QCD spectroscopy and quark mass renormalisation in external magnetic fields with Wilson fermions

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1. Why spectroscopy in external magnetic fields?

External magnetic fields: Physical relevance

In the past few years: Investigation of the effect of QED fields on QCD observables has attracted a lot of attention.

- Motivation for QCD+QED:
  - For high precision observables (e.g. spectrum) QED effects become visible with current accuracy.
- Strong external magnetic fields appear in:
  - Non-central heavy Ion collisions ($\sim 10^{18}$ G $\rightarrow 0.02$ GeV^2)
    [Kharzeev, PLB 633 (2006); Kharzeev, McLerran, Warringa, NPA 803 (2008); Skokov, Illarionov, Toneev, IJMPA 24 (2009)]
  - Surface and interior of magnetars ($\sim 10^{15}$ to $\sim 10^{20}$ G $\rightarrow \lesssim 1.96$ GeV^2)
    [Review: Ferrer, de la Incera, LNP 871 (2013)]
  - The early universe ($\sim 10^{9}$ G – at $T_C^{\rm QCD}$)
    (due to gradients in the VEV of the Higgs field after phase transitions)
    [Vachaspati, PLB 265 (1991); Enqvist, Olesen, PLB 319 (1993)]

$\Rightarrow$ Understanding the properties of QCD in external magnetic fields is important!

External magnetic fields: Physical relevance

Effects of external fields:

- Influence the phase diagram of QCD!

  [Review: Andersen, Naylor, Tranberg, arXiv:1411.7176]

- Affect the spectrum (masses) of the theory!
  - Field has a direct influence on masses of charged particles.
  - Uncharged particles influenced indirectly. (subleading effect?)

For many applications both effects are of relevance!

- Phase diagram is by now rather well understood from LQCD!

  [e.g. Bali, *et al*, JHEP 1202 (2012); Endrödi, arXiv:1504.08280]

- For the spectrum there are only some first quenched studies.

  [SU(2): Braguta, *et al*, PLB 718 (2012); Luschevskaya, Larina, NPB 884 (2014)]

  [SU(3): Hidaka, Yamamoto, PRD 87 (2013); Luschevskaya, Teryaev, Kotchetkov, arXiv:1411.4284]

- Charged vector meson condensation: [Muller, Schramm, Schramm, MPLA 07 (1992)]
  Naively: $m_\rho^\pm(\mathbf{B}_{\rm cr}) = 0$ for $\mu > 0$ (μ: proj. of magn. moment on $\mathbf{B}$)

2. Quenched spectrum with Wilson fermions

Lattice setup – External field

- Introduce external field by minimal coupling to gauge potential A_μ:

$$D_\mu = D_\mu^{\mathrm{QCD}} + iq_f A_\mu \qquad q_f\text{: charge of flavour } f$$

Choice:
$\mathbf{B} = B\hat{z} \;\Leftrightarrow\; A_0(x) = A_3(x) = 0\,,\; A_1(x) = -(B/2)x_2\,,\; A_2(x) = (B/2)x_1$

- On the lattice: Replace gluonic links $U_\mu^{\mathrm{G}}(x)$ in Dirac matrix M by:

$$U_\mu(x) = U_\mu^{\mathrm{G}}(x)u_\mu(x) \in \mathrm{U}(3) \qquad u_\mu(x) = \exp\big(iaq_f A_\mu(x)/2\big)\text{: EM links}$$

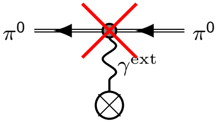
Magnetic field quantisation via $qB\,L_x L_y = 2\pi N_b$.

- Fermion matrix M becomes flavour-dependent! $\Rightarrow$ $N_f = 1+1$-setup
- Here: Use (for moment unimproved) Wilson fermions!
- Quenched test setup: (Neglect sea quark effects!)
  - $48x16^3$ Lattice with $a \approx 0.09$ fm.
  - ~ 200 meas (1 inversion per config)
  - Solver: DFL-SAP-GCR ([Lüscher, CPC 156 (2004); JHEP 0707 (2007)])
  - Focus on $m_\pi \approx 400$ MeV (several masses to check dependence)

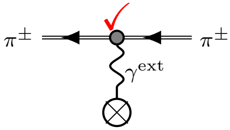
Results for the spectrum – Pions

Theoretical expectation (for small $|B|$):

π^0: (spin 0, $q = 0$)

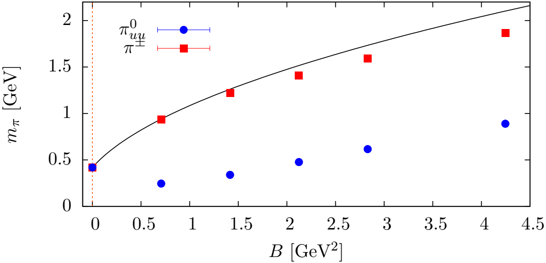


$\pi^{\pm}$: (spin 0, $q = \pm 1$)



$\Rightarrow$ Not affected directly!

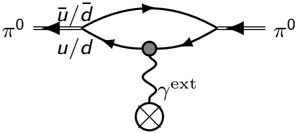
$\Rightarrow$ Energies: $E^2 = m_\pi^2 + (1 + 2n)|qB|$



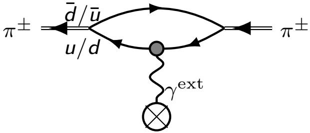
Results for the spectrum – Pions

Theoretical expectation (for small $|B|$):

Internal structure:

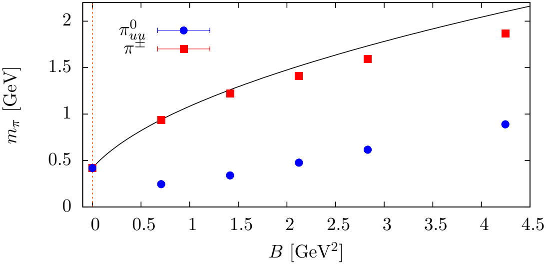


Internal structure:



$\Rightarrow$ Affected by subleading effects!

$\Rightarrow$ Additional subleading effects!



Results for the spectrum – ρ-mesons $s_z = 0$

External photons enable decays:

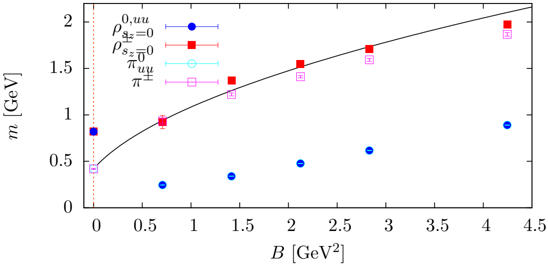
$$\rho^{0,\pm}_{s_z=0} \to \gamma\pi^{0,\pm}$$

$\Rightarrow$ Pion states appear in $\langle V_3 V_3 \rangle$ correlation functions.

Groundstate: Pion state!

Possible method to solve the problem:

- GEVP via a correlation matrix in some operator basis.
- Problem: Multi-π states with $E < m_\rho$. (especially for small m_π)
  $\Rightarrow$ Needs large correlation matrices.



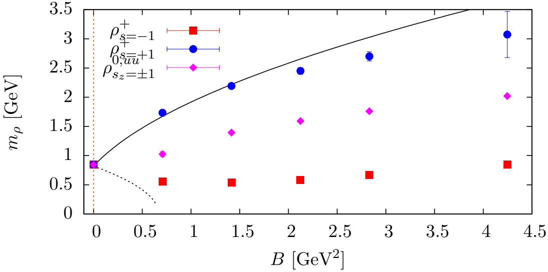
Results for the spectrum – ρ-mesons $s_z = \pm 1$

- $\rho^0_{s_z=\pm1}$: (spin 1, $q = 0$)
  $\Rightarrow$ Only indirect effects!
- $\rho^\pm_{s_z=\pm1}$: (spin 1, $q = \pm1$)
  $\Rightarrow$ Direct coupling to **B**.

  Energies:
  $E^2 = m_\rho^2 + (1 + 2n + g\mu)|qB|$
  $\mu = \pm1$, g: g-factor of the particle

ρ-meson condensation:

- $E = 0$ when $\mu = -1$ and $eB = m_\rho^2$.
- System becomes superconducting!
  [Chernodub, PRL 106 (2011)]
- QCD inequalities:
  Condensation cannot occur!
  [Hidaka, Yamamoto, PRD 87 (2013)]



Comparison of π^0 masses to other results

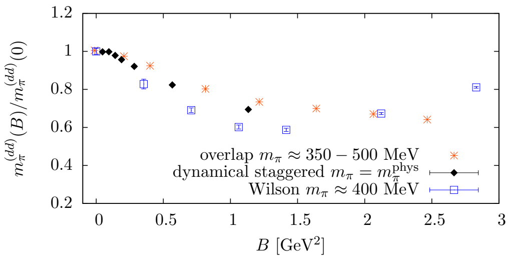
π^0 shows a (relatively) small dependence on the magnetic field.

But: Behaviour is non-monotonous!

- Consistent with other results from Wilson fermions.
  [Hidaka, Yamamoto, PRD 87 (2013)]
- Overlap results look different. [Luschevskaya, Teryaev, Kotchetkov, arXiv:1411.4284]
- Staggered results?

Is this a physical effect?

(Also: What about disconnected contributions?)



3. Additive quark mass renormalisation in QCD+QED

What we ignored up to now!

Appears to be a discrepancy between overlap and Wilson results!

(However: Many systematic effects with unknown impact!)

One effect which has been ignored in Wilson studies:

Change of κ_c with **B**!

I.e.: Keeping κ fixed leads not to a line of constant physics!

- κ_c is an artefact due to the introduction of the Wilson term.
  $\Rightarrow$ A change of the operator means a change in κ_c.
- The change in κ_c is significant for QCD+QED.

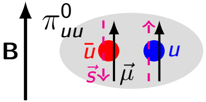
  [e.g. Borsanyi, *et al* Science 347 (2015)]
- What happens for external magnetic fields?

First: Check the effect in the free case.

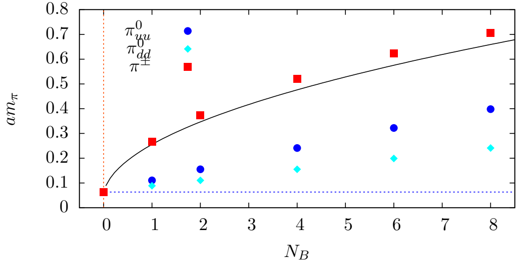
- Easiest way: Look at pole in the quark propagator.
- Problem: Magnetic field spoils applicability of Fourier transformation!

Free case

Alternative: Look at the energy levels of free “pions”!
(in practice: two quarks in a box with imposed π-quantum numbers)



$\Rightarrow \quad E_{uu/dd} = 2m_{u/d}$ $\qquad \Rightarrow \quad E_{ud} = m_u + \sqrt{m_d + 2|q_d B|}$



$\Rightarrow$ Quark mass changes with magnetic field!

Free case – adjusted κ

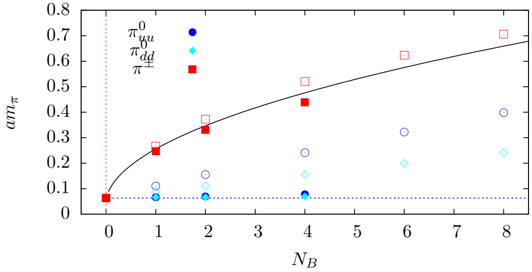
Here: Change in the quark masses can only be due to a change in κ_c!

⇒ Recompute κ_c with condition

$$E_{uu/dd}(\mathbf{B}) \sim \frac{1}{\kappa} - \frac{1}{\kappa_{c,u/d}(\mathbf{B})} = \mathrm{const}$$

We find: $\Delta m_{c,f} = \frac{6\pi}{N_x N_y} q_f N_b$ (free case)

With retuned masses: $\kappa_f \to \left(\kappa^{-1} - \Delta m_{c,f}\right)^{-1}$



Tuning of quark masses in the interacting theory

- Standard method for QCD+QED:
  Adjust $\bar{m}_{u/d}$ so that pseudo-pions ($\pi^0_{uu/dd}$) masses remain constant.
  [BMW, PRL 111 (2013); Science 347 (2015)]
- Advantage: No renormalisation needed.
- Problem: Disconnected diagrams present. $\Rightarrow$ Typically neglected!

Application to external magnetic fields?

- Method can be applied when we know the physical mass in this situation!
  But: Masses will change with the magnetic field (objects of interest)!
- $\Rightarrow$ Method cannot be applied!

Alternatives to determine κ_c:

- Use fact: $m_{\pi^0}(\mathbf{B}) \to 0$ for $m_{u/d} \to 0$.
- Extract m_f from Ward identities (WIs), determine $\kappa_{c,f}$ via $m_f \to 0$!

Ward identities for QCD+QED

- The WIs are obtained in the standard way!
  Including QED: covariant derivative does not commute with τ^i!
  (because the links are flavour matrices)
  $\Rightarrow$ New terms appear in WIs!
- Continuum WIs:

$$\partial_\mu (J_V)^j_\mu(x) = i\epsilon_{3jk}\left\{(m_u - m_d)\bar{\psi}(x)\frac{\tau^k}{2}\psi(x) + i\bar{\psi}(x)\gamma_\mu A_\mu(x)\frac{\tau^k}{2}\psi(x)\right\}$$

$$\partial_\mu (J_A)^j_\mu(x) = (m_u + m_d)\bar{\psi}(x)\gamma_5\frac{\tau^j}{2}\psi(x) + \delta_{j3}\frac{1}{2}(m_u - m_d)\bar{\psi}(x)\gamma_5\mathbf{1}\psi(x) - \epsilon_{3jk}\bar{\psi}(x)A_\mu(x)\gamma_\mu\gamma_5\frac{\tau^k}{2}\psi(x)$$

  (see also [Blum, *et al* PRD 82 (2010)])
- On the lattice with Wilson fermions:
  Similar WIs including new dimension 5 operators!
- WIs potentially provide clean definition for quark masses in QCD+QED!

Ward identities for QCD+QED – quark masses

The above WIs can be used to define current quark masses:

- Charged WIs ($\bar{d} \dots u$ and $\bar{u} \dots d$):
  - Advantage: Disconnected diagrams do not appear!
  - Disadvantage: Vector and axial WI are needed for individual quark masses!
- "Neutral" WIs ($\bar{u} \dots u$ and $\bar{d} \dots d$):

  Define current quark masses associated with pions $\pi^0_{uu/dd}$ via:

$$m^{\mathrm{AWI}}_{uu/dd} = \frac{\nabla^0_x \left\langle \left(\widetilde{A}_{uu/dd}\right)^3_0 P_{uu/dd} \right\rangle}{\left\langle P_{uu/dd} \, P_{uu/dd} \right\rangle}$$

  - Advantage: Easy to compute (like standard PCAC masses)
  - Disadvantages:

    Disconnected diagrams and neutral pion mixings are ignored.

    $\Rightarrow$ Leads to systematic effects of unknown size!

Note: For now ignore all subjects associated with multiplicative renormalisation.

4. A first test of quark mass tuning for external magnetic fields

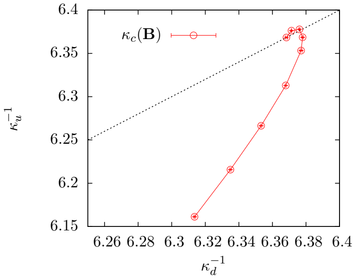
Determination of $\kappa_{c,u/d}$

First naive strategy:

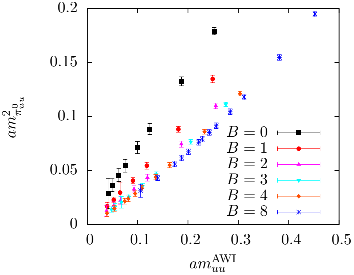
Perform a (linear) chiral extrapolation of $m^{\mathrm{AWI}}_{uu/dd}$ to determine $\kappa_{c,u/d}$!

(Problems have been discussed above – but lets see how it works)

Results in the $(\bar{m}_u, \bar{m}_d)$-plane:



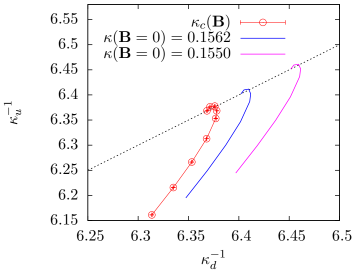
Compatibility with pseudo-pions:



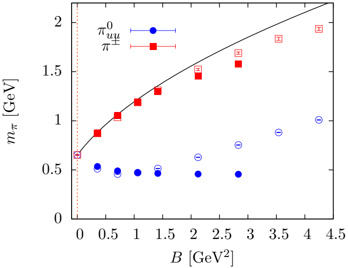
(Results have been checked with higher orders in $(\bar{m} - \bar{m}_c)$.)

Results with adjusted κ-values – Pions

Adjust $\kappa_{u/d}$-values:



Results for pions:



$\Rightarrow$ Non-monotonic behaviour appears to be gone!

Next step: Look at behaviour of ρ-mesons!

Summary

- Presented status of exploratory study of the QCD spectrum in external magnetic fields.
- In lattice QCD ρ-mesons do not seem to condense!
- But: observe inconsistent results between Wilson and overlap fermions!
  $\Rightarrow$ Are the results for ρ-mesons correct?
- One systematic effect neglected so far:
  Additive quark mass renormalisation for Wilson fermions!
  Effect is present even for free quarks!
- We introduced (and briefly discussed) methods how to tune the quark mass!
  However, determining κ_c is conceptually challenging in the presence of external magnetic fields!
  - One method: Use neutral pseudo-pions, $\pi^0_{uu/dd}$.
    Conceptually problematic: Disconnected diagrams and mixings neglected!
  - A generically clean way to define quark masses: Ward identities!
    $\Rightarrow$ We have derived them for the case of QCD+QED.
- First test: Look at tuning via masses associated with pseudo-pions. $\Rightarrow$
  Inconsistencies seem to disappear!

Perspectives

- What happens to ρ-mesons after tuning?
- Several issues have been ignored:
  - disconnected diagrams
  - mixings between neutral pions

  $\Rightarrow$ Is the tuning actually correct?
- Plans for the future:
  - Compare the results to results obtained with staggered fermions.
  - Compute disconnected diagrams relevant for neutral pions and look at their importance.
  - Look at the quark masses obtained with charged Ward identities:
    No disconnected diagrams are needed!
    Might provide clean definitions for quark masses in QCD+QED.
    Problem: Need accurate results for vector WIs.
  - Look at lattice artifacts and finite size effects.
- Final goal: Extract the spectrum in full QCD.

Thank you for your attention!

Lattice setup – Parameters

- Test setup:
  - Use 48×16^3 lattice.
  - Standard Wilson action for gluons with $\beta = 6.00$.
    $\Rightarrow \quad a \approx 0.09$ fm [Bhattacharya, *et al*, PRD 53 (1996)]
  - Statistic ~ 200 configurations
- Inversions:
  - Use DFL-SAP-GCR solver [Lüscher, CPC 156 (2004); JHEP 0707 (2007)]
  - We use one inversion per configuration.
  - Point-smeared correlation functions.
- Quark masses:
  - Use around 8 (degenerate) κ-values to study the quark mass dependence with $0.153 \leq \kappa \leq 0.1563$.
    ($\kappa_c \approx 0.1571$ [Bhattacharya, *et al*, PRD 53 (1996)])
  - Mostly: Focus on $\kappa_{u,d} = 0.1562$
    $\Rightarrow \quad m_\pi(\mathbf{B} = 0) \approx 400$ MeV