Phenomenology with Lattice NRQCD b Quarks

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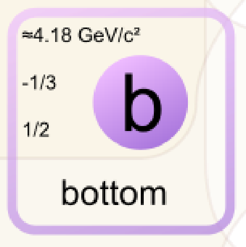
Our approaches to b quarks

In Glasgow, we take two complementary approaches to b quarks: Nonrelativistic QCD and heavy HISQ.

Here I will focus exclusively on NRQCD (for b quarks). So why NRQCD?

- b quark can be simulated at its physical mass.
- NRQCD proceeds with a relatively straightforward evolution equation $\rightarrow$ computationally inexpensive.



Gluon Field Configurations [1212.4768]

Gluon field configurations provided by MILC Collaboration with $2+1+1$ flavours of HISQ quarks in the sea. Those marked (*) are ensembles with *physical* light quark masses.

Set	β	am_l	am_s	am_c	$L/a \times T/a$	$n_{\rm cfg}$
1	5.80	0.013	0.065	0.838	16×48	1020
2	5.80	0.0064	0.064	0.828	24×48	1000
3^*	5.80	0.00235	0.0647	0.831	32×48	1000
4	6.00	0.0102	0.0509	0.635	24×64	1052
5	6.00	0.00507	0.0507	0.628	32×64	1000
6^*	6.00	0.00184	0.0507	0.628	48×64	1000
7	6.30	0.0074	0.037	0.440	32×96	1008
8^*	6.30	0.0012	0.0363	0.432	64×96	621
9	6.72	0.0048	0.024	0.286	48×144	1000

NRQCD

We use the following NRQCD Hamiltonian:

$$e^{-aH} = \left(1 - \frac{a\delta H}{2}\right)\left(1 - \frac{aH_0}{2n}\right)^n U_t^\dagger \left(1 - \frac{aH_0}{2n}\right)\left(1 - \frac{a\delta H}{2}\right)$$

where we include terms up to $\mathcal{O}(v^4)$

$$\begin{aligned}
aH_0 &= -\frac{\Delta^{(2)}}{2am_b} \\
a\delta H &= -c_1\frac{(\Delta^{(2)})^2}{8(am_b)^3} + c_2\frac{i}{8(am_b)^2}\left(\nabla\cdot\tilde{\mathbf{E}} - \tilde{\mathbf{E}}\cdot\nabla\right) \\
&\quad - c_3\frac{g}{8\left(am_b\right)^2}\sigma\cdot\left(\tilde{\nabla}\times\tilde{\mathbf{E}} - \tilde{\mathbf{E}}\times\tilde{\nabla}\right) \\
&\quad - c_4\frac{g}{2am_b}\sigma\cdot\tilde{\mathbf{B}} + c_5\frac{a^2\Delta^{(4)}}{24am_b} - c_6\frac{a\left(\Delta^{(2)}\right)^2}{16n\left(am_b\right)^2}
\end{aligned}$$

with most c_i coefficients $\mathcal{O}(\alpha_s)$ improved. [1110.6887],[1105.5309]

Υ Decay Constant & Leptonic Width [1408.5768]

Leptonic Width:

$$\Gamma(\Upsilon^{(n)} \to e^+e^-) = \frac{4\pi}{3}\alpha_{\rm em}^2 e_b^2 \frac{f_{\Upsilon^{(n)}}^2}{M_{\Upsilon^{(n)}}}$$

Decay constant:

$$\langle 0|J_{V,i}|\Upsilon_j^{(n)}\rangle = f_{\Upsilon^{(n)}} M_{\Upsilon^{(n)}} \delta_{ij}$$

The Υ leptonic width is experimentally well measured, so a determination of this quantity on the lattice is a good test of QCD, and also of our approach to b quark physics.

Time moments [1408.5768]

We need to renormalise our NRQCD currents,

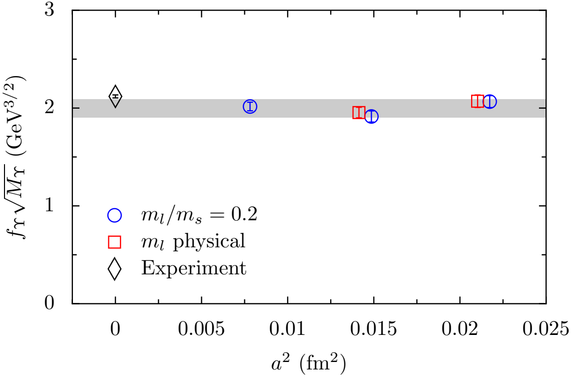
$$J_V = Z_V \left(J^{(0)}_{V,\mathrm{NRQCD}} + k_1 J^{(1)}_{V,\mathrm{NRQCD}} \right)$$

by determining both k_1 and overall normalisation Z_V. We do this by comparing NRQCD to continuum QCD perturbation theory for time moments of the correlators:

$$G_n^V = \frac{g_n^V(\alpha_s, \mu/m_b)}{[a\overline{m}_b(\mu)]^{n-2}}$$

$$G_n^V = Z_V^2 G_n^{V,\mathrm{NRQCD}}$$

Υ Leptonic Width [1408.5768]



Decay Constant

$$f_\Upsilon = 0.649(31)\ \text{GeV}$$

Leptonic Width

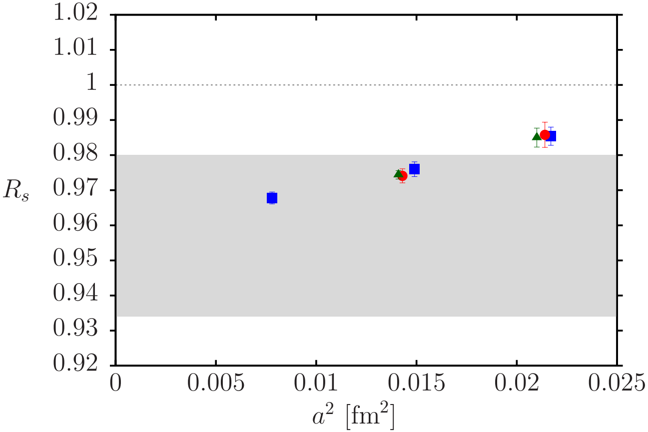
$$\Gamma(\Upsilon \to e^+e^-) = 1.19(11)\ \text{keV}$$

B Meson Decay Constants [1503.05762]

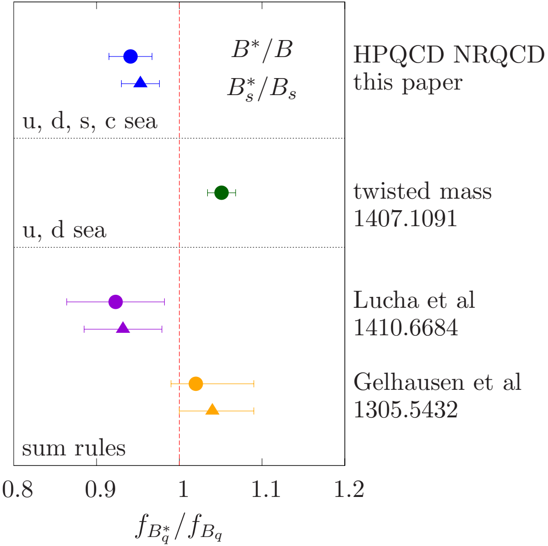
We have updated our picture of B meson decay constants:

- B^*, B_s^* and B_c^* as well as B, B_s and B_c

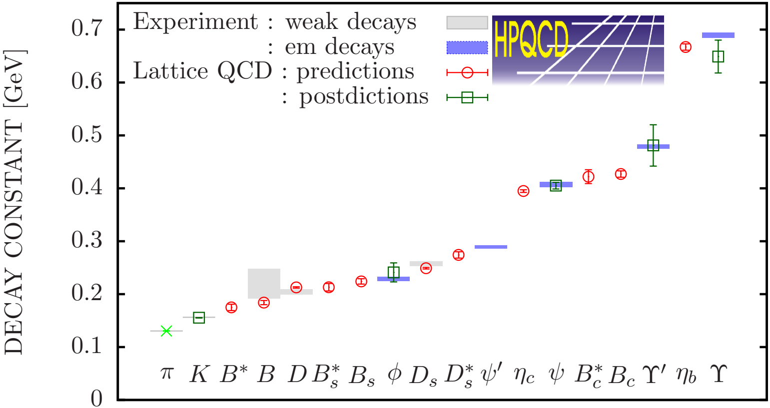
$$R_s = \frac{f_{B_s^*}\sqrt{M_{B_s^*}}}{f_{B_s}\sqrt{M_{B_s}}}$$



$f_{B_q^*}/f_{B_q}$ [1503.05762]



Decay Constants: summary



Determination of m_b [1408.5768]

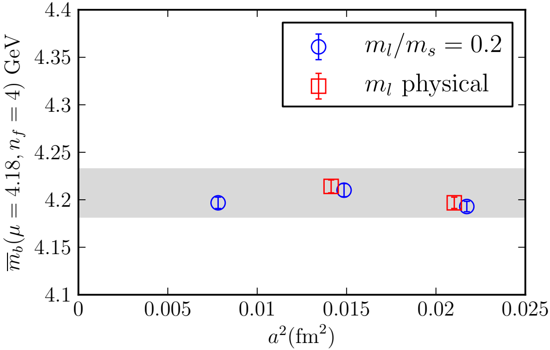
m_b is an important parameter in the Standard Model, for example in accurately determining Higgs branching fraction to $b\bar{b}$.

Recall:

$$G_n^V = \frac{g_n^V(\alpha_s, \mu/m_b)}{[a\overline{m}_b(\mu)]^{n-2}}$$

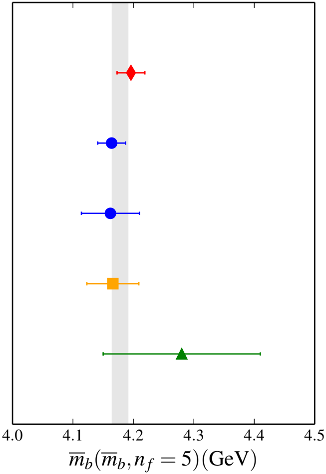
so we can make a determination of m_b using NRQCD time moments.

Determination of m_b [1408.5768]



$$\overline{m}_b(\mu = \overline{m}_b, n_f = 5) = 4.196(23)\ \mathrm{GeV}$$

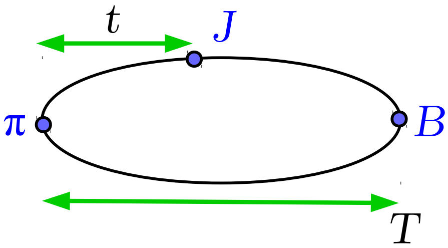
m_b: lattice summary



HPQCD NRQCD JJ	[1408.5768]
HPQCD HISQ JJ $n_f = 3$	[1004.4285]
HPQCD HISQ ratio $n_f = 4$	[1408.4169]
HPQCD NRQCD E_0	[1302.3739]
ETMC ratio	[1311.2837]

Semileptonic decays

We use NRQCD b quarks and HISQ light quarks in our calculation of $B \to \pi \ell \nu$.



We pick multiple values of T and fit B and π 2-point correlators simultaneously with 3-point correlators.

$B \to \pi$ at zero recoil

For B and π at rest, $q^2_{\max} \approx 26.5 \text{ GeV}^2$,

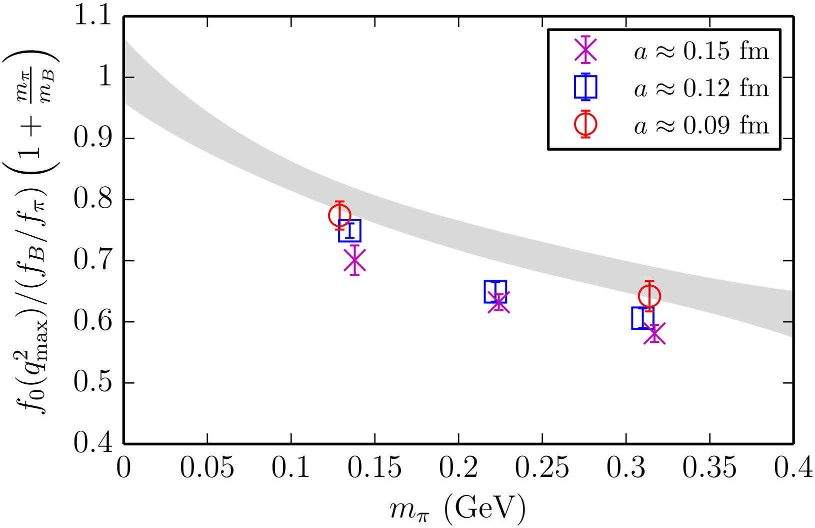
$$\langle \pi | V^0 | B \rangle = f_0\left(q^2_{\max}\right)(m_B + m_\pi)$$

Soft pion theorem relates this to decay constants in $m_\pi \to 0$ limit:

$$f_0(q^2_{\max}) = f_B / f_\pi$$

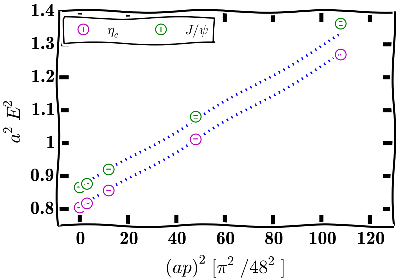
This relation was previously studied in lattice QCD, but it did not seem to hold. We now have the opportunity to study with light quarks at physical mass.

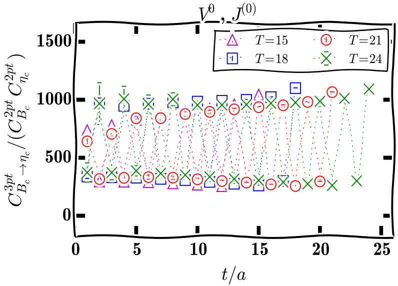
Soft Pion Theorem



A brief look at the future...

We are optimistic about being able to explore the full q^2 range in $B_c \to \eta_c \ell \nu$ decays on superfine lattices ($a \approx 0.06$ fm). $p_{\eta_c} = 2.4$ GeV in the B_c rest frame at $q^2 = 0$





Summary

Bottomonium:

- Using time moments from the lattice and continuum perturbation theory we have:
  - calculated the Υ and Υ' leptonic widths using NRQCD b quarks.
  - made an accurate determination of $\overline{m}_b(\overline{m}_b)$.

B Physics

- Decay constants have been calculated for B_q^* in addition to B_q
- We have calculated $B \to \pi$ at $q^2_{\rm max}$; we find results consistent with soft pion theorem.
- This program is now being extended and includes a full range of q^2 for $B_c \to \eta_c$ on superfine ensembles.

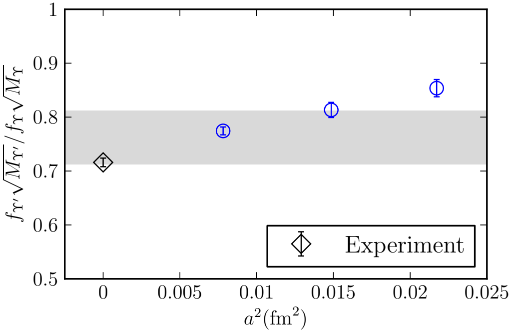

OSLO
"We have time for just one long-winded, self-indulgent question that relates to nothing we've been talking about."

Thank you!

Backup Slides

Υ' Leptonic Width [1408.5768]

$$A = \frac{\langle 0|J_V|\Upsilon'\rangle}{\langle 0|J_V|\Upsilon\rangle} = \frac{f_{\Upsilon'}}{f_{\Upsilon}}\sqrt{\frac{M_{\Upsilon'}}{M_{\Upsilon}}}$$



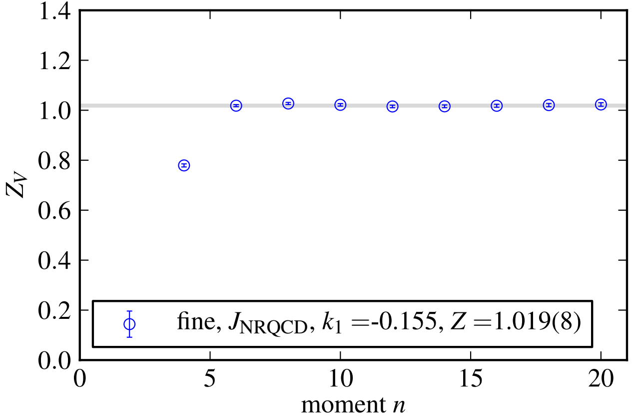
Decay Constant

$$f_{\Upsilon'} = 0.481(39)\ \mathrm{GeV}$$

Leptonic Width

$$\Gamma(\Upsilon' \to e^+e^-) = 0.69(9)\ \mathrm{keV}$$

Z_V [1408.5768]



1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
Z_V
0
5
10
15
20
moment n
fine, J_{NRQCD}, k_1 =-0.155, Z =1.019(8)

Time moments [1408.5768]

Time moments of our NRQCD correlators defined as,

$$G_n^{V,\mathrm{NRQCD}} = 2\sum_t (t/a)^n \, C_{V,\mathrm{NRQCD}}(t) e^{(\overline{E_0} - \overline{M}_{\mathrm{kin}})t}.$$

for $n = 4, 6, 8, \ldots$

Fit forms

Bottomonium

$$
\begin{aligned}
h(a, m_{\text{sea}}) =& h_{\text{phys}} \Bigg[1 + b_l \delta m_{\text{sea}}/(10 m_s) + \sum_{j=1}^{3} c_j (a\Lambda)^{2j} \\
&+ \sum_{j=1}^{2} (a\Lambda)^{2j} \left[c_{jb} \delta x_m + c_{jbb} \right] (\delta x_m)^2 \Bigg]
\end{aligned}
$$

$B \to \pi$

$$
\begin{aligned}
\Gamma(a, m_\pi) =& \Gamma_{\text{phys}} \Bigg[1 + \sum_{j=1}^{3} c_j (a\Lambda)^{2j} + b_j m_\pi^j \\
&+ \left(\frac{\Lambda}{m_b} \right)^2 \left[(a\Lambda)^2 c_b \delta x_m + (a\Lambda)^2 (\delta x_m)^2 \right] \\
&+ d a^2 m_\pi^2 - l \left(\frac{m_\pi^2}{1.6} \right) \log m_\pi^2 \Bigg]
\end{aligned}
$$

Determination of m_b [1408.5768]

$$R_n^V = r_n^V(\alpha_{\overline{\text{MS}}}, \mu/m_b)\left[\frac{m_b}{\overline{m}_b(\mu)}\right]^{n-2}$$

$$\left[\frac{R_n r_{n-2}}{R_{n-2} r_n}\right]^{1/2}\frac{\overline{M}_{\text{kin}}}{2m_b} = \frac{\overline{M}_{\Upsilon,\eta_b}}{2\overline{m}_b(\mu)}$$

$$\overline{m}_b(\mu) = \frac{\overline{M}_{\Upsilon,\eta_b}}{2}\left[\frac{R_{n-2} r_n}{R_n r_{n-2}}\right]^{1/2}\frac{2m_b}{\overline{M}_{\text{kin}}}$$

Determination of m_b [1408.5768]