The Rho Resonance from $N_f = 2+1+1$ Twisted Mass Lattice QCD

C. Helmes, C. Jost, B. Knippschild, L. Liu, C. Urbach, Z. Wang, M. Werner

Introduction

- Most hadrons are resonances described by their mass and decay width
- Scattering lengths are fundamental quantities in QCD and ingredients for EFTs and interesting for nuclear physics
- Lattice QCD: Non-perturbative description from first principles

- Scattering a priori defined in continuous Minkowski space and cannot be calculated directly from the lattice [1]
- Use Lüscher Method [2]: Scattering parameters can be calculated from effective mass and energy plateaus of two particles in a box
- Extensions to moving reference frames [3], [4]
- Want to calculate energy levels for multitude of reference frames and irreducible representations of octohedral groups

- Laplacian Heaviside smearing of quark fields [5]
- Eigendecomposition of Laplace matrix and truncation to the first N_ν eigenvectors on each timeslice
- Scaling problem on large lattices
- Stochastical approach: Introduce random vectors , ρ, in T, D and V_s
- Dilution of random vectors, $P^{(b)}\rho$, to reduce variance
- Quark line $\mathcal{Q}$ stochastical estimate for the propagator

$$\mathcal{Q} = \sum_b E\left(V_s V_s^\dagger D^{-1} V_s P^{(b)} \rho (V_s P^{(b)} \rho)^\dagger\right)$$

- Save perambulators and randomvectors to disk
- Correlation functions by performing Wick contractions for the desired operators
- Expensive method, but propagators can be stored and reused

Overview of the used ETMC Ensembles

name	L_s	L_t	#conf	a[fm]	m_π[MeV]
A30.32	32	64	161	0.086	284
A40.32	32	64	251	0.086	324
A40.24	24	48	405	0.086	332
A40.20	20	48	281	0.086	342
A60.24	24	48	314	0.086	396
A80.24	24	48	307	0.086	455
A100.24	24	48	313	0.086	510
B55.32	32	64	256	0.082	372
D45.32	32	64	300	0.062	384

All Ensembles are generated by the European Twisted Mass Collaboration [2] with $N_f = 2+1+1$ Clover-fermions

Calculation of the phase shift

- We used moving frames with lattices momenta up to $\vec{P} = (2,0,0)$ in the A_1-irreducible representation of the octohedral group
- For each moving frame, generalized eigenvalue problem:

$$C_{2x2}(t_2 - t_1) = \begin{pmatrix} \langle(\pi\pi)(t_1)(\pi\pi)^\dagger(t_2)\rangle & \langle(\pi\pi)(t_1)\rho^\dagger(t_2)\rangle \\ \langle\rho(t_1)(\pi\pi)^\dagger(t_2)\rangle & \langle\rho(t_1)\rho^\dagger(t_2)\rangle \end{pmatrix}$$

- with operators

$$(\pi\pi)(t) = (\bar{u}(t)\gamma_5 \exp(-i\vec{P}\vec{x})d(t))(\bar{u}(t+1)\gamma_5 \exp(+i\vec{P}\vec{x})d(t+1))$$
$$(\rho)(t) = \bar{u}(t)\gamma_i \exp(-i\vec{P}\vec{x})u(t) + \bar{d}(t)\gamma_i \exp(-i\vec{P}\vec{x})d(t)$$

- Quarks in $(\pi\pi)(t)$ on different timeslices to avoid Fierz rearrangement
- Obtain phaseshift from relation

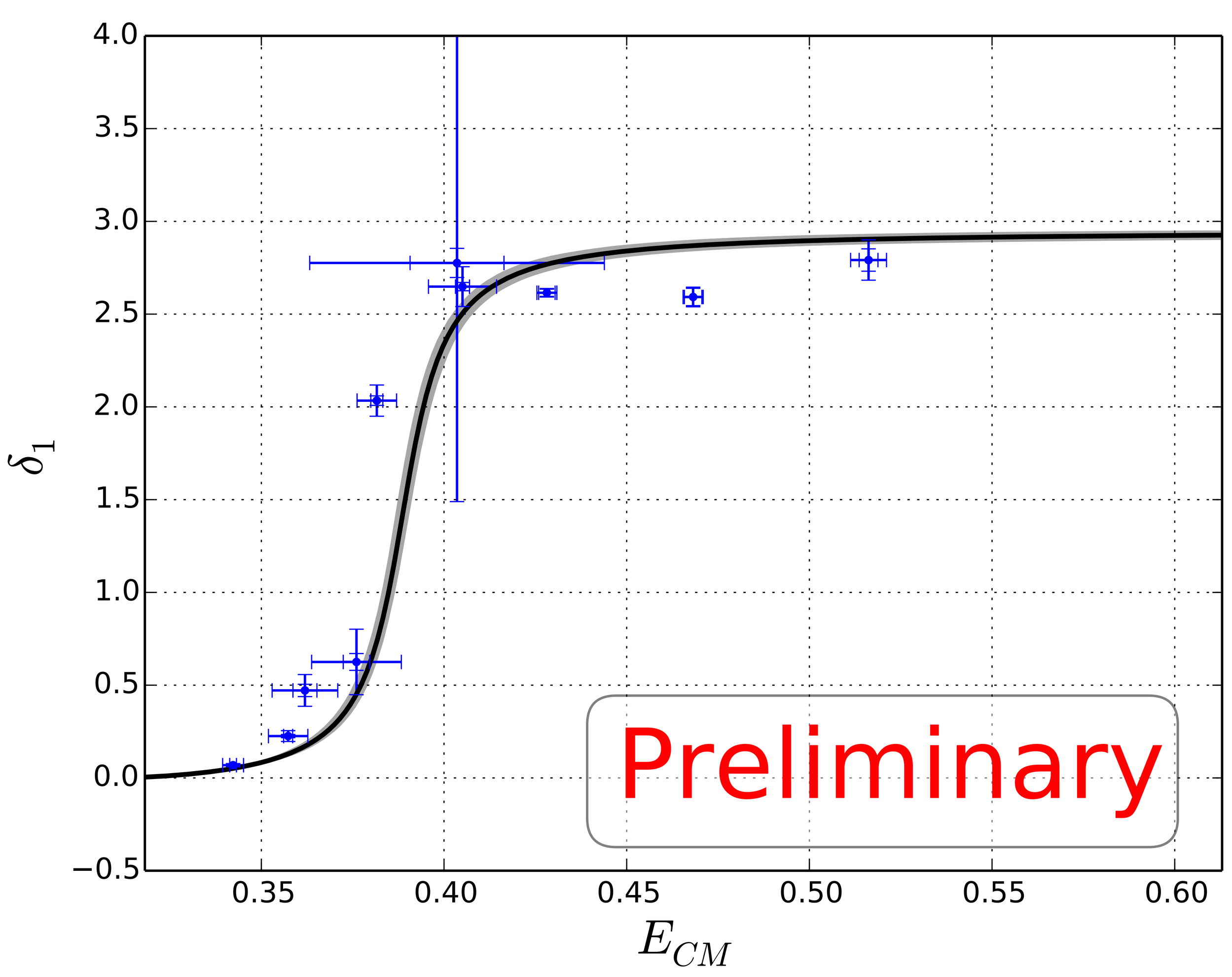
$$\det\left(e^{2i\delta_1}(\mathrm{M} - i\mathbb{1}) - (\mathrm{M} + i\mathbb{1})\right) = 0 \quad (1)$$

- Neglect mixing with higher partial waves
- M analytically from solutions of the generalized eigenvalue problem, pion mass and total momentum of the moving frame

Conclusion and Future Work

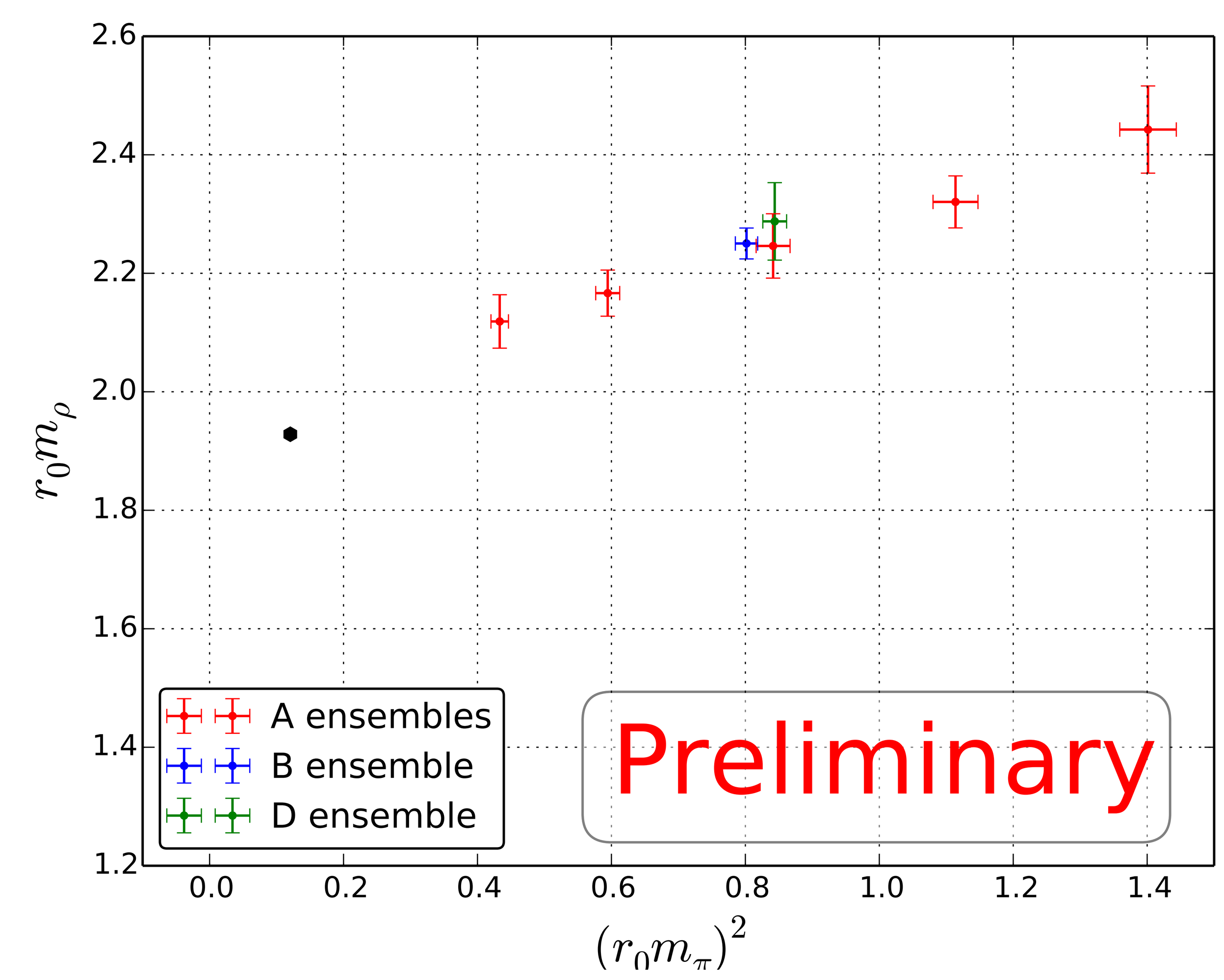
- The preliminary results visually reproduce the expected behavior
- Outliers can be explained by still neglected systematic effects
- 4pt functions $\langle(\pi\pi)(t_1)(\pi\pi)^\dagger(t_2)\rangle$ contain finite size contributions Those "'thermal states"' must be accounted for
- Investigate systematics of phase shift fits
- Generalized eigenvalue problem can be extended by more irreducible representations and perambulators with twisted boundary conditions

$I = 1$ $\pi\pi$ scattering phase shift



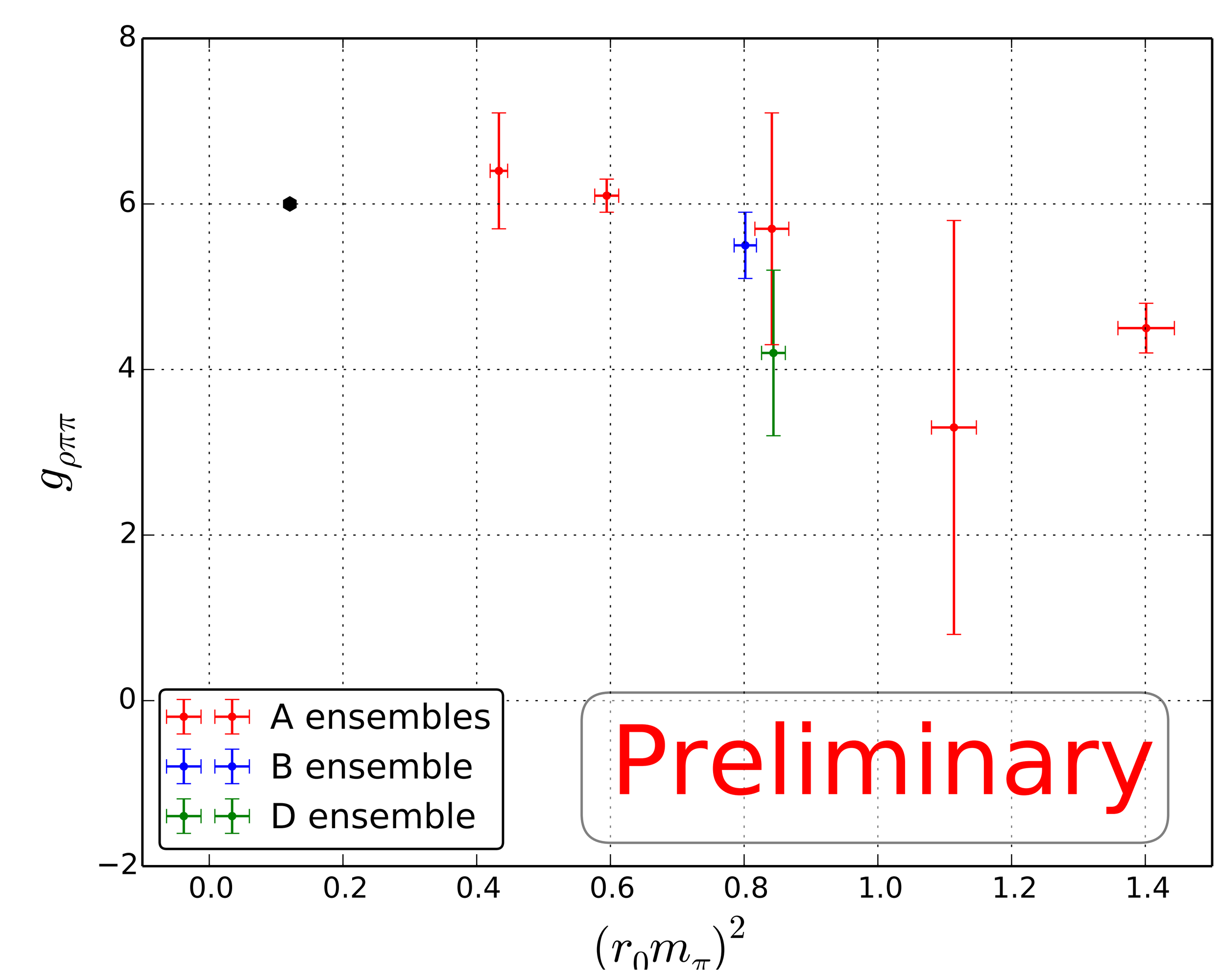
$I = 1$ $\pi\pi$ scattering phase shift for the B55 ensemble. The black line is a fit of $\tan(\delta_1) = \frac{g_{\rho\pi\pi}^2}{6\pi} \frac{\sqrt{E_{CM}^2/4 - m_\pi^2}^3}{E_{CM}(m_\rho^2 - E_{CM}^2)}$ to the data, where $g_{\rho\pi\pi}$ is the $\rho \to \pi\pi$ coupling and m_ρ is the rho mass.

Results for m_ρ



Dependence of the rho mass $r_0 m_\rho$ on the pion mass $r_0 m_\pi$. r_0 is the Sommer parameter. The black hexagon is the corresponding PDG value. [7]

Results for $g_{\rho\pi\pi}$



Dependence of the $\rho \to \pi\pi$ coupling on the pion mass $r_0 m_\pi$. r_0 is the Sommer parameter. The black hexagon is the corresponding PDG value. [7]

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