Study of high density phase transition in lattice QCD with canonical approach

Yusuke Taniguchi (University of Tsukuba)

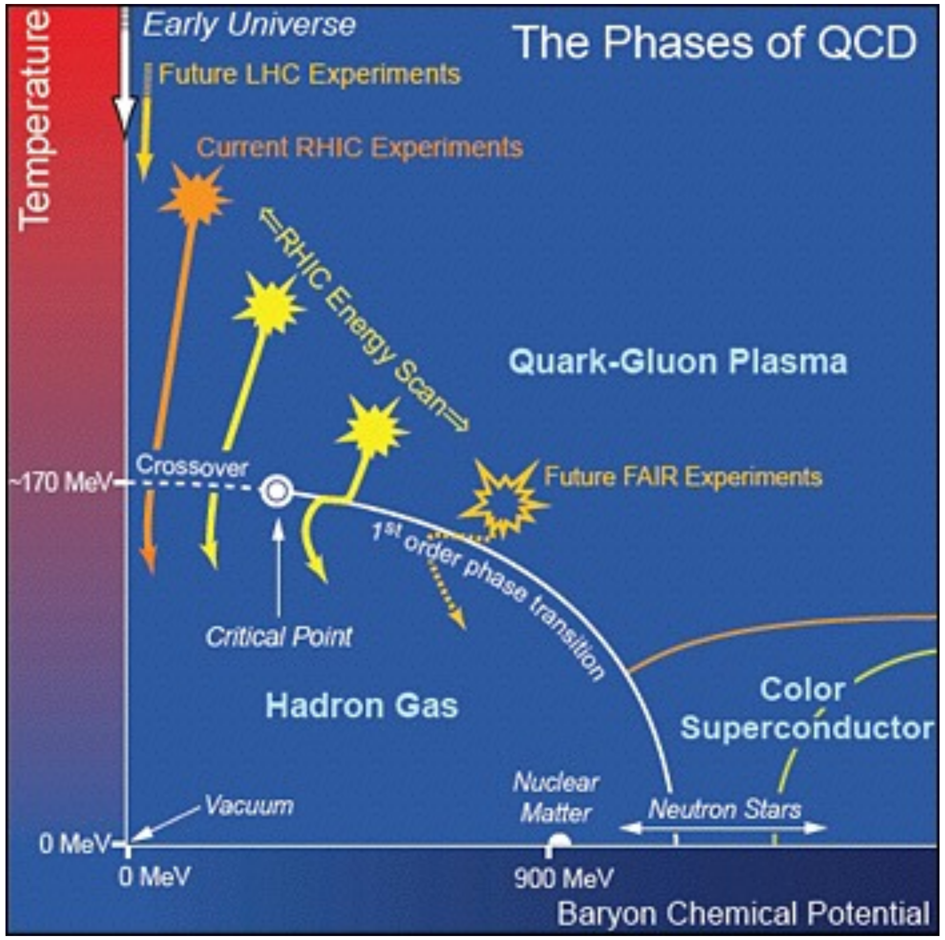
for

Zn Collaboration

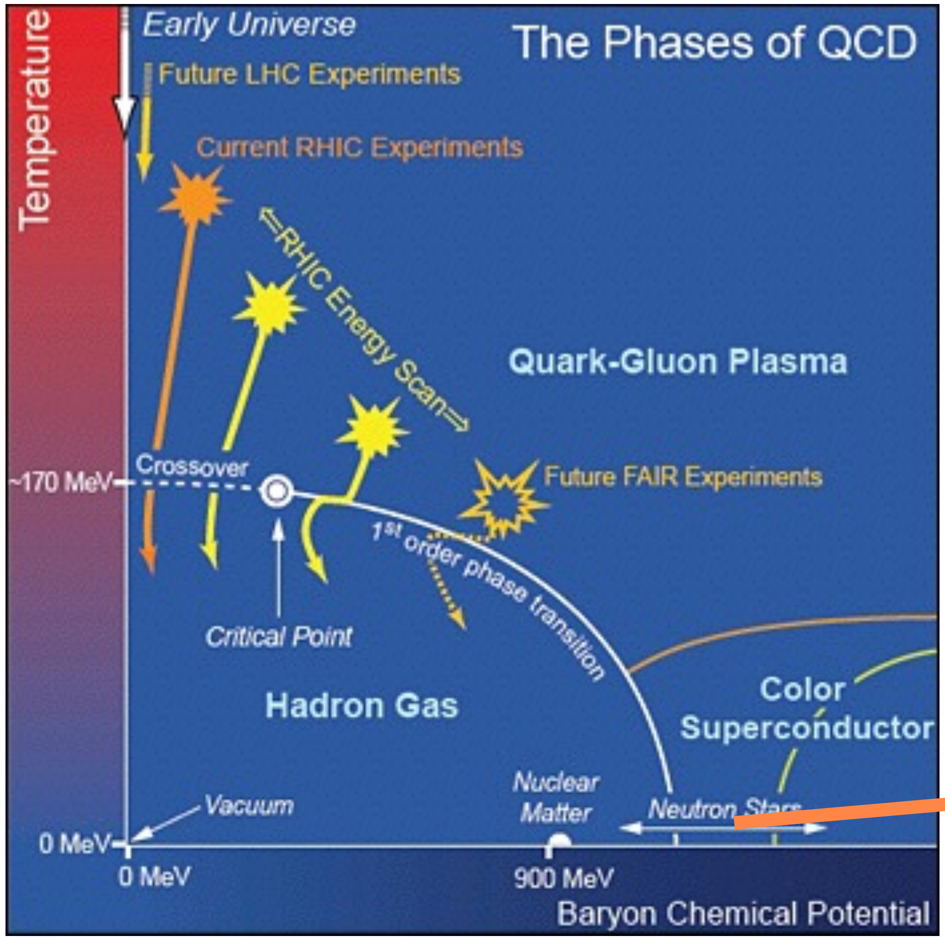
R.Fukuda (Tokyo) A.Nakamura (RCNP) S.Oka (Rikkyo) A.Suzuki (Tsukuba)

het-lat/1504.04471

Finite density QCD

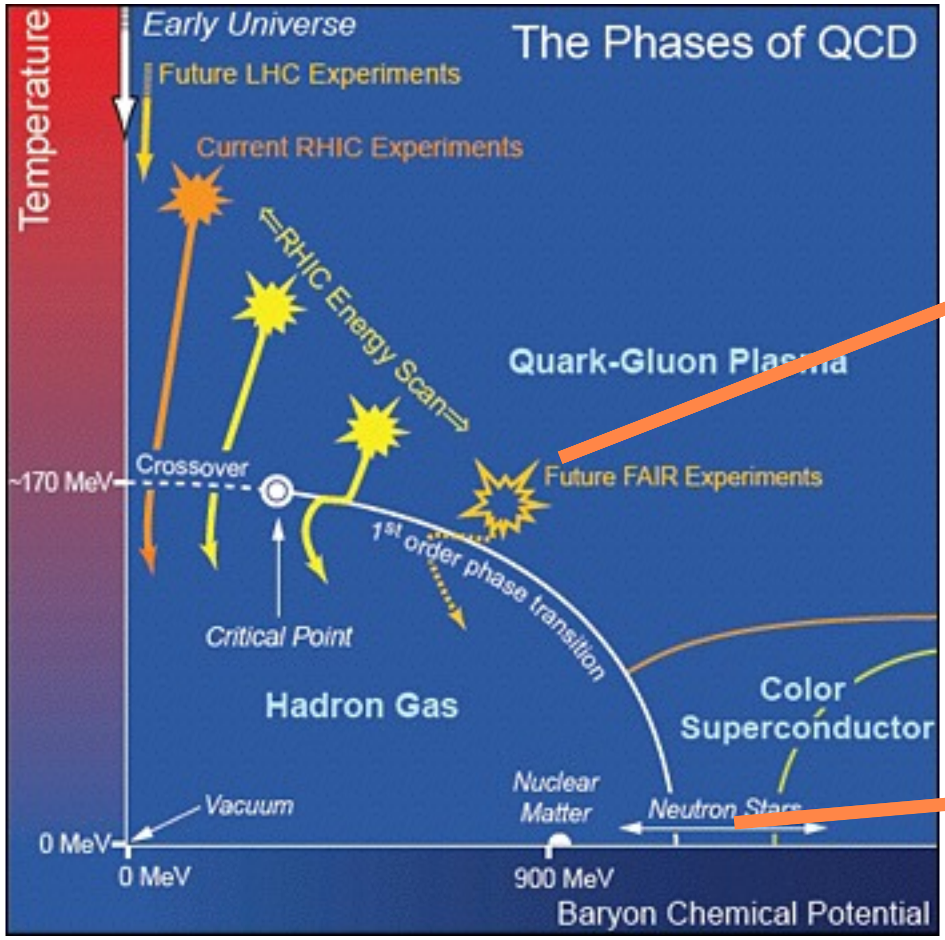


Finite density QCD



Neutron start with
2 x Solar mass

Finite density QCD



High density region
by experiments

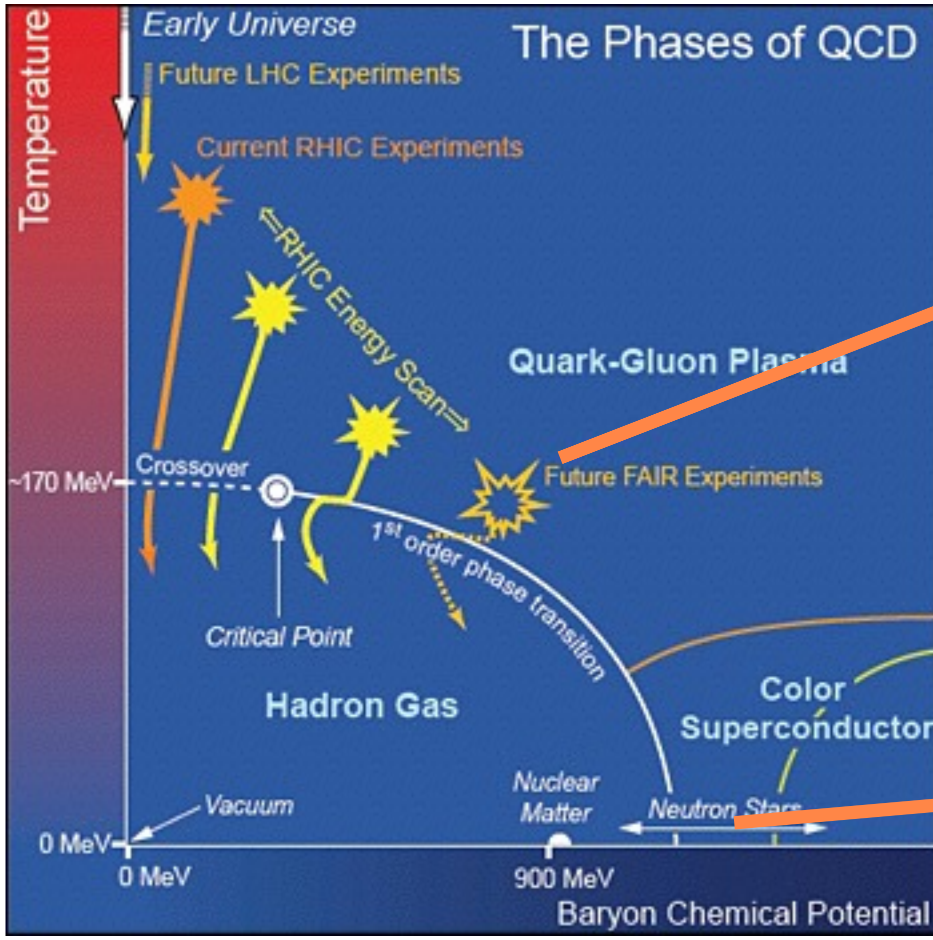
J-PARC

RIKEN-RIBF

GSI-FAIR

Neutron start with
2 x Solar mass

Finite density QCD



High density region
by experiments

J-PARC

RIKEN-RIBF

GSI-FAIR

Neutron start with
2 x Solar mass

Our tool: Lattice QCD

Complex action

Sign problem

Solution = Canonical approach

Solution = Canonical approach

Grand canonical partition function

$$Z_G(T,\mu,V) = \mathrm{Tr}\left[\exp\left(-\frac{1}{T}\left(\hat{H}-\mu\hat{N}\right)\right)\right]$$

Solution = Canonical approach

Grand canonical partition function

$$Z_G(T,\mu,V) = \mathrm{Tr}\left[\exp\left(-\frac{1}{T}\left(\hat{H}-\mu\hat{N}\right)\right)\right]$$

for every energy and number of particles

Solution = Canonical approach

Grand canonical partition function

$$Z_G(T,\mu,V) = \mathrm{Tr}\left[\exp\left(-\frac{1}{T}\left(\hat{H} - \mu\hat{N}\right)\right)\right]$$

for every energy and number of particles

For QCD $\left[\hat{H},\hat{N}\right] = 0$

Solution = Canonical approach

Grand canonical partition function

$$Z_G(T,\mu,V) = \mathrm{Tr}\left[\exp\left(-\frac{1}{T}\left(\hat{H} - \mu\hat{N}\right)\right)\right]$$

$$= \sum_n \sum_E \left\langle E, n \middle| \exp\left(-\frac{\hat{H}}{T} + \frac{\mu}{T}n\right) \middle| E, n \right\rangle$$

Solution = Canonical approach

Grand canonical partition function

$$Z_G(T,\mu,V) = \mathrm{Tr}\left[\exp\left(-\frac{1}{T}\left(\hat{H}-\mu\hat{N}\right)\right)\right]$$

$$= \sum_n \sum_E \left\langle E,n \middle| \exp\left(-\frac{\hat{H}}{T}+\frac{\mu}{T}n\right) \middle| E,n \right\rangle$$

$$= \sum_n Z_C(T,n,V)\xi^n$$

Solution = Canonical approach

Grand canonical partition function

$$Z_G(T,\mu,V) = \mathrm{Tr}\left[\exp\left(-\frac{1}{T}\left(\hat{H}-\mu\hat{N}\right)\right)\right]$$

$$= \sum_n \sum_E \left\langle E,n \middle| \exp\left(-\frac{\hat{H}}{T}+\frac{\mu}{T}n\right) \middle| E,n \right\rangle$$

$$= \sum_n Z_C(T,n,V)\xi^n$$

Fugacity $\xi = e^{\frac{\mu}{T}}$

Solution = Canonical approach

Grand canonical partition function

$$Z_G(T,\mu,V) = \mathrm{Tr}\left[\exp\left(-\frac{1}{T}\left(\hat{H}-\mu\hat{N}\right)\right)\right]$$

$$= \sum_n \sum_E \left\langle E,n \middle| \exp\left(-\frac{\hat{H}}{T}+\frac{\mu}{T}n\right) \middle| E,n \right\rangle$$

$$= \sum_n Z_C(T,n,V)\xi^n$$

Fugacity $\xi = e^{\frac{\mu}{T}}$

Canonical partition function

$$Z_C(T,n,V) = \sum_E \left\langle E,n \middle| \exp\left(-\frac{\hat{H}}{T}\right) \middle| E,n \right\rangle$$

Solution = Canonical approach

Grand canonical partition function

$$Z_G(T,\mu,V) = \mathrm{Tr}\left[\exp\left(-\frac{1}{T}\left(\hat{H}-\mu\hat{N}\right)\right)\right]$$

$$= \sum_n \sum_E \left\langle E,n \middle| \exp\left(-\frac{\hat{H}}{T}+\frac{\mu}{T}n\right) \middle| E,n \right\rangle$$

$$= \sum_n \underline{Z_C(T,n,V)}\xi^n$$

Fugacity $\xi = e^{\frac{\mu}{T}}$

Fugacity expansion

Canonical partition function

$$Z_C(T,n,V) = \sum_E \left\langle E,n \middle| \exp\left(-\frac{\hat{H}}{T}\right) \middle| E,n \right\rangle$$

Solution = Canonical approach

Grand canonical partition function

$$Z_G(T,\mu,V) = \mathrm{Tr}\left[\exp\left(-\frac{1}{T}\left(\hat{H}-\mu\hat{N}\right)\right)\right]$$

$$= \sum_n \sum_E \left\langle E,n \middle| \exp\left(-\frac{\hat{H}}{T}+\frac{\mu}{T}n\right) \middle| E,n \right\rangle$$

$$= \sum_n Z_C(T,n,V)\xi^n$$

Fugacity expansion

Fugacity $\xi = e^{\frac{\mu}{T}}$

Canonical partition function

$$Z_C(T,n,V) = \sum_E \left\langle E,n \middle| \exp\left(-\frac{\hat{H}}{T}\right) \middle| E,n \right\rangle$$

Real, positive!

Hopping parameter expansion

Fugacity expansion is embedded in hopping parameter expansion

Hopping parameter expansion

Fugacity expansion is embedded in hopping parameter expansion

Wilson Dirac operator

$$D_W(\mu) = 1 - \kappa(Q_s + T) - \kappa e^{\mu a} Q_4^+ - \kappa e^{-\mu a} Q_4^-$$

Hopping parameter expansion

Fugacity expansion is embedded in hopping parameter expansion

Wilson Dirac operator

$$D_W(\mu) = 1 - \kappa(Q_s + T) - \kappa e^{\mu a} Q_4^+ - \kappa e^{-\mu a} Q_4^-$$

$$\left(Q_\mu^+\right)_{nm} = (1 - \gamma_\mu)\, U_\mu(n) \delta_{m,n+\hat{\mu}}$$

$$\left(Q_\mu^-\right)_{nm} = (1 + \gamma_\mu)\, U_\mu^\dagger(m) \delta_{m,n-\hat{\mu}}$$

Hopping parameter expansion

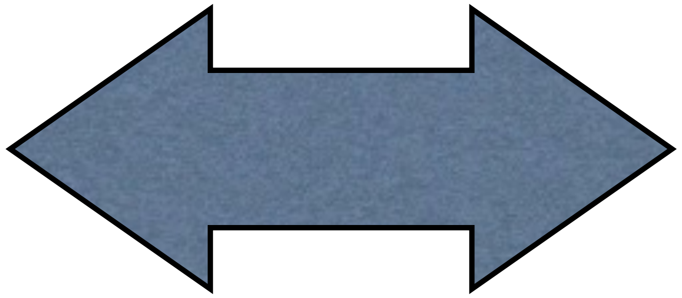
Fugacity expansion is embedded in hopping parameter expansion

Wilson Dirac operator

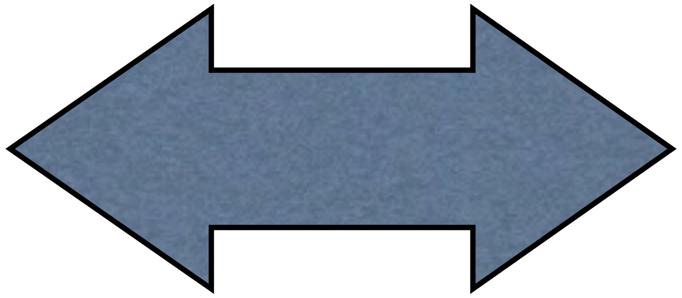
$$D_W(\mu) = 1 - \kappa(Q_s + T) - \kappa e^{\mu a} Q_4^+ - \kappa e^{-\mu a} Q_4^-$$

$$\left(Q_\mu^+\right)_{nm} = (1 - \gamma_\mu)\, U_\mu(n) \delta_{m,n+\hat{\mu}}$$

$$\left(Q_\mu^-\right)_{nm} = (1 + \gamma_\mu)\, U_\mu^\dagger(m) \delta_{m,n-\hat{\mu}}$$

expansion in $\kappa = \dfrac{1}{2(ma+4)}$  expansion in $e^{\pm \mu a}$

Hopping parameter expansion

expansion in $\kappa = \dfrac{1}{2(ma+4)}$  expansion in $e^{\pm\mu a}$

Hopping parameter expansion

expansion in $\kappa = \dfrac{1}{2(ma+4)}$ $\Longleftrightarrow$ expansion in $e^{\pm\mu a}$

Instead of Det, expand $\mathrm{TrLog} D_W(\mu)$

$$\mathrm{TrLog}(1-\kappa Q(\mu)) = -\sum_n \frac{\kappa^n}{n} \mathrm{Tr}\left(Q_s + e^{\mu a} Q_4^+ + e^{-\mu a} Q_4^-\right)^n$$

Hopping parameter expansion

expansion in $\kappa = \dfrac{1}{2(ma+4)}$ ⟺ expansion in $e^{\pm\mu a}$

Instead of Det, expand $\mathrm{TrLog} D_W(\mu)$

$$\mathrm{TrLog}(1-\kappa Q(\mu)) = -\sum_n \frac{\kappa^n}{n} \mathrm{Tr}\left(Q_s + e^{\mu a} Q_4^+ + e^{-\mu a} Q_4^-\right)^n$$

Hopping parameter expansion

expansion in $\kappa = \frac{1}{2(ma+4)}$ ⟺ expansion in $e^{\pm\mu a}$

Instead of Det, expand $\mathrm{TrLog} D_W(\mu)$

$$\mathrm{TrLog}(1-\kappa Q(\mu)) = -\sum_n \frac{\kappa^n}{n} \mathrm{Tr}\left(Q_s + e^{\mu a} Q_4^+ + e^{-\mu a} Q_4^-\right)^n$$

quark hopping need to make a loop

Hopping parameter expansion

expansion in $\kappa = \frac{1}{2(ma+4)}$ ⟷ expansion in $e^{\pm\mu a}$

Instead of Det, expand $\mathrm{TrLog} D_W(\mu)$

$$\mathrm{TrLog}(1-\kappa Q(\mu)) = -\sum_n \frac{\kappa^n}{n} \mathrm{Tr}\left(Q_s + e^{\mu a} Q_4^+ + e^{-\mu a} Q_4^-\right)^n$$

quark hopping need to make a loop

$\left(e^{\pm\mu N_t a}\right)^m$ survives

Hopping parameter expansion

expansion in $\kappa = \dfrac{1}{2(ma+4)}$ ⟺ expansion in $e^{\pm\mu a}$

Instead of Det, expand $\mathrm{TrLog} D_W(\mu)$

$$\mathrm{TrLog}(1-\kappa Q(\mu)) = -\sum_n \frac{\kappa^n}{n} \mathrm{Tr}\left(Q_s + e^{\mu a} Q_4^+ + e^{-\mu a} Q_4^-\right)^n$$

quark hopping need to make a loop

$\left(e^{\pm\mu N_t a}\right)^m$ survives = Fugacity $\xi^m = \left(e^{\pm\mu/T}\right)^m$

Hopping parameter expansion

expansion in $\kappa = \dfrac{1}{2(ma+4)}$ ⟺ expansion in $e^{\pm\mu a}$

Instead of Det, expand $\mathrm{TrLog} D_W(\mu)$

$$\mathrm{TrLog}(1-\kappa Q(\mu)) = -\sum_n \frac{\kappa^n}{n} \mathrm{Tr}\left(Q_s + e^{\mu a}Q_4^+ + e^{-\mu a}Q_4^-\right)^n$$

quark hopping need to make a loop

$\left(e^{\pm\mu N_t a}\right)^m$ survives = Fugacity $\xi^m = \left(e^{\pm\mu/T}\right)^m$

Re-sum the expansion in temporal winding

Hopping parameter expansion

expansion in $\kappa = \dfrac{1}{2(ma+4)}$ ⟺ expansion in $e^{\pm\mu a}$

Instead of Det, expand $\mathrm{TrLog} D_W(\mu)$

$$\mathrm{TrLog}(1-\kappa Q(\mu)) = -\sum_n \frac{\kappa^n}{n} \mathrm{Tr}\left(Q_s + e^{\mu a} Q_4^+ + e^{-\mu a} Q_4^-\right)^n$$

quark hopping need to make a loop

$\left(e^{\pm\mu N_t a}\right)^m$ survives = Fugacity $\xi^m = \left(e^{\pm\mu/T}\right)^m$

Re-sum the expansion in temporal winding

⇒ **Fugacity expansion of Dirac determinant Det D(ξ)**

Meng et al. (Kentucky)

Canonical partition function Zc(n)

Kentucky '08

Evaluation of Det Dw(μ) ← re-weighting

Canonical partition function Zc(n)

Kentucky '08

Evaluation of Det Dw(μ) ← re-weighting

$$Z_G(\mu) = \int DU \frac{\mathrm{Det} D_W(\mu)}{\mathrm{Det} D_W(\mu_0)} \mathrm{Det} D_W(\mu_0) e^{-S_G}$$

Canonical partition function Zc(n)

Kentucky '08

Evaluation of Det Dw(μ) ← re-weighting

$$Z_G(\mu) = \int DU \frac{\mathrm{Det} D_W(\mu)}{\mathrm{Det} D_W(\mu_0)} \mathrm{Det} D_W(\mu_0) e^{-S_G}$$

↓ 0 or imaginary

Canonical partition function Zc(n)

Kentucky '08

Evaluation of Det Dw(μ) ← re-weighting

$$Z_G(\mu) = \int DU \frac{\mathrm{Det} D_W(\mu)}{\mathrm{Det} D_W(\mu_0)} \mathrm{Det} D_W(\mu_0) e^{-S_G}$$

↓ 0 or imaginary

$$= \left\langle \frac{\mathrm{Det} D_W(\mu)}{\mathrm{Det} D_W(\mu_0)} \right\rangle_0 Z_G(\mu_0)$$

Canonical partition function Zc(n)

Kentucky '08

Evaluation of Det Dw(μ) ← re-weighting

$$Z_G(\mu) = \int DU \frac{\mathrm{Det} D_W(\mu)}{\mathrm{Det} D_W(\mu_0)} \mathrm{Det} D_W(\mu_0) e^{-S_G}$$

↓ 0 or imaginary

$$= \left\langle \frac{\mathrm{Det} D_W(\mu)}{\mathrm{Det} D_W(\mu_0)} \right\rangle_0 Z_G(\mu_0)$$

↓ hopping parameter exp.

$$= \left\langle \frac{\exp\left(\sum_{k=-\infty}^{\infty} W_k \xi^k\right)}{\mathrm{Det} D_W(\mu_0)} \right\rangle_0 Z_G(\mu_0)$$

Canonical partition function Zc(n)

Kentucky '08

Evaluation of Det Dw(μ) ← re-weighting

$$Z_G(\mu) = \int DU \frac{\mathrm{Det} D_W(\mu)}{\mathrm{Det} D_W(\mu_0)} \mathrm{Det} D_W(\mu_0) e^{-S_G}$$

↓ 0 or imaginary

$$= \left\langle \frac{\mathrm{Det} D_W(\mu)}{\mathrm{Det} D_W(\mu_0)} \right\rangle_0 Z_G(\mu_0)$$

↓ hopping parameter exp.

$$\sum_{n=-\infty}^{\infty} Z_C(n) \xi^n = \left\langle \frac{\exp\left(\sum_{k=-\infty}^{\infty} W_k \xi^k\right)}{\mathrm{Det} D_W(\mu_0)} \right\rangle_0 Z_G(\mu_0)$$

Canonical partition function Zc(n)

Kentucky '08

Evaluation of Det Dw(μ) ← re-weighting

$$Z_G(\mu) = \int DU \frac{\mathrm{Det}\, D_W(\mu)}{\mathrm{Det}\, D_W(\mu_0)} \mathrm{Det}\, D_W(\mu_0) e^{-S_G}$$

↓ 0 or imaginary

$$= \left\langle \frac{\mathrm{Det}\, D_W(\mu)}{\mathrm{Det}\, D_W(\mu_0)} \right\rangle_0 Z_G(\mu_0)$$

↓ hopping parameter exp.

$$\sum_{n=-\infty}^{\infty} Z_C(n) \xi^n = \left\langle \frac{\exp\left(\sum_{k=-\infty}^{\infty} W_k \xi^k\right)}{\mathrm{Det}\, D_W(\mu_0)} \right\rangle_0 Z_G(\mu_0)$$

Canonical partition function Zc(n)

Kentucky '08

Evaluation of Det Dw(μ) ← re-weighting

$$Z_G(\mu) = \int DU \frac{\mathrm{Det} D_W(\mu)}{\mathrm{Det} D_W(\mu_0)} \mathrm{Det} D_W(\mu_0) e^{-S_G}$$

↓ 0 or imaginary

$$= \left\langle \frac{\mathrm{Det} D_W(\mu)}{\mathrm{Det} D_W(\mu_0)} \right\rangle_0 Z_G(\mu_0)$$

↓ hopping parameter exp.

$$\sum_{n=-\infty}^{\infty} Z_C(n) \xi^n = \left\langle \frac{\exp\left(\sum_{k=-\infty}^{\infty} W_k \xi^k\right)}{\mathrm{Det} D_W(\mu_0)} \right\rangle_0 Z_G(\mu_0)$$

Need to check the overlap problem

Plan of the talk

✔ 1. Introduction

✔ 2. Hopping parameter expansion

3. Numerical setup

4. Canonical partition function Zn

5. Hadronic observables

6. Conclusion

Numerical setup

★ Iwasaki gauge action

★ Clover fermion Nf=2

- APE stout smeared gauge link $c_{\mathrm{SW}} = 1.1$

★ Box sizes $8^3 \times 4$ $12^3 \times 4$

β	T/Tc	κ	$m\pi/m\rho$
0.9	0.64	0.137	0.8978(55)
1.1	0.67	0.133	0.9038(56)
1.3	0.71	0.138	0.809(12)
1.5	0.81	0.136	0.756(13)
1.7	1	0.129	0.770(13)
1.9	1.7	0.125	0.714(15)
2.1	3.4	0.122	0.836(47)

Numerical setup

★ Iwasaki gauge action

★ Clover fermion Nf=2

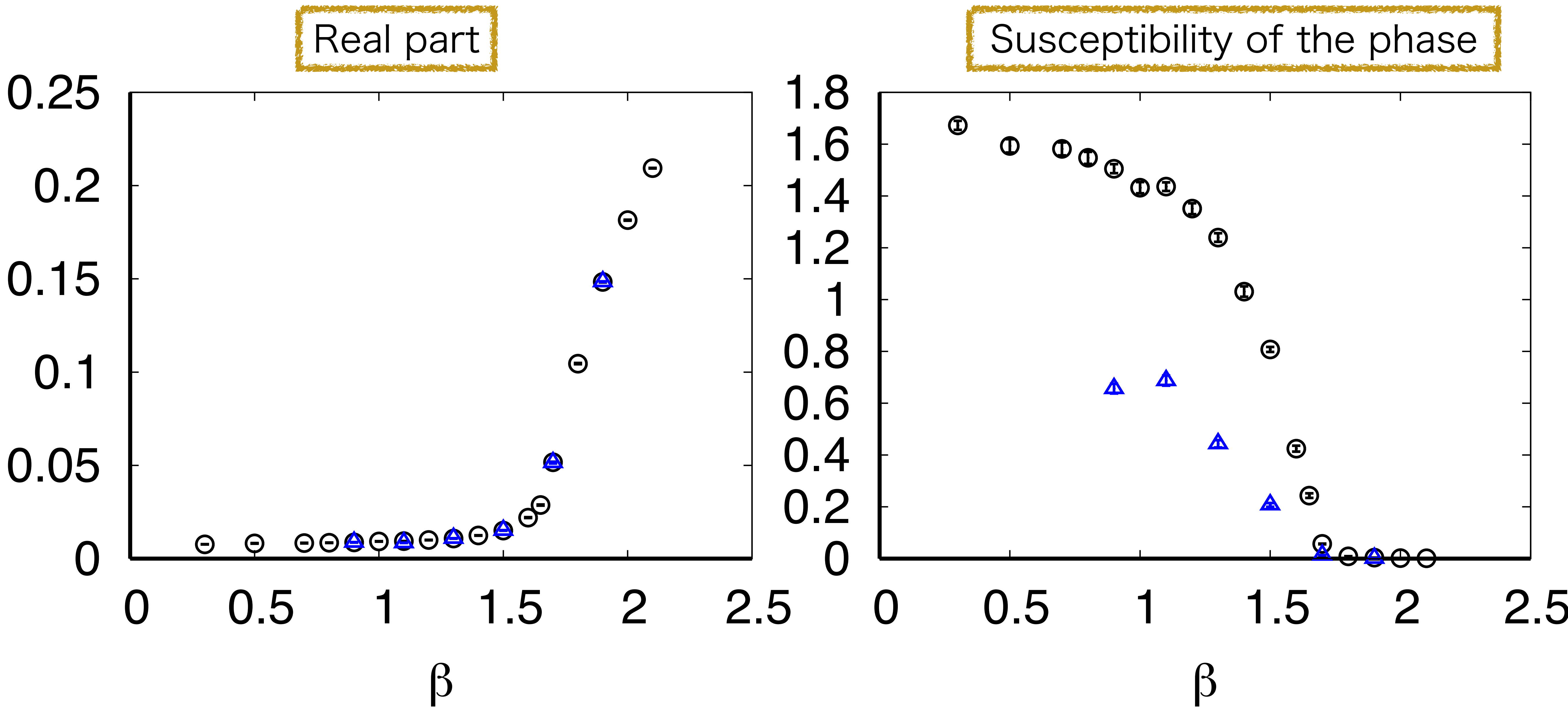
- APE stout smeared gauge link $c_{\mathrm{SW}} = 1.1$

★ Box sizes $8^3 \times 4$ $12^3 \times 4$

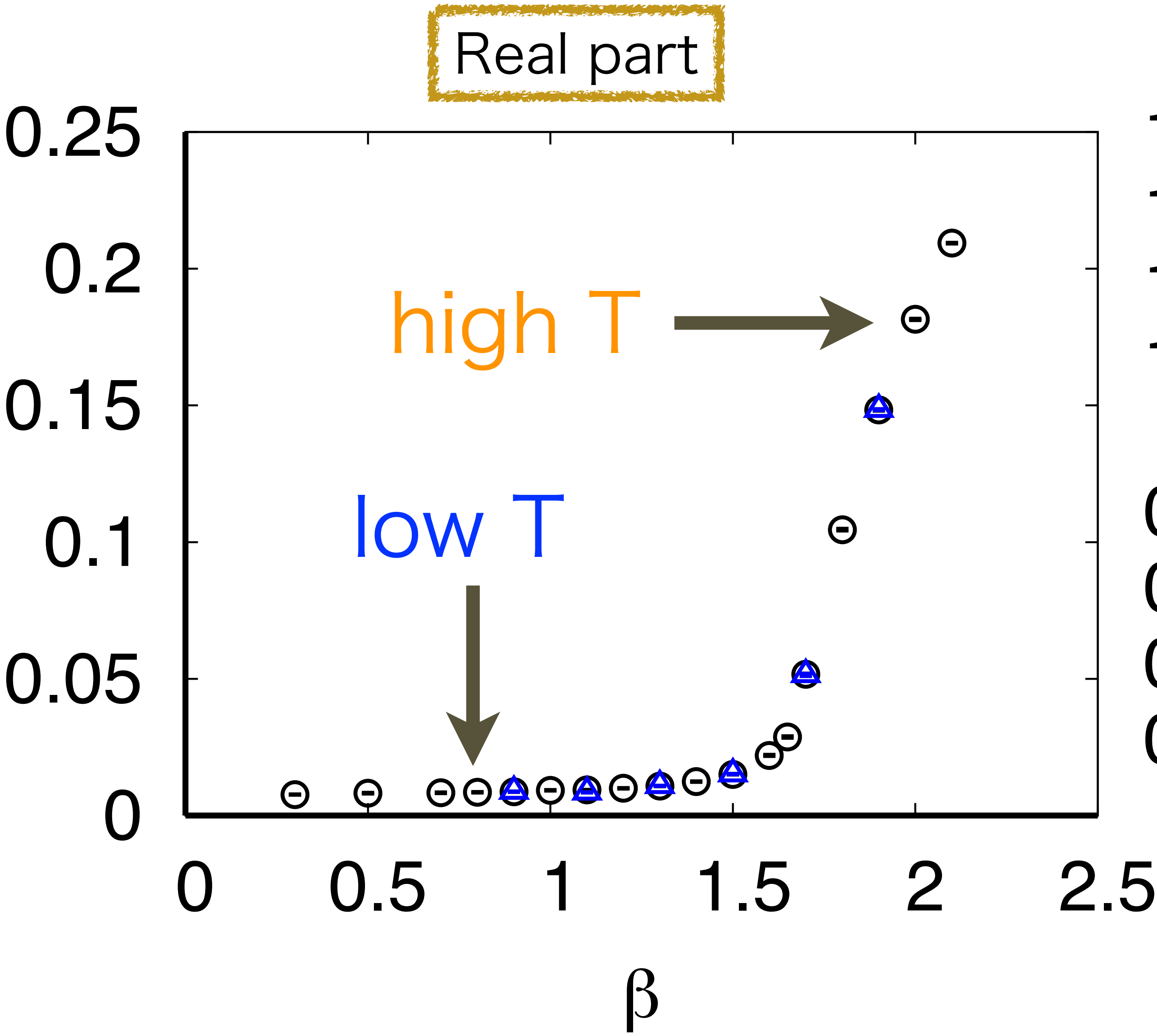
Tc∼220MeV

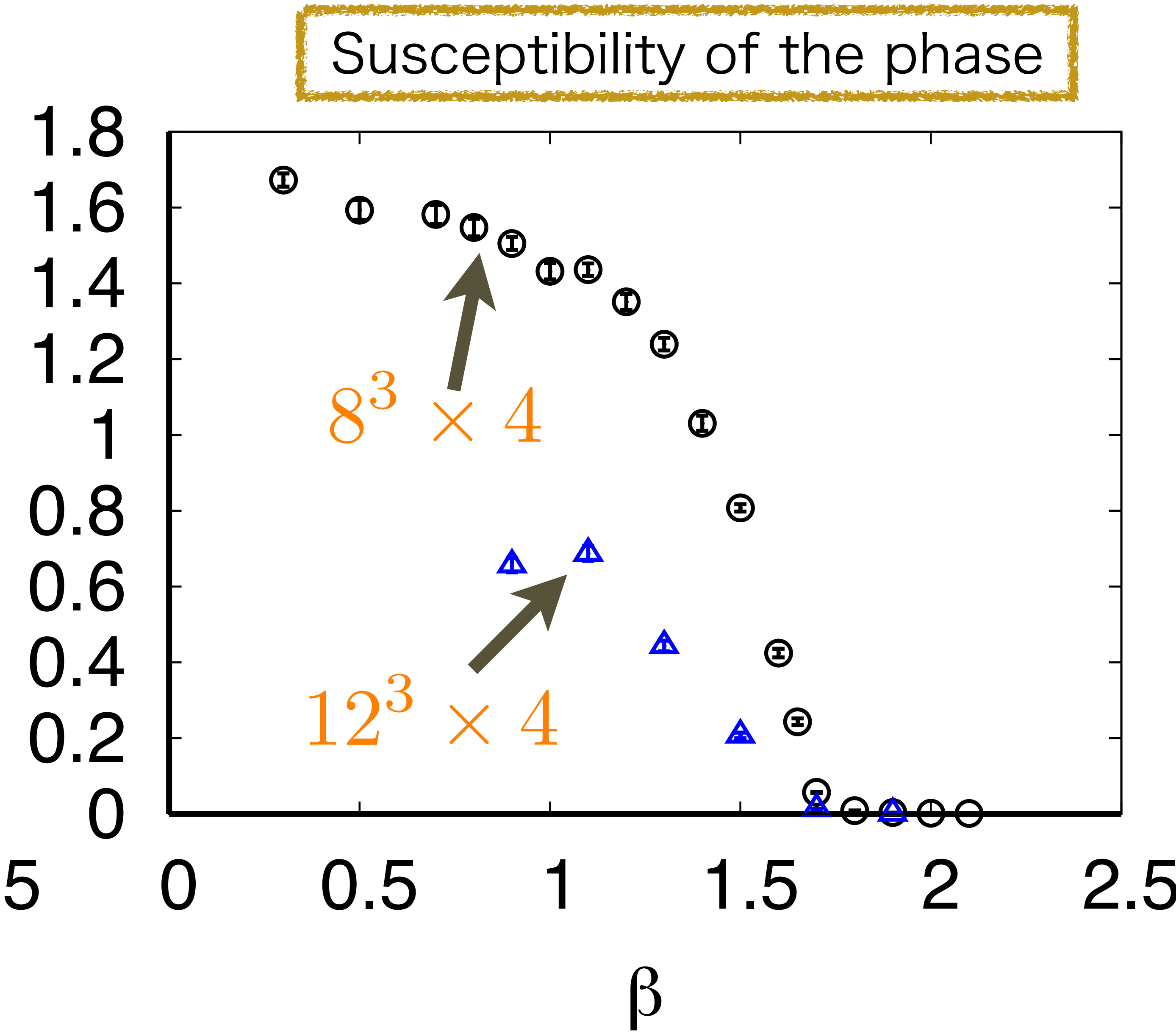
β	T/Tc	κ	mπ/mρ
0.9	0.64	0.137	0.8978(55)
1.1	0.67	0.133	0.9038(56)
1.3	0.71	0.138	0.809(12)
1.5	0.81	0.136	0.756(13)
1.7	1	0.129	0.770(13)
1.9	1.7	0.125	0.714(15)
2.1	3.4	0.122	0.836(47)

Polyakov loop

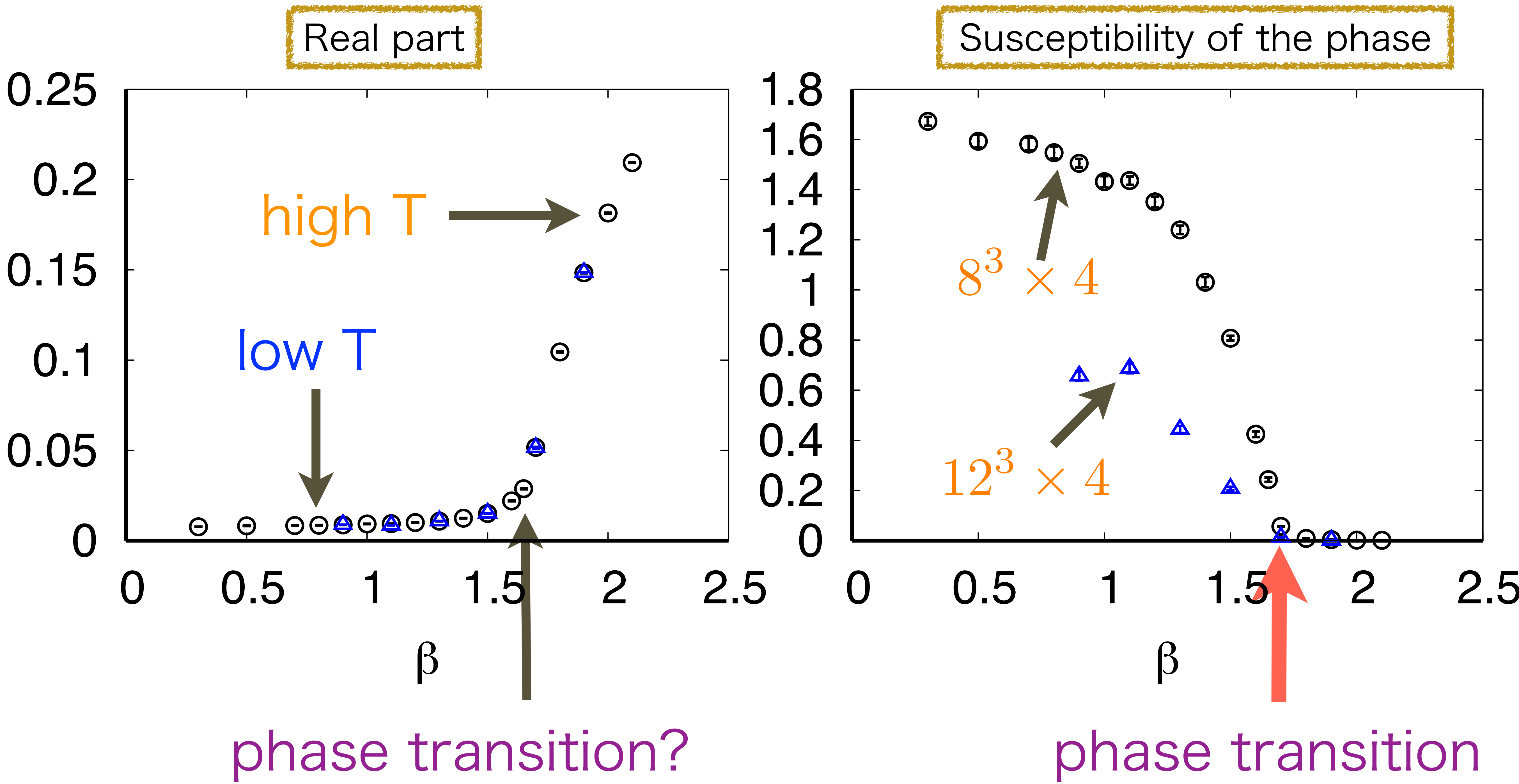


Polyakov loop





Polyakov loop



Plan of the talk

✔ 1. Introduction

✔ 2. Hopping parameter expansion

✔ 3. Numerical setup

4. Canonical partition function Zn

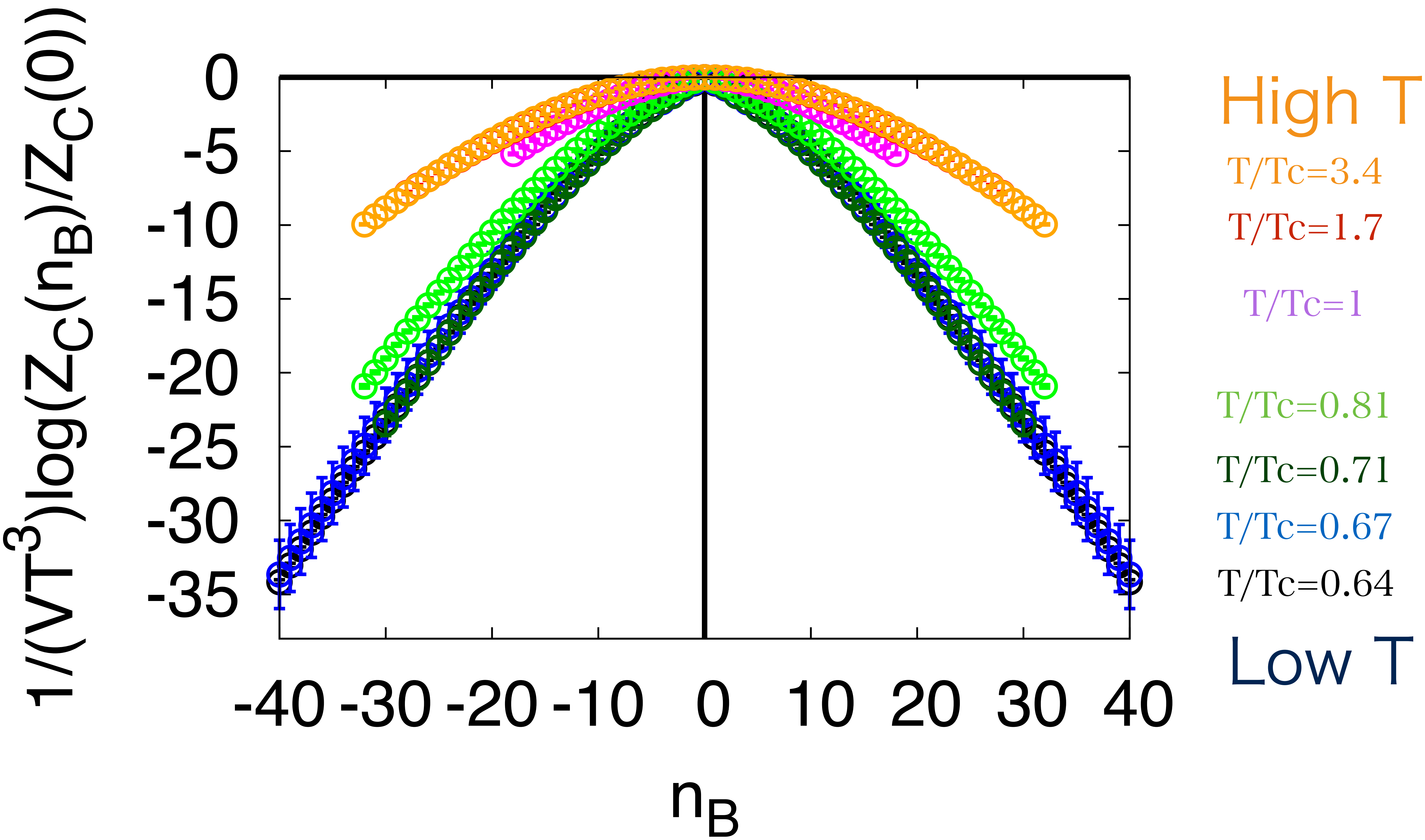
5. Hadronic observables

6. Conclusion

Canonical |Zc(n)|

canonical partition fn.

$$Z_C\,(T,n,V) = |Z_C(\beta,n)|e^{i\theta(\beta,n)}$$



Use of Zc(n)

Get the grand partition function $$Z(\mu) = \sum_{n=-\infty}^{\infty} |Z_n| \xi^n$$

Use of Zc(n)

Get the grand partition function $$Z(\mu) = \sum_{n=-\infty}^{\infty} |Z_n| \xi^n$$

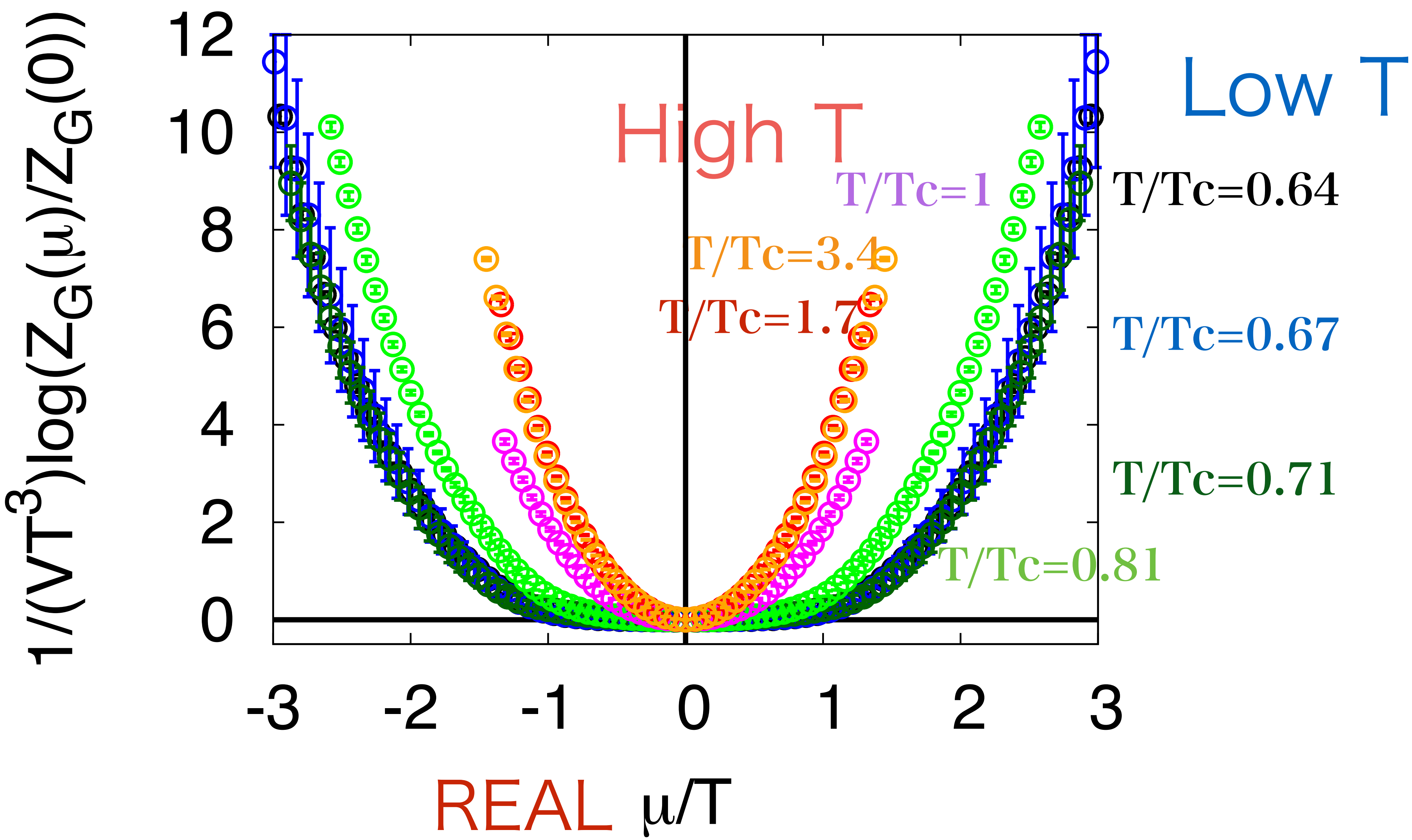
1. The pressure $$\frac{P}{T^4} = \frac{\log Z(\mu)}{VT^3}$$

Use of Zc(n)

Get the grand partition function $Z(\mu) = \sum_{n=-\infty}^{\infty} |Z_n| \xi^n$

1. The pressure $\frac{P}{T^4} = \frac{\log Z(\mu)}{VT^3}$

$\frac{P}{T^4}$

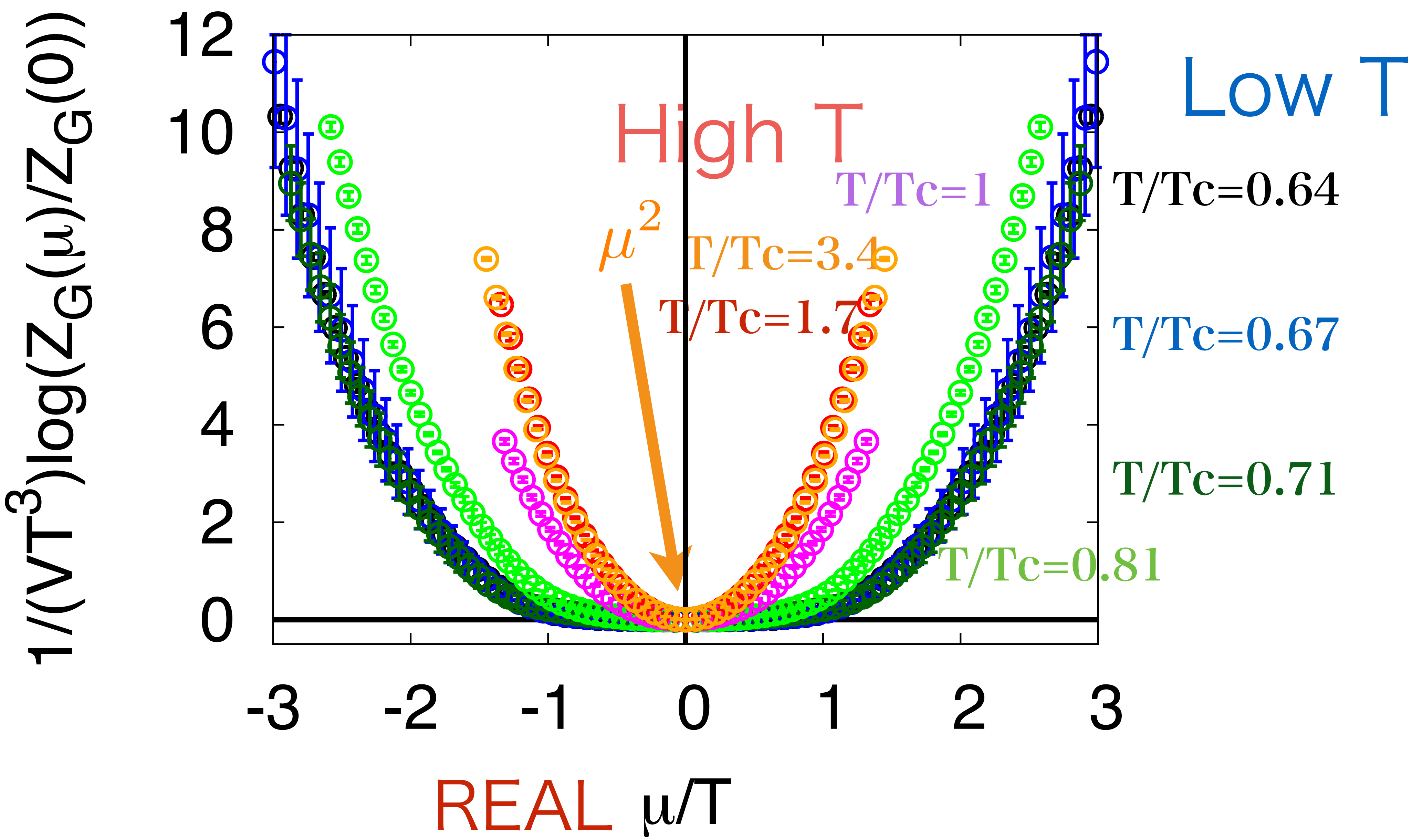


Use of Zc(n)

Get the grand partition function $Z(\mu) = \sum_{n=-\infty}^{\infty} |Z_n| \xi^n$

1. The pressure $\frac{P}{T^4} = \frac{\log Z(\mu)}{VT^3}$

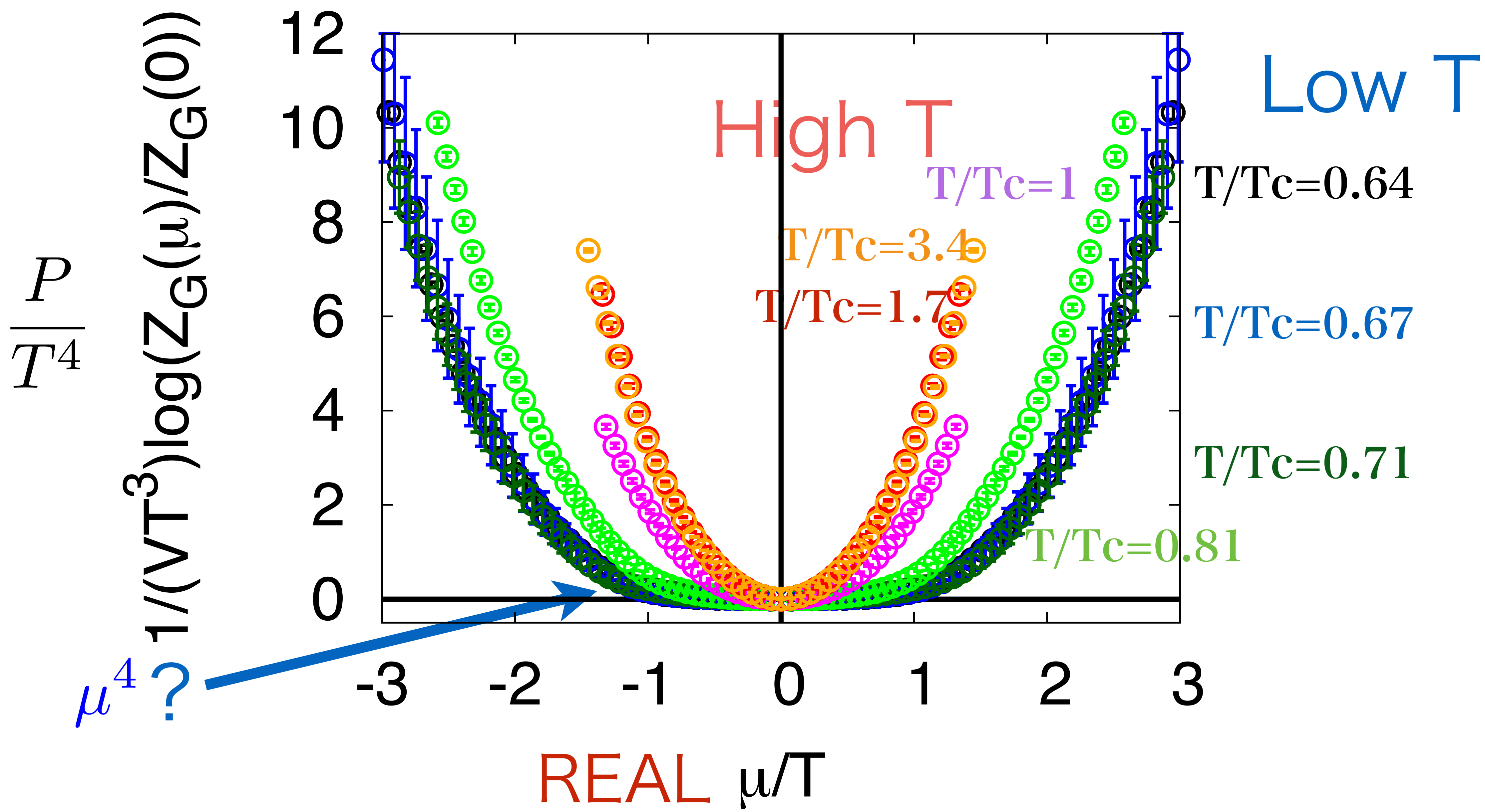
$\frac{P}{T^4}$



Use of Zc(n)

Get the grand partition function $Z(\mu) = \sum_{n=-\infty}^{\infty} |Z_n| \xi^n$

1. The pressure $\frac{P}{T^4} = \frac{\log Z(\mu)}{VT^3}$

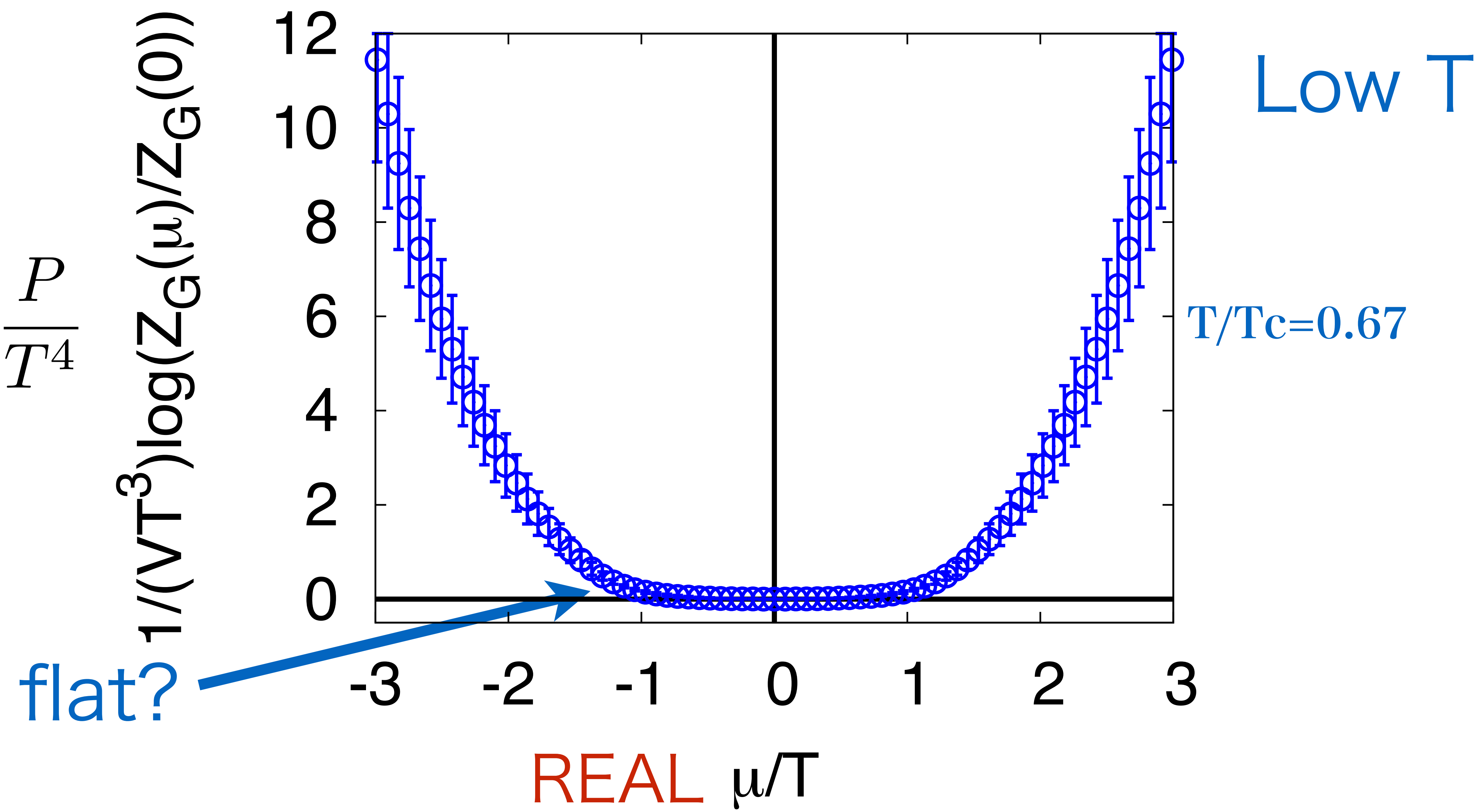


Use of Zc(n)

Get the grand partition function $Z(\mu) = \sum_{n=-\infty}^{\infty} |Z_n| \xi^n$

1. The pressure $\frac{P}{T^4} = \frac{\log Z(\mu)}{VT^3}$

$\frac{P}{T^4}$



Use of Zc(n)

We have the canonical partition function Z_n

Use of Zc(n)

We have the canonical partition function Z_n

2. quark number density

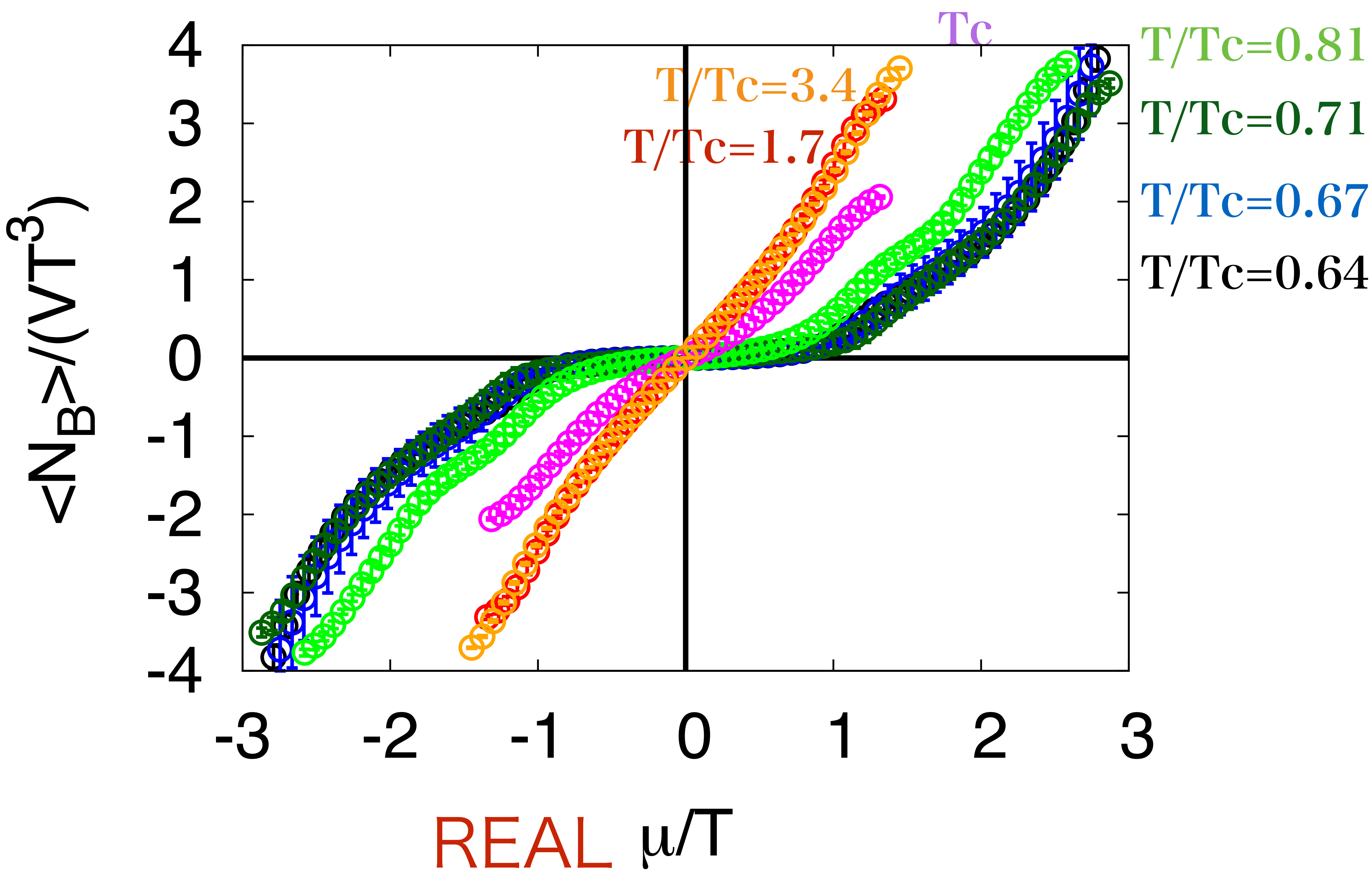
$$\langle N \rangle = \frac{1}{Z(\mu)} \sum_{n=-\infty}^{\infty} n|Z_n|\xi^n$$

Use of Zc(n)

We have the canonical partition function Z_n

2. quark number density

$$\langle N \rangle = \frac{1}{Z(\mu)} \sum_{n=-\infty}^{\infty} n|Z_n|\xi^n$$

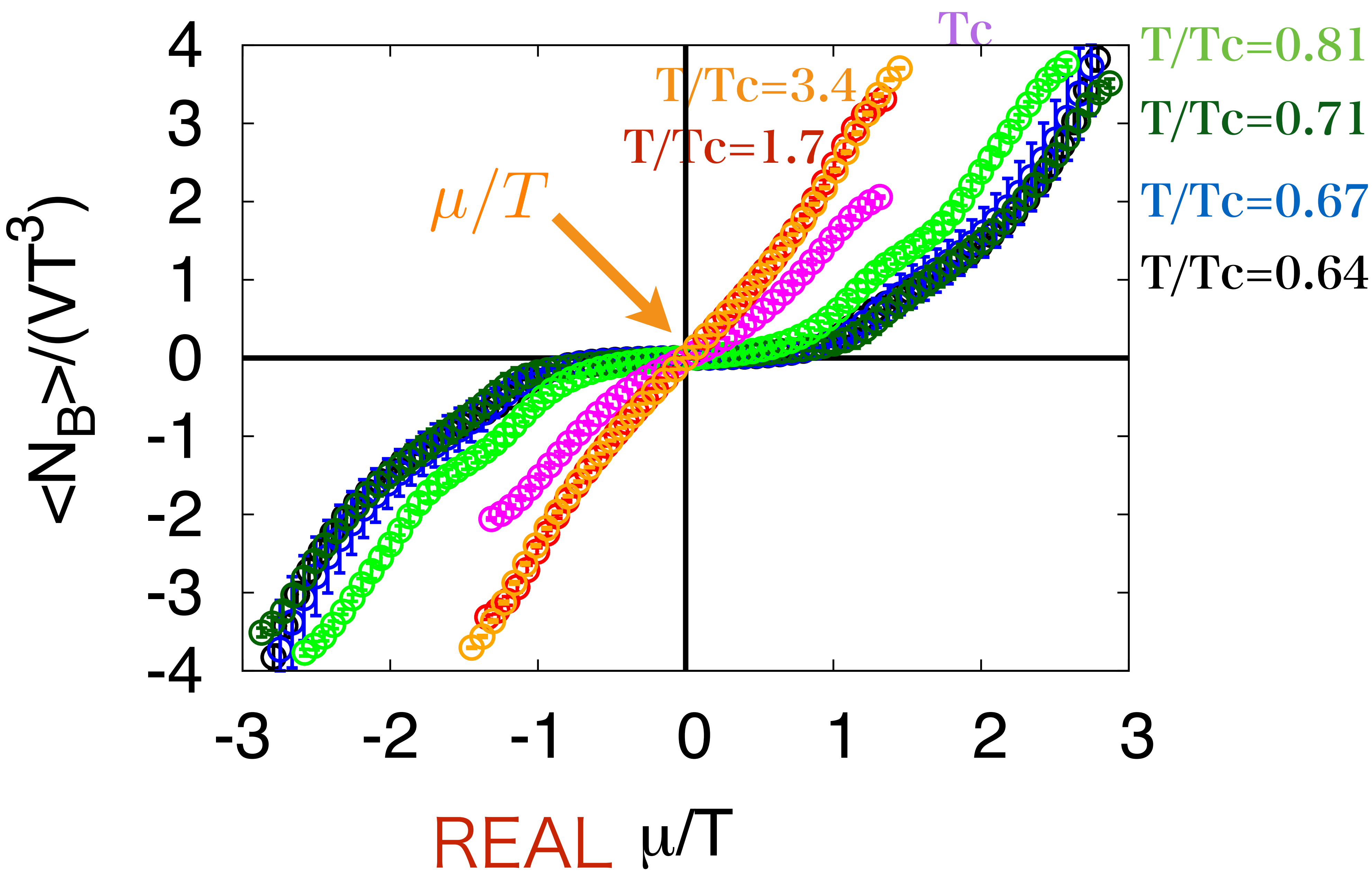


Use of Zc(n)

We have the canonical partition function Z_n

2. quark number density

$$\langle N \rangle = \frac{1}{Z(\mu)} \sum_{n=-\infty}^{\infty} n|Z_n|\xi^n$$

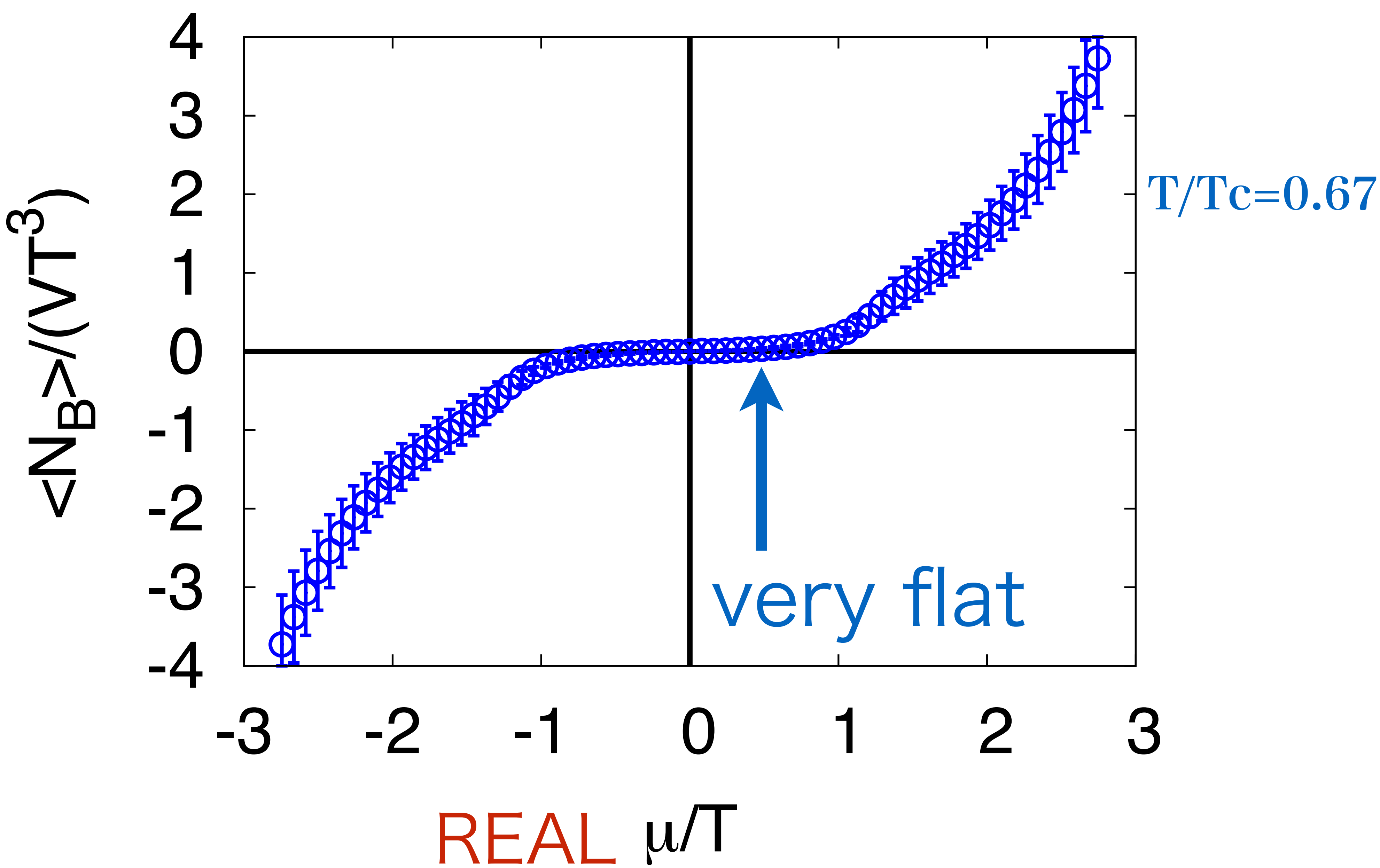


Use of Zc(n)

We have the canonical partition function Z_n

2. quark number density

$$\langle N \rangle = \frac{1}{Z(\mu)} \sum_{n=-\infty}^{\infty} n|Z_n|\xi^n$$



Use of Zc(n)

We have the canonical partition function Z_n

Use of Zc(n)

We have the canonical partition function Z_n

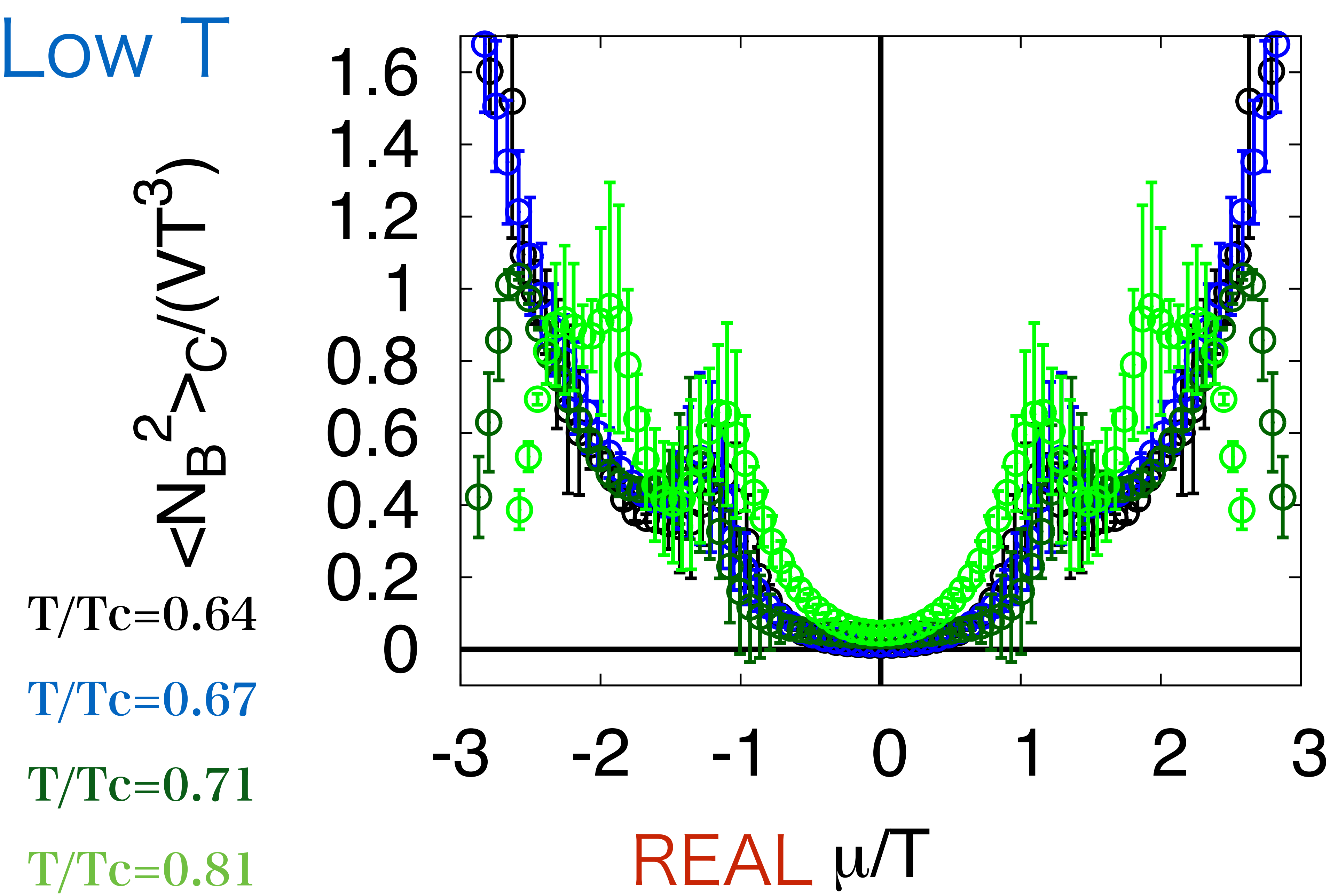
3. cumulant of quark number density $\langle N^2 \rangle_c = \left(\xi \frac{\partial}{\partial \xi} \right)^2 \log Z(\xi)$

Use of Zc(n)

We have the canonical partition function Z_n

3. cumulant of quark number density $\langle N^2 \rangle_c = \left(\xi \frac{\partial}{\partial \xi} \right)^2 \log Z(\xi)$

Low T

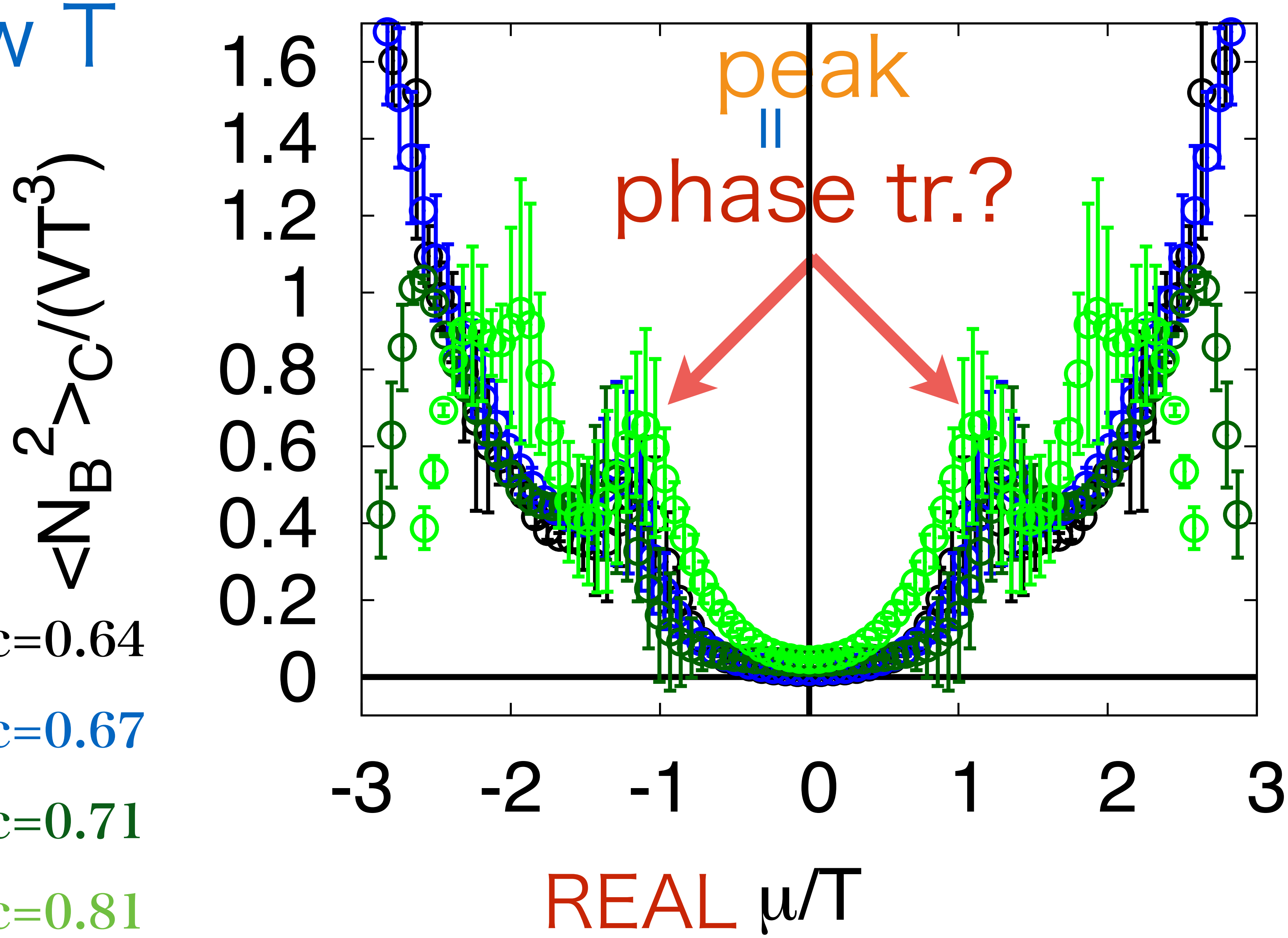


Use of Zc(n)

We have the canonical partition function Z_n

3. cumulant of quark number density $\langle N^2 \rangle_c = \left(\xi \frac{\partial}{\partial \xi} \right)^2 \log Z(\xi)$

Low T



T/Tc=0.64

T/Tc=0.67

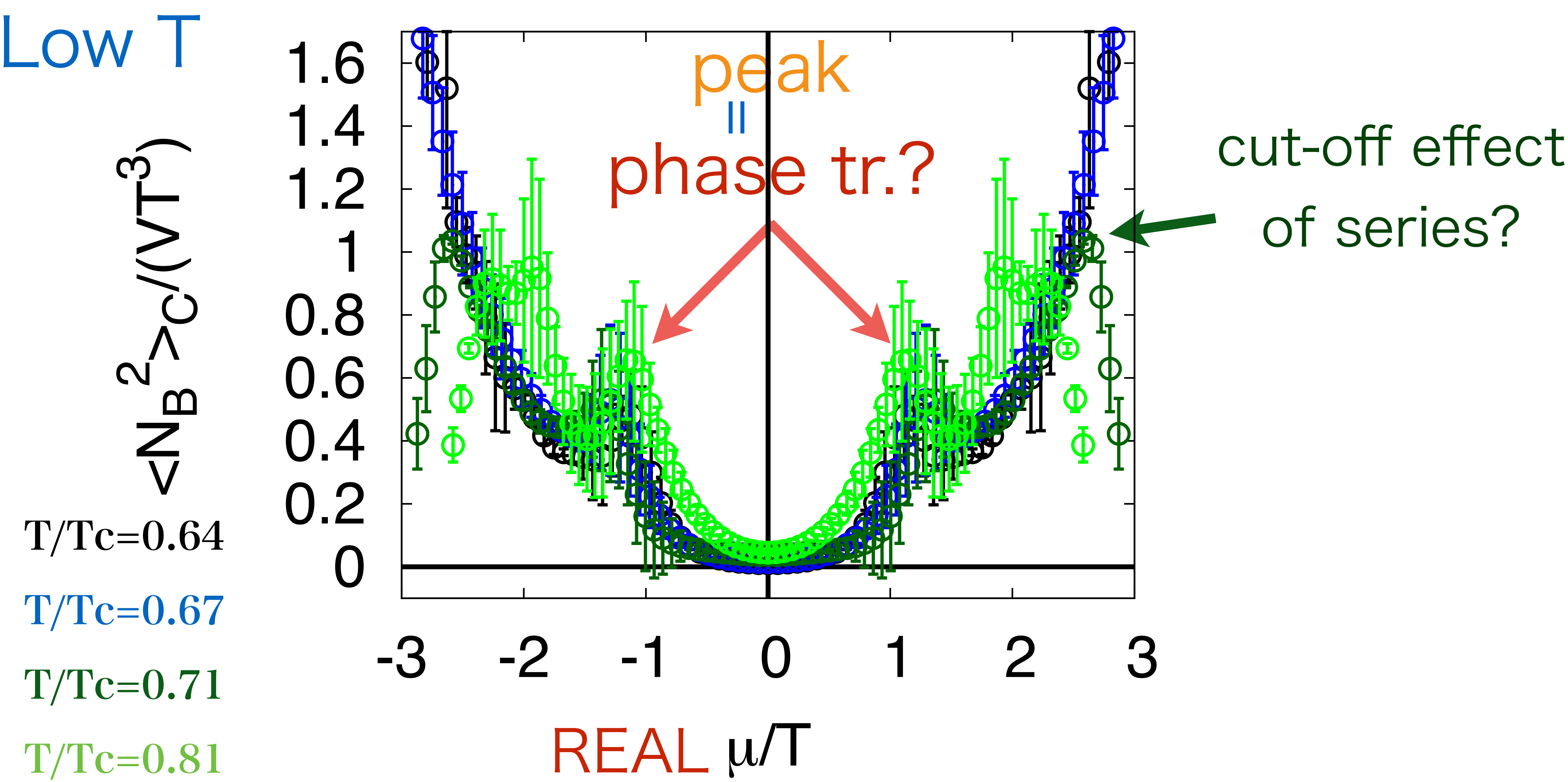
T/Tc=0.71

T/Tc=0.81

Use of Zc(n)

We have the canonical partition function Z_n

3. cumulant of quark number density $\langle N^2 \rangle_c = \left(\xi \frac{\partial}{\partial \xi} \right)^2 \log Z(\xi)$

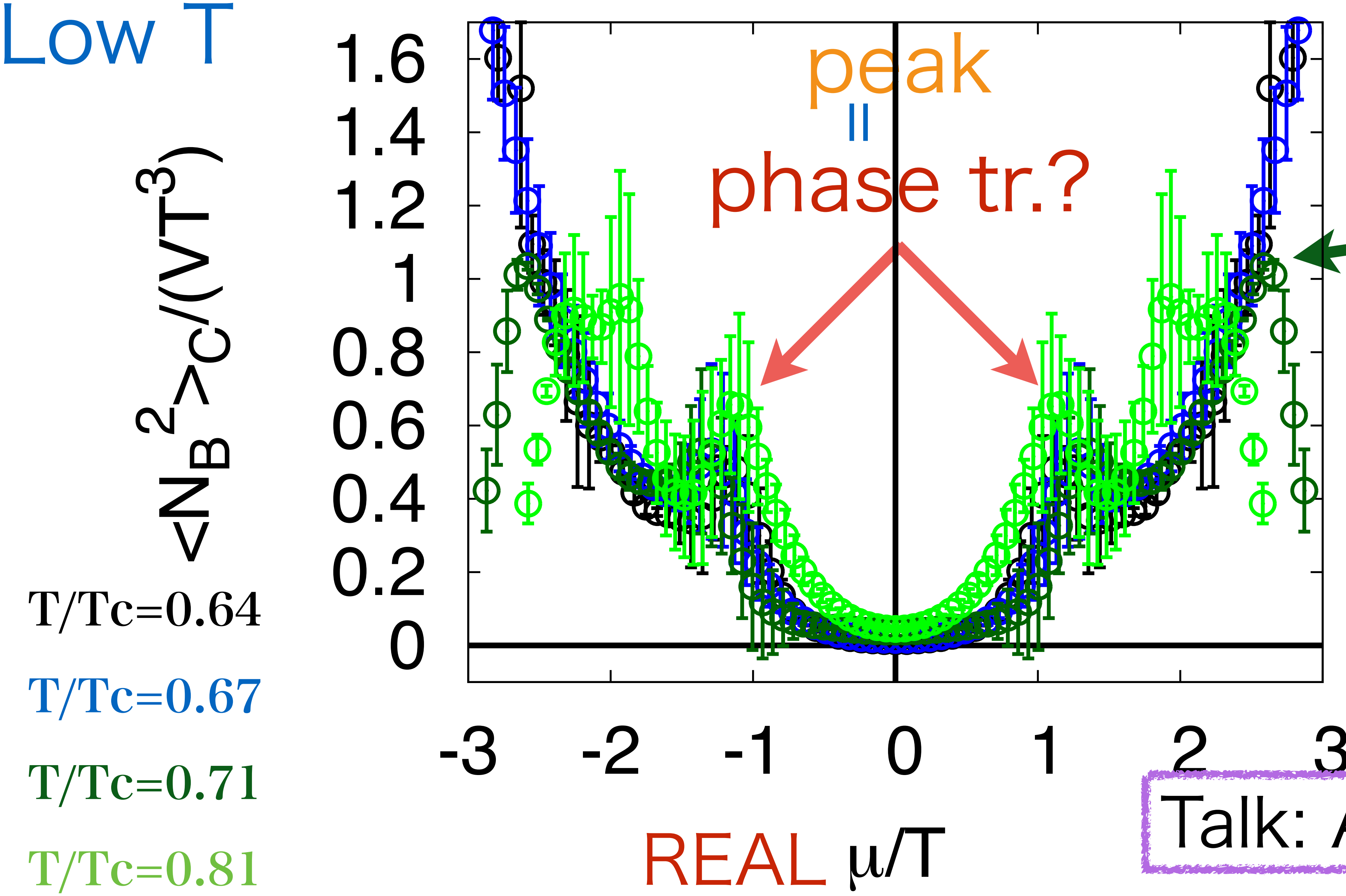


Use of Zc(n)

We have the canonical partition function Z_n

3. cumulant of quark number density $\langle N^2 \rangle_c = \left(\xi \frac{\partial}{\partial \xi} \right)^2 \log Z(\xi)$

Low T



cut-off effect of series?

Talk: A.Suzuki (9:30)

Use of Zc(n)

We have the canonical partition function Z_n

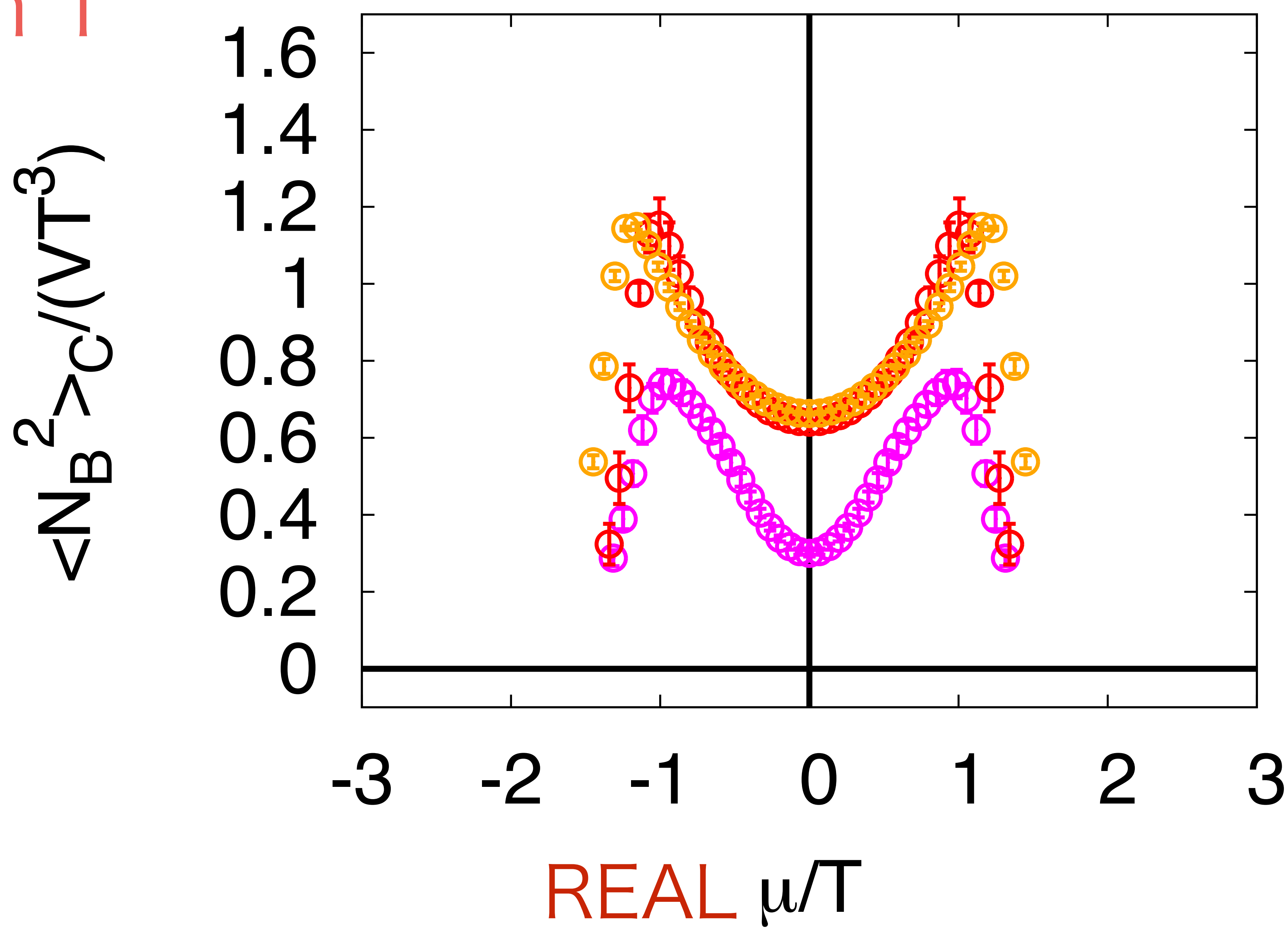
3. cumulant of quark number density $\langle N^2 \rangle_c = \left(\xi \frac{\partial}{\partial \xi} \right)^2 \log Z(\xi)$

Use of Zc(n)

We have the canonical partition function Z_n

3. cumulant of quark number density $$\langle N^2 \rangle_c = \left(\xi \frac{\partial}{\partial \xi} \right)^2 \log Z(\xi)$$

High T

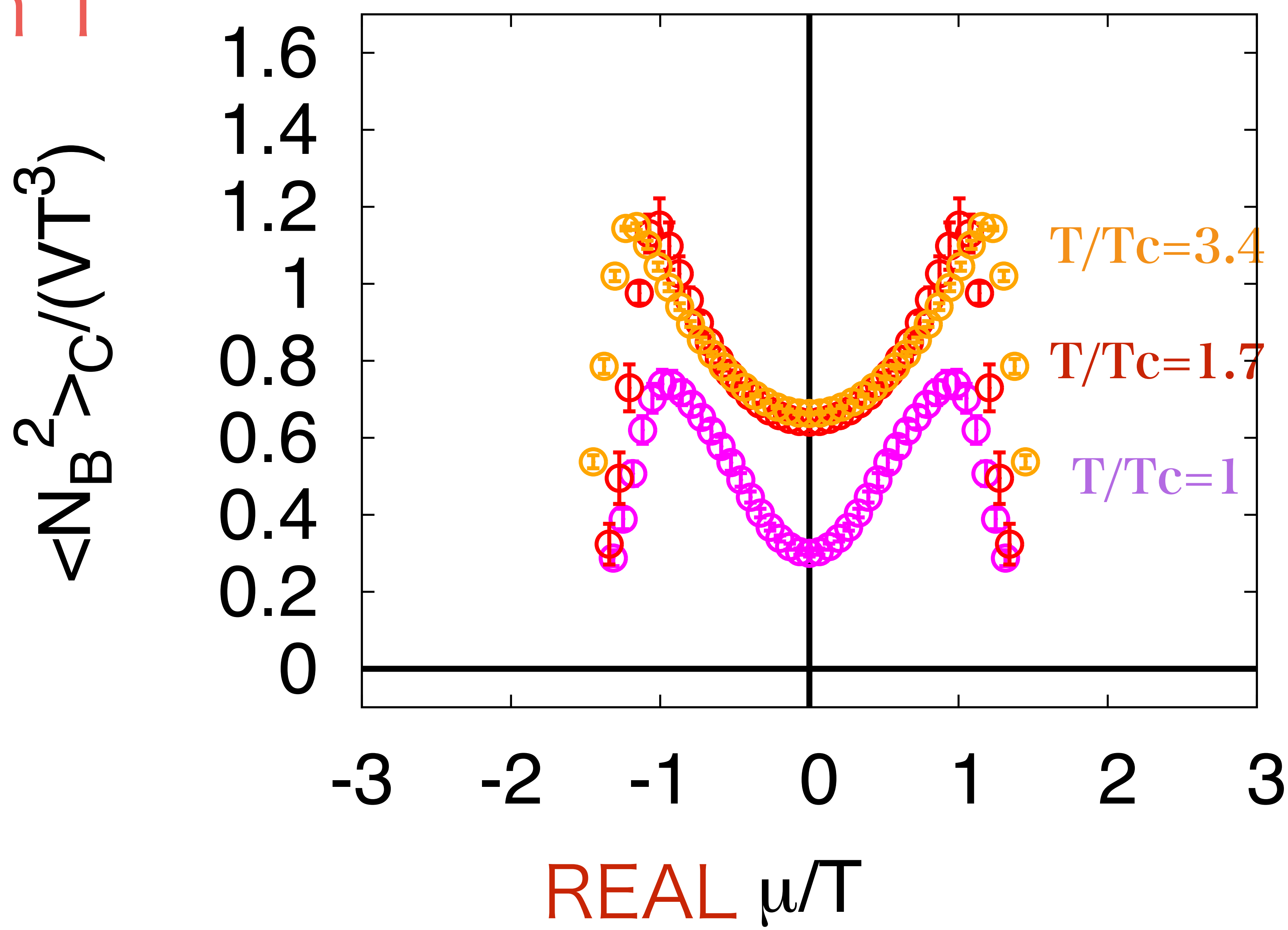


Use of Zc(n)

We have the canonical partition function Z_n

3. cumulant of quark number density $\langle N^2 \rangle_c = \left(\xi \frac{\partial}{\partial \xi} \right)^2 \log Z(\xi)$

High T

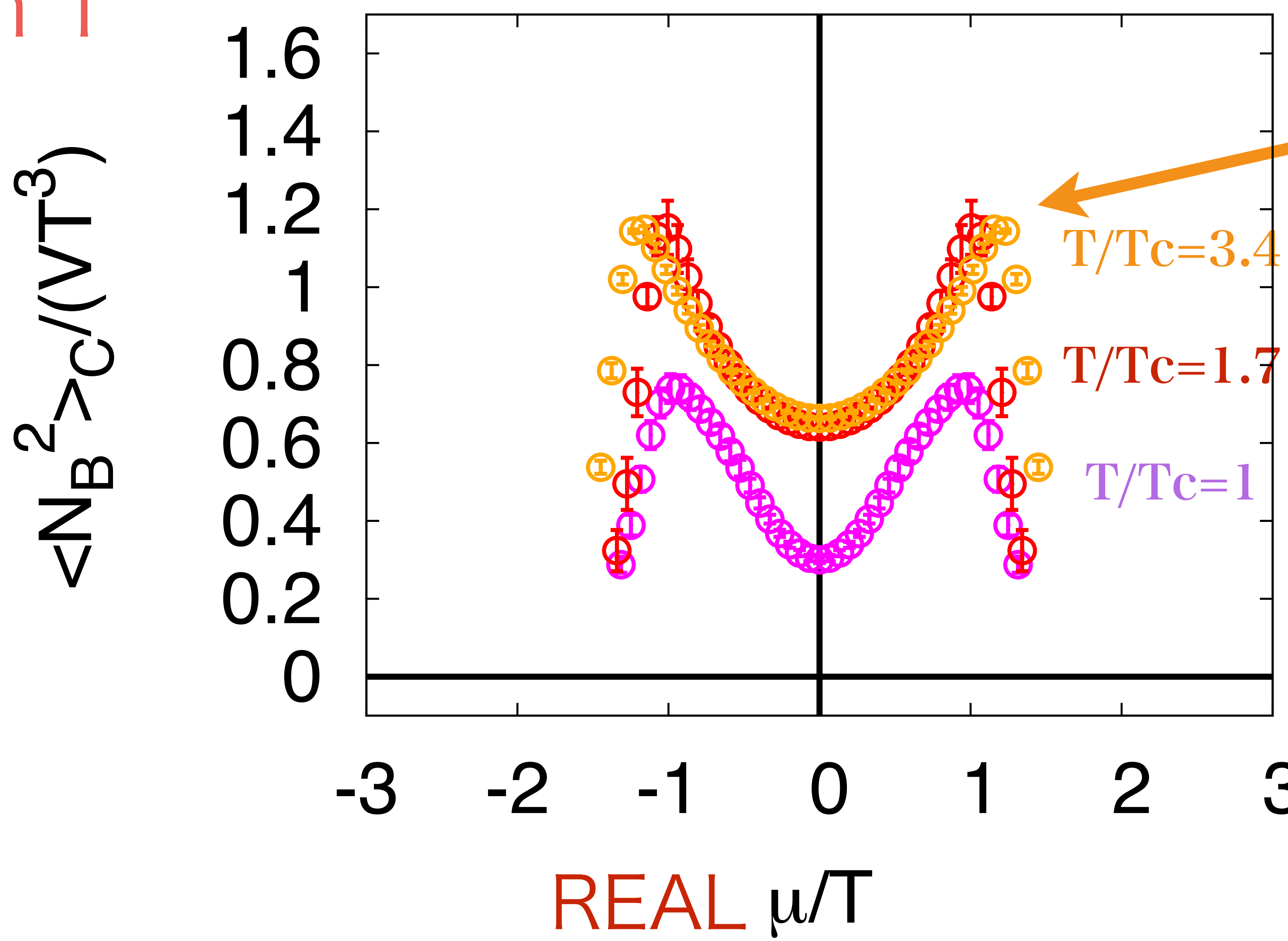


Use of Zc(n)

We have the canonical partition function Z_n

3. cumulant of quark number density $\langle N^2 \rangle_c = \left(\xi \frac{\partial}{\partial \xi} \right)^2 \log Z(\xi)$

High T



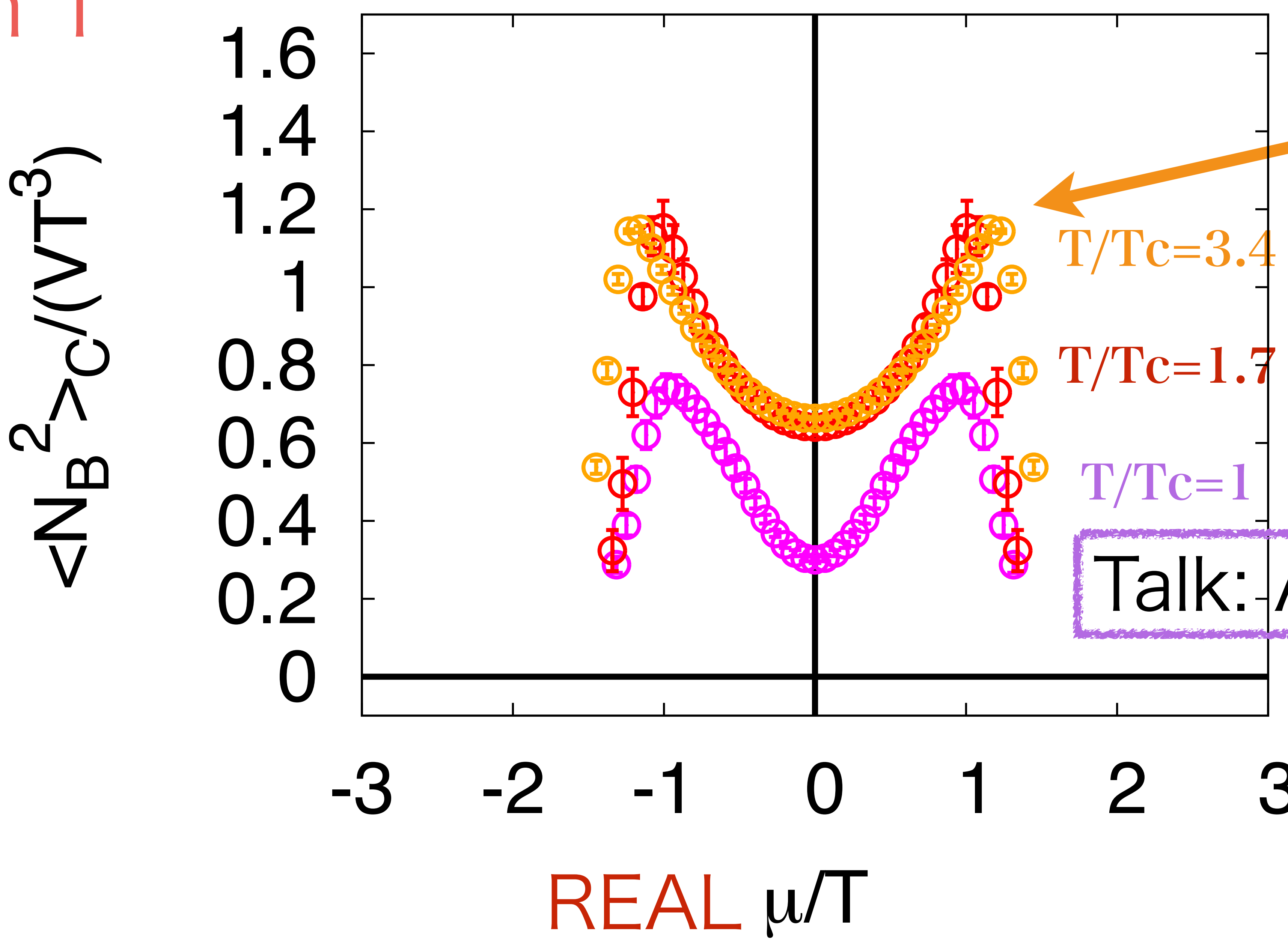
cut-off effect of series?

Use of Zc(n)

We have the canonical partition function Z_n

3. cumulant of quark number density $\langle N^2 \rangle_c = \left(\xi \frac{\partial}{\partial \xi} \right)^2 \log Z(\xi)$

High T



cut-off effect of series?

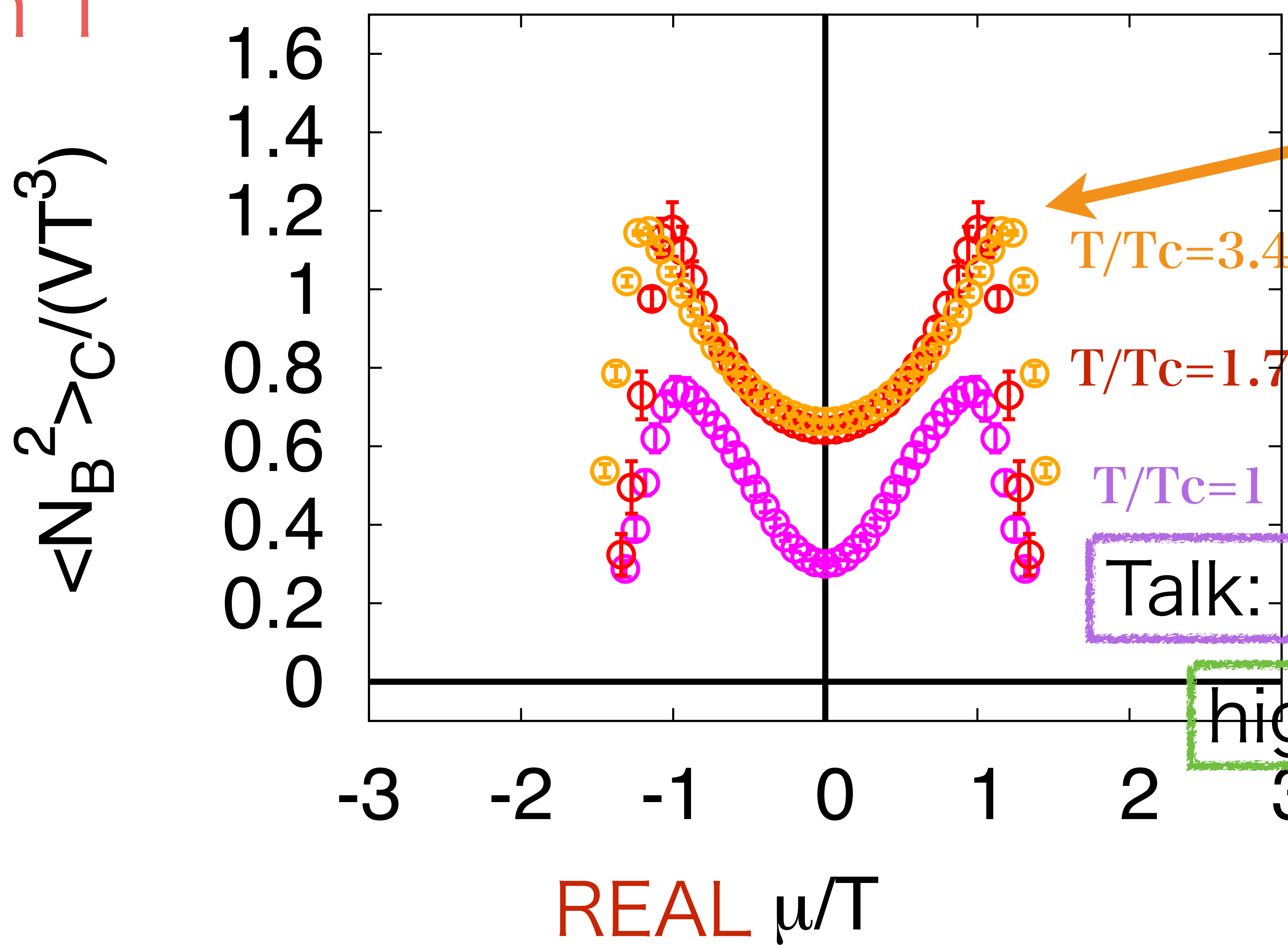
Talk: A.Suzuki (9:30)

Use of Zc(n)

We have the canonical partition function Z_n

3. cumulant of quark number density $\langle N^2 \rangle_c = \left(\xi \frac{\partial}{\partial \xi} \right)^2 \log Z(\xi)$

High T



cut-off effect of series?

Talk: A.Suzuki (9:30)

higher cumulants

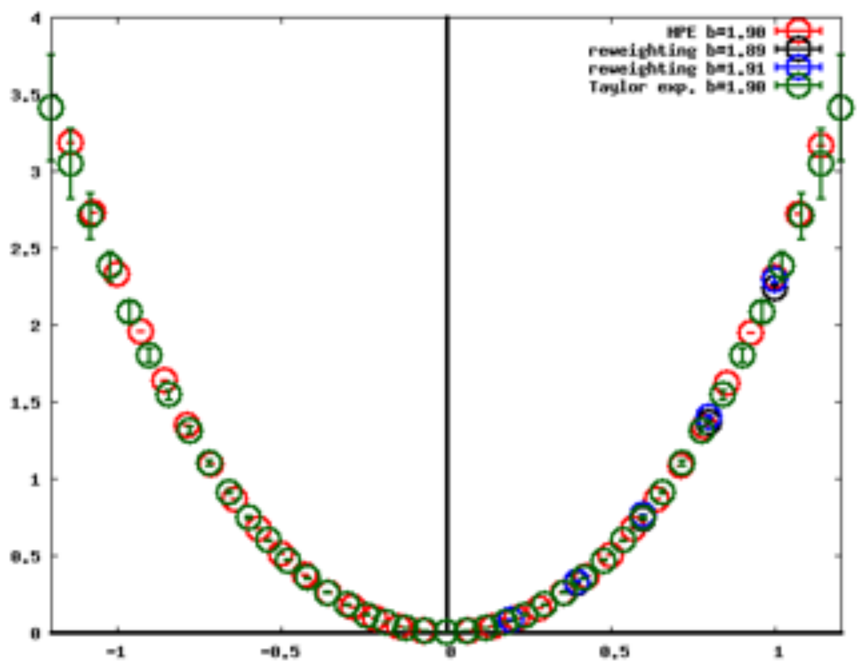
Comparison with other methods

canonical approach ⟷ re-weighting
Taylor expansion

OK with reduction formula (Nagata-Nakamura 2012)

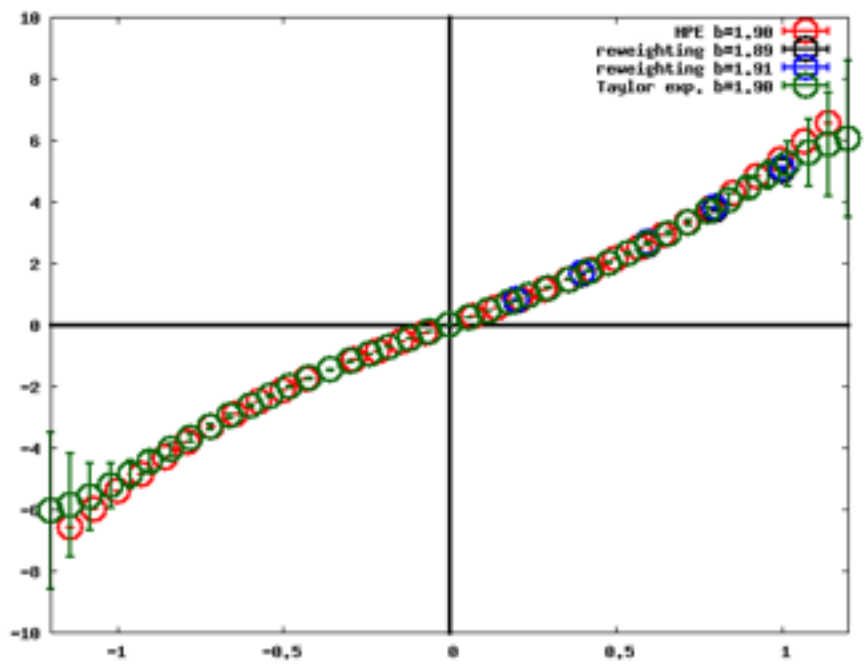
Validity of hopping parameter expansion: R.Fukuda

pressure/T^4



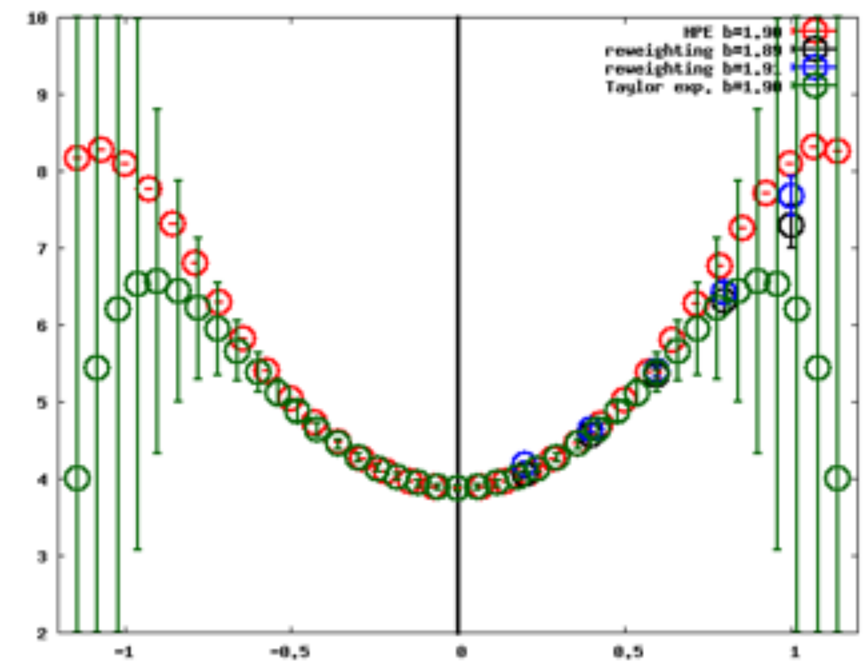
μ_B/T

quark number



μ_B/T

susceptibility

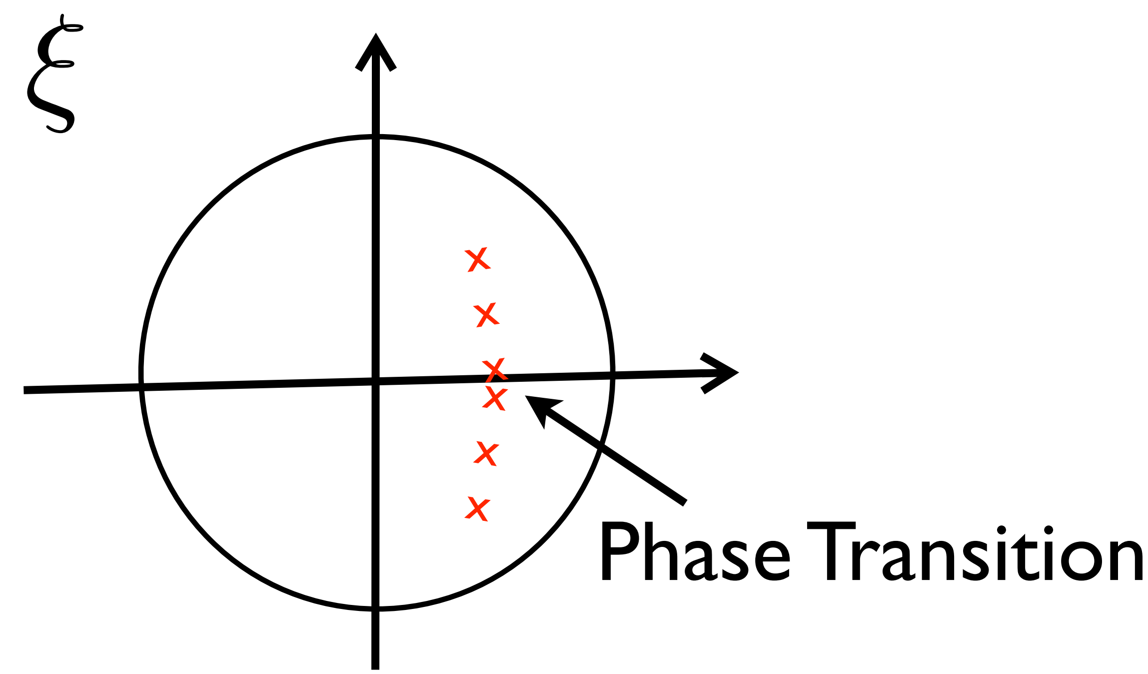


μ_B/T

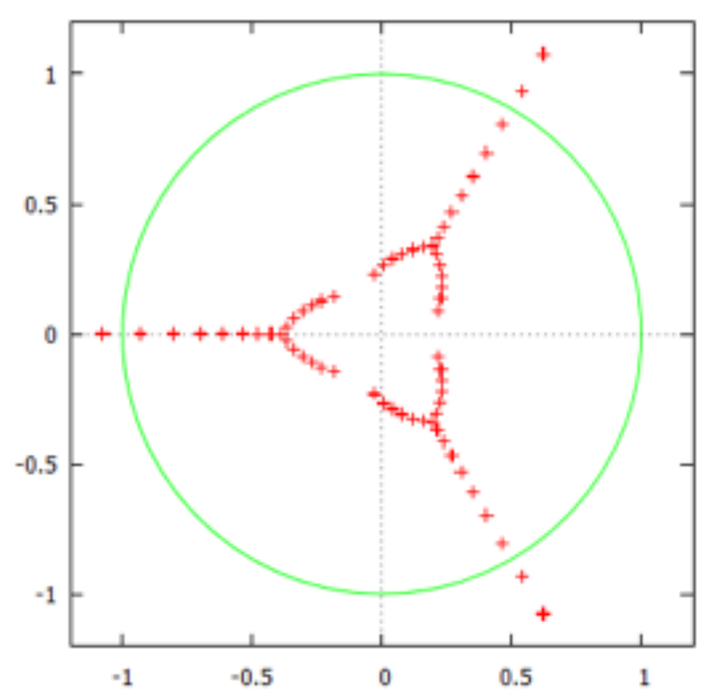
Use of Zc(n)

We have the analytic form of $Z(\mu) = \sum_{n=-\infty}^{\infty} Z_n \xi^n$

3. Zeros of $Z(\xi)$ in complex ξ plane → **Lee-Yang Zeros**



$\beta = 1.9$: T>Tc



S.Oka : next talk!

Plan of the talk

✔ 1. Introduction

✔ 2. Hopping parameter expansion

✔ 3. Numerical setup

✔ 4. Canonical partition function Zn

5. Hadronic observables

6. Conclusion

Hadronic observables

Fugacity expansion of EV of GC observables

$$\langle\hat{O}\rangle_G(\beta,\mu,V)=\frac{\mathrm{Tr}\left[\hat{O}\exp\left(-\beta\left(\hat{H}-\mu\hat{N}\right)\right)\right]}{\mathrm{Tr}\left[\exp\left(-\beta\left(\hat{H}-\mu\hat{N}\right)\right)\right]}$$

Hadronic observables

Fugacity expansion of EV of GC observables

$$\langle\hat{O}\rangle_G(\beta,\mu,V)=\frac{\mathrm{Tr}\left[\hat{O}\exp\left(-\beta\left(\hat{H}-\mu\hat{N}\right)\right)\right]}{\mathrm{Tr}\left[\exp\left(-\beta\left(\hat{H}-\mu\hat{N}\right)\right)\right]}$$

$$\mathrm{Numerator}=\sum_{n=-\infty}^{\infty}\sum_{E}\left\langle E,n\left|\hat{O}e^{-\beta\hat{H}}\right|E,n\right\rangle\xi^n$$

Hadronic observables

Fugacity expansion of EV of GC observables

$$\langle\hat{O}\rangle_G(\beta,\mu,V) = \frac{\mathrm{Tr}\left[\hat{O}\exp\left(-\beta\left(\hat{H}-\mu\hat{N}\right)\right)\right]}{\mathrm{Tr}\left[\exp\left(-\beta\left(\hat{H}-\mu\hat{N}\right)\right)\right]}$$

$$\mathrm{Numerator} = \sum_{n=-\infty}^{\infty}\sum_{E}\left\langle E,n\left|\hat{O}e^{-\beta\hat{H}}\right|E,n\right\rangle\xi^n \equiv \sum_{n=-\infty}^{\infty} O_n\xi^n$$

Hadronic observables

Fugacity expansion of EV of GC observables

$$\langle\hat{O}\rangle_G(\beta,\mu,V) = \frac{\mathrm{Tr}\left[\hat{O}\exp\left(-\beta\left(\hat{H}-\mu\hat{N}\right)\right)\right]}{\mathrm{Tr}\left[\exp\left(-\beta\left(\hat{H}-\mu\hat{N}\right)\right)\right]}$$

$$\text{Numerator} = \sum_{n=-\infty}^{\infty}\sum_{E}\left\langle E,n\left|\hat{O}e^{-\beta\hat{H}}\right|E,n\right\rangle\xi^n \equiv \sum_{n=-\infty}^{\infty} O_n\xi^n$$

$$O_n = \oint\frac{d\xi}{2\pi i}\xi^{-n-1}\left\langle O(D_W(\xi))\frac{\mathrm{Det}\,D_W(\xi)}{\mathrm{Det}\,D_W(\mu_0)}\right\rangle_0 Z_G(\mu_0)$$

Hadronic observables

Fugacity expansion of EV of GC observables

$$\langle \hat{O} \rangle_G(\beta, \mu, V) = \frac{\mathrm{Tr}\left[\hat{O}\exp\left(-\beta\left(\hat{H}-\mu\hat{N}\right)\right)\right]}{\mathrm{Tr}\left[\exp\left(-\beta\left(\hat{H}-\mu\hat{N}\right)\right)\right]}$$

$$\text{Numerator} = \sum_{n=-\infty}^{\infty}\sum_{E}\left\langle E, n\left|\hat{O}e^{-\beta\hat{H}}\right|E, n\right\rangle \xi^n \equiv \sum_{n=-\infty}^{\infty} O_n \xi^n$$

$$O_n = \oint \frac{d\xi}{2\pi i}\xi^{-n-1}\left\langle \underline{O(D_W(\xi))}\frac{\mathrm{Det}\,D_W(\xi)}{\mathrm{Det}\,D_W(\mu_0)}\right\rangle_0 Z_G(\mu_0)$$

function of ξ

Hadronic observables

Fugacity expansion of EV of GC observables

$$\langle \hat{O} \rangle_G(\beta, \mu, V) = \frac{\mathrm{Tr}\left[\hat{O} \exp\left(-\beta\left(\hat{H} - \mu \hat{N}\right)\right)\right]}{\mathrm{Tr}\left[\exp\left(-\beta\left(\hat{H} - \mu \hat{N}\right)\right)\right]}$$

$$\mathrm{Numerator} = \sum_{n=-\infty}^{\infty} \sum_{E} \left\langle E, n \left| \hat{O} e^{-\beta \hat{H}} \right| E, n \right\rangle \xi^n \equiv \sum_{n=-\infty}^{\infty} O_n \xi^n$$

$$O_n = \oint \frac{d\xi}{2\pi i} \xi^{-n-1} \left\langle O(D_W(\xi)) \frac{\mathrm{Det} D_W(\xi)}{\mathrm{Det} D_W(\mu_0)} \right\rangle_0 Z_G(\mu_0)$$

function of ξ HPE

Hadronic observables

Fugacity expansion of EV of GC observables

$$\langle \hat{O} \rangle_G(\beta, \mu, V) = \frac{\mathrm{Tr}\left[\hat{O}\exp\left(-\beta\left(\hat{H}-\mu\hat{N}\right)\right)\right]}{\mathrm{Tr}\left[\exp\left(-\beta\left(\hat{H}-\mu\hat{N}\right)\right)\right]}$$

$$\mathrm{Numerator} = \sum_{n=-\infty}^{\infty}\sum_{E}\left\langle E, n \left| \hat{O} e^{-\beta \hat{H}} \right| E, n \right\rangle \xi^n \equiv \sum_{n=-\infty}^{\infty} O_n \xi^n$$

$$O_n = \oint \frac{d\xi}{2\pi i}\xi^{-n-1}\left\langle O(D_W(\xi))\frac{\mathrm{Det}D_W(\xi)}{\mathrm{Det}D_W(\mu_0)}\right\rangle_0 Z_G(\mu_0)$$

function of ξ — HPE

$$\bar{\psi}\psi = -\mathrm{tr}\left(\frac{1}{D_W}\right) = -\mathrm{tr}\left(\frac{1}{1-\kappa Q}\right) = \sum_{m=0}^{\infty}\kappa^m \mathrm{tr} Q^m$$

Hadronic observables

$$O_n = \sum_E \left\langle E, n \left| \hat{O} e^{-\beta \hat{H}} \right| E, n \right\rangle$$

$$Z_n = \sum_E \left\langle E, n \left| e^{-\beta \hat{H}} \right| E, n \right\rangle$$

Hadronic observables

$$O_n = \sum_E \left\langle E, n \left| \hat{O} e^{-\beta \hat{H}} \right| E, n \right\rangle$$

$$Z_n = \sum_E \left\langle E, n \left| e^{-\beta \hat{H}} \right| E, n \right\rangle$$

VEV in canonical ensamble

Hadronic observables

$$O_n = \sum_E \left\langle E, n \left| \hat{O} e^{-\beta \hat{H}} \right| E, n \right\rangle$$

$$Z_n = \sum_E \left\langle E, n \left| e^{-\beta \hat{H}} \right| E, n \right\rangle$$

VEV in canonical ensamble

$$\langle \hat{O} \rangle_C(\beta, n, V) = \frac{O_n}{Z_n}$$

Hadronic observables

$$O_n = \sum_E \left\langle E, n \left| \hat{O} e^{-\beta \hat{H}} \right| E, n \right\rangle$$

$$Z_n = \sum_E \left\langle E, n \left| e^{-\beta \hat{H}} \right| E, n \right\rangle$$

VEV in canonical ensamble

$$\langle \hat{O} \rangle_C (\beta, n, V) = \frac{O_n}{Z_n}$$

VEV in the REAL μ!

Hadronic observables

$$O_n = \sum_E \left\langle E, n \left| \hat{O} e^{-\beta \hat{H}} \right| E, n \right\rangle$$

$$Z_n = \sum_E \left\langle E, n \left| e^{-\beta \hat{H}} \right| E, n \right\rangle$$

VEV in canonical ensamble

$$\langle \hat{O} \rangle_C (\beta, n, V) = \frac{O_n}{Z_n}$$

VEV in the REAL μ !

$$O(\mu) = \sum_{n=-\infty}^{\infty} O_n \xi^n$$

$$Z(\mu) = \sum_{n=-\infty}^{\infty} Z_n \xi^n$$

Hadronic observables

$$O_n = \sum_E \left\langle E, n \left| \hat{O} e^{-\beta \hat{H}} \right| E, n \right\rangle$$

$$Z_n = \sum_E \left\langle E, n \left| e^{-\beta \hat{H}} \right| E, n \right\rangle$$

VEV in canonical ensamble

$$\langle \hat{O} \rangle_C(\beta, n, V) = \frac{O_n}{Z_n}$$

VEV in the REAL μ!

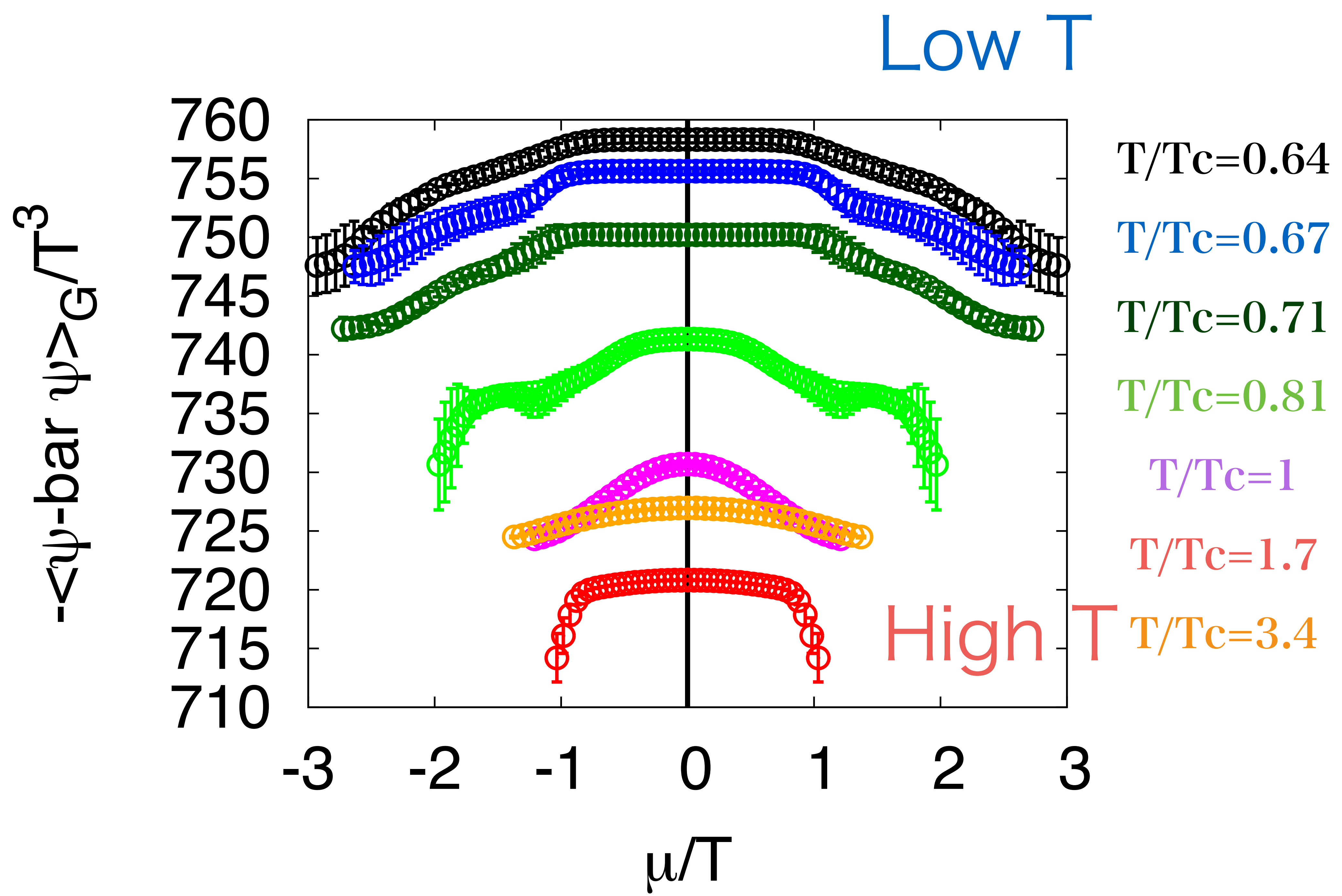
$$O(\mu) = \sum_{n=-\infty}^{\infty} O_n \xi^n$$

$$Z(\mu) = \sum_{n=-\infty}^{\infty} Z_n \xi^n$$

$$\longrightarrow \quad \langle \hat{O} \rangle_G(\beta, \mu, V) = \frac{O(\mu)}{Z(\mu)}$$

Chiral restoration?

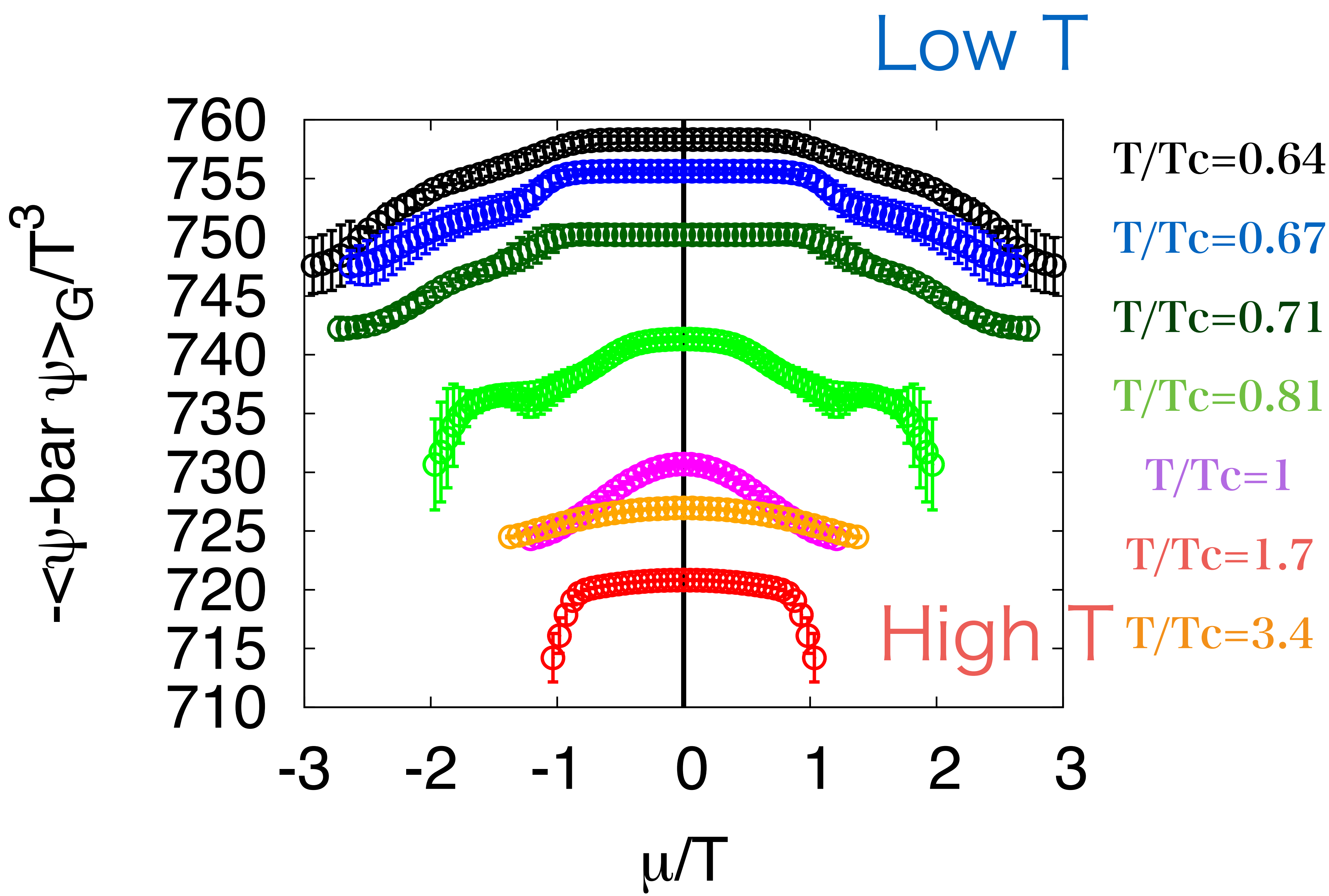
Grand canonical chiral condensate



Chiral restoration?

Grand canonical chiral condensate

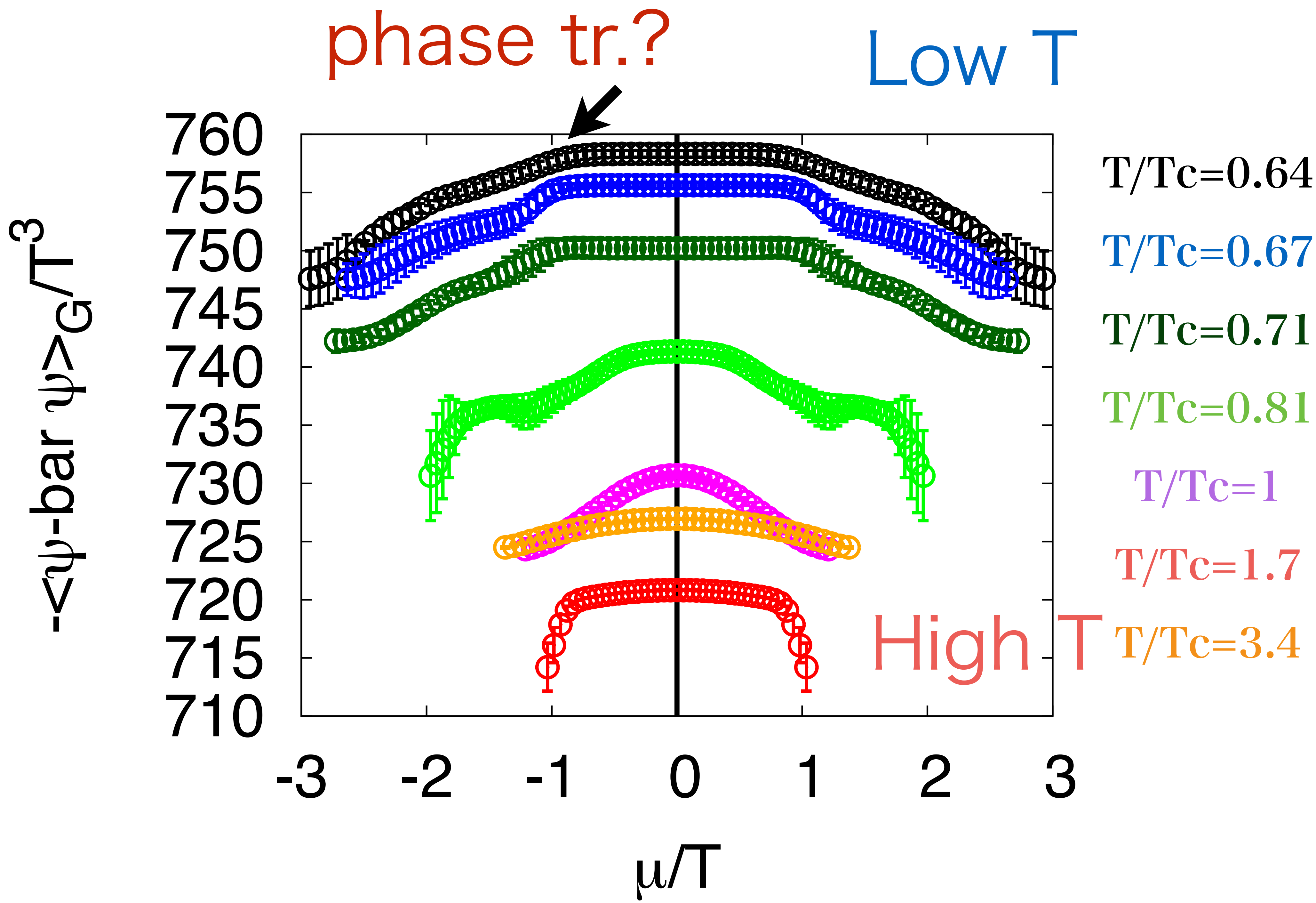
No renormalization! No subtraction! Sorry...



Chiral restoration?

Grand canonical chiral condensate

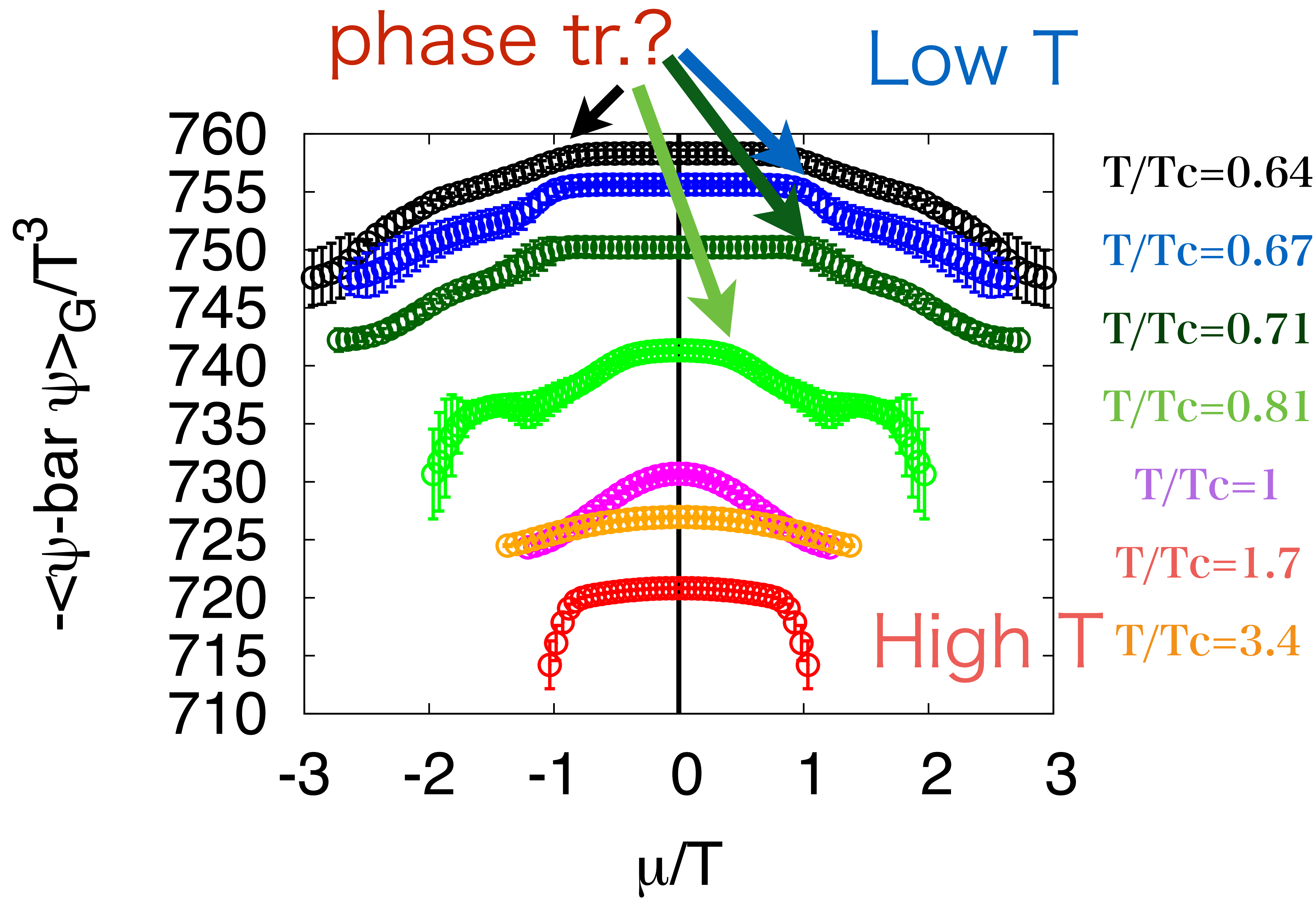
No renormalization! No subtraction! Sorry...



Chiral restoration?

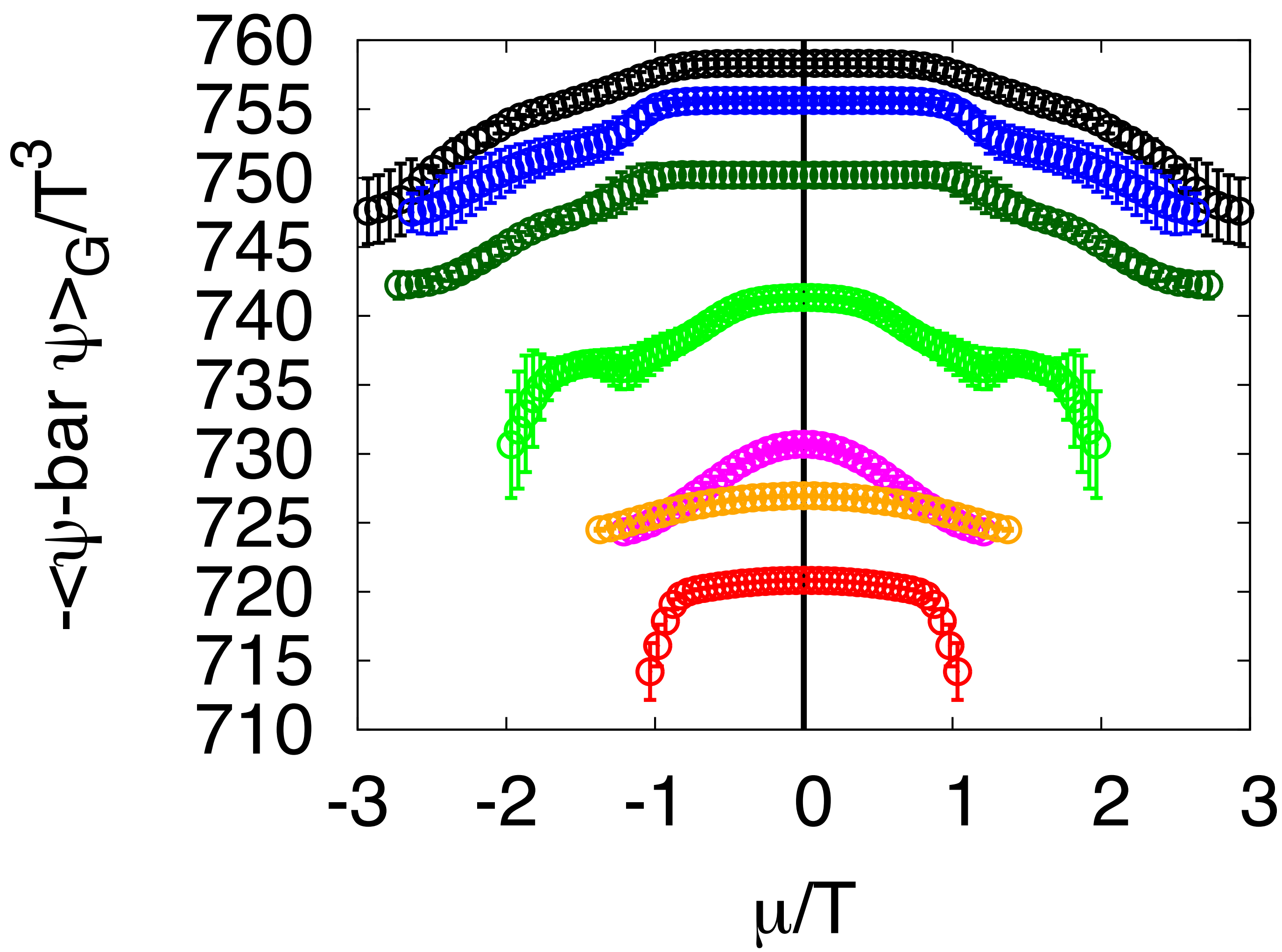
Grand canonical chiral condensate

No renormalization! No subtraction! Sorry...



Chiral restoration?

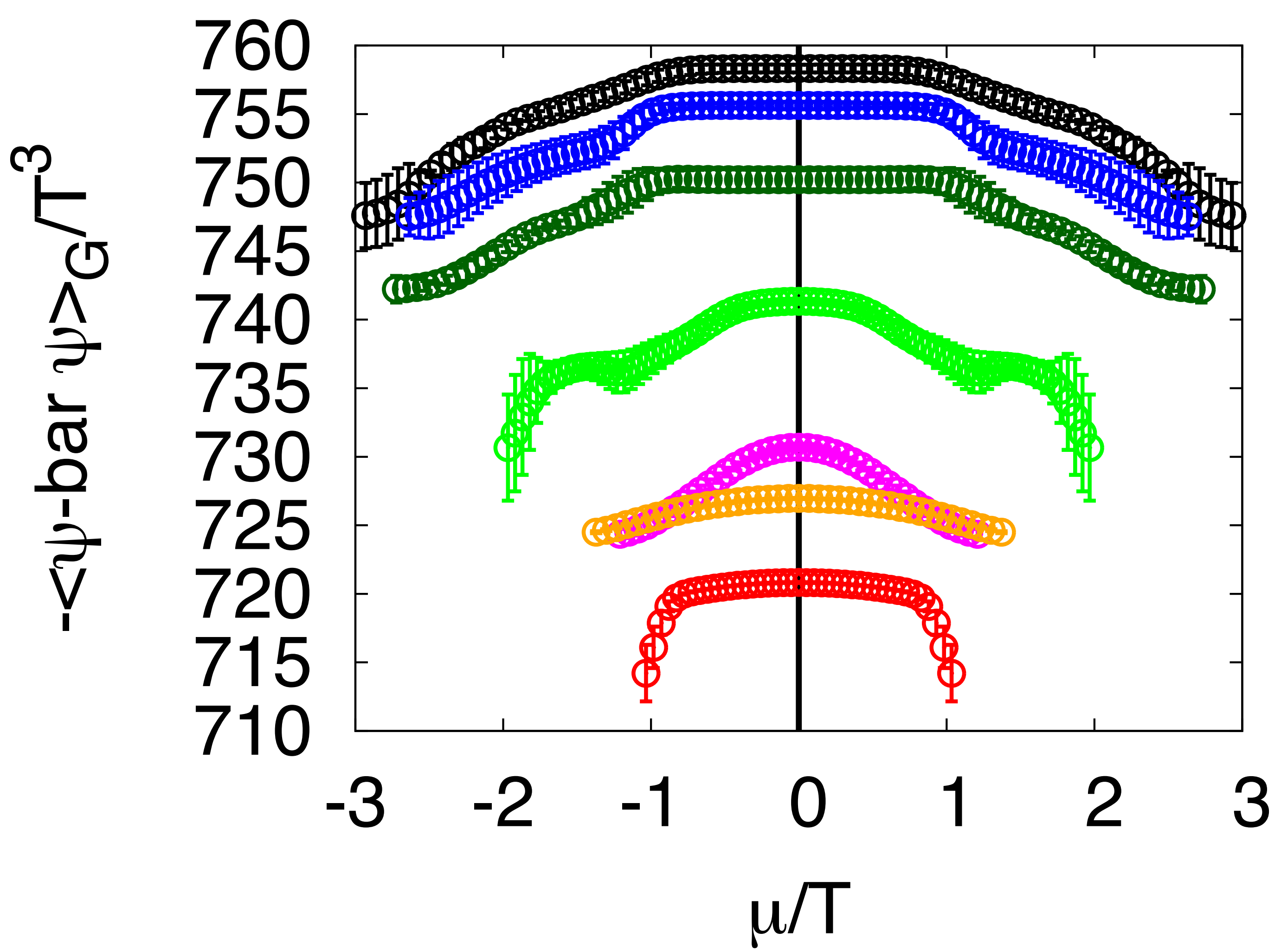
Grand canonical chiral condensate

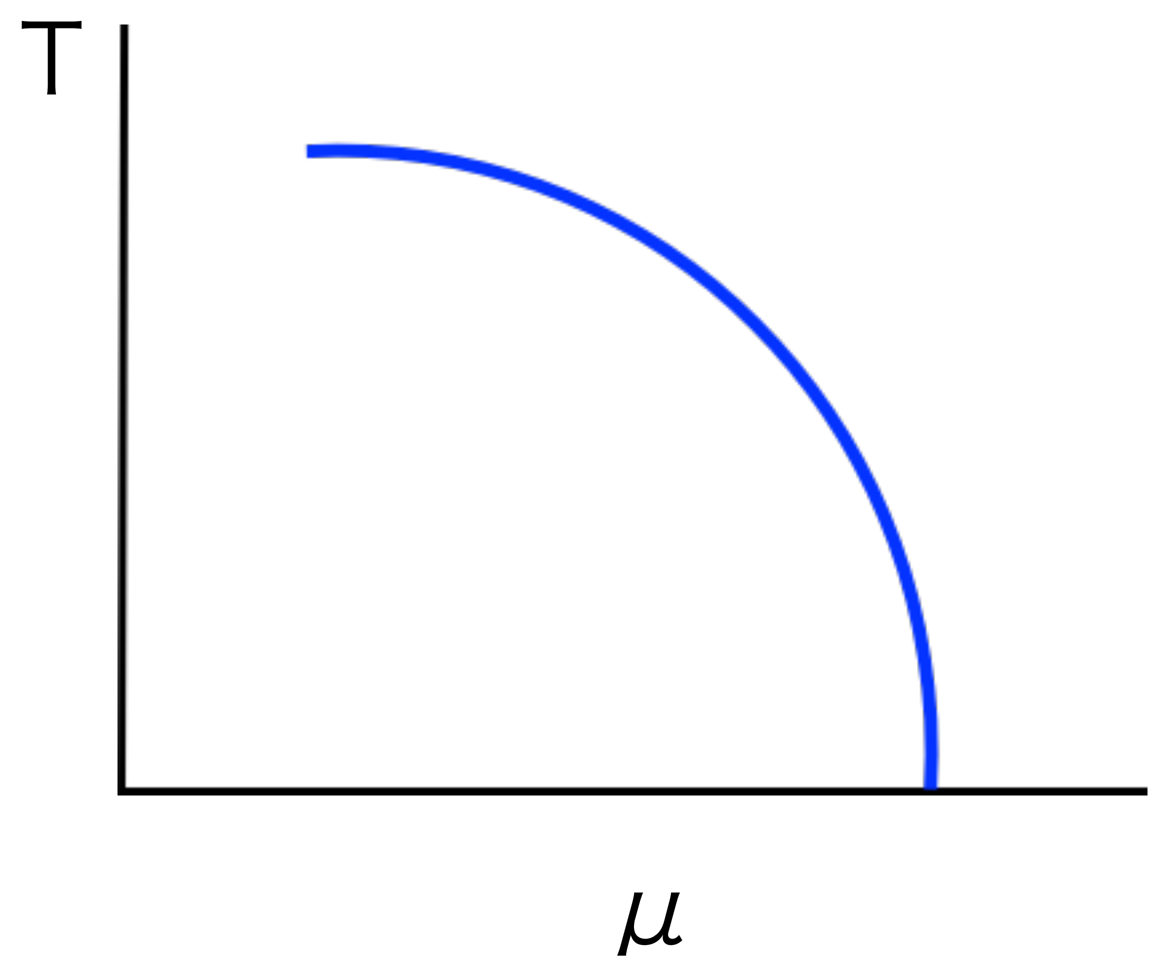


Chiral restoration?

Grand canonical chiral condensate

Correspondence with QCD phase diagram?

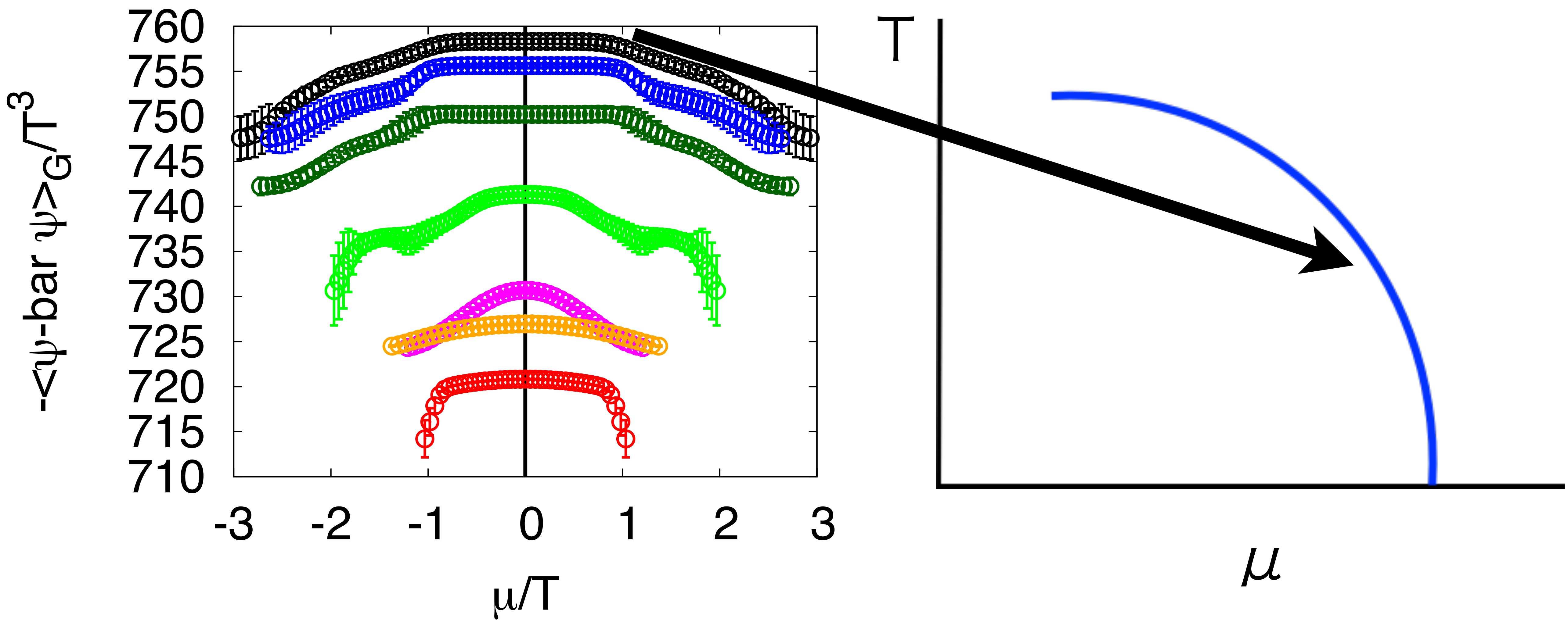




Chiral restoration?

Grand canonical chiral condensate

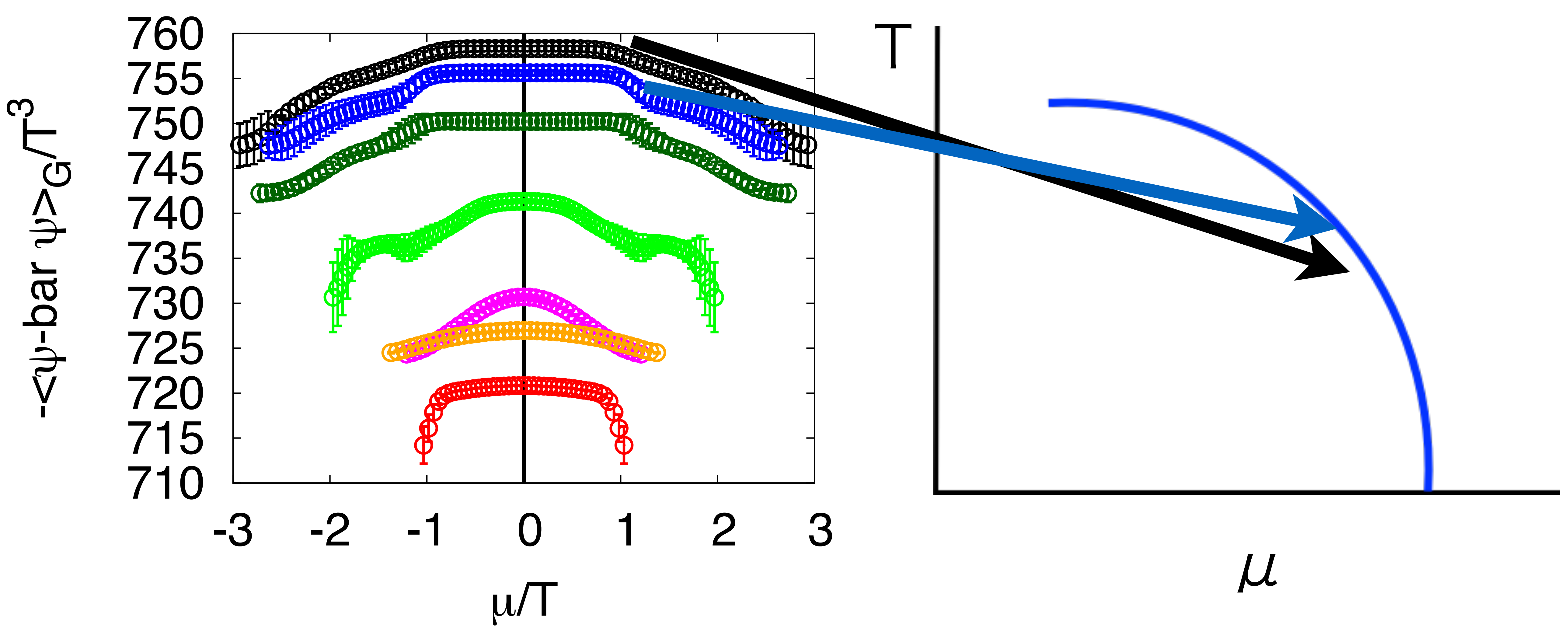
Correspondence with QCD phase diagram?



Chiral restoration?

Grand canonical chiral condensate

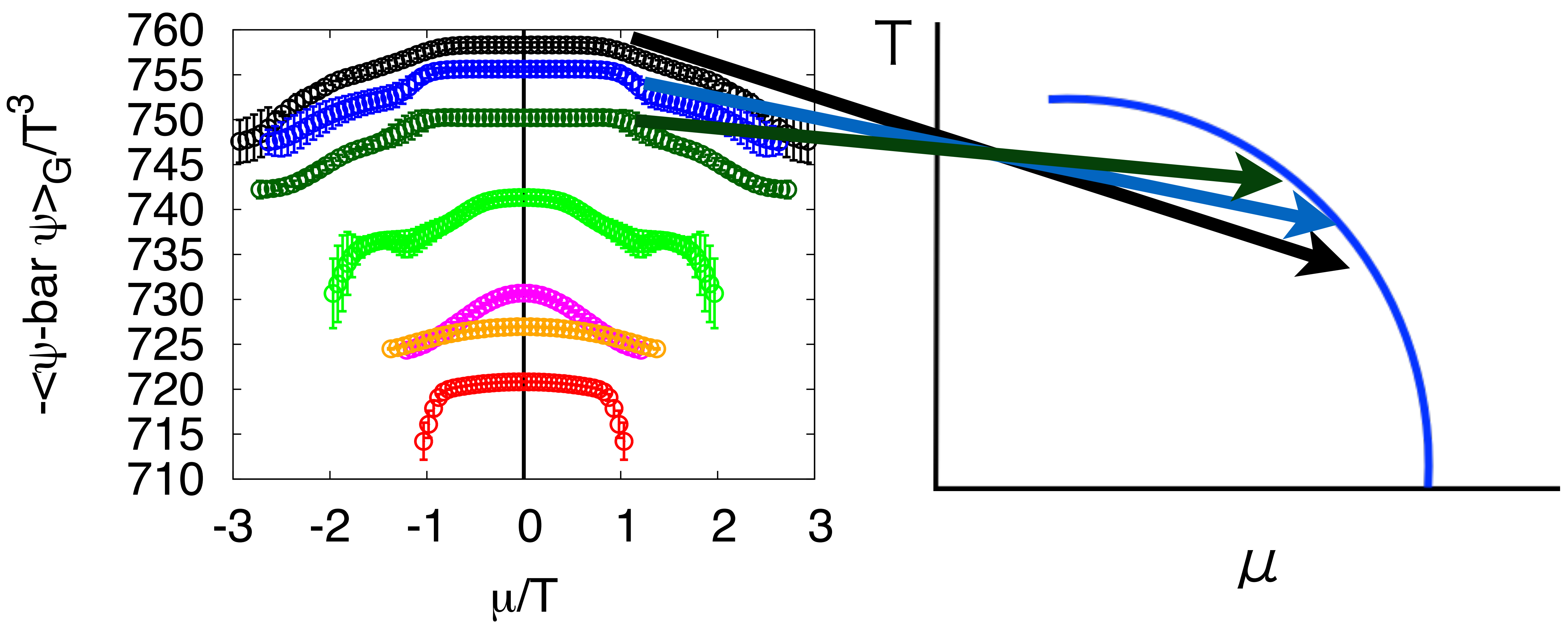
Correspondence with QCD phase diagram?



Chiral restoration?

Grand canonical chiral condensate

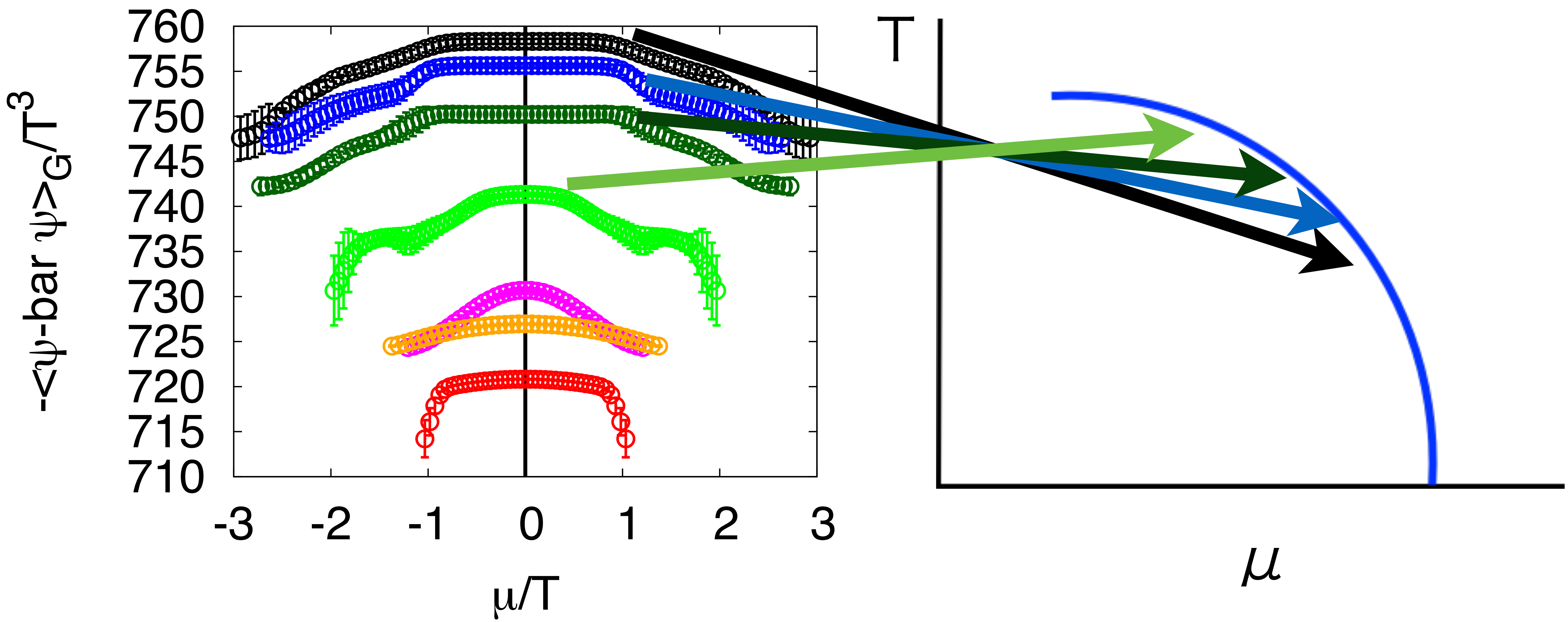
Correspondence with QCD phase diagram?



Chiral restoration?

Grand canonical chiral condensate

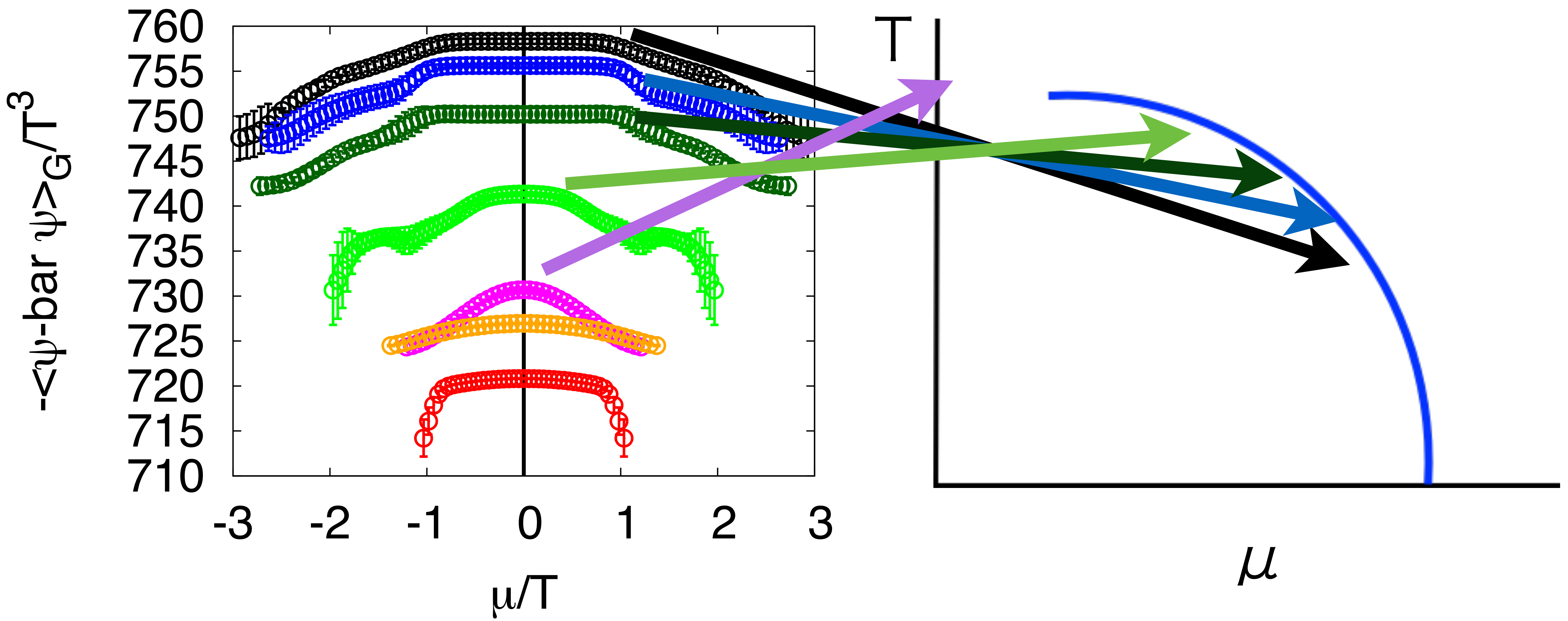
Correspondence with QCD phase diagram?



Chiral restoration?

Grand canonical chiral condensate

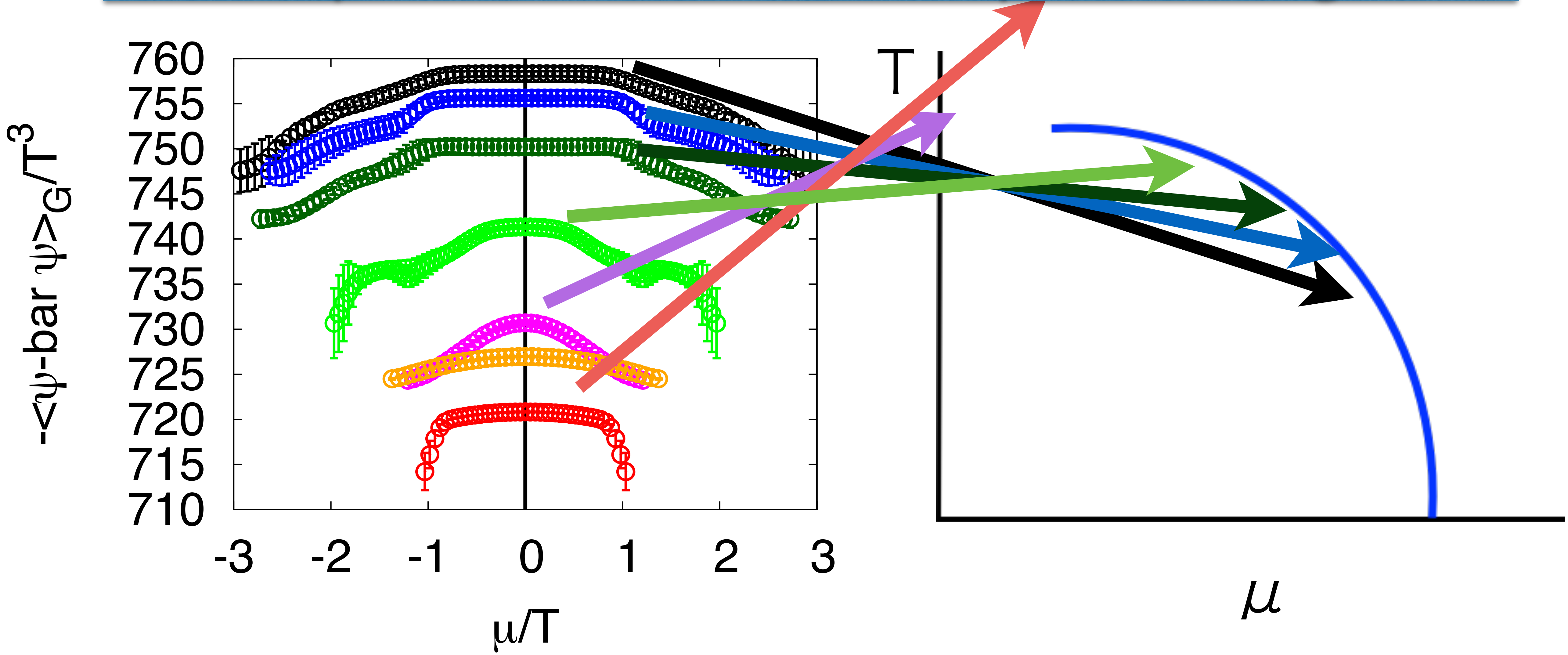
Correspondence with QCD phase diagram?



Chiral restoration?

Grand canonical chiral condensate

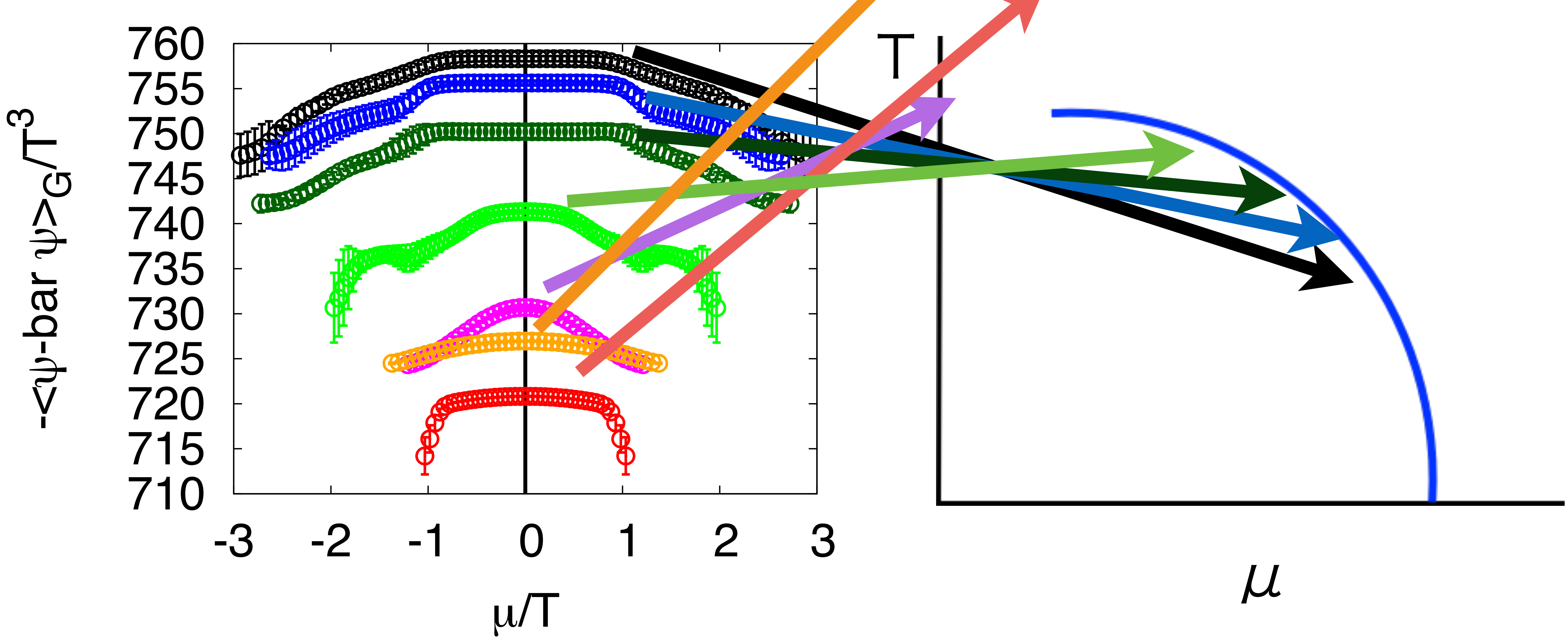
Correspondence with QCD phase diagram?



Chiral restoration?

Grand canonical chiral condensate

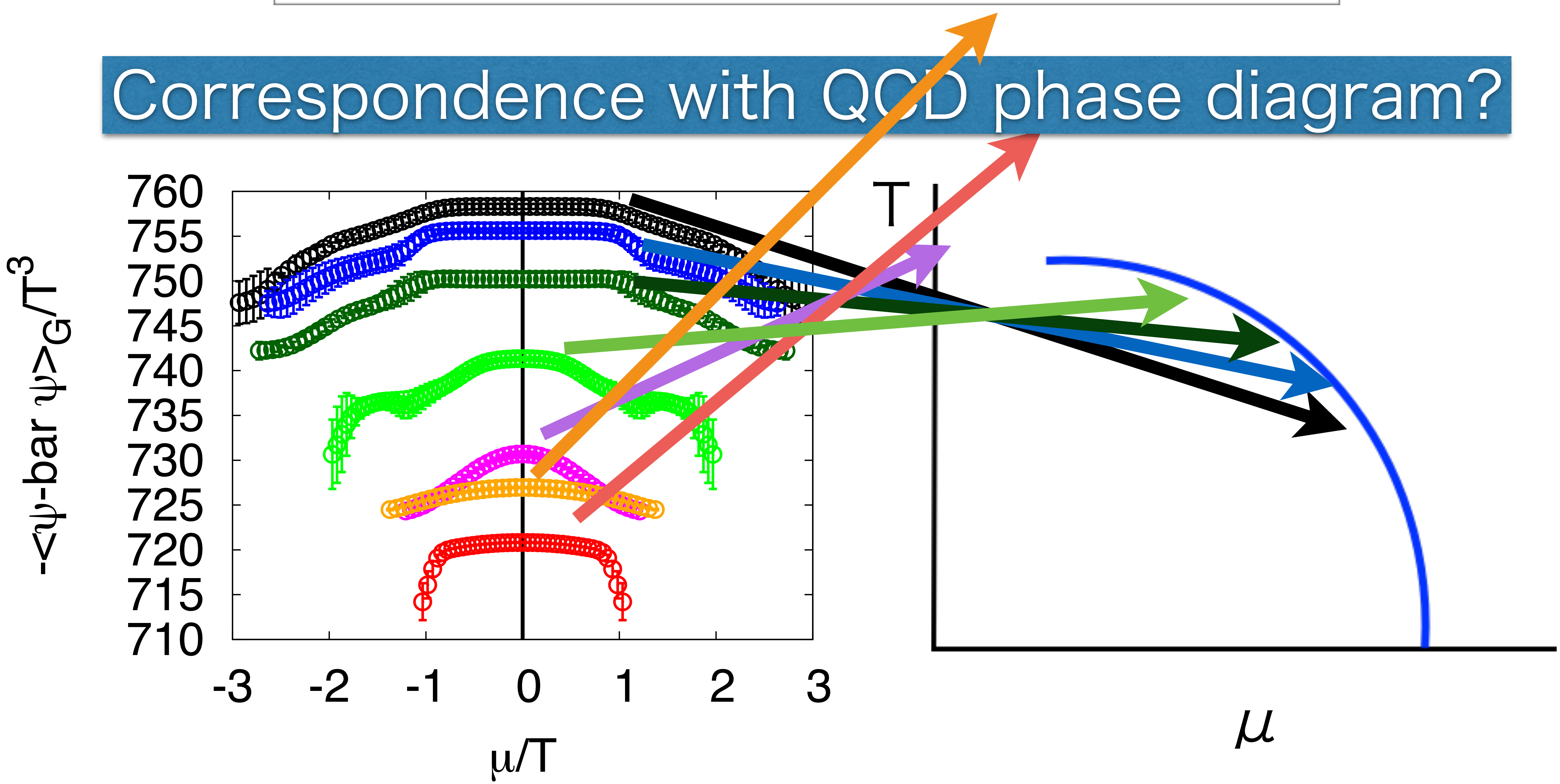
Correspondence with QCD phase diagram?



Chiral restoration?

Grand canonical chiral condensate

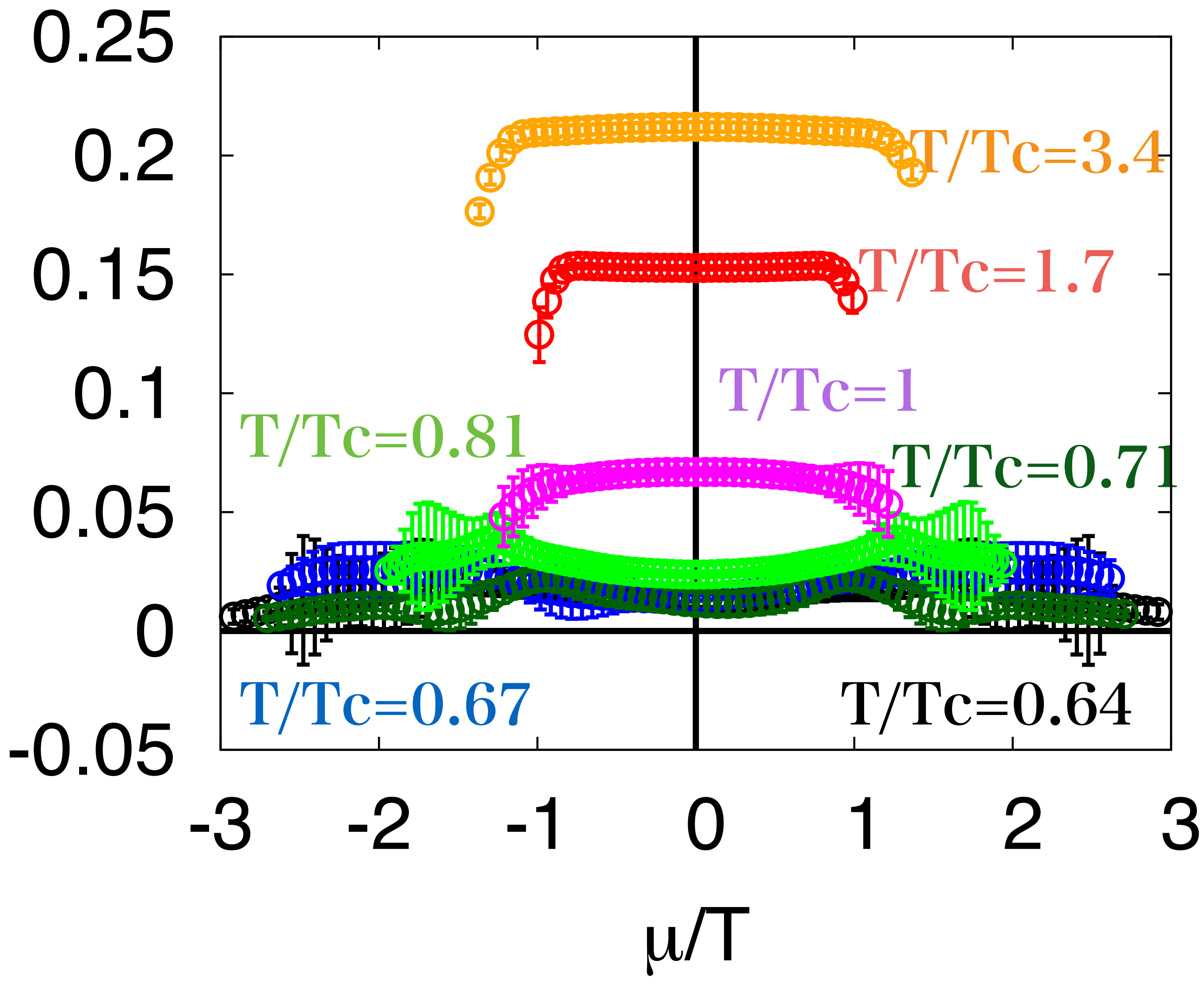
Correspondence with QCD phase diagram?



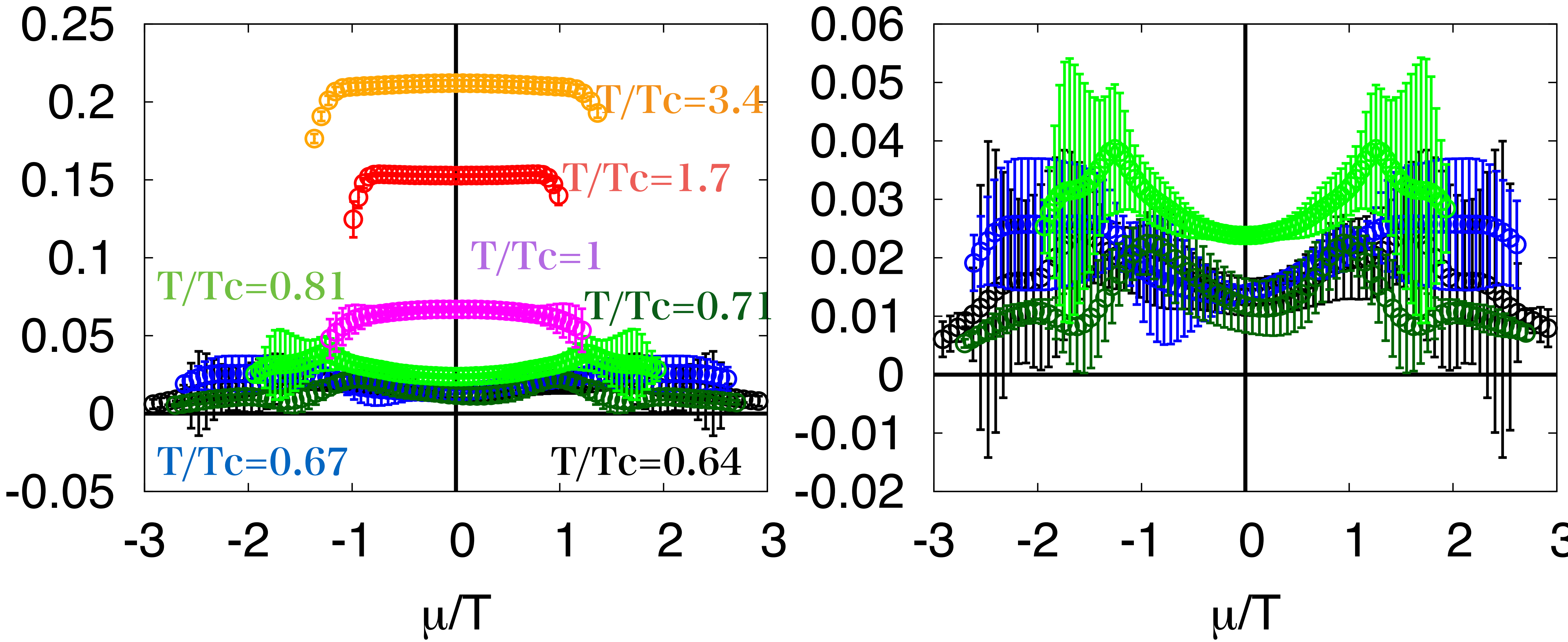
NOTE:

- μ/T vs μ
- mass is not the same.

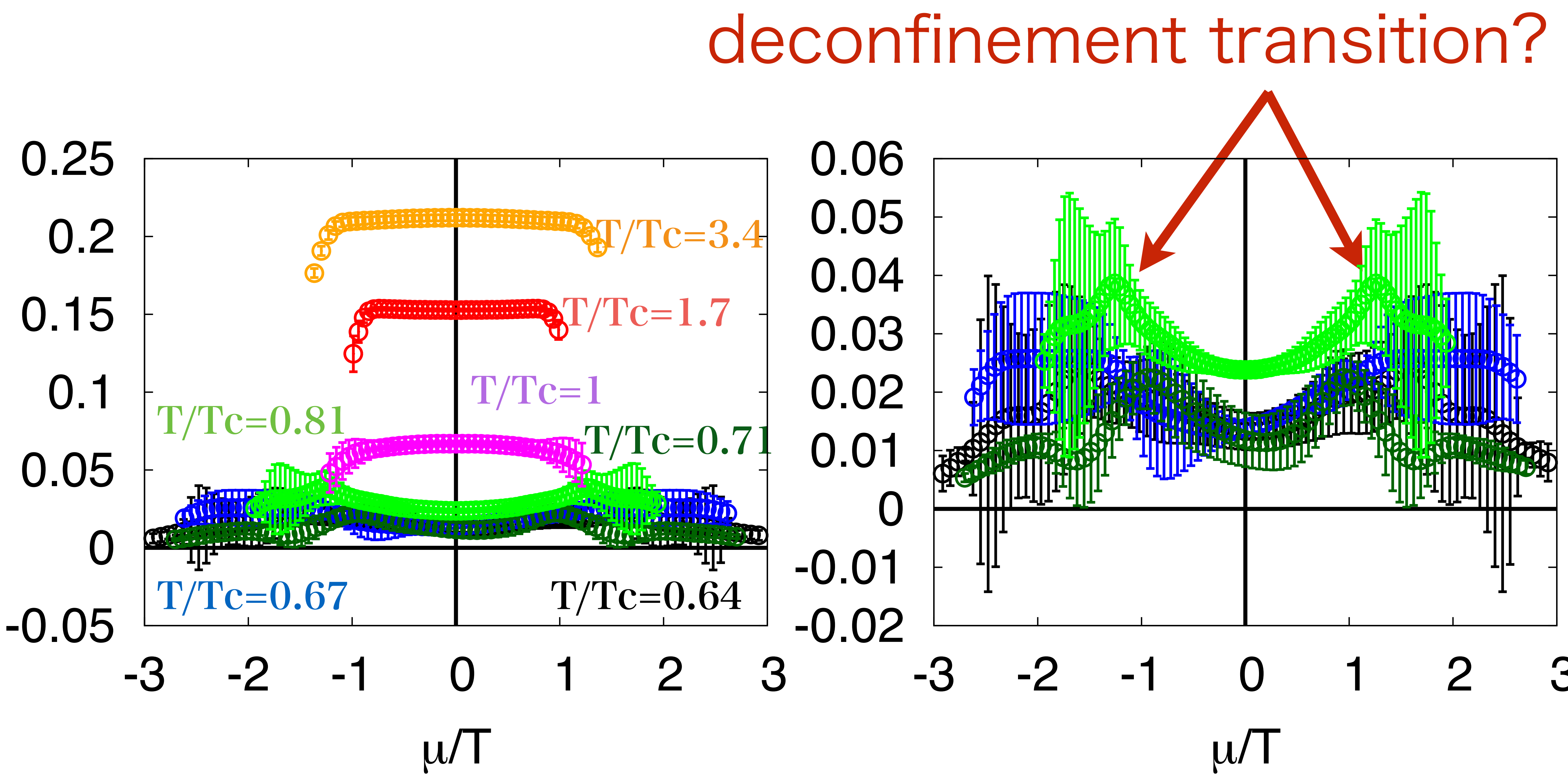
Polyakov loop



Polyakov loop



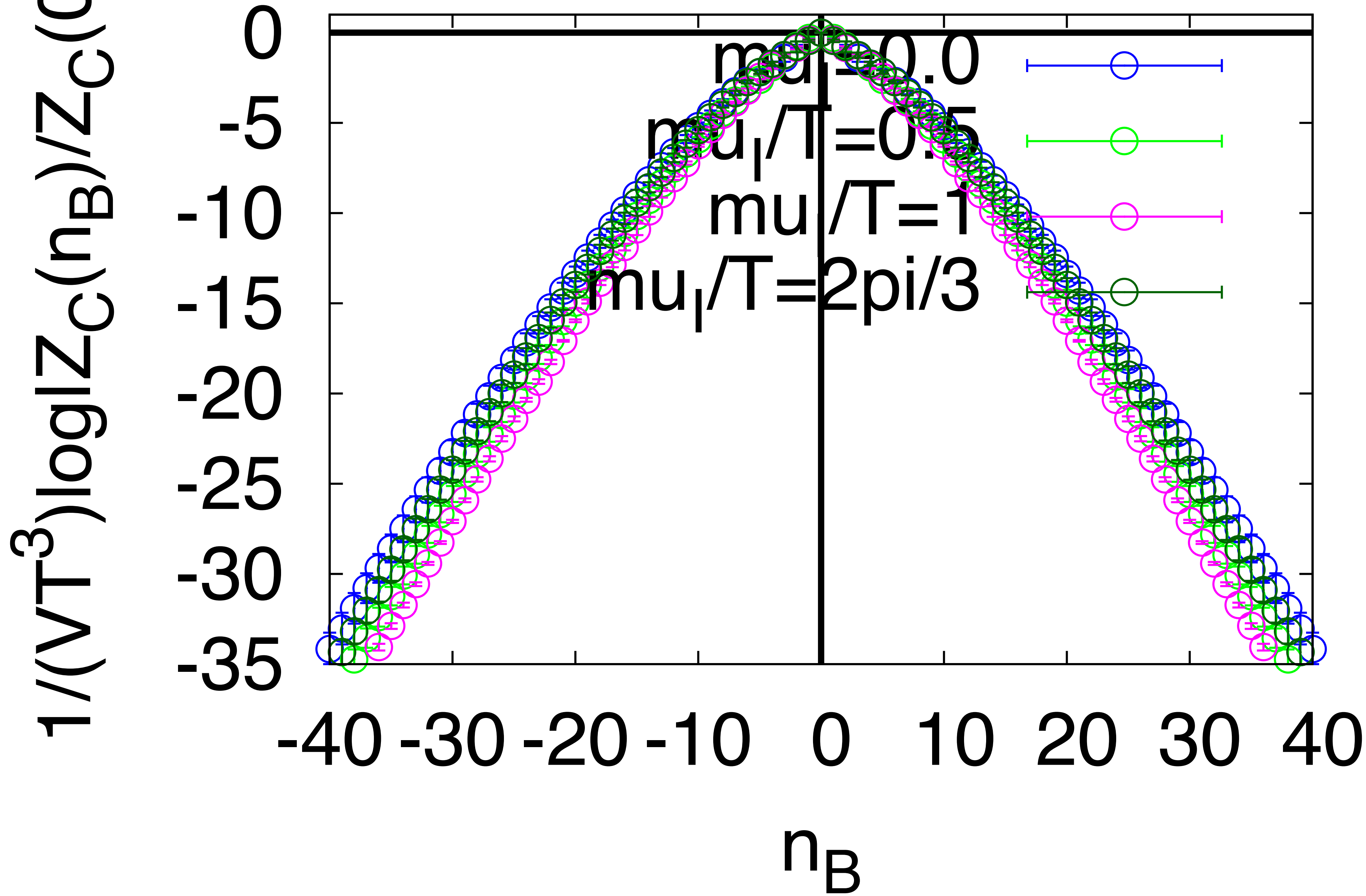
Polyakov loop



Overlap problem?

Comparison between $\mu_I/T = 0,\ 0.5,\ 1,\ 2\pi/3$

T/Tc=0.67



canonical

Zn

Overlap problem?

Comparison between $\mu_I/T = 0,\ 0.5,\ 1,\ 2\pi/3$

T/Tc=0.67

Overlap problem?

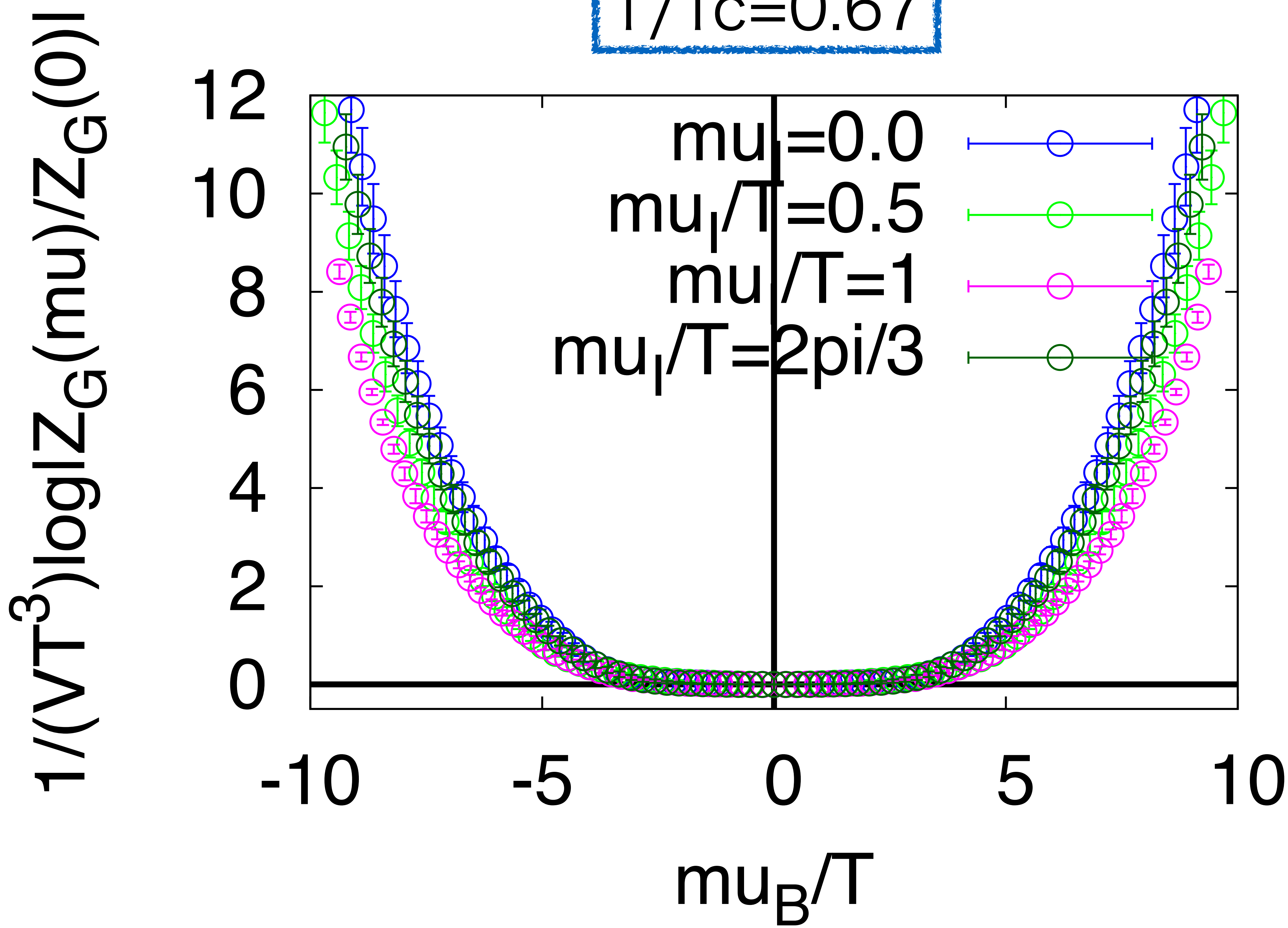
Comparison between $\mu_I/T = 0,\ 0.5,\ 1,\ 2\pi/3$

T/Tc=0.67

Overlap problem?

Comparison between $\mu_I/T = 0,\ 0.5,\ 1,\ 2\pi/3$

T/Tc=0.67

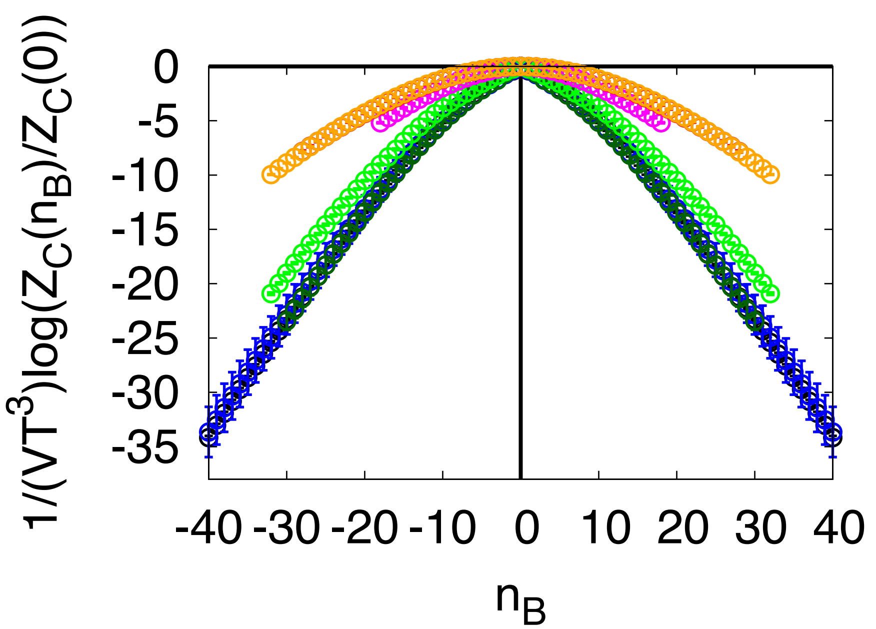


grand canonical $Z(\mu)$

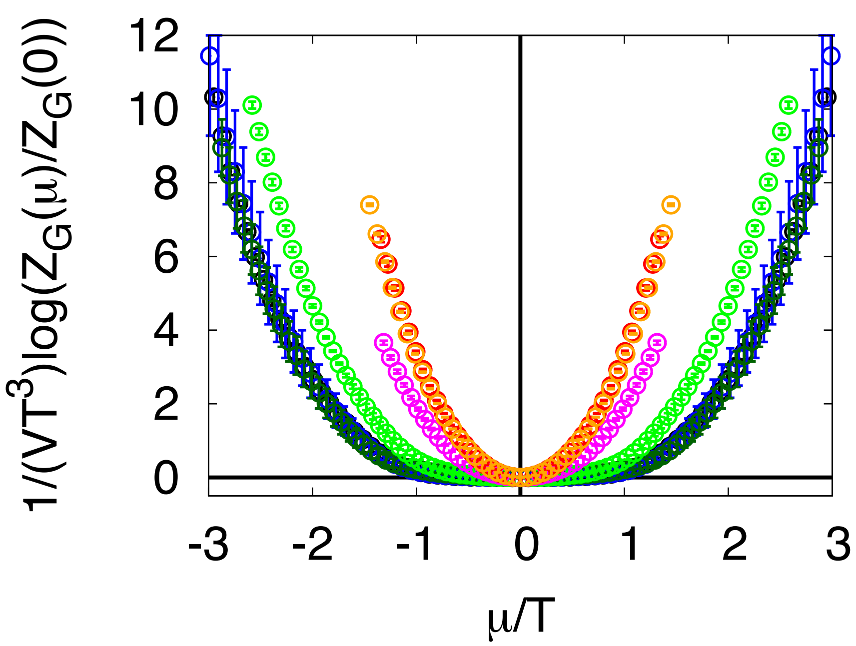
Conclusion

- Canonical approach is a good choice for finite density QCD.
- Hopping parameter expansion works more than we expected.
- We may observe the deconfinement phase transition.
- We may observe the chiral restoration.

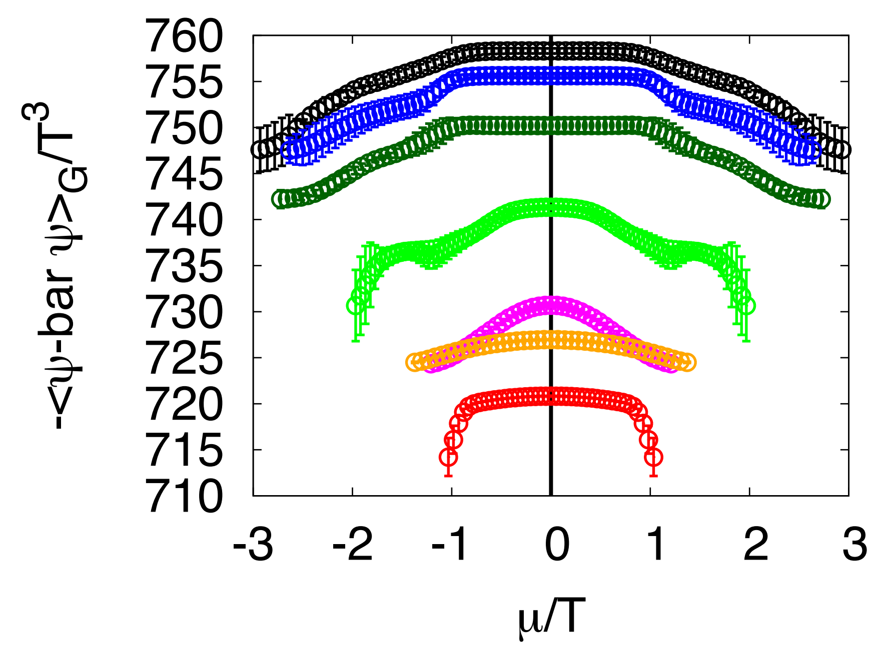
Zn



pressure



chiral condensate



T/Tc=0.64 0.67 0.71 0.81 1.0 1.7 3.4

Where can we apply HPE?

Convergence radius

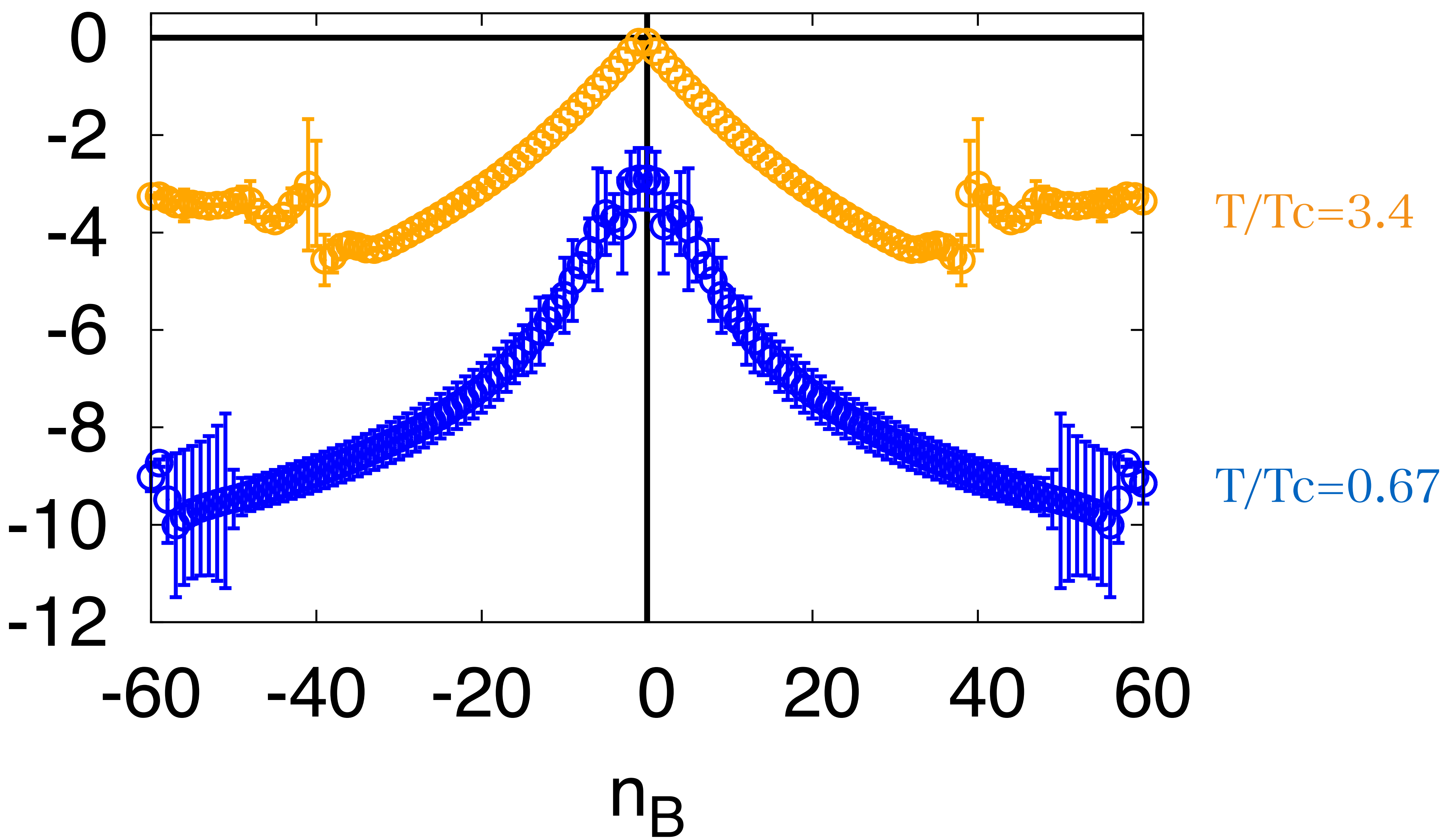
$$\sum_{n=-\infty}^{\infty} Z_n \xi^n = Z_0 + Z_1 \xi + Z_2 \xi^2 + \cdots$$
$$+ Z_{-1} \xi^{-1} + Z_{-2} \xi^{-2} + \cdots$$

Where can we apply HPE?

Convergence radius

$$\sum_{n=-\infty}^{\infty} Z_n \xi^n = Z_0 + Z_1\xi + Z_2\xi^2 + \cdots$$
$$+ Z_{-1}\xi^{-1} + Z_{-2}\xi^{-2} + \cdots$$

$$\log \left| \frac{Z_{n+1}}{Z_n} \right|$$

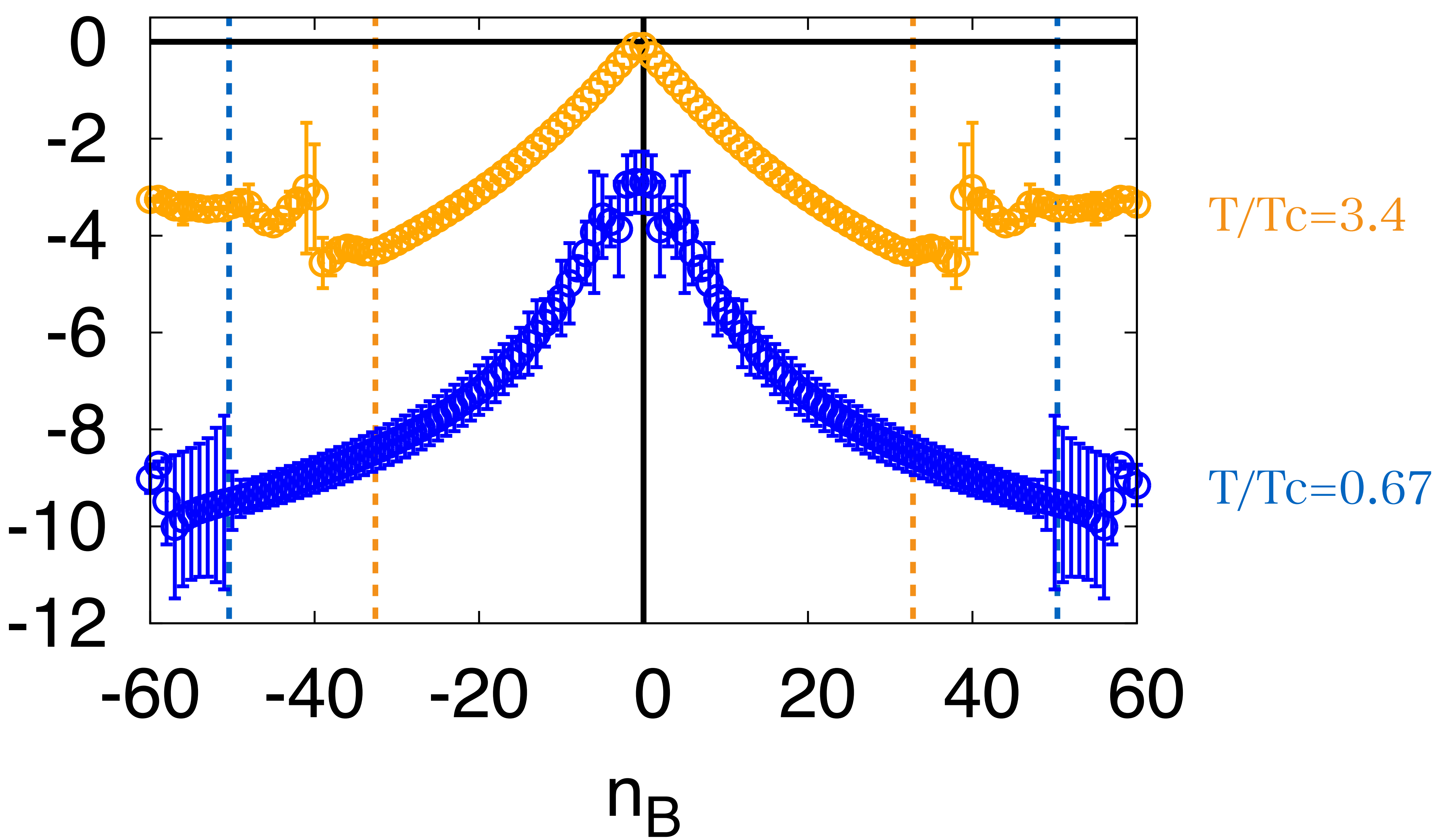


Where can we apply HPE?

Convergence radius

$$\sum_{n=-\infty}^{\infty} Z_n \xi^n = Z_0 + Z_1\xi + Z_2\xi^2 + \cdots$$
$$+ Z_{-1}\xi^{-1} + Z_{-2}\xi^{-2} + \cdots$$

$\log \left| \frac{Z_{n+1}}{Z_n} \right|$



Where can we apply HPE?

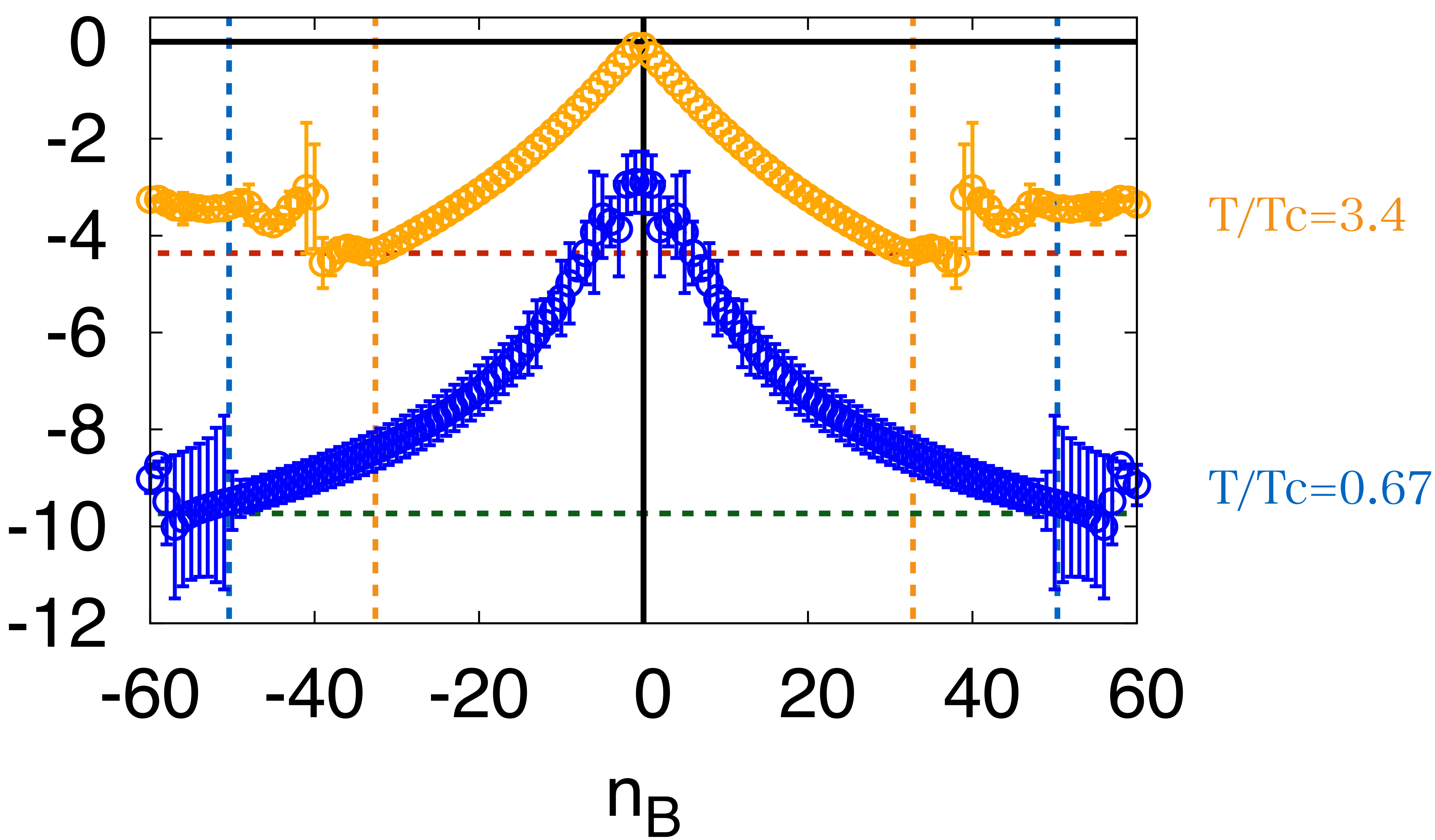
Convergence radius

$$\sum_{n=-\infty}^{\infty} Z_n \xi^n = Z_0 + Z_1\xi + Z_2\xi^2 + \cdots$$
$$+ Z_{-1}\xi^{-1} + Z_{-2}\xi^{-2} + \cdots$$

$\log\left|\frac{Z_{n+1}}{Z_n}\right|$

$\downarrow$

$\log\left|\frac{1}{\xi_c}\right|$



Where can we apply HPE?

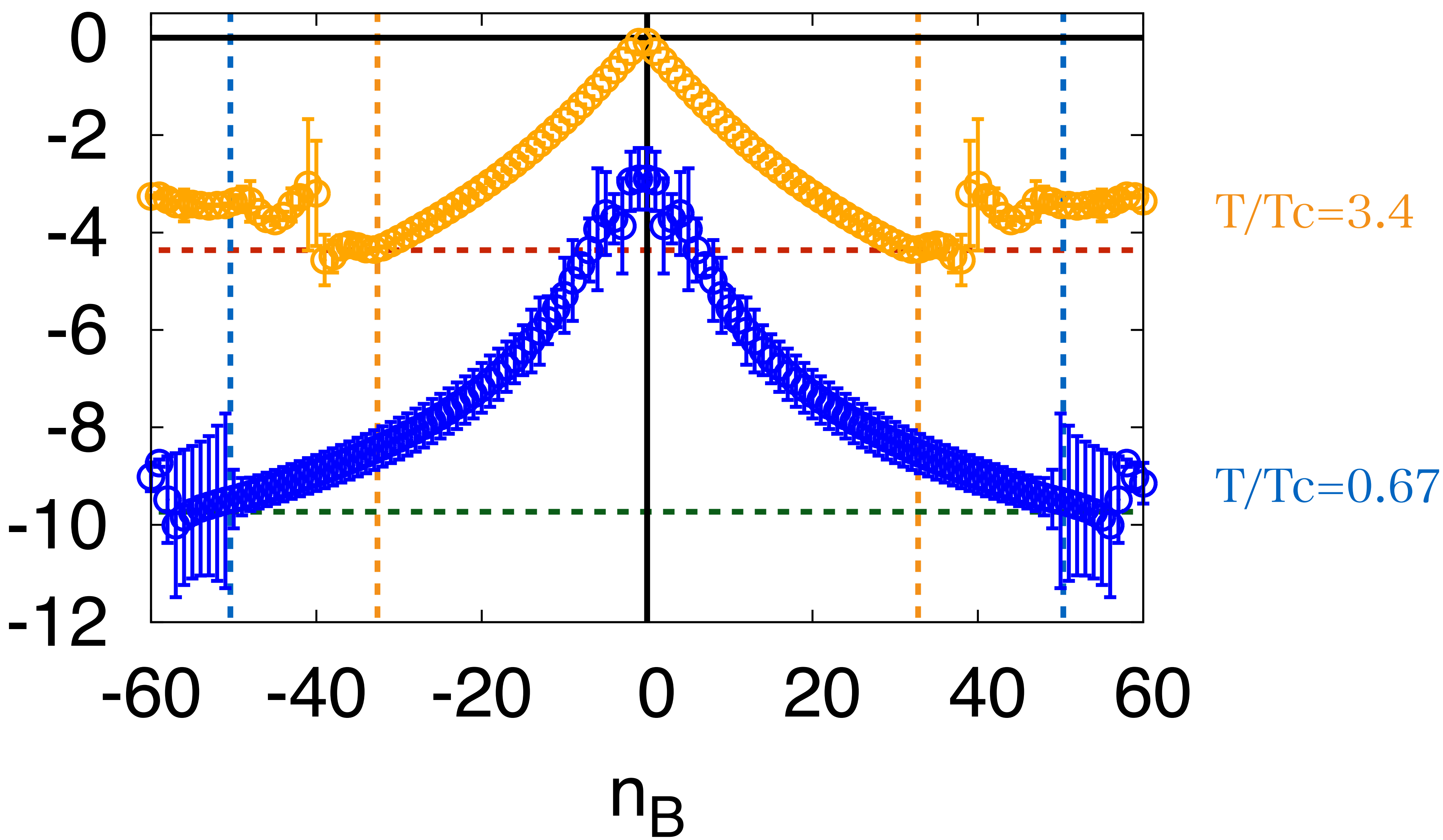
Convergence radius

$$\sum_{n=-\infty}^{\infty} Z_n \xi^n = Z_0 + Z_1\xi + Z_2\xi^2 + \cdots$$
$$+ Z_{-1}\xi^{-1} + Z_{-2}\xi^{-2} + \cdots$$

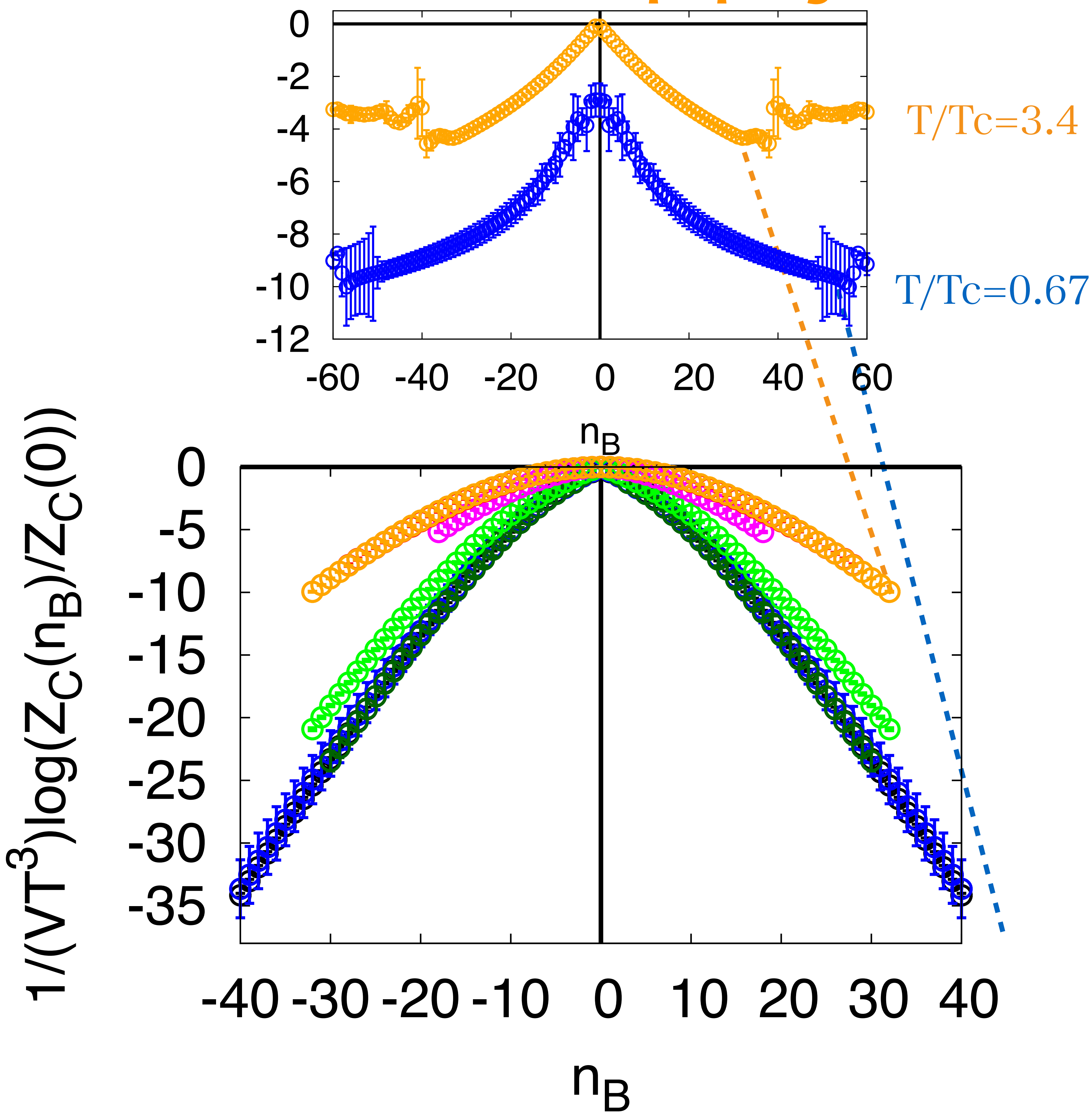
$\pm\mu_c/T$

$\log\left|\frac{Z_{n+1}}{Z_n}\right|$

$\log\left|\frac{1}{\xi_c}\right|$



Where can we apply HPE?



Chiral restoration?

2nd cumulant of chiral condensate

No renormalization! No subtraction! Sorry...

Preliminary!