Improving the lattice axial vector current

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Lattice 2015, Kobe, Japan, July 14 - 18, 2015

Outline

Introduction 1

- Axial (non-singlet) vector current – important quantity to study hadronic structures as quark masses and meson decay constants

$$A_\mu(x) = \bar{\psi}(x)\gamma_\mu\gamma_5\psi(x)$$

In the continuum that current satisfies the axial Ward identity

$$\partial_\mu A_\mu(x) = 2mP(x)\,, \quad P(x) = \bar{\psi}(x)\gamma_5\psi(x)$$

- In most applications with Wilson/clover fermions the local axial vector (plus $O(a)$ improvement) is used

Renormalised and improved (local) axial current

$$A^{\overline{\mathrm{MS}}}_{\mu,\mathrm{loc}} = Z_{A^{\mathrm{loc}}}(1 + b_A am)\left[A_{\mu,\mathrm{loc}} + ac_A\partial_\mu P\right]^{\mathrm{lat}}$$

$A_{\mu,\mathrm{loc}}$ is used to determine e.g. the nucleon axial charge

- To what extent the (continuum) axial Ward identity is fulfilled? How to handle lattice corrections?

Introduction 2

- Proposal: use an one-link axial vector current whose divergence exactly satisfies a lattice Ward identity, involving the pseudoscalar density and a number of irrelevant operators
- Such operators naturally appear in derivation of lattice Ward identities
  see e.g. Bochicchio, Maiani, Martinelli, Rossi, Testa NPB262 (1985), also Reisz, Rothe PRD62 (2000), recent Bhattacharya et al., arXiv:1502.07325
  Reisz and Rothe performed a proof of the Ward identity in perturbation theory for Wilson like fermions
- We study such an axial Ward identity for clover fermions and check it both perturbatively and nonperturbatively

Construction I

- Start from an identity on every configuration (i, j, k, x - site positions, no implicit summation)

$$M^{-1}_{jx}\,\gamma_5\,\delta_{xi} + \delta_{jx}\,\gamma_5\,M^{-1}_{xi} = \sum_k \left(M^{-1}_{jx}\,\gamma_5\,M_{xl} M^{-1}_{ki} + M^{-1}_{jk} M_{kx}\,\gamma_5\,M^{-1}_{xi} \right)$$

- Split the clover fermion matrix M

$$M_{ij} = D_{ij} + W_{ij} + C_{ij} + m_B \delta_{ij} I$$

$D \sim \gamma_\mu$, W Wilson term, $C \sim c_{sw}$ clover term
bare mass m_B vanishing for $\kappa = 1/8$

$$am_B = \frac{1}{2\kappa} - 4 \quad \text{in MC}\,, \quad am_B = am - \frac{C_F g^2}{16\pi^2}\Sigma_0 \quad \text{in 1-loop LPT}$$

Construction II

- Averaging that expression over configurations, we get

$$S_{jx}\gamma_5\delta_{xi} + \delta_{jx}\gamma_5 S_{xi} = -G_{jxi}[\partial A] + 2m_B G_{jxi}[P] + 2G_{jxi}[O_C] + G_{jxi}[O_W]$$

S_{jx} fermion propagator (2-point function)
$G_{jxi}[O]$ fermion-line connected 3-point function of operator O

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$$a\partial A = \sum_\mu [A_\mu(x) - A_\mu(x - a\hat{\mu})]$$

$$A_\mu(x) = \frac{1}{2}\left[\bar{\psi}_x\gamma_\mu\gamma_5 U_\mu(x)\psi_{x+a\hat{\mu}} + \bar{\psi}_{x+a\hat{\mu}}\gamma_\mu\gamma_5 U^\dagger_\mu(x)\psi_x\right]$$

$$P = \bar{\psi}_x\gamma_5\psi_x\,, \quad aO_C = \bar{\psi}_x\gamma_5 C_{xx}\psi_x$$

$$aO_W = 8P - \frac{1}{2}\sum_\mu\left[\bar{\psi}_x\gamma_5 U_\mu(x)\psi_{x+a\hat{\mu}} + \bar{\psi}_{x+a\hat{\mu}}\gamma_5 U^\dagger_\mu(x)\psi_x + (x \to x - a\hat{\mu})\right]$$

Construction III

- Rearrange

$$G_{jxi}[\partial A] = 2m_B G_{jxi}[P] - S_{jx}\gamma_5\delta_{xi} - \delta_{jx}\gamma_5 S_{xi} + 2G_{jxi}[O_C] + G_{jxi}[O_W]$$

axial Ward identity (WI) contact term to make WI hold offshell
tree level Wilson term: delta function in position space to cancel lattice artefacts in propagators
in interaction case mixing of O_W and O_C with lower dimensional operators

- Forward case

$$G_{jxi}[\partial A] = 0$$

we are left with an identity linking the propagators to the 3-point functions of P, O_W, O_C
could be used to find κ_c measuring the 3-point functions at different κ on-shell

Perturbative checks

- Use of Wilson gauge action and clover fermions with stout smeared U in A_μ and O_W, unsmeared U in O_C
- Check of identity in 1-loop LPT for amputated Green functions in momentum space [$q = p_2 - p_1$, $\Lambda_O = \Lambda_O(p_2, p_1)$]

$$\gamma_5 S^{-1}(p_1) + S^{-1}(p_2)\gamma_5 = -\Lambda_{\partial A} + 2m_B\Lambda_P + 2\Lambda_C + \Lambda_W$$

- Feynman rules for all operators
  Forward case: 1-loop up to $O(a^2)$ for arbitrary masses (check against available results of Cyprus group, technique differs)
  Non-forward case 1-loop up to $O(am)$

Typical diagrams

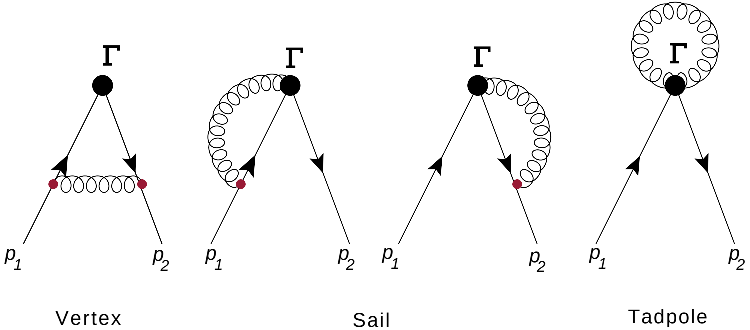


Figure: Typical diagrams

Nonvanishing contributions
A, ∂A, O_W: vertex, sails, tadpole , P: vertex , O_C: sails
results are functions of $ap_1, ap_2, am, \alpha, c_{sw}, \omega$

Renormalisation factors: one-link Z_A

- κ_c derivation from mass independent contributions of the WI (valid both forward and nonforward)
- Wave function and mass renormalisation

$$S^{\overline{\mathrm{MS}}}(p, m^{\overline{\mathrm{MS}}}) = Z_2^{-1}\, S^{\mathrm{lat}}(p, Z_m m)$$

- Forward/nonforward amputated Green's functions

$$\Lambda_O^{\overline{\mathrm{MS}}}(p_2, p_1) = Z_O\, \Lambda_O^{\mathrm{lat}}(p_2, p_1)\,, \quad O = (P, A, \partial A)$$

  Z_P checked against Cyprus group

- New:

$$Z_A = Z_{\partial A} = 1 + \frac{g^2 C_F}{16\pi^2}\Big[-8.66279 + 0.9116267 c_{sw} + 2.00070 c_{sw}^2 + 85.9927\omega - 4.77316 c_{sw}\omega - 282.301\omega^2 \Big]$$

- Naive WI not fulfilled: $Z_m Z_P - Z_A = O(g^2)$

Renormalisation invariance – forward identity

- Improved identity in forward case
  l.h.s.: Propagators depends only on Z_2

$$(\gamma_5 S^{-1} + S^{-1}\gamma_5)^{\overline{\rm MS}} = Z_2(\gamma_5 S^{-1} + S^{-1}\gamma_5)^{\rm lat}$$

  r.h.s.: The combined operator

$$2m_B\bar{P} \equiv 2m_B P + 2O_C + O_W$$

  is renormalisation invariant

$$(2m\Lambda_P)^{\overline{\rm MS}} = Z_2(2m_B\Lambda_{\bar{P}})^{\rm lat}$$

- The local operator $2mP$ would need a finite renormalisation $Z_m Z_P$
  Operator with m_B and "irrelevant" terms needs neither additive nor multiplicative renormalisation

Renormalisation invariance - nonforward case

- The nonforward case contains the operator ∂A with the one-link axial vector

  LPT result

$$\left(-\partial A + 2mP\right)^{\overline{\mathrm{MS}}} = \left(-\partial A + 2m_B\bar{P}\right)^{\mathrm{lat}}$$

  $\Rightarrow$ the constructed axial WI is renormalisation invariant

- Form of the renormalisation mixing matrix

$$\begin{pmatrix} -\partial A \\ 2mP \end{pmatrix}^{\overline{\mathrm{MS}}} = \begin{pmatrix} Z_A & 0 \\ 1 - Z_A & 1 \end{pmatrix} \begin{pmatrix} -\partial A \\ 2m_B\bar{P} \end{pmatrix}^{\mathrm{latt}}$$

- Lattice axial Ward identity passed all tests in 1-loop LPT

Lattice setup

- Gluon action: tree-level Symanzik improved
  Fermion action: $n_f = 2 + 1$ Wilson fermions with clover term
  analytic stout smeared links in the Dirac kinetic and mass terms,
  no smearing in the clover term
  $32^3 \times 64$, $c_{sw} = 2.65$, $\omega = 0.1$, $\beta = 5.5$ [$a = 0.074(2)$ fm]
- (κ_l, κ_s) choices: Flavor symmetric point and along a line of constant singlet quark mass corresponding to pion masses

  465, 360, 310 MeV
- Measure two point correlation functions $C_{\pi O}(t)$ as function of "time" t
  source: smeared pion at rest
  sink: O - smeared pion or operators at t
  no contribution from "contact" terms

First results – Correlators at $m_\pi = 465$ MeV

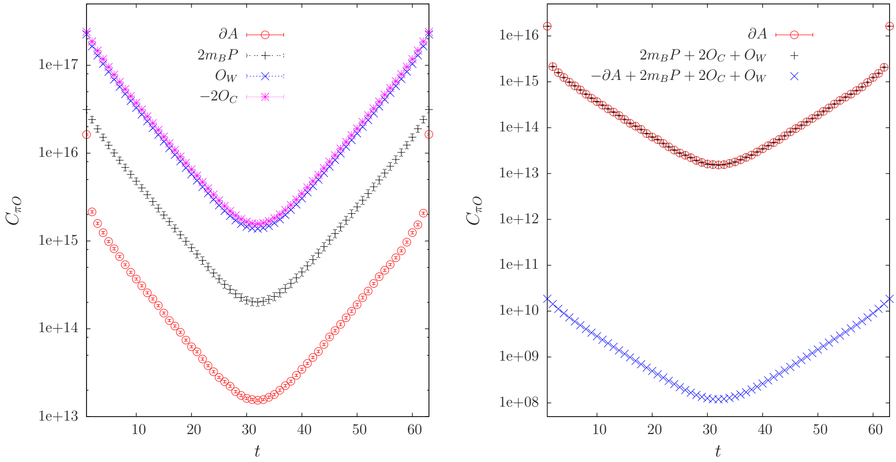


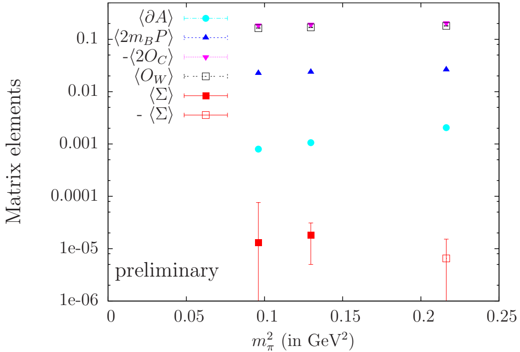
Figure: Correlation functions: (l) individual, (r) differences

First results – Ward identity

- Fit separately the correlators to get the amplitudes
  Combine amplitudes at bootstrap level to obtain bootstrap errors

$$\langle \Sigma \rangle = \langle -\partial A + 2m_B P + 2O_C + O_W \rangle$$

Result



Amazing precision: Ward identity passes nonperturbative test

First results – (unrenormalised) pion decay constant

- $\langle \partial A \rangle = (af_\pi)(am_\pi)^2$ (statistics will be significantly improved)

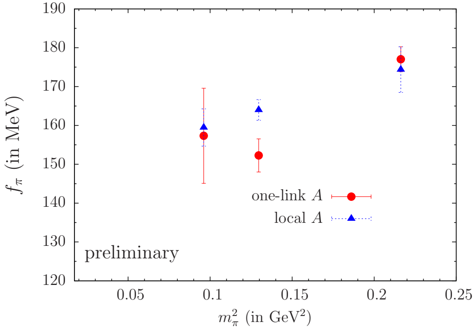


Figure: Unrenormalised pion decay constant as function of m_π

- $Z_{A^{\rm loc}} = 0.873$ (PR D91)
  Hope: Z_A for the one-link current is close to one
  LPT (Wilson gauge action, $g^2 = 1, c_{sw} = 1, \omega = 0.1$) $Z_A = 0.9962$

Short summary and outlook

- We have proposed to use an one-link axial vector present in a lattice axial Ward identity

$$G_{jxi}[\partial A] = 2m_B G_{jxi}[P] + S_{jx}\gamma_5\delta_{xi} + \delta_{jx}\gamma_5 S_{xi} - 2G_{jxi}[O_C] - G_{jxi}[O_W]$$

- Check of WI in 1-loop LPT [partly up to $O(a^2)$]
  Z factor issues and way of realisation of renormalisation invariance are discussed
- Nonperturbative check using correlators of pion at rest with the operators in the WI
  f_π determination started
- Outlook
  Check the axial WI at still lighter pion masses and other β's
  Determine quantities as f_π with more statistics
  Find Z_A nonperturbatively using the FH approach
  Alternative κ_c determination
  Study problems with nonzero momentum transfer (formfactors)