LATTICE INPUT ON THE τ V_{us} PUZZLE

with R. J. Hudspith, R. Lewis, C. Wolfe, J. Zanotti

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OUTLINE

- *Background/context: V_{us} from inclusive FB τ FESRs*
- *The "conventional" implementation and the $> 3\sigma$ low $|V_{us}|$ puzzle*
- *OPE issues and investigations using lattice data*
- *A new implementation strategy (and $|V_{us}|$ results)*

CONTEXT/BACKGROUND I

- Object of interest: the us CKM matrix element V_{us}

  * From 3-family unitarity: $|V_{ud}|$ from $0^+ \rightarrow 0^+$ nuclear β decays (Hardy-Towner'14) $\Rightarrow$ $|V_{us}| = 0.2258(9)(?)$

  * From $K_{\ell 3}$: lattice $f_+(0)$ $\Rightarrow$

    $$|V_{us}| = 0.2233(5)_{exp}(9)_{latt} \quad (RBC/UKQCD\ 2015)$$
    $$|V_{us}| = 0.2238(5)_{exp}(9)_{latt} \quad (FNAL/MILC\ 2012)$$

  * From $\Gamma[K_{\mu 2}]/\Gamma[\pi_{\mu 2}]$: FLAG $n_f = 2+1+1$ f_K/f_π input $\Rightarrow$ $|V_{us}| = 0.2250(4)_{exp}(9)_{latt}$

- C.f. “conventional” FB τ kinematic-weight FESR determination, inclusive ud, us BF input [Gamiz et al.]

  * With HFAG input: $|V_{us}| = 0.2176\,(19)_{exp}(10??)_{th}$ (Passemar, CKM14 τ $|V_{us}|$ summary)

  * 3.4σ low c.f. 3-family unitarity expectations

  * Interesting **if real**, but theory systematics?

- **This talk:**

  * Lattice data to clarify continuum OPE input/treatment issues, quantify OPE errors

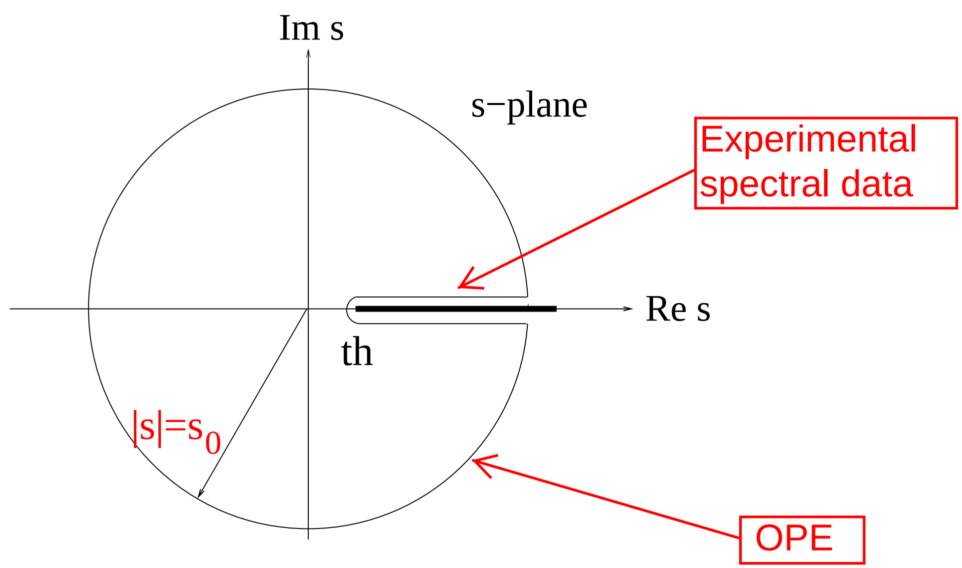
  * Re-visit implementation of FB FESR approach with lattice lessons in mind

CONTEXT/BACKGROUND II

- Basic theoretical tool: FESRs (*Cauchy's Thm*)

- For any s_0, analytic $w(s)$, kinematic-singularity-free Π:

$$\int_{s_{th}}^{s_0} ds\, w(s)\rho(s) = \frac{-1}{2\pi i}\oint_{|s|=s_0} ds\, w(s)\Pi(s)$$



- Inclusive FB τ $|V_{us}|$ determination: From FESRs for V,A current 2-pt function polarizations $\Pi^{(J)}_{ud-us;V+A}(Q^2)$

  * Experimental spectral input: Scaled spectral functions $|V_{ij}|^2\rho^{(J)}_{ij;V/A}(s)$ from $dR_{ij;V/A}/ds$, with $R_{ij;V/A} \equiv \frac{\Gamma[\tau\rightarrow\nu_\tau \,\text{hadrons}_{ij;V/A}(\gamma)]}{\Gamma[\tau^-\rightarrow\nu_\tau e^-\bar{\nu}_e(\gamma)]}$ (+ small us $J=0$ subtraction) [SM "kinematic weight" w_τ]

  * $R^w_{ij;V/A}(s_0)$: Re-weighted $R_{ij;V/A}$ analogue, integrated to variable upper endpoint s_0 in spectrum

$$R^w_{ij;V/A}(s_0) \sim \int_{th}^{s_0} ds\, \frac{dR_{ij;V/A}}{ds}\, \frac{w(s/s_0)}{w_\tau(s/m_\tau^2)}$$

* FESR for $\delta R^w \equiv \frac{R^w_{ud;V+A}}{|V_{ud}|^2} - \frac{R^w_{us;V+A}}{|V_{us}|^2}$ yields

$$\boxed{|V_{us}| = \sqrt{\frac{R^w_{us;V+A}(s_0)}{\frac{R^w_{ud;V+A}(s_0)}{|V_{ud}|^2} - [\delta R^w(s_0)]^{OPE}}}}$$

valid for arbitrary analytic $w(y)$, $s_0 \leq m_\tau^2$

* "Self-consistency tests" :

- $|V_{us}|$ independent of s_0, w (tests control, understanding of OPE, experimental systematics)

- E.g., integrated $D = 2k + 2$ OPE $\sim 1/s_0^k$: errors in higher D treatment $\leftrightarrow$ s_0-instability

- Variable-s_0 FESRs below for $\Delta\Pi_\tau \equiv \Pi^{(0+1)}_{ud-us;V+A}$

- The conventional FB τ FESR implementation:

  * $w = w_\tau$, $s_0 = m_\tau^2 \Rightarrow R_{ud,us;V+A}$ from inclusive ud, us BFs

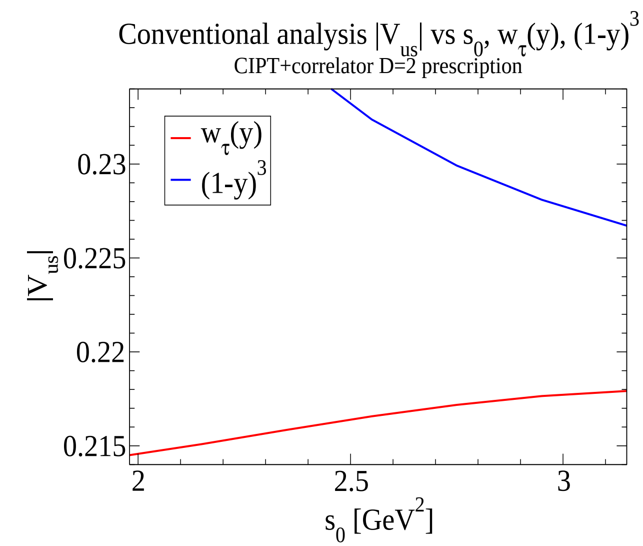
  * $[\delta R^w(s_0)]^{OPE}$ $(D \geq 2)$: for $w = w_\tau$, $s_0 = m_\tau^2$, conventional estimates a few % of individual $R_{ud,us;V+A}$

    $\Rightarrow$ modest OPE accuracy enough for precision $|V_{us}|$

  * w_τ degree 3 $\Rightarrow$ OPE contributions up to $D = 8$

  * OPE: $D = 2$, $D = 4$ known, $D = 6$ OPE estimated with "VSA" (very small), $D = 8$ neglected

- Conventional implementation: self-consistency tests
  - * Variable $s_0 \leq m_\tau^2$, s_0-stability check
  - * Compare $|V_{us}|$ from $\hat{w}$, w_τ $\Delta\Pi_\tau$ FESRs for weights $w_\tau(y) = 1 - 3y^2 + 2y^3$, $\hat{w}(y) = 1 - 3y + 3y^2 - y^3$ $(y = s/s_0)$
    - ∘ $D = 6$, $D = 8$ for $\hat{w}$ are -1, $-1/2$ $\times$ those for w_τ
    - ∘ $D > 4$ assumptions OK for w_τ $\Rightarrow$ also for $\hat{w}$ $\Rightarrow$ $|V_{us}|$ agreement
    - ∘ If NOT, opposite-sign s_0-instabilities for w_τ, $\hat{w}$, decreasing with s_0 for both



It's pretty clear which of the two scenarios is actually realized

- Candidate self-consistency problem sources:

  * Experiment: The less-well-known us distribution

  * Theory: Conventional $D > 4$ assumptions (w_τ, $\hat{w}$ comparison); slow $D = 2$ OPE convergence

- The $D = 2,4$ series and slow $D = 2$ convergence

$$\left[\Delta\Pi_\tau(Q^2)\right]^{OPE}_{D=2} = \frac{3}{2\pi^2}\frac{\bar{m}_s^2}{Q^2}\left[1 + 2.333\bar{a} + 19.933\bar{a}^2 + 208.746\bar{a}^3 + \cdots\right]$$

$$\left[\Delta\Pi_\tau(Q^2)\right]^{OPE}_{D=4} = \frac{\left[\langle m_\ell\bar{\ell}\ell\rangle - \langle m_s\bar{s}s\rangle\right]}{Q^4}\left(2 - 2\,\bar{a} - \frac{26}{3}\,\bar{a}^2\right)$$

with running $\overline{MS}$ quantities $\bar{a} = \frac{\alpha_s(Q^2)}{\pi}$, $\bar{m}_s = m_s(Q^2)$

* $\bar{a}(m_\tau^2) > 0.10 \Rightarrow$ slowly converging $D = 2$ series at ALL scales accessible in τ decay

* Slow $D = 2$ convergence a potential issue for conventional $D = 2$ truncation error estimates

LATTICE RE $D = 2$ SERIES, $D > 4$ OPE ISSUES

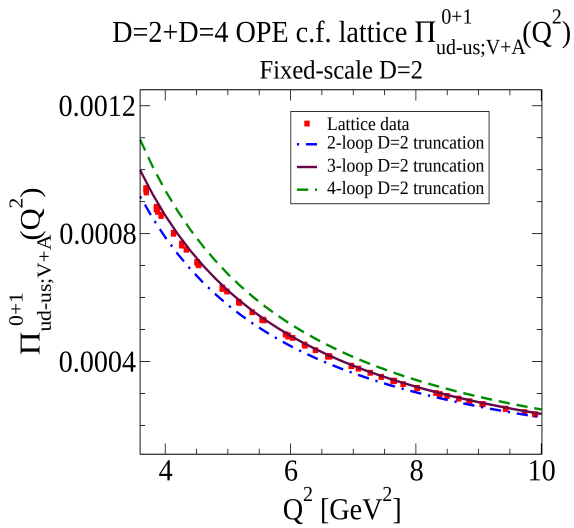
- OPE c.f. RBC/UKQCD lattice $\Delta\Pi_\tau(Q^2)$

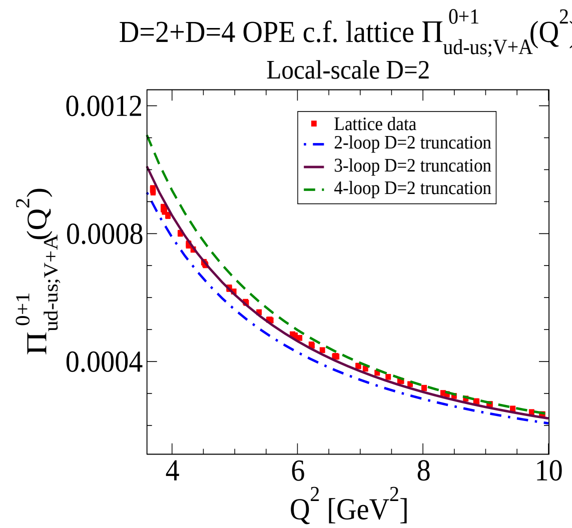
  * Here: lightest m_π, fine ($1/a = 2.38\ GeV$) $32^3 \times 64$ $2+1$ ensemble ($m_\pi \sim 300\ MeV$, $m_\pi L \sim 4.1$)

  * Cylinder cut for continuum correlator behavior [See Randy Lewis' α_s talk]

  * Large Q^2: $D = 2+4$ OPE exploration; lower Q^2: re possible non-negligible $D > 4$ at τ decay scales

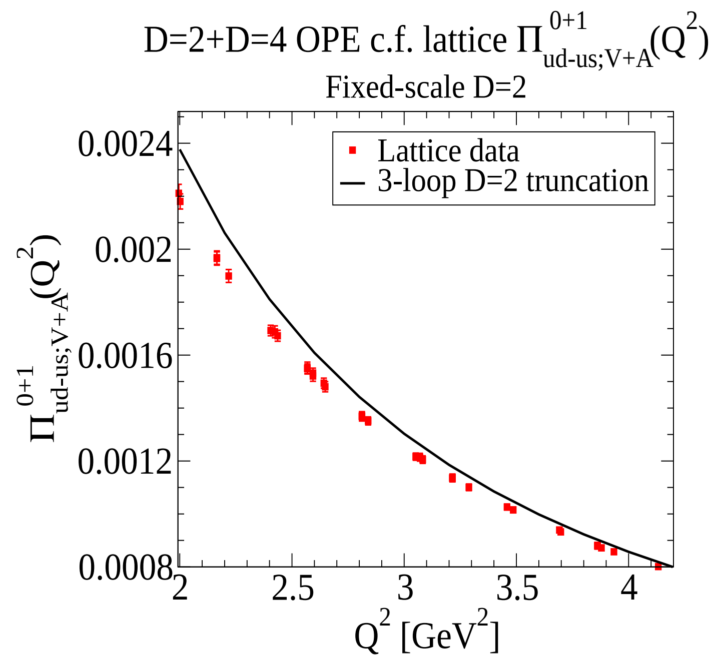
  * Fixed- or local-scale $D = 2$? [c.f. "FOPT" vs "CIPT" issue for FESRs]



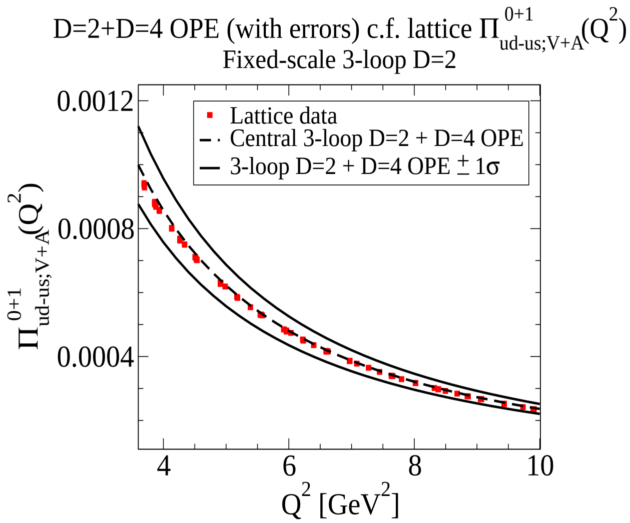


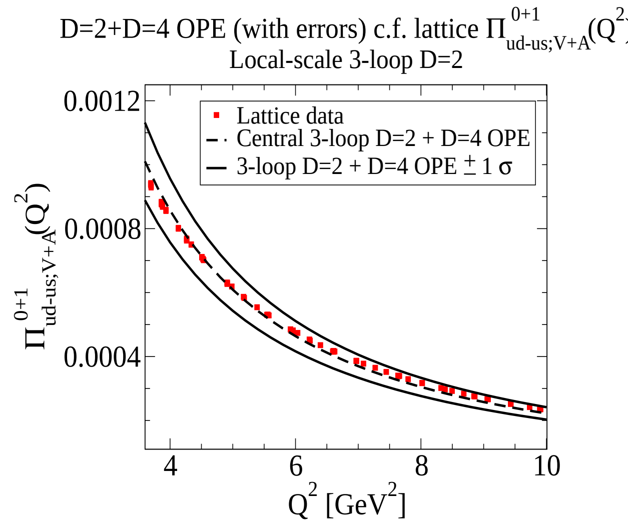
- Higher Q^2: best (excellent) lattice vs $D = 2 + 4$ OPE match for 3-loop-truncated, fixed-scale $D = 2$

- Fixed scale suggests FOPT for FESR $D = 2$



- Onset of $D > 4$ contributions below $\sim 4\ GeV^2$





- Standard $D = 2$, $D = 4$ error estimates conservative, despite very slow convergence of the $D = 2$ series

A NEW IMPLEMENTATION STRATEGY BASED ON LATTICE/CONTINUUM LESSONS

- No assumptions re $D > 4$; include contributions and fit effective condensates C_D as part of analysis

- 3-loop-truncated FOPT for $D = 2$ contribution; standard error estimates for $D = 2 + 4$

- Use range of s_0 and fit both $|V_{us}|$, C_D

- Fits simplest for $w(y) = w_N(y) = 1 - \frac{y}{N-1} + \frac{y^N}{N-1}$ (C_{2N+2}, $|V_{us}|$ as only fit parameters)

- Self-consistency check: $|V_{us}|$ from different w_N

OPE, SPECTRAL INPUT

- PDG, FLAG, HPQCD input for $D = 2, 4$ OPE
- ud V+A spectral data from ALEPH 2013
- us V+A spectral data from sum over exclusive modes [$>$ 90% of B_{us}^{TOT} from $K_{\ell 2}$, Belle, BaBar $K\pi$, $K\pi\pi$, 3K results; residual: 1999 ALEPH]
- Here, for brevity, with $K\pi$ normalization including preliminary BaBar $B[\tau \to K^- \pi^0 \nu_\tau]$ update

RESULTS OF THE NEW ANALYSES

- Fitted $|V_{us}|$ (as expected) between w_τ and $\hat{w}$ conventional implementation results

- Fitted C_D show FB cancellation in comparison to fitted ud V+A analogues (qualitative self-consistency test)

- Excellent stability of both $|V_{us}|$, C_{2N+2}, wrt variation of fit window size; excellent agreement of central $|V_{us}|$ from different w_N

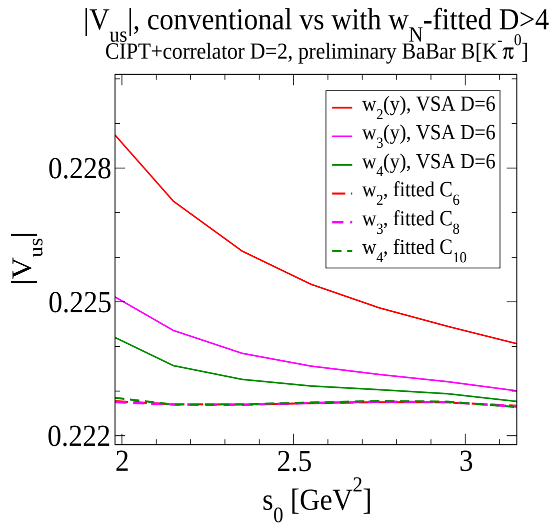
- Results using preliminary $K\pi$ BF update favored by lattice

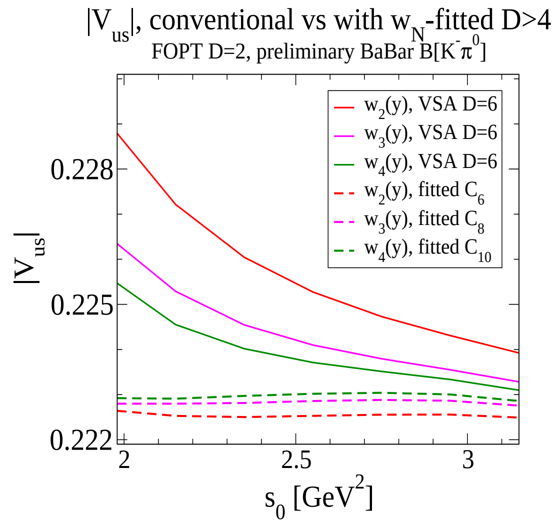
Weight	$\lvert V_{us}\rvert$ CIPT+corr $D=2$	$\lvert V_{us}\rvert$ FOPT $D=2$
w_2	0.22271(228)	0.22252(228)
w_3	0.22271(228)	0.22282(228)
w_4	0.22271(229)	0.22296(229)

Error budget, 3-loop-truncated FOPT $D=2$ fits

Source	$\delta\lvert V_{us}\rvert$ (w_2 FESR)	$\delta\lvert V_{us}\rvert$ (w_3 FESR)	$\delta\lvert V_{us}\rvert$ (w_4 FESR)
$\delta\alpha_s$	0.00001	0.00004	0.00004
$\delta m_s(2\ GeV)$	0.00017	0.00019	0.00019
$\delta\langle m_s\bar{s}s\rangle$	0.00035	0.00035	0.00035
$\delta(long\ corr)$	0.00009	0.00009	0.00009
ud exp	0.00027	0.00028	0.00028
us exp	0.00226	0.00227	0.00227

Fit quality: s_0-stability for conventional implementation, except with fitted C_{2N+2}





SUMMARY/CONCLUSIONS

- Continuum, lattice $\Rightarrow$ conventional $D > 4$ assumptions unreliable, must fit $D > 4$ effective condensates

- Fitting $D > 4$ resolves s_0-, $w(y)$-dependence problems

- Lattice $\Rightarrow$ 3-loop-truncated FOPT for $D = 2$ series

- Lattice $\Rightarrow$ $D = 2 + 4$ OPE error estimate conservative

- $|V_{us}|$ errors then strongly dominated by us experimental uncertainties, subject to experimental improvement

- $D > 4$ contributions, when included, raise $|V_{us}|$ by ~ 0.0020, reduce previous discrepancy

- New preliminary $K^- \pi^0$ BF normalization, 3-weight combined fit result $|V_{us}| = 0.2228(23)_{exp}(5)_{th}$ in agreement with other determinations (especially $K_{\ell 3}$)

  (c.f. $0.2200(23)_{exp}(5)_{th}$ with 2014 HFAG $B[K^- \pi^0]$)

- Theory error ~ 0.0005 c.f. 0.0009 for $K_{\ell 3}$, $\Gamma[K_{\mu 2}]/\Gamma[\pi_{\mu 2}]$ (lattice crucial here) $\Rightarrow$ **method competitive once us experimental errors sufficiently improved**

- **Old τ $|V_{us}|$ puzzle almost certainly resolved**

BACKUP SLIDES

- Spectral functions from hadronic τ decay distributions: With $R_{ij;V/A} \equiv \frac{\Gamma[\tau \to \nu_\tau\, \text{hadrons}_{ij;V/A}\, (\gamma)]}{\Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e (\gamma)]}$

$$\frac{dR_{V/A;ij}}{ds} = \frac{12\pi^2\, |V_{ij}|^2 S_{EW}}{m_\tau^2} \left[w_\tau \left(\frac{s}{m_\tau^2} \right) \rho^{(0+1)}_{ij;V/A}(s) + w_L \left(\frac{s}{m_\tau^2} \right) \rho^{(0)}_{ij;V/A}(s) \right]$$

$$w_\tau(y) = (1-y)^2(1+2y), w_L(y) = -2y(1-y)^2$$

More on the $D > 6$ VSA guesstimate

- VSA VERY crude: sizable (e.g. $\sim 4 - 5$), channel-dependent violation in ud V, A channels

- Crudeness $\Rightarrow$ double very close cancellation (in individual ud, us V+A sums, and in FB $ud - us$ difference) dangerous to rely on

- Integrated $D > 4 << $ than integrated $D = 2$ for w_τ (absence of $O(y)$ term) but comparable for other $w(y)$ $\Rightarrow$ small doubly-cancelled $D = 6$ VSA estimate again suspect

MORE ON THE us DATA

- K pole via $f_K|V_{us}|$ from $K_{\ell 2}$

- Rather precise unit-normalized $K^-\pi^0$, $\bar{K}^0\pi^-$, $K^-\pi^+\pi^-$, $\bar{K}^0\pi^-\pi^0$, $3K$ distributions from Belle, BaBar (main uncertainties from BFs)

- K, B-factory modes over 90% of B_{us}^{TOT}

- Residual us exclusive mode contributions from 1999 ALEPH data, covariances

Alternative $K\pi$ BF normalizations

- Existing HFAG $B[(K^-\pi^0 + \bar{K}^0\pi^-)\nu_\tau] = 0.0126$

- Existing $B[K^-\pi^0\nu_\tau] = 0.00431(15)$ value $\rightarrow$ preliminary BaBar (Adametz thesis) result 0.00500(15) yields $B[(K^-\pi^0 + \bar{K}^0\pi^-)\nu_\tau] = 0.0133$

- Central $B[(K^-\pi^0+\bar{K}^0\pi^-)\nu_\tau]$ from $K_{\ell 3}$, dispersion rel'n expectations [ACLP13] also 0.0133

- 0.07% difference "small" but represents $\sim 2.4\%$ of B_{us}^{TOT}, hence $\sim 1.2\%$ increase in $|V_{us}|$

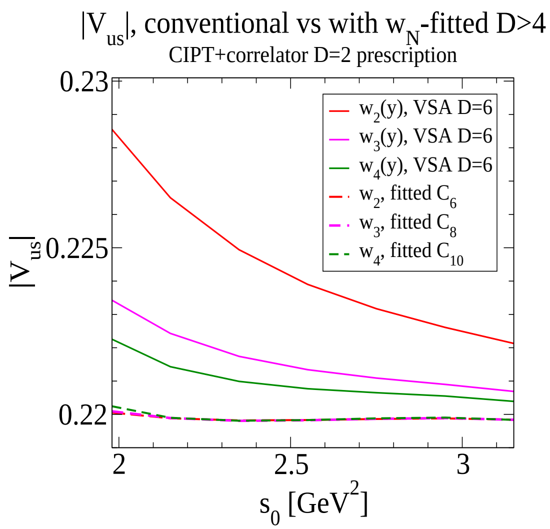
Results for $|V_{us}|$ for current $K\pi$ BFs:

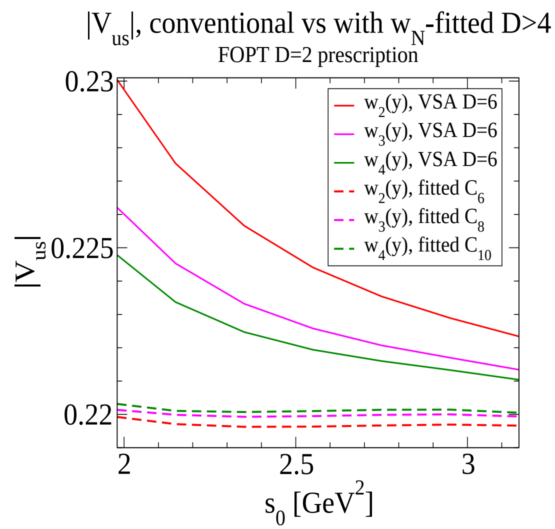
Weight	$\lvert V_{us}\rvert$ CIPT+corr $D=2$	$\lvert V_{us}\rvert$ FOPT $D=2$
w_2	0.21985(230)	0.21966(230)
w_3	0.21985(231)	0.21966(231)
w_4	0.21985(231)	0.22009(231)

Error budget, existing $K\pi$ BFs

Source	$\delta\lvert V_{us}\rvert$ (w_2 FESR)	$\delta\lvert V_{us}\rvert$ (w_3 FESR)	$\delta\lvert V_{us}\rvert$ (w_4 FESR)
$\delta\alpha_s$	0.00001	0.00003	0.00005
$\delta m_s(2\ GeV)$	0.00017	0.00018	0.00020
$\delta\langle m_s\bar{s}s\rangle$	0.00034	0.00034	0.00034
$\delta(long\ corr)$	0.00009	0.00009	0.00009
ud exp	0.00027	0.00027	0.00027
us exp	0.00229	0.00229	0.00230

Stability of $|V_{us}|$ with fitted C_{2N+2} input, existing $K\pi$ BF normalization





FUTURE PROSPECTS/COMMENTS

- Errors dominated by us spectral integral errors

- For exclusive us modes measured by Belle, BaBar, these errors dominated by BF normalization uncertainties $\Rightarrow$ improvements in BF errors highly desirable

- Updated ud V and A distributions from Belle and BaBar also most welcome, including 4π (where unexpectedly large CVC violations c.f. $e^+e^- \to 4\pi$ still not resolved)

- Full inclusive us distribution from B-factories (or Belle II) also a highly desirable long-term goal