Connected contribution to hadron correlation functions

Alfonso Sastre
Budapest - Marseille - Wuppertal

Bergische Universität Wuppertal

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Outline

Introduction

- Anomalous Magnetic Momentum: an open window to BSM?

$$a_\mu^{Exp} - a_\mu^{Th} = (28.7 \pm 8.0) \times 10^{-10} (3.6\sigma) \text{[Davier, 11; Prades, 09]}$$
$$a_\mu^{Exp} - a_\mu^{Th} = (25.0 \pm 8.6) \times 10^{-10} (2.9\sigma) \text{[Hagiwara, 11; Nyffeler, 10]}$$

- The experimental error is going to be reduced by FNAL and J-PARC by a factor ~ 4.
- The theoretical error is dominated by the LO Hadron Vacuum Polarization (HVP)

$$a_\mu^{\mathrm{LO-HVP}} = 692.3(4.2) \times 10^{-10} \text{[Prades, 09]}$$
$$a_\mu^{\mathrm{LO-HVP}} = 694.9(4.3) \times 10^{-10} \text{[Nyffeler, 10]}$$

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$\Rightarrow$ Can we do an ab initio calculation on the lattice?

Introduction - $a_\mu^{\mathrm{LO-HVP}}$ from LQCD

- The $a_\mu^{\mathrm{LO-HVP}}$ in Euclidean space [Lautrup,74; Blum,02]:

$$a_\mu^{\mathrm{LO-HVP}} = 4\pi\alpha \sum_f Q_f^2 \int_0^\infty dq^2 F(q^2) \hat{\Pi}^f(q^2),$$

$$F(q^2) = \frac{m_\mu^2 q^2 A(1-q^2 A)}{1+m_\mu^2 q^2 A^2},\ A = \frac{\sqrt{q^4+4m_\mu^2 q^2}-q^2}{2m_\mu^2 q^2}$$

- $\Pi(q^2)$ can be computed using LQCD

$$\Pi_{\mu\nu}(q^2) = (q_\mu q_\nu - \delta_{\mu\nu} q^2)\Pi(q^2) = \sum e^{iqx} C_{\mu\nu}(x) \quad (1)$$

$$C_{\mu\nu}(x) = < j_\mu(x) j_\nu(0) >$$

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$\Rightarrow$ $\hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$, but $\Pi(0)$ cannot be computed using Eq. (1)

- Current lattice data without information about $\Pi(0) \Rightarrow$ large extrapolation errors.

HVP - Low and High Momentum

- We use the hybrid approach proposed by Golterman, Maltnan and Peris (GMP)

$$a_\mu = 4\pi\alpha \sum_f Q_f^2 \left(\int_0^{q^2_{\rm min}} dq^2 F(q^2)\hat{\Pi}^f(q^2) + \int_{q^2_{\rm min}}^{\infty} dq^2 F(q^2)\hat{\Pi}^f(q^2) \right)$$

- $q^2 < q^2_{\rm min} \Rightarrow \hat{\Pi}(q^2)$ is approximated by a Padé function
- $q^2 > q^2_{\rm min} \Rightarrow$ Lattice data are integrated numerically.
- According with [GMP,14]
  - $q^2_{\rm min} = 0.2\,\mathrm{GeV}^2$: Padé[1,1] (Padé[2,1]) is enough.
  - We need: Π_1 and Π_2 (Π_3) ($\Pi_n \equiv \partial^{2n}_{q^2}\Pi(q^2)|_{q^2=0}$)

$$\hat{\Pi}^{\rm lo}(q^2) = \frac{\Pi_1 q^2}{1 - \frac{\Pi_2}{\Pi_1} q^2}$$

- We use two independent methods to compute Π_1 and Π_2.

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- We use two independent methods to compute Π_1 and Π_2.

$\Rightarrow$ The final error is dominated by the error of $\Pi_1^{(\mathrm{ud})}$ [De Rafael, 14]

Determination of Π_n: Derivative Method

- We use the method proposed by [ETMC,13; HPQCD,14]

$$\hat{\Pi}(q^2) = \sum_n q^{2n}\Pi_n$$

$$\Pi_n = \frac{(-1)^{(n+1)}}{(2(n+1))!}\frac{1}{12}\sum_{\mu\neq\nu}\sum_{x_\mu} x_\mu^{2(n+1)} C_{\nu\nu}(\vec{x},t)$$

- With our data, we can compute Π_1 and Π_2 with reasonable errors.
- Π_3 and Π_4 were computed by [HPQCD,14] for the strange and the charm contribution.

HVP - Low q: Padé fitting

- We can fit the lattice data using a Padé approximation
- In order to obtain the renormalized quantity, $\hat{\Pi}(q^2)$, we can use the method proposed by [Bernecker and Meyer,11; ETMC,13; Lehner,14;Malak,14]

$$\Pi(q^2) = \begin{cases} \frac{1}{3}\sum_t \frac{e^{iqt}-1}{q^2}\sum_{\vec{x},a} C_{aa}(\vec{x},t), & q_0 \\ \frac{1}{6}\sum_{i\neq j,x_i} \frac{e^{iqx_i}-1}{q^2}\sum_{x_j,t}\sum_{a\neq i} C_{aa}(\vec{x},t), & q_i \\ \frac{1}{3}\sum_{i\neq j,x_i} \frac{e^{iqx_i}-1}{q^2}\sum_{x_j,t} C_{00}(\vec{x},t), & q_i \end{cases}$$

$\Pi(0)$ is determined as Π_0.

- According with [GMP,14]: Padé[2,1] to reach the 1% error.

$$\hat{\Pi}(q^2) = \frac{\Pi_1 q^2 + \frac{(\Pi_2^2 - \Pi_1\Pi_3)}{\Pi_2} q^4}{1 - \frac{\Pi_3}{\Pi_2} q^2}$$

Extrapolation to the Physical Point

- We consider for each Π_n $(n > 0)$:

$$\Pi_n^{\mathrm{lat}} = \Pi_n \left(1 + Aa^2 + B(M_\pi^2 - M_{\pi,\phi}^2) + C(M_K^2 - M_{K,\phi}^2)\right) \qquad (2)$$

- Derivative method $\Rightarrow$ we compute $\Pi_{1,2}$ for each ensemble and extrapolate to the Physical point using Eq. 2.
- Padé fit method $\Rightarrow$ we compute $\Pi_{1,2,3}$ by fitting all the ensembles together:

$$\Pi^{\mathrm{lat}}(q^2) = \Pi_0^{lat} + \frac{\Pi_1^{lat} q^2 + \frac{(\Pi_2^{lat})^2 - \Pi_1^{lat}\Pi_3^{lat}}{\Pi_2^{lat}} q^4}{1 - \frac{\Pi_3^{lat}}{\Pi_2^{lat}} q^2}$$

$$\chi^2 = \sum_i^{\mathrm{ens}} \sum_{l,m} (\Pi_{i,m}^{\mathrm{lat}} - \Pi_{i,m}^{\mathrm{lat}})(\Pi_{i,n}^{\mathrm{lat}} - \Pi_{i,n}^{\mathrm{lat}})(S_i^{-1})_{m,n}$$

Extrapolation to the Physical Point - High q^2 contribution

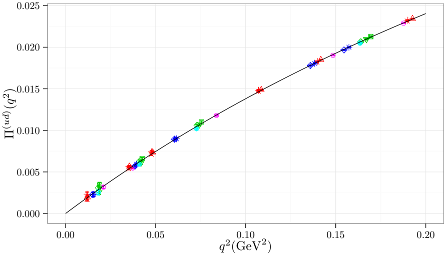
- We compute the large q^2 contribution for each ensemble and extrapolate to the Physical value using:

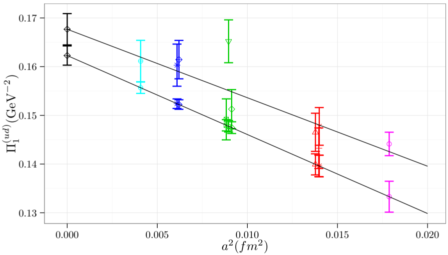
$$a_\mu^{\mathrm{hi,Lat}} = a_\mu^{\mathrm{hi}} \left(1 + A_i a^2 + B(M_\pi^2 - M_{\pi,\phi}^2) + C(M_K^2 - M_{K,\phi}^2)\right)$$

Lattice Details

- Gauge action: Symanzik tree level.
- Fermions: 4 step stout smeared staggered fermions (2+1+1).
- 10 ensembles with 5 different lattice spacings ($a \sim 0.063 - 0.133\,\mathrm{fm}$).
- Volume: $\sim$ 6 fm.
- $\sim$ 1000 configurations are used for each lattice spacing.
- All the ensembles are generated around the Physical masses.
- Conserved vector current is computed using 768 sources for light quarks and 64 for the strange and the charm.
- The lattice spacing is computed using the Wilson Flow, w_0 [BMWc,12].
- Pion and Kaon masses are computed as in [BMWc,13].

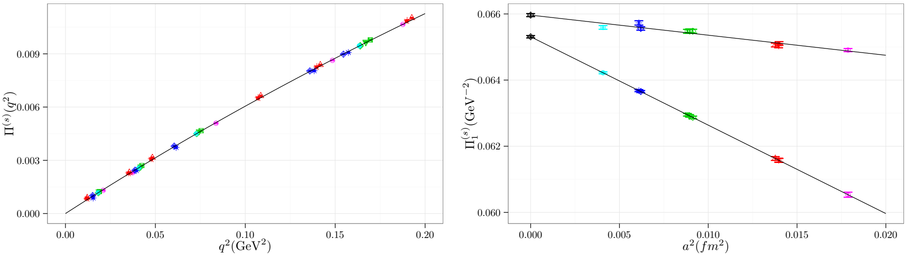
Light quarks - Preliminary Results





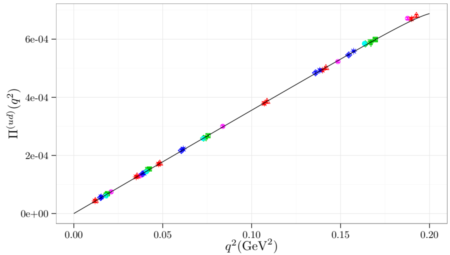
	Fit	Derivative
$\Pi_1^{ud}(\mathrm{GeV}^{-2})$	0.162(2)	0.167(3)
$\Pi_2^{ud}(\mathrm{GeV}^{-4})$	-0.29(2)	-0.34(4)
$a_\mu^{\mathrm{hi},ud}$	$52.05(8) \times 10^{-10}$	

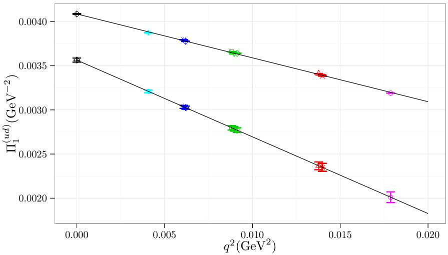
Strange quark - Preliminary Results



	Fit	Derivative
$\Pi_1^s(\mathrm{GeV}^{-2})$	0.06531(4)	0.06568(3)
$\Pi_2^s(\mathrm{GeV}^{-4})$	-0.0520(3)	-0.05350(3)
$a_\mu^{\mathrm{hi},s}$	$5.387(6) \times 10^{-10}$	

Charm quark - Preliminary Results





	Fit	Derivative
$\Pi_1^c(\mathrm{GeV}^{-2})$	0.00356(3)	0.00408(1)
$\Pi_2^c(\mathrm{GeV}^{-4})$	-0.0001(20)	0.00027(7)
$a_\mu^{\mathrm{hi},c}$	$1.603(8) \times 10^{-10}$	

Conclusions

- LQCD results for $a_\mu^{\mathrm{LO-HVP}}$ start to reach percent level.
- Understand discrepancies between the two methods.
- FV effects have to be under control. [Francis,13; Malak,14; Golterman, Tuesday]
- A lot of work has to be done to reach 0.1 percent level.
  - Reduce statistical errors
  - Isospin violation effects.
  - EM ...

Tensor index dependence