The $U_A(1)$ anomaly in high temperature QCD with chiral fermions on the lattice

Sayantan Sharma

BROOKHAVEN NATIONAL LABORATORY

July 18, 2015

In collaboration with:

V. Dick, F. Karsch, E. Laermann, S. Mukherjee.

Outline

1. The $U_A(1)$ puzzle in QCD
2. Background
3. Our results
4. Conclusions

Outline

The $U_A(1)$ puzzle

- Origin:
  Anomalous $U_A(1)$ not an exact symmetry of QCD yet may the order of phase transition for $N_f = 2$ [Pisarki & Wilczek, 83].

- In model QFT, it is not possible to quantify the $U_A(1)$ effects in observables.

- Need lattice studies with fermions having exact chiral/flavour symmetry and correct anomaly on the lattice.

The $U_A(1)$ puzzle

- Origin:
  Anomalous $U_A(1)$ not an exact symmetry of QCD yet may the order of phase transition for $N_f = 2$ [Pisarki & Wilczek, 83].

- In model QFT, it is not possible to quantify the $U_A(1)$ effects in observables.

- Need lattice studies with fermions having exact chiral/flavour symmetry and correct anomaly on the lattice.

The $U_A(1)$ puzzle

- Origin:
  Anomalous $U_A(1)$ not an exact symmetry of QCD yet may the order of phase transition for $N_f = 2$ [Pisarki & Wilczek, 83].

- In model QFT, it is not possible to quantify the $U_A(1)$ effects in observables.

- Need lattice studies with fermions having exact chiral/flavour symmetry and correct anomaly on the lattice.

What are the constituents of the hot QCD medium?

- At $T = 0$, anomaly effects related to instantons [t'Hooft, 76].
- Near chiral crossover transition T_c, a medium consisting of interacting instantons can explain chiral symmetry breaking $\Rightarrow$ Instanton Liquid Model [Shuryak, 82].
- At $T >> T_c$, medium is like a dilute gas of instantons [Gross, Pisarski & Yaffe, 81].
- What is the medium made up of for $T_c \leq T \leq 2T_c$?

Spectral density when chiral symmetry is restored

- Very little known. Only recently there are very interesting results [Aoki, Fukaya & Taniguchi, 12].

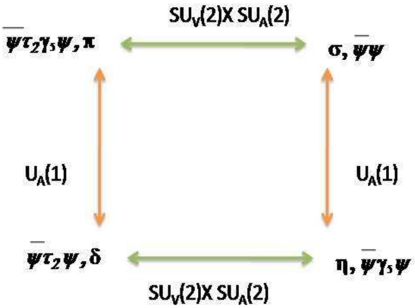
- Assuming $\rho(\lambda, m)$ to be analytic in m^2, look at chiral Ward identities of n-point function of scalar & pseudo-scalar currents.

- $\rho(\lambda, m \to 0) \sim \lambda^3 \Rightarrow U_A(1)$ breaking effects invisible in these sectors for upto 6-point functions.

What to look for: Non-analyticities in eigenvalue spectrum

$U_A(1)$ Not an exact symmetry$\rightarrow$ what observables to look for?

Degeneracy of the correlators with specific quantum numbers in meson channels [Shuryak, 94]



What to look for: Non-analyticities in eigenvalue spectrum

- Either look at the difference of the integrated correlators

$$\chi_\pi - \chi_\delta = \int d^4x \ \left[\langle i\pi^+(x) i\pi^-(0)\rangle - \langle \delta^+(x)\delta^-(0)\rangle\right]$$

- Equivalently study $\rho(\lambda, m_f)$ of the Dirac operator.

$$\chi_\pi - \chi_\delta \overset{V\to\infty}{\rightarrow} \int_0^\infty d\lambda \frac{4m_f^2 \ \rho(\lambda, m_f)}{(\lambda^2 + m_f^2)^2} \ , \ \langle\bar{\psi}\psi\rangle \overset{V\to\infty}{\rightarrow} \int_0^\infty d\lambda \frac{2m_f \ \rho(\lambda, m_f)}{(\lambda^2 + m_f^2)}$$

- If chiral symmetry restored: $\lim_{m_f\to 0}\lim_{V\to\infty} \rho(0, m_f) \to 0$.
- A gap in the infrared spectrum $\Rightarrow U_A(1)$ restored

- chiral symmetry restored + $U_A(1)$ broken if: $\lim_{\lambda\to 0} \rho(\lambda, m_f) \to \delta(\lambda) m_f^\alpha \ , 1 < \alpha < 2$... Look for non-analyticities in eigenvalue spectrum

Chiral fermions on the lattice

- Only two well defined chiral fermion formulations on the lattice that satisfy Ginsparg Wilson relation $\{\gamma_5, D\} = aD\gamma_5 D$ [Ginsparg & Wilson, 82]
- Overlap fermions [Narayanan & Neuberger, 94, Neuberger, 98] have exact chiral symmetry on the lattice.

$$D_{ov} = M(1 + \gamma_5 \,\mathbf{sgn}(\gamma_5 D_W(-M)))\ ,\ \mathbf{sgn}(A) = A/\sqrt{A.A}.$$

- Domain wall fermions [Kaplan 92, Shamir 95] in the limit $N_5 \to \infty$

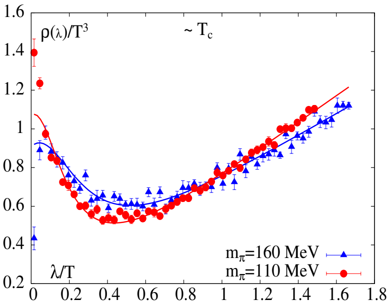
$$D_{DW} = M(1 - \gamma_5 \,\mathbf{sgn}(\ln|T|))\ ,\ T = (1 + a_5\gamma_5 D_W P_+)^{-1}(1 - a_5\gamma_5 D_W P_-).$$

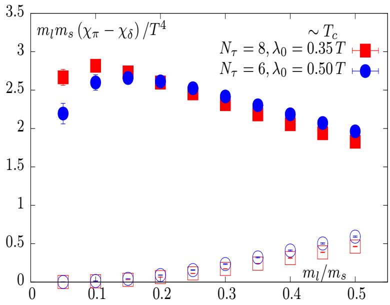
- For finite a_5, infrared spectra of D_{DW} different from D_{ov}.

Outline

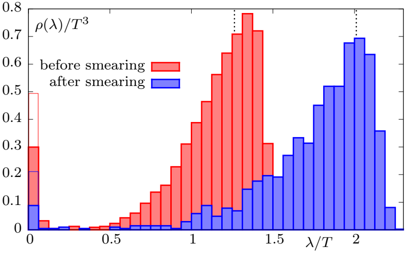
What did we know so far

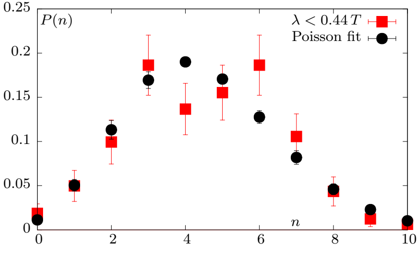
- D_{ov} has an exact index theorem like in the continuum $\Rightarrow$ the zero modes of D_{ov} related to topological structures of the underlying gauge field. [Hasenfratz, Laliena & Niedermeyer, 98].
- Used overlap as valence operator to probe the infrared spectrum of Highly Improved Staggered Quarks(HISQ).
- $U_A(1)$ broken near T_c and near-zero modes primarily responsible for it.





- HYP smearing [Hasenfratz & Knechtli, 02] expected to eliminate dislocations





- Smearing does not eliminate the near zero modes.
- At 1.5 T_c, QCD medium is a dilute gas of small instantons
  $r = 0.23$ fm, $\rho = 0.15\text{fm}^{-4}$

Outline

Our Set-up

- We study the eigenspectrum of large volume Möbius domain wall configurations using the overlap operator.
- Previous independent study at $m_\pi = 200$ MeV found hints for the presence of a near-zero peak [Columbia-BNL-LLNL, 13].
- We look at how robust the peak is..in particular to lowering lower pion mass
  and larger volumes.

Numerical details

- Möbius domain wall fermions on 5D hypercube with $N = 32$ sites along each spatial 4-dim, $N_5 = 16$ and $N_\tau = 8$ sites along temporal dim.
- Volumes, $V = N^3 a^3$, Temperature, $T = \frac{1}{N_\tau a}$, a is the lattice spacing.
- Box size: $m_\pi \ V^{1/3} > 4$
- 2 light+1 heavy flavour
- Input m_s physical ≈ 100 MeV and $m_s/m_l = 27, 11$ $\Rightarrow m_\pi = 135, 200$ MeV.
- Temperatures and configurations [Columbia-BNL-LLNL, 13,14]:

T	# configurations	m_π(MeV)	# eigenvalues/config.
1.08 T_c	140	135	50
1.08 T_c	100	200	50
1.2 T_c	150	135	50
1.2 T_c	110	200	50

Numerical details

Implementing the overlap operator

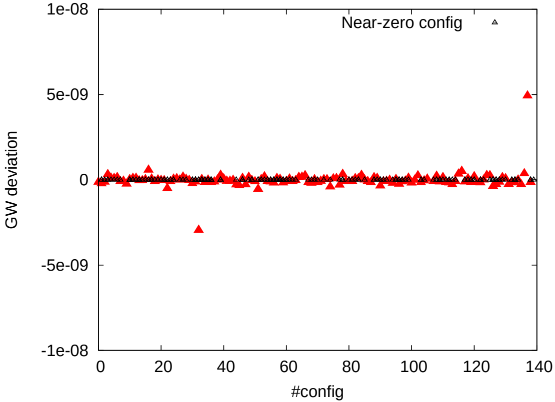
- Matrix sign function non-trivial!
- For the lowest modes sign function was computed explicitly from eigenvalues of D_W.
- For the higher modes, sign function approximated as a Zolotarev Rational Polynomial with 15 terms.
- The sign function is computed as precise as 10^{-8}.

Eigenvalue computation

- The Kalkreuter-Simma Ritz algorithm for eigenvalues of $D_{ov}^{\dagger} D_{ov}$.
- Convergence criterion: $\epsilon^2 < 10^{-8}$.

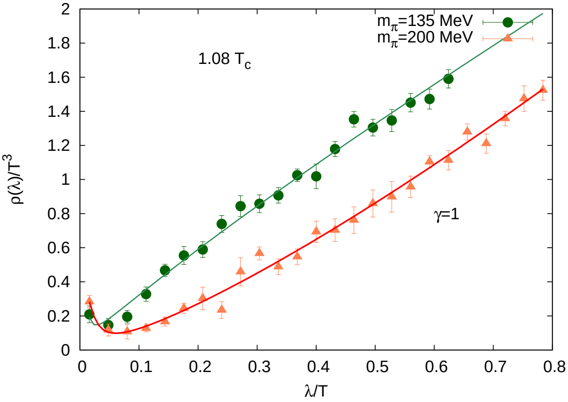
Ginsparg-Wilson relation

- A few configurations with near-zero modes have larger GW violations than average.
- No general trend observed



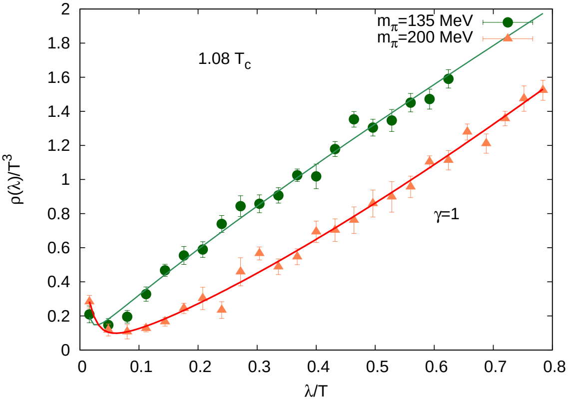
Eigenvalue distribution near T_c

- General features: Near zero mode peak +bulk
- We fit to the ansatz: $\rho(\lambda) = \frac{A\epsilon}{\lambda^2+A} + B\lambda^\gamma$
- Bulk rises linearly as λ,no gap seen.
- No gap even when quark mass reduced!



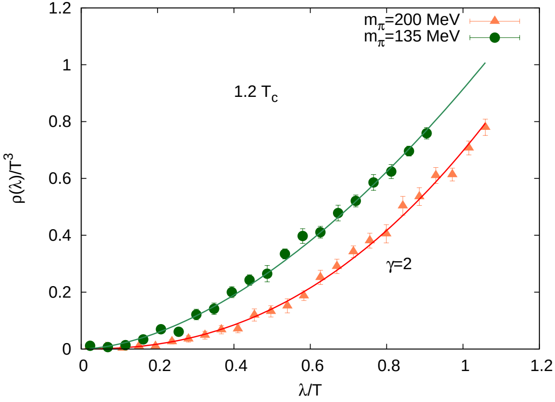
Eigenvalue distribution near T_c

- General features: Near zero mode peak +bulk
- We fit to the ansatz: $\rho(\lambda) = \frac{A\epsilon}{\lambda^2 + A} + B\lambda^{\gamma}$
- Bulk rises linearly as λ,no gap seen.
- No gap even when quark mass reduced!



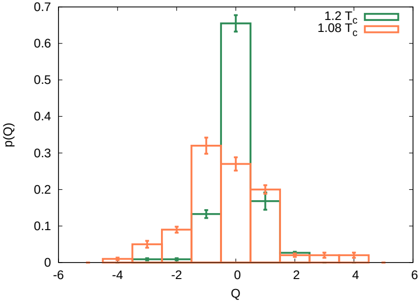
At higher temperatures..

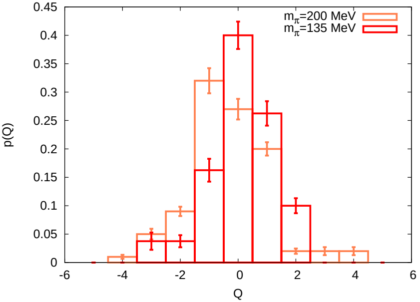
- The rise of the bulk is $\gamma \sim 2$.
- Infrared modes becomes rarer with a small peak.



Topological charge distribution

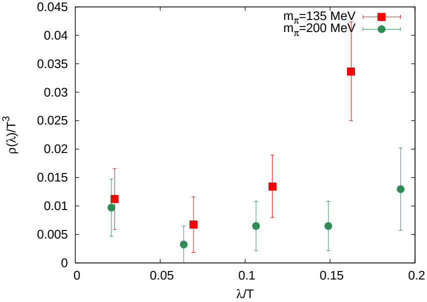
- The higher Q configurations suppressed with temperature.
- Not sensitive to the quark mass.

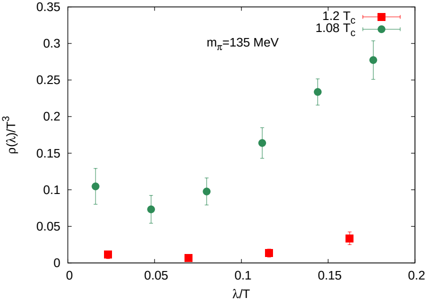




A closer look at the near-zero modes

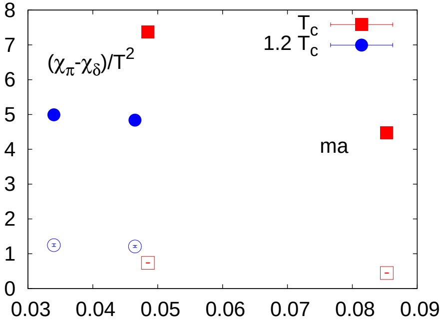
- The near-zero modes sensitive to the sea quark mass $\rightarrow$ sparse when m_π heavier but the peak survives!
- Falls by more than a third at $1.2\,T_c$.





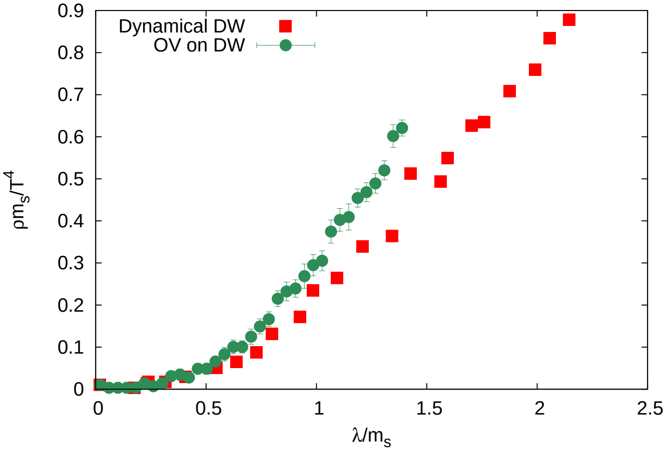
Near zero modes and $U_A(1)$

- m_s tuned by matching RG invariant combination $\frac{m_s\langle\bar{\psi}\psi\rangle_l - m_l\langle\bar{\psi}\psi\rangle_s}{T^4}$.
- Significant contribution comes from the near zero modes than the bulk $\Rightarrow$ Near zero modes primarily responsible for $U_A(1)$ breaking



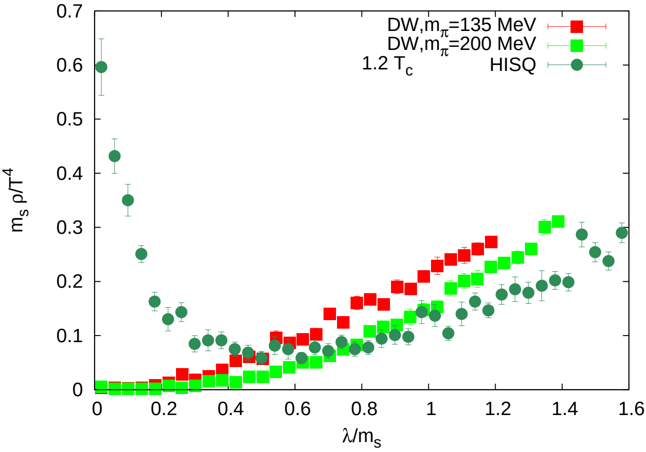
Comparing with earlier results

- The renormalized spectra of dynamical Domain wall fermions [Columbia-BNL-LLNL, 13] agrees very well with what we measured with the overlap.

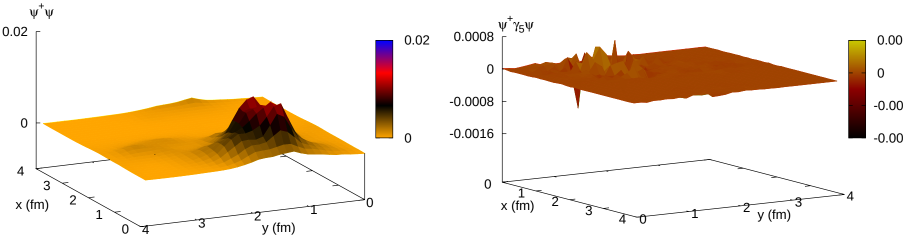


Eigenvalue spectra of HISQ vs Domain wall fermions

- The bulk HISQ spectra with Goldstone pion mass 160MeV consistent with DW with $m_\pi = 200$ MeV at $1.2\,T_c$.
- More near-zero states in HISQ than domain wall..taste effects?



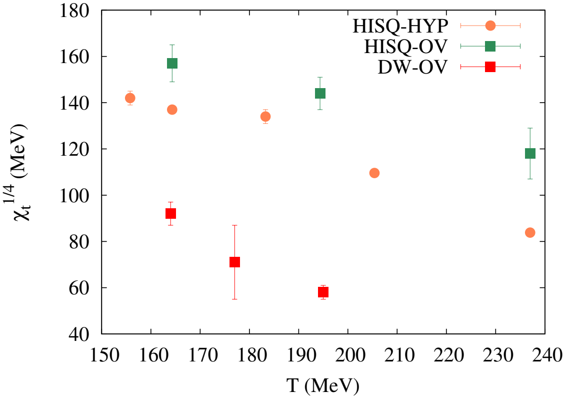
A closer look at near-zero modes



Near-zero modes due to an interacting instanton-antiinstanton pair.

Topological susceptibility across T_c

- The topological susceptibility changes gradually compared to pure gauge theory.



Outline

Conclusions

- We looked at the low-lying eigenspectrum of Möbius domain wall fermions with the overlap operator.
- On large volume lattice we found that $U_A(1)$ broken for $T \leq 1.2\,T_c$. The fermion near-zero modes are mainly responsible for its breaking.
- At 1.2 T_c the instanton-antiinstanton pair start separating $\rightarrow$ towards a dilute gas?