Neutral B-meson and D-meson mixing bag parameters from $2+1$ flavor lattice QCD

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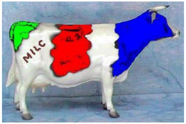
J.N. Simone, and R. Van de Water

representing the
Fermilab Lattice and MILC collaborations

16 July 2015



Combine precise matrix elements from theory

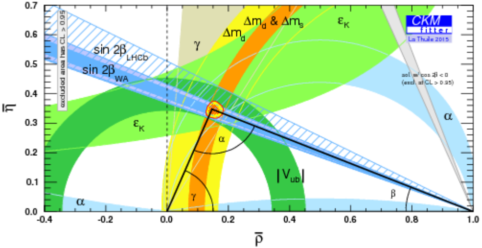
. . . with precise experimental measurements:

SM B.R. for rare decays $B_s^0 \to \mu^+\mu^-$ and $B^+ \to \tau^+\nu$ (seen at LHCb and B-factories) depend upon $|f_{B_q}|^2$.

Establish limits on the scale of BSM physics from B-mixing and D-mixing.

B mixing rates yield CKM elements $|V_{td}|$ and $|V_{ts}|$.

ξ and $\Delta M_d/\Delta M_s$ constrain the apex of CKM unitarity triangle.



XXIX Rencontres de Physique de la Vallée d'Aoste 3/2015

Neutral meson mixing

Matrix elements for B_s^0, B^0, and D^0 mixing in SM and beyond.

Update LAT14 report, [C. Bouchard,E. Freeland,*et al.* arXiv:1412.5097].

New. . .

Chiral-continuum extrapolations account for most of the systematic effects, e.g. FV, r_1/a, LQ masses.

“Mostly nonperturbative” matching plus addition of $o(\alpha_3)$ term to fit function.

Analysis of both statistics and systematics effects is finished.

Matrix element determinations and error budgets nearly final.

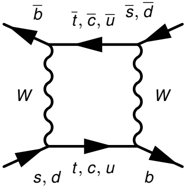
Results are still considered PRELIMINARY.

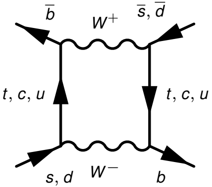
Coordinated bootstrap analysis of mixing MEs and decay constants to obtain bag parameters accounting for correlations.

B mixing operators

OPE to integrate out heavy d.o.f.s ...

SM box diags:





$$\Delta M_q = (\text{known}) \left| V_{tq}^* V_{tb} \right|^2 \left\langle B_q^0 \mid \mathcal{O}_{1q} \mid \bar{B}_q^0 \right\rangle$$

BSM particle contributions in loops?

SM ΔM_q: 1 and $\Delta\Gamma_q$: 1,2,3

$$\mathcal{O}_{1q} = \left(\bar{b}^\alpha \, \gamma_\mu L \, q^\alpha\right) \, \left(\bar{b}^\beta \, \gamma_\mu L \, q^\beta\right)$$

$$\mathcal{O}_{2q} = \left(\bar{b}^\alpha \, L \, q^\alpha\right) \, \left(\bar{b}^\beta \, L \, q^\beta\right)$$

$$\mathcal{O}_{3q} = \left(\bar{b}^\alpha \, L \, q^\beta\right) \, \left(\bar{b}^\beta \, L \, q^\alpha\right)$$

BSM:

$$\mathcal{O}_{4q} = \left(\bar{b}^\alpha \, L \, q^\alpha\right) \, \left(\bar{b}^\beta \, R \, q^\beta\right)$$

$$\mathcal{O}_{5q} = \left(\bar{b}^\alpha \, L \, q^\beta\right) \, \left(\bar{b}^\beta \, R \, q^\alpha\right)$$

Bag parameters:

$$\left\langle \mathcal{O}_{iq} \right\rangle (\mu) = \epsilon_i \, f_{B_q}^2 \, M_{B_q}^2 \, B_{B_q}^{(i)}(\mu)$$

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MILC asqtad $N_f = 2 + 1$

Same gauge ensembles used for both mixing and decay constants.

id	a [fm]	beta	m_l/m_h	am_h	m_h/m_s	r_1/a	N_{config}	N_{tsrc}
en24	0.043	7.81	0.2	0.014	1.079	7.208	801	4
en18	0.059	7.46	0.1	0.018	1.019	5.307	827	4
en19	0.058	7.465	0.139	0.018	1.024	5.330	801	4
en20	0.058	7.47	0.2	0.018	1.028	5.353	673	8
en23	0.058	7.48	0.4	0.018	1.037	5.399	593	4
en12	0.083	7.075	0.05	0.031	1.255	3.738	791	4
en13	0.083	7.08	0.1	0.031	1.256	3.755	1015	4
en14	0.083	7.085	0.15	0.031	1.262	3.772	984	4
en15	0.082	7.09	0.2	0.031	1.267	3.789	1931	4
en17	0.081	7.11	0.4	0.031	1.290	3.858	1996	4
en04	0.11	6.76	0.1	0.05	1.489	2.739	2099	4
en05	0.11	6.76	0.14	0.05	1.489	2.739	2110	4
en06	0.11	6.76	0.2	0.05	1.489	2.739	2259	4
en09	0.11	6.79	0.4	0.05	1.534	2.821	2052	4
en00	0.14	6.572	0.2	0.0484	1.156	2.222	631	24

- Sea quarks near to physical: $m_l/m_h = 1/20$ (en12).
- Four lattice spacings used in analysis, a fifth coarser spacing (en00) used only as a check.
- Fits of 2- and 3-point correlators covered in LAT14 report!

Mixing chiral continuum extrapolation

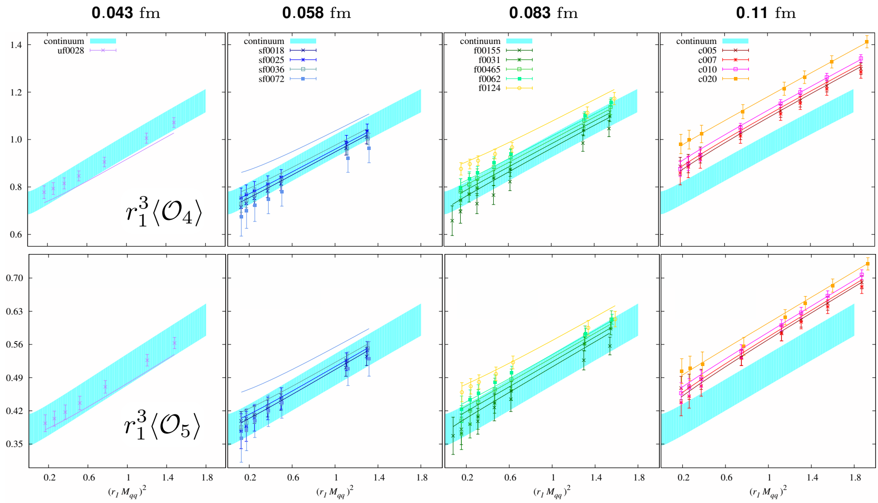
Ops $i = (1,2,3)$ and $(4,5)$ mix; simultaneous fit of all five ME:

$$\langle \mathcal{O}_i \rangle = \beta_i \left[1 + \boxed{\chi \text{ logs}}_i + \boxed{\text{"w.s." logs}}_i + \boxed{\text{analy.}}_i \right] + \beta'_i \boxed{\text{"wrong spin" logs}}_i$$
$$+ \boxed{\text{HQ mass tuning}}_i + \boxed{\text{HQ discr.}}_i + \boxed{\text{H.O. matching}}_{ij}$$

- ✓ Chiral logarithms + FV + staggered taste effects $+1/M$ splittings.
- ✓ Mixing with staggered "wrong spin" terms.
- ✓ NNLO analytic; and as check: N^3LO.
- ✓ Adjust for mis-tuning of simulation HQ masses.
- ✓ Heavy quark discretization effects: $O(\alpha_s a \Lambda), O(a^2\Lambda^2)$.
- ✓ "Mostly NP" matching: $\langle \mathcal{O}_i \rangle = Z_{V^4_{QQ}} Z_{V^4_{qq}} \left[\delta_{ij} + \alpha_s \zeta_{ij} \right] \langle \mathcal{O}_j \rangle^{\text{lat}}$.
- ✓ H.O. matching: $\alpha_s^2 \, \xi_{ij} \, \beta_j$ plus $\alpha_s^3 \, \psi_{ij} \, \beta_j$ as check.

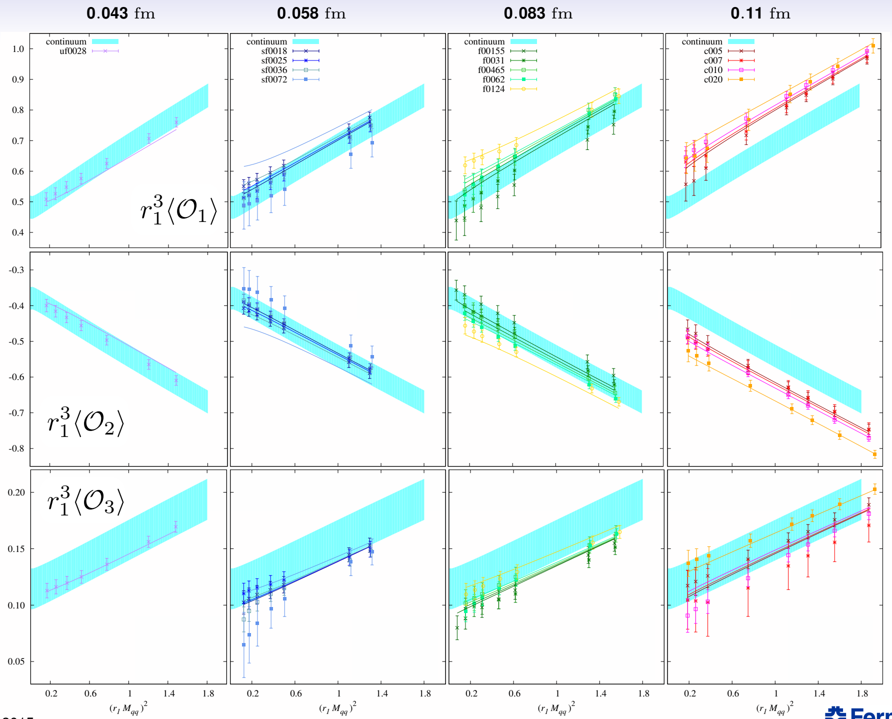
HMSχPT [C. Bernard arXiv:1303.0435]

B fit: $r_1^3 \langle \mathcal{O}_i \rangle$ vs $(r_1 M_{qq})^2$ by lattice spacing



Band in Cyan indicates the continuum limit. Data points and the fit curves at finite lattice spacing shown in other colors.

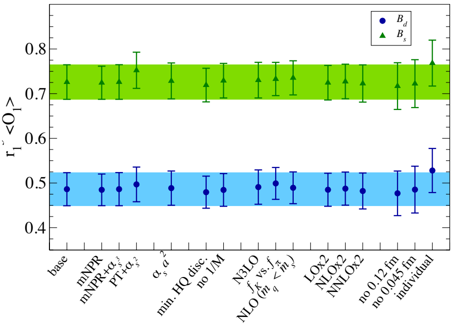
Fermilab

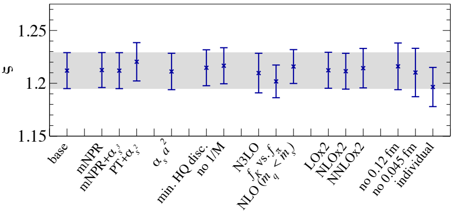

0.043 fm
0.058 fm
0.083 fm
0.11 fm
continuum
uf0028
sf0018
sf0025
sf0036
sf0072
f00155
f0031
f00465
f0062
f0124
c005
c007
c010
c020
$r_1^3\langle\mathcal{O}_1\rangle$
$r_1^3\langle\mathcal{O}_2\rangle$
$r_1^3\langle\mathcal{O}_3\rangle$
$(r_1 M_{qq})^2$

Fermilab

B mixing fit stability: $\langle \mathcal{O}_1 \rangle$ and ξ

base: mNPR+$o(\alpha_s^2)$ matching; HQ disc.;$1/M$ (B^*-B + flavor splittings); N^{2}LO analy.; f_π





matching

mNPR: drop $o(\alpha_s^2)$ from matching

mNPR+α_s^3: include $o(\alpha_s^3)$ in matching

PT+o(α_s^2): perturbative only matching

discretization

$\alpha_s a^2$: add additional discr. term

min. HQ discr.: keep only the lowest order HQ discr. term

χPT truncation

no $1/$M: no B^*-B splittings

N3LO: include N^{3}LO analytic terms

f_π vs f_K: decay constant in χPT

NLO($m_q < m_s$): only NLO analytic but drop heavier m_q

LOx2: double prior widths for LO analytic

NLOx2: double NLO prior widths

NNLOx2: double N^{2}LO widths

other

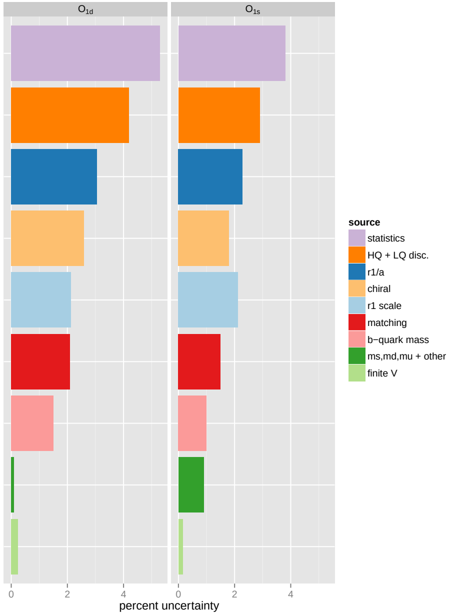
no 0.12 fm: drop coarsest lattice spacing

no 0.045 fm: drop finest spacing

individual: separate fit for $\mathcal{O}_{4,5}$

B mixing ME and error budget

$\langle \mathcal{O}_{1d} \rangle$ and $\langle \mathcal{O}_{1s} \rangle$ error breakdown:



PRELIMINARY

	BBGLN	BMU
$M_{B^0}\langle \mathcal{O}_{1d} \rangle$	0.654(56)	
$M_{B^0_s}\langle \mathcal{O}_{1s} \rangle$	0.992(61)	
$M_{B^0}\langle \mathcal{O}_{2d} \rangle$	-0.545(49)	-0.517(47)
$M_{B^0_s}\langle \mathcal{O}_{2s} \rangle$	-0.832(53)	-0.782(50)
$M_{B^0}\langle \mathcal{O}_{3d} \rangle$	0.154(24)	0.155(25)
$M_{B^0_s}\langle \mathcal{O}_{3s} \rangle$	0.225(25)	0.226(25)
$M_{B^0}\langle \mathcal{O}_{4d} \rangle$	0.993(72)	
$M_{B^0_s}\langle \mathcal{O}_{4s} \rangle$	1.390(78)	
$M_{B^0}\langle \mathcal{O}_{5d} \rangle$	0.516(48)	
$M_{B^0_s}\langle \mathcal{O}_{5s} \rangle$	0.725(51)	

in GeV^4; $\overline{\mathrm{MS}}$-NDR, $\mu = m_b$

BBGLN = Beneke, Buchalla, Greub, Lenz, Nierste

BMU = Buras, Misiak, Urban

$\Delta M_{B_s^0} / \Delta M_{B^0}$

Uncertainties tend to cancel in

$$\xi = \frac{M_{B^0}}{M_{B_s^0}} \sqrt{\frac{\langle \mathcal{O}_{1s} \rangle}{\langle \mathcal{O}_{1d} \rangle}}$$

$$\left| \frac{V_{td}}{V_{ts}} \right| = \xi \sqrt{\frac{\Delta M_{B^0} / M_{B^0}}{\Delta M_{B_s^0} / M_{B_s^0}}}$$

Our result with total error

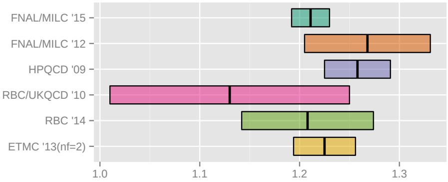
$$\xi = 1.211(19)$$

World averages [PDG 9/2014]:

$$\Delta M_{B^0} = 0.510(3)\ \mathrm{ps}^{-1}$$

$$\Delta M_{B_s^0} = 17.761(21)_{\mathrm{stat}}(7)_{\mathrm{syst}}\ \mathrm{ps}^{-1}$$

Comparison with other results:



FNAL/MILC: arXiv:1205.7013 HPQCD: arXiv:0902.1815 – neglects "wrong spin"
RBC/UKQCD†: arXiv:1001.2023 RBC† arXiv:1406.6192 † static b quark
ETMC(nf=2): arXiv:1308.1851

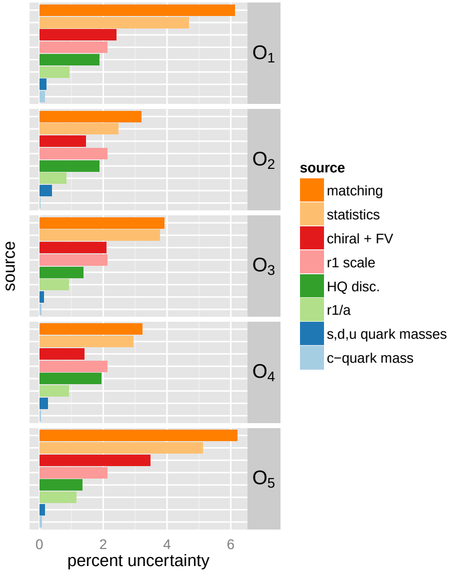
Our result:

$$\left| \frac{V_{td}}{V_{ts}} \right| = 0.2069(6)_{\mathrm{exp}}(32)_{\mathrm{thy}}$$

vs FNAL/MILC '12 . . .

PDG '14: $0.2166(7)_{\mathrm{exp}}(108)_{\mathrm{thy}}$

D^0 mixing error budget and results

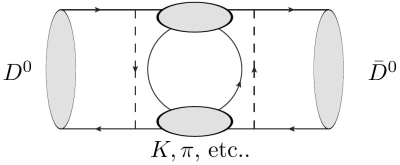


PRELIMINARY

	BBGLN	BMU
$M_{D^0}\langle\mathcal{O}_1\rangle$	0.072(6)	
$M_{D^0}\langle\mathcal{O}_2\rangle$	-0.143(7)	-0.133(7)
$M_{D^0}\langle\mathcal{O}_3\rangle$	0.056(4)	0.056(4)
$M_{D^0}\langle\mathcal{O}_4\rangle$	0.29(2)	
$M_{D^0}\langle\mathcal{O}_5\rangle$	0.11(1)	

in GeV^4; $\overline{\mathrm{MS}}$-NDR, $\mu = 3\,\mathrm{GeV}$

Long distance SM contributions from. . .



Fermilab

Decay constants

Decay constants for D and B mesons from asqtad three flavor.

Fermilab clover HQ – same as for mixing ME.

HISQ HQ on HISQ $N_f = 2 + 1 + 1$: [C. Detar (previous talk)].

Analysis of systematics nearing completion.

Central values will remain blinded until error budget is finalized.

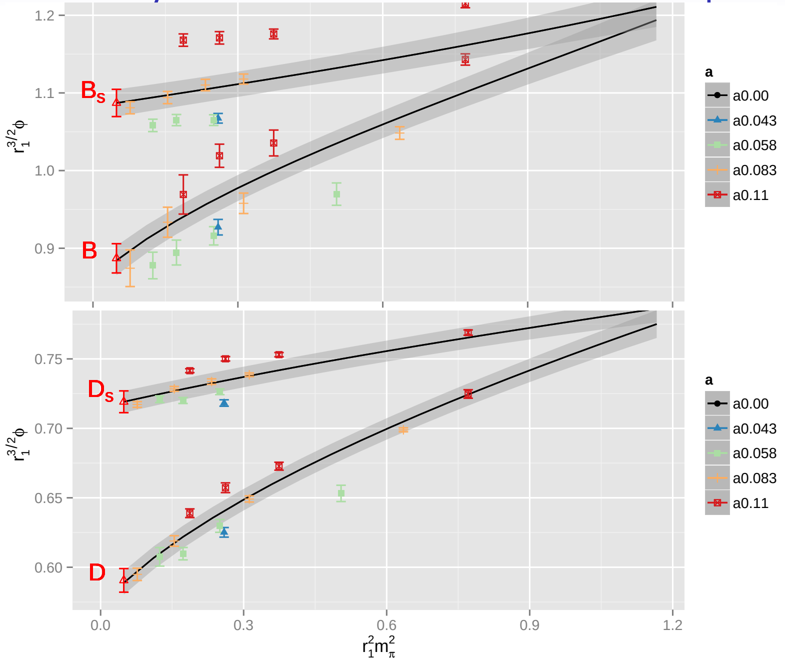
$\Longrightarrow$ **PRELIMINARY** – bag parameters t.b.d.

Correlations of statistical and systematic effects will be preserved between mixing matrix elements and decay constants.

Chiral fit function:

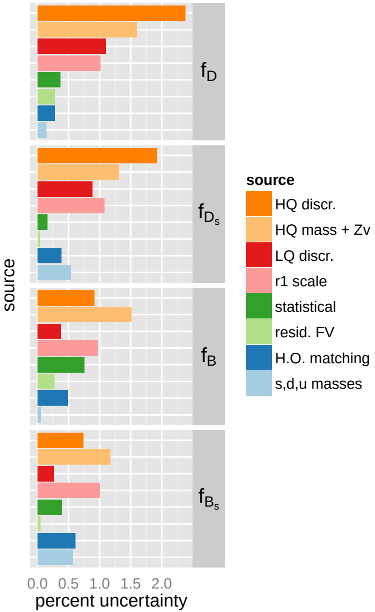
$$f_{H_q}\sqrt{M_{H_q}} = \Phi_0 \left[1 + \boxed{\chi \text{ logs}} + \boxed{\text{analy.}} + \boxed{\text{HQ discr.}} \right]$$

Decay constant chiral continuum extrapolations



Decay constants error budget

PRELIMINARY!



Compare to Fermilab/MILC ’11 [arXiv:1112.3051]. . .

Higher statistics.

Discretization errors smaller with addition of finer $a = 0.058$ and $0.043\,\mathrm{fm}$ lattice spacings.

Chiral extrapolation uncertainties smaller with nearly physical light quark ensemble.

Better tuning of HQ mass.

Fermilab

Outlook

Mixing ME determinations for B_s^0, B^0, and D^0 being finalized.

Uncertainties in ME and ξ have been reduced compared to the FNAL/MILC '12 calculation.

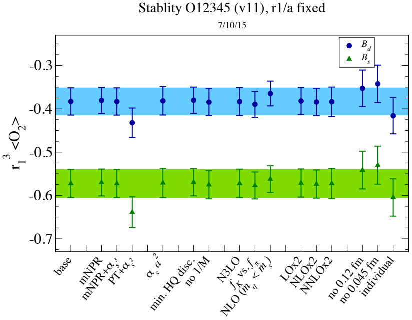
New preliminary result $\xi = 1.211(19)$ reduces theory error for the resulting $|V_{td}/V_{ts}|$ by about a factor of three compared to our 2012 result.

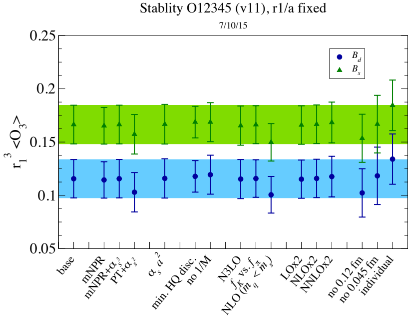
Central values for decay constants will remain blinded pending completion of error budget.

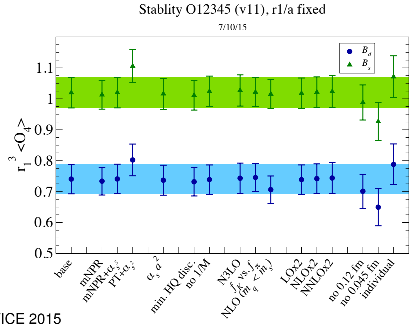
We expect to finalize all results, including bag parameters in the next few months.

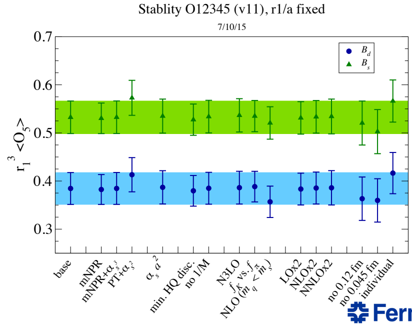
BACKUP SLIDES

B mixing fit stability $\langle \mathcal{O}_j \rangle$ $j = 2, 3, 4, 5$









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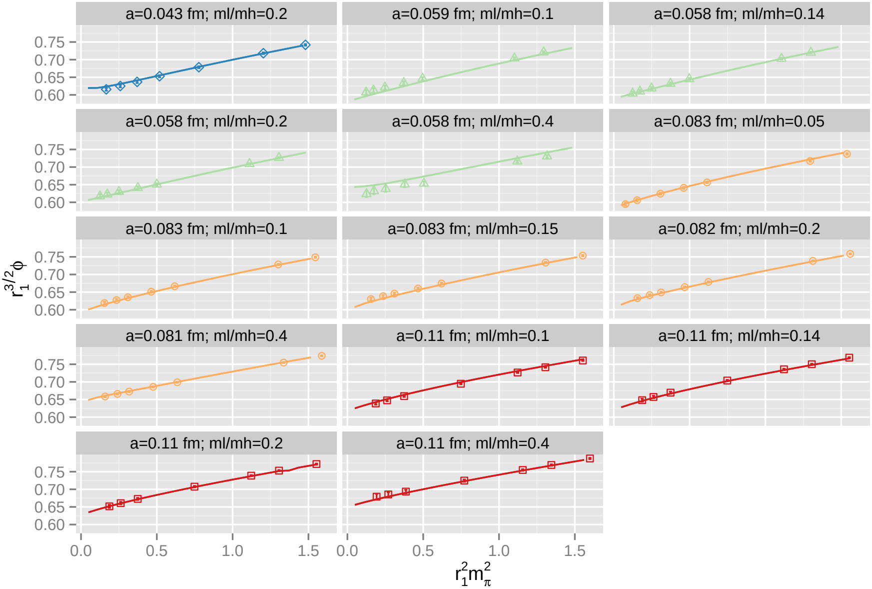
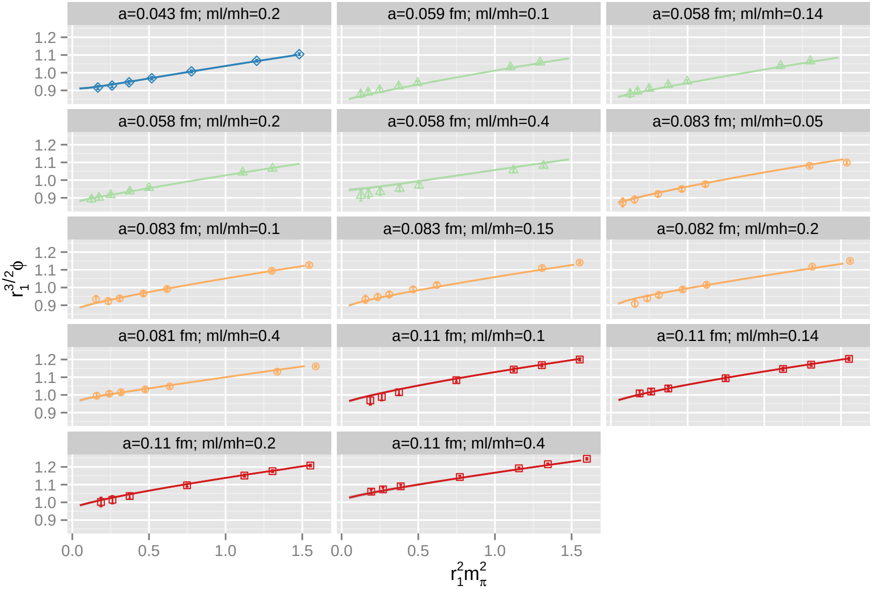
D decay constant chiral fit



$\chi^2_{\rm aug} = 97.6$ for 98 points; $\chi^2/dof = 92.7/77$; $p_{\rm fit} = 0.107$

B decay constant chiral fit



$$\chi^2_{\rm aug} = 46.3 \text{ for 98 points}; \quad \chi^2/dof = 43.5/77; \quad p_{\rm fit} = 0.999$$