Nucleon-pion-state contributions in the determination of the nucleon axial charge

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Outline

- ❐ Introduction
- ❐ ...
- ❐ ...

Lattice 2015 - Kobe

15 July 2015

Lattice simulations close to the physical pion mass

- Advantage: No or short chiral extrapolation
  - ➡ Systematic uncertainties are much better controlled

- Disadvantage: Some problems get more severe for small pion masses
  - Signal-to-noise problem
  - Presence of multi-particle state contamination in correlation functions

Excited-state contribution to g_A

- Compute $C_3^A(t,t') = \Gamma'_{\alpha\beta}\langle N_\beta(t) A(t') \overline{N}_\alpha(0)\rangle$

  $C_2(t) = \langle N_\alpha(t)\overline{N}_\alpha(0)\rangle$ ➡ $R = \dfrac{C_3^A(t,t')}{C_2(t)}$

  $N_\alpha, \overline{N}_\alpha$: nucleon fields $\quad A$: axial vector current

- Consider large times

  excited-state contributions $\quad \Delta E_1 = E_1 - M_N$

$$R \xrightarrow{t \gg t' \gg 0} g_A + b_1 e^{-\Delta E_1 (t-t')} + \tilde{b}_1 e^{-\Delta E_1 t'} + c_1 e^{-\Delta E_1 t} + \ldots$$

- For small pion masses and large volumes (i.e. small momenta)

$$E_1 \approx E_N + E_\pi \quad < \quad E_2 \approx M_N + 2M_\pi \quad < \quad E_3 \approx M_{N^*}$$

  - $N\pi$-state contribution has the smallest exponential suppression
  - $b_1, \tilde{b}_1, c_1$ are volume suppressed and expected to be small $\quad$ How small?

Topic of this talk

$$R \xrightarrow{t \gg t' \gg 0} g_A + b_1 e^{-\Delta E_1 (t-t')} + \tilde{b}_1 e^{-\Delta E_1 t'} + c_1 e^{-\Delta E_1 t} + \ldots$$

- Here: Computation of $b_1, \tilde{b}_1, c_1$ due to $N\pi$-states in ChPT

  ➥ provides good estimate for this particular excited-state contamination

- Important: LO ChPT makes a <u>definite prediction</u> for the coefficients i.e. they do not depend on LECs associated with interpolating fields

- Details and full results for the 2pt-function in arXiv:1503.03649

Baryon ChPT

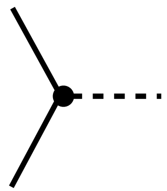
- Framework: Covariant Baryon ChPT

  Gasser et al 1988
  Becher, Leutwyler 1998
  ...

  Use literature expressions for Lagrangian and currents
  Example: leading interaction term

$$\mathcal{L}^{(1)}_{\mathrm{int,LO}} = \frac{i g_A}{2f} \overline{\Psi} \gamma_\mu \gamma_5 \sigma^a \Psi \, \partial_\mu \pi^a \qquad \Psi(x) = \begin{pmatrix} p(x) \\ n(x) \end{pmatrix}$$

doublet of nucleon spinor fields



nucleon-pion Vertex

- We assume
  - isospin symmetry
  - a finite box with spatial extent L , $\Rightarrow$ discrete spatial momenta
  - infinite time extent T (for simplicity)

Nucleon interpolating fields

- Local 3-quark interpolating fields without derivatives

Ioffe 1981, Espriu et al 1983

$$N_1 = (\tilde{q}q)q \qquad N_2 = (\tilde{q}\gamma_5 q)\gamma_5 q \qquad q = \begin{pmatrix} u \\ d \end{pmatrix} \qquad \tilde{q} = q^{\mathrm{T}} C\gamma_5 (i\sigma_2)$$

charge conjugation

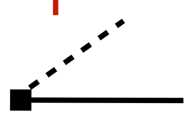
antisym sum over color indices suppressed !

- Map interpolating fields to ChPT based on their symmetry properties

Nagata et al 2008
P. Wein et.al. 2011

- Expand in powers of the pion fields

$$N_i = \tilde{\alpha}_i \left(\Psi + \frac{i}{2f}\pi^a \sigma^a \gamma_5 \Psi + \ldots \right)$$



nucleon-pion coupling

LEC, only difference between N_1 and N_2 (at this order)

Smeared interpolating fields

- Smeared interpolating fields build from smeared quark fields

$$q_{\rm sm}(x) = \int \mathrm{d}^4 y K(x-y) q(y)$$

$$\begin{aligned} N_{1,\rm sm} &= (\tilde{q}_{\rm sm} q_{\rm sm}) q_{\rm sm} \\ N_{2,\rm sm} &= (\tilde{q}_{\rm sm} \gamma_5 q_{\rm sm}) \gamma_5 q_{\rm sm} \end{aligned}$$

- The gauge covariant kernel K
  - depends on smearing (Gaussian, Gradient flow, ...)
  - is essentially zero for $|x-y| > R_{\rm smear}$ (``*smearing radius*”)
  - is diagonal in spinor space

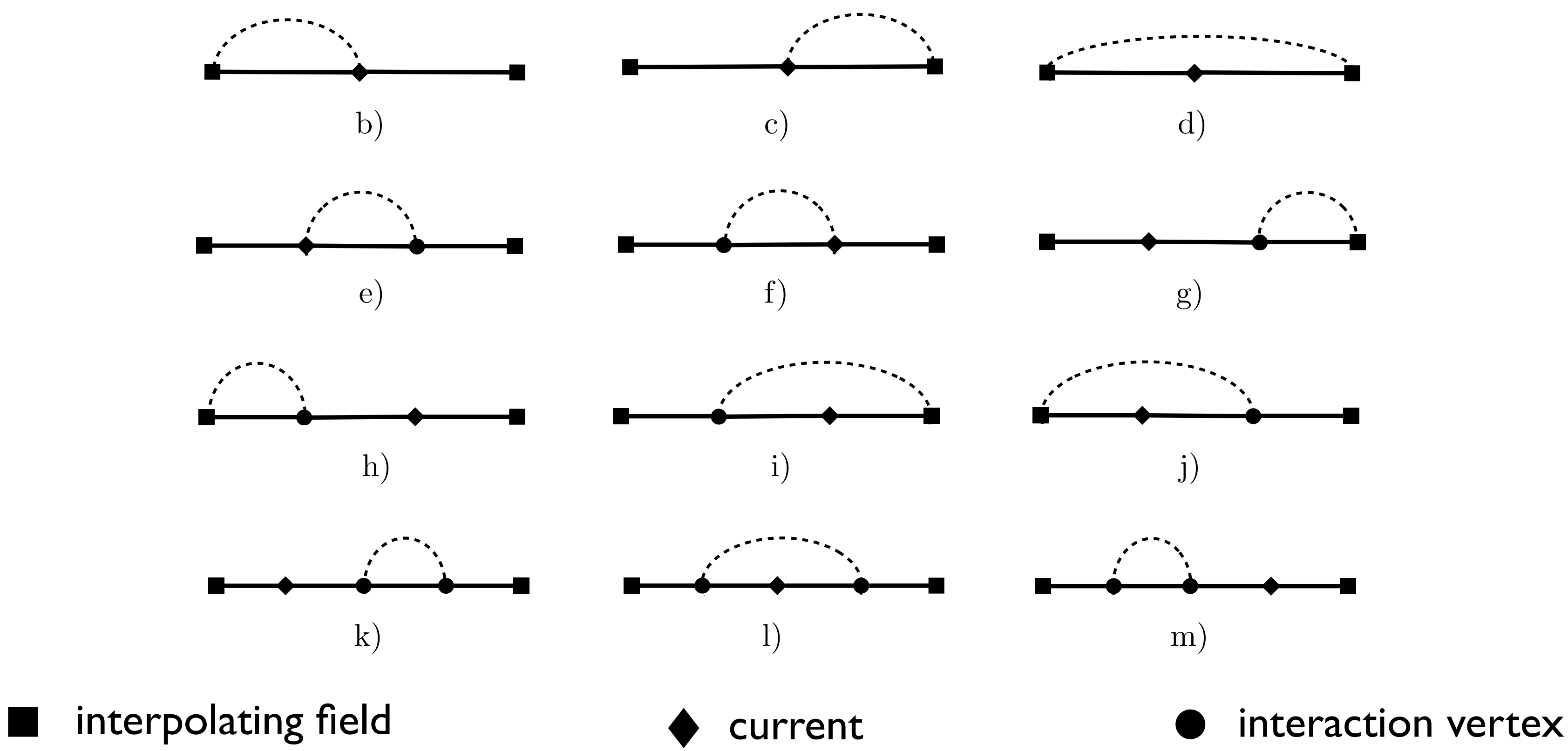
Güsken 1989,
Alexandrou et. al. 1991
Lüscher 2013

$\Rightarrow$ Smeared fields
- transform as unsmeared ones
- map to the same pointlike fields as their unsmeared counterparts provided $R_{\rm smear} \ll \frac{1}{M_\pi}$

  Different LECs only !

Correlators in ChPT

- Remaining task: Standard PT calculation of the 2- and 3-pt functions
- Feynman diagrams for the 3-pt function



- 4 diagrams for the 2-pt function

OB 2015

$N\pi$ contribution to ratio R

$$\Delta E_n = E_n - M_N$$

$$R = g_A + \sum_{p_n} \left(b_n e^{-\Delta E_n (t-t')} + \tilde{b}_n e^{-\Delta E_n t'} + c_n e^{-\Delta E_n t} \right)$$

General remarks concerning the result

- We find $b_n = \tilde{b}_n$ (as expected by symmetry)
- The coefficients $b_n, \tilde{b}_n, c_n$ depend only on $f/M_N \quad g_A \quad M_\pi/M_N \quad M_\pi L$
- No dependence on the LECs associated with the interpolating fields (these enter at higher order) !
- For estimates use $f/M_N \approx f_{\rm exp}/M_{N,\rm exp} \qquad g_A \approx g_{A,\rm exp}$

➥ LO ChPT makes a definite prediction for these coefficients as a function of M_π/M_N and $M_\pi L$

$N\pi$ contribution to g_A

$$\Delta E_n = E_n - M_N$$

$$R = g_A + \sum_{p_n} \left(b_n e^{-\Delta E_n (t-t')} + \tilde{b}_n e^{-\Delta E_n t'} + c_n e^{-\Delta E_n t} \right)$$

General remarks concerning the result

- Separate trivial prefactor

  and non-trivial ChPT result

multiplicity of momentum states
$m_1 = 6, m_2 = 12, \ldots$

$$b_n = \frac{m_n}{16(fL)^2 E_\pi L} \bar{b}_n^{\mathrm{3pt}}$$

non-trivial ChPT result

$$c_n = c_n^{\mathrm{3pt}} - c_n^{\mathrm{2pt}}$$

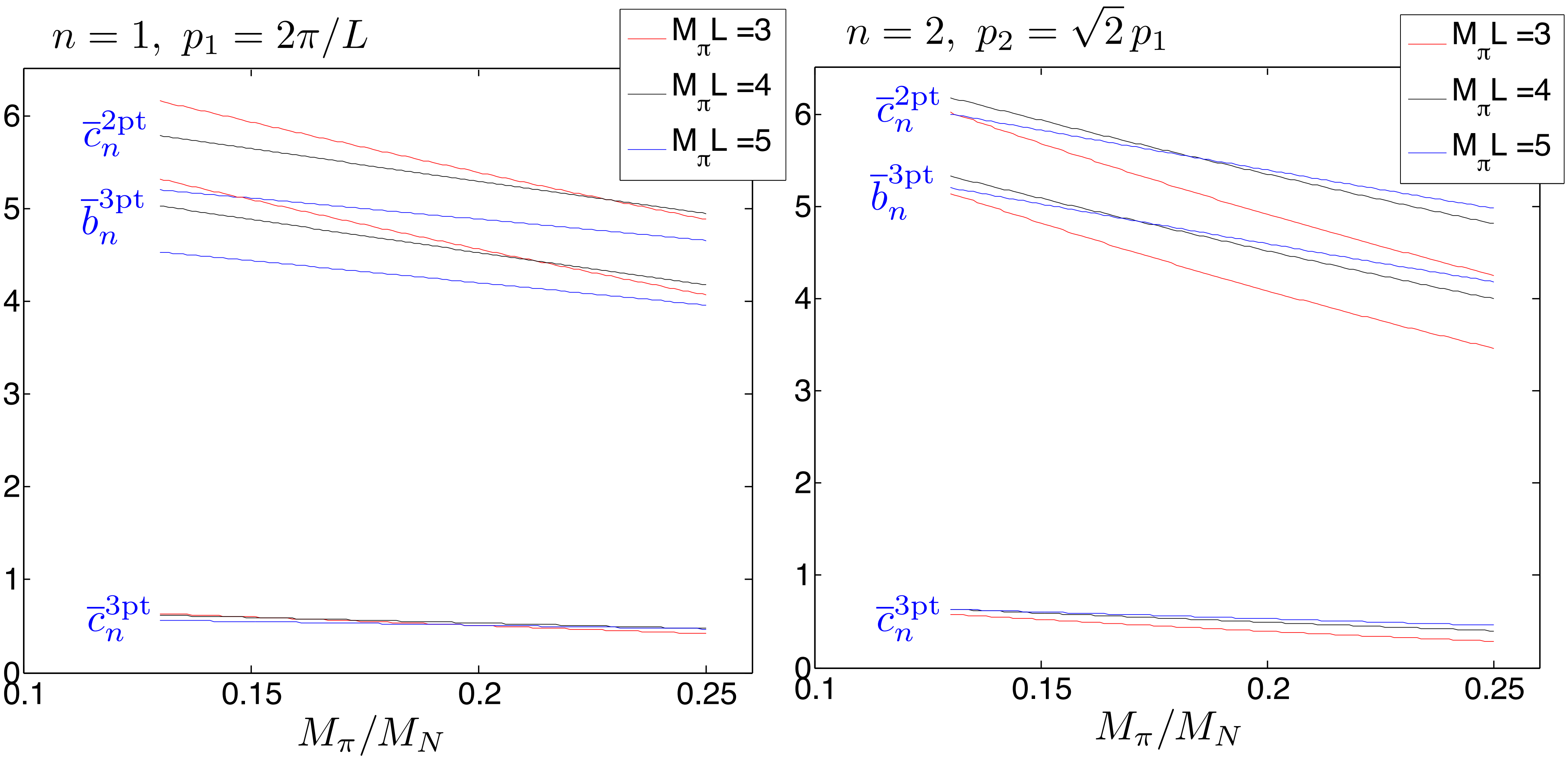
$$= \frac{m_n}{16(fL)^2 E_\pi L} \left(\bar{c}_n^{\mathrm{3pt}} - \bar{c}_n^{\mathrm{2pt}} \right)$$

Comment: Expand in $\frac{p_n}{M_N}$ and keep the leading term only

➥ we reproduce the results obtained with Heavy Baryon ChPT

Tiburzi 2015

Results for the coefficients



Observations:

$$\bar{c}_n^{\rm 3pt} - \bar{c}_n^{\rm 2pt} \approx -\bar{c}_n^{\rm 2pt} < 0$$

$$\bar{b}_n^{\rm 3pt} \approx -(\bar{c}_n^{\rm 3pt} - \bar{c}_n^{\rm 2pt}) > 0$$

$N\pi$ contribution to g_A

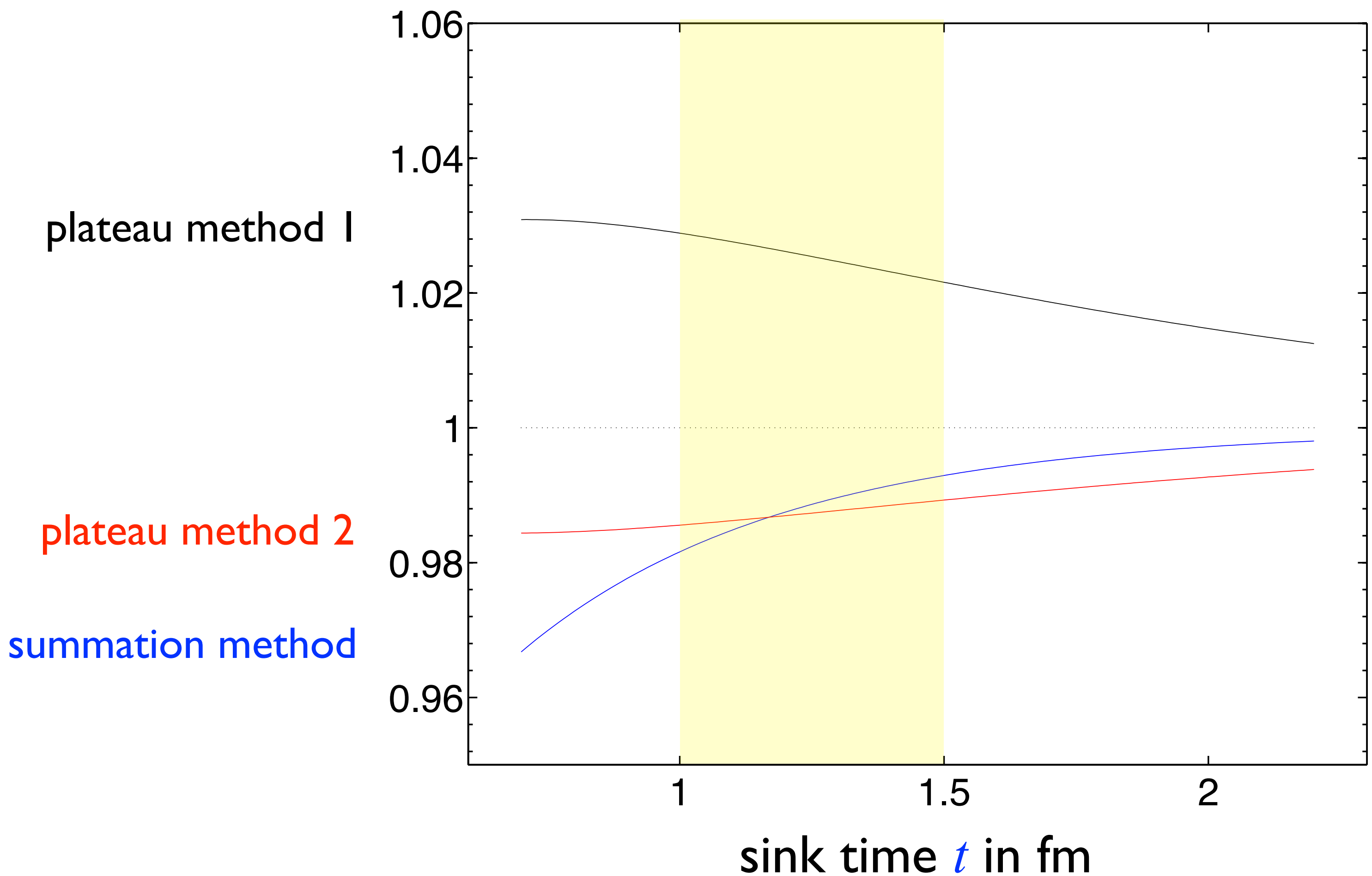
$$\Delta E_n = E_n - M_N$$

$$R = g_A + \sum_{p_n} \left(b_n e^{-\Delta E_n (t-t')} + \tilde{b}_n e^{-\Delta E_n t'} + c_n e^{-\Delta E_n t} \right)$$

- Consider
  - plateau method 1 $\quad g_A \approx R(t, t' = \frac{t}{2})$
  - summation method
    Maiani et al 1987
    $$S(t) = \int_0^t dt' R(t, t') = \text{const} + t\ m(t) + \ldots$$
    ➥ $g_A \approx m(t)$
  - plateau method 2 $\quad g_A \approx \frac{C_3^A(t, t' = t/2)}{C_3^V(t, t' = t/2)}$ ← 3pt-function with vector current

as a function of sink time t

$N\pi$ contribution to g_A



Here:

$M_\pi = 150\text{MeV}$

$M_\pi L = 4$

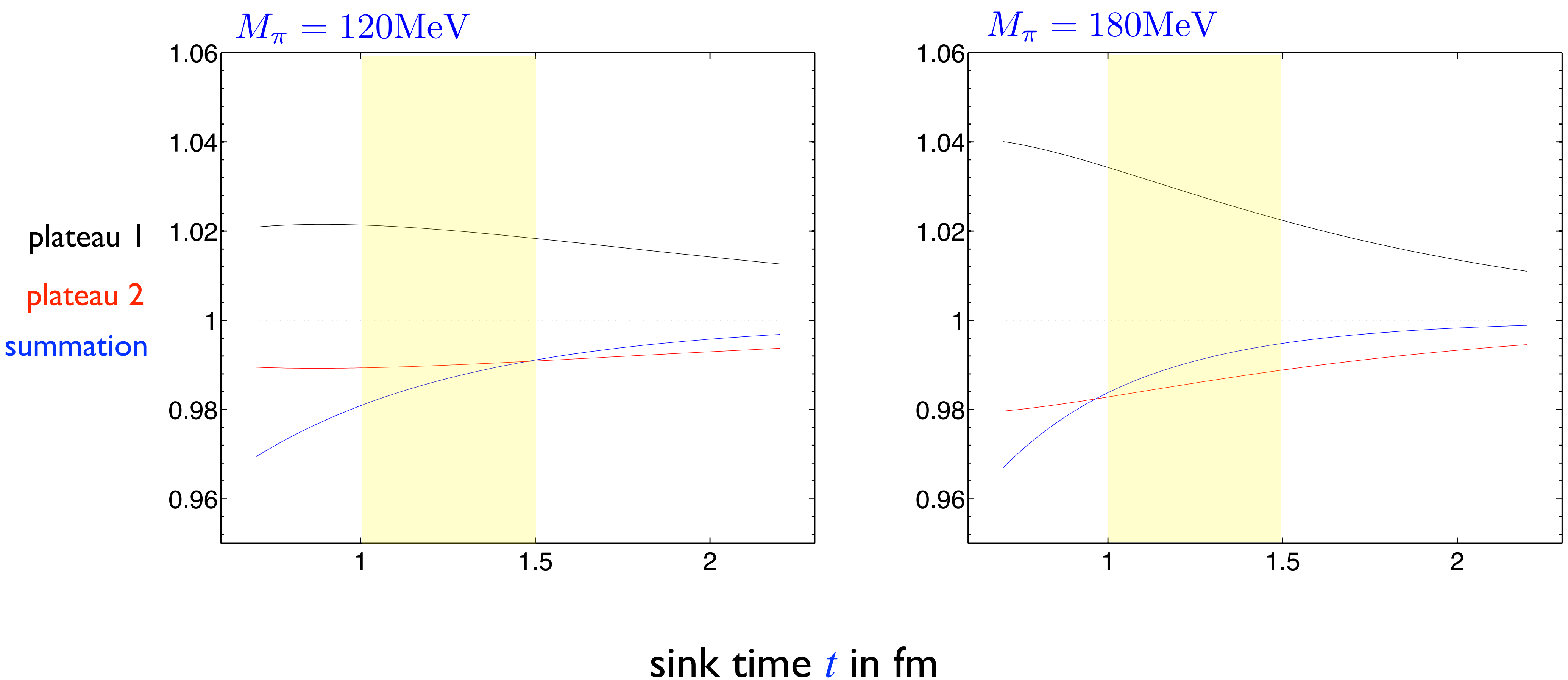
$n \leq 3$

Observations:

- +2 to +3% shift for plateau method 1
- -0.5 to -2% shift for summation method and plateau method 2
- Substantial cancellation effect in the plateau methods due to opposite signs of the coefficients

$N\pi$ contribution to g_A

Vary pion mass:

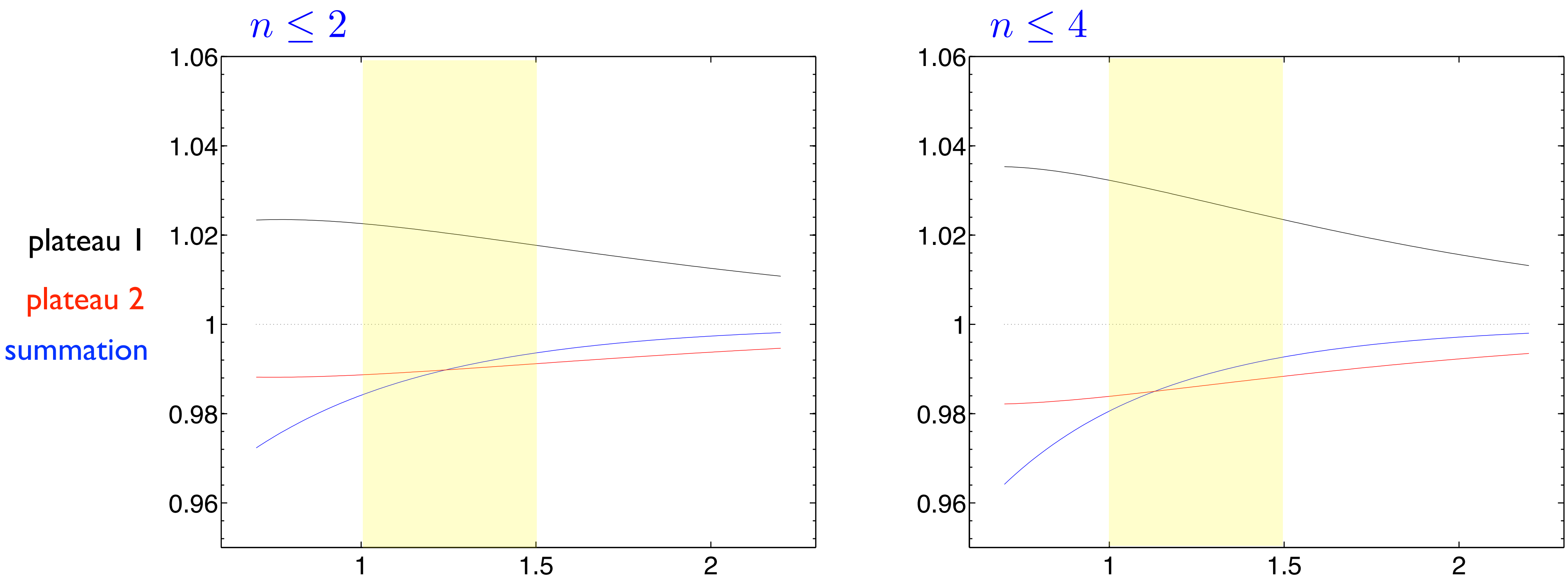


$n \leq 3$

$M_\pi L = 4$

$N\pi$ contribution to g_A

Vary number of included momentum states:



sink time t in fm

$M_\pi = 150\text{MeV}$

$M_\pi L = 4$

Errors

- LO calculation only ➡ error ?
  ➥ perform next-order calculation (straightforward)
- Smeared fields mapped on point-like ChPT fields ➡ error $\mathrm{O}(\frac{1}{M_\pi R_{\mathrm{smear}}})$?

➥ Crude error estimate for the LO result: 50 -100 % (??)

- Comment: Impact of 3-particle $N\pi\pi$ state:
  - suppressed by additional factor $\frac{1}{2(fL)^2 M_\pi L}$
  - no momentum degeneracy

➥ probably too small to play any role

Summary and outlook

- ChPT provides estimates for the $N\pi$-state contributions in nucleon correlation functions

- $N\pi$-state contamination to g_A is at the few-percent level

- Outlook:
  - Compute the next order
  - Other observables: Other nucleon charges
    Momentum fraction $< x >_{u\text{-}d}$
    Electromagnetic form factors
    ...

Backup slides

$N\pi$ contribution to g$_A$ - analytical results

$$\Delta E_n = E_n - M_N$$

$$R = g_A + \sum_{p_n} \left(b_n e^{-\Delta E_n (t-t')} + \tilde{b}_n e^{-\Delta E_n t'} + c_n e^{-\Delta E_n t} \right)$$

$$c_n = c_n^{3\mathrm{pt}} - c_n^{2\mathrm{pt}} = \frac{m_n}{16(fL)^2 E_\pi L} \left(\overline{c}_n^{3\mathrm{pt}} - \overline{c}_n^{2\mathrm{pt}} \right)$$

$$\overline{c}_{n,3\mathrm{pt}} = \frac{1}{3} g_A \left(\overline{g}_{A,n} - 1 \right)^2 \left(1 - \frac{M_N}{E_{N,n}} \right) \left(\frac{2M_N}{E_{N,n}} - 1 \right)$$

$$\overline{c}_{n,2\mathrm{pt}} = 3 g_A \left(\overline{g}_{A,n} - 1 \right)^2 \left(1 - \frac{M_N}{E_{N,n}} \right)$$

$$\overline{g}_A = g_A \frac{E_{\mathrm{tot},n} + M_N}{E_{\mathrm{tot},n} - M_N}$$

$N\pi$ contribution to g_A - analytical results

$$\Delta E_n = E_n - M_N$$

$$R = g_A + \sum_{p_n} \left(b_n e^{-\Delta E_n (t-t')} + \tilde{b}_n e^{-\Delta E_n t'} + c_n e^{-\Delta E_n t} \right)$$

$\tilde{b}_n = b_n$

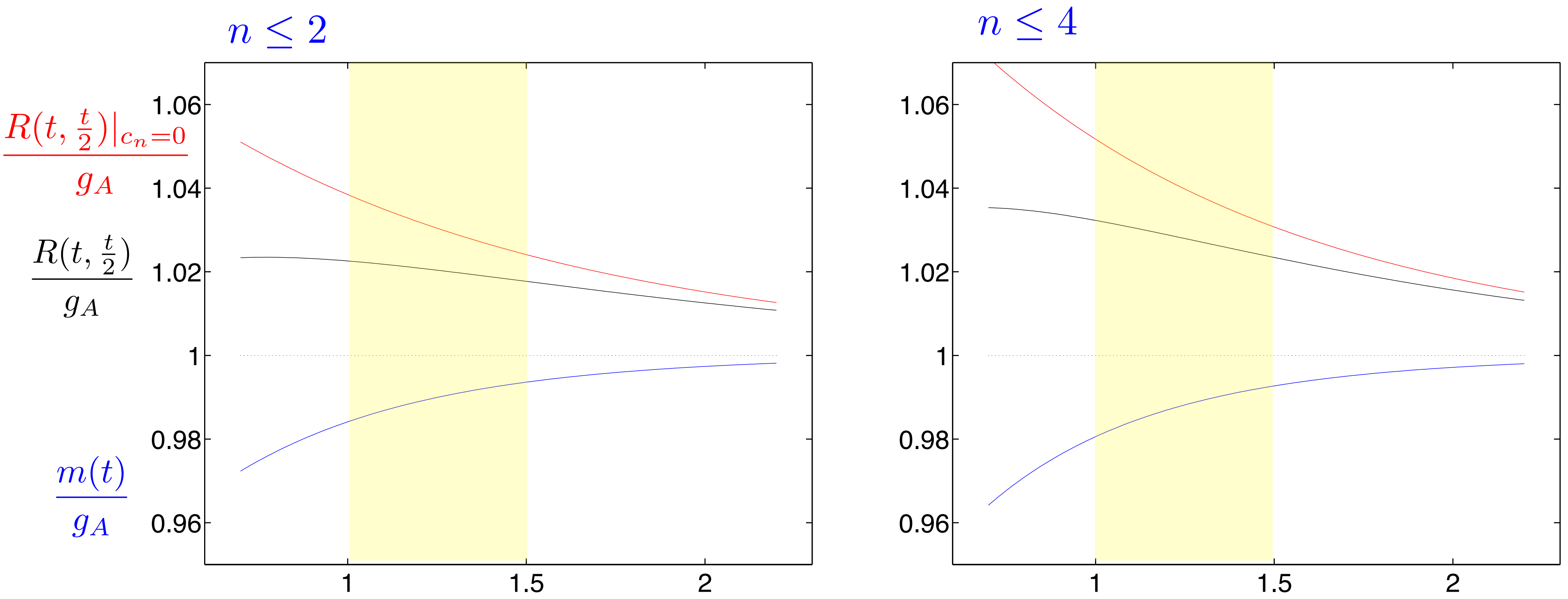
$$b_n = \frac{m_n}{16(fL)^2 E_\pi L} \bar{b}_n^{\mathrm{3pt}}$$

$$\bar{b}_n = (\bar{g}_{A,n} - 1) \left[\bar{g}_{A,n} g_A \left(3 + \frac{1}{3} \left(1 - \frac{2M_N}{E_N} \right) \right) \dots \right.$$

$$\left. \dots + \frac{2}{3} g_A^2 \left(1 + \frac{M_N}{E_{N,n}} \right) \frac{E_{N,n} - E_{\pi,n} - M_N}{E_{N,n} - E_{\pi,n} + M_N} - 4 \right]$$

$$\bar{g}_A = g_A \frac{E_{\mathrm{tot},n} + M_N}{E_{\mathrm{tot},n} - M_N}$$

$N\pi$ contribution to g_A

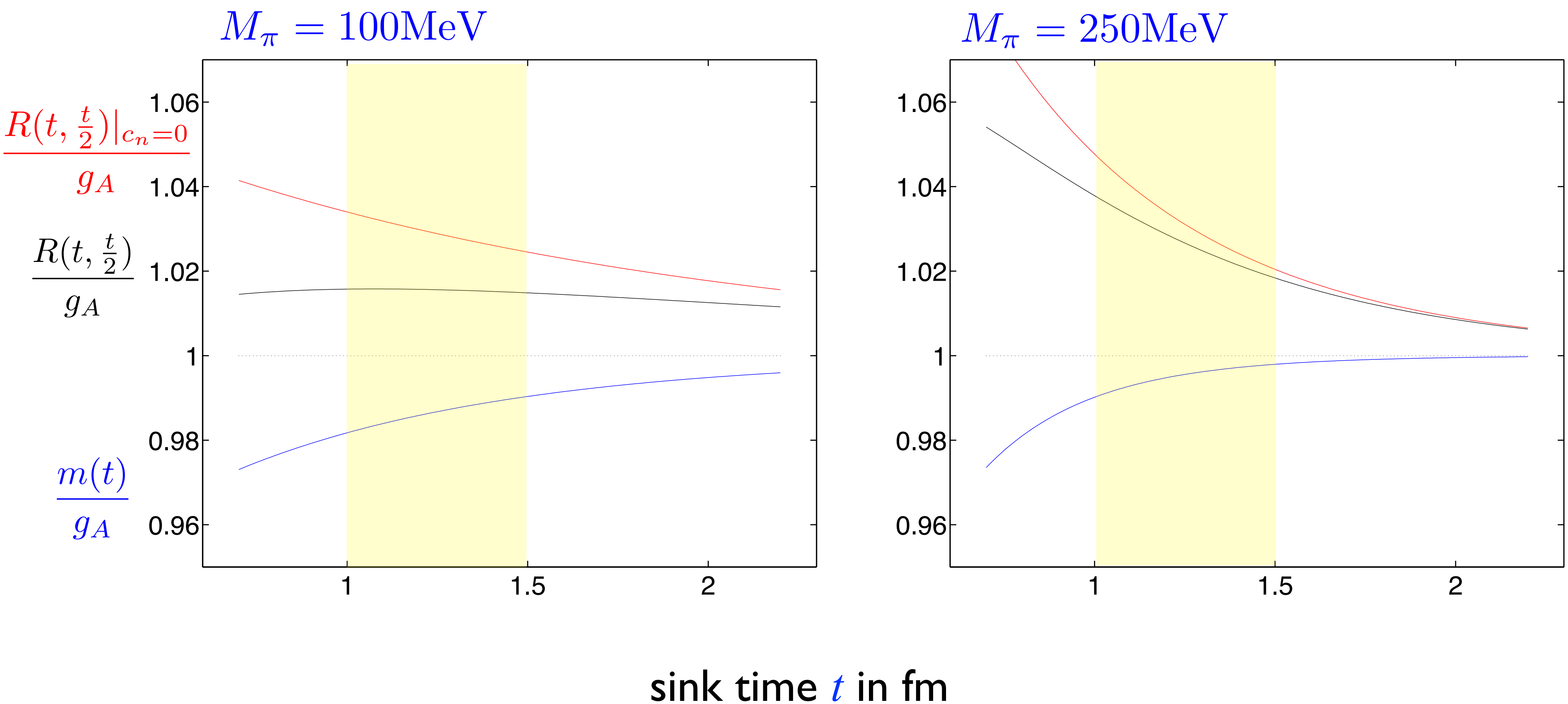


sink time t in fm

$M_\pi = 150\text{MeV}$

$M_\pi L = 4$

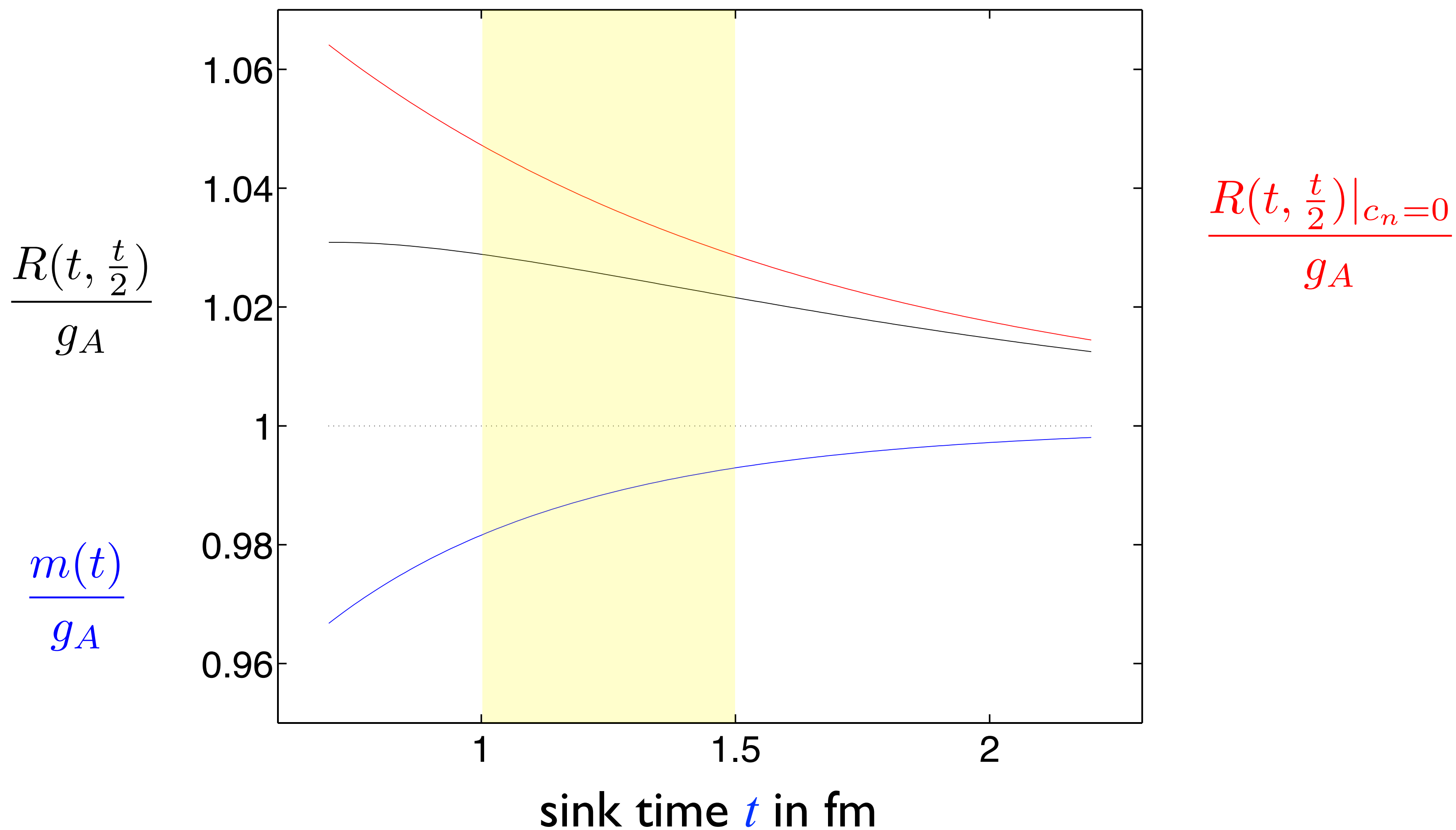
$N\pi$ contribution to g_A



$n \leq 3$

$M_\pi L = 4$

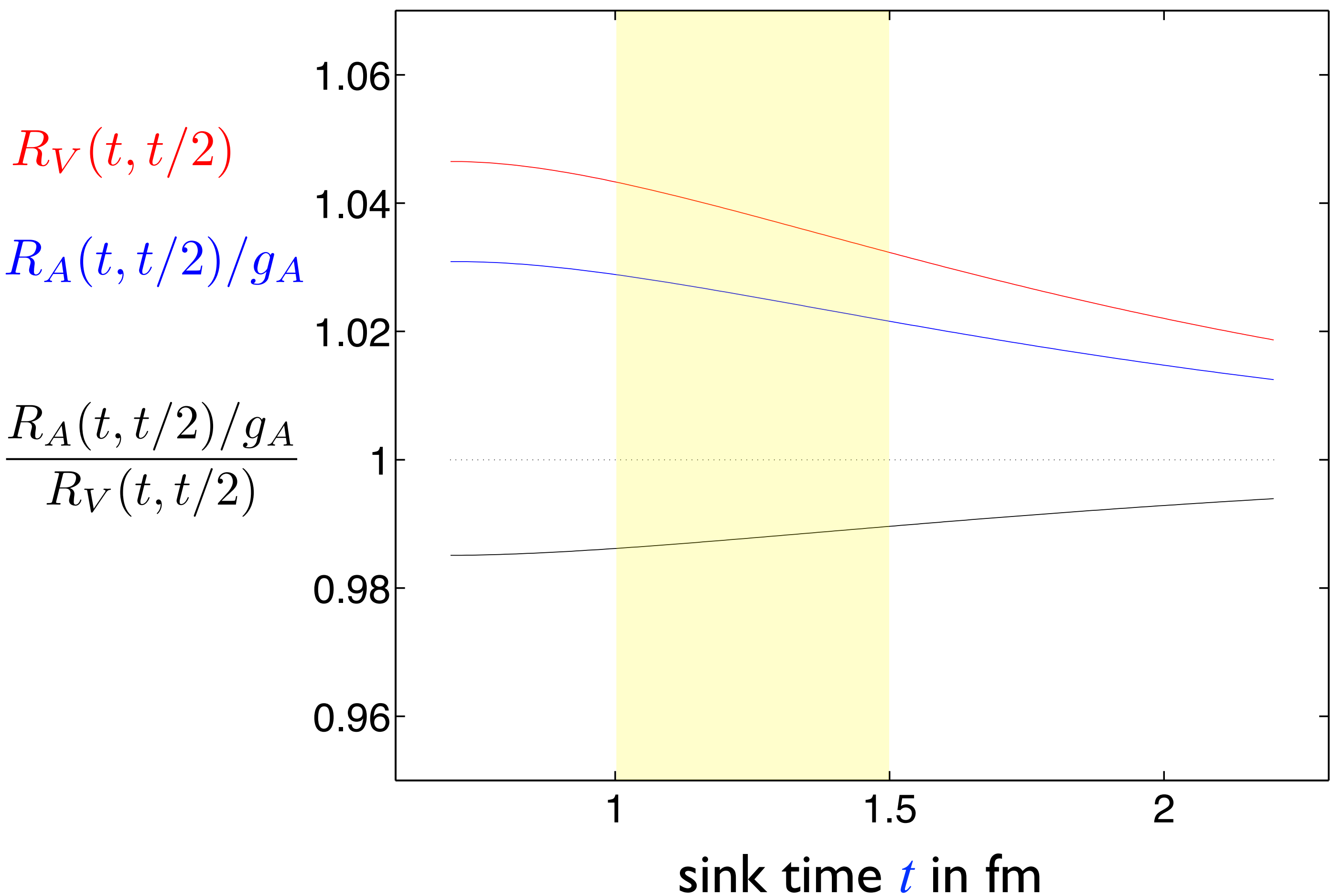
$N\pi$ contribution to g_A



Observations:

- 2 - 3% shift upwards for plateau method
- 0.5 - 2% shift downwards for summation method

$N\pi$ contribution to g_A



Comparison with finite volume effects

$$R = g_A \left(1 + \delta_{\mathrm{N}\pi}(M_\pi L, M_\pi / M_N) \right)$$

↑

$$g_A(L) = g_A(\infty) \left(1 + \delta_{\mathrm{FV}}(M_\pi L) \right)$$

combined finite volume/excited state effects