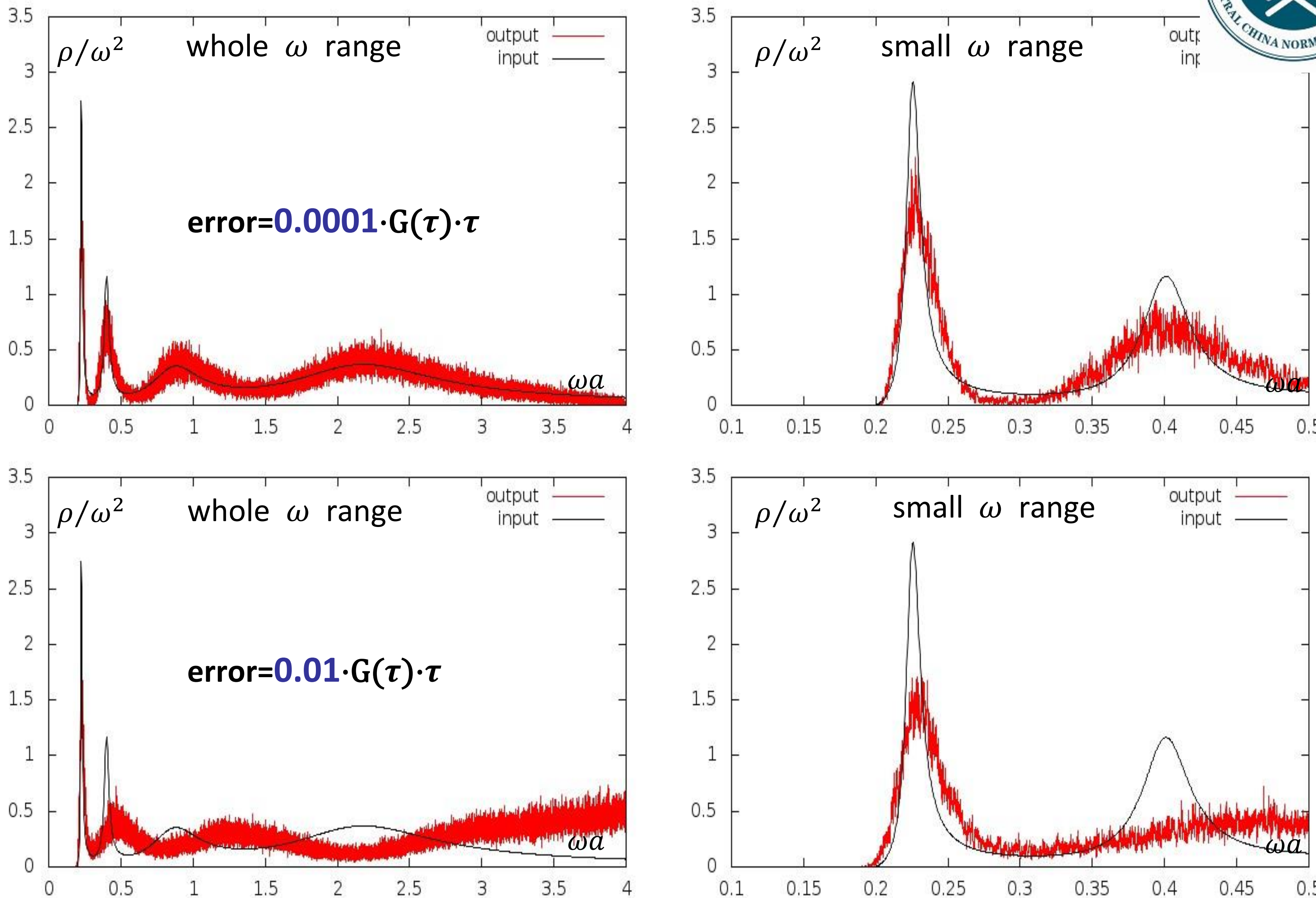


MOCK DATA TEST on 2 sharp resonances +continuum with N_τ=96



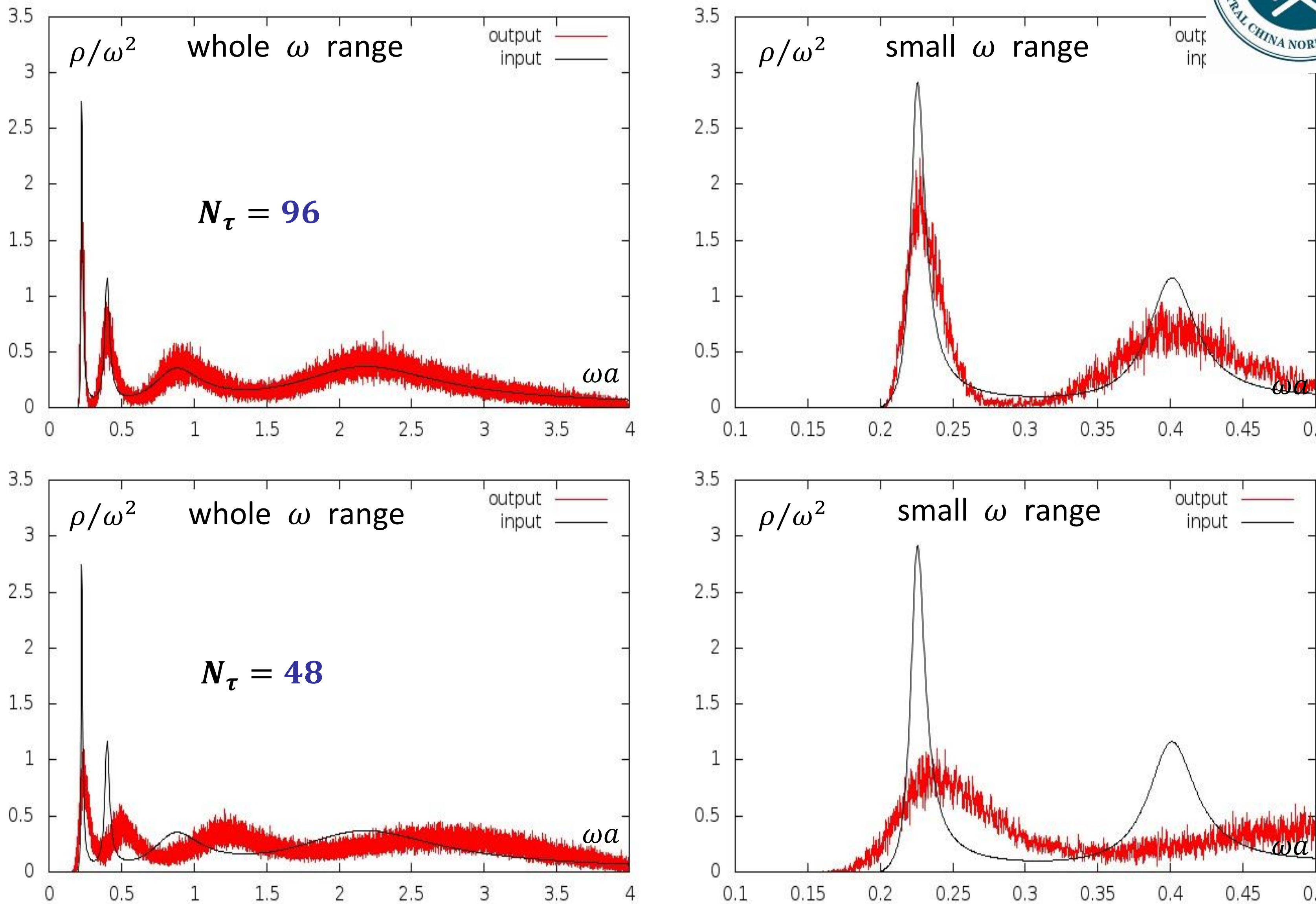
MOCK DATA TEST: dependence on errors with N_τ=96

1 possible solution



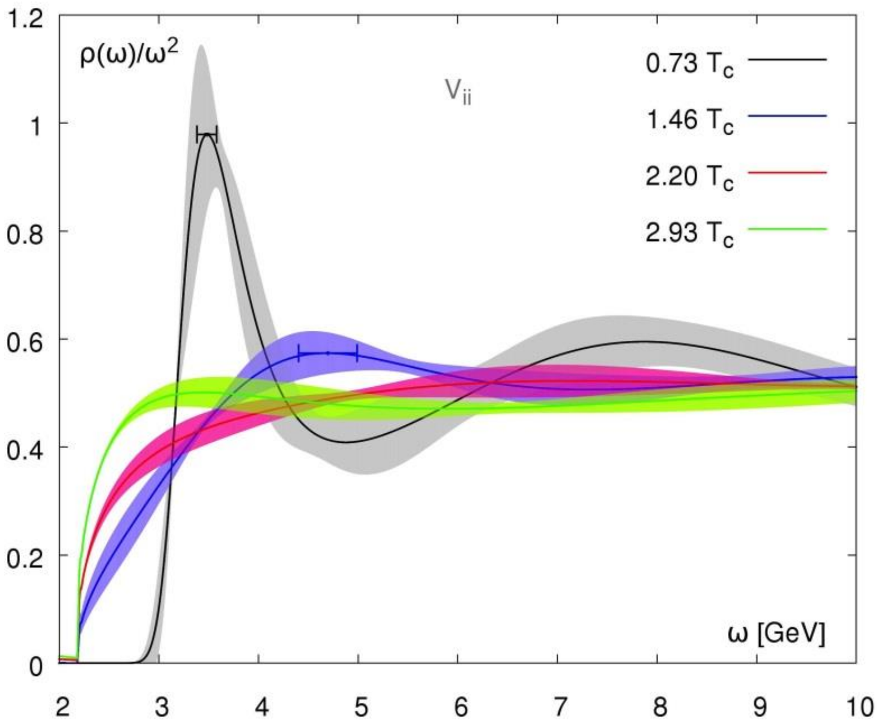
MOCK DATA TEST: dependence on N_τ

1 possible solution

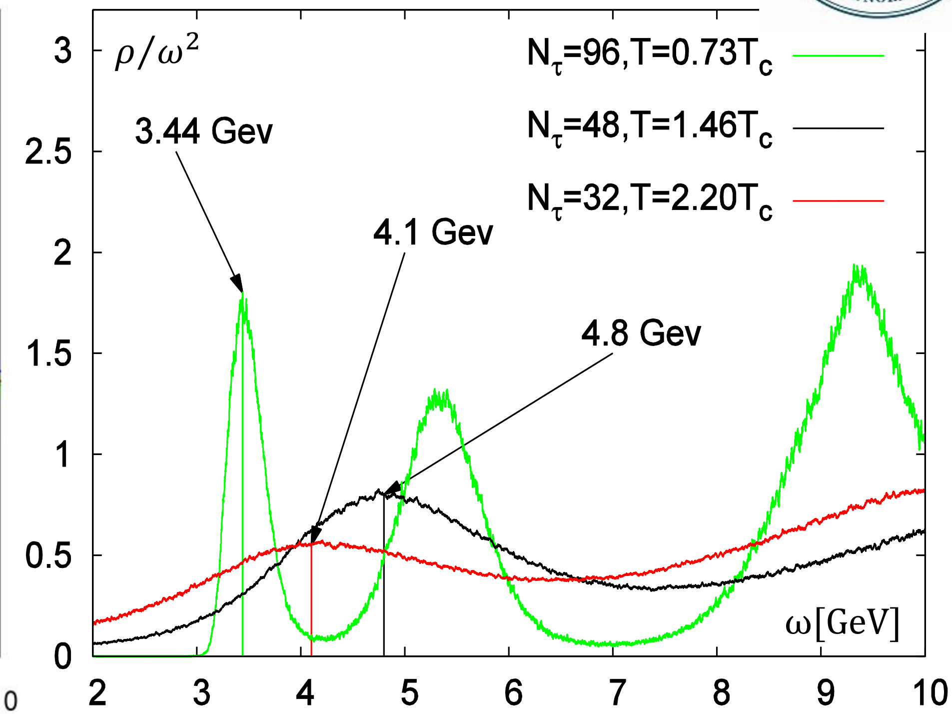


RESULTS FROM LATTICE DATA:

Charmonium states in the vector channels at different temperatures



[H.T. Ding et al., Phys. Rev D 86, 014509 (2012)]



average over 100 possible solutions

Comparison: peak-location (in [GeV]**) and peak-height of the first resonance by SOM and MEM .The screening mass (pole mass) is** 3.472 GeV **obtained from G(z)**.

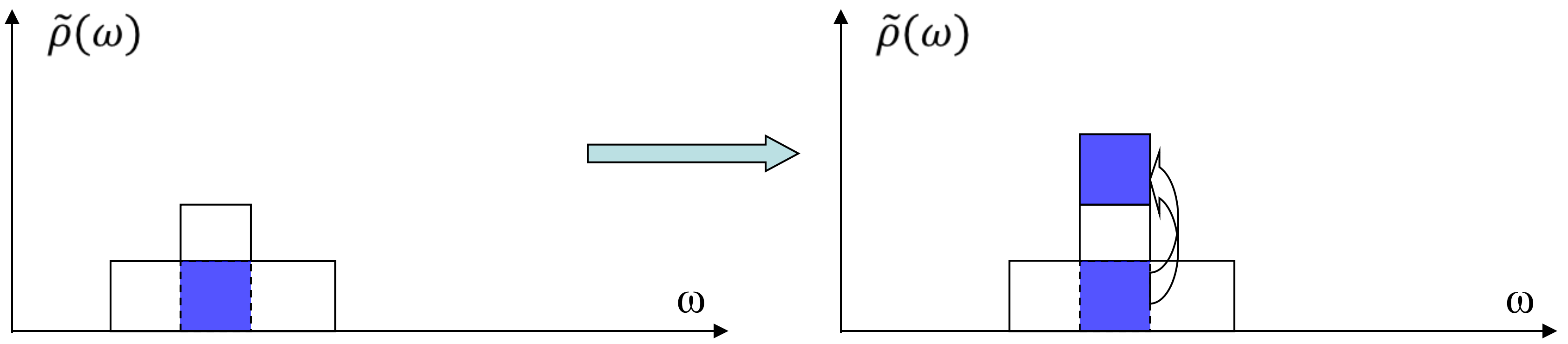
	$0.73T_c$	$1.46T_c$	$2.20T_c$
SOM	3.44/1.75	4.8/0.72	4.1/0.57
MEM	3.48/0.98	4.7/0.58	???

Summary & Outlook

- We have introduced Stochastic Optimization Method to extract spectral function from temporal correlation function in Lattice QCD
- SOM is found to work well for transport peaks and resonance peaks by using mock data
- Results obtained from charmonium correlation functions calculated from LQCD by SOM are quite similar to those by MEM
- Improvement is needed to obtain very sharp peaks

Back up

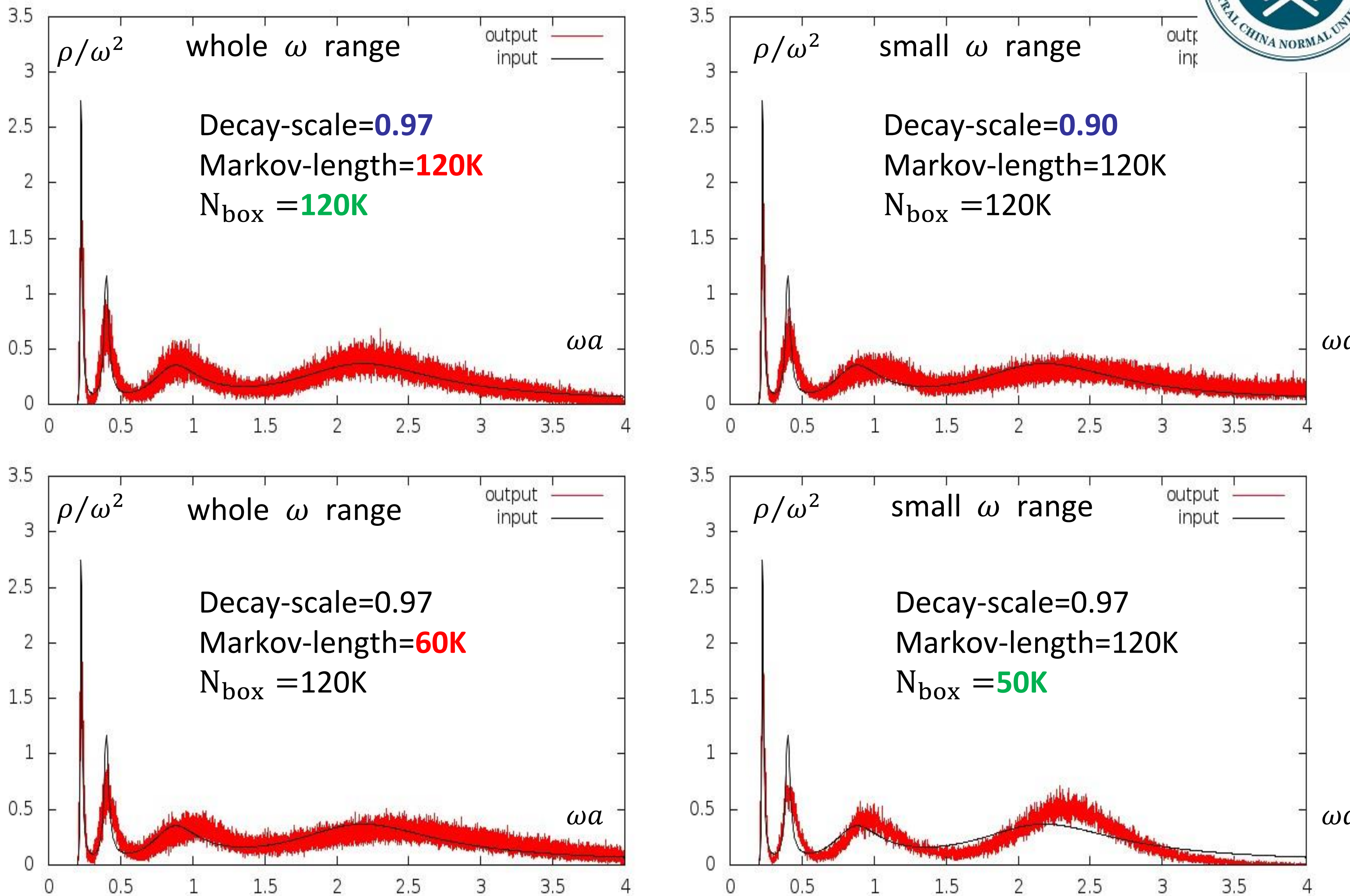
How the overlap of boxes is understood



Back up

MOCK DATA TEST: dependence on SOM parameters

1 possible solution



Back up

MOCK DATA TEST: dependence on SAA

1 possible solution