Flavor Filtered Fermions

James C. Osborn

with Xiao-Yong Jin

Argonne Leadership Computing Facility

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Naive fermions

- Naive Dirac operator (T_μ covariant shift/parallel transporter)

$$D_N = \frac{1}{2}\gamma_\mu \left(T_\mu - T_\mu^\dagger\right)$$

- Naive propagator

$$\frac{1}{D_N + m}$$

- Has doubler poles at momenta (in the free, massless case)

$$p_\mu \in \{0, \pi\}$$

Basic idea

- Naive propagator

$$\frac{1}{D_N + m}$$

- Filter out doublers

$$F \frac{1}{D_N + m} F^{\dagger}$$

- F: “flavor filter”, simple example (free case, momentum space)

$$F = \prod_{\mu} \left(1 + \cos p_{\mu}\right)/2$$

$$F \to \begin{cases} 1, p = \{0,0,0,0\} \\ 0, \text{any } p_{\mu} = \pi \end{cases}$$

Flavor Filtered Fermions

- Zeros of F → poles in D → nonlocal D in real space
- Use Ginsparg-Wilson relation to provide way out

$$D^{-1}\gamma_5 + \gamma_5 D^{-1} = 2\rho\gamma_5$$

- (massless) Flavor Filtered Fermion

$$D_F^{-1} = \rho + F\frac{1}{D_N}F^\dagger$$

$$D_F = \frac{1}{\rho}\left[1 - F\frac{1}{\rho D_N + F^\dagger F}F^\dagger\right]$$

Mass term

- Simplest version

$$D_F^{-1} = \rho + F \frac{1}{D_N + m} F^\dagger$$

- Can spin diagonalize inverse → 4 staggered solves

- Could use other operators for mass term $\quad m \to mG$
  - Give heavier mass to doublers
  - Make operator in denominator more expensive, but improve condition number

Action

- Rewrite determinant

$$
\begin{aligned}
|D_F| &= \left| \frac{1}{\rho} \left[1 - F \frac{1}{\rho(D_N + m) + F^\dagger F} F^\dagger \right] \right| \\
&= \frac{|D_N + m|}{|\rho(D_N + m) + F^\dagger F|}
\end{aligned}
$$

- Can view as partial quenching of 15 flavors
  - $F^\dagger F$ mass term → 15 light, 1 heavy state

Tuning filter

- Free case: simplest version

$$F = \prod_\mu \left(1+\cos p_\mu\right)/2$$
$$= \prod_\mu \left(2+T_\mu+T_\mu^\dagger\right)/4$$

- Interacting case: link renormalization requires tuning filter
  Symmetrize product over permutations

$$F = \mathrm{Sym}\prod_\mu \left[(1-\alpha)+\alpha(T_\mu+T_\mu^\dagger)/2\right]$$

- α=0 : naive fermions
  α=0.5: untuned filter
  α>0.5: tuned filter

Tests

- Testing on quenched lattices
- Plaquette action
- Volume: 32^3 x 64
- 3 couplings
- Scale from Wilson flow

beta	w0 *	“a” (fm, Nf=2+1) *
6.0	1.766(2)	0.099(1)
6.2	2.442(4)	0.072(1)
6.4	3.273(13)	0.054(1)

* BMW [S. Borsányi,et al., JHEP09(2012)010]

Meson spectrum

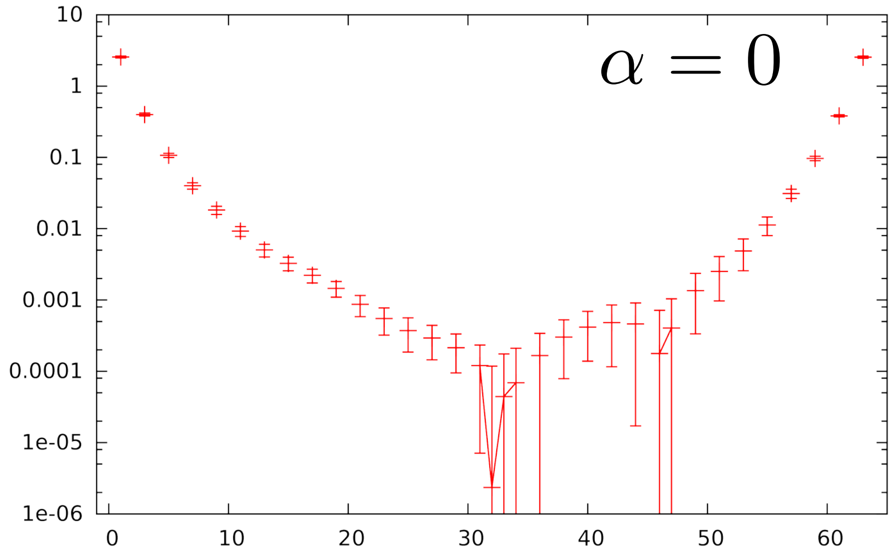
- Local mesons:
  - Treat like Wilson/Overlap fermions: insert appropriate gamma
  - Watch oscillating "time doubler" disappear as alpha tuned from naive fermions to filtered case
- Non-local mesons:
  - Gives complete information on suppression of doublers
  - Useful to think in terms of conversion from naive to staggered basis

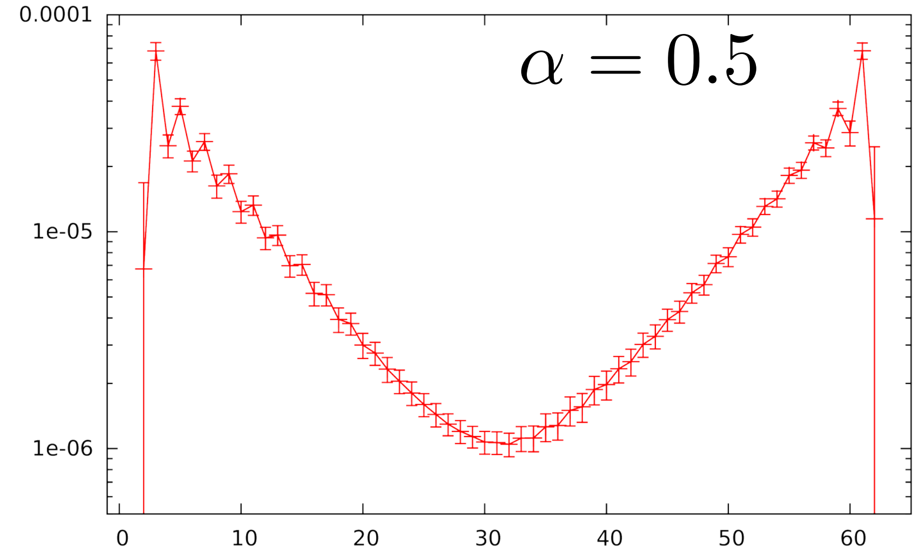
$$
\begin{aligned}
\gamma_n &\rightarrow \gamma_n(\gamma_n \otimes \xi_n) \\
T_n &\rightarrow \gamma_n(1 \otimes \xi_n) \\
(-1)^{x+x_n} &\rightarrow 1(\gamma_n \otimes \xi_n)
\end{aligned}
$$

staggered flavor

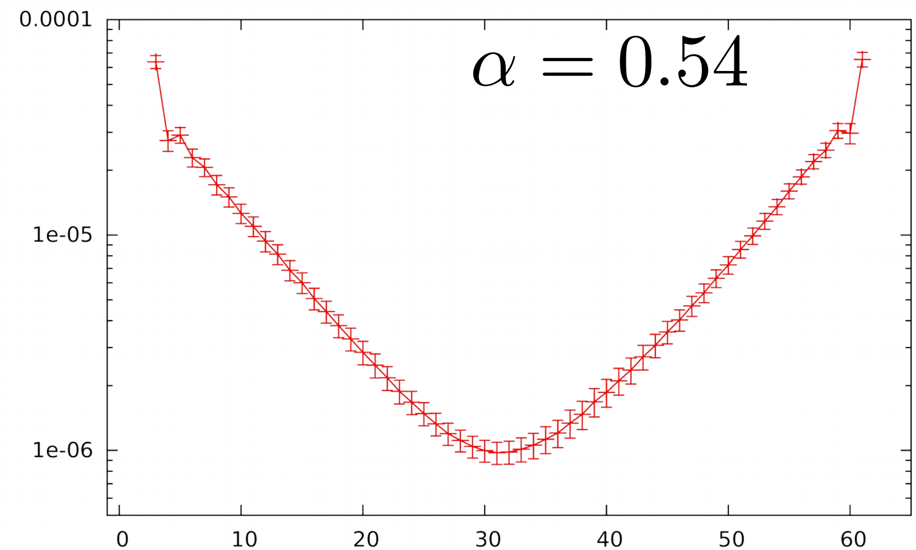
staggered spin, taste

Local meson correlator





Oscillating state goes negative, not shown



- β=6.4, m=0.0069
- Local meson operator γ_{45}

Meson spectrum

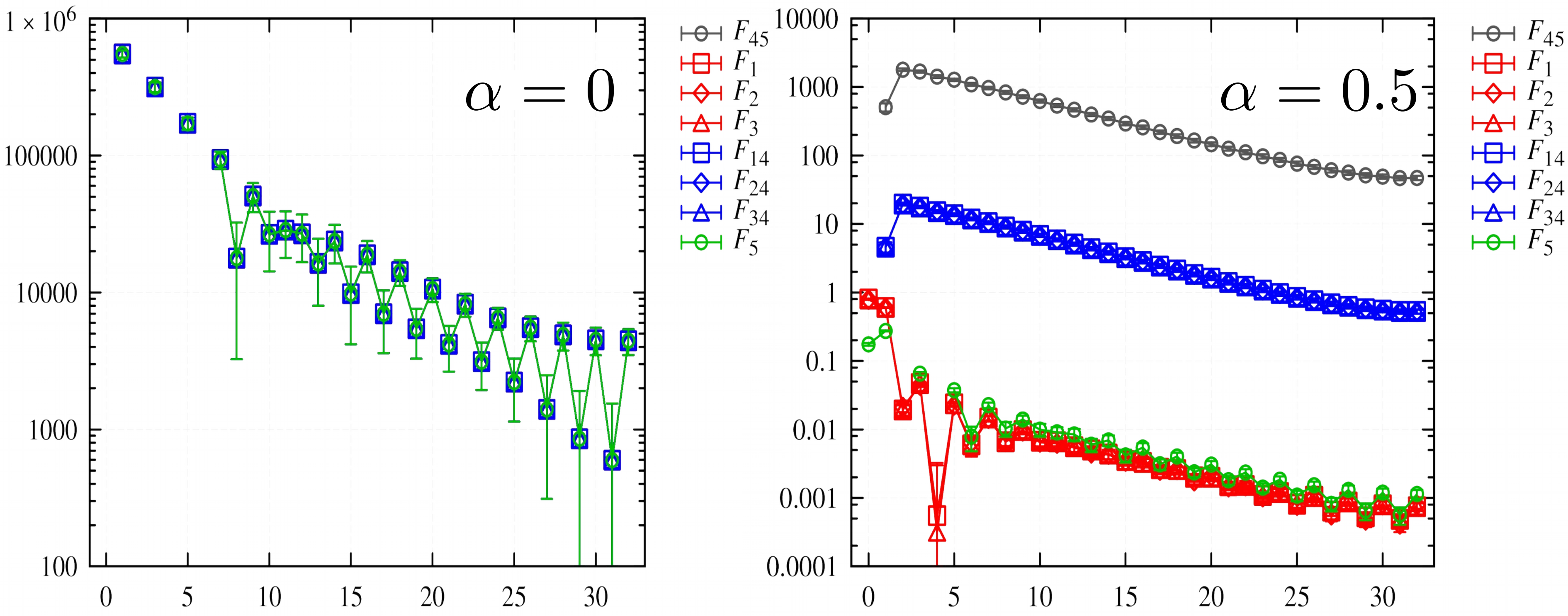
- Local mesons:
  - Treat like Wilson/Overlap fermions: insert appropriate gamma
  - Watch oscillating “time doubler” disappear as alpha tuned from naive fermions to optimally filtered case
- Non-local mesons:
  - Gives complete information on suppression of doublers
  - Useful to think in terms of conversion from naive to staggered basis

$$
\begin{aligned}
\gamma_n &\rightarrow \gamma_n(\gamma_n \otimes \xi_n) \\
T_n &\rightarrow \gamma_n(1 \otimes \xi_n) \\
(-1)^{x+x_n} &\rightarrow 1(\gamma_n \otimes \xi_n)
\end{aligned}
$$

staggered flavor

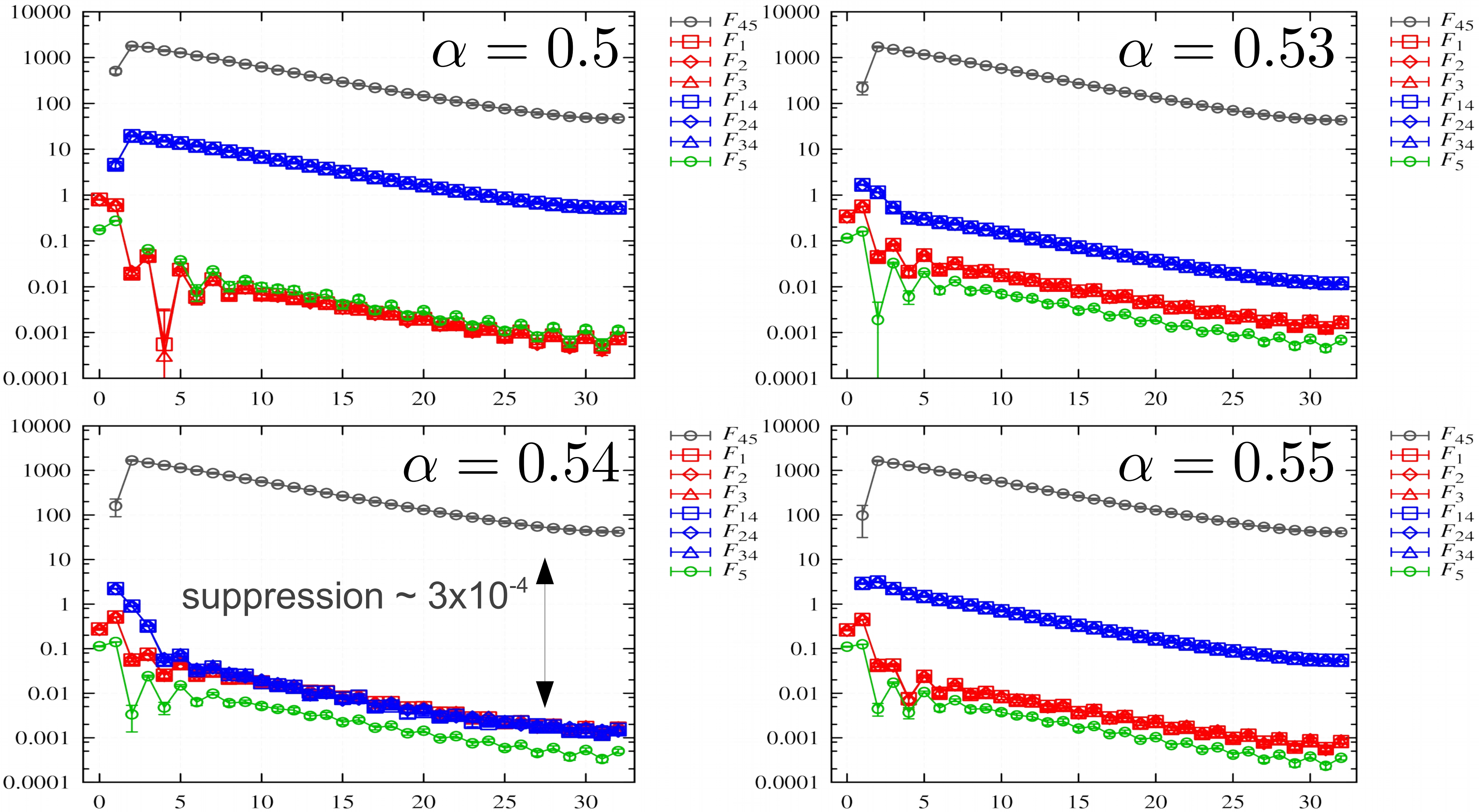
staggered spin, taste

Non-local meson correlator



- β=6.4, m=0.0069
- Correlator of staggered states: $\gamma_F(\gamma_{45} \otimes \xi_{45})$

Non-local meson correlator



Tuning filter

- Untuned filter (α = 0.5)
  - Suppression ~ 1e-2
- Coarse tuning (α = 0.54)
  - Suppression ~ 3e-4
- Finer tuning (α = 0.5352)
  - Suppression ~ 1e-4
- Stout smearing (2 steps)
  - Reduces need for tuning
  - Untuned suppression (α = 0.5) ~ 3e-4
  - Fine tuning suppression (α = 0.50615) ~ 1e-6
- Still exploring optimal tuning

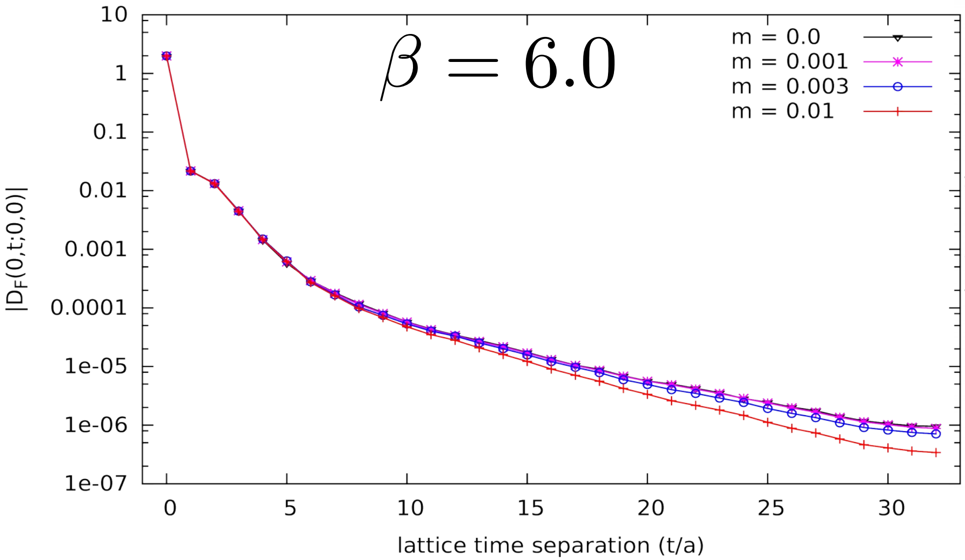
Fine tuning

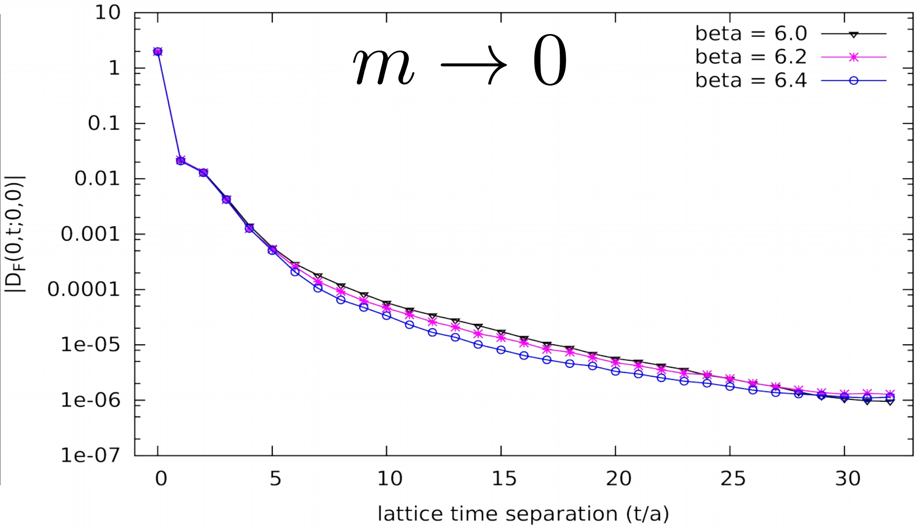
- Simple filter with one parameter

$$F = \mathrm{Sym} \prod_\mu \left[(1-\alpha) + \alpha (T_\mu + T_\mu^\dagger)/2 \right]$$

- Expands into 5 distinct operators,
  one for each number of hops (0,1,2,3,4)
- Could tune each coefficient separately
  (only ratio matters, so really 4 parameters to tune)
- Use non-local meson spectrum as guide
- Tuning 1 fairly easy, maybe 2 ok, 4 probably difficult

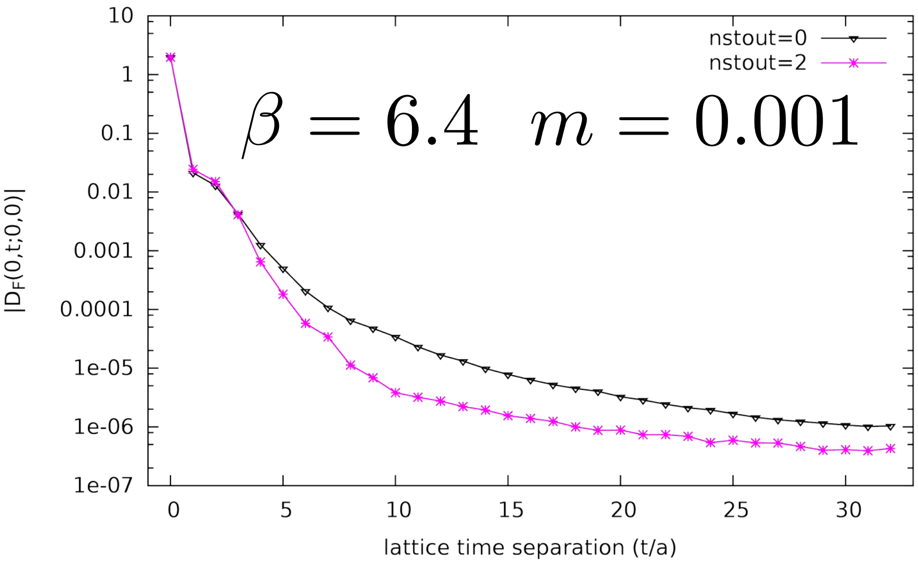
Dirac operator locality (α=0.5)





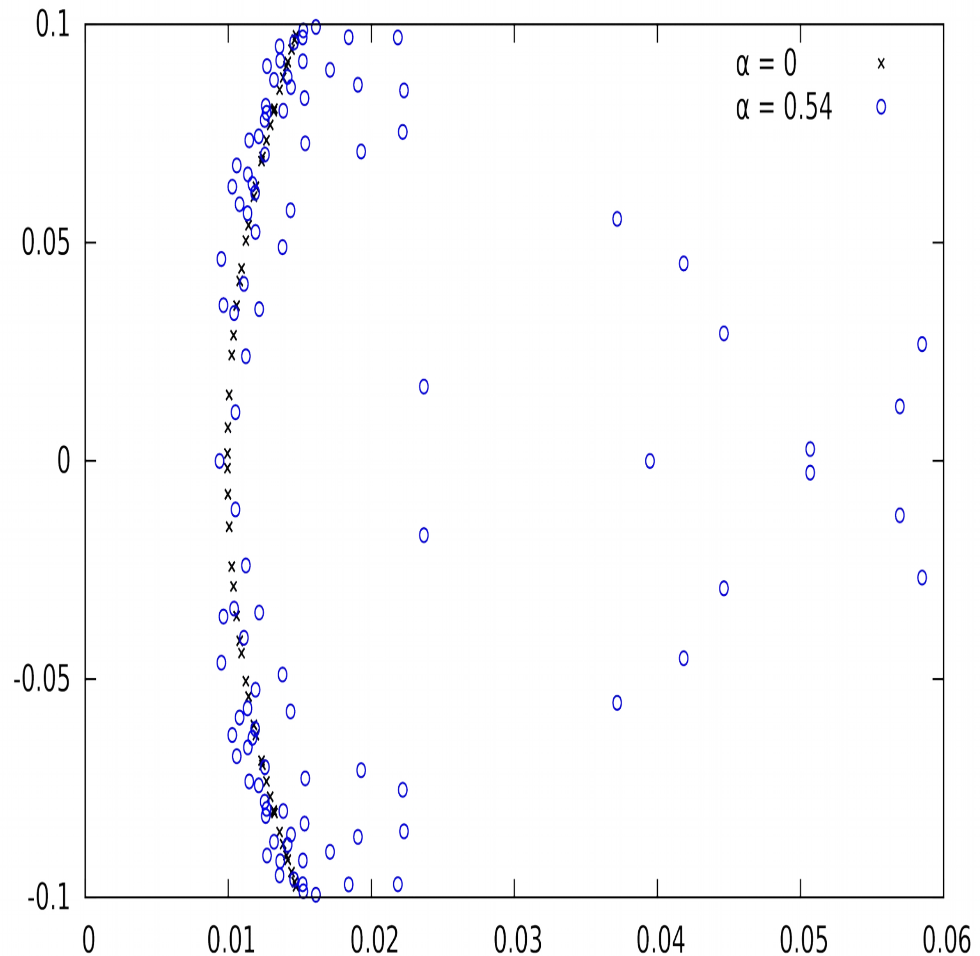
- Tail grows with m (effective mass gets lighter); reaches finite limit as m→0
- More local at intermediate distances as 'a' decreased
- Larger distances requires better filtering to reduce contribution

$$D_F = \frac{1}{\rho}\left[1 - F\frac{1}{\rho(D_N + m) + F^\dagger F}F^\dagger\right]$$



Dirac operator topology

- Eigenvalues of D_F on 8^4 lattice
- β=5.8, m=0.01
- α=0.54 eigenvalues scaled to roughly match α=0
- 15 eigenvalues move to larger real values, leaving 1 behind
- Effect goes away for smaller quark mass
- Need to check limit a → 0 first, then m → 0

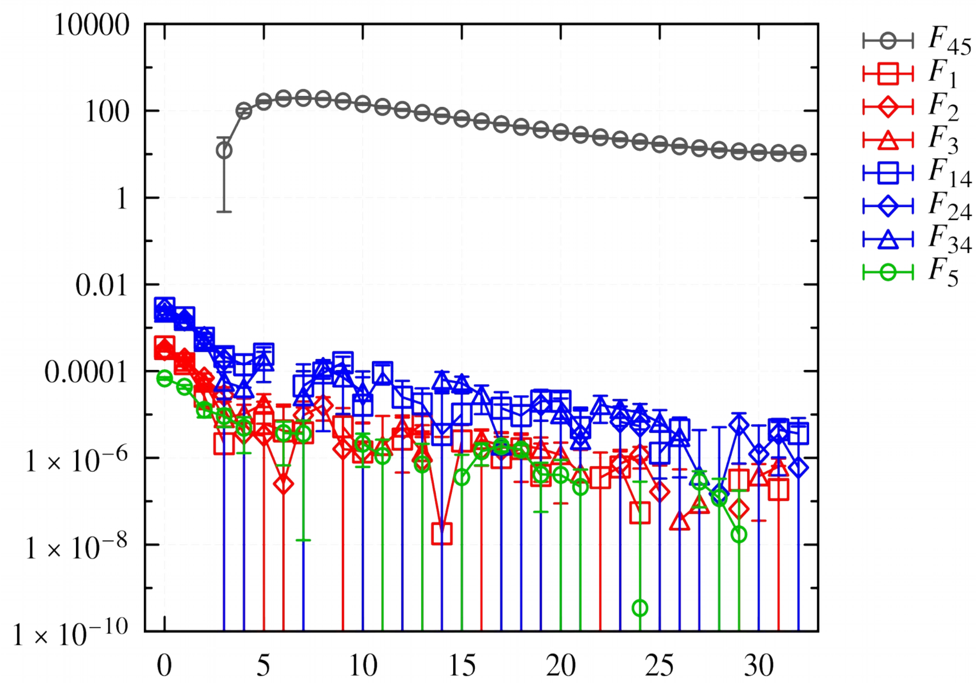


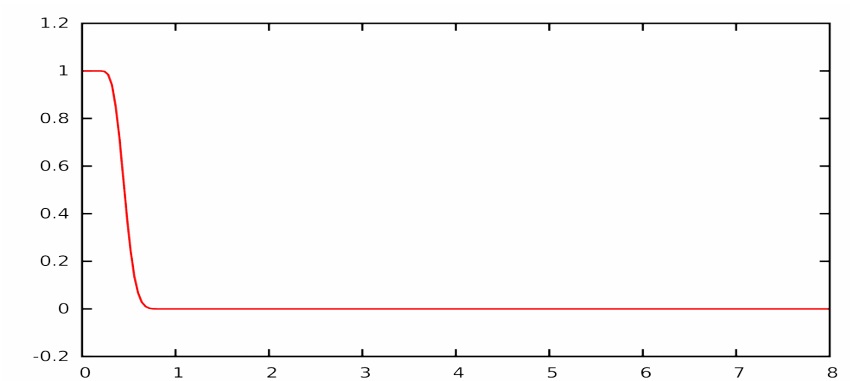
Alternative filters

- Use Wilson term to form filter

$$W = 4 - \frac{1}{2}\sum_{\mu}\left(T_{\mu} + T_{\mu}^{\dagger}\right)$$

- Keep small eigenvalues of W, remove large
- Step function would introduce nonlocality
- Try smooth function
- Polynomial (order 64)





- β=6.4, m=0.0069

$$\gamma_F\left(\gamma_{45} \otimes \xi_{45}\right)$$

- suppression < 10^{-6}
  (too noisy to measure accurately)

Alternative filters

- Is there an optimal filter (at any cost)?
- What about something like:

$$\sim \frac{D_N}{D_{overlap}}$$

- Ratio preserves physical mode, cancels doublers
- Expensive (no better than just using overlap)
- Maybe a good enough approximation is cheaper?

Flavor (taste) Filtered Staggered?

- Start with staggered, filter out bad tastes

$$F \frac{1}{D_S + m} F^\dagger$$

- 2 flavor (taste)

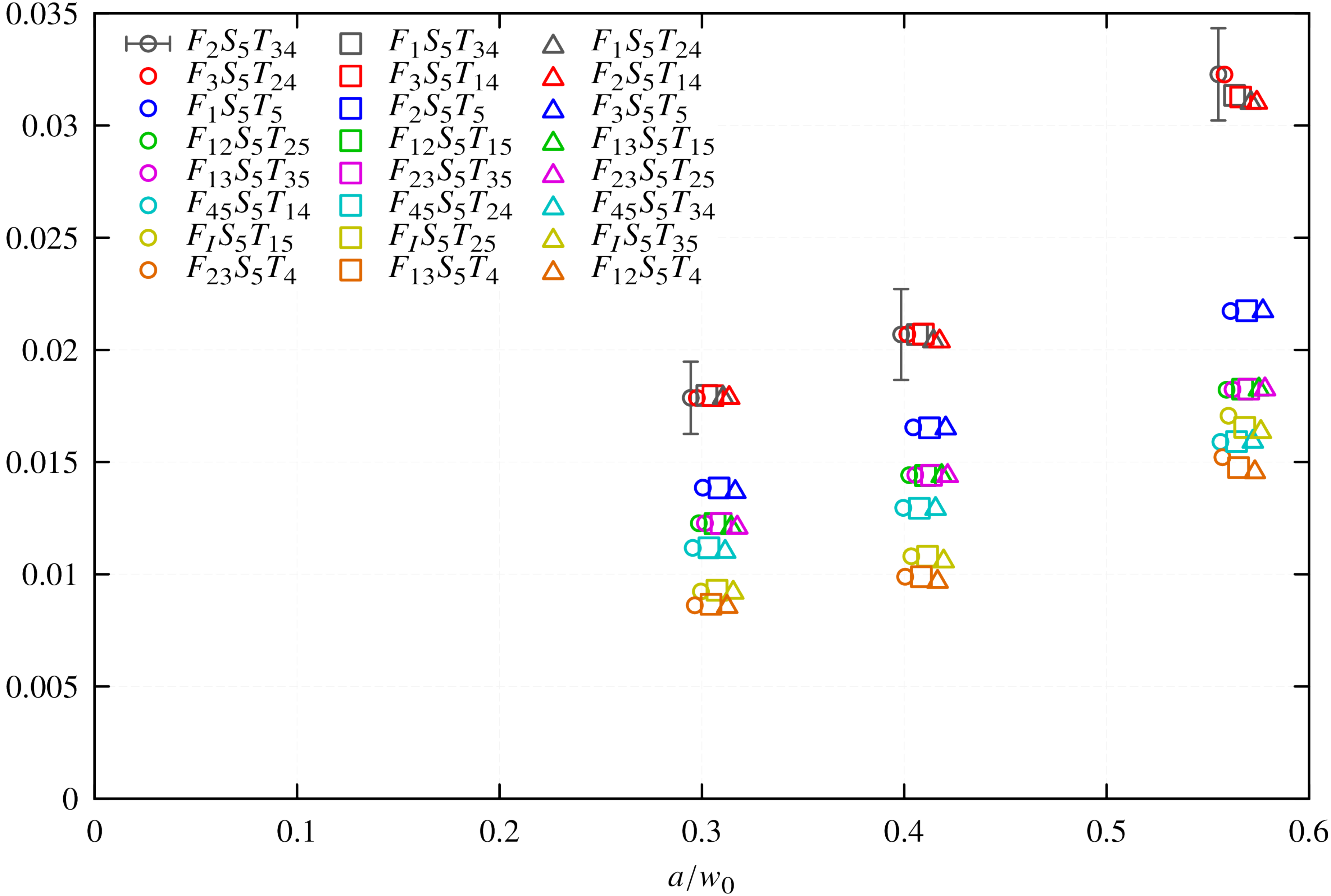
$$F = (1 - \alpha) + \alpha(1 \otimes \xi_5)$$

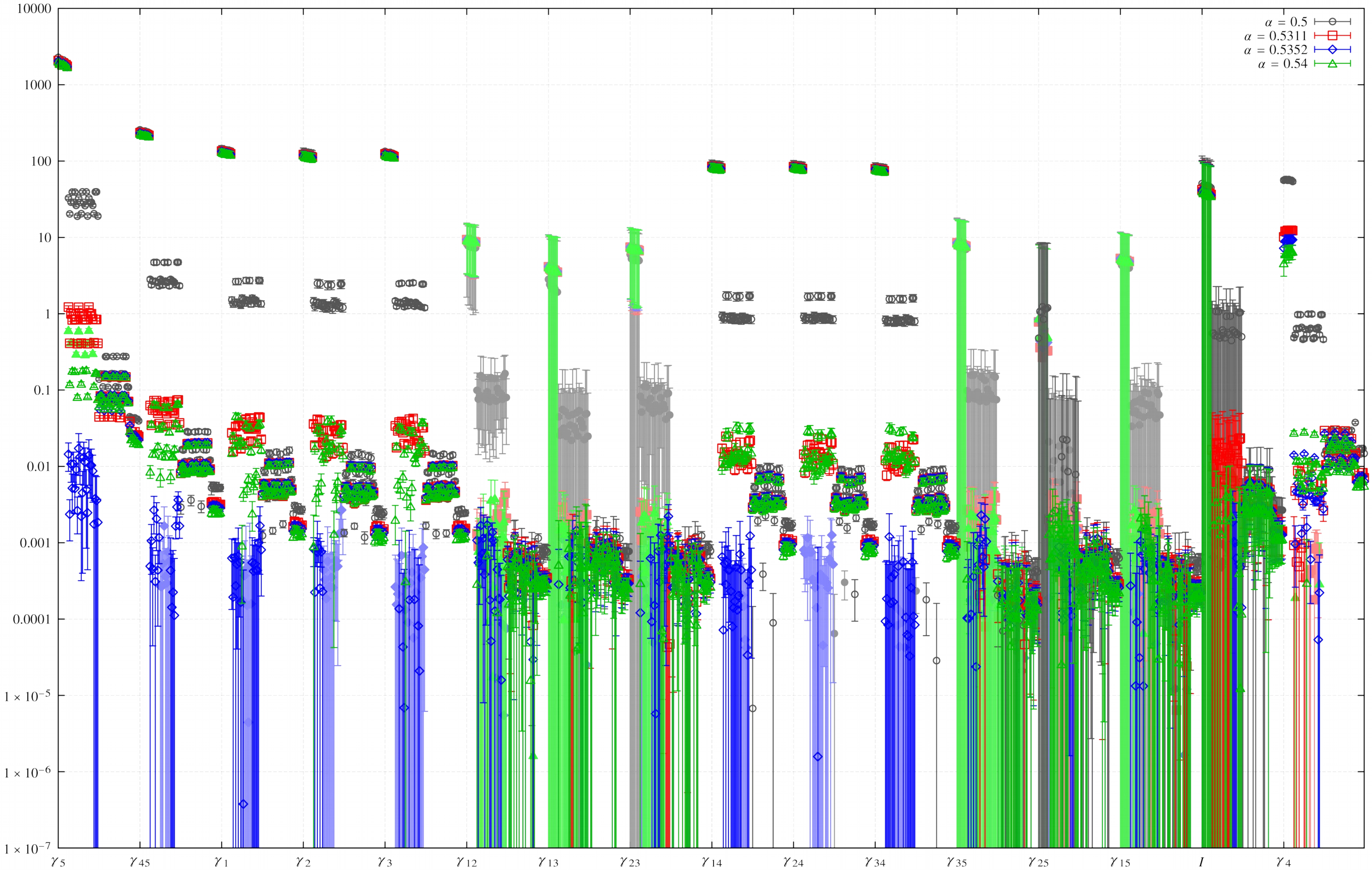
- 1 flavor possible, but breaks rotational symmetry (see staggered-wilson literature)
  - Is there a variation that makes this feasible?

Summary

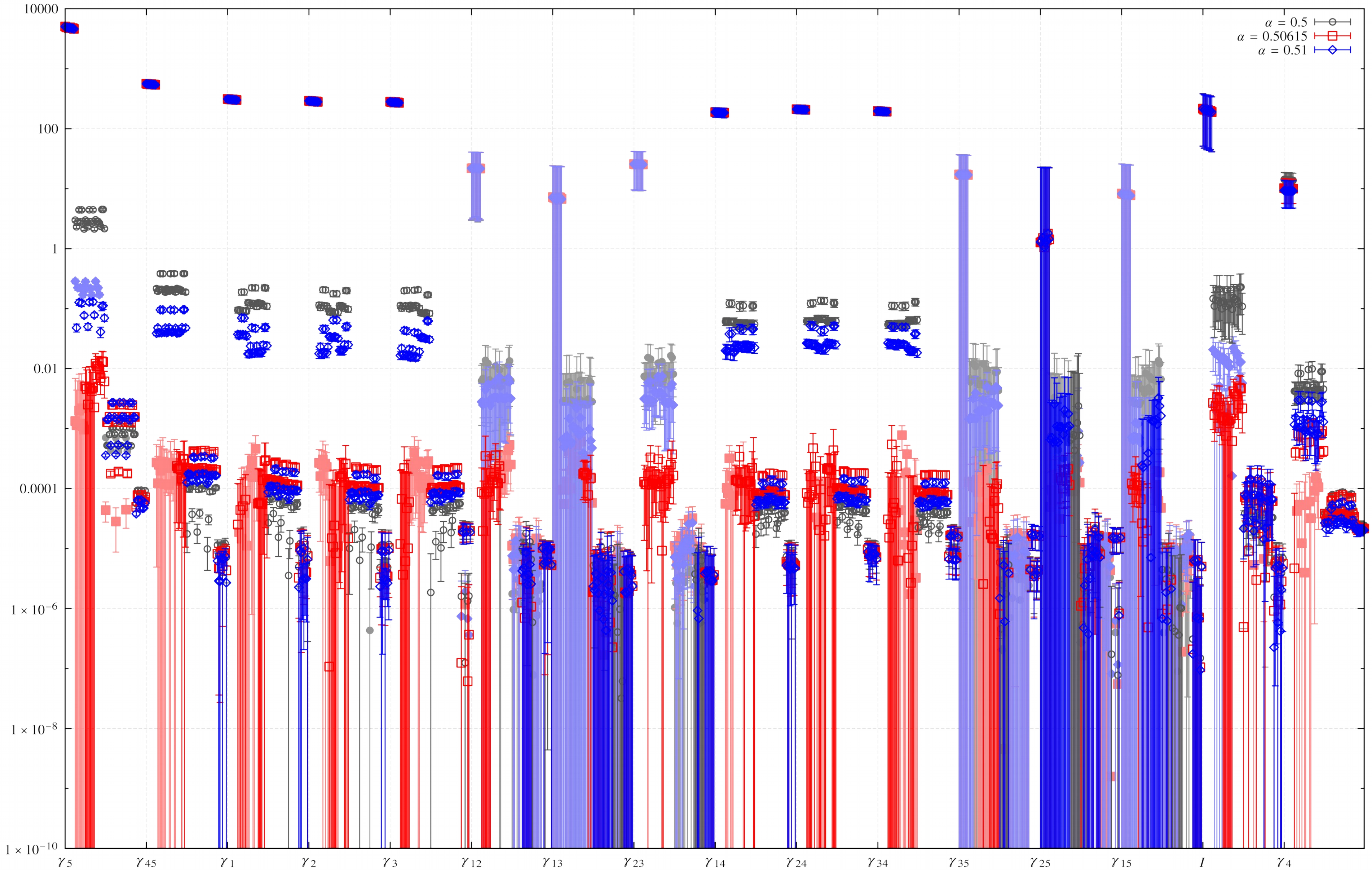
- Constructed new lattice Dirac operator
  - Satisfies Ginsparg-Wilson relation
  - Approximately 1 flavor (needs more study)
  - Depending on choice of filter,
    can probably satisfy basic locality requirement
  - Signs of emerging topological zero modes
- Cheaper than overlap/DW (in simplest form)
- Comparable expense to Wilson
  - Wilson has effective MG
- Staggered still cheaper
  - Can do Nf=2 flavor filtered staggered

Extra slides

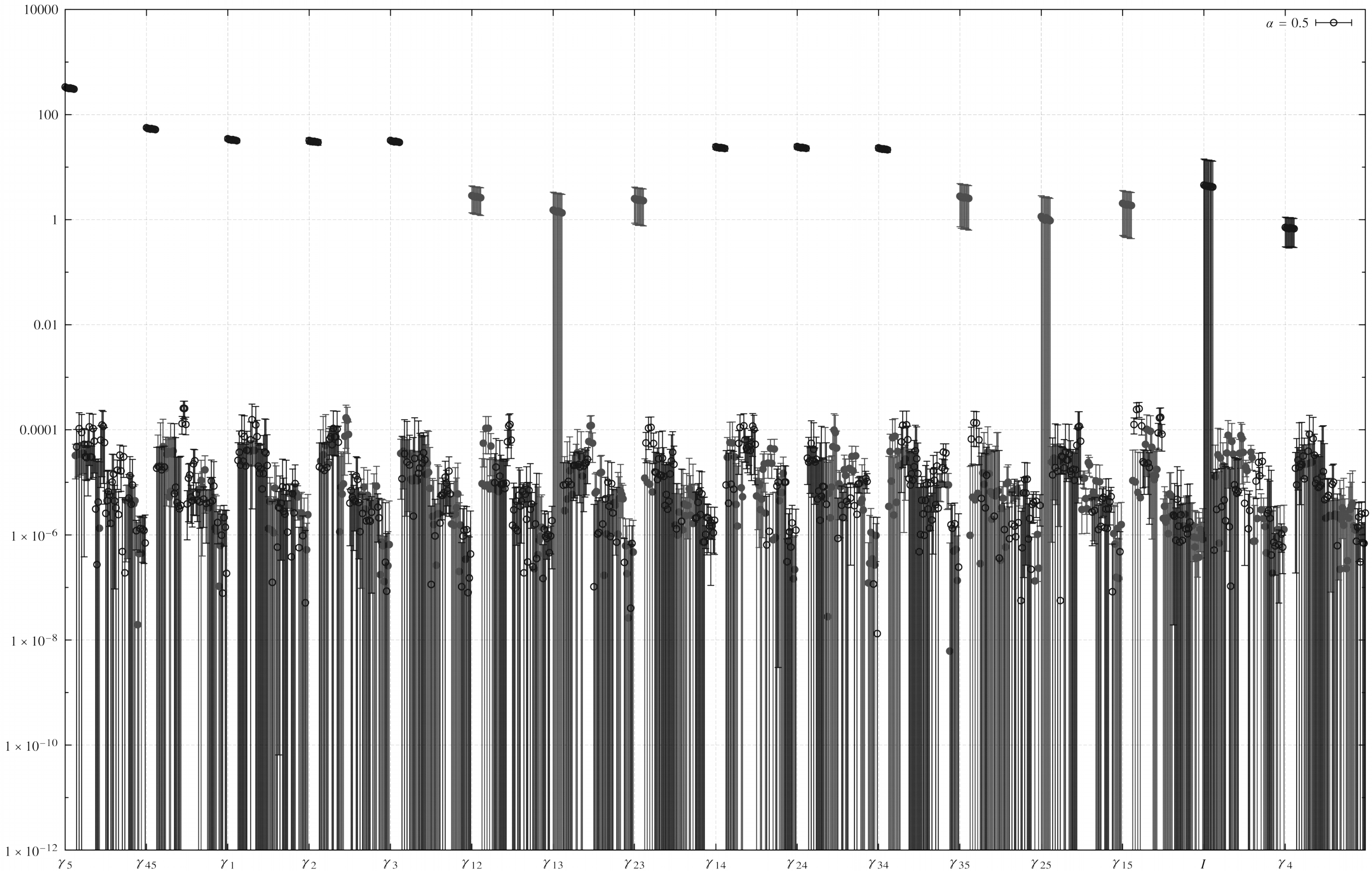

$F_2S_5T_{34}$
$F_3S_5T_{24}$
$F_1S_5T_5$
$F_{12}S_5T_{25}$
$F_{13}S_5T_{35}$
$F_{45}S_5T_{14}$
$F_IS_5T_{15}$
$F_{23}S_5T_4$
$F_1S_5T_{34}$
$F_3S_5T_{14}$
$F_2S_5T_5$
$F_{12}S_5T_{15}$
$F_{23}S_5T_{35}$
$F_{45}S_5T_{24}$
$F_IS_5T_{25}$
$F_{13}S_5T_4$
$F_1S_5T_{24}$
$F_2S_5T_{14}$
$F_3S_5T_5$
$F_{13}S_5T_{15}$
$F_{23}S_5T_{25}$
$F_{45}S_5T_{34}$
$F_IS_5T_{35}$
$F_{12}S_5T_4$
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0
0.1
0.2
0.3
0.4
0.5
0.6
a/w_0



10000
1000
100
10
1
0.1
0.01
0.001
0.0001
1 × 10^−5
1 × 10^−6
1 × 10^−7
α = 0.5
α = 0.5311
α = 0.5352
α = 0.54
γ5
γ45
γ1
γ2
γ3
γ12
γ13
γ23
γ14
γ24
γ34
γ35
γ25
γ15
I
γ4



10000
100
1
0.01
0.0001
1×10^{-6}
1×10^{-8}
1×10^{-10}
α = 0.5
α = 0.50615
α = 0.51
γ_5 γ_{45} γ_1 γ_2 γ_3 γ_{12} γ_{13} γ_{23} γ_{14} γ_{24} γ_{34} γ_{35} γ_{25} γ_{15} I γ_4



$\alpha = 0.5$
10000
100
1
0.01
0.0001
1×10^{-6}
1×10^{-8}
1×10^{-10}
1×10^{-12}
γ_5 γ_{45} γ_1 γ_2 γ_3 γ_{12} γ_{13} γ_{23} γ_{14} γ_{24} γ_{34} γ_{35} γ_{25} γ_{15} I γ_4