$Q\bar{Q}$ dynamics with external magnetic fields

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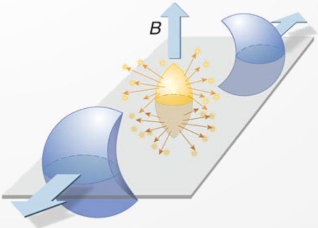
QCD with external B fields

QCD with B fields at the strong scale $\rightarrow eB \simeq m_\pi^2$
Relevant in many contexts

- Neutron stars, $\mathbf{B} \sim \mathbf{10^{10}}$ **T** [Duncan and Thompson, 1992]
- Off-central heavy ion collisions, $\mathbf{B} \sim \mathbf{10^{15}}$ **T** [Skokov et al., 2009]
- Early universe, $\mathbf{B} \sim \mathbf{10^{16}}$ **T** [Vachaspati, 1991]

$10^{15} e$ Tesla $\simeq 0.06$ GeV^2

- LHC eB factory: expected fields up to $eB \simeq 0.3$ GeV^2
- Lifetime of eB after collisions still under debate



eB effects on gluon dynamics

eB couples with quarks. Nevertheless, at this scale electromagnetic interactions lead to non trivial non perturbative effects also in the gluon sector

Lattice results:

- *Sea* and *Valence* contributions to the chiral condensate [D'Elia, Negro (2011)]
- Inverse catalysis [Bruckmann et al. (2013)]
- Anisotropies in the plaquettes [Ilgenfritz et al. (2014); Bali et al. (2013)]
- Anisotropy of the static $\bar{Q}Q$ potential at $T = 0$ [Bonati et al. (2014)]

Outline

$T = 0$ results

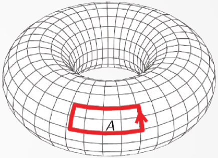
- Updated results for $V_{\bar{Q}Q}(B)$ at $T = 0$ with continuum extrapolations
- Magnetic angular dependence $\rightarrow$ anisotropy coefficients

Finite T results

- Preliminary study of Debye screening masses above T_c
- Dependence on eB at $T = 200, 250$ MeV for fields up to $eB = 1.03$ GeV^2

Magnetic field on the lattice

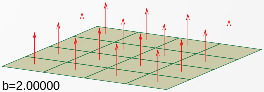
- Gauge fields interactions enter through the covariant derivative.
- Continuum: $\partial_\mu + igA_\mu^a T^a \to \partial_\mu + igA_\mu^a T^a + ieA_\mu$
- On the lattice: add proper $u(1)$ phases to the $SU(3)$ links :

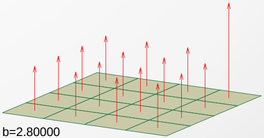


- $U_\mu(n) \to U_\mu(n) \exp\left(iqa_\mu(n)\right)$
- Periodic boundary conditions
  $e^{iqBA} = e^{iqB(A - L_x L_y a^2)}$
  Quantization condition
  $qB = \frac{2\pi b}{L_x L_y a^2}$ con $b \in \mathbb{Z}$
- $\vec{B} = B\hat{z} \to$ gauge fixing $a_y = Bx$, then:

$$u_y^{(q)}(n) = e^{ia^2 qBn_x}$$

$$u_x^{(q)}(n)|_{n_x = L_x} = e^{-i\ a^2 qL_x Bn_y}$$





- For $b \notin \mathbb{Z}$ string become visible. Non-uniform B

Static $Q\bar{Q}$ potential

Interaction between NR quarks $\bar{Q}Q$ well described by the so called Cornell potential

$$V(r) = C + \frac{\alpha}{r} + \sigma r; \quad \sigma \to \text{string tension} \quad \alpha \to \text{Coulomb term}$$

Good effective description of meson spectra and confinement

- The static potential have been largely investigated from first principles on the lattice
- One consider the Wilson loop $W(r,T) \to$ related to the energy to create a couple $\bar{Q}Q$ at distance r annihilated after a time T

$$V(\hat{r}) = \lim_{\hat{T}\to\infty} \log\left(\frac{W(\hat{r},\hat{T}+1)}{W(\hat{r},\hat{T})}\right)$$

- In PRD 89 (2014) 114502 we investigated the effects induced by external magnetic fields on the potential at $T = 0$

Static $Q\bar{Q}$ potential

- Fixed magnetic field direction $\vec{B} = B\hat{z}$

- Defined orthogonal (W_{XY}) and parallel (W_Z) Wilson loops according to the spatial direction of the loop

$$\langle W_{XY}(|r|,T)\rangle = \frac{\langle W(r\hat{x},T) + W(r\hat{y},T)\rangle}{2}; \qquad \langle W_Z(|r|,T)\rangle = \langle W(r\hat{z},T)\rangle$$

- Kept separeted these components and measured the potential from them

$$V_{XY}(\hat{r}) = c_{XY} + \frac{\alpha_{XY}}{\hat{r}} + \hat{\sigma}_{XY}\hat{r}^2$$

$$V_Z(\hat{r}) = c_Z + \frac{\alpha_Z}{\hat{r}} + \hat{\sigma}_Z\hat{r}^2$$

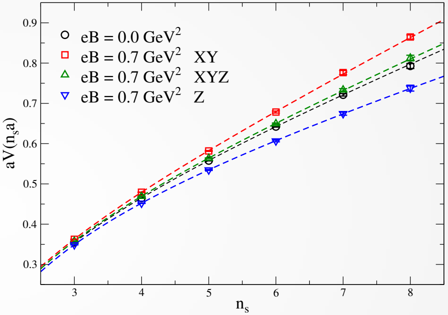
Anisotropic static $Q\bar{Q}$ potential

We used

- Symanzik improved gauge action
- Staggered fermions with 2-level sout smearing improvement
- Phyisical quark masses
- 1 HYP smearing on T-direction
- 24 APE smearing on spatial directions (36 APE for new data)

Lattice setups

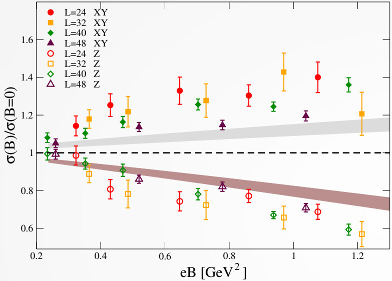
L	a [fm]	b	
24	0.2173	12 - 40	old data
32	0.1535	12 - 40	old data
40	0.1249	8 - 40	old data
48	0.0989	8 - 64	⟵ NEW

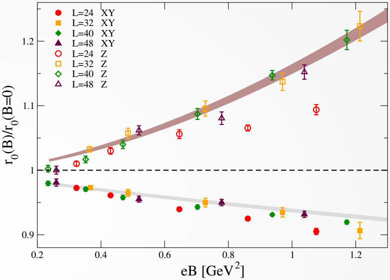


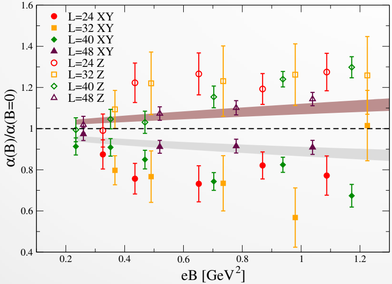
- Anisotropies washed out averaging over all directions
- Compute $\mathcal{O}(eB)/\mathcal{O}(0)$ for $\mathcal{O} = \alpha, r_0, \sigma$

Anisotropic static $Q\bar{Q}$ potential

Continuum extrapolations







- Continuum extrapolations confirm anisotropies
- $\sigma_{XY} > \sigma_Z$ while $\alpha_Z > \alpha_{XY}$
- For σ expected deviations larger than 10% above 1 GeV^2

Anisotropic static $Q\bar{Q}$ potential

Electromagnetism: medium with anisotropic dielectric constant

$$\frac{e}{r} \to \frac{e}{\sqrt{\epsilon^{(x)}\epsilon^{(y)}\epsilon^{(z)}}\sqrt{\frac{x^2}{\epsilon^{(x)}} + \frac{y^2}{\epsilon^{(y)}} + \frac{z^2}{\epsilon^{(z)}}}}$$

$$\frac{\alpha}{r} \to \frac{\alpha}{\sqrt{\epsilon_{xy}^{(\alpha)}(x^2+y^2) + \epsilon_z^{(\alpha)} z^2}}$$

$$\sigma r \to \sigma\sqrt{\epsilon_{xy}^{(\sigma)}(x^2+y^2) + \epsilon_z^{(\sigma)} z^2}$$

Can be reformulated as

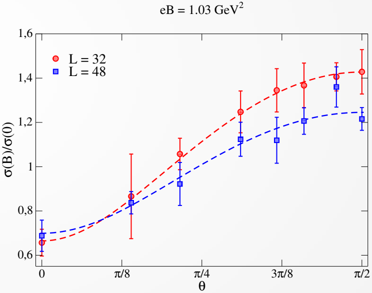
$$V(r,\theta,B) = -\frac{\alpha(\theta,B)}{r} + \sigma(\theta,B)r$$

$$\sigma(\theta,B) = \sigma\epsilon_1^{(\sigma)}(B)\sqrt{1+\epsilon_2^{(\sigma)}(B)\sin^2(\theta)}$$

$$\alpha(\theta,B) = \frac{\alpha}{\epsilon_1^{(\alpha)}\sqrt{1+\epsilon_2^{(\alpha)}(B)\sin^2(\theta)}}$$

$$\epsilon_1^{\mathcal{O}} = \sqrt{\epsilon_z^{\mathcal{O}}} \qquad \epsilon_2^{\mathcal{O}} = \epsilon_x y^{\mathcal{O}}/\epsilon_z^{\mathcal{O}} - 1$$

$\theta \to$ angle between eB and $W(r,T)$



$L = 48$

$\epsilon_1^{(\sigma)} = 0.70(5) \qquad \epsilon_2^{(\sigma)} = 2.17(52)$

Finite T with external B fields

- Above T_C strongly interacting matter deconfine $\rightarrow$ Quark Gluon Plasma
- Color screening melts heavy quark bound states
- One can define screening masses for the chromo-electric and chromo-magnetic gauge fields $\rightarrow m_E$ and m_M.
- Screening masses defined non perturbatively by inverse correlation lengths of proper gauge invariant operators
- Previous studies on the lattice
  - Pure gauge [Kacmareck et al., PRD **62** (2000) 034021]
  - $N_f = 2$ dynamical Wilson fermions [Maezawa et al., PRD **81** (2010) 091591]
  - $N_f = 2 + 1$ dynamical staggered fermions [Borsányi et al., JHEP04 (2015) 138]

Debye screening masses

To separate m_E and m_M one can decompose Polyakov loops correlators into combinations symmetric under Eucledean time reflection and $\mathcal{C}$

$$\left.\begin{aligned} L_M &= \frac{L+L^\dagger}{2} \\ L_E &= \frac{L-L^\dagger}{2} \end{aligned}\right\}$$

M (E) even (odd) under Euclidean time reflection

$$\begin{aligned} A_4(t,x) &\rightarrow -A_4(-t,x) \\ A_i(t,x) &\rightarrow A_i(-t,x) \end{aligned}$$

$$\left.\begin{aligned} L_{M\pm} &= \frac{L_M \pm L_M^*}{2} \\ L_{E\pm} &= \frac{L_E \pm L_E^*}{2} \end{aligned}\right\}$$

$+$ $(-)$ even (odd) under $\mathcal{C}$

$$A_\mu(t,x) \rightarrow -A_\mu^*(t,x)$$

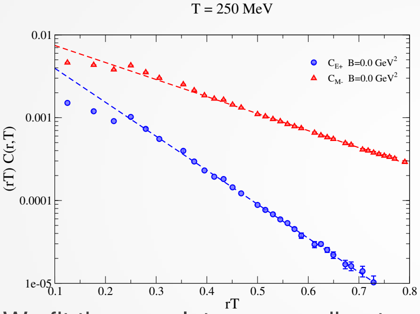
Because Tr $L_{E+} =$ Tr $L_{M-} = 0$ one considers only:

$$\begin{aligned} C_{M+}(r,T) &= \langle \textstyle\sum_x \mathrm{Tr} L_{M+}(x) \mathrm{Tr} L_{M+}(x+r)\rangle - |\langle \textstyle\sum_x \mathrm{Tr} L(x)\rangle|^2 \\ C_{E-}(r,T) &= -\langle \textstyle\sum_x \mathrm{Tr} L_{E-}(x) \mathrm{Tr} L_{E-}(x+r)\rangle \end{aligned}$$

Debye screening masses defined from the exponential fall off of these correlators

Debye screening masses

Preliminary study



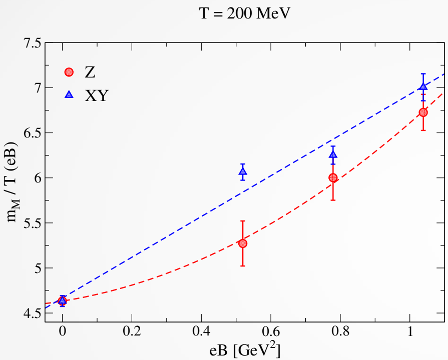
- Improved action and physical quark masses
- Used our smallest lattice spacing $a = 0.0989$ fm with $L = 48$ for $N_T = 8, 10$
- 36 steps APE smering on spatial directions

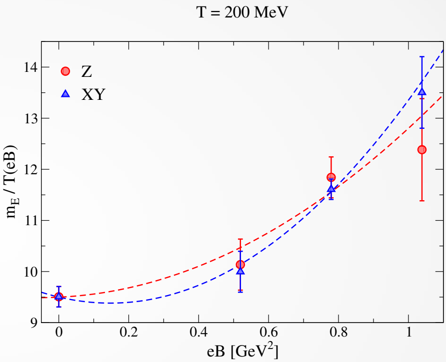
We fit the correlators according to

$$
\begin{aligned}
C_{M+}(r,T) &\to a_M(T)\frac{e^{-m_M(T)r}}{r} \\
C_{E-}(r,T) &\to a_E(T)\frac{e^{-m_E(T)r}}{r}
\end{aligned}
$$

For $eB \neq 0$ we keep XY and Z directions separate to look for anisotropies in the screening masses.

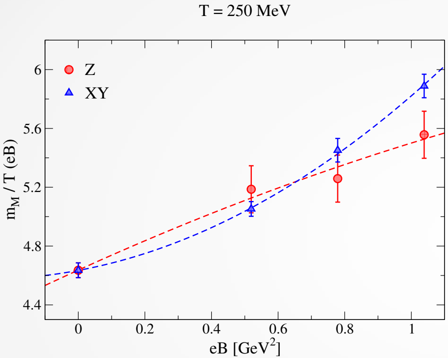
Debye screening masses

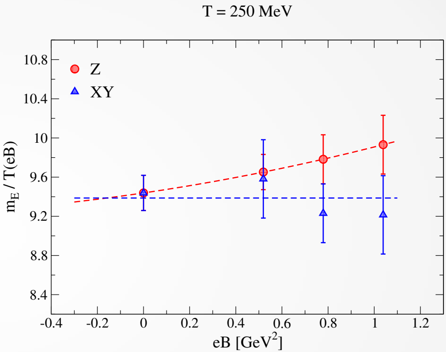




- $B = 0$ results at $T = 200$ MeV: $m_M/T = 4.68(6)$ and $m_E/T = 9.52(21)$ (good agreement with JHEP04 (2015) 138 at the same lattice spacing)
- XY, Z results compatible with each other at present precision
- Increase of both masses with eB at $T = 200$ MeV

Debye screening masses

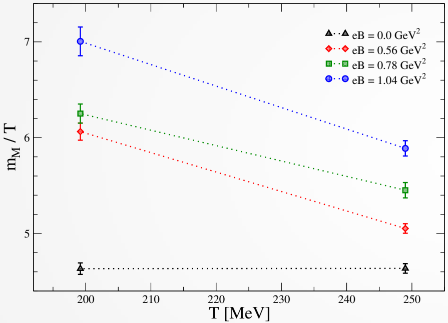


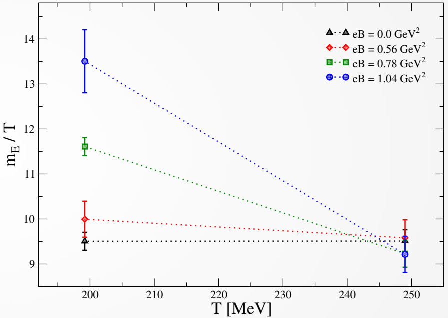


- $B = 0$ results at $T = 250$ MeV: $m_M/T = 4.64(5)$ and $m_E/T = 9.44(18)$ (good agreement with JHEP04 (2015) 138 at the same lattice spacing)
- XY, Z results compatible with each other at present precision
- m_E stay constant with eB, m_M still increases.

Debye screening masses

Magnetic corrections to the screening masses decrease with Tc





- Extend this study to other temperatures above T_c
- Continuum extrapolation
- Work in progress: measurement of Polyakov loop correlators for $T < T_c$

Conclusions

Summary

- Continuum extrapolation confirms anisotropies in the static $\bar{Q}Q$ potential
- Angular dependence of the observables agrees with an anisotropic description of the mediuum
- In the deconfined phase, we found magnetic dependence in the screeneng masses m_E, m_M near T_c
- No relevant anisotropies above T_c

Future studies

- Precision improvement and continuum limit for the anisotropy coefficients
- Extend our investigations above T_c to determine T dependence of the magnetic corrections to the screening masses
- Extend our investigation at finite temperature for $T < T_c$

THANK YOU