

Chiral phase transition of $N_f=3$ and $2+1$ QCD at $\mu_B = 0$

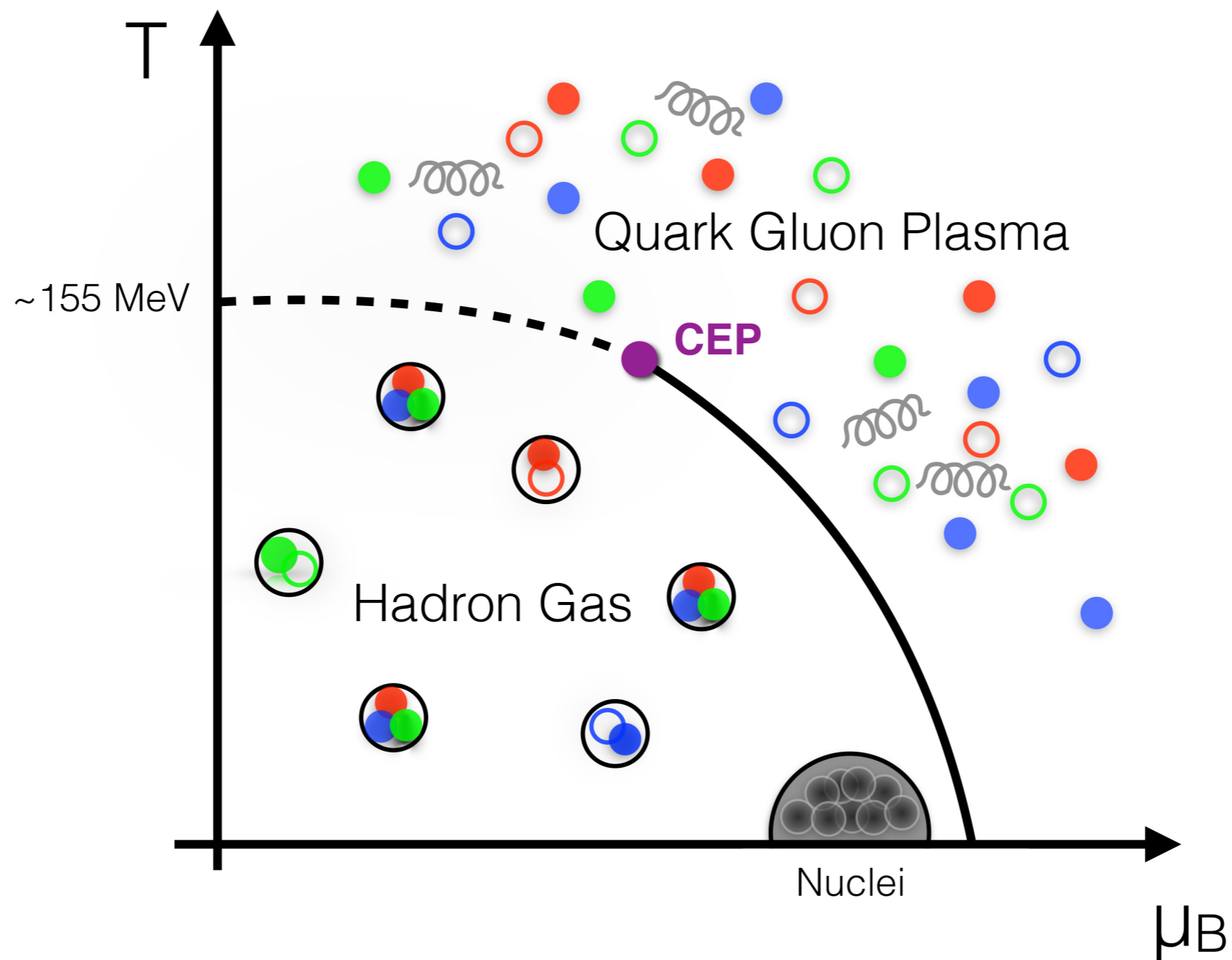
Heng-Tong Ding
for the Bielefeld-BNL-CCNU collaboration

Central China Normal University



14-18 July, 2015, Lattice 2015, Kobe, Japan

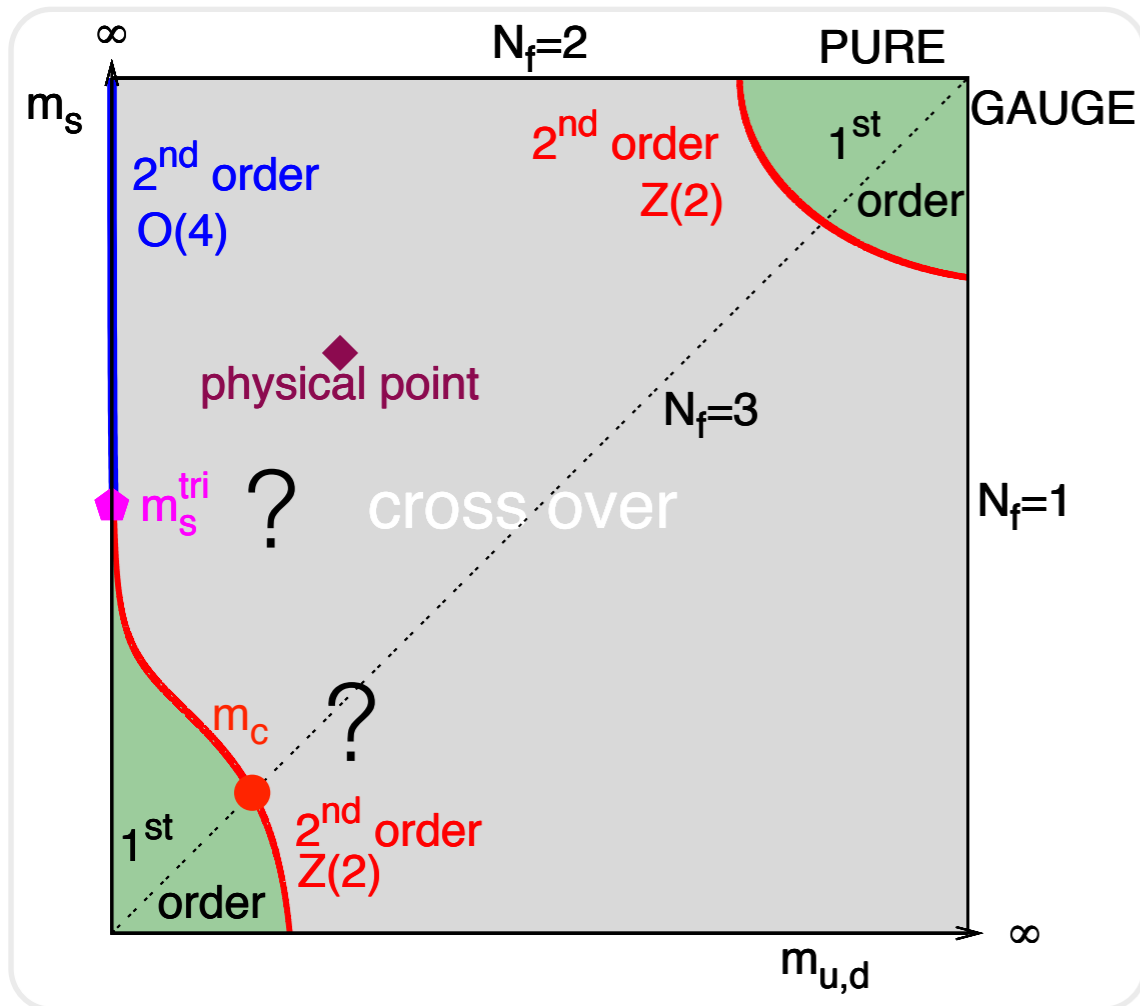
QCD phase diagram



HTD, F. Karsch, S. Mukherjee, arXiv:1504.0527

QCD phase diagram at $\mu_B=0$

columbia plot:



At physical point: cross over,
 $T_{pc} \sim 155 \text{ MeV}$ HotQCD '12, BMW '10

$N_f=2(+1)$: $U_A(1)$ remains broken at $T_{\chi SB}$

JLQCD '13,'14,'15, HotQCD '13,'14

Critical lines of second order transition

Pisarski & Wilczek PRD '84

$N_f=2$: $O(4)$ universality class Kogut & Sinclair, PRD '06

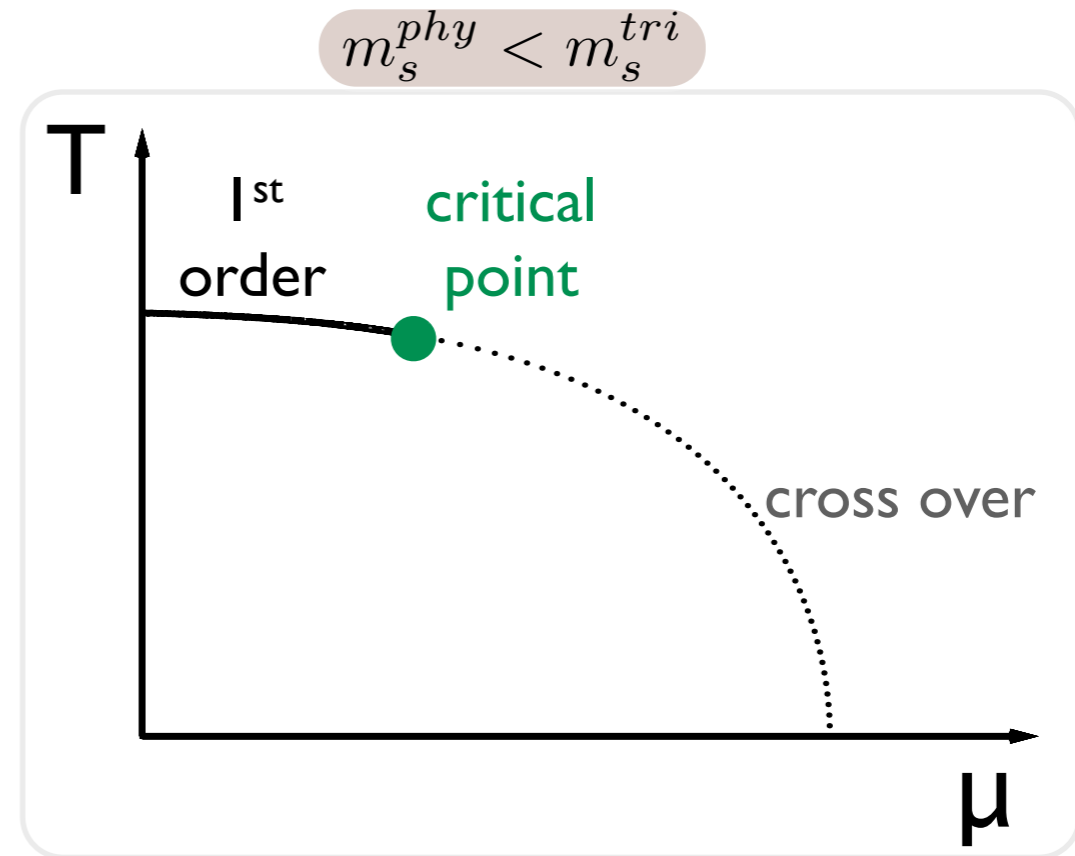
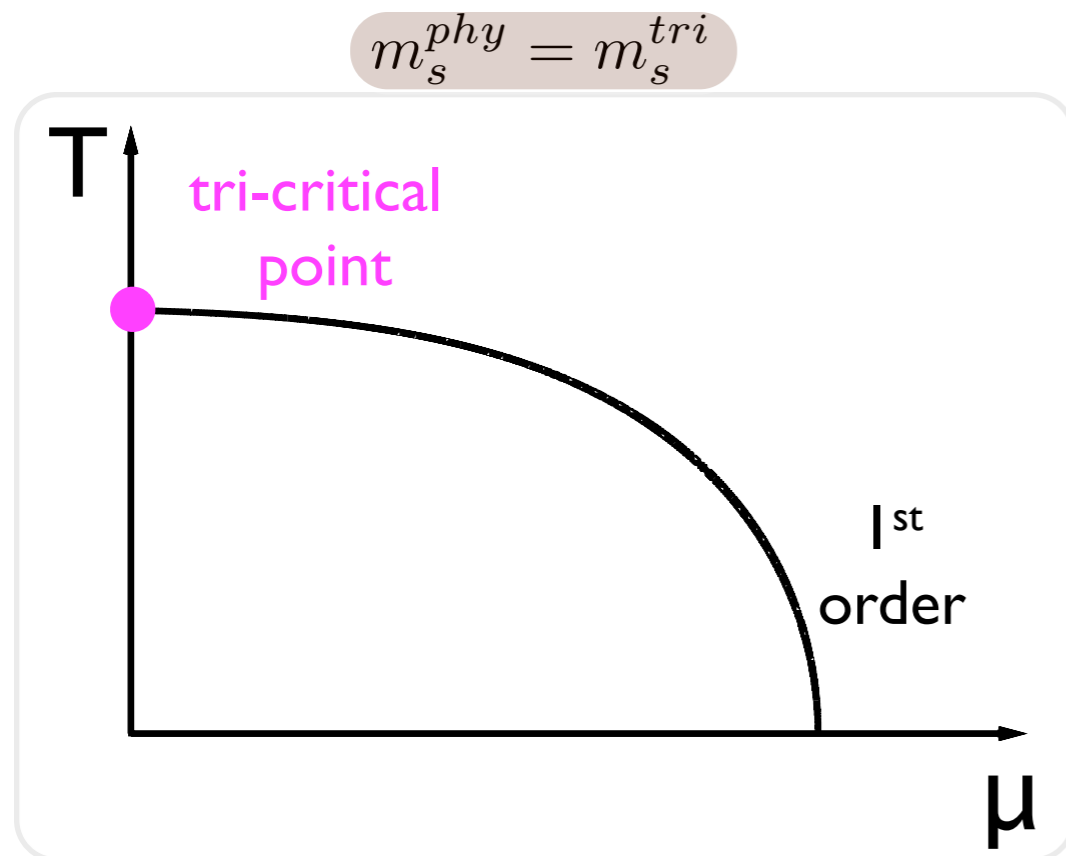
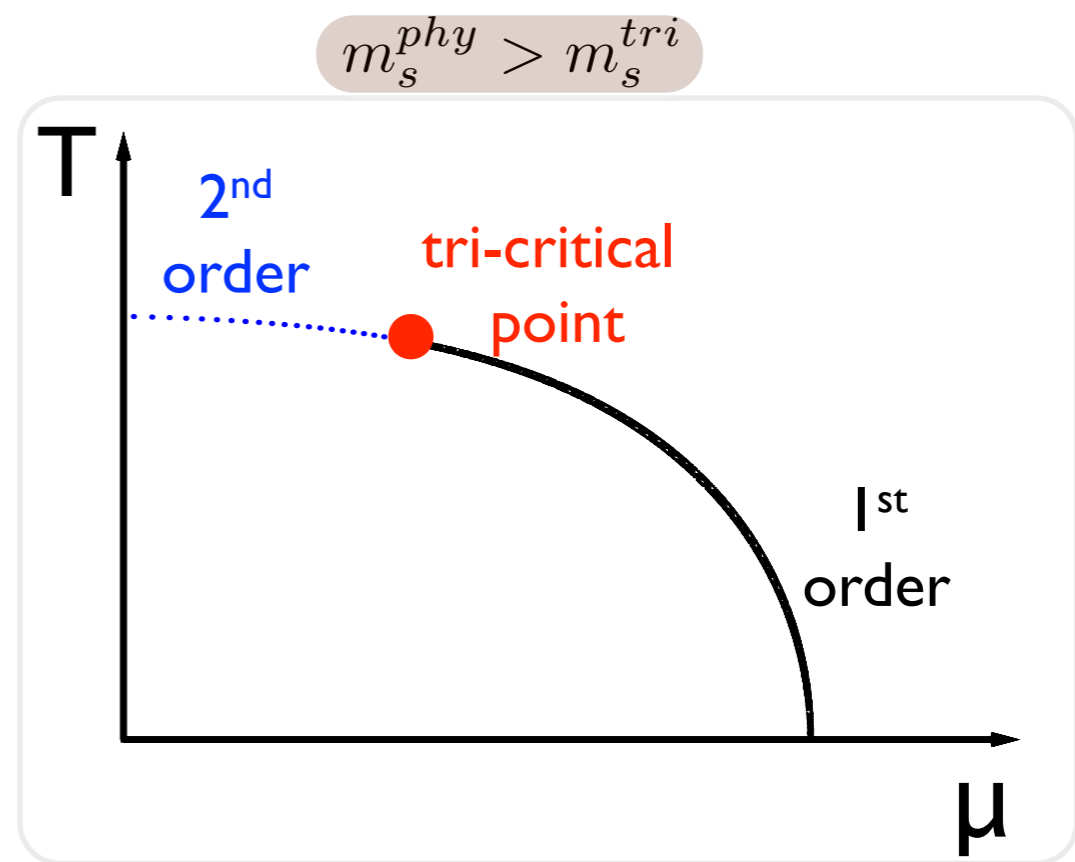
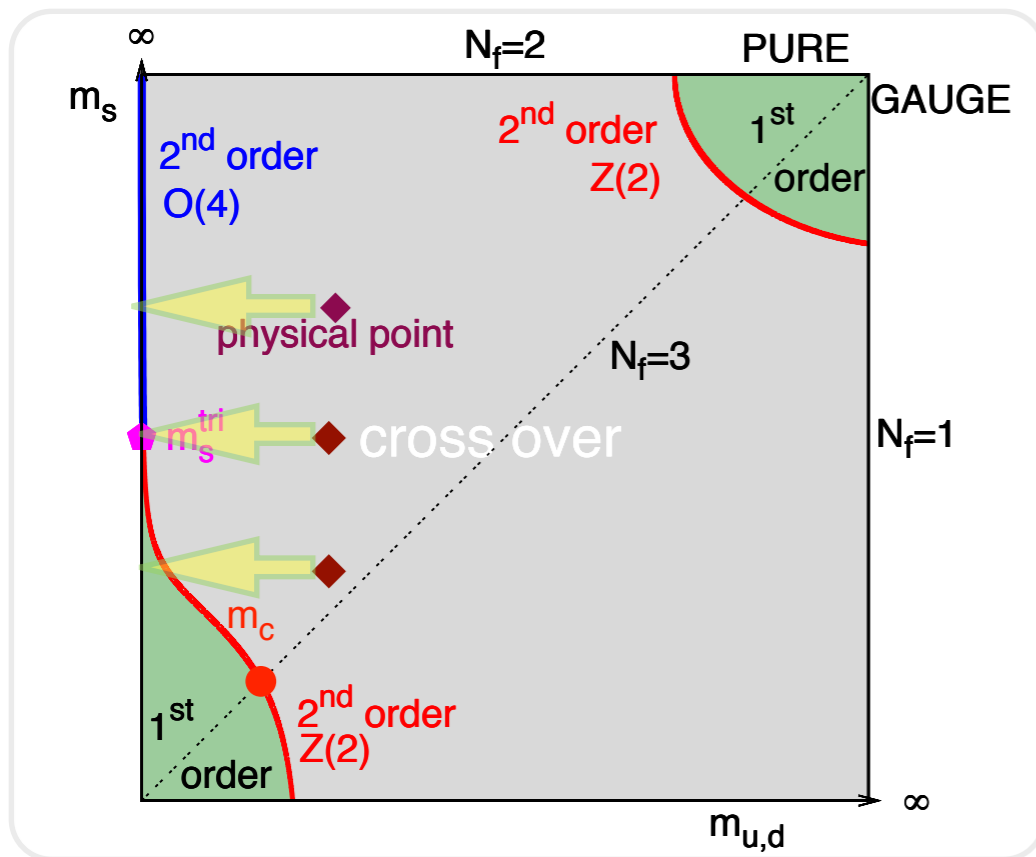
$N_f=3$: Ising universality class Karsch, Laermann, Schmidt PLB '04,...

Towards the chiral limit:

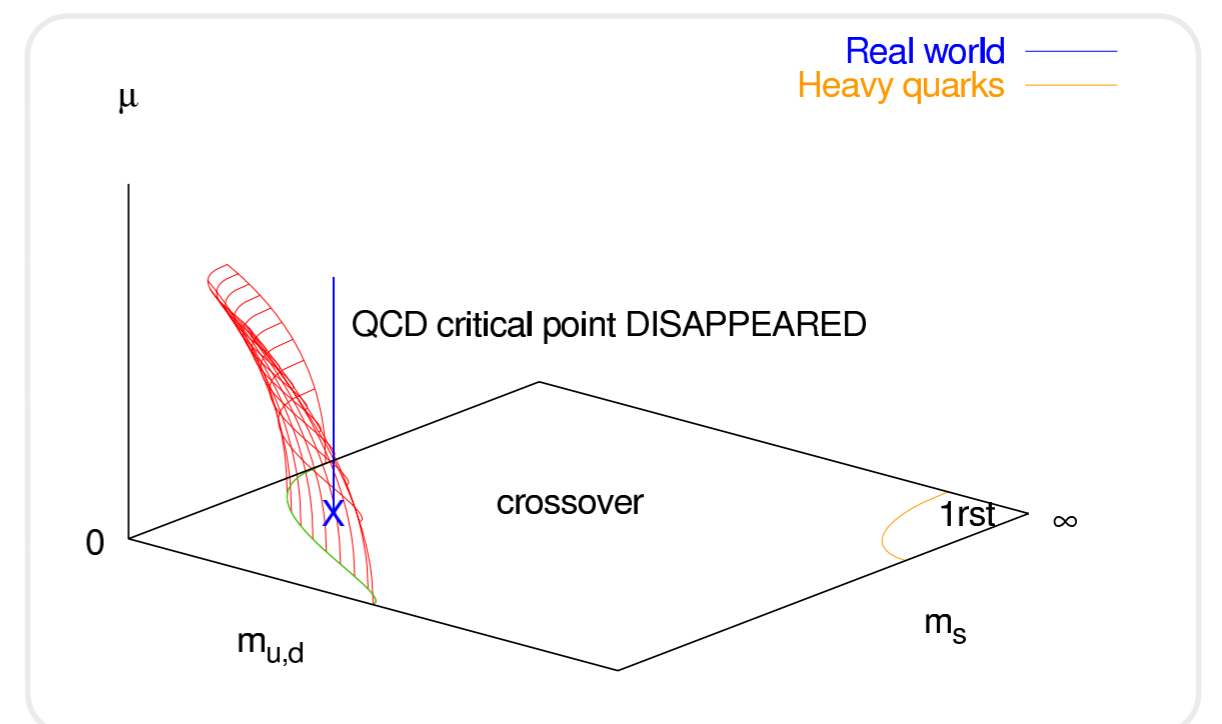
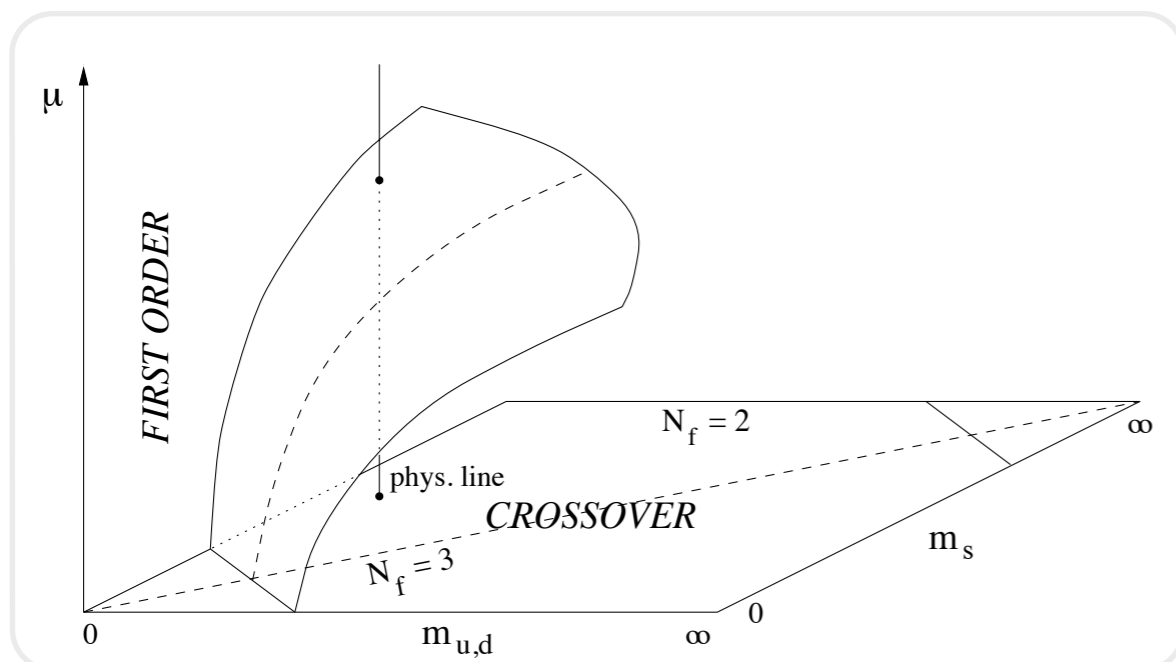
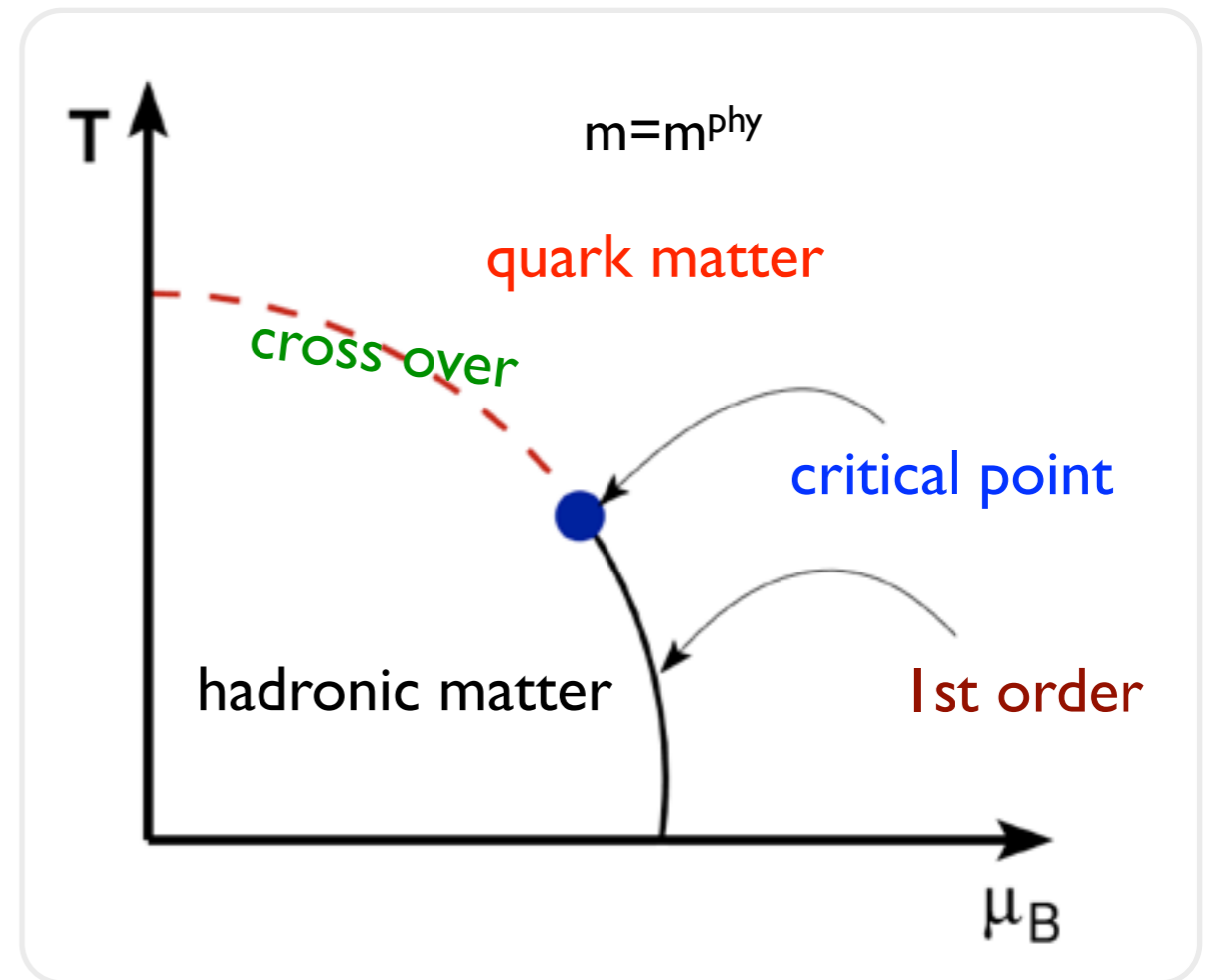
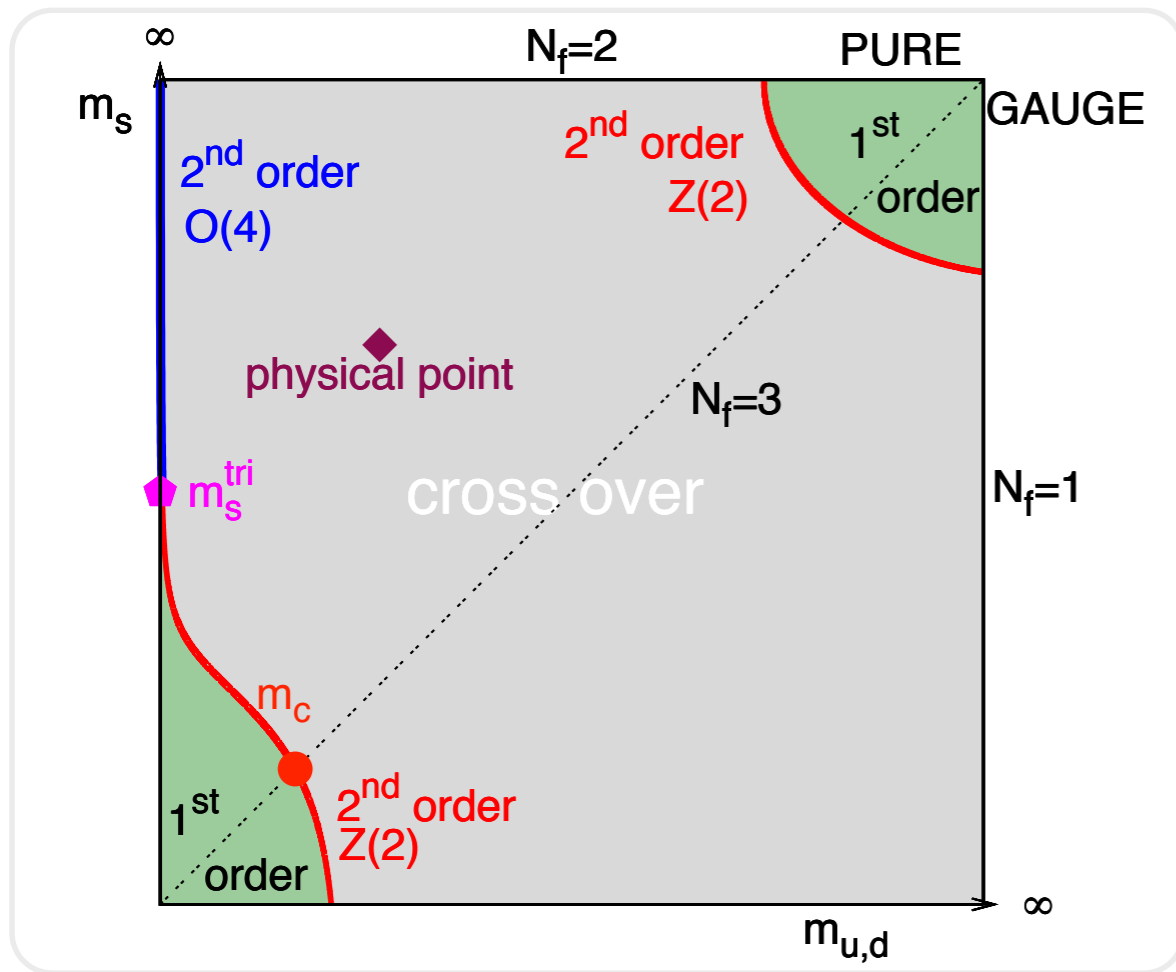
$N_f=2+1$ QCD: m_s^{tri} ? m_s^{phy}

$N_f=3$ QCD: critical mass m_c ?

scenarios of QCD phase transition at $m_l=0$



QCD phase transition at the physical point



Phase transition in the chiral limit

$N_f=2+1$:

- standard staggered action: $N_\tau = 4 \Rightarrow m_s^{\text{tri}} > m_s^{\text{phy}}$
de Forcrand & Philipsen, JHEP 0701(2007)077
- p4fat3 action: $N_\tau = 4 \Rightarrow m_s^{\text{tri}} < m_s^{\text{phy}}$
S. Ejiri et al., Phys.Rev. D80 (2009) 094505

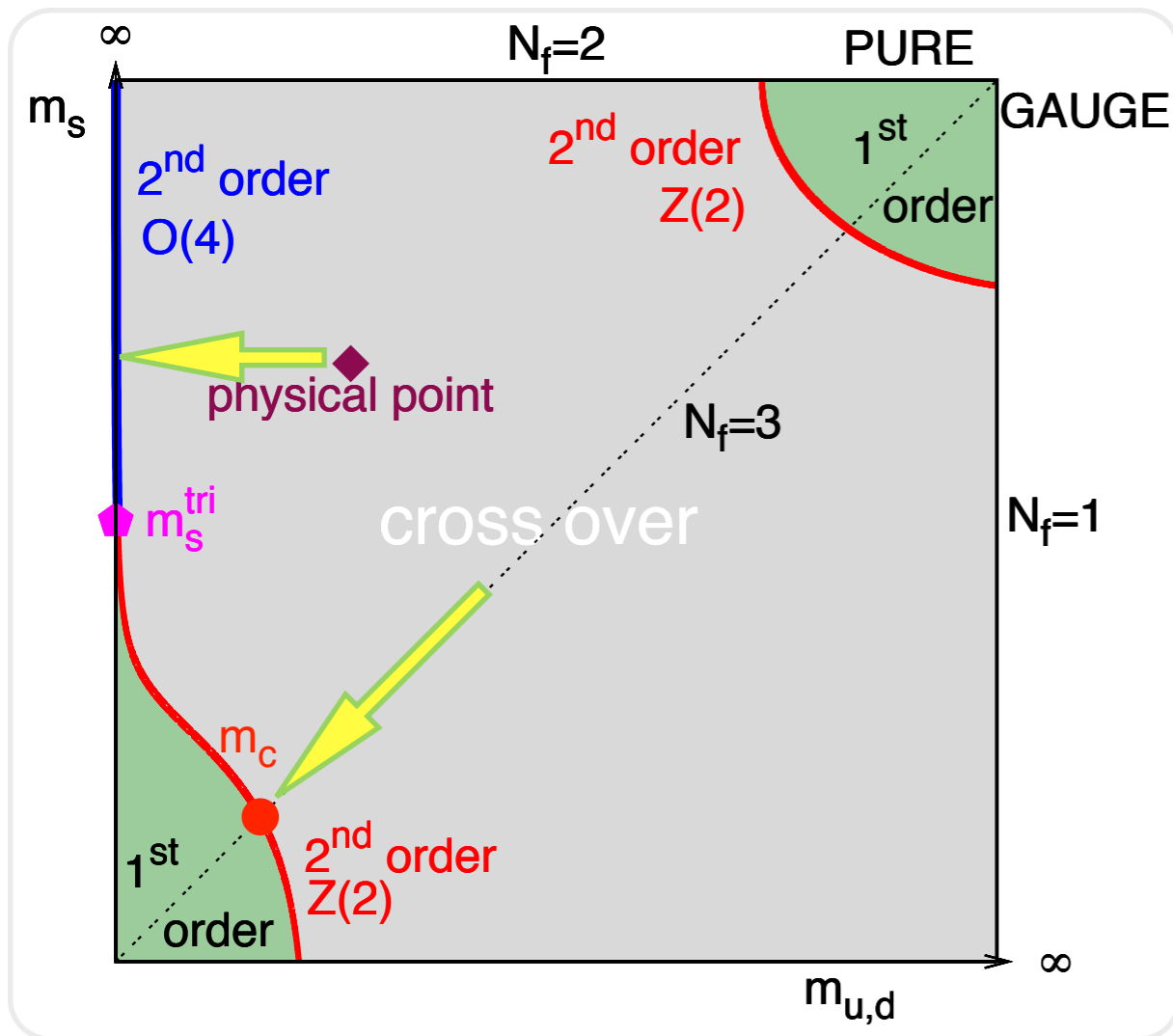
$N_f=3$:

- Standard staggered action: $N_\tau = 4 \Rightarrow m_\pi^c \approx 290 \text{ MeV}$
F. Karsch et al., Nucl.Phys.Proc.Suppl. 129 (2004) 614
- Standard staggered action: $N_\tau = 6 \Rightarrow m_\pi^c \approx 140 \text{ MeV}$
P. de Forcrand et al, PoS LATTICE2007 (2007) 178
- p4fat3 action: $N_\tau = 4 \Rightarrow m_\pi^c \approx 67 \text{ MeV}$
F. Karsch et al., Nucl.Phys.Proc.Suppl. 129 (2004) 614
- stout action: $N_\tau = 6 \Rightarrow m_\pi^c \approx 50 \text{ MeV}$
G. Endrodi et al., PoS LAT2007 (2007) 228
- HISQ action: $N_\tau = 6 \Rightarrow m_\pi^c \approx 45 \text{ MeV}$
HTD et al., '13
- Wilson-clover fermion: $N_\tau = 4,6,8 \Rightarrow m_\pi^c \approx 304 \text{ MeV}$
X.-Y.Jin et al., Phys.Rev. D91 (2015) 1, 014508

$N_f=2$:

- standard staggered action: $N_\tau = 4 \Rightarrow 1^{\text{st}}$ order
D'Elia et al., Phys.Rev. D72 (2005) 114510
- standard staggered action: $N_\tau = 6 \Rightarrow 2^{\text{nd}}$ order, $O(2)$
Kogut & Sinclair, Phys.Rev. D73 (2006) 074512

$N_f=2+1$ & $N_f=3$ QCD simulations



📍 HISQ/tree action on $N_t=6$ lattices

📍 **$N_f=2+1$:**

☑ $m_s^{\text{phy}} / m_l = 20, 27, 40, 60, 80$

$m_\pi \approx 160, 140, 110, 90, 80$ MeV

☑ $m_\pi L > 3$

📍 **$N_f=3$:**

☑ $m_s^{\text{phy}} / m_q = 10, 20, 30, 40, 60, 80$

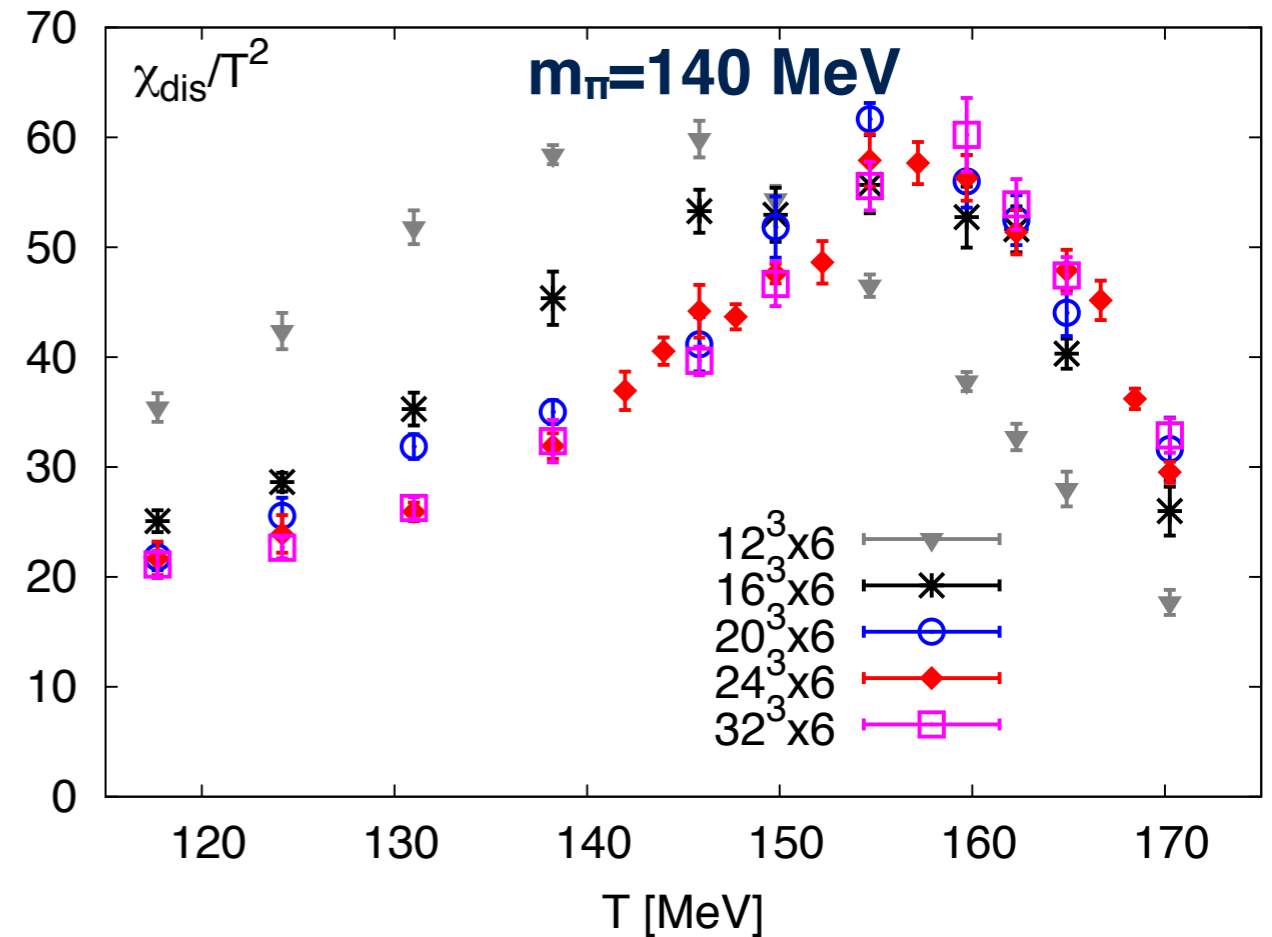
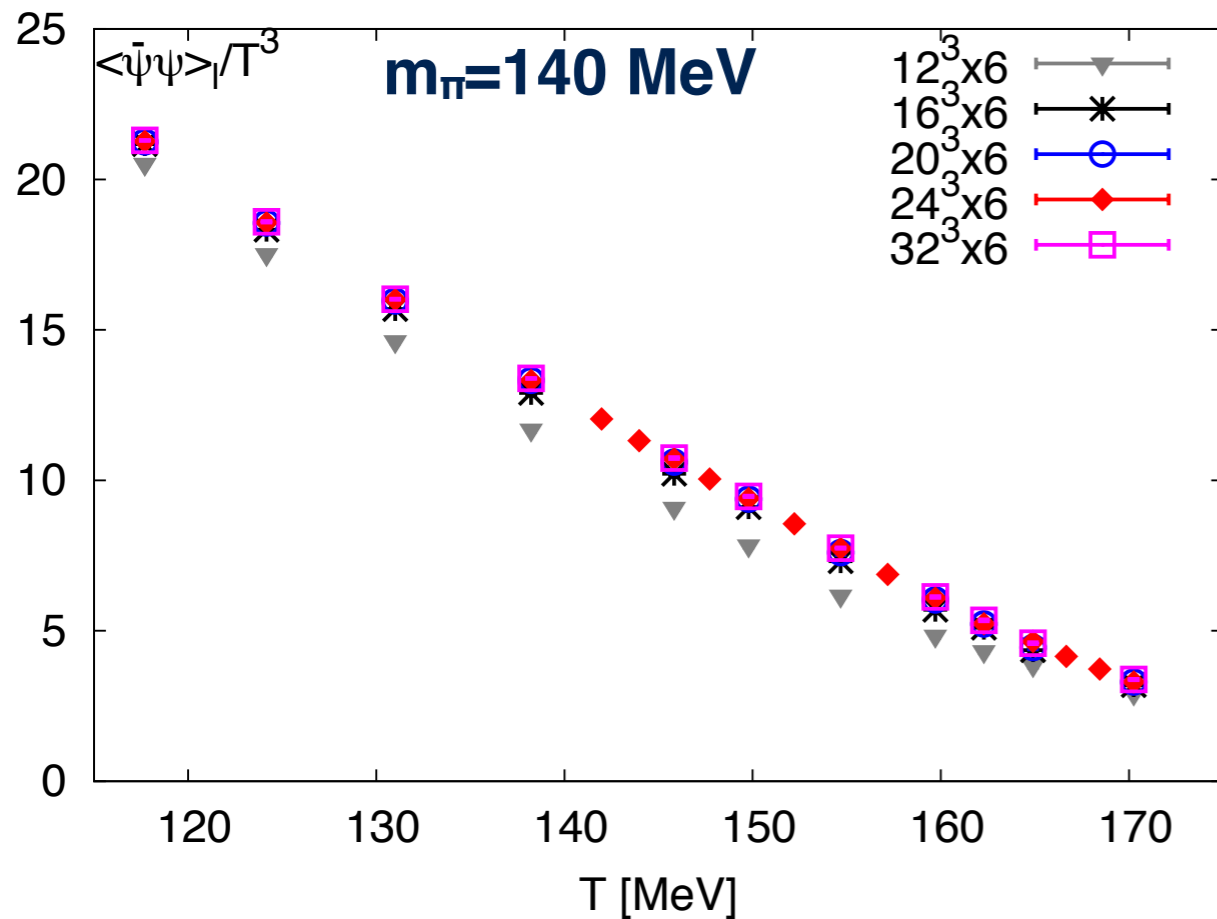
$m_\pi \approx 230, 160, 140, 110, 90, 80$ MeV

☑ $m_\pi L > 3$

update results of 1312.0119 and 1302.5740

$$N_f = 2 + 1$$

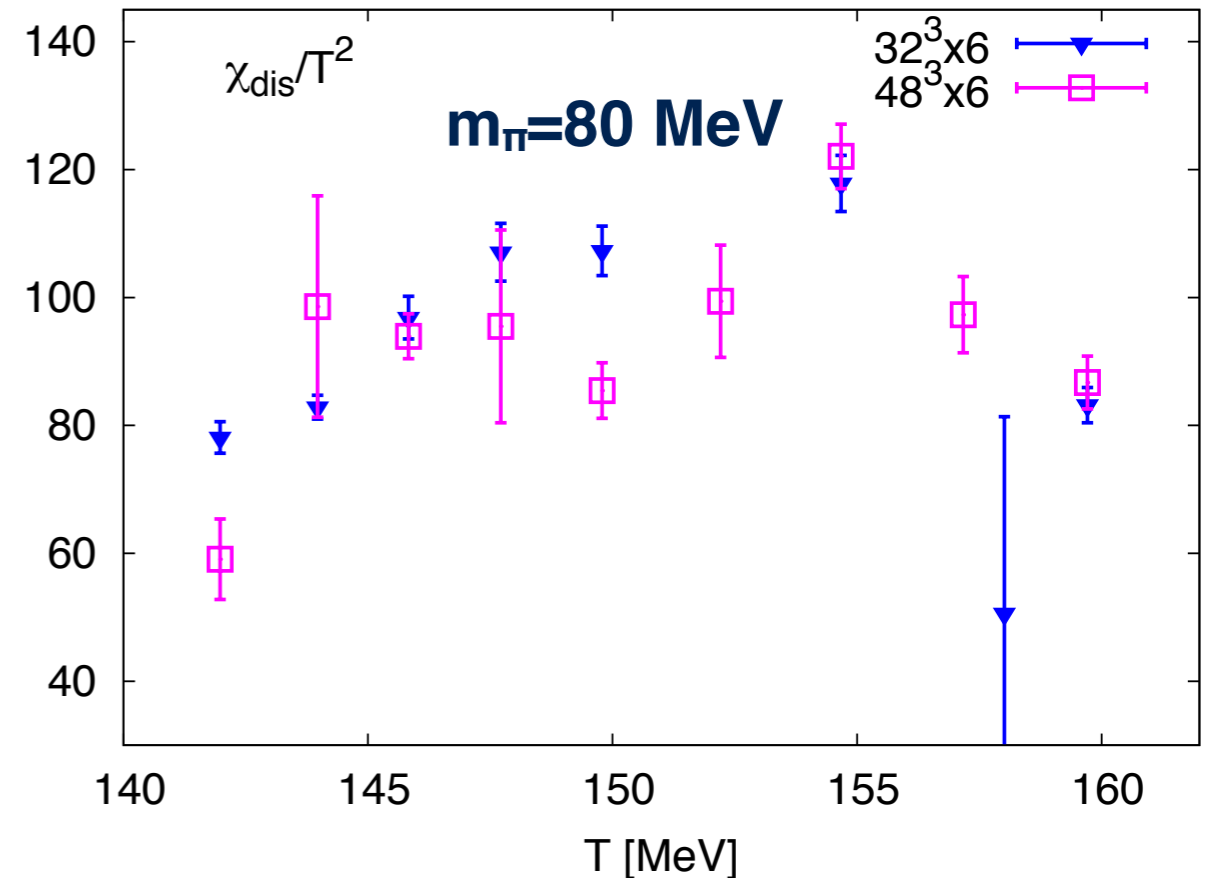
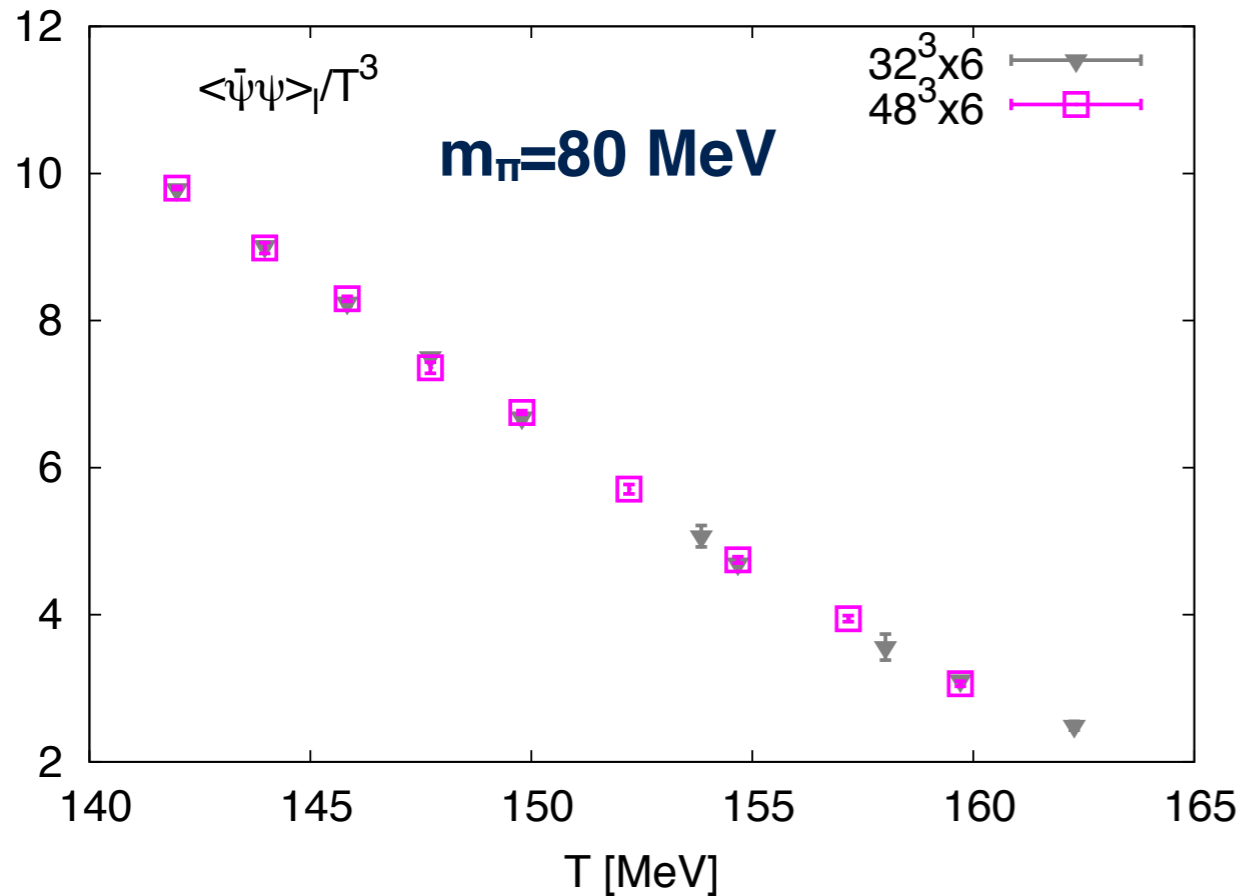
Volume dependence of chiral observables



- volume effects are small in 3 largest volume
- $m_\pi L > 4$ is ensured in the following other datasets

$48^3 \times 6$ with $m_\pi = 80$ MeV,
 $40^3 \times 6$ with $m_\pi = 90$ MeV,
 $32^3 \times 6$ with $m_\pi = 110$ MeV,
 $24^3 \times 6$ with $m_\pi = 160$ MeV

Volume dependence of chiral observables



- Mild volume dependence is seen from chiral observables
- No evidence of linear volume scaling as signatures of first order phase transition

chiral phase transition and universal scaling

Behavior of the free energy close to critical lines

$$f(m, T) = h^{1+1/\delta} f_s(z) + f_{\text{reg}}(m, T), \quad z = t/h^{1/\beta\delta}$$

h : external field, t : reduced temperature, β, δ : universal critical exponents

$f_s(z)$: universal scaling function, O(N) etc.

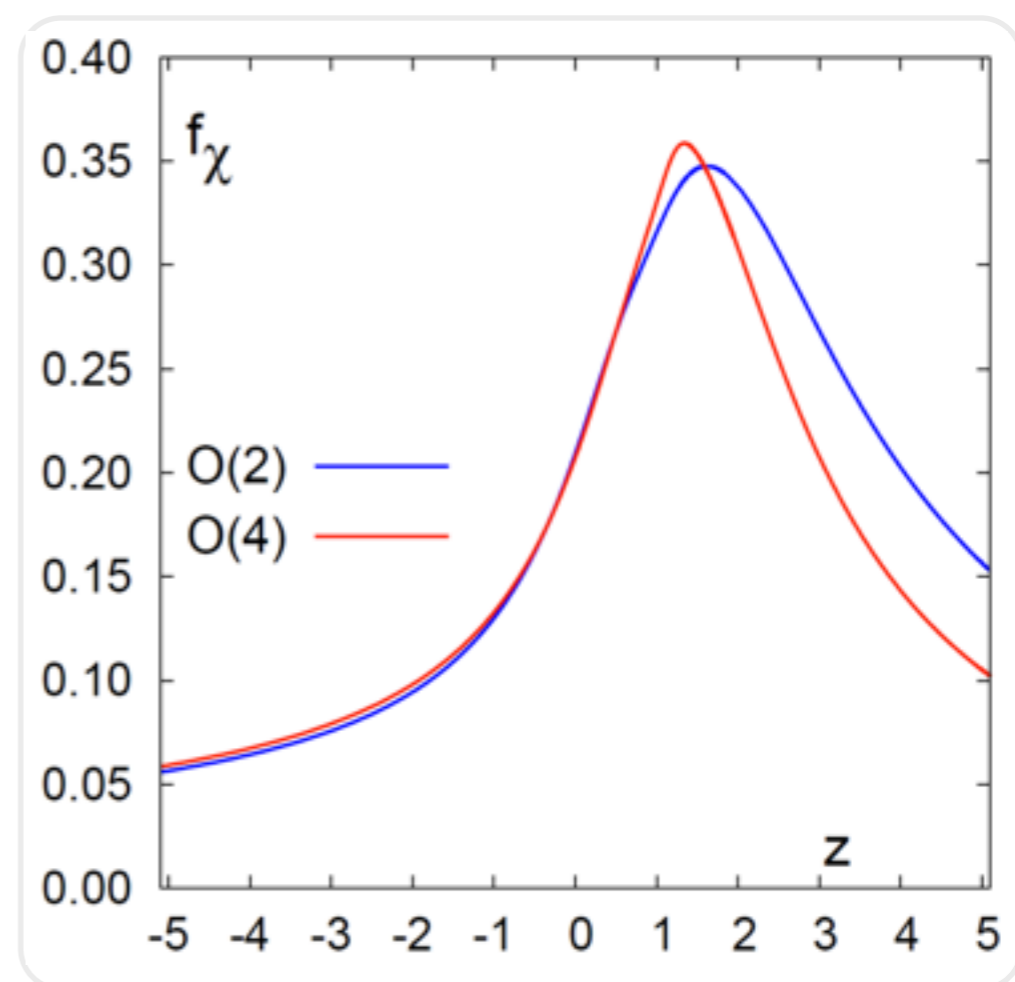
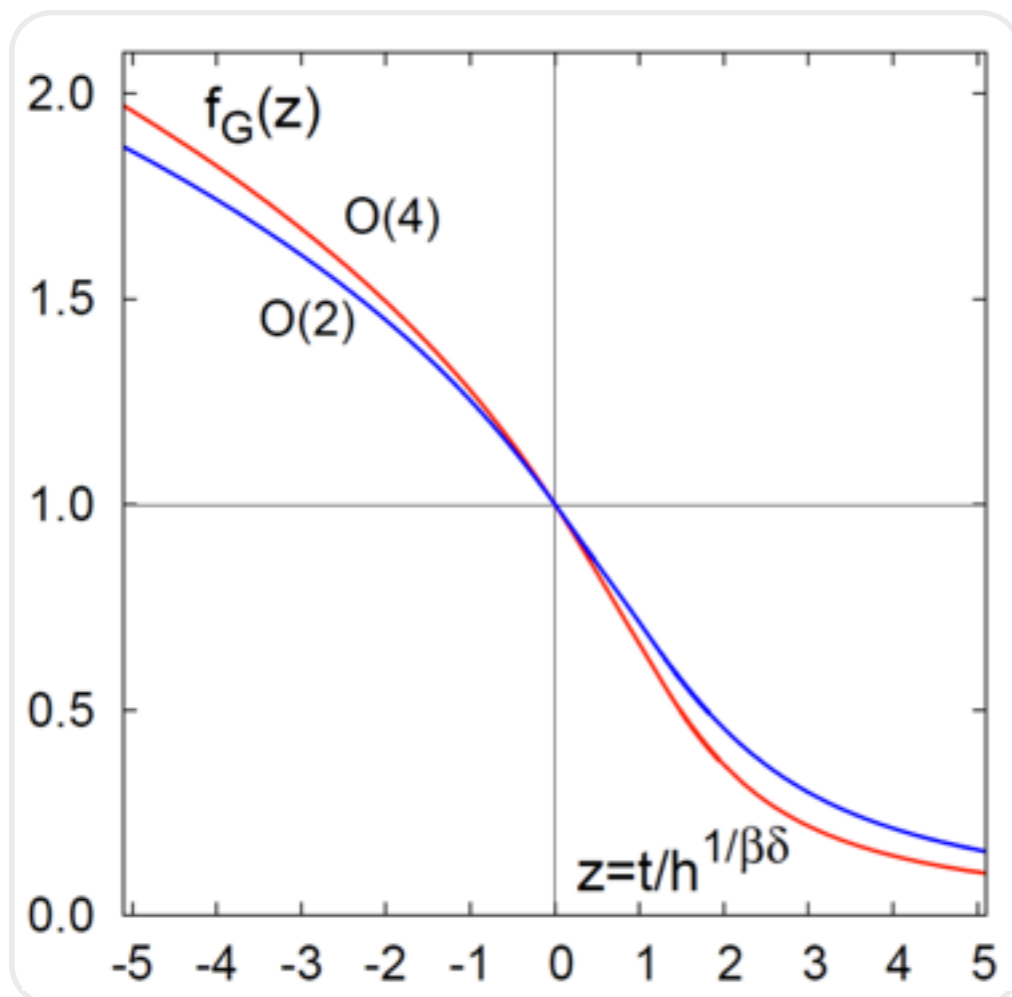
$$h = \frac{I}{h_0} \frac{m_I}{m_s}$$

$$t = \frac{1}{t_0} \frac{T - T_c}{T_c}$$

Magnetic Equation of State (MEoS):

$$M = -\partial f_s(t, h) / \partial h = h^{1/\delta} f_G(z)$$

$$f_\chi(z) = h_0^{1/\delta} (m_I/m_s)^{1-1/\delta} \partial M / \partial h$$



chiral phase transition and universal scaling

Behavior of the free energy close to critical lines

$$f(m, T) = h^{1+1/\delta} f_s(z) + f_{reg}(m, T), \quad z = t/h^{1/\beta\delta}$$

h : external field, t : reduced temperature, β, δ : universal critical exponents

$f_s(z)$: universal scaling function, O(N) etc.

$$h = \frac{I m_l}{h_0 m_s}$$

$$t = \frac{I}{t_0} \frac{T - T_c}{T_c}$$

Magnetic Equation of State (MEoS):

$$M = -\partial f_s(t, h) / \partial h = h^{1/\delta} f_G(z)$$

$$f_\chi(z) = h_0^{1/\delta} (m_l/m_s)^{1-1/\delta} \partial M / \partial h$$

Comparison with QCD

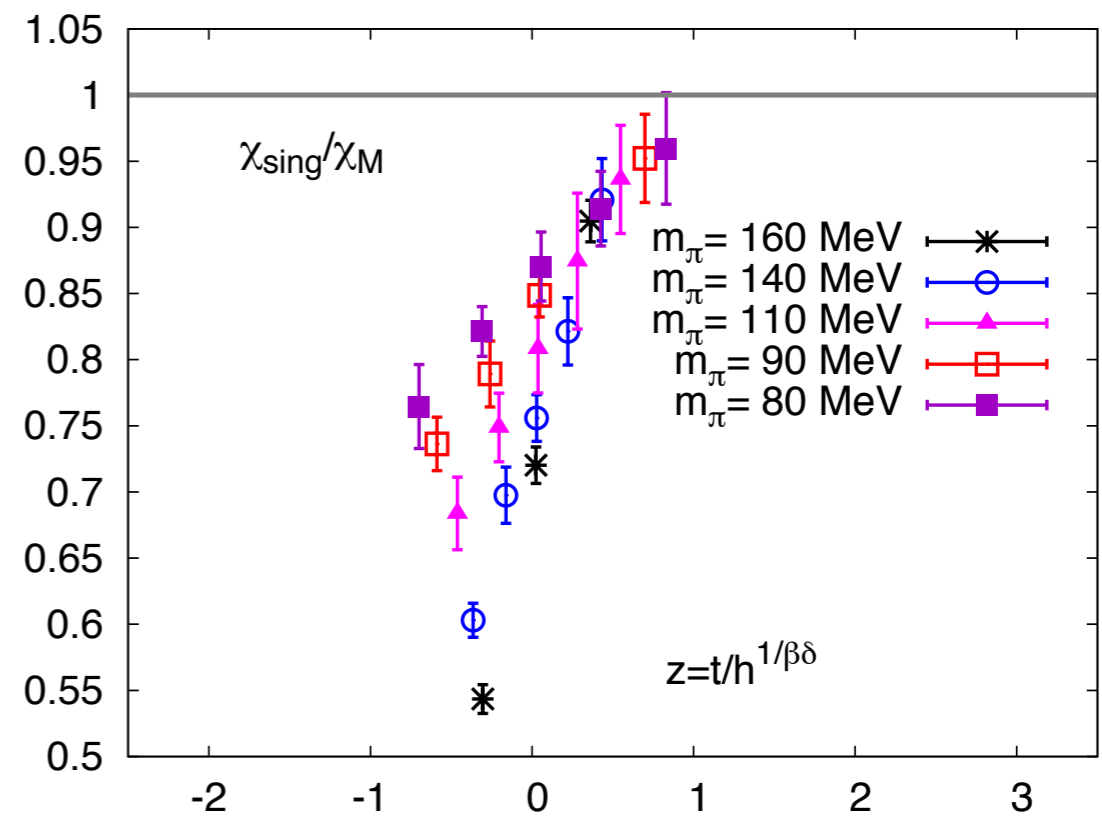
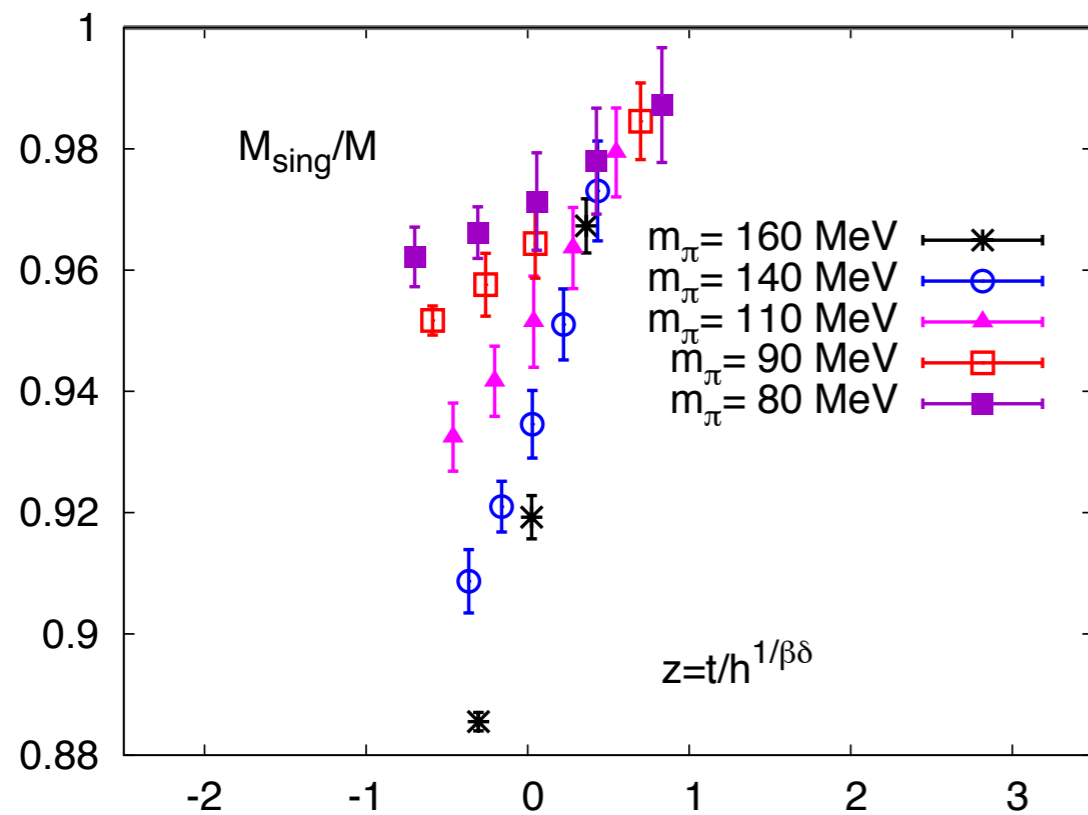
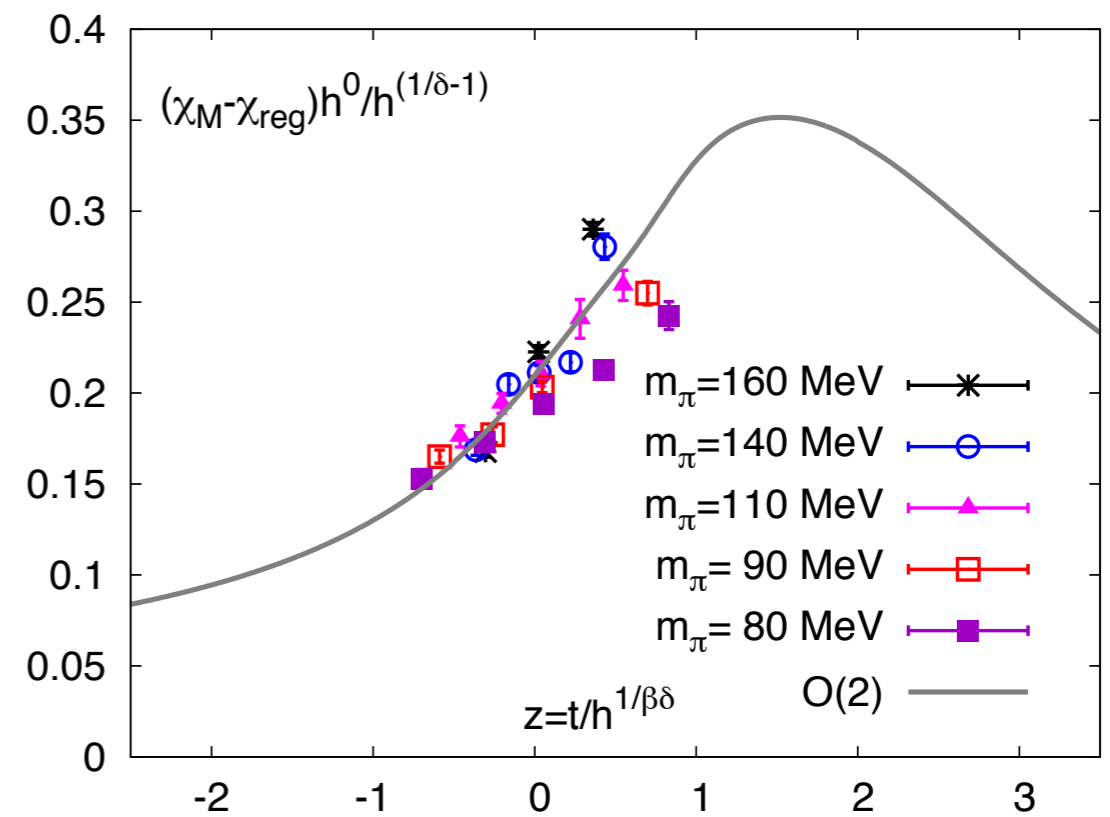
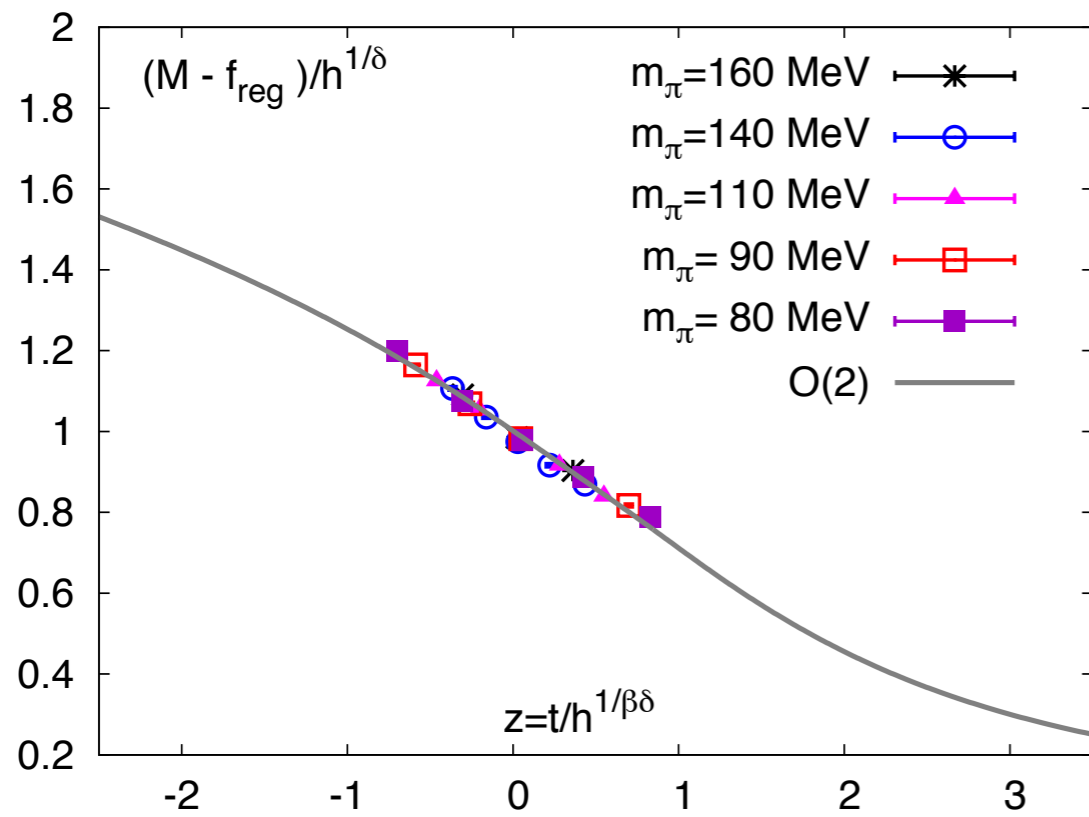
$$M = m_s \langle \bar{\psi} \psi \rangle_l / T^4, \quad \chi_M = m_s^2 \chi_{tot} / T^4$$

Contributions from the regular term

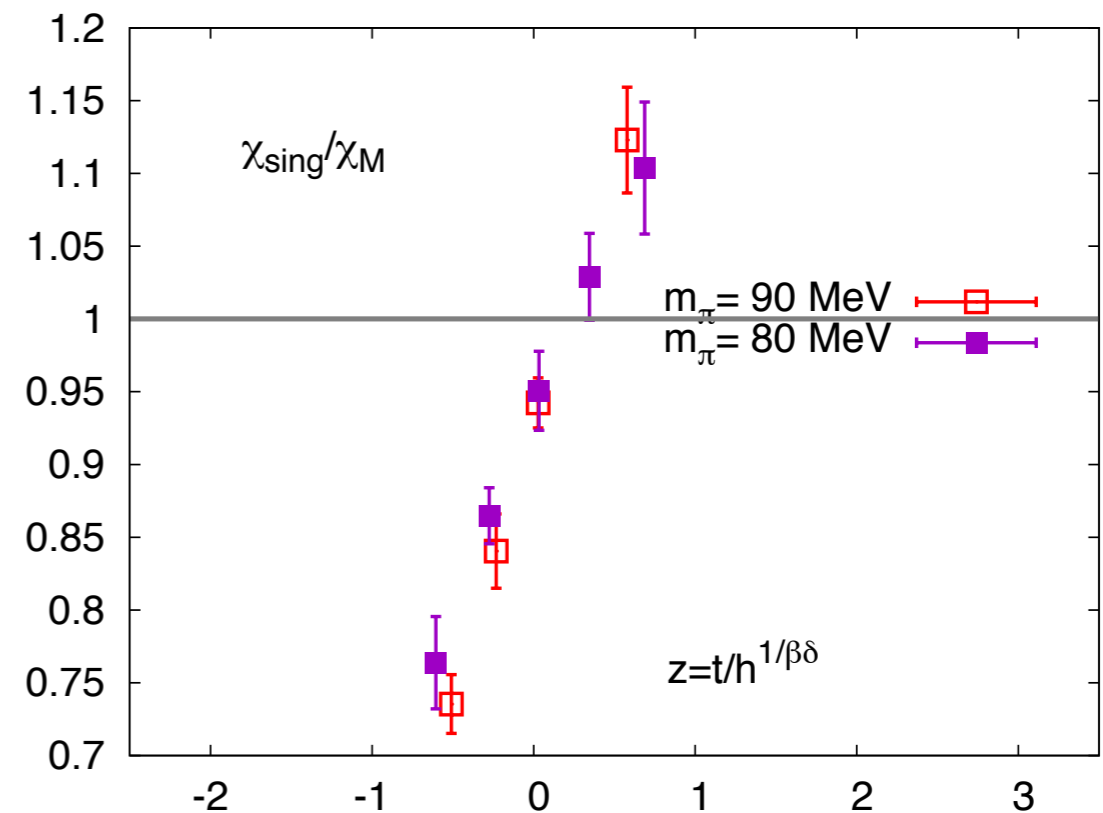
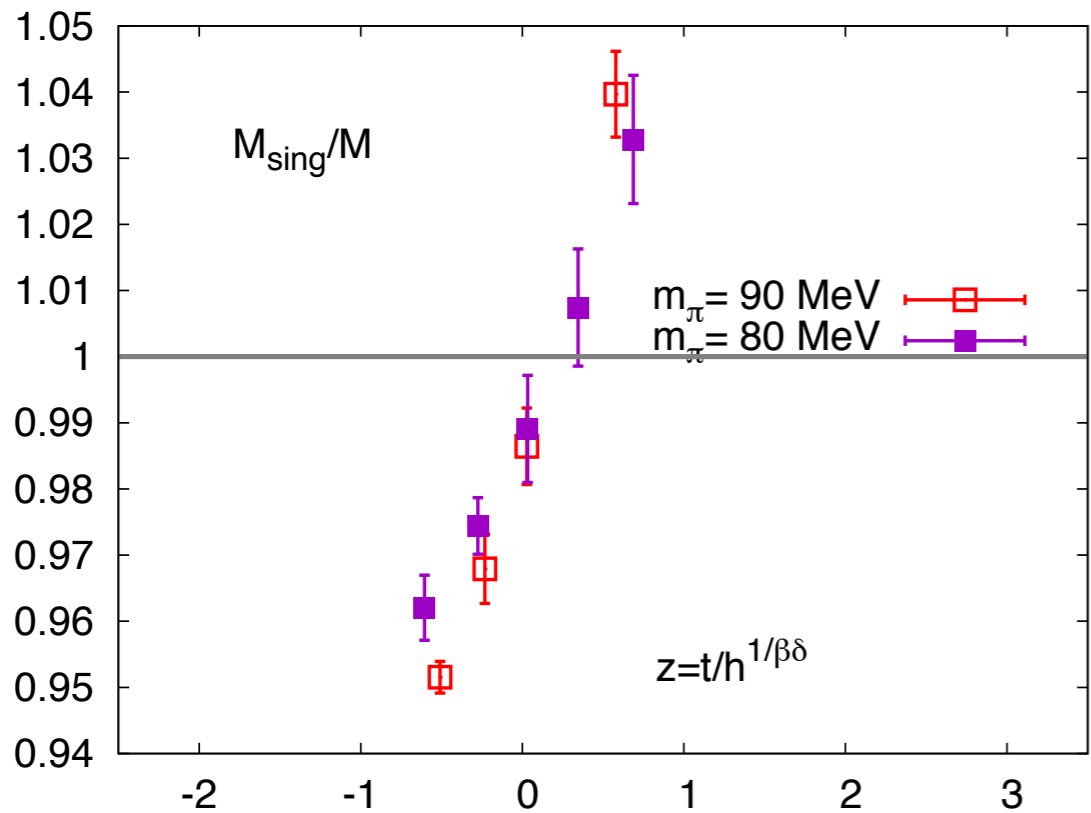
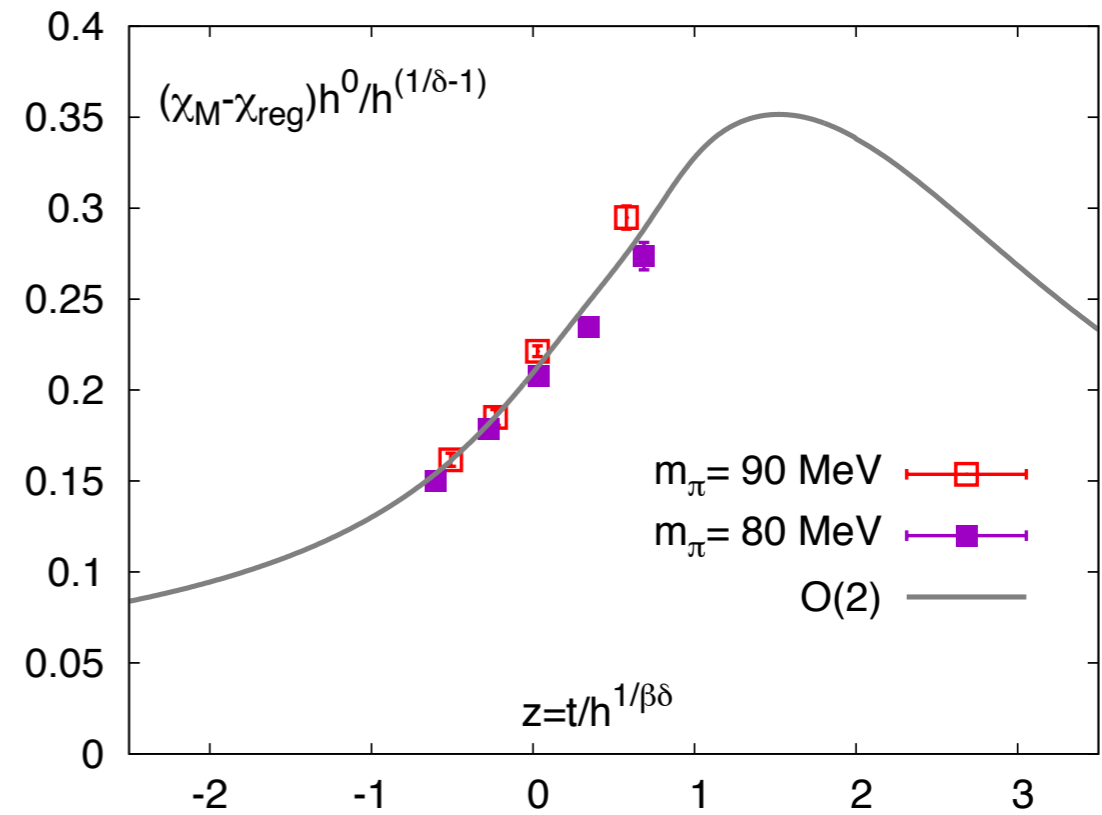
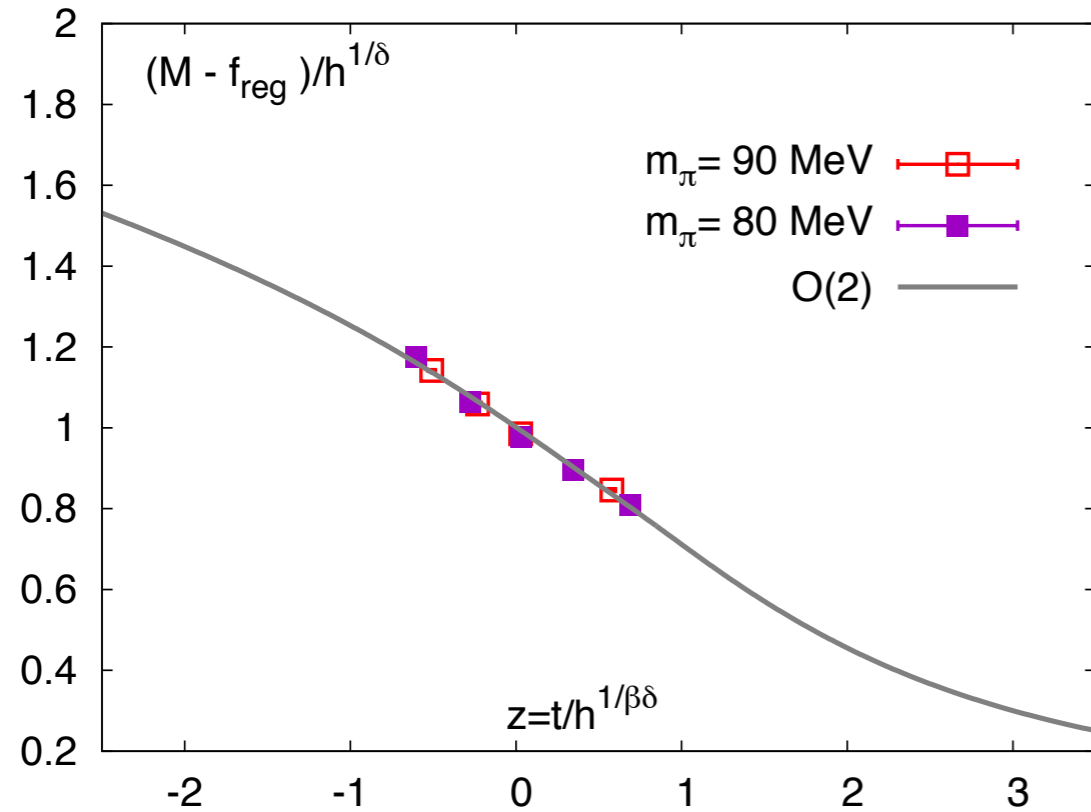
$$M = h^{1/\delta} f_G(z) + f_{reg}, \quad f_{reg} = \left(a_0 + a_1 \frac{T - T_c}{T_c} \right) \frac{m_l}{m_s}$$

$$\chi_M = h_0^{-1} h^{1/\delta-1} f_\chi(z) + f'_{reg}, \quad f'_{reg} = a_0 + a_1 \frac{T - T_c}{T_c}$$

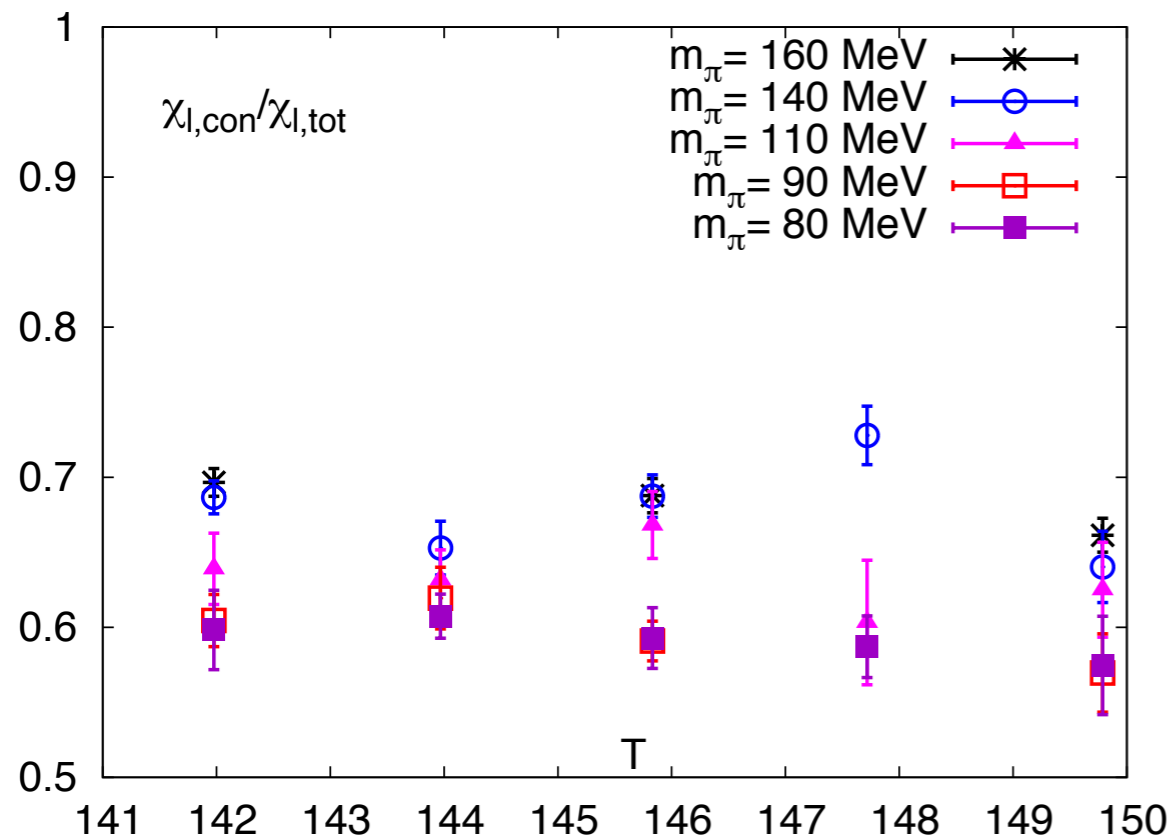
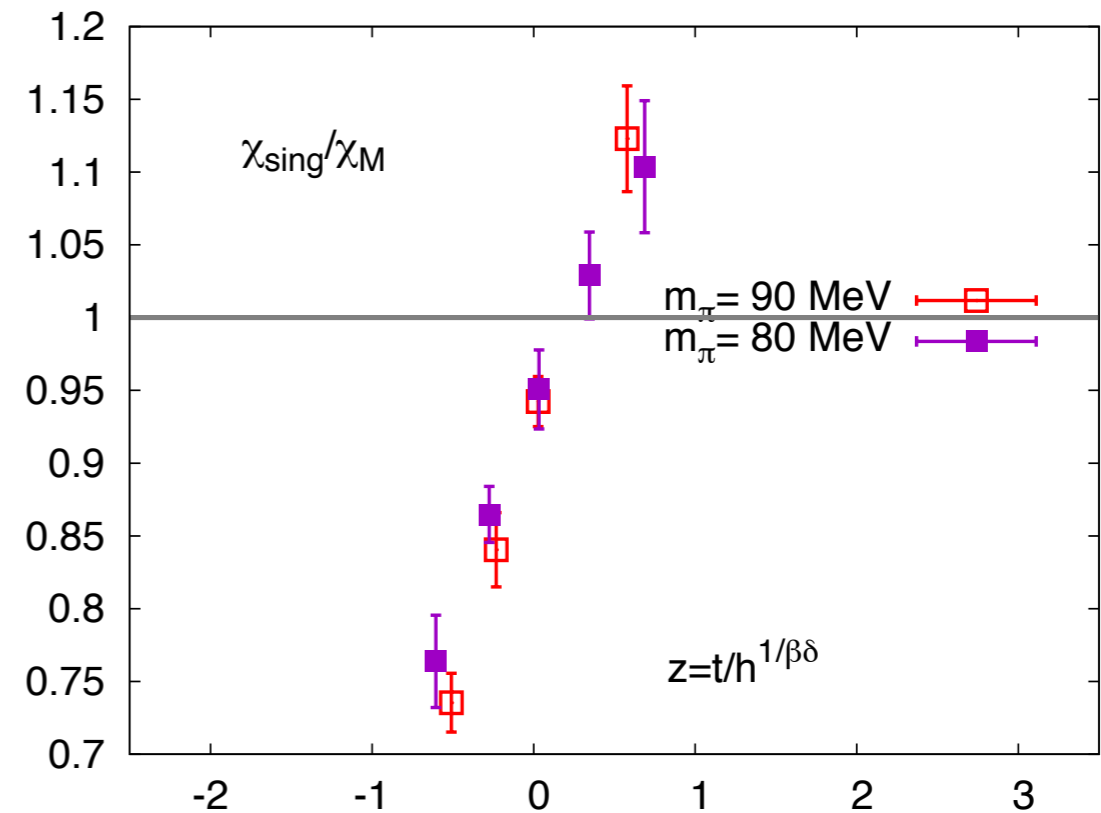
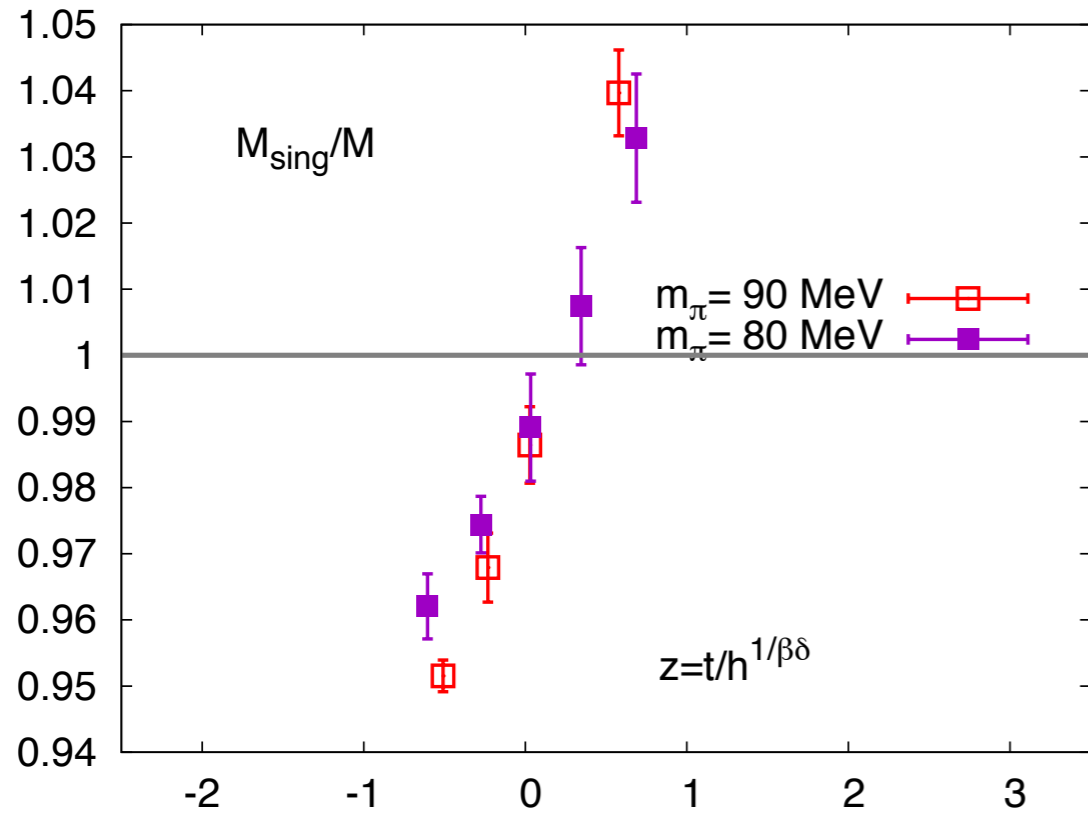
O(2) scaling fit to chiral susceptibilities & condensates



O(2) scaling fit to chiral susceptibilities & condensates



O(2) scaling fit to chiral susceptibilities & condensates

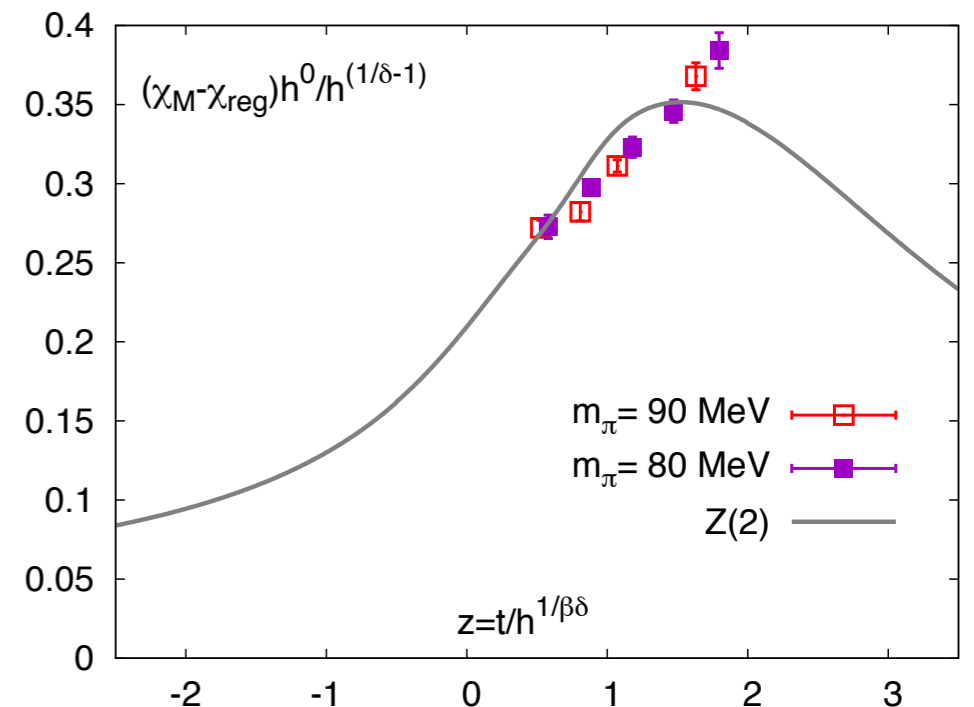
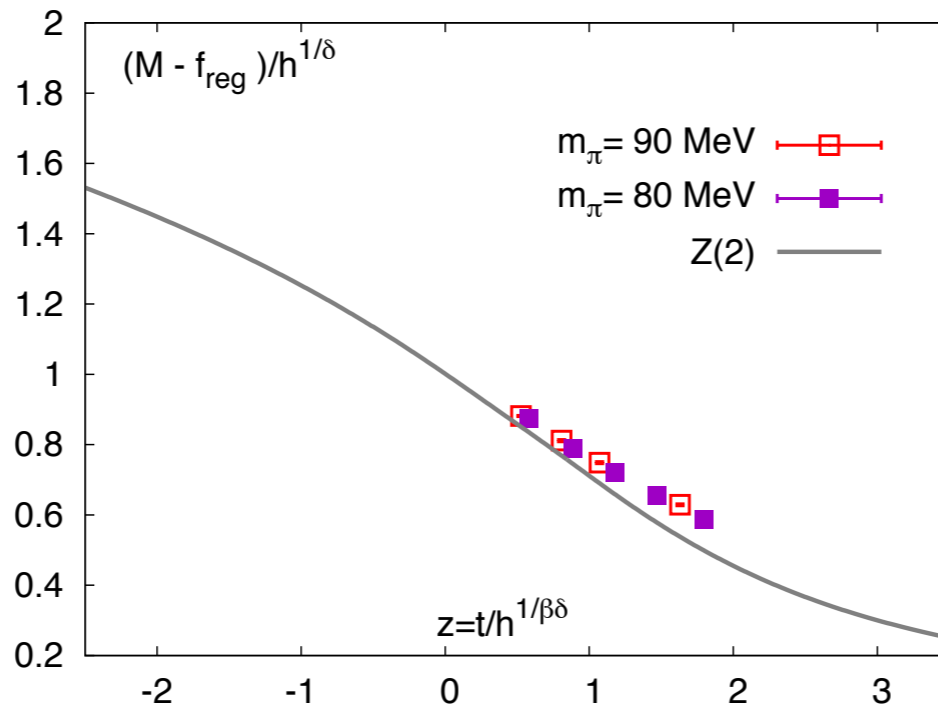
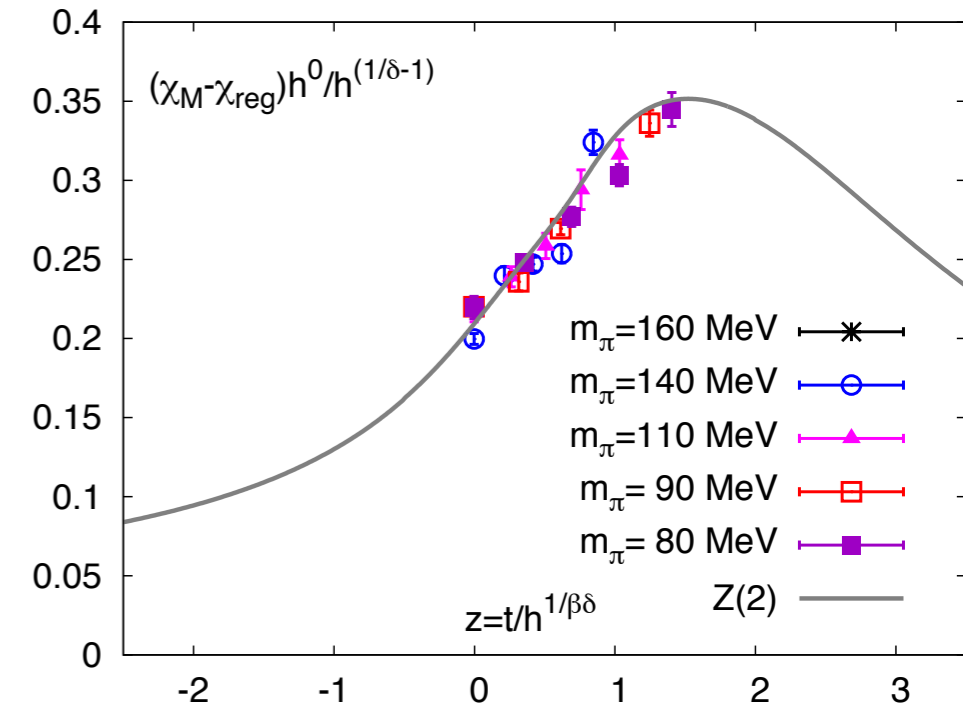
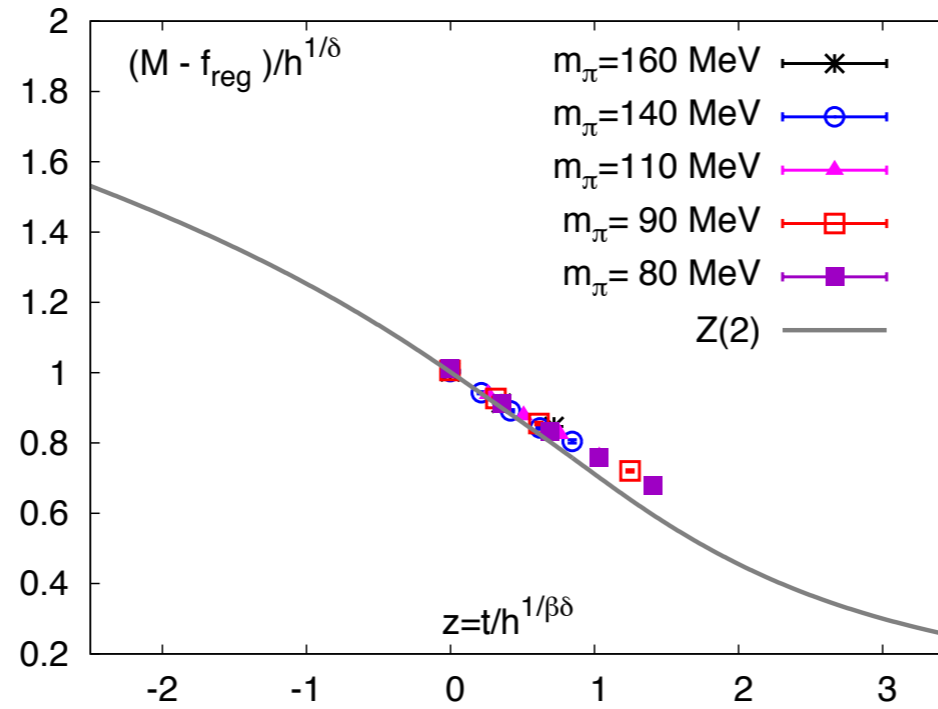


A good fit to chiral observables using O(2) scaling function is obtained

Z(2) scaling fit to chiral susceptibilities & condensates

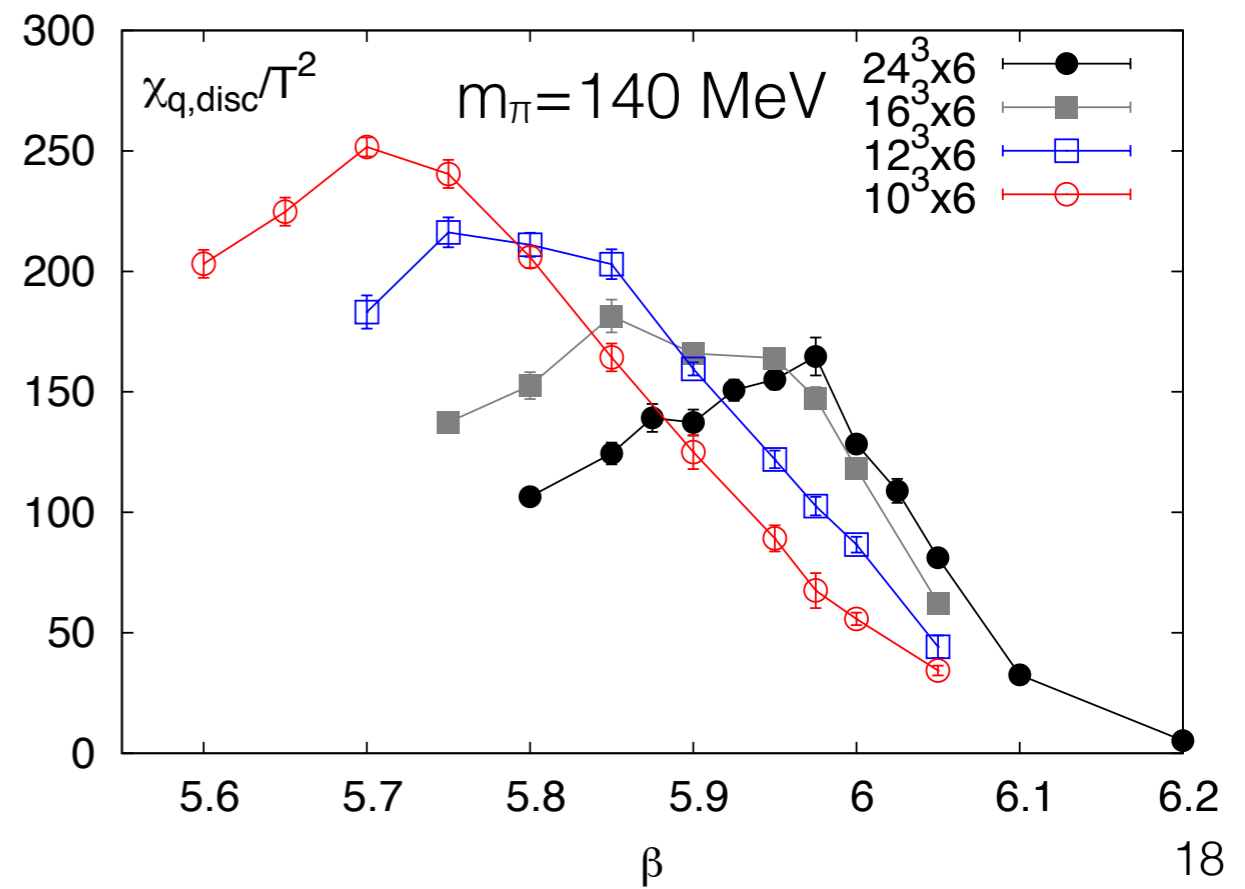
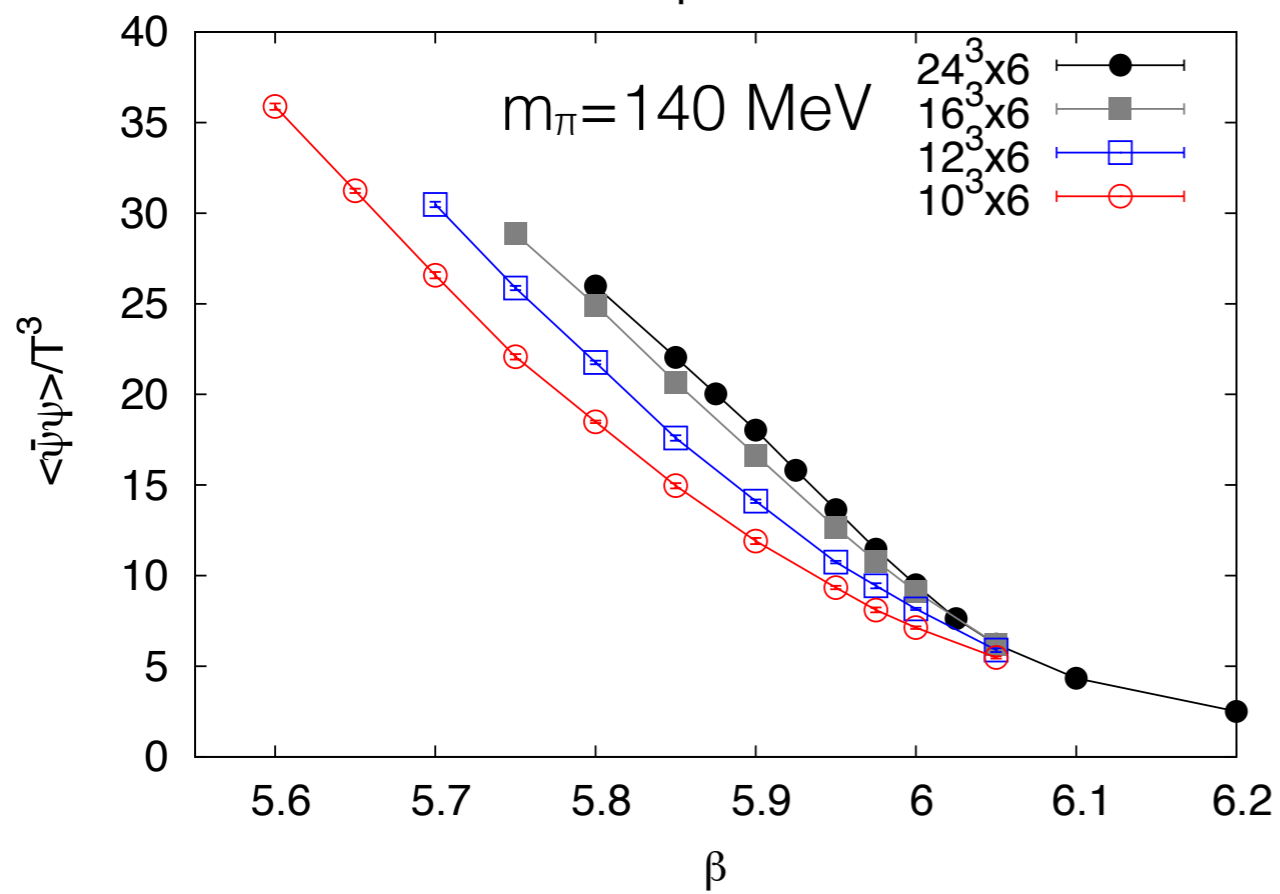
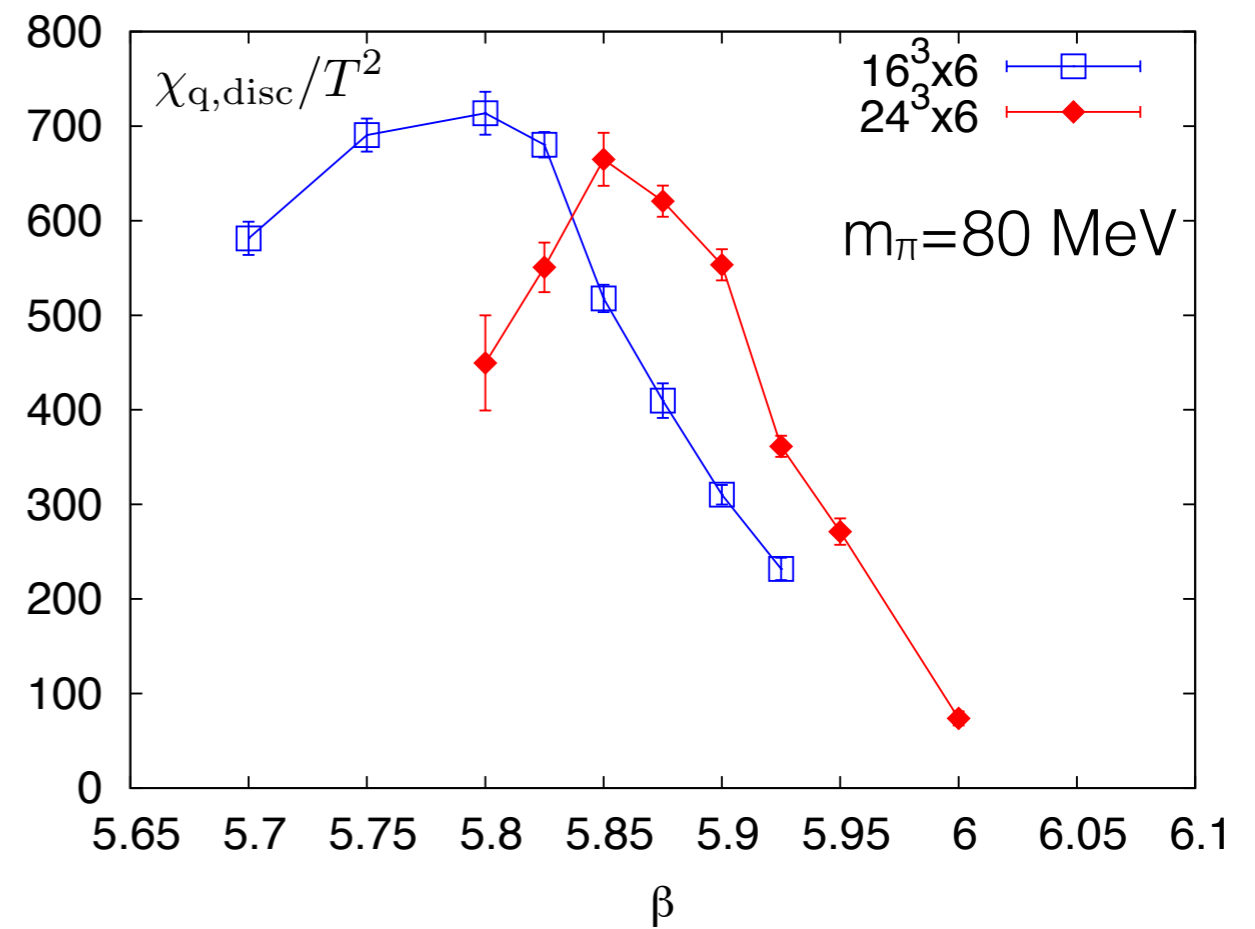
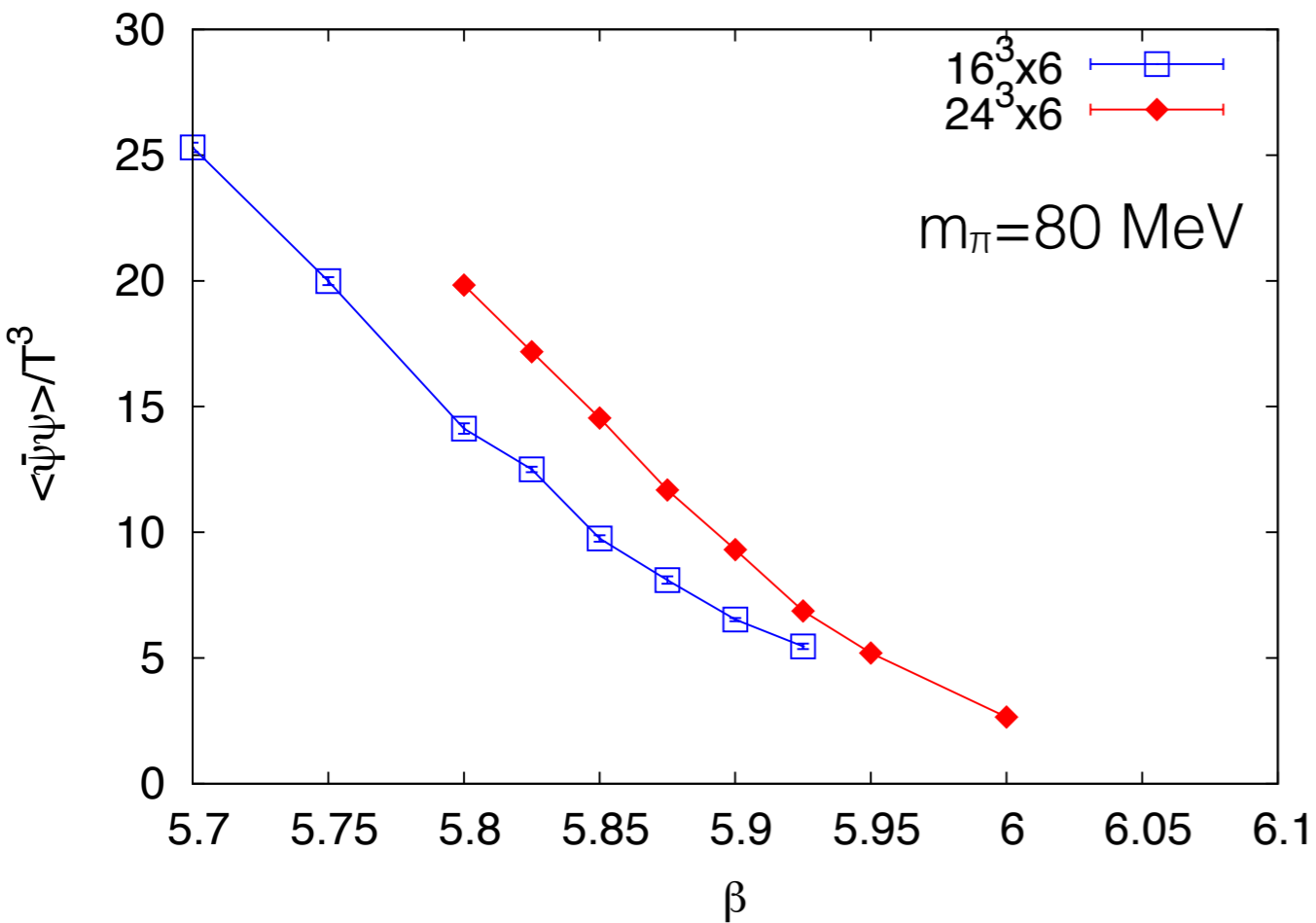
O(2) $h = 1/h_0 m_l/m_s$ \longrightarrow $h = 1/h_0 (m_l - m_c)/m_s$ **Z(2)**

A small and negative value of m_c is favored from fit

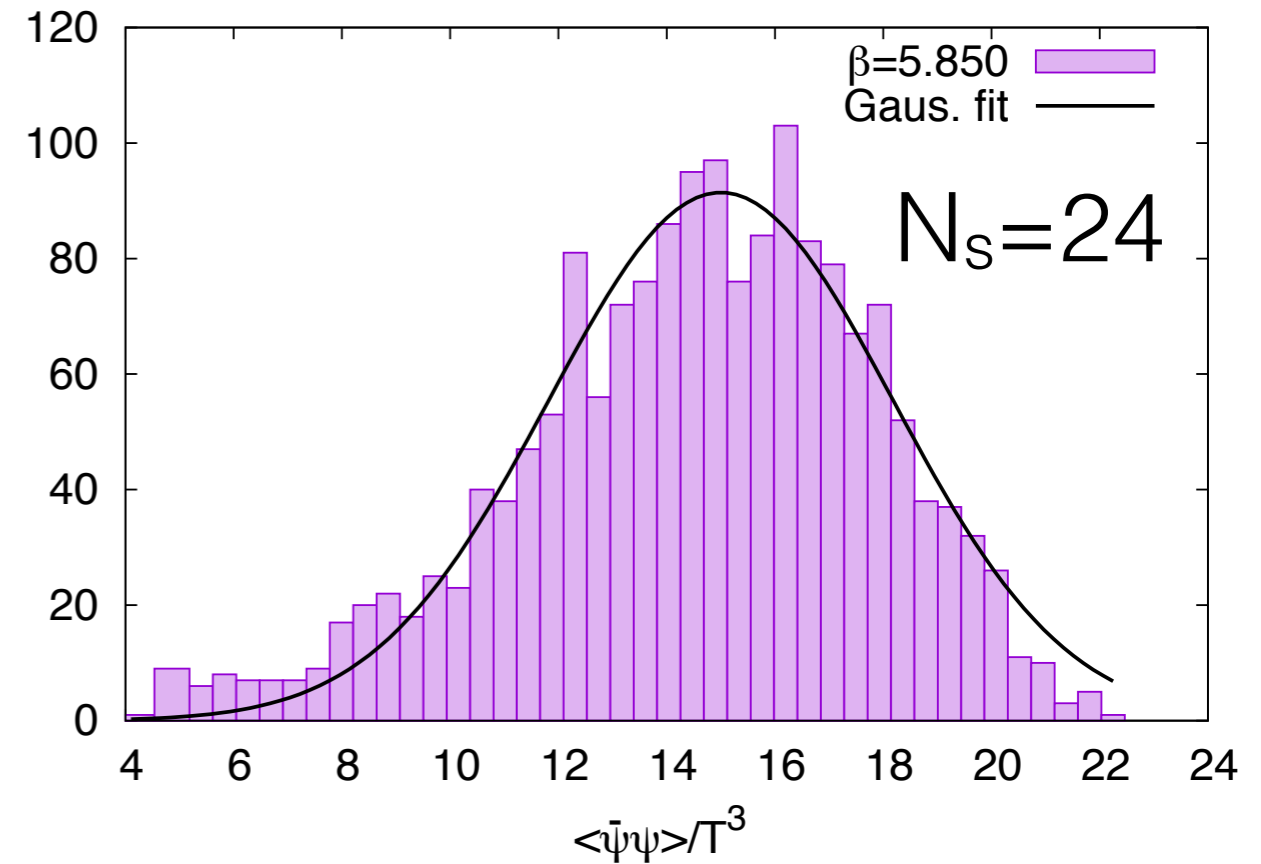
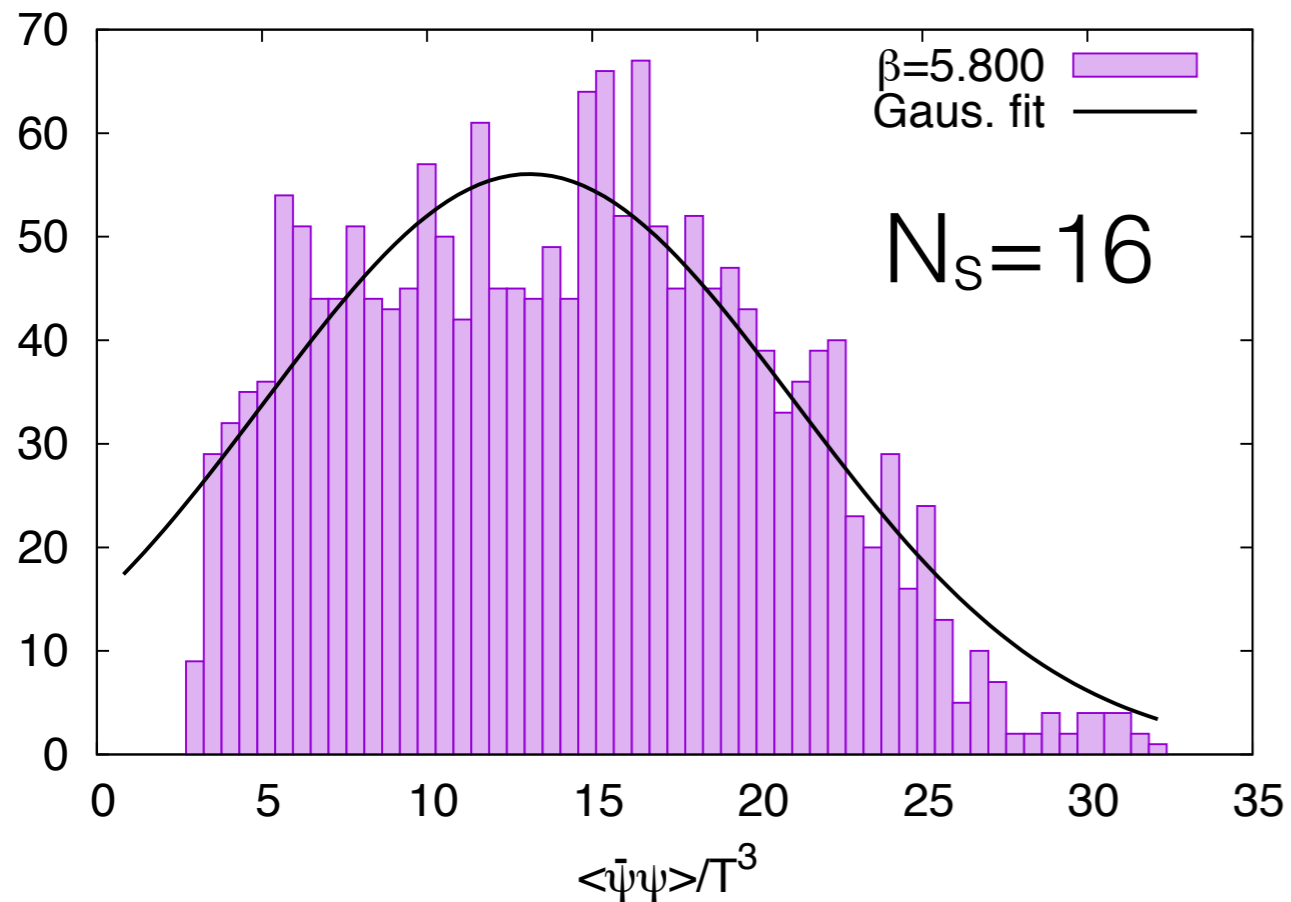


$$N_f = 3$$

Volume dependence of chiral observables



time history of chiral condensate near β_c at lowest quark mass



No double peak structure is observed

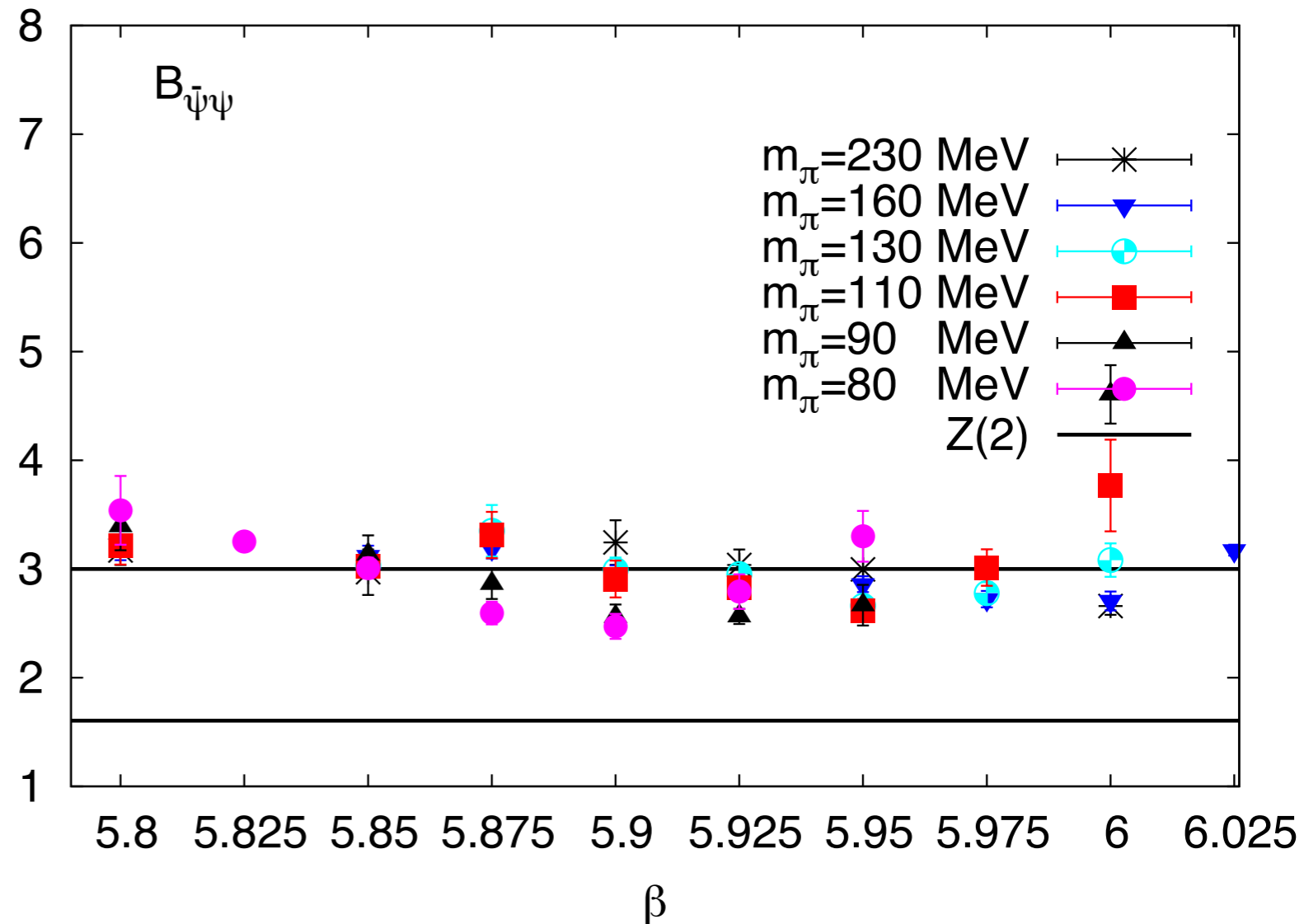
Binder cumulants of chiral condensates

$$B_{\bar{\psi}\psi} \equiv \frac{\langle (\delta\bar{\psi}\psi)^4 \rangle}{\langle (\delta\bar{\psi}\psi)^2 \rangle^2}$$

In the crossover region $B=3$

2nd order transition in the Ising universal class $B=1.604$

1st order transition $B=1$



**No first order phase transition is observed
in the current pion mass window: [80, 230] MeV**

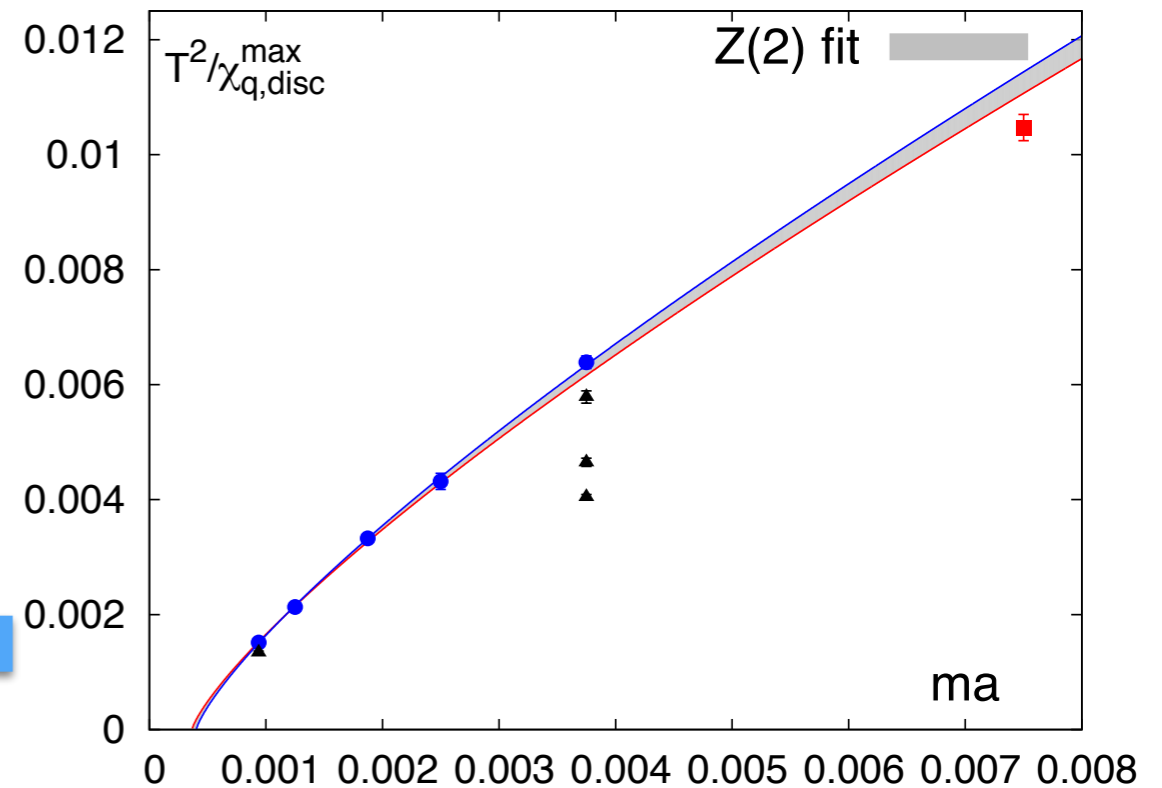
estimate of critical mass m_c

At the pseudo critical temperature

$$M_p = h^{1/\delta} f_G(z_p), \quad h = (m_q - m_c)/h_0$$

$$\chi_M^p = h_0^{-1} h^{1/\delta-1} f_\chi(z_p)$$

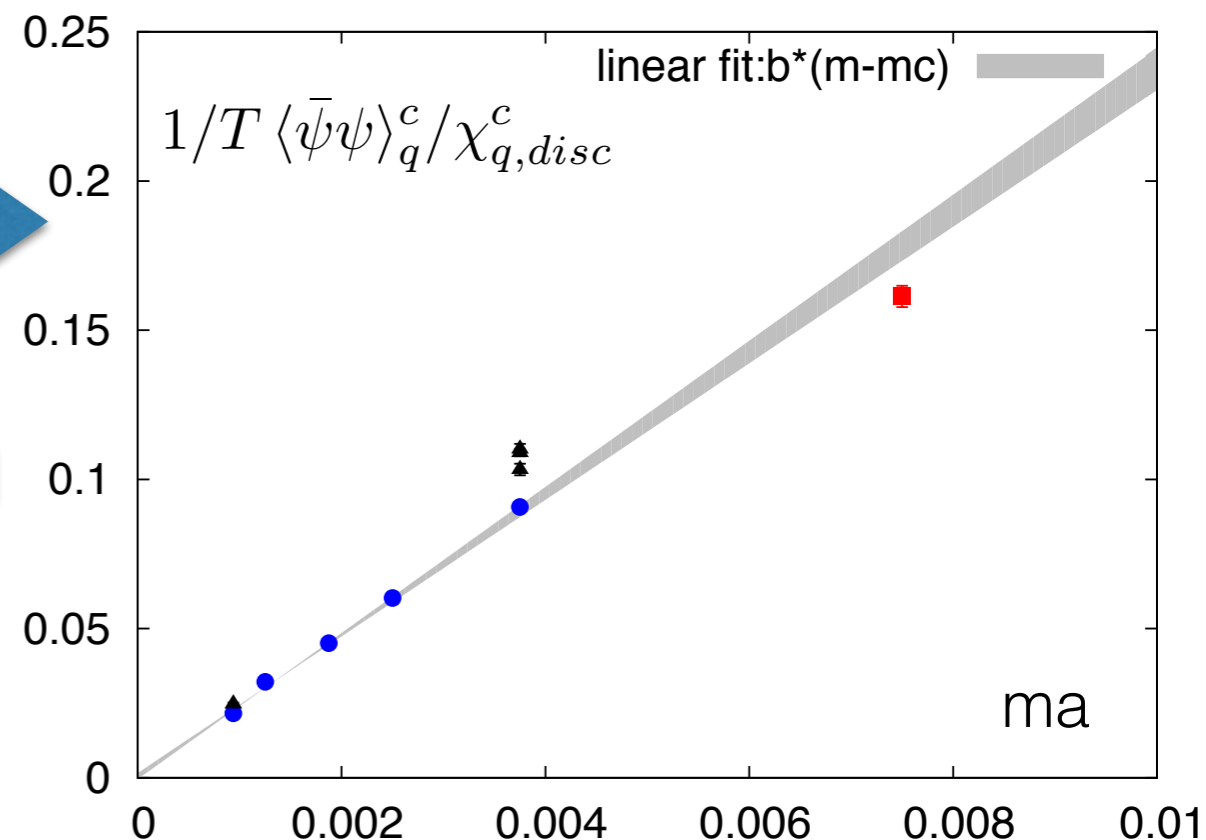
$$m_\pi^c \approx 50 \text{ MeV} \Leftrightarrow m^c a = 3.9(1)e-04$$



independent of universal exponents

$$M_p / \chi_M^p = (m_q - m_c) f_G(z_P) / f_\chi(z_p)$$

$$m^c a = -6(8)e-05$$

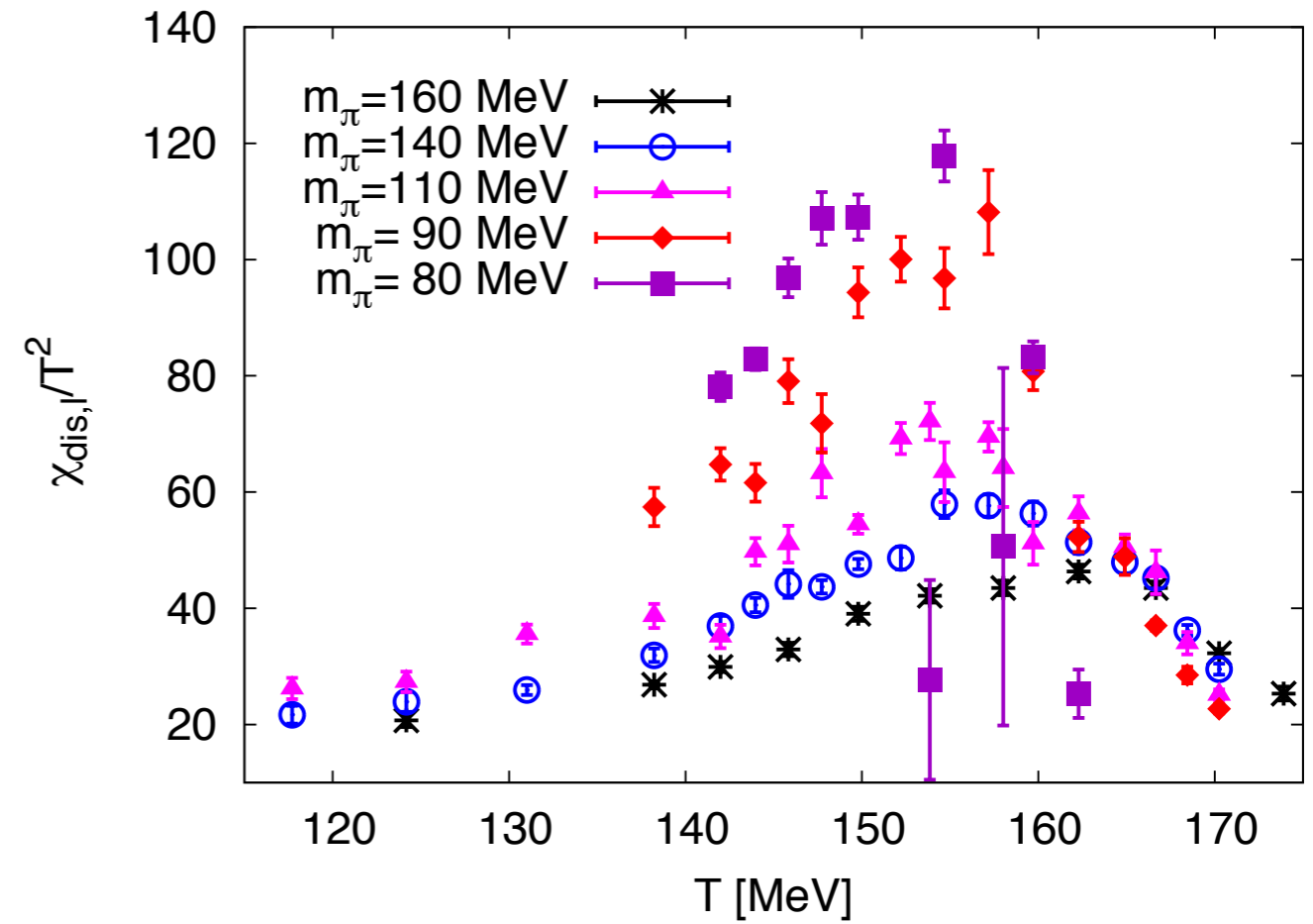
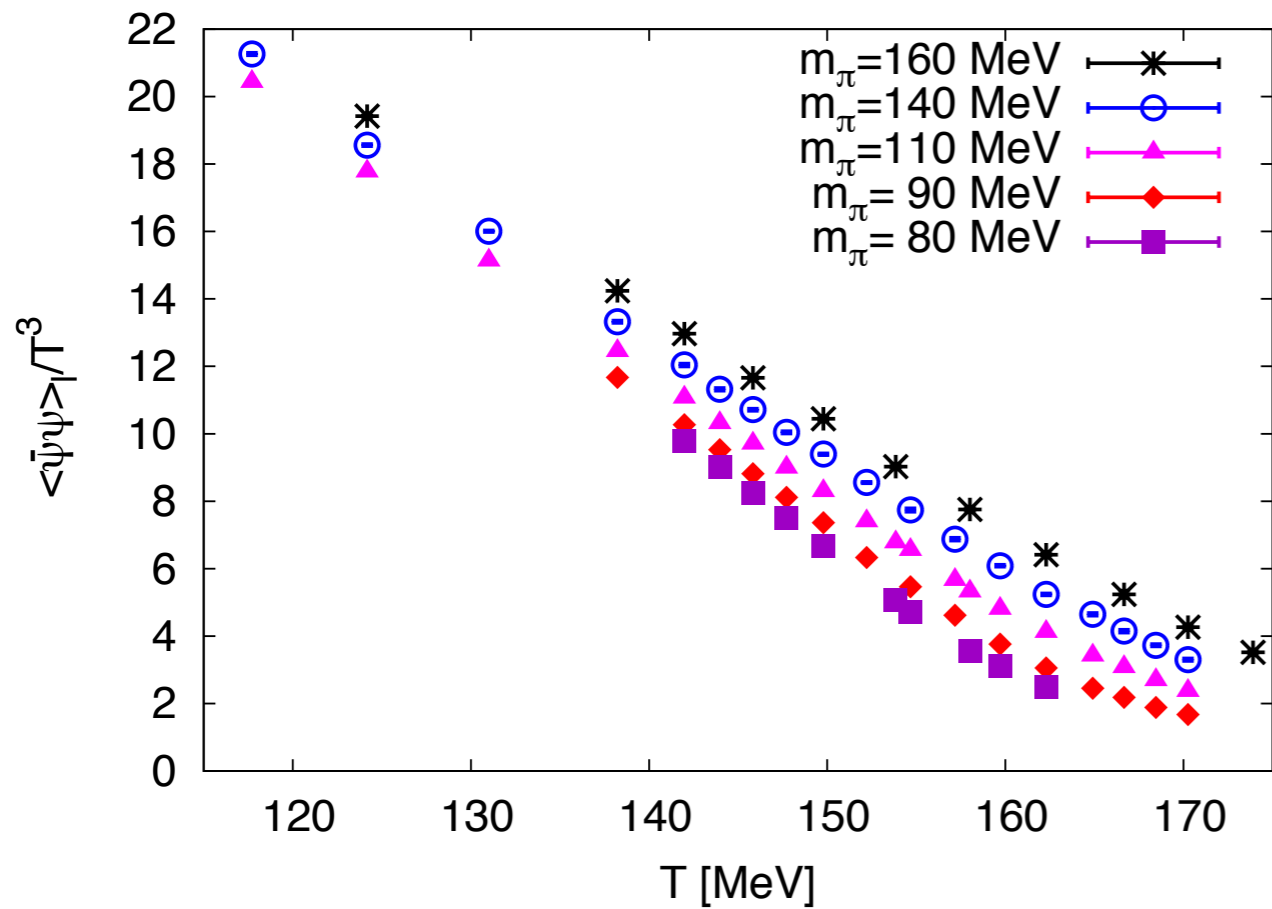


Summary & Outlook

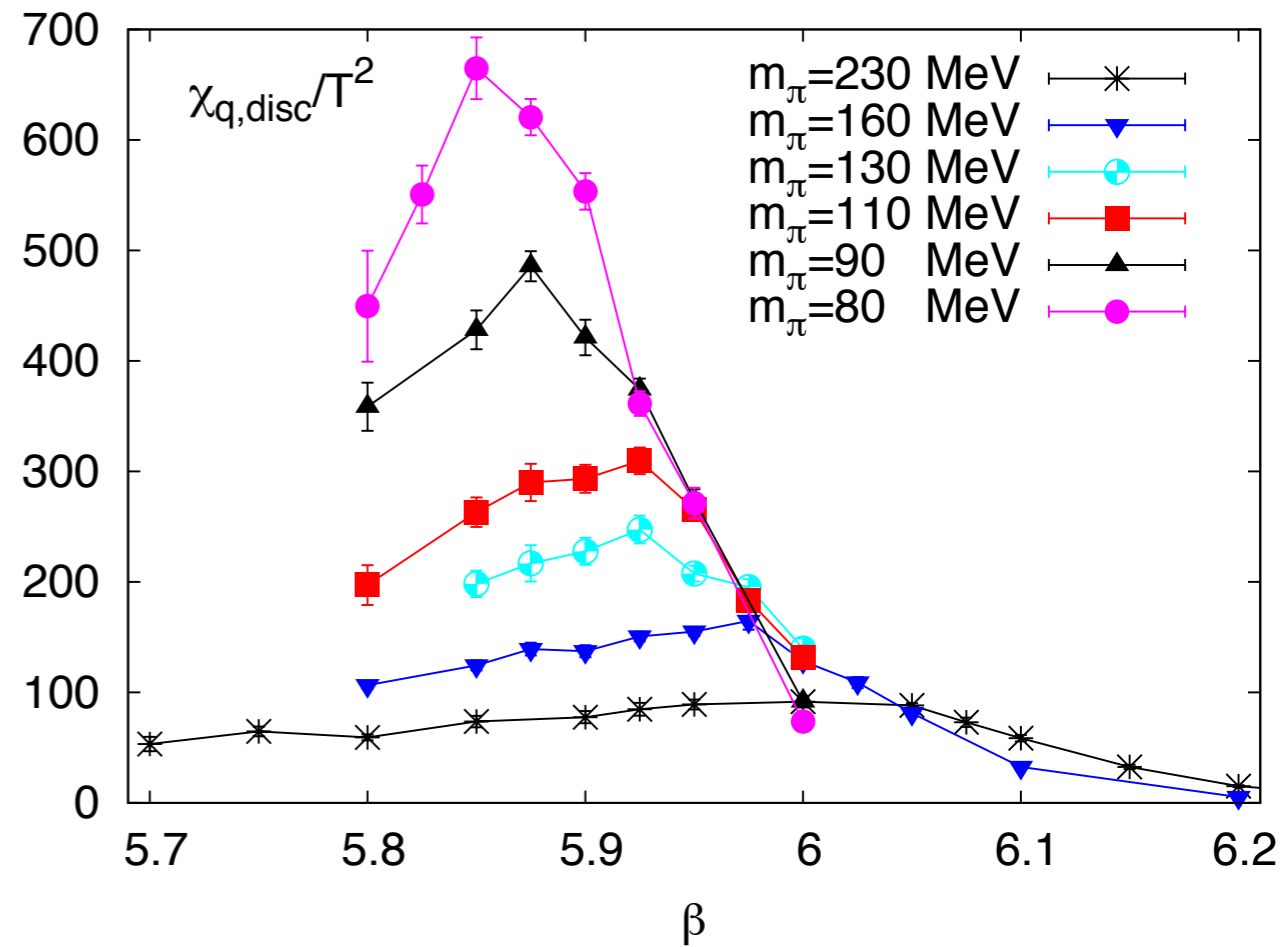
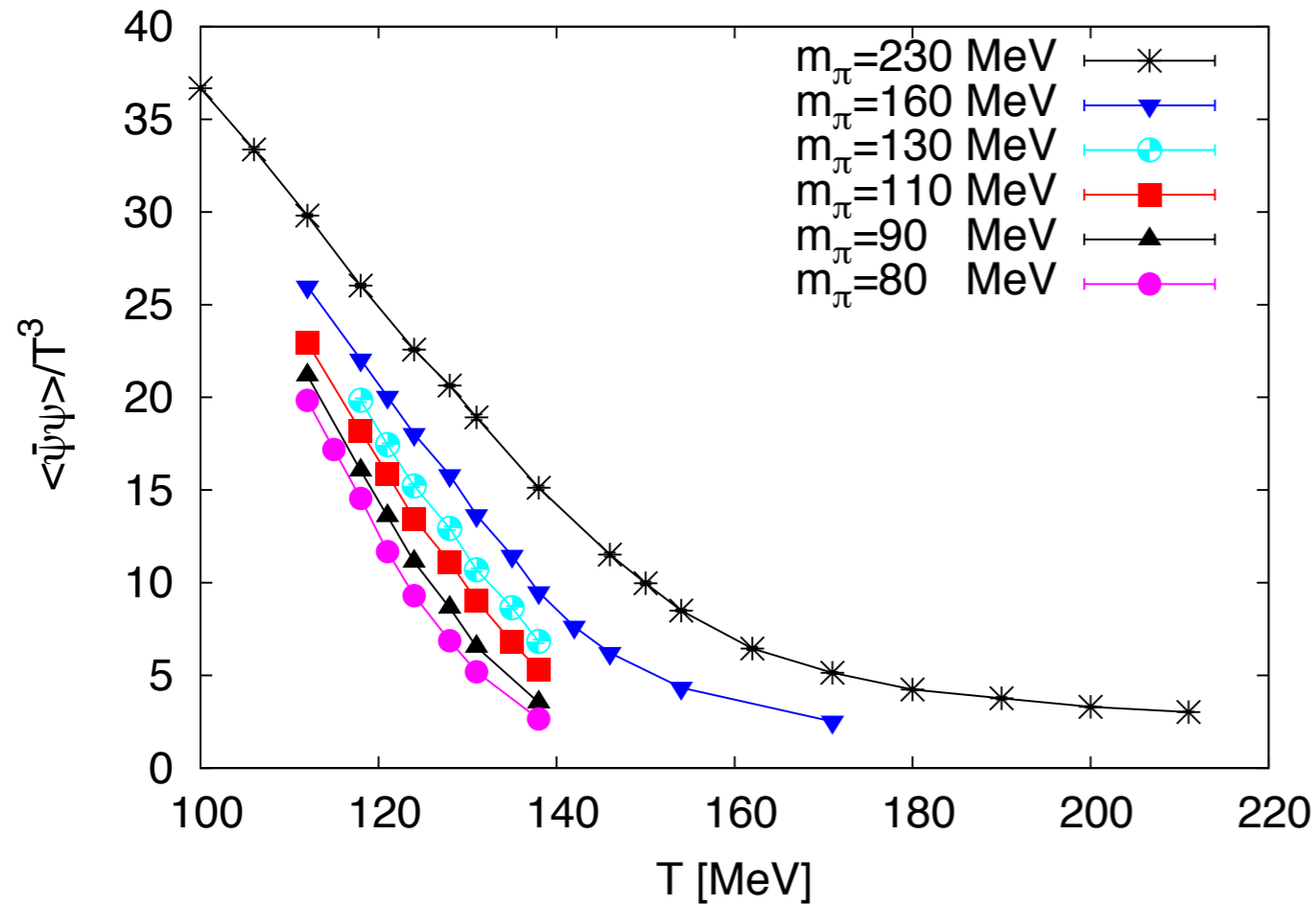
- We have performed simulations on $N_t=6$ lattices using HISQ/tree action in $N_f=2+1$ & 3 QCD at 5 & 6 different values of quark masses.
 - We found no evidence of a first order phase transition region in $230 \text{ MeV} \lesssim m_\pi \lesssim 80 \text{ MeV}$ in $N_f=3$ QCD
 - Critical pion mass is estimated: $m_\pi \lesssim 50 \text{ MeV}$
 - A good description of chiral observables in $N_f=2+1$ QCD is provided by the $O(2)$ scaling function
 - Our results indicate $m_s^{\text{tric}} < m_s^{\text{phy}}$
- $N_t=8$ calculations are on the way

$N_f=2+1$:

chiral condensates and disconnected sus.



$N_f=3$: chiral condensates & susceptibilities



Nf=3: Binder cumulants of chiral condensates

