The Three-Quark Potential and Perfect Abelian Dominance in SU(3) Lattice QCD

H. Suganuma (Kyoto U.) and N. Sakumichi (Ochanomizu U.)

Abstract:

We study the static three-quark (3Q) potential for more than 300 different patters of 3Q systems with high statistics in SU(3) quenched lattice QCD. For all the distances, the 3Q potential is found to be well described by the Y-Ansatz, i.e., *one-gluon-exchange Coulomb plus Y-type linear potential*. Remarkably, quark confinement forces in both QQbar and 3Q systems can be described only with Abelian variables in the maximally Abelian gauge, which we call "*perfect Abelian dominance*" of quark confinement.

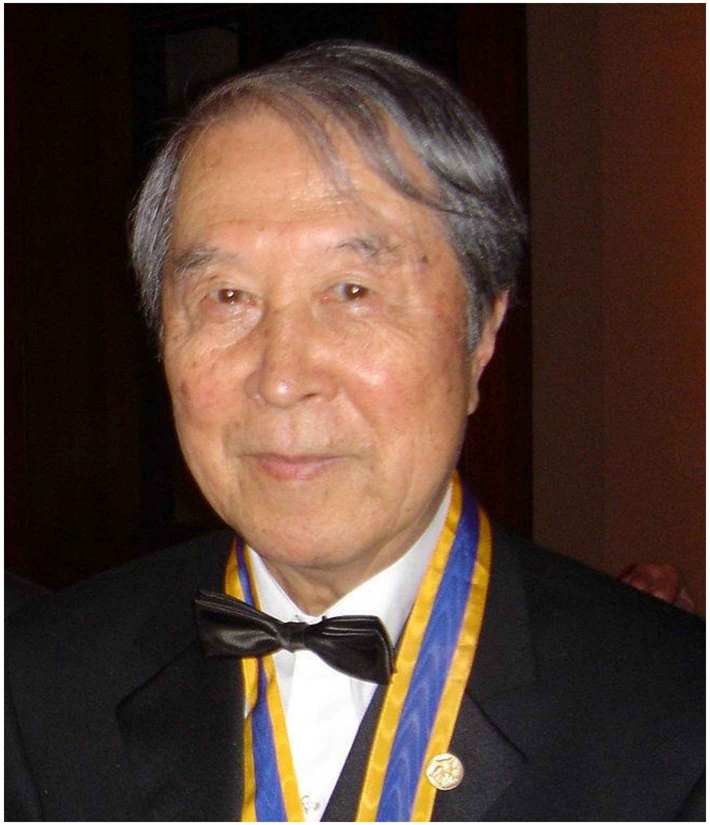
Reference:

[1] N.Sakumichi and H.S., Physical Review D90 Rapid Communication 111501 (2014), "Perfect Abelian Dominance of Quark Confinement in SU(3) QCD".

[2] N.Sakumichi and H.S., arXiv: 1501.07596 [hep-lat], "Perfect Abelian Dominance of Quark Confinement in Baryonic Three-Quark Potential"

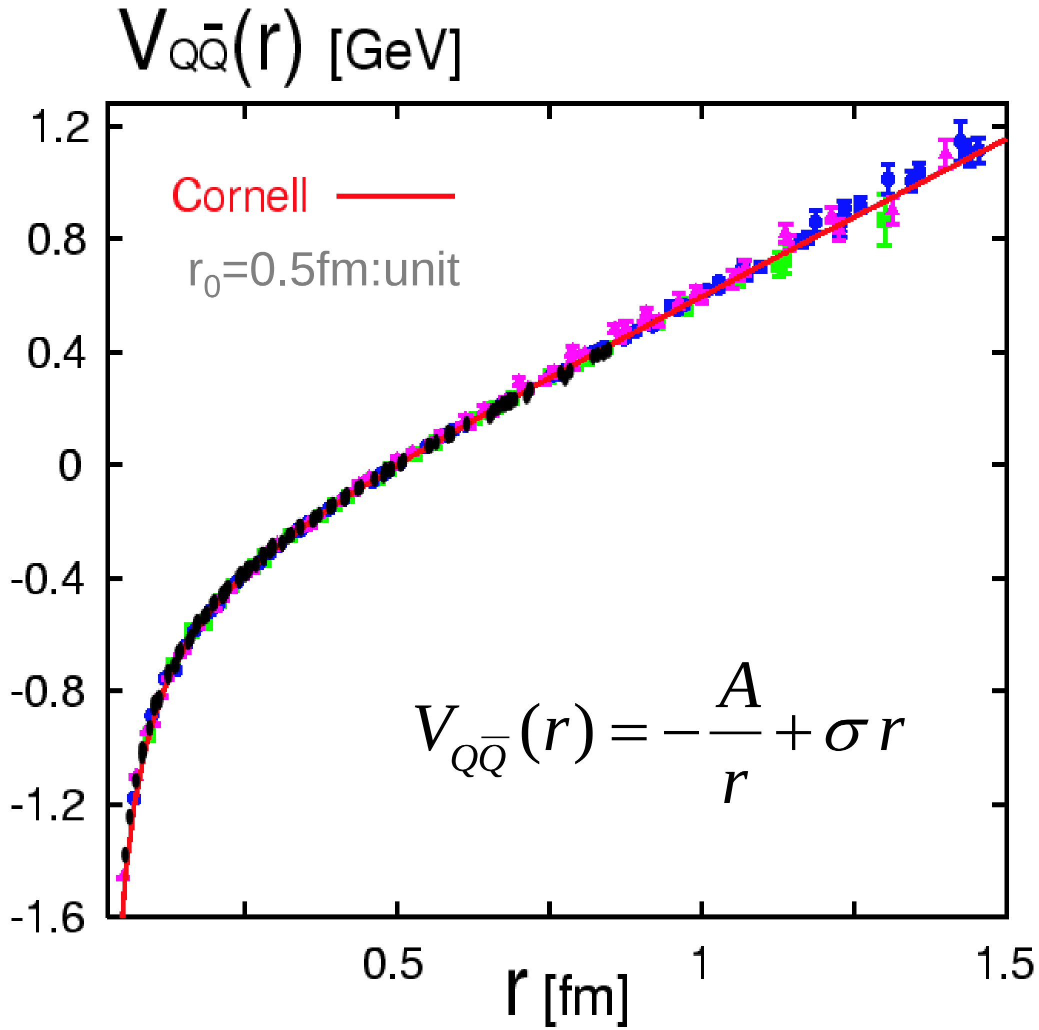
Lattice 2015, July 14-18, 2015, Kobe

Very sadly, Great Professor Yoichiro Nambu
passed away on 5th July 2015.



I sincerely thank Prof. Nambu for his interests and several valuable suggestions on our confinement studies, and also for his many great works in physics.

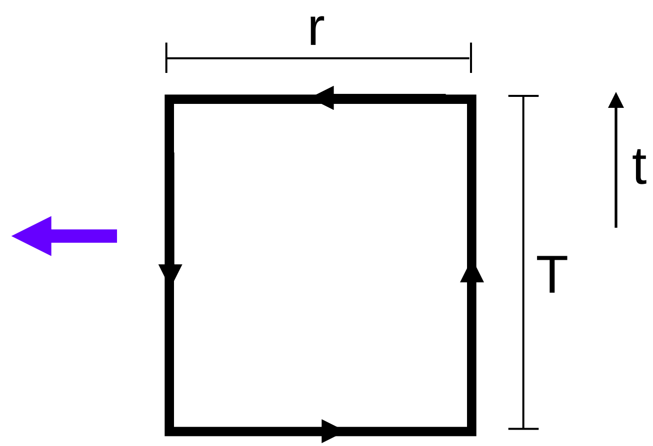
Quark–antiquark static potential in Lattice QCD



M.Creutz (1979,80)

quark **anti-quark**

Summarized lattice QCD data
G.S.Bali (2001)
T.T.Takahashi, H.S. et al. (2002)
JLQCD (2003)

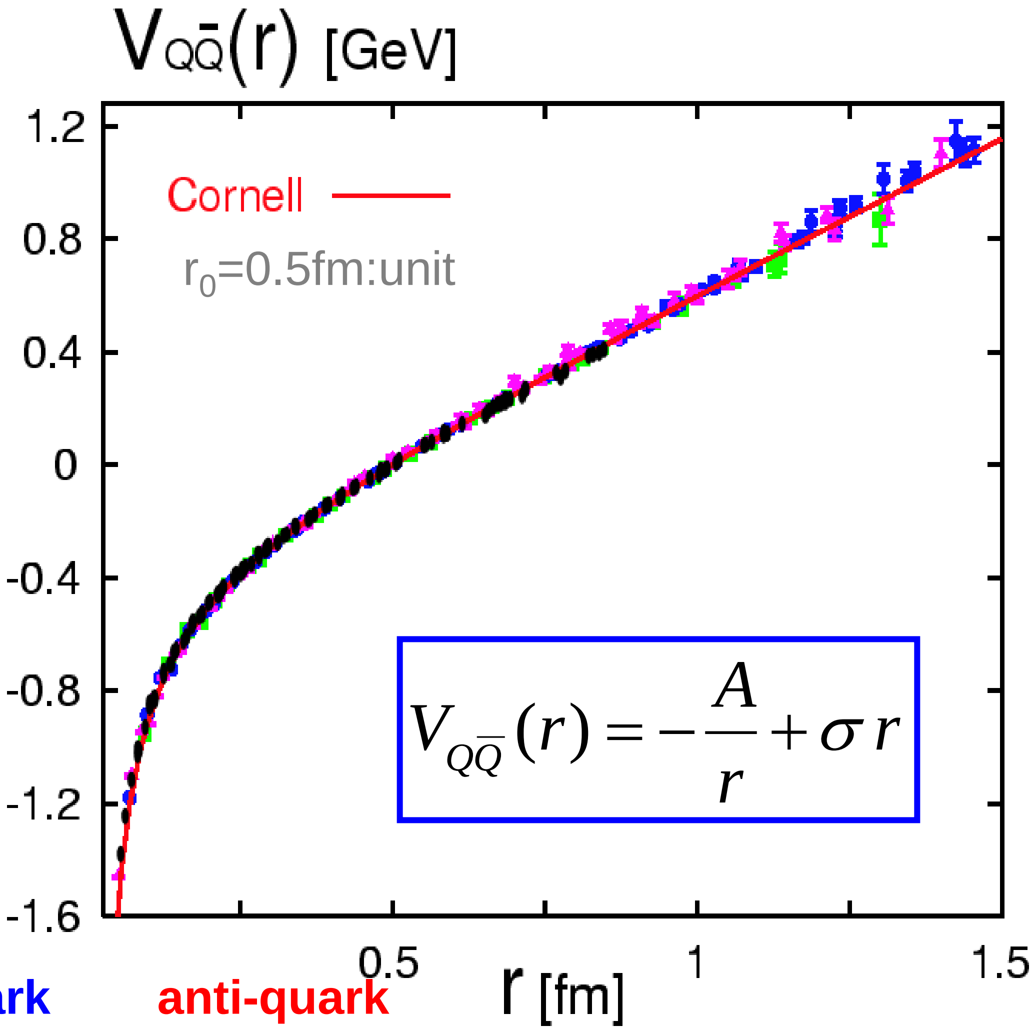


Wilson loop

$$V_{Q\bar{Q}}(r) = -\lim_{T\to\infty} \frac{1}{T} \ln \langle W \rangle_T$$

The *inter-quark potential* is obtained from the *Wilson loop* in lattice QCD.

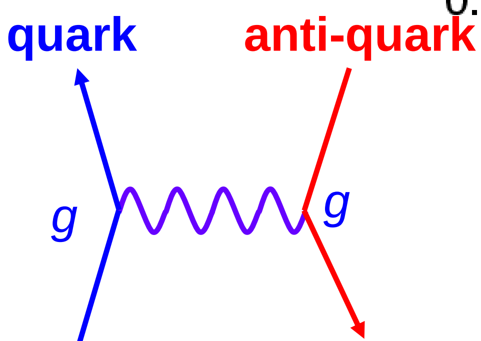
Quark-antiquark static potential in Lattice QCD



M.Creutz (1979,80)

Summarized lattice QCD data
G.S.Bali (2001)
T.T.Takahashi, H.S. et al. (2002)
JLQCD (2003)

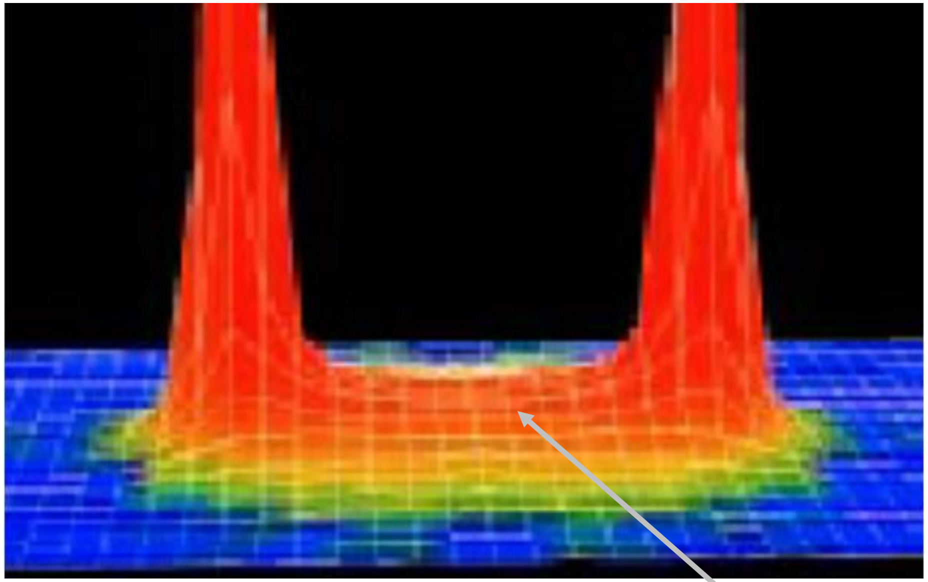
Quantitatively, the QQbar potential is well described by a simple sum of Coulomb plus linear.



One-dimensional squeezing of color flux between q and $\overline{q}$

Flux-tube formation for QQbar system in Lattice QCD

G.S. Bali



Color-Electric Flux Tube

Non-perturbative

As for the flux-tube formation, it been observed in lattice QCD, and such one-dimensional squeezing of color flux leads to linear confinement potential.

Dual Superconductor Picture for Confinement

Historical Overview

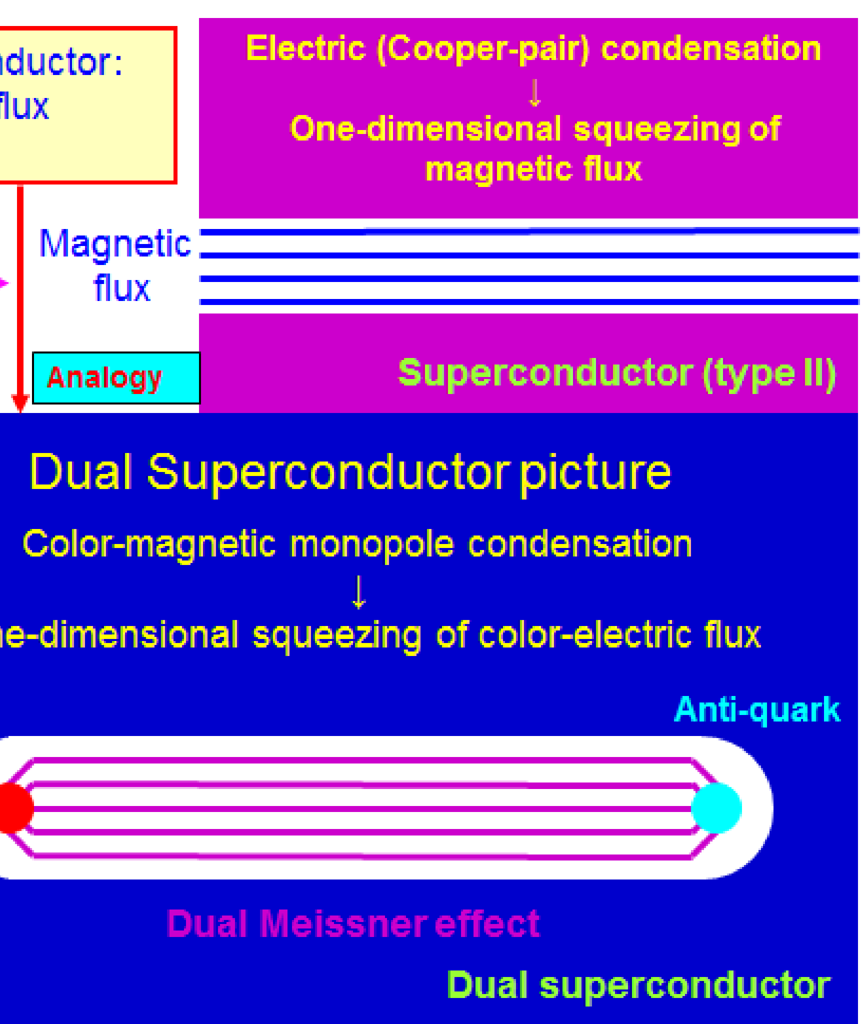
In 1970's, *Nambu, 't Hooft, Mandalstam* proposed *Dual Superconductor picture* for quark confinement, based on the analogy between Abrikosov vortex in Type-II superconductor and flux-tube/string picture for hadrons.

Meissner effect in Superconductor:
Exclusion of magnetic flux
⇒ Abrikosov vortex

Electromagnetic Duality of Maxwell equations

$$\begin{cases} \partial_\mu F^{\mu\nu} = j^\nu \\ \partial_\mu \tilde{F}^{\mu\nu} = k^\nu \end{cases}$$

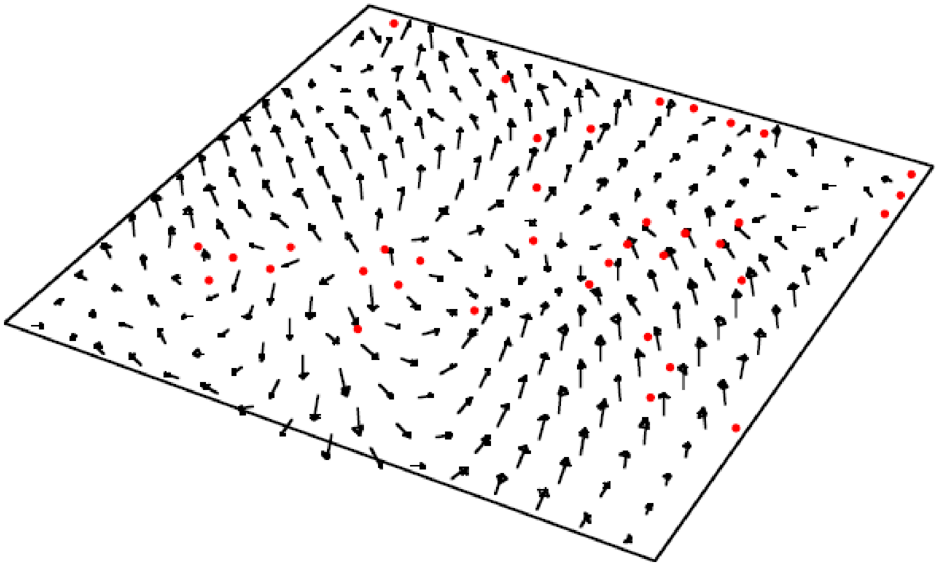
Nambu, 'tHooft, Mandelstam
(1974,75,76)



In *maximally Abelian (MA) gauge*, QCD is reduced into an *Abelian gauge theory* with *magnetic monopoles*. [G. 't Hooft, NPB190(1981)]

1. *Non-Abelian* gauge symmetry SU(3) is reduced into *Abelian* gauge symmetry $U(1)^2$, i.e., $SU(3) \rightarrow U(1)^2$. (maximal torus subgroup of SU(3))

2. There appear *magnetic monopoles* from hedgehog singularity, corresponding to the *Nontrivial Homotopy Group* $\Pi_2(SU(3)/U(1)^2)=\mathbf{Z}^2$, similar to the appearance of 't Hooft-Polyakov or GUT monopoles.



Monopoles appear around hedgehog singularities in gluon field in MA gauge (SU(2) Lattice QCD)
H. Ichie and H.S., NPB574 (2000) 70.

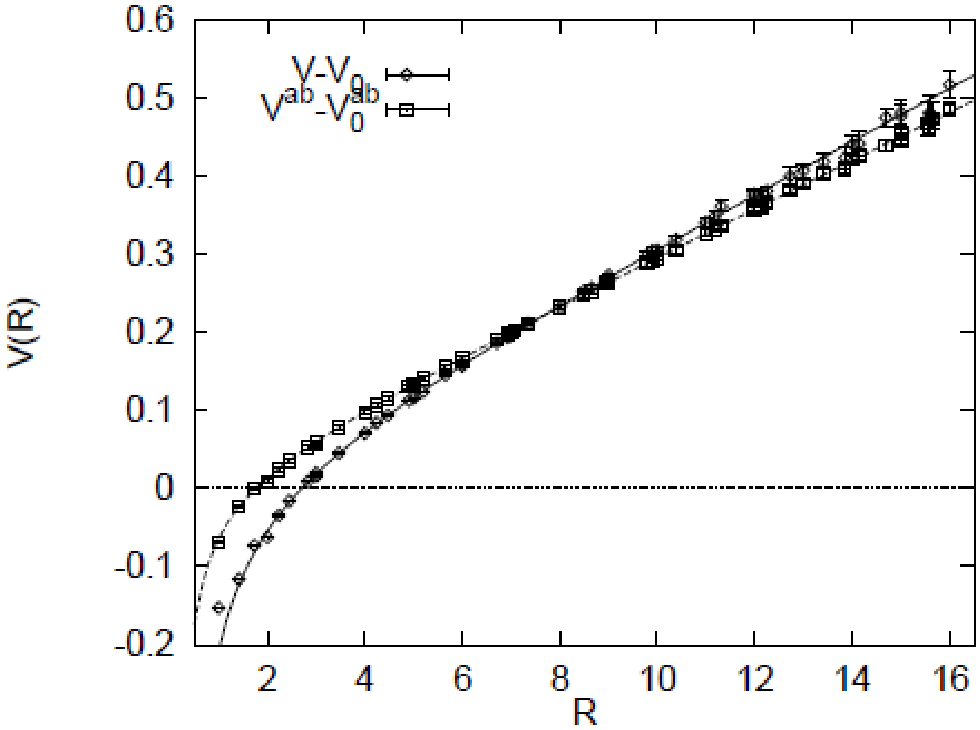
Thus, in MA gauge, QCD can be dual superconductor theory, if *Abelian dominance (inactiveness of off-diagonal gluon)* and *monopole condensation* are realized.

Abelian Dominance in SU(2) Lattice QCD

Abelian dominance has been studied mainly in SU(2) Lattice QCD.

G.S. Bali, V. Bornyakov, M. Mueller-Preussker, K. Schilling, PRD54 (1996) 2863.

M.-I. Polikarpov's Plenary Review Talk at LATTICE 1996.



$\sigma_{Abel} \fallingdotseq 0.92\ \sigma_{SU(2)}$

MA projection of Inter-Quark Potential in Quenched SU(2) Lattice QCD
at β=2.5115 ($a \fallingdotseq$ 0.1fm) on 32^4 ($L \fallingdotseq$ 3 fm)

Perfect Abelian Dominance for Confinement for QQbar Potential in SU(3) Lattice QCD

Maximally Abelian (MA) Gauge

In continuum Euclidean QCD, MA gauge is defined by minimizing off-diagonal gluon-field amplitude using SU(3) gauge transformation.

$$R_{\rm off}[A_\mu(\cdot)] \equiv \int d^4x \; {\rm tr}\left\{[\hat{D}_\mu, \vec{H}][\hat{D}_\mu, \vec{H}]^\dagger\right\} \propto \int d^4x \sum_\alpha |A_\mu^\alpha(x)|^2.$$

In lattice QCD, MA gauge is defined by maximizing "Abelian part ",

$$R\left[U_\mu(s)\right] \equiv {\rm Re} \sum_{s,\mu} {\rm Tr}\left(U_\mu^\dagger(s)\, \vec{H}\, U_\mu(s)\, \vec{H}\right)$$

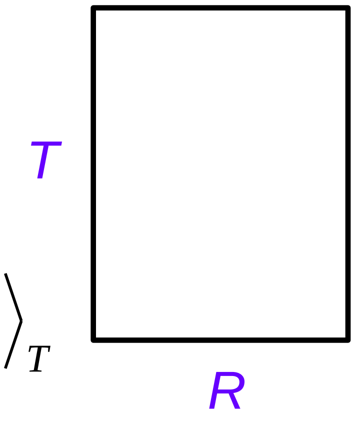
After MA gauge fixing, SU(3) link variables are factorized into $U(1)^2$ and $SU(3)/U(1)^2$.

SU(3) link-variable $\quad U_\mu(s) = M_\mu(s) u_\mu(s) \in SU(3)$

Abelian link-variable $\quad u_\mu(s) = e^{i\vec{\theta}_\mu(s)\cdot\vec{H}} \in U(1)^2$

off-diagonal link-variable $\quad M_\mu(s) = e^{i\theta_\mu^\alpha(s)E^{-\alpha}} \in SU(3)/U(1)^2$

Abelian Wilson loop and MA-projected quark potential



ordinary SU(3) Wilson loop

$$W[U_\mu] = Tr \prod_{i=1}^{2(R+T)} U_{\mu_i}(s_i)$$

→

SU(3) quark potential

$$V(R) = -\lim_{T\to\infty} \frac{1}{T} \ln \left\langle W[U_\mu] \right\rangle_T$$

Abelian Wilson loop

$$W[u_\mu] = Tr \prod_{i=1}^{2(R+T)} u_{\mu_i}(s_i)$$

→

MA-projected quark potential

$$V_{Abel}(R) = -\lim_{T\to\infty} \frac{1}{T} \ln \left\langle W[u_\mu] \right\rangle_T$$

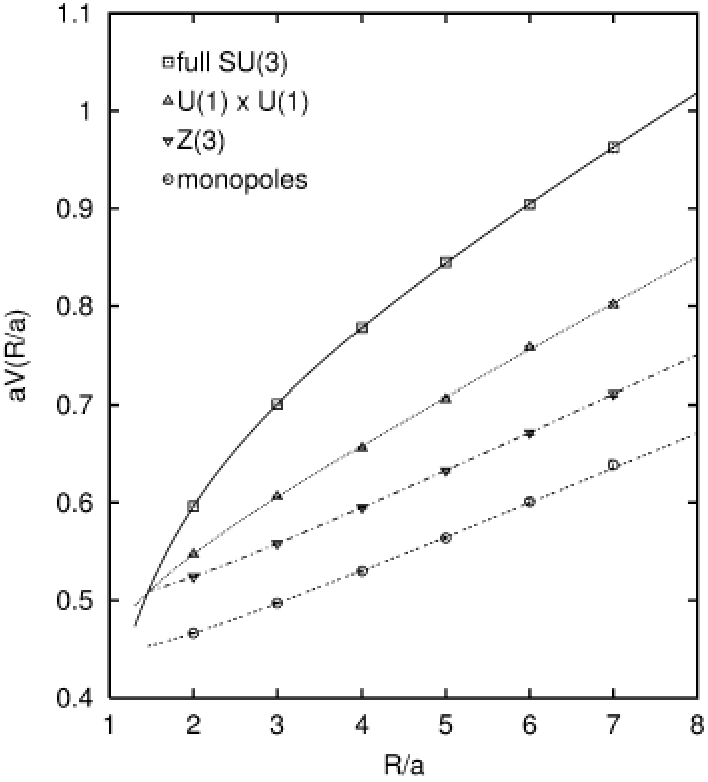
Similar to the extraction of the quark potential from the Wilson loop,
MA-projected quark potential can be obtained from Abelian Wilson loop.

N.B. The Abelian Wilson loop and MA-projected quark potential
are invariant under residual Abelian gauge transformation.

Pioneering work in SU(3) Lattice QCD 1

J.D. Stack, W.W. Tucker, R.J. Wensley, NPB639 (2002) 2013.

The maximal abelian gauge, monopoles, and vortices in SU(3) lattice gauge theory



SU(3)

Abelian

Z_3

monopole

Analysis with
Coulomb + linear Ansatz

$\sigma_{Abel} \fallingdotseq 0.9\ \sigma_{SU(3)}$

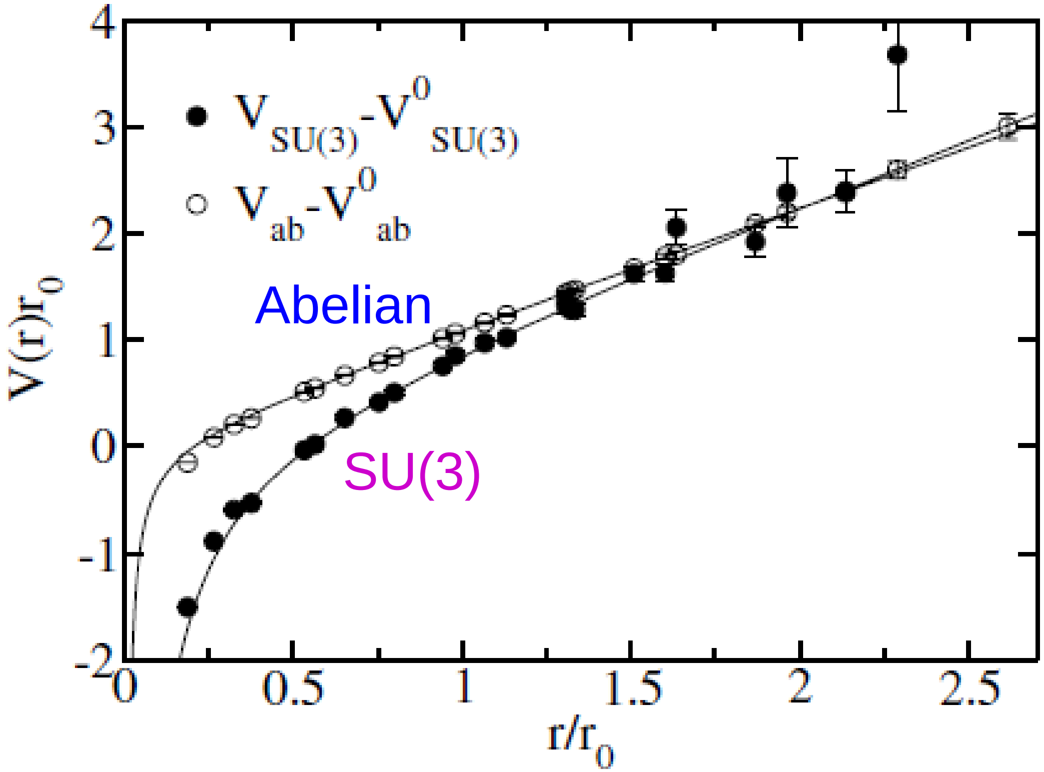
MA projection of Inter-Quark Potential in Quenched SU(3) Lattice QCD
at β=6.0 ($a \fallingdotseq$ 0.1fm) on 16^4 ($L \fallingdotseq$ 1.6fm)

Pioneering work in SU(3) Lattice QCD 2

V.G. Bornyakov et al. (DIK collaboration) PRD70 (2004) 074511.

Dynamics of monopoles and flux tubes in two-flavor dynamical QCD

V. G. Bornyakov,[1,2,3] H. Ichie,[3,4] Y. Koma,[5] Y. Mori,[3] Y. Nakamura,[3] D. Pleiter,[6] M. I. Polikarpov,[2] G. Schierholz,[6,7]
T. Streuer,[6,8] H. Stüben,[9] and T. Suzuki[3]



Analysis with
Coulomb + linear Ansatz

$\sigma_{Abel} \fallingdotseq 0.83\ \sigma_{SU(3)}$

Simulated annealing algorithm was used to avoid Gribov copy effect.

MA projection of Inter-Quark Potential in Quenched SU(3) Lattice QCD
at β=6.0 ($a\fallingdotseq$0.1fm) on 16^3 x 32 ($L\fallingdotseq$1.6fm)

(They also investigated full QCD and flux-tube formation.)

Our study

Numerical condition for QQbar potential calculation

- SU(3) standard plaquette action at quenched level
- various lattice parameter: β = 5.8~6.4, corresponding to lattice spacing: a = 0.058~0.148fm
- various lattice size: La = 2~3fm for main calculations
- large number (200~600) of gauge configurations
- over-relaxation method for MA gauge fixing
- smearing method for accurate measurement

β	L^3L_t	$N_{\rm con}$	a [fm]	La [fm]	$\sigma_{\rm Abel}/\sigma$
6.4	32^4	200	0.0582(2)	1.86(1)	1.015(09)
6.0	32^4	200	0.1022(5)	3.27(1)	1.009(10)
5.8	$16^3 32$	600	0.148(1)	2.37(2)	1.00(2)
6.0	$16^3 32$	600	0.102(1)	1.64(1)	0.94(1)
6.0	$12^3 32$	400	0.104(1)	1.25(4)	0.94(3)
6.2	$16^3 32$	400	0.075(1)	1.20(1)	0.95(2)

MA gauge fixing and Gribov copy effect

In MA gauge, we maximize

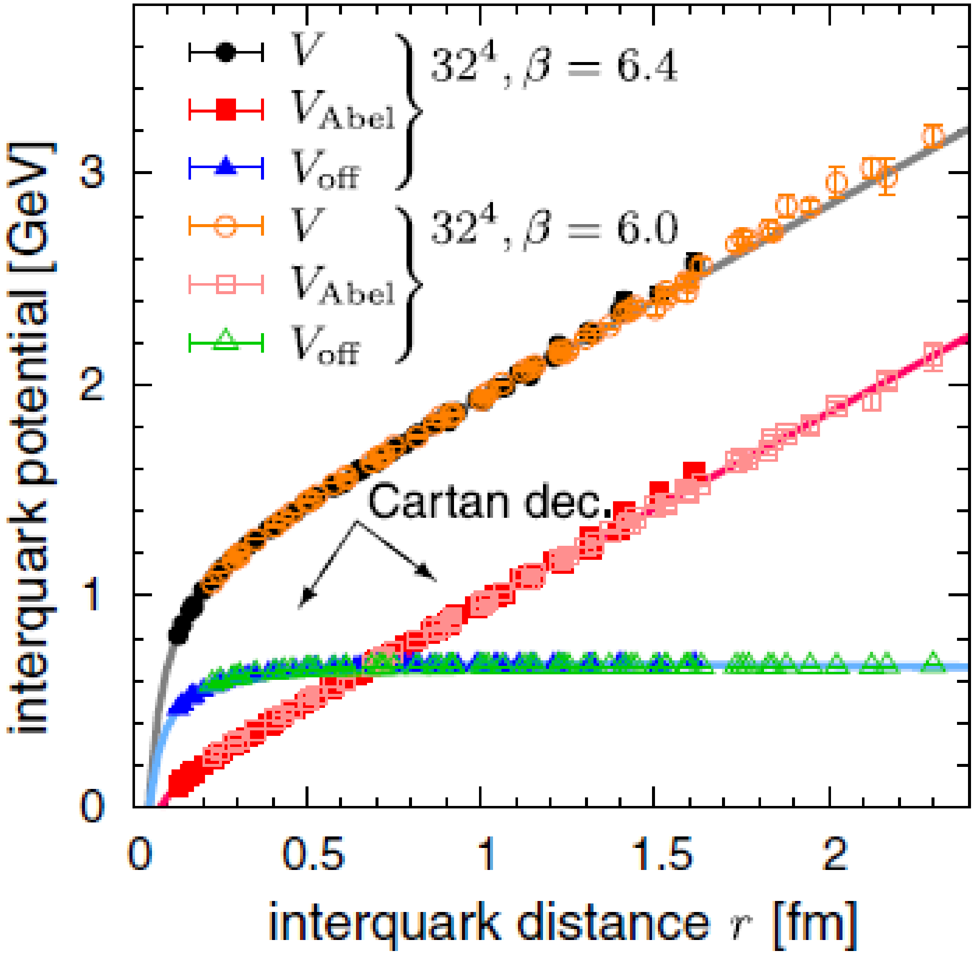
$$R\left[U_\mu(s)\right] \equiv \mathrm{Re} \sum_{s,\mu} \mathrm{Tr}\left(U_\mu^\dagger(s)\, \vec{H}\, U_\mu(s)\, \vec{H} \right)$$

In our over-relaxation method, the maximized value of *R* is almost the same over 200~600 gauge configurations.

Actually, the converged values of *R* are 0.7072(6) with $16^3 32$ at β=5.8; 0.7321(11), 0.7322(7), and 0.7318(3) with $12^3 32$, $16^3 32$, and 32^4 at β=6.0; 0.7510(7) with $16^3 32$ at β=6.2; and 0.7656(3) with 32^4 at β=6.4. Here, the values in parentheses denote the standard deviation.

In fact, our procedure seems to escape bad local minima, where *R* is relatively small. Then, we expect that the Gribov copy effect is not so significant in our calculation.

Abelian Dominance for Confinement in QQbar Potential

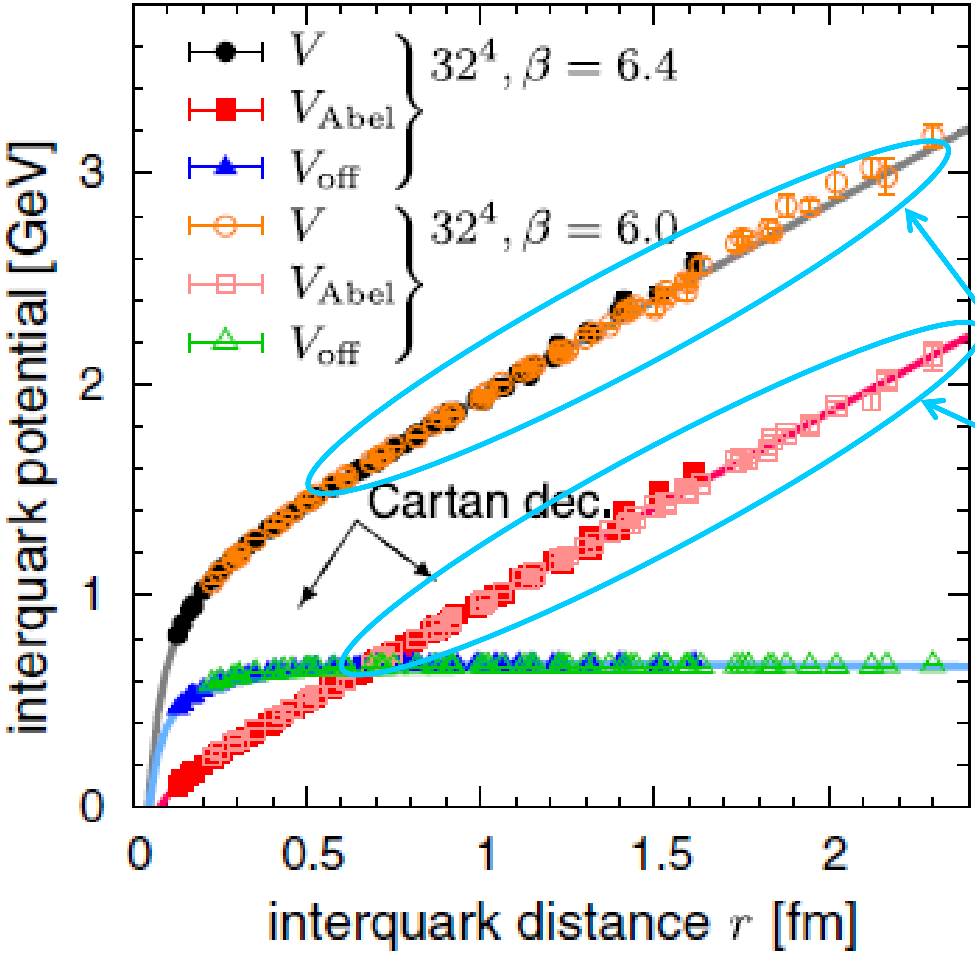


SU(3) potential V(r)

Abelian part $V_{\rm Abel}(r)$

SU(3) potential and Abelian-projected potential in MA gauge in lattice QCD with β=6.0~6.4 and 32^4.

Abelian Dominance for Confinement in QQbar Potential



SU(3) potential V(r)

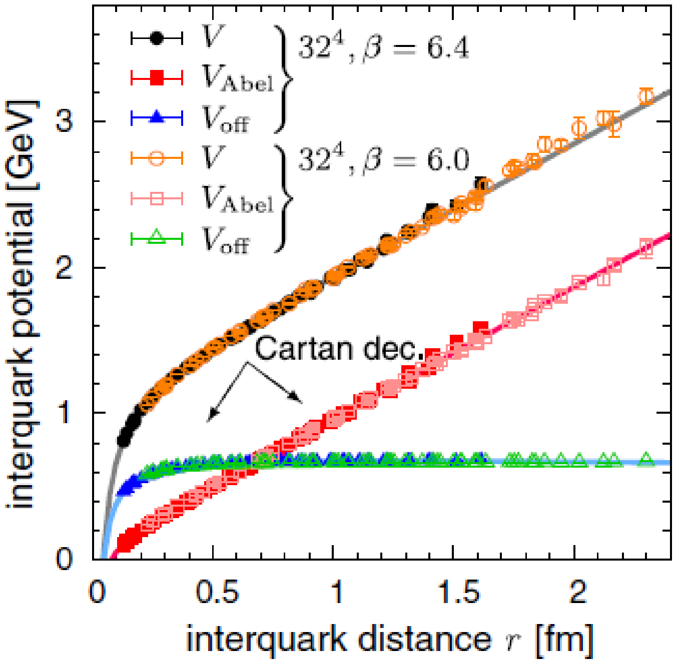
Abelian part $V_{\rm Abel}(r)$

SU(3) potential and its Abelian part have almost the same slope at large distance.

⇒ Abelian Dominance for Confinement

SU(3) potential and Abelian-projected potential in MA gauge in lattice QCD with β=6.0~6.4 and 32^4.

Quantitative Analysis on Abelian Dominance for Confinement



SU(3) potential V(r)

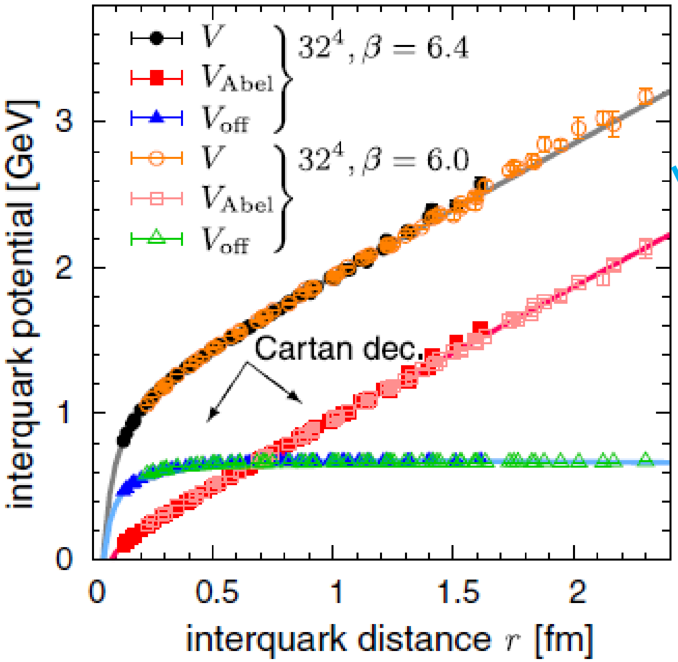
Abelian part $V_{Abel}(r)$

Fit analysis with

Coulomb-plus-linear Ansatz

$$V(r) = -\frac{A}{r} + \sigma r + C$$

Quantitative Analysis on Abelian Dominance for Confinement



SU(3) potential V(r)

Abelian part $V_{Abel}(r)$

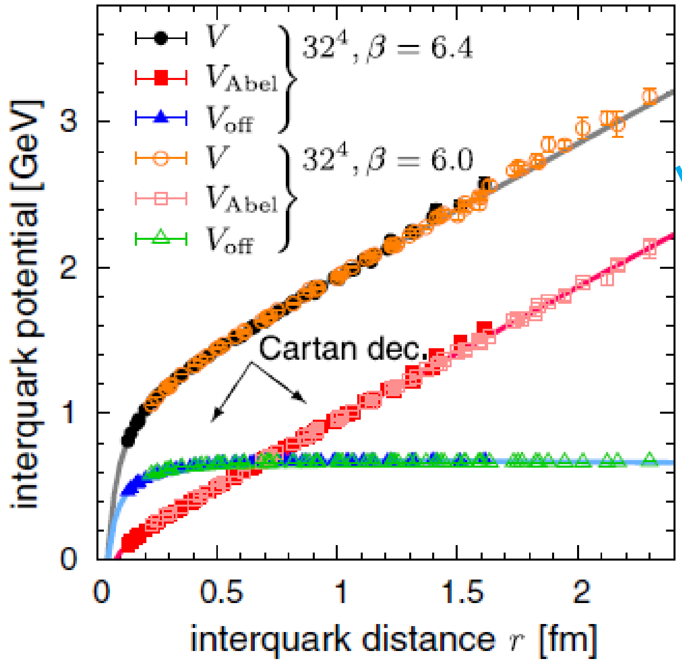
Fit analysis with
Coulomb-plus-linear Ansatz

$$V(r) = -\frac{A}{r} + \sigma r + C$$

	$32^4, \beta = 6.4$			$32^4, \beta = 6.0$		
	σ	A	C	σ	A	C
V	0.01528(12)	0.265(3)	0.598(1)	0.0471(4)	0.290(7)	0.659(4)
$V_{\rm Abel}$	0.01550(06)	0.056(1)	0.167(1)	0.0475(2)	0.044(3)	0.178(2)

string tension | Coulomb coefficient | irrelevant constant

Quantitative Analysis on Abelian Dominance for Confinement



SU(3) potential V(r)

Abelian part $V_{\rm Abel}(r)$

Fit analysis with
Coulomb-plus-linear Ansatz

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$V_{\rm Abel}$	0.01550(06)	0.056(1)	0.167(1)	0.0475(2)	0.044(3)	0.178(2)

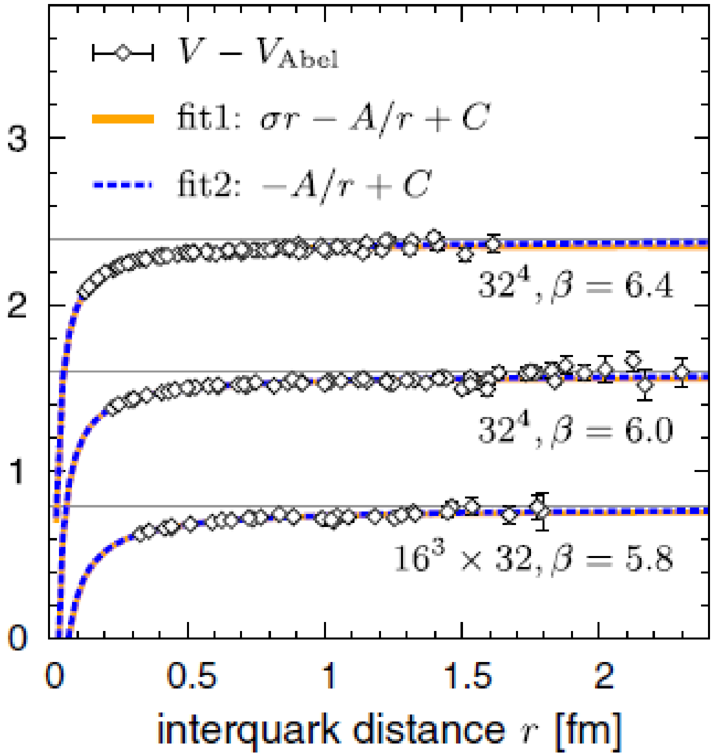
$\sigma_{\rm Abel} = \sigma_{\rm SU(3)}$

$\Rightarrow$ Perfect Abelian Dominance for Confinement

Quantitative Analysis on Abelian Dominance for Confinement

Difference between SU(3) potential $V_{SU(3)}$ and Abelian part V_{Abel}

$V_{SU(3)}(r) - V_{Abel}(r)$



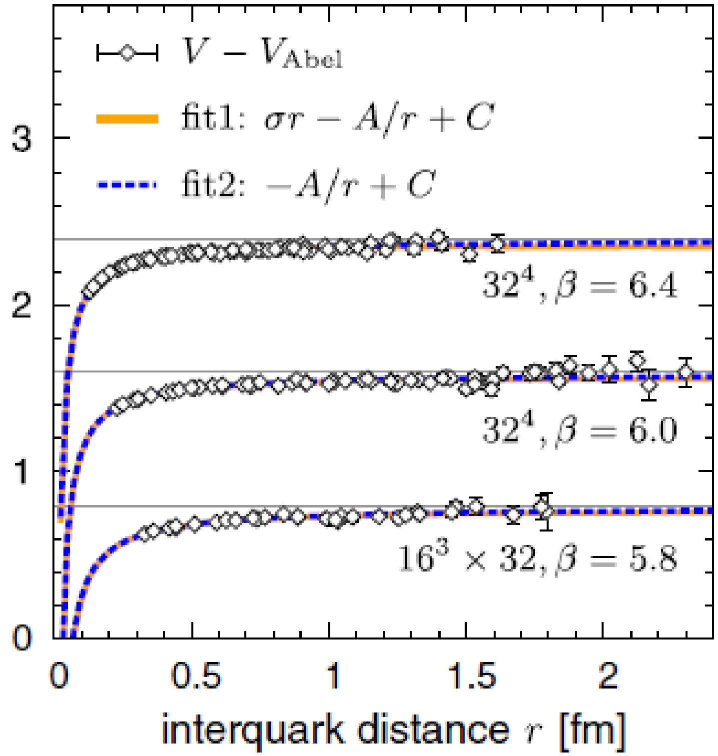
- No string tension in the difference $V_{SU(3)}(r) - V_{Abel}(r)$.

	$32^4, \beta = 6.4$			$32^4, \beta = 6.0$			$16^3 32, \beta = 5.8$		
	σ	A	C	σ	A	C	σ	A	C
V	0.01528(12)	0.265(3)	0.598(1)	0.0471(4)	0.290(7)	0.659(4)	0.0988(19)	0.315(25)	0.679(15)
$V_{\rm Abel}$	0.01550(06)	0.056(1)	0.167(1)	0.0475(2)	0.044(3)	0.178(2)	0.0988(08)	0.039(10)	0.183(06)
$V - V_{\rm Abel}$	−0.00024(11)	0.209(3)	0.432(1)	−0.0005(3)	0.247(6)	0.481(3)	−0.0010(17)	0.285(21)	0.502(12)
$V - V_{\rm Abel}$	0	0.205(1)	0.429(1)	0	0.240(3)	0.476(1)	0	0.273(09)	0.494(03)

Quantitative Analysis on Abelian Dominance for Confinement

Difference between SU(3) potential $V_{SU(3)}$ and Abelian part V_{Abel}

$V_{SU(3)}(r) - V_{Abel}(r)$



- No string tension in the difference $V_{SU(3)}(r) - V_{Abel}(r)$.

- The difference $V_{SU(3)}(r) - V_{Abel}(r)$ can be well fitted by pure Coulomb potential.

	$32^4, \beta = 6.4$			$32^4, \beta = 6.0$			$16^3 32, \beta = 5.8$		
	σ	A	C	σ	A	C	σ	A	C
V	0.01528(12)	0.265(3)	0.598(1)	0.0471(4)	0.290(7)	0.659(4)	0.0988(19)	0.315(25)	0.679(15)
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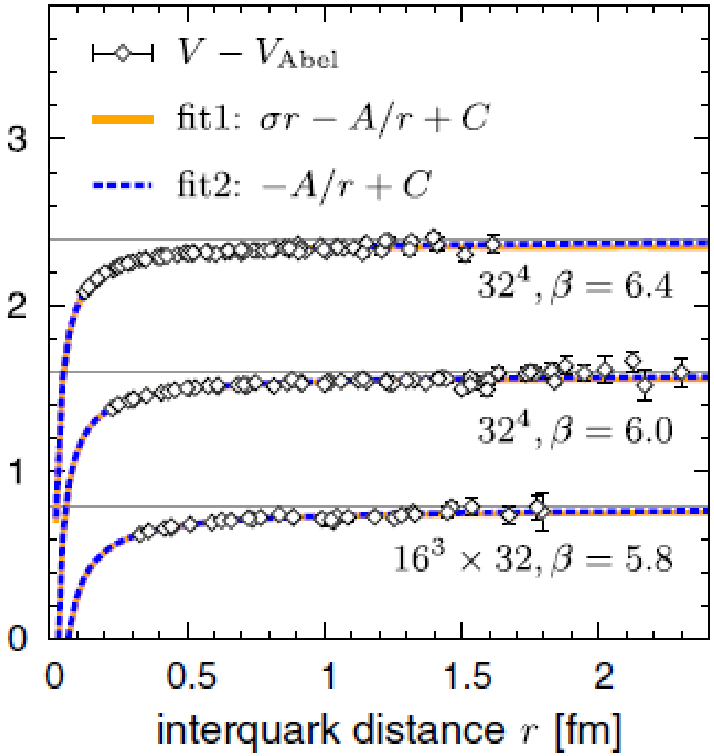
Quantitative Analysis on Abelian Dominance for Confinement

Difference between SU(3) potential $V_{SU(3)}$ and Abelian part V_{Abel}

$V_{SU(3)}(r) - V_{Abel}(r)$



- No string tension in the difference $V_{SU(3)}(r) - V_{Abel}(r)$.

- The difference $V_{SU(3)}(r) - V_{Abel}(r)$ can be well fitted by pure Coulomb potential.

⇒ This also suggests perfect Abelian dominance for confinement

	$32^4, \beta = 6.4$			$32^4, \beta = 6.0$			$16^3 32, \beta = 5.8$		
	σ	A	C	σ	A	C	σ	A	C
V	0.01528(12)	0.265(3)	0.598(1)	0.0471(4)	0.290(7)	0.659(4)	0.0988(19)	0.315(25)	0.679(15)
V_{Abel}	0.01550(06)	0.056(1)	0.167(1)	0.0475(2)	0.044(3)	0.178(2)	0.0988(08)	0.039(10)	0.183(06)
$V - V_{\mathrm{Abel}}$	−0.00024(11)	0.209(3)	0.432(1)	−0.0005(3)	0.247(6)	0.481(3)	−0.0010(17)	0.285(21)	0.502(12)
$V - V_{\mathrm{Abel}}$	0	0.205(1)	0.429(1)	0	0.240(3)	0.476(1)	0	0.273(09)	0.494(03)

Quantitative Analysis on Abelian Dominance for Confinement

Physical spatial-size dependence of $\sigma_{Abel} / \sigma_{SU(3)}$

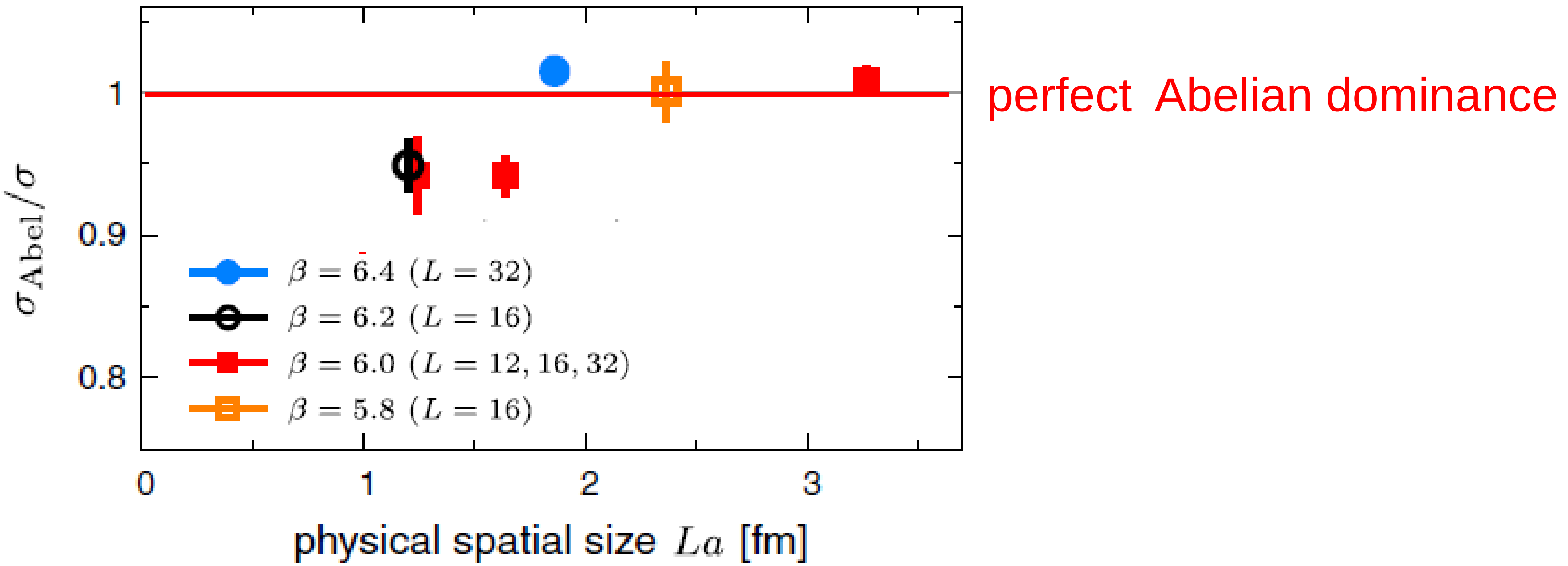
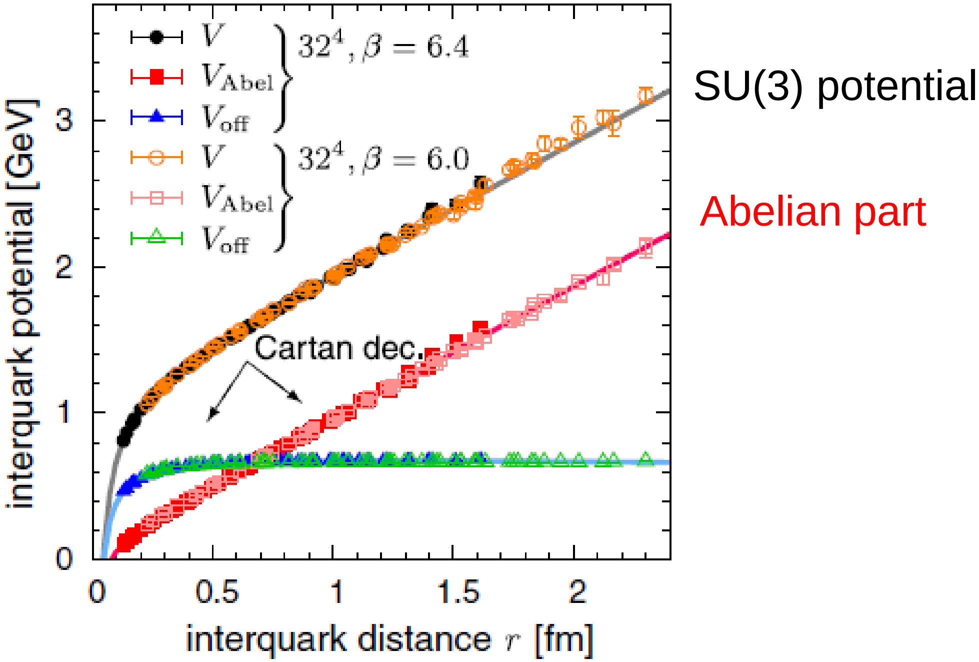


FIG. 4 (color online). Physical spatial-size dependence of σ_{Abel}/σ. Perfect Abelian dominance ($\sigma_{Abel}/\sigma \simeq 1$) seems to be realized when the spatial size La is sufficiently large.

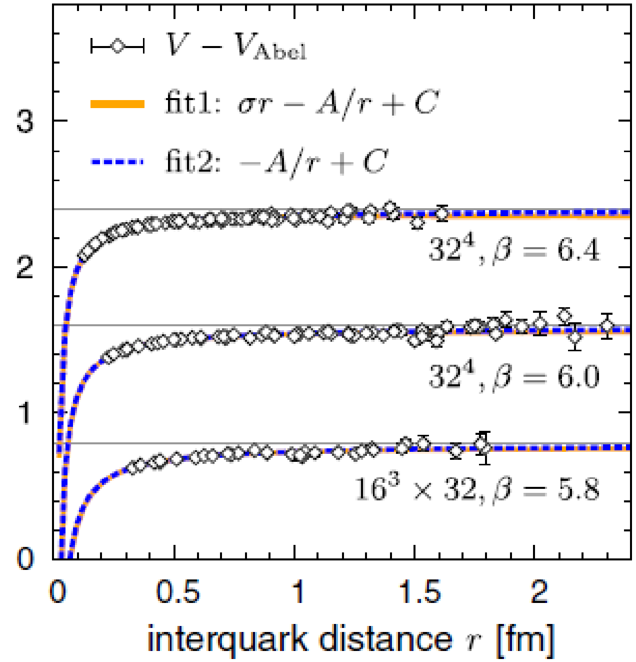
When physical spatial size La is larger than 2 fm,
Perfect Abelian dominance $\sigma_{Abel} / \sigma_{SU(3)}$ ≒1 is realized.
~One of key quantities is the physical spatial volume.

Perfect Abelian Dominance for Quark Confinement in SU(3) Lattice QCD

Summary of 1st part: We study MA projection of Q-Qbar potential in SU(3) quenched lattice QCD with large physical-volume lattices, and find *almost perfect Abelian dominance* of quark confinement.



SU(3) potential and the Abelian part in MA gauge in lattice QCD at β=6.0~6.4 and 32^4. They have almost the same slope at large distance.

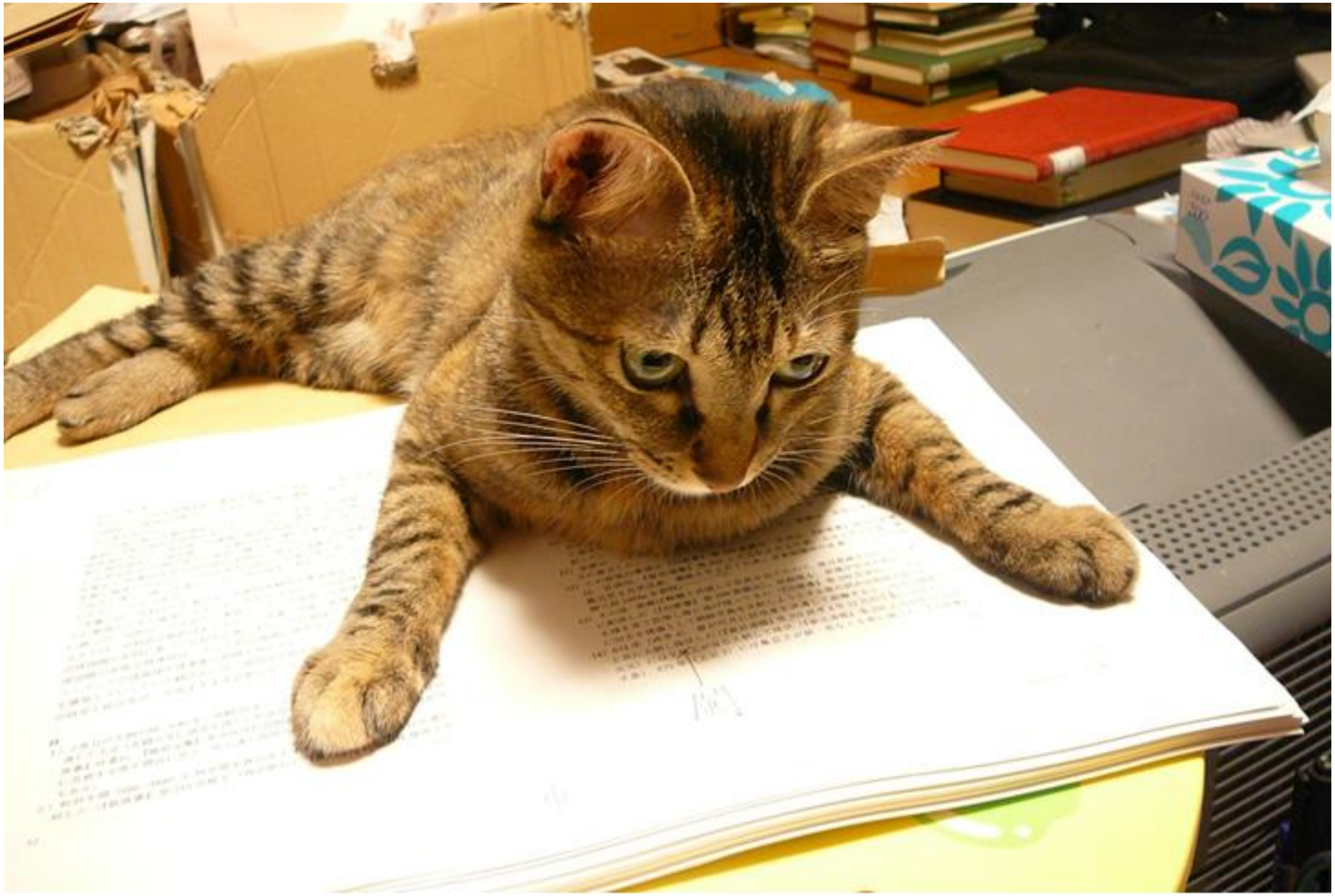


Difference between SU(3) potential and the Abelian part is almost pure Coulomb form, which suggests perfect Abelian dominance of confinement.

Reference:

[1] N.Sakumichi and H.S., Physical Review D90 Rapid Communication 111501 (2014), "Perfect Abelian Dominance of Quark Confinement in SU(3) QCD".

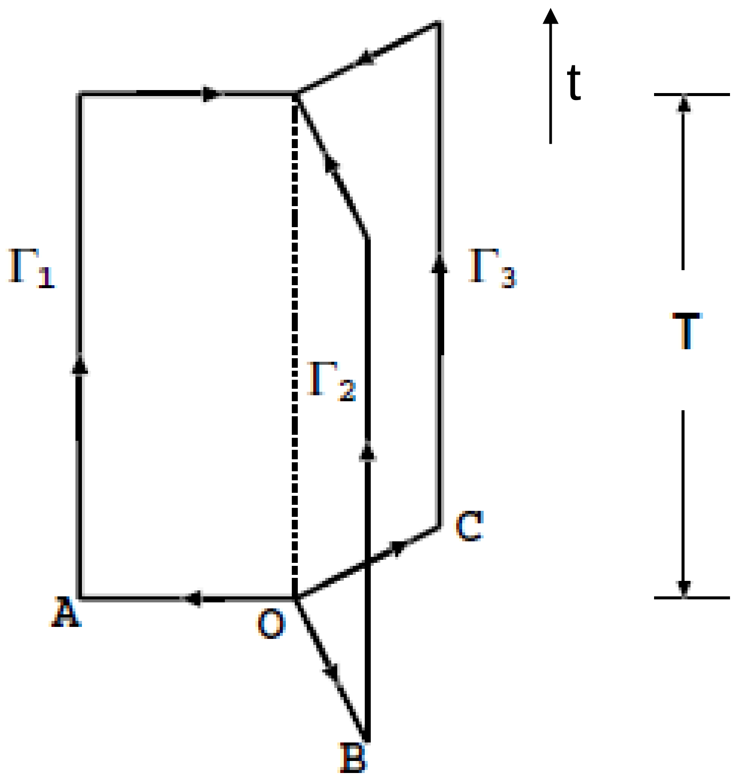
Baryonic Three-Quark Potential and Perfect Abelian Dominance for Confinement



The 3Q potential is responsible to baryon properties.

Baryonic Three-Quark Potential in QCD

Similar to the QQbar potential, the 3Q potential can be calculated with the 3Q Wilson loop defined on the contour of three large staples as



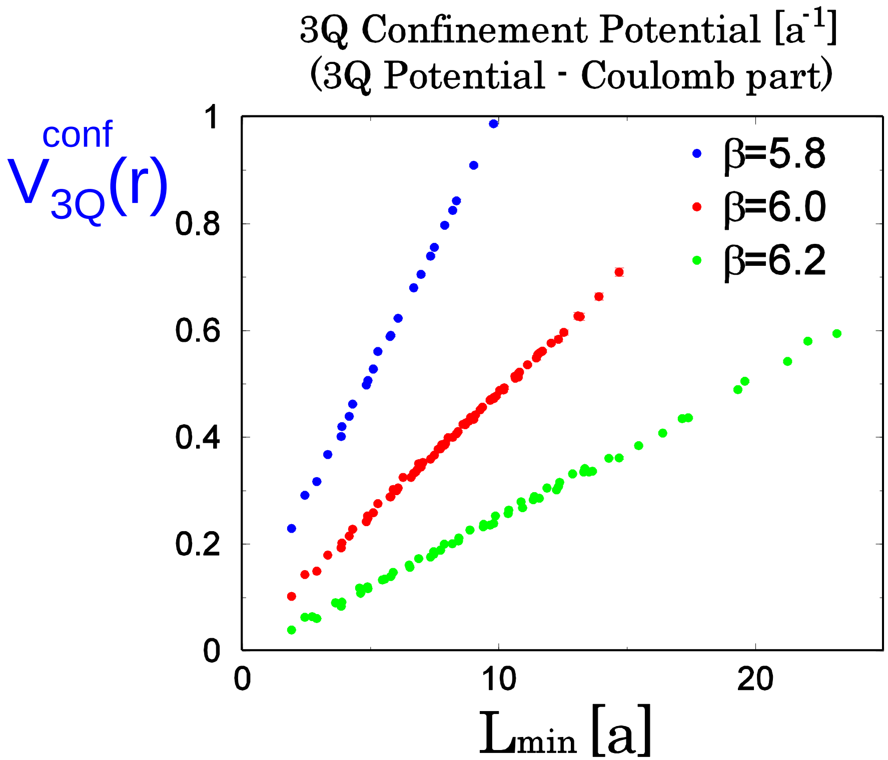
T.T.Takahashi, H.S. et al. PRL 86 (2001) 18.
T.T.Takahashi, H.S. et al. PRD65 (2002)114509.

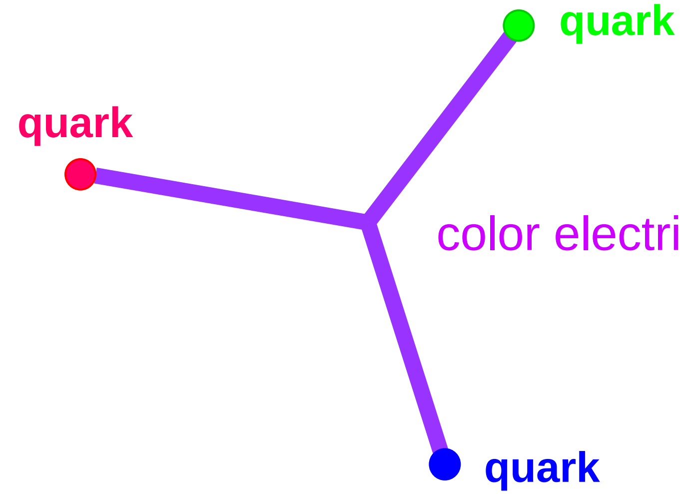
$$V_{3Q} = -\lim_{T\to\infty} \frac{1}{T} \ln \langle W_{3Q} \rangle_T$$

The 3Q Wilson loop physically means that gauge-invariant 3Q state is created at $t = 0$ and is annihilated at $t = T$ with the three quarks spatially fixed in $\mathbf{R}^3$ for $0 < t < T$.

Baryonic Three-Quark Potential in Lattice QCD

T.T.Takahashi, H.S. et al. PRL 86 (2001) 18.
T.T.Takahashi, H.S. et al. PRD65 (2002)114509.
T.T.Takahashi, H.S. Phys. Rev. Lett. 90 (2003).
F.Okiharu, H.S. et al., PRD72 (2005) 014505.





L_{min} : *total length of string linking three valence quarks*

$$V_{3Q} = -A_{3Q} \sum_{i<j} \frac{1}{\left|\vec{r}_i - \vec{r}_j\right|} + \sigma L_{min}$$

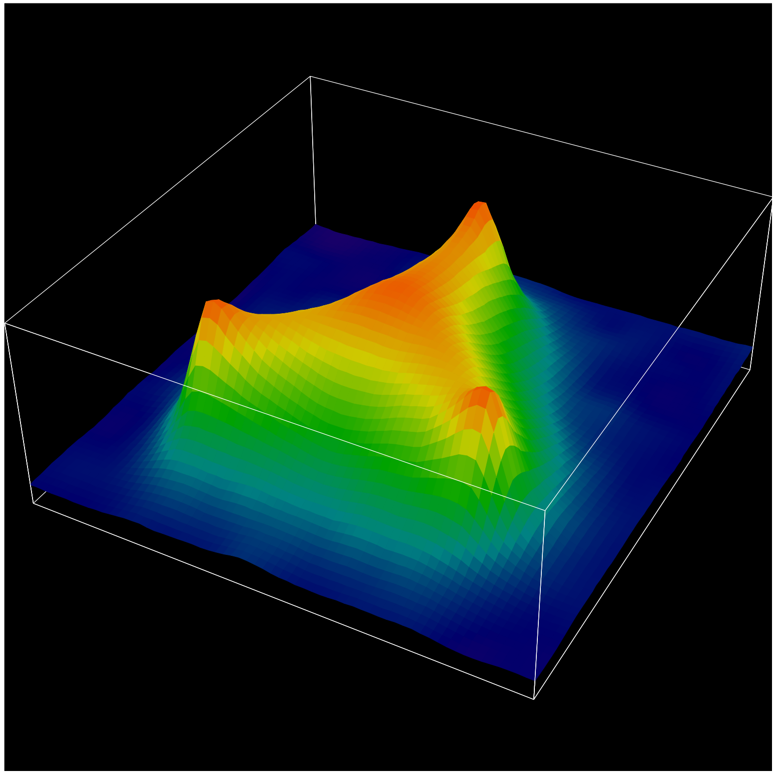
one-gluon-exchange Coulomb potential

Y-type linear potential

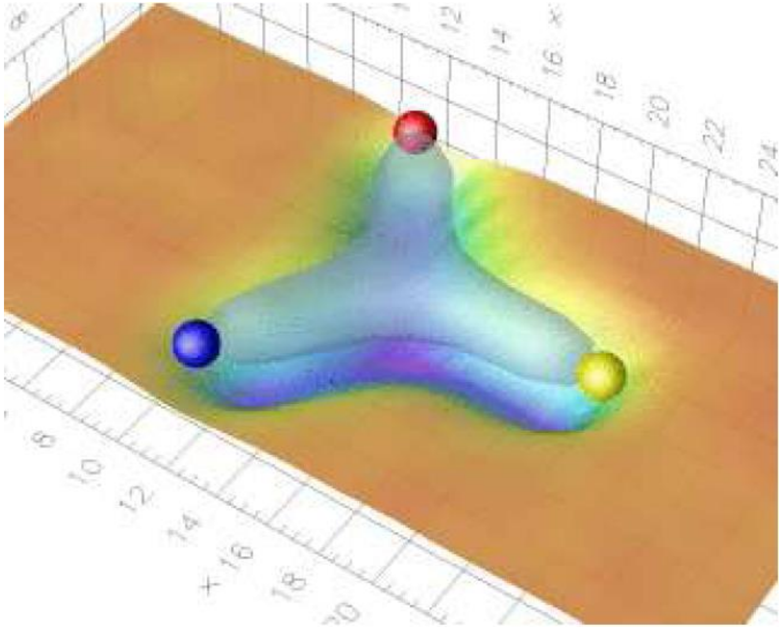
The 3Q potential is well described by Y-Ansatz, i.e., sum of one-gluon-exchange Coulomb and Y-type linear potential.

Y-shaped Flux-tube formation for 3Q systems in Lattice QCD

Y−shaped Flux Tube



H. Ichie et al., Nucl. Phys. A721, 899 (2003).
V.G. Bornyakov et al., PRD70, 054506 (2004).
F. Bissey et al., Phys.Rev.D76,114512 (2007).



For 3Q systems, Y-shaped flux-tube formation has been observed in lattice QCD, and such one-dimensional squeezing of color flux leads to Y-type linear confinement potential.

Our study

Numerical condition for 3Q potential calculation

- SU(3) standard plaquette action at quenched level

β	$L^3 L_t$	$N_{\rm con}$	a [fm]	La [fm]
5.8	$16^3 32$	2000	0.148(2)	2.37(3)
6.0	$20^3 32$	1000	0.1022(5)	2.05(1)

- lattice parameter: β = 5.8, 6.0,
  corresponding to lattice spacing: a = 0.148, 0.102fm
- lattice size: La = 2.37, 2.05fm
- large number (2000, 1000) of gauge configurations
- over-relaxation method for MA gauge fixing
- smearing method for accurate measurement

SU(3) Lattice QCD calculation for 3Q potential

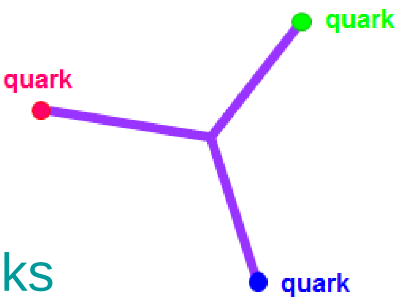
For more than 300 different patterns of 3Q systems,

(101 3Q-systems at $\beta = 5.8$, 211 3Q-systems at $\beta = 6.0$)

The 3Q potential is well described by Y-Ansatz, i.e., a sum of one-gluon-exchange (OGE) Coulomb and Y-type linear potential.

Y-Ansatz:

$$V_{3Q} = -A_{3Q} \sum_{i<j} \frac{1}{|\vec{r}_i - \vec{r}_j|} + \sigma L_{\min} + C$$



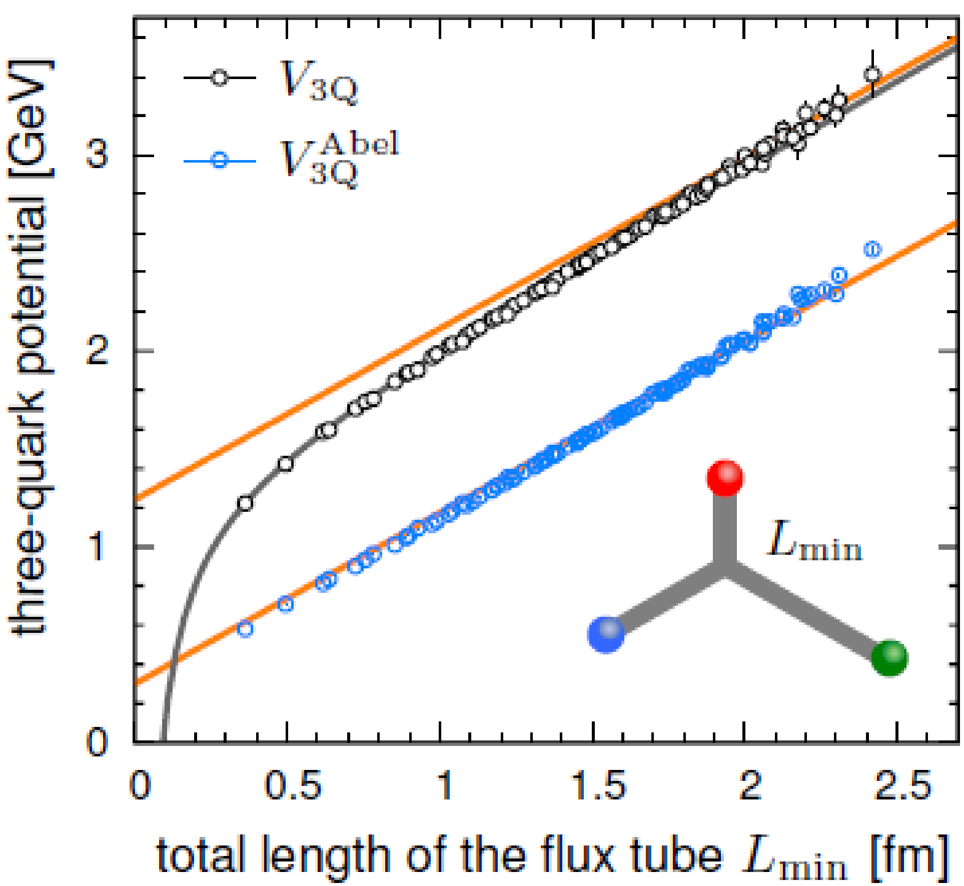
$L_{\min}$: total length of string linking three valence quarks

TABLE II. Fit analysis of interquark potentials in lattice units at $\beta = 5.8$ and 6.0 (i.e., $a \simeq 0.15$ and $a \simeq 0.10$ fm). The best-fit parameter sets (σ, A, C) of the $Q\bar{Q}$ potential V and the Abelian part $V^{\rm Abel}$ are listed with the functional form (1). The best-fit parameter sets $(\sigma_{3Q}, A_{3Q}, C_{3Q})$ of the 3Q potential V_{3Q} and the Abelian part $V_{3Q}^{\rm Abel}$ are listed with the Y-ansatz (2). The label of (equi. triangle) means the fit analysis only with the lattice data of equilateral-triangle 3Q configurations. N_Q is the number of different patterns of $Q\bar{Q}$ or 3Q systems. The string tension ratio $\sigma^{\rm Abel}/\sigma$ is listed at the last column.

			SU(3)			Abelian part			
β		N_Q	σ	A	C	$\sigma^{\rm Abel}$	$A^{\rm Abel}$	$C^{\rm Abel}$	$\sigma^{\rm Abel}/\sigma$
5.8	$Q\bar{Q}$	26	0.099(2)	0.30(3)	0.67(2)	0.098(1)	0.043(12)	0.187(7)	0.99(3)
	3Q (equi. triangle)	5	0.097(1)	0.118(3)	0.93(1)	0.098(3)	−0.001(8)	0.19(2)	1.01(3)
	3Q	101	0.0997(4)	0.109(1)	0.905(4)	0.0967(5)	0.006(2)	0.213(5)	0.97(1)
6.0	$Q\bar{Q}$	39	0.0472(6)	0.289(10)	0.658(5)	0.0457(2)	0.050(3)	0.183(2)	0.97(1)
	3Q (equi. triangle)	8	0.0471(10)	0.121(3)	0.936(9)	0.0455(12)	0.014(4)	0.233(12)	0.97(3)
	3Q	211	0.0480(3)	0.113(1)	0.917(3)	0.0456(2)	0.013(1)	0.232(2)	0.95(1)

3Q Potential and Abelian Dominance

(b) MA projection of the 3Q potential

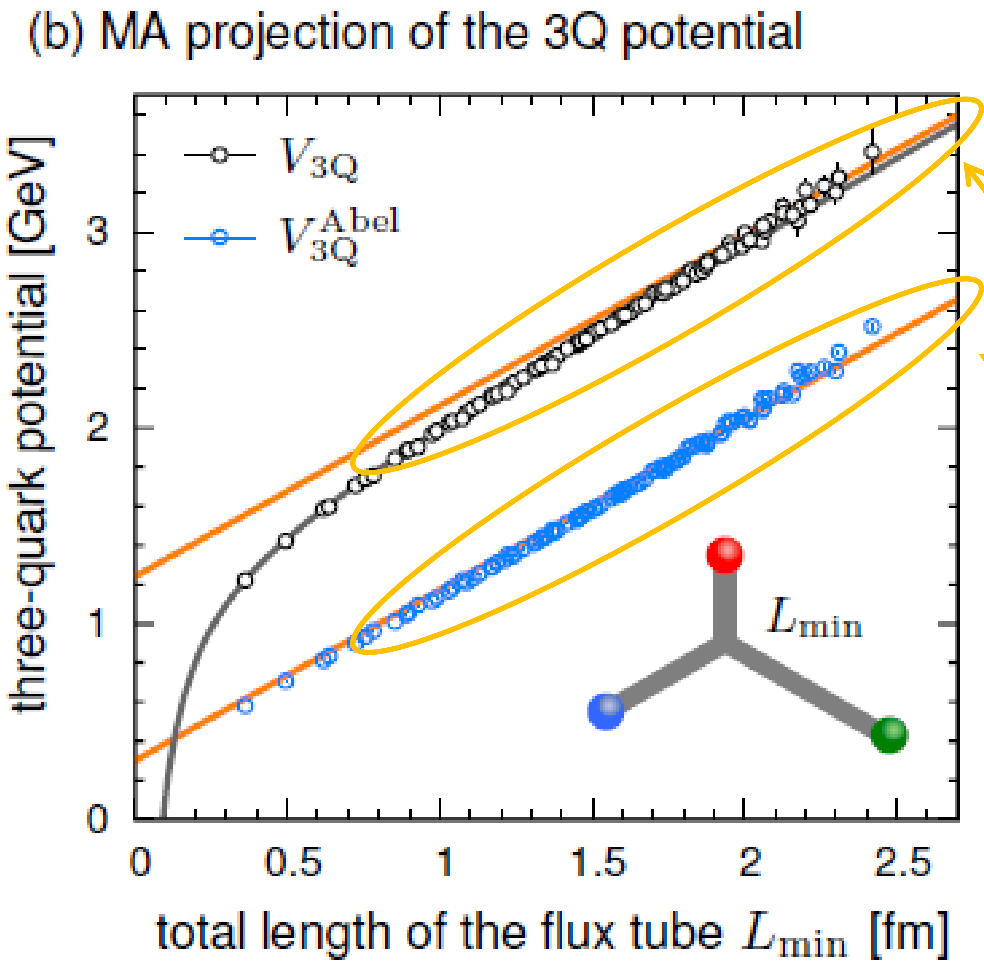


3Q potential $V_{3Q}(r)$

Abelian part $V_{3Q}^{Abel}(r)$

3Q potential and Abelian-projected 3Q potential plotted against minimal linking length L_{min} in MA gauge in SU(3) lattice QCD with β=5.8, $16^3 32$.

3Q Potential and Abelian Dominance



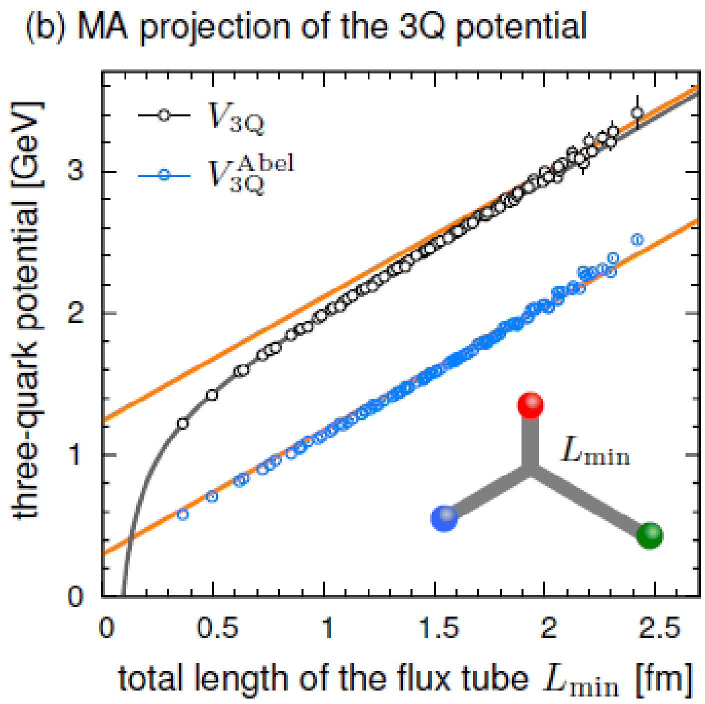
3Q potential and its Abelian part have almost the same slope at large distance.

⇒

Abelian Dominance for Confinement

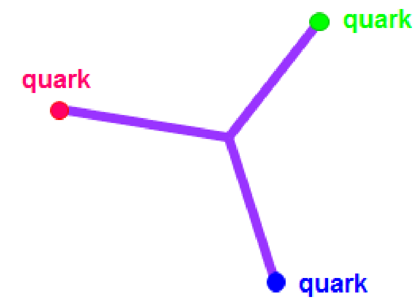
3Q potential and Abelian-projected 3Q potential plotted against minimal linking length L_{min} in MA gauge in SU(3) lattice QCD with β=5.8, $16^3 32$.

Quantitative Analysis on Abelian Dominance for 3Q Confinement



3Q potential $V_{3Q}(r)$

Abelian part $V_{3Q}^{\rm Abel}(r)$



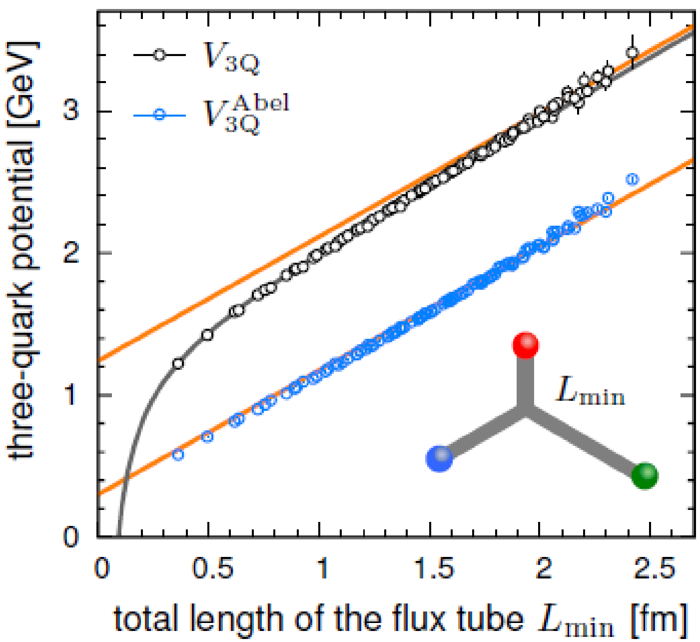
Fit analysis with Y-Ansatz for 3Q potential

$$V_{3Q} = -A_{3Q} \sum_{i<j} \frac{1}{|\vec{r}_i - \vec{r}_j|} + \sigma L_{\rm min} + C$$

$L_{\rm min}$: total length of string linking three valence quarks

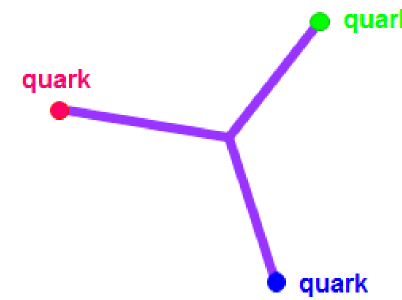
Quantitative Analysis on Abelian Dominance for 3Q Confinement

(b) MA projection of the 3Q potential



3Q potential $V_{3Q}(r)$

Abelian part $V_{3Q}^{\rm Abel}(r)$



Fit analysis with Y-Ansatz for 3Q potential

$$V_{3Q} = -A_{3Q}\sum_{i<j}\frac{1}{|\vec{r}_i-\vec{r}_j|}+\sigma L_{\rm min}+C$$

$L_{\rm min}$: total length of string linking three valence quarks

β		N_Q	SU(3) σ	SU(3) A	SU(3) C	Abelian part $\sigma^{\rm Abel}$	Abelian part $A^{\rm Abel}$	Abelian part $C^{\rm Abel}$	$\sigma^{\rm Abel}/\sigma$
5.8	$Q\bar{Q}$	26	0.099(2)	0.30(3)	0.67(2)	0.098(1)	0.043(12)	0.187(7)	0.99(3)
	3Q (equi. triangle)	5	0.097(1)	0.118(3)	0.93(1)	0.098(3)	−0.001(8)	0.19(2)	1.01(3)
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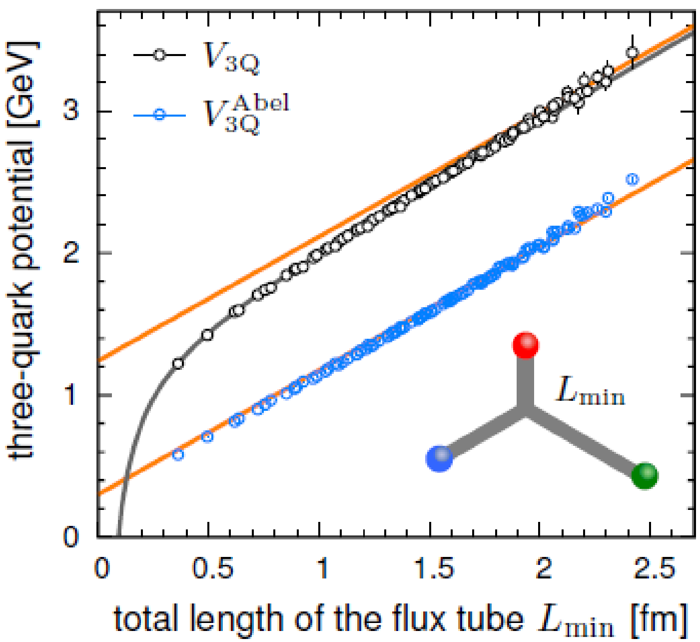
6.0	$Q\bar{Q}$	39	0.0472(6)	0.289(10)	0.658(5)	0.0457(2)	0.050(3)	0.183(2)	0.97(1)
	3Q (equi. triangle)	8	0.0471(10)	0.121(3)	0.936(9)	0.0455(12)	0.014(4)	0.233(12)	0.97(3)
	3Q	211	0.0480(3)	0.113(1)	0.917(3)	0.0456(2)	0.013(1)	0.232(2)	0.95(1)

string tension — Coulomb coefficient — irrelevant constant

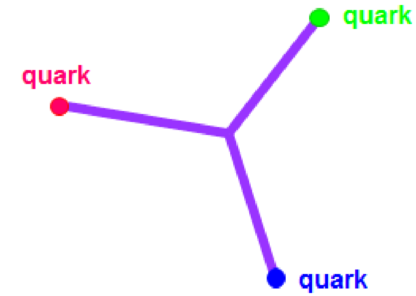
Quantitative Analysis on Abelian Dominance for 3Q Confinement

(b) MA projection of the 3Q potential



3Q potential $V_{3Q}(r)$

Abelian part $V_{3Q}^{\rm Abel}(r)$



Fit analysis with Y-Ansatz for 3Q potential

$$V_{3Q} = -A_{3Q} \sum_{i<j} \frac{1}{|\vec{r}_i - \vec{r}_j|} + \sigma L_{\rm min} + C$$

$L_{\rm min}$: total length of string linking three valence quarks

β		N_Q	SU(3)			Abelian part			$\sigma^{\rm Abel}/\sigma$
			σ	A	C	$\sigma^{\rm Abel}$	$A^{\rm Abel}$	$C^{\rm Abel}$	
5.8	$Q\bar{Q}$	26	0.099(2)	0.30(3)	0.67(2)	0.098(1)	0.043(12)	0.187(7)	0.99(3)
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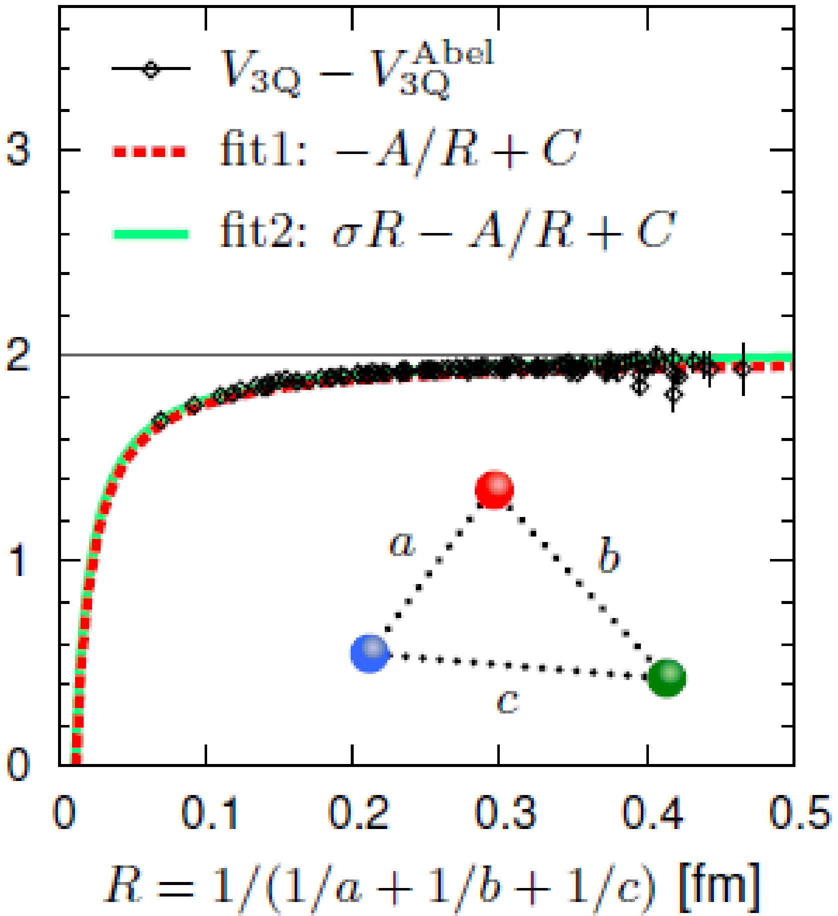
	3Q	211	0.0480(3)	0.113(1)	0.917(3)	0.0456(2)	0.013(1)	0.232(2)	0.95(1)

$\sigma^{\rm Abel} = \sigma$ ⇒ Perfect Abelian Dominance for 3Q Confinement

Quantitative Analysis on Abelian Dominance for 3Q Confinement

Difference between 3Q potential V_{3Q} and Abelian part V_{3Q}^{Abel}

$V_{3Q}(r) - V_{3Q}^{Abel}(r)$



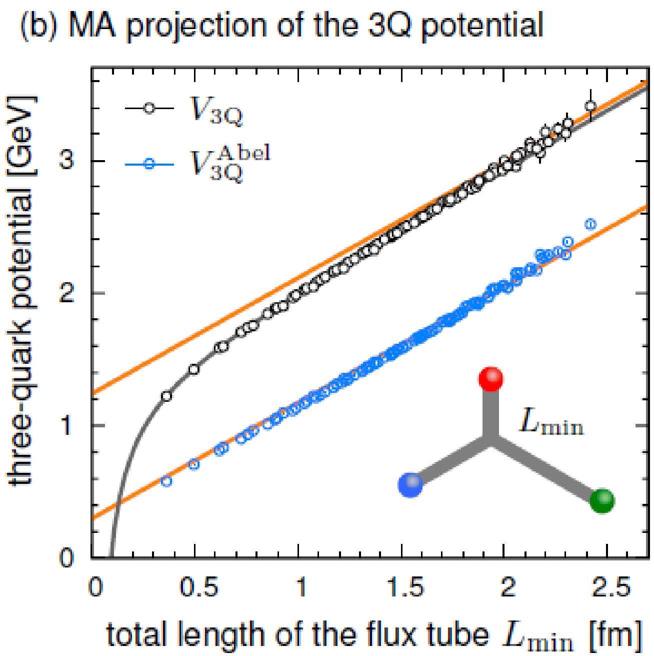
- No string tension in the difference $V_{3Q}(r) - V_{3Q}^{Abel}(r)$.

- The difference $V_{3Q}(r) - V_{3Q}^{Abel}(r)$ can be well fitted by 2-body pure Coulomb potential.

⇒ This also suggests perfect Abelian dominance for 3Q confinement

3Q potential and Perfect Abelian Dominance for 3Q Confinement

Summary of 2nd part: 3Q potential is well described by Y-Ansatz, i.e., OGE Coulomb plus Y-type linear potential.
In MA gauge, the Abelian part of 3Q potential has almost the same string tension (perfect Abelian dominance).

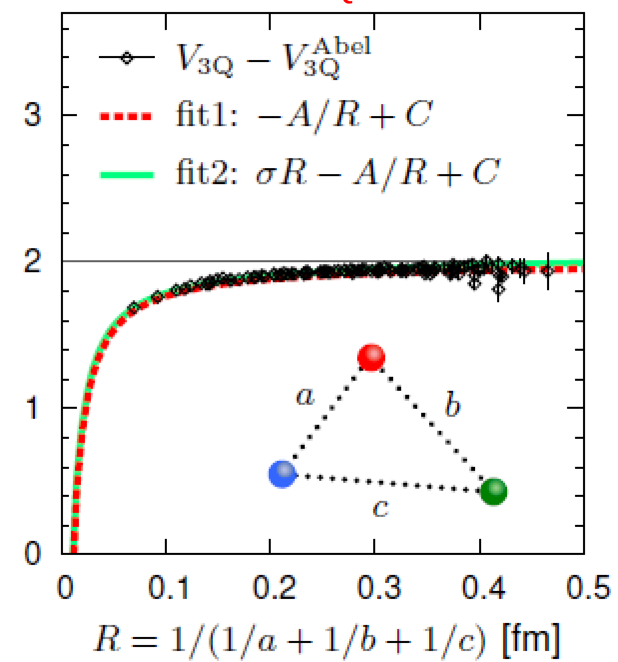


3Q potential

Abelian part

3Q potential and its Abelian part obtained from SU(3) lattice QCD at β=5.8, 6.0. They have almost the same slope at large distance.

$V_{3Q}(r) - V_{3Q}^{\rm Abel}(r)$



Difference between 3Q potential and the Abelian part is well described by pure Coulomb form, which suggests perfect Abelian dominance for 3Q confinement.

Summary and Conclusion

We have studied the static three-quark (3Q) potential for more than 300 different patters of 3Q systems with high statistics in SU(3) quenched lattice QCD.

For all the distances, the 3Q potential is found to be well described by the Y-Ansatz, i.e., *one-gluon-exchange Coulomb plus Y-type linear potential*.

As a remarkable fact, quark confinement forces in both QQbar and 3Q systems can be described only with Abelian variables in the maximally Abelian (MA) gauge, which we call "*perfect Abelian dominance*" of the quark confinement.

Reference:

[1] N.Sakumichi and H.S., Physical Review D90 Rapid Communication 111501 (2014), "Perfect Abelian Dominance of Quark Confinement in SU(3) QCD".

[2] N.Sakumichi and H.S., arXiv: 1501.07596 [hep-lat], "Perfect Abelian Dominance of Quark Confinement in Baryonic Three-Quark Potential"

Thank you!