Lattice calculation of static quark correlators at finite temperature

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in collaboration with
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- Overview & introduction
- Static quark free energy
- Onset of thermal effects
- Screening mass
- Summary & outlook

Commonly known facts

- Asymptotic freedom: QCD in thermal medium at short distances vacuum-like with weak coupling
- Hard thermal loop approximation: QCD in thermal medium for large distances and weak coupling: Debye screening of colour charges
- For weak coupling: description of heavy quarkonia in NREFTs

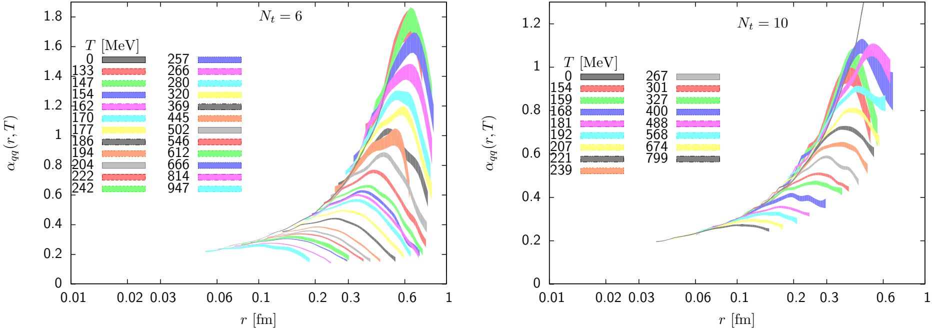
Transition between vacuum-like and screening regions

- Quarkonium suppression indicates temperature in heavy-ion collisions
- Distance of the transition between regions?
- $Q\bar{Q}$ free energies are observables sensitive to the regions
- Determine screening mass in screening region

Our lattice setup

- HISQ 2+1 flavours: $N_\tau = 4, 6, 8, 10, 12$, aspect ratio 4
- Temperature range $115\,\text{MeV} \lesssim T \lesssim 1.4\,\text{GeV}$, $M_\pi \approx 160\,\text{MeV}$

Effective coupling α_{qq}



- Effective coupling $\alpha_{qq} = 3/4r^2 \frac{\partial E(r)}{\partial r}$, $E(r) = \{F_1(T,r), V_0(r)\}$
- $\max \alpha_{qq} \gtrsim 0.5$ for $T \lesssim 300$ MeV indicates strongly coupled plasma
- Regions separated by $\max \alpha_{qq}$: vacuum physics or Debye screening

Static $Q\bar{Q}$ free energy

- Wilson lines represent static $Q, \bar{Q}$: $\psi(\tau, \mathbf{r}) = W(0, \tau; \mathbf{r})\psi(0, \mathbf{r}), \ldots$
- Polyakov loop correlator gives exponentiated free energy

$$P_c = e^{-(F_{Q\bar{Q}} - F_0)/T} = \frac{\langle \mathrm{Tr}\, W(1/T, 0; \mathbf{r}_1)\ \mathrm{Tr}\, W(0, 1/T; \mathbf{r}_2) \rangle}{N_c^2}$$

- P_c formally splits into singlet and octet contributions

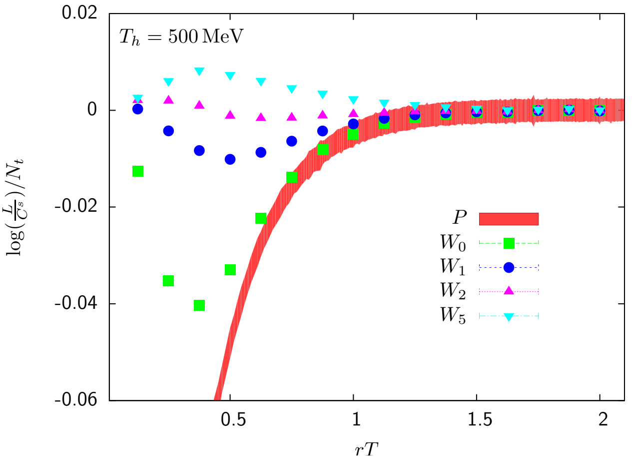
$$P_c = \frac{\left\langle \mathrm{Tr}\left[W(\tfrac{1}{T}, 0; \mathbf{r}_1) W(0, \tfrac{1}{T}; \mathbf{r}_2) \right] \right\rangle}{N_c^3} + \frac{\left\langle \mathrm{Tr}\left[W(\tfrac{1}{T}, 0; \mathbf{r}_1) T^a W(0, \tfrac{1}{T}; \mathbf{r}_2) T^a \right] \right\rangle}{T_F N_c^2}$$

$$= \frac{1}{N_c^2} \exp\left[-F_1/T\right] + \frac{N_c^2 - 1}{N_c^2} \exp\left[-F_8/T\right]$$

Singlet and octet free energies

- $\exp[-F_1/T]$ and $\exp[-F_8/T]$ undergo mixing, gauge-dependent
- For lattice QCD, singlet (octet) free energy from
  1. Cyclic Wilson loop
     - loop closed by spatial Wilson lines (gauge invariant)
     - path dependence leads to extra divergences
  2. Coulomb gauge Wilson line correlator (aka singlet free energy correlator)
     - no spatial lines required
     - gauge dependence leaves physical interpretation questionable
- Both correlators agree with static energy only at leading order

Cyclic Wilson loop



- Logarithm of ratios over Coulomb gauge Wilson line correlator C^s
- Different iterations of spatial HYP smearing for cyclic Wilson loops W_N
- Singlet fraction in C^s and W_N decreases for larger rT (cf. P_c)

Perturbative predictions

- HTL at one loop for $rT > 1$

$$F_1(T,r) = -\frac{N_c^2-1}{2N_c}\alpha_s m_D - \frac{N_c^2-1}{2N_c}\alpha_s \frac{\exp(-m_D r)}{r}$$

$$F_8(T,r) = -\frac{N_c^2-1}{2N_c}\alpha_s m_D + \frac{1}{2N_c}\alpha_s \frac{\exp(-m_D r)}{r}$$

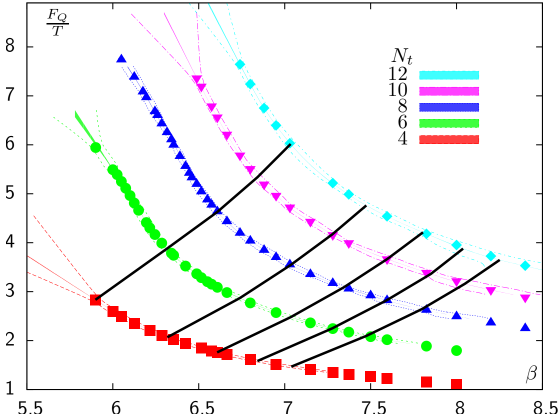
$$F_{Q\bar{Q}}(T,r) = -\frac{N_c^2-1}{2N_c}\alpha_s m_D - \frac{1}{N_c^2}\alpha_s^2 \frac{\exp(-2m_D r)}{r^2 T}$$

- Magnetic mass contributes at even larger distances in EQCD:

$$\frac{F_{Q\bar{Q}}(T,r) - 2F_{Q\bar{Q}}(T,r\to\infty)}{T} \sim \#_1 \frac{\exp(-m_{A_0} r)}{r} + \#_2 \frac{\exp(-m_M r)}{r}$$

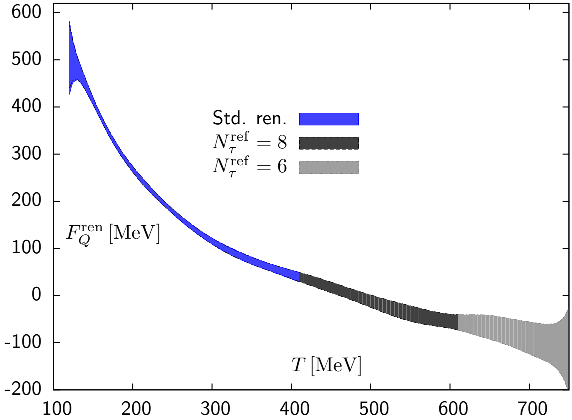
$$m_{A_0} < m_M \ \left[\text{despite power counting } m_{A_0} \sim 2m_D \sim gT,\ m_M \sim g^2 T\right]$$

Bare single quark free energy F_Q



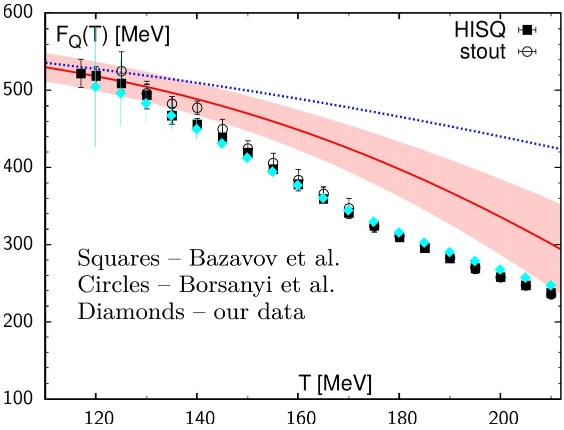
- Quark free energy from Polyakov loop: $-F_Q^{\mathrm{bare}}/T = \frac{1}{N_\tau} \log \mathrm{Tr}\ W(0, \frac{1}{T})$

Single quark free energy F_Q



- Combine three renormalisation schemes & extend F_Q to $T \approx 700\,\mathrm{MeV}$

Single quark free energy F_Q



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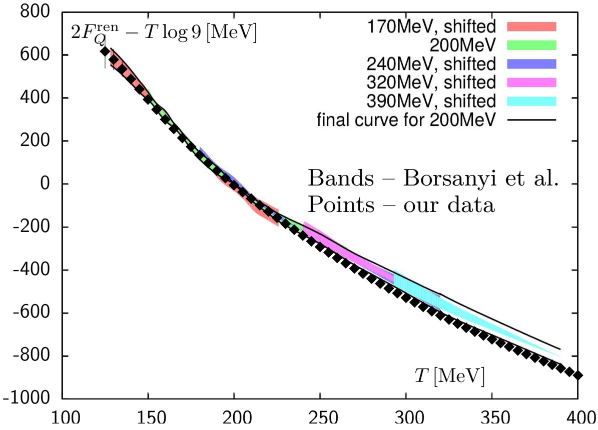
Single quark free energy F_Q



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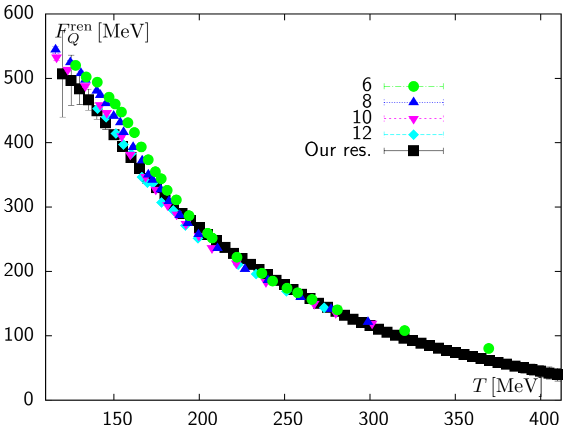
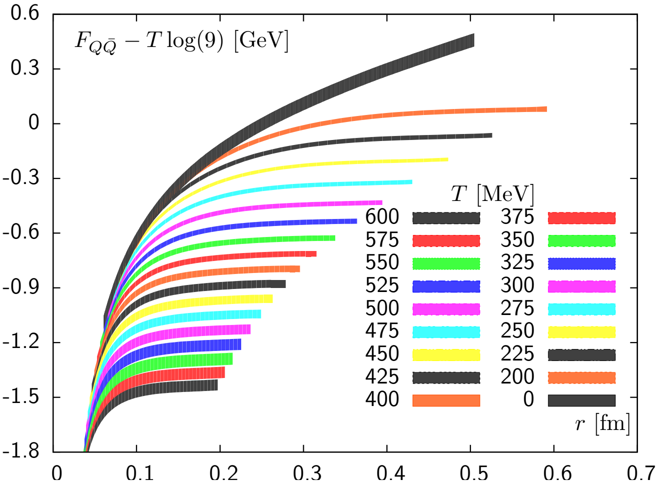
- Bazavov and Petreczky, Phys. Rev. D **87** (2013) 9 : consistent
- Borsanyi et al., JHEP **1504** (2015) 138 : for $T = 200\,\mathrm{MeV}$ set to zero

Single quark free energy F_Q with gradient flow



- F_Q w. gradient flow (finite N_τ) from H.-P. Schadler (Wed. 15th, 17:50)
- Constant flow time in physical units, same for all temperatures

Quark-Antiquark free energy $F_{Q\bar{Q}}$



- Short distance & low temperature: reproduce static energy ($T = 0$)

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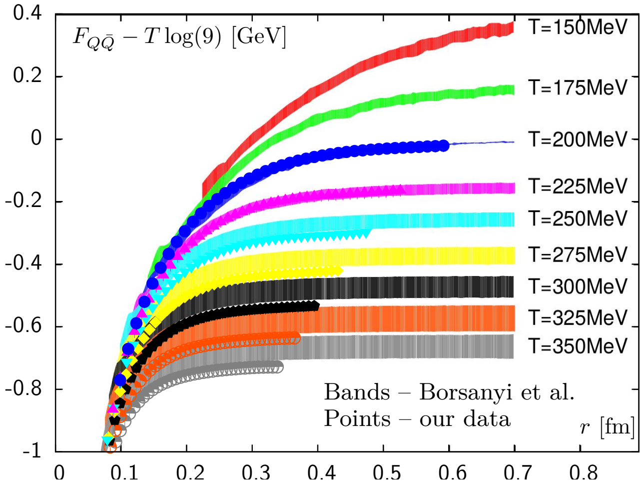

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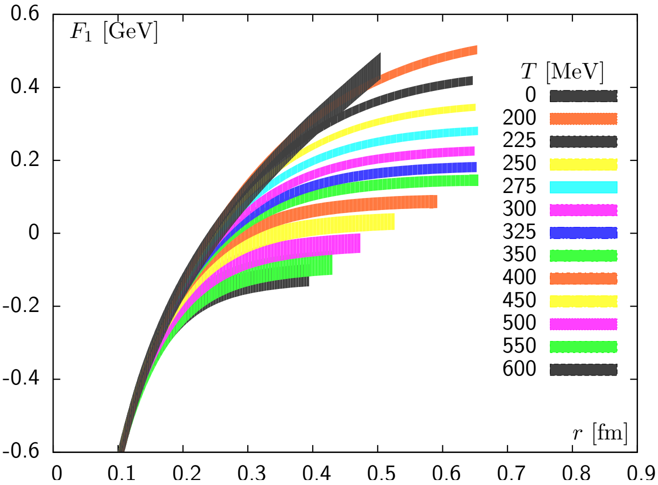
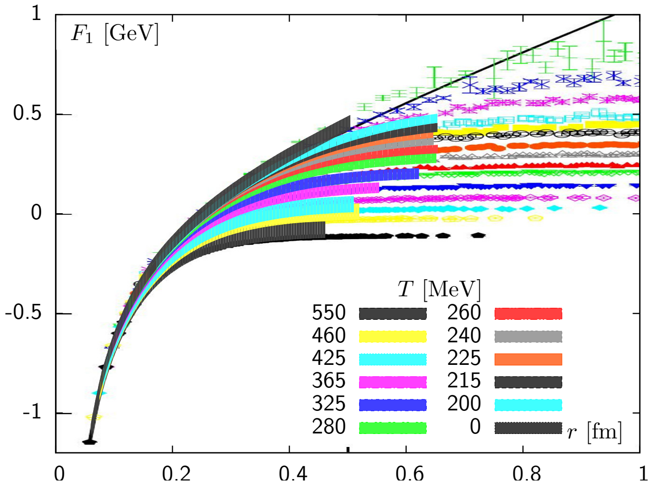
Singlet free energy F_1



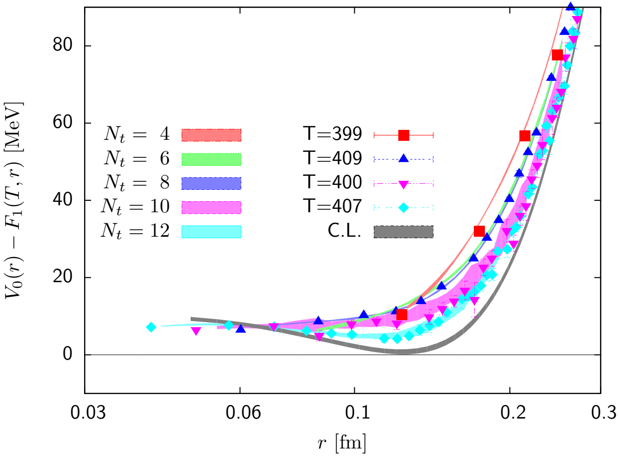
- Reproduce static energy ($T = 0$): larger distances & higher T

Singlet free energy F_1



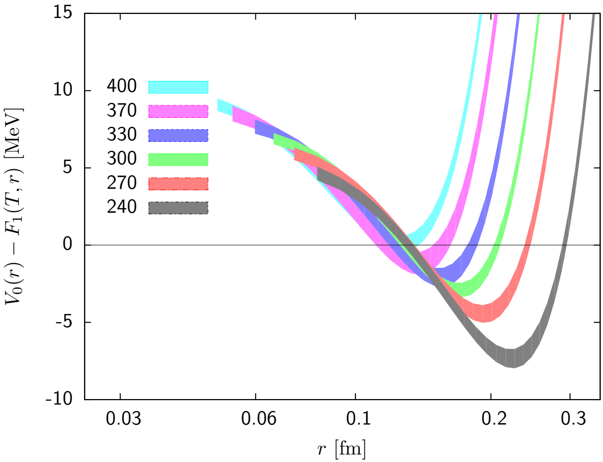
- Reproduce static energy ($T = 0$): larger distances & higher T
- Kaczmarek, PoS CPOD **07** (2007) 043, $N_\tau = 4, 6$, $M_\pi \sim 220$MeV
- Our $N_\tau = 6$ and continuum results are higher: chiral or cutoff effects?

Onset of thermal effects: $V_0(r) - F_1(T,r)$



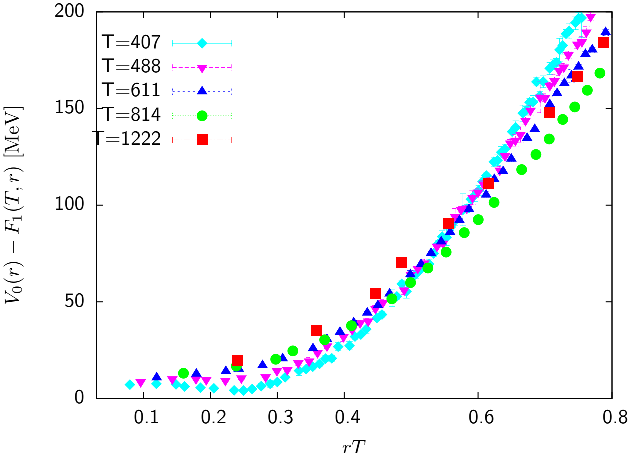
- Static ($T = 0$) and singlet free energies: $v(T,r) = V_0(r) - F_1(T,r)$
- Large cutoff effects, minimum visible only for $N_\tau \geq 10$
- Continuum extrapolation with only $N_\tau = 8, 10, 12$

Onset of thermal effects: $V_0(r) - F_1(T,r)$



- Static ($T = 0$) and singlet free energies: $v(T,r) = V_0(r) - F_1(T,r)$
- T independent falling slope at $rT \approx 0.15$, minimum at $rT \approx 0.25$

Onset of thermal effects: $V_0(r) - F_1(T, r)$



- Static ($T = 0$) and singlet free energies: $v(T, r) = V_0(r) - F_1(T, r)$
- Estimate cutoff effects as $\sim$ 10-20 MeV from data at $T \approx 410$ MeV
- For $rT \lesssim 0.3$ almost constant, for $rT > 0.3$ rapid rise

Screening mass

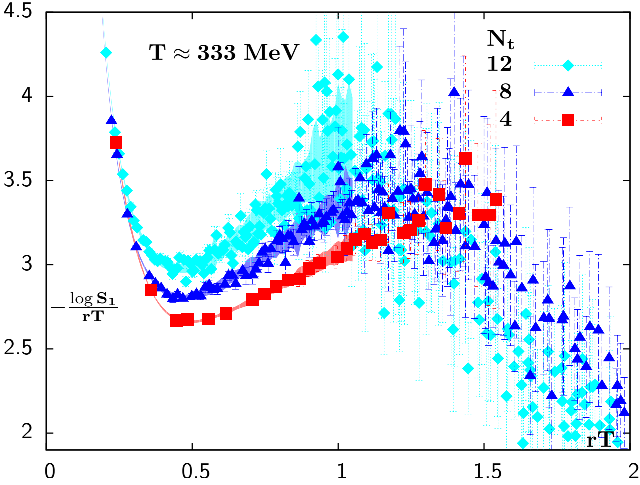
- At large distance & weak coupling $F_1(T,\infty) = F_{Q\bar{Q}}(T,\infty) = 2F_Q(T)$
- Subtract asymptotic constant to define screening functions

$$S_1(T,rT) = rT\,\frac{F_1(T,r) - 2F_Q(T)}{T} \overset{HTL}{\rightarrow} -\frac{N_c^2-1}{2N_c}\alpha_s \exp\left(-m_D r\right),$$

$$S_{\mathrm{avg}}(T,rT) = (rT)^2\,\frac{F_{Q\bar{Q}}(T,r) - 2F_Q(T)}{T} \overset{HTL}{\rightarrow} -\frac{1}{N_c^2}\alpha_s^2 \exp\left(-2m_D r\right)$$

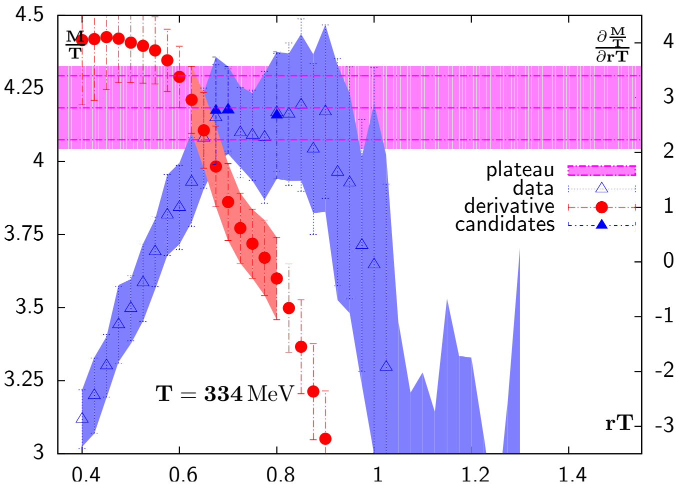
- Theoretically ideal: extract m_D from $F_{Q\bar{Q}}$, but data is too noisy
- Singlet free energy: extra rT dependendence due to Z^{ren}
  1. Cyclic Wilson loop extra linear divergence due to self-energy
  2. Coulomb gauge correlator: gauge dependence, but cleanest probe

Screening mass: extraction



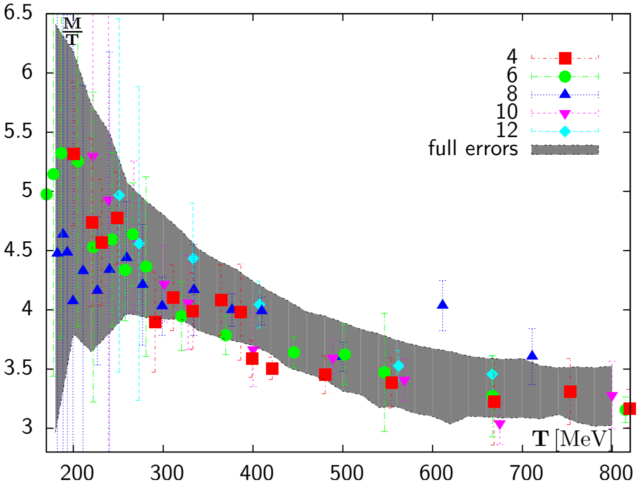
- $rT \gg 1$, weak coupling: $\log S_1(T,r) = A - \frac{M}{T}\, rT \overset{HTL}{\rightarrow} \log\left(C_F \alpha_s\right) - \frac{m_D}{T}\, rT$

Screening mass: extraction



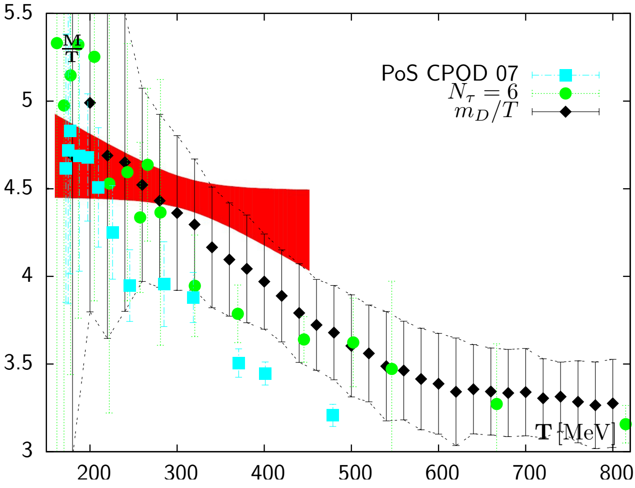
- $rT \gg 1$, weak coupling: $\log S_1(T,r) = A - \frac{M}{T}\,rT \overset{HTL}{\longrightarrow} \log\left(C_F\alpha_s\right) - \frac{m_D}{T}\,rT$
- Estimate systematical uncertainty with derivative $\partial(\frac{M}{T})/\partial(rT)$

Screening mass: extraction



- Largest rT before signal loss, increase errors w. systematic effects
- Jackknife estimate of extraction method dependence w. 8×4 methods

Screening mass: comparison with other lattice results



- Kaczmarek, PoS CPOD **07** (2007) 043: lower due to heavier M_π
- Borsanyi et al., JHEP **1504** (2015) 138: magnetic mass $m_M(T)$

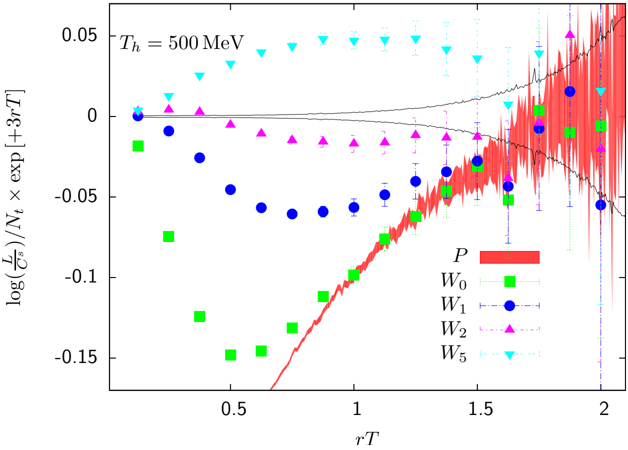
Summary

- Extend numerical results for static quark free energy to $T \approx 700\,\text{MeV}$ and $Q\bar{Q}$ free energy to $T \approx 600\,\text{MeV}$
- Study the onset of screening with new observable $V_0(r) - F_1(T, r)$, field-theoretically cleaner than $\alpha_{qq}(T, r)$
- Extract screening mass in screening region

Outlook

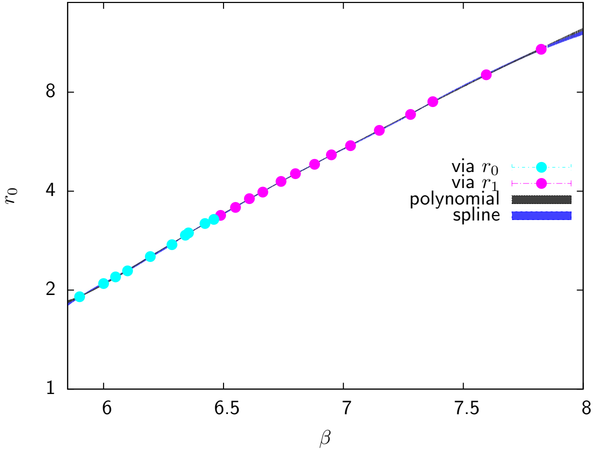
- Comparison of $V_0(r) - F_1(T, r)$ with weak coupling (pNRQCD)
- Need finer zero temperature lattices $[a \sim 0.025, 0.03, 0.035\,\text{fm}]$
- Extract spectral function from static correlators, extract imaginary part of potentialwith 3 dynamical flavours

Cyclic Wilson loop



- Boost w. factor $\exp(3rT)$, $\log P_c/C^s$ w. error reduced by factor 1/20
- Small rT: negative values indicate larger octet fraction than C^s (cf. P_c)
- Unsmeared Wilson loop on top of Polyakov loop correlator for $rT \gtrsim 1$

Scale r_0 and renormalisation constant



- Use $T = 0$ data from Bazavov et al., Phys. Rev. D **90** (2014) 9
- Scale setting with r_1 (r_0) for fine (coarse) lattices

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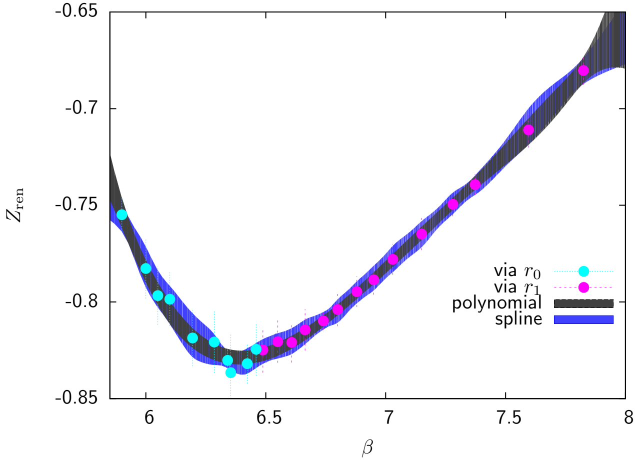
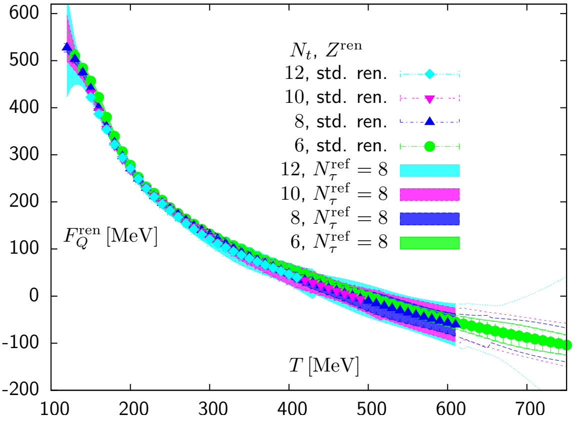


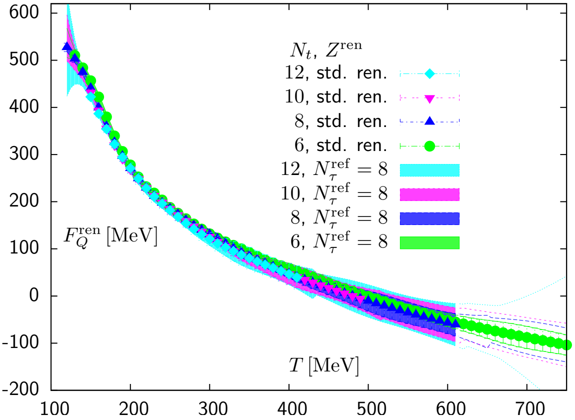
- Use $T = 0$ data from Bazavov et al., Phys. Rev. D **90** (2014) 9
- Scale setting with r_1 (r_0) for fine (coarse) lattices
- Renormalisation constant $Z^{\rm ren}(\beta)$ from $T = 0$ static energy

Renormalisation schemes for F_Q



- Old (standard) scheme: use $Z^{\mathrm{ren}}(\beta)$ (rescaled with N_τ) for each N_τ

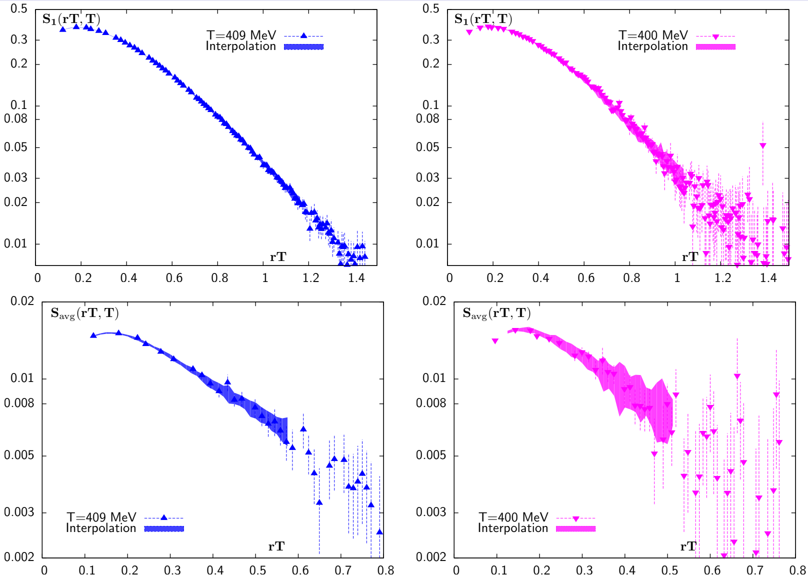
Renormalisation schemes for F_Q



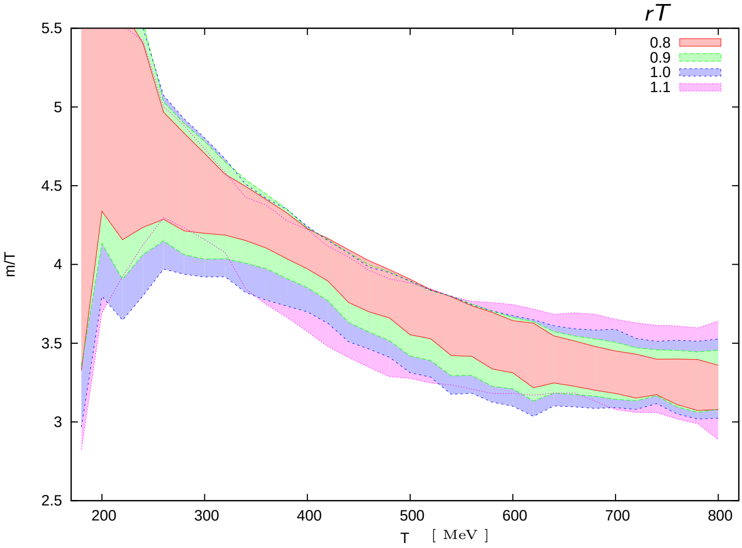
- Old (standard) scheme: use $Z^{\rm ren}(\beta)$ (rescaled with N_τ) for each N_τ
- New scheme: avoid extrapolated $Z^{\rm ren}(\beta)$ assuming small cutoff effects:

$$Z^{\rm ren}(T,N_\tau) = Z^{\rm ren}(T,N_\tau^{\rm ref}) + 2\left(F_Q^{\rm bare}(T,N_\tau) - F_Q^{\rm bare}(T,N_\tau^{\rm ref})\right)$$

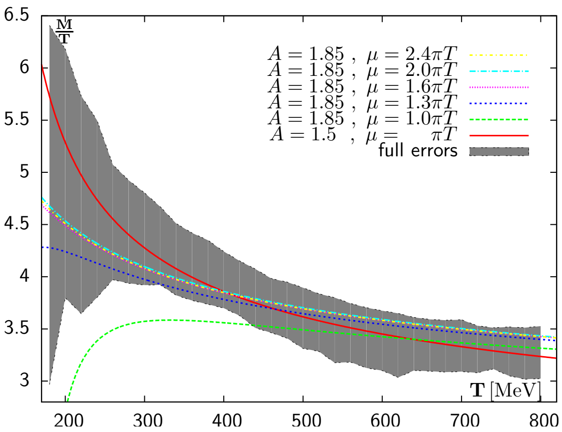
Screening functions



rT dependence of screening mass



Screening mass: comparison with weak coupling calculations



- Braaten and Nieto, Phys. Rev. D **53** (1996) 3421: electric screening mass

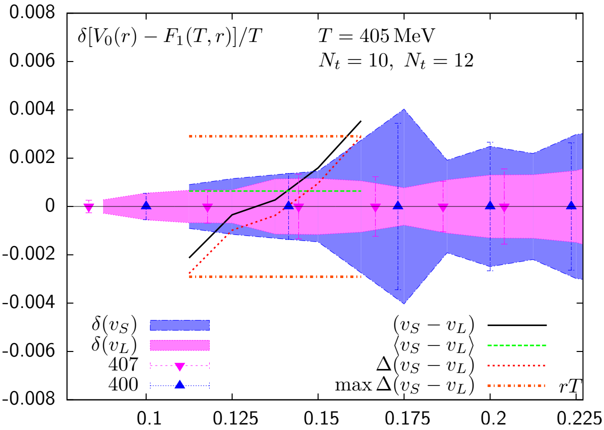
$$m_{\rm el}^2 = 4\pi\alpha_s T^2 \left(\frac{N_c}{3} + \frac{N_f}{6}\right) + \mathcal{O}(\alpha_s^2)$$

- Plot $A\sqrt{m_{\rm el}^2[T,\mu,g(\mu)]}/T$ w. $m_{\rm el}$ at one or two-loop, $\alpha_s(\mu)$ at two-loop

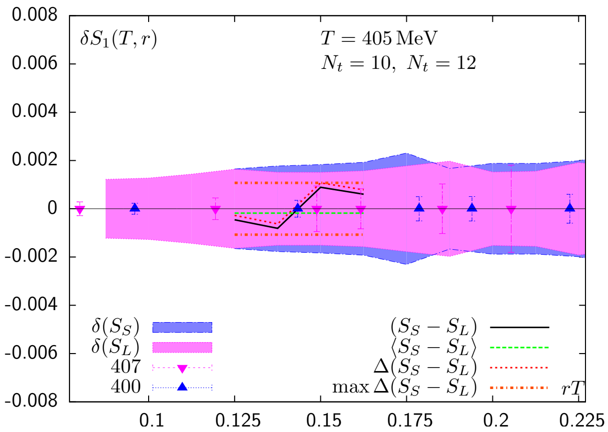
Short distance cutoff effects

- Data with equal temperature and neighbouring N_τ at short distance
- Estimate systematic uncertainties by studying rT-dependence of cutoff effect
- Compute difference of correlators $C_S - C_L$ and subtract interval average
- Absolute maximum of remainder is considered as systematical uncertainty and applied to all distances up to largest rT of interval
- Quadratically added to statistical errors
- Significant only for $V_0(r) - F_1(T, r)$

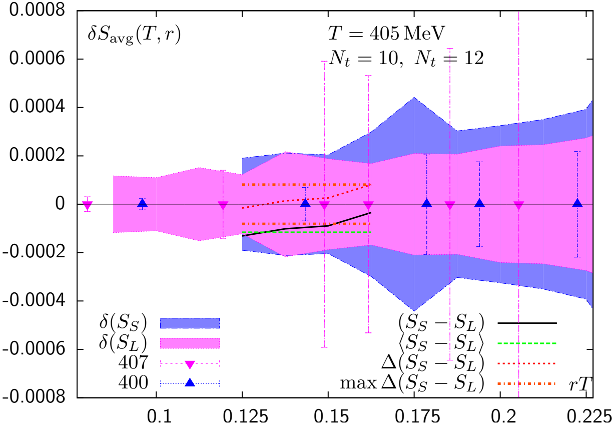
Cutoff effects in $V_0(r) - F_1(T, r)$



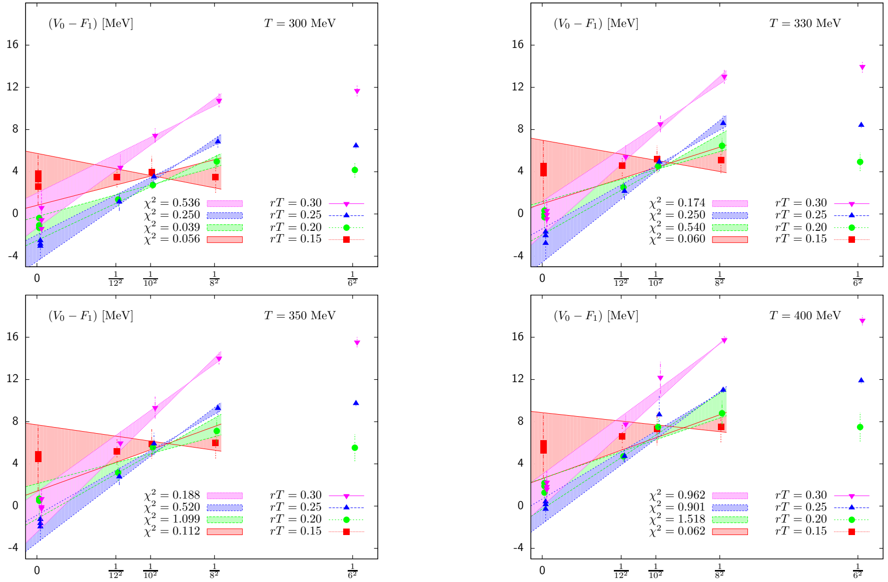
Cutoff effects in $S_1(T, rT)$



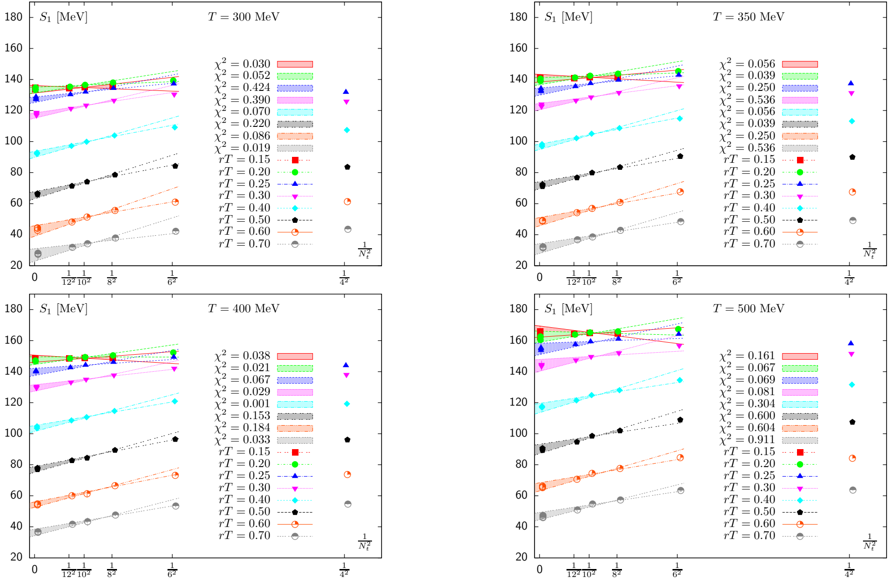
Cutoff effects in $S_{\rm avg}(T, rT)$



Scaling behaviour: $V_0(r) - F_1(T, r)$



Scaling behaviour: $S_1(T, rT)$



Scaling behaviour: $S_{\mathrm{avg}}(T, rT)$